A mathematical approach to financial allocation strategies

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Declaration

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

Signature: ______________________________

Date: ______________________________

Executive summary

Ultimately, an investment professional stands or falls as a result of his ability to consistently outperform his competitors. Apart from employing new technology to assist in making swifter, smarter moves (so as to improve profits), investment managers are increasingly moving towards structuring their decision–making and implementation processes to accommodate risk and return.

Within the South African emerging market context, special attention is given in this study to certain extreme market conditions. Within these periods, the use of certain mathematical techniques (such as optimisation techniques and ranking methods), are examined to ascertain their potential with respect to capital allocation decision support. It is shown that a mathematical approach (under appropriate assumptions) towards capital allocation may contribute largely to understanding the complex interactions between underlying assets of a portfolio. Furthermore, it is demonstrated that in some instances ranking methods, which are computationally much simpler than full optimisation methods, may be used as an alternative to these time-consuming procedures. The latter approach may add significant value to structured decision–making in rapidly changing financial markets. Throughout the study, human attitude towards risk is incorporated in the analyses so as to investigate its effect on decisions (regarding the allocation of capital). It is demonstrated how this information (regarding different risk–profiles) may be used in assessing the risk and return characteristics of underlying assets.

It is believed that the mathematical decision support procedures suggested in this study, may in actual fact contribute largely to understanding the nature of different financial assets under different circumstances in a very practical way.

Opsomming

‘n Beleggingsbestuurders staan of val by die vermoë om konsekwent sy konkurente in terme van prestasie te klop. Naas die gebruik van nuwe tegnologie om vinniger en beter skuiwe te maak (en sodoende winste te verbeter), is daar ook ‘n toenemend groter erns te bespeur in die strukturering van besluitneming– en implimenteringsprosesse om risiko en opbrengs in ag te neem.

Binne die Suid–Afrikaanse ontwikkelende mark konteks, word aandag in hierdie studie gegee aan tydperke wat gekenmerk word aan sekere ekstreme marktoestande. Binne hierdie tydperke word die gebruik van sekere wiskundige tegnieke (soos optimeringstegnieke en rangskikkingsmetodes) ondersoek. Die doel van die ondersoek is om te bepaal tot watter mate die metodes ‘n bydrae kan lewer tot kapitaal–allokasie in ‘n batebestuursomgewing. Daar sal aangetoon word dat ‘n wiskundige benadering (onder geskikte aannames) grootliks tot kapitaal–allokasie kan bydra en kan lei tot ‘n beter begrip van die komplekse interaksie van die onderliggende bates van ‘n portefeulje. Verder word daar gedemonstreer dat, in sekere gevalle, rangskikkingsmetode, wat eenvoudiger berekenbaar is as volle optimeringstegnieke, gebruik kan word as alternatief tot meer tydrowende optimeringsprosedures; so ‘n benadering mag waarde toevoeg tot gestruktureerde besluitneming in vinnig veranderende finansiële markte. In hierdie studie word menslike voorkeur teenoor risiko deurgaans in die analises geïnkorporeer om die invloed daarvan op die besluitneming aangaande die allokasie van kapitaal te ondersoek. Dit word aangetoon hoe die informasie wat verkry word uit die toedeling van kapitaal vir verskillende risiko–profiele gebruik kan word om die risiko– en opbrengs eienskappe van die onderliggende bates te ondersoek.

Daar word gehoop dat die wiskundige metodes wat in hierdie studie as ondersteuning tydens kapitaal–allokasie besluitneming voorgestel word, grootliks en op ‘n praktiese wyse mag bydra tot die begrip van finansiële bates in verskillende omstandighede.

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Table of Contents

List of Figures

List of Tables

Glossary

Asset allocation The process of specifying an appropriate combination of investment assets.

Bear/bull trend (Definition 14) Any decrease/increase in the value of the All Share Index, and therefore the equity market from one point to a subsequent point 4 weeks later.

Bear/bull trend (Definition 15) Any 4 week period during which the equity market (All Share Index) falls/rises by more than 10%.

Comfort/satisfaction function A function that attempts the quantification of the dynamics of investment decision–making as for a specific investor.

Diversification The elimination of unsystematic risk by investing in shares of a number of different companies with different internal factors influencing their movements. Market risk cannot be eliminated in this manner, as all shares are subjected to the same fluctuations.

Dominating set A set $S \subseteq V$ of vertices in a graph $G = (V, E)$ is called a *dominating set* if every vertex $v \in V$ is either an element of S or is adjacent to an element of S.

Graph A graph $G = (V, E)$ is a finite nonempty set $V(G)$ of objects called vertices and a (possibly empty) set $E(G)$ of 2–element subsets of $V(G)$ called edges. Two vertices u and v in a graph $G = (V, E)$ are adjacent if uv is an element of the edge set $E(G)$.

Homogeneous time windows A set of time windows are homogeneous if their vertices in a ranking graph are adjacent.

Indifference contour A contour in the risk–return plane joining those combinations of return and standard deviation with which a particular investor will be equally comfortable/satisfied.

Lower domination number The lower domination number $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set in G.

Market capitalisation The size of a company in terms of the number of company shares outstanding, as well as the price of these shares in such a way that

$$\text{Market capitalisation} = \text{Number of shares} \times \text{Price per share.}$$

Market indicator A quantitative measure of qualitative or quantitative factors which may contain information regarding the state of one or more financial market sectors or of the aggregate market at specific points in time.

Market (systematic) risk The risk that market fluctuations may influence the price of a share. All shares are exposed to this risk associated with prevailing market conditions.

Performance attribution A retrospective procedure with a purpose to trace the impact of all decisions made with respect to the construction of a portfolio.

Ranking graph A graph whose vertices represent the time windows identified by a specific market indicator. The vertices (windows) are adjacent if and only if their ranking combinations either *coincide* or are *similar*. The rankings of two windows *coincide* if and only if the ranking combinations are the same for both windows. The rankings of two windows are *similar* if the ranking numbers of exactly two sectors differ by only one. More formally, the ranking combinations of two windows, $\underline{w}_1 = [w_{11}, \ldots, w_{1n}]$ and $\underline{w}_2 = [w_{21}, \ldots, w_{2n}]$ are similar if and only if the Euclidean distance between them, is $d(\underline{w}_1, \underline{w}_2) = 2$.

Risk averters Individuals who are comfortable with an increase in risk exposure only if the compensation for the additional risk is proportionally greater than the addition of risk.

Risk–free asset An asset for which there is no uncertainty regarding future returns, so that the variability of expected future returns, and therefore the standard deviation of expected returns, is zero.

Risk neutral individuals Individuals who are comfortable with an increase in risk exposure if the compensation for the additional risk is equal to the additional risk taken.

Risk seekers Individuals who are comfortable with an increase in risk exposure even if the compensation for the additional risk is proportionally less than the addition of risk. They are therefore willing to give up some compensation, just for the enjoyment of being exposed to risk.

Risk tolerance A measure of an investor's willingness to take on additional risk in order to achieve added return.

Risky asset An asset about which there is uncertainty regarding future returns. This uncertainty may be measured by the standard deviation of expected returns.

Set of indifference contours A set of contours in the risk–return domain representing an investor's attitude towards all levels of risk and return.

Shock An extreme market condition quantified by a specific state of a market indicator. It is defined as the event that occurs when the value of the market indicator time series is either above or below one standard deviation from its mean. A shock above the one standard deviation band will be called an *upward* shock, while a shock below the one standard deviation band will be called a *downward* shock.

Specific or unsystematic risk The risk associated with the share price movements of a specific company and is caused by the internal activities of the company. This type of risk is present, independent of the concurrent market conditions.

Time window A period during which a shock prevails.

Chapter 1

Introduction

The financial field of investments has experienced many changes since the early 1980's. This is due, in part, to an abundance of newly designed securities, in part to the creation of new trading strategies that would have been impossible without concurrent advances in computer technology, and in part to rapid advances in the theory of investments that have emerged from the academic community. It is believed that since the 1990's the transmission of theory to real–world practice is in no other field as rapid as in the investment industry. These developments place new burdens on investment practitioners far beyond what was required previously. Furthermore, since the late 1980's, investment companies have shifted their marketing focus from institutions to the historically unexploited individual investor. A vigorous campaign was launched to convince both individual and institutional investors of the accessibility of financial markets. Unfortunately, the public was inundated with so much investment advice that many investors did not know whom or what to believe [50]. Nevertheless, in due course the campaign generally produced educated investors who demanded service and performance from financial services companies in exchange for choosing the investment medium of one financial company above that of another.

At the same time investors were increasingly applying ethical or social tests to their investment choices, i.e. they realised that they had a social responsibility and wanted to select alternatives to fulfil this obligation [16, 36]. The focus of investment professionals shifted globally to pleasing individual investors [11]. Intermediaries and advisers took far more responsibility and an open approach to tailoring investments to the needs of clients [4]. It also became necessary for the asset managers of financial services companies to be at the forefront of development in order to enhance the performance of their assets under management above that of their competition, while providing socially responsible products and hence redistributing some of the profits to certain target groups. Financial services companies hoped that they may secure and expand their client base in this way. With the idea of embracing this equity culture and investor–focus came the development of a more sophisticated financial market environment [11].

The advocation of accessibility and appropriateness of the financial markets for all kinds of investors brought a responsibility to the industry to provide investment alternatives for investors with different appetites for risk (tolerances for uncertainty). So–called *life–cycle funds* came to the fore which were designed to allow investors to choose an investment based on their age and tolerance for risk and were soon followed by *lifestyle funds*, which were even more finely tailored to their holder's requirements [7, 86]. Nearly simultaneously, specialised investment mediums, focusing on investment in specific market sectors, were introduced [86]. These two aspects resulted in more emphasis being placed on the risks associated with different market sectors. In particular, a need arose to ensure that the actual risks associated with investments in the specialised sectors match the risk profiles contained in the investment mandates (prescribed rules to which managers should keep while making investment decisions) of the portfolios. Reinebach [76] accentuates this point by suggesting that cognisance of risk is finally back (and there to stay) at the forefront of decision–making, after being underplayed for several years as investors vigorously chased incremental returns around the globe.

The reasons cited above are not the only factors contributing to the importance of above–average performance of investment portfolios, associated with acceptable levels of risk. Since the mid–1990's the multi–manager approach has also attracted considerable attention (and capital). This approach entails that a portfolio manager entrusts the management of the different asset classes (cash, bonds, equities) or financial sectors (such as Mining Resources, Financials, Industrials, Consumers and Information Technology) within his portfolio to specialist managers or teams of various asset management companies. Two main characteristics make multi–manager investments so attractive. First, most multi–manager investments are designed to synchronise with an investor's age, investment horizon, or tolerance for risk. This makes them a popular choice by individuals who do not have the time, inclination, or ability to choose from a potentially bewildering array of investment options to fit their profiles [64]. Secondly, the client's investment is not exposed to the investment views of a single individual or team, nor even to the investment views of a single company, but to the views of the best specialist individuals or teams in the industry [86]. However, the success of such an investment is obviously very dependent upon the correct decisions of a number of role players [61]; through the well–known *Long Term Capital* debacle, history has shown that even the management team who could legitimately claim the best reputation in the world for financial modelling may falter [15].

The task of the portfolio manager within the multi–manager environment is to make the very crucial asset class and financial sector allocation decisions, select the appropriate specialists to manage the proportion allocated to each asset class and financial sector according to the specialist's past and expected future performances, and to ascertain whether the level of risk associated with the portfolio as a whole coincides with its mandated risk profile. In turn, it is expected of the specialists to select those securities that will turn out to be top–performers, without exposing the investment proportion to excessive risk. If the seemingly unpredictable and sometimes extremely volatile financial markets are taken into account, being successful becomes a challenging task. In order to meet expectation, the specialist managers in practice rely heavily on the research and subsequent recommendations of the analyst teams within their companies. The more efficient this interaction within the decision–making process, the better the chance of meeting expectations of outperformance.

Asset management companies typically house a number of different private, segregated, pension and unit trust portfolios, each with different mandates and risk profiles. It is common in the industry that surveys are conducted by influential independent institutions on the separate fund mandates [70], but that results are also combined so that the management companies are judged according to all assets under their management. The ranking results of these surveys are published quarterly and received with great interest by the industry. It is important for asset managers to be reflected upon positively in these surveys – their results are unbiased indicators of success and may influence decisions regarding new investments and place doubt in the minds of the existing clients of lower ranking companies.

Ultimately, an investment company stands or falls as a result of its ability to consistently outperform its competitors. Outperformance should empower an asset management company to compete for opportunities that will secure more assets under management, not only from institutions and individuals, but also from other asset management companies by means of the multi–manager approach. Hann [32] also comments on the fact that as assets under management became a major measure of success for financial services companies, asset managers increasingly looked to technology for aid in making swifter, smarter moves in order to improve trading profits. Apart from employing new technology to attempt this quest for more assets under management, asset management companies are also forced to structure their decision–making and implementation processes and examine these with regards to risk and return, which in turn may lead to outperformance.

The evolution and advancement of the theory and practice of portfolio management have been shaped by many factors including growth in the number and size of investments, enhancement in technology and large direct and indirect costs of shortfalls in meeting portfolio objectives, as discussed above. While it is unknown whether these same factors will be principal shapers of the future investment environment or whether an entirely different set of influences will come into play, change will still almost certainly occur in the industry. Given the enhanced sophistication and competitiveness of the asset management community, reaction to change is expected to be increasingly swift and resolute [56]. It seems impossible to deny the importance of a well–developed, competitive investment decision–making strategy and that all the facets of decision–making within the strategy as a whole should be efficient in order to keep up with change. However, the volatile and complex nature of the environment in which such a strategy should be implemented and maintained, increases the challenge.

1.1 The economic environment

Unequivocal evidence exists of a relationship between the state of securities markets and that of the economy. Chen *et al.* [12] examined a set of macroeconomic variables (such as growth in industrial production, inflation and interest rates) and concluded that some of these factors were important in explaining returns of the securities markets. Heathcotte and Apilade [34] examined the relationship between a short list of economic indicators and the S&P500 index (an index compiled by Standard & Poor to include the 500 largest companies in the United States of America). For their study, they constructed a portfolio and managed it actively, but they based their decisions solely upon information they gathered from the movement of a subset of the economic indicators. They compared the results of their actively managed portfolio to that of an investment that followed a buy–and–hold strategy and found that, as long as they had perfect foresight of the correct subset of indicators to use, they were able to beat the buy–and–hold strategy. Numerous studies have been dedicated to finding possible economic factors contributing to financial market conditions [12, 17, 34, 72, 75]. Some conclusions were branded as speculative, but on the contributions of certain factors consensus was reached.

Two of these factors, which are believed to carry particular importance, are a country's monetary and fiscal policies [3, 17, 60, 72]. Within the South African economic environment, these policies are prescribed by the South African Reserve Bank (SARB) and the South African government respectively. SARB affects economic activity through its impact on the supply of money and credit. The government affects the economy through taxation, expenditures and how its deficit is financed. As a result of the impact of the monetary and fiscal policies on interest rates and company earnings, the prices of securities may be altered. Unfortunately, for the performance of many assets, the linkage between the asset's price and the general economy is very complex. The complexity of such an interaction may be illustrated by example.

Example 1 *An expanding economy may increase the demand for a company's product, which may in turn lead to higher earnings. The higher earnings may permit increased dividends or growth, since the company has more capital at its disposal. Therefore, if other factors are held constant, an expanding economy should result in higher share prices. Unfortunately, other factors cannot be assumed to remain constant. An expanding economy may result in higher wages and salaries, higher interest rates to counter inflation and increased competition. All of these factors may have an adverse effect on the specific company. The additional factors may offset the positive impact of the higher earnings or may even cause the increase in earnings never to materialise [60] p.349.*

The relationship between the state of the economy and investment alternatives is made more difficult when one realises that security prices are a harbinger (indicator) of future economic activity, and not a mirror of current activity. Changes in security prices tend to precede changes in economic activity. If a portfolio manager waits until the economic circumstances have changed before making and executing a particular decision, the action is often too late as security prices tend to anticipate the changes [60].

While this discussion suggests that it is far from simple to link financial markets, and the ultimate investment selection from the available alternatives, to the state of the economy, there are some economic scenarios (indicators), such as changes in interest rates, inflation/deflation or economic growth, that may suggest broad frameworks for investment decisions. None of these economic scenarios prescribe one simple strategy to take advantage of resulting circumstances. Within a single scenario, there may be many possible investments, ranging from extremely conservative to aggressive. Some alternatives may not be appropriate for an investor whose willingness to bear risk differs from the risk exposure generated by a set of risky alternatives. However, the same set of alternatives may be combined differently, so as to suit the investor's requirements and willingness to bear risk.

It may be concluded that it is not sufficient to base investment decisions on the current state of an economy. It is the future economic environment that is crucial, which helps to explain why portfolio decisions, based on the anticipated state of the economy, are among the most difficult decisions facing an asset management team [37, 60]. Nevertheless, Beller, *et al.* [9], like others [19] advocate that their analyses provide strong evidence that some forecasting models in the investment industry may be useful in predicting some aspects of an economy and financial markets. Compounding the complexity of the decision–making problem is the fact that investors have different appetites for risk and that financial markets are not influenced by economic factors alone. Irrational investor behaviour may also have an effect on the state of financial markets [1, 69].

Although it is not necessarily clear how macroeconomic conditions interact with the securities markets, it is exceedingly clear that the economic environment should be considered seriously when making investment decisions. Furthermore, the apparent unpredictability of financial securities market conditions seems irreconcilable with the ungovernable need of investors for consistent above–average performance. This adds to the necessity of paying special attention to a typical investment process and to examine how the complexities of investment decision–making are tackled in practice.

1.2 The investment decision–making process

It is believed that most asset management companies have established strategies to evaluate performance of, and source of risk within, their investment portfolio construction process. However, enhancing different aspects of the process, as pointed out by evaluations as being inefficient, seems to be an ongoing engagement. So too is the changing of aspects of the process to pre–empt expected structural changes in the South African economy. This may be a result of extensive research being conducted throughout the industry in an attempt at better understanding market conditions and at applying this knowledge to reduce uncertainty.

In its most general form, the investment process is an integrated set of steps undertaken in a consistent manner to create and maintain appropriate combinations of investment assets. Each asset management company has a preferred operation course that is uniquely its own, which is believed to ensure competitiveness within the industry. This study makes no judgement and voices no opinions about how the process should be organised. Instead, the basic steps of these processes that are common to all asset management companies, will be discussed.

In a broad sense, the role of analysts, economists and market strategists is to research, examine and interpret market information from the perspective of their various fields and prepare the results for implementation. The role of portfolio managers is to assimilate these inputs and proceed systematically through the orderly process of converting the raw material into a portfolio that maximises expected return relative to the investor's ability to bear risk. This view sees portfolio management as the art of packaging and maintaining a proper set of securities to deliver satisfying returns, given the objectives of a typical client. Portfolio management is therefore not seen as a set of separate events induced by intuition or inspiration, but as an integrated whole in which every decision moves the portfolio down the process path [56].

In conformance with the view that the investment process may be seen as an integration of decisions, the term *asset allocation* will be defined as follows in this study.

Definition 1 *Asset allocation is the process of specifying an appropriate choice and weighting of investment assets.*

The asset allocation process is subdivided into a number of distinct decision stages, namely

- **Asset class allocation** – an allocation among the main asset classes, namely *cash*, *bonds* and *equities.*
- **financial sector/style allocation** – an allocation of the equity portion to specific financial sectors (like the industrial or mining resources sectors) or styles. *Styles* refer to an alternative way to classify the shares trading on an exchange according to certain criteria. Four main style categories exist within the South African context, namely *large capitalisation* (choose shares of large companies), *small capitalisation* (choose shares of smaller companies), *growth* (choose shares of companies with potential sustainable growth in profits in excess of that of the average company) and *value* (choose shares that are trading at a discount to their companies' fundamental inherent values).
- **Specialist selection** (when engaged in the multi–manager approach).
- **Share selection** – specific selection of shares within the financial sectors or styles identified in the previous stage.

Integral to each of the above decision stages is the risk tolerance of an individual or group of investors, which should be honoured at every stage. Decisions made in every phase rely on one of two strategies, namely *bottom–up fundamental* or *top–down fundamental* analyses.

A bottom–up allocation approach uses a variety of fundamental economic and financial factors to determine whether a single security is priced properly. At the extreme of this approach the analyst or team does not focus on the different industries or state of the economy at all, but in more moderate cases, the approach may be represented graphically as in Figure 1.1. This approach assumes that undervalued shares will eventually be recognised by all market participants, irrespective of the state of the economy or factors influencing market conditions. The portfolio should be constructed in such a way that the securities held are selling below their intrinsic value. The financial statements of the companies are typically analysed and various evaluations are done in order to calculate the intrinsic value, and as soon as these securities exceed their intrinsic value, they are sold. Two major problems, namely how to select investment assets and when to buy and sell them, are directly overcome by this approach. A third problem, namely that of how to go about packaging the selected assets, may be solved via a portfolio optimisation process. The optimisation approach mathematically determines the best allocation of resources (capital) amongst the set of analysed assets, given their expected returns, risk, and interaction of movement.

A top–down approach also attempts to include assets trading below their intrinsic value (as this means that the asset is trading for less than its true value), but goes about it in a different way. The aggregate local and international economy is analysed first, industries are identified as being prosperous relative to the overall economic cycle next, and then companies within the industries or sectors are selected, based on the extent to which they are assumed to be mispriced (again see Figure 1.1). After the appropriate assets are selected, the capital may also be allocated among them by using an optimisation technique. In practice, a combination of these two approaches is usually used in the asset allocation process as a whole [56, 86].

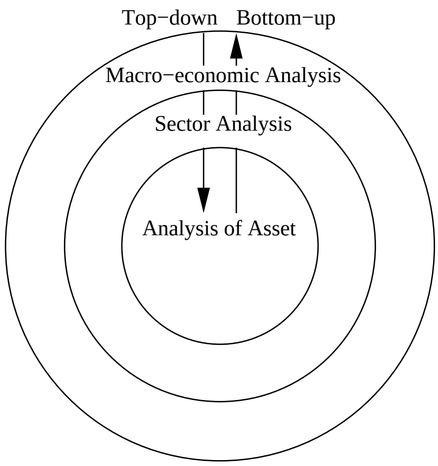


Figure 1.1: Top–down and bottom–up approaches to investment decision–making

It is again stressed that, in practice, it is the result of the process in its entirety that is judged; i.e. the performance of the resulting portfolios relative to expectations and comparison standards. Therefore, *performance attribution* is such an important aspect of portfolio management. To emphasise this point, Hann [32] states categorically that performance attribution systems are one of the hot tools on asset managers' technology wish lists.

Definition 2 *Performance attribution is a retrospective procedure whose purpose is to trace the impact of all decisions made with respect to the construction of a portfolio.*

As was mentioned previously, the set of decisions include the asset class allocation decision, the financial sector allocation decision, the specialist selection decision and the security selection decision. For the purposes of the attribution analysis, decisions are separated in such a way that their influences are isolated in order to be examined independently. The results indicate whether a specific decision, made in anticipation of certain circumstances, contributed positively or negatively to the overall performance in view of what actually happened after some specified time. The effect of the decisions within a simple asset allocation process is illustrated by example.

Allocation decision	Asset class with Superior Performance	Asset class with Poor Performance	Sector with Superior Performance	Sector with Poor Performance	Security with Superior Performance	Security with Poor Performance
Overweight	+	−	+	−	+	−
Underweight	−	+	−	+	−	+

Table 1.1: Performance attribution

Example 2 *Suppose the mandate of a certain portfolio is that of an enhanced benchmark tracker [2, 56], which means that the default position of the portfolio is that of a certain fixed portfolio (mostly a financial market index). If the manager sees no opportunities in the market to add value for the investors, the investors' performance will be the same as that of the benchmark portfolio. However, the manager may alter the allocation of the portfolio to deviate from the benchmark in order to add value. The manager may add value (outperform the benchmark) by being overweight (investing in excess of what is prescribed by the benchmark) in a specific asset class, financial sector or security, if he anticipates that the asset class, financial sector or security will perform well. Alternatively, the manager may add value by being underweight (investing less than what is prescribed by the benchmark) in a specific asset class, financial sector or security, if he anticipates that the asset class, financial sector or security will perform poorly. As was mentioned previously, the outcome of such decisions is known only at the end of a specific period. Regardless of the manager's anticipations (and decisions made accordingly), the asset class, financial sector or security may have had superior or poor performance for the specific period in question.*

The impacts of these decisions upon the performance of a portfolio are given in Table 1.1. A "+" indicates that the decision of being overweight/underweight enhanced the performance of the portfolio in view of what realised in the market, while a "−" indicates that the decision detracted from the performance – in retrospect the decision was therefore the wrong one.

With the above broad understanding of the investment process and the importance of the efficiency thereof, the aim of this study may now be stated.

1.3 Aims of this study

The considerable attention still given to various investment opportunities within the South African financial context for all the reasons discussed in this chapter so far (and many more), is causing the market to become increasingly more efficient. Efficiency in terms of the efficient market hypothesis (EMH) is decribed in terms of three levels, namely the weak, semi-strong and strong forms of efficiency. The weak form proclaims that all information regarding past price movements of securities is reflected in current prices and can therefore not be used to obtain excess returns. The semi-strong form proclaims that all publicly available information is reflected in security prices, while the strong form proclaims that all information, whether it is public or non-public is reflected in security prices and therefore cannot aid investors to obtain excess returns. Subsequently, managers have no chance of outperforming competitors [54]. Although the

EMH (and the belief that some markets are completely efficient in all three forms) has never been widely accepted by the global community of professional portfolio managers [10, 30], it is not denied that some markets show signs of near–efficiency in one or all three forms from time to time. This implies that inefficiencies still occur in the market. However, the frequency of occurrence of such inefficiencies may subside and it may be necessary to employ scientifically more sophisticated methods to exploit them. The need for higher precision in the identification of market movements and subsequent reactions on the part of investment professionals may be fulfilled to some extent by quantitative assessments, attempting to point out marginal differences in the possible outcomes of different investment alternatives as decision–making tools.

This study will attempt to identify homogeneous periods of extreme market conditions by examining certain states of the very complex financial market structure (Chapter 3). The main reason for this is to examine the use of certain basic mathematical techniques as quantitative tools during these periods (Chapters 4 and 5) to ascertain their potential to aid in the decision–making process within the financial asset management environment. An attempt will be made to present the work so that it is relevant within the South African emerging market context.

In order to achieve this, a rigorous comparison process of possible portfolio allocations is presented in Chapter 3, incorporating the identification of these periods of similar market conditions (via general bear and bull classifications, consumer sentiment, business confidence, etc.). This is followed by an examination of different mathematical techniques for the allocation of capital in Chapters 4 and 5.

The study will specifically focus on allocation decisions pertaining to main financial sectors of the JSE Securities Exchange, but the procedure suggested may easily be extended to the security selection and asset class allocation stages. A simplified top–down approach will be followed in the sense that historical macroeconomic scenarios will be identified in order to categorise the environment within which the allocation decisions occur. The comparison process will first entail the examination of a capital allocation method by means of optimisation (Chapter 4). The aim is to point out that, by formulating an optimisation problem under appropriate assumptions, such a method may contribute largely to understanding the nature of the underlying assets.

Thereafter, certain risk–adjusted ranking methods will be introduced in Chapter 5. Sectoral rankings, each pertaining to a historical period during which a specific economic scenario prevailed, will be examined in order to conclude whether they display significantly similar characteristics. Chapter 5 will conclude by determining whether the results of ranking methods may legitimately be used as an approximation for the results of optimisation procedures, as is often done in practice.

Throughout the decision support procedures (optimisation and ranking), different tolerances for risk will be assumed in order to investigate the effect of risk tolerance on the subsequent portfolio efficiency. Whether or not this effect seems to vary in different economic environments, will also be examined. It is believed that this study and the procedures that are discussed herein may contribute to understanding the nature of different financial sectors under different circumstances in a very practical way. The suggested procedures may not only be used by multi–managers for this purpose, but also by the specialists they employ to manage underlying portions of a total portfolio.

Chapter 2

Theoretical Background

In this chapter, the theoretical history of investments will be touched upon in order to understand the development of some measures, which will be used in the analyses that will follow in the rest of the study. First, in §2.1 the nature of certainty and risk will be discussed, together with their impact on investment decision–making.

In §2.2, the development of a measure of risk by Markowitz, which laid the foundation for the so–called modern portfolio theory, is discussed. The quantification of risk broadened the extent to which portfolios could be assessed. The quantification of an investor's tolerance of risk followed, where concepts from the field of utility theory (a mathematical component of behavioural science) were employed to describe the attitude of investors toward risk and return. This allowed for a much more scientific platform for decision–making with regards to specific types of investors.

In §2.3, indifference contours, which are assumptions that join all the investment alternatives with which a particular investor will be equally comfortable, is discussed. In §§2.4–2.5, it is shown how Markowitz incorporated these assumptions of investor behaviour and his method for quantifying risk to develop a rigorous model for the selection of a risky portfolio to fit the requirements of a specific investor.

The development within the field of investments continued with the realisation of the capital market theory, as discussed in §2.6. This theory was based on the assumptions made by Markowitz regarding risk and return and its purpose was to extend portfolio theory to a model that may be used for the pricing of all risky assets. The result was the Capital Asset Pricing Model (CAPM).

At the stage of development of the CAPM, there were no composite measures of risk and return available in the investment industry. However, the CAPM assumptions lend themselves to incorporate these two factors, which gave rise to the risk–adjusted return measures, as discussed in §2.7.

2.1 The nature of certainty and risk

When referring to the expectations of an investor with respect to an investment, we distinguish between the following two concepts.

1. **Certainty**: Perfect certainty refers to cases where the investment professional's expectation is single–valued; that is, an individual or group views prospective profits in terms of one particular outcome, and not in terms of a range of alternative outcomes. Investments that may be classified in this category include savings accounts, call accounts and negotiable cash deposits. These investment vehicles, whose outcomes are known with certainty, are usually called *riskless assets*.

2. **Risk**: The term risk is used to describe an investment choice of which the return is not known with absolute certainty, but for which an estimated array of alternative returns together with their probabilities are available; in other words, for which the distribution of expected returns is known. This distribution may have been estimated on the basis of objective or subjective probabilities. Investments that are classified in this category include all market–linked investments, like government bonds, shares and unit trusts. Investment vehicles that are exposed to risk are usually called *risky assets*.

An example of a frequency distribution (a survey of the number of occurrences of a specific event, in this case the number of occurrences of a certain return being rendered) is given in Table 2.1, which summarises the historical record of rates of return for a hypothetical investment over a period of 40 years. The histogram of the distribution is given in Figure 2.1. Historical data like those in Table 2.1 are often available for financial investments, but even where a long time series of past rates of return is available, the investment decision procedure remains complex. There may often be no reason why the future distribution of returns should resemble their distributions in the past. Furthermore, even if the distribution is expected to remain unchanged, realising positive returns (the right hand side of the histogram) or negative returns (the left–hand side) in any particular year is random and therefore largely a matter of luck. In other words, even if it were a known fact that the past will be representing the future, investing for one year is equivalent to drawing one observation from the distribution of a random variable. Because of this, future probability beliefs with respect to financial investments are almost invariably assumed to be subjective.

2.2 Markowitz Portfolio Theory

In the 1950's and early 1960's a large segment of the investment community became concerned with the risk of investment portfolios. At that time, evaluation of portfolio performance was almost entirely done based on the rate of return. The existence of risk was acknowledged, but it was not clear how this entity should be quantified. In 1952, Markowitz [58] laid down the foundation of modern portfolio management by introducing a set of financial and economic assumptions to typify the investment process. During the

Interval	Rates of Return (%)	Frequency (number of years)
1	−30.00 to −20.01	2
2	−20.00 to −10.01	3
3	−10.00 to −0.01	5
4	0.00 to 9.99	10
5	10.00 to 19.99	9
6	20.00 to 29.99	6
7	30.00 to 39.99	3
8	40.00 to 49.99	2
	TOTAL	40

Table 2.1: An example of a frequency distribution of rates of return for a hypothetical investment over a 40 year period

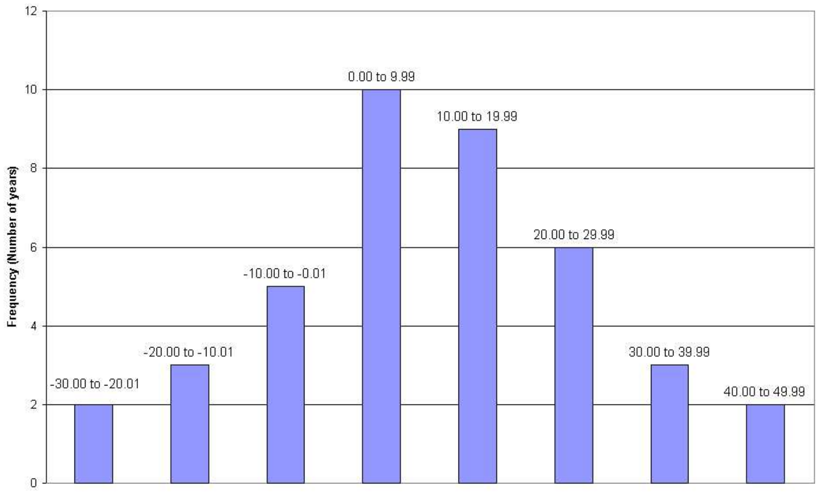


Figure 2.1: Histogram for Table 2.1

late 1950's Markowitz [58, 59] showed that the variance (or standard deviation) of the rate of return of a portfolio was a meaningful measure of risk under a reasonable set of assumptions and also derived a formula for computing the variance (σ_p^2) of a portfolio of risky assets as

$$\sigma_p^2 = \sum_{j=1}^{n}\sum_{i=1}^{n} w_i w_j \mathrm{Cov}(\underline{r}_i, \underline{r}_j) \tag{2.2.1}$$

$$= \sum_{j=1}^{n}\sum_{i=1}^{n} w_i w_j \rho_{\underline{r}_i,\underline{r}_j} \sigma_{\underline{r}_i} \sigma_{\underline{r}_j}, \tag{2.2.2}$$

where

$$\begin{aligned} n &= \text{the number of risky assets in the portfolio,} \\ w_i &= \text{weight allocated to risky asset } i \text{ within the portfolio,} \\ \underline{r}_i &= [r_{i1}, r_{i2}, \ldots, r_{ik}], \text{the series of length } k \text{ of returns for risky asset } i, \\ \rho_{ij} &= \text{coefficient of correlation of the returns of risky assets } i \text{ and } j, \\ \sigma_i &= \text{standard deviation of returns of risky asset } i. \end{aligned}$$

Covariance is an *absolute* measure of the extent to which two sets of numbers move together over time. In this regard, "moves together" means that the corresponding data entries in the sets of numbers are generally above their means or below their means at the same time. Covariance between sets $\underline{r}_i = [r_{i1}, r_{i2}, \ldots, r_{in}]$ and $\underline{r}_j = [r_{j1}, r_{j2}, \ldots, r_{jn}]$, both of length n is defined as

$$\text{Cov}(\underline{r}_i, \underline{r}_j) = \frac{\sum_{k=1}^{n}(r_{ik} - \hat{\underline{r}}_i)(r_{jk} - \hat{\underline{r}}_j)}{n}, \tag{2.2.3}$$

where $\hat{\underline{r}}_i$ and $\hat{\underline{r}}_j$ are the respective means of the series. If the corresponding numbers within the sets are consistently above or below their individual means at the same time, their products will be positive, and the covariance–average will have a positive value. In contrast, if the value of r_{ik} is below its mean when the value of r_{jk} is above its mean, or vice versa, their products will be negative and the covariance may be negative. As was noted, covariance is an absolute measure of the relationship of two series and therefore can range from $-\infty$ to $+\infty$. To obtain a normalised indication of the covariance measure, the correlation coefficient ($\rho_{\underline{r}_i,\underline{r}_j}$) is used, where

$$\rho_{\underline{r}_i,\underline{r}_j} = \frac{\text{Cov}(\underline{r}_i, \underline{r}_j)}{\sigma_{\underline{r}_i}\sigma_{\underline{r}_j}}. \tag{2.2.4}$$

If it is noted that the covariance of a series (say the series $\underline{r}_i$) with itself is its variance, it becomes clear that if two series move completely together, the covariance would equal $\sigma^2_{\underline{r}_i}$ and

$$\frac{\text{Cov}(\underline{r}_i, \underline{r}_i)}{\sigma_{\underline{r}_i}\sigma_{\underline{r}_i}} = 1.$$

The correlation coefficient would equal unity in this case, and the two series are said to be perfectly correlated. Similarly, if two series move in totally opposite directions, in other words, if all corresponding entries have the same absolute values, but have opposite signs,

$$\frac{\text{Cov}(\underline{r}_i, \underline{r}_j)}{\sigma_{\underline{r}_i}\sigma_{\underline{r}_j}} = -1.$$

From (2.2.4) it follows that

$$\text{Cov}(\underline{r}_i, \underline{r}_j) = \rho_{\underline{r}_i,\underline{r}_j}\sigma_{\underline{r}_i}\sigma_{\underline{r}_j}, \tag{2.2.5}$$

which has been used in (2.2.2). The formula in (2.2.2) not only indicated the importance of diversification to reduce risk (the smaller the correlation between the returns of two portfolios, the smaller the risk measure), but also showed how to diversify a portfolio efficiently (choose the risky assets with the lowest correlation of returns).

Within his foundation assumptions, Markowitz in [58] assumed that an investor makes his/her investment decisions based solely on risk and return considerations. However, he did not suggest how this relationship should be quantified [75]. Substantial work has been done in the field of behavioural science, focussing specifically on applications in the investment arena [24, 27, 51, 63, 69, 91]. However, gathering reliable information regarding investors' tastes and their consequent reactions to events is likely to remain one of the central problems of investment analysis for some time to come [54].

Definition 3 *A comfort/satisfaction function is an attempt at the quantification of the dynamics of investment decision–making as for a specific investor.*

Such a function mathematically combines the attitude of an investor towards risk and the subsequent return from exposing the investment to risk. It renders an indication of the comfort/satisfaction of an investor with a specific risky investment alternative. One way suggested by financial theorists in [10] to describe the risk–return relationship of an investor succinctly results by using a comfort/satisfaction function of the form

$$U_{ck} = R_c - \frac{a_k}{s}\sigma_c^2, \qquad (2.2.6)$$

where

$$\begin{aligned} U_{ck} &= \text{investment comfort/satisfaction of investor } k, \\ &\quad\ \text{associated with riskreturn combination } c, \\ R_c &= \text{return of riskreturn combination } c, \\ a_k &= \text{risk tolerance factor for investor } k, \\ s &= \text{scaling constant}, \\ \sigma_c^2 &= \text{variance of riskreturn combination } c. \end{aligned}$$

The concept of *risk tolerance* is a particularly useful measure of an investor's willingness to take on additional risk in order to achieve added return. It is incorporated in a comfort/satisfaction function so as to simplify the interpretation of the latter [56].

Definition 4 *Risk tolerance is the added standard deviation that offsets one unit of added expected return, providing the same investment comfort/satisfaction for the investor in question.*

A portion of the final term in equation (2.2.6), namely $a_k\sigma_c^2$, may be considered a *risk penalty* [56]. It increases with an increase in the standard deviation of the investment alternative or a decrease in the investor risk tolerance. Correspondingly, the investor's comfort/satisfaction may be considered an indication of risk–adjusted return, where the adjustment is done in such a way that it encompasses the individual's perception and subsequent choice dynamics. The constant s in the last term is employed to scale the term for subtraction from the first. Although the formula in (2.2.6) may provide a powerful tool for rationalising investors' behaviour, it does not, by itself provide the proverbial dividing rod for choosing a risky portfolio. Investment choice remains a difficult problem in the absence of more precise knowledge regarding the reaction of investors to uncertainty. Kritzman and Rich [51] assumed that investor behaviour suggests that they care about multiple dimensions of risk and developed a risk containment model in which investor comfort/satisfaction is explicitly contingent on more than one random variable. McMullen and Strong [62] also suggest that investor comfort/satisfaction is, in addition to risk and expected return, reliant upon a combination of other factors, such as brokerage costs, capital constraints, and expense ratios. In this study, it is assumed that the partial information on the investor's preferences, contained in (2.2.6) is adequate to add some value to the process of choosing a risky portfolio and it is demonstrated how this information may be useful in decision making.

Within the extremely broad spectrum of investor attitudes towards risk, it is common to group investors into the following three basic types.

1. **Risk averters** – these individuals are comfortable with an increase in risk exposure only if the compensation for the additional risk is proportionally *greater* than the addition of risk.

2. **Risk neutral individuals** – these individuals are comfortable with an increase in risk exposure if the compensation for the additional risk is *equal* to the additional risk taken.

3. **Risk seekers** – these individuals are comfortable with an increase in risk exposure even if the compensation for the additional risk is proportionally *less* than the addition of risk. They are therefore willing to give up some compensation, just for the enjoyment of being exposed to risk.

By closely examining the characteristics of the risk tolerance factor a_k in (2.2.6), it becomes apparent that it may be used as indicator to classify individuals within two of the three basic investor types. For the purposes of this study, we shall assume the following.

Definition 5 *A risk averter has a risk tolerance factor* $a_k \geq 0$.

Definition 6 *A risk seeker has a risk tolerance factor* $a_k < 0$.

The comfort/satisfation function of a risk–neutral investor may be portrayed against his/her risk tolerance factor by a straight line with slope one.

Evidence that most investors are risk–averse may be found in their purchase of various types of insurance, such as health/life insurance and car/house insurance. Insurance is in essence a current certain outlay of a given amount to guard against an uncertain, possibly larger outlay in the future. Individuals are therefore willing to pay to avoid the risk of potentially large future losses. However, the foregoing does not imply that the vast majority of individuals are risk–averse regarding all their financial commitments. It is a common occurrence all over the world that individuals buy insurance and also gamble at the race tracks, where it is known that the expected returns are negative, which means that participants are willing to pay for the excitement of the risk involved [54]. Friedman and Savage [24] constructed a hypothesis to explain this apparent inconsistency in human behaviour. They proposed that these investors' tolerence for risk is dependent upon the level of risk of the investment. If the risk is less than a certain *tolerance divide* σ^* (the magnitude of σ^* will differ from investor to investor), the investor will be willing to give up some compensation, just for the enjoyment of being exposed to risk and is therefore a risk seeker. However, if the risk exceeds σ^*, the investor becomes comfortable with an increase in risk exposure only if the compensation for the additional risk is proportionally greater than the addition of risk, which makes him/her risk averse.

This combination of risk preference and risk aversion may be explained by extending the comfort/satisfaction function (2.2.6) to accommodate a dual risk tolerance factor a_k^* in such a way that

$$U_{ck}^* = R_c - a_k^* \sigma_c^2,$$

where

$$a_k^* = \begin{cases} a_{k_1}, & \text{if} \quad \sigma_c < \sigma^*, \\ a_{k_2}, & \text{if} \quad \sigma_c \geq \sigma^*, \end{cases}$$

and $a_{k_1} < 0$, $a_{k_2} \geq 0$.

Taking into account the suggestions of Friedman and Savage regarding human nature, it may be true that individuals cannot necessarily be categorised into a single investor type. However, those investors whose capital is exposed to the tradable securities market, generally invest in this specific medium for a chance of higher returns on their capital than is offered by a (riskless) bank account [87]. Their foremost requirement for this specific investment is to be compensated with a premium to the risk they are allowing their investment to endure. This makes the vast majority of investors risk averse regarding their financial market investments, although the degree of this aversity may vary from investor to investor [54].

The main aim of trying to quantify the decision–making dynamics of an investor by considering his/her risk tolerance, is ultimately to suggest a best portfolio of risky alternatives to fit the tolerance of the investor. A first step in attaining this goal is to eliminate those risky alternatives that are not suitable for the needs of the investor. In order to do this, the comfort/satisfaction of an investor with a risk–free investment is set as benchmark and serves as a first criterion for assessing a risky alternative. A risky investment with a risk–return combination that renders a lower comfort/satisfaction level than the risk–free benchmark, should be discarded, as the combination does not fit the requirements of the investor in question. For the choice of any other risky alternative, the investor would like to maximise his comfort/satisfaction.

In the following section, it will be discussed how Markowitz incorporated his assumptions of investor behaviour and method for quantifying risk (equation (2.2.1)) in order to develop a rigorous model for the selection of a risky portfolio to fit the requirements of a specific investor.

2.3 Indifference contours

Markowitz assumed that all investors are *rational* individuals [58]. In other words, with all else being equal, these individuals prefer more return to less return. This characteristic is in essence quantified by the *mean–variance criterion* in the sense that a rational investor will always prefer a *dominating portfolio.*

Definition 7 *A portfolio F dominates (is preferred to) a portfolio G by the mean–variance criterion if and only if*

$$R_F \geq R_G \tag{2.3.1}$$

$$\text{and} \quad \sigma_F^2 \leq \sigma_G^2, \tag{2.3.2}$$

where R denotes return and σ^2 the variance of the portfolio, on the condition that at least one strict inequality holds.

In this study, standard deviation, σ, instead of σ^2 is the assumed measure of risk. For the practicality of implementation of the criterion in this thesis, (2.3.2) is substituted by

$$\sigma_F \leq \sigma_G, \tag{2.3.3}$$

without altering whether the condition is satisfied or not. The mean–variance criterion is by its very nature two–dimensional, so that the Markowitz efficiency analysis using this criterion readily lends itself to graphical representation. In view of this, the preferences of an investor may now be investigated graphically, by referring to comfort/satisfaction function (2.2.6). Whenever a portfolio such as M in Figure 2.2 is replaced by an alternative located in the direction of the arrow marked a, the investor's comfort/satisfaction is increased, as any movement along the line Ma raises the return, without altering the standard deviation, and therefore the risk of the investment. Similarly, any movement in the direction of arrow b reduces the investor's comfort/satisfaction with the investment, since the standard deviation is thereby increased without any change in the return of the investment. Since any movement in the direction of arrow b reduces the investor's comfort/satisfaction, while any movement in the direction of arrow a increases the comfort/satisfaction, a point may be found in a direction between those of a and b (say, K) at which the investor's comfort/satisfaction towards the investment is neither increased nor decreased. If alternative K is substituted for portfolio M, both the return and standard deviation of the investment is increased, but since by assumption the investor's investment comfort/satisfaction remains unchanged, the increased return is exactly offset by the increased standard deviation, so that the investor is indifferent to the choice of these investments. By examining the investor's specific comfort/satisfaction function, other combinations of return and standard deviation may also be found to render the same level of satisfaction/comfort as alternatives M and K. In principle all such combinations may be plotted along a so called *indifference contour* such as I.

Definition 8 *An indifference contour joins those combinations of return and standard deviation with which a particular investor will be equally comfortable/satisfied.*

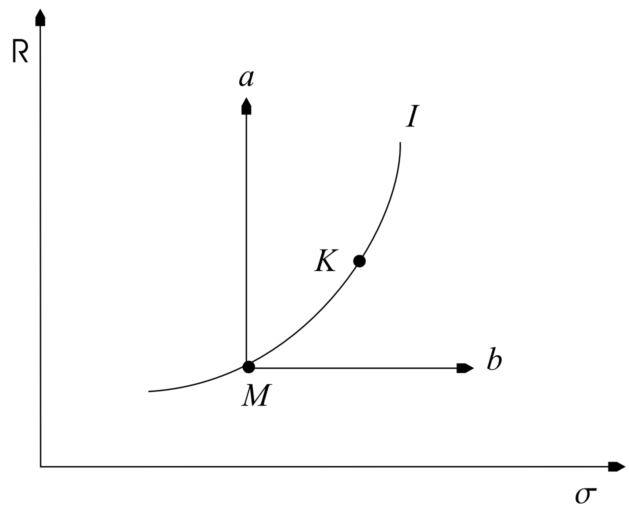


Figure 2.2: Indifference contours in the risk–return domain

One characteristic of an indifference contour is that it represents a lower level of comfort/satisfaction than the contour that lies above it. This again results from the fact that any movement in the direction of arrow b reduces the investor's comfort/satisfaction, while any movement in the direction of arrow a increases the comfort/satisfaction as stated previously. Another characteristic of indifference contours is that they are continuous and independent of the actual investment alternatives available. The situation may occur that a return–standard deviation combination renders a comfort/satisfaction value, but no actual investment choice exists with the specific return–standard deviation combination. However, if such an alternative existed, the investor would have been indifferent to this investment and any other existing portfolio represented on the contour. The availability of investment alternatives and the assessment of these will be discussed in §2.4.

By assuming different levels of comfort/satisfaction for an investor and calculating all risk–return combinations that render the same comfort/satisfaction in accordance with the specific investor's comfort/satisfaction function, a *landscape or set of indifference contours* is generated.

Definition 9 *A set of indifference contours represents the investor's attitude towards all levels of risk and return.*

Another property of an indifference contour is that it will always be positively sloping in order to accommodate an increased (decreased) standard deviation to be offset by an increased (decreased) return. No two contours within the set of indifference contours of a single investor can intersect. This may be proved via a simple contradiction argument by examining Figure 2.3, in which two different curves I_1 and I_2 for the same individual intersect at point R. The characteristics of indifference contours I_1 and I_2 prescribe that an investor will be equally comfortable/satisfied with all investments represented on curve I_1 and equally comfortable/satisfied with all investments represented on curve I_2. The fact that the two contours intersect, implies that the level of comfort/satisfaction associated with them should be equal in order to accommodate the mutual portfolio. However, the rationality assumption (which implies the preference for a dominating portfolio) prescribes that the investor will always prefer R_1 to R_2, because of the higher return rendered, but this contradicts the assumption that the investor is indifferent to all the portfolios represented on the two contours.

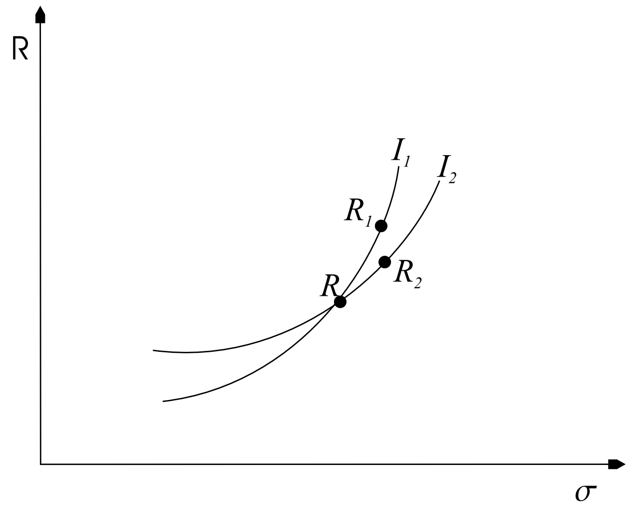


Figure 2.3: Intersecting indifference contours in the risk–return domain

As implied by (2.2.1), different combinations of risky assets provide alternative combinations of risk and return, which in turn provide alternative levels of comfort/satisfaction for an investor. Therefore, investor preferences present only half of the ingredients required for an optimal risky portfolio decision. Financial market conditions and the risky opportunities they produce represent the other half [56]. In the next section, attention will be given to finding efficient investment opportunities out of those presented by current market conditions, so that they may be assessed for possible inclusion in an investment portfolio.

2.4 The Markowitz efficient frontier

Markowitz continued by using his formula for calculating risk of a set of risky assets and the mean–variance criterion to find a set of combinations that should be selected above any other. He supposed that portfolios are compiled by considering all combinations of available risky assets. If his train of thought is represented graphically, it may be shown that, giving its expected return and standard deviation, any portfolios may be represented by a point on the risk–return plane as indicated in Figure 2.4. Markowitz applied the mean–variance criterion rules to the set of feasible portfolio values in order to isolate the set of efficient investments. Only the portfolios represented by the points in the risk–return plane falling on the curve AB in Figure 2.4 are efficient (undominated). This may be shown by first examining all portfolios on an arbitrary vertical line L. For every portfolio represented by a point on L, the return of the portfolio represented by point C is greater, while the standard deviations are equal, thus satisfying both inequalities (2.3.1) and (2.3.3) of which (2.3.1) is strict. Hence, no portfolio on L, but off the segment AB can possibly be efficient.

For any two portfolios on the AB segment, the inequalities (2.3.1) and (2.3.3) will always be satisfied, but because of the convexity of the curve, neither the returns nor the standard deviations will be equal. The condition that at least one strict inequality holds is therefore not satisfied and none of the portfolios on the segment are dominated by another on the segment, resulting in the segment being called the *efficient frontier* (also see [23, 79]).

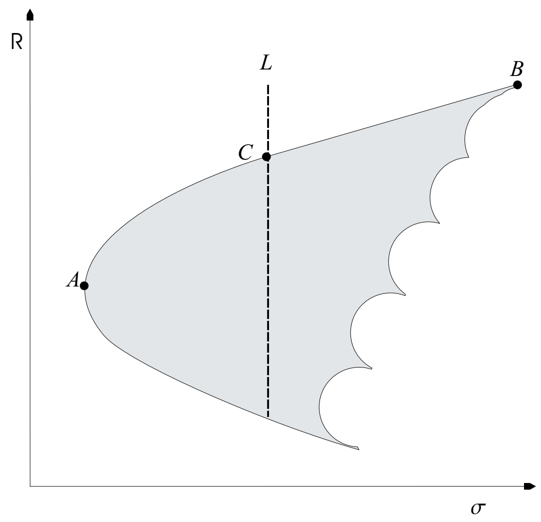


Figure 2.4: Markowitz efficient frontier in the risk–return domain

The efficient frontier not only contains information on current financial market conditions, but its existance also simplifies the process of selecting risky portfolios to some extent. If the risk–return preferences of a investor, implied by his/her set of indifference contours, and information obtained from the efficient frontier is merged, an attempt is made to choose the best risky investment opportunity for the specific investor. In essence, the process of combining the information simply relies on the rationale that an investor desires an existing, risky opportunity mix, which renders the highest comfort/satisfaction level. Figure 2.5 superimposes the individual's set of indifference contours on the opportunity set of investments. The investor would prefer an alternative whose risk–return combination places it on indifference contour I_5, but no investment opportunity of this kind exists (the shaded opportunity set holds all risk–return combinations attainable by combining the risky investments available, and I_5 does not intersect this set). The most comfortable/satisfactory investment this investor can make, given the alternatives available, is to choose the portfolio with risk–return combination a on the efficient set AB which is on indifference contour I_3. Should the investor choose an alternative risky portfolio out of the efficient set, say point c, the investor's level of comfort/satisfaction will drop, as the alternative permits the investor to reach indifference contour I_2, which lies below I_3 and therefore represents a lower comfort/satisfaction level. Therefore portfolio a is optimal for the investor with the set of indifference contours presented in Figure 2.5.

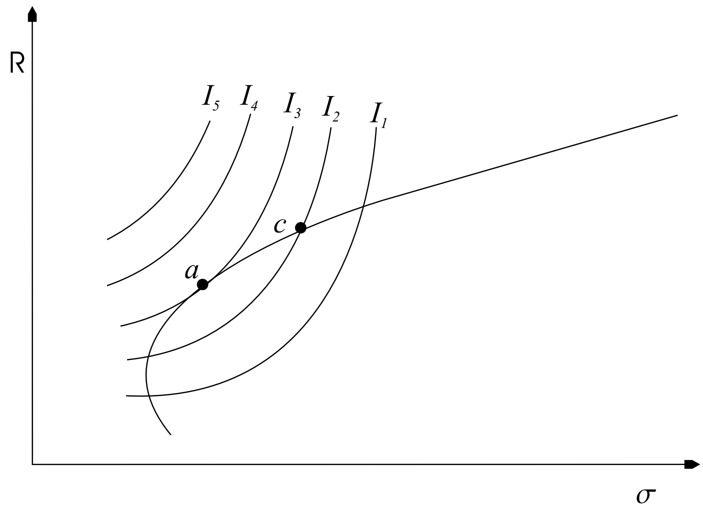


Figure 2.5: Investor preferences and the efficient frontier in the risk–return domain

However, it cannot be inferred from this analysis that portfolio c will never be an optimal choice. Since the indifference contours are induced by the comfort/satisfaction functions, their shapes are also prescribed by these functions. It is quite conceivable that another investor may have indifference contours representing his/her preferences, which are tangent to the efficient set at point c rather than point a. This is illustrated in Figure 2.6, which depicts the indifference contours of two different investors. The indifference contour of the investor who selects portfolio a (I_1) in Figure 2.6 is steeper than that of the investor who prefers investment c (I_2). This means that when the standard deviation is increased by one unit, the former investor (the one with indifference contour I_1) requires a greater increase in return to offset the increased standard deviation than the latter individual (whose preferences are portrayed by I_2). However, this is exactly the definition of risk tolerance in Definition 4. Therefore, the higher the risk tolerance of the investor, the less return is needed to offset the increased risk and hence the gentler the slope. A different slope can be accommodated at any point on the efficient locus AB, therefore any one of these points may represent an optimal portfolio for *some* investor.

2.5 Asset allocation via optimisation techniques

In §2.2 the process for choosing an optimal portfolio in accordance with the preferences of a specific investor was described graphically. In this section it is shown how this optimisation problem may be stated mathematically. Firstly, the aim of the investor is to maximise his/her comfort/satisfaction. By again referring to (2.2.6), the aim is stated

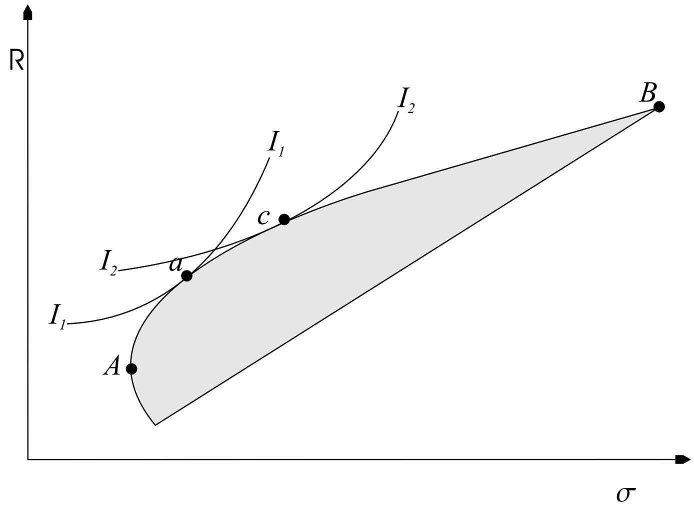


Figure 2.6: Optimal portfolios for different investors in the risk–return domain

as

$$\text{Maximise } U = R_c - \frac{a_k}{s}\sigma_c^2. \tag{2.5.1}$$

However, the comfort/satisfaction may only be maximised taking existing alternatives into account. The combinations available in the opportunity set (R_c, σ_c) are prescribed by the formulae (as set out by Markowitz, see (2.2.1))

$$R_c = \sum_{i=1}^{n} \hat{\underline{r}}_i w_i \quad \text{and} \tag{2.5.2}$$

$$\sigma_c = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \text{Cov}(\underline{r}_i, \underline{r}_j)}, \tag{2.5.3}$$

where

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0. \tag{2.5.4}$$

Here $\hat{\underline{r}}_i$ denotes the average return from risky investment i, compared to $\underline{r}_i$ which is a set of historical returns for asset i, while w_i and $\text{Cov}(\underline{r}_i, \underline{r}_j)$ have the same meaning as in (2.2.1).

By introducing the opportunity set included in (2.5.2), (2.5.3) and (2.5.4) into the objective function (2.5.1), the comfort/satisfaction function is restricted to consider only those combinations that actually exist. Hence we obtain the nonlinear optimisation problem

$$\left.\begin{aligned} \text{Maximise} \quad U &= \sum_{i=1}^{n} r_i w_i - \frac{a_k}{s} \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \operatorname{Cov}(\underline{r}_i, \underline{r}_j) \\ \text{subject to} \quad \sum_{i=1}^{n} w_i &= 1, \\ w_i &\geq 0 \text{ for all } i = 1, \ldots, n. \end{aligned}\right\} \quad (2.5.5)$$

Nonlinear programming approaches of which one is described in (2.5.5) are widely used during asset allocation, since unlike other approaches, they are able to deal effectively with the fact that risk depends on interactions among the risky securities held [56]. Hill [37] supports the use of mean–variance optimisation as a tool to help investors build the necessary wealth in order to reach long–term financial goals. She adds that the construction of an optimal portfolio is a complex one. One of the most critical contributions involves the development of risk, return and correlation assumptions to approximate future asset class behaviour. Beller, *et al.* [9] also examined Markowitz's mean–variance optimisation. Their analysis provided strong evidence that this method, in conjunction with forecasting models for industry returns, is a useful tool for portfolio selection.

A possible alternative to mean–variance optimisation, namely *data envelopment analysis (DEA)* is discussed in [62] by McMullen and Strong. Despite the prospects of this method, McMullen and Strong, emphasize that the purpose of their study was not to prescribe an alternative to mean–variance optimisation, but rather to demonstrate the DEA methodology. DEA is an operations research tool potentially useful in portfolio selection when an investor's expected comfort/satisfaction function contains more than two attributes. Basso and Funari [8] also suggest DEA to determine the overall return on an investment as, in addition to the traditional mean–variance terms, such a tool may incorporate other investment attributes such as brokerage costs, capital constraints, and expense ratios in a coordinated set of linear programming problems. Ultimately the results may be used to determine portfolios with the best combination of attributes for any multi–attribute comfort/satisfaction function.

2.6 Capital Market Theory

In §1.3 it was stated that one aim of the thesis is to attempt to understand the risk–return characteristics of the financial market sectors by means of risk–adjusted ranking methods. In order to sketch the background for the use of these methods, it is important to pay some more attention to capital market theory, which is a framework within which the market may be viewed and interpreted. Twelve years after the Markowitz–revolution, the Capital Asset Pricing Model (CAPM), which is based on a set of predictions concerning expected returns on risky assets and encapsulates the dynamics of the capital market theory, was developed by Sharpe [80], Lintner [55] and Mossin [65]. The time required for this gestation indicates that the leap from Markowitz's portfolio selection model to the CAPM was not trivial.

2.6.1 Basic assumptions

The assumptions made by Markowitz in order to establish the mean–variance portfolio theory made it possible to derive a generalised theory of capital asset pricing under conditions of uncertainty. The purpose of capital market theory was to extend portfolio theory to a model that may be used for the pricing of all risky assets. The final product was the capital asset pricing model (CAPM). The model has generally been attributed to Sharpe [80], but similar independent derivations were made by Lintner [55] and Mossin [65]. Consequently, in the literature, it is sometimes referred to as the Sharpe–Lintner–Mossin (SLM) capital asset pricing model.

The assumptions that form the basis of the capital market theory framework are as follows [75]:

1. All investors are Markowitz efficient investors (or rational investors), which means that they want to maximise their comfort/satisfaction and hence they choose a risk–return combination on the efficient frontier, where their comfort/satisfaction contour is tangent to the frontier of risky alternatives.
2. There are many investors, each with a wealth that is small compared to the total wealth of all investors. Investors are price–takers, in that they act as though security prices are unaffected by their own trades.
3. It is possible for investors to borrow or lend any amount of money at a fixed rate.
4. All investors plan for an identical holding period.
5. Investments are limited to a universe of publicly traded financial assets, such as shares and bonds, and to riskless borrowing or lending arrangements.
6. Investors pay no taxes on returns and no transaction costs (commissions and service charges) on trades of securities.
7. All investors analyse securities in the same way and share the same economic view (also referred to as homogeneous expectations).
8. Capital markets are in equilibrium, therefore all securities are priced correctly. This assumption relates to the efficient market hypothesis (EMH) discussed in [10, 30].

The usefulness of some of these assumptions has been debated [75] and since the development of the theory, some of the assumptions have been relaxed, which has provided a number of new models that allow for market dependence [68].

Some of the models that maintain the rational investor assumption include:

1. The *Competence–Difficulty Gap Model* by Heiner [35]. This model states that the greater the spread between competence of investors and the complexity of information, the greater the increase in the market dependence. The investors are rational. They simply cannot react quickly enough to provide an efficient market.

2. The *Rational Belief Equilibrium Model* by Kurz [52]. This model states that different market participants hold different beliefs about future events even though they share the same information.

3. The *Different Investment Horizon Theory (Fractal Market Theory)* by Peters [73] (pre–CAPM). This model states that different investors have different investment horizons and will stay in their "preferred habitat" no matter what expectations are in the current information set. The model also states that the market requires investors with different time horizons to trade and maintain liquidity.

Rationality of investors are rejected by the following models:

1. The *Utility Satisfying Model* by Simon [83]. This model states that, rather than paying the heavy information costs to be completely informed and to maximise utility, individuals will make decisions using incomplete information in order to achieve satisfactory utility.

2. The *Irrational Investors Model* by Arrow [5]. This model states that very few participants in the market are rational. Most markets include irrational investor behaviour leading to disequilibrium prices.

Other models characterise the market as an information processing system that generates disequilibrium prices:

1. The *Sequential Information Arrival Model* by Copeland [18]. This model states that information arrives in the marketplace sequentially. Since the information set is incomplete until the full information set arrives, trades will take place at disequilibrium prices.

2. The *Bifurcation Theory* by Nawrocki [67]. This model states that the market is an adaptive information assimilation process. As the price moves away from the equilibrium price, the market will institute numerous structural changes (bifurcations) in order to obtain information necessary to bring the price back to equilibrium. The larger the disequilibrium–equilibrium disparity, the larger the structural change required. The stress of continuing large disequilibrium–equilibrium disparities can result in the catastrophic breakdown of the market.

However, because the aim of this section is to discuss the fundamentals in the development of the CAPM and not the validity of the theory itself, further attention will not be given to the above extentions of the CAPM assumptions. Apart from the assumptions of the capital market theory, which set the environment for the development of the CAPM, a major factor in the development of this model was the concept of a risk–free asset.

2.6.2 Risk–free rate of return

In §2.1, both the concepts of risky and risk–free assets were discussed. However, following the suggestion of Markowitz to represent the risk of an asset by its variability or standard deviation (§2.2), their definitions may be extended.

1. A *risky asset* is one about which there is uncertainty regarding expected future returns. This uncertainty may be measured by the standard deviation of returns.
2. A *risk–free asset* f, is one for which there is no uncertainty regarding expected future returns, so that the variability of future returns, and therefore the standard deviation of returns, is zero ($\sigma_f = 0$).

The covariance between two sets of returns

$$\begin{aligned} \underline{r}_f &= [r_{f1}, r_{f2}, \ldots, r_{fk}, \ldots, r_{fn}] \\ \underline{r}_j &= [r_{j1}, r_{j2}, \ldots, r_{jk}, \ldots, r_{jn}] \end{aligned}$$

is given by

$$\text{Cov}(\underline{r}_f, \underline{r}_j) = \frac{\sum_{k=1}^{n}(r_{fk} - \mathcal{E}[\underline{r}_f])(r_{jk} - \mathcal{E}[\underline{r}_j])}{n}.$$

Here the scalar $\mathcal{E}[\underline{r}_j]$ represents expected return for security j by taking the information obtained from the series of historical returns $\underline{r}_j$ into account. It is sometimes assumed that the expected return of a security equals its average return over a given historic period. This assumption is followed in the rest of this discussion, but the expectancy operator may easily be defined to combine the information differently. If $\underline{r}_f$ denotes the set of returns of a risk–free asset, $\mathcal{E}[\underline{r}_f] = r_{fk} = r_f$ (say), as the variability of returns is zero. Consequently, when computing the covariance of a risk–free asset with any risky asset or portfolio of assets, $r_{fk} - \mathcal{E}[\underline{r}_f] = 0$ and hence $\text{Cov}(\underline{r}_f, \underline{r}_j) = 0$.

For the development of the capital market theory, Sharpe, Lintner and Mossin examined the rates of return and standard deviations of portfolios where a risk–free asset f and a portfolio of risky assets (one that exists on the Markowitz efficient frontier and with series of returns $\underline{r}_j$ and standard deviation $\sigma_{\underline{r}_j}$) are combined. They concluded that the expected return for such a combined portfolio, say p, is simply the weighted average of the two returns,

$$\mathcal{E}[r_p] = w_f\mathcal{E}[\underline{r}_f] + (1 - w_f)\mathcal{E}[\underline{r}_j] = w_f r_f + (1 - w_f)\mathcal{E}[\underline{r}_j], \tag{2.6.1}$$

where w_f is the proportion of the combined portfolio invested in the risk–free asset f. Therefore, the expected return of the combined portfolio is a linear combination of the returns of the relevant assets' returns.

For the examination of the standard deviation of the combined portfolio, the Markowitz suggestion for the calculation of variance and standard deviation (mathematically defined in §2.2) were applied in the reduced, two–asset portfolio form,

$$\sigma_p^2 = w_1^2\sigma_{\underline{r}_1}^2 + w_2^2\sigma_{\underline{r}_2}^2 + 2w_1w_2\text{Cov}(\underline{r}_1, \underline{r}_2). \tag{2.6.2}$$

Substituting the risk–free and risky assets of (2.6.1) for the first and second security respectively, (2.6.2) becomes

$$\sigma_p^2 = w_f^2\sigma_{r_f}^2 + (1 - w_f)^2\sigma_{\underline{r}_j}^2 + 2w_f(1 - w_f)\text{Cov}(\underline{r}_f, \underline{r}_j). \tag{2.6.3}$$

It has already been established that the standard deviation of a risk–free asset (σ_{r_f}), as well as the covariance of such an asset with any risky asset equals zero. Therefore, (2.6.3) becomes

$$\sigma_p^2 = (1 - w_f)^2\sigma_{\underline{r}_j}^2.$$

The standard deviation of a portfolio that combines a risk–free asset and a risky asset portfolio is therefore

$$\sigma_p = (1 - w_f)\sigma_{\underline{r}_j},$$

which is a linear combination of the standard deviations of the two sets of assets.

Since both the expected return and the standard deviation for such a combined portfolio are linear combinations, reliant upon the proportion allocated to each asset (or asset portfolio), the portfolio's returns and risks are represented by a straight line between the two assets on the risk–return plain. A graph depicting portfolio possibilities when a risk–free asset is combined with risky portfolios on the Markowitz efficient frontier is shown in Figure 2.7.

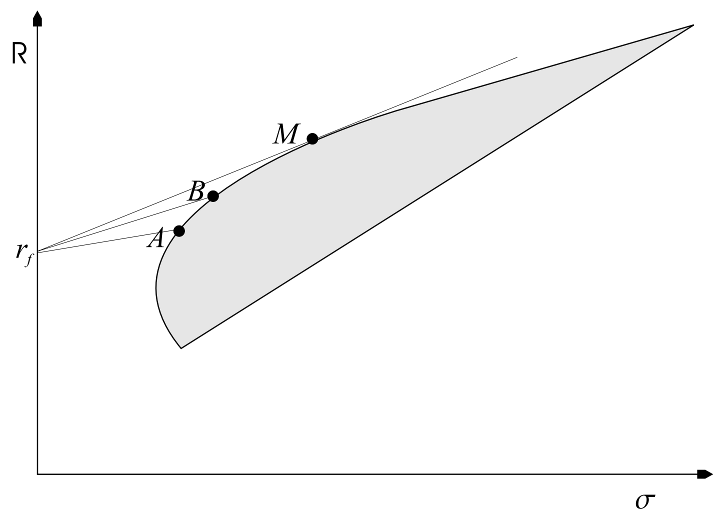


Figure 2.7: Combining a risk–free asset and alternative risky portfolios on an efficient frontier in the risk–return domain

In the scenario illustrated by Figure 2.7, it is possible to attain a portfolio with risk–return combination anywhere along the straight line between r_f and A, by investing some portion of the capital in the risk–free asset and the remainder in the risky asset portfolio A on the efficient frontier. This set of portfolio possibilities dominates (in terms of the rules of the mean–variance criterion discussed in §2.3) all risky asset portfolios below A on the efficient frontier, as for each of these risky asset portfolios, there is a portfolio on the straight line with an equal standard deviation, but higher return. Likewise, it is possible to attain any point along the straight line between r_f and B by investing in some combination of the risk–free asset and the risky asset portfolio B. Again, these combinations dominate all portfolio possibilities below B, including those on the straight line between r_f and A. A number of these lines may be drawn from r_f to the efficient frontier at higher and higher points, until the point of tangency between the straight line and the efficient frontier is reached, which coincides with the risk–return combination of risky portfolio M. The set of possibilities on the line between r_f and M dominates all alternatives below it, which makes them a new efficient set in terms of the capital market theory and is called the *capital market line (CML)*.

Within the South African context, there are many risk–free investments, ranging from bank accounts to fixed term deposits. Each of these investments quotes a different risk–free rate of return and the question is posed which one of these rates should be used in calculations regarding the CML. In the following section, it will be discussed why the rate of a 90–day Negotiable Certificate of Deposit (NCD) may be considered in calculations as the risk–free rate of return.

2.6.3 NCD as risk–free asset

An NCD is a fixed deposit receipt issued by a bank that is negotiable in the secondary market as a financial asset. The issuer undertakes to pay the amount of the deposit plus the interest to the holder of the certificate on maturity date. NCDs are usually issued for periods of 30, 60, 90, 180, 360 days or up to three years. When issued for periods of less than one year, interest is usually payable at the end of the period. When issued for longer than one year, interest may be payable either at the end of the period or six–monthly in arrears, but usually the latter. NCDs are issued at variable rates, usually linked to the wholesale prime rate.

A very important characteristic of NCDs is that they offer the same security as a term deposit with the bank in question, but are fully negotiable before the date of maturity. In order to ensure that these securities are fully negotiable, it is important to have an active secondary market for these to trade on. An active secondary market is important for a number of reasons. Firstly, an active secondary market provides investors in NCDs with the assurance that they will be able to dispose of their NCDs if they so desire. Secondly, a secondary market provides the basis for determining the yields that have to be offered on new issues and should be used for the fair pricing of existing ones. Thirdly, a secondary market registers changing market conditions rapidly, thereby indicating the receptiveness of the market for new primary issues.

Within the capital market, a distinction is made between a *deposit* and an *investment*. Any capital market security with a term shorter than 90 days is deemed a deposit and holds no risk, while any security with a term longer than 90 days is seen as an investment that may hold risk. In respect of its term, the 90–day NCD is therefore a link between a riskless deposit and a risky investment. However, because of the fact that NCDs have the same security as a bank deposit, coupled with the trading activity on the secondary market, which allows NCDs to incorporate current capital market conditions rapidly, the yield on the 90–day NCDs is a very good indication of the best risk–free rate of return. A graphical representation of the historic yields on the 90–day NCD, which will be used in the calculations of this study as the risk–free rate of return, is given in Figure 2.8.

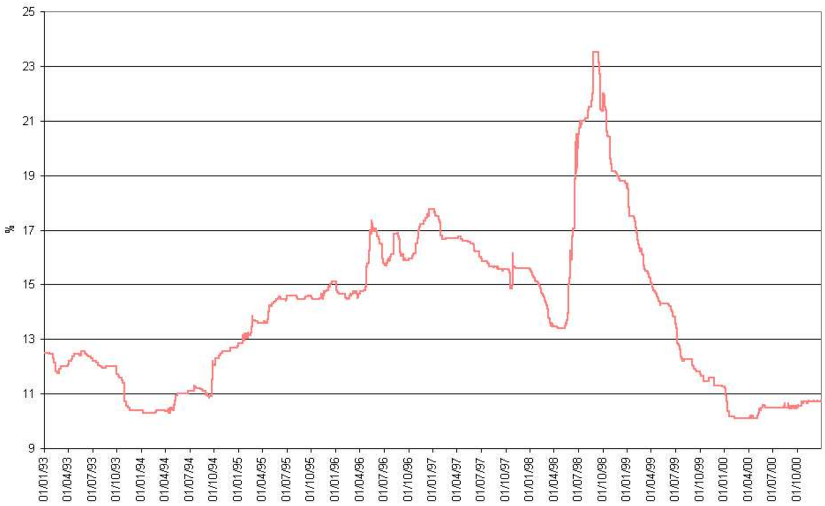


Figure 2.8: Yield on the 90–day NCD

2.6.4 The market portfolio

As was concluded in §2.6.2, portfolio M in Figure 2.7 is the tangent portfolio that renders the highest portfolio possibility line or CML. In the assumptions that serve as basis for the capital market theory, it was stated that all investors are Markowitz efficient investors, which means that all will invest their portion in risky assets somewhere on the efficient frontier. Because the CML suggests that a combination with portfolio M dominates the combination with any other portfolio on the efficient frontier, all investors will invest the risky asset portion of their total portfolio in risky asset portfolio M. This leads to an important feature of this portfolio, namely that it *holds all risky assets*. This feature is a result of the fact that the market is believed to be in equilibrium, in other words market demand equals market supply. If an asset existed that was not held by the risky asset portfolio M, there would be no demand for this specific asset (all investors are only interested in the assets within risky asset portfolio M), the value of the asset will fall to zero and the asset will cease to exist. The portfolio M of all risky assets is referred to as the *market portfolio*. Since the market portfolio contains all risky assets, it is completely *diversified*.

In order to describe the term *diversification*, two other concepts, namely *market risk* and *specific risk* should first be discussed.

Definition 10 *Market or systematic risk is the risk that market fluctuations may influ-*

ence the price of an asset. All assets are exposed to the risk associated with prevailing market conditions. Specific or unsystematic risk is the risk associated with the price movements of a specific asset and is caused by the internal activities of that asset. This type of risk is present, independent of the concurrent market conditions.

The notion of diversification may now be discussed.

Definition 11 *Diversification is the elimination of unsystematic risk by investing in different assets with different internal factors influencing their price movements. Market risk cannot be eliminated in this manner, as all shares are subjected to the same fluctuations.*

It is important to note that capital market theory implies that the market rewards investors only for bearing systematic (market) risk, not unsystematic risk. In order to ensure a possibility of returns with a lower degree of risk, it is extremely important to eliminate the unsystematic risk by diversification. The level of diversification of a portfolio may be judged on the basis of the correlation between the portfolio returns and the returns for a market portfolio (in the South African context the JSE Securities Exchange's All Share Index is used as the market portfolio). A completely diversified portfolio is perfectly correlated with the completely diversified market portfolio.

2.6.5 Capital Asset Pricing Model (CAPM)

In the Markowitz portfolio discussion, it was noted that the relevant risk consideration for a security, when it is added to a portfolio, is its weighted average covariance with all other assets in the portfolio. In the case of capital market theory, it has been established in §2.6.4 that all investors hold the market portfolio; thus the only relevant portfolio is this portfolio M. Therefore the only risk consideration for a security is its weighted average covariance with the market portfolio. In Figure 2.9, the segment AB representing the efficient frontier consists of portfolios of risky assets and M is the efficient market portfolio corresponding to the tangency line $r_f M$. Now let us continue to create a portfolio p which is a mix (a linear combination) of some risky security i and the market portfolio M. Thus, the new portfolio, with rate of return r_p is given by

$$r_p = w_i r_i + (1 - w_i) r_M$$

where

$$\begin{aligned} w_i &= \text{the proportion invested in security } i, \\ 1 - w_i &= \text{the proportion invested in portfolio } M, \\ r_i &= \text{the rate of return on security } i, \\ r_M &= \text{the rate of return on the market portfolio } M. \end{aligned}$$

By changing the proportion w_i of security i in portfolio p, the curve ij is obtained which describes all portfolios consisting of a mix between the two assets. (Note that the segment

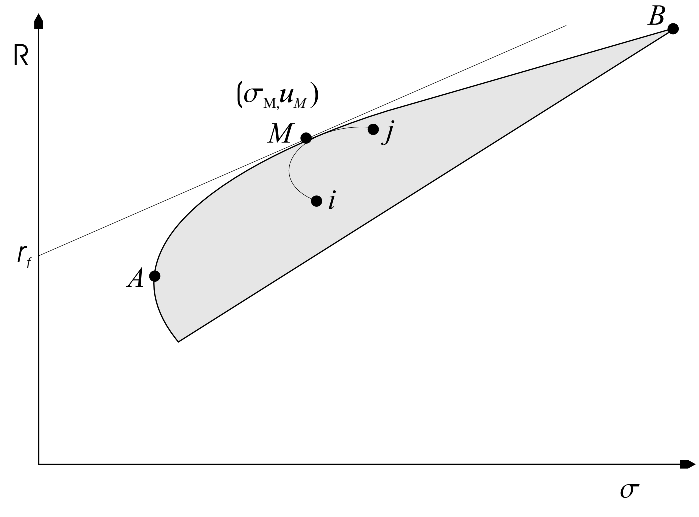


Figure 2.9: Derivation of the beta–measure

Mj corresponds to a strategy of short selling security i and investing more than 100% of the capital in the market portfolio M, but because short selling is not taken into account in this study, no further attention will be given to segment Mj.)

The basic claim that is integral to the derivation of the CAPM is that, at point M, the curves ij and AB (the efficient frontier) have the same tangent whose slope is equal to the slope of the straight line $r_f M$. Thus the slope of curve ij at this point of tangency is given by

$$\frac{\mu_M - r_f}{\sigma_M}, \tag{2.6.4}$$

where $\mu_M = \mathcal{E}[r_M]$ and σ_M are the expected rate of return and the standard deviation of the market portfolio M respectively. The expected rate of return and the variance of portfolio p consisting of a mix of security i and market portfolio M are given by

$$\begin{aligned} \mu_p &= w_i \mu_i + (1 - w_i)\mu_M \quad \text{and} \\ \sigma_p^2 &= w_i^2 \sigma_i^2 + (1 - w_i)^2 \sigma_M^2 + 2w_i(1 - w_i)\sigma_{iM} \end{aligned}$$

respectively, where μ_p and σ_p^2 are the parameters of any portfolio lying on the curve ij, μ_i and σ_i^2 are the parameters of security i and σ_{iM} is the covariance of return of security i and portfolio M. Recall that at point M, the curves ij and AB coincide, hence the proportion w_i invested in security i is zero. If one takes the derivatives $\frac{\partial \mu_p}{\partial w_i}$ and $\frac{\partial \sigma_p}{\partial w_i}$, and finds their values at the point M where $w_i = 0$, then

$$\frac{\partial \mu_p}{\partial w_i} = \mu_i - \mu_M \tag{2.6.5}$$

and

$$\frac{\partial \sigma_p}{\partial w_i} = \frac{1}{2\sigma_p}[2w_i\sigma_i^2 - 2(1 - w_i)\sigma_M^2 + 2\sigma_{iM} - 4w_i\sigma_{iM}]. \tag{2.6.6}$$

But, for $w_i = 0$ (at point M), $\sigma_p = \sigma_M$ and hence (2.6.6) reduces to

$$\frac{\partial \sigma_p}{\partial M_i} = \frac{\sigma_{iM} - \sigma_M^2}{\sigma_M}. \tag{2.6.7}$$

By the chain rule of differentiation,

$$\frac{\partial \mu_p}{\partial w_i} = \frac{\partial \mu_p}{\partial \sigma_p} \frac{\partial \sigma_p}{\partial w_i},$$

which may be written as

$$\frac{\partial \mu_p}{\partial \sigma_p} = \frac{\partial \mu_p}{\partial w_i} \bigg/ \frac{\partial \sigma_p}{\partial w_i}. \tag{2.6.8}$$

The left hand side of (2.6.8) is in fact the slope of curve ij at point M, given in (2.6.4). Thus, using (2.6.4), (2.6.5) and (2.6.7), (2.6.8) may be rewritten as

$$\frac{\mu_M - r_f}{\sigma_M} = \frac{(\mu_i - \mu_M)\sigma_M}{\sigma_{iM} - \sigma_M^2}. \tag{2.6.9}$$

By cross–multipying (2.6.9), the CAPM risk–return relationship is obtained to be

$$\mu_i = r_f + (\mu_M - r_f)\frac{\sigma_{iM}}{\sigma_M^2}. \tag{2.6.10}$$

If beta (β) is formally defined as

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}, \tag{2.6.11}$$

then (2.6.10) becomes the CAPM model

$$\mathcal{E}[\underline{r}_i] = r_f + (\mathcal{E}[\underline{r}_M] - r_f)\beta_i. \tag{2.6.12}$$

Note that since the same derivation may be used for any security i, the risk–return relationship (2.6.12) holds for all risky assets so that any individual asset's expected return may be described in terms of the expected return of the market portfolio. With this, Sharpe, Lintner and Mossin were successful in their quest to extend portfolio theory to a model that may be used for the pricing of all risky assets.

The CAPM states that the risk premium on individual assets will be proportional to the risk premium on the market portfolio, M, and the beta coefficient of the security relative to the market portfolio. Therefore, a *beta–value* measures the extent to which the returns on the security (or set of securities) and the market moves together and therefore serves as an alternative, comparable measure of risk [79].

The establishment of the CAPM, and especially the beta–value as standardised measure of systematic risk for a risky asset or portfolio of risky assets, opened up yet another dimension of portfolio management. It implied that if it were possible for an investment team to anticipate market movements, investments should be made in high beta–valued portfolios during a bull market and in low beta–valued portfolios as well as money market instruments in a bear market, thereby achieving an above–average risk–adjusted return. This is a result of the fact that if the market as a whole is positive, a portfolio with a high beta–value will be positively correlated with the market, which leads to a favourable outcome. Similarly, in negative market conditions, a portfolio with a low beta–value will have a low correlation with the market returns, which again leads to outperformance of the market. If the beta–value is larger than one, the portfolio will react more aggressively than the market will. A portfolio with a beta–value larger than one will therefore be able to take advantage of an up–market. If the beta–value is less than one, the portfolio will react less severely to changes relative to the market. A low–beta portfolio will therefore be less affected by the down–market.

The total variance of a portfolio is not normally used as a measure of risk if the portfolio is believed to be well–diversified. The relevant risk in this case is the portfolio's beta–value, as the only risk applicable to the portfolio is that of the market. Suppose the performance of a number of portfolios is compared. Using the beta–value as the risk measure, it is explicitly assumed that, in addition to the portfolio in question, investors also diversify their holdings by investing in other shares or unit trust funds, so that overall, each investor holds the market portfolio. This assumption is also true for any individual shares. Since σ_M^2 in (2.6.11) is common to all the stocks, the covariance $\mathrm{Cov}(\underline{r}_i, \underline{r}_M)$ is the only risk factor which distinguishes the risk of one portfolio from that of another. However, $\mathrm{Cov}(\underline{r}_i, \underline{r}_M)$ measures the co–movement of the rates of return of portfolio i and the market portfolio M. This co–movement constitutes an appropriate risk measure only if investors actually hold the market portfolio M. If the investors hold a different portfolio k, then for this set of investors, the relevant risk measure would be

$$\left[\frac{\mathrm{Cov}(\underline{r}_i, \underline{r}_k)}{\sigma_k^2}\right],$$

where $\underline{r}_k$ and σ_k^2 are the rate of return and the variance of return of portfolio k (distinct from market portfolio M) respectively.

Consider an alternative case where investors hold only one security in their portfolio. For example, assume that a set of investors hold only units of (say) Security A. For these investors the beta–value of Security A measured against the market portfolio (or any other portfolio) is completely irrelevant as a risk measure, since the investors simply do not hold such a diversified portfolio. The appropriate risk measure in this extreme case is the variability of the rates of return on Security A, measured by the portfolio's variance or standard deviation.

Consider a more realistic case where some investors hold only one security in their portfolio (say Security A), another set hold a (small) number of securities, including Security A, and a third set hold the entire market portfolio. In this case, there is no single measure which properly measures the risk of Security A for all investors. The proper risk measure is Security A's variance σ_A^2 for non–diversifiers, β_A for the market portfolio holders, and some complex combination of σ_A^2 and β_A for the investors holding a diversified portfolio in which the diversification falls short of that required by the CAPM. Therefore, the appropriate risk measure depends on the degree of diversification.

Although the CAPM and some of its extentions are still widely used in the industry, the linearity of the relationship between risk and return has caused some concern [54, 56]. Arguably one of the more interesting of the proposed suggestions for improving risk–return measurement is an approach based on the arbitrage pricing model (APT) [56, 75]. The major critique of CAPM is that the relationship between risk and return is more complex than suggested by the single–index model. In an APT model, returns are explained by a variety of factors, that may include (but are not limited to) the overall market. Roll and Ross [78] based a model on the APT approach and assert that the risk elements that influence security returns are unanticipated changes in four economic variables, namely inflation, industrial production, risk premiums and the slope of the term structure of interest rates. Subsequent to research regarding the most consequent measure of risk within markets, Fama and French [20], found that collectively, P/E ratio (price–earnings ratio), P/B ratio (price–book value ratio) and market capitalisation explain the cross–sectional variation in equity returns better than the beta–value.

Methods of risk–adjusted performance evaluation following similar argument as discussed above came on stage simultaneously with the capital asset pricing model. Treynor [90], Sharpe [81] and Jensen [44] recognised immediately the implications of the CAPM for rating the performance of portfolios [10].

2.7 Risk–adjusted measures of return

In the following sub-sections (§§2.7.1—2.7.2), the development and some uses of two risk-adjusted measures, namely the Sharpe and Treynor measures, will be discussed. The section will conclude with a discussion of further developments in the field of risk-adjusted return measures in §2.8.

2.7.1 The Treynor measure

One of the first composite measures of portfolio performance was believed to be developed by Treynor [90], who recognised that one of the major problems in evaluating portfolio performance was taking a measure of risk into account in the evaluation process [75]. To identify a type of risk that he felt to be a suitable measure, he firstly used the concept of a risk–free asset in the calculation of a risk premium (the expected excess return as a result of additional risk tolerated) for a specific portfolio. Secondly, he introduced the so–called *characteristic line*, which defines the relationship between the rates of return for a portfolio over time and the rates of return for an appropriate market portfolio. Treynor presented this relationship graphically (see Figure 2.10) and noted that the *slope* of the characteristic line measures the *relative volatility* of the portfolio's return in relation to the aggregate market returns. By rewriting (2.6.12) to

$$\beta_i = \frac{\mathcal{E}[r_i] - r_f}{\mathcal{E}[r_M] - r_f},$$

one recognises the slope Treynor refers to in Figure 2.10 as the portfolio's *beta coefficient* in terms of the CAPM model. The higher the slope (or beta–value), the more sensitive the portfolio is to market returns and the greater its market risk.

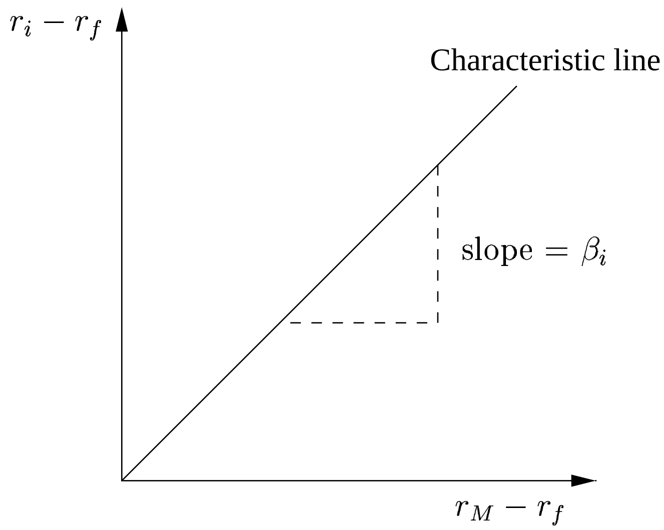


Figure 2.10: Treynor's characteristic line

Treynor was interested in a measure that would apply to all investors regardless of their risk profile and introduced the Treynor risk–adjusted measure for performance,

$$T_i = \frac{r_i - r_f}{\beta_i},$$

where

r_i	=	the average rate of return for portfolio i during a specific time period,
r_f	=	the average rate of return of a risk–free investment during the same time period,
β_i	=	the slope of the portfolio's characteristic line computed during that time period, which indicates the portfolio's relative volatility.

Since the numerator of this ratio $(r_i - r_f)$ is the risk premium, and the denominator is a measure of risk, the expression indicates the portfolio's *return per unit risk*, and all risk–averse investors would prefer to maximise this value. Thus, the larger the T_i value, the more preferable the portfolio is for investors. By selecting the systematic risk variable (β_i), as measure of risk, the Treynor formula implicitly assumes complete diversification.

If the Treynor–ratio for a portfolio is negative because the β–value is negative, but the risk premium is positive, it is an indication of outperformance over the market, despite inverse reaction of the portfolio relative to the market. Alternatively, if the Treynor–ratio is negative because of a negative risk–premium and positive β–value, underperformance of the portfolio is indicated. Finally, if the Treynor–ratio is positive, either because both the risk premium and β–value are positive, or both are negative, it is an indication that the portfolio has performed at least in line with market expectations, but the value of the ratio relative to that of other portfolios should be investigated for performance evaluation.

2.7.2 The Sharpe measure

The Sharpe composite measure is closely related to Sharpe's earlier work on the Capital Asset Pricing Model (CAPM) [80]. In the CAPM, the assumption is made that all investors are able to borrow or lend at the risk–free rate and share the same set of expectations, which implies that all efficient portfolios will coincide with a straight line of the form

$$\mathcal{E}[r] = r_f + \left(\frac{\mathcal{E}[\underline{r}_i] - r_f}{\sigma_i}\right)\sigma,$$

when expected return ($\mathcal{E}[r]$, the dependent variable) is plotted against risk (σ, the independent variable). Here

$\mathcal{E}[\underline{r}_i]$ = the expected rate of return on portfolio i,
r_f = the risk–free rate of return, as discussed previously,
σ_i = standard deviation of the returns for portfolio i.

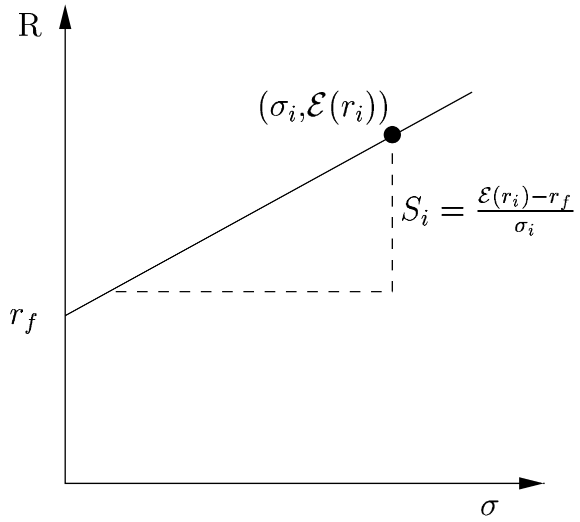


Figure 2.11: Derivation of the Sharpe ratio

This means that any portfolio will give rise to a complete linear set of (σ, $\mathcal{E}[r]$) combinations, and the best portfolio will be the portfolio with the highest $\frac{\mathcal{E}[r_i]-r_f}{\sigma_i}$ ratio. In order to use this theory to test historic returns, it was necessary for Sharpe to progress from expected forward rates to historic average rates of return and the actual standard deviation of returns of the alternative portfolios. Therefore, in practice, the Sharpe measure is formulated as

$$S_i = \frac{r_i - r_f}{\sigma_i},$$

where

r_i = the rate of return for the portfolio i during a historic time period,
r_f = the risk–free rate that prevailed during the time period,
σ_i = the standard deviation of the rate of return for portfolio i during the time period.

The Sharpe measure uses the standard deviation (total risk) of returns as the measure of risk, while the Treynor measure employs the beta–value (systematic risk). Comparing the two measures, therefore implicitly evaluates the portfolio on the basis of return, but also considers how well–diversified the portfolio was during the specific period. If a portfolio is perfectly diversified (does not contain any unsystematic risk), the two measures would give identical results, because the total variance of the portfolio would be the systematic variance. Alternatively, if a portfolio is poorly diversified, it can have a high Treynor value based on the Treynor measure, but a much lower value in terms of the Sharpe measure, with the difference directly attributed to poor diversification. The two performance measures therefore provide complementary information. Because Sharpe thought that variability due to unsystematic risk was probably short–term, he believed that the Treynor measure might be a better measure for predicting future performance.

Friend and Blume [25] reviewed the composite measures of Treynor and Sharpe and pointed out that, at least theoretically, the measures of performance should be independent of corresponding risk measures, since they are risk–adjusted measures. The authors analysed the relationship between the composite measurements of performance and risk for 200 random portfolios of 788 common stocks listed on the New York Stock Exchange. The three composite measures were regressed against the two standard measures of portfolio risk (the beta–value and the standard deviation of the rate of returns of a portfolio). In all cases, there were significant inverse relationships between the performance measures and the measure of risk (the risk–adjusted performance of low–risk portfolios was better than the comparable performance of high–risk portfolios).

A later paper by Klemkosky [49] examined the relationship between composite performance measures and risk measures, using actual general portfolio data in contrast to the random portfolio data used by Friend and Blume [25]. The author calculated the composite measures for the relative portfolios and the results indicated a positive relationship between the composite performance by the general portfolios and the risk involved. This was especially true for the Treynor measures. It was concluded that while there might be a relationship between the measures of performance and risk, it was not clear whether this relationship was inverse or positive.

In [29] Grinblatt and Titman also discussed the traditional use of performance measures and noted that traditional methods (like the Treynor and Sharpe measures) do not utilise information that is often readily available about the composition of the evaluated portfolio. When the composition of the evaluated portfolio is used in evaluations, the evaluator may eliminate the need to compare returns to a benchmark portfolio. Consequently, Grinblatt and Titman recognised this and introduced a new measure of portfolio performance which correlates the weighting of the assets of the evaluated portfolio with a constant weight portfolio.

2.8 Further work on performance evaluation

Taking in account the substantial amount of literature still being published on new and improved performance measures [13, 14, 22, 53, 82, 84] it is clear that the race for finding generally acceptable unit trust performance measures is still on. In this section some of these articles are discussed.

Leland [53] acknowledged the fact that most practitioners use the CAPM to measure investment performance. What concerned the author was that the CAPM either assumes that all asset returns are normally distributed (and thus symmetrical) or that investors have mean–variance preferences (and thus ignore skewness). Both of these assumptions seemed suspect to Leland. Subsequently, he assumed that only the rate of return on the market portfolio is independently and identically distributed and used this to show that the CAPM market portfolio is mean–variance inefficient. Furthermore, he concluded that, with the traditional CAPM performance measures, strategies with positively skew returns (such as strategies limiting downside risk), will be incorrectly underrated by resulting risk–adjusted rankings. He proposed a simple modification of the CAPM beta–measure which should produce a correct risk measurement for portfolios with arbitrary return distributions, without requiring any additional information.

In estimating abnormal excess performance for a sample of pension fund portfolio managers, Christopherson and Turner [14] concluded that past performance of a portfolio provides little or no useful information about expected future performance. Their study, however, relied upon *unconditional* performance measures, in other words those measures whose estimates of future performance ignore information about the changing nature of the economy. Thus, the authors noted that unconditional measures may incorrectly measure expected excess returns when portfolio managers react to market information and engage in dynamic trading strategies.

Ferson and Schadt [22] retained the CAPM framework, but advocated *conditional* performance evaluations (CPE) to form expectations about excess return and risk more accurately. Subsequent to the work of Ferson and Schadt on CPE, Christopherson, *et al.* [13] expanded on the concept by using

- a broader sample. They applied CPE to a larger sample of pension funds – 261 vs. 185.
- a longer time period. The time period was extended from January 1990 to June 1996.
- different market cycles.
- simpler instruments. They implemented a simpler set of instruments to represent public information.

The results contained in [13] confirm that the conditional measures are more informative about future performances than unconditional measures. Therefore, the conclusion is that the use of conditional measures may improve upon the current practice of performance measurement.

In 1998 the Dutch government passed a law requiring all pension funds to construct a benchmark that must be used to evaluate risk and performance. Sortino *et al.* [84] offer a framework for a new performance measure based on behavioural finance theory constructed for the purposes of abiding by the Dutch law. The authors argue that the average return above a designated minimal acceptable return is a proxy for upside potential, a valuable new way of measuring return. Style analysis plays a very important role in the discussed method. Unlike the information ratio, the "upside potential ratio" that the authors propose does not penalise the manager for performance above the manager's benchmark. An analysis of Dutch mutual portfolios in [84] indicates that the upside potential ratio approach is applicable to both small and large markets.

In [82] Sharpe also discusses the fact that portfolio performance measures are typically based on one or more summary statistics of past performance. Measures that attempt to take risk into account incorporate a measure of historical return and historical variability. However, because investment decisions affect only the future, the use of historical results involves an implicit assumption that the statistics derived from past performance have at least some predictive content for future performance. Sharpe alleges that the evidence is ample that, although measures of historical variability can be useful for predicting future levels of risk, measures of average and cumulative return are, at best highly imperfect predictors of expected future returns. He addresses this problem by examining the properties of Morningstar's measure (a measure that computes relative performances and relative risk by dividing the performance and risk of a portfolio by a denominator used for all portfolios in a specified peer group) under the assumption that statistics from historical frequency distributions are reliable predictors of corresponding statistics from a probability distribution of future returns. The aim of [82] is not only to relate alternative performance measures to likely investment decisions, but ultimately the intent is also to use all relevant information to make unbiased forecasts of expected return.

Sharpe concludes that Morningstar's measure has a number of drawbacks. It is complex, and it has poor statistical qualities. More significantly, it fails to capture an important aspect of investor preference – the desire for portfolios that are neither the least nor the most risky available. Fortunately, the inherent disadvantages are considerably mitigated by Morningstar's practice of adjusting the risk aversion implicit to the measure in order to equal the ratio of return to risk for each peer group over a specified period, although this adjustment is only made if the peer group performance has been modest or poor. The study finds that Morningstar varies one of the measure's parameters in a manner that frequently produces results similar to the results of using the excess–return Sharpe ratio. Finally the argument is presented that neither Morningstar's measure nor the excess–return Sharpe ratio is an efficient tool for choosing mutual portfolios within peer groups for a multi-manager portfolio.

Apart from the ongoing work on the modification of existing measures or the formulation of new alternatives to existing measures, substantial work has also been done on explicit factors influencing performance. Indro, *et al.* [40] raise the question whether the rise in net assets of a portfolio under management have an adverse impact on the portfolio's investment performance. The perception that size can impede performance is a valid concern in a financial market, where information acquisition and trading are costly. In such a market, the incentive for active management is that the economic gains to information compensate the portfolio manager for the cost of research and trading.

The empirical results of prior studies that examine the relationship between risk–adjusted mutual portfolio returns, the cost of research (the expense ratio) and cost of trading (turnover) that are associated with active investment management are conflicting [42]. On the one hand, Sharpe [81] observed that mutual portfolios with higher reward–to–volatility ratios tend to be those with lower expenses. On the other hand, Friend *et al.* [26] reported an insignificant negative correlation between risk–adjusted mutual portfolio returns and expense ratios, as well as a slight positive relationship with turnover. Similarly, Ippolito [41] found that risk–adjusted mutual portfolio returns are unrelated to expense ratios and turnover.

However, the authors of [40] studied 683 non–indexed US equity portfolios over the 1993–1995 period and found that portfolio size (net assets under management) does affect mutual portfolio performance. They concluded that mutual portfolios must attain a minimum portfolio size in order to achieve sufficient returns to justify their costs of acquiring and trading on information. Furthermore, they found that there are diminishing marginal returns to information acquisition and trading with a growth in size of a portfolio, and the marginal returns become negative when the mutual portfolio exceeds its optimal portfolio size.

Chapter 3

Extreme Market Conditions

In practice, various allocation strategies are implemented as part of the daily decision making processes of investment professionals in search of portfolio outperformance on a risk–adjusted basis. In this study, an attempt is made to shed some light on the use of some of these strategies in various market conditions. Particular emphasis will be placed on investment in *extreme market conditions*. In order to achieve this, a rigorous process incorporating the identification of periods of similar (extreme) market conditions, the isolation of data pertaining to these periods, the allocation process of capital using the relevant information and the ultimate comparison of different allocation strategies will be presented in this chapter.

In short, this chapter is dedicated to identifying these sets of data to be used in the allocation decision analyses carried out in this study. The information serving as source for this identification process is relevant within the South African context. Therefore, it seems apt to open the chapter by paying attention to the broad structure of the South African equity market, after which the focus will be narrowed down to the particular detail relevant in this study. All data used in the computations in this study are listed in Appendix A.

3.1 The JSE Securities Exchange

The cornerstone of the South African financial market is contained in the formal structure of the JSE Securities Exchange. The main function of an exchange is believed to be the raising of primary capital by re–channelling cash resources into productive economic activity via a listed company, thus building an economy while enhancing job opportunities and wealth creation. Such an exchange is an essential cog in the functioning of a capitalist economy and provides an orderly market for dealing in securities, thereby creating new investment opportunities in a country. Generally, the advantages of a company being listed on an exchange are that the exchange provides an orderly marketplace for buyers and sellers of its shares, fair price determination, accurate and continuous reporting on sales and quotations, information on listed companies and strict regulations for the protection of minority shareholders from fraudulent activity on the part of majority shareholders or directors of companies. Because of the main function of a country's exchange, as stated above, the success of the exchange is globally seen as a measure of financial prosperity of the listed companies. It is therefore also a factor taken into account when considering economic well–being of the country itself.

Within the South African context, there are approximately 500 listed companies (as in March 2002). The JSE/Actuaries Indices are a set of indices reflecting the performance of these companies in the South African market. Within the suite of indices, the entire spectrum of listed companies is covered. The All Share Index measures the performance of the overall market, the sectoral indices measure the performance of companies operating within the same industry, while the size indices measure the performance of companies of similar size. The purpose of these indices is to provide a tool to describe the market at a point in time in terms of price levels, dividend yields and earnings yields. In this study, particular attention will be paid to these sets of shares contained in the sectoral indices.

3.1.1 Sectoral indices

All the shares that are listed on the JSE Securities Exchange are classified according to the specific industry serviced by the corresponding companies. The five main sector classifications are *Mining Resources*, *Non–Mining Resources*, *Financials*, *Industrials* and *Real Estate*, and covers all shares listed on the JSE Securities Exchange. Each of these categories is further divided into sub–categories (sub–sectors), but for the purpose of this study, attention will only be paid to the five main sector (industry) classifications. Figure 3.1 gives a graphical representation of the JSE Securities Exchange sectoral classification structure.

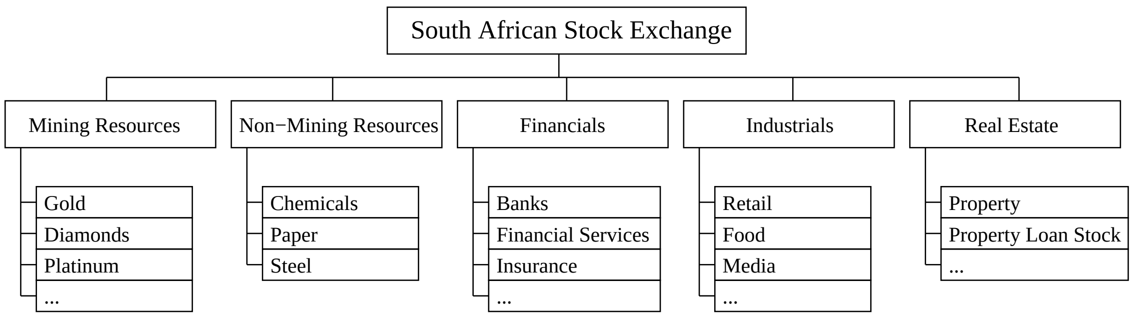


Figure 3.1: JSE Securities Exchange sector classification

Each of the sectors is represented by a JSE/Actuaries Index, which includes all the constituents of the specific sector and gives an indication of the price–movement of the sector as a whole. Index constituents are removed from an index if they delist, if the company control passes over to another that is also in the index or if a liquidator has been appointed for the company. Changes to the classification of a company or the composition of the sectors are also accommodated in the indices (for example, if the sectoral classification of a constituent changes, the security is deleted from the old sector and is tested for eligibility in a new sector [46]).

The securities exchange concerns itself only with that part of a company's capital raised from the primary market or capital linked to trades on the secondary market. Therefore the size of a company within this context is expressed in terms of its *market capitalisation.*

Definition 12 *The market capitalisation of a company i at time t takes into account the number of company shares outstanding, as well as the price of these shares in such a way that*

$$\text{Market capitalisation, } c_{it} = \text{Number of shares} \times \text{Price per share, } p_{it}. \text{ at time } t.$$

The concept of the market capitalisation of a company is an important one, as it serves as basis for the structure of all the JSE/Actuaries Indices. These indices are all arithmetic averages of the prices of their constituent securities, weighted by the constituents' market capitalisation, or

$$I_t = \frac{\sum_{i=1}^{n} c_{it} p_{it}}{\sum_{i=1}^{n} c_{it}}, \tag{3.1.1}$$

where

$$\begin{aligned} I_t &= \text{index value at any time } t \\ p_{it} &= \text{share price of company } i \text{ at time } t \\ n &= \text{number of constituents in index.} \end{aligned}$$

This implies that the share price movement of a larger company (by market capitalisation), would have a greater effect on the value of an index than that of a smaller company. As was mentioned already, the market capitalisations of the various companies are recalculated on a quarterly basis, and therefore do not change with the same frequency (usually daily) as does the index value.

In essence, the structure of an index, such as the one described in (3.1.1), lends itself to be viewed as a portfolio of shares. The percentage weight of each share in the portfolio will equal its market capitalisation as a percentage of the combined market capitalisation of the set of shares. Weighing of the portfolio this way, ensures that, if one invests in this portfolio, 100% of one's capital will be distributed across the constituent shares.

As the JSE Securities Exchange sectoral indices are all priced daily, the main sector indices may be seen as portfolios of non–mining resources, mining resources, financial, industrial and real estate shares respectively, for which daily historical pricing is available (a graphical representation of these price histories is given in Figure 3.2). Some of these portfolio values will be used as input parameters to the allocation strategies discussed in Chapters 4 and 5, and hence will play an integral part in the analyses throughout the rest of this study.

3.2 Market indicators

In §1.1 it was acknowledged that financial markets as a whole are fairly unpredictable. They are driven by numerous factors, some qualitative, some quantitative. It was argued

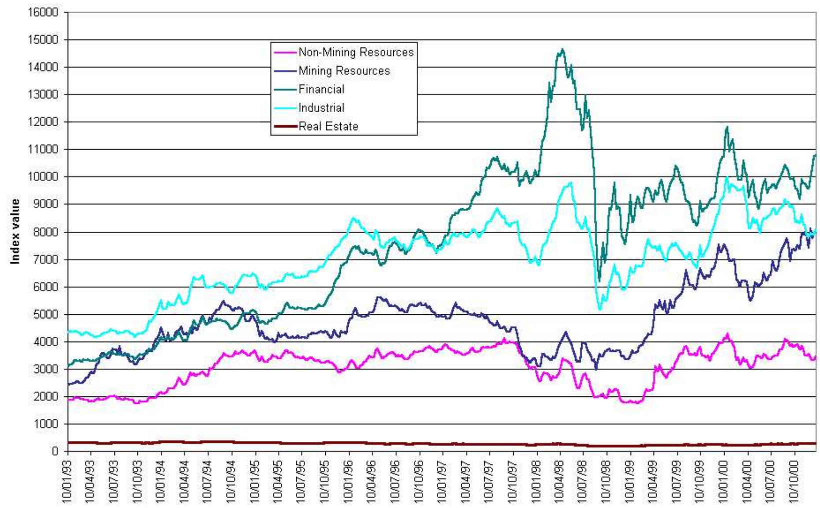


Figure 3.2: Main sectoral indices

that, in the absence of evidence to disprove the assumption, a number of macro economic indicators may possibly be used to describe certain market conditions, which in turn may influence share and financial sector performance. In addition to this, certain financial (company) data ([54]) or a sample of personal opinions ([88], [89]) may contain information which describes current or predicts future market conditions. Within this study, the term *market indicator* will be used in the following way.

Definition 13 *A market indicator is formally defined as a quantitative measure of qualitative or quantitative factors which may contain information regarding the state of one or more financial market sectors or of the aggregate market at specific points in time.*

In practice, absolute values of some indicators, as well as the rates of change in these values are widely used to characterise market conditions. In this study, the same will be done. As the interested lies in historic periods during which similar conditions prevailed, the indicators' data series will be examined according to specific criteria in order to identify such periods. In turn, these periods will be used in the process of ascertaining which portfolio compositions were best suited for these specific, predetermined conditions. When selecting indicators for this study, an attempt was made to choose them in such a way that most of the different types of contributing factors (macro–economic factors, a sample of personal opinions about market conditions, a combination of financial market and economic factors and technical financial market factors) are represented. In the sections that follow, the perceived significance of each of the selected indicators within the South African market environment will be discussed, but at no point in this thesis an attempt is made to conclude to what extent the indicators explain market reactions. The indicators that will be used in this study, are:

- R150 Bond yields
- Weekly change in R150 Bond yields

- South African business confidence
- Monthly change in South African business confidence
- United States consumer sentiment
- Monthly change in United States consumer sentiment
- Bear and bull market classifications.

Each of these indicators is represented either by an index recognised by the industry to render reliable historical and representative current information (these will be called the absolute indicators) or is calculated from such an index (these will be called the percentage change indicators). The indices indicating change are calculated as the percentage change of one data point to the next at the frequency at which the original index's data is available. The absolute indicator data, as well as the calculated percentage change indicator data are given in §§3.3–3.5. The bear/bull market indicators are identified by applying criteria to the South African equity market as a whole in §3.6. The values of some of these indices are available at real–time, others are updated monthly. Data for the period from 1 January 1993 to 31 December 2000 is used within this study.

3.3 R150 Bond Yields

Government bonds are the means by which the South African government borrows money directly from institutions or very high net–worth members of the public. Bonds are issued on a primary bond market. By issuing a bond paper, the government pledges to repay the face value (the size of the loan made to the government) of the bond at the date of maturity, plus a (usually) bi–annual coupon for the lifetime of the bond.

After they have been issued, bond papers may be sold and bought on a secondary market. The holder of the paper at any given time is entitled to the remaining coupon payments, as well as the payment of the face value at the date of maturity. When selling the bond paper, all rights to any further payment is transferred to the new owner. In assessing a fair price, investors contemplating the purchase of a bond should be concerned with the rate of return that the bond offers if held to maturity [54]. This rate of return, when a bond is held to redemption, is called its *yield to maturity* and is given by an application of the familiar formula for the internal rate of return,

$$\begin{aligned} P_0 &= \frac{C_1}{1+R} + \frac{C_2}{(1+R)^2} + \cdots + \frac{C_n}{(1+R)^n} + \frac{P_n}{(1+R)^n} \\ &= \sum_{t=1}^{n} \frac{C_t}{(1+R)^t} + \frac{P_n}{(1+R)^n}, \end{aligned}$$

where

$$\begin{aligned} C_t &= \text{the interest coupon during time period } t \\ P_0 &= \text{the current price of the bond} \\ P_n &= \text{the redemption value of the bond at maturity} \\ R &= \text{the yield to maturity of the bond.} \end{aligned}$$

The yield of a coupon–bearing bond is therefore the discount rate that equates the cash flows on the bonds to its market value [39]. Although the yield to maturity is widely used in practice as a convenient proxy of the overall profitability of a bond bought at a specific price, it has some practical flaws [54]. The reason is that coupon–bearing bonds pay their interest coupons over the entire lifetime of the bond, and the investor may be unable to re–invest the interim cash receipts at the same rate of return R, as is assumed by the formula. The price should ideally incorporate the expectation of changes in future interest rates for the lifetime of the bond; as these interest rate expectations change, so could the price of the bond.

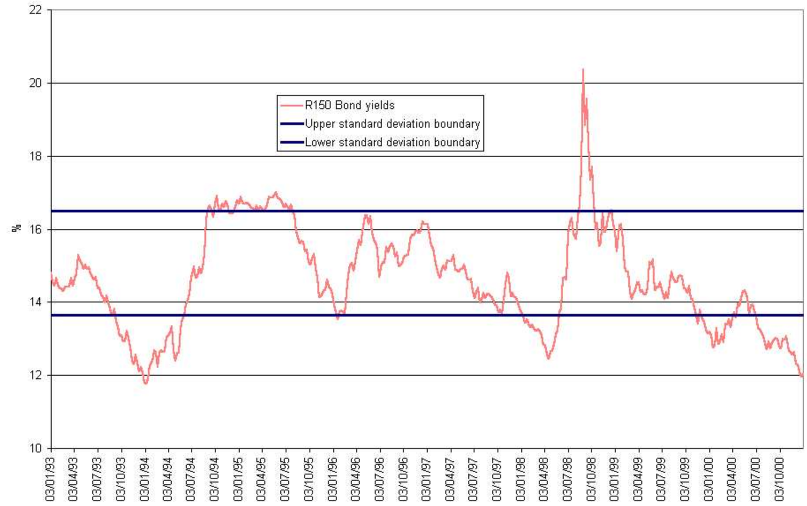
The price of a government bond is seen by some as an indication of the well–being of a government and the policies they implement. The success of these policies filter through to the well–being of the companies of the country, which in turn influences their share prices and the strength of the equity market [31]. For this reason, this indicator includes a possible effect of macro–economic policies on the South African equity market.

The most liquid and traded government bond, the R150, maturing on 28 February 2005, will be used as indicator of bond yields for the period from January 1993 to December 2000. A graphical representation of the yield index and percentage change index is given in Figures 3.3(a) and (b) respectively.

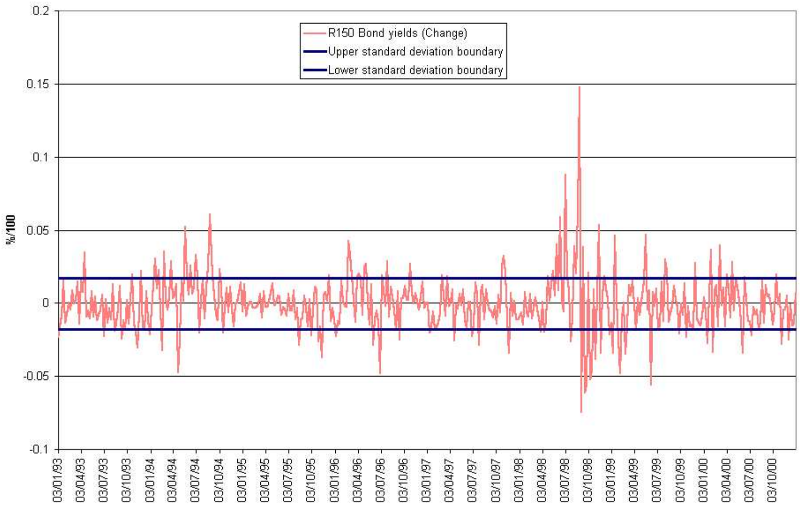
3.4 South African Business Confidence

The Business Confidence Index (BCI) of the South African Chamber of Business (SACOB) incorporates developments in the financial markets as well as in the real economy in order to quantify the local business mood. The resulting data may be used as an indication of the overall well–being of the South African economy relative to previous periods and may identify driving forces behind then current circumstances. The levels of the following sub–indices are incorporated into the BCI in such a way that their relevance portrays business mood:

1. Manufacturing production
2. Vehicle sales
3. Core inflation
4. Real yield on government bonds
5. Weighted exchange rate of the Rand
6. Weighted US dollar gold and platinum prices
7. JSE Securities Exchange All Share price index
8. Real value of building plans passed
9. Real retail sales



(a) The R150 Bond yield

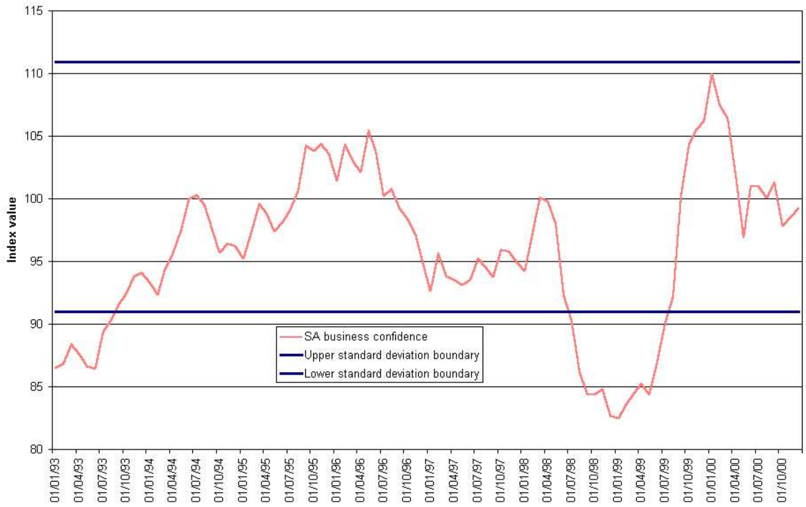


(b) Percentage weekly change in R150 Bond yield

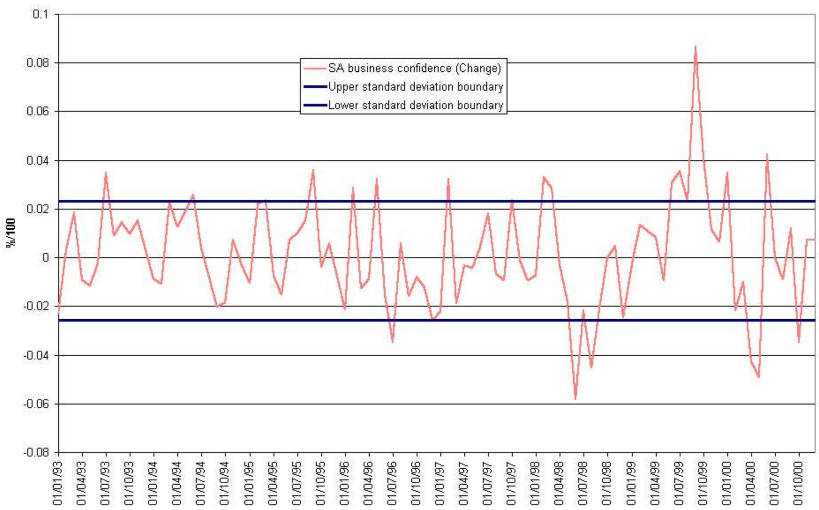
Figure 3.3: The R150 Bond

10. Volume of merchandise imports
11. Volume of merchandise exports
12. Liquidations
13. Real credit extended to the private sector [85].

The BCI figures are computed monthly and are usually interpreted together with the individual data of the various sub–indices in order to assess the contribution of these factors to confidence. A graphical representation of the BCI values and rates of change in the BCI values are given in Figures 3.4(a) and (b) respectively.



(a) The SACOB Business Confidence Index



(b) Percentage monthly change in the SACOB Business Confidence Index

Figure 3.4: SA business confidence

3.5 United States Survey of Consumers

The Survey of Consumers is conducted by the Survey Research Center (SRC) at the University of Michigan. Founded in 1946, the surveys have long stressed the important influence of consumer spending and saving decisions in determining the course of the national economy. Each monthly survey contains approximately 50 core questions, each of which tracks a different aspect of consumer attitudes and expectations. The surveys use a national sample of dwelling units selected by area probability sampling that is representative of the adult population of the United States and each month a minimum of 500 interviews are conducted by telephone [89]. The core questions of the surveys cover three broad areas of consumer sentiment, namely personal finance, business conditions and buying conditions. In each area, consumers are not only asked to give their overall opinions, but are also asked to describe in their own words their reasons for holding these views. These questions reflect a growing interest in not only projecting how consumers will act, but also in understanding why consumers make certain spending and saving decisions [88].

Economic optimism promotes consumer confidence and a willingness to make large expenditures and debt commitments, while economic uncertainty breeds pessimism and a desire to curtail expenditure and rebuild financial reserves. When many people change from an optimistic to a pessimistic view of economic prospects at the same time, it has repeatedly been found that a widespread shift toward postponement of expenditure follows. It is in this respect that the economic optimism and confidence of individual families exert their influence on the course of the aggregate economy.

The SRC believes that changes in the consumers' willingness to buy are best assessed by using the answers to all questions asked in the survey, especially the open–ended questions that probe underlying reasons [88] (see questions 3 and 4 below). Nevertheless, in order to make available a summary measure of change in consumer sentiment, the centre uses the answers to five specific questions to calculate and Index of Consumer Sentiment (ICS). The five questions that are used for this purpose are

1. "We are interested in how people are getting along financially theses days. Would you say that you (and your family living there) are <u>better off</u> or <u>worse off</u> financially than you were <u>a year ago</u>?"

2. "Now looking ahead – do you think that a year from now you (and your family living there) will be <u>better off</u> financially, or <u>worse off</u>, or just about the same as now?"

3. "Now turning to business conditions in the country as a whole – do you think that during the <u>next twelve months</u> we'll have <u>good</u> times financially, or <u>bad</u> times, or what?"

4. "Looking ahead, which would you say is more likely – that in the country as a whole we'll have continuous good times during the <u>next five years</u> or so, or that we will have periods of widespread <u>un</u>employment or depression, or what?"

5. "About the big things people buy for their homes – such as furniture, a refrigerator, stove, television, and things like that. Generally speaking, do you think now is a good or bad time for people to buy major household items?"

The importance of consumer optimism and confidence in shaping the course of the economy has been recognised in many countries. Other countries that now regularly monitor consumer sentiment through studies patterned after the Survey of Consumers include among others, Austria, Australia, Great Britain, Sweden and Taiwan.

The monthly data for the United States Consumer Sentiment Index from 1 January 1993 to 31 December 2000 are given in Figure 3.5(a), while the time series representing the monthly change in Consumer Sentiment for the same period is given in Figure 3.5(b).

3.6 Bear and bull trends

Bear and bull trend conditions is a term used to indicate a clear trend in the direction of the specific market being examined. Although it is defined in [71] that a bear trend is *a persistent and prolonged decline in price* and therefore a downward trend in the price movement of the securities being examined (in our case the All Share Index) and a bull trend is defined as *a rising market* or a market with an upward trend, the magnitude of these trends or absolute values of the movement are not formally defined in practice. In this study this particular hurdle is overcome by providing two different definitions of bear/bull trends, each of which will be used as a separate indicator.

Definition 14 *Any decrease/increase in the value of the All Share Index, and therefore the equity market from one point to a subsequent point 4 weeks later, is defined as a bear/bull trend.*

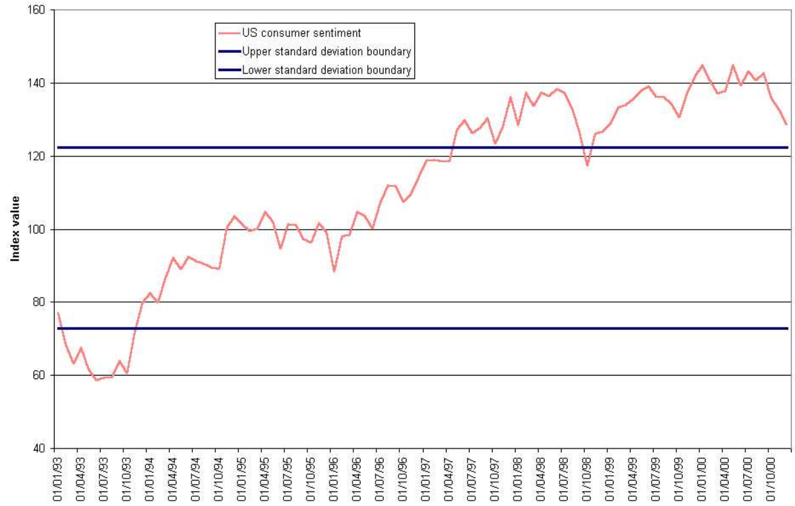
The second definition that will be used in this study is as follows.

Definition 15 *Any 4 week period during which the equity market (All Share Index) falls/rises by more than 10% is defined as a bear/bull trend.*

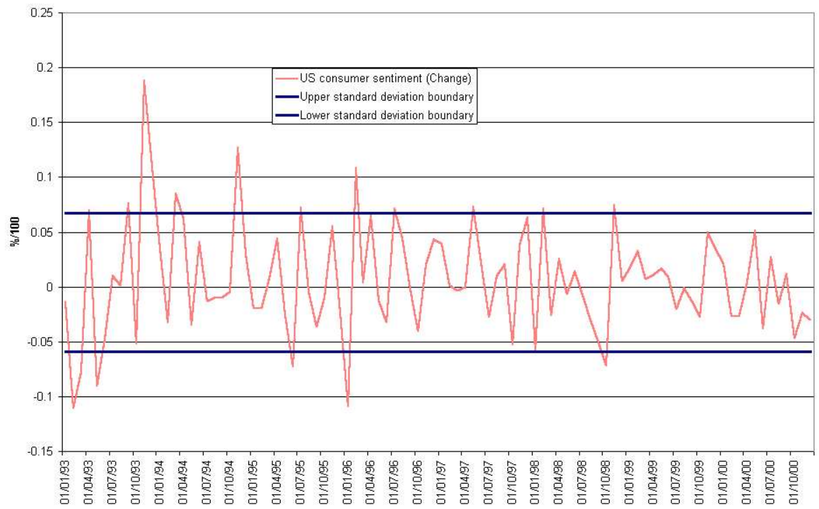
These indicators are slightly different from the other indicators discussed. The indicators discussed previously, are quoted independently of the equity market, but may contain some information pertaining to the equity market as a whole. Bear/bull equity trend indicators are different in that the reaction of the equity market as a whole becomes an indicator to contain information regarding the inherent state of the aggregate market. The bear and bull trend indicator identified by means of Definitions 14 and 15 for the period 1 January 1993 to 31 December 2000 are given in Figure 3.6. The frequency of observation of the time series (in this case the All Share Index) is weekly.

3.7 Extreme events

Periods dictated by extreme market conditions are arguably the most difficult times to make investment decisions. Investor sentiment, (the effect of which on financial markets)



(a) The US Consumer Sentiment Index



(b) Percentage monthly change in the US Consumer Sentiment Index

Figure 3.5: US consumer sentiment

is extremely difficult to quantify, and is believed to play an active role during such times. Different market sectors whose price movements are generally uncorrelated, become correlated. Also in these times, visibility of future market movements may be reduced substantially.

The main task at hand in this study is not only to investigate the appropriateness of different allocation strategies within specific extreme market conditions, but also to gain some insight into the characteristics of the periods through analysing the allocation results. As the assumption is made that the market indicators contain information regarding market

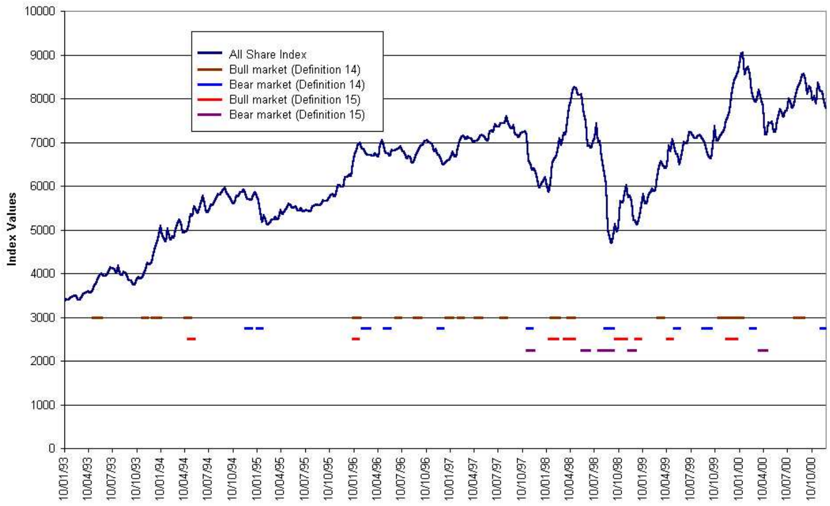


Figure 3.6: Bear and bull trend indicators

conditions, their information is fundamental to setting the framework within which the rest of the study will develop. An attempt will be made to use the information contained in the indicator data to identify those states of market indicators that should represent extreme market conditions (or extreme events). Consequently, the notion of a *shock* is defined.

Definition 16 *A shock is an extreme market condition quantified by a specific state of a market indicator. It is defined as the event that occurs when the value of an indicator time series is either above or below one standard deviation from its mean. A shock above the one standard deviation band will be called an upward shock, while a shock below the one standard deviation band will be called a downward shock. (Although two standard deviations from the mean could also be used, one standard deviation is very often used in the financial industry as measure to portray change.)*

In the financial industry, one For the sake of simplicity, upward and downward shocks of a single indicator will be considered as two different extreme events and will therefore be investigated as such. In order to identify the extreme events relevant to the various indicators, Definition 16 was applied to the data of the indicators introduced in §§3.3–3.5. The resulting shocks in these different market indicator data are those above and below the one standard deviation band, shown graphically in Figures 3.3–3.5.

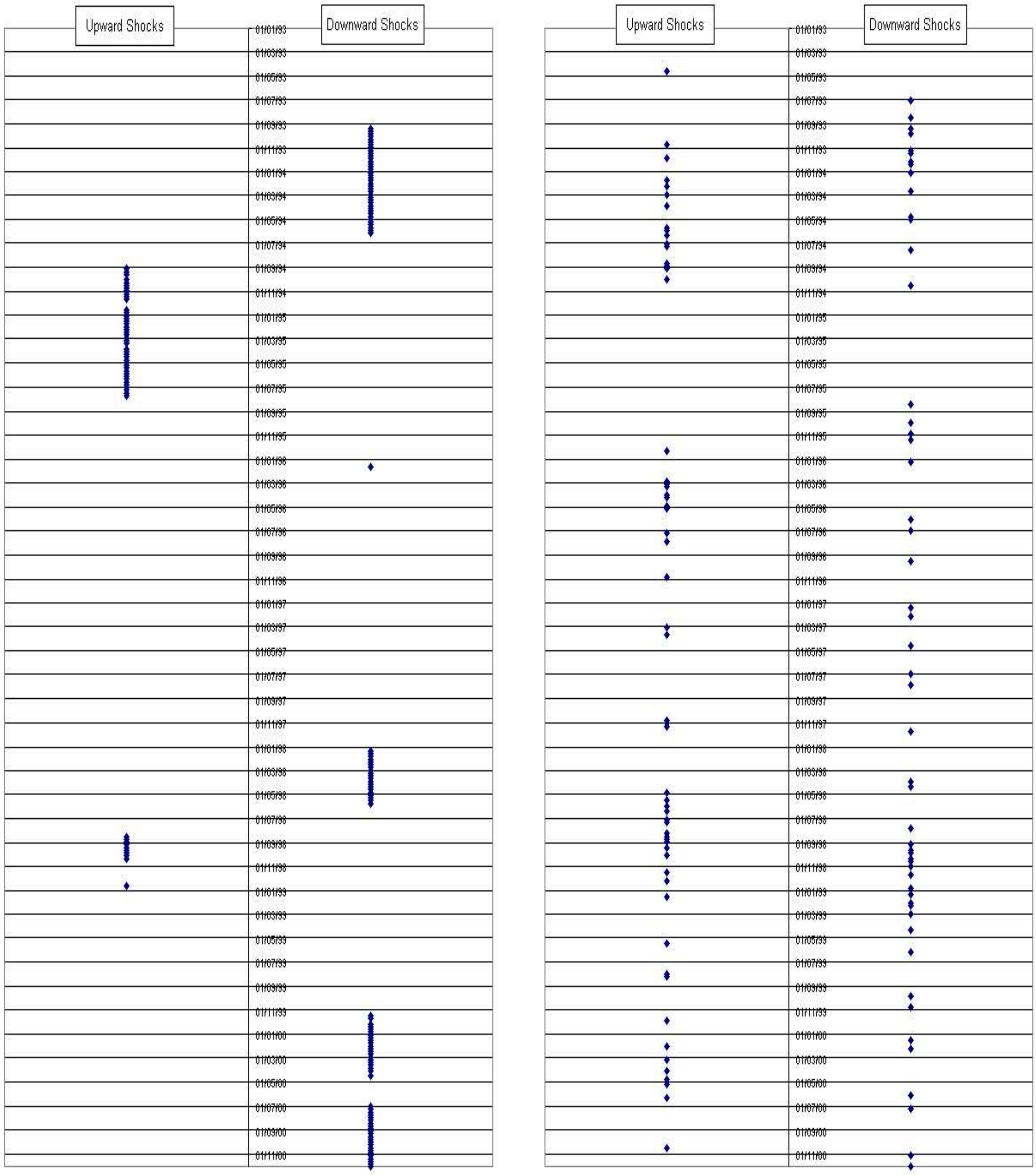
It is important to note that the definition of a shock, as given in Definition 16, does not pertain to the bear or bull market indicators, as their definitions already imply extreme events in the movement of the South African market. In other words, the indicator in itself is an extreme event. Therefore, the graphical representation of the bear and bull markets indicators in Figure 3.6 are the graphical representations of the extreme events as well.

In the following section, it will be outlined how the identification of shocks, dictated by a specific indicator, will be relayed to the identification of sets of data (called time windows) to be used during allocation strategies.

3.8 Identification of time windows

For the purposes of this study *time windows* are defined as periods during which extreme market conditions prevail. To this end, the period from 1 January 1993 to 31 December 2000 will be examined to identify those sub–periods called time windows. Because a number of different extreme events are considered in this study (upward and downward shocks in four market indicators as defined in §§3.2 and 3.7), all the time windows included in the total period under consideration and resulting from the same extreme market condition are combined in an independent set of time windows. The identified time windows pertaining to R150 Bond yields, SA business confidence and US consumer sentiment are given in Figures 3.7 – 3.9, while the time windows pertaining to bear and bull market conditions have already been identified in Figure 3.6.

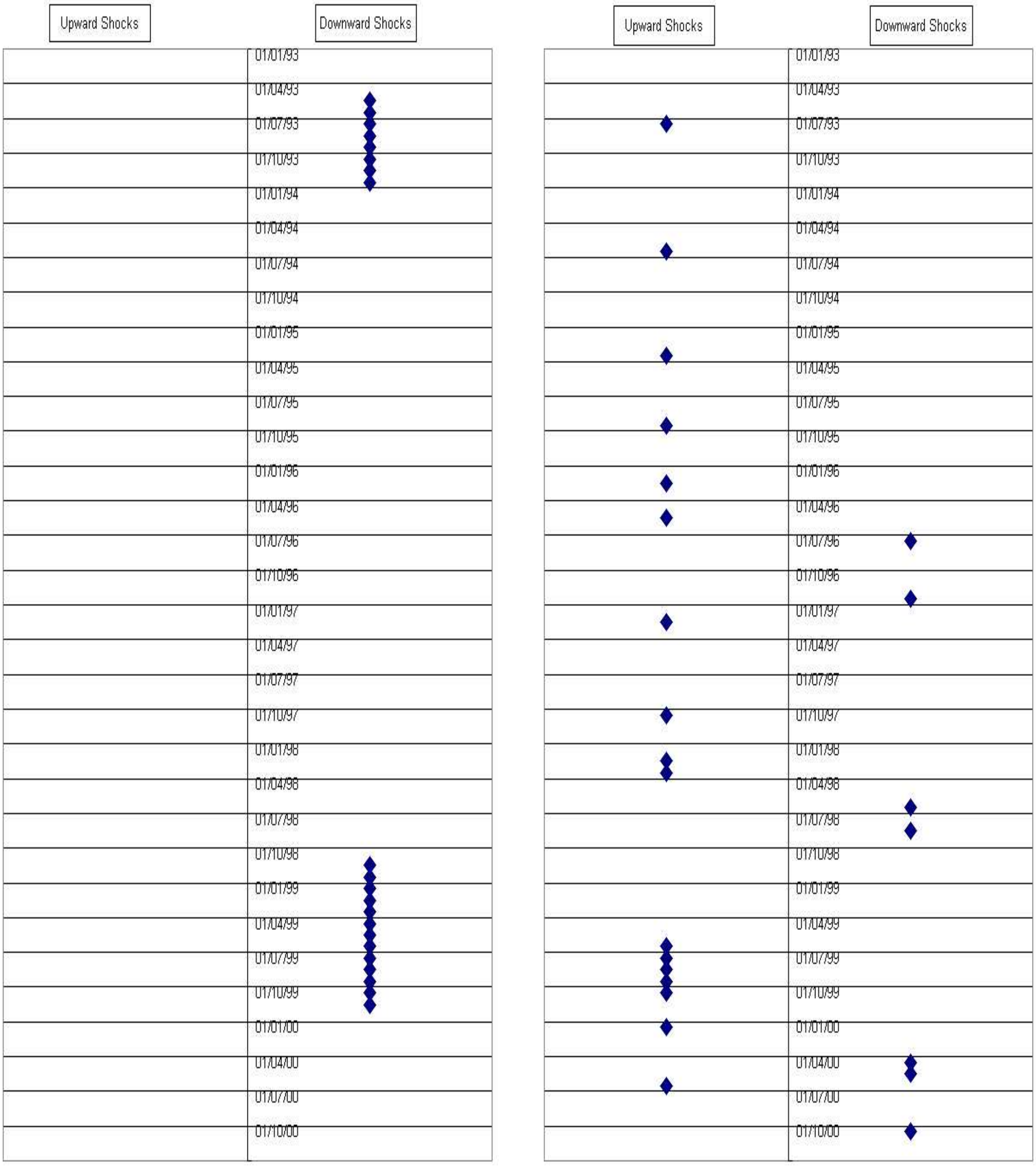
The sets of time windows pertaining to the different extreme events are mapped onto the data of the main sector indices. Each data set produced in this manner represents the historical reactions of the five main sectors to the extreme event in the market indicator data. These data sets will be used in the process of finding optimal allocations (Chapter 4) and risk–adjusted rankings (Chapter 5) across different sectors during the economic conditions described by the shocks in the different market indicators.



(a) R150 Bond yields

(b) Percentage weekly change in R150 Bond yields

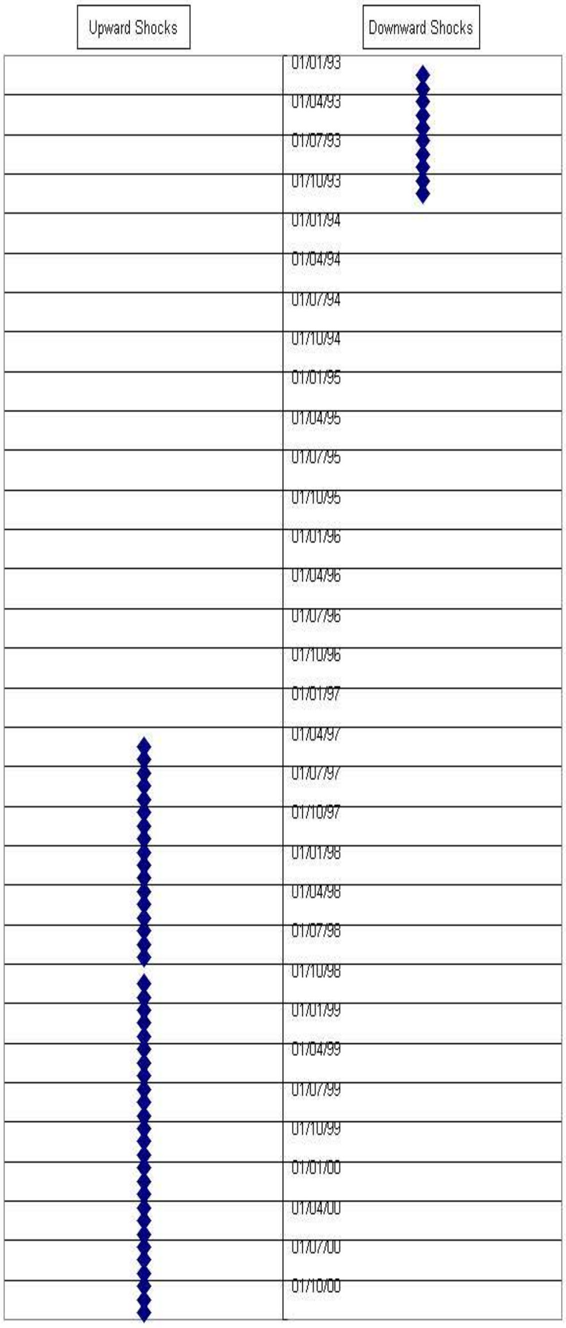
Figure 3.7: Time windows: The R150 Bond indicators



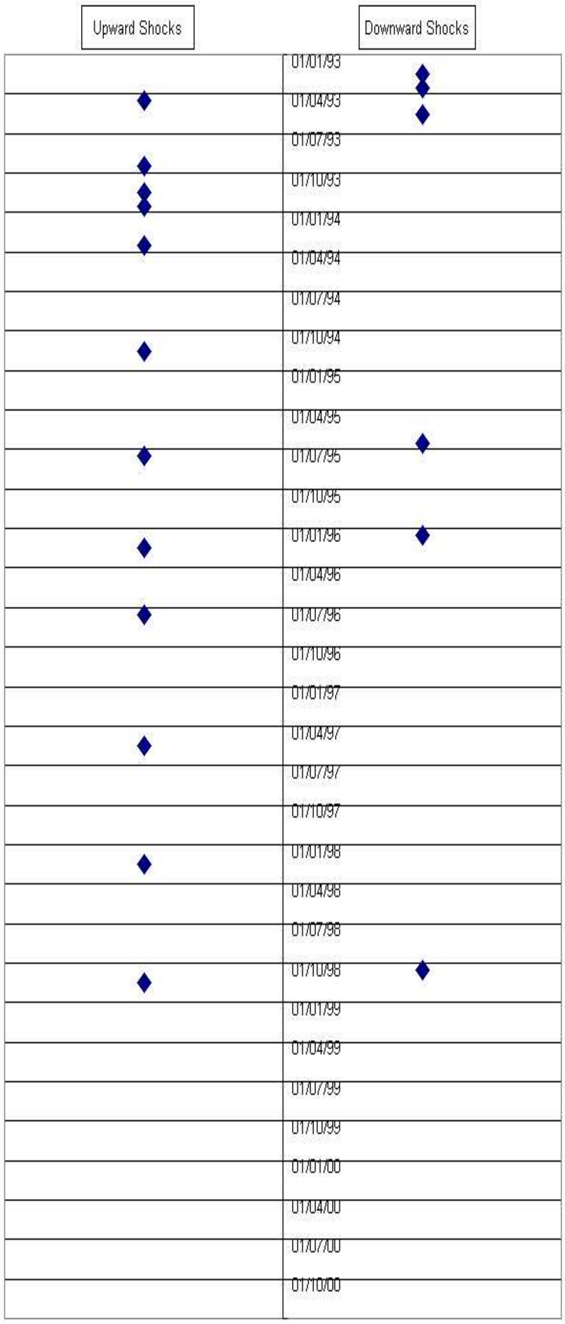
(a) Absolute values of SA business confidence

(b) Percentage monthly change in SA business confidence

Figure 3.8: Time windows: SA business confidence



(a) Absolute values of US consumer sentiment



(b) Percentage monthly change in US consumer sentiment

Figure 3.9: Time windows: US consumer sentiment

Chapter 4

Asset Allocation via optimisation

In §2.2, a measure of investment comfort/satisfaction was introduced, which combines the expected return and volatility of an investment. In the process a risk–aversion constant was also introduced. With the introduction of this concept, no attempt was made to explore possible values of the constant so as to represent the tolerance for risk of different investors. In the following section, more attention will be given to the risk–aversion constant and values will be proposed to represent the tolerance for risk of three broad classes of risk–averse investors, namely conservative, moderate and aggressive investors.

Also in §2.2, the assumptions and rationale behind Markowitz Portfolio Theory were outlined. It was shown how optimisation plays an integral role in this approach, which suggests the maximisation of a specific investor's comfort/satisfaction in order to find the most suitable alternative out of all available investment alternatives. In this chapter, the proposed risk–aversion factors for conservative, moderate and aggressive investors are incorporated into the objective function of the model in (2.5.5). The resulting optimisation problems are then resolved for the various (extreme) economic conditions as defined by a variety of indicator shocks.

4.1 A risk–aversion constant

Recall, from §2.2, that a representation of an investor's comfort/satisfaction for the risk–return combination of an investment alternative may be given by

$$U = R_c - \frac{a}{s}\sigma_c^2 \tag{4.1.1}$$

and that the goal of Markowitz Portfolio Theory is to maximise this comfort/satisfaction of an investor. Integral to this equation is the aversion constant, a. Firstly, the idea behind suggesting possible risk aversion values is not to create the impression that it is deemed possible to determine how much risk an investor can stand and to represent this risk tolerance by a single constant. Risk tolerance is seen by many as variable and by some as unquantifiable, yet it is acknowledged by most players in the investment industry that different investors have different tolerances for risk and that their investment choices should fit their risk profiles [10]. To this end, many companies, like

Merrill Lynch, Rothchilds, Sanlam Personal Portfolios and Galaxy have designed risk questionnaires to help investors determine whether they are conservative, moderate or aggressive. Typically, these risk questionnaires include ten to twenty questions about an investor's investment experience, financial security and tendency to make risky or conservative choices. The benefit of these questionnaires is that they are an objective resource investors may use to obtain at least a rough idea of their own risk tolerance, by attributing weights to the different questions and scores to the possible answers. Many experts warn, however, that the questionnaires should only be used as a first indication of risk tolerance [10]. Since the possible contribution of this study is more in terms of the identification of homogeneous periods, characterised by the economic environment and allocating capital during these periods, emphasis will not be placed on finding the most suitable or accurate risk–aversion factor per risk profile. Instead, the following example of a questionnaire, given in [10], is considered.

Example 3 *The risk tolerance questionnaire in [10] focuses mainly on the investment horizon of the investor and his tendency to make risky or conservative choices. It includes nine questions pertaining to these issues, each with three possible answers. In order to illustrate the nature of these questions, two are extracted from the questionnaire and given below.*

1. *Just 60 days after you put money into an investment, its price falls by 20%. Assuming none of the fundamentals has changed, what would you do?*
   - *(a) Sell to avoid further worry and try something else.*
   - *(b) Do nothing and wait for the investment to come back.*
   - *(c) Buy more. It was a good investment before; now it is a cheap investment too.*
2. *A good investment opportunity just came along, but you have to borrow money to get in. Would you take out a loan?*
   - *(a) Definitely not.*
   - *(b) Perhaps.*
   - *(c) Yes.*

The first answer to each question portrays a very conservative reaction, the second answer depicts a reaction that is a bit more risky, while the third is the most daring of the three. The risk tolerance score associated with the first answer is 3, the risk tolerance score of the second answer is 2, while the score of the third is 1. After completing the questionnaire, the investor may add his scores and compare the result to the bounds dictating the specific risk profiles, namely

Conservative:	$22 - 27$ *points*
Moderate:	$15 - 21$ *points*
Aggressive:	$9 - 14$ *points*

If it is assumed that a typical moderate investor is most comfortable with the second answer to each of the nine questions, his total score will equal 18, placing it in the middle of the moderate region (and the entire scale). If it is assumed that a very aggressive investor chooses the third answer for all of the nine questions, his score is placed at the extreme point of the aggressive scale. Conversely, if the investor answered all nine questions conservatively, the score would have matched the extreme point of the conservative scale.

In accordance with the decision that the focus of this thesis should not be the quantification of risk–aversion, it was decided to assume the above suggested profile boundaries as a basis for determining the risk aversion constants. The risk aversion factors to represent the risk tolerances of conservative, moderate and aggressive investors in this study, were calculated as the average, rounded score within the boundaries suggested by the risk questionnaire in [10]. The resulting factors are as follows:

$$\begin{array}{ll} \text{Conservative:} & a = 25, \\ \text{Moderate:} & a = 18, \\ \text{Aggressive:} & a = 12. \end{array}$$

4.2 Optimal allocation across market sectors

Theoretically if one believes Markowitz Portfolio Theory to be a close approximation of the investment environment, the results emerging from solving the optimisation problem in §2.5 should be accurate and practically viable, as it follows the process of Markowitz mean–variance optimisation very closely. However, Markowitz' mean–variance model is simplistic in that some of the underlying assumptions are not met in practice and it also ignores some practical considerations. While Markowitz established the basis for portfolio optimisation, difficulty of incorporating real–world constraints and dilemmas into his classical theory has limited its use in its purest form. The challenge for mathematicians is to enrich the mean–variance model with additional real–world constraints and generate practical insights from the solutions of these more realistic models. Some suggestions made by theorists and investment professionals regarding possible alternative strategies, such as data envelopment analysis (DEA), has already been mentioned in §2.5. To capture the realism of portfolio planning, a number of discrete restrictions have been considered by different researchers. Some of these are summarised below.

1. A *buy–in threshold* or *floor constraint* is defined as the minimum level below which an asset or set of assets is not purchased. This requirement eliminates unrealistically small trades that may otherwise be included in an optimal portfolio. Very small weightings of an asset will only add to administrative and monitoring costs; thus floor or minimum weightings are commonly employed. Similarly, very high weightings in any one asset (or set of assets as in the case of this study) introduce excessive exposure to the idiosyncrasies of the asset (even though the portfolio's overall risk may appear acceptable); this problem is similarly overcome by introducing a *ceiling constraint*, which restricts excessive exposure to certain securities.

2. *Cardinality constraints* are introduced if investors wish to specify the number of securities in their portfolios for the purpose of monitoring and control.

3. *A roundlot* is defined as a discrete quantity of a security which is taken as the basic unit of investment in this security. Investors are restricted to making transactions only in multiples of these roundlots. This overcomes the problem of infinite divisibility of assets inherent to the mean–variance assumptions [45].

Although the above–mentioned adjustments to the optimisation problem based on Markowitz Portfolio Theory are extremely important to transform it into a real–world problem, any additional restriction to the problem disguises the true nature of the interaction between the underlying securities (or groups of securities). It is important not to ignore the fact that additional constraints are needed to ensure that the theory is practically viable. However, as the main objective of the optimisation in this study is to analyse the nature of the five main sectors during extreme market conditions, no additional restrictions will be incorporated in this study.

It seems more important rather to incorporate into the theory an additional measure of risk. An integral assumption of Markowitz Portfolio Theory is that variance is the most appropriate measure of risk. All investment professionals do not necessarily agree with this assumption. In the following section, an alternative measure of risk, namely *downside risk* is introduced and an attempt is made to incorporate this measure in a comfort/satisfaction function to be maximised in a similar manner to the mean–variance procedures, resulting in an alternative optimal mix of securities.

4.3 Downside Risk

The appropriate measure of risk is often dictated by the needs of the investor. Traditionally, standard deviation has been the most common measure of risk in asset allocation programmes. One reason is that most sophisticated investors have come to accept volatility as a measure of riskiness and have therefore become comfortable with standard deviation as a quantification of this risk. The use of standard deviation as risk measure implies that risk lies in the variability of returns. However, instead of using a measure that analyses any deviation from expectation, some believe that investors should be concerned only with returns below expectations; in other words deviations below the mean value [48]. A measure for this is semi–variance, denoted by σ_s^2, and quantified as follows. Let $\underline{r}_i = [r_{i1}, r_{i2}, \ldots, r_{in}]$ be a vector of returns of length n, then

$$\sigma_s^2 = \frac{\sum_{k=1}^{n} \min\{0, (r_{ik} - \hat{\underline{r}}_i)\}^2}{n}, \tag{4.3.1}$$

where $\hat{\underline{r}}_i$ is the arithmetic mean of returns of vector $\underline{r}_i$. This measure assumes that the risk for an investor is the damage to the portfolio caused by below–average returns. Obviously, investors would welcome above–average returns, so these are not considered when measuring risk [75].

Markowitz [59] also recognised the importance of this idea. He realised that investors may be interested in downside risk for two reasons:

1. only downside risk or the safety of his investment is relevant to an investor and
2. security distributions may not necessarily be normally distributed and therefore standard deviation (which assumes that the deviations are evenly spread around the mean) may not be the most accurate measure of risk.

In [59], Markowitz shows that when distributions are normal, both the downside risk measure and the variance provide the same answer. However, if distributions are not normal, Markowitz claims that only the downside risk measure provides an accurate measure of risk. Markowitz derived a formula similar to (2.2.1) for variance, for the semi–variance $\sigma^2_{s_p}$ of a portfolio of risky assets as

$$\sigma^2_{s_p} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \text{Cov}_\text{s}(\underline{r}_i, \underline{r}_j), \tag{4.3.2}$$

where

$$\text{Cov}_\text{s}(\underline{r}_i, \underline{r}_j) = \frac{\sum_{k=1}^{n}(\min\{0, (r_{ik} - \hat{\underline{r}}_i)\})(r_{jk} - \hat{\underline{r}}_j)}{n}, \tag{4.3.3}$$

and

$$\begin{aligned} n &= \text{the number of risky assets in the portfolio,} \\ w_i &= \text{weight allocated to risky asset } i. \end{aligned}$$

The vector $\underline{r}_i$ and value $\hat{\underline{r}}_i$ are defined as in (4.3.1).

After proposing the semi–variance measure, Markowitz still preferred the variance measure because it was computationally simpler. The semi–variance optimisation models, using a non–symmetric semi–covariance matrix, require double the number of data inputs compared to a variance model. The lack of cost–effective computer–power at that time and the fact that the variance model was already mathematically very complex, was a consideration until the 1980's with the advent of the microcomputer. However, research on semi–variance did continue in the 1960's and 1970's. Quirk and Saposnik [74] demonstrated the theoretical superiority of the semi–variance versus the variance. Mao [57] provided a strong argument that investors will only be interested in downside risk and that semi–variance should be used. A number of researchers found that the below–mean semi–variance is helpful in testing for skewed probability distributions. By dividing the variance by the below–mean semi–variance, a ratio is obtained. If the distribution is symmetric, the ratio should equal two. If the ratio is not equal to two, there is evidence that the distribution is skew or asymmetric. When the skewness of an asset return is positive (ratio > 2), then the upside returns will have a larger magnitude than the downside returns, in other words when gains occur, they will tend to be greater, and when losses occur they will tend to be smaller. Conversely, if the skewness of an asset return is negative (ratio < 2), then the downside returns will have a larger magnitude than the upside returns, in other words when gains occur, they will tend to be smaller, and when losses occur they will tend to be greater [66].

Kazemi and Martin [48] also researched the attributes of downside risk and referred to studies that show the use of semi–variance not to affect optimal asset allocation drastically when the distribution of returns is approximately symmetric. However, when the return distributions are not symmetric, the use of semi–variance as measure of risk may introduce significant changes in the optimal allocation. Therefore, this measure of risk may be more appropriate when portfolios whose returns tend to be skewed, are part of the allocation programme [48].

Downside risk may satisfy some of the critics of standard deviation as a measure of risk, by only focusing on undesirable returns. However, Kazemi and Martin [48] state that there are also a number of problems with this measure of risk.

- First and foremost, any quantitative measure of risk has to be predictable. It is one thing to use historical data to measure the past performance of a portfolio, but it is another thing to forecast the riskiness of a portfolio. Statistical properties of semi–variance are not well understood and models for forecasting are not well developed.

- The standard deviation of a portfolio is related to the standard deviation and correlation of the securities that are included in the portfolio. The semi–variance of a portfolio is not related to the semi–variance and correlation of the underlying assets in a well understood way.

- Semi–variance and other measures of downside risk rely on about half the data points (because only the below–mean returns are used) and therefore, much longer series of returns are needed to obtain accurate estimates.

The argument for or against the measure of downside risk becomes one of a trade–off between simplicity and tractability on the one hand and realism on the other. It should again be stressed that this study will not engage itself in the debate of what the most appropriate risk measure might be. However, the influence of a measure of downside risk on the decision making process may be examined by incorporating it in a portfolio optimisation problem, as will be done in the rest of the section, or in a risk–adjusted return ratio and comparing the two strategies. This will be addressed in Chapter 5.

The main objective of this section is to put forward a set of allocation suggestions, which are optimal within a specific (extreme) context. In order to attain these results, it is necessary to formulate a mathematical problem so as to encapsulate the natural steps in the allocation process, and then resolve this problem. However, before the rest of the section is devoted to these two aspects, it seems important to reiterate the broader context in which the results should be interpreted by defining a framework for optimisation.

Definition 17 *Attention will be paid to specific (extreme) economic conditions or scenarios, while finding optimal allocations among the five main sectors of the JSE, namely Mining Resources, Non–Mining Resources, Financials, Industrials, and Real Estate for these specific conditions. The allocation should maximise the comfort/satisfaction of an investor with a specific risk profile in terms of return and variance [semi–variance]. The risk profiles to be taken into account are those of conservative, moderate and aggressive investors.*

If the five main market sectors are represented by an index in the set $\{1, 2, \ldots, 5\}$ so that

1 denotes the Mining Resources sector (CI09),
2 denotes the Non–Mining Resources sector (CI10),
3 denotes the Financials sector (CI24),
4 denotes the Industrial sector (CI27) and
5 denotes the Real Estate sector (CI70),

the goal stated in Definition 17 may be accomplished by

$$\text{Maximising} \quad U = \sum_{k=1}^{5} \breve{r}_k w_k - a \sum_{k=1}^{5} \sum_{m=1}^{5} w_k w_m \frac{\text{Cov}(\underline{r}_k, \underline{r}_m)}{\hat{\underline{r}}_k \hat{\underline{r}}_k} \tag{4.3.4}$$

or

$$\text{Maximising} \quad U = \sum_{k=1}^{5} \breve{r}_k w_k - a \sum_{k=1}^{5} \sum_{m=1}^{5} w_k w_m \frac{\text{Cov}_s(\underline{r}_k, \underline{r}_m)}{\hat{\underline{r}}_k \hat{\underline{r}}_k}, \tag{4.3.5}$$

both subject to

$$\begin{cases} \sum_{k=1}^{5} w_k & = 1, \\ w_k & \geq 0. \end{cases}$$

The variables and constants used in (4.3.4), (4.3.5) and (4.3.6) may be defined in such a way that

w_k denotes the percentage weighting of the optimal portfolio in sector k,
$\breve{r}_k$ the percentage return of sector k for the period pertaining to the specific time window,

and

$$\begin{aligned} \text{Cov}(\underline{r}_k, \underline{r}_m) &= \frac{\sum_{i=1}^{n} (r_{ki} - \hat{\underline{r}}_k)(r_{mi} - \hat{\underline{r}}_m)}{n} \quad \text{from (2.2.3)}, \\ \text{Cov}_s(\underline{r}_k, \underline{r}_m) &= \frac{\sum_{i=1}^{n} (\min\{0, (r_{ki} - \hat{\underline{r}}_k)\})(r_{mi} - \hat{\underline{r}}_m)}{n} \quad \text{from (4.3.3)}, \end{aligned}$$

where $\hat{\underline{r}}_k$ is the average return for sector k for the period pertaining to the specific time window. In the objective functions (4.3.4) and (4.3.5), the respective covariances and semi–covariances are divided by the average returns of the sectors that are being correlated. The averages act as scaling constants to enable the risk to be subtracted from returns.

It will be noticed that the covariance in (4.3.4) was merely exchanged for semi–covariance in the objective function pertaining to downside risk in (4.3.5). This is in part because of the resemblance of formulas (2.2.1) and (4.3.2), but furthermore, Jobst *et al.* [45] found the work of Hanoch and Levy [33] to show that the mean–variance criterion is a valid efficiency criterion for many individuals' utility functions. A study comparing alternative utility functions also appears in Kallberg and Ziemba [47]. They conclude that mean–variance type analyses are justified for any general concave utility function.

4.4 The optimisation procedure

A very important theorem within the scope of the optimisation procedures employed in this study is stated next.

Theorem 1 *If the feasible region S for a non–linear programming problem is a convex set and the objective function $f(x)$ is concave, then a local maximum for the optimisation problem is also an optimal solution to the problem.*

For the proof of this theorem see [94]. To explain why this theorem has particular significance within the context of this study, a few definitions are needed. First, the definition of a *convex set* is given.

Definition 18 *A set of points S is a convex set if the line segment joining any pair of points in S is wholly contained in S.*

Secondly, a local maximum is defined as follows.

Definition 19 *A point $\underline{x} = (x_1, x_2, \ldots, x_n)$ is a local maximum for a function f if $f(\underline{x}) \geq f(\underline{x}')$, for all feasible points $\underline{x}' = (x_1', x_2', \ldots, x_n')$ in a neighbourhood of $\underline{x}$.*

It follows from Theorem 1 that, in order to ensure that a solution obtained via solving the optimisation problems is in actual fact globally optimal, it is sufficient to show that the relevant objective function is concave on its domain. In order to do this, the notion of a *concave function* should be defined.

Definition 20 *A function $f(\underline{x})$ is a concave function on a convex set S if for any $\underline{x}' \in S$ and $\underline{x}'' \in S$*

$$f(c\underline{x}' + (1-c)\underline{x}'') \geq cf(\underline{x}') + (1-c)f(\underline{x}'')$$

holds for all $0 \leq c \leq 1$.

In order to conclude whether a function is concave, the function's *Hessian matrix* may be examined.

Definition 21 *The Hessian of $f(\underline{x})$, denoted by $H(\underline{x})$, where $\underline{x} = (x_1, x_2, \ldots, x_n)$, is the $n \times n$ matrix whose (ij)–th entry is given by*

$$\frac{\partial^2 f(\underline{x})}{\partial x_i \partial x_j}.$$

The Hessian of an example function $f(x_1, x_2)$ is given below.

Example 4 *If $f(x_1, x_2) = x_1^3 + 2x_1x_2 + x_2^2$, then*

$$H(x_1, x_2) = \begin{bmatrix} 6x_1 & 2 \\ 2 & 2 \end{bmatrix}.$$

A definition for ascertaining whether a function is concave will now be given.

Definition 22 *A function $f(\underline{x})$ with continuous second–order partial derivatives for each point $\underline{x} = (x_1, x_2, \ldots, x_n) \in S$ is a concave function on S if and only if for each $x \in S$ and $k = 1, 2, \ldots, n$, all non–zero principal minors of its Hessian $H(\underline{x})$ have the same sign as $(-1)^k$ [94],*

where the *k–th principal minor* of a matrix is defined as follows.

Definition 23 *A kth principal minor of an $n \times n$ matrix is the determinant of any $k \times k$ matrix obtained by deleting $n-k$ rows and the corresponding $n-k$ columns of the matrix.*

In the following example, the first and second principal minors of a 2×2 matrix are calculated.

Example 5 *Thus for the matrix*

$$\begin{bmatrix} -2 & -1 \\ -1 & -4 \end{bmatrix}$$

the first principal minors are -2 and -4 (for any matrix, the first principal minors are just the diagonal entries of the matrix), and the second principal minor is the determinant $(-2)(-4) - (-1)(-1) = 7$.

By considering the above information, an example function may thus be shown to be concave in the following manner.

Example 6 *The function $f(x_1, x_2) = -x_1^2 - x_1x_2 - 2x_2^2$ can be shown to be concave on $\mathcal{R}^2$ by examining its Hessian,*

$$H(x_1, x_2) = \begin{bmatrix} -2 & -1 \\ -1 & -4 \end{bmatrix}.$$

The first principal minors are the diagonal entries of the Hessian (-2 and -4). These are both non–positive. The second principal minor is the determinant of $H(x_1, x_2)$ and equals $-2(-4) - (-1)(-1) = 7$, which is non–negative. Thus, $f(x_1, x_2)$ is a concave function on $\mathcal{R}^2$.

The constraints of the optimisation problems in (4.3.4) and (4.3.5) in combination with (4.3.6) are linear and therefo4re results in a convex feasible region. It only remains to be proved that the objective functions used in this study are concave. This will be illustrated by means of applying the above–mentioned theories to a general two–asset portfolio. Although a similar (but somewhat tedious) procedure may be followed to prove the concavity of the objective function in the case of an n–asset portfolio, it will not be done here.

The objective function of the maximisation problem, where variance is assumed the most appropriate measure of risk, is given by

$$U(w_1, w_2) = w_1 r_1 + w_2 r_2 - w_1^2 \sigma_1^2 - w_2^2 \sigma_2^2 - w_1 w_2 \mathrm{Cov}(\underline{r}_1, \underline{r}_2) - w_1 w_2 \mathrm{Cov}(\underline{r}_2, \underline{r}_1). \quad (4.4.1)$$

The Hessian for this function U is

$$H(w_1, w_2) = \begin{bmatrix} -2\sigma_1^2 & -\mathrm{Cov}(\underline{r}_1, \underline{r}_2) - \mathrm{Cov}(\underline{r}_2, \underline{r}_1) \\ -\mathrm{Cov}(\underline{r}_1, \underline{r}_2) - \mathrm{Cov}(\underline{r}_2, \underline{r}_1) & -2\sigma_2^2 \end{bmatrix}. \quad (4.4.2)$$

The first principal minors ($-2\sigma_1^2$ and $-2\sigma_2^2$) are both non–positive. The second principal minor, $H_2(w_1, w_2)$, is given by

$$\begin{aligned} H_2(w_1, w_2) &= (-2\sigma_1^2)(-2\sigma_2^2) - [-\mathrm{Cov}(\underline{r}_1, \underline{r}_2) - \mathrm{Cov}(\underline{r}_2, \underline{r}_1)][-\mathrm{Cov}(\underline{r}_1, \underline{r}_2) - \mathrm{Cov}(\underline{r}_2, \underline{r}_1)] \\ &= 4\sigma_1^2\sigma_2^2 - [\mathrm{Cov}(\underline{r}_1, \underline{r}_2) + \mathrm{Cov}(\underline{r}_2, \underline{r}_1)]^2 \quad (4.4.3) \end{aligned}$$

It is recalled, from (2.2.5), that $\mathrm{Cov}(\underline{r}_1, \underline{r}_2) = \rho_{12}\sigma_1\sigma_2$, and hence (4.4.3) becomes

$$H_2(w_1, w_2) = 4\sigma_1^2\sigma_2^2 - \rho_{12}^2\sigma_1^2\sigma_2^2 - 2\rho_{12}\rho_{21}\sigma_1^2\sigma_2^2 - \rho_{21}^2\sigma_1^2\sigma_2^2. \quad (4.4.4)$$

However, $\mathrm{Cov}(\underline{r}_1, \underline{r}_2) = \mathrm{Cov}(\underline{r}_2, \underline{r}_1)$ so that $\rho_{12} = \rho_{21}$ and therefore (4.4.4) reduces to

$$H_2(w_1, w_2) = 4\sigma_1^2\sigma_2^2 - 4\rho_{12}^2\sigma_1^2\sigma_2^2.$$

As $-1 \leq \rho_{12} \leq 1$,

$$H_2(w_1, w_2) \geq 0.$$

The objective function is therefore concave.

Where semi–variance is used as measure of risk, the objective function resembles (4.4.1), except for the covariance that is exchanged for semi-covariance. Therefore, the Hessian also resembles (4.4.2), except for the covariance that is exchanged for semi-covariance. The first principal minors ($-2\sigma_{s_1}^2$ and $-2\sigma_{s_2}^2$) are also non–positive, while the second principal minor ($H_{s_2}(w_1, w_2)$) may be calculated as

$$H_{s_2}(w_1, w_2) = 4\sigma_{s_1}^2\sigma_{s_2}^2 - \rho_{12}^2\sigma_{s_1}^2\sigma_{s_2}^2 - 2\rho_{12}\rho_{21}\sigma_{s_1}^2\sigma_{s_2}^2 - \rho_{21}^2\sigma_{s_1}^2\sigma_{s_2}^2.$$

Here $-1 \leq \rho_{12}, \rho_{21} \leq 1$ and therefore

$$H_{s_2}(w_1, w_2) \geq 2\sigma_{s_1}^2\sigma_{s_2}^2 - 2\rho_{12}\rho_{21}\sigma_{s_1}^2\sigma_{s_2}^2.$$

If $\rho_{12}\rho_{21} \leq 0$, then

$$H_{s_2}(w_1, w_2) \geq 2\sigma_{s_1}^2\sigma_{s_2}^2 > 0,$$

else, if $\rho_{12}\rho_{21} > 0$, then

$$H_{s_2}(w_1, w_2) \geq 0.$$

In both cases $H_{s_2}(w_1, w_2)$ is non–negative, which implies that the objective function, where semi–variance is assumed the most appropriate measure of risk, is concave.

For the purposes of this study, a variance–based and semi–variance–based optimisation problem was solved in Excel 2000 with the Standard Solver package per economic scenario per time window per risk profile. Excel uses a generalised reduced gradient method to solve non–linear optimisation problems, which solves any problem of the form

$$\begin{array}{lll} \text{Maximise} & f(\underline{x}) & \underline{x} \in \mathcal{R}^n \\ \text{Subject to} & h_i(\underline{x}) = \underline{0} & i = 1, \ldots, m \\ & L_j \leq x_j \leq U_j & j = 1, \ldots, n. \end{array}$$

Inequality constraints $g_i(\underline{x})$ may be accommodated by subtracting non–negative slack variables from the inequality constraints, i.e.

$$h_i(\underline{x}) = g_i(\underline{x}) - v_i^2 = 0,$$

while permitting the bounds on the v_i's to be $-\infty < v_i < \infty$. (In this case the v_i's are added to the set of n variables.)

The algorithm pertaining to the generalised reduced gradient method implemented by Excel 2000 is an extension of Wolfe's reduced gradient method (which in turn was proposed to extend the simplex procedures to resolve problems with non–linear (convex) objective functions) so as to incorporate non–linear constraints. The generalised reduced gradient method has proven one of the most robust and reliable approaches to solving intricate non–linear programming problems [28]. The constraints of the problems solved in this study are all linear and therefore the generalised reduced gradient is reduced to Wolfe's reduced gradient method. As was already mentioned, Wolfe's method [93] was proposed to extend the simplex procedure to problems with non–linear convex objective functions. As in simplex procedures, the variables are also partitioned into basic and non–basic variables.

In order to examine Wolfe's algorithm, we consider the maximisation of $M(\underline{x})$ subject to the linear constraints

$$A\underline{x} - \underline{b} = 0 \quad and \quad \underline{x} \geq \underline{0}, \tag{4.4.5}$$

where A is an $m \times n$ matrix and $m < n$. The basic variables are denoted $\underline{x}_B = [x_{B_1}, x_{B_2}, \ldots, x_{B_m}]^T$, while the non–basic variables are $\underline{x}_N = [x_{N_1}, x_{N_2}, \ldots, x_{N_{n-m}}]^T$. If the non–singular $m \times m$ matrix B consists of the columns of A representing the coefficients of the basic variables $\underline{x}_B$, then, from (4.4.5),

$$\underline{x}_B = -B^{-1}C\underline{x}_N + B^{-1}\underline{b}, \tag{4.4.6}$$

where C is the $m \times (n - m)$ matrix which consists of the columns of A related to the non–basic variables $\underline{x}_N$.

In this case, the Kuhn–Tucker conditions for optimality are

$$\begin{array}{rcl} \underline{v} & = & \nabla_{\underline{x}} M(\underline{x}) - A^T \underline{u} \leq \underline{0}, \\ \underline{v}^T \underline{x} & = & 0, \\ \underline{x} & \geq & \underline{0}, \\ A\underline{x} & = & \underline{b}. \end{array}$$

Since the basic variables are positive, $\underline{x}_B > \underline{0}$; therefore the corresponding part of $\underline{v}$ is

$$\underline{v}_B = \nabla_{\underline{x}_B} M(\underline{x}) - B^T \underline{u} = \underline{0},$$

from which we deduce that

$$\underline{u} = (B^T)^{-1} \nabla_{\underline{x}_B} M(\underline{x}). \tag{4.4.7}$$

Furthermore, from (4.4.6) and (4.4.7) it follows that

$$\begin{aligned} dM(\underline{x}) &= [\nabla_{\underline{x}_B} M(\underline{x})]^T d\underline{x}_B + [\nabla_{\underline{x}_N} M(\underline{x})]^T d\underline{x}_N \\ &= [\nabla_{\underline{x}_N} M(\underline{x}) - C^T (B^T)^{-1} \nabla_{\underline{x}_B} M(\underline{x})]^T d\underline{x}_N \\ &= [\nabla_{\underline{x}_N} M(\underline{x}) - C^T \underline{u}]^T d\underline{x}_N \\ &= \underline{v}_N^T d\underline{x}_N, \end{aligned} \tag{4.4.8}$$

where $\underline{v}_N$ is that part of $\underline{v}$ corresponding to the non–basic variables $\underline{x}_N$. From (4.4.8) the reduced gradient

$$\frac{dM(\underline{x})}{d\underline{x}_N} = \underline{v}_N$$

is obtained, which is used to define the search direction $\underline{s}^k$ during the k–th iteration

$$\underline{x}^{k+1} = \underline{x}^k + \lambda^k \underline{s}^k \tag{4.4.9}$$

as

$$s^k_{N_i} = 0 \;\text{ if }\; x^k_{N_i} = 0 \;\text{ and }\; v^k_{N_i} < 0 \tag{4.4.10}$$

(where i is an index for the vector entries), otherwise,

$$s^k_{N_i} = -v^k_{N_i} \tag{4.4.11}$$

and

$$\underline{s}^k_B = -B^{-1} C \underline{s}^k_N, \tag{4.4.12}$$

where λ^k is the steplength during iteration k.

From (4.4.9) and (4.4.12) it follows that

$$\begin{aligned} A\underline{x}^{k+1} &= B\underline{x}^{k+1}_B + C\underline{x}^{k+1}_N \\ &= B\underline{x}^k_B + \lambda^k B \underline{s}^k_B + C\underline{x}^k_N + \lambda^k C \underline{s}^k_N \\ &= B\underline{x}^k_B - \lambda^k C \underline{s}^k_N + C\underline{x}^k_N + \lambda^k C \underline{s}^k_N \\ &= B\underline{x}^k_B + C\underline{x}^k_N \\ &= A\underline{x}^k \\ &= \underline{b}, \end{aligned}$$

which indicates that if $\underline{x}^k$ satisfies the linear constraints $A\underline{x} = \underline{b}$, then $\underline{x}^{k+1}$ also satisfies the constraints.

Relationships (4.4.10) through (4.4.12) yield the search direction $\underline{s}^k \neq \underline{0}$ unless $\underline{v}^T \underline{s}^k = 0$. If the latter equality holds, then the Kuhn–Tucker conditions are satisfied and $\underline{x}^k$ is optimal [43].

For further information on the generalised reduced gradient method, see [38]. Although testing of the generalised reduced gradient method has shown that in some instances the algorithm may converge to a local optimum instead of a global one, we are certain from the discussion above that the solutions found by the algorithms in the case of this study, are global optima, because both objective functions are concave.

4.5 Optimisation Results

The results of both the variance– and semi–variance based optimisation problems as per economic scenario and risk profile provide a basis for finding possible trends, similarities and dissimilarities within comparable sets of time windows. These results are given in Appendix B. In this section, certain trends in the results will be discussed by referring to specific cases. The objective value for each time window, which is given in all the

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Bull	1	0.1181	0	1	0	0	0	−0.0632	0.0000	−0.0415	−0.0527	−0.0648	0.0630
	2	0.1372	0.0904	0.9096	0	0	0	0.0000	0.0000	−0.0423	−0.0529	−0.0478	0.0378
	3	0.0783	0	0.5762	0	0.4238	0	−0.0062	0.0000	−0.0229	0.0000	−0.0574	0.0595
	4	0.1375	0	1	0	0	0	−0.0526	0.0000	−0.0405	−0.0141	−0.0263	0.0399
	5	0.0883	1	0	0	0	0	0.0000	−0.0080	−0.0137	−0.0022	−0.1166	0.0013
	6	0.1786	0	1	0	0	0	−0.0880	0.0000	−0.0839	−0.0885	−0.1313	0.1335
	7	0.0494	0	0	1	0	0	−0.0437	−0.0668	0.0000	−0.0237	−0.0019	0.0471
	8	0.0734	0	0	1	0	0	−0.0096	−0.0126	0.0000	−0.0299	−0.0722	0.0540
	9	0.0808	0	0	1	0	0	−0.0127	−0.0719	0.0000	−0.0272	−0.0819	0.0716
	10	0.0937	0	0.3336	0.6664	0	0	−0.1125	0.0000	0.0000	−0.0495	−0.0347	0.0853
	11	0.0471	0	0	1	0	0	−0.0009	−0.0414	0.0000	−0.0213	−0.0300	0.0354
	12	0.0505	0	0	0	1	0	−0.0008	−0.0432	−0.0155	0.0000	−0.0218	0.0449
	13	0.1977	0	0	1	0	0	−0.0597	−0.0057	0.0000	−0.0175	−0.0437	0.0627
	14	0.1539	0	0.9637	0.0363	0	0	−0.0685	0.0000	0.0000	−0.0330	−0.0464	0.0618
	15	0.0935	0.5351	0.0871	0	0.3778	0	0.0000	0.0000	−0.0202	0.0000	−0.0191	0.0164
	16	0.1356	0	0	0.7268	0	0.2732	−0.0399	−0.3267	0.0000	−0.6431	0.0000	0.0253
	17	0.1200	0	1	0	0	0	−0.0116	0.0000	−0.0331	−0.0429	−0.0366	0.0574
Bear	1	−0.0277	0	0	0	0	1	−0.0006	−0.0320	−0.0411	−0.0320	0.0000	−0.0296
	2	0.0044	0	0	1	0	0	−0.0640	−0.0901	0.0000	−0.0071	−0.0227	0.0044
	3	−0.0245	0	0	0	0	1	−0.0759	−0.1465	−0.0816	−0.0647	0.0000	−0.0254
	4	0.0254	1	0	0	0	0	0.0000	−0.0598	−0.0288	−0.0278	−0.0255	0.0076
	5	−0.0226	0	1	0	0	0	−0.0571	0.0000	−0.0317	−0.0546	−0.0555	−0.0244
	6	0.0289	1	0	0	0	0	0.0000	−0.0598	−0.0670	−0.0614	−0.0422	0.0254
	7	−0.0429	0	0	0	1	0	−0.0208	−0.0288	−0.0172	0.0000	−0.0306	−0.0477
	8	−0.0256	0	0	0	0	1	−0.1115	−0.2242	−0.0272	−0.0895	0.0000	−0.0271
	9	−0.0979	0	0.7691	0	0	0.2309	−0.1527	0.0000	−0.5081	−0.3151	0.0000	−0.1555
	10	−0.0264	0	0	0	0	1	−0.1474	−0.0720	−0.0554	−0.0570	0.0000	−0.0295
	11	−0.0345	0	0	0	0	1	−0.0606	−0.0522	−0.0708	−0.0119	0.0000	−0.0370
	12	0.0153	0	0	0	0	1	−0.1957	−0.1485	−0.1394	−0.0301	0.0000	0.0147
	13	−0.0053	0	0	0	0	1	−0.0778	−0.0620	−0.0274	−0.0867	0.0000	−0.0055

Table 4.1: Optimisation: Standard deviation as risk and bull and bear markets (Definition 14) as indicator; conservative investment

tables, is an indicator of comfort/satisfaction of an investor with the optimal portfolio. If it is recalled, from §2.3, that an investor is indifferent to all investments with the same comfort/satisfaction value, the objective value may be interpreted as that risk–free rate at which capital should be invested for the investor so as to be indifferent between the two choices. This provides a simple way of evaluating the attractiveness of the optimal portfolios across different time windows for the same risk profile. For instance, if the objective values associated with the optimal portfolios for bull markets and conservative investments in Table 4.1 are compared, the optimal portfolio of time window 13 with (0.1977 or 19.77%) rendered the most satisfaction, while conservative investors would have been the least satisfied with the portfolio of time window 11 (an objective value of 0.0471 or 4.71%).

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment													
Bull	1	0.1336	0	1	0	0	0	−0.106	0.000	−0.076	−0.087	−0.098	0.094
	2	0.1697	0	1	0	0	0	−0.044	0.000	−0.076	−0.090	−0.104	0.084
	3	0.0837	0	0.4866	0	0.5134	0	−0.006	0.000	−0.027	0.000	−0.066	0.070
	4	0.1648	0	1	0	0	0	−0.070	0.000	−0.062	−0.029	−0.045	0.095
	5	0.1126	1	0	0	0	0	0.000	−0.044	−0.030	−0.019	−0.159	0.050
	6	0.1913	0	1	0	0	0	−0.114	0.000	−0.098	−0.104	−0.155	0.159
	7	0.0501	0	0	1	0	0	−0.045	−0.068	0.000	−0.024	−0.001	0.048
	8	0.0788	0	0	1	0	0	−0.010	−0.020	0.000	−0.033	−0.080	0.065
	9	0.0834	0	0	1	0	0	−0.013	−0.077	0.000	−0.027	−0.088	0.077
	10	0.0969	0	0.5826	0.4174	0	0	−0.113	0.000	0.000	−0.050	−0.035	0.086
	11	0.0504	0	0	1	0	0	−0.003	−0.049	0.000	−0.024	−0.037	0.042
	12	0.0521	0	0	0	1	0	−0.001	−0.046	−0.018	0.000	−0.023	0.048
	13	0.2356	0	0	1	0	0	−0.119	−0.070	0.000	−0.052	−0.095	0.138
	14	0.1813	0	1	0	0	0	−0.054	0.000	−0.042	−0.052	−0.102	0.111
	15	0.1198	1	0	0	0	0	0.000	−0.008	−0.033	−0.001	−0.026	0.040
	16	0.1780	0	0	0.9365	0	0.0635	−0.043	−0.329	0.000	−0.645	0.000	0.028
	17	0.1376	0	1	0	0	0	−0.011	0.000	−0.053	−0.068	−0.075	0.092
Bear	1	−0.0272	0	0	0	0	1	−0.001	−0.032	−0.039	−0.031	0.000	−0.029
	2	0.0044	0	0	1	0	0	−0.064	−0.090	0.000	−0.007	−0.023	0.004
	3	−0.0242	0	0	0	0	1	−0.073	−0.143	−0.079	−0.063	0.000	−0.025
	4	0.0304	1	0	0	0	0	0.000	−0.074	−0.043	−0.043	−0.039	0.018
	5	−0.0220	0	1	0	0	0	−0.056	0.000	−0.032	−0.054	−0.054	−0.023
	6	0.0299	1	0	0	0	0	0.000	−0.062	−0.072	−0.066	−0.045	0.027
	7	−0.0416	0	0	0	1	0	−0.020	−0.029	−0.016	0.000	−0.031	−0.045
	8	−0.0252	0	0	0	0	1	−0.108	−0.218	−0.025	−0.086	0.000	−0.026
	9	−0.0787	0	0.9842	0	0	0.0158	−0.146	0.000	−0.428	−0.270	0.000	−0.139
	10	−0.0256	0	0	0	0	1	−0.143	−0.071	−0.052	−0.056	0.000	−0.028
	11	−0.0338	0	0	0	0	1	−0.060	−0.053	−0.070	−0.013	0.000	−0.036
	12	0.0155	0	0	0	0	1	−0.199	−0.151	−0.141	−0.031	0.000	0.015
	13	−0.0052	0	0	0	0	1	−0.077	−0.063	−0.028	−0.087	0.000	−0.005

Table 4.2: Optimisation: Standard deviation as risk and bull and bear markets (Definition 14) as indicator; moderate investment

If the objective value for a specific time window is lower than the general risk–free investment rate available during that period, it was, in hindsight not the best investment, as the investor could have invested in the risk–free asset (thus in an alternative with a higher comfort/satisfaction value). However, the nature of risky assets is such that it provides the investor with a chance of higher returns, without necessarily performing to expectation. For both types of optimisation problems and both Definitions of bear markets (see Tables 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7 and 4.8), the objective values are predominantly negative. If the objective values for a conservative, moderate and aggressive investment are compared for a time window where the optimal mix is exactly the same, (see, for example, time window 1 pertaining to a bull market in Table 4.1), the objective values will always be higher the more aggressive the investment (0.1181 vs. 0.1336 vs. 0.1468). This is a confirmation of the fact that the more aggressive the investor, the more comfortable he is with taking risk. The Lagrange multipliers (λ) given in the last column of Tables 4.1–4.8, as well as all other tables containing optimisation results in Appendix B, are associated with the constraint that 100% of the capital should be distributed at all times ($\sum_{i=1}^{n} w_i = 1$) and give an indication of the sensitivity of the optimal value U^* of the objective function U (the comfort/satisfaction function of an investor) to changes

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
							Aggressive Investment						
Bull	1	0.1468	0	1	0	0	0	−0.142	0.000	−0.106	−0.117	−0.126	0.120
	2	0.1983	0	1	0	0	0	−0.100	0.000	−0.117	−0.137	−0.173	0.141
	3	0.0886	0	0.3266	0	0.6734	0	−0.006	0.000	−0.031	0.000	−0.073	0.078
	4	0.1882	0	1	0	0	0	−0.085	0.000	−0.080	−0.041	−0.061	0.141
	5	0.1335	1	0	0	0	0	0.000	−0.075	−0.044	−0.034	−0.195	0.092
	6	0.2021	0	1	0	0	0	−0.137	0.000	−0.111	−0.118	−0.175	0.180
	7	0.0506	0	0	1	0	0	−0.047	−0.069	0.000	−0.025	−0.001	0.050
	8	0.0835	0	0	1	0	0	−0.011	−0.026	0.000	−0.036	−0.087	0.074
	9	0.0856	0	0	1	0	0	−0.013	−0.081	0.000	−0.028	−0.094	0.081
	10	0.1024	0	1	0	0	0	−0.115	0.000	0.000	−0.051	−0.036	0.087
	11	0.0532	0	0	1	0	0	−0.005	−0.056	0.000	−0.027	−0.044	0.048
	12	0.0535	0	0	0	1	0	−0.002	−0.048	−0.021	0.000	−0.025	0.051
	13	0.2680	0	0	1	0	0	−0.170	−0.125	0.000	−0.082	−0.139	0.203
	14	0.2049	0	1	0	0	0	−0.041	0.000	−0.083	−0.070	−0.156	0.158
	15	0.1464	1	0	0	0	0	0.000	−0.037	−0.061	−0.019	−0.058	0.093
	16	0.2355	0	0	1	0	0	−0.066	−0.367	0.000	−0.644	−0.108	0.118
	17	0.1526	0	1	0	0	0	−0.011	0.000	−0.070	−0.089	−0.108	0.123
Bear	1	−0.0268	0	0	0	0	1	−0.001	−0.032	−0.038	−0.030	0.000	−0.028
	2	0.0044	0	0	1	0	0	−0.064	−0.090	0.000	−0.007	−0.023	0.004
	3	−0.0240	0	0	0	0	1	−0.071	−0.139	−0.077	−0.061	0.000	−0.024
	4	0.0347	1	0	0	0	0	0.000	−0.085	−0.056	−0.056	−0.050	0.026
	5	−0.0216	0	1	0	0	0	−0.056	0.000	−0.033	−0.053	−0.053	−0.022
	6	0.0308	1	0	0	0	0	0.000	−0.065	−0.076	−0.069	−0.047	0.029
	7	−0.0405	0	0	0	1	0	−0.020	−0.029	−0.016	0.000	−0.031	−0.043
	8	−0.0248	0	0	0	0	1	−0.105	−0.213	−0.022	−0.083	0.000	−0.026
	9	−0.0581	0	1	0	0	0	−0.168	0.000	−0.443	−0.294	−0.032	−0.099
	10	−0.0248	0	0	0	0	1	−0.139	−0.071	−0.049	−0.055	0.000	−0.026
	11	−0.0332	0	0	0	0	1	−0.059	−0.053	−0.070	−0.014	0.000	−0.034
	12	0.0157	0	0	0	0	1	−0.202	−0.154	−0.143	−0.031	0.000	0.015
	13	−0.0051	0	0	0	0	1	−0.077	−0.064	−0.028	−0.088	0.000	−0.005

Table 4.3: Optimisation: Standard deviation as risk and bull and bear markets (Definition 14) as indicator; agressive investment

in the constraint's right–hand side value. If the right–hand side value of the constraint is increased by Δ, so that

$$\sum_{i=1}^{n} w_i = 1 + \Delta,$$

then the optimal value of the investor's comfort/satisfaction function will increase by $\lambda\Delta$.

Although the right–hand side value of this specific constraint is non–negotiable within the context of this study, an interpretation of the Lagrange multiplier for the various problems solved, may contain valuable information. For instance, if $\lambda > 0$ for a specific problem, the comfort/satisfaction of the investor would have increased if the investor were allowed to invest more than 100% (by borrowing for instance). Conversely, if $\lambda < 0$, the investor would have preferred to invest less than 100%, but as the unary constraint must be satisfied, the investor had to give up some of his satisfaction/comfort. The case where $\lambda = 0$ is more theoretical than practical, as it indicates that the optimal weight allocation by itself sums to unity, without being forced by the constraint to do so; this is highly unlikely to occur in practice. The size of λ gives an indication of how favourably an increase or decrease in the right–hand side of the constraint would be received by an investor.

Negative objective function values for time windows pertaining to bear markets in Tables 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7 and 4.8 mostly coincide with negative Lagrange multipliers. However, this trend is not only inherent to bear markets, as it is noticed throughout tables pertaining to the various shocks and across different risk–profiles. The occurrence of negative (or even low) objective function values coupled with negative Lagrange multipliers

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Bull	1	0.1490	0	1	0	0	0	−0.0602	0.0000	−0.0711	−0.0801	−0.1051	0.0856
	2	0.1375	0	1	0	0	0	−0.0526	0.0000	−0.0405	−0.0141	−0.0263	0.0399
	3	0.1373	0	0	1	0	0	−0.0091	−0.0889	0.0000	−0.0142	−0.0707	0.0891
	4	0.1613	0	1	0	0	0	−0.0734	0.0000	−0.0783	−0.0764	−0.1307	0.1342
	5	0.0728	0	0	0.7295	0	0.2705	−0.0887	−0.0415	0.0000	−0.0297	0.0000	0.0033
	6	0.1252	0.1217	0.3934	0.4849	0	0	0.0000	0.0000	0.0000	−0.0025	−0.0445	0.0400
	7	0.1356	0	0.2570	0.0391	0.7039	0	−0.1168	0.0000	0.0000	0.0000	−0.0112	0.0413
	8	0.0691	0	0	0	1	0	−0.0234	−0.0402	−0.0054	0.0000	−0.0130	0.0057
	9	0.0935	0.5351	0.0871	0	0.3778	0	0.0000	0.0000	−0.0202	0.0000	−0.0191	0.0164
	10	0.1530	1	0	0	0	0	0.0000	−0.0457	−0.0605	−0.0144	−0.1148	0.1063
Bear	1	−0.0245	0	0	0	0	1	−0.0759	−0.1465	−0.0816	−0.0647	0.0000	−0.0254
	2	−0.0356	0	0	0.4354	0	0.5646	−0.1068	−0.2437	0.0000	−0.0861	0.0000	−0.0428
	3	−0.1164	0	0	0	0	1	−0.3475	−0.1555	−0.0257	−0.1008	0.0000	−0.1435
	4	−0.2142	0	0.5922	0	0	0.4078	−0.2451	0.0000	−0.6418	−0.4112	0.0000	−0.2852
	5	−0.0633	0	1	0	0	0	−0.2227	0.0000	−0.2068	−0.1001	−0.0166	−0.0826
	6	0.1774	0	1	0	0	0	−0.0422	0.0000	−0.0897	−0.0739	−0.0530	0.0652
	7	0.0148	0	0	0	0	1	−0.0892	−0.1466	−0.0450	−0.1728	0.0000	0.0122
Moderate Investment													
Bull	1	0.1668	0	1	0	0	0	−0.0998	0.0000	−0.0978	−0.1080	−0.1465	0.1211
	2	0.1648	0	1	0	0	0	−0.0700	0.0000	−0.0615	−0.0286	−0.0451	0.0945
	3	0.1507	0	0	1	0	0	−0.0251	−0.1043	0.0000	−0.0129	−0.0884	0.1161
	4	0.1689	0	1	0	0	0	−0.0824	0.0000	−0.0855	−0.0830	−0.1442	0.1494
	5	0.0981	0	0	1	0	0	−0.0972	−0.0460	0.0000	−0.0352	−0.0136	0.0255
	6	0.1562	0.0339	0.8611	0.1050	0	0	0.0000	0.0000	0.0000	−0.0051	−0.0541	0.0508
	7	0.1717	0	0.0637	0.2438	0.6925	0	−0.1204	0.0000	0.0000	0.0000	−0.0120	0.0441
	8	0.0906	0	0	0.4778	0.5222	0	−0.0310	−0.0467	0.0000	0.0000	−0.0166	0.0156
	9	0.1198	1	0	0	0	0	0.0000	−0.0083	−0.0326	−0.0014	−0.0262	0.0400
	10	0.1660	1	0	0	0	0	0.0000	−0.0453	−0.0510	−0.0119	−0.1429	0.1324
Bear	1	−0.0242	0	0	0	0	1	−0.0731	−0.1427	−0.0791	−0.0625	0.0000	−0.0249
	2	−0.0331	0	0	0.6283	0	0.3717	−0.1033	−0.2355	0.0000	−0.0830	0.0000	−0.0413
	3	−0.1088	0	0	0	0	1	−0.3062	−0.1379	−0.0198	−0.0879	0.0000	−0.1284
	4	−0.1939	0	0.6918	0	0	0.3082	−0.2055	0.0000	−0.5278	−0.3448	0.0000	−0.2479
	5	−0.0579	0	1	0	0	0	−0.2037	0.0000	−0.1942	−0.0949	−0.0204	−0.0718
	6	0.2089	0.04024	0.9598	0	0	0	0.0000	0.0000	−0.1276	−0.1290	−0.1005	0.1233
	7	0.0156	0	0	0	0	1	−0.0880	−0.1432	−0.0449	−0.1702	0.0000	0.0137
Aggressive Investment													
Bull	1	0.1820	0	1	0	0	0	−0.1339	0.0000	−0.1207	−0.1319	−0.1819	0.1516
	2	0.1882	0	1	0	0	0	−0.0849	0.0000	−0.0795	−0.0410	−0.0612	0.1414
	3	0.1623	0	0	1	0	0	−0.0388	−0.1175	0.0000	−0.0117	−0.1035	0.1392
	4	0.1754	0	1	0	0	0	−0.0901	0.0000	−0.0916	−0.0886	−0.1557	0.1624
	5	0.1223	0	0	1	0	0	−0.1183	−0.0597	0.0000	−0.0487	−0.0440	0.0739
	6	0.1958	0.0904	0.9096	0	0	0	0.0000	0.0000	−0.0377	−0.0273	−0.1212	0.1151
	7	0.2331	0	0	0.9831	0.0169	0	−0.1293	−0.0120	0.0000	0.0000	−0.0145	0.0556
	8	0.1246	0	0	1	0	0	−0.0586	−0.0731	0.0000	−0.0118	−0.0344	0.0476
	9	0.1464	1	0	0	0	0	0.0000	−0.0370	−0.0613	−0.0192	−0.0576	0.0932
	10	0.1772	1	0	0	0	0	0.0000	−0.0450	−0.0428	−0.0097	−0.1670	0.1548
Bear	1	−0.0240	0	0	0	0	1	−0.0708	−0.1393	−0.0769	−0.0607	0.0000	−0.0244
	2	−0.0294	0	0	0.9728	0	0.0272	−0.1003	−0.2285	0.0000	−0.0804	0.0000	−0.0399
	3	−0.1023	0	0	0	0	1	−0.2709	−0.1229	−0.0148	−0.0768	0.0000	−0.1153
	4	−0.1748	0	0.8696	0	0	0.1304	−0.1715	0.0000	−0.4301	−0.2878	0.0000	−0.2160
	5	−0.0532	0	1	0	0	0	−0.1874	0.0000	−0.1834	−0.0905	−0.0237	−0.0625
	6	0.2507	0.6911	0.3089	0	0	0	0.0000	0.0000	−0.1300	−0.1336	−0.1031	0.1274
	7	0.0162	0	0	0	0	1	−0.0869	−0.1403	−0.0448	−0.1679	0.0000	0.0149

Table 4.4: Optimisation: Standard deviation as risk and bull and bear markets (Definition 15) as indicator

makes a case for *flexible investment mandates* and/or *short selling*. Investment products with flexible mandates allow managers to move from nearly fully invested in risky assets to nearly fully invested in risk–free cash in times when risky assets are expected to underperform cash investments. The question is whether investment professionals can predict when such periods will occur and act in time to add value to their clients' investments. Short selling is a strategy of borrowing shares from large institutions, selling these shares in anticipation of a fall in their value and buying them back later at lower prices in order to return the borrowed shares to the institution who actually owns them. This way the investment is still fully invested in risky assets, but it will have a positive return if the value of the risky assets fall. This strategy is not yet allowed in the management of pension funds or unit trusts in South Africa.

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Bull	1	0.1446	0	1	0	0	0	−0.1372	0.0000	−0.1010	−0.1130	−0.1215	0.1159
	2	0.1895	0	1	0	0	0	−0.0876	0.0000	−0.1033	−0.1191	−0.1544	0.1234
	3	0.0929	0	0	0	1	0	−0.0050	−0.0063	−0.0320	0.0000	−0.0785	0.0841
	4	0.1830	0	1	0	0	0	−0.0695	0.0000	−0.0685	−0.0293	−0.0520	0.1309
	5	0.1389	1	0	0	0	0	0.0000	−0.0726	−0.0442	−0.0323	−0.2019	0.1026
	6	0.2029	0	1	0	0	0	−0.1453	0.0000	−0.1144	−0.1237	−0.1774	0.1819
	7	0.0506	0	0	1	0	0	−0.0465	−0.0689	0.0000	−0.0245	−0.0009	0.0493
	8	0.0852	0	0	1	0	0	−0.0129	−0.0321	0.0000	−0.0389	−0.0891	0.0775
	9	0.0868	0	0	1	0	0	−0.0132	−0.0828	0.0000	−0.0281	−0.0958	0.0835
	10	0.1008	0	0.9046	0.0954	0	0	−0.1143	0.0000	0.0000	−0.0522	−0.0359	0.0857
	11	0.0536	0	0	1	0	0	−0.0073	−0.0571	0.0000	−0.0273	−0.0444	0.0484
	12	0.0532	0.0517	0	0	0.9483	0	0.0000	−0.0464	−0.0199	0.0000	−0.0247	0.0503
	13	0.2818	0	0	1	0	0	−0.2257	−0.1871	0.0000	−0.1041	−0.1616	0.2309
	14	0.1980	0	1	0	0	0	−0.0292	0.0000	−0.0738	−0.0681	−0.1412	0.1439
	15	0.1300	1	0	0	0	0	0.0000	−0.0212	−0.0461	−0.0050	−0.0332	0.0603
	16	0.2554	0	0	1	0	0	−0.0875	−0.4121	0.0000	−0.6575	−0.1458	0.1580
	17	0.1439	0	1	0	0	0	−0.0027	0.0000	−0.0586	−0.0749	−0.0904	0.1050
Bear	1	−0.0268	0	0	0	0	1	−0.0010	−0.0320	−0.0379	−0.0298	0.0000	−0.0277
	2	0.0044	0	0	1	0	0	−0.0639	−0.0902	0.0000	−0.0071	−0.0227	0.0044
	3	−0.0241	0	0	0	0	1	−0.0712	−0.1407	−0.0775	−0.0613	0.0000	−0.0246
	4	0.0360	1	0	0	0	0	0.0000	−0.0880	−0.0581	−0.0578	−0.0523	0.0287
	5	−0.0218	0	1	0	0	0	−0.0542	0.0000	−0.0323	−0.0525	−0.0534	−0.0230
	6	0.0303	1	0	0	0	0	0.0000	−0.0629	−0.0746	−0.0679	−0.0457	0.0281
	7	−0.0407	0	0	0	1	0	−0.0202	−0.0298	−0.0160	0.0000	−0.0322	−0.0433
	8	−0.0246	0	0	0	0	1	−0.1025	−0.2094	−0.0203	−0.0803	0.0000	−0.0251
	9	−0.0708	0	1	0	0	0	−0.1390	0.0000	−0.4104	−0.2675	−0.0035	−0.1248
	10	−0.0253	0	0	0	0	1	−0.1399	−0.0739	−0.0494	−0.0546	0.0000	−0.0272
	11	−0.0332	0	0	0	0	1	−0.0573	−0.0509	−0.0686	−0.0138	0.0000	−0.0345
	12	0.0158	0	0	0	0	1	−0.2029	−0.1546	−0.1433	−0.0314	0.0000	0.0156
	13	−0.0051	0	0	0	0	1	−0.0772	−0.0639	−0.0279	−0.0877	0.0000	−0.0052

Table 4.5: Optimisation: Downside risk, bull and bear markets (Definition 14) as indicator and a conservative investment

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment													
Bull	1	0.1526	0	1	0	0	0	−0.1591	0.0000	−0.1189	−0.1308	−0.1386	0.1320
	2	0.2080	0	1	0	0	0	−0.1221	0.0000	−0.1305	−0.1501	−0.1984	0.1604
	3	0.0954	0	0	0	1	0	−0.0047	−0.0068	−0.0342	0.0000	−0.0828	0.0890
	4	0.1976	0	1	0	0	0	−0.0822	0.0000	−0.0817	−0.0395	−0.0636	0.1601
	5	0.1491	1	0	0	0	0	0.0000	−0.0904	−0.0517	−0.0410	−0.2204	0.1229
	6	0.2087	0	1	0	0	0	−0.1555	0.0000	−0.1202	−0.1297	−0.1878	0.1937
	7	0.0509	0	0	1	0	0	−0.0473	−0.0695	0.0000	−0.0248	−0.0006	0.0500
	8	0.0873	0	0	1	0	0	−0.0127	−0.0339	0.0000	−0.0399	−0.0924	0.0818
	9	0.0877	0	0	1	0	0	−0.0131	−0.0845	0.0000	−0.0281	−0.0983	0.0853
	10	0.1056	0	1	0	0	0	−0.1245	0.0000	−0.0055	−0.0580	−0.0422	0.0930
	11	0.0551	0	0	1	0	0	−0.0075	−0.0607	0.0000	−0.0284	−0.0477	0.0513
	12	0.0540	0	0	0	1	0	−0.0007	−0.0480	−0.0213	0.0000	−0.0255	0.0518
	13	0.2961	0	0	1	0	0	−0.2385	−0.2003	0.0000	−0.1144	−0.1800	0.2594
	14	0.2131	0	1	0	0	0	−0.0248	0.0000	−0.0991	−0.0788	−0.1753	0.1742
	15	0.1495	1	0	0	0	0	0.0000	−0.0417	−0.0664	−0.0190	−0.0576	0.0993
	16	0.2826	0	0	1	0	0	−0.0980	−0.4274	0.0000	−0.6532	−0.2144	0.2125
	17	0.1547	0	1	0	0	0	−0.0051	0.0000	−0.0713	−0.0906	−0.1138	0.1268
Bear	1	−0.0266	0	0	0	0	1	−0.0013	−0.0317	−0.0371	−0.0295	0.0000	−0.0272
	2	0.0044	0	0	1	0	0	−0.0638	−0.0901	0.0000	−0.0071	−0.0227	0.0044
	3	−0.0240	0	0	0	0	1	−0.0697	−0.1384	−0.0761	−0.0601	0.0000	−0.0243
	4	0.0380	1	0	0	0	0	0.0000	−0.0939	−0.0645	−0.0644	−0.0580	0.0328
	5	−0.0215	0	1	0	0	0	−0.0542	0.0000	−0.0329	−0.0525	−0.0525	−0.0223
	6	0.0309	1	0	0	0	0	0.0000	−0.0646	−0.0771	−0.0702	−0.0471	0.0293
	7	−0.0400	0	0	0	1	0	−0.0197	−0.0296	−0.0156	0.0000	−0.0319	−0.0419
	8	−0.0245	0	0	0	0	1	−0.1014	−0.2073	−0.0197	−0.0794	0.0000	−0.0248
	9	−0.0557	0	1	0	0	0	−0.1609	0.0000	−0.4313	−0.2904	−0.0309	−0.0945
	10	−0.0247	0	0	0	0	1	−0.1376	−0.0727	−0.0478	−0.0542	0.0000	−0.0261
	11	−0.0329	0	0	0	0	1	−0.0575	−0.0517	−0.0688	−0.0143	0.0000	−0.0338
	12	0.0158	0	0	0	0	1	−0.2045	−0.1557	−0.1440	−0.0316	0.0000	0.0157
	13	−0.0051	0	0	0	0	1	−0.0769	−0.0646	−0.0281	−0.0879	0.0000	−0.0052

Table 4.6: Optimisation: Downside risk, bull and bear markets (Definition 14) as indicator and a moderate investment

Throughout the solutions of different time windows, it is noticed that, for low levels of

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
							Aggressive Investment						
Bull	1	0.1595	0	1	0	0	0	–0.1780	0.0000	–0.1342	–0.1461	–0.1533	0.1457
	2	0.2239	0	1	0	0	0	–0.1516	0.0000	–0.1538	–0.1766	–0.2362	0.1922
	3	0.0975	0	0	0	1	0	–0.0045	–0.0072	–0.0362	0.0000	–0.0865	0.0933
	4	0.2101	0	1	0	0	0	–0.0930	0.0000	–0.0930	–0.0482	–0.0736	0.1851
	5	0.1578	1	0	0	0	0	0.0000	–0.1056	–0.0582	–0.0484	–0.2363	0.1404
	6	0.2138	0	1	0	0	0	–0.1643	0.0000	–0.1253	–0.1349	–0.1968	0.2037
	7	0.0512	0	0	1	0	0	–0.0479	–0.0700	0.0000	–0.0250	–0.0003	0.0506
	8	0.0891	0	0	1	0	0	–0.0125	–0.0354	0.0000	–0.0408	–0.0953	0.0855
	9	0.0885	0	0	1	0	0	–0.0130	–0.0859	0.0000	–0.0281	–0.1003	0.0869
	10	0.1099	0	1	0	0	0	–0.1362	0.0000	–0.0118	–0.0648	–0.0494	0.1014
	11	0.0563	0	0	1	0	0	–0.0077	–0.0638	0.0000	–0.0294	–0.0505	0.0538
	12	0.0547	0	0	0	1	0	–0.0015	–0.0497	–0.0228	0.0000	–0.0261	0.0533
	13	0.3083	0	0	1	0	0	–0.2495	–0.2116	0.0000	–0.1233	–0.1957	0.2839
	14	0.2261	0	1	0	0	0	–0.0210	0.0000	–0.1208	–0.0880	–0.2046	0.2001
	15	0.1662	1	0	0	0	0	0.0000	–0.0592	–0.0839	–0.0310	–0.0786	0.1327
	16	0.3060	0	0	1	0	0	–0.1069	–0.4405	0.0000	–0.6495	–0.2731	0.2592
	17	0.1641	0	1	0	0	0	–0.0071	0.0000	–0.0821	–0.1041	–0.1339	0.1454
Bear	1	–0.0263	0	0	0	0	1	–0.0016	–0.0315	–0.0363	–0.0292	0.0000	–0.0268
	2	0.0044	0	0	1	0	0	–0.0638	–0.0901	0.0000	–0.0071	–0.0227	0.0044
	3	–0.0238	0	0	0	0	1	–0.0685	–0.1365	–0.0749	–0.0591	0.0000	–0.0241
	4	0.0398	1	0	0	0	0	0.0000	–0.0990	–0.0700	–0.0700	–0.0628	0.0363
	5	–0.0212	0	1	0	0	0	–0.0543	0.0000	–0.0334	–0.0524	–0.0517	–0.0218
	6	0.0314	1	0	0	0	0	0.0000	–0.0660	–0.0794	–0.0722	–0.0483	0.0304
	7	–0.0394	0	0	0	1	0	–0.0194	–0.0295	–0.0152	0.0000	–0.0316	–0.0406
	8	–0.0244	0	0	0	0	1	–0.1003	–0.2056	–0.0192	–0.0786	0.0000	–0.0246
	9	–0.0427	0	1	0	0	0	–0.1797	0.0000	–0.4492	–0.3099	–0.0544	–0.0686
	10	–0.0243	0	0	0	0	1	–0.1356	–0.0717	–0.0466	–0.0539	0.0000	–0.0252
	11	–0.0326	0	0	0	0	1	–0.0578	–0.0525	–0.0690	–0.0147	0.0000	–0.0332
	12	0.0159	0	0	0	0	1	–0.2058	–0.1567	–0.1446	–0.0317	0.0000	0.0158
	13	–0.0050	0	0	0	0	1	–0.0767	–0.0651	–0.0282	–0.0881	0.0000	–0.0051

Table 4.7: Optimisation: Downside risk, bull and bear markets (Definition 14) as indicator and an aggressive investment

risk–aversion (in other words a low value for a in the objective function and an aggressive investment), the optimal portfolios tend to consist of only one or two sectors, and those with the highest returns. The reason for this may be found in the fact that high risk is not penalised extensively in the objective function. Therefore one can afford to include the higher–returning sectors in the solution without having to pay too much attention to the associated higher risk. The conservative investments usually have a larger mix of different sectors. It is also noticed that within a set of time windows, the optimal portfolios may differ extensively from one period to the next.

A very good example of how the risk–return characteristics of time windows within the same set differ may be seen in the optimal portfolios pertaining to upward shocks in the change in US consumer sentiment, given in Table 4.9 and Figures 4.2(a)–(c). If attention is focused on the optimal allocations of time windows 3 and 4 for a conservative investment, it seems that there is a correspondence in the risk–return attributes of the time windows, with a very similar optimal mix between Non–Mining Resources (CI10) and Industrial (CI27) shares. However, in time windows 3 and 4 of the moderate investment in Figure 4.2(b), the picture changes and the same conclusion cannot necessarily be reached. In time window 3 the exposure to Non–Mining Resources is reduced in favour of the Industrial shares, while in time window 4 the exposure to Non–Mining Resources is increased to 100%. In the optimal aggressive investment in Figure 4.2(c), this trend of reducing the exposure to Non–Mining Resources to ensure optimality of the investment during the period pertaining to time window 3 is continued. From this it can be showed that the exposure to Non–Mining Resources in time window 3 serves to reduce the total risk of the optimal portfolio and the more aggressive the investment becomes, the less need there is

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Bull	1	0.1821	0	1	0	0	0	−0.1353	0.0000	−0.1208	−0.1306	−0.1815	0.1516
	2	0.1830	0	1	0	0	0	−0.0695	0.0000	−0.0685	−0.0293	−0.0520	0.1309
	3	0.1614	0	0	1	0	0	−0.0482	−0.1174	0.0000	−0.0157	−0.1034	0.1374
	4	0.1755	0	1	0	0	0	−0.0996	0.0000	−0.0933	−0.0904	−0.1569	0.1626
	5	0.1138	0	0	1	0	0	−0.1184	−0.0792	0.0000	−0.0511	−0.0367	0.0569
	6	0.2130	1	0	0	0	0	0.0000	−0.0072	−0.0621	−0.0435	−0.1452	0.1419
	7	0.2166	0	0	0.5698	0.4302	0	−0.1494	−0.0365	0.0000	0.0000	−0.0054	0.0771
	8	0.1356	0	0	1	0	0	−0.0745	−0.0818	0.0000	−0.0175	−0.0637	0.0695
	9	0.1300	1	0	0	0	0	0.0000	−0.0212	−0.0461	−0.0050	−0.0332	0.0603
	10	0.1746	1	0	0	0	0	0.0000	−0.0449	−0.0420	−0.0099	−0.1602	0.1497
Bear	1	−0.0241	0	0	0	0	1	−0.0712	−0.1407	−0.0775	−0.0613	0.0000	−0.0246
	2	−0.0264	0	0	1	0	0	−0.0900	−0.2066	0.0000	−0.0697	−0.0015	−0.0344
	3	−0.1013	0	0	0	0	1	−0.2659	−0.1194	−0.0176	−0.0769	0.0000	−0.1133
	4	−0.1771	0	0.5581	0	0	0.4419	−0.1498	0.0000	−0.4593	−0.3067	0.0000	−0.2097
	5	−0.0556	0	1	0	0	0	−0.1902	0.0000	−0.1885	−0.0924	−0.0203	−0.0672
	6	0.2135	0.0897	0.9103	0	0	0	0.0000	0.0000	−0.1235	−0.1246	−0.0995	0.1265
	7	0.0159	0	0	0	0	1	−0.0913	−0.1438	−0.0453	−0.1712	0.0000	0.0144
Moderate Investment													
Bull	1	0.1906	0	1	0	0	0	−0.1539	0.0000	−0.1336	−0.1444	−0.2015	0.1687
	2	0.1976	0	1	0	0	0	−0.0822	0.0000	−0.0817	−0.0395	−0.0636	0.1601
	3	0.1681	0	0	1	0	0	−0.0532	−0.1248	0.0000	−0.0139	−0.1119	0.1509
	4	0.1791	0	1	0	0	0	−0.1012	0.0000	−0.0963	−0.0931	−0.1630	0.1698
	5	0.1298	0	0	1	0	0	−0.1301	−0.0814	0.0000	−0.0580	−0.0558	0.0888
	6	0.2329	1	0	0	0	0	0.0000	−0.0075	−0.0854	−0.0572	−0.1883	0.1817
	7	0.2671	0	0	1	0	0	−0.1713	−0.0777	0.0000	−0.0162	−0.0411	0.1214
	8	0.1541	0	0	1	0	0	−0.1075	−0.1157	0.0000	−0.0320	−0.0787	0.1065
	9	0.1495	1	0	0	0	0	0.0000	−0.0417	−0.0664	−0.0190	−0.0576	0.0993
	10	0.1816	1	0	0	0	0	0.0000	−0.0447	−0.0376	−0.0086	−0.1756	0.1637
Bear	1	−0.0240	0	0	0	0	1	−0.0697	−0.1384	−0.0761	−0.0601	0.0000	−0.0243
	2	−0.0242	0	0	1	0	0	−0.0927	−0.2100	0.0000	−0.0725	−0.0061	−0.0299
	3	−0.0980	0	0	0	0	1	−0.2475	−0.1119	−0.0140	−0.0707	0.0000	−0.1066
	4	−0.1673	0	0.6816	0	0	0.3184	−0.1385	0.0000	−0.4006	−0.2720	0.0000	−0.1945
	5	−0.0523	0	1	0	0	0	−0.1803	0.0000	−0.1810	−0.0894	−0.0231	−0.0607
	6	0.2474	0.6444	0.3556	0	0	0	0.0000	0.0000	−0.1237	−0.1249	−0.0996	0.1265
	7	0.0164	0	0	0	0	1	−0.0895	−0.1411	−0.0451	−0.1690	0.0000	0.0153
Aggressive Investment													
Bull	1	0.1979	0	1	0	0	0	−0.1699	0.0000	−0.1445	−0.1561	−0.2185	0.1833
	2	0.2101	0	1	0	0	0	−0.0930	0.0000	−0.0930	−0.0482	−0.0736	0.1851
	3	0.1739	0	0	1	0	0	−0.0575	−0.1311	0.0000	−0.0124	−0.1192	0.1624
	4	0.1822	0	1	0	0	0	−0.1027	0.0000	−0.0988	−0.0953	−0.1683	0.1760
	5	0.1434	0	0	1	0	0	−0.1402	−0.0833	0.0000	−0.0639	−0.0722	0.1161
	6	0.2499	1	0	0	0	0	0.0000	−0.0078	−0.1053	−0.0690	−0.2252	0.2158
	7	0.3157	0	0	1	0	0	−0.2211	−0.1689	0.0000	−0.0547	−0.1231	0.2185
	8	0.1699	0	0	1	0	0	−0.1358	−0.1447	0.0000	−0.0444	−0.0915	0.1382
	9	0.1662	1	0	0	0	0	0.0000	−0.0592	−0.0839	−0.0310	−0.0786	0.1327
	10	0.1876	1	0	0	0	0	0.0000	−0.0446	−0.0339	−0.0076	−0.1888	0.1756
Bear	1	−0.0238	0	0	0	0	1	−0.0685	−0.1365	−0.0749	−0.0591	0.0000	−0.0241
	2	−0.0222	0	0	1	0	0	−0.0950	−0.2130	0.0000	−0.0749	−0.0100	−0.0261
	3	−0.0951	0	0	0	0	1	−0.2317	−0.1055	−0.0109	−0.0653	0.0000	−0.1008
	4	−0.1570	0	0.9021	0	0	0.0979	−0.1289	0.0000	−0.3503	−0.2423	0.0000	−0.1814
	5	−0.0495	0	1	0	0	0	−0.1718	0.0000	−0.1746	−0.0868	−0.0255	−0.0551
	6	0.2995	1	0	0	0	0	0.0000	−0.0261	−0.1739	−0.1856	−0.1550	0.1874
	7	0.0167	0	0	0	0	1	−0.0879	−0.1389	−0.0449	−0.1672	0.0000	0.0160

Table 4.8: Optimisation: Downside risk and bull and bear markets (Definition 15) as indicator

for this sector to be included. However, in time window 4 the Industrial exposure serves this purpose. Therefore there is a definite difference in the risk–return characteristics of the two time windows, although it was not necessarily clear from evaluating the optimal allocations for a conservative investment only. In some cases, like in time windows 1, 2, 6 and 7 for all risk profiles in Figure 4.2 it is much easier to notice a difference in risk–return characteristics, because of the obvious difference in optimal compositions.

Another important and insightful aspect that comes to light in Figure 4.2(c) is that of diversification and the possible effect thereof on an optimal portfolio. The fact that both the Non–Mining Resources and Industrial sectors are included in even an aggressive investment (time window 3 of Figure 4.2), where little emphasis is placed on risk, is a good example of where the effect of risk diversification is so great that it even outplays the less–penalised effect of higher returns. This observation may be explained by considering

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Upward	1	0.0835	0	1	0	0	0	−0.0867	0.0000	−0.0952	−0.1063	−0.1001	0.0760
	2	−0.0246	0	0	0	0	1	−0.0719	−0.0659	−0.0424	−0.0331	0.0000	−0.0272
	3	0.0650	0	0.6741	0	0.3259	0	−0.0062	0.0000	−0.0280	0.0000	−0.0604	0.0594
	4	0.0854	0	0.6780	0	0.3220	0	−0.0283	0.0000	−0.0276	0.0000	−0.0149	0.0477
	5	0.1112	0	1	0	0	0	−0.0140	0.0000	−0.0791	−0.0679	−0.0893	0.1012
	6	0.0811	0	0	1	0	0	−0.0580	−0.0892	0.0000	−0.0201	−0.0923	0.0666
	7	0.0029	0	0	0	1	0	−0.0388	−0.0270	−0.0232	0.0000	−0.0178	0.0026
	8	−0.0122	0	0	0	0	1	−0.0304	−0.0149	−0.0101	−0.0219	0.0000	−0.0130
	9	−0.0068	0	0	0	0	1	−0.0414	−0.0211	−0.0314	−0.0348	0.0000	−0.0069
	10	0.0423	0	0	1	0	0	−0.0490	−0.0505	0.0000	−0.0495	−0.0761	0.0398
	11	0.1292	0	0.3150	0.6850	0	0	−0.0173	0.0000	0.0000	−0.0309	−0.0799	0.1206
	12	0.0085	0	0.6547	0	0.2555	0.0898	−0.1161	0.0000	−0.0106	0.0000	0.0000	0.0074
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0336	0.0000	−0.0403	−0.0590	0.0313
	2	0.0607	0	1	0	0	0	−0.0380	0.0000	−0.0257	−0.0388	−0.0247	0.0245
	3	0.1439	0	1	0	0	0	−0.0498	0.0000	−0.0595	−0.0750	−0.0993	0.0797
	4	0.0144	0	1	0	0	0	−0.0555	0.0000	−0.0079	−0.0264	−0.0311	0.0120
	5	0.1613	0	1	0	0	0	−0.0734	0.0000	−0.0783	−0.0764	−0.1307	0.1342
	6	0.0469	0	0.8155	0.1124	0.0721	0	−0.0730	0.0000	0.0000	0.0000	−0.0154	0.0342
Moderate Investment													
Upward	1	0.0856	0	1	0	0	0	−0.0881	0.0000	−0.0998	−0.1125	−0.1062	0.0802
	2	−0.0239	0	0	0	0	1	−0.0694	−0.0650	−0.0410	−0.0321	0.0000	−0.0257
	3	0.0670	0	0.5038	0	0.4962	0	−0.0060	0.0000	−0.0288	0.0000	−0.0611	0.0602
	4	0.0985	0	1	0	0	0	−0.0330	0.0000	−0.0332	−0.0044	−0.0205	0.0607
	5	0.1140	0	1	0	0	0	−0.0138	0.0000	−0.0841	−0.0717	−0.0932	0.1068
	6	0.0852	0	0	1	0	0	−0.0639	−0.1024	0.0000	−0.0231	−0.1032	0.0747
	7	0.0030	0	0	0	1	0	−0.0392	−0.0271	−0.0235	0.0000	−0.0182	0.0028
	8	−0.0120	0	0	0	0	1	−0.0305	−0.0136	−0.0104	−0.0220	0.0000	−0.0125
	9	−0.0068	0	0	0	0	1	−0.0412	−0.0211	−0.0313	−0.0346	0.0000	−0.0069
	10	0.0429	0	0	1	0	0	−0.0527	−0.0521	0.0000	−0.0517	−0.0783	0.0412
	11	0.1324	0	0.1612	0.8388	0	0	−0.0174	0.0000	0.0000	−0.0311	−0.0804	0.1211
	12	0.0089	0	0.5563	0	0.3144	0.1293	−0.1152	0.0000	−0.0110	0.0000	0.0000	0.0075
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0337	0.0000	−0.0403	−0.0589	0.0314
	2	0.0709	0	1	0	0	0	−0.0644	0.0000	−0.0408	−0.0543	−0.0455	0.0448
	3	0.1619	0	1	0	0	0	−0.0886	0.0000	−0.0846	−0.1018	−0.1415	0.1157
	4	0.0151	0	1	0	0	0	−0.0570	0.0000	−0.0090	−0.0277	−0.0326	0.0133
	5	0.1689	0	1	0	0	0	−0.0824	0.0000	−0.0855	−0.0830	−0.1442	0.1494
	6	0.0517	0	0.7763	0.2237	0	0	−0.0724	0.0000	0.0000	−0.0012	−0.0172	0.0351
Aggressive Investment													
Upward	1	0.0874	0	1	0	0	0	−0.0893	0.0000	−0.1037	−0.1177	−0.1114	0.0838
	2	−0.0233	0	0	0	0	1	−0.0672	−0.0643	−0.0399	−0.0312	0.0000	−0.0245
	3	0.0703	0	0.1996	0	0.8004	0	−0.0059	0.0000	−0.0294	0.0000	−0.0617	0.0609
	4	0.1111	0	1	0	0	0	−0.0481	0.0000	−0.0473	−0.0166	−0.0335	0.0859
	5	0.1164	0	1	0	0	0	−0.0136	0.0000	−0.0884	−0.0751	−0.0966	0.1116
	6	0.0886	0	0	1	0	0	−0.0689	−0.1138	0.0000	−0.0256	−0.1126	0.0817
	7	0.0030	0	0	0	1	0	−0.0395	−0.0272	−0.0238	0.0000	−0.0185	0.0029
	8	−0.0118	0	0	0	0	1	−0.0306	−0.0126	−0.0106	−0.0222	0.0000	−0.0122
	9	−0.0067	0	0	0	0	1	−0.0410	−0.0210	−0.0312	−0.0344	0.0000	−0.0068
	10	0.0435	0	0	1	0	0	−0.0558	−0.0536	0.0000	−0.0537	−0.0802	0.0424
	11	0.1377	0	0	1	0	0	−0.0210	−0.0053	0.0000	−0.0335	−0.0837	0.1254
	12	0.0096	0	0.3807	0	0.4194	0.1999	−0.1145	0.0000	−0.0113	0.0000	0.0000	0.0075
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0589	0.0314
	2	0.0796	0	1	0	0	0	−0.0870	0.0000	−0.0537	−0.0675	−0.0633	0.0622
	3	0.1773	0	1	0	0	0	−0.1219	0.0000	−0.1062	−0.1249	−0.1777	0.1465
	4	0.0156	0	1	0	0	0	−0.0583	0.0000	−0.0099	−0.0287	−0.0339	0.0145
	5	0.1754	0	1	0	0	0	−0.0901	0.0000	−0.0916	−0.0886	−0.1557	0.1624
	6	0.0597	0	0.6444	0.3556	0	0	−0.0719	0.0000	0.0000	−0.0031	−0.0195	0.0364

Table 4.9: Optimisation: Standard deviation as risk and change in US Consumer Sentiment as indicator

the objective function for optimisation. The return of the optimal portfolio is a linear combination of the returns of all the sectors in the solution. Because of this linearity, the combined portfolio's return will never exceed that of the highest returning sector. For the objective value to give up an 100% allocation to the highest returning sector in order to include another sector, it must be compensated by a resulting reduction in risk by means of diversification. Furthermore, as the risk contribution in the objective function is penalised by the risk tolerance factor, the contribution of diversification must be even higher to justify the reduction in total return to include an additional sector in the optimal solution.

Finally, a question about the resemblance of the solutions obtained from solving the

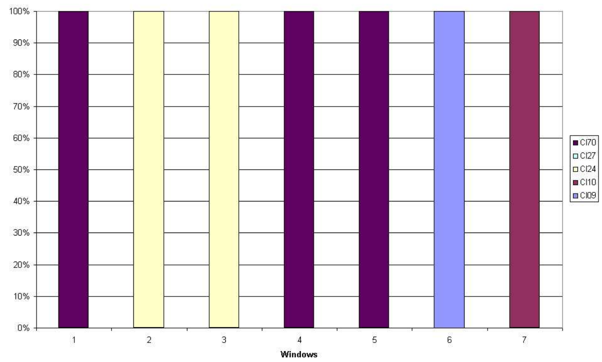


Figure 4.1: Optimisation: Downward shock in change in SA business confidence (for all risk profiles and both optimisation problems)

variance– and semi–variance based optimisation problems for the same set of time windows is raised. This question is answered by referring to two specific cases. The optimal conservative portfolios in times of upward shocks in US consumer sentiment in Table 4.10 (where variance is the assumed measure of risk) differ in amongst others, time windows 7, 10, 13, 23, 27, 29, 38, 39 and 40 from the portfolios in the same time windows of Table 4.11 (where semi–variance is the assumed measure of risk) in that the first optimal portfolio is exposed to two sectors, and the second only to one. The fact that more than one sector is included in the optimal portfolios of the identified time windows when variance is used as measure of risk, points to a negative correlation between the two sectors to the extent that the reduction of risk increases comfort/satisfaction.

When semi–variance is used as measure of risk, the optimal portfolios are reduced to include only one sector. One of the following reasons may prove true – either the **downside returns** of the sectors in the optimal mix of Table 4.10 are actually more positively correlated than when variance is used as measure of risk, increasing the total risk of the combined portfolio and reducing the associated comfort/satisfaction within the semi–variance based optimisation environment, or the **downside risk** of the single sector is already so low that the added effect of diversification by another sector does not justify the reduction in total return (thus increasing the comfort/satisfaction) when this sector is included.

On the other hand, solving the two optimisation problems renders exactly the same optimal portfolios for all three risk–profiled investments pertaining to downward shocks in the change in SA business confidence (a graphical representation of the solution for all three risk profiles and both optimisation problems is given in Figure 4.1). Therefore it is concluded that resemblances in solutions obtained from solving the two optimisation problems differ from time window set to time window set and should be evaluated on a case by case basis.

Solution								Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Upward	1	0.042	0	0	1	0	0	−0.0490	−0.0505	0.0000	−0.0495	−0.0761	0.039
	2	0.073	0	0	1	0	0	−0.0296	−0.0497	0.0000	−0.0215	−0.0677	0.046
	3	0.033	0	0	0	1	0	−0.0207	−0.0857	−0.0051	0.0000	−0.0094	0.029
	4	0.032	1	0	0	0	0	0.0000	−0.0475	−0.0613	−0.0369	−0.0213	0.025
	5	−0.011	1	0	0	0	0	0.0000	−0.0230	−0.0080	−0.0333	−0.0123	−0.020
	6	0.041	0	0	1	0	0	−0.0548	−0.0503	0.0000	−0.0147	−0.0286	0.035
	7	−0.040	0	0	0.7541	0	0.2459	−0.1125	−0.1241	0.0000	−0.0659	0.0000	−0.043
	8	−0.011	0	0	0	0	1	−0.0373	−0.0032	−0.0128	−0.0451	0.0000	−0.011
	9	0.057	0	0	1	0	0	−0.1741	−0.1164	0.0000	−0.0465	−0.0038	0.042
	10	0.129	0	0.3150	0.6850	0	0	−0.0173	0.0000	0.0000	−0.0309	−0.0799	0.120
	11	0.124	0	0	1	0	0	−0.1639	−0.1032	0.0000	−0.0162	−0.0567	0.094
	12	0.150	0	1	0	0	0	−0.0450	0.0000	−0.0869	−0.0283	−0.1046	0.104
	13	−0.056	0	0	0.8220	0.1780	0	−0.0997	−0.0876	0.0000	0.0000	−0.0183	−0.065
	14	−0.101	0	0	1	0	0	−0.1193	−0.0494	0.0000	−0.0377	−0.0045	−0.132
	15	0.128	0	1	0	0	0	−0.0662	0.0000	−0.1394	−0.1470	−0.2160	0.111
	16	−0.159	0	0	0	0	1	−0.2494	−0.1335	−0.4430	−0.2719	0.0000	−0.197
	17	0.167	0	1	0	0	0	−0.1065	0.0000	−0.1583	−0.1606	−0.1521	0.151
	18	0.008	0	0.6547	0	0.2555	0.0898	−0.1161	0.0000	−0.0106	0.0000	0.0000	0.007
	19	0.002	0	0	0	0	1	−0.0829	−0.0872	−0.1096	−0.0497	0.0000	−0.001
	20	0.100	0	0	0	1	0	−0.0819	−0.0243	−0.0009	0.0000	−0.0310	0.083
	21	0.061	0	1	0	0	0	−0.0249	0.0000	−0.0266	−0.0356	−0.0112	0.038
	22	0.161	1	0	0	0	0	0.0000	−0.0715	−0.0854	−0.0660	−0.0950	0.142
	23	0.073	0	0.4651	0	0	0.5348	−0.0440	0.0000	−0.0600	−0.0345	0.0000	0.013
	24	−0.005	0	0	0	0	1	−0.0906	−0.0437	−0.0378	−0.0544	0.0000	−0.007
	25	0.112	1	0	0	0	0	0.0000	−0.0517	−0.0216	−0.0307	−0.1363	0.096
	26	0.151	1	0	0	0	0	0.0000	−0.0437	−0.0919	−0.1141	−0.1146	0.118
	27	0.025	0	0.3591	0	0	0.6409	−0.0705	0.0000	−0.0720	−0.0077	0.0000	0.010
	28	−0.005	0.4713	0.5287	0	0	0	0.0000	0.0000	−0.0926	−0.0784	−0.0005	−0.006
	29	0.091	0	0	0	0.7656	0.2344	−0.1010	−0.0238	−0.0226	0.0000	0.0000	0.076
	30	0.081	0	0	0	1	0	−0.0376	−0.0482	−0.0330	0.0000	−0.1036	0.056
	31	0.095	0	0	0.3633	0.6367	0	−0.0092	−0.0211	0.0000	0.0000	−0.0601	0.059
	32	0.012	0.6195	0	0	0.2507	0.1298	0.0000	−0.0443	−0.0088	0.0000	0.0000	0.001
	33	0.015	0	0	0	1	0	−0.2151	−0.1215	−0.0711	0.0000	−0.0491	0.015
	34	0.017	0.3194	0	0	0	0.6806	0.0000	−0.0259	−0.0115	−0.0081	0.0000	0.011
	35	0.008	0	0	0	0	1	−0.1120	−0.1034	−0.0790	−0.1406	0.0000	0.005
	36	0.093	1	0	0	0	0	0.0000	−0.0018	−0.1407	−0.1382	−0.0422	0.074
	37	0.071	0	0	0	1	0	−0.0806	−0.0507	−0.0295	0.0000	−0.0134	0.070
	38	0.034	0	0.6631	0	0	0.3369	−0.0184	0.0000	−0.0136	−0.0214	0.0000	0.026
	39	0.089	0.0471	0.9529	0	0	0	0.0000	0.0000	−0.0102	−0.0260	−0.0218	0.042
	40	0.005	0.0656	0	0	0	0.9344	0.0000	−0.0578	−0.0280	−0.0045	0.0000	0.001
	41	0.064	0	1	0	0	0	−0.0915	0.0000	−0.1356	−0.1261	−0.0559	0.062
	42	0.056	0	1	0	0	0	−0.1203	0.0000	−0.0069	−0.1223	−0.0585	0.054
	43	0.174	0	0	1	0	0	−0.1045	−0.1460	0.0000	−0.1077	−0.1151	0.123
Downward	1	0.031	0	0	1	0	0	−0.0036	−0.0336	0.0000	−0.0403	−0.0590	0.031
	2	0.060	0	1	0	0	0	−0.0380	0.0000	−0.0257	−0.0388	−0.0247	0.024
	3	0.083	0	1	0	0	0	−0.0867	0.0000	−0.0952	−0.1063	−0.1001	0.076
	4	0.143	0	1	0	0	0	−0.0498	0.0000	−0.0595	−0.0750	−0.0993	0.079
	5	0.047	1	0	0	0	0	0.0000	−0.0541	−0.0149	−0.0179	−0.0251	0.038
	6	0.040	0	0	0	0	1	−0.0770	−0.0145	−0.0678	−0.0680	0.0000	0.037
	7	0.021	0	0	0	1	0	−0.0299	−0.0431	−0.0091	0.0000	−0.0255	0.021
	8	−0.024	0	0	0	0	1	−0.0719	−0.0659	−0.0424	−0.0331	0.0000	−0.027
	9	0.056	0	0	1	0	0	−0.0303	−0.0006	0.0000	−0.0139	−0.0666	0.052
	10	0.065	0	0.6741	0	0.3259	0	−0.0062	0.0000	−0.0230	0.0000	−0.0604	0.059

Table 4.10: Optimisation: Conservative investment, standard deviation as risk and US consumer sentiment as indicator

4.6 Conclusion

The optimal allocations found in Appendix B (of which some were discussed in §4.5) are largely reliant upon the form of the comfort/satisfaction function and the associated risk tolerance factor. Therefore, cognisance should be taken of the fact that if the function is not truly representative of the risk–return appetite of a specific investor, the optimal allocation will most probably not be entirely suitable for the investor either. From this, one can again enter into the argument as to whether human behaviour or attitude towards risk may indeed be captured fully in a mathematical utility function. Maybe the outcome of this argument is not that crucial to the usefulness of the optimisation method. Simon [83] reckons utility only has to be “satisfied” not maximised and that it does not have to be applied exactly – if the utility function is a fairly close approximation of the attitude

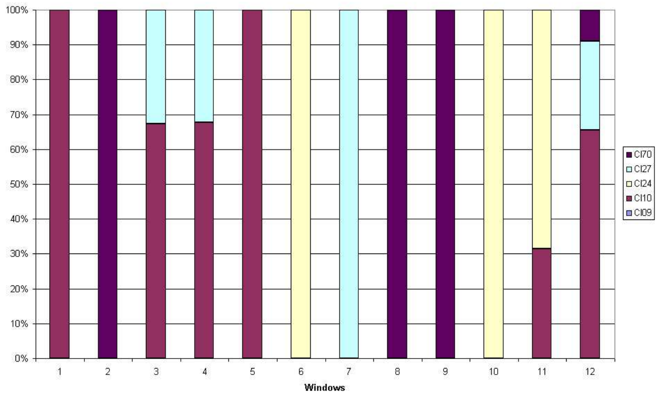
			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Upward	1	0.043	0	0	1	0	0	−0.0554	−0.0533	0.0000	−0.0534	−0.0797	0.042
	2	0.084	0	0	1	0	0	−0.0453	−0.0730	0.0000	−0.0272	−0.0914	0.068
	3	0.036	0	0	0	1	0	−0.0234	−0.0954	−0.0055	0.0000	−0.0110	0.034
	4	0.034	1	0	0	0	0	0.0000	−0.0493	−0.0661	−0.0417	−0.0271	0.029
	5	−0.004	1	0	0	0	0	0.0000	−0.0247	−0.0136	−0.0352	−0.0210	−0.006
	6	0.045	0	0	1	0	0	−0.0660	−0.0679	0.0000	−0.0213	−0.0382	0.043
	7	−0.036	0	0	1	0	0	−0.1158	−0.1311	0.0000	−0.0698	−0.0028	−0.041
	8	−0.011	0	0	0	0	1	−0.0381	−0.0038	−0.0133	−0.0458	0.0000	−0.011
	9	0.067	0	0	1	0	0	−0.2141	−0.1423	0.0000	−0.0621	−0.0175	0.062
	10	0.139	0	0	1	0	0	−0.0281	−0.0094	0.0000	−0.0364	−0.0884	0.129
	11	0.141	0	0	1	0	0	−0.1918	−0.1166	0.0000	−0.0313	−0.0869	0.129
	12	0.172	0	1	0	0	0	−0.0186	0.0000	−0.1285	−0.0498	−0.1603	0.148
	13	−0.052	0	0	1	0	0	−0.0972	−0.0839	0.0000	−0.0021	−0.0223	−0.056
	14	−0.077	0	0	1	0	0	−0.0969	−0.0558	0.0000	−0.0336	−0.0212	−0.085
	15	0.132	0	1	0	0	0	−0.0625	0.0000	−0.1404	−0.1495	−0.2176	0.119
	16	−0.138	0	0	0	0	1	−0.1670	−0.1145	−0.3562	−0.2243	0.0000	−0.155
	17	0.174	0	1	0	0	0	−0.1196	0.0000	−0.1726	−0.1792	−0.1626	0.164
	18	0.010	0	0.2806	0	0.5199	0.1995	−0.1216	0.0000	−0.0089	0.0000	0.0000	0.007
	19	0.004	0	0	0	0	1	−0.0872	−0.0928	−0.1077	−0.0494	0.0000	0.002
	20	0.111	0	0	0.5670	0.4330	0	−0.0964	−0.0297	0.0000	0.0000	−0.0494	0.099
	21	0.067	0	1	0	0	0	−0.0252	0.0000	−0.0502	−0.0459	−0.0220	0.050
	22	0.167	1	0	0	0	0	0.0000	−0.0795	−0.0932	−0.0718	−0.1014	0.153
	23	0.125	0	1	0	0	0	−0.0253	0.0000	−0.0589	−0.0358	−0.0053	0.018
	24	−0.004	0	0	0	0	1	−0.0841	−0.0446	−0.0334	−0.0523	0.0000	−0.006
	25	0.120	1	0	0	0	0	0.0000	−0.0737	−0.0280	−0.0405	−0.1522	0.111
	26	0.167	1	0	0	0	0	0.0000	−0.0647	−0.1431	−0.1607	−0.1401	0.151
	27	0.036	0	1	0	0	0	−0.0653	0.0000	−0.0779	−0.0148	−0.0017	0.016
	28	−0.005	0.5037	0.4963	0	0	0	0.0000	0.0000	−0.0907	−0.0766	−0.0008	−0.006
	29	0.099	0	0	0	1	0	−0.1074	−0.0306	−0.0263	0.0000	−0.0018	0.091
	30	0.093	0	0	0	1	0	−0.0411	−0.0555	−0.0509	0.0000	−0.1332	0.080
	31	0.115	0	0	0.8765	0.1235	0	−0.0213	−0.0451	0.0000	0.0000	−0.0834	0.086
	32	0.021	0.4696	0	0	0.5304	0	0.0000	−0.0542	−0.0047	0.0000	−0.0181	0.012
	33	0.015	0	0	0	1	0	−0.2093	−0.1197	−0.0681	0.0000	−0.0484	0.015
	34	0.021	0.5497	0	0	0	0.4503	0.0000	−0.0322	−0.0050	−0.0102	0.0000	0.015
	35	0.009	0	0	0	0	1	−0.1122	−0.1000	−0.0765	−0.1359	0.0000	0.007
	36	0.098	1	0	0	0	0	0.0000	−0.0156	−0.1597	−0.1459	−0.0535	0.085
	37	0.072	0	0	0	1	0	−0.0810	−0.0504	−0.0307	0.0000	−0.0121	0.071
	38	0.045	0	1	0	0	0	−0.0304	0.0000	−0.0213	−0.0286	−0.0102	0.036
	39	0.109	0	1	0	0	0	−0.0153	0.0000	−0.0330	−0.0582	−0.0634	0.081
	40	0.007	0	0	0	0	1	−0.0038	−0.0655	−0.0315	−0.0082	0.0000	0.004
	41	0.065	0	1	0	0	0	−0.0923	0.0000	−0.1399	−0.1317	−0.0572	0.065
	42	0.056	0	1	0	0	0	−0.1189	0.0000	−0.0071	−0.1199	−0.0593	0.055
	43	0.197	0	0	1	0	0	−0.1620	−0.1810	0.0000	−0.1461	−0.1527	0.170
Downward	1	0.031	0	0	1	0	0	−0.0036	−0.0337	0.0000	−0.0402	−0.0588	0.031
	2	0.078	0	1	0	0	0	−0.0843	0.0000	−0.0535	−0.0667	−0.0607	0.059
	3	0.087	0	1	0	0	0	−0.0891	0.0000	−0.1047	−0.1187	−0.1122	0.084
	4	0.171	0	1	0	0	0	−0.1108	0.0000	−0.0972	−0.1140	−0.1651	0.134
	5	0.051	1	0	0	0	0	0.0000	−0.0606	−0.0156	−0.0205	−0.0290	0.046
	6	0.041	0	0	0	0	1	−0.0825	−0.0192	−0.0730	−0.0725	0.0000	0.040
	7	0.022	0	0	0	1	0	−0.0309	−0.0455	−0.0097	0.0000	−0.0271	0.022
	8	−0.023	0	0	0	0	1	−0.0690	−0.0638	−0.0402	−0.0317	0.0000	−0.024
	9	0.057	0	0.3344	0.6656	0	0	−0.0295	0.0000	0.0000	−0.0136	−0.0675	0.052
	10	0.069	0	0.2040	0	0.7960	0	−0.0059	0.0000	−0.0229	0.0000	−0.0591	0.059

Table 4.11: Optimisation: Conservative investment, downside risk and US consumer sentiment as indicator

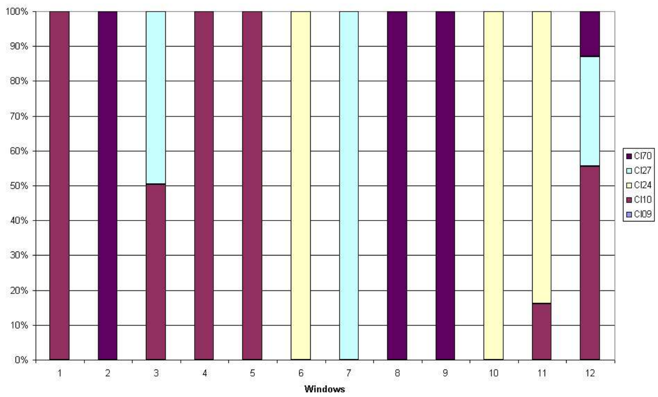
of an investor, the resulting optimal allocation should be an acceptable approximation of the best investment for the investor over a specific period. However, keeping in mind the inherent unpredictability of human nature, it is deemed presumptuous and naive to proclaim the optimisation method as the ultimate stand–alone investment answer. Still, it should not be ignored as a powerful investment tool.

In these problems an allocation of 100% of the available capital was allowed in a single sector (in other words the cardinality of the optimal portfolios were unconstrained). It was furthermore not taken into account that the five sectors differ in size in terms of market capitalisation and that some of the sectors, like Real Estate and Non–Mining Resources combined only represent approximately 10% of the total market spectrum. Investing 100% of an investment in only one sector firstly exposes an entire investment to the idiosyncrasies of one sector. Secondly, if an optimal investment in one of the smaller sectors pertains to a large number of investors, it may cause the demand for securities in the specific sector to be so high, that there exist no more sellers of securities at reasonable prices. The prices of securities are therefore increased artificially – this phenomenon is sometimes referred to as a "bubble". The risk of such an investment is that prices are increased to irrational levels at which point there are no more buyers in the market; the prices will have to return to more reasonable levels, resulting in rapid negative growth. The introduction of cardinality and floor and ceiling constraints may make the results practically more feasible. However, the results of the unconstrained problem should not be discarded as they provide a true optimality, which is an important basis for further decisions. Any additional constraints will cause a reduction in the comfort/satisfaction of the optimal mix as they may force an exposure to lower–returning assets or assets with different risk characteristics from those in the unconstrained optimal mix.

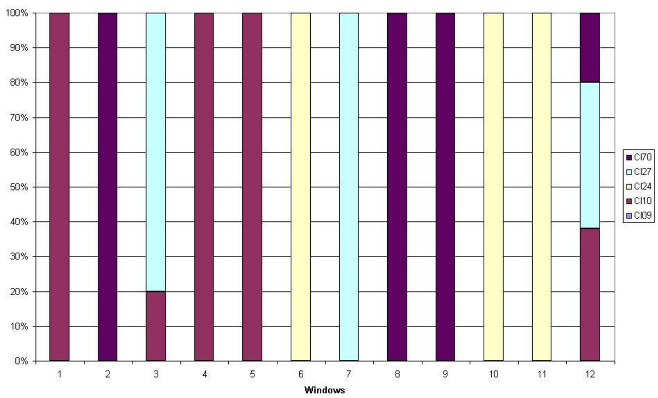
The actual risk and return values of the five main sectors have been used to obtain the optimal results in an attempt to understand some of the market dynamics during specific (extreme) periods and their influence on different risk–profiled investments. However, the historical optimal allocations are not necessarily a true indication of future optimal mixes. In order for this optimisation approach to be used as a tool for forecasting, the coefficients in the objective function may be replaced by expected values which are derived from more sophisticated forecasting methods.



(a) Conservative Investment



(b) Moderate Investment



(c) Aggressive Investment

Figure 4.2: Optimisation: Standard deviation as risk, Upward shocks in the change in US consumer sentiment as indicator

Chapter 5

Risk Adjusted Measures

As was already mentioned in §2.2, the evaluation of risky alternatives was almost entirely done based on the rate of return before the revolution in portfolio management during the early 1960's. Although Markowitz succeeded in quantifying risk, this strategy was not sufficient, since there were no composite measures in use to take both performance factors (return and risk) into account simultaneously: at that stage it was still necessary to consider the two factors separately [26].

Twelve years after the Markowitz–revolution, the Capital Asset Pricing Model (CAPM), which is a set of predictions concerning expected returns on risky assets, was developed by Sharpe [80], Lintner [55] and Mossin [65]. The time required for this gestation indicates that the leap from Markowitz's portfolio selection model to the CAPM was not trivial. The CAPM uses Markowitz's suggestion of variance as measure of market risk and states that the risk premium on individual assets will be proportional to the risk premium on the market portfolio. The nature of this dependency is prescribed by the so–called *beta coefficient or beta–value* §2.6.5.

From the discussion in §2.8 it seems evident that a culture came to the fore within the financial arena of questioning and adjusting existing risk measures. Maybe the reason why the most efficient measure for describing risk still seems to elude investment professionals can be found in the following statement by Kazemi and Martin [48]: *One can argue that no single variable can serve as an accurate measure of risk. Risk has many dimensions and therefore a number of measures should be used.* Nevertheless, regardless of what measure for determining risk is believed to be most efficient, the industry seems to agree that it is important to relate the risk of an investment to its subsequent return. This should be done in order to determine whether the expected return is sufficient to reward the investor for the degree of risk incurred [56]. One way of relating return to risk is by means of a risk–return ratio, or risk–adjusted return ratio, which indicates the return per unit risk of a specific investment, as was discussed briefly in §2.7. An advantage of such a ratio is that it may be used to compare the degree of success of different investments to compensate investors for the risk incurred – the higher the risk–adjusted return ratio of an investment, the better return per unit risk is rendered. This information may subsequently be used for ranking risky alternatives in terms of their risk–adjusted return ratios, to be used as a decision support tool.

Methods of risk–adjusted evaluation came on stage simultaneously with the CAPM. Treynor [90], Sharpe [81] and Jensen [44] recognised immediately the implications of the CAPM, and beta–coefficient, for ranking the adjusted returns of risky alternatives [10]. In the context of this study, Treynor and Sharpe measures will be implemented to evaluate risk–return attributes of portfolios, but they will not be viewed as products of the CAPM model as such. They will rather be seen as risk–adjusted return ratios where either the beta–value or standard deviation is assumed the most appropriate measure of risk to be used for the adjustment. Within this environment a third measure of risk, namely downside risk (which was discussed in Chapter 4) will be introduced to serve as yet another means to adjust returns.

In this chapter, characteristics of different risk–adjusted return ratios will be utilised, each representing the use of a different measure of risk to adjust returns. Within each time window as described in §3.8, the five main sectoral portfolios will be ranked according to their risk–return desirability and assigned a ranking number from 1 to 5, with 1 indicating the most desirable portfolio. The aim of this chapter is first to use the information obtained from the ranking–procedures to understand more of the risk–return characteristics of the market (in terms of the five main sectors), in times of extreme market conditions. Secondly, we would like to ascertain how similar or diverse the risk characteristics of a set of time windows pertaining to the extreme market conditions discussed in this study are. This will be done in §§5.2–5.8. Finally, the results of the optimisation method in Chapter 4 and those of the ranking methods will be compared and discussed in §5.9.

5.1 Ranking by means of risk–adjusted measures

In Chapter 4, the goal was to find an optimal allocation of the sectoral portfolios for each of the periods during which a specific shock prevailed. The goal of risk–adjusted ranking procedures is slightly different in the sense that risk–adjusted return ratios give an indication of the relative attractiveness (in terms of risk and return) of the different sectoral portfolios during a specific period, but at no point do they suggest the make–up of an optimal mix. However, this does not necessarily mean that risk–adjusted rankings are obsolete within the investment environment. Investment decisions must be made daily, and within the fast moving equity market, it is not always practical to follow a complete optimisation procedure. Although the ranking methods sacrifice information on exactly how much more attractive one sector is to the next, their results still provide a quick reference of the order of attractiveness for an investment decision to be made. The reason for wanting to compare the optimisation results with that of the ranking methods in §5.9 is to investigate whether it is in fact wise to use ranking procedures as interim approximation for optimisation results.

In order to calculate the risk–adjusted measures, let $\underline{x} = [x_1, x_2, \ldots, x_n]$ be the portfolio price–value of the market portfolio (in this case the All Share Index) pertaining to a specific time window with n data points, and let $\underline{y} = [y_1, y_2, \ldots, y_n]$ be the portfolio price–values of any other portfolio under evaluation (say the portfolio with Financial shares) over the same period. If y_0 is the last portfolio price–value of the portfolio described by $\underline{y}$ before the time window starts, then the *return* of this portfolio, namely r_y, is calculated as the percentage change in the price history from y_0 to y_n by means of the formula

$$r_y = \frac{y_n - y_0}{y_0}.$$

The standard deviation (σ) that will be used in this study is the normal standard deviation of the set of portfolio price–values $\underline{y}$ (where it is assumed that the arguments are the entire population) as a percentage of the mean value of the set (calculated by $\sum_{i=1}^{n} y_i/n$). For the downside risk measure (σ_s), the formula in (4.3.1) is used, where $\underline{r}_i$ now equals $\underline{y}$ and $\hat{\underline{r}}_i = (\sum_{i=1}^{n} y_i)/n$. Finally, the beta–measure (β) is calculated as

$$\beta = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n}, y_i)^2}.$$

The risk–free rate of return (r_f) necessary to calculate the excess return of the risky portfolio (as discussed in §2.6.2) is assumed as the average 90–day NCD rate over the period under evaluation. Therefore, the σ–, σ_s– and β–adjusted return measures for a portfolio with price–values $\underline{y}$ in a specific time window is calculated as

$$\frac{r_y - r_f}{\sigma}, \quad \frac{r_y - r_f}{\sigma_s} \quad \text{and} \quad \frac{r_y - r_f}{\beta}$$

respectively.

For every time window generated by the different market indicators (as identified in §3.8) these risk–adjusted return measures are calculated for every sectoral portfolio in the time window. The portfolios within every time window are then ranked in descending order according to each risk–adjusted measure. The portfolio with the highest risk–adjusted measure will be the most attractive, as it renders better excess return per unit of risk. Throughout the following sections, reference will be made to the sectoral ranking of a time window, which refers to the ranking numbers of the Mining Resources, Non–Mining Resources, Financial, Industrial, and Real Estate portfolios, in one time window in that specific order with respect to one of the three risk–adjusted return measures described.

As the risk–adjusted measures render returns per unit of risk generated by all portfolios, ranking these values gives the relative attractiveness of one portfolio to another. This information may be used to determine in what proportion different portfolios (sectors) should be included in an investment. However, sectoral rankings may actually be employed for more than allocating funds. If ranking results of different time windows by the same risk–adjusted measure or the ranking results of the same time window by different risk–adjusted measures are compared, the interpretation (while keeping in mind how the rankings were calculated) may aid significantly in understanding the characteristics of the portfolios within the time windows being evaluated. This strategy will be explored further in the rest of this section and then used to interpret some of the attributes of the time windows in §§5.2–5.8.

First, attention will be focused on the comparison of sectoral rankings of **different** time windows by the **same** risk–adjusted measure. If δ is any of the three risk measure discussed, then let $\underline{r}_{\delta j}$ be the vector of δ–adjusted returns for time window j,

$$\underline{r}_{\delta j} \;=\; [\frac{r_{1j} - r_f}{\delta_{1j}}, \quad \frac{r_{2j} - r_f}{\delta_{2j}}, \; \ldots, \; \frac{r_{5j} - r_f}{\delta_{5j}}]$$

inducing the sectoral ranking

$$\underline{w}_{\delta j} \quad = \quad [w_{1j}, w_{2j}, \ldots, w_{5j}].$$

From one time window to the next, the return, associated risk or both could change for any or all of the portfolios. If, for time windows i and j it is true that $\underline{w}_{\delta i} = \underline{w}_{\delta j}$, this suggests that, for any changes from time window i to j in one portfolio's risk and/or return attributes, the other portfolios have adjusted accordingly to the extent that the sectoral ranking stayed the same. Therefore, repeated rankings may be seen as a possible first indication that some time windows within a set may possess similar risk–return characteristics. However, caution should be taken that not too much emphasis is placed on the correspondence of sectoral rankings alone in the assessment of these characteristics. The nature of ranking methods is such that the relative size of one risk–adjusted return to another is discarded when the ranking number is assigned. This may cause relative positions of the different portfolios' risk–adjusted returns (and thus the sectoral rankings of time windows) to remain the same without the relative sizes necessarily remaining exactly the same as well. Therefore, there may be some degree of variance in the risk–return characteristics of two time windows without a difference in their sectoral rankings.

In order to obtain more information on the properties of the various sets of time windows, it is important to compare the rankings obtained from **different** risk–adjusted measures. The inherent nature of the ranking methods to position the portfolios relative to each other according to certain criteria, encapsulates an important characteristic of the methods that will play an integral part in the discussion to follow. The particular characteristic of concern is that the ranking of a specific time window by one risk–adjusted measure compared to the ranking of the same time window by another risk–adjusted measure will only produce the same order if the proportion of the portfolios' risk measures by the respective risk measures are fairly similar. This information will be used to compare the results of the three risk–adjusted measures and to arrive at certain conclusions regarding the nature of the time windows being discussed.

The above–mentioned characteristic may be explained further by considering two sets of risk–adjusted rankings produced by two arbitrary risk measures, say δ and ψ, for the same time window. The case where the ordering of portfolios by two different risk measures coincide, and by this produce time windows in which the rankings of the δ– and ψ–adjusted measures coincide, may be analysed as follows. If the proportion of the two risk–measures (δ and ψ) remains constant throughout the portfolios in a certain time window, the sectoral rankings of the δ– and ψ–adjusted methods will coincide. Therefore, the δ– and ψ–adjusted rankings of a time window j will coincide if

$$\frac{\delta_{ij}}{\psi_{ij}} = k, \tag{5.1.1}$$

where δ_{ij} and ψ_{ij} are the respective risk–values for portfolio $i \in \{1, \ldots, 5\}$ and where k is constant. This statement may be confirmed as follows.

Let $\underline{r}_{\delta_j}$ be the series of δ–adjusted returns for time window j, which produces a sectoral ranking $\underline{w}_{\delta_j}$ so that

$$\underline{r}_{\delta_j} \quad = \quad \left[\frac{r_{1j} - r_f}{\delta_{1j}}, \quad \frac{r_{2j} - r_f}{\delta_{2j}}, \ \ldots, \ \frac{r_{5j} - r_f}{\delta_{5j}} \right]$$

induces the sectoral ranking

$$\underline{w}_{\delta_j} \quad = \quad w_{\delta_{1j}}, w_{\delta_{2j}}, \ldots, w_{\delta_{5j}}.$$

By taking (5.1.1) into account and introducing the vector r_{ψ_j} of ψ–adjusted returns, it follows that

$$\begin{aligned} \underline{r}_{\psi_j} &= \left[\frac{r_{1j}-r_f}{\psi_{1j}}, \quad \frac{r_{2j}-r_f}{\psi_{2j}}, \ \ldots, \ \frac{r_{5j}-r_f}{\psi_{5j}}\right] \\ &= \left[k\frac{r_{1j}-r_f}{\delta_{1j}}, \quad k\frac{r_{2j}-r_f}{\delta_{2j}}, \ \ldots, \ k\frac{r_{5j}-r_f}{\delta_{5j}}\right] \\ &= k \ \underline{r}_{\delta_j}. \end{aligned}$$

This implies that $\underline{w}_{\psi_j} = \underline{w}_{\delta_j}$. Conversely, if

$$\underline{w}_{\psi_j} \quad = \quad \underline{w}_{\delta_j},$$

it is not necessarily true that

$$\begin{aligned} \underline{r}_{\psi_j} &= \left[\frac{r_{1j}-r_f}{\psi_{1j}}, \quad \frac{r_{2j}-r_f}{\psi_{2j}}, \ \ldots, \ \frac{r_{5j}-r_f}{\psi_{5j}}\right] \\ &= \left[k\frac{r_{1j}-r_f}{\delta_{1j}}, \quad k\frac{r_{2j}-r_f}{\delta_{2j}}, \ \ldots, \ k\frac{r_{5j}-r_f}{\delta_{5j}}\right], \end{aligned}$$

implying that

$$\begin{aligned} [\psi_{1j}, \ \psi_{2j}, \ \ldots, \ \psi_{5j}] &\neq [k\delta_{1j}, \quad k\delta_{2j}, \ \ldots, \ k\delta_{5j}] \\ &= k[\delta_{1j}, \quad \delta_{2j}, \ \ldots, \ \delta_{5j}]. \end{aligned}$$

For instance, if

$$\frac{r_{ij}-r_f}{\delta_{ij}} \gg \frac{r_{mj}-r_f}{\delta_{mj}}$$

for some portfolios i and m during time window j, with

$$\frac{r_{ij}-r_f}{\psi_{ij}} > \frac{r_{mj}-r_f}{\psi_{mj}},$$

it may be that $\frac{\delta_{ij}}{\psi_{ij}} \neq k$, but that the sectoral rankings of the two measures do not differ. It may, therefore be more acceptable to introduce a constant

$$k_i = \frac{\delta_{ij}}{\psi_{ij}}$$

for portfolios $i = 1, \ldots, 5$, with k_i in the vacinity of some common value k, to the extent that the sectoral rankings do not change. Coinciding sectoral rankings may be used as a first indication of this very specific risk–characteristic. The actual risk values should be examined further to conclude whether the relation is in fact constant or not; rankings cannot by themselves provide this information.

Certain similarities between specific time windows may be identified by the interpretations discussed in the previous paragraphs, but another question we would like to answer is whether the reactions of the main sectors of the South African equity market showed signs of similarity within the periods typified by extreme events, in other words whether the set of time windows generated by (say) an upward shock in R150 Bond yields showed signs of homogeneity. The sectoral ranking interpretation described above provides a first indication as to whether certain time windows share similar risk–return characteristics and is helpful in comparing the characteristics of specific time windows. Therefore, the line of interpretation will be applied to the time windows in §§5.2–5.8 to understand some of their characteristics. However, it is difficult to conclude from the above–described analyses exactly how homogeneous a set of time windows is as a whole.

In order to investigate the ranking–characteristics of a set of time windows as a whole, the study ventures into the field of graph theory for the necessary tools to examine similarities in the rankings of the same risk–adjusted measure across the different time windows. In particular, attention will be given to *dominating sets in graphs* and in order to introduce its specific application, it seems that a short discussion of its theory by means of a few definitions is in order.

Definition 24 *A graph $G = (V, E)$ is a finite non–empty set $V(G)$ of objects called vertices and a (possibly empty) set $E(G)$ of 2–element subsets of $V(G)$ called edges. Two vertices u and v in a graph $G = (V, E)$ are adjacent if uv is an element of the edge set $E(G)$.*

Every graph may be represented by a diagram. The vertices are represented by points and two points are joined by a line whenever the corresponding pair of vertices are adjacent. Figure 5.1 gives an example of a graph with vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ and edge set $E(G) = \{v_1v_5, v_2v_6, v_2v_9, v_2v_{10}, v_6v_7, v_6v_9, v_6v_{10}, v_7v_9, v_7v_{10}, v_9v_{10}\}$.

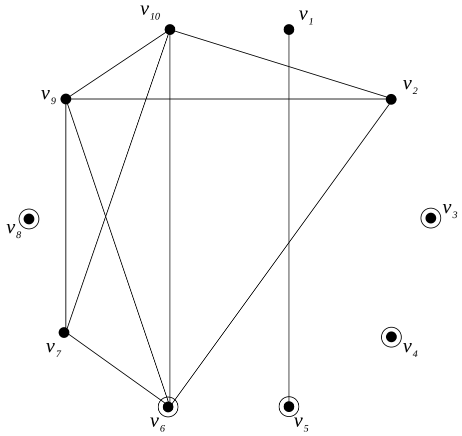
Definition 25 *A set $S \subseteq V$ of vertices in a graph $G = (V, E)$ is called a* dominating set *if every vertex $v \in V$ is either an element of S or is adjacent to an element of S.*

Definition 26 *The* lower domination number *$\gamma(G)$ of a graph G is the minimum cardinality of a dominating set in G.*

To utilise the theory of graph domination to analyse the sectoral rankings rendered by a risk–adjusted measure, the ranking dynamics of the time windows should be captured in a graph. Therefore, the time windows identified by a specific market indicator are represented by the vertices of a graph, which we call a *ranking graph*. The numbering of the vertices of this graph corresponds to the numbers of the time windows they represent. The essential feature of such a graph is:

Definition 27 *The vertices (time windows) in a ranking graph are adjacent if and only if their sectoral rankings either* coincide *or are* similar.

By the terms *coincide* and *similar* the following is meant.



(a)

Vertex	Sectoral ranking				
v_1	5	2	3	4	1
v_2	2	3	4	5	1
v_3	2	5	3	4	1
v_4	1	2	3	4	5
v_5	5	2	4	3	1
v_6	2	3	5	4	1
v_7	3	2	5	4	1
v_8	4	5	3	2	1
v_9	2	3	5	4	1
v_{10}	2	3	5	4	1

(b)

Figure 5.1: (a) Ranking graph with lower domination number 5 and dominating set $\{v_3, v_4, v_5, v_6, v_8\}$, as indicated by the circled vertices, (b) Sectoral rankings pertaining to vertices

Definition 28 *The sectoral rankings of two time windows* coincide *if and only if the sectoral rankings are the same for both time windows.*

Definition 29 *The sectoral rankings of two time windows are* similar *if the ranking numbers of exactly two sectors differ by only one. More formally, the sectoral rankings ($\underline{w}_1 = [w_{11}, \ldots, w_{15}]$ and $\underline{w}_2 = [w_{21}, \ldots, w_{25}]$) of two time windows are similar if and only if the Euclidean distance between them, is $d(\underline{w}_1, \underline{w}_2) = 2$.*

An example of a ranking graph is given in Figure 5.1. This graph represents a time window set of 10 windows, where the sectoral rankings of adjacent vertices (time windows) coincide or are similar. The time window set $\{v_3, v_4, v_5, v_6, v_8\}$, circled in the graph, is a dominating set with cardinality equal to the lower domination number, 5.

Earlier mention was made of the question of possible homogeneity of time windows, but the term was not defined formally. Within the scope of the study of rankings, homogeneity of time windows in terms of their risk–return characteristics is defined as follows.

Definition 30 *A set of time windows are homogeneous if their rankings either coincide or are similar.*

This definition is a relaxation of the criterion used earlier in this section to compare different time windows. It not only encapsulates the fact that time windows share risk–return characteristics if their sectoral rankings are exactly the same, as was done earlier, but now also accommodates the fact that rankings, which are nearly the same, may also share risk–return characteristics.

Within a ranking graph $G = (V, E)$ of order n (number of time windows in the set), the lower domination number $\gamma(G)$ and an associated dominating set S of cardinality $\gamma(G)$

may prove to contain valuable information regarding the ranking–characteristics of the time windows. By Definitions 27 and 30 it follows that two time windows are adjacent if they are homogeneous. Furthermore, any time window is either in a dominating set or adjacent to a time window that is in a dominating set (by Definition 25). Therefore, a dominating set with cardinality equal to the lower domination number of the ranking graph encapsulates the minimum number of rankings needed to represent the risk–characteristics of the set of time windows as whole. Furthermore, the smaller the lower domination number, the more homogeneous the risk–characteristics of the time windows are, as fewer different sectoral rankings are needed to represent the risk–characteristics of the complete set of time windows.

In order to find the lower domination number, the following combinatorial minimisation problem should be solved:

$$\left.\begin{array}{ll} \text{Minimise} & z = \sum_{i=1}^{n} x_i \\[2ex] \text{subject to} & \sum_{i=1}^{n} A_{ij} x_i \geq 1, \quad j = 1, \ldots, n \\[2ex] & x_i \text{ is binary }, i = 1, \ldots, n. \end{array}\right\}$$

Here

$$x_i = \begin{cases} 1 & \text{if } v_i \in S \\ 0 & \text{otherwise} \end{cases}$$

is a decision variable dictating whether or not v_i forms part of a dominating set S of minimal cardinality, while $A = [A_{ij}]$ is the adjacency matrix for the graph $G = (V, E)$, following the convention that the entries on the main diagonal are ones, so that

$$A_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if } i \neq j \text{ and } v_i v_j \in E \\ 0 & \text{otherwise.} \end{cases}$$

The optimal solution of the minimisation problem not only renders the lower domination number $\gamma(G)$ as the value of the objective function, but also a dominating set S of minimum cardinality, which includes all vertices whose indices correspond to the decision variables of unary value. The sectoral rankings rendered by all three risk–adjusted methods were evaluated within the discussed graph theory context to produce the lower domination numbers and associated dominating sets of minimal cardinality. Most of the minimisation problems were solved with the Standard Solver package of Excel 2000, which uses the branch and bound method to solve integer programming problems. However, because the Standard Solver package can only handle problems with a maximum of 100 constraints, some of the larger problems were solved with the large-scale Linear Programming package on the Premium Solver Platform for Excel 2000 [28]. The dominating sets with minimal cardinality resulting from solving these problems per market indicator time window set per risk–adjusted measure are given and discussed in the following sections.

5.2 R150 Bond yields

	σ					σ_s					β				
Upward	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
1	2	1	4	3	5	2	1	3	4	5	5	4	2	3	1
2	2	1	4	3	5	2	1	4	3	5	2	1	4	3	5
3	3	4	1	2	5	3	4	1	2	5	3	4	1	2	5
4	2	1	4	3	5	2	1	4	3	5	3	2	5	4	1
5	2	1	4	3	5	2	1	4	3	5	2	1	4	3	5
6	4	2	1	3	5	4	2	1	3	5	5	3	2	4	1
7	3	1	2	4	5	1	2	4	3	5	4	2	3	5	1
8	4	5	2	3	1	4	5	2	3	1	4	5	2	3	1
9	4	2	1	3	5	4	3	1	2	5	5	3	2	4	1
10	4	3	1	2	5	4	3	1	2	5	5	4	2	3	1
11	4	1	2	3	5	4	1	2	3	5	5	3	2	4	1
12	3	4	1	2	5	3	4	2	1	5	4	5	2	3	1
13	2	1	3	4	5	2	1	3	4	5	2	1	3	4	5
14	4	3	2	5	1	4	3	2	5	1	3	2	5	4	1
15	3	1	4	2	5	4	1	2	3	5	2	1	4	3	5
16	2	4	3	1	5	1	3	4	2	5	3	4	2	1	5
17	4	3	2	1	5	3	4	1	2	5	4	3	2	1	5
18	2	4	3	1	5	2	3	4	1	5	2	4	3	1	5
19	2	1	4	5	3	2	1	3	4	5	2	1	4	5	3
20	3	4	1	2	5	1	4	3	2	5	3	4	1	2	5
21	2	3	4	1	5	2	3	4	1	5	2	3	4	1	5
22	1	3	4	2	5	1	4	3	2	5	3	5	2	4	1
23	4	1	2	3	5	4	2	1	3	5	5	3	2	4	1
24	4	1	3	2	5	3	1	4	2	5	4	2	5	3	1
25	3	2	4	1	5	4	3	2	1	5	4	3	5	2	1
26	2	5	3	4	1	1	5	2	4	3	2	5	3	4	1
27	1	4	2	3	5	1	4	2	3	5	3	2	4	5	1
28	4	2	1	3	5	4	2	1	3	5	4	5	2	3	1
29	3	1	2	4	5	2	1	4	3	5	5	1	4	3	2
30	5	2	3	4	1	5	2	3	4	1	4	2	3	5	1
31	2	1	4	3	5	2	1	4	3	5	3	2	4	5	1
32	4	3	1	2	5	4	3	1	2	5	5	4	2	3	1
33	2	4	3	1	5	3	2	4	1	5	2	4	3	1	5
34	3	2	4	1	5	1	3	4	2	5	5	2	3	4	1
35	2	4	1	3	5	2	4	1	3	5	2	5	3	4	1
36	3	4	1	2	5	3	4	1	2	5	3	4	1	2	5
37	1	4	2	3	5	1	4	2	3	5	4	2	5	3	1
38	1	3	2	4	5	1	2	3	4	5	1	3	2	4	5
39	3	4	2	1	5	3	4	2	1	5	3	4	2	1	5
40	2	4	3	1	5	2	4	3	1	5	3	5	4	2	1
41	2	1	4	3	5	2	1	4	3	5	3	2	5	4	1
42	3	5	4	2	1	3	5	4	2	1	2	4	3	1	5
43	3	4	2	1	5	2	4	3	1	5	2	4	3	1	5
44	1	4	2	3	5	1	4	2	3	5	1	4	2	3	5
45	4	1	2	3	5	4	1	2	3	5	4	1	2	3	5
46	1	2	3	4	5	1	2	3	4	5	4	1	2	3	5
47	2	3	4	5	1	2	3	4	5	1	4	1	2	3	5
48	5	2	3	4	1	5	2	4	3	1	5	2	3	4	1
49	4	5	3	2	1	2	5	4	3	1	2	5	4	3	1
50	1	4	3	2	5	1	4	3	2	5	5	4	3	2	1
51	1	4	2	3	5	1	4	2	3	5	4	3	1	2	5
52	2	3	1	4	5	3	1	2	4	5	4	2	3	1	5
Downward															
1	1	3	4	2	5	1	2	4	3	5	1	4	3	2	5
2	1	2	4	3	5	1	2	3	4	5	5	3	1	4	2
3	3	4	2	1	5	1	4	3	2	5	3	4	2	1	5
4	5	4	2	3	1	5	4	3	2	1	2	1	3	4	5
5	1	3	2	4	5	1	3	2	4	5	2	3	4	5	1
6	4	2	3	5	1	3	2	4	5	1	4	2	3	5	1
7	1	3	4	2	5	1	3	4	2	5	2	3	4	1	5
8	1	3	4	2	5	2	4	3	1	5	3	2	5	4	1
9	3	5	2	4	1	2	5	3	4	1	4	5	2	3	1
10	3	1	4	2	5	3	1	4	2	5	3	1	4	2	5
11	5	2	3	4	1	5	2	3	4	1	5	2	3	4	1
12	1	2	4	3	5	1	2	4	3	5	1	3	4	2	5
13	3	5	4	2	1	3	5	4	2	1	3	5	4	2	1
14	2	5	4	3	1	2	5	4	3	1	2	5	4	3	1
15	2	3	5	4	1	3	2	5	4	1	2	3	4	5	1
16	5	2	3	4	1	5	2	3	4	1	5	2	3	4	1
17	5	2	3	4	1	3	2	4	5	1	5	2	3	4	1
18	4	2	5	3	1	4	3	5	2	1	4	2	5	3	1
19	2	5	4	3	1	2	5	4	3	1	1	4	3	2	5
20	5	2	3	4	1	5	2	3	4	1	1	3	4	2	5
21	4	5	3	2	1	3	5	4	2	1	2	4	1	3	5
22	3	1	5	4	2	3	1	5	4	2	3	1	5	4	2
23	4	1	2	3	5	4	1	2	3	5	4	1	2	3	5
24	1	4	3	2	5	1	4	3	2	5	2	4	5	3	1

Table 5.1: Rankings: R150 Bond yields as indicator

There were 52 time windows within the period 1 January 1993 to 31 December 2000 during which upward shocks occurred in the R150 Bond yields, and 108 during which downward shocks occurred. The sectoral rankings of all these time windows are given in Tables 5.1–5.2.

By firstly examining the three risk–adjusted measures in isolation, some information is gathered. With the σ–adjusted return method, the sectoral rankings of 12 upward shock time windows, namely $\{1, 3, 4, 6, 7, 10, 11, 16, 25, 27, 30, 39\}$ are repeated across other time windows. At the same time, the sectoral rankings of downward shock time windows $\{1, 2, 3, 4, 6, 9, 10, 11, 13, 14, 15, 21, 22, 23, 24, 31, 32, 35, 41, 45, 56, 61, 66, 67, 70, 73, 82, 83, 86, 90, 94, 95, 96\}$ are repeated across other time windows.

When the returns within the different time windows are adjusted by the downside risk, σ_s, sectoral rankings are also repeated, so that the set $\{1, 2, 3, 6, 9, 11, 12, 16, 18, 20, 27, 29, 38, 40\}$ [$\{1, 2, 3, 6, 9, 10, 11, 13, 14, 15, 19, 23, 30, 32, 34, 35, 38, 42, 44, 45, 46, 47, 55, 58, 67, 73, 74, 75, 86\}$] contains time windows with coinciding rankings elsewhere in the time window set of upward shocks [downward shocks]. The same is true for the sets of upward shock time windows $\{1, 2, 3, 4, 6, 7, 8, 16, 18, 24, 27, 34, 42, 45\}$ and downward shock time windows $\{1, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 18, 22, 23, 27, 28, 31, 32, 41, 44, 46, 48, 54, 61, 73, 87\}$, where the rankings were produced by adjusting the returns within a time window by the β–measure and the combinations were repeated.

From the rankings in Tables 5.1–5.2, the time window set $\{2, 3, 4, 5, 6, 8, 10, 11, 13, 14, 21, 27, 28, 30, 31, 32, 35, 36, 37, 39, 40, 41, 42, 44, 45, 46, 47, 50, 51\}$ [$\{5, 7, 10, 11, 12, 13, 14, 16, 19, 20, 22, 23, 24, 26, 27, 29, 32, 33, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 60, 61, 62, 63, 66, 67, 68, 70, 71, 73, 75, 77, 78, 79, 80, 83, 85, 86, 88, 90, 91, 92, 93, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 108\}$] is identified to contain all the upward shock [downward shock] time windows for which the σ– and σ_s–adjusted rankings correspond. From the discussion in §5.1, it may be derived that the portfolios within each of these time windows have similar downside risk to total risk characteristics.

If the attention is shifted to comparing the σ– and β–adjusted rankings, a subset of upward shock time windows, namely $\{2, 3, 5, 8, 13, 17, 18, 19, 20, 21, 26, 33, 36, 38, 39, 44, 45, 48\}$, and downward shock time windows $\{3, 6, 10, 11, 13, 14, 16, 17, 18, 22, 23, 26, 30, 31, 32, 33, 34, 36, 40, 41, 42, 44, 51, 52, 55, 58, 60, 63, 64, 67, 73, 79, 87, 91, 96, 98, 100, 101, 103, 107\}$ are found to correspond fully. It was noted earlier (in §2.7.2) that, if a set of portfolios are completely diversified, their σ– and β–adjusted rankings will coincide. Completely diversified portfolios pose a special case where the proportion of total risk (σ) to market risk (β) for every portfolio equals one. By the discussion in §5.1, it can be showed that if the proportion of total risk to market risk remains constant throughout the portfolios in a certain time window, the sectoral rankings of the σ– and β–adjusted methods will coincide, which proves the statement made in §2.7.2 that diversified portfolios have the same σ– and β–adjusted sectoral rankings. It should be kept in mind that the converse is not necessarily true and therefore, the portfolios in the time window sets above are not necessarily diversified. It may be concluded that the time windows in each set share some risk–characteristics, but further information is needed to determine whether each of the time windows are fully diversified or not. The correspondence in the σ–and β–adjusted rankings per time window provides no information on similarities across the different time windows in the identified sets.

	σ					σ_s					β				
Downward	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
25	4	1	2	3	5	3	1	2	4	5	3	1	4	2	5
26	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
27	5	2	3	4	1	5	2	3	4	1	3	2	4	5	1
28	5	2	3	4	1	4	2	5	3	1	5	2	4	3	1
29	2	5	4	3	1	2	5	4	3	1	5	4	2	1	3
30	1	3	4	2	5	4	2	3	1	5	1	3	4	2	5
31	4	1	3	2	5	4	2	3	1	5	4	1	3	2	5
32	2	1	4	3	5	2	1	4	3	5	2	1	4	3	5
33	5	2	3	4	1	5	2	3	4	1	5	2	3	4	1
34	4	2	3	5	1	4	2	3	5	1	4	2	3	5	1
35	4	5	2	3	1	4	5	2	3	1	3	4	2	1	5
36	4	2	3	5	1	4	2	3	5	1	4	2	3	5	1
37	2	5	4	3	1	2	5	4	3	1	1	4	3	2	5
38	2	1	3	4	5	2	1	3	4	5	3	1	4	5	2
39	2	3	4	5	1	3	4	2	5	1	2	1	3	4	5
40	4	1	2	3	5	4	1	2	3	5	4	1	2	3	5
41	3	4	1	2	5	4	2	1	3	5	3	4	1	2	5
42	3	4	1	2	5	3	4	1	2	5	3	4	1	2	5
43	3	5	4	2	1	3	5	4	2	1	4	2	5	3	1
44	2	3	5	4	1	2	3	5	4	1	2	3	5	4	1
45	5	2	4	3	1	5	2	4	3	1	3	4	2	1	5
46	4	5	2	3	1	4	5	3	2	1	2	4	3	1	5
47	3	4	2	1	5	3	4	2	1	5	2	5	4	3	1
48	4	5	3	2	1	4	5	3	2	1	5	2	4	3	1
49	3	4	2	1	5	3	4	2	1	5	4	5	3	2	1
50	2	1	5	4	3	2	1	5	4	3	2	1	4	3	5
51	3	4	2	1	5	3	4	2	1	5	3	4	2	1	5
52	3	2	5	4	1	3	2	5	4	1	3	2	5	4	1
53	3	1	5	4	2	3	2	5	4	1	2	1	4	3	5
54	1	2	4	3	5	1	2	4	3	5	5	3	2	1	4
55	2	1	4	3	5	2	1	3	4	5	2	1	4	3	5
56	1	4	3	2	5	1	4	3	2	5	5	3	2	1	4
57	1	2	4	3	5	1	2	4	3	5	3	4	2	5	1
58	4	3	2	1	5	4	3	2	1	5	4	3	2	1	5
59	3	4	2	1	5	4	3	2	1	5	4	5	3	2	1
60	4	3	1	2	5	4	3	1	2	5	4	3	1	2	5
61	4	5	3	2	1	4	5	3	2	1	5	1	4	3	2
62	3	1	4	2	5	3	1	4	2	5	4	2	5	3	1
63	2	1	4	3	5	2	1	4	3	5	2	1	4	3	5
64	3	1	4	2	5	3	2	1	4	5	3	1	4	2	5
65	3	2	1	4	5	2	3	1	4	5	4	2	3	5	1
66	5	4	2	3	1	5	4	2	3	1	5	4	3	2	1
67	3	5	2	4	1	3	5	2	4	1	3	5	2	4	1
68	2	1	4	3	5	2	1	4	3	5	3	2	5	4	1
69	5	4	2	3	1	4	5	3	2	1	2	1	3	4	5
70	4	1	2	3	5	4	1	2	3	5	3	4	1	2	5
71	4	5	2	3	1	4	5	2	3	1	3	4	1	2	5
72	3	4	2	1	5	3	4	1	2	5	4	5	3	2	1
73	1	4	2	3	5	1	4	2	3	5	1	4	2	3	5
74	2	5	4	3	1	3	5	4	2	1	4	1	5	3	2
75	3	4	2	1	5	3	4	2	1	5	4	5	3	2	1
76	2	1	4	3	5	2	1	3	4	5	3	1	5	4	2
77	4	5	3	2	1	4	5	3	2	1	3	4	1	2	5
78	2	1	4	3	5	2	1	4	3	5	3	2	5	4	1
79	5	2	3	4	1	5	2	3	4	1	5	2	3	4	1
80	2	5	4	3	1	2	5	4	3	1	1	4	3	2	5
81	4	5	2	3	1	4	5	3	2	1	3	4	1	2	5
82	2	5	3	4	1	3	5	2	4	1	3	2	4	5	1
83	4	5	2	3	1	4	5	2	3	1	5	2	3	4	1
84	2	1	4	3	5	3	1	4	2	5	3	2	5	4	1
85	5	1	4	2	3	5	1	4	2	3	5	1	4	3	2
86	4	1	3	2	5	4	1	3	2	5	4	2	5	3	1
87	4	3	2	5	1	5	2	3	4	1	4	3	2	5	1
88	3	1	4	2	5	3	1	4	2	5	4	2	5	3	1
89	3	5	2	4	1	2	5	3	4	1	4	3	2	5	1
90	4	1	2	3	5	4	1	2	3	5	4	2	3	5	1
91	4	1	3	2	5	4	1	3	2	5	4	1	3	2	5
92	1	4	3	2	5	1	4	3	2	5	2	5	4	3	1
93	3	4	2	1	5	3	4	2	1	5	3	4	1	2	5
94	3	5	4	2	1	4	5	3	2	1	2	4	3	1	5
95	2	3	5	4	1	2	4	5	3	1	3	2	4	5	1
96	1	4	2	3	5	1	4	2	3	5	1	4	2	3	5
97	2	1	3	4	5	2	1	3	4	5	2	5	3	4	1
98	2	3	5	4	1	2	3	5	4	1	2	3	5	4	1
99	3	5	4	2	1	3	5	4	2	1	3	5	4	1	2
100	3	2	5	4	1	3	2	5	4	1	3	2	5	4	1
101	1	4	2	3	5	1	4	3	2	5	1	4	2	3	5
102	1	4	2	3	5	1	4	2	3	5	1	5	3	2	4
103	4	1	2	3	5	4	1	2	3	5	4	1	2	3	5
104	5	2	4	3	1	5	2	4	3	1	4	1	3	2	5
105	2	5	3	4	1	2	5	3	4	1	2	1	3	4	5
106	4	5	2	3	1	4	5	2	3	1	3	5	4	2	1
107	2	4	3	5	1	3	4	1	5	2	2	4	3	5	1
108	3	1	4	2	5	3	1	4	2	5	4	2	5	3	1

Table 5.2: Rankings: R150 Bond yields as indicator (continue)

For the periods pertaining to the upward shock time windows $\{2, 3, 5, 8, 13, 21, 36, 39, 44, 45\}$ and downward shock time windows $\{10, 11, 13, 14, 16, 22, 23, 26, 32, 33, 34, 36, 40, 42, 51, 52, 58, 60, 63, 67, 73, 79, 91, 96, 98, 100, 103\}$, adjusting returns by any of the three measure have resulted in the same sectoral rankings.

A summary of homogeneity in terms of the different risk–adjusted measures for the set of upward and downward shock time windows is given in Table 5.3. If the domination percentage, which is the minimum percentage of all time windows needed to represent the ranking–characteristics of a set of time windows as a whole (lower domination number/total number of windows) is considered, it seems that, using the σ_s–measure for adjusting returns for risk during upward shocks in the R150 Bond yields and the σ–measure for downward shocks, result in the most homogeneous set of sectoral rankings, with a domination percentage of 23% for upward shocks, and 13% for downward shocks.

Shock	Risk measure	Total windows	Domination number	Domination percentage	Dominating set
Upward	σ	52	19	37%	{3,4,8,14,19,23,25,26,29,33,34,35,39,40,46,47,48,49,50}
	σ_s	52	12	23%	{6,8,12,13,14,16,24,26,37,47,48,49}
	β	52	13	25%	{15,18,19,26,29,30,32,36,37,44,47,49,52}
Downward	σ	108	14	13%	{1,5,6,14,18,22,32,35,39,40,41,47,65,85}
	σ_s	108	15	14%	{1,5,8,22,25,28,35,36,37,44,60,66,85,86,107}
	β	108	15	14%	{1,2,4,12,13,31,33,41,44,49,54,74,82,87,97}

Table 5.3: Dominating sets: R150 Bond yields as indicator

5.3 Weekly change in R150 Bond yields

The sectoral rankings produced by adjusting returns over periods pertaining to shocks in the change in the R150 Bond yields are given in Tables 5.4–5.5. The set of time windows of which the σ–adjusted sectoral rankings are repeated across other time windows pertaining to upward shocks [downward shocks] is $\{3, 6, 7, 9, 10, 11, 12, 13, 14, 20, 24, 25, 29\}$ [$\{4, 5, 9, 13, 14, 16, 17, 22, 23, 24, 28, 34, 38, 39\}$]. Although these repetitions may indicate similar risk–return characteristics for the portfolios in time windows with similar sectoral rankings, there may be some heterogeneity in these characteristics which is discarded by the ranking–method followed.

Also, because of repetition of the σ_s–adjusted sectoral rankings of upward shock time windows $\{3, 7, 9, 10, 11, 12, 25, 26, 29, 31, 33\}$ [downward shock time windows $\{3, 4, 5, 6, 8, 9, 14, 16, 17, 22, 23, 24, 26, 28, 30, 34\}$], it may be deduced that the time windows within the set have similar downside risk to return characteristics. However, it may again be dangerous to assume complete resemblance of the time windows produced by rapid change in bond yields.

When one pays attention to the β–adjusted sectoral rankings, the rankings of the set of time windows $\{1, 2, 3, 5, 8, 10, 13, 15, 17, 29, 30, 38, 41, 42\}$ representing upward shocks and the set of time windows $\{3, 4, 6, 8, 9, 13, 14, 17, 21, 23, 28, 35\}$ representing downward shocks are repeated across other time windows.

The correspondence within the rankings of the different risk–adjusted measures has been

	σ					σ_s					β				
Upward	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
1	4	2	1	3	5	2	3	1	4	5	4	3	2	5	1
2	1	3	4	2	5	1	3	4	2	5	2	3	4	1	5
3	1	2	4	3	5	1	2	4	3	5	1	3	4	2	5
4	5	2	3	4	1	5	2	3	4	1	1	3	4	2	5
5	3	1	5	4	2	3	1	5	4	2	3	1	5	4	2
6	4	1	2	3	5	3	1	2	4	5	3	1	4	2	5
7	2	5	4	3	1	2	5	4	3	1	5	4	2	1	3
8	2	5	4	3	1	2	5	4	3	1	1	4	3	2	5
9	2	1	3	4	5	2	1	3	4	5	3	1	4	5	2
10	3	2	4	5	1	3	2	4	5	1	3	2	4	5	1
11	3	4	2	1	5	4	1	2	3	5	4	1	3	2	5
12	3	4	1	2	5	3	4	1	2	5	2	3	4	1	5
13	2	1	4	3	5	2	1	3	4	5	3	2	5	4	1
14	3	2	5	4	1	2	3	5	4	1	3	2	5	4	1
15	2	1	4	3	5	2	1	3	4	5	5	4	2	3	1
16	2	1	4	3	5	2	1	4	3	5	3	2	5	4	1
17	1	3	2	4	5	1	2	3	4	5	2	3	4	5	1
18	2	1	3	4	5	2	1	4	3	5	2	1	4	3	5
19	2	1	3	4	5	3	1	4	2	5	2	1	3	4	5
20	3	1	4	2	5	2	1	4	3	5	1	4	3	2	5
21	2	1	4	3	5	2	1	4	3	5	3	2	5	4	1
22	2	4	1	3	5	3	4	1	2	5	2	4	1	3	5
23	1	2	4	3	5	1	2	3	4	5	1	2	4	3	5
24	1	4	3	2	5	1	2	4	3	5	1	3	4	2	5
25	1	4	2	3	5	1	4	2	3	5	3	2	4	5	1
26	3	4	1	2	5	3	4	2	1	5	3	4	2	1	5
27	2	1	4	3	5	3	1	4	2	5	1	4	3	2	5
28	1	5	2	3	4	1	5	2	3	4	1	5	2	3	4
29	4	3	1	2	5	4	3	1	2	5	3	4	1	2	5
30	4	3	1	2	5	4	3	1	2	5	4	3	1	2	5
31	4	3	1	2	5	4	3	2	1	5	4	3	1	2	5
32	5	3	2	4	1	5	3	2	4	1	5	3	2	4	1
33	1	4	3	2	5	1	4	3	2	5	5	3	2	1	4
34	3	4	2	1	5	4	3	2	1	5	4	5	3	2	1
35	3	4	1	2	5	3	4	1	2	5	3	4	1	2	5
36	3	4	1	2	5	3	4	1	2	5	3	4	1	2	5
37	1	2	3	4	5	2	1	4	3	5	2	3	4	5	1
38	4	1	3	2	5	4	1	3	2	5	5	2	4	3	1
39	4	3	1	2	5	4	3	1	2	5	4	3	1	2	5
40	3	4	2	1	5	2	4	3	1	5	2	4	3	1	5

Table 5.4: Rankings: Change in R150 Bond yields as indicator

discussed in §5.1, but for all three risk–adjusted measures the rankings of a number of time windows actually do not coincide. This occurrence also provides some information on the nature of the time windows. If the ranking by the same risk–adjusted measure differs from one time window to the next, it indicates that the risk–return ratio of at least one portfolio within the first time window changed in relation to the risk return ratio of the other portfolios from the first time window to the next. However, it cannot be derived from the sectoral rankings, which portfolio (or portfolios) caused the rankings to change, and whether the change in the ratio was because of a change in returns, risk, or both.

The results regarding corresponding sectoral rankings of upward shock [downward shock] time windows by the σ– and σ_s–adjusted rankings and σ– and β–adjusted rankings are $\{2, 3, 4, 5, 7, 8, 9, 10, 12, 16, 21, 25, 28, 29, 30, 32, 33, 35, 36, 38, 39, 41, 42, 43, 44, 45, 47, 49, 52, 54, 55, 58\}$ [$\{1, 2, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 34, 35, 36, 37, 41, 42, 43, 45, 46, 51, 52\}$] and $\{5, 10, 14, 19, 22, 23, 28, 30, 31, 32, 35, 36, 39, 41, 42, 45, 46, 49, 55, 57\}$ [$\{4, 6, 7, 8, 9, 11, 12, 13, 15, 17, 25, 28, 30, 32, 35, 36, 39, 41, 46, 51\}$] respectively.

In the periods pertaining to upward shocks [downward shocks], the time windows $\{5, 10, 28, 30, 32, 35, 36, 39, 41, 42, 45, 49, 55\}$ [downward shock time windows $\{2, 6, 7, 8, 11, 12, 13, 17, 25, 28, 35, 36, 41, 46, 51\}$] rendered the same risk adjusted rankings, regardless of which of the three risk measures were applied.

By investigating Table 5.6 for the information on the homogeneity of the time windows

	σ					σ_s					β				
Upward	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
41	1	4	2	3	5	1	4	2	3	5	1	4	2	3	5
42	4	1	2	3	5	4	1	2	3	5	4	1	2	3	5
43	2	3	4	5	1	2	3	4	5	1	4	1	2	3	5
44	1	4	3	2	5	1	4	3	2	5	5	4	3	2	1
45	1	4	2	3	5	1	4	2	3	5	1	4	2	3	5
46	3	4	1	2	5	2	4	1	3	5	3	4	1	2	5
47	1	4	3	2	5	1	4	3	2	5	4	3	2	1	5
48	3	4	1	2	5	3	4	2	1	5	5	4	2	3	1
49	2	4	3	5	1	2	4	3	5	1	2	4	3	5	1
50	3	2	5	4	1	3	2	4	5	1	4	3	2	5	1
51	3	1	4	2	5	3	2	4	1	5	5	2	4	3	1
52	4	5	2	3	1	4	5	2	3	1	3	4	1	2	5
53	2	1	4	3	5	2	1	3	4	5	3	1	5	4	2
54	2	5	4	3	1	2	5	4	3	1	1	4	3	2	5
55	4	3	1	2	5	4	3	1	2	5	4	3	1	2	5
56	2	3	5	4	1	3	2	4	5	1	5	3	1	2	4
57	3	2	4	5	1	2	3	5	4	1	3	2	4	5	1
58	2	1	3	4	5	2	1	3	4	5	2	5	3	4	1
Downward															
1	2	3	5	4	1	2	3	5	4	1	4	5	1	3	2
2	2	4	3	1	5	2	4	3	1	5	2	4	3	1	5
3	1	3	4	2	5	1	2	4	3	5	1	4	3	2	5
4	3	4	2	1	5	1	4	3	2	5	3	4	2	1	5
5	3	5	2	4	1	2	5	3	4	1	4	5	2	3	1
6	3	1	4	2	5	3	1	4	2	5	3	1	4	2	5
7	3	5	4	2	1	3	5	4	2	1	3	5	4	2	1
8	2	5	4	3	1	2	5	4	3	1	2	5	4	3	1
9	5	2	3	4	1	3	2	4	5	1	5	2	3	4	1
10	1	4	3	2	5	1	4	3	2	5	2	4	5	3	1
11	5	2	3	4	1	5	2	3	4	1	5	2	3	4	1
12	4	2	3	5	1	4	2	3	5	1	4	2	3	5	1
13	3	2	4	5	1	3	2	4	5	1	3	2	4	5	1
14	4	2	1	3	5	4	2	1	3	5	5	3	2	4	1
15	3	2	4	1	5	3	1	4	2	5	3	2	4	1	5
16	4	5	3	2	1	4	5	3	2	1	3	4	2	1	5
17	3	4	1	2	5	3	4	1	2	5	3	4	1	2	5
18	2	5	3	4	1	2	5	3	4	1	4	3	5	2	1
19	2	1	3	4	5	1	3	2	4	5	3	2	4	5	1
20	4	1	2	3	5	4	1	2	3	5	5	2	3	4	1
21	4	5	3	2	1	4	5	3	2	1	4	3	2	1	5
22	5	4	2	3	1	5	4	2	3	1	5	3	2	4	1
23	4	1	3	2	5	4	1	3	2	5	5	2	4	3	1
24	4	5	2	3	1	4	5	2	3	1	3	4	1	2	5
25	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1
26	5	2	4	3	1	5	2	4	3	1	1	2	4	3	5
27	4	2	1	3	5	4	2	1	3	5	3	1	4	2	5
28	3	2	5	4	1	3	2	5	4	1	3	2	5	4	1
29	1	2	4	3	5	1	2	4	3	5	5	3	2	1	4
30	3	4	1	2	5	3	4	2	1	5	3	4	1	2	5
31	1	2	3	4	5	1	2	3	4	5	4	1	2	3	5
32	5	2	3	4	1	5	2	4	3	1	5	2	3	4	1
33	4	5	3	2	1	2	5	4	3	1	2	5	4	3	1
34	1	4	2	3	5	1	4	2	3	5	4	3	1	2	5
35	4	1	3	2	5	4	1	3	2	5	4	1	3	2	5
36	5	2	4	3	1	5	2	4	3	1	5	2	4	3	1
37	3	4	2	1	5	3	4	2	1	5	4	3	2	1	5
38	5	4	3	2	1	5	4	2	3	1	2	3	5	4	1
39	2	1	4	3	5	1	2	4	3	5	2	1	4	3	5
40	3	4	2	1	5	1	2	4	3	5	1	4	3	2	5
41	3	5	2	4	1	3	5	2	4	1	3	5	2	4	1
42	2	1	4	3	5	2	1	4	3	5	5	2	4	3	1
43	3	2	4	5	1	3	2	4	5	1	4	2	5	3	1
44	4	1	3	2	5	3	1	4	2	5	5	2	4	3	1
45	1	4	2	3	5	1	4	2	3	5	3	4	1	2	5
46	5	4	3	2	1	5	4	3	2	1	5	4	3	2	1
47	5	4	2	3	1	4	5	3	2	1	2	1	3	4	5
48	3	4	2	1	5	3	4	1	2	5	4	5	3	2	1
49	4	5	1	2	3	4	5	3	1	2	4	2	1	3	5
50	4	5	2	3	1	4	5	2	3	1	5	2	3	4	1
51	3	2	5	4	1	3	2	5	4	1	3	2	5	4	1
52	5	2	4	3	1	5	2	4	3	1	4	1	3	2	5

Table 5.5: Rankings: Change in R150 Bond yields as indicator (continue)

pertaining to upward and downward shocks by different risk–adjusted measures, both the σ_s and β– adjusted sectoral rankings of portfolios during upward shocks render a lower domination number of 12 out of 58 time windows (21%). Although their domination numbers are equal, their dominating sets differ. It should be kept in mind that the solution of the minimisation problem described in §5.1 is not necessarily unique so that more than one dominating set may be found with cardinality equal to the lower domination number. This is significant if we would like to ascertain whether specific time windows might be identified to represent the characteristics of the time window set as a whole, regardless of which risk measure is used to adjust the returns. In order to find such a set (only if the lower domination numbers of the sectoral rankings from different risk–adjusted measures are equal, there is a possibility that such a representative set may be found), one would have to engage in a search–and–compare evaluation of all the optimal solutions across the different risk–adjusted rankings. This study places more emphasis on finding the lower domination number than on the composition of the dominating sets, and although it takes note of the significance of possible results, it does not engage in such an evaluation.

Shock	Risk measure	Total windows	Domination number	Domination percentage	Dominating set
Upward	σ	29	15	52%	{1,3,5,6,7,8,9,11,15,17,22,26,27,28,29}
	σ_s	29	10	34%	{3,4,6,7,9,19,22,26,28,29}
	β	29	10	34%	{1,5,6,7,15,18,24,25,28,29}
Downward	σ	26	13	50%	{1,2,3,8,9,13,17,20,22,23,24,25}
	σ_s	26	15	58%	{1,5,8,9,10,12,13,14,15,19,20,21,22,24,26}
	β	26	9	35%	{3,5,8,15,16,18,20,25,26}

Table 5.6: Dominating sets: Change in R150 Bond yields as indicator

5.4 South African Business Confidence

The time windows pertaining to shocks in South African Business Confidence were all extreme events to the downside (see Table 5.7). After adjusting the returns of the various portfolios by the different risk measures, some similar results as discussed in the previous sections were found.

	σ					σ_s					β				
Downward	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
1	4	2	1	3	5	3	2	1	4	5	5	2	3	4	1
2	2	3	1	4	5	2	3	1	4	5	4	2	1	3	5
3	4	1	2	3	5	5	1	2	3	4	2	3	4	5	1
4	2	1	5	4	3	2	1	5	4	3	5	4	2	3	1
5	4	1	2	3	5	4	1	3	2	5	5	2	3	4	1
6	1	5	3	2	4	1	5	3	2	4	2	5	3	4	1
7	3	2	4	5	1	3	2	5	4	1	4	1	3	2	5
8	4	3	2	1	5	4	3	2	1	5	5	4	3	2	1
9	2	1	3	4	5	2	1	4	5	3	2	1	3	4	5
10	1	4	2	3	5	4	3	1	2	5	4	3	1	2	5
11	2	1	4	5	3	2	1	4	5	3	1	5	2	3	4
12	5	1	3	2	4	5	1	3	2	4	5	2	4	3	1
13	4	5	3	2	1	4	5	3	2	1	5	1	4	3	2
14	5	4	2	3	1	4	3	2	5	1	4	5	1	2	3
15	5	4	3	2	1	5	4	3	2	1	4	1	3	2	5
16	3	1	5	4	2	2	3	4	5	1	1	2	3	4	5
17	1	3	2	4	5	3	2	1	5	4	2	3	4	5	1
18	3	2	4	5	1	3	2	4	5	1	3	4	5	2	1
19	3	4	1	5	2	4	3	2	5	1	4	3	1	2	5
20	3	4	2	1	5	2	4	3	1	5	2	5	4	3	1
21	2	1	4	5	3	2	1	4	5	3	4	3	1	2	5

Table 5.7: Rankings: SA business confidence as indicator

The σ–adjusted sectoral rankings of the time window set $\{1, 7, 9, 16\}$ were repeated across other time windows; so were the σ_s–adjusted sectoral rankings of time windows $\{1, 5, 16\}$ and the β–adjusted sectoral rankings of time windows $\{1, 3, 7, 10\}$. The σ– and σ_s–adjusted rankings of time windows $\{1, 2, 3, 4, 6, 8, 10, 11, 12, 16, 17, 18, 19, 20, 21\}$ coincide and so does the σ–and β–adjusted rankings of time windows $\{6, 9, 10, 12, 17\}$.

In conclusion to this section, the results of the optimisation procedure to find the lower domination numbers for the different risk–adjusted measures are given in Table 5.8. It does not seem that the sectoral rankings by any one of the risk–adjusted measures is especially homogeneous with domination percentages of 43% for the σ–adjusted and 48% for both the σ_s– and β–adjusted measures.

Shock	Risk measure	Total windows	Domination number	Domination percentage	Dominating set
Upward	σ	-	-	-	-
	σ_s	-	-	-	-
	β	-	-	-	-
Downward	σ	21	9	43%	{2,6,10,11,13,15,16,19,20}
	σ_s	21	10	48%	{2,3,5,6,8,12,13,14,16,20}
	β	21	10	48%	{2,8,11,12,14,15,16,17,18,20}

Table 5.8: Dominating sets: SA business confidence as indicator

5.5 Monthly change in SA Business Confidence

In Table 5.9 the results of adjusting the returns of the portfolios by the three measures of risk and ranking them, rendered a repetition of the σ–adjusted sectoral rankings for upward shock time windows $\{10, 11\}$ and downward shock time windows $\{1, 2\}$. Four upward shock time windows, namely $\{4, 9, 11, 15\}$ have σ_s–adjusted sectoral rankings that are repeated across other time windows, compared to the one downward shock time window ($\{1\}$) of which the sectoral ranking is repeated. Adjusting returns by the β–measure, resulted in the sectoral rankings of upward shock time windows $\{1, 9\}$ and downward shock time window $\{1\}$ to be repeated across other time windows.

	σ					σ_s					β				
Upward	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
1	3	2	4	5	1	3	2	5	4	1	4	1	3	2	5
2	4	1	3	5	2	4	3	2	5	1	4	2	3	5	1
3	3	5	2	1	4	3	4	2	1	5	1	4	2	3	5
4	5	4	3	2	1	5	4	3	2	1	1	5	3	4	2
5	3	1	2	5	4	3	1	2	4	5	4	1	2	3	5
6	1	2	5	3	4	3	1	4	5	2	3	2	4	1	5
7	5	3	2	4	1	5	1	4	2	3	2	3	4	5	1
8	5	4	2	1	3	5	3	1	2	4	3	2	1	4	5
9	1	2	4	3	5	5	1	3	2	4	2	5	4	3	1
10	5	4	2	3	1	5	4	2	3	1	4	3	2	1	5
11	3	4	2	1	5	2	4	3	1	5	2	5	4	3	1
12	2	1	4	5	3	2	1	4	5	3	4	3	1	2	5
13	3	1	5	4	2	3	2	5	4	1	3	1	4	2	5
14	2	1	5	4	3	2	1	5	4	3	5	2	3	4	1
15	5	3	4	1	2	5	2	4	1	3	4	1	3	2	5
16	1	5	3	2	4	2	5	3	1	4	2	5	4	3	1
17	5	4	3	1	2	5	4	3	1	2	5	4	3	2	1
Downward															
1	4	1	2	3	5	4	1	2	3	5	4	3	1	2	5
2	2	4	1	3	5	2	3	1	4	5	4	5	2	3	1
3	3	5	1	2	4	3	5	2	4	1	4	3	1	2	5
4	1	4	2	3	5	4	3	1	2	5	5	4	2	3	1
5	5	2	3	4	1	5	3	2	4	1	5	4	1	3	2
6	2	1	5	4	3	3	1	5	4	2	4	1	3	2	5
7	3	1	5	4	2	3	1	5	4	2	4	5	3	2	1

Table 5.9: Rankings: Change in SA business confidence as indicator

By investigating the correspondence of their σ– and σ_s–adjusted sectoral rankings, it seems that, upward shock [downward shock] time windows $\{3, 4, 5, 11, 12, 13, 15, 16\}$ [{1, 3, 4, 5, 6}], display similar downside risk to total risk characteristics of their portfolios.

There is also a correspondence between the σ– and β–adjusted rankings of upward shock time windows $\{2, 3, 5, 6, 17\}$ and downward shock time window $\{1\}$. The fact that there are very few repetitions of sectoral rankings across the downward shock time windows, indicates that the risk characteristics of the portfolios within a time window vary significantly across the different periods.

For the periods associated with time windows $\{3, 5\}$ and time window $\{1\}$ pertaining to upward and downward shocks in the rate of change in SA Business Confidence respectively, the portfolio rankings are the same, regardless of the measure of risk used to adjust returns.

The evaluation of homogeneity of the time windows pertaining to upward shocks in Table 5.10 indicates that 47% of the time windows were needed to represent the ranking characteristics of the σ–adjusted measure, 53% to represent the characteristics inherent to the σ_s–adjusted measure and 59% for the β–adjusted measure. The 59% for the last measure suggests that the characteristics of the time windows in terms of the β–risk measure is rather diverse. When the time windows pertaining to downward shocks in the rate of change in SA business confidence are investigated as a whole in search for a minimum number of time windows to represent the ranking characteristics of the different risk–adjusted measures, the results in Table 5.10 imply that the ranking characteristics of the different time windows for all three measures are very diverse.

Shock	Risk measure	Total windows	Domination number	Domination percentage	Dominating set
Upward	σ	17	8	47%	{5,6,7,8,9,14,16,17}
	σ_s	17	9	53%	{1,2,4,5,7,8,11,12,14}
	β	17	10	59%	{3,4,5,7,8,12,13,14,16,17}
Downward	σ	7	4	57%	{1,5,6,7}
	σ_s	7	4	57%	{3,5,6,7}
	β	7	4	57%	{3,4,6,7}

Table 5.10: Dominating sets: Change in SA business confidence as indicator

5.6 United States Consumer Sentiment

Sectoral rankings of the time windows pertaining to shocks in US consumer sentiment is given in Table 5.11. For periods pertaining to upward shocks, the σ–adjusted, σ_s–adjusted and β–adjusted sectoral rankings of the respective time window sets $\{1, 2, 6, 7, 8, 9, 11, 15, 21, 25, 27\}$, $\{1, 2, 3, 5, 6, 8, 10, 11, 12, 20, 21, 25\}$ and $\{2, 3, 7, 9, 11, 13, 15, 20, 21\}$ are repeated. For time windows $\{1, 2, 4, 13, 15, 17, 18, 22, 23, 24, 25, 26, 27, 28, 30, 33, 34, 35, 36, 37, 39, 40, 43\}$, from the set of time windows pertaining to upward shocks, the σ–adjusted ranking corresponds to the σ_s–adjusted rankings. From the information at hand there seems to be a similarity in the downside risk to total risk characteristics of the portfolios within the identified time windows.

For six out of the ten time windows typified by downward shocks in US consumer sentiment, namely $\{1, 2, 3, 5, 7, 9\}$, the σ–adjusted ranking corresponds to the σ_s–adjusted rankings. The practical interpretation of the corresponding σ– and σ–adjusted rankings may be as follows. Because the South African markets (together with other emerging

	σ					σ_s					β				
upward	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
1	2	4	1	3	5	4	5	1	3	2	4	2	3	1	5
2	3	4	1	2	5	3	4	2	1	5	2	5	4	3	1
3	4	5	2	1	3	4	5	2	1	3	5	2	4	3	1
4	1	3	5	4	2	1	3	5	4	2	5	3	2	1	4
5	1	3	2	4	5	1	4	2	5	3	1	4	2	3	5
6	5	4	2	1	3	5	3	1	2	4	3	2	1	4	5
7	4	2	1	5	3	3	4	1	5	2	4	3	1	2	5
8	2	1	3	4	5	3	1	2	5	4	5	3	2	4	1
9	4	5	2	3	1	4	5	1	3	2	5	4	2	3	1
10	1	2	4	3	5	5	1	3	2	4	2	5	4	3	1
11	5	4	2	3	1	5	4	2	3	1	5	4	3	2	1
12	3	2	4	1	5	4	2	3	1	5	3	2	5	4	1
13	3	1	4	2	5	2	4	5	1	3	4	3	2	1	5
14	3	5	1	2	4	3	5	2	4	1	4	3	1	2	5
15	2	1	3	4	5	2	1	4	5	3	2	1	3	4	5
16	1	4	2	3	5	4	3	1	2	5	4	3	1	2	5
17	2	1	4	5	3	2	1	4	5	3	1	5	2	3	4
18	4	5	3	2	1	4	5	3	2	1	5	1	4	3	2
19	5	4	2	3	1	4	3	2	5	1	4	5	1	2	3
20	5	4	3	2	1	5	4	3	2	1	4	1	3	2	5
21	3	1	5	4	2	2	3	4	5	1	1	2	3	4	5
22	1	3	2	4	5	3	2	1	5	4	2	3	4	5	1
23	3	2	4	5	1	3	2	4	5	1	3	4	5	2	1
24	3	4	1	5	2	4	3	2	5	1	4	3	1	2	5
25	3	4	2	1	5	2	4	3	1	5	2	5	4	3	1
26	2	1	4	5	3	2	1	4	5	3	4	3	1	2	5
27	3	1	5	4	2	3	2	5	4	1	4	2	5	3	1
28	2	1	5	4	3	2	1	5	4	3	5	2	3	4	1
29	5	3	4	1	2	5	2	4	1	3	5	2	4	3	1
30	4	3	2	1	5	4	2	3	1	5	2	5	4	3	1
31	2	4	3	1	5	2	4	3	1	5	2	4	5	3	1
32	1	5	3	2	4	2	5	3	1	4	2	5	4	3	1
33	4	3	2	1	5	3	2	4	1	5	4	3	2	1	5
34	2	5	3	4	1	3	4	2	5	1	2	5	3	4	1
35	5	2	3	4	1	5	3	2	4	1	5	4	2	3	1
36	2	1	5	4	3	3	1	5	4	2	4	1	3	2	5
37	5	4	3	1	2	5	4	3	1	2	5	4	3	2	1
38	5	2	1	4	3	5	2	3	4	1	2	1	3	4	5
39	2	3	1	4	5	3	2	1	5	4	1	2	3	4	5
40	2	4	5	3	1	2	4	5	3	1	1	4	3	2	5
41	3	1	5	4	2	3	1	5	4	2	3	5	2	1	4
42	3	1	2	4	5	4	1	2	3	5	4	1	2	3	5
43	5	4	1	2	3	5	4	1	2	3	3	5	2	4	1
Downward															
1	2	3	1	4	5	2	3	1	4	5	4	2	1	3	5
2	4	1	2	3	5	5	1	2	3	4	2	3	4	5	1
3	2	1	5	4	3	2	1	5	4	3	5	4	2	3	1
4	4	1	2	3	5	4	1	3	2	5	5	2	3	4	1
5	1	5	3	2	4	1	5	3	2	4	2	5	3	4	1
6	3	2	4	5	1	3	2	5	4	1	4	1	3	2	5
7	4	3	2	1	5	4	3	2	1	5	5	4	3	2	1
8	1	5	3	4	2	1	5	4	2	3	4	3	2	1	5
9	4	3	2	1	5	4	3	2	1	5	4	5	3	2	1
10	3	1	4	2	5	3	1	4	2	5	2	1	3	4	5

Table 5.11: Rankings: US consumer sentiment as indicator

markets) are very dependent upon the well–being of the American economy, it may indicate that, in times of low US consumer sentiment, all portfolios in a particular time window are affected to approximately the same extent, thus explaining the similarity in downside risk characteristics.

Except for the σ–adjusted sectoral ranking of time window $\{7\}$ pertaining to downward shocks that is repeated once, no ranking of any of the risk–adjusted measures are repeated across other time windows. Furthermore, there is very little correspondence between the rankings obtained from the σ– and β–risk adjusted measures (only the rankings of time windows $\{5, 8, 10\}$ coincide). These factors may be an indication of volatility in the reaction (and therefore unpredictability) of South African markets during these times and the diversity of reactions is confirmed by the high domination percentages for the downward shocks in Table 5.12. The sectoral rankings of the σ–adjusted measure for time windows pertaining to upward shocks stand out to be most homogeneous with a domination percentage of 21%.

Shock	Risk measure	Total windows	Domination number	Domination percentage	Dominating set
Upward	σ	43	9	21%	{2,10,13,18,28,38,40,41,42}
	σ_s	43	15	42%	{2,4,9,17,19,23,24,25,27,30,34,37,38,40,41}
	β	43	13	31%	{5,6,9,12,19,22,29,31,33,34,39,41,42}
Downward	σ	10	6	60%	{1,4,5,6,7,10}
	σ_s	10	7	70%	{1,2,4,5,6,8,9}
	β	10	8	80%	{1,2,4,5,6,7,8,10}

Table 5.12: Dominating set: US consumer sentiment as indicator

5.7 Monthly change in US Consumer Sentiment

	σ					σ_s					β				
Upward	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
1	2	1	5	4	3	2	1	5	4	3	5	4	2	3	1
2	1	5	3	4	2	1	5	4	2	3	4	3	2	1	5
3	3	1	4	2	5	3	1	4	2	5	2	1	3	4	5
4	5	3	4	1	2	1	4	5	2	3	2	3	5	4	1
5	2	1	5	3	4	2	1	5	3	4	3	2	5	4	1
6	3	4	1	2	5	3	4	2	1	5	3	4	2	1	5
7	2	5	4	1	3	2	4	3	1	5	3	5	4	2	1
8	3	1	2	5	4	3	1	2	4	5	4	1	2	3	5
9	4	1	2	3	5	4	1	2	3	5	4	3	1	2	5
10	2	4	1	3	5	4	5	1	3	2	4	2	3	1	5
11	1	2	4	3	5	5	1	3	2	4	2	5	4	3	1
12	4	5	3	2	1	4	5	3	2	1	5	1	4	3	2
Downward															
1	2	3	1	4	5	2	3	1	4	5	4	2	1	3	5
2	4	1	2	3	5	5	1	2	3	4	2	3	4	5	1
3	4	1	2	3	5	4	1	3	2	5	5	2	3	4	1
4	4	1	2	3	5	3	1	2	4	5	4	1	2	3	5
5	4	1	2	3	5	4	1	2	3	5	1	2	3	4	5
6	5	1	3	2	4	5	1	3	2	4	5	2	4	3	1

Table 5.13: Rankings: Change in US consumer sentiment as indicator

For time windows pertaining to shocks in the rate of change of US consumer sentiment, the sectoral rankings after adjusting the returns of the portfolios by the three measures of risk are given in Table 5.13. For time windows pertaining to upward [downward] shocks, the σ–adjusted sectoral rankings of time windows $\{1, 3, 5\}$ $[\{2\}]$ are repeated, so are the σ_s–adjusted rankings of time windows $\{5, 6\}$ $[\{2\}]$. None of the β–adjusted sectoral rankings are repeated, which indicates that the market risk characteristics of the portfolios are rather diverse.

From Table 5.14 none of the risk–adjusted rankings stand out to produce especially homogeneous sectoral rankings.

Shock	Risk measure	Total windows	Domination number	Domination percentage	Dominating set
Upward	σ	12	6	50%	{2,3,5,7,8,12}
	σ_s	12	7	58%	{2,3,4,5,8,10,12}
	β	12	7	58%	{1,2,3,5,7,8,12}
Downward	σ	6	3	50%	{1,3,6}
	σ_s	6	3	50%	{1,5,6}
	β	6	4	67%	{2,3,4,5}

Table 5.14: Dominating set: Change in US consumer sentiment as indicator

	σ					σ_s					β				
Bull	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
1	4	1	2	3	5	4	1	2	3	5	2	3	5	4	1
2	2	1	3	4	5	1	2	3	4	5	2	3	4	5	1
3	3	1	4	2	5	3	4	2	1	5	1	2	3	4	5
4	5	1	4	2	3	5	2	4	1	3	2	3	4	5	1
5	1	4	3	2	5	2	4	3	1	5	1	4	2	3	5
6	4	1	2	3	5	4	1	2	3	5	1	2	3	4	5
7	4	5	1	3	2	4	5	1	3	2	2	5	3	4	1
8	2	3	1	4	5	3	4	1	2	5	1	2	3	4	5
9	2	4	1	3	5	2	4	1	3	5	2	5	3	4	1
10	5	4	2	3	1	5	4	1	3	2	3	4	2	5	1
11	2	4	1	3	5	2	5	1	3	4	3	2	4	5	1
12	3	5	1	2	4	1	5	2	3	4	2	5	3	4	1
13	5	4	1	2	3	5	4	1	2	3	3	2	5	4	1
14	4	2	1	3	5	2	3	1	4	5	3	2	4	5	1
15	2	1	4	3	5	3	2	5	1	4	2	3	5	4	1
16	2	3	1	5	4	2	3	1	5	4	2	4	3	5	1
17	3	1	2	4	5	1	4	2	3	5	2	3	4	5	1
Bear															
1	5	4	1	2	3	5	4	1	3	2	4	3	2	1	5
2	2	3	4	1	5	2	3	4	1	5	4	3	5	2	1
3	1	4	3	2	5	1	4	3	2	5	4	3	2	1	5
4	1	5	3	2	4	1	5	4	3	2	1	4	3	2	5
5	1	2	5	4	3	3	1	4	5	2	3	2	4	1	5
6	1	4	3	2	5	1	3	4	2	5	3	4	2	1	5
7	3	5	1	2	4	4	5	2	3	1	3	4	1	2	5
8	3	4	1	2	5	4	2	1	3	5	3	4	1	2	5
9	5	1	2	3	4	5	1	2	3	4	4	3	1	2	5
10	4	3	1	5	2	4	3	1	5	2	4	3	1	2	5
11	5	4	2	1	3	4	5	2	1	3	4	3	2	1	5
12	3	2	4	5	1	5	4	3	2	1	4	2	1	3	5
13	2	1	5	3	4	5	1	4	2	3	5	2	4	3	1

Table 5.15: Rankings: Bear and bull markets (Definition 14) as indicator

5.8 Bear and bull markets

In the time windows pertaining to a bull and bear market by Definition 14 given in Table 5.15, there is a correspondence between the σ–adjusted rankings and the β–adjusted rankings for bull market time windows $\{3, 4, 5, 8, 12, 15\}$ and bear market time windows $\{1, 3, 4, 5, 8, 11\}$, which indicates a similarity in the total risk to market risk characteristics of the portfolios in each time window. However, this also points to the fact that, for the time windows not in the identified sets, some or all of the portfolios are not diversified. If all were diversified, their standard deviations would have contained no unsystematic (unique) risk, only market risk (denoted by the beta–measure), the two risk–measures (σ and β) would have been equal and hence result in exactly the same risk–adjusted rankings. This issue was discussed in further detail in §5.1. A closely related reason for the risk–adjusted measures to differ is that the portfolios are not only non–diversified, but the levels of diversification also differ. This may cause the rankings to vary substantially. In a bull market, it is common for one or two portfolios (sectors) to have a higher beta–value than the rest. These portfolios are often the driving forces behind the bull market conditions. It should be noted that the driving sectors for the bull market might differ from time window to time window. During bear markets, most sectors tend to fall in value, but often there are drivers to the decline.

In the time windows pertaining to a bear market according to definition 14, the rankings of the σ– and σ_s–adjusted returns correspond in time windows $\{3, 4, 7, 9, 10, 11, 13\}$. For the rest of the time windows pertaining to a bear market, the various portfolios within the same time window do not have similar downside risk characteristics. Nevertheless, the rankings combinations of the σ_s–adjusted measure for bear markets are very homogeneous with a domination percentage of 31% in Table 5.16. This indicates that the diversity in the downside risk characteristics of portfolios are repeated across various time windows. In the specific case of a bull market by Definition 14, the β–adjusted measure in Table 5.16 stands out to produce a number of coinciding and similar sectoral rankings so that its lower domination number is the lowest of all risk–adjusted methods. Only 4 different sectoral rankings (or 24% of time windows) are necessary to represent the ranking characteristics of the set of time windows as a whole.

Shock	Risk measure	Total windows	Domination number	Domination percentage	Dominating set
Upward	σ	17	8	47%	{1,4,5,7,8,10,12,13}
	σ_s	17	6	35%	{1,3,12,13,14,15}
	β	17	4	24%	{3,5,13,16}
Downward	σ	13	5	38%	{2,6,7,12,13}
	σ_s	13	4	31%	{1,3,12,13,14,15}
	β	13	6	46%	{2,4,5,6,12,13}

Table 5.16: Dominating sets: Bear and bull markets (Definition 14) as indicator

	σ					σ_s					β				
Bull	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
1	4	1	2	3	5	4	1	2	3	5	5	2	3	4	1
2	5	1	4	2	3	5	2	4	1	3	2	3	4	5	1
3	1	5	2	3	4	4	5	1	2	3	2	5	3	4	1
4	4	1	2	3	5	4	1	2	3	5	1	2	3	4	5
5	5	4	1	3	2	5	4	1	3	2	5	2	4	3	1
6	4	3	1	2	5	1	2	4	3	5	2	3	5	4	1
7	5	4	2	1	3	5	4	3	2	1	3	2	5	4	1
8	4	5	2	1	3	4	5	2	1	3	4	5	3	2	1
9	2	1	4	3	5	3	2	5	1	4	2	3	5	4	1
10	1	3	4	2	5	1	3	4	2	5	2	3	5	4	1
Bear															
1	1	4	3	2	5	1	4	3	2	5	4	3	2	1	5
2	3	4	1	2	5	5	3	1	2	4	3	4	1	2	5
3	2	4	1	3	5	4	5	1	2	3	4	3	1	2	5
4	3	4	1	2	5	4	3	1	2	5	3	4	1	2	5
5	2	1	5	4	3	4	1	3	2	5	4	3	1	2	5
6	2	1	4	5	3	4	2	3	5	1	2	3	4	5	1
7	3	5	2	4	1	3	4	2	5	1	5	4	2	3	1

Table 5.17: Rankings: Bear and bull markets (Definition 15) as indicator

Except for the fact that the time windows pertaining to a bull market by Definition 15 are less homogeneous (see Table 5.18) than those by Definition 14, no new information is obtained from analysing this second set of time windows. However, if the attention is shifted to the time windows pertaining to a bear market in terms of Definition 15, the following interesting results are observed.

For 5 out of the 7 time windows pertaining to downward shocks ($\{1,3,5,6,7\}$), the sectoral rankings for all 3 risk–adjusted measures coincide. For the other time windows ($\{2,4\}$), the rankings combinations are similar, in the sense of Definition 29.

If we refer to Table 5.18 for information regarding the homogeneity of sectoral rankings of different time windows, it is found that the characteristics of time windows pertaining to a bull market are fairly diverse, regardless of the risk–adjusted measure being used. Within the time windows pertaining to bear markets, the characteristics of time windows are marginally more homogeneous, with a domination percentage of 43%, the similarities are not outstanding.

Shock	Risk measure	Total windows	Domination number	Domination percentage	Dominating set
Bull market	σ	10	8	80%	{1,3,4,5,6,7,8,9}
	σ_s	10	7	70%	{2,3,4,5,6,7,8}
	β	10	5	50%	{3,4,5,6,8}
Bear market	σ	7	3	43%	{5,6,7}
	σ_s	7	3	43%	{1,6,7}
	β	7	3	43%	{3,6,7}

Table 5.18: Dominating sets: Bear and bull markets (Definition 15) as indicator

5.9 Optimisation versus Ranking Methods

It was already mentioned in §5.1 that risk–adjusted ranking results are often used in practice as a quick reference of the order of attractiveness of one sector (or share) to another. This information serves as input to the investment decision–making process, without a complete optimisation procedure being followed. Although this is common practice, the question should still be posed whether it is in fact wise to use the rankings as interim approximations for optimisation results. Therefore, the validity of such approximations will be examined by means of correlation in this section.

Firstly, the optimisation results are translated to rankings by ranking the sector with the highest weighting first, the one with the second highest weighting second, *et cetera.* Sectors with the same weightings are attributed the same ranking numbers.

It will be recalled from Chapter 4 that the optimisation problems were solved for three different risk–profiles. The optimisation problems were also solved by assuming either variance or semi–variance as most appropriate measures of risk. In this section, the optimisation results with variance as measure of risk for all three risk profiles will be compared to the ranking results of the σ–adjusted measure, while the optimisation results for all three risk profiles using semi–variance as measure of risk will be compared to the σ_s–adjusted ranking measure. The optimisation results will be compared by sector (indicated by the vectors v_{ij} in Table 5.19(a)) to the corresponding risk–adjusted rankings (the r_j–vectors in Table 5.19(a)). This is done in order to conclude to what extent the risk–adjusted rankings per sector track the relative changes in the rankings of the sectors obtained from the optimisation results from one time window to the next. As the entries of a single vector are attained from different time windows that are assumed independent, they may be assumed to be independent as well. A linear correlation measure ρ which is calculated as

$$\rho_{\underline{v}_{ij},\underline{r}_j} = \frac{1}{n}\frac{\sum_{k=1}^{n}(v_{ij,k} - \hat{\underline{v}}_{ij})(r_{jk} - \hat{\underline{r}}_{j,k})}{\sigma_{\underline{v}_{ij}}\sigma_{\underline{r}_j}}$$

where $\underline{v}_{ij} = [v_{ij,1}, \ldots, v_{ij,n}]$, $\underline{r}_j = [r_{j,1}, \ldots, r_{j,n}]$ and $\hat{\underline{v}}_{ij}$, $\hat{\underline{r}}_j$ are the averages of $\underline{v}_{ij}$ and $\underline{r}_j$, is therefore deemed appropriate.

Table 5.19(b) gives the correlation results for both measures of risk, all risk profiles and extreme conditions in all eight market indicators. Wherever "NA" appears in the table, one of the vectors in the calculation had no variance (therefore a σ of zero), causing the calculation of ρ to include a divide–by–zero–operation, which is not allowed.

From the contents of Table 5.19(b) it is clear that there are vast differences in the resemblances of the two sets of results across the various sectors, risk profiles and extreme conditions. In some instances, where the correlation values are negative, substituting optimisation by ranking methods is inappropriate. In other cases, where the correlations are at least positive, the practitioner should first consider how accurate the approximation should be, before a ranking method is considered as substitute for an optimisation procedure.

5.10 Conclusion

In Chapters 1 and 2 the concept of risk first came to the fore. From discussions concerning the relaxations of assumptions regarding some measures of risk in §2.6 and subsequent development of further measures, even in recent times, (§§2.7.2–2.8) it may be concluded that investment professionals and academics alike have not yet laid the much debated issue of risk to rest. The fact that risk–adjusted measures incorporate the quantification of risk, the uncertainty regarding this quantification, as well as the question surrounding the attributes of a good measure of risk, is automatically imposed on the risk–adjusted measures.

The difficulty of deciding which measure of risk is the most appropriate one, and the fact that information of relative sizes of risk–adjusted returns is discarded by ranking methods, are only two of the reasons rankings are only used as a tool in decision–making, and not as the final decision–making tool. Ranking methods are computationally simpler in the sense that they do not require the much debated quantification of investor risk–tolerances or attitude towards risk to render results. At the same time, they do not provide an allocation across different sectors, only an indication of attractiveness. In order to use rankings in portfolio construction, some degree of subjective decision–making is required.

Regardless of the shortcomings of information obtained from ranking methods, they are still very much needed in practice. In §5.1, rankings have been proven valuable in understanding the nature of different sectors. Also, in §5.9 it was shown that in some instances, ranking results may be used as an alternative to optimisation and add value to the investment decision by being quicker to calculate in the fast moving equity market.

In this study historical returns and risk measures were used to obtain ranking results, but these may easily be substituted with estimated values in order for the results to be used in forecasting.

(a) Standard deviation σ, Upward Shock, Bull market (Definition 15)

	Optimisation															Ranking results				
Window	Conservative Investment					Moderate Investment					Aggressive Investment									
	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
1	2	1	2	2	2	2	1	2	2	2	2	1	2	2	2	4	1	2	3	5
2	2	1	2	2	2	2	1	2	2	2	2	1	2	2	2	2	3	4	5	1
3	2	2	1	2	2	2	2	1	2	2	2	2	1	2	2	2	5	3	4	1
4	2	1	2	2	2	2	1	2	2	2	2	1	2	2	2	2	1	3	4	5
5	3	3	1	3	2	2	2	1	2	2	2	2	1	2	2	5	2	4	3	1
6	3	2	1	4	4	3	1	2	4	4	2	1	3	3	3	1	2	4	3	5
7	4	2	3	1	4	4	3	2	1	4	3	3	1	2	3	3	4	5	2	1
8	2	2	2	1	2	3	3	2	1	3	2	2	1	2	2	4	5	3	2	1
9	1	3	4	2	4	1	2	2	2	2	1	2	2	2	2	2	3	5	4	1
10	1	2	2	2	2	1	2	2	2	2	1	2	2	2	2	1	2	4	3	5
$\underline{v}_{ij}/\underline{r}_j \rightarrow$	$\underline{v}_{11}\uparrow$	$\underline{v}_{12}\uparrow$	$\underline{v}_{13}\uparrow$	$\underline{v}_{14}\uparrow$	$\underline{v}_{15}\uparrow$	$\underline{v}_{21}\uparrow$	$\underline{v}_{22}\uparrow$	$\underline{v}_{23}\uparrow$	$\underline{v}_{24}\uparrow$	$\underline{v}_{25}\uparrow$	$\underline{v}_{31}\uparrow$	$\underline{v}_{32}\uparrow$	$\underline{v}_{33}\uparrow$	$\underline{v}_{34}\uparrow$	$\underline{v}_{35}\uparrow$	$\underline{r}_1\uparrow$	$\underline{r}_2\uparrow$	$\underline{r}_3\uparrow$	$\underline{r}_4\uparrow$	$\underline{r}_5\uparrow$
$\rho_{v_{ij},r_j} \rightarrow$	0.34	0.29	0.50	0.23	-0.09	0.25	0.73	0.11	0.29	0.00	0.38	0.60	0.02	-0.11	0.10					

(b) Standard deviation (σ)

		Conservative Investment					Moderate Investment					Aggressive Investment				
Shock	Indicator	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
Upward	Bond	0.50	0.71	0.51	-0.12	0.00	0.48	0.69	0.50	-0.04	0.10	0.58	0.66	0.49	0.12	0.09
	Bond (Change)	0.53	0.63	0.44	0.10	0.13	0.51	0.65	0.44	0.18	0.13	0.51	0.65	0.49	0.22	0.07
	SACOB	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	SACOB (Change)	0.56	0.40	0.14	0.49	0.26	0.56	0.55	0.18	0.51	0.11	0.68	0.55	0.18	0.46	0.20
	USCONF	0.36	0.57	0.39	0.56	0.10	0.40	0.51	0.35	0.51	0.02	0.55	0.56	0.33	0.45	-0.02
	USCONF (Change)	0.08	0.40	0.32	0.59	-0.62	0.23	0.40	0.20	0.59	-0.50	0.40	0.37	0.20	0.32	-0.37
	Bull (Definition 14)	0.12	0.58	0.45	0.21	-0.18	0.14	0.56	0.12	0.00	-0.11	0.02	0.65	0.12	-0.22	0.18
	Bull (Definition 15)	0.34	0.29	0.50	0.23	-0.09	0.25	0.73	0.11	0.29	0.00	0.38	0.60	0.02	-0.11	0.10
Downward	Bond	0.47	0.53	0.21	0.29	0.15	0.47	0.56	0.21	0.30	0.13	0.46	0.59	0.24	0.28	0.16
	Bond (Change)	0.57	0.50	0.32	0.24	0.07	0.53	0.46	0.32	0.30	0.17	0.51	0.46	0.32	0.30	0.20
	SACOB	0.54	0.47	0.30	0.39	-0.23	0.58	0.32	0.30	0.39	-0.37	0.67	0.45	0.23	0.36	-0.37
	SACOB (Change)	0.42	0.76	0.47	NA	-0.11	0.42	0.76	0.47	NA	-0.11	0.42	0.76	0.47	NA	-0.11
	USCONF	0.00	0.84	0.50	0.48	0.37	-0.02	0.84	0.50	0.00	0.40	-0.02	0.39	0.49	-0.18	0.40
	USCONF (Change)	0.67	0.88	0.63	-0.45	-1.00	0.67	0.88	0.63	-0.45	-1.00	0.67	0.88	0.63	-0.45	-1.00
	Bear (Definition 14)	0.77	0.51	-0.22	0.21	0.27	0.77	0.51	-0.22	0.21	0.27	0.91	0.51	-0.25	-0.02	0.27
	Bear (Definition 15)	-0.15	0.71	-0.28	-0.23	-0.09	0.15	0.71	0.50	0.42	-0.26	0.33	0.64	0.50	0.42	-0.26

Downside risk (σ_s)

		Conservative Investment					Moderate Investment					Aggressive Investment				
Shock	Indicator	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70
Upward	Bond	0.57	0.65	0.37	0.13	0.12	0.57	0.61	0.37	0.13	0.12	0.57	0.61	0.38	0.03	0.10
	Bond (Change)	0.50	0.63	0.49	0.23	0.13	0.50	0.63	0.49	0.23	0.13	0.54	0.63	0.49	0.27	0.13
	SACOB	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	SACOB (Change)	0.73	0.44	0.20	0.34	0.20	0.67	0.59	0.20	0.34	0.20	0.67	0.59	0.20	0.34	0.20
	USCONF	0.36	0.50	0.23	0.44	0.00	0.42	0.62	0.26	0.49	–0.05	0.44	0.64	0.31	0.45	0.05
	USCONF (Change)	0.10	0.17	0.21	0.39	–0.37	0.37	0.35	0.15	0.39	–0.41	0.37	0.35	0.19	0.39	–0.41
	Bull (Definition 14)	0.39	0.84	0.45	0.11	–0.49	0.16	0.82	0.48	–0.02	NA	0.16	0.82	0.48	–0.02	NA
	Bull (Definition 15)	0.61	0.55	0.11	NA	–0.27	0.77	0.53	0.11	NA	NA	0.77	0.53	0.11	NA	NA
Downward	Bond	0.45	0.59	0.28	0.25	0.16	0.46	0.59	0.23	0.25	0.16	0.47	0.63	0.26	0.25	0.17
	Bond (Change)	0.46	0.40	0.27	0.26	0.18	0.48	0.40	0.24	0.18	0.18	0.46	0.49	0.24	0.14	0.20
	SACOB	0.58	0.48	0.13	0.38	–0.33	0.58	0.59	0.19	0.48	–0.33	0.59	0.66	0.14	0.48	–0.26
	SACOB (Change)	0.28	0.76	0.45	NA	–0.11	0.28	0.76	0.45	NA	–0.11	0.28	0.76	0.45	NA	–0.11
	USCONF	–0.08	0.77	0.48	–0.18	0.40	0.20	0.51	0.56	–0.18	0.37	0.15	0.51	0.56	0.18	0.38
	USCONF (Change)	0.67	0.88	0.63	–0.45	–1.00	0.67	0.88	0.63	–0.45	–1.00	0.67	0.93	–0.13	–0.45	–1.00
	Bear (Definition 14)	0.86	0.69	–0.49	0.00	0.27	0.86	0.69	–0.49	0.00	0.27	0.86	0.69	–0.49	0.00	0.27
	Bear (Definition 15)	0.07	0.35	0.50	0.58	–0.26	0.64	0.26	0.50	0.58	–0.26	0.64	0.26	0.00	–0.15	0.09

Table 5.19: (a) Example of vectors to be correlated (b) Correlation of optimisation and ranking results

Chapter 6

Conclusion

The aim of this final chapter is to revisit some of the concepts touched upon throughout the study, to discuss where the study is believed to have contributed to investment decision–making, and within which context the suggested applications are most suited for.

In Chapter 3, the concept of a market indicator, as well as the subsequent identification of extreme events was discussed. The analyses in Chapters 4 and 5 suggested ways of interpreting the interaction of different groups of securities in order to understand the characteristics of the underlying assets under the conditions suggested by the indicator. The results are in actual fact a reflection of the characteristics innate to the extreme market condition as well. Therefore, if the indicators and accompanying extreme events are identified for the purpose of extracting characteristics, it makes sense to assess the indicators separately (as was done in this study) in order to obtain a clear and uninfluenced understanding of a specific indicator. However, if the states of different market indicators are continuously followed in order to keep track of then current market conditions and if investment decisions are made accordingly, the isolation of indicators is not necessarily optimal. Due to the complexity of financial market movements, reality does not necessarily lend itself to be assessed via a single indicator and this should be a serious consideration if market indicators and shocks are used in real–time decision–making or forecasting.

Appropriate market indicators may be incorporated in a multi–factor model to represent the aggregate state of the market, which may in turn be used to identify shocks as discussed in §3.7. There is no reason for these multi–factor models to be used only for real–time decision–making or forecasting. They may also be used retrospectively in order to test for similarities across different historic periods (which may in turn shed some light on the future.)

It was already mentioned in §4.6 that the optimal allocations obtained from the optimisation procedures are, to a great extent, reliant upon the form of the comfort/satisfaction function (from §2.2) and the associated risk tolerance factor (§4.1). Apart from the fact that, if the comfort/satisfaction function is not truly representative of the risk–return appetite of a specific investor, the optimal allocation will most probably not be entirely suitable for the investor either: the less representative a function becomes, the less practical value it holds. The comfort/satisfaction function (2.2.6) was only a first attempt at quantifying an individual's risk–return preferences. The function is not necessarily representative enough to be of practical value, but, (as was discussed in §4.3) may be substituted by any concave function deemed more appropriate by the reader.

Together with the numerous debates and diverse opinions regarding the extent to which investor behaviour may be quantified, the weight the resulting functions carry in decision–making, is just as diverse. The spectrum varies from practitioners who expend extensive resources to perfect the comfort/satisfaction functions they employ in their analyses to those who do not believe that a comfort/satisfaction function may add value to their decision–making processes at all. Instead of trying to identify the risk–return combinations with which an investor is equally comfortable and attempting to quantify this interaction in a not–too–complex function, the latter group of practitioners identify bands of volatility for the main risk–profiles within which a typical investor would be comfortable. By mixing results from optimisation procedures and subjective interpretation, a mix on the efficient frontier is identified per risk profile. The part of this study pertaining to optimisation is aimed at those who believe that value may be added to the decision–making process by quantifying an investor's risk–return preferences. It suggests a way to employ these functions in order to understand the interaction of securities in a portfolio meant for a specific risk–profile.

When our attention is shifted to the actual optimisation problems (4.3.4) and (4.3.5) used in this study, it should be mentioned that the introduction of additional constraints, like cardinality, floor and ceiling constraints may make the results practically more acceptable. However, the results of the unconstrained problem should not be discarded as these provide a picture of true optimality, which is an important basis for understanding the nature of interaction between the underlying assets and for further decisions.

In this study, the ranking methods investigated in Chapter 5 are suggested as a tool in decision–making, and not as the final decision–making tool. Ranking methods are computationally simpler than full optimisation procedures in the sense that they do not require the much debated quantification of investor risk–tolerances or attitude towards risk to render results. At the same time, they do not provide an allocation across different sectors, only an indication of attractiveness. In order to use rankings in portfolio construction, some degree of subjective decision–making is required (which is one of the factors that appeal to many practitioners). Regardless of the restrictions of the information obtained from ranking methods, they are still very much needed in practice. In §5.1, rankings were proven invaluable in understanding the nature of price movements in different sectors. Also, in §5.9 it was shown that only in some instances, ranking results may be used as an alternative to optimisation, in which cases they add value to the investment decision by being quicker to calculate in the fast moving equity market. This latter result is believed to be a contribution of this study: testing the validity of an existing decision–making strategy which is sometimes employed, without questioning the theory behind the method.

Although the main JSE market sectors were used in the analyses of the allocation strategies in this study, there is no reason for the underlying shares to be limited to such a sectoral classification. For instance, allocations may be refined further by using the sector sub–categories of the JSE as indication for the groups of securities and other asset classes like cash and bonds may be added to be used in the process. Alternatively, some asset managers rely on proprietary groupings of the shares on the South African market for allocation purposes and refer to these groups as clusters. No adjustment to the suggested process is needed to incorporate different sets of securities or assets. However, cognisance should be taken of the number of different groups included in the optimisation procedures of Chapter 4. The standard Excel Solver package can accommodate up to 200 variables (and may therefore be used to allocate capital across 200 groups of securities (or single shares). If the number of groups exceed this, the standard Solver should be upgraded to the Solver Premium Platform with the Large–Scale Non–linear programming package, or else alternative software, like Lingo, may be used.

Closely related to the issue of how the securities are grouped in the analyses, is the issue of the accompanying input data of the groups for the analyses. In this study, the actual risk and return values of the five main sectors have been used to obtain the optimal results in an attempt to understand some of the market dynamics during specific periods and their influence on different risk–profiled investments. However, it is acknowledged that historical optimal allocations are not necessarily a true indication of future optimal mixes. In order for the optimisation and ranking approaches to be used as tools for forecasting, the coefficients in the objective functions or risk–adjusted return measures may be replaced by expected values. Maybe the scenario regarding true or forecasted values is best described by the following quotation.

"*The academic is nearly always looking backward seeking to understand and explain. The practitioner is looking forward seeking to forecast [66]*".

The hope is that this study not only suggests an academic approach that will assist the investment professional in understanding the complex system with which he interacts (and from this understanding, derive still better forecasting techniques), but that the study also contributes to bringing the two fields closer together.

Appendix A

Data

The data used in this study was obtained from I-Graph (a product from the I-Net Bridge range) and is given in this appendix.

Date	BCI	USCONF	Date	BCI	USCONF	Date	BCI	USCONF	Date	BCI	USCONF
31/01/93	86.5	77	31/01/95	95.2	101.4	31/01/97	92.6	118.7	31/01/99	82.5	128.9
28/02/93	86.8	68.5	28/02/95	97.3	99.4	28/02/97	95.6	118.9	28/02/99	83.6	133.1
31/03/93	88.4	63.2	31/03/95	99.6	100.2	31/03/97	93.8	118.5	31/03/99	84.5	134
30/04/93	87.6	67.6	30/04/95	98.9	104.6	30/04/97	93.5	118.5	30/04/99	85.2	135.5
31/05/93	86.6	61.5	31/05/95	97.4	102	31/05/97	93.1	127.1	31/05/99	84.4	137.7
30/06/93	86.4	58.6	30/06/95	98.1	94.6	30/06/97	93.5	129.9	30/06/99	87	139
31/07/93	89.4	59.2	31/07/95	99.1	101.4	31/07/97	95.2	126.3	31/07/99	90.1	136.2
31/08/93	90.2	59.3	31/08/95	100.6	101	31/08/97	94.6	127.6	31/08/99	92.2	136
30/09/93	91.5	63.8	30/09/95	104.2	97.3	30/09/97	93.7	130.2	30/09/99	100.2	134.2
31/10/93	92.4	60.5	31/10/95	103.8	96.3	31/10/97	95.9	123.4	31/10/99	104.3	130.5
30/11/93	93.8	71.9	30/11/95	104.4	101.6	30/11/97	95.8	128.1	30/11/99	105.5	137
31/12/93	94.1	79.8	31/12/95	103.6	99.2	31/12/97	94.9	136.2	31/12/99	106.2	141.7
31/01/94	93.3	82.6	31/01/96	101.4	88.4	31/01/98	94.2	128.3	31/01/00	109.9	144.7
28/02/94	92.3	79.9	29/02/96	104.3	98	28/02/98	97.3	137.4	29/02/00	107.5	140.8
31/03/94	94.4	86.7	31/03/96	103	98.4	31/03/98	100.1	133.8	31/03/00	106.4	137.1
30/04/94	95.6	92.1	30/04/96	102.1	104.8	30/04/98	99.8	137.2	30/04/00	101.9	137.7
31/05/94	97.4	88.9	31/05/96	105.4	103.5	31/05/98	98	136.3	31/05/00	96.9	144.7
30/06/94	99.9	92.5	30/06/96	103.8	100.1	30/06/98	92.3	138.2	30/06/00	101	139.2
31/07/94	100.3	91.3	31/07/96	100.2	107.2	31/07/98	90.3	137.2	31/07/00	101	143
31/08/94	99.5	90.4	31/08/96	100.8	112	31/08/98	86.2	133.1	31/08/00	100.1	140.8
30/09/94	97.5	89.5	30/09/96	99.2	111.8	30/09/98	84.4	126.4	30/09/00	101.3	142.5
31/10/94	95.7	89.1	31/10/96	98.4	107.3	31/10/98	84.4	117.3	31/10/00	97.8	135.8
30/11/94	96.4	100.4	30/11/96	97.2	109.5	30/11/98	84.8	126	30/11/00	98.5	132.6
31/12/94	96.2	103.4	31/12/96	94.7	114.2	31/12/98	82.7	126.7	31/12/00	99.2	128.6

Table A.1: Data: SA Business Confidence Index (BCI) and US Consumer Sentiment Index (USCONF) (monthly) 31/01/93 – 31/12/00

Date	Bond	NCD	Date	Bond	NCD	Date	Bond	NCD	Date	Bond	NCD
31/01/93	14.47	12.45	20/06/93	14.61	12.35	07/11/93	12.77	10.55	27/03/94	13.04	10.4
07/02/93	14.37	12.15	27/06/93	14.69	12.3	14/11/93	12.38	10.5	03/04/94	13.08	10.35
14/02/93	14.37	11.8	04/07/93	14.36	12.2	21/11/93	12.3	10.4	10/04/94	13.16	10.35
21/02/93	14.3	11.75	11/07/93	14.4	12.2	28/11/93	12.57	10.4	17/04/94	13.33	10.3
28/02/93	14.42	11.9	18/07/93	14.2	12.15	05/12/93	12.34	10.4	24/04/94	12.7	10.5
07/03/93	14.41	12	25/07/93	14.15	12.05	12/12/93	12.11	10.4	01/05/94	12.39	10.4
14/03/93	14.45	12	01/08/93	13.99	12	19/12/93	12.21	10.4	08/05/94	12.6	10.75
21/03/93	14.66	12	08/08/93	14.17	11.95	26/12/93	12.07	10.4	15/05/94	12.62	10.95
28/03/93	14.45	12	15/08/93	13.9	11.95	02/01/94	11.81	10.4	22/05/94	13.28	11
04/04/93	14.67	12.2	22/08/93	13.71	12	09/01/94	11.76	10.3	29/05/94	13.54	11
11/04/93	14.78	12.2	29/08/93	13.66	12	16/01/94	11.91	10.3	05/06/94	13.62	11
18/04/93	15.29	12.35	05/09/93	13.82	12	23/01/94	12.27	10.3	12/06/94	13.97	11
25/04/93	15.15	12.45	12/09/93	13.48	12	30/01/94	12.39	10.3	19/06/94	14.06	11
02/05/93	15.07	12.45	19/09/93	13.32	12	06/02/94	12.67	10.3	26/06/94	14.22	11
09/05/93	14.91	12.45	26/09/93	13.07	12	13/02/94	12.62	10.3	03/07/94	14.69	11.1
16/05/93	15.03	12.4	03/10/93	13.1	11.7	20/02/94	12.21	10.3	10/07/94	14.97	11.1
23/05/93	14.89	12.55	10/10/93	12.91	11.6	27/02/94	12.64	10.4	17/07/94	14.67	11.1
30/05/93	14.96	12.55	17/10/93	12.95	11.6	06/03/94	12.68	10.4	24/07/94	14.71	11.25
06/06/93	14.78	12.45	24/10/93	13.2	11.4	13/03/94	12.62	10.4	31/07/94	14.95	11.2
13/06/93	14.68	12.4	31/10/93	13.02	10.7	20/03/94	12.68	10.4	07/08/94	14.79	11.2

Table A.2: Data: R150 Bond and 90–day NCD yields (weekly) 31/01/93 – 07/08/94

Date	Bond	NCD	Date	Bond	NCD	Date	Bond	NCD	Date	Bond	NCD
14/08/94	14.95	11.15	24/03/96	14.86	14.6	02/11/97	14.5	15.65	13/06/99	14.42	14.2
21/08/94	15.32	11.15	31/03/96	15.13	14.65	09/11/97	14.81	15.6	20/06/99	14.39	14
28/08/94	16.25	11.1	07/04/96	15.43	14.75	16/11/97	14.65	15.6	27/06/99	14.53	13.8
04/09/94	16.56	10.95	14/04/96	15.69	14.75	23/11/97	14.15	15.6	04/07/99	14.28	13.65
11/09/94	16.65	10.9	21/04/96	15.56	14.8	30/11/97	14.25	15.6	11/07/99	14.07	12.85
18/09/94	16.52	10.9	28/04/96	15.95	15.65	07/12/97	14.12	15.6	18/07/99	14.28	12.7
25/09/94	16.32	10.9	05/05/96	16.38	15.8	14/12/97	14.14	15.6	25/07/99	14.1	12.3
02/10/94	16.7	12.05	12/05/96	16.38	17	21/12/97	13.99	15.6	01/08/99	14.52	12.2
09/10/94	16.91	12.3	19/05/96	16.14	17.35	28/12/97	13.83	15.6	08/08/99	14.84	12.25
16/10/94	16.57	12.3	26/05/96	16.35	16.95	04/01/98	13.71	15.6	15/08/99	14.65	12.25
23/10/94	16.53	12.4	02/06/96	15.88	17.05	11/01/98	13.62	15.35	22/08/99	14.56	12.25
30/10/94	16.7	12.5	09/06/96	15.71	16.75	18/01/98	13.42	15.25	29/08/99	14.53	12.25
06/11/94	16.59	12.55	16/06/96	15.63	16.5	25/01/98	13.52	15.15	05/09/99	14.7	12.05
13/11/94	16.77	12.55	23/06/96	15.43	16.5	01/02/98	13.44	15.1	12/09/99	14.74	11.9
20/11/94	16.7	12.55	30/06/96	14.69	15.85	08/02/98	13.31	15	19/09/99	14.7	11.85
27/11/94	16.43	12.55	07/07/96	14.97	15.7	15/02/98	13.39	14.7	26/09/99	14.39	11.8
04/12/94	16.42	12.7	14/07/96	15.1	15.75	22/02/98	13.25	14.6	03/10/99	14.38	11.8
11/12/94	16.43	12.7	21/07/96	15.07	15.85	01/03/98	13.21	14.35	10/10/99	14.26	11.65
18/12/94	16.55	12.7	28/07/96	15.5	16.1	08/03/98	13.26	14.25	17/10/99	14.45	11.45
25/12/94	16.78	12.7	04/08/96	15.35	16.1	15/03/98	13.21	13.75	24/10/99	14.11	11.45
01/01/95	16.68	12.85	11/08/96	15.52	16.85	22/03/98	13.08	13.6	31/10/99	14.08	11.45
08/01/95	16.89	12.85	18/08/96	15.59	16.85	29/03/98	12.82	13.5	07/11/99	13.85	11.6
15/01/95	16.74	12.85	25/08/96	15.52	16.8	05/04/98	12.83	13.45	14/11/99	13.64	11.6
22/01/95	16.7	12.95	01/09/96	15.24	16.2	12/04/98	12.58	13.45	21/11/99	13.41	11.6
29/01/95	16.71	13.2	08/09/96	15.36	16.1	19/04/98	12.43	13.4	28/11/99	13.78	11.3
05/02/95	16.72	13.05	15/09/96	14.97	15.9	26/04/98	12.66	13.4	05/12/99	13.63	11.3
12/02/95	16.66	13.15	22/09/96	15.02	15.9	03/05/98	12.68	13.4	12/12/99	13.46	11.3
19/02/95	16.6	13.15	29/09/96	15.07	15.9	10/05/98	12.85	13.4	19/12/99	13.25	11.3
26/02/95	16.56	13.7	06/10/96	15.25	15.95	17/05/98	13.13	13.6	26/12/99	13.14	11.25
05/03/95	16.55	13.7	13/10/96	15.28	15.95	24/05/98	13.2	13.7	02/01/00	13.19	11.25
12/03/95	16.65	13.65	20/10/96	15.32	16.1	31/05/98	13.73	15	09/01/00	13.11	11
19/03/95	16.48	13.6	27/10/96	15.73	16.15	07/06/98	13.85	15.9	16/01/00	12.75	10.35
26/03/95	16.61	13.6	03/11/96	15.84	16.5	14/06/98	14.66	17.05	23/01/00	12.82	10.15
02/04/95	16.57	13.6	10/11/96	15.83	16.9	21/06/98	14.69	17.8	30/01/00	13.29	10.15
09/04/95	16.52	13.65	17/11/96	15.94	17.2	28/06/98	14.6	19.1	06/02/00	12.84	10.15
16/04/95	16.54	13.6	24/11/96	15.96	17.2	05/07/98	15.88	20.5	13/02/00	12.95	10.1
23/04/95	16.66	13.7	01/12/96	15.89	17.35	12/07/98	16.17	20.75	20/02/00	13.11	10.1
30/04/95	16.88	14.25	08/12/96	15.97	17.45	19/07/98	16.31	21	27/02/00	12.9	10.1
07/05/95	16.85	14.25	15/12/96	16.2	17.6	26/07/98	15.88	21	05/03/00	13.41	10.1
14/05/95	16.85	14.35	22/12/96	16.14	17.75	02/08/98	15.73	21.1	12/03/00	13.39	10.1
21/05/95	16.93	14.35	29/12/96	16.14	17.75	09/08/98	16.34	21.3	19/03/00	13.53	10.1
28/05/95	17	14.45	05/01/97	16.14	17.75	16/08/98	16.67	21.5	26/03/00	13.31	10.1
04/06/95	16.86	14.55	12/01/97	15.81	17.6	23/08/98	17.73	21.5	02/04/00	13.57	10.1
11/06/95	16.81	14.5	19/01/97	15.56	17.5	30/08/98	20.35	23.5	09/04/00	13.72	10.2
18/06/95	16.74	14.45	26/01/97	15.47	17.4	06/09/98	18.83	23.5	16/04/00	13.58	10.2
25/06/95	16.59	14.45	02/02/97	15.19	16.8	13/09/98	19.55	23.5	23/04/00	13.96	10.1
02/07/95	16.69	14.4	09/02/97	14.95	16.65	20/09/98	18.35	22.7	30/04/00	13.89	10.1
09/07/95	16.61	14.6	16/02/97	14.75	16.65	27/09/98	17.34	21.4	07/05/00	14.13	10.15
16/07/95	16.52	14.6	23/02/97	14.65	16.7	04/10/98	17.7	22	14/05/00	14.29	10.45
23/07/95	16.66	14.6	02/03/97	14.93	16.7	11/10/98	16.77	21.5	21/05/00	14.31	10.5
30/07/95	16.47	14.6	09/03/97	15.01	16.7	18/10/98	16	20.8	28/05/00	14.14	10.6
06/08/95	16.41	14.6	16/03/97	14.88	16.7	25/10/98	16.15	20.4	04/06/00	13.66	10.55
13/08/95	15.94	14.45	23/03/97	15.15	16.7	01/11/98	15.53	19.75	11/06/00	13.9	10.5
20/08/95	15.76	14.45	30/03/97	15.12	16.7	08/11/98	15.65	19.15	18/06/00	13.94	10.5
27/08/95	15.6	14.45	06/04/97	15.15	16.7	15/11/98	16.48	19.15	25/06/00	13.77	10.5
03/09/95	15.68	14.45	13/04/97	15.29	16.75	22/11/98	15.92	19.1	02/07/00	13.6	10.5
10/09/95	15.64	14.45	20/04/97	14.9	16.65	29/11/98	16.02	18.95	09/07/00	13.3	10.5
17/09/95	15.39	14.55	27/04/97	14.88	16.6	06/12/98	16.31	18.8	16/07/00	13.26	10.5
24/09/95	15.43	14.6	04/05/97	14.82	16.6	13/12/98	16.48	18.8	23/07/00	13.16	10.5
01/10/95	15.09	14.55	11/05/97	14.91	16.6	20/12/98	16.52	18.8	30/07/00	13.1	10.5
08/10/95	15.02	14.45	18/05/97	14.89	16.55	27/12/98	16.16	18.8	06/08/00	12.87	10.5
15/10/95	15.19	14.45	25/05/97	15.03	16.5	03/01/99	15.94	18.55	13/08/00	12.71	10.5
22/10/95	15.32	14.45	01/06/97	14.86	16.5	10/01/99	15.39	17.75	20/08/00	12.92	10.5
29/10/95	14.92	14.45	08/06/97	14.61	16.3	17/01/99	16.1	17.5	27/08/00	12.74	10.5
05/11/95	14.68	14.5	15/06/97	14.62	16.2	24/01/99	16.13	17.5	03/09/00	12.9	10.45
12/11/95	14.13	14.65	22/06/97	14.63	16.2	31/01/99	15.73	17.3	10/09/00	12.96	10.5
19/11/95	14.21	14.75	29/06/97	14.3	16	07/02/99	14.97	16.9	17/09/00	13.03	10.55
26/11/95	14.32	14.75	06/07/97	14.1	15.85	14/02/99	14.82	16.5	24/09/00	12.99	10.45
03/12/95	14.34	14.8	13/07/97	14.3	15.85	21/02/99	14.82	16.25	01/10/00	12.8	10.5
10/12/95	14.61	14.95	20/07/97	14.39	15.85	28/02/99	14.3	16.15	08/10/00	12.74	10.55
17/12/95	14.43	15.05	27/07/97	13.98	15.8	07/03/99	14.09	15.75	15/10/00	12.99	10.55
24/12/95	14.38	15.1	03/08/97	14.09	15.7	14/03/99	14.26	15.5	22/10/00	12.98	10.65
31/12/95	14.22	15.1	10/08/97	14.26	15.65	21/03/99	14.33	15.45	29/10/00	13.06	10.7
07/01/96	13.9	14.8	17/08/97	14.1	15.65	28/03/99	14.55	15.25	05/11/00	12.69	10.7
14/01/96	13.73	14.7	24/08/97	14.23	15.65	04/04/99	14.54	15.05	12/11/00	12.63	10.7
21/01/96	13.53	14.65	31/08/97	14.22	15.65	11/04/99	14.27	14.8	19/11/00	12.57	10.7
28/01/96	13.74	14.65	07/09/97	14.15	15.55	18/04/99	14.32	14.75	26/11/00	12.63	10.7
04/02/96	13.76	14.65	14/09/97	14.08	15.55	25/04/99	14.2	14.6	03/12/00	12.31	10.7
11/02/96	13.74	14.5	21/09/97	13.94	15.5	02/05/99	14.22	14.4	10/12/00	12.3	10.7
18/02/96	13.7	14.45	28/09/97	13.91	15.55	09/05/99	14.43	14.3	17/12/00	12.11	10.75
25/02/96	14.28	14.6	05/10/97	13.71	15.55	16/05/99	15.1	14.3	24/12/00	11.95	10.7
03/03/96	14.75	14.65	12/10/97	13.79	15.55	23/05/99	15.03	14.3	31/12/00	12.03	10.7
10/03/96	15.03	14.75	19/10/97	13.7	15.4	30/05/99	15.16	14.3			
17/03/96	15.06	14.7	26/10/97	14.05	14.85	06/06/99	14.31	14.3			

Table A.3: Data: R150 Bond and 90–day NCD yields (weekly) 14/08/94 – 31/12/00

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
04/01/93	3253	2325.3	1756.3	3004.6	4185.5	335.8	29/04/93	3748	3220.3	1885.2	3308.6	4170.2	309.4
05/01/93	3249	2316.1	1771	3007.3	4183.2	335.3	30/04/93	3733	3174.8	1883.6	3337.2	4178.5	307.4
06/01/93	3280	2345.1	1806.7	3012.4	4211.9	334.4	03/05/93	3777	3249.1	1884.3	3338.9	4191.8	306.8
07/01/93	3334	2377.6	1846.8	3075	4274	334	04/05/93	3781	3251.6	1901	3342.2	4185.7	306.4
08/01/93	3382	2418.1	1877.3	3096.7	4322.4	333.3	05/05/93	3760	3221.8	1912.5	3335.4	4176.2	306.1
11/01/93	3411	2447	1895.2	3119.3	4345.3	333.3	06/05/93	3766	3223.8	1928.3	3349.5	4180.6	304.9
12/01/93	3408	2435.9	1895.4	3130.7	4349.2	332.4	07/05/93	3771	3228.7	1929.5	3364.2	4175.9	304.3
13/01/93	3402	2424.5	1884.1	3143	4348.9	333	10/05/93	3746	3214.3	1890	3364.2	4160.4	303.7
14/01/93	3418	2438.7	1894.7	3144.4	4352.1	333	11/05/93	3747	3197.3	1892.4	3373.7	4170.7	303
15/01/93	3415	2435.1	1874.6	3148	4365.5	333.4	12/05/93	3764	3225.4	1875.2	3379.1	4203.3	302.2
18/01/93	3415	2432.5	1864.2	3159	4376.3	333.1	13/05/93	3888	3432.6	1883	3398.3	4238.9	301.6
19/01/93	3382	2392.4	1838.7	3155.5	4369.6	333.5	14/05/93	3904	3474	1875.8	3416.3	4223.2	302
20/01/93	3360	2375.5	1843.7	3140	4350.7	332	17/05/93	3892	3445.3	1874.2	3397.9	4236.7	302
21/01/93	3382	2404.3	1846.4	3137.9	4354.1	332.9	18/05/93	3873	3400.1	1866.3	3400.4	4261.5	302.5
22/01/93	3404	2439.3	1861.5	3144.9	4362.5	332	19/05/93	3984	3598.9	1885.6	3422.3	4283.8	301
25/01/93	3399	2439.4	1872.5	3161.7	4339.8	332.3	21/05/93	3986	3584.8	1881.5	3446.5	4296	300.4
26/01/93	3405	2448.5	1893.5	3173	4324.5	332.9	24/05/93	4032	3665.5	1897.7	3459.5	4298.8	299.1
27/01/93	3406	2453.2	1886.4	3176.3	4322.2	332.5	25/05/93	4010	3626.3	1873.3	3454.9	4306	297.6
28/01/93	3415	2454.7	1880.1	3200.2	4327.3	333.2	26/05/93	3999	3601.4	1886.8	3466.7	4310.6	297
29/01/93	3433	2476.8	1878.9	3205.1	4345.3	332.7	27/05/93	3984	3565.7	1886.7	3490.6	4303.1	297.4
01/02/93	3433	2475.7	1881.6	3208.7	4346.8	332.6	28/05/93	3992	3571.9	1898.5	3488.8	4314.1	299.9
02/02/93	3452	2479.1	1905.9	3259.6	4361.2	333.4	01/06/93	3981	3564.3	1896.4	3463.7	4303.7	298.3
03/02/93	3457	2487	1910.2	3275.3	4354.7	333.7	02/06/93	3899	3410	1901	3454.3	4281.8	298.5
04/02/93	3478	2493.2	1941.7	3287.2	4385.8	332.9	03/06/93	3942	3482.8	1910	3457.1	4288	299.1
05/02/93	3473	2469.5	1951.1	3307.5	4390.5	332.1	04/06/93	3958	3493.9	1911.2	3466.7	4312.6	298.9
08/02/93	3452	2443.9	1951.8	3303.3	4373.1	332.2	07/06/93	3933	3435	1931	3480.9	4308.3	300.9
09/02/93	3474	2465.3	1960.7	3302.4	4388	331.9	08/06/93	3927	3409.9	1955.5	3494.2	4305.1	300
10/02/93	3541	2553.5	1956.7	3347.2	4420.6	331.9	09/06/93	3891	3330.4	1939.7	3494.3	4310.7	301.4
11/02/93	3550	2606.5	1935.3	3336.6	4404.3	332	10/06/93	3932	3376.7	1952.1	3507.3	4336.1	299.9
12/02/93	3492	2529.8	1923.9	3313.2	4388.5	331.7	11/06/93	3949	3392	1953.6	3541.6	4344.8	301.4
15/02/93	3474	2518.6	1908	3314.8	4371.8	329.9	14/06/93	3904	3313.6	1955	3549.5	4339.6	301.5
16/02/93	3507	2564	1935.6	3311.1	4394.9	330.7	15/06/93	3932	3328.1	1995.3	3562	4363.4	301.9
17/02/93	3483	2560.9	1917.9	3302.2	4355.8	330.3	16/06/93	3953	3357.7	1991.3	3569.5	4370.8	302.2
18/02/93	3482	2558.4	1913.7	3289.9	4360.4	329	17/06/93	3978	3384.5	1996.7	3580	4393.8	302.5
19/02/93	3488	2558.5	1919.1	3299.7	4358	329.1	18/06/93	3982	3398.8	1994	3596.4	4390.8	303.1
22/02/93	3463	2534.3	1910.6	3285.3	4340.8	327	21/06/93	4011	3447.3	2003.6	3608.4	4392.8	303.2
23/02/93	3447	2513.5	1905	3287.2	4336.1	325.7	22/06/93	4006	3422.2	2007.8	3617.8	4414.8	303.2
24/02/93	3438	2494.6	1920.4	3299.2	4329.7	323.2	23/06/93	3993	3392.8	2002.8	3621.7	4427.3	304.1
25/02/93	3426	2478.6	1918.4	3296.4	4318.4	322.9	24/06/93	4027	3462	1999.7	3617.7	4436.2	305.6
26/02/93	3418	2470.5	1931.3	3305.9	4307.4	323.6	25/06/93	4057	3514.1	2006.6	3629.8	4450.3	306.5
01/03/93	3423	2472.3	1955.5	3295.5	4305.8	323.1	28/06/93	4074	3556.9	2006.6	3623.6	4447	306.4
02/03/93	3397	2446.8	1928	3283	4287.6	322.8	29/06/93	4054	3537.3	1992.7	3618	4437.4	308
03/03/93	3367	2422.5	1905	3270.6	4263.2	323	30/06/93	4078	3581.3	2005.6	3621.6	4444.1	309.3
04/03/93	3349	2408.3	1905.7	3253.3	4237.6	322.7	01/07/93	4099	3626.1	2003.9	3616.3	4430.5	309.8
05/03/93	3385	2467.2	1901.5	3255.8	4230.2	323	02/07/93	4147	3723.6	2007.9	3619.7	4420.5	311.7
08/03/93	3391	2466.6	1894.1	3257.4	4243.6	322.5	05/07/93	4174	3747	2039.6	3619.2	4428.4	311.9
09/03/93	3425	2502.1	1889.5	3267.7	4270.8	321.9	06/07/93	4210	3819.6	2050.1	3600.9	4438.6	313.1
10/03/93	3419	2485.8	1899.6	3272.9	4275.3	320.9	07/07/93	4213	3836.2	2060.8	3597.7	4434	313.4
11/03/93	3439	2507	1903.2	3310.5	4279.5	321.6	08/07/93	4145	3727.5	2048.1	3580.2	4412.3	315.9
12/03/93	3450	2528.5	1906.5	3333.4	4279.4	323.2	09/07/93	4117	3699.4	2012.3	3555	4388.1	315.9
15/03/93	3459	2540.1	1909.5	3319.5	4288.5	323.6	12/07/93	4103	3692.9	1996.9	3558.9	4374.3	316.7
16/03/93	3454	2569.6	1906.5	3295	4261.3	322.9	13/07/93	4090	3677	1987.5	3548.8	4366.6	317.1
17/03/93	3451	2567.8	1901.6	3288.3	4265.6	323	14/07/93	4134	3744.5	1994.5	3560.4	4368.3	317.5
18/03/93	3477	2592.5	1850.9	3300	4306.6	321.8	15/07/93	4113	3708.2	1994.6	3555	4367.1	318.5
19/03/93	3528	2665.8	1851.4	3328	4341.2	321.7	16/07/93	4106	3693.8	1986.4	3544.8	4371.5	319.8
22/03/93	3529	2676.8	1854	3335.1	4322	321.3	19/07/93	4089	3682.4	1973.1	3527.2	4352.6	320.3
23/03/93	3522	2663.4	1836	3357.2	4317.2	322.2	20/07/93	4055	3647.1	1947	3515.3	4340.6	320.4
24/03/93	3516	2653	1844.8	3363.2	4307.2	323.9	21/07/93	4028	3609.3	1954.2	3512.2	4319.7	320.5
25/03/93	3563	2718.8	1857.5	3361.5	4335.1	324.6	22/07/93	3991	3552.5	1934.7	3498.5	4310.4	321
26/03/93	3556	2710.1	1863.2	3363.8	4320.5	323	23/07/93	3999	3576.7	1914	3499.6	4296.1	319.8
29/03/93	3506	2662.5	1842.8	3351.1	4281.3	322.5	26/07/93	3986	3564.9	1908.2	3489.6	4281.1	319.5
30/03/93	3535	2705.1	1852.3	3346.8	4296.5	322.7	27/07/93	4023	3623	1908.8	3509.5	4281.8	319.8
31/03/93	3560	2769	1819.2	3354.7	4296.8	322.5	28/07/93	4050	3664.9	1916.4	3505.9	4290.1	319.8
01/04/93	3544	2756.9	1810.9	3324	4283.9	322.9	29/07/93	4085	3711.9	1932.4	3504.8	4324.1	319.4
02/04/93	3571	2828.9	1802.4	3302.4	4270.7	321.2	30/07/93	4177	3837.7	1937.3	3562.4	4351.2	319.8
05/04/93	3586	2890.4	1791.7	3289.6	4238.1	320.6	02/08/93	4163	3821.8	1941.1	3522.6	4363.6	320
07/04/93	3565	2874.4	1788.1	3280.7	4212.9	319.6	03/08/93	4124	3754.8	1932.6	3526.2	4357.5	321.1
08/04/93	3589	2894.8	1820.9	3283	4215.6	320	04/08/93	4131	3754.4	1940.4	3550.8	4354	322.1
13/04/93	3530	2815.4	1803.8	3253	4199.9	318.5	05/08/93	4060	3638.4	1921.6	3541.1	4335.6	321.9
14/04/93	3498	2768.3	1790.6	3249.5	4193.2	318.7	06/08/93	4010	3546.7	1902.2	3519.6	4329.6	321.3
15/04/93	3545	2825.7	1796.6	3261.5	4215.5	318.1	09/08/93	4041	3591.9	1900.9	3524.3	4348.4	321.3
16/04/93	3563	2844.8	1816.9	3306.5	4205.1	317.7	10/08/93	4042	3619.7	1877.6	3487.2	4348.7	320.9
19/04/93	3558	2861.8	1813.3	3293.1	4175.2	315	11/08/93	4042	3613.4	1882	3500.3	4345.7	320.6
20/04/93	3582	2892.1	1827.4	3298.1	4181.6	315.1	12/08/93	4032	3587	1882.6	3498.6	4349.7	321.2
21/04/93	3572	2867.8	1860.9	3291	4176.1	315	13/08/93	3961	3472.4	1877.9	3497.9	4323	319.7
22/04/93	3565	2863.2	1860.6	3298	4158.3	314.3	16/08/93	3990	3535.3	1883.4	3498.2	4313	317.2
23/04/93	3610	2956.6	1861.8	3294.7	4159.5	313	17/08/93	4007	3560.2	1892	3507.6	4313.9	318
26/04/93	3686	3112.7	1854.2	3290.9	4167	312.4	18/08/93	4025	3568.9	1898.8	3515.4	4338.4	317.7
27/04/93	3692	3115.4	1882.5	3287.1	4171.8	311	19/08/93	4075	3618.5	1916.8	3534	4377.5	317.5
28/04/93	3693	3114.4	1887.7	3281.2	4165.1	311.4	20/08/93	4044	3562.5	1923	3536.3	4370.4	317.2

Table A.4: Data: Equity indices (daily) 04/01/93 – 20/08/93

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
23/08/93	4049	3564.7	1940.6	3537.3	4375.3	316.9	13/12/93	4547	4057.5	1929.7	3749.7	4948.9	315.5
24/08/93	4051	3572.6	1942.3	3537.3	4371.5	316.7	14/12/93	4594	4105.2	1940.5	3796.2	4975.5	316
25/08/93	4045	3551.7	1920.7	3542.1	4384.3	317.2	15/12/93	4635	4129.3	1961.8	3822.5	5002.4	318.7
26/08/93	4054	3566.2	1895.7	3541	4397.9	317	17/12/93	4610	4092.7	1956.2	3838.4	4995	324.8
27/08/93	4017	3497.2	1898.8	3545.4	4393.9	318	20/12/93	4647	4158	1965.4	3833.5	4999.4	324.5
30/08/93	4010	3481.4	1892.4	3547.1	4395.9	318	21/12/93	4608	4086.9	1986.9	3841.2	4991.3	325
31/08/93	4034	3515	1903.7	3541.7	4406.8	317.8	22/12/93	4612	4065.2	1991.4	3854.3	5016.4	326.5
01/09/93	4027	3508.6	1900.7	3540.8	4403.5	318	23/12/93	4641	4074	2015.4	3862.2	5073.9	328.5
02/09/93	4010	3478.8	1897.9	3544.5	4393.9	319	24/12/93	4720	4135.6	2045.3	3921.2	5151.4	329.3
03/09/93	3955	3381.9	1887.6	3532.5	4378.2	319.4	27/12/93	4789	4197.1	2078.1	3941.4	5224.1	331.3
06/09/93	3952	3387.6	1879.4	3513.6	4374.9	318.8	28/12/93	4797	4185.5	2094.2	3970.6	5260.9	335.4
07/09/93	3907	3320.5	1869	3486.8	4371	319.1	29/12/93	4801	4155.7	2091.2	3993.5	5291.2	335.6
08/09/93	3804	3167	1832	3447.1	4326.2	318.5	30/12/93	4889	4279.4	2111.1	4014.2	5326.4	339.6
09/09/93	3841	3246.9	1836.4	3451.6	4310.2	317.2	31/12/93	4893	4277.4	2111.7	4046.5	5347.5	343.9
10/09/93	3846	3246.8	1860	3454.9	4319.1	317	03/01/94	4909	4294.3	2114.1	4041.8	5367.8	344.1
13/09/93	3824	3201.9	1870.1	3446.3	4312.8	316	04/01/94	5088	4524.2	2166	4173.7	5466.6	346.4
14/09/93	3830	3211.9	1869	3444.1	4304.1	315.6	05/01/94	5086	4537.5	2157.7	4131.9	5443.8	347.5
15/09/93	3858	3253.6	1874.1	3448.6	4316.2	314.7	06/01/94	5062	4470.8	2173.2	4151.3	5465.5	348.8
16/09/93	3862	3299	1866	3444.2	4295.8	313.7	07/01/94	5082	4495.2	2166.3	4139	5500	349.1
17/09/93	3848	3290.9	1851.8	3437	4287.5	313.6	10/01/94	5044	4447.3	2162.5	4116.7	5378.2	349.7
20/09/93	3831	3276	1805.8	3436.1	4279.4	312.8	11/01/94	4961	4335.2	2149.6	4112.6	5335.6	349
21/09/93	3811	3283.8	1755.5	3418.4	4240.1	312	12/01/94	4958	4350.5	2149	4110.5	5323.7	348.5
22/09/93	3769	3265.8	1736.4	3372	4176.2	311.3	13/01/94	4834	4198.1	2116.3	4075.9	5219.7	348.7
23/09/93	3734	3223.2	1735	3349.9	4147.9	311.3	14/01/94	4861	4299.1	2093.8	4042	5179	351.3
24/09/93	3730	3200.7	1737.4	3334.2	4167.4	311	17/01/94	4865	4302.1	2092.9	4059.7	5181.2	351
27/09/93	3795	3252.3	1770.3	3352.3	4228.2	311.3	18/01/94	4852	4280.6	2095.1	4063.8	5177.8	352
28/09/93	3771	3205.5	1757.7	3380.7	4224.6	311.5	19/01/94	4820	4225.9	2113.7	4063	5154.5	353.2
29/09/93	3792	3208.5	1761.5	3397.4	4259.6	311.7	20/01/94	4855	4257.3	2135.3	4091.2	5178.3	354.5
30/09/93	3770	3166.2	1747.9	3405.9	4280.5	312.7	21/01/94	4784	4109.2	2139.8	4087.4	5179.9	354.6
01/10/93	3763	3163.3	1731.7	3413.7	4261	311.9	24/01/94	4739	4049.5	2128.9	4075.9	5154	354.3
04/10/93	3741	3155.1	1719.1	3399.8	4228.3	311.4	25/01/94	4762	4105	2137.5	4081.6	5151	353.3
05/10/93	3714	3111.5	1701.5	3397.9	4216.1	310.7	26/01/94	4756	4104.6	2124.9	4073.1	5158	354.9
06/10/93	3755	3147.3	1739.6	3421.4	4243.5	310.2	27/01/94	4792	4144.6	2129.8	4098.5	5192.7	357.3
07/10/93	3819	3249.4	1751	3459.5	4256.7	310.2	28/01/94	4725	4022.2	2126.6	4084.6	5210.3	358
08/10/93	3870	3315.3	1754.1	3492.5	4277.4	308.7	31/01/94	4755	4054.6	2126.1	4067.1	5243.4	356
11/10/93	3909	3378.1	1747.4	3503	4293.4	309.9	01/02/94	4772	4078.1	2140.2	4062.6	5260.1	356.4
12/10/93	3907	3361.6	1766.5	3514.7	4303.6	310.9	02/02/94	4899	4197.3	2180.5	4169	5354.7	357.7
13/10/93	3937	3419.3	1782.8	3517.5	4307.9	311	03/02/94	4933	4231.3	2207.6	4155.7	5378.2	358.3
14/10/93	3925	3401.9	1795	3515.8	4297.4	311.2	04/02/94	5021	4308.3	2279.9	4186.5	5438	359.2
15/10/93	3915	3378.7	1820.2	3526.4	4296.5	311.2	07/02/94	4933	4191.2	2243.1	4175.9	5379.9	358.5
18/10/93	3901	3370	1815.6	3519.7	4286.3	311.2	08/02/94	4891	4120.3	2247.2	4185.3	5369.5	358.5
19/10/93	3916	3401.9	1826.5	3523	4295.4	310.8	09/02/94	4854	4076	2249.2	4140.7	5348	356.3
20/10/93	3910	3405.6	1837.5	3518.6	4283.7	309.5	10/02/94	4845	4085.3	2270.6	4102.1	5352.4	355.8
21/10/93	3905	3401.8	1797.4	3515.8	4293.1	307.4	11/02/94	4777	4009.2	2276.4	4096.6	5298.8	355.6
22/10/93	3872	3343.8	1786.8	3519.2	4293.7	307	14/02/94	4692	3937.1	2210.2	4025.9	5232.8	355.4
25/10/93	3891	3380.9	1790.8	3506.2	4300.9	306.5	15/02/94	4742	4004.4	2223.1	3991.7	5269.9	354.8
26/10/93	3881	3373.5	1794.3	3509.4	4289.9	306.1	16/02/94	4782	4059.7	2239.7	4041.4	5260.2	353.7
27/10/93	3899	3397.5	1795.5	3527.2	4307	304.6	17/02/94	4887	4191.9	2268.2	4070.3	5329.6	353.1
28/10/93	3894	3407.7	1783.6	3513.6	4302.8	305	18/02/94	4852	4120	2271.5	4082.3	5349.8	353.8
29/10/93	3916	3419.5	1792.5	3535.9	4341.1	305.6	21/02/94	4831	4077.6	2281.2	4076.4	5355.9	354.5
01/11/93	3907	3399.7	1778.8	3549.2	4352.2	305.2	22/02/94	4823	4062.6	2275.7	4088.1	5358.1	354.5
02/11/93	3914	3395.5	1779.6	3560.8	4373.7	306.1	23/02/94	4816	4054.8	2270.8	4083.9	5364.8	353.5
03/11/93	3946	3451.8	1795.9	3566.4	4376.1	306	24/02/94	4754	3982.8	2257.9	4080.7	5307.7	354.2
04/11/93	3978	3524	1801	3567.2	4380	305.6	25/02/94	4798	4049	2282.7	4086.6	5304.4	354.1
05/11/93	4004	3585.4	1800	3566.7	4376.3	306.4	28/02/94	4846	4169.9	2277.3	4051.7	5283.4	353.7
08/11/93	4020	3523.3	1812.3	3565.6	4397.8	306.7	01/03/94	4880	4213.9	2299.4	4060.7	5279.7	353.9
09/11/93	3997	3510.2	1796.8	3537.2	4389.9	305.8	02/03/94	4881	4188.4	2335.4	4074	5288.1	354.1
10/11/93	4022	3541.3	1793.4	3516.3	4415.3	305.6	03/03/94	4931	4241.9	2363.9	4084.8	5314.6	354
11/11/93	4081	3622.6	1827.1	3535.1	4442.3	306.3	04/03/94	5010	4299.9	2372.4	4143.4	5389.2	354.4
12/11/93	4109	3659.2	1836	3551.7	4453	306.3	07/03/94	5072	4349.7	2406.3	4169.7	5451.3	354.6
15/11/93	4103	3636.8	1840.1	3542.4	4472.1	305.5	08/03/94	5118	4352.2	2502.8	4209.7	5530	355
16/11/93	4126	3630	1860.4	3573.3	4522.9	307.2	09/03/94	5088	4280.3	2555.4	4227.2	5549.7	355.8
17/11/93	4164	3605.1	1897.4	3636.4	4610	308	10/03/94	5137	4338.1	2548	4264.1	5601.1	355.6
18/11/93	4205	3633.9	1932.1	3691	4671.8	308.6	11/03/94	5124	4362.2	2500.9	4248.1	5569.1	357.3
19/11/93	4239	3657.3	1961.4	3736.4	4730.5	309.4	14/03/94	5156	4391.6	2543.8	4267.4	5592.4	357.1
22/11/93	4253	3705.8	1972.4	3733.1	4718.3	307.5	15/03/94	5255	4472.1	2594.2	4365.2	5662.7	356.7
23/11/93	4224	3673.1	1965.2	3706.8	4708.6	308.2	16/03/94	5202	4412.6	2602.8	4349.9	5628.4	357.1
24/11/93	4176	3594.7	1943.9	3707.3	4701.8	306.2	17/03/94	5188	4359.3	2654.1	4333.5	5661.2	358
25/11/93	4193	3621.9	1934.2	3712	4696	306.4	18/03/94	5234	4411.4	2657.6	4351.1	5697.9	359.3
26/11/93	4204	3639.6	1933.6	3697	4703.5	305.8	21/03/94	5230	4431.1	2614.5	4340.6	5693	359.2
29/11/93	4183	3607.8	1912.3	3700.8	4689.9	306.3	22/03/94	5231	4470.9	2576.6	4318.1	5666.6	359.2
30/11/93	4164	3575.2	1898.7	3669.6	4704.2	305.2	23/03/94	5183	4476.3	2515.1	4258.1	5587.4	359.7
01/12/93	4209	3611.3	1913.8	3681.9	4749	306.6	24/03/94	5241	4576.8	2527.6	4243.7	5604.7	358.6
02/12/93	4273	3690.9	1917.3	3692	4801.8	309.2	25/03/94	5171	4539.9	2529.2	4185.4	5514.1	360.4
03/12/93	4262	3671.1	1921.2	3700.7	4799	310.1	28/03/94	5026	4404.3	2479.1	4068.2	5407.3	359.7
06/12/93	4277	3705.8	1911.9	3695.8	4791.3	310.2	29/03/94	4968	4281.8	2487	4080.7	5417.7	359.3
07/12/93	4296	3717.6	1911.2	3709.9	4815.9	311.1	30/03/94	4935	4270.3	2436.3	4028.1	5373.3	358.1
08/12/93	4390	3830.6	1917.7	3758.3	4867.2	314.3	31/03/94	4939	4299.9	2428.2	4028.1	5332.8	357.1
09/12/93	4491	3969.6	1926.7	3779.4	4917.2	314.9	05/04/94	4849	4171.4	2378.5	4000.6	5300.5	318.2
10/12/93	4467	3952.7	1921.3	3740.6	4906.8	315.5	07/04/94	4858	4175.6	2407.9	3999.5	5308.7	317.9

Table A.5: Data: Equity indices (daily) 23/08/93 – 07/04/94

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
08/04/94	4953	4257.4	2449.8	4048	5399.9	318.8	05/08/94	5728	5069.1	3195.6	4738.3	6056.6	353.7
11/04/94	5009	4353.5	2447.4	4030.4	5437	318.9	08/08/94	5758	5094.3	3232.2	4742.1	6091	354.3
12/04/94	4937	4249.3	2441.9	4064.4	5381.5	318.4	09/08/94	5823	5157.2	3298.4	4751.8	6142.2	357.2
13/04/94	5023	4378	2481.2	4071.9	5416.5	318.5	10/08/94	5918	5294.8	3299.4	4788.3	6184.8	357.2
14/04/94	4954	4298.9	2465.9	4023	5379.8	318.5	11/08/94	5883	5256.6	3250.5	4812.6	6155.4	357.8
15/04/94	4968	4317.7	2519.2	4027.1	5386.4	315.8	12/08/94	5812	5166.6	3192.9	4832.3	6116	358.5
18/04/94	4970	4309.1	2539	4016.5	5405.2	316.5	15/08/94	5790	5129.4	3217.7	4816.2	6108.8	359.6
19/04/94	5080	4363.7	2619.3	4149.4	5508.9	317.4	16/08/94	5763	5098.2	3191	4806	6111	358.7
20/04/94	5045	4265	2640.3	4201	5530.4	317.8	17/08/94	5824	5172.8	3231.6	4811.3	6130.4	358.1
21/04/94	5037	4246.9	2647.8	4202.5	5547.9	318.7	18/08/94	5813	5155.7	3256.3	4807.8	6133.2	357.3
22/04/94	5040	4225	2663.2	4224.6	5576.2	318.8	19/08/94	5799	5151.3	3276.1	4774.5	6111.8	356.2
25/04/94	5068	4241.1	2675.3	4243.4	5617.1	318.6	22/08/94	5847	5239.7	3309.4	4783.1	6123	356
26/04/94	5240	4371.2	2769.5	4357.9	5803	321.1	23/08/94	5858	5237.4	3337.8	4799.9	6136.3	358
29/04/94	5359	4468.1	2853.7	4447.5	5930.4	324	24/08/94	5897	5297	3392.3	4786.8	6157.7	359.6
02/05/94	5363	4460.3	2805.5	4461.6	5978.8	323.4	25/08/94	5916	5322.4	3405.7	4814.3	6176.5	359.5
03/05/94	5291	4353.7	2784.5	4465.5	5970.2	323.1	26/08/94	5859	5273.2	3380.9	4782.5	6148.3	359.5
04/05/94	5254	4291.7	2756.1	4455.2	5994.3	322.7	29/08/94	5823	5230	3371.6	4769.5	6129	358.8
05/05/94	5232	4264.6	2745.6	4470.4	5986.7	328	30/08/94	5846	5302.6	3356.4	4767.8	6095.2	359.2
06/05/94	5309	4274.7	2782.6	4548.4	6129.5	328.9	31/08/94	5834	5303.8	3350.8	4744.5	6089.1	358.5
09/05/94	5440	4391.1	2815.4	4736.6	6214.2	329.3	01/09/94	5868	5354.4	3395.6	4757.3	6079.9	358.6
11/05/94	5456	4404	2815.4	4738.2	6250.9	329.6	02/09/94	5915	5406.1	3446.5	4755.5	6094.6	355.8
13/05/94	5534	4497.1	2819.6	4773.7	6334.8	332.1	05/09/94	5956	5470.2	3484.2	4742.1	6102.6	357
16/05/94	5497	4461.1	2832.8	4730.2	6339.2	332.5	06/09/94	6009	5548.5	3553.3	4720.5	6126.1	356.2
17/05/94	5457	4448.9	2798.7	4707	6275.9	333.2	07/09/94	6054	5592.9	3604.6	4772.9	6126.7	356.3
18/05/94	5388	4384	2748.6	4631.4	6257.7	333.4	08/09/94	6008	5540.5	3586.8	4754.4	6100.3	356.3
19/05/94	5440	4493.2	2749.2	4623.4	6239.9	333.4	09/09/94	5955	5481	3560.7	4728.5	6091.4	355.1
20/05/94	5474	4515.8	2745.2	4636.8	6293.2	335	12/09/94	5883	5384.4	3520.6	4719.3	6071.4	354.9
23/05/94	5525	4596.6	2791.5	4669.3	6302.8	335.2	13/09/94	5858	5364.3	3500	4699.9	6048.9	355.3
24/05/94	5518	4589.6	2825.2	4662.4	6292.1	335.3	14/09/94	5874	5387.6	3507	4694.3	6047.1	356
25/05/94	5486	4514.4	2827.9	4666.9	6307.5	335.9	15/09/94	5841	5338.6	3490.2	4704.1	6030.5	356.5
26/05/94	5391	4392.8	2808.6	4627.5	6266.8	337.1	16/09/94	5831	5316.9	3511.2	4718.6	6021.1	353.8
27/05/94	5382	4397.1	2804	4604.1	6250.6	335.5	19/09/94	5764	5238.9	3481.9	4715.5	5967.6	354.5
30/05/94	5396	4433	2807.9	4605.7	6227.7	336.9	20/09/94	5801	5332.5	3494.5	4709.1	5943.4	352
01/06/94	5420	4472.6	2811.9	4601.5	6237.1	337.2	21/09/94	5783	5359	3477.3	4639.3	5904.6	352.1
02/06/94	5473	4570.8	2801.4	4613.6	6245.8	337.6	22/09/94	5754	5332.4	3447.5	4617.1	5878.1	352.2
03/06/94	5500	4602.7	2815.9	4641.2	6265.8	336.5	23/09/94	5755	5322.3	3461.5	4624.2	5880.7	351.4
06/06/94	5462	4561.6	2804.1	4633.4	6241.9	337.5	26/09/94	5617	5218.7	3379.2	4497.8	5771.2	350.6
07/06/94	5518	4659.6	2820.5	4623.2	6227	337	27/09/94	5583	5175.7	3380.2	4459.9	5728.6	348.8
08/06/94	5583	4727.2	2834.3	4663	6275.4	338.3	28/09/94	5639	5237.2	3450.3	4467	5754.7	349.1
09/06/94	5585	4725.6	2839.9	4676.1	6269.8	339.5	29/09/94	5636	5215	3466.3	4479.9	5766	348.6
10/06/94	5644	4788.9	2856.5	4695.6	6317.7	342.5	30/09/94	5676	5267.8	3458.3	4513.3	5786.2	348.4
13/06/94	5721	4869.9	2874.7	4740.1	6389.8	343.1	03/10/94	5672	5258	3470.4	4502.3	5793.3	348.2
14/06/94	5763	4930.6	2874	4781.5	6410	343.5	04/10/94	5654	5222	3496	4504.7	5786.4	347.3
15/06/94	5746	4864.8	2877.5	4843.8	6443.7	342.9	05/10/94	5616	5184	3475.9	4475.3	5772.3	346.4
16/06/94	5774	4926	2874.6	4856.2	6426.1	346	06/10/94	5595	5145.5	3469	4475.3	5770.1	346.7
17/06/94	5785	4950.8	2886.1	4883.2	6417.5	350.1	07/10/94	5598	5168.6	3460.7	4441	5759.8	345.9
20/06/94	5712	4897.8	2828.3	4888.6	6327	351.8	11/10/94	5525	5045.1	3440.8	4422.5	5749.1	345.8
21/06/94	5721	4921.9	2814.5	4870.6	6294.9	351.4	12/10/94	5559	5093.5	3438.3	4439.2	5764.3	342
22/06/94	5706	4909.5	2817.2	4871.7	6264.6	351.8	13/10/94	5612	5141	3503.4	4469.3	5788.6	342.1
23/06/94	5688	4869.8	2847.5	4863.9	6259.5	351.7	14/10/94	5627	5134.3	3509.2	4509.8	5818.3	341.6
24/06/94	5592	4799.5	2825.9	4772.3	6162	353.3	17/10/94	5655	5149.9	3559.6	4521.2	5846.5	340.6
27/06/94	5453	4685.9	2721.7	4672.6	6012	353.6	18/10/94	5682	5167	3591.1	4542	5899.2	338.9
28/06/94	5487	4695.6	2766.3	4702.5	6052.1	354.5	19/10/94	5691	5164	3629.7	4525.3	5922.6	336.9
29/06/94	5422	4637.5	2751.1	4665.8	5988.3	352.7	20/10/94	5743	5187.1	3644.8	4548.7	6014.4	334.5
30/06/94	5404	4627	2759.3	4638.5	5969.3	351.8	21/10/94	5775	5227.6	3645.5	4565	6029.9	332.6
01/07/94	5401	4649	2756	4608.3	5940.5	351.3	24/10/94	5751	5191.2	3606	4576.5	6030.2	333.4
04/07/94	5454	4694.7	2779.7	4644.5	5978.7	350.8	25/10/94	5701	5145.7	3548	4559.7	5993.3	332.8
05/07/94	5451	4702.7	2753.3	4663.2	5968.7	349.8	26/10/94	5707	5132.6	3559.8	4562.3	6020.5	333.4
06/07/94	5404	4658.5	2739.9	4637.8	5928.3	350.2	27/10/94	5735	5173.7	3553.7	4572.3	6035.5	331.6
07/07/94	5408	4638.1	2742.3	4694.1	5940.2	348.5	28/10/94	5750	5195.5	3548.1	4583.6	6041	333.5
08/07/94	5405	4639.9	2739.1	4665	5948.9	348.8	31/10/94	5724	5136	3550.5	4594	6053.7	332.7
11/07/94	5420	4649.5	2781.7	4662.6	5946.7	349	01/11/94	5708	5118.3	3521.8	4607.6	6036.3	333.5
12/07/94	5455	4709.5	2810.7	4680.6	5934.2	350.2	02/11/94	5737	5145.9	3545.3	4636.6	6046.7	330.9
13/07/94	5474	4743.2	2851.1	4672.8	5926.6	351.5	03/11/94	5840	5234.1	3583.6	4669.8	6157.9	330.7
14/07/94	5506	4780.6	2935	4665.7	5932.1	354.1	04/11/94	5862	5245	3588.4	4730	6195.7	330.2
15/07/94	5577	4850.3	3012.7	4716	5976.1	354.4	07/11/94	5840	5211.8	3567.8	4747.2	6193.8	331.1
18/07/94	5588	4856.4	3024.3	4741.3	5981.3	355.2	08/11/94	5805	5154.8	3556.4	4773.1	6185	330.8
19/07/94	5549	4813	2987.1	4710.6	5958.1	355.5	09/11/94	5850	5199.5	3580.7	4811.3	6218.2	331
20/07/94	5579	4869	2983.8	4710.2	5963.9	354.6	10/11/94	5849	5190.4	3571.6	4838.4	6221.6	331.1
21/07/94	5553	4826.9	2975.8	4710.9	5978.3	353.8	11/11/94	5866	5200.9	3596.3	4849.9	6247.2	329.1
22/07/94	5548	4802.4	2991.8	4724.5	5989.7	353.9	14/11/94	5861	5185.4	3602.1	4880.7	6242.1	328.2
25/07/94	5568	4807.8	2989.7	4730	6036.5	355	15/11/94	5865	5194.1	3617.6	4874.3	6252.2	327.8
26/07/94	5608	4870.6	3042.4	4743.6	6038.3	353.7	16/11/94	5898	5209.9	3644.8	4918.8	6290.8	326.5
27/07/94	5659	4973.6	3040.4	4741.5	6055.2	353.9	17/11/94	5942	5229.8	3704.2	4977.1	6346.6	326.6
28/07/94	5672	5019.6	3056	4736.1	6037.5	353.7	18/11/94	5924	5171.9	3684.8	4997	6394	327.3
29/07/94	5652	5004.1	3067.8	4723.1	6018.1	351.7	21/11/94	5901.8	5123.5	3675.5	5015.2	6409.4	327.5
01/08/94	5652	5016.5	3054.3	4701.1	6013.1	351.8	22/11/94	5908.9	5100.1	3679.1	5029.2	6456.2	325.6
02/08/94	5684	5052.8	3098	4708.2	6028.6	352.9	23/11/94	5799.4	4979.4	3600.8	4986.1	6375.6	324.5
03/08/94	5697	5069.1	3109.8	4715.2	6023.7	352.8	24/11/94	5866.3	5032.3	3624.4	5040.2	6441.2	323
04/08/94	5689	5036	3133.1	4714.7	6038.5	353.7	25/11/94	5831.4	4983.4	3606.9	5021.8	6433.5	321.6

Table A.6: Data: Equity indices (daily) 08/04/94 – 25/11/94

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
28/11/94	5833.9	4989.1	3598.1	5017.9	6441.5	320.9	24/03/95	5231.2	3975.8	3439.7	4816.2	6136.3	309
29/11/94	5749	4862.3	3541.8	4993.4	6417.6	320.8	27/03/95	5200.6	3945.9	3437.7	4815.9	6088.1	309.1
30/11/94	5756.3	4867.2	3566.1	4992.7	6426.4	321.1	28/03/95	5212.4	3954.6	3443.7	4818.5	6108.4	307.7
01/12/94	5745.8	4820.7	3584.5	5024.9	6423.6	322.4	29/03/95	5211.5	3963.4	3443.5	4803.6	6100.8	307.5
02/12/94	5720.2	4774.3	3559.5	5028.8	6414.1	321.9	30/03/95	5217.9	3957.6	3448	4825.7	6116.4	305.5
05/12/94	5719.3	4742.8	3576.3	5033.3	6433.6	322.5	31/03/95	5281.9	4051	3447.5	4868.5	6150.4	310
06/12/94	5722.1	4740.7	3586.1	5040.9	6438.9	322.1	03/04/95	5327	4142.2	3434.9	4882.7	6153.9	308.1
07/12/94	5774	4833.4	3575	5054.7	6449.3	322.6	04/04/95	5395.7	4221.7	3451.9	4942.1	6210	308.5
08/12/94	5758.1	4812.2	3501.8	5064.9	6444.9	322.7	05/04/95	5432.8	4296.4	3394.7	4976	6219.8	306.1
09/12/94	5694.5	4747.5	3493.5	5018.3	6403.1	322.4	06/04/95	5455.6	4350.4	3370.2	4998.7	6208.1	306.6
12/12/94	5648.9	4694.8	3462.3	5008.5	6380.6	320.7	07/04/95	5447.1	4321.5	3373.6	5015.3	6214.3	307
13/12/94	5659.9	4722.1	3447.6	5005.8	6365.7	321.2	10/04/95	5385.3	4194.3	3378.5	5005.7	6214.3	306.8
14/12/94	5637.3	4714.4	3442.8	4995.4	6324.3	321.7	11/04/95	5342.9	4128.5	3380.4	5004	6181.5	307.3
15/12/94	5667.4	4729.2	3466.5	5019	6377	321.3	12/04/95	5352.4	4154.5	3374	5006	6174.2	307
19/12/94	5696.9	4757.1	3524.2	5030.8	6381	319.7	13/04/95	5361.9	4168.3	3380.4	5027.8	6170.1	305.9
20/12/94	5747.1	4818	3553.8	5083.8	6389.1	320.6	18/04/95	5404.9	4232.9	3426.3	5047.2	6180.2	305.3
21/12/94	5804.5	4900.2	3566.5	5112	6412.6	320.9	19/04/95	5439.8	4292.3	3436.8	5070	6184.6	304.6
22/12/94	5798.7	4898.7	3561.4	5113.5	6407.7	320.7	20/04/95	5418.1	4243.2	3445.7	5065.4	6186.4	303.4
23/12/94	5802.7	4893.2	3582	5111.3	6417	320.8	21/04/95	5414.9	4225.8	3467.7	5061.2	6195.1	304.2
27/12/94	5809.8	4885.2	3584	5161.9	6417.1	319.6	24/04/95	5434.4	4237.6	3514.7	5084.1	6207.7	304.2
28/12/94	5840.1	4915.6	3605.4	5174.2	6438.7	320.7	25/04/95	5459.9	4237.4	3551.7	5108.6	6258.3	304.7
29/12/94	5857.5	4952.3	3622.3	5157.8	6447.3	321.6	26/04/95	5465	4217.6	3566.4	5134.7	6283.3	305
30/12/94	5866.9	4968.3	3621.4	5160	6467.8	326.4	28/04/95	5479.1	4218.8	3583.6	5159.5	6304.3	305.3
03/01/95	5837	4930.5	3616.7	5142.7	6460.9	324.6	02/05/95	5536.6	4270.2	3638.7	5182	6372	304.8
04/01/95	5778.5	4839	3616.2	5129.8	6448.1	324.7	03/05/95	5637.6	4319.2	3674.6	5387.7	6489.1	304.5
05/01/95	5751.9	4792.9	3647.9	5110.9	6436.6	324.8	04/05/95	5634.4	4315.9	3667.2	5395.4	6483.3	304.5
06/01/95	5768	4811.2	3658.9	5114	6447.7	323.6	05/05/95	5599.2	4279.7	3674	5358.6	6445.2	305.5
09/01/95	5716.9	4721.5	3657.8	5098.2	6425.6	324.1	08/05/95	5571.5	4236.5	3667.3	5365.9	6422.5	305.5
10/01/95	5674	4693.1	3601.1	5056.5	6400.6	323.7	09/05/95	5550.3	4201.1	3660.9	5360.5	6415.3	304.7
11/01/95	5635.6	4666.1	3547.4	5029.5	6360	323.5	10/05/95	5536.8	4169.3	3658.8	5360.1	6421.3	304.7
12/01/95	5701.6	4770.2	3545.5	5055.3	6382.4	323.5	11/05/95	5568.8	4185.6	3683.1	5386.8	6471.9	304.4
13/01/95	5669.6	4717.1	3542.4	5047.9	6382.5	322.9	12/05/95	5576.5	4194.5	3681.6	5404.2	6474.2	304
16/01/95	5596.2	4599.4	3504.9	5038.3	6365.5	322.8	15/05/95	5569	4197	3661.3	5382	6467.4	305.2
17/01/95	5515	4531.6	3424.1	4954.1	6309.2	322.1	16/05/95	5560	4203	3645.1	5337	6461.9	303.3
18/01/95	5524.2	4600.1	3411.6	4932	6248.5	321.7	17/05/95	5544.5	4161.6	3637.3	5336.3	6477.7	301.9
19/01/95	5434.8	4514.8	3372.9	4872.4	6171.6	321.5	18/05/95	5540.3	4163.7	3619.1	5335	6469.6	301.4
20/01/95	5410.4	4484.1	3394	4826.6	6154	320.8	19/05/95	5494.9	4133.6	3581.6	5291.3	6412.8	301.3
23/01/95	5337.8	4447.7	3321.1	4758.9	6069	320.9	22/05/95	5488.9	4115.8	3585.5	5273.3	6427.9	301.1
24/01/95	5314.2	4383.2	3352.8	4744.8	6073.2	321.2	23/05/95	5490.3	4114.5	3613.5	5229.9	6446.7	300.5
25/01/95	5261.9	4315	3332.2	4703	6041.6	320.1	24/05/95	5497.4	4123.8	3614.1	5229.4	6455.1	300.5
26/01/95	5163.2	4209.4	3273.3	4656.7	5952	319.3	25/05/95	5496.4	4129.1	3613.2	5226.5	6446.4	300.9
27/01/95	5163	4191.9	3296.8	4664	5946.7	318.7	26/05/95	5506.2	4157.5	3600.4	5221.6	6443.8	300.2
30/01/95	5092.4	4095.2	3260.1	4639.9	5896.6	318.5	29/05/95	5497.3	4170.5	3559	5211.7	6419.3	300.2
31/01/95	5054.1	4073.9	3235.5	4595.3	5853	318.1	30/05/95	5482	4157.9	3553.4	5194.1	6401.9	300.7
01/02/95	5161	4154.8	3314	4656.5	5957.8	319.4	31/05/95	5471.4	4131.4	3540.6	5192.5	6413.2	300.1
02/02/95	5261.2	4262.3	3377.3	4721.1	6029.7	319.4	01/06/95	5492.6	4149.5	3560.9	5209.4	6435.8	298.8
03/02/95	5340.1	4362.9	3442.7	4736.1	6077.2	319.5	02/06/95	5526.8	4181.2	3581.9	5257.6	6461.5	297.6
06/02/95	5332.7	4333.5	3452.9	4757.4	6087.7	319.3	05/06/95	5527.9	4180.6	3568.8	5282.4	6457.5	297.7
07/02/95	5213.2	4191.1	3372.2	4714	6026.7	319.4	06/06/95	5543.9	4198.9	3596.9	5307.5	6456.2	297.2
08/02/95	5187.5	4180.5	3335	4706.8	5966.7	318.5	07/06/95	5535.8	4183.4	3583.8	5313.2	6455.9	296.7
09/02/95	5252.5	4276.4	3329.2	4717.9	5998.1	315.6	08/06/95	5497.5	4160.4	3545.1	5264.8	6413.3	296.8
10/02/95	5234.1	4250.5	3327.4	4723.2	5984.3	314.8	09/06/95	5447.3	4113.7	3486.7	5231.3	6368.2	295.9
13/02/95	5213	4219.8	3331	4704.4	5972	313.1	12/06/95	5426	4115.6	3432.2	5194.7	6341.7	294.9
14/02/95	5184	4200.2	3308.5	4674.7	5941.6	312.5	13/06/95	5406.7	4103.6	3430.2	5148.5	6325.9	294.8
15/02/95	5191	4202.9	3303.8	4683.7	5947.8	312.3	14/06/95	5411	4105.3	3418.7	5164.3	6332.5	294.8
16/02/95	5146.1	4150.3	3280	4663.5	5924.4	311.8	15/06/95	5435.2	4157.6	3416.7	5192.7	6317.2	294.4
17/02/95	5117.3	4096.5	3263.9	4671.7	5903.4	310.7	19/06/95	5469.4	4206.1	3436.7	5201.7	6338.8	295.4
20/02/95	5107.7	4092.4	3231.1	4670.1	5886.6	310	20/06/95	5518.8	4262.7	3473	5255.9	6365.4	296
21/02/95	5101.8	4069.4	3224.5	4666.8	5890.2	310	21/06/95	5501.6	4235.8	3463.1	5225.4	6371.5	295.9
22/02/95	5101.5	4066.1	3208.2	4659.5	5901.7	309.4	22/06/95	5500.8	4233.9	3447.9	5242.4	6369.1	294.8
23/02/95	5126.5	4092.8	3219.4	4658.2	5917.9	308.7	23/06/95	5492.6	4227.4	3441.8	5248	6353.2	294.3
24/02/95	5175	4155	3254.9	4632.7	5941.2	307.8	26/06/95	5508.5	4233.9	3443.5	5262.9	6383.3	294.3
27/02/95	5141.4	4118.5	3215.9	4605.5	5929.4	307.2	27/06/95	5500	4221	3441.2	5265.6	6375.4	294.3
28/02/95	5147.1	4100.1	3237.9	4612.9	5940.1	305.8	28/06/95	5465.4	4178.9	3418.7	5231.9	6357.5	292.7
01/03/95	5234.4	4145.8	3343.8	4661.7	6031	306.2	29/06/95	5438.4	4151.5	3404.8	5210.2	6331.6	292.2
02/03/95	5242.8	4124.3	3345.9	4699.2	6073	306.2	30/06/95	5420.7	4130.5	3399.7	5204.6	6312.2	292.7
03/03/95	5231.2	4081.3	3360.3	4749.4	6073	307	03/07/95	5367.8	4046.5	3391.1	5196.6	6278.4	292.4
06/03/95	5239.7	4079.3	3396.6	4744.6	6090.4	306.7	04/07/95	5403.3	4110.9	3384.7	5205.6	6292.8	292
07/03/95	5268.3	4110.7	3406.9	4756.4	6113.1	307.3	05/07/95	5379.2	4059.7	3348.9	5201.9	6308.4	291.2
08/03/95	5264.2	4106.3	3369.8	4777.7	6117.8	307.4	06/07/95	5381.5	4072.8	3341	5197	6303	290.3
09/03/95	5222.4	4041	3372.2	4772.1	6087.6	308.4	07/07/95	5442.8	4150.1	3369	5217.3	6358.7	289.6
10/03/95	5224	4058.5	3378.9	4742.2	6073.4	307.4	10/07/95	5434.5	4132.9	3385.3	5202.6	6359.3	289.3
13/03/95	5237.7	4033.6	3437.8	4780.8	6093	306.4	11/07/95	5408.3	4091.8	3382.1	5199	6341.5	288.9
14/03/95	5268	4065.4	3459.3	4797.6	6117.6	306.6	12/07/95	5434.6	4136.9	3402.7	5182.4	6359.5	287.5
15/03/95	5277.4	4090.8	3428.6	4807.1	6118.2	308	13/07/95	5468.2	4185	3431.5	5204.9	6369.6	288.3
16/03/95	5273.9	4056.5	3481.8	4808.8	6136.7	308.2	14/07/95	5450.1	4159.7	3439.2	5187	6357.4	288
17/03/95	5300.8	4081.6	3496.9	4841.3	6159	309	17/07/95	5476.4	4197.1	3446.7	5197.1	6376.1	288.3
20/03/95	5309.8	4087.7	3516.2	4835.4	6173.6	308.7	18/07/95	5522.5	4244.4	3468.2	5235.1	6420	289.1
22/03/95	5268	4014	3485	4829.9	6168.5	308.2	19/07/95	5509	4235	3424.6	5220.2	6417.1	288.9
23/03/95	5260.5	4015.5	3456.2	4837.3	6150.3	309.9	20/07/95	5459.6	4183.9	3402.2	5179.4	6370.1	288.7

Table A.7: Data: Equity indices (daily) 28/11/94 – 20/07/95

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
21/07/95	5439.7	4152.5	3387.8	5169.2	6364.6	289.4	15/11/95	5980.7	4283.5	3155.3	6170.7	7343.9	325.7
24/07/95	5450.5	4182.3	3380.1	5161.3	6360.9	289.1	16/11/95	6007.4	4282.3	3136.2	6208.4	7414.8	325.1
25/07/95	5447.7	4174.3	3361	5161.2	6371.2	289.7	17/11/95	6021.1	4290.3	3099.5	6256.7	7433.4	327
26/07/95	5427.7	4148	3320	5156.7	6365.9	290.2	20/11/95	6052.7	4303.4	3053	6331.3	7490.6	326.1
27/07/95	5433.5	4157.4	3336.4	5158.8	6363.3	289.6	21/11/95	6045.4	4302.7	2923.6	6369.9	7502.5	324.4
28/07/95	5415.2	4125	3314.4	5137.1	6373.4	289.6	22/11/95	6014.8	4273	2850.2	6346.2	7493	324.5
31/07/95	5438.5	4161.1	3328.3	5128	6392.8	289.3	23/11/95	5966.9	4225.4	2852.9	6293.4	7441.7	324.1
01/08/95	5443.3	4141.9	3336.6	5148.9	6419.5	289.3	24/11/95	5986.4	4228.9	2917.4	6308.8	7460.6	324.8
02/08/95	5510.2	4206.2	3401.3	5196.7	6480.6	289.4	27/11/95	5961.1	4218.5	2890.2	6273.6	7428.4	325.2
03/08/95	5515.9	4223.9	3382.6	5199.5	6479.5	289.2	28/11/95	6005.8	4260.9	2909.1	6312.7	7475.1	327.2
04/08/95	5526.5	4236.4	3392.3	5204.4	6487.4	288.9	29/11/95	5983	4226.2	2881.4	6321.4	7460.9	327.3
07/08/95	5544.2	4250.4	3408.1	5200.1	6517.5	288.8	30/11/95	5972.1	4191.3	2877	6341.4	7467.4	326.9
08/08/95	5580.7	4286.3	3423.3	5220.7	6560.8	289	01/12/95	5969.8	4146	2862.7	6412.9	7490.5	326.4
10/08/95	5581.2	4263.8	3433.9	5223.9	6587.1	289.1	04/12/95	5982.7	4161.6	2832.6	6435.8	7508.8	325.5
11/08/95	5563.2	4252.4	3408.5	5217.3	6562.4	289.1	05/12/95	6045.9	4209.5	2814	6528.1	7592	325.4
14/08/95	5543.5	4229.4	3402.8	5192.7	6550.2	289.2	06/12/95	6147.8	4293.7	2850.5	6714.5	7669.1	325.1
15/08/95	5581.4	4292.9	3405.2	5195.7	6573.8	289.7	07/12/95	6240	4354.1	2952.5	6824.9	7764.8	324.5
16/08/95	5561.8	4260.9	3400.3	5196.9	6560.2	289.6	08/12/95	6210.6	4368.3	2918.7	6753.1	7709.5	324.1
17/08/95	5582.4	4294.1	3416.9	5211.6	6562.2	289.9	11/12/95	6243.5	4419.8	2923.8	6783.7	7720.4	323.7
18/08/95	5585.7	4314.2	3408	5202.1	6552.3	291.1	12/12/95	6283.3	4430.9	2947.4	6845.5	7782.1	323.9
21/08/95	5596.1	4342.6	3410.6	5200	6544.3	291.4	13/12/95	6262.1	4387.3	2949.1	6846.5	7777.6	322.2
22/08/95	5578.2	4325.4	3395.3	5171.2	6536.1	290.5	14/12/95	6232.9	4333.8	2941	6830.5	7775.1	321.3
23/08/95	5567.5	4306.1	3384.7	5176.2	6532.3	290.1	15/12/95	6213.6	4308.2	2922.5	6808.6	7771.5	320.5
24/08/95	5576.9	4318	3363.8	5203.9	6538.3	290	18/12/95	6240	4334.4	2933.4	6813.5	7807.3	320.9
25/08/95	5565.9	4306.1	3337.9	5187.7	6540.1	291.1	19/12/95	6198.9	4304.8	2932.1	6752.9	7756.7	320.5
28/08/95	5551.3	4287.3	3297.5	5192	6534.5	291.1	20/12/95	6218.7	4313.2	2956.9	6771.9	7784.2	320.9
29/08/95	5548.1	4281.5	3306.2	5189.9	6530.7	290.9	21/12/95	6256.2	4342.3	2963.7	6852.9	7811.6	320.6
30/08/95	5540.4	4265.3	3289.2	5205.2	6528.6	291.2	22/12/95	6257.4	4335.4	2993.5	6848.4	7814.6	320.6
31/08/95	5543.4	4271	3281.8	5204.7	6532.7	291.8	27/12/95	6247.5	4321.8	2971.9	6848.1	7812.5	320.4
01/09/95	5546.6	4265.7	3269.5	5222.4	6544.5	292.5	28/12/95	6237.2	4299.6	2978.4	6863.9	7801	320.4
04/09/95	5528.8	4236.4	3260.1	5218	6537.2	292.8	29/12/95	6228.4	4267.9	3000.5	6860.3	7810	321.3
05/09/95	5520	4213.2	3238.8	5230.3	6544.8	292.7	02/01/96	6250.7	4290.4	3005.7	6881.7	7833	321.1
06/09/95	5546.7	4239.4	3254.7	5262.9	6565.2	293.3	03/01/96	6407.5	4448	3091.8	6968.8	8008	320.9
07/09/95	5580.6	4278.4	3305	5284.3	6580.7	295.1	04/01/96	6466.3	4526.6	3131.9	7005.5	8041.8	320.3
08/09/95	5598.5	4301.5	3322.7	5278.9	6597.8	296.5	05/01/96	6524.8	4621.4	3120.1	7076.1	8055.3	320.9
11/09/95	5614.7	4303	3340.6	5290	6630.8	297.1	08/01/96	6603.7	4739.4	3184.3	7130.2	8076.5	321.4
12/09/95	5630	4324.9	3338.1	5304.3	6640	297.6	09/01/96	6628.9	4768.2	3212.2	7156.3	8084.9	321.9
13/09/95	5650	4347.4	3318.7	5320.4	6668.4	298	10/01/96	6638.7	4797.5	3212.6	7118.4	8093.9	322.3
14/09/95	5668.3	4356.9	3309.2	5328.8	6709.3	298.3	11/01/96	6661.3	4850.9	3217.9	7107.8	8086.7	323.3
15/09/95	5674.1	4356	3303.4	5340.6	6723.1	299.9	12/01/96	6723.4	4841.5	3261.2	7208.2	8215.6	323.3
18/09/95	5661.1	4346.7	3291.1	5331.8	6705.4	301.5	15/01/96	6710.4	4773.9	3290.8	7223.1	8244.6	322.4
19/09/95	5666.4	4373.3	3290	5320.9	6690.6	302.5	16/01/96	6794.4	4857.8	3337.1	7242.1	8350.7	322.7
20/09/95	5658.6	4352	3271.4	5307.7	6710.5	302.8	17/01/96	6836.7	4914.4	3327.4	7300.7	8375.6	324.2
21/09/95	5658.4	4348.1	3269.5	5313.6	6712.4	303.3	18/01/96	6816.7	4856.4	3324.6	7312.7	8391.7	324.8
22/09/95	5663.7	4372.9	3254.1	5295.5	6709.8	303.4	19/01/96	6820.6	4830	3311.1	7370.8	8415.3	324
26/09/95	5664.4	4379.5	3261.8	5295.6	6699.7	303.7	22/01/96	6905.6	4955.9	3324.9	7418.9	8468.7	324.5
27/09/95	5645.6	4341.2	3255	5285.8	6702	306.7	23/01/96	6935.2	4997.8	3314.2	7431.4	8497.7	325.3
28/09/95	5632.6	4306.4	3239.8	5285	6716.5	308.1	24/01/96	6931.1	4988.4	3254.8	7445.6	8515.8	324.6
29/09/95	5657.3	4326.3	3225.5	5306.5	6757.2	308	25/01/96	6950.1	5009.2	3271.1	7457.3	8531.3	325.2
02/10/95	5651.9	4330.6	3206.2	5301.8	6745.4	308.2	26/01/96	6961.8	5072.2	3249.3	7440.4	8501.5	324.4
03/10/95	5650	4327.8	3193.1	5299	6749.5	309.8	29/01/96	6937.2	5036.3	3241.6	7419.9	8494.3	323.7
04/10/95	5663.2	4344	3183.3	5311	6763.4	311.3	30/01/96	6930.2	5050.7	3228.8	7385.6	8473	325.5
05/10/95	5705.8	4368.4	3204.5	5337.2	6836	310.8	31/01/96	6870.9	4975	3148.7	7388.9	8425.9	325.6
06/10/95	5731.9	4371.3	3236.1	5366.3	6881.1	312.5	01/02/96	6898.7	5003.6	3163.4	7444.8	8434.8	325.6
09/10/95	5768.4	4404.1	3271.8	5375.9	6926	312.9	02/02/96	6993.5	5223.1	3126.3	7467.7	8424	324.2
10/10/95	5723.3	4354.5	3254.8	5364.9	6871.5	312.6	05/02/96	6950.2	5194.2	3080.4	7418.8	8379	323.7
11/10/95	5753	4377.6	3274.6	5401.4	6900.7	314.3	06/02/96	6921.9	5156.8	3059.6	7372.2	8378.1	323.9
12/10/95	5784.1	4375.6	3273.7	5490.1	6950.4	314.5	07/02/96	6960.3	5205.1	3079.8	7410.9	8401	324.2
13/10/95	5798.7	4353.9	3282.2	5548.6	6988.8	315.6	08/02/96	6882.5	5091	3051.2	7386.5	8348.5	323.5
16/10/95	5804.8	4337	3285.2	5577.9	7013.4	315.8	09/02/96	6873.7	5082.2	3040.4	7369.8	8345.5	323.6
17/10/95	5816.1	4302.9	3306.6	5605.4	7069	318.8	12/02/96	6800.1	4954.3	3017.4	7355.1	8315.7	323.3
18/10/95	5878.9	4342.4	3318.7	5654.2	7170.7	321.6	13/02/96	6763.5	4876.5	2991.9	7368.3	8317.6	321.3
19/10/95	5852.4	4281.8	3305.4	5672.8	7170.2	320.9	14/02/96	6777.8	4843.5	2996.9	7398.5	8386.9	320.8
20/10/95	5826.7	4249.1	3299.1	5667.4	7143.5	320.8	15/02/96	6854	4920.3	3027.5	7479	8455.7	320.1
23/10/95	5833.2	4263	3293.1	5673.3	7143.6	320.4	16/02/96	6842.4	4939	3032.4	7463.9	8401.3	320
24/10/95	5820.7	4242.5	3249.6	5694.1	7140.6	320.4	19/02/96	6791	4911	3019.5	7412.3	8319.2	320.2
25/10/95	5858.2	4202.9	3266.4	5752.7	7268.7	320.8	20/02/96	6731.9	4860	3021.3	7328.6	8252.9	320.7
26/10/95	5818.5	4160	3245.6	5766.9	7209.2	320.3	21/02/96	6649.3	4777.6	3025.3	7236.8	8165	319.7
27/10/95	5765.7	4113.4	3209.1	5727.3	7149.6	319.2	22/02/96	6753.6	4938.8	3076.9	7263.9	8222.7	320.3
30/10/95	5753	4081.4	3214.4	5724.3	7154.6	318.9	23/02/96	6767.5	4961	3112.2	7273.9	8214.2	320.7
31/10/95	5789.1	4112.6	3240.3	5736.8	7202.3	320.1	26/02/96	6786.6	4999.9	3155.8	7271.5	8204	319.6
02/11/95	5812.9	4146.3	3232	5751	7223.4	320	27/02/96	6722.8	4932.5	3076.7	7221.8	8163.2	318.7
03/11/95	5848.4	4177.1	3218.4	5822.9	7256	319.9	28/02/96	6703.3	4874.1	3138.2	7229.2	8155.9	317.7
06/11/95	5886.4	4182.2	3242.6	5884.7	7318.9	322	29/02/96	6705.1	4906.4	3128	7235.1	8118.9	317
07/11/95	5932	4203.3	3234.1	5962.9	7385.6	322.3	01/03/96	6712.5	4909	3156.1	7239	8123.7	317.4
08/11/95	5973	4284.4	3204.2	6046.5	7365.7	322.7	04/03/96	6718.6	4888.8	3171.4	7235.1	8165.5	317.3
09/11/95	6000.8	4332.3	3170.7	6089.8	7373.8	324.8	05/03/96	6652	4774.1	3207.7	7213.2	8118.3	316.8
10/11/95	6020.7	4404.1	3143.8	6078.8	7352.3	325.6	06/03/96	6678.8	4828.4	3230.9	7209.7	8117.6	316.4
13/11/95	6027.3	4408.9	3153.7	6110.4	7345	325.8	07/03/96	6677.4	4843.5	3276.9	7187.1	8086.6	315.6
14/11/95	6008.5	4334.3	3172	6157.5	7357.5	326.2	08/03/96	6697	4879.5	3261.7	7186.2	8103.6	314.7

Table A.8: Data: Equity indices (daily) 21/07/95 – 08/03/96

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
11/03/96	6554.8	4760.6	3184	7025.5	7956	314	08/07/96	6873.3	5322.5	3513.2	7572.4	7739	284.2
12/03/96	6661.3	4884.2	3271.6	7137.2	8014.3	311.1	09/07/96	6913.2	5357.2	3523.1	7602.6	7790.7	284.8
13/03/96	6662.1	4885.6	3288	7141.4	8007.2	309.4	10/07/96	6937.9	5389.2	3526.5	7659	7790.6	284.9
14/03/96	6663.9	4887.4	3291.2	7173.2	7993	307.9	11/07/96	6891.3	5348.6	3496.3	7632.3	7733.3	284.8
15/03/96	6703.7	4918.9	3296.1	7347.6	7975.6	307.7	12/07/96	6802.8	5280.6	3447	7549.6	7625.5	283.2
18/03/96	6652	4858.3	3281.3	7297.6	7936.5	305.9	15/07/96	6808	5275.7	3453.3	7576.7	7630.4	284.1
19/03/96	6645.8	4848.3	3312	7294	7920.7	306.7	16/07/96	6628.4	5160.1	3378.4	7332.1	7410	282.7
20/03/96	6633.5	4834.4	3330.9	7267.1	7909.6	305.8	17/07/96	6684.2	5149.5	3441.4	7421	7517.8	282.9
22/03/96	6690.5	4931.5	3364.3	7239	7950.9	304.2	18/07/96	6715.1	5165.9	3466.5	7442.3	7565	283.1
25/03/96	6735.5	4990.2	3447.9	7251.7	7965.1	304.1	19/07/96	6789.1	5229.7	3524.9	7496.9	7645.4	283.8
26/03/96	6741	5037	3442.1	7224	7936	302.5	22/07/96	6752.2	5209.8	3497.1	7457.6	7594.1	283.8
27/03/96	6770.6	5084.6	3467.8	7225.4	7949	302.3	23/07/96	6752.6	5233.8	3481.1	7422.9	7587.5	284.4
28/03/96	6721.1	5051.8	3453.3	7182.2	7873.8	302.7	24/07/96	6628.6	5126.1	3406.3	7309.6	7450.3	283.6
29/03/96	6748.6	5065.5	3471.2	7210.5	7916.5	300.6	25/07/96	6655.7	5134.2	3429.7	7333.2	7497.4	284.5
01/04/96	6710.8	4988.1	3460.6	7204.5	7915.5	300.5	26/07/96	6635.1	5122.9	3434.3	7320.2	7456.4	284
02/04/96	6699	4985.9	3470	7189.5	7888.1	299.5	29/07/96	6657.8	5143.6	3461.5	7339.6	7475.4	283.9
03/04/96	6683.9	5013.2	3430	7201.4	7817.6	297.7	30/07/96	6632.9	5112.8	3456.4	7307.2	7462.3	283.2
04/04/96	6700.6	5042.1	3424.5	7209.4	7826.2	298.3	31/07/96	6606.9	5114.4	3434.2	7277.2	7408	283.6
09/04/96	6689.3	5076.8	3415.8	7187.4	7760.9	298.7	01/08/96	6616.9	5134.8	3452.6	7253.4	7414.2	284.2
10/04/96	6654.3	5061.4	3375.6	7151.9	7712.6	298	02/08/96	6693.1	5185.5	3499.1	7339.2	7509.1	284.1
11/04/96	6654.2	5080.3	3400	7140.3	7683.5	297.5	05/08/96	6776	5254.3	3509	7435	7609.4	285.3
12/04/96	6669.6	5104	3419.1	7155.7	7682.1	296.7	06/08/96	6760.5	5219.5	3482.1	7469.8	7601.3	285.8
15/04/96	6699.4	5129.2	3494.2	7122	7724	296.7	07/08/96	6721.1	5174.5	3456.1	7455.4	7562.6	285.7
16/04/96	6815.3	5255.7	3546.1	7151.4	7862.2	297.3	08/08/96	6665.9	5109.8	3423.7	7415	7520.3	284.7
17/04/96	6859.8	5268.5	3576.3	7227.8	7926.1	295.8	12/08/96	6668.5	5137	3427.6	7388	7506.2	283
18/04/96	6895.3	5292.5	3603.5	7267.8	7967.3	296.4	13/08/96	6691.8	5180	3431	7366.4	7527.8	282.4
19/04/96	6972.7	5349.4	3685.5	7335.3	8053.3	295.4	14/08/96	6621.4	5099.5	3412.3	7303.8	7466.9	281.4
22/04/96	7070.2	5409.1	3764.4	7455.3	8168.9	295.1	15/08/96	6564.3	5065.4	3390	7235.4	7387.3	281
23/04/96	7013.2	5389.6	3708.5	7369.7	8092.1	295.5	16/08/96	6527.4	5038.3	3353.5	7187	7355	279.7
24/04/96	6980.1	5373.5	3665.9	7353.8	8041.2	294.7	19/08/96	6521.5	5023.5	3355.4	7193.3	7354.2	277.6
25/04/96	7048.9	5496.8	3740.4	7325.3	8064.2	295.8	20/08/96	6445.8	4974.3	3335.1	7076.4	7265.7	275.1
26/04/96	7047	5601	3725.9	7218.7	7980.6	295	21/08/96	6421	4976.2	3311.4	7054.4	7212.2	272.3
29/04/96	6937.4	5533.9	3689.7	7083.8	7829.9	294.4	22/08/96	6479	4999.3	3344.3	7168.6	7281.2	272
30/04/96	6976.3	5597.1	3681.6	7051.7	7881.3	293.8	23/08/96	6564.2	5097.7	3348.3	7294.3	7336	273.2
02/05/96	6992.3	5638.5	3686.9	7002.8	7898	292	26/08/96	6557.4	5112.7	3332.8	7246	7327.1	272.3
03/05/96	6927.2	5608	3634.7	6933.3	7803.8	290	27/08/96	6616.1	5169.4	3356.8	7287	7394.6	271.8
06/05/96	6864.6	5537.2	3586.2	6922.2	7738.4	289.7	28/08/96	6679.3	5244.1	3397.5	7368	7425.1	271.3
07/05/96	6876.1	5560.2	3584.6	6911.5	7748.9	287.8	29/08/96	6704	5297	3394.5	7358.4	7433.9	271.2
08/05/96	6795.6	5607.7	3475.5	6776.9	7559.6	287.9	30/08/96	6689.4	5291.5	3417.2	7328	7404.1	271.2
09/05/96	6692.3	5585.8	3418.4	6640.9	7376.3	286.8	02/09/96	6735	5317	3473.1	7359.3	7464.9	272.6
10/05/96	6745.4	5596.8	3464.3	6765.5	7435.8	286.6	03/09/96	6705.1	5306.9	3462.1	7314.6	7417.1	272
13/05/96	6794.1	5609.4	3468.7	6827.8	7530.5	284.9	04/09/96	6771.9	5352.5	3492.9	7401.7	7496.8	271.3
14/05/96	6845	5626.1	3491.7	6902.5	7612.9	282	05/09/96	6789.6	5357.6	3525	7418.3	7520.7	271.2
15/05/96	6826	5602.1	3458.1	6952.2	7575.9	281.7	06/09/96	6786.1	5340.6	3500.2	7445	7528.6	272.1
16/05/96	6793	5591.9	3410.6	6943.9	7514.2	283.1	09/09/96	6817.1	5356.8	3532.7	7479.3	7568.9	271.7
17/05/96	6741.6	5552.1	3388.1	6863.6	7465	282.1	10/09/96	6789.2	5291	3513.4	7487.6	7561.2	272.7
20/05/96	6670.8	5500.9	3382.6	6806.4	7355.8	280.8	11/09/96	6780	5288.3	3476.3	7499.2	7545.8	272.6
21/05/96	6653.8	5466.7	3424.6	6769.9	7354.6	278.2	12/09/96	6814.6	5296	3500	7566.6	7593.7	272.2
22/05/96	6652.8	5462.1	3447.9	6764	7352.4	276.5	13/09/96	6870.4	5334	3532.2	7640.4	7655.2	274.6
23/05/96	6671	5484.8	3427.5	6801.6	7365	275.5	16/09/96	6926.3	5345.7	3548.3	7765.6	7733.9	275.3
24/05/96	6688.4	5485.2	3446.2	6820.5	7398	274.1	17/09/96	6947.1	5361.9	3547.5	7776.1	7767.1	276.3
27/05/96	6689.4	5474.9	3469.8	6841.3	7394.5	274.4	18/09/96	6911.2	5327	3532.4	7773.4	7714.5	276.6
28/05/96	6749.8	5519.4	3474.5	6929	7468.2	273.7	19/09/96	6939.3	5340.1	3579.5	7855.2	7716.8	277.1
29/05/96	6765.6	5520.3	3482	6926.3	7514.1	270.9	20/09/96	6935.3	5311.9	3623.6	7853.9	7724.7	277.4
30/05/96	6752.3	5517.5	3468	6899.1	7498.2	270.4	23/09/96	6917.9	5282.5	3631.3	7870.2	7701.7	276.4
31/05/96	6818.5	5547.1	3471.8	6979.4	7612.3	270.4	25/09/96	6959.8	5330	3645.6	7933.3	7724.9	275.9
03/06/96	6818.1	5540.3	3471	7002.7	7608.7	269.6	26/09/96	6934.5	5281.5	3632.4	7932	7721.8	274.9
04/06/96	6838.5	5527.8	3516.8	7080.2	7626.7	269.5	27/09/96	6933.2	5275.3	3615.7	7963.2	7712.8	276.5
05/06/96	6815	5486.9	3508.4	7097.3	7605.9	269.8	30/09/96	6878	5212.7	3598.3	7895.9	7672.1	277.4
06/06/96	6828.1	5439.8	3516.2	7232.8	7633.5	269.8	01/10/96	6896.2	5236.2	3614.9	7929.3	7670.3	276.7
07/06/96	6814.8	5425	3537	7263.2	7589.1	271	02/10/96	6997.1	5336	3617.1	8085.9	7754.9	277
10/06/96	6826.4	5395.1	3558.6	7362.2	7603.4	271.9	03/10/96	7014.5	5347.1	3617.6	8103.1	7781.3	277.5
11/06/96	6778	5300.7	3541.3	7363.1	7592.5	272.8	04/10/96	7020.6	5371.9	3638.8	8081.7	7769.1	277.6
12/06/96	6820.3	5316.6	3564.1	7419	7657.3	275	07/10/96	7036.3	5375.1	3652.9	8117.3	7787.5	277.5
13/06/96	6840.1	5312.3	3562.9	7452.7	7705	274.7	08/10/96	7046.9	5390	3648	8116.5	7800.4	278.7
14/06/96	6819.8	5274.8	3564.9	7457.3	7691.1	276.5	09/10/96	7059.1	5409.5	3647.4	8110.9	7813.1	279.4
18/06/96	6822.9	5251.8	3617.4	7458.5	7708.7	276.5	10/10/96	7031.5	5399.4	3637.7	8040.6	7782.5	279.4
19/06/96	6863.5	5321.3	3607.2	7511.5	7712.8	276.8	11/10/96	7054.5	5414.2	3646.4	8055.3	7818.2	280.1
20/06/96	6839	5303	3600.3	7502.1	7671.5	277.9	14/10/96	7071.5	5430.2	3643.8	8064.3	7845.1	280.1
21/06/96	6855.3	5326.8	3616.1	7507.7	7678.6	278.2	15/10/96	7073.1	5427.9	3661.5	8058.9	7847.9	279.3
24/06/96	6859.1	5336.4	3620	7508.1	7671.5	279.5	16/10/96	7022.9	5377.2	3653.4	7991.1	7806.9	277.7
25/06/96	6891.8	5367.9	3625	7547.5	7704.3	279.2	17/10/96	7007.6	5352.4	3649.5	7983.6	7801.8	276.4
26/06/96	6913.9	5362	3643.8	7613.7	7735.1	280.8	18/10/96	7003.8	5327	3650.7	8018.6	7804	276.6
27/06/96	6910.5	5341.3	3627.6	7629.4	7751.6	280.1	21/10/96	7011.5	5334.2	3642.6	8016.6	7819.2	276.2
28/06/96	6878.7	5268.1	3604.5	7606.4	7772.9	285.9	22/10/96	7027.8	5370.6	3649.6	7990.8	7826.5	276.2
01/07/96	6844.9	5255.7	3572.3	7570.7	7720.7	285.1	23/10/96	7036.4	5383.2	3669.4	7974.5	7833.8	276
02/07/96	6891.7	5315.9	3571.7	7573.9	7777.3	285	24/10/96	7016.4	5359.3	3694.4	7932.5	7819.8	274.7
03/07/96	6913.7	5355.7	3555.9	7591.1	7787.3	284.2	25/10/96	6985.2	5320.5	3711.6	7884.2	7796.3	275.1
04/07/96	6916.5	5338.8	3560.1	7636.3	7792.8	285.5	28/10/96	6964.3	5318.4	3711	7862.3	7750.8	273.8
05/07/96	6905.1	5320.1	3546	7639.2	7788	285.1	29/10/96	6868	5261.1	3684.6	7695.9	7642.3	272.4

Table A.9: Data: Equity indices (daily) 11/03/96 – 29/10/96

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
30/10/96	6891.8	5298.1	3702.3	7733.4	7642.1	271.5	25/02/97	7202.8	5351.1	3654.4	8736.4	8000.8	274
31/10/96	6975.3	5371.5	3786.1	7816.8	7711.1	274.6	26/02/97	7205.7	5392.6	3621.2	8747.1	7963.1	274.3
01/11/96	6971.4	5378.7	3751.1	7807.5	7708.5	275	27/02/97	7196	5426.2	3621.4	8720.4	7904.5	275.2
04/11/96	6952.9	5364.1	3751.5	7766.1	7694.7	273.1	28/02/97	7145.2	5393.5	3564.9	8697.9	7832.6	275.3
05/11/96	6924	5343.4	3767.6	7719.7	7654.8	272.7	03/03/97	7153.8	5408	3606.6	8672.3	7835.3	274.8
06/11/96	6873.9	5291.1	3776	7695.6	7585.2	273.2	04/03/97	7132.6	5332.6	3647	8697	7843.8	272.9
07/11/96	6802.9	5218.1	3753.4	7627.7	7515.7	274.8	05/03/97	7070.4	5233	3639.9	8673.9	7807.4	272.3
08/11/96	6793.5	5216.7	3764.8	7642.1	7476.1	275	06/03/97	7084.2	5230.3	3636.9	8693	7842.6	272
11/11/96	6732.7	5192.5	3735.7	7556.2	7386.2	275	07/03/97	7061.9	5209.6	3635	8674.2	7815.7	270.7
12/11/96	6762.6	5233.5	3747.9	7552.1	7416.8	273.8	10/03/97	7091.3	5200.3	3638.8	8691.6	7904.1	269
13/11/96	6755.3	5234.2	3760.1	7524.7	7402.7	275.2	11/03/97	7162	5277.2	3667.8	8763.7	7960.8	271.2
14/11/96	6816.2	5293.1	3780.4	7578.9	7469.2	275.7	12/03/97	7167.9	5291.1	3649.7	8801.1	7947.9	271.4
15/11/96	6842.6	5287.1	3818.8	7605.6	7525.4	276.1	13/03/97	7171.6	5284.4	3652	8823.7	7954.9	271.6
18/11/96	6915.3	5345	3866.5	7688.7	7603.2	274.1	14/03/97	7127.1	5240.3	3621.1	8792.5	7910.9	270.9
19/11/96	6866	5290.3	3841.2	7651.3	7560.5	272.3	17/03/97	7112.4	5197.7	3642.4	8783.7	7919.4	271.3
20/11/96	6850.9	5272.9	3814	7618.1	7566.4	272.5	18/03/97	7085.5	5135.5	3625	8822.2	7909.2	270.7
21/11/96	6774.4	5216.6	3771	7540.4	7473.6	271.2	19/03/97	7099.5	5151.7	3598	8822.4	7940.5	269.1
22/11/96	6726.7	5147	3760.4	7495.6	7453.1	269.7	20/03/97	7098.5	5169.7	3578	8811.4	7929	267
25/11/96	6691.3	5082.6	3756	7451	7460.5	267.9	24/03/97	7088.9	5143.2	3563.8	8822.3	7935.1	268.1
26/11/96	6721.3	5076.5	3769.7	7520.6	7516.3	267	25/03/97	7089.5	5133.8	3571.8	8832.3	7939.6	270
27/11/96	6749.4	5090.4	3788.1	7576.8	7544.1	267.3	26/03/97	7059.6	5105.6	3535.7	8807.8	7914.6	271.5
28/11/96	6714.1	5060.3	3757.6	7572	7494	266.3	27/03/97	7094.8	5147.8	3523.2	8817	7960.5	275.2
29/11/96	6713.9	5073.4	3749.6	7543.3	7495.1	264.8	01/04/97	6962.4	5054.6	3442.5	8648.3	7813.6	275.1
02/12/96	6699.7	5071.1	3752.1	7534.1	7488.4	265.5	02/04/97	6981.1	5071.3	3450.4	8703	7816.3	273.3
03/12/96	6699.5	5058.6	3765.3	7563.9	7483.4	264.6	03/04/97	6981.1	5051.6	3482	8718.9	7822.5	271.3
04/12/96	6713.2	5112.4	3722.4	7541.4	7479.9	264.4	04/04/97	7015.8	5075.7	3498.7	8780.9	7854.9	269.3
05/12/96	6717.6	5099.6	3758.9	7570.1	7480.1	264.7	07/04/97	7074.7	5114.7	3536.6	8927.4	7885.9	270.4
06/12/96	6601.2	5045	3671.4	7351	7328.9	265.7	08/04/97	7044.1	5091.5	3534.8	8891.7	7846.9	268.7
09/12/96	6615.2	5050	3708	7367.4	7340.3	265.5	09/04/97	7048.5	5098.1	3568.8	8890.5	7839	267.8
10/12/96	6666.4	5061.8	3747.5	7466.1	7409	264.8	10/04/97	7045.1	5105.6	3583.5	8860.9	7827.1	270.9
11/12/96	6639.1	5044	3722.2	7455.9	7369.9	262	11/04/97	7022	5098.6	3568.4	8807.8	7802.2	271.2
12/12/96	6607	5026	3677.1	7398.2	7347.1	260	14/04/97	6966.4	5055.7	3543.6	8695.7	7762.8	271.4
13/12/96	6482.5	4930.7	3601.8	7205.8	7238.1	257	15/04/97	6997.3	5051.4	3599.5	8741.6	7813.6	270.6
17/12/96	6384	4831.7	3577.3	7159.7	7130.8	255.2	16/04/97	6976.8	5013.3	3568.7	8732.6	7819.4	271.1
18/12/96	6414.9	4831.5	3625.6	7193.2	7185.6	253.9	17/04/97	6976	4993.5	3544.8	8772.7	7831.8	270.3
19/12/96	6468	4834.1	3701.1	7305.2	7250.6	254.6	18/04/97	7022.2	5004.7	3598.6	8836.2	7897.9	271.3
20/12/96	6540.4	4877.7	3703.7	7453.4	7329.2	254.8	21/04/97	7040	5009.9	3621.8	8866.6	7918.8	271.5
23/12/96	6552.6	4901.2	3681.2	7463.5	7336.8	254.8	22/04/97	7065.9	5023.6	3659.2	8901.3	7944.1	271.1
24/12/96	6562.1	4934.2	3683.6	7464.9	7320.1	254.9	23/04/97	7080.2	4996.1	3675.4	8960.5	7983.8	272.2
27/12/96	6583.1	4946	3698.9	7494.6	7344.2	256	24/04/97	7087.9	4990.2	3688.4	9009.4	7983.2	273
30/12/96	6628.4	4960.3	3717.2	7560.8	7416.3	257.1	25/04/97	7072	4981.2	3703.9	8982.7	7956.3	272.7
31/12/96	6657.5	4986.4	3732.8	7587.2	7446.8	259.4	29/04/97	7107	4979.2	3730.5	9109	7987.4	268.6
02/01/97	6616.9	4963.8	3706.4	7538.4	7392.8	259.1	30/04/97	7130.5	4973.8	3756.6	9163.3	8024.9	268.8
03/01/97	6597.5	4912.2	3724.3	7534.3	7400	256.2	02/05/97	7149	4960.1	3770.8	9203.1	8070.2	269.6
06/01/97	6576.2	4858.2	3714.7	7531.3	7413.5	255.2	05/05/97	7190.4	5000.2	3764.1	9277	8103.5	269.5
07/01/97	6614.5	4855.8	3760.8	7620	7465.8	253.9	06/05/97	7211.9	5020	3745.5	9327.7	8121.1	270.1
08/01/97	6651.2	4866.4	3805.2	7684.5	7510.1	252.6	07/05/97	7160.7	4975.1	3696.5	9293.4	8067.8	268.3
09/01/97	6647.9	4847.9	3842.5	7668.4	7518.9	251.6	08/05/97	7162.9	4996.9	3683.5	9255.2	8071.4	268.4
10/01/97	6674.6	4886.4	3832.1	7671.7	7548.3	252.1	09/05/97	7179.4	5018.8	3676.5	9297.4	8070.8	270
13/01/97	6758.6	4964.9	3889.2	7759	7623.5	253.4	12/05/97	7190.3	5050.9	3663.4	9339.1	8045.1	268.2
14/01/97	6751.5	4949	3884.8	7769.1	7619.7	254	13/05/97	7193.1	5045.3	3649.6	9392.4	8037.1	269
15/01/97	6751.4	4925.7	3873.1	7810.9	7633.8	253.5	14/05/97	7214.5	5045.1	3631.2	9489.9	8056.2	269.1
16/01/97	6779	4941.6	3881.6	7824.7	7683.5	252.8	15/05/97	7173.1	5001	3609.3	9485.1	8004.4	267.4
17/01/97	6782.9	4937.1	3880.1	7854.9	7685.8	253.3	16/05/97	7142.8	4958.9	3602.6	9446.9	7992.6	267.9
20/01/97	6777.4	4917.9	3890.5	7872.8	7681.5	253	19/05/97	7116.6	4939.7	3604.1	9400.5	7966.4	265
21/01/97	6738.7	4893.4	3852.8	7854.7	7624.2	253.8	20/05/97	7106.2	4956.6	3602.9	9366.7	7932.5	264.6
22/01/97	6756.8	4879.8	3876.5	7922.5	7649.8	255.5	21/05/97	7127.2	4969.2	3637.1	9405.1	7944	264.3
23/01/97	6717.9	4853.2	3869.7	7923.5	7573.4	255	22/05/97	7096.1	4946.5	3643.5	9383.8	7891.5	263.4
24/01/97	6684.6	4834.8	3856.8	7862.9	7536.1	254.7	23/05/97	7071.7	4922.8	3639.5	9384.1	7853.2	262
27/01/97	6689	4834.6	3836.9	7893.1	7541	256	26/05/97	7067.5	4916.6	3652.2	9364.4	7855.2	260
28/01/97	6736	4904.4	3812.2	7932.9	7576.2	256.4	27/05/97	7065.3	4916.5	3647.3	9362.3	7852.5	259.4
29/01/97	6687.5	4855.7	3749.5	7914.4	7531.7	257.7	28/05/97	7040.8	4885.3	3637.8	9369.7	7821	259.7
30/01/97	6658.2	4840.4	3670	7891	7507.8	260	29/05/97	7074.7	4913.2	3659.6	9414.1	7854.3	256.8
31/01/97	6676.1	4823.1	3718.5	7951.2	7532.3	261.1	30/05/97	7021.7	4886.4	3629.2	9331.9	7788.3	257.2
03/02/97	6713.5	4819.7	3714.9	8031.7	7605.1	263.5	02/06/97	7061.3	4896.4	3654	9416.5	7835.9	259.5
04/02/97	6766	4861.7	3768.6	8123.3	7634.9	264.9	03/06/97	7112	4932.4	3673.2	9529.9	7871.4	259.2
05/02/97	6804.1	4848.8	3700.7	8315.5	7690.7	267.3	04/06/97	7172.8	4958.4	3734.5	9616.1	7946.4	260.2
06/02/97	6863.6	4903.3	3690.6	8416.7	7744.2	269.9	05/06/97	7218.5	4977	3746.7	9727.3	7994.1	259.5
07/02/97	6879.6	4898.9	3725.7	8462	7758.4	272	06/06/97	7262.2	5032.2	3767.7	9819.4	7993.8	259.1
10/02/97	6956	4958.6	3743.7	8568.2	7841.6	273.4	09/06/97	7296.5	5037.2	3800.7	9904.6	8029.6	260.8
11/02/97	6975	4987.6	3742.7	8586	7850.5	273.6	10/06/97	7265.5	4991.7	3812.5	9876.3	8007.5	262.3
12/02/97	6966.4	4990.2	3717	8591.9	7828.8	274.4	11/06/97	7286.1	5017.2	3817	9913.4	8013.7	262
13/02/97	7041.5	5086.2	3728.6	8626.8	7900.3	275.2	12/06/97	7292.7	5015.1	3815.4	9955.9	8015	260.7
14/02/97	7020.7	5080	3682.9	8585.9	7886.7	275.1	13/06/97	7271.5	4977.4	3785.6	9966.2	8009.7	258.7
17/02/97	7042.5	5126.1	3661.6	8588.4	7898.1	274.5	17/06/97	7192.1	4871.7	3763.7	9889.3	7964.4	257.3
18/02/97	7094.2	5189.9	3691	8603.9	7946.6	274.6	18/06/97	7218.1	4914.2	3715.5	9939.2	7979.1	255.7
19/02/97	7107.7	5220.3	3682.5	8607.4	7947.6	274.5	19/06/97	7209.1	4883.1	3706.2	9956.3	7988.4	255.2
20/02/97	7102.6	5207.3	3675	8587.3	7961.9	275.9	20/06/97	7234.1	4833.4	3760.8	9992.9	8087.7	253.6
21/02/97	7124.1	5268.1	3660.9	8581.9	7955	274.6	23/06/97	7286.3	4865.9	3765	10127.7	8125.6	254.8
24/02/97	7152.3	5301.7	3701	8641.9	7947.9	272.9	24/06/97	7333.6	4887.9	3774.8	10235.2	8175.4	254.8

Table A.10: Data: Equity indices (daily) 30/10/96 – 24/06/97

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
25/06/97	7354	4873.8	3766.1	10275.4	8236.2	255.1	16/10/97	7335.3	4556.5	4002.6	10404.2	8434.7	259.7
26/06/97	7402.7	4893.8	3777.8	10343.9	8312.5	255.5	17/10/97	7257	4513.2	3938	10279.5	8353	257.2
27/06/97	7411	4912.2	3787.3	10328.3	8317.3	255.3	20/10/97	7294.3	4494.9	3932.6	10398.6	8426.6	257
30/06/97	7420	4901	3830.1	10309.1	8350	258	21/10/97	7384.4	4520	3928	10656.9	8524	257.1
01/07/97	7420.1	4879	3817	10342.2	8368.4	255.8	22/10/97	7420	4455.1	3974.9	10818.8	8609.8	256.7
02/07/97	7416.7	4852.9	3820	10356.1	8384.4	254.5	23/10/97	7094.2	4217.6	3840.1	10322.5	8277.3	253.6
03/07/97	7403.2	4844	3814.8	10309.1	8383	254.4	24/10/97	7168.8	4220.5	3871.2	10542.4	8363	256.2
04/07/97	7368	4795.1	3791.1	10293.4	8360.9	254.5	27/10/97	6763.2	3922.9	3670.6	9932.6	7961.6	253.7
07/07/97	7293.2	4698.8	3738	10251.2	8310.7	253.4	28/10/97	6007.4	3520	3287.6	8706	7062.5	248.5
08/07/97	7310.3	4695.8	3748.7	10327.4	8319.7	255	29/10/97	6423.4	3736.2	3557.3	9302.1	7585.2	249.4
09/07/97	7299.2	4655	3752.3	10359.3	8320.9	257.4	30/10/97	6294.9	3640.7	3452.7	9126.8	7464.5	249.4
10/07/97	7337.9	4693.7	3757.3	10386.3	8367.3	258.6	31/10/97	6589.1	3826.9	3576	9655.7	7758.7	251.9
11/07/97	7348	4705.2	3767.6	10396.3	8373.2	258	03/11/97	6729.9	3882	3685.7	9990.7	7881.2	253.4
14/07/97	7380.6	4731.9	3762.4	10448.1	8409	257.7	04/11/97	6749.7	3887.7	3725	10042.3	7890.1	253.1
15/07/97	7390.7	4707.6	3807.6	10473.6	8440	257.4	05/11/97	6780.5	3906	3713.2	10130.8	7914.3	253.1
16/07/97	7422.8	4705.6	3824.3	10558.1	8486.5	257.6	06/11/97	6749.3	3852.4	3713.6	10163.2	7874.5	254.5
17/07/97	7437	4690.2	3812.9	10676.5	8491.7	257.5	07/11/97	6524.8	3709.8	3576.7	9880.2	7603.3	252.9
18/07/97	7424.9	4676.7	3812.9	10634.7	8493.2	261.2	10/11/97	6605.2	3721.7	3636.2	10030.9	7721.4	253.8
21/07/97	7431.9	4661.9	3843.6	10608.3	8535.2	261.4	11/11/97	6659.5	3739.5	3632.1	10219.4	7759.2	254.7
22/07/97	7430.2	4650.4	3843.2	10611.3	8543.2	262.3	12/11/97	6540.9	3664.9	3537.9	10026.6	7646.7	253
23/07/97	7446.9	4639.4	3853.7	10630.4	8593.1	262	13/11/97	6512.6	3625.7	3519.6	10041.4	7614.1	253.1
24/07/97	7427	4593.9	3837.8	10621.8	8604.7	262.2	14/11/97	6373.7	3492.4	3455.8	9845.4	7508.3	251
25/07/97	7444	4590.7	3835.9	10673.2	8632.5	262.1	17/11/97	6427.8	3506.8	3472.8	9961.2	7581.4	250.2
28/07/97	7490.8	4615.9	3817.3	10717.4	8719.7	263.5	18/11/97	6419	3499.8	3498.6	9966.3	7552.3	249.9
29/07/97	7473.8	4586.6	3822	10733.4	8697.3	264.8	19/11/97	6392	3482.5	3481.1	9953	7510.2	248.1
30/07/97	7473.7	4573.4	3815	10743.6	8712.3	263.7	20/11/97	6422.8	3485.5	3481.2	10035.5	7553.5	247.7
31/07/97	7484.5	4598.6	3779	10663.5	8764.6	267.5	21/11/97	6405.9	3442.3	3473.3	10090.5	7533.6	247.9
01/08/97	7456.8	4574.3	3805.8	10590	8743.3	267	24/11/97	6397.2	3495.6	3445.6	10055.4	7469	246.8
04/08/97	7445.2	4605.9	3791.4	10524.8	8707.8	266	25/11/97	6379.2	3513	3382.5	10047	7422.4	246.2
05/08/97	7492.7	4623.4	3852.1	10593.2	8766.2	264.6	26/11/97	6344.5	3430.8	3270.3	10140.5	7419.6	246.3
06/08/97	7546.8	4644	3919.7	10699.9	8815.9	265.6	27/11/97	6378.6	3446.3	3267.6	10271	7432.5	244.8
07/08/97	7614.4	4723.6	3962.2	10725.6	8879.2	266.4	28/11/97	6326.3	3418.6	3202.4	10201.2	7377.1	243.4
08/08/97	7599.2	4719.7	3967.5	10714.2	8843.5	265.4	01/12/97	6324.5	3395.1	3173.6	10151.9	7438.3	243.9
11/08/97	7583.1	4742.5	3980.4	10644.9	8798	264.8	02/12/97	6341.6	3425.9	3235.5	10209.6	7397	241.3
12/08/97	7579.9	4700.1	3993.5	10674.7	8823.7	265.1	03/12/97	6296.8	3390.4	3263	10144.3	7337.1	240.3
13/08/97	7538	4646.4	3985.9	10630	8797	266	04/12/97	6262.7	3373.8	3224.8	10121.6	7284.9	240.3
14/08/97	7530.3	4635.3	3978.9	10632.1	8791.8	264.8	05/12/97	6160.1	3322.2	3145.6	10023.6	7132.2	240.3
15/08/97	7439.8	4572.1	3941.2	10522.6	8681.7	263	08/12/97	6156.1	3331.3	3149.1	10043.3	7097.6	239.7
18/08/97	7361.3	4523	3914.9	10371.9	8605.6	260.8	09/12/97	6158.8	3336.4	3138.9	10061.7	7092.9	240.1
19/08/97	7397.2	4550.9	3901	10423.6	8653	260.9	10/12/97	6097.4	3302.6	3122.4	9968.1	7012.4	240.4
20/08/97	7437.8	4560.2	3928.4	10502.4	8709.3	259	11/12/97	5953.8	3220.5	3007.3	9795.7	6831.8	240.7
21/08/97	7463.1	4574	3934.5	10549.6	8738.7	258.2	12/12/97	5951.5	3222.4	2989.4	9754.9	6849.5	241.3
22/08/97	7347.9	4485.3	3903.3	10354.6	8630.8	258.8	15/12/97	6058.9	3283.4	3049.3	9927.1	6972.5	240.8
25/08/97	7398.6	4521.9	3935.9	10420	8684.5	259.2	17/12/97	6170	3344.7	3132.4	10086.3	7103.1	240.5
26/08/97	7378.6	4523	3960.2	10349.7	8651.4	260.3	18/12/97	6168.8	3367.7	3116.6	10081	7078	240.4
27/08/97	7391.2	4531.3	4003.4	10389.8	8639.1	261.3	19/12/97	6049.9	3314.1	3046.8	9869.9	6937.5	239.2
28/08/97	7380.4	4545.4	4043.7	10363.8	8587.6	260	22/12/97	6042.3	3329.7	3047.6	9865.6	6896.7	240.4
29/08/97	7307	4498.8	3983.9	10257.7	8510	262.4	23/12/97	6104.6	3359.7	3060.1	9973.3	6978.5	240.4
01/09/97	7255.5	4494.8	3987.4	10086.1	8453.9	260.9	24/12/97	6091	3364.2	3042.5	9947.4	6953.5	240.7
02/09/97	7273.2	4500.1	4041.8	10133.3	8453.2	261.2	29/12/97	6130.3	3375.9	3043.7	10058.9	6991.9	242.1
03/09/97	7317.8	4503.2	4100.2	10246.2	8498.2	261.1	30/12/97	6169.3	3363.6	3066.6	10170.8	7051.7	244
04/09/97	7336.9	4508.3	4123.6	10293.8	8512.7	261.9	31/12/97	6202.3	3367.3	3063.7	10249.8	7098.4	250.4
05/09/97	7400.2	4540.5	4128.6	10477.6	8558.5	261.8	05/01/98	6190.4	3384	3082.1	10184.7	7070.8	249.8
08/09/97	7393.2	4529.9	4129.2	10534.1	8522	261.6	06/01/98	6154.9	3332.6	3036.7	10202	7037.9	249.8
09/09/97	7353.3	4521.5	4069.1	10463.5	8475.6	262	07/01/98	6073.9	3264.7	2972.2	10091.6	6969	250.1
10/09/97	7311.3	4503.5	3997.6	10411.9	8430.6	261.5	08/01/98	6057.2	3232	2932.3	10101.2	6970.8	249.6
11/09/97	7172	4405.8	3910.8	10215	8287.1	259.9	09/01/98	5990.6	3171	2796.9	10048	6932.1	247.6
12/09/97	7168.7	4401.8	3937.8	10196.2	8282	260.2	12/01/98	5596.6	2953.8	2547.4	9373.9	6510.2	246.4
15/09/97	7179.5	4396.5	3940.6	10263.3	8284.3	260.4	13/01/98	5691.6	2957.9	2629.6	9599.3	6631	246.7
16/09/97	7145.8	4362.8	3933.1	10226.3	8253.1	258.4	14/01/98	5807.9	3024.7	2672.3	9865.2	6726	248.1
17/09/97	7191.8	4369.8	3966.1	10302.7	8326.9	258.1	15/01/98	5813.3	3021	2635	9946.6	6715.6	247.3
18/09/97	7191.8	4349.9	3964.9	10317.4	8345.8	258.3	16/01/98	5868.4	3075	2607.5	10067.3	6752.6	247.3
19/09/97	7177.2	4350.7	3953.2	10345.1	8292.1	256.9	19/01/98	5938.3	3069.4	2623.5	10249.6	6860.3	247.5
22/09/97	7155.4	4349.5	3981.8	10228.4	8280.3	256.6	20/01/98	6012.4	3098.8	2602.3	10445.4	6938.9	249.4
23/09/97	7057.8	4270.5	3950.7	10089.3	8183.1	255.3	21/01/98	6074.7	3104.6	2595.7	10638.6	7009.4	250.2
25/09/97	7094.7	4324.7	3984.5	10114.5	8193.5	256	22/01/98	6077.6	3119.5	2599.7	10659.9	6983.9	253.7
26/09/97	7101.3	4372.1	3976.1	10071.3	8176.4	255.4	23/01/98	6073	3102.8	2558.1	10665	7005	252.5
29/09/97	7084.4	4359.3	3919	10049	8177	255.4	26/01/98	6146.1	3195.3	2548.2	10743.3	7061.9	253.6
30/09/97	7123.4	4403.5	3939.4	10071.2	8212.6	259.5	27/01/98	6183.8	3187.4	2555.3	10849.9	7121.8	254.5
01/10/97	7130.2	4465.9	3901.3	10010.7	8197.3	258.4	28/01/98	6344.3	3270.3	2630.8	11154.8	7294.2	255.9
02/10/97	7174.3	4495.1	3928	10086.5	8238.3	258.7	29/01/98	6508	3449.7	2724.2	11310.5	7428.4	257
03/10/97	7214	4500.2	3974.4	10157.4	8292.9	259.9	30/01/98	6550.3	3530.3	2769.4	11306.6	7434.7	258.7
06/10/97	7237.4	4513.1	4025.3	10165.2	8321.2	259.9	02/02/98	6644.7	3605.6	2824.9	11429.7	7529	257.1
07/10/97	7262.1	4516	4035.1	10181.3	8378	259.7	03/02/98	6569.4	3560	2857	11242.4	7456.2	257.6
08/10/97	7296.6	4552	4022.7	10261.5	8394.9	259.1	04/02/98	6506.7	3480.1	2843.6	11212.7	7391.8	259.9
09/10/97	7249	4530.3	3965.6	10174.2	8351.3	259	05/02/98	6609.9	3568.3	2848.7	11356.6	7502.2	260.8
10/10/97	7237.6	4521.2	3953.5	10178.3	8332.1	259.3	06/02/98	6620.5	3548.7	2860.2	11411.1	7523.7	261.4
13/10/97	7297.1	4557.1	3983.9	10290.7	8389.3	258.2	09/02/98	6664.6	3535.8	2821.2	11568.3	7598.5	261.5
14/10/97	7302.7	4540.4	3986.5	10345	8397.9	259.1	10/02/98	6664.3	3485.7	2846.4	11615	7625.3	262.7
15/10/97	7282.4	4523.7	3976.2	10357.5	8356.9	259.4	11/02/98	6679.1	3421.2	2858.3	11736.1	7679.6	262

Table A.11: Data: Equity indices (daily) 25/06/97 – 11/02/98

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
12/02/98	6631.8	3371.1	2833.6	11740.2	7610.1	264.5	10/06/98	7139.6	3356.4	2521.8	12799	8644.5	241.7
13/02/98	6657	3436.2	2825.9	11763.7	7590.6	263.9	11/06/98	7084.7	3328.4	2486	12755.8	8554.8	241.8
16/02/98	6697.2	3430.8	2860	11894.3	7630.7	263	12/06/98	6912	3255.2	2365.7	12473.2	8338.9	239.9
17/02/98	6761.2	3455.5	2883.9	12047.7	7694.6	263	15/06/98	6720.4	3173.9	2363.4	12053.5	8114.3	237
18/02/98	6813.2	3431	2913.4	12274.7	7743.7	261.7	17/06/98	7018	3325.6	2407.6	12606.3	8473.1	237.6
19/02/98	6800.9	3427.3	2894	12254.2	7729.7	263	18/06/98	7043.3	3319.5	2371.8	12754.2	8485.6	238.6
20/02/98	6802.5	3447.8	2832.5	12264.3	7721.4	264.5	19/06/98	6919.9	3299.1	2290	12465.6	8339.8	237.6
23/02/98	6826	3380.1	2834.3	12379.7	7805.7	267.1	22/06/98	6684.2	3219.7	2210.8	11942.9	8066.4	233.2
24/02/98	6788	3323.4	2755.6	12406.6	7777.7	265.3	23/06/98	6777.5	3248.6	2258.2	12102.4	8200.2	229.8
25/02/98	6873.8	3315.4	2767.3	12755.7	7843.6	266.9	24/06/98	6952	3272.9	2375.5	12633.2	8345.5	228.3
26/02/98	6996.2	3324.4	2758.8	13129.8	7989.1	270.4	25/06/98	6936	3250.1	2420.4	12601.2	8327.7	228.6
27/02/98	7095.7	3378	2767	13417.3	8047.9	273.4	26/06/98	6874.3	3241.9	2415	12445.3	8241.7	226.9
02/03/98	7166.7	3424.3	2716.9	13461.3	8191.9	273.5	29/06/98	6768.2	3352.7	2383.3	11985.4	8053	224.9
03/03/98	7122.3	3393.5	2700.5	13315.5	8187.4	270.5	30/06/98	6771.6	3395	2388.5	11999.6	8000.9	222.8
04/03/98	7052	3338.6	2684.7	13101.2	8173.8	271.2	01/07/98	6822.2	3577.7	2434.4	11765.3	8032.6	221.1
05/03/98	6861.8	3278.2	2570.5	12592.5	8008.4	271.4	02/07/98	6921.2	3786.2	2538.4	11670.4	8076.5	221
06/03/98	6922.8	3319.6	2570.6	12711.6	8067.7	273.6	03/07/98	7004	3931.7	2647.3	11686.2	8088.9	222.1
09/03/98	6998.4	3366.6	2673.1	12837.7	8129.9	277	06/07/98	7101.4	4179.1	2761.7	11472.6	8148.6	220.2
10/03/98	7056.8	3415.9	2705.8	12889.9	8199.9	277.9	07/07/98	7198.1	4078.9	2805.7	11883.5	8314.5	219.1
11/03/98	7178.6	3442.5	2654.3	13408.8	8256.6	278.2	08/07/98	7175.5	3969.7	2807	11968.1	8335.6	219.1
12/03/98	7200.6	3412.1	2695.9	13456.9	8323	274.6	09/07/98	7128	3896.6	2793.3	11959.5	8295	219.9
13/03/98	7204.7	3453.1	2734.1	13302.5	8354.5	274.7	10/07/98	7116.3	3949.5	2813.1	11853.2	8247.8	220.7
16/03/98	7282.3	3494.2	2795.4	13465.3	8418.9	275.1	13/07/98	7082.7	3854.7	2819.4	11886.9	8246.2	220.5
17/03/98	7293.9	3478.8	2747	13472.4	8483.6	274.8	14/07/98	7164.5	3801.8	2825.3	12272	8335.5	220.3
18/03/98	7210.1	3386.9	2705.9	13381.2	8420.5	273.2	15/07/98	7309.4	3860.9	2820.7	12696	8450.4	220.4
19/03/98	7244.5	3431.1	2688.3	13431.6	8443.2	272.9	16/07/98	7392.5	3914.8	2821.7	12878.7	8517.4	218.9
20/03/98	7179.7	3382.4	2664.8	13307.3	8391.1	272.5	17/07/98	7431	3944.8	2825.6	12974	8537.1	218.7
23/03/98	7166.1	3367.1	2686.7	13266.6	8384.7	272.6	20/07/98	7332.5	3947.4	2815.9	12687.3	8401.8	217.2
24/03/98	7230.2	3460.5	2730.5	13354.8	8390.5	272.9	21/07/98	7285	3902.6	2768.7	12586.1	8385.8	214.9
25/03/98	7314.2	3454.8	2679.9	13696.3	8475.4	275.5	22/07/98	7201.4	3867.9	2695.6	12410.1	8311.9	212
26/03/98	7349.6	3449.4	2693.4	13869.3	8486.7	276.9	23/07/98	7119.8	3801.6	2685.8	12297.8	8212.4	211.9
27/03/98	7473.1	3538.5	2718.1	14160.4	8570.1	275.9	24/07/98	6993.6	3711	2606.2	12134.3	8076.1	209.5
30/03/98	7531	3605.5	2758.4	14190.4	8626.2	276.7	27/07/98	6883.7	3643.6	2550.1	11949.5	7963.4	209.3
31/03/98	7578.9	3633.4	2792.5	14237.4	8688.9	281.1	28/07/98	6882.9	3623.2	2543.3	11964.5	7980	209.1
01/04/98	7710.8	3698.9	2777.3	14483.6	8866.1	277.6	29/07/98	6905.7	3614.2	2569	12018.9	8021	208
02/04/98	7778	3729.3	2786.7	14463.1	9032.1	279.5	30/07/98	7022.2	3634.4	2624.8	12346.5	8136.4	208.7
03/04/98	7828	3779.7	2803	14479.5	9099.8	277.4	31/07/98	7020.4	3609.9	2611.9	12438.3	8114.4	206.2
06/04/98	7940.3	3906.1	2830	14596.5	9195.4	277.8	03/08/98	6887.7	3518	2509.2	12272.3	7963.6	208.2
07/04/98	7982.1	3920.6	2868.5	14648.3	9258.8	278.3	04/08/98	6956	3542	2495.1	12458.6	8031.7	207
08/04/98	7948.9	3932.6	2861.8	14517.1	9222.6	277.1	05/08/98	6780.4	3510	2396.9	12017.4	7832.2	207.3
09/04/98	7936.9	3938.3	2922.2	14395.1	9226.2	274.9	06/08/98	6721.8	3499.6	2364.1	11881.8	7759.9	203.3
14/04/98	8048.9	3968.4	3039.7	14619.7	9352.9	273.1	07/08/98	6643.3	3509	2284.3	11623.9	7690.1	199.1
15/04/98	8199.9	4066.9	3230.3	14819.3	9492.5	273.9	11/08/98	6303.7	3365.5	2078.4	10800.2	7410.1	192.5
16/04/98	8201.4	4082.2	3283	14771.8	9481.9	273.1	12/08/98	6332.7	3342.9	2144	10951.3	7404.2	192.7
17/04/98	8194.7	4113.1	3371.7	14648.1	9458.7	272.3	13/08/98	6200.3	3206.3	2090.6	10779.6	7292.1	192.2
20/04/98	8358.7	4197	3558.8	14794.5	9687.3	273.1	14/08/98	6291	3287.8	2141.4	10886	7373.8	192.6
21/04/98	8282.2	4087.5	3470.6	14775	9643.3	273	17/08/98	6176.8	3229.6	2078.7	10655.1	7260.4	191.6
22/04/98	8310.1	4151.2	3384.1	14766.2	9679.8	272.6	18/08/98	6273.4	3268.4	2069	10883.8	7368.9	192.4
23/04/98	8286.3	4207.3	3301.7	14577	9672.3	269.5	19/08/98	6384.4	3398.6	2128.4	11037.8	7434.4	192.5
24/04/98	8258.6	4234.8	3300.7	14476.5	9612.9	269.5	20/08/98	6314.4	3414.9	2086.8	10820.5	7347.9	192.3
28/04/98	8138.8	4154.1	3278.2	14256.3	9496	265.5	21/08/98	6039.2	3339.7	1962.1	10162.8	7050	190.4
29/04/98	8181.2	4221	3304.5	14165.2	9578.4	264.7	24/08/98	5916.8	3277.7	2003.1	9897.7	6905.1	189.5
30/04/98	8235.5	4352.6	3320.9	14048.2	9629.5	263.4	25/08/98	5897.6	3261.1	2024.7	9887.2	6865.8	189.5
04/05/98	8264.2	4262.7	3420.9	14133.2	9743.1	263.7	26/08/98	5532	3063.3	1997.5	9137.2	6482	187.9
05/05/98	8123.3	4202.9	3307.5	13832.2	9611	262.1	27/08/98	5218.6	3071.2	2025.9	8241.3	6076.2	186.4
06/05/98	8120.2	4206.5	3318.9	13797.3	9613.2	263.1	28/08/98	4986.3	2948	1971.9	7743.3	5855.6	184.1
07/05/98	8054.1	4113.7	3272.3	13639.5	9642.9	262.9	31/08/98	4923.4	2985.2	2028.3	7478.8	5768.4	179.3
08/05/98	8075.5	4167.7	3270.8	13613.7	9654.2	262.2	01/09/98	4807	3021.6	2055.9	7123.7	5579.3	177.5
11/05/98	8170.5	4221.5	3298.3	13757.8	9775.8	260.5	02/09/98	4911.5	3121.6	2062.8	7235.5	5693.1	178.3
12/05/98	8016	4126	3180.7	13499.4	9624.8	262.2	03/09/98	4791.1	3138.4	1978.3	6898.2	5537.2	178.5
13/05/98	8029.4	4049.7	3261.8	13558.1	9700.5	262.7	04/09/98	4826.9	3312.5	1983.1	6736.6	5516.9	178.9
14/05/98	8019.3	3985	3239.8	13654.7	9705.6	260.9	07/09/98	4954.3	3477.2	2138.2	6791.7	5604.1	180.7
15/05/98	8071.7	4033.4	3222.8	13760.5	9742.8	260.9	08/09/98	5050	3546.3	2306.4	6926.2	5667.9	180.6
18/05/98	8079.4	4009.9	3218.2	13859.6	9743.8	258.7	09/09/98	4975.3	3436.1	2274.6	6927.2	5597.8	180.8
19/05/98	8121.6	4014.5	3261	13979.6	9780.6	257.4	10/09/98	4852.7	3473.2	2107.9	6708.4	5377.9	181.8
20/05/98	8085.5	3984.4	3194.3	13961.7	9739.1	258.4	11/09/98	4694.4	3549.1	2000.3	6196.6	5145.9	181.8
21/05/98	8127.4	3986	3196.8	14083.7	9792	260	14/09/98	4780.8	3514.4	2044	6449.1	5286.2	181.5
22/05/98	8098.8	3939.6	3182.6	14058.9	9786.1	260.4	15/09/98	4846.3	3547.7	2093.4	6552.2	5360.9	181.9
25/05/98	7958.4	3854.5	3122.8	13810	9639.6	257.8	16/09/98	5009.5	3539.4	2185.1	7010.4	5556.8	184.7
26/05/98	7842.1	3820.8	3064.1	13543	9511.4	254.8	17/09/98	4869	3491	2083.2	6830.2	5344.4	182.3
27/05/98	7452.2	3617.4	2933.7	12800.9	9084.4	252.9	18/09/98	4844.4	3435.1	2074.5	6922.4	5290.9	182.9
28/05/98	7465.6	3687.2	2880.5	12796	9059.6	251.9	21/09/98	4751.6	3389.9	2016.1	6810.5	5156.3	183.1
29/05/98	7629.6	3716.8	2874.5	13380.7	9177.5	250.6	22/09/98	4906.7	3349.6	2117.8	7207.6	5401.4	186.3
01/06/98	7487.9	3666.2	2824.7	13147.7	8973	249.6	23/09/98	5068.9	3332.9	2126.6	7617.1	5665.8	187
02/06/98	7527.9	3661.6	2814.1	13322.4	9004.2	249.4	25/09/98	5138.5	3490.1	2078.1	7613.1	5695.8	186.1
03/06/98	7569.9	3624.3	2768.8	13455.3	9115.3	249.2	28/09/98	5293.4	3506.3	2052.8	7905.1	5965.5	186.2
04/06/98	7516.3	3563.1	2715.8	13401.4	9084.1	248.6	29/09/98	5194.4	3514.1	2034.9	7652	5817.8	184.5
05/06/98	7491.1	3499.3	2650.1	13499.1	9060.3	247.7	30/09/98	5098.6	3445	1992.5	7516.7	5711	188
08/06/98	7446.2	3437.1	2657	13426.9	9042.8	247.8	01/10/98	4927	3489.5	1920.1	7051.5	5444.4	186.9
09/06/98	7386.3	3487.4	2653.6	13231.7	8915.7	244.1	02/10/98	4958.5	3629.7	1934.4	6844.6	5478.1	185.9

Table A.12: Data: Equity indices (daily) 12/02/98 – 02/10/98

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
05/10/98	5019.8	3641.6	1934.2	6993.8	5559.5	184.4	28/01/99	5766.8	3568	1762.6	8953.6	6723.4	202.6
06/10/98	5116.6	3657	1948.5	7288.8	5653.7	186.8	29/01/99	5799.1	3591	1791.2	9003.2	6753.9	203.3
07/10/98	5130.8	3664.8	1961.2	7420.5	5607.8	188.7	01/02/99	5824.3	3572	1777.5	9067.8	6808.7	204.2
08/10/98	5003	3683	1906.2	7128.9	5395.5	186	02/02/99	5786.2	3565.7	1782	8999.9	6743.5	206.8
09/10/98	5061.8	3683.7	1924.9	7263.3	5486.3	185.4	03/02/99	5759.3	3540	1740.2	8994.9	6709	207.4
12/10/98	5212.8	3716.4	1976.3	7584.1	5690.1	184.4	04/02/99	5809.5	3567.6	1770.9	9108.2	6747	208.7
13/10/98	5197.4	3651.9	1973.5	7626.5	5700.5	184.1	05/02/99	5849.3	3603.5	1747.1	9287.6	6725	208.6
14/10/98	5275.5	3634.2	2019.2	7896.8	5784.3	182.9	08/02/99	5912.1	3646.9	1740.4	9443.6	6767.2	209.8
15/10/98	5375.4	3668.5	2094.6	8190.8	5844.4	182.5	09/02/99	5893.4	3693.5	1751.7	9300.7	6744	211.1
16/10/98	5653.3	3733.8	2242.3	8861	6151.5	184.7	10/02/99	5837.5	3712.2	1736	9139.8	6664.2	208.7
19/10/98	5615.1	3634.8	2206.8	8906.3	6137.2	186.9	11/02/99	5887.1	3770.9	1763.8	9161.1	6723	206.5
20/10/98	5764.9	3699	2259.3	9177	6323.9	187.6	12/02/99	5931.9	3846.9	1810.7	9158.4	6757	206.8
21/10/98	5734.9	3692.1	2191.4	9065.4	6331.3	189.4	15/02/99	5901.5	3834.4	1804.4	9060.8	6743.2	207
22/10/98	5683.6	3623.3	2204.2	8954.5	6321.1	189.8	16/02/99	5896.8	3861.2	1853.3	8989.5	6728.2	208.3
23/10/98	5622	3641.5	2144.4	8793.2	6234.6	189.7	17/02/99	5871.3	3830.5	1876.6	8952.5	6702.4	209.4
26/10/98	5625	3619.3	2144.7	8820.2	6253.8	189.3	18/02/99	5897.8	3854	1867.4	8942.3	6761.9	211
27/10/98	5732.1	3648.3	2188.8	9038.2	6391.5	188.7	19/02/99	5871.7	3859.9	1836.1	8865.5	6736.2	210.8
28/10/98	5709.2	3658.5	2147.3	8986	6355.9	189.7	22/02/99	5898.3	3869.9	1845.4	8927.6	6763	208.2
29/10/98	5747.3	3657.4	2171.9	9066.3	6412.2	191.7	23/02/99	5938	3849.3	1873.1	9040.6	6825.9	205.6
30/10/98	5828.3	3667.1	2180	9247.8	6528.3	196.1	24/02/99	5941.8	3874.4	1866.1	9005.9	6831.7	208.4
02/11/98	5977.5	3710.1	2218	9587.5	6699.1	203.3	25/02/99	5940.7	3896	1874.7	8949.7	6837.4	207.7
03/11/98	5959	3685	2280.1	9532.9	6680.9	206.8	26/02/99	5914.6	3893.6	1882	8886.3	6802.9	209.1
04/11/98	6061.3	3688.3	2322.8	9810	6805.1	205.6	01/03/99	5948	3935	1877	8899	6846.9	209.7
05/11/98	6030.7	3665.5	2231.5	9832.7	6760.8	206.1	02/03/99	5980.2	3975.3	1864.7	8955.2	6866.3	209.8
06/11/98	6024.3	3647	2265.7	9780.7	6777.7	205.9	03/03/99	6087.5	4048.2	1893.5	9142.6	6975.9	211.2
09/11/98	5916.1	3588.9	2236.7	9552.6	6669	206.7	04/03/99	6130.7	4050	1938.9	9200.9	7051.9	210.2
10/11/98	5783.9	3553.6	2225.5	9194.2	6536.4	204.5	05/03/99	6221.6	4075	2086.7	9372.1	7138.8	212.5
11/11/98	5821.6	3598	2231.3	9201.3	6586.2	205.1	08/03/99	6275.2	4057.1	2129.2	9472.1	7239	212.8
12/11/98	5727.4	3596.3	2223.6	8934.2	6470.9	204.6	09/03/99	6286.6	4014.3	2143.1	9496.4	7298.3	216.8
13/11/98	5730.7	3677.1	2244.9	8815.5	6447.2	204.7	10/03/99	6279.5	3996.4	2134.8	9473.4	7314	218
16/11/98	5753.6	3650.8	2253.9	8874.6	6507.7	200.5	11/03/99	6385.1	4069.2	2190.8	9595.1	7456.2	217.6
17/11/98	5735.7	3640.5	2174.9	8870	6499.7	197	12/03/99	6440.1	4156.2	2242.4	9604.7	7495.6	220.3
18/11/98	5703.9	3662	2195.7	8727.2	6456.2	196.4	15/03/99	6409.8	4138.1	2214.2	9544.6	7472	221.8
19/11/98	5731.6	3699	2222.5	8741.8	6476.8	196.5	16/03/99	6505.6	4236.1	2234.5	9692.4	7548.6	219.5
20/11/98	5767.8	3679.6	2212.6	8909.9	6512.1	195.8	17/03/99	6419.8	4270.8	2187.9	9652.1	7590	220
23/11/98	5825.3	3662.3	2160.6	9156.8	6574.2	196.7	18/03/99	6479.6	4219	2182	9565.4	7582.6	222
24/11/98	5770	3667.4	2099.9	9097.5	6462.7	194.9	19/03/99	6574.9	4266.1	2202.6	9582.9	7711	227.5
25/11/98	5702.1	3662.1	2024.5	8945.5	6383.2	194.1	23/03/99	6547.4	4263.7	2250.1	9653.6	7654.1	229
26/11/98	5706.3	3680.5	2011.4	8919.9	6391.9	192.8	24/03/99	6463.3	4190.5	2240.2	9547	7559.5	225.3
27/11/98	5655.7	3666.9	1945.4	8848.1	6323.6	192.5	25/03/99	6519.2	4244.5	2252.3	9647.6	7598.1	226.4
30/11/98	5620.9	3688.1	1915.3	8733.8	6272.4	192.8	26/03/99	6460.6	4237.4	2222.1	9500.5	7535.6	224.7
01/12/98	5387.7	3575.2	1874.1	8251.6	6023.2	191.8	29/03/99	6439	4252.6	2221	9361.7	7539.9	224.7
02/12/98	5391.6	3567.5	1936.1	8223.2	6041.7	190.9	30/03/99	6422.9	4218.8	2273.1	9350.9	7520.4	225.2
03/12/98	5299.2	3521.4	1887	8030	5956.1	192	31/03/99	6382.5	4222.6	2250.9	9250.6	7467.4	225.8
04/12/98	5211.7	3456.9	1836	7853.3	5895	190.8	01/04/99	6402.6	4278.1	2235.4	9226.4	7483.7	227.8
07/12/98	5247.6	3510.1	1832.6	7828.5	5952.1	191.6	06/04/99	6406.8	4293.1	2259.2	9234.8	7466.8	229.1
08/12/98	5216.3	3566.9	1808.2	7677.4	5892.6	191.1	07/04/99	6404	4346.7	2266.5	9163.7	7444.6	229.3
09/12/98	5236	3567.3	1851.6	7754.8	5893.2	191.1	08/04/99	6403.4	4445.9	2271.2	9051.7	7403.6	229.9
10/12/98	5251.4	3523.2	1836.2	7833.3	5945.4	191.5	09/04/99	6474	4596.1	2304.5	9084.8	7416.9	231.1
11/12/98	5215.4	3486.7	1815.3	7768.7	5926.4	190.6	12/04/99	6498.2	4602.2	2349.2	9135.5	7436.8	231.2
14/12/98	5160.1	3465.5	1736.2	7681.3	5865	190	13/04/99	6602.1	4662.1	2539.6	9280.1	7527.7	231.3
15/12/98	5171.7	3492.5	1786.8	7646.6	5876.6	187.7	14/04/99	6672.8	4710.4	2663.7	9413.5	7561.8	232.5
17/12/98	5135.6	3447.8	1785.1	7555	5875.9	188.5	15/04/99	6785.4	4959.4	2950.7	9455	7513.4	233.4
18/12/98	5108.7	3386.1	1784.2	7534	5878.6	187.3	16/04/99	6932.3	5333.1	3082.4	9485	7482	235.2
21/12/98	5157.7	3390.1	1802.6	7681	5921.3	189.5	19/04/99	6992.7	5362.3	3143.5	9576.6	7550.3	234.3
22/12/98	5182.1	3335.4	1772.7	7795.5	5990.2	190	20/04/99	6922.2	5358.9	3038.7	9439.7	7463.8	235.8
23/12/98	5180.7	3289.8	1763.7	7871.7	5989.5	192.8	21/04/99	6786	5137.6	2902.2	9408.3	7366.5	235
24/12/98	5238.8	3333.5	1782.9	7978.8	6039.4	193.7	22/04/99	6740.5	5069.3	2865.7	9403.9	7322.2	233.2
28/12/98	5348.8	3366.1	1789.3	8236.9	6163.1	194.5	23/04/99	6782.2	5224.1	2884.6	9344.8	7309	234.8
29/12/98	5396.5	3366	1774.2	8383.7	6214.4	197.1	26/04/99	6815.4	5219.6	2925.1	9444.2	7336.8	234.8
30/12/98	5438.3	3381.7	1765.3	8457.2	6274.5	200.2	28/04/99	6922.7	5300.9	3007.5	9578.4	7454.9	236.6
31/12/98	5430.5	3372.4	1797.2	8427.9	6265	206.2	29/04/99	6967	5351.2	3050.5	9671.8	7462.3	235.4
04/01/99	5404.9	3348.7	1788.8	8358.9	6262.6	203.1	30/04/99	7064.7	5517.1	3155.6	9716.7	7507.5	239.8
05/01/99	5425.2	3335.4	1774.1	8459	6279.2	203.3	03/05/99	7009.6	5476.7	3154.9	9642.1	7436.8	239.5
06/01/99	5680.7	3388.8	1819.4	9092.2	6563.7	204.8	04/05/99	7024.3	5436.8	3164.3	9728.5	7466.6	240.5
07/01/99	5727.4	3443.4	1816.9	9124.1	6621	208.7	05/05/99	6916.4	5301.1	3044.2	9655.7	7381.2	239
08/01/99	5819.1	3482.8	1832.1	9335.5	6711.5	206.7	06/05/99	6963.5	5366	3039	9776.5	7378.7	239.8
11/01/99	5880.3	3526.8	1884.3	9311.6	6836.8	207.5	07/05/99	6918.5	5281.8	3002.9	9882.1	7286.3	241.1
12/01/99	5863	3613.2	1900.4	9161.2	6781.3	206.3	10/05/99	6890	5171.6	2943.6	9943.2	7301.1	238.4
13/01/99	5615.6	3601	1846.1	8521.5	6485.8	203.8	11/05/99	6883	5105.9	2935.5	9954.8	7345.1	239.1
14/01/99	5666.3	3643.6	1844.5	8544.6	6569.9	203.7	12/05/99	6792.8	5062.5	2898.8	9734.5	7272.7	242.4
15/01/99	5614.8	3672.9	1816.8	8353.7	6511.3	203.5	13/05/99	6802.3	5168.9	2897.4	9641.5	7246.7	241.5
18/01/99	5718.7	3635.3	1843.3	8654.4	6661.7	205	14/05/99	6753.9	5151.8	2833.4	9616.8	7161.2	240.1
19/01/99	5755.5	3626	1855.4	8770.1	6705.3	205.3	17/05/99	6642.7	5188.1	2787.4	9309.4	7007.9	236.3
20/01/99	5722.3	3574.3	1849.5	8683	6718.6	204.8	18/05/99	6678	5315	2750.6	9291.1	6997	236.4
21/01/99	5670.1	3578.2	1811.1	8597	6629.3	204.3	19/05/99	6645.4	5324	2686.7	9203.9	6962.2	239.7
22/01/99	5589.6	3564.6	1779.7	8441.6	6515.3	204.2	20/05/99	6659.1	5416.9	2694.3	9146.6	6938	239.6
25/01/99	5580.9	3550.9	1745.2	8496.4	6484.9	201.7	21/05/99	6721.7	5472	2775.2	9208.2	7001.4	238
26/01/99	5592.2	3567.2	1754.9	8479.7	6507	203.1	24/05/99	6671.2	5316.1	2782.9	9199.5	7025.2	234.6
27/01/99	5709	3595.9	1782.3	8751.1	6641.7	201.8	25/05/99	6610.6	5294.2	2712.6	9049.1	6984.4	235

Table A.13: Data: Equity indices (daily) 05/10/98 – 25/05/99

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
26/05/99	6549.9	5203.5	2671.9	8968	6968.3	235.4	20/09/99	6657.5	5857.8	3563.5	8403.4	6772.3	237.5
27/05/99	6479.2	5008.8	2701.6	8986.9	6950	235.6	21/09/99	6616.4	5879	3526.2	8303.9	6706.5	237
28/05/99	6483.4	5004.5	2666.6	9062.6	6932.7	234.2	22/09/99	6601.3	6016.9	3482.7	8166.5	6627	234.8
31/05/99	6488.8	4979.5	2697.9	9071.2	6960.8	234.3	23/09/99	6621.1	6033.2	3475.6	8199	6648.5	236.2
01/06/99	6669.1	5078.7	2742.5	9390	7167.1	235.9	27/09/99	6757.5	6393.4	3442.1	8170	6707.9	235.7
03/06/99	6772.4	5170.8	2821.8	9429.3	7322.4	233.2	28/09/99	6876.6	6681.1	3552	8135.2	6747.8	239.7
04/06/99	6769.3	5109.1	2853.8	9488.7	7332.4	233.2	29/09/99	6869	6548.6	3622.6	8210.9	6793.8	239.8
07/06/99	6887.5	5183.6	2885.8	9667.8	7476.9	231.1	30/09/99	6855.5	6379.9	3671	8259.2	6879.1	242.1
08/06/99	6922.1	5238.4	2885.8	9723.2	7484.6	232.3	01/10/99	6923.6	6401.5	3696	8367	6975.3	246.6
09/06/99	6942.3	5284.3	2929	9771	7452.4	232.3	04/10/99	7096.5	6493.2	3779.8	8611.9	7210.9	246
10/06/99	6943.5	5374.7	2864	9686.4	7432	233.4	05/10/99	7183.9	6640	3786.7	8745.5	7233.8	243.1
11/06/99	7004.3	5491.4	2904.9	9722.3	7452.3	231	06/10/99	7186.8	6639.5	3759.1	8730	7259.3	242.9
14/06/99	6991.4	5470.2	2911.1	9721.9	7435.5	233.9	07/10/99	7324.3	6737	3780.4	8920.9	7423.6	250.5
15/06/99	6990	5431.5	2945.7	9773.9	7432.7	232.2	08/10/99	7365.9	6659.5	3801.8	9111.7	7496.5	251.1
17/06/99	6991.5	5309.2	3032.9	9864.1	7486.8	233	11/10/99	7413.2	6628.4	3707.8	9305	7568.7	250.4
18/06/99	6998.1	5236.8	3039.6	9893.9	7562	234.5	12/10/99	7389.9	6634.3	3751.7	9237.1	7530.8	247.2
21/06/99	7093.7	5265.9	3059.7	10192.2	7622.5	233.3	13/10/99	7291.2	6580.2	3741.3	9087.8	7398.6	245.1
22/06/99	7064.6	5169.1	3056.9	10246.5	7608.4	233.7	14/10/99	7213	6570	3583	8996.3	7292.9	236.2
23/06/99	7007.5	5095.8	3017.7	10220.9	7549.7	233.6	15/10/99	7034.6	6454.8	3480.5	8728.5	7082	239.6
24/06/99	6981.1	5178.8	2988.5	10004.7	7529.5	229.4	18/10/99	7001	6422.6	3482.9	8649.8	7062.3	242.5
25/06/99	6963	5225.6	3010.1	9933.6	7474.7	223.2	19/10/99	7071.5	6329	3504.5	8897.4	7196.3	246.2
28/06/99	7004.4	5274.8	3006	9980.5	7510.2	226.9	20/10/99	7093.5	6374.8	3478.3	8906.2	7204	253.4
29/06/99	6984.9	5245.4	3035.8	9920.2	7524.5	218.9	21/10/99	6987.7	6268.8	3437.5	8765.4	7109.7	249.7
30/06/99	7048	5304.2	3086.9	10048.4	7561.1	215.4	22/10/99	7049	6282.9	3479.3	8855.5	7204.2	251.1
01/07/99	7174.8	5456.3	3213	10243.2	7617.8	216.3	25/10/99	7034.6	6224.6	3495	8881.7	7193.2	256
02/07/99	7224.9	5527.1	3259.8	10395	7585.6	217.9	26/10/99	7067.8	6253.7	3463.8	8863.6	7278	257.9
05/07/99	7339.5	5603.3	3461.1	10559.6	7682.9	217.5	27/10/99	7049	6245.5	3458.2	8829.3	7259.8	253.9
06/07/99	7290.9	5554.4	3423.3	10519.2	7628.6	218.4	28/10/99	7109.3	6363.2	3473.8	8853	7297	253.8
07/07/99	7243.9	5514.9	3381	10443	7586.4	221.9	29/10/99	7153.1	6414.8	3404.4	8880	7362.6	260.3
08/07/99	7239.1	5566.1	3352.9	10427.3	7523.2	230.6	01/11/99	7159.3	6442.6	3419.3	8899.2	7341.7	256.8
09/07/99	7231.3	5611.9	3327.6	10332.8	7513.5	232.7	02/11/99	7135.8	6474.3	3379	8823.3	7307.5	251.1
12/07/99	7288.1	5654.2	3376.4	10441.7	7554	233.4	03/11/99	7129.1	6455.9	3391.6	8814.1	7311.5	250.5
13/07/99	7188.5	5653.4	3400.3	10207.9	7406.6	232.9	04/11/99	7222.9	6493.1	3433.9	8934.4	7459.8	251.4
14/07/99	7216.2	5633.5	3424	10294.2	7446.6	233.1	05/11/99	7222.8	6398.6	3469.2	8994.8	7513.6	249.7
15/07/99	7217.1	5616.7	3566.5	10277.9	7435.5	235.4	08/11/99	7245.4	6361	3390.3	9100.2	7571.5	249.8
16/07/99	7166	5655.3	3499	10134.8	7363.5	231.2	09/11/99	7361.3	6393	3338.5	9352.8	7727.4	251.9
19/07/99	7175.9	5662.1	3534.9	10120.7	7384.9	231.8	10/11/99	7283.1	6309.2	3271.7	9301.8	7634.3	251.7
20/07/99	7117.6	5602.3	3499.6	10057.2	7326.7	231.3	11/11/99	7307.4	6338.5	3330.9	9228.8	7707	251.6
21/07/99	7068.6	5592.1	3518.9	9937.1	7267.8	230.1	12/11/99	7286.3	6340.3	3275.6	9142.2	7714	251
22/07/99	7087.6	5672	3540.3	9867.2	7281	230.8	15/11/99	7364	6360.2	3334.8	9245	7841	250.7
23/07/99	7112.5	5740.2	3564.8	9831.9	7301.7	230.6	16/11/99	7401.6	6346.7	3365.3	9251.2	7956	249.4
26/07/99	7051.7	5760	3502.9	9674	7221.8	231	17/11/99	7434.1	6546.5	3384.9	9108.1	7932.6	248.9
27/07/99	7065.1	5755.1	3567.5	9742.9	7201.6	232.1	18/11/99	7475.8	6677.4	3475.3	9104.3	7899.5	248.4
28/07/99	7084.5	5744.3	3570	9803.3	7229	233.4	19/11/99	7524.6	6630	3531.9	9210	8009.9	248.4
29/07/99	7065.1	5796.9	3532.5	9715.4	7182	235.4	22/11/99	7503.7	6548.5	3523.7	9245.7	8013.8	246.6
30/07/99	7095.9	5828.7	3547.1	9772.5	7199.8	235.9	23/11/99	7507	6492.3	3520.2	9295	8051.7	246.5
02/08/99	7071	5844	3629.6	9654.7	7165.8	234.8	24/11/99	7522.9	6607.7	3534.5	9220.8	8026.8	244.1
03/08/99	7124	5926.1	3671	9665.3	7219.6	234.7	25/11/99	7501.7	6550.4	3580	9165.6	8047.2	244.5
04/08/99	7212.1	6082.3	3730.3	9693.9	7265.8	246.3	26/11/99	7590.7	6670.3	3577	9220.8	8149.3	245.2
05/08/99	7180.6	6170.6	3801.1	9536	7159.3	249.9	29/11/99	7570.7	6652.9	3600.4	9198.7	8117.6	244.7
06/08/99	7141.6	6177.9	3803.8	9429.3	7105.3	250.2	30/11/99	7552.6	6694.2	3593.8	9091.5	8103	238.2
10/08/99	7088.3	6227.3	3761	9265.1	7016.6	250	01/12/99	7613.3	6677.9	3633.6	9220.4	8204.4	239
11/08/99	7094.1	6278.8	3723.6	9236.5	7018.6	245.1	02/12/99	7707.6	6775.8	3678.7	9334.9	8290.8	241.2
12/08/99	7188.3	6565.1	3812.1	9189.7	7008.8	243.1	03/12/99	7714.8	6786.8	3659.6	9296.9	8324.7	244.8
13/08/99	7177.1	6599.8	3819.9	9124.6	6979.5	244	06/12/99	7789.2	6811	3700	9437.9	8424	240.2
16/08/99	7160.4	6466.7	3818.7	9191.8	7028.4	241.1	07/12/99	7850.2	6833.7	3807.7	9520.3	8498.1	241.3
17/08/99	7150.2	6370.1	3651.1	9233.9	7120.3	239.6	08/12/99	7943.7	6941.1	3888.1	9594.2	8591.8	242.5
18/08/99	7188.3	6407	3606.8	9309.6	7156.7	241.2	09/12/99	8007.4	6970.5	3853.2	9779	8640.5	241.9
19/08/99	7114.4	6306.3	3658.9	9188.9	7106	241.6	10/12/99	8066.8	7068.7	3778.2	9921.5	8641.2	242
20/08/99	7091.9	6305.4	3691	9102.3	7087.8	241.7	13/12/99	8163.8	7167.7	3735.5	10120.5	8710.1	240.4
23/08/99	7095	6250.2	3672.9	9099	7164.9	239.6	14/12/99	8113.2	7059.3	3791.9	10120.6	8657.7	240.9
24/08/99	7056.9	6210.4	3622.3	9025.2	7157.1	238.8	15/12/99	8115.1	7086.4	3826	10108.2	8625.6	246.4
25/08/99	7065.7	6194	3619.1	9017.2	7205.6	239.3	17/12/99	8320.3	7522.1	3922.1	10238.5	8675.3	242.1
26/08/99	7026.7	6046.8	3534.8	9065.4	7234	239.9	20/12/99	8495.1	7585.1	3953.1	10622.2	8865.6	245.1
27/08/99	7022.7	6062	3497	9054.3	7227.9	238.1	21/12/99	8397.5	7364.9	3967.2	10553.3	8850.2	243.7
30/08/99	7014	6120.7	3496.2	8999.1	7179.8	236.7	22/12/99	8434.2	7318.2	3991.2	10596.9	8973.4	243.4
31/08/99	6938.1	6050.2	3527.4	8844.5	7120.4	237.4	23/12/99	8422.6	7223.2	3968.5	10613.1	9030.2	245.5
01/09/99	6960	6063.6	3540.8	8855.4	7160.6	237.8	24/12/99	8458.9	7241.4	3944.7	10576.1	9148.8	245.3
02/09/99	6903.6	6051	3499.2	8766.4	7081.4	235	28/12/99	8448.8	7217.2	3903.9	10615.7	9129.1	244.5
03/09/99	6902.9	6052.7	3539.9	8762.3	7069.4	234.5	29/12/99	8542.8	7325	3929.4	10724.4	9211.8	247.1
06/09/99	6909.8	6073.2	3530.7	8743.6	7083.5	234.5	04/01/00	8516.3	7338.9	3878.5	10632.7	9196.3	243.7
07/09/99	6898.4	6117.6	3522.9	8652.8	7063.7	236.3	05/01/00	8422.2	7227.9	3835.2	10532	9112.8	242.5
08/09/99	6817.8	6140.1	3518.7	8448.9	6935.7	237.9	06/01/00	8432.5	7348.9	3909.3	10413.8	9074.5	241.3
09/09/99	6769.5	6094.3	3481.3	8404.2	6877.1	240.2	07/01/00	8653.6	7521.2	4122	10732.6	9288.2	239.5
10/09/99	6731.3	6065.2	3494.6	8322.4	6847.1	238.1	10/01/00	8963.6	7612.6	4237.8	11460.6	9599.3	244.3
13/09/99	6687.1	5988.1	3519.3	8299.8	6805.8	236.6	11/01/00	9003	7514.7	4089.3	11738	9681.1	241.8
14/09/99	6768	6009.6	3591.2	8463.5	6896.2	237	12/01/00	8946.3	7444.9	4068.9	11644.7	9647.3	243.3
15/09/99	6787.6	5998.6	3651.2	8548.8	6892.9	237.1	13/01/00	8953.1	7407.6	4024.3	11610.7	9742.7	242.3
16/09/99	6701.7	5904.3	3595.4	8429.1	6824.3	240.9	14/01/00	9035.2	7443.4	4088.4	11676	9886.9	242.9
17/09/99	6669.4	5875.9	3582.1	8404.2	6779	240.2	17/01/00	9226.5	7483.6	4256.7	11941.1	10196	244.2

Table A.14: Data: Equity indices (daily) 26/05/99 – 17/01/00

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
18/01/00	9187.3	7378.3	4159.7	12022.1	10172.3	238.9	16/05/00	7558.5	6142.5	3553.1	9296.3	8609.5	241.1
19/01/00	9151.1	7401	4225.3	11849.8	10139.1	238	17/05/00	7461.4	6017	3474.3	9268	8492.6	242.4
20/01/00	9151.4	7422.8	4262	11836.2	10124.3	231.9	18/05/00	7420.3	6053.1	3413.3	9182.2	8403.5	244
21/01/00	9043.1	7285.1	4273.3	11806.8	9966.7	230.5	19/05/00	7254.9	6028	3480.6	8813	8164.5	240.4
24/01/00	9012.8	7221.3	4272.4	11718.2	9996.8	232.5	22/05/00	7211.4	6052.7	3460.4	8747.5	8059.8	240.6
25/01/00	8781.8	7105.4	4157.1	11400.5	9671.6	231.5	23/05/00	7241	6145.4	3449.3	8824.3	8009.4	240.9
26/01/00	8656.5	6964.4	4153	11176.4	9597.9	229.3	24/05/00	7126.1	6100.1	3367.8	8686.7	7830	241
27/01/00	8656.7	6943.9	4138.8	11208.1	9594.5	234.8	25/05/00	7260.6	6095	3443.9	8982.8	8015.6	247.7
28/01/00	8537.6	6955.3	4057.4	10904.5	9439.3	240.6	26/05/00	7236.5	6078.8	3435.5	9034.3	7935.8	247.4
31/01/00	8475.3	6873.7	4034.9	10866.7	9366.2	243.3	29/05/00	7309.7	6075.6	3467.4	9214.6	8027.2	251.9
01/02/00	8763.4	7069.4	4040.3	11333.4	9711.9	242.6	30/05/00	7372.3	6058.2	3490.1	9425.7	8092.5	252.4
02/02/00	8749.9	6982.5	4027.4	11377.5	9739.2	241.5	31/05/00	7364.2	5957	3431.4	9444	8177.8	250.7
03/02/00	8795.3	6946.5	4070.1	11456.1	9851.3	240.7	01/06/00	7395	6068.3	3349.8	9438	8180.6	249.6
04/02/00	8691.6	6933.3	4019.2	11240	9712.7	239.3	02/06/00	7506.9	6159.4	3391.7	9534.2	8343.1	248
07/02/00	8862.8	7237	4041.2	11498.9	9727.3	240.1	05/06/00	7537.9	6197.1	3413.9	9617.3	8328.6	254.6
08/02/00	8889.8	7109.8	3977.9	11563	9916.9	239	06/06/00	7652.6	6333.4	3430.4	9763.8	8430.4	253.1
09/02/00	8944.8	7152.3	3966.5	11673.8	9971.7	235.3	07/06/00	7703.8	6353.1	3485	9836.9	8495.2	256.5
10/02/00	8826.4	7079.8	3898.7	11527.4	9816.4	231.5	08/06/00	7720.2	6374.6	3546.4	9796.9	8529.2	255.8
11/02/00	8723.7	7001.7	3907.9	11369.8	9692.9	231.8	09/06/00	7753.3	6512	3536.7	9795.7	8489.9	256.5
14/02/00	8607.4	6857.8	3862	11119.6	9675	230.3	12/06/00	7834.2	6592.3	3564.3	9900.3	8560.8	263.7
15/02/00	8512.4	6834.6	3881.1	10844.9	9593.1	229.4	13/06/00	7781.8	6489.9	3540.5	9919.9	8507.5	264.5
16/02/00	8424.8	6828.3	3866.8	10686.5	9447.8	229	14/06/00	7735.3	6386.7	3519.7	9892.5	8500.9	263.4
17/02/00	8573.8	7062.4	3812.3	10755.5	9608.8	233.1	15/06/00	7695.6	6295.4	3520.3	9925	8460.9	263.1
18/02/00	8544.3	6973.1	3767.6	10721.9	9650.7	232.7	19/06/00	7700.3	6295.1	3461.6	9779.2	8566.2	268.5
21/02/00	8366.5	6832.2	3682.4	10564.3	9397	231.1	20/06/00	7726.6	6310.8	3406.7	9801	8638.2	260.8
22/02/00	8189.9	6625.5	3606.4	10303.6	9282.2	230.3	21/06/00	7656.3	6287.4	3406.9	9647.4	8548.7	261.7
23/02/00	8270	6641.9	3525.4	10435.7	9441.3	229	22/06/00	7619.1	6240.1	3419	9531.6	8551.4	264.5
24/02/00	8273.5	6503.6	3445.7	10473.3	9591.9	232.8	23/06/00	7574.5	6223.6	3410.9	9408.7	8517.6	263.5
25/02/00	8161.4	6277.7	3314.1	10383.7	9592.2	232.8	26/06/00	7633	6270.9	3443.5	9499.6	8574.2	264.1
28/02/00	8031.8	6073.1	3219.5	10231.1	9550.1	230.9	27/06/00	7637	6267.4	3457.8	9504.3	8580.6	265.6
29/02/00	7992.4	6040.2	3304.8	10159.8	9491.4	230.4	28/06/00	7656.2	6334.3	3444.9	9538.1	8548.2	268.2
01/03/00	8109.4	6243.8	3355.5	10188.6	9594.2	230.4	29/06/00	7640.4	6365.3	3372	9498.5	8511.6	269.6
02/03/00	8012.2	6205.3	3399.4	9942.8	9495	229.8	30/06/00	7709.7	6416.9	3359.1	9588.3	8605.4	271.4
03/03/00	8014.5	6223.4	3411.9	9896.6	9505.1	232.9	03/07/00	7741.5	6483.4	3306.9	9622.7	8618.4	273.9
06/03/00	8062.4	6314.6	3372.5	9994.9	9494.9	236.5	04/07/00	7782.8	6532.5	3315.9	9653.5	8664.6	275.1
07/03/00	8074.4	6327.8	3360.7	10050.6	9488.8	234.9	05/07/00	7825.8	6462.3	3266.8	9762.5	8803.8	276
08/03/00	8018	6238.8	3311.3	10028.2	9446	234.5	06/07/00	7770.5	6431.2	3334.8	9711.2	8691.5	275.7
09/03/00	7948.6	6060.7	3182.7	9972.5	9486.4	238.8	07/07/00	7723.6	6417.2	3345.4	9625.7	8618.7	275.7
10/03/00	7925.9	6001.9	3155.8	9894.7	9538	235.5	10/07/00	7775.2	6540.7	3407.4	9557.8	8679.7	268.5
13/03/00	7668.5	5729	2990.1	9548.9	9327	234.9	11/07/00	7770.4	6588.6	3392.1	9497	8659	268.4
14/03/00	7829.8	5793.8	3119.4	9740.8	9577.1	234.4	12/07/00	7888	6798.7	3373.5	9632.8	8712.4	268.3
15/03/00	7823.9	5834.5	3081	9662.6	9575.8	233.5	13/07/00	7908	6778.3	3389.2	9662.4	8770.9	265.9
16/03/00	7937.8	6139.3	3287.8	9646.5	9559.7	227.6	14/07/00	8003.6	6948.2	3443.9	9720.3	8821.9	269
17/03/00	7997.6	6206.6	3247.2	9872.7	9539.7	229.3	17/07/00	8000.4	6976.4	3480.3	9731.8	8776	264.9
20/03/00	8014.8	6284.7	3206.9	9931.5	9483.7	230.4	18/07/00	7978.4	7005.8	3454.6	9691.8	8714.9	265.5
22/03/00	8080.6	6325.4	3258.2	10167.8	9472.2	233.1	19/07/00	7990.2	6979.7	3455.8	9761.3	8729.8	266.8
23/03/00	8138.3	6274	3313.9	10291.7	9600.4	234.1	20/07/00	7930.3	6807.7	3441.8	9710.9	8759.5	267
24/03/00	8209.1	6218.8	3427.4	10579.8	9653.8	237.2	21/07/00	7927.3	6701.9	3489.6	9707.5	8839.4	269.5
27/03/00	8243.8	6205.6	3417	10583.9	9768.8	236.6	24/07/00	7908.4	6679.9	3510.4	9665.6	8828	268.5
28/03/00	8090.6	6005.8	3353.8	10422.7	9641.8	237.6	25/07/00	7872.4	6642.4	3584.9	9628.3	8769.6	266.1
29/03/00	8122.1	6142.9	3335.5	10373.9	9625.1	239.3	26/07/00	7854.7	6617	3525.6	9635.9	8754.1	266.3
30/03/00	7944.6	6075.3	3216.3	10222.8	9303.8	241.6	27/07/00	7825.7	6575.6	3505	9615.6	8724.5	269.6
31/03/00	7957.2	6225.3	3257.6	10193.1	9196.9	240.8	28/07/00	7778	6557.5	3469	9575.3	8640.8	269.2
03/04/00	7982.3	6232.8	3306.9	10225.5	9237.3	236	31/07/00	7737.6	6471.5	3435.3	9599.4	8606	268.3
04/04/00	8017.8	6341.8	3357.7	10276.8	9184	237.5	01/08/00	7745.6	6474.6	3412.8	9682.3	8583.4	268.2
05/04/00	7739.2	6159.4	3322.2	9948.5	8776.2	236.7	02/08/00	7823.7	6617.8	3450	9721.2	8631.5	269.2
06/04/00	7848.9	6233.6	3297	10036	8971.4	236	03/08/00	7788.6	6584.1	3445.9	9693.9	8579.4	271.3
07/04/00	7831.5	6175.4	3311.4	10029.8	8977.1	237.3	04/08/00	7888.4	6656.3	3504.2	9777.6	8719.1	276.4
10/04/00	7856.6	6181.2	3288.3	10021	9056.2	237.4	07/08/00	7894.9	6600.1	3576.2	9760.7	8784.1	275.4
11/04/00	7709.8	6167.8	3217.9	9817.9	8788.7	237.9	08/08/00	7931.6	6704.1	3575	9747.5	8797.7	274.3
12/04/00	7628.4	6066.2	3199.4	9706.4	8730.2	237.7	10/08/00	8065.9	6898	3675.4	9864.5	8894.1	275.3
13/04/00	7377.2	5868.3	3165.8	9461.8	8363.7	238.4	11/08/00	8103.3	7020	3702.3	9882.7	8867.2	273.5
14/04/00	7162.7	5665.4	3107.7	9244.2	8099.1	238.5	14/08/00	8189.1	7087.2	3715.9	10056	8941.2	272.8
17/04/00	6632.6	5270.6	2822.3	8635.1	7432.6	228.7	15/08/00	8196.6	7158.5	3685.2	9984.7	8938.2	275.5
18/04/00	6908.7	5314.6	2943.1	8996.1	7927.5	231	16/08/00	8262.1	7265.1	3683.6	10044.4	8984.2	276.4
19/04/00	7028.6	5310.9	2954.3	9349.7	8057.6	233.7	17/08/00	8246.3	7305.2	3677.2	10064.9	8892.9	276.8
20/04/00	7179.1	5480.6	3028.4	9604.8	8147.1	233.3	18/08/00	8260.5	7350.8	3690.1	10070.5	8884	276.3
25/04/00	7332.9	5552.1	3104	9840.7	8353.8	234.2	21/08/00	8304.8	7343.7	3844.3	10119	8945.6	277.3
26/04/00	7400.1	5590.3	3101.6	9943.2	8442.5	237.8	22/08/00	8406.9	7562.7	3884.6	10188.8	8972.8	276.8
28/04/00	7445.1	5707.1	3090.3	9878.5	8493	239.9	23/08/00	8406	7649.1	3843.4	10198.3	8894.9	275.9
02/05/00	7456.2	5798.8	2999.5	9763.2	8524.3	239.5	24/08/00	8409.7	7584.2	3880.8	10260.3	8922.6	275.4
03/05/00	7502.1	6088.6	3151.1	9661.7	8388.7	239.4	25/08/00	8365.4	7460.3	3835.2	10225.6	8951.1	273.3
04/05/00	7461.1	6100.2	3198.9	9494.8	8347.1	239.5	28/08/00	8377.5	7387.3	3871	10285.4	9009.9	273.4
05/05/00	7431.4	6159.8	3259	9308.7	8293.5	242.9	29/08/00	8417.8	7421.3	3924.2	10338.8	9047.7	272.4
08/05/00	7453.3	6203.3	3380.9	9263.4	8310.2	241.6	30/08/00	8398.5	7403	3953.8	10287.8	9034.9	270.7
09/05/00	7570.7	6389.6	3476.7	9248.2	8443.2	242.1	31/08/00	8489.1	7459.5	4045.7	10370.6	9157.7	273.7
10/05/00	7601.9	6554.8	3544.1	9180.3	8385.9	245	01/09/00	8550.5	7570.6	4104.5	10440.5	9170.3	272.5
11/05/00	7487	6324.6	3525	9195.9	8289.3	240.8	04/09/00	8655.7	7712.5	4187.9	10497.2	9274	272.7
12/05/00	7465.7	6129.6	3492.1	9170.8	8443.3	243.2	05/09/00	8578.9	7627.3	4102.5	10459.8	9184.1	273.1
15/05/00	7492.9	6135.3	3568.7	9203.1	8477.1	242.7	06/09/00	8570.2	7667.1	4058.8	10414.6	9158.1	273.5

Table A.15: Data: Equity indices (daily) 18/01/00 – 06/09/00

Date	CI01	CI10	CI09	CI24	CI27	CI70	Date	CI01	CI10	CI09	CI24	CI27	CI70
07/09/00	8558.8	7635.4	4102.1	10385.9	9161	273.5	02/11/00	8417.7	7977.9	3906.7	9969.1	8712.5	280.6
08/09/00	8556.8	7755.5	4063.7	10330.7	9086.3	271.5	03/11/00	8363.1	7957.7	3843.5	9899.8	8632.8	281.5
11/09/00	8521.5	7691.6	4016.1	10272.7	9099.1	267.7	06/11/00	8321.6	7926.6	3768.7	9879.9	8579.1	279.8
12/09/00	8440.6	7516.3	3989.1	10158.1	9114.8	265.8	07/11/00	8389.3	8069.8	3729.4	9947.3	8601.7	279.9
13/09/00	8477	7617.2	4001.4	10242.3	9067.3	268	08/11/00	8369.7	8067.8	3769.8	9848.7	8593.8	280.1
14/09/00	8509.2	7598.4	4032	10300.5	9136.9	265.8	09/11/00	8296.1	8036.8	3710.9	9825.4	8443.9	281.5
15/09/00	8454.1	7562.6	3996.2	10210.7	9078.6	264.9	10/11/00	8171.3	7899.2	3699	9764.7	8268.9	279.4
18/09/00	8375.2	7403.1	3982.3	10074.9	9090.1	265.4	13/11/00	8079.6	7841.9	3659.4	9607.4	8171.6	276.4
19/09/00	8274.5	7206.7	3965.7	9985.3	9049.5	266.8	14/11/00	8210.3	7942.9	3686	9792.4	8323.7	278.4
20/09/00	8247.3	7091.1	3882.2	10074.8	9049.2	270.1	15/11/00	8239.2	8001.8	3630.6	9793.5	8360.9	278.2
21/09/00	8151.6	6927.7	3811.3	9971.1	9009	273.9	16/11/00	8227.1	8060.3	3600.2	9667.3	8350	278
22/09/00	8099.9	6926.3	3817	9915.4	8894.2	276.4	17/11/00	8163.8	7968.4	3484.7	9749.2	8244.9	277.8
26/09/00	8177.4	7029.8	3809.8	9951.7	8993.2	275.3	20/11/00	8097.3	7849.6	3464.6	9729.8	8188.1	279.9
27/09/00	8232.8	7108.3	3912.5	9967.8	9037.4	275.7	21/11/00	8127.9	7969.2	3489.9	9768	8127.4	281.2
28/09/00	8294.9	7324.7	3902.3	9989.4	8990.7	277	22/11/00	7981.9	7885.9	3472.6	9672.5	7861.6	279.8
29/09/00	8274.2	7347	3890.4	9917.3	8955	277.2	23/11/00	7896.4	7798.3	3453.5	9613.5	7748.2	279
02/10/00	8340.8	7496	3991.1	9946	8957.6	276	24/11/00	7937.3	7812	3491	9627.1	7830.9	278.1
03/10/00	8389.2	7479.1	3985.4	10050.3	9053.4	274.7	27/11/00	8049.3	7841.2	3545.3	9740.9	8036.4	278.3
04/10/00	8355.6	7500.6	3919.4	10010.8	8978.3	274.7	28/11/00	8044.5	7894.7	3597.3	9770	7943.1	276.7
05/10/00	8276.7	7374.9	3886.1	9955.7	8920	274.2	29/11/00	8052.7	7922.6	3648.4	9745.5	7938.6	277.4
06/10/00	8237.2	7370.3	3794.1	9939.8	8848.2	273.7	30/11/00	7804.5	7567.6	3585.7	9581	7697.7	277.8
09/10/00	8180.2	7380.9	3823.7	9882.5	8708.4	270.9	01/12/00	7783	7390.9	3531.5	9568.4	7826.5	280.1
10/10/00	8133.1	7371.2	3831	9771.7	8646.1	271.8	04/12/00	7837	7543.1	3468.6	9661.5	7792.3	281.4
11/10/00	8024.8	7297.2	3855.8	9631.1	8491.1	270.5	06/12/00	8036.9	7813.1	3416.7	9900.8	7966.9	281.4
12/10/00	8001.4	7306.5	3821.6	9633	8420	273.8	07/12/00	8007.7	7942.4	3365.6	9793.5	7834.6	281.5
13/10/00	7962.5	7316	3860.1	9574.4	8328.1	272.6	08/12/00	8144.5	8100.4	3423.4	9958.4	7955.4	281.2
16/10/00	8077.2	7465.7	3887.7	9700.1	8425.4	272.8	11/12/00	8343.3	8364.1	3446.6	10147	8133.6	287.5
17/10/00	8021.5	7492.8	3867.2	9485.6	8371.1	272.8	12/12/00	8338	8401.3	3341.6	10205.9	8078.2	286.6
18/10/00	7792.9	7361.9	3771.8	9069	8117.3	274.6	13/12/00	8237.9	8146.9	3328.4	10225.9	8034.6	287.8
19/10/00	7986.3	7463.4	3861	9416.9	8338.8	275	14/12/00	8150.7	7956.8	3293.4	10279	7955.3	286.8
20/10/00	8052	7500.4	3890.3	9545.8	8401.1	278.4	15/12/00	8064.8	7753.4	3323.7	10289.6	7898.6	286.5
23/10/00	8074	7567.2	3870.5	9465.3	8450.6	276.5	18/12/00	8152.4	7818.6	3331.6	10438.7	7990.7	289
24/10/00	8052.4	7519.3	3829.7	9376.2	8495.6	276.2	19/12/00	8192.7	7894	3334.3	10498.5	7997.7	287
25/10/00	7946.3	7456	3768.9	9249.2	8344.8	278.4	20/12/00	8132.3	7822.7	3338.3	10495.7	7898.3	286.8
26/10/00	7904.1	7453.8	3755.8	9189.7	8269.7	276.6	21/12/00	8094.6	7877.7	3300.2	10455.8	7773.8	285
27/10/00	7884.3	7386.6	3711.5	9163	8300.2	279	22/12/00	8239.3	7960	3343.9	10751.8	7916.5	287.3
30/10/00	7889	7395.1	3717.6	9125.1	8325.4	278.9	27/12/00	8338.5	8061.6	3441.5	10858.8	8011.2	285.8
31/10/00	8111.5	7699.1	3822.7	9407.7	8464.7	280.7	28/12/00	8404.1	8083.4	3478.3	10924.7	8126.1	285.9
01/11/00	8239.1	7782.4	3863.3	9689.2	8573.7	279.1	29/12/00	8326.2	7990.4	3459.3	10778.3	8084.2	286.7

Table A.16: Data: Equity indices (daily) 07/09/00 – 29/12/00

Appendix B

Optimisation Results

The results of solving optimisation problems of the form (4.3.4), as well as the form (4.3.5) per market indicator, per time window and per risk profile are contained in this appendix.

	Solution						Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
1	0.022	1	0	0	0	0	0.000	−0.004	−0.024	−0.027	−0.028	0.019
2	0.030	0	1	0	0	0	−0.016	0.000	−0.033	−0.027	−0.028	0.026
3	−0.002	0	0	1	0	0	−0.027	−0.012	0.000	−0.009	−0.001	−0.002
4	−0.004	0	1	0	0	0	−0.005	0.000	−0.017	−0.010	0.000	−0.007
5	0.000	0	1	0	0	0	−0.019	0.000	−0.017	−0.005	−0.007	0.000
6	0.014	0	0	1	0	0	−0.022	−0.002	0.000	−0.004	−0.024	0.013
7	0.037	0	1	0	0	0	−0.018	0.000	−0.023	−0.003	−0.057	0.034
8	0.004	0	0	1	0	0	−0.010	−0.031	0.000	−0.002	−0.001	0.004
9	0.029	0	0	1	0	0	−0.022	−0.018	0.000	−0.006	−0.035	0.026
10	0.024	0	0	1	0	0	−0.031	−0.021	0.000	−0.015	−0.025	0.022
11	0.028	0	0	1	0	0	−0.031	−0.006	0.000	−0.006	−0.030	0.025
12	0.000	0	0	1	0	0	−0.004	−0.008	0.000	−0.004	−0.003	0.000
13	0.033	0	1	0	0	0	0.000	0.000	−0.015	−0.026	−0.034	0.032
14	0.016	0	0	0	0	1	−0.001	−0.004	−0.005	−0.007	0.000	0.014
15	0.010	0	1	0	0	0	−0.039	0.000	−0.017	−0.012	−0.017	0.009
16	−0.002	0	0	0	0	1	−0.017	−0.030	−0.011	−0.008	0.000	−0.002
17	−0.007	0	0	0	0	1	−0.043	−0.036	−0.039	−0.030	0.000	−0.007
18	−0.007	0	0	0	0	1	−0.061	−0.023	−0.028	−0.028	0.000	−0.007
19	0.031	0	1	0	0	0	−0.007	0.000	−0.015	−0.012	−0.017	0.018
20	−0.003	0	0	1	0	0	−0.025	−0.032	0.000	−0.013	−0.012	−0.003
21	−0.011	0	0	1	0	0	−0.026	−0.009	0.000	−0.003	−0.002	−0.011
22	0.013	1	0	0	0	0	0.000	−0.016	−0.019	−0.006	−0.020	0.011
23	0.024	0	0.9366	0.0634	0	0	−0.034	0.000	0.000	−0.003	−0.018	0.016
24	0.005	0	1	0	0	0	−0.011	0.000	−0.006	−0.005	−0.003	0.005
25	0.000	0	0	0	0	1	−0.026	−0.016	−0.005	−0.003	0.000	0.000
26	0.017	1	0	0	0	0	0.000	−0.012	−0.005	−0.013	−0.012	0.014
27	0.059	1	0	0	0	0	0.000	−0.065	−0.029	−0.044	−0.059	0.051
28	0.002	0	0	1	0	0	−0.038	0.000	0.000	−0.009	−0.006	0.002
29	0.025	0	1	0	0	0	−0.011	0.000	−0.018	−0.021	−0.030	0.025
30	0.032	0	1	0	0	0	−0.032	0.000	−0.013	−0.015	−0.028	0.031
31	0.032	0	0	1	0	0	−0.014	−0.003	0.000	−0.009	−0.024	0.025
32	0.008	0	0	1	0	0	−0.028	−0.006	0.000	−0.004	−0.013	0.008
33	−0.009	0	0	0	0.2328	0.7672	−0.006	−0.019	−0.012	0.000	0.000	−0.010
34	0.005	0.8312	0.1688	0	0	0	0.000	0.000	−0.018	0.000	−0.009	0.005
35	0.006	0	0	1	0	0	−0.001	−0.012	0.000	−0.004	−0.014	0.006
36	−0.006	0	0	0.5084	0	0.4916	−0.011	−0.023	0.000	−0.009	0.000	−0.006
37	0.010	1	0	0	0	0	0.000	−0.029	−0.017	−0.017	−0.014	0.009
38	0.016	1	0	0	0	0	0.000	−0.009	−0.006	−0.010	−0.016	0.016
39	−0.006	0	0	0	0	1	−0.018	−0.007	−0.003	−0.001	0.000	−0.006
40	0.007	0	0	0	1	0	−0.003	−0.015	−0.004	0.000	−0.016	0.006
41	0.020	0	1	0	0	0	−0.018	0.000	−0.024	−0.019	−0.024	0.018
42	0.005	0	0	0	0	1	−0.006	−0.020	−0.008	−0.004	0.000	0.005
43	−0.033	0	0	0	0	1	−0.031	−0.030	−0.031	−0.009	0.000	−0.033
44	0.004	1	0	0	0	0	0.000	−0.081	−0.065	−0.041	−0.005	−0.008
45	0.003	0	1	0	0	0	−0.129	0.000	−0.261	−0.184	−0.036	0.000
46	0.090	1	0	0	0	0	0.000	−0.031	−0.129	−0.093	−0.086	0.057
47	0.067	1	0	0	0	0	0.000	−0.046	−0.130	−0.123	−0.048	0.063
48	0.092	0	0	1	0	0	−0.091	−0.053	0.000	−0.054	−0.068	0.066
49	0.048	0.1158	0	0.8842	0	0	0.000	−0.034	0.000	−0.012	−0.003	0.007
50	0.032	1	0	0	0	0	0.000	−0.086	−0.100	−0.052	−0.023	0.025
51	0.051	0	0	1	0	0	−0.027	−0.052	0.000	−0.046	−0.050	0.041
52	−0.008	0	0	0	1	0	−0.021	−0.010	−0.022	0.000	−0.009	−0.008

Table B.1: Optimisation: Standard deviation as risk, upward shock in Bond yields as indicator and a conservative investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
1	−0.008	0	0	0	0	1	−0.033	−0.007	−0.015	−0.006	0.000	−0.008
2	0.010	1	0	0	0	0	0.000	−0.009	−0.011	−0.012	−0.015	0.006
3	−0.008	0	0	0	0	1	−0.020	−0.055	−0.023	−0.021	0.000	−0.009
4	0.023	0	0	1	0	0	−0.030	−0.023	0.000	−0.001	−0.019	0.022
5	0.034	1	0	0	0	0	0.000	−0.019	−0.010	−0.022	−0.028	0.020
6	0.033	0	1	0	0	0	−0.010	0.000	−0.020	−0.024	−0.021	0.028
7	−0.001	0	0	0	1	0	−0.010	−0.017	−0.001	0.000	−0.013	−0.001
8	0.022	1	0	0	0	0	0.000	−0.018	−0.017	−0.011	−0.026	0.021
9	0.037	1	0	0	0	0	0.000	−0.027	−0.019	−0.019	−0.024	0.026
10	0.018	0	1	0	0	0	−0.002	0.000	−0.021	0.000	−0.017	0.016
11	0.054	0	1	0	0	0	−0.043	0.000	−0.012	−0.002	−0.035	0.041
12	−0.006	0.0577	0	0	0.9423	0	0.000	−0.009	−0.005	0.000	−0.006	−0.006
13	0.018	0	0	0	1	0	−0.012	−0.023	−0.015	0.000	−0.004	0.016
14	0.055	1	0	0	0	0	0.000	−0.038	−0.034	−0.026	−0.026	0.034
15	0.034	1	0	0	0	0	0.000	−0.017	−0.009	−0.017	−0.005	0.033
16	0.041	0	1	0	0	0	−0.025	0.000	−0.020	−0.013	−0.026	0.036
17	0.040	0	0	0	0	1	−0.008	−0.007	−0.010	−0.003	0.000	0.036
18	0.041	1	0	0	0	0	0.000	−0.014	−0.018	−0.010	−0.020	0.031
19	0.006	0	0	0	0	1	−0.050	−0.038	−0.029	−0.063	0.000	0.006
20	0.020	0	1	0	0	0	−0.056	0.000	−0.008	−0.018	−0.010	0.018
21	0.009	0	0	0	0	1	−0.029	−0.014	−0.009	−0.004	0.000	0.008
22	0.058	1	0	0	0	0	0.000	−0.001	−0.034	−0.017	−0.046	0.045
23	−0.002	0	1	0	0	0	−0.063	0.000	−0.016	−0.021	−0.006	−0.003
24	0.016	1	0	0	0	0	0.000	−0.017	−0.014	−0.002	−0.007	0.004
25	0.005	0	1	0	0	0	−0.023	0.000	−0.003	−0.013	−0.003	0.004
26	0.059	1	0	0	0	0	0.000	−0.024	−0.046	−0.044	−0.056	0.056
27	0.043	0	1	0	0	0	−0.013	0.000	−0.013	−0.007	−0.025	0.032
28	0.056	0	1	0	0	0	−0.037	0.000	−0.029	−0.031	−0.046	0.050
29	0.026	1	0	0	0	0	0.000	−0.065	−0.055	−0.050	−0.019	0.022
30	−0.009	0	0	0	0	1	−0.044	−0.032	−0.029	−0.024	0.000	−0.010
31	0.011	0	0	0	1	0	−0.023	−0.005	−0.006	0.000	−0.117	0.009
32	0.025	0	1	0	0	0	−0.010	0.000	−0.026	−0.024	−0.030	0.022
33	0.050	0	1	0	0	0	−0.059	0.000	−0.009	−0.017	−0.036	0.043
34	0.054	0	1	0	0	0	−0.007	0.000	−0.009	−0.003	−0.030	0.037
35	0.031	0	0	0	1	0	−0.068	−0.054	−0.010	0.000	−0.016	0.029
36	0.050	0.4913	0	0.5087	0	0	0.000	−0.035	0.000	−0.018	−0.040	0.048
37	0.009	0	0	0	0	1	−0.005	−0.034	−0.036	−0.015	0.000	0.008
38	0.021	0	1	0	0	0	−0.047	0.000	−0.028	−0.027	−0.019	0.020
39	0.041	1	0	0	0	0	0.000	−0.031	−0.029	−0.034	−0.032	0.035
40	0.023	0	0	0	1	0	−0.026	−0.008	−0.001	0.000	−0.020	0.022
41	−0.012	0	0	0	0	1	−0.049	−0.080	−0.009	−0.012	0.000	−0.012
42	−0.001	0	0	0.0415	0	0.9585	−0.031	−0.068	0.000	−0.026	0.000	−0.002
43	0.053	0	0	1	0	0	−0.042	−0.062	0.000	−0.016	−0.032	0.047
44	0.094	1	0	0	0	0	0.000	−0.037	−0.030	−0.031	−0.040	0.051
45	0.032	0	1	0	0	0	−0.026	0.000	−0.022	−0.020	−0.022	0.032
46	0.030	0	0	1	0	0	−0.056	−0.041	0.000	−0.020	−0.020	0.029
47	0.039	0	0	1	0	0	−0.031	−0.034	0.000	−0.021	−0.032	0.035
48	0.069	0	0	1	0	0	−0.066	−0.059	0.000	−0.023	−0.027	0.044
49	0.001	0	0	0	0.5314	0.4686	−0.021	−0.077	−0.059	0.000	0.000	0.000
50	0.061	0	1	0	0	0	−0.020	0.000	−0.012	−0.026	−0.053	0.058
51	0.004	0	0	0	1	0	−0.026	−0.030	−0.004	0.000	−0.012	0.004
52	0.050	0	0	1	0	0	−0.004	−0.016	0.000	−0.023	−0.029	0.035
53	0.061	1	0	0	0	0	0.000	−0.026	−0.038	−0.009	−0.047	0.053
54	0.042	1	0	0	0	0	0.000	0.000	−0.046	−0.028	−0.050	0.041
55	0.118	0	1	0	0	0	−0.064	0.000	−0.069	−0.067	−0.091	0.083
56	0.026	1	0	0	0	0	0.000	−0.035	−0.030	−0.005	−0.030	0.022
57	0.018	1	0	0	0	0	0.000	−0.008	−0.044	−0.002	−0.035	0.009
58	0.002	0	0	0	1	0	−0.045	−0.020	−0.035	0.000	−0.006	0.001
59	0.009	0	0	1	0	0	−0.041	−0.025	0.000	0.000	−0.012	0.008

Table B.2: Optimisation: Standard deviation as risk, downward shocks in Bond yields as indicator and a conservative investment

		Solution					Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
60	0.021	0	0	1	0	0	−0.042	−0.032	0.000	−0.016	−0.023	0.020
61	0.025	0	0	0	1	0	−0.033	−0.077	−0.009	0.000	−0.019	0.024
62	0.067	0	1	0	0	0	−0.029	0.000	−0.045	−0.022	−0.062	0.055
63	0.021	0	1	0	0	0	−0.005	0.000	−0.015	−0.002	−0.022	0.019
64	0.059	0	0	1	0	0	−0.021	−0.023	0.000	−0.021	−0.064	0.051
65	0.048	1	0	0	0	0	0.000	−0.011	−0.006	−0.029	−0.029	0.031
66	0.051	0	0	0	1	0	−0.076	−0.041	−0.014	0.000	−0.035	0.047
67	0.013	0	0	1	0	0	−0.003	−0.017	0.000	−0.007	−0.007	0.013
68	0.029	0	1	0	0	0	−0.002	0.000	−0.019	−0.011	−0.033	0.010
69	0.086	0	0	1	0	0	−0.093	−0.088	0.000	−0.022	−0.069	0.085
70	0.043	0	1	0	0	0	−0.062	0.000	−0.027	−0.031	−0.088	0.041
71	0.037	0	0	0	0	1	−0.071	−0.072	−0.095	−0.075	0.000	0.030
72	0.022	0	0	1	0	0	−0.023	−0.025	0.000	0.000	−0.017	0.014
73	0.010	0	0	1	0	0	0.000	−0.037	0.000	−0.014	−0.041	0.008
74	0.003	0	0	0	0	1	−0.010	−0.034	−0.057	−0.007	0.000	0.001
75	−0.001	0	0	0	0	1	−0.093	−0.112	−0.030	−0.007	0.000	−0.002
76	0.018	0	1	0	0	0	−0.027	0.000	−0.042	−0.015	−0.008	0.007
77	0.010	0	0	0	0	1	−0.042	−0.080	−0.009	−0.006	0.000	0.009
78	0.009	0.7223	0	0	0.2777	0	0.000	−0.006	−0.005	0.000	−0.005	−0.006
79	0.059	0	0	1	0	0	−0.039	−0.018	0.000	−0.042	−0.023	0.046
80	0.014	0	0	0	0	1	−0.011	−0.053	−0.043	−0.051	0.000	0.012
81	0.005	0	0	0	0	1	−0.085	−0.065	−0.081	−0.099	0.000	0.005
82	0.029	1	0	0	0	0	0.000	−0.037	−0.009	−0.017	−0.001	0.027
83	0.016	0	0	0	0	1	−0.015	−0.020	−0.012	−0.015	0.000	0.015
84	0.071	1	0	0	0	0	0.000	−0.032	−0.057	−0.042	−0.083	0.058
85	0.013	0	1	0	0	0	−0.046	0.000	−0.013	−0.011	−0.011	0.012
86	−0.002	0	0	0	0	1	−0.018	−0.001	−0.011	−0.019	0.000	−0.003
87	0.024	0	0	0	0	1	−0.010	−0.015	−0.002	−0.014	0.000	0.020
88	0.050	0	1	0	0	0	−0.008	0.000	−0.038	−0.031	−0.054	0.044
89	0.043	1	0	0	0	0	0.000	−0.040	−0.020	−0.035	−0.031	0.038
90	0.039	0	1	0	0	0	−0.025	0.000	−0.023	−0.031	−0.049	0.038
91	0.059	0	1	0	0	0	−0.042	0.000	−0.032	−0.031	−0.051	0.048
92	0.023	1	0	0	0	0	0.000	−0.033	−0.032	−0.031	−0.025	0.022
93	−0.001	0	0	0	1	0	−0.023	−0.016	−0.011	0.000	−0.023	−0.001
94	0.038	0	0	0	0	1	−0.097	−0.063	−0.057	−0.047	0.000	0.032
95	0.052	1	0	0	0	0	0.000	−0.030	−0.042	−0.034	−0.043	0.043
96	0.002	0.4080	0	0.5920	0	0	0.000	−0.030	0.000	−0.015	−0.014	0.001
97	0.017	0	1	0	0	0	−0.023	0.000	−0.052	−0.073	−0.020	0.016
98	0.024	1	0	0	0	0	0.000	−0.019	−0.032	−0.018	−0.002	0.023
99	0.002	0	0	0	0	1	−0.015	−0.045	−0.040	−0.012	0.000	0.001
100	0.059	1	0	0	0	0	0.000	−0.025	−0.003	−0.021	−0.035	0.040
101	−0.008	0	0	0	0	1	0.000	−0.030	−0.006	−0.035	0.000	−0.008
102	0.007	1	0	0	0	0	0.000	−0.060	−0.008	−0.011	−0.011	0.005
103	0.001	0	0.9638	0	0	0.0362	−0.021	0.000	−0.014	−0.052	0.000	0.001
104	0.009	0	0.6354	0	0	0.3646	−0.067	0.000	−0.015	−0.009	0.000	0.008
105	0.079	1	0	0	0	0	0.000	−0.084	−0.034	−0.053	−0.058	0.062
106	0.033	0	0	1	0	0	−0.068	−0.058	0.000	−0.037	−0.013	0.032
107	0.042	0	0	1	0	0	−0.015	−0.034	0.000	−0.037	−0.036	0.039
108	0.034	0	1	0	0	0	−0.030	0.000	−0.032	−0.014	−0.036	0.034

Table B.3: Optimisation: Standard deviation as risk, downward shock in Bond yields as indicator and a conservative investment (continue)

	Solution						Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment												
1	0.023	1	0	0	0	0	0.000	−0.005	−0.026	−0.029	−0.030	0.021
2	0.031	0	1	0	0	0	−0.017	0.000	−0.035	−0.029	−0.030	0.028
3	−0.002	0	0	1	0	0	−0.028	−0.012	0.000	−0.009	−0.001	−0.002
4	−0.003	0	1	0	0	0	−0.006	0.000	−0.019	−0.011	−0.002	−0.006
5	0.000	0	1	0	0	0	−0.019	0.000	−0.017	−0.005	−0.008	0.000
6	0.014	0	0	1	0	0	−0.022	−0.002	0.000	−0.005	−0.025	0.013
7	0.037	0	1	0	0	0	−0.019	0.000	−0.024	−0.003	−0.059	0.036
8	0.004	0	0	1	0	0	−0.010	−0.031	0.000	−0.002	−0.001	0.004
9	0.030	0	0	1	0	0	−0.022	−0.019	0.000	−0.006	−0.037	0.028
10	0.024	0	0	1	0	0	−0.032	−0.022	0.000	−0.016	−0.026	0.023
11	0.028	0	0	1	0	0	−0.032	−0.006	0.000	−0.006	−0.032	0.027
12	0.000	0	0	1	0	0	−0.004	−0.008	0.000	−0.004	−0.003	0.000
13	0.033	0.1787	0.8213	0	0	0	0.000	0.000	−0.015	−0.026	−0.034	0.032
14	0.016	0	0	0	0	1	−0.001	−0.005	−0.005	−0.008	0.000	0.015
15	0.010	0	1	0	0	0	−0.040	0.000	−0.018	−0.012	−0.018	0.009
16	−0.002	0	0	0	0	1	−0.017	−0.030	−0.011	−0.008	0.000	−0.002
17	−0.007	0	0	0	0	1	−0.043	−0.036	−0.038	−0.030	0.000	−0.007
18	−0.007	0	0	0	0	1	−0.061	−0.022	−0.028	−0.028	0.000	−0.007
19	0.035	0	1	0	0	0	−0.006	0.000	−0.019	−0.015	−0.024	0.025
20	−0.003	0	0	1	0	0	−0.024	−0.032	0.000	−0.013	−0.012	−0.003
21	−0.011	0	0	1	0	0	−0.026	−0.008	0.000	−0.003	−0.002	−0.011
22	0.013	1	0	0	0	0	0.000	−0.016	−0.020	−0.006	−0.021	0.012
23	0.026	0	1	0	0	0	−0.038	0.000	−0.002	−0.005	−0.023	0.020
24	0.005	0	1	0	0	0	−0.011	0.000	−0.006	−0.005	−0.003	0.005
25	0.000	0	0	0	0	1	−0.026	−0.016	−0.005	−0.003	0.000	0.000
26	0.017	1	0	0	0	0	0.000	−0.013	−0.006	−0.014	−0.013	0.016
27	0.061	1	0	0	0	0	0.000	−0.072	−0.031	−0.047	−0.064	0.056
28	0.002	0	0	1	0	0	−0.038	0.000	0.000	−0.009	−0.006	0.002
29	0.025	0	1	0	0	0	−0.011	0.000	−0.019	−0.021	−0.030	0.025
30	0.033	0	1	0	0	0	−0.033	0.000	−0.014	−0.015	−0.028	0.032
31	0.034	0	0	1	0	0	−0.017	−0.006	0.000	−0.011	−0.028	0.029
32	0.008	0	0	1	0	0	−0.028	−0.006	0.000	−0.004	−0.013	0.008
33	−0.009	0	0	0	0.0425	0.9575	−0.006	−0.019	−0.012	0.000	0.000	−0.010
34	0.006	0.9967	0.0033	0	0	0	0.000	0.000	−0.018	0.000	−0.009	0.005
35	0.007	0	0	1	0	0	−0.001	−0.012	0.000	−0.004	−0.014	0.006
36	−0.006	0	0	0.7554	0	0.2446	−0.011	−0.022	0.000	−0.009	0.000	−0.006
37	0.010	1	0	0	0	0	0.000	−0.030	−0.018	−0.017	−0.015	0.010
38	0.016	1	0	0	0	0	0.000	−0.009	−0.006	−0.011	−0.017	0.016
39	−0.006	0	0	0	0	1	−0.018	−0.007	−0.003	−0.001	0.000	−0.006
40	0.007	0	0	0	1	0	−0.003	−0.015	−0.004	0.000	−0.017	0.007
41	0.020	0	1	0	0	0	−0.018	0.000	−0.025	−0.020	−0.024	0.019
42	0.005	0	0	0	0	1	−0.006	−0.020	−0.008	−0.004	0.000	0.005
43	−0.033	0	0	0	0	1	−0.031	−0.030	−0.031	−0.009	0.000	−0.033
44	0.007	1	0	0	0	0	0.000	−0.086	−0.070	−0.046	−0.011	−0.001
45	0.003	0	1	0	0	0	−0.127	0.000	−0.256	−0.181	−0.037	0.001
46	0.099	1	0	0	0	0	0.000	−0.055	−0.164	−0.118	−0.104	0.075
47	0.069	1	0	0	0	0	0.000	−0.051	−0.136	−0.127	−0.050	0.066
48	0.099	0	0	1	0	0	−0.108	−0.061	0.000	−0.064	−0.080	0.080
49	0.062	0	0	1	0	0	−0.015	−0.048	0.000	−0.014	−0.019	0.025
50	0.035	1	0	0	0	0	0.000	−0.093	−0.111	−0.059	−0.028	0.029
51	0.054	0	0	1	0	0	−0.033	−0.056	0.000	−0.050	−0.054	0.046
52	−0.008	0	0	0	1	0	−0.021	−0.009	−0.022	0.000	−0.009	−0.008

Table B.4: Optimisation: Standard deviation as risk, upward shock in Bond yields as indicator and a moderate investment

Downward Shock	Objective Value	Solution CI09	Solution CI10	Solution CI24	Solution CI27	Solution CI70	Reduced Gradient CI09	Reduced Gradient CI10	Reduced Gradient CI24	Reduced Gradient CI27	Reduced Gradient CI70	Lagrange Multiplier
Moderate Investment												
1	−0.008	0	0	0	0	1	−0.033	−0.007	−0.015	−0.006	0.000	−0.008
2	0.011	1	0	0	0	0	0.000	−0.012	−0.013	−0.015	−0.018	0.008
3	−0.008	0	0	0	0	1	−0.020	−0.054	−0.022	−0.020	0.000	−0.008
4	0.023	0	0	1	0	0	−0.032	−0.024	0.000	−0.001	−0.020	0.022
5	0.038	1	0	0	0	0	0.000	−0.023	−0.014	−0.028	−0.036	0.028
6	0.034	0	1	0	0	0	−0.012	0.000	−0.022	−0.026	−0.023	0.031
7	−0.001	0	0	0	1	0	−0.010	−0.017	−0.001	0.000	−0.013	−0.001
8	0.022	1	0	0	0	0	0.000	−0.018	−0.017	−0.011	−0.026	0.022
9	0.040	1	0	0	0	0	0.000	−0.032	−0.025	−0.025	−0.030	0.032
10	0.018	0	1	0	0	0	−0.001	0.000	−0.022	−0.001	−0.018	0.017
11	0.058	0	1	0	0	0	−0.050	0.000	−0.013	−0.003	−0.042	0.048
12	−0.006	0.1358	0	0	0.8642	0	0.000	−0.009	−0.005	0.000	−0.006	−0.006
13	0.019	0	0	0	1	0	−0.012	−0.024	−0.016	0.000	−0.005	0.017
14	0.061	1	0	0	0	0	0.000	−0.049	−0.043	−0.034	−0.035	0.046
15	0.035	1	0	0	0	0	0.000	−0.017	−0.009	−0.017	−0.005	0.034
16	0.042	0	1	0	0	0	−0.028	0.000	−0.021	−0.013	−0.028	0.039
17	0.041	0	0	0	0	1	−0.008	−0.009	−0.011	−0.004	0.000	0.039
18	0.044	1	0	0	0	0	0.000	−0.017	−0.021	−0.013	−0.024	0.037
19	0.006	0	0	0	0	1	−0.050	−0.039	−0.029	−0.064	0.000	0.006
20	0.020	0	1	0	0	0	−0.059	0.000	−0.009	−0.019	−0.011	0.019
21	0.009	0	0	0	0	1	−0.030	−0.015	−0.010	−0.004	0.000	0.009
22	0.062	1	0	0	0	0	0.000	0.000	−0.037	−0.020	−0.052	0.052
23	−0.002	0	1	0	0	0	−0.064	0.000	−0.017	−0.022	−0.007	−0.003
24	0.019	1	0	0	0	0	0.000	−0.021	−0.019	−0.007	−0.014	0.011
25	0.005	0	1	0	0	0	−0.023	0.000	−0.003	−0.013	−0.004	0.004
26	0.060	1	0	0	0	0	0.000	−0.023	−0.047	−0.045	−0.057	0.058
27	0.046	0	1	0	0	0	−0.021	0.000	−0.018	−0.011	−0.031	0.038
28	0.058	0	1	0	0	0	−0.041	0.000	−0.032	−0.033	−0.049	0.053
29	0.027	1	0	0	0	0	0.000	−0.068	−0.058	−0.053	−0.021	0.024
30	−0.009	0	0	0	0	1	−0.044	−0.031	−0.029	−0.024	0.000	−0.009
31	0.011	0	0	0	1	0	−0.023	−0.005	−0.007	0.000	−0.118	0.010
32	0.026	0	1	0	0	0	−0.011	0.000	−0.028	−0.026	−0.032	0.024
33	0.052	0	1	0	0	0	−0.065	0.000	−0.009	−0.018	−0.039	0.047
34	0.059	0	1	0	0	0	−0.009	0.000	−0.012	−0.004	−0.037	0.047
35	0.032	0	0	0	1	0	−0.071	−0.055	−0.010	0.000	−0.017	0.030
36	0.050	0.8796	0	0.1204	0	0	0.000	−0.035	0.000	−0.018	−0.040	0.048
37	0.009	0	0	0	0	1	−0.005	−0.034	−0.037	−0.015	0.000	0.009
38	0.021	0	1	0	0	0	−0.047	0.000	−0.028	−0.028	−0.019	0.021
39	0.043	1	0	0	0	0	0.000	−0.034	−0.032	−0.037	−0.035	0.038
40	0.023	0	0	0	1	0	−0.026	−0.008	−0.002	0.000	−0.021	0.022
41	−0.011	0	0	0	0	1	−0.049	−0.079	−0.009	−0.012	0.000	−0.012
42	−0.001	0	0	0.0838	0	0.9162	−0.031	−0.068	0.000	−0.026	0.000	−0.002
43	0.055	0	0	1	0	0	−0.044	−0.066	0.000	−0.018	−0.034	0.051
44	0.106	1	0	0	0	0	0.000	−0.042	−0.044	−0.043	−0.061	0.075
45	0.032	0	1	0	0	0	−0.026	0.000	−0.023	−0.020	−0.022	0.032
46	0.030	0	0	1	0	0	−0.058	−0.042	0.000	−0.021	−0.020	0.029
47	0.040	0	0	1	0	0	−0.033	−0.036	0.000	−0.022	−0.035	0.037
48	0.076	0	0	1	0	0	−0.080	−0.076	0.000	−0.031	−0.036	0.058
49	0.001	0	0	0	0.6480	0.3520	−0.021	−0.076	−0.058	0.000	0.000	0.001
50	0.062	0	1	0	0	0	−0.021	0.000	−0.013	−0.026	−0.055	0.060
51	0.004	0	0	0	1	0	−0.025	−0.030	−0.004	0.000	−0.012	0.004
52	0.054	0	0	1	0	0	−0.008	−0.024	0.000	−0.029	−0.035	0.043
53	0.063	1	0	0	0	0	0.000	−0.029	−0.040	−0.008	−0.052	0.058
54	0.042	1	0	0	0	0	0.000	0.000	−0.047	−0.028	−0.050	0.042
55	0.128	0	1	0	0	0	−0.076	0.000	−0.088	−0.084	−0.111	0.103
56	0.027	1	0	0	0	0	0.000	−0.039	−0.033	−0.008	−0.032	0.024
57	0.021	1	0	0	0	0	0.000	−0.012	−0.048	−0.008	−0.039	0.014
58	0.002	0	0	0	1	0	−0.045	−0.019	−0.034	0.000	−0.007	0.002
59	0.010	0	0	1	0	0	−0.042	−0.025	0.000	−0.001	−0.013	0.009

Table B.5: Optimisation: Standard deviation as risk, downward shock in Bond yields as indicator and a moderate investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment												
60	0.021	0	0	1	0	0	−0.043	−0.032	0.000	−0.016	−0.023	0.021
61	0.026	0	0	0	1	0	−0.034	−0.079	−0.009	0.000	−0.020	0.025
62	0.070	0	1	0	0	0	−0.030	0.000	−0.052	−0.027	−0.070	0.062
63	0.022	0	1	0	0	0	−0.005	0.000	−0.015	−0.002	−0.023	0.020
64	0.061	0	0	1	0	0	−0.022	−0.026	0.000	−0.023	−0.068	0.055
65	0.052	1	0	0	0	0	0.000	−0.015	−0.013	−0.037	−0.039	0.040
66	0.052	0	0	0	1	0	−0.081	−0.043	−0.016	0.000	−0.037	0.049
67	0.014	0	0	1	0	0	−0.003	−0.018	0.000	−0.007	−0.007	0.013
68	0.035	0	1	0	0	0	−0.008	0.000	−0.027	−0.020	−0.046	0.021
69	0.087	0	0	1	0	0	−0.094	−0.090	0.000	−0.023	−0.071	0.086
70	0.044	0	1	0	0	0	−0.063	0.000	−0.029	−0.032	−0.090	0.042
71	0.039	0	0	0	0	1	−0.076	−0.078	−0.102	−0.081	0.000	0.034
72	0.025	0	0	1	0	0	−0.026	−0.029	0.000	−0.001	−0.022	0.019
73	0.010	0	0	1	0	0	−0.001	−0.038	0.000	−0.014	−0.042	0.009
74	0.003	0	0	0	0	1	−0.009	−0.035	−0.058	−0.007	0.000	0.002
75	0.000	0	0	0	0	1	−0.095	−0.114	−0.031	−0.007	0.000	−0.001
76	0.022	0	1	0	0	0	−0.030	0.000	−0.052	−0.022	−0.014	0.014
77	0.010	0	0	0	0	1	−0.043	−0.082	−0.009	−0.006	0.000	0.010
78	0.015	1	0	0	0	0	0.000	−0.005	−0.006	−0.001	−0.008	−0.004
79	0.062	0	0	1	0	0	−0.047	−0.018	0.000	−0.047	−0.027	0.053
80	0.014	0	0	0	0	1	−0.012	−0.057	−0.046	−0.054	0.000	0.013
81	0.005	0	0	0	0	1	−0.086	−0.065	−0.082	−0.100	0.000	0.005
82	0.030	1	0	0	0	0	0.000	−0.040	−0.010	−0.018	−0.001	0.028
83	0.016	0	0	0	0	1	−0.015	−0.020	−0.012	−0.015	0.000	0.016
84	0.074	1	0	0	0	0	0.000	−0.038	−0.061	−0.047	−0.089	0.065
85	0.013	0	1	0	0	0	−0.047	0.000	−0.014	−0.011	−0.011	0.012
86	−0.002	0	0	0	0	1	−0.019	−0.002	−0.011	−0.020	0.000	−0.002
87	0.024	0	0	0	0	1	−0.011	−0.015	−0.003	−0.015	0.000	0.022
88	0.052	0	1	0	0	0	−0.006	0.000	−0.040	−0.033	−0.057	0.048
89	0.044	1	0	0	0	0	0.000	−0.043	−0.022	−0.038	−0.033	0.041
90	0.039	0	1	0	0	0	−0.025	0.000	−0.023	−0.031	−0.049	0.038
91	0.062	0	1	0	0	0	−0.045	0.000	−0.037	−0.035	−0.057	0.054
92	0.024	1	0	0	0	0	0.000	−0.033	−0.033	−0.032	−0.026	0.023
93	−0.001	0	0	0	1	0	−0.024	−0.016	−0.011	0.000	−0.023	−0.001
94	0.039	0	0	0	0	1	−0.106	−0.070	−0.061	−0.052	0.000	0.035
95	0.054	1	0	0	0	0	0.000	−0.033	−0.047	−0.040	−0.047	0.048
96	0.002	0.5711	0	0.4289	0	0	0.000	−0.029	0.000	−0.015	−0.015	0.002
97	0.017	0	1	0	0	0	−0.024	0.000	−0.052	−0.074	−0.021	0.017
98	0.024	1	0	0	0	0	0.000	−0.018	−0.031	−0.018	−0.003	0.024
99	0.002	0	0	0	0	1	−0.016	−0.046	−0.040	−0.013	0.000	0.002
100	0.064	1	0	0	0	0	0.000	−0.030	−0.002	−0.026	−0.044	0.051
101	−0.008	0	0	0	0	1	0.000	−0.030	−0.006	−0.035	0.000	−0.008
102	0.007	1	0	0	0	0	0.000	−0.062	−0.008	−0.011	−0.012	0.006
103	0.001	0	1	0	0	0	−0.021	0.000	−0.014	−0.052	0.000	0.001
104	0.009	0	0.7902	0	0	0.2098	−0.067	0.000	−0.015	−0.009	0.000	0.008
105	0.084	1	0	0	0	0	0.000	−0.096	−0.040	−0.061	−0.068	0.072
106	0.033	0	0	1	0	0	−0.070	−0.059	0.000	−0.038	−0.013	0.032
107	0.043	0	0	1	0	0	−0.016	−0.036	0.000	−0.039	−0.038	0.041
108	0.034	0	1	0	0	0	−0.030	0.000	−0.032	−0.014	−0.036	0.034

Table B.6: Optimisation: Standard deviation as risk, downward shock in Bond yields as indicator and a moderate investment (continue)

	Solution						Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
1	0.024	1	0	0	0	0	0.000	−0.005	−0.028	−0.030	−0.032	0.022
2	0.032	0	1	0	0	0	−0.018	0.000	−0.036	−0.031	−0.032	0.030
3	−0.002	0	0	1	0	0	−0.028	−0.012	0.000	−0.009	−0.001	−0.002
4	−0.002	0	1	0	0	0	−0.007	0.000	−0.020	−0.013	−0.004	−0.004
5	0.001	0	1	0	0	0	−0.019	0.000	−0.017	−0.005	−0.008	0.000
6	0.015	0	0	1	0	0	−0.022	−0.002	0.000	−0.005	−0.026	0.014
7	0.038	0	1	0	0	0	−0.019	0.000	−0.025	−0.003	−0.061	0.037
8	0.004	0	0	1	0	0	−0.010	−0.031	0.000	−0.002	−0.001	0.004
9	0.031	0	0	1	0	0	−0.022	−0.019	0.000	−0.006	−0.038	0.029
10	0.025	0	0	1	0	0	−0.032	−0.022	0.000	−0.016	−0.027	0.024
11	0.029	0	0	1	0	0	−0.034	−0.006	0.000	−0.007	−0.033	0.028
12	0.000	0	0	1	0	0	−0.004	−0.008	0.000	−0.004	−0.003	0.000
13	0.033	0.5294	0.4706	0	0	0	0.000	0.000	−0.015	−0.026	−0.034	0.032
14	0.017	0	0	0	0	1	−0.002	−0.005	−0.006	−0.008	0.000	0.016
15	0.010	0	1	0	0	0	−0.040	0.000	−0.018	−0.013	−0.018	0.010
16	−0.002	0	0	0	0	1	−0.017	−0.030	−0.011	−0.008	0.000	−0.002
17	−0.007	0	0	0	0	1	−0.043	−0.036	−0.038	−0.030	0.000	−0.007
18	−0.007	0	0	0	0	1	−0.060	−0.022	−0.028	−0.028	0.000	−0.007
19	0.038	0	1	0	0	0	−0.005	0.000	−0.022	−0.017	−0.030	0.032
20	−0.003	0	0	1	0	0	−0.024	−0.032	0.000	−0.013	−0.012	−0.003
21	−0.011	0	0	1	0	0	−0.026	−0.008	0.000	−0.003	−0.002	−0.011
22	0.014	1	0	0	0	0	0.000	−0.016	−0.021	−0.007	−0.022	0.013
23	0.028	0	1	0	0	0	−0.042	0.000	−0.004	−0.007	−0.027	0.024
24	0.005	0	1	0	0	0	−0.011	0.000	−0.006	−0.005	−0.004	0.005
25	0.000	0	0	0	0	1	−0.026	−0.016	−0.005	−0.004	0.000	0.000
26	0.018	1	0	0	0	0	0.000	−0.015	−0.007	−0.015	−0.014	0.017
27	0.063	1	0	0	0	0	0.000	−0.077	−0.033	−0.050	−0.068	0.059
28	0.002	0	0	1	0	0	−0.038	0.000	0.000	−0.009	−0.006	0.002
29	0.026	0	1	0	0	0	−0.011	0.000	−0.019	−0.021	−0.031	0.025
30	0.033	0	1	0	0	0	−0.034	0.000	−0.014	−0.016	−0.029	0.032
31	0.035	0	0	1	0	0	−0.019	−0.008	0.000	−0.013	−0.031	0.032
32	0.008	0	0	1	0	0	−0.028	−0.006	0.000	−0.004	−0.013	0.008
33	−0.009	0	0	0	0	1	−0.006	−0.018	−0.012	0.000	0.000	−0.009
34	0.006	1	0	0	0	0	0.000	0.000	−0.018	−0.001	−0.009	0.005
35	0.007	0	0	1	0	0	−0.001	−0.012	0.000	−0.004	−0.015	0.006
36	−0.005	0	0	1	0	0	−0.011	−0.022	0.000	−0.009	0.000	−0.006
37	0.010	1	0	0	0	0	0.000	−0.030	−0.018	−0.018	−0.015	0.010
38	0.017	1	0	0	0	0	0.000	−0.009	−0.006	−0.011	−0.017	0.016
39	−0.006	0	0	0	0	1	−0.018	−0.007	−0.003	−0.001	0.000	−0.006
40	0.007	0	0	0	1	0	−0.003	−0.016	−0.005	0.000	−0.017	0.007
41	0.020	0	1	0	0	0	−0.018	0.000	−0.025	−0.020	−0.025	0.020
42	0.005	0	0	0	0	1	−0.007	−0.020	−0.008	−0.004	0.000	0.005
43	−0.033	0	0	0	0	1	−0.031	−0.030	−0.031	−0.009	0.000	−0.033
44	0.010	1	0	0	0	0	0.000	−0.091	−0.074	−0.051	−0.016	0.005
45	0.004	0	1	0	0	0	−0.126	0.000	−0.251	−0.179	−0.037	0.003
46	0.108	1	0	0	0	0	0.000	−0.076	−0.194	−0.139	−0.120	0.091
47	0.070	1	0	0	0	0	0.000	−0.055	−0.141	−0.131	−0.052	0.068
48	0.105	0	0	1	0	0	−0.122	−0.067	0.000	−0.072	−0.091	0.093
49	0.075	0	0	1	0	0	−0.038	−0.064	0.000	−0.017	−0.040	0.050
50	0.036	1	0	0	0	0	0.000	−0.098	−0.121	−0.066	−0.032	0.033
51	0.056	0	0	1	0	0	−0.037	−0.059	0.000	−0.053	−0.057	0.051
52	−0.008	0	0	0	1	0	−0.021	−0.009	−0.022	0.000	−0.009	−0.008

Table B.7: Optimisation: Standard deviation as risk, upward shock in Bond yields as indicator and an aggressive investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
1	−0.008	0	0	0	0	1	−0.033	−0.007	−0.015	−0.006	0.000	−0.008
2	0.012	1	0	0	0	0	0.000	−0.014	−0.015	−0.017	−0.020	0.010
3	−0.008	0	0	0	0	1	−0.019	−0.054	−0.022	−0.020	0.000	−0.008
4	0.023	0	0	1	0	0	−0.033	−0.025	0.000	−0.001	−0.020	0.023
5	0.041	1	0	0	0	0	0.000	−0.027	−0.018	−0.034	−0.044	0.035
6	0.035	0	1	0	0	0	−0.014	0.000	−0.024	−0.029	−0.025	0.033
7	−0.001	0	0	0	1	0	−0.010	−0.018	−0.001	0.000	−0.013	−0.001
8	0.022	1	0	0	0	0	0.000	−0.019	−0.018	−0.011	−0.026	0.022
9	0.043	1	0	0	0	0	0.000	−0.036	−0.030	−0.030	−0.035	0.038
10	0.019	0	1	0	0	0	−0.001	0.000	−0.023	−0.001	−0.019	0.018
11	0.062	0	1	0	0	0	−0.057	0.000	−0.014	−0.004	−0.047	0.055
12	−0.006	0.2754	0	0	0.7246	0	0.000	−0.009	−0.005	0.000	−0.006	−0.006
13	0.019	0	0	0	1	0	−0.012	−0.025	−0.017	0.000	−0.005	0.018
14	0.067	1	0	0	0	0	0.000	−0.058	−0.050	−0.041	−0.043	0.056
15	0.035	1	0	0	0	0	0.000	−0.017	−0.009	−0.017	−0.005	0.034
16	0.043	0	1	0	0	0	−0.030	0.000	−0.022	−0.013	−0.029	0.041
17	0.042	0	0	0	0	1	−0.009	−0.010	−0.011	−0.005	0.000	0.040
18	0.046	1	0	0	0	0	0.000	−0.020	−0.023	−0.016	−0.028	0.042
19	0.006	0	0	0	0	1	−0.050	−0.039	−0.029	−0.064	0.000	0.006
20	0.021	0	1	0	0	0	−0.061	0.000	−0.009	−0.020	−0.011	0.020
21	0.009	0	0	0	0	1	−0.030	−0.015	−0.010	−0.004	0.000	0.009
22	0.065	0.8382	0.1618	0	0	0	0.000	0.000	−0.040	−0.022	−0.057	0.059
23	−0.002	0	1	0	0	0	−0.065	0.000	−0.018	−0.023	−0.007	−0.002
24	0.022	1	0	0	0	0	0.000	−0.024	−0.023	−0.010	−0.020	0.016
25	0.005	0	1	0	0	0	−0.022	0.000	−0.004	−0.013	−0.004	0.005
26	0.061	1	0	0	0	0	0.000	−0.023	−0.047	−0.045	−0.058	0.059
27	0.049	0	1	0	0	0	−0.027	0.000	−0.021	−0.014	−0.036	0.043
28	0.060	0	1	0	0	0	−0.044	0.000	−0.034	−0.035	−0.052	0.056
29	0.028	1	0	0	0	0	0.000	−0.071	−0.061	−0.056	−0.023	0.026
30	−0.009	0	0	0	0	1	−0.044	−0.031	−0.029	−0.024	0.000	−0.009
31	0.012	0	0	0	1	0	−0.023	−0.004	−0.007	0.000	−0.118	0.011
32	0.027	0	1	0	0	0	−0.012	0.000	−0.030	−0.028	−0.034	0.025
33	0.054	0	1	0	0	0	−0.069	0.000	−0.009	−0.019	−0.042	0.050
34	0.063	0	1	0	0	0	−0.011	0.000	−0.014	−0.005	−0.043	0.055
35	0.032	0	0	0	1	0	−0.073	−0.056	−0.010	0.000	−0.017	0.031
36	0.051	1	0	0	0	0	0.000	−0.036	−0.001	−0.018	−0.041	0.049
37	0.009	0	0	0	0	1	−0.005	−0.034	−0.037	−0.015	0.000	0.009
38	0.021	0	1	0	0	0	−0.047	0.000	−0.028	−0.028	−0.019	0.021
39	0.044	1	0	0	0	0	0.000	−0.037	−0.034	−0.039	−0.038	0.041
40	0.024	0	0	0	1	0	−0.027	−0.008	−0.002	0.000	−0.021	0.023
41	−0.011	0	0	0	0	1	−0.048	−0.078	−0.009	−0.012	0.000	−0.011
42	−0.001	0	0	0.1593	0	0.8407	−0.031	−0.068	0.000	−0.026	0.000	−0.001
43	0.056	0	0	1	0	0	−0.046	−0.070	0.000	−0.019	−0.035	0.054
44	0.117	1	0	0	0	0	0.000	−0.047	−0.055	−0.054	−0.078	0.096
45	0.033	0	1	0	0	0	−0.027	0.000	−0.023	−0.020	−0.022	0.032
46	0.030	0	0	1	0	0	−0.059	−0.042	0.000	−0.021	−0.021	0.030
47	0.041	0	0	1	0	0	−0.035	−0.037	0.000	−0.023	−0.037	0.039
48	0.082	0	0	1	0	0	−0.091	−0.089	0.000	−0.038	−0.044	0.070
49	0.001	0	0	0	0.8564	0.1436	−0.021	−0.076	−0.058	0.000	0.000	0.001
50	0.062	0	1	0	0	0	−0.022	0.000	−0.015	−0.027	−0.056	0.061
51	0.004	0	0	0	1	0	−0.025	−0.030	−0.004	0.000	−0.012	0.004
52	0.057	0	0	1	0	0	−0.011	−0.031	0.000	−0.033	−0.041	0.050
53	0.065	1	0	0	0	0	0.000	−0.031	−0.042	−0.008	−0.055	0.061
54	0.042	0.9698	0.0302	0	0	0	0.000	0.000	−0.047	−0.028	−0.050	0.042
55	0.137	0	1	0	0	0	−0.087	0.000	−0.104	−0.099	−0.129	0.120
56	0.028	1	0	0	0	0	0.000	−0.043	−0.036	−0.010	−0.035	0.026
57	0.023	1	0	0	0	0	0.000	−0.015	−0.051	−0.014	−0.043	0.019
58	0.002	0	0	0	1	0	−0.045	−0.019	−0.034	0.000	−0.007	0.002
59	0.010	0	0	1	0	0	−0.042	−0.025	0.000	−0.001	−0.014	0.009

Table B.8: Optimisation: Standard deviation as risk, downward shock in Bond yields as indicator and an aggressive investment

		Solution					Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
60	0.021	0	0	1	0	0	−0.044	−0.033	0.000	−0.017	−0.023	0.021
61	0.026	0	0	0	1	0	−0.035	−0.080	−0.010	0.000	−0.020	0.025
62	0.073	0	1	0	0	0	−0.031	0.000	−0.059	−0.031	−0.076	0.067
63	0.022	0	1	0	0	0	−0.005	0.000	−0.015	−0.002	−0.023	0.021
64	0.063	0	0	1	0	0	−0.023	−0.029	0.000	−0.025	−0.072	0.059
65	0.056	1	0	0	0	0	0.000	−0.019	−0.020	−0.045	−0.047	0.048
66	0.053	0	0	0	1	0	−0.084	−0.045	−0.018	0.000	−0.038	0.051
67	0.014	0	0	1	0	0	−0.003	−0.018	0.000	−0.007	−0.007	0.013
68	0.040	0	1	0	0	0	−0.013	0.000	−0.034	−0.027	−0.057	0.030
69	0.087	0	0	1	0	0	−0.096	−0.092	0.000	−0.023	−0.072	0.086
70	0.044	0	1	0	0	0	−0.064	0.000	−0.031	−0.034	−0.092	0.043
71	0.041	0	0	0	0	1	−0.080	−0.084	−0.108	−0.086	0.000	0.037
72	0.027	0	0	1	0	0	−0.028	−0.033	0.000	−0.001	−0.027	0.023
73	0.011	0	0	1	0	0	−0.001	−0.038	0.000	−0.014	−0.042	0.010
74	0.003	0	0	0	0	1	−0.009	−0.037	−0.059	−0.008	0.000	0.003
75	0.000	0	0	0	0	1	−0.097	−0.117	−0.031	−0.007	0.000	−0.001
76	0.024	0	1	0	0	0	−0.033	0.000	−0.060	−0.027	−0.019	0.019
77	0.011	0	0	0	0	1	−0.044	−0.083	−0.010	−0.007	0.000	0.010
78	0.021	1	0	0	0	0	0.000	−0.005	−0.016	−0.012	−0.025	0.009
79	0.065	0	0	1	0	0	−0.055	−0.017	0.000	−0.051	−0.030	0.059
80	0.014	0	0	0	0	1	−0.013	−0.059	−0.048	−0.057	0.000	0.014
81	0.005	0	0	0	0	1	−0.086	−0.066	−0.082	−0.101	0.000	0.005
82	0.030	1	0	0	0	0	0.000	−0.042	−0.011	−0.019	−0.001	0.029
83	0.016	0	0	0	0	1	−0.015	−0.020	−0.012	−0.014	0.000	0.016
84	0.077	1	0	0	0	0	0.000	−0.043	−0.065	−0.051	−0.095	0.071
85	0.013	0	1	0	0	0	−0.047	0.000	−0.014	−0.011	−0.011	0.013
86	−0.001	0	0	0	0	1	−0.019	−0.003	−0.012	−0.020	0.000	−0.002
87	0.025	0	0	0	0	1	−0.011	−0.016	−0.004	−0.016	0.000	0.024
88	0.054	0	1	0	0	0	−0.005	0.000	−0.042	−0.035	−0.061	0.051
89	0.045	1	0	0	0	0	0.000	−0.045	−0.024	−0.040	−0.034	0.043
90	0.039	0	1	0	0	0	−0.024	0.000	−0.023	−0.031	−0.050	0.039
91	0.065	0	1	0	0	0	−0.049	0.000	−0.041	−0.039	−0.063	0.059
92	0.024	1	0	0	0	0	0.000	−0.033	−0.034	−0.032	−0.027	0.023
93	−0.001	0	0	0	1	0	−0.024	−0.016	−0.011	0.000	−0.023	−0.001
94	0.041	0	0	0	0	1	−0.113	−0.076	−0.065	−0.056	0.000	0.038
95	0.056	1	0	0	0	0	0.000	−0.036	−0.052	−0.044	−0.051	0.052
96	0.002	0.8623	0	0.1377	0	0	0.000	−0.029	0.000	−0.015	−0.015	0.002
97	0.017	0	1	0	0	0	−0.024	0.000	−0.053	−0.075	−0.021	0.017
98	0.025	1	0	0	0	0	0.000	−0.018	−0.030	−0.017	−0.003	0.024
99	0.002	0	0	0	0	1	−0.016	−0.047	−0.041	−0.013	0.000	0.002
100	0.068	1	0	0	0	0	0.000	−0.034	0.000	−0.030	−0.052	0.059
101	−0.008	0	0	0	0	1	0.000	−0.030	−0.006	−0.035	0.000	−0.008
102	0.008	1	0	0	0	0	0.000	−0.063	−0.009	−0.011	−0.013	0.007
103	0.002	0	1	0	0	0	−0.021	0.000	−0.014	−0.052	0.000	0.001
104	0.010	0	1	0	0	0	−0.067	0.000	−0.016	−0.009	0.000	0.008
105	0.088	1	0	0	0	0	0.000	−0.106	−0.045	−0.067	−0.076	0.080
106	0.033	0	0	1	0	0	−0.072	−0.060	0.000	−0.039	−0.014	0.033
107	0.044	0	0	1	0	0	−0.017	−0.037	0.000	−0.040	−0.039	0.042
108	0.034	0	1	0	0	0	−0.030	0.000	−0.032	−0.013	−0.036	0.034

Table B.9: Optimisation: Standard deviation as risk, downward shock in Bond yields as indicator and an aggressive investment (continue)

	Solution						Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
1	0.006	0	0	1	0	0	−0.025	−0.009	0.000	−0.007	−0.012	0.005
2	−0.001	0	0	0	1	0	−0.010	−0.017	−0.001	0.000	−0.013	−0.001
3	−0.006	0.0577	0	0	0.9423	0	0.000	−0.009	−0.005	0.000	−0.006	−0.006
4	0.020	0	1	0	0	0	−0.056	0.000	−0.008	−0.018	−0.010	0.018
5	0.058	1	0	0	0	0	0.000	−0.001	−0.034	−0.017	−0.046	0.045
6	0.005	0	1	0	0	0	−0.023	0.000	−0.003	−0.013	−0.003	0.004
7	0.026	1	0	0	0	0	0.000	−0.065	−0.055	−0.050	−0.019	0.022
8	0.009	0	0	0	0	1	−0.005	−0.034	−0.036	−0.015	0.000	0.008
9	0.021	0	1	0	0	0	−0.047	0.000	−0.028	−0.027	−0.019	0.020
10	0.034	1	0	0	0	0	0.000	−0.018	−0.019	−0.022	−0.013	0.027
11	−0.006	0	0	0	0	1	−0.026	−0.018	−0.029	−0.031	0.000	−0.006
12	0.012	0	0	1	0	0	−0.013	−0.017	0.000	−0.010	−0.018	0.011
13	0.024	0	1	0	0	0	−0.026	0.000	−0.033	−0.023	−0.027	0.022
14	0.029	0	1	0	0	0	−0.006	0.000	−0.025	−0.022	−0.019	0.026
15	0.022	1	0	0	0	0	0.000	−0.004	−0.024	−0.027	−0.028	0.019
16	−0.004	0	1	0	0	0	−0.005	0.000	−0.017	−0.010	0.000	−0.007
17	0.045	1	0	0	0	0	0.000	−0.031	−0.003	−0.017	−0.041	0.036
18	0.023	0	1	0	0	0	−0.020	0.000	−0.042	−0.040	−0.017	0.019
19	0.012	0	1	0	0	0	−0.021	0.000	−0.015	−0.021	−0.020	0.010
20	0.030	0	1	0	0	0	−0.033	0.000	−0.032	−0.027	−0.034	0.027
21	0.032	0	1	0	0	0	−0.005	0.000	−0.035	−0.035	−0.043	0.031
22	0.004	0	0	1	0	0	−0.006	−0.012	0.000	−0.010	−0.018	0.004
23	0.041	1	0	0	0	0	0.000	−0.026	−0.044	−0.040	−0.036	0.035
24	0.000	1	0	0	0	0	0.000	−0.023	−0.037	−0.022	−0.015	−0.001
25	0.009	1	0	0	0	0	0.000	−0.024	−0.004	−0.007	−0.010	0.008
26	0.001	0	0	0	0	1	−0.021	−0.026	−0.024	−0.025	0.000	0.001
27	0.015	0	1	0	0	0	−0.015	0.000	−0.029	−0.015	−0.019	0.014
28	0.022	1	0	0	0	0	0.000	−0.043	−0.008	−0.035	−0.019	0.021
29	0.002	0	0	0	1	0	−0.016	−0.014	0.000	0.000	−0.016	0.002
30	0.018	0	0	1	0	0	−0.088	−0.036	0.000	−0.021	−0.018	0.011
31	−0.018	0	0	0	0	1	−0.087	−0.069	−0.081	−0.066	0.000	−0.020
32	0.021	0	0	1	0	0	−0.056	−0.024	0.000	−0.044	−0.015	0.018
33	0.026	1	0	0	0	0	0.000	−0.035	−0.030	−0.005	−0.030	0.022
34	0.009	0	0	1	0	0	−0.041	−0.025	0.000	0.000	−0.012	0.008
35	−0.040	0	0	0	0	1	−0.023	−0.070	−0.016	−0.031	0.000	−0.043
36	−0.035	0	0	0	0	1	−0.043	−0.091	−0.053	−0.057	0.000	−0.038
37	0.118	1	0	0	0	0	0.000	−0.049	−0.048	−0.050	−0.031	0.022
38	0.062	0	1	0	0	0	−0.050	0.000	−0.051	−0.043	−0.067	0.060
39	−0.028	1	0	0	0	0	0.000	−0.102	−0.041	−0.026	−0.007	−0.029
40	−0.033	0	0	0	0	1	−0.031	−0.030	−0.031	−0.009	0.000	−0.033
41	0.004	1	0	0	0	0	0.000	−0.081	−0.065	−0.041	−0.005	−0.008
42	0.003	0	1	0	0	0	−0.129	0.000	−0.261	−0.184	−0.036	0.000
43	0.067	1	0	0	0	0	0.000	−0.046	−0.130	−0.123	−0.048	0.063
44	0.032	1	0	0	0	0	0.000	−0.086	−0.100	−0.052	−0.023	0.025
45	0.005	1	0	0	0	0	0.000	−0.013	−0.091	−0.047	−0.008	0.002
46	−0.009	0	0	0	0	1	−0.051	−0.048	−0.108	−0.061	0.000	−0.010
47	0.050	1	0	0	0	0	0.000	−0.046	−0.127	−0.065	−0.056	0.045
48	−0.005	0	0	0	0	1	−0.017	−0.049	−0.018	−0.010	0.000	−0.006
49	0.021	0	0	0	0	1	−0.006	−0.025	−0.027	−0.033	0.000	0.020
50	0.064	0	1	0	0	0	−0.016	0.000	−0.083	−0.065	−0.020	0.055
51	0.017	0	0	0	1	0	−0.011	−0.004	−0.014	0.000	−0.028	0.016
52	0.037	0	0	0	0	1	−0.071	−0.072	−0.095	−0.075	0.000	0.030
53	0.018	0	1	0	0	0	−0.027	0.000	−0.042	−0.015	−0.008	0.007
54	0.014	0	0	0	0	1	−0.011	−0.053	−0.043	−0.051	0.000	0.012
55	−0.001	0	0	0.9449	0	0.0551	−0.019	−0.034	0.000	−0.020	0.000	−0.038
56	0.066	1	0	0	0	0	0.000	−0.032	−0.093	−0.064	−0.043	0.052
57	0.051	1	0	0	0	0	0.000	−0.012	−0.022	−0.032	−0.013	0.045
58	0.017	0	1	0	0	0	−0.023	0.000	−0.052	−0.073	−0.020	0.016

Table B.10: Optimisation: Standard deviation as risk, upward shocks in rate of change of bond yields as indicator and a conservative investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
1	0.051	1	0	0	0	0	0.000	−0.044	−0.045	−0.048	−0.030	0.043
2	−0.002	0	0	0	1	0	−0.021	−0.011	−0.004	0.000	−0.003	−0.002
3	−0.008	0	0	0	0	1	−0.033	−0.007	−0.015	−0.006	0.000	−0.008
4	−0.008	0	0	0	0	1	−0.020	−0.055	−0.023	−0.021	0.000	−0.009
5	0.037	1	0	0	0	0	0.000	−0.027	−0.019	−0.019	−0.024	0.026
6	0.018	0	1	0	0	0	−0.002	0.000	−0.021	0.000	−0.017	0.016
7	0.018	0	0	0	1	0	−0.012	−0.023	−0.015	0.000	−0.004	0.016
8	0.055	1	0	0	0	0	0.000	−0.038	−0.034	−0.026	−0.026	0.034
9	0.040	0	0	0	0	1	−0.008	−0.007	−0.010	−0.003	0.000	0.036
10	0.016	1	0	0	0	0	0.000	−0.017	−0.014	−0.002	−0.007	0.004
11	0.050	0	1	0	0	0	−0.059	0.000	−0.009	−0.017	−0.036	0.043
12	0.054	0	1	0	0	0	−0.007	0.000	−0.009	−0.003	−0.030	0.037
13	0.078	0	1	0	0	0	−0.032	0.000	−0.050	−0.055	−0.049	0.057
14	0.014	0	0	1	0	0	−0.022	−0.002	0.000	−0.004	−0.024	0.013
15	0.011	0	0	0	1	0	−0.007	−0.007	−0.009	0.000	−0.010	0.011
16	0.014	0	0	0	0	1	−0.022	−0.021	−0.011	−0.007	0.000	0.013
17	0.010	0	0	1	0	0	−0.038	−0.035	0.000	−0.009	−0.014	0.009
18	0.045	1	0	0	0	0	0.000	−0.048	−0.003	−0.023	−0.022	0.036
19	0.064	1	0	0	0	0	0.000	−0.026	−0.028	−0.029	−0.047	0.046
20	0.027	0	0	0	1	0	−0.015	−0.017	−0.004	0.000	−0.036	0.025
21	0.026	0	0	0	0	1	−0.032	−0.026	−0.012	−0.013	0.000	0.024
22	0.025	0	0	1	0	0	−0.024	−0.014	0.000	−0.008	−0.015	0.023
23	0.025	0	1	0	0	0	−0.027	0.000	−0.007	−0.005	−0.034	0.021
24	0.024	0	0	0	0	1	−0.023	−0.052	−0.011	−0.022	0.000	0.022
25	0.032	0	0	1	0	0	−0.015	−0.024	0.000	−0.006	−0.024	0.031
26	0.016	0	0	0	1	0	−0.033	−0.009	−0.012	0.000	−0.012	0.015
27	0.024	0	0	1	0	0	−0.036	−0.018	0.000	−0.020	−0.035	0.023
28	0.050	0	0	1	0	0	−0.004	−0.016	0.000	−0.023	−0.029	0.035
29	0.042	1	0	0	0	0	0.000	0.000	−0.046	−0.028	−0.050	0.041
30	−0.046	0	0	0	0	1	−0.022	−0.044	−0.025	−0.012	0.000	−0.050
31	0.090	1	0	0	0	0	0.000	−0.031	−0.129	−0.093	−0.086	0.057
32	0.092	0	0	1	0	0	−0.091	−0.053	0.000	−0.054	−0.068	0.066
33	0.048	0.1158	0	0.8842	0	0	0.000	−0.034	0.000	−0.012	−0.003	0.007
34	0.051	0	0	1	0	0	−0.027	−0.052	0.000	−0.046	−0.050	0.041
35	0.135	0	0	1	0	0	−0.051	−0.027	0.000	−0.012	−0.056	0.050
36	0.046	0	0	1	0	0	−0.036	−0.028	0.000	−0.004	−0.015	0.040
37	0.010	0	0	0	1	0	−0.009	−0.024	0.000	0.000	−0.054	0.010
38	0.054	0	0	1	0	0	−0.060	−0.047	0.000	−0.027	−0.021	0.049
39	0.059	0	0	1	0	0	−0.011	−0.016	0.000	−0.002	−0.027	0.011
40	0.051	0	0	1	0	0	−0.031	−0.037	0.000	−0.019	−0.041	0.035
41	0.028	0	0	1	0	0	−0.024	−0.048	0.000	−0.029	−0.001	0.025
42	0.024	0	1	0	0	0	−0.015	0.000	−0.020	−0.014	−0.031	0.023
43	0.057	1	0	0	0	0	0.000	−0.019	−0.048	−0.046	−0.030	0.040
44	0.058	0	1	0	0	0	−0.037	0.000	−0.015	−0.010	−0.046	0.045
45	0.022	1	0	0	0	0	0.000	−0.041	−0.035	−0.032	−0.032	0.018
46	0.042	0	0	0	0	1	−0.059	−0.035	−0.027	−0.023	0.000	0.036
47	0.086	0	0	1	0	0	−0.093	−0.088	0.000	−0.022	−0.069	0.085
48	0.022	0	0	1	0	0	−0.023	−0.025	0.000	0.000	−0.017	0.014
49	0.052	0	0	1	0	0	−0.037	−0.058	0.000	−0.004	−0.044	0.049
50	0.016	0	0	0	0	1	−0.015	−0.020	−0.012	−0.015	0.000	0.015
51	0.059	1	0	0	0	0	0.000	−0.025	−0.003	−0.021	−0.035	0.040
52	0.009	0	0.6354	0	0	0.3646	−0.067	0.000	−0.015	−0.009	0.000	0.008

Table B.11: Optimisation: Standard deviation as risk, downward shocks in rate of change of bond yields as indicator and a conservative investment

		Solution					Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment												
1	0.0063	0	0	1	0	0	−0.0246	−0.0088	0.0000	−0.0080	−0.0123	0.0054
2	−0.0007	0	0	0	1	0	−0.0096	−0.0174	−0.0014	0.0000	−0.0127	−0.0007
3	−0.0057	0.1358	0	0	0.8642	0	0.0000	−0.0090	−0.0049	0.0000	−0.0060	−0.0059
4	0.0204	0	1	0	0	0	−0.0590	0.0000	−0.0087	−0.0187	−0.0108	0.0189
5	0.0618	1	0	0	0	0	0.0000	−0.0002	−0.0375	−0.0200	−0.0518	0.0524
6	0.0047	0	1	0	0	0	−0.0226	0.0000	−0.0033	−0.0130	−0.0036	0.0044
7	0.0267	1	0	0	0	0	0.0000	−0.0682	−0.0581	−0.0534	−0.0212	0.0243
8	0.0086	0	0	0	0	1	−0.0048	−0.0341	−0.0367	−0.0150	0.0000	0.0085
9	0.0210	0	1	0	0	0	−0.0472	0.0000	−0.0279	−0.0275	−0.0191	0.0206
10	0.0356	1	0	0	0	0	0.0000	−0.0199	−0.0219	−0.0248	−0.0156	0.0307
11	−0.0059	0	0	0	0	1	−0.0259	−0.0186	−0.0291	−0.0306	0.0000	−0.0061
12	0.0120	0	0	1	0	0	−0.0132	−0.0175	0.0000	−0.0102	−0.0185	0.0117
13	0.0245	0	1	0	0	0	−0.0272	0.0000	−0.0343	−0.0239	−0.0285	0.0230
14	0.0299	0	1	0	0	0	−0.0064	0.0000	−0.0266	−0.0230	−0.0200	0.0279
15	0.0230	1	0	0	0	0	0.0000	−0.0046	−0.0261	−0.0288	−0.0301	0.0208
16	−0.0032	0	1	0	0	0	−0.0062	0.0000	−0.0188	−0.0112	−0.0024	−0.0055
17	0.0472	1	0	0	0	0	0.0000	−0.0322	−0.0022	−0.0193	−0.0468	0.0409
18	0.0236	0	1	0	0	0	−0.0207	0.0000	−0.0447	−0.0424	−0.0190	0.0209
19	0.0125	0	1	0	0	0	−0.0218	0.0000	−0.0163	−0.0220	−0.0212	0.0110
20	0.0310	0	1	0	0	0	−0.0346	0.0000	−0.0348	−0.0297	−0.0360	0.0285
21	0.0316	0	1	0	0	0	−0.0048	0.0000	−0.0352	−0.0357	−0.0429	0.0314
22	0.0038	0	0	1	0	0	−0.0058	−0.0119	0.0000	−0.0097	−0.0183	0.0037
23	0.0427	1	0	0	0	0	0.0000	−0.0291	−0.0494	−0.0446	−0.0398	0.0383
24	0.0004	1	0	0	0	0	0.0000	−0.0237	−0.0378	−0.0223	−0.0156	−0.0004
25	0.0091	1	0	0	0	0	0.0000	−0.0243	−0.0045	−0.0072	−0.0111	0.0084
26	0.0007	0	0	0	0	1	−0.0212	−0.0264	−0.0243	−0.0255	0.0000	0.0007
27	0.0158	0	1	0	0	0	−0.0157	0.0000	−0.0301	−0.0156	−0.0198	0.0149
28	0.0227	1	0	0	0	0	0.0000	−0.0449	−0.0088	−0.0360	−0.0198	0.0215
29	0.0022	0	0	0	1	0	−0.0157	−0.0139	−0.0001	0.0000	−0.0164	0.0022
30	0.0204	0	0	1	0	0	−0.0889	−0.0379	0.0000	−0.0218	−0.0211	0.0153
31	−0.0178	0	0	0	0	1	−0.0841	−0.0660	−0.0771	−0.0633	0.0000	−0.0189
32	0.0214	0	0	1	0	0	−0.0551	−0.0240	0.0000	−0.0439	−0.0161	0.0196
33	0.0268	1	0	0	0	0	0.0000	−0.0394	−0.0331	−0.0076	−0.0324	0.0241
34	0.0097	0	0	1	0	0	−0.0416	−0.0252	0.0000	−0.0008	−0.0128	0.0087
35	−0.0394	0	0	0	0	1	−0.0216	−0.0673	−0.0144	−0.0290	0.0000	−0.0412
36	−0.0337	0	0	0	0	1	−0.0414	−0.0867	−0.0503	−0.0547	0.0000	−0.0360
37	0.1442	1	0	0	0	0	0.0000	−0.0681	−0.1115	−0.1010	−0.0876	0.0757
38	0.0619	0	1	0	0	0	−0.0525	0.0000	−0.0499	−0.0429	−0.0672	0.0611
39	−0.0283	1	0	0	0	0	0.0000	−0.1004	−0.0398	−0.0257	−0.0070	−0.0286
40	−0.0327	0	0	0	0	1	−0.0309	−0.0302	−0.0310	−0.0086	0.0000	−0.0327
41	0.0074	1	0	0	0	0	0.0000	−0.0865	−0.0697	−0.0460	−0.0111	−0.0010
42	0.0032	0	1	0	0	0	−0.1275	0.0000	−0.2557	−0.1814	−0.0369	0.0014
43	0.0685	1	0	0	0	0	0.0000	−0.0509	−0.1363	−0.1271	−0.0497	0.0656
44	0.0345	1	0	0	0	0	0.0000	−0.0927	−0.1113	−0.0594	−0.0282	0.0291
45	0.0060	1	0	0	0	0	0.0000	−0.0140	−0.0955	−0.0499	−0.0093	0.0037
46	−0.0091	0	0	0	0	1	−0.0502	−0.0476	−0.1069	−0.0606	0.0000	−0.0094
47	0.0512	1	0	0	0	0	0.0000	−0.0509	−0.1360	−0.0702	−0.0602	0.0479
48	−0.0048	0	0	0	0	1	−0.0182	−0.0497	−0.0190	−0.0110	0.0000	−0.0055
49	0.0218	0	0	0	0	1	−0.0065	−0.0261	−0.0273	−0.0342	0.0000	0.0207
50	0.0661	0	1	0	0	0	−0.0147	0.0000	−0.0897	−0.0709	−0.0179	0.0599
51	0.0168	0	0	0	1	0	−0.0114	−0.0044	−0.0149	0.0000	−0.0289	0.0161
52	0.0389	0	0	0	0	1	−0.0761	−0.0784	−0.1022	−0.0813	0.0000	0.0340
53	0.0215	0	1	0	0	0	−0.0303	0.0000	−0.0518	−0.0217	−0.0140	0.0136
54	0.0141	0	0	0	0	1	−0.0120	−0.0566	−0.0458	−0.0544	0.0000	0.0130
55	0.0102	0	0	1	0	0	−0.0320	−0.0415	0.0000	−0.0229	−0.0150	−0.0186
56	0.0696	1	0	0	0	0	0.0000	−0.0301	−0.1054	−0.0751	−0.0500	0.0599
57	0.0527	1	0	0	0	0	0.0000	−0.0126	−0.0241	−0.0343	−0.0155	0.0483
58	0.0171	0	1	0	0	0	−0.0237	0.0000	−0.0523	−0.0741	−0.0206	0.0167

Table B.12: Optimisation: Standard deviation as risk, upward shocks in rate of change of bond yields as indicator and a moderate investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment												
1	0.0535	1	0	0	0	0	0.0000	−0.0479	−0.0501	−0.0529	−0.0339	0.0474
2	−0.0016	0	0	0	1	0	−0.0204	−0.0112	−0.0045	0.0000	−0.0034	−0.0017
3	−0.0076	0	0	0	0	1	−0.0331	−0.0073	−0.0147	−0.0062	0.0000	−0.0078
4	−0.0084	0	0	0	0	1	−0.0195	−0.0544	−0.0223	−0.0204	0.0000	−0.0084
5	0.0404	1	0	0	0	0	0.0000	−0.0319	−0.0247	−0.0253	−0.0302	0.0324
6	0.0185	0	1	0	0	0	−0.0012	0.0000	−0.0220	−0.0010	−0.0176	0.0170
7	0.0186	0	0	0	1	0	−0.0120	−0.0243	−0.0162	0.0000	−0.0047	0.0169
8	0.0614	1	0	0	0	0	0.0000	−0.0492	−0.0427	−0.0342	−0.0355	0.0461
9	0.0414	0	0	0	0	1	−0.0085	−0.0087	−0.0106	−0.0042	0.0000	0.0386
10	0.0191	1	0	0	0	0	0.0000	−0.0208	−0.0188	−0.0066	−0.0139	0.0106
11	0.0521	0	1	0	0	0	−0.0647	0.0000	−0.0089	−0.0182	−0.0391	0.0471
12	0.0590	0	1	0	0	0	−0.0092	0.0000	−0.0120	−0.0041	−0.0367	0.0465
13	0.0843	0	1	0	0	0	−0.0380	0.0000	−0.0609	−0.0660	−0.0588	0.0687
14	0.0145	0	0	1	0	0	−0.0218	−0.0018	0.0000	−0.0046	−0.0250	0.0135
15	0.0113	0	0	0	1	0	−0.0074	−0.0066	−0.0088	0.0000	−0.0104	0.0110
16	0.0146	0	0	0	0	1	−0.0234	−0.0220	−0.0119	−0.0073	0.0000	0.0140
17	0.0099	0	0	1	0	0	−0.0392	−0.0357	0.0000	−0.0095	−0.0142	0.0092
18	0.0477	1	0	0	0	0	0.0000	−0.0563	−0.0052	−0.0283	−0.0263	0.0411
19	0.0696	1	0	0	0	0	0.0000	−0.0308	−0.0346	−0.0352	−0.0571	0.0563
20	0.0274	0	0	0	1	0	−0.0158	−0.0184	−0.0045	0.0000	−0.0375	0.0258
21	0.0263	0	0	0	0	1	−0.0340	−0.0274	−0.0126	−0.0137	0.0000	0.0249
22	0.0251	0	0	1	0	0	−0.0253	−0.0152	0.0000	−0.0085	−0.0155	0.0239
23	0.0261	0	1	0	0	0	−0.0289	0.0000	−0.0081	−0.0060	−0.0370	0.0233
24	0.0240	0	0	0	0	1	−0.0245	−0.0548	−0.0120	−0.0229	0.0000	0.0230
25	0.0325	0	0	1	0	0	−0.0159	−0.0249	0.0000	−0.0055	−0.0251	0.0315
26	0.0161	0	0	0	1	0	−0.0332	−0.0096	−0.0124	0.0000	−0.0124	0.0157
27	0.0244	0	0	1	0	0	−0.0372	−0.0186	0.0000	−0.0204	−0.0357	0.0239
28	0.0536	0	0	1	0	0	−0.0083	−0.0237	0.0000	−0.0288	−0.0354	0.0432
29	0.0418	1	0	0	0	0	0.0000	−0.0002	−0.0469	−0.0278	−0.0502	0.0416
30	−0.0449	0	0	0	0	1	−0.0206	−0.0419	−0.0240	−0.0123	0.0000	−0.0477
31	0.0995	1	0	0	0	0	0.0000	−0.0554	−0.1640	−0.1178	−0.1044	0.0753
32	0.0987	0	0	1	0	0	−0.1077	−0.0608	0.0000	−0.0640	−0.0803	0.0802
33	0.0622	0	0	1	0	0	−0.0148	−0.0476	0.0000	−0.0138	−0.0188	0.0247
34	0.0538	0	0	1	0	0	−0.0327	−0.0558	0.0000	−0.0500	−0.0542	0.0465
35	0.1588	0	0	1	0	0	−0.0948	−0.0349	0.0000	−0.0364	−0.1028	0.0977
36	0.0475	0	0	1	0	0	−0.0385	−0.0299	0.0000	−0.0038	−0.0160	0.0433
37	0.0099	0	0	0.0607	0.9393	0	−0.0088	−0.0244	0.0000	0.0000	−0.0535	0.0097
38	0.0556	0	0	1	0	0	−0.0641	−0.0507	0.0000	−0.0281	−0.0223	0.0521
39	0.0729	0	0	1	0	0	−0.0292	−0.0366	0.0000	−0.0114	−0.0490	0.0381
40	0.0551	0	0	1	0	0	−0.0388	−0.0438	0.0000	−0.0220	−0.0492	0.0437
41	0.0291	0	0	1	0	0	−0.0253	−0.0500	0.0000	−0.0307	−0.0020	0.0266
42	0.0242	0	1	0	0	0	−0.0150	0.0000	−0.0210	−0.0142	−0.0314	0.0234
43	0.0621	1	0	0	0	0	0.0000	−0.0258	−0.0595	−0.0564	−0.0385	0.0498
44	0.0613	0	1	0	0	0	−0.0405	0.0000	−0.0174	−0.0108	−0.0540	0.0523
45	0.0236	1	0	0	0	0	0.0000	−0.0455	−0.0398	−0.0360	−0.0354	0.0204
46	0.0437	0	0	0	0	1	−0.0636	−0.0387	−0.0291	−0.0253	0.0000	0.0394
47	0.0868	0	0	1	0	0	−0.0943	−0.0901	0.0000	−0.0226	−0.0706	0.0856
48	0.0247	0	0	1	0	0	−0.0258	−0.0292	0.0000	−0.0007	−0.0221	0.0187
49	0.0531	0	0	1	0	0	−0.0383	−0.0606	0.0000	−0.0040	−0.0466	0.0508
50	0.0157	0	0	0	0	1	−0.0152	−0.0196	−0.0120	−0.0145	0.0000	0.0156
51	0.0639	1	0	0	0	0	0.0000	−0.0299	−0.0016	−0.0256	−0.0441	0.0506
52	0.0094	0	0.7902	0	0	0.2098	−0.0670	0.0000	−0.0154	−0.0088	0.0000	0.0081

Table B.13: Optimisation: Standard deviation as risk, downward shocks in rate of change of bond yields as indicator and a moderate investment

		Solution					Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
1	0.0066	0	0	1	0	0	−0.0245	−0.0090	0.0000	−0.0086	−0.0130	0.0060
2	−0.0007	0	0	0	1	0	−0.0096	−0.0175	−0.0014	0.0000	−0.0128	−0.0007
3	−0.0057	0.2754	0	0	0.7246	0	0.0000	−0.0090	−0.0049	0.0000	−0.0060	−0.0059
4	0.0209	0	1	0	0	0	−0.0614	0.0000	−0.0094	−0.0198	−0.0114	0.0199
5	0.0649	0.8382	0.1618	0	0	0	0.0000	0.0000	−0.0402	−0.0224	−0.0570	0.0585
6	0.0047	0	1	0	0	0	−0.0225	0.0000	−0.0035	−0.0132	−0.0037	0.0046
7	0.0275	1	0	0	0	0	0.0000	−0.0713	−0.0611	−0.0561	−0.0228	0.0259
8	0.0087	0	0	0	0	1	−0.0047	−0.0344	−0.0369	−0.0151	0.0000	0.0086
9	0.0211	0	1	0	0	0	−0.0474	0.0000	−0.0281	−0.0277	−0.0194	0.0209
10	0.0372	1	0	0	0	0	0.0000	−0.0220	−0.0242	−0.0273	−0.0179	0.0340
11	−0.0058	0	0	0	0	1	−0.0258	−0.0187	−0.0290	−0.0305	0.0000	−0.0059
12	0.0121	0	0	1	0	0	−0.0136	−0.0178	0.0000	−0.0104	−0.0188	0.0119
13	0.0251	0	1	0	0	0	−0.0278	0.0000	−0.0356	−0.0249	−0.0298	0.0240
14	0.0306	0	1	0	0	0	−0.0071	0.0000	−0.0278	−0.0240	−0.0209	0.0292
15	0.0237	1	0	0	0	0	0.0000	−0.0050	−0.0277	−0.0305	−0.0319	0.0223
16	−0.0025	0	1	0	0	0	−0.0073	0.0000	−0.0202	−0.0125	−0.0041	−0.0040
17	0.0494	1	0	0	0	0	0.0000	−0.0328	−0.0017	−0.0210	−0.0515	0.0451
18	0.0245	0	1	0	0	0	−0.0211	0.0000	−0.0470	−0.0444	−0.0207	0.0227
19	0.0131	0	1	0	0	0	−0.0227	0.0000	−0.0172	−0.0231	−0.0223	0.0120
20	0.0318	0	1	0	0	0	−0.0362	0.0000	−0.0368	−0.0318	−0.0380	0.0302
21	0.0316	0	1	0	0	0	−0.0047	0.0000	−0.0354	−0.0358	−0.0431	0.0315
22	0.0038	0	0	1	0	0	−0.0058	−0.0120	0.0000	−0.0098	−0.0183	0.0038
23	0.0441	1	0	0	0	0	0.0000	−0.0314	−0.0539	−0.0484	−0.0426	0.0412
24	0.0007	1	0	0	0	0	0.0000	−0.0244	−0.0388	−0.0227	−0.0165	0.0001
25	0.0094	1	0	0	0	0	0.0000	−0.0249	−0.0049	−0.0074	−0.0116	0.0089
26	0.0007	0	0	0	0	1	−0.0212	−0.0264	−0.0243	−0.0255	0.0000	0.0007
27	0.0161	0	1	0	0	0	−0.0164	0.0000	−0.0312	−0.0163	−0.0205	0.0155
28	0.0230	1	0	0	0	0	0.0000	−0.0466	−0.0093	−0.0370	−0.0203	0.0223
29	0.0023	0	0	0	1	0	−0.0157	−0.0140	−0.0001	0.0000	−0.0165	0.0022
30	0.0222	0	0	1	0	0	−0.0894	−0.0394	0.0000	−0.0226	−0.0239	0.0187
31	−0.0175	0	0	0	0	1	−0.0816	−0.0639	−0.0739	−0.0607	0.0000	−0.0182
32	0.0220	0	0	1	0	0	−0.0547	−0.0237	0.0000	−0.0437	−0.0172	0.0208
33	0.0278	1	0	0	0	0	0.0000	−0.0431	−0.0358	−0.0095	−0.0349	0.0259
34	0.0101	0	0	1	0	0	−0.0421	−0.0253	0.0000	−0.0011	−0.0138	0.0094
35	−0.0388	0	0	0	0	1	−0.0207	−0.0646	−0.0131	−0.0275	0.0000	−0.0400
36	−0.0330	0	0	0	0	1	−0.0404	−0.0831	−0.0484	−0.0525	0.0000	−0.0345
37	0.1671	1	0	0	0	0	0.0000	−0.0843	−0.1656	−0.1445	−0.1364	0.1214
38	0.0621	0	1	0	0	0	−0.0544	0.0000	−0.0494	−0.0430	−0.0678	0.0616
39	−0.0282	1	0	0	0	0	0.0000	−0.0994	−0.0390	−0.0253	−0.0068	−0.0284
40	−0.0327	0	0	0	0	1	−0.0307	−0.0301	−0.0309	−0.0086	0.0000	−0.0327
41	0.0102	1	0	0	0	0	0.0000	−0.0908	−0.0739	−0.0506	−0.0165	0.0046
42	0.0038	0	1	0	0	0	−0.1257	0.0000	−0.2515	−0.1791	−0.0373	0.0026
43	0.0695	1	0	0	0	0	0.0000	−0.0548	−0.1414	−0.1310	−0.0515	0.0676
44	0.0364	1	0	0	0	0	0.0000	−0.0982	−0.1212	−0.0657	−0.0325	0.0327
45	0.0067	1	0	0	0	0	0.0000	−0.0152	−0.0993	−0.0523	−0.0109	0.0052
46	−0.0090	0	0	0	0	1	−0.0496	−0.0475	−0.1058	−0.0601	0.0000	−0.0092
47	0.0524	1	0	0	0	0	0.0000	−0.0549	−0.1439	−0.0750	−0.0635	0.0501
48	−0.0046	0	0	0	0	1	−0.0190	−0.0506	−0.0203	−0.0117	0.0000	−0.0051
49	0.0222	0	0	0	0	1	−0.0069	−0.0267	−0.0279	−0.0351	0.0000	0.0214
50	0.0682	0	1	0	0	0	−0.0139	0.0000	−0.0956	−0.0758	−0.0158	0.0641
51	0.0170	0	0	0	1	0	−0.0114	−0.0045	−0.0153	0.0000	−0.0294	0.0166
52	0.0405	0	0	0	0	1	−0.0805	−0.0837	−0.1082	−0.0865	0.0000	0.0373
53	0.0242	0	1	0	0	0	−0.0329	0.0000	−0.0600	−0.0274	−0.0190	0.0189
54	0.0144	0	0	0	0	1	−0.0127	−0.0593	−0.0478	−0.0571	0.0000	0.0137
55	0.0198	0	0	1	0	0	−0.0452	−0.0492	0.0000	−0.0263	−0.0303	0.0006
56	0.0729	1	0	0	0	0	0.0000	−0.0283	−0.1160	−0.0844	−0.0556	0.0664
57	0.0542	1	0	0	0	0	0.0000	−0.0132	−0.0260	−0.0361	−0.0180	0.0513
58	0.0172	0	1	0	0	0	−0.0241	0.0000	−0.0529	−0.0748	−0.0209	0.0170

Table B.14: Optimisation: Standard deviation as risk, upward shocks in rate of change of bond yields as indicator and an aggressive investment

		Solution					Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
1	0.0556	1	0	0	0	0	0.0000	−0.0516	−0.0542	−0.0574	−0.0368	0.0515
2	−0.0016	0	0	0	1	0	−0.0201	−0.0112	−0.0045	0.0000	−0.0034	−0.0017
3	−0.0076	0	0	0	0	1	−0.0329	−0.0072	−0.0146	−0.0062	0.0000	−0.0077
4	−0.0083	0	0	0	0	1	−0.0194	−0.0541	−0.0221	−0.0202	0.0000	−0.0084
5	0.0431	1	0	0	0	0	0.0000	−0.0360	−0.0297	−0.0303	−0.0355	0.0377
6	0.0190	0	1	0	0	0	−0.0006	0.0000	−0.0227	−0.0015	−0.0185	0.0180
7	0.0192	0	0	0	1	0	−0.0119	−0.0251	−0.0173	0.0000	−0.0052	0.0180
8	0.0665	1	0	0	0	0	0.0000	−0.0583	−0.0504	−0.0409	−0.0434	0.0563
9	0.0424	0	0	0	0	1	−0.0090	−0.0098	−0.0112	−0.0049	0.0000	0.0405
10	0.0219	1	0	0	0	0	0.0000	−0.0238	−0.0229	−0.0104	−0.0201	0.0162
11	0.0538	0	1	0	0	0	−0.0694	0.0000	−0.0087	−0.0195	−0.0420	0.0505
12	0.0632	0	1	0	0	0	−0.0108	0.0000	−0.0142	−0.0054	−0.0429	0.0549
13	0.0895	0	1	0	0	0	−0.0435	0.0000	−0.0702	−0.0758	−0.0671	0.0791
14	0.0148	0	0	1	0	0	−0.0219	−0.0017	0.0000	−0.0048	−0.0260	0.0142
15	0.0114	0	0	0	1	0	−0.0076	−0.0067	−0.0089	0.0000	−0.0106	0.0112
16	0.0148	0	0	0	0	1	−0.0242	−0.0226	−0.0123	−0.0076	0.0000	0.0144
17	0.0101	0	0	1	0	0	−0.0403	−0.0364	0.0000	−0.0096	−0.0147	0.0097
18	0.0499	1	0	0	0	0	0.0000	−0.0634	−0.0070	−0.0326	−0.0297	0.0455
19	0.0740	1	0	0	0	0	0.0000	−0.0349	−0.0402	−0.0406	−0.0661	0.0652
20	0.0279	0	0	0	1	0	−0.0165	−0.0195	−0.0049	0.0000	−0.0392	0.0268
21	0.0268	0	0	0	0	1	−0.0356	−0.0286	−0.0132	−0.0143	0.0000	0.0258
22	0.0255	0	0	1	0	0	−0.0260	−0.0158	0.0000	−0.0088	−0.0160	0.0247
23	0.0271	0	1	0	0	0	−0.0306	0.0000	−0.0090	−0.0070	−0.0396	0.0252
24	0.0244	0	0	0	0	1	−0.0255	−0.0569	−0.0126	−0.0238	0.0000	0.0237
25	0.0329	0	0	1	0	0	−0.0164	−0.0254	0.0000	−0.0054	−0.0257	0.0322
26	0.0162	0	0	0	1	0	−0.0337	−0.0099	−0.0125	0.0000	−0.0126	0.0160
27	0.0246	0	0	1	0	0	−0.0379	−0.0190	0.0000	−0.0208	−0.0362	0.0242
28	0.0571	0	0	1	0	0	−0.0115	−0.0305	0.0000	−0.0335	−0.0408	0.0502
29	0.0418	0.9698	0.0302	0	0	0	0.0000	0.0000	−0.0471	−0.0279	−0.0504	0.0417
30	−0.0440	0	0	0	0	1	−0.0195	−0.0398	−0.0236	−0.0122	0.0000	−0.0458
31	0.1075	1	0	0	0	0	0.0000	−0.0763	−0.1939	−0.1390	−0.1203	0.0914
32	0.1048	0	0	1	0	0	−0.1215	−0.0672	0.0000	−0.0723	−0.0905	0.0925
33	0.0747	0	0	1	0	0	−0.0378	−0.0644	0.0000	−0.0169	−0.0400	0.0497
34	0.0563	0	0	1	0	0	−0.0372	−0.0592	0.0000	−0.0532	−0.0574	0.0514
35	0.1792	0	0	1	0	0	−0.1320	−0.0416	0.0000	−0.0572	−0.1431	0.1384
36	0.0489	0	0	1	0	0	−0.0406	−0.0317	0.0000	−0.0041	−0.0166	0.0461
37	0.0100	0	0	0.3597	0.6403	0	−0.0088	−0.0243	0.0000	0.0000	−0.0535	0.0097
38	0.0567	0	0	1	0	0	−0.0676	−0.0537	0.0000	−0.0293	−0.0232	0.0544
39	0.0845	0	0	1	0	0	−0.0445	−0.0538	0.0000	−0.0197	−0.0677	0.0613
40	0.0589	0	0	1	0	0	−0.0456	−0.0492	0.0000	−0.0246	−0.0564	0.0513
41	0.0299	0	0	1	0	0	−0.0262	−0.0520	0.0000	−0.0324	−0.0032	0.0283
42	0.0245	0	1	0	0	0	−0.0154	0.0000	−0.0216	−0.0145	−0.0319	0.0239
43	0.0662	1	0	0	0	0	0.0000	−0.0317	−0.0695	−0.0653	−0.0456	0.0580
44	0.0642	0	1	0	0	0	−0.0434	0.0000	−0.0193	−0.0114	−0.0608	0.0583
45	0.0247	1	0	0	0	0	0.0000	−0.0492	−0.0436	−0.0393	−0.0381	0.0225
46	0.0451	0	0	0	0	1	−0.0673	−0.0419	−0.0306	−0.0271	0.0000	0.0423
47	0.0871	0	0	1	0	0	−0.0956	−0.0921	0.0000	−0.0229	−0.0716	0.0864
48	0.0267	0	0	1	0	0	−0.0285	−0.0328	0.0000	−0.0010	−0.0268	0.0227
49	0.0538	0	0	1	0	0	−0.0396	−0.0631	0.0000	−0.0040	−0.0487	0.0523
50	0.0158	0	0	0	0	1	−0.0154	−0.0197	−0.0120	−0.0144	0.0000	0.0157
51	0.0684	1	0	0	0	0	0.0000	−0.0339	−0.0001	−0.0295	−0.0522	0.0595
52	0.0100	0	1	0	0	0	−0.0671	0.0000	−0.0157	−0.0092	−0.0004	0.0084

Table B.15: Optimisation: Standard deviation as risk, downward shocks in rate of change of bond yields as indicator and an aggressive investment

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Bull	1	0.1181	0	1	0	0	0	−0.0632	0.0000	−0.0415	−0.0527	−0.0648	0.0630
	2	0.1372	0.0904	0.9096	0	0	0	0.0000	0.0000	−0.0423	−0.0529	−0.0478	0.0378
	3	0.0783	0	0.5762	0	0.4238	0	−0.0062	0.0000	−0.0229	0.0000	−0.0574	0.0595
	4	0.1375	0	1	0	0	0	−0.0526	0.0000	−0.0405	−0.0141	−0.0263	0.0399
	5	0.0883	1	0	0	0	0	0.0000	−0.0080	−0.0137	−0.0022	−0.1166	0.0013
	6	0.1786	0	1	0	0	0	−0.0880	0.0000	−0.0839	−0.0885	−0.1313	0.1335
	7	0.0494	0	0	1	0	0	−0.0437	−0.0668	0.0000	−0.0237	−0.0019	0.0471
	8	0.0734	0	0	1	0	0	−0.0096	−0.0126	0.0000	−0.0299	−0.0722	0.0540
	9	0.0808	0	0	1	0	0	−0.0127	−0.0719	0.0000	−0.0272	−0.0819	0.0716
	10	0.0937	0	0.3336	0.6664	0	0	−0.1125	0.0000	0.0000	−0.0495	−0.0347	0.0853
	11	0.0471	0	0	1	0	0	−0.0009	−0.0414	0.0000	−0.0213	−0.0300	0.0354
	12	0.0505	0	0	0	1	0	−0.0008	−0.0432	−0.0155	0.0000	−0.0218	0.0449
	13	0.1977	0	0	1	0	0	−0.0597	−0.0057	0.0000	−0.0175	−0.0437	0.0627
	14	0.1539	0	0.9637	0.0363	0	0	−0.0685	0.0000	0.0000	−0.0330	−0.0464	0.0618
	15	0.0935	0.5351	0.0871	0	0.3778	0	0.0000	0.0000	−0.0202	0.0000	−0.0191	0.0164
	16	0.1356	0	0	0.7268	0	0.2732	−0.0399	−0.3267	0.0000	−0.6431	0.0000	0.0253
	17	0.1200	0	1	0	0	0	−0.0116	0.0000	−0.0331	−0.0429	−0.0366	0.0574
Bear	1	−0.0277	0	0	0	0	1	−0.0006	−0.0320	−0.0411	−0.0320	0.0000	−0.0296
	2	0.0044	0	0	1	0	0	−0.0640	−0.0901	0.0000	−0.0071	−0.0227	0.0044
	3	−0.0245	0	0	0	0	1	−0.0759	−0.1465	−0.0816	−0.0647	0.0000	−0.0254
	4	0.0254	1	0	0	0	0	0.0000	−0.0598	−0.0288	−0.0278	−0.0255	0.0076
	5	−0.0226	0	1	0	0	0	−0.0571	0.0000	−0.0317	−0.0546	−0.0555	−0.0244
	6	0.0289	1	0	0	0	0	0.0000	−0.0598	−0.0670	−0.0614	−0.0422	0.0254
	7	−0.0429	0	0	0	1	0	−0.0208	−0.0288	−0.0172	0.0000	−0.0306	−0.0477
	8	−0.0256	0	0	0	0	1	−0.1115	−0.2242	−0.0272	−0.0895	0.0000	−0.0271
	9	−0.0979	0	0.7691	0	0	0.2309	−0.1527	0.0000	−0.5081	−0.3151	0.0000	−0.1555
	10	−0.0264	0	0	0	0	1	−0.1474	−0.0720	−0.0554	−0.0570	0.0000	−0.0295
	11	−0.0345	0	0	0	0	1	−0.0606	−0.0522	−0.0708	−0.0119	0.0000	−0.0370
	12	0.0153	0	0	0	0	1	−0.1957	−0.1485	−0.1394	−0.0301	0.0000	0.0147
	13	−0.0053	0	0	0	0	1	−0.0778	−0.0620	−0.0274	−0.0867	0.0000	−0.0055
Moderate Investment													
Bull	1	0.1336	0	1	0	0	0	−0.106	0.000	−0.076	−0.087	−0.098	0.094
	2	0.1697	0	1	0	0	0	−0.044	0.000	−0.076	−0.090	−0.104	0.084
	3	0.0837	0	0.4866	0	0.5134	0	−0.006	0.000	−0.027	0.000	−0.066	0.070
	4	0.1648	0	1	0	0	0	−0.070	0.000	−0.062	−0.029	−0.045	0.095
	5	0.1126	1	0	0	0	0	0.000	−0.044	−0.030	−0.019	−0.159	0.050
	6	0.1913	0	1	0	0	0	−0.114	0.000	−0.098	−0.104	−0.155	0.159
	7	0.0501	0	0	1	0	0	−0.045	−0.068	0.000	−0.024	−0.001	0.048
	8	0.0788	0	0	1	0	0	−0.010	−0.020	0.000	−0.033	−0.080	0.065
	9	0.0834	0	0	1	0	0	−0.013	−0.077	0.000	−0.027	−0.088	0.077
	10	0.0969	0	0.5826	0.4174	0	0	−0.113	0.000	0.000	−0.050	−0.035	0.086
	11	0.0504	0	0	1	0	0	−0.003	−0.049	0.000	−0.024	−0.037	0.042
	12	0.0521	0	0	0	1	0	−0.001	−0.046	−0.018	0.000	−0.023	0.048
	13	0.2356	0	0	1	0	0	−0.119	−0.070	0.000	−0.052	−0.095	0.138
	14	0.1813	0	1	0	0	0	−0.054	0.000	−0.042	−0.052	−0.102	0.111
	15	0.1198	1	0	0	0	0	0.000	−0.008	−0.033	−0.001	−0.026	0.040
	16	0.1780	0	0	0.9365	0	0.0635	−0.043	−0.329	0.000	−0.645	0.000	0.028
	17	0.1376	0	1	0	0	0	−0.011	0.000	−0.053	−0.068	−0.075	0.092
Bear	1	−0.0272	0	0	0	0	1	−0.001	−0.032	−0.039	−0.031	0.000	−0.029
	2	0.0044	0	0	1	0	0	−0.064	−0.090	0.000	−0.007	−0.023	0.004
	3	−0.0242	0	0	0	0	1	−0.073	−0.143	−0.079	−0.063	0.000	−0.025
	4	0.0304	1	0	0	0	0	0.000	−0.074	−0.043	−0.043	−0.039	0.018
	5	−0.0220	0	1	0	0	0	−0.056	0.000	−0.032	−0.054	−0.054	−0.023
	6	0.0299	1	0	0	0	0	0.000	−0.062	−0.072	−0.066	−0.045	0.027
	7	−0.0416	0	0	0	1	0	−0.020	−0.029	−0.016	0.000	−0.031	−0.045
	8	−0.0252	0	0	0	0	1	−0.108	−0.218	−0.025	−0.086	0.000	−0.026
	9	−0.0787	0	0.9842	0	0	0.0158	−0.146	0.000	−0.428	−0.270	0.000	−0.139
	10	−0.0256	0	0	0	0	1	−0.143	−0.071	−0.052	−0.056	0.000	−0.028
	11	−0.0338	0	0	0	0	1	−0.060	−0.053	−0.070	−0.013	0.000	−0.036
	12	0.0155	0	0	0	0	1	−0.199	−0.151	−0.141	−0.031	0.000	0.015
	13	−0.0052	0	0	0	0	1	−0.077	−0.063	−0.028	−0.087	0.000	−0.005
Aggressive Investment													
Bull	1	0.1468	0	1	0	0	0	−0.142	0.000	−0.106	−0.117	−0.126	0.120
	2	0.1983	0	1	0	0	0	−0.100	0.000	−0.117	−0.137	−0.173	0.141
	3	0.0886	0	0.3266	0	0.6734	0	−0.006	0.000	−0.031	0.000	−0.073	0.078
	4	0.1882	0	1	0	0	0	−0.085	0.000	−0.080	−0.041	−0.061	0.141
	5	0.1335	1	0	0	0	0	0.000	−0.075	−0.044	−0.034	−0.195	0.092
	6	0.2021	0	1	0	0	0	−0.137	0.000	−0.111	−0.118	−0.175	0.180
	7	0.0506	0	0	1	0	0	−0.047	−0.069	0.000	−0.025	−0.001	0.050
	8	0.0835	0	0	1	0	0	−0.011	−0.026	0.000	−0.036	−0.087	0.074
	9	0.0856	0	0	1	0	0	−0.013	−0.081	0.000	−0.028	−0.094	0.081
	10	0.1024	0	1	0	0	0	−0.115	0.000	0.000	−0.051	−0.036	0.087
	11	0.0532	0	0	1	0	0	−0.005	−0.056	0.000	−0.027	−0.044	0.048
	12	0.0535	0	0	0	1	0	−0.002	−0.048	−0.021	0.000	−0.025	0.051
	13	0.2680	0	0	1	0	0	−0.170	−0.125	0.000	−0.082	−0.139	0.203
	14	0.2049	0	1	0	0	0	−0.041	0.000	−0.083	−0.070	−0.156	0.158
	15	0.1464	1	0	0	0	0	0.000	−0.037	−0.061	−0.019	−0.058	0.093
	16	0.2355	0	0	1	0	0	−0.066	−0.367	0.000	−0.644	−0.108	0.118
	17	0.1526	0	1	0	0	0	−0.011	0.000	−0.070	−0.089	−0.108	0.123
Bear	1	−0.0268	0	0	0	0	1	−0.001	−0.032	−0.038	−0.030	0.000	−0.028
	2	0.0044	0	0	1	0	0	−0.064	−0.090	0.000	−0.007	−0.023	0.004
	3	−0.0240	0	0	0	0	1	−0.071	−0.139	−0.077	−0.061	0.000	−0.024
	4	0.0347	1	0	0	0	0	0.000	−0.085	−0.056	−0.056	−0.050	0.026
	5	−0.0216	0	1	0	0	0	−0.056	0.000	−0.033	−0.053	−0.053	−0.022
	6	0.0308	1	0	0	0	0	0.000	−0.065	−0.076	−0.069	−0.047	0.029
	7	−0.0405	0	0	0	1	0	−0.020	−0.029	−0.016	0.000	−0.031	−0.043
	8	−0.0248	0	0	0	0	1	−0.105	−0.213	−0.022	−0.083	0.000	−0.026
	9	−0.0581	0	1	0	0	0	−0.168	0.000	−0.443	−0.294	−0.032	−0.099
	10	−0.0248	0	0	0	0	1	−0.139	−0.071	−0.049	−0.055	0.000	−0.026
	11	−0.0332	0	0	0	0	1	−0.059	−0.053	−0.070	−0.014	0.000	−0.034
	12	0.0157	0	0	0	0	1	−0.202	−0.154	−0.143	−0.031	0.000	0.015
	13	−0.0051	0	0	0	0	1	−0.077	−0.064	−0.028	−0.088	0.000	−0.005

Table B.16: Optimisation: Standard deviation as risk and bull and bear markets (Definition 14) as indicator

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Bull	1	0.1490	0	1	0	0	0	−0.0602	0.0000	−0.0711	−0.0801	−0.1051	0.0856
	2	0.1375	0	1	0	0	0	−0.0526	0.0000	−0.0405	−0.0141	−0.0263	0.0399
	3	0.1373	0	0	1	0	0	−0.0091	−0.0889	0.0000	−0.0142	−0.0707	0.0891
	4	0.1613	0	1	0	0	0	−0.0734	0.0000	−0.0783	−0.0764	−0.1307	0.1342
	5	0.0728	0	0	0.7295	0	0.2705	−0.0887	−0.0415	0.0000	−0.0297	0.0000	0.0033
	6	0.1252	0.1217	0.3934	0.4849	0	0	0.0000	0.0000	0.0000	−0.0025	−0.0445	0.0400
	7	0.1356	0	0.2570	0.0391	0.7039	0	−0.1168	0.0000	0.0000	0.0000	−0.0112	0.0413
	8	0.0691	0	0	0	1	0	−0.0234	−0.0402	−0.0054	0.0000	−0.0130	0.0057
	9	0.0935	0.5351	0.0871	0	0.3778	0	0.0000	0.0000	−0.0202	0.0000	−0.0191	0.0164
	10	0.1530	1	0	0	0	0	0.0000	−0.0457	−0.0605	−0.0144	−0.1148	0.1063
Bear	1	−0.0245	0	0	0	0	1	−0.0759	−0.1465	−0.0816	−0.0647	0.0000	−0.0254
	2	−0.0356	0	0	0.4354	0	0.5646	−0.1068	−0.2437	0.0000	−0.0861	0.0000	−0.0428
	3	−0.1164	0	0	0	0	1	−0.3475	−0.1555	−0.0257	−0.1008	0.0000	−0.1435
	4	−0.2142	0	0.5922	0	0	0.4078	−0.2451	0.0000	−0.6418	−0.4112	0.0000	−0.2852
	5	−0.0633	0	1	0	0	0	−0.2227	0.0000	−0.2068	−0.1001	−0.0166	−0.0826
	6	0.1774	0	1	0	0	0	−0.0422	0.0000	−0.0897	−0.0739	−0.0530	0.0652
	7	0.0148	0	0	0	0	1	−0.0892	−0.1466	−0.0450	−0.1728	0.0000	0.0122
Moderate Investment													
Bull	1	0.1668	0	1	0	0	0	−0.0998	0.0000	−0.0978	−0.1080	−0.1465	0.1211
	2	0.1648	0	1	0	0	0	−0.0700	0.0000	−0.0615	−0.0286	−0.0451	0.0945
	3	0.1507	0	0	1	0	0	−0.0251	−0.1043	0.0000	−0.0129	−0.0884	0.1161
	4	0.1689	0	1	0	0	0	−0.0824	0.0000	−0.0855	−0.0830	−0.1442	0.1494
	5	0.0981	0	0	1	0	0	−0.0972	−0.0460	0.0000	−0.0352	−0.0136	0.0255
	6	0.1562	0.0339	0.8611	0.1050	0	0	0.0000	0.0000	0.0000	−0.0051	−0.0541	0.0508
	7	0.1717	0	0.0637	0.2438	0.6925	0	−0.1204	0.0000	0.0000	0.0000	−0.0120	0.0441
	8	0.0906	0	0	0.4778	0.5222	0	−0.0310	−0.0467	0.0000	0.0000	−0.0166	0.0156
	9	0.1198	1	0	0	0	0	0.0000	−0.0083	−0.0326	−0.0014	−0.0262	0.0400
	10	0.1660	1	0	0	0	0	0.0000	−0.0453	−0.0510	−0.0119	−0.1429	0.1324
Bear	1	−0.0242	0	0	0	0	1	−0.0731	−0.1427	−0.0791	−0.0625	0.0000	−0.0249
	2	−0.0331	0	0	0.6283	0	0.3717	−0.1033	−0.2355	0.0000	−0.0830	0.0000	−0.0413
	3	−0.1088	0	0	0	0	1	−0.3062	−0.1379	−0.0198	−0.0879	0.0000	−0.1284
	4	−0.1939	0	0.6918	0	0	0.3082	−0.2055	0.0000	−0.5278	−0.3448	0.0000	−0.2479
	5	−0.0579	0	1	0	0	0	−0.2037	0.0000	−0.1942	−0.0949	−0.0204	−0.0718
	6	0.2089	0.04024	0.9598	0	0	0	0.0000	0.0000	−0.1276	−0.1290	−0.1005	0.1233
	7	0.0156	0	0	0	0	1	−0.0880	−0.1432	−0.0449	−0.1702	0.0000	0.0137
Aggressive Investment													
Bull	1	0.1820	0	1	0	0	0	−0.1339	0.0000	−0.1207	−0.1319	−0.1819	0.1516
	2	0.1882	0	1	0	0	0	−0.0849	0.0000	−0.0795	−0.0410	−0.0612	0.1414
	3	0.1623	0	0	1	0	0	−0.0388	−0.1175	0.0000	−0.0117	−0.1035	0.1392
	4	0.1754	0	1	0	0	0	−0.0901	0.0000	−0.0916	−0.0886	−0.1557	0.1624
	5	0.1223	0	0	1	0	0	−0.1183	−0.0597	0.0000	−0.0487	−0.0440	0.0739
	6	0.1958	0.0904	0.9096	0	0	0	0.0000	0.0000	−0.0377	−0.0273	−0.1212	0.1151
	7	0.2331	0	0	0.9831	0.0169	0	−0.1293	−0.0120	0.0000	0.0000	−0.0145	0.0556
	8	0.1246	0	0	1	0	0	−0.0586	−0.0731	0.0000	−0.0118	−0.0344	0.0476
	9	0.1464	1	0	0	0	0	0.0000	−0.0370	−0.0613	−0.0192	−0.0576	0.0932
	10	0.1772	1	0	0	0	0	0.0000	−0.0450	−0.0428	−0.0097	−0.1670	0.1548
Bear	1	−0.0240	0	0	0	0	1	−0.0708	−0.1393	−0.0769	−0.0607	0.0000	−0.0244
	2	−0.0294	0	0	0.9728	0	0.0272	−0.1003	−0.2285	0.0000	−0.0804	0.0000	−0.0399
	3	−0.1023	0	0	0	0	1	−0.2709	−0.1229	−0.0148	−0.0768	0.0000	−0.1153
	4	−0.1748	0	0.8696	0	0	0.1304	−0.1715	0.0000	−0.4301	−0.2878	0.0000	−0.2160
	5	−0.0532	0	1	0	0	0	−0.1874	0.0000	−0.1834	−0.0905	−0.0237	−0.0625
	6	0.2507	0.6911	0.3089	0	0	0	0.0000	0.0000	−0.1300	−0.1336	−0.1031	0.1274
	7	0.0162	0	0	0	0	1	−0.0869	−0.1403	−0.0448	−0.1679	0.0000	0.0149

Table B.17: Optimisation: Standard deviation as risk and bull and bear markets (Definition 15) as indicator

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
1	0.0313	0	0	1	0	0	−0.027	−0.009	0.000	−0.023	−0.029	0.028
2	0.0314	0	0	1	0	0	−0.004	−0.034	0.000	−0.040	−0.059	0.031
3	0.0607	0	1	0	0	0	−0.038	0.000	−0.026	−0.039	−0.025	0.024
4	0.0835	0	1	0	0	0	−0.087	0.000	−0.095	−0.106	−0.100	0.076
5	0.1439	0	1	0	0	0	−0.050	0.000	−0.060	−0.075	−0.099	0.080
6	0.0479	1	0	0	0	0	0.000	−0.054	−0.015	−0.018	−0.025	0.039
7	0.0406	0	0	0	0	1	−0.077	−0.015	−0.068	−0.068	0.000	0.038
8	0.0219	0	0	0	1	0	−0.030	−0.043	−0.009	0.000	−0.026	0.021
9	0.1280	0	1	0	0	0	−0.066	0.000	−0.139	−0.147	−0.216	0.111
10	−0.1591	0	0	0	0	1	−0.249	−0.134	−0.443	−0.272	0.000	−0.197
11	0.1678	0	1	0	0	0	−0.107	0.000	−0.158	−0.161	−0.152	0.152
12	0.0469	0	0.815	0.112	0.072	0	−0.073	0.000	0.000	0.000	−0.015	0.034
13	0.0085	0	0.655	0	0.256	0.090	−0.116	0.000	−0.011	0.000	0.000	0.007
14	0.0027	0	0	0	0	1	−0.083	−0.087	−0.110	−0.050	0.000	−0.001
15	0.1006	0	0	0	1	0	−0.082	−0.024	−0.001	0.000	−0.031	0.083
16	0.0613	0	1	0	0	0	−0.025	0.000	−0.027	−0.036	−0.011	0.038
17	0.1617	1	0	0	0	0	0.000	−0.071	−0.085	−0.066	−0.095	0.143
18	0.0730	0	0.465	0	0	0.535	−0.044	0.000	−0.060	−0.035	0.000	0.014
19	−0.0052	0	0	0	0	1	−0.091	−0.044	−0.038	−0.054	0.000	−0.008
20	0.1124	1	0	0	0	0	0.000	−0.052	−0.022	−0.031	−0.136	0.096
21	0.1514	1	0	0	0	0	0.000	−0.044	−0.092	−0.114	−0.115	0.119
Moderate Investment												
1	0.0323	0	0	1	0	0	−0.0290	−0.0091	0.0000	−0.0251	−0.0312	0.0296
2	0.0314	0	0	1	0	0	−0.0036	−0.0337	0.0000	−0.0403	−0.0589	0.0314
3	0.0709	0	1	0	0	0	−0.0644	0.0000	−0.0408	−0.0543	−0.0455	0.0448
4	0.0856	0	1	0	0	0	−0.0881	0.0000	−0.0998	−0.1125	−0.1062	0.0802
5	0.1619	0	1	0	0	0	−0.0886	0.0000	−0.0846	−0.1018	−0.1415	0.1157
6	0.0504	1	0	0	0	0	0.0000	−0.0594	−0.0153	−0.0200	−0.0278	0.0439
7	0.0414	0	0	0	0	1	−0.0805	−0.0176	−0.0710	−0.0708	0.0000	0.0394
8	0.0221	0	0	0	1	0	−0.0302	−0.0436	−0.0093	0.0000	−0.0263	0.0215
9	0.1327	0	1	0	0	0	−0.0660	0.0000	−0.1479	−0.1520	−0.2175	0.1207
10	−0.1485	0	0	0	0	1	−0.2138	−0.1198	−0.3863	−0.2388	0.0000	−0.1758
11	0.1723	0	1	0	0	0	−0.1131	0.0000	−0.1702	−0.1747	−0.1580	0.1608
12	0.0517	0	0.7763	0.2237	0	0	−0.0724	0.0000	0.0000	−0.0012	−0.0172	0.0351
13	0.0089	0	0.5563	0	0.3144	0.1293	−0.1152	0.0000	−0.0110	0.0000	0.0000	0.0075
14	0.0037	0	0	0	0	1	−0.0848	−0.0900	−0.1081	−0.0501	0.0000	0.0011
15	0.1057	0	0	0.0714	0.9286	0	−0.0906	−0.0270	0.0000	0.0000	−0.0414	0.0923
16	0.0677	0	1	0	0	0	−0.0274	0.0000	−0.0464	−0.0472	−0.0237	0.0512
17	0.1670	1	0	0	0	0	0.0000	−0.0773	−0.0927	−0.0680	−0.0981	0.1533
18	0.0960	0	0.7105	0	0	0.2895	−0.0449	0.0000	−0.0603	−0.0347	0.0000	0.0136
19	−0.0044	0	0	0	0	1	−0.0857	−0.0425	−0.0350	−0.0529	0.0000	−0.0063
20	0.1170	1	0	0	0	0	0.0000	−0.0609	−0.0247	−0.0363	−0.1473	0.1052
21	0.1606	1	0	0	0	0	0.0000	−0.0555	−0.1206	−0.1402	−0.1248	0.1370
Aggressive Investment												
1	0.0332	0	0	1	0	0	−0.0307	−0.0097	0.0000	−0.0266	−0.0331	0.0314
2	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0589	0.0314
3	0.0796	0	1	0	0	0	−0.0870	0.0000	−0.0537	−0.0675	−0.0633	0.0622
4	0.0874	0	1	0	0	0	−0.0893	0.0000	−0.1037	−0.1177	−0.1114	0.0838
5	0.1773	0	1	0	0	0	−0.1219	0.0000	−0.1062	−0.1249	−0.1777	0.1465
6	0.0526	1	0	0	0	0	0.0000	−0.0640	−0.0157	−0.0218	−0.0302	0.0483
7	0.0421	0	0	0	0	1	−0.0835	−0.0203	−0.0738	−0.0732	0.0000	0.0407
8	0.0223	0	0	0	1	0	−0.0303	−0.0441	−0.0094	0.0000	−0.0270	0.0219
9	0.1367	0	1	0	0	0	−0.0658	0.0000	−0.1551	−0.1563	−0.2188	0.1287
10	−0.1394	0	0	0	0	1	−0.1832	−0.1080	−0.3377	−0.2104	0.0000	−0.1576
11	0.1762	0	1	0	0	0	−0.1187	0.0000	−0.1804	−0.1869	−0.1630	0.1685
12	0.0597	0	0.6444	0.3556	0	0	−0.0719	0.0000	0.0000	−0.0031	−0.0195	0.0364
13	0.0096	0	0.3807	0	0.4194	0.1999	−0.1145	0.0000	−0.0113	0.0000	0.0000	0.0075
14	0.0045	0	0	0	0	1	−0.0865	−0.0924	−0.1069	−0.0504	0.0000	0.0028
15	0.1104	0	0	0.2370	0.7630	0	−0.0980	−0.0286	0.0000	0.0000	−0.0503	0.1001
16	0.0732	0	1	0	0	0	−0.0294	0.0000	−0.0633	−0.0571	−0.0344	0.0622
17	0.1716	1	0	0	0	0	0.0000	−0.0824	−0.0990	−0.0697	−0.1008	0.1624
18	0.1357	0	1	0	0	0	−0.0329	0.0000	−0.0819	−0.0606	−0.0212	0.0384
19	−0.0038	0	0	0	0	1	−0.0814	−0.0415	−0.0325	−0.0515	0.0000	−0.0051
20	0.1209	1	0	0	0	0	0.0000	−0.0688	−0.0274	−0.0410	−0.1568	0.1130
21	0.1685	1	0	0	0	0	0.0000	−0.0656	−0.1452	−0.1626	−0.1336	0.1528

Table B.18: Optimisation: Standard deviation as risk and SA business confidence as indicator

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Upward	1	0.0406	0	0	0	0	1	−0.0770	−0.0145	−0.0678	−0.0680	0.0000	0.0378
	2	0.0749	0	1	0	0	0	−0.0613	0.0000	−0.0448	−0.0823	−0.0241	0.0582
	3	0.0506	1	0	0	0	0	0.0000	−0.0864	−0.0116	−0.0164	−0.0427	0.0444
	4	0.0377	0	0	0	0	1	−0.0569	−0.0241	−0.0164	−0.0142	0.0000	0.0331
	5	−0.0122	0	0	0	0	1	−0.0304	−0.0149	−0.0101	−0.0219	0.0000	−0.0130
	6	−0.0226	0	1	0	0	0	−0.0571	0.0000	−0.0317	−0.0546	−0.0555	−0.0244
	7	0.0683	0	0.1897	0.4129	0	0.3974	−0.0947	0.0000	0.0000	−0.0224	0.0000	0.0508
	8	0.0412	0	0	1	0	0	−0.0548	−0.0503	0.0000	−0.0147	−0.0286	0.0357
	9	0.1292	0	0.3150	0.6850	0	0	−0.0173	0.0000	0.0000	−0.0309	−0.0799	0.1206
	10	0.1248	0	0	1	0	0	−0.1639	−0.1032	0.0000	−0.0162	−0.0567	0.0949
	11	0.1124	1	0	0	0	0	0.0000	−0.0517	−0.0216	−0.0307	−0.1363	0.0960
	12	0.1514	1	0	0	0	0	0.0000	−0.0437	−0.0919	−0.1141	−0.1146	0.1186
	13	0.0259	0	0.3591	0	0	0.6409	−0.0705	0.0000	−0.0720	−0.0077	0.0000	0.0109
	14	−0.0058	0.4713	0.5287	0	0	0	0.0000	0.0000	−0.0926	−0.0784	−0.0005	−0.0063
	15	0.0913	0	0	0	0.7656	0.2344	−0.1010	−0.0238	−0.0226	0.0000	0.0000	0.0764
	16	0.0124	0.6195	0	0	0.2507	0.1298	0.0000	−0.0443	−0.0088	0.0000	0.0000	0.0017
	17	0.0718	0	0	0	1	0	−0.0806	−0.0507	−0.0295	0.0000	−0.0134	0.0702
Downward	1	−0.0068	0	0	0	0	1	−0.0770	−0.0145	−0.0678	−0.0680	0.0000	0.0378
	2	−0.0067	0	0	1	0	0	−0.0613	0.0000	−0.0448	−0.0823	−0.0241	0.0582
	3	−0.1011	0	0	1	0	0	0.0000	−0.0864	−0.0116	−0.0164	−0.0427	0.0444
	4	−0.1591	0	0	0	0	1	−0.0569	−0.0241	−0.0164	−0.0142	0.0000	0.0331
	5	0.0084	0	0	0	0	1	−0.0304	−0.0149	−0.0101	−0.0219	0.0000	−0.0130
	6	0.0930	1	0	0	0	0	−0.0571	0.0000	−0.0317	−0.0546	−0.0555	−0.0244
	7	0.0646	0	1	0	0	0	−0.0947	0.0000	0.0000	−0.0224	0.0000	0.0508
Moderate Investment													
Upward	1	0.0414	0	0	0	0	1	−0.0770	−0.0145	−0.0678	−0.0680	0.0000	0.0378
	2	0.0795	0	1	0	0	0	−0.0613	0.0000	−0.0448	−0.0823	−0.0241	0.0582
	3	0.0523	1	0	0	0	0	0.0000	−0.0864	−0.0116	−0.0164	−0.0427	0.0444
	4	0.0389	0	0	0	0	1	−0.0569	−0.0241	−0.0164	−0.0142	0.0000	0.0331
	5	−0.0120	0	0	0	0	1	−0.0304	−0.0149	−0.0101	−0.0219	0.0000	−0.0130
	6	−0.0220	0	1	0	0	0	−0.0571	0.0000	−0.0317	−0.0546	−0.0555	−0.0244
	7	0.0744	0	0.1999	0.8001	0	0	−0.0947	0.0000	0.0000	−0.0224	0.0000	0.0508
	8	0.0428	0	0	1	0	0	−0.0548	−0.0503	0.0000	−0.0147	−0.0286	0.0357
	9	0.1324	0	0.1612	0.8388	0	0	−0.0173	0.0000	0.0000	−0.0309	−0.0799	0.1206
	10	0.1331	0	0	1	0	0	−0.1639	−0.1032	0.0000	−0.0162	−0.0567	0.0949
	11	0.1170	1	0	0	0	0	0.0000	−0.0517	−0.0216	−0.0307	−0.1363	0.0960
	12	0.1606	1	0	0	0	0	0.0000	−0.0437	−0.0919	−0.1141	−0.1146	0.1186
	13	0.0306	0	0.5080	0	0	0.4920	−0.0705	0.0000	−0.0720	−0.0077	0.0000	0.0109
	14	−0.0057	0.4513	0.5487	0	0	0	0.0000	0.0000	−0.0926	−0.0784	−0.0005	−0.0063
	15	0.0955	0	0	0	0.8473	0.1527	−0.1010	−0.0238	−0.0226	0.0000	0.0000	0.0764
	16	0.0165	0.6434	0	0	0.3566	0	0.0000	−0.0443	−0.0088	0.0000	0.0000	0.0017
	17	0.0722	0	0	0	1	0	−0.0806	−0.0507	−0.0295	0.0000	−0.0134	0.0702
Downward	1	−0.0068	0	0	0	0	1	−0.0770	−0.0145	−0.0678	−0.0680	0.0000	0.0378
	2	−0.0049	0	0	1	0	0	−0.0613	0.0000	−0.0448	−0.0823	−0.0241	0.0582
	3	−0.0924	0	0	1	0	0	0.0000	−0.0864	−0.0116	−0.0164	−0.0427	0.0444
	4	−0.1485	0	0	0	0	1	−0.0569	−0.0241	−0.0164	−0.0142	0.0000	0.0331
	5	0.0092	0	0	0	0	1	−0.0304	−0.0149	−0.0101	−0.0219	0.0000	−0.0130
	6	0.0982	1	0	0	0	0	−0.0571	0.0000	−0.0317	−0.0546	−0.0555	−0.0244
	7	0.0652	0	1	0	0	0	−0.0947	0.0000	0.0000	−0.0224	0.0000	0.0508
Aggressive Investment													
Upward	1	0.0421	0	0	0	0	1	−0.0770	−0.0145	−0.0678	−0.0680	0.0000	0.0378
	2	0.0835	0	1	0	0	0	−0.0613	0.0000	−0.0448	−0.0823	−0.0241	0.0582
	3	0.0538	1	0	0	0	0	0.0000	−0.0864	−0.0116	−0.0164	−0.0427	0.0444
	4	0.0401	0	0	0	0	1	−0.0569	−0.0241	−0.0164	−0.0142	0.0000	0.0331
	5	−0.0118	0	0	0	0	1	−0.0304	−0.0149	−0.0101	−0.0219	0.0000	−0.0130
	6	−0.0216	0	1	0	0	0	−0.0571	0.0000	−0.0317	−0.0546	−0.0555	−0.0244
	7	0.0800	0	0.1510	0.8490	0	0	−0.0947	0.0000	0.0000	−0.0224	0.0000	0.0508
	8	0.0441	0	0	1	0	0	−0.0548	−0.0503	0.0000	−0.0147	−0.0286	0.0357
	9	0.1377	0	0	1	0	0	−0.0173	0.0000	0.0000	−0.0309	−0.0799	0.1206
	10	0.1403	0	0	1	0	0	−0.1639	−0.1032	0.0000	−0.0162	−0.0567	0.0949
	11	0.1209	1	0	0	0	0	0.0000	−0.0517	−0.0216	−0.0307	−0.1363	0.0960
	12	0.1685	1	0	0	0	0	0.0000	−0.0437	−0.0919	−0.1141	−0.1146	0.1186
	13	0.0363	0	0.7741	0	0	0.2259	−0.0705	0.0000	−0.0720	−0.0077	0.0000	0.0109
	14	−0.0056	0.4156	0.5844	0	0	0	0.0000	0.0000	−0.0926	−0.0784	−0.0005	−0.0063
	15	0.0993	0	0	0	0.9932	0.0068	−0.1010	−0.0238	−0.0226	0.0000	0.0000	0.0764
	16	0.0210	0.5563	0	0	0.4437	0	0.0000	−0.0443	−0.0088	0.0000	0.0000	0.0017
	17	0.0726	0	0	0	1	0	−0.0806	−0.0507	−0.0295	0.0000	−0.0134	0.0702
Downward	1	−0.0067	0	0	0	0	1	−0.0770	−0.0145	−0.0678	−0.0680	0.0000	0.0378
	2	−0.0033	0	0	1	0	0	−0.0613	0.0000	−0.0448	−0.0823	−0.0241	0.0582
	3	−0.0849	0	0	1	0	0	0.0000	−0.0864	−0.0116	−0.0164	−0.0427	0.0444
	4	−0.1394	0	0	0	0	1	−0.0569	−0.0241	−0.0164	−0.0142	0.0000	0.0331
	5	0.0099	0	0	0	0	1	−0.0304	−0.0149	−0.0101	−0.0219	0.0000	−0.0130
	6	0.1027	1	0	0	0	0	−0.0571	0.0000	−0.0317	−0.0546	−0.0555	−0.0244
	7	0.0656	0	1	0	0	0	−0.0947	0.0000	0.0000	−0.0224	0.0000	0.0508

Table B.19: Optimisation: Standard deviation as risk and rate of change in SA business confidence as indicator

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Upward	1	0.0423	0	0	1	0	0	−0.0490	−0.0505	0.0000	−0.0495	−0.0761	0.0398
	2	0.0738	0	0	1	0	0	−0.0296	−0.0497	0.0000	−0.0215	−0.0677	0.0469
	3	0.0337	0	0	0	1	0	−0.0207	−0.0857	−0.0051	0.0000	−0.0094	0.0295
	4	0.0320	1	0	0	0	0	0.0000	−0.0475	−0.0613	−0.0369	−0.0213	0.0255
	5	−0.0110	1	0	0	0	0	0.0000	−0.0230	−0.0080	−0.0333	−0.0123	−0.0201
	6	0.0412	0	0	1	0	0	−0.0548	−0.0503	0.0000	−0.0147	−0.0286	0.0357
	7	−0.0401	0	0	0.7541	0	0.2459	−0.1125	−0.1241	0.0000	−0.0659	0.0000	−0.0436
	8	−0.0113	0	0	0	0	1	−0.0373	−0.0032	−0.0128	−0.0451	0.0000	−0.0116
	9	0.0575	0	0	1	0	0	−0.1741	−0.1164	0.0000	−0.0465	−0.0038	0.0429
	10	0.1292	0	0.3150	0.6850	0	0	−0.0173	0.0000	0.0000	−0.0309	−0.0799	0.1206
	11	0.1248	0	0	1	0	0	−0.1639	−0.1032	0.0000	−0.0162	−0.0567	0.0949
	12	0.1506	0	1	0	0	0	−0.0450	0.0000	−0.0869	−0.0283	−0.1046	0.1044
	13	−0.0562	0	0	0.8220	0.1780	0	−0.0997	−0.0876	0.0000	0.0000	−0.0183	−0.0650
	14	−0.1011	0	0	1	0	0	−0.1193	−0.0494	0.0000	−0.0377	−0.0045	−0.1323
	15	0.1280	0	1	0	0	0	−0.0662	0.0000	−0.1394	−0.1470	−0.2160	0.1114
	16	−0.1591	0	0	0	0	1	−0.2494	−0.1335	−0.4430	−0.2719	0.0000	−0.1970
	17	0.1678	0	1	0	0	0	−0.1065	0.0000	−0.1583	−0.1606	−0.1521	0.1518
	18	0.0085	0	0.6547	0	0.2555	0.0898	−0.1161	0.0000	−0.0106	0.0000	0.0000	0.0074
	19	0.0027	0	0	0	0	1	−0.0829	−0.0872	−0.1096	−0.0497	0.0000	−0.0008
	20	0.1006	0	0	0	1	0	−0.0819	−0.0243	−0.0009	0.0000	−0.0310	0.0830
	21	0.0613	0	1	0	0	0	−0.0249	0.0000	−0.0266	−0.0356	−0.0112	0.0383
	22	0.1617	1	0	0	0	0	0.0000	−0.0715	−0.0854	−0.0660	−0.0950	0.1426
	23	0.0730	0	0.4651	0	0	0.5348	−0.0440	0.0000	−0.0600	−0.0345	0.0000	0.0136
	24	−0.0052	0	0	0	0	1	−0.0906	−0.0437	−0.0378	−0.0544	0.0000	−0.0078
	25	0.1124	1	0	0	0	0	0.0000	−0.0517	−0.0216	−0.0307	−0.1363	0.0960
	26	0.1514	1	0	0	0	0	0.0000	−0.0437	−0.0919	−0.1141	−0.1146	0.1186
	27	0.0259	0	0.3591	0	0	0.6409	−0.0705	0.0000	−0.0720	−0.0077	0.0000	0.0109
	28	−0.0058	0.4713	0.5287	0	0	0	0.0000	0.0000	−0.0926	−0.0784	−0.0005	−0.0063
	29	0.0913	0	0	0	0.7656	0.2344	−0.1010	−0.0238	−0.0226	0.0000	0.0000	0.0764
	30	0.0818	0	0	0	1	0	−0.0376	−0.0482	−0.0330	0.0000	−0.1036	0.0568
	31	0.0953	0	0	0.3633	0.6367	0	−0.0092	−0.0211	0.0000	0.0000	−0.0601	0.0590
	32	0.0124	0.6195	0	0	0.2507	0.1298	0.0000	−0.0443	−0.0088	0.0000	0.0000	0.0017
	33	0.0156	0	0	0	1	0	−0.2151	−0.1215	−0.0711	0.0000	−0.0491	0.0151
	34	0.0177	0.3194	0	0	0	0.6806	0.0000	−0.0259	−0.0115	−0.0081	0.0000	0.0117
	35	0.0084	0	0	0	0	1	−0.1120	−0.1034	−0.0790	−0.1406	0.0000	0.0053
	36	0.0930	1	0	0	0	0	0.0000	−0.0018	−0.1407	−0.1382	−0.0422	0.0742
	37	0.0718	0	0	0	1	0	−0.0806	−0.0507	−0.0295	0.0000	−0.0134	0.0702
	38	0.0347	0	0.6631	0	0	0.3369	−0.0184	0.0000	−0.0136	−0.0214	0.0000	0.0265
	39	0.0891	0.0471	0.9529	0	0	0	0.0000	0.0000	−0.0102	−0.0260	−0.0218	0.0421
	40	0.0057	0.0656	0	0	0	0.9344	0.0000	−0.0578	−0.0280	−0.0045	0.0000	0.0011
	41	0.0646	0	1	0	0	0	−0.0915	0.0000	−0.1356	−0.1261	−0.0559	0.0628
	42	0.0561	0	1	0	0	0	−0.1203	0.0000	−0.0069	−0.1223	−0.0585	0.0545
	43	0.1745	0	0	1	0	0	−0.1045	−0.1460	0.0000	−0.1077	−0.1151	0.1233
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0336	0.0000	−0.0403	−0.0590	0.0313
	2	0.0607	0	1	0	0	0	−0.0380	0.0000	−0.0257	−0.0388	−0.0247	0.0245
	3	0.0835	0	1	0	0	0	−0.0867	0.0000	−0.0952	−0.1063	−0.1001	0.0760
	4	0.1439	0	1	0	0	0	−0.0498	0.0000	−0.0595	−0.0750	−0.0993	0.0797
	5	0.0479	1	0	0	0	0	0.0000	−0.0541	−0.0149	−0.0179	−0.0251	0.0389
	6	0.0406	0	0	0	0	1	−0.0770	−0.0145	−0.0678	−0.0680	0.0000	0.0378
	7	0.0219	0	0	0	1	0	−0.0299	−0.0431	−0.0091	0.0000	−0.0255	0.0210
	8	−0.0246	0	0	0	0	1	−0.0719	−0.0659	−0.0424	−0.0331	0.0000	−0.0272
	9	0.0565	0	0	1	0	0	−0.0303	−0.0006	0.0000	−0.0139	−0.0666	0.0525
	10	0.0650	0	0.6741	0	0.3259	0	−0.0062	0.0000	−0.0230	0.0000	−0.0604	0.0594

Table B.20: Optimisation: Standard deviation as risk, US consumer sentiment as indicator and a conservative investment

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
						Moderate Investment							
Upward	1	0.0429	0	0	1	0	0	−0.0527	−0.0521	0.0000	−0.0517	−0.0783	0.0412
	2	0.0813	0	0	1	0	0	−0.0381	−0.0646	0.0000	−0.0271	−0.0840	0.0620
	3	0.0349	0	0	0	1	0	−0.0219	−0.0907	−0.0049	0.0000	−0.0099	0.0319
	4	0.0339	1	0	0	0	0	0.0000	−0.0506	−0.0659	−0.0413	−0.0258	0.0292
	5	−0.0085	1	0	0	0	0	0.0000	−0.0239	−0.0103	−0.0344	−0.0158	−0.0150
	6	0.0428	0	0	1	0	0	−0.0600	−0.0590	0.0000	−0.0173	−0.0328	0.0388
	7	−0.0388	0	0	0.8875	0	0.1125	−0.1120	−0.1247	0.0000	−0.0662	0.0000	−0.0433
	8	−0.0113	0	0	0	0	1	−0.0377	−0.0037	−0.0131	−0.0455	0.0000	−0.0115
	9	0.0616	0	0	1	0	0	−0.1901	−0.1257	0.0000	−0.0516	−0.0092	0.0511
	10	0.1324	0	0.1612	0.8388	0	0	−0.0174	0.0000	0.0000	−0.0311	−0.0804	0.1211
	11	0.1331	0	0	1	0	0	−0.1726	−0.1103	0.0000	−0.0242	−0.0720	0.1116
	12	0.1635	0	1	0	0	0	−0.0275	0.0000	−0.1114	−0.0414	−0.1369	0.1303
	13	−0.0538	0	0	0.7978	0.2022	0	−0.0949	−0.0841	0.0000	0.0000	−0.0196	−0.0601
	14	−0.0924	0	0	1	0	0	−0.1111	−0.0518	0.0000	−0.0361	−0.0102	−0.1149
	15	0.1327	0	1	0	0	0	−0.0660	0.0000	−0.1479	−0.1520	−0.2175	0.1207
	16	−0.1485	0	0	0	0	1	−0.2138	−0.1198	−0.3863	−0.2388	0.0000	−0.1758
	17	0.1723	0	1	0	0	0	−0.1131	0.0000	−0.1702	−0.1747	−0.1580	0.1608
	18	0.0089	0	0.5563	0	0.3144	0.1293	−0.1152	0.0000	−0.0110	0.0000	0.0000	0.0075
	19	0.0037	0	0	0	0	1	−0.0848	−0.0900	−0.1081	−0.0501	0.0000	0.0011
	20	0.1057	0	0	0.0714	0.9286	0	−0.0906	−0.0270	0.0000	0.0000	−0.0414	0.0923
	21	0.0677	0	1	0	0	0	−0.0274	0.0000	−0.0464	−0.0472	−0.0237	0.0512
	22	0.1670	1	0	0	0	0	0.0000	−0.0773	−0.0927	−0.0680	−0.0981	0.1533
	23	0.0960	0	0.7105	0	0	0.2895	−0.0449	0.0000	−0.0603	−0.0347	0.0000	0.0136
	24	−0.0044	0	0	0	0	1	−0.0857	−0.0425	−0.0350	−0.0529	0.0000	−0.0063
	25	0.1170	1	0	0	0	0	0.0000	−0.0609	−0.0247	−0.0363	−0.1473	0.1052
	26	0.1606	1	0	0	0	0	0.0000	−0.0555	−0.1206	−0.1402	−0.1248	0.1370
	27	0.0306	0	0.5080	0	0	0.4920	−0.0677	0.0000	−0.0791	−0.0156	0.0000	0.0168
	28	−0.0057	0.4513	0.5487	0	0	0	0.0000	0.0000	−0.0915	−0.0773	−0.0010	−0.0061
	29	0.0955	0	0	0	0.8473	0.1527	−0.1091	−0.0298	−0.0235	0.0000	0.0000	0.0844
	30	0.0888	0	0	0	1	0	−0.0428	−0.0535	−0.0429	0.0000	−0.1207	0.0708
	31	0.1071	0	0	0.8757	0.1243	0	−0.0151	−0.0322	0.0000	0.0000	−0.0699	0.0703
	32	0.0165	0.6434	0	0	0.3566	0	0.0000	−0.0451	−0.0084	0.0000	−0.0028	0.0034
	33	0.0158	0	0	0	1	0	−0.2107	−0.1193	−0.0691	0.0000	−0.0490	0.0154
	34	0.0197	0.4242	0	0	0	0.5758	0.0000	−0.0286	−0.0098	−0.0097	0.0000	0.0139
	35	0.0092	0	0	0	0	1	−0.1114	−0.1007	−0.0786	−0.1381	0.0000	0.0070
	36	0.0982	1	0	0	0	0	0.0000	−0.0143	−0.1565	−0.1492	−0.0529	0.0847
	37	0.0722	0	0	0	1	0	−0.0806	−0.0504	−0.0302	0.0000	−0.0120	0.0711
	38	0.0378	0	0.8431	0	0	0.1569	−0.0194	0.0000	−0.0138	−0.0217	0.0000	0.0269
	39	0.1027	0	1	0	0	0	−0.0083	0.0000	−0.0261	−0.0463	−0.0489	0.0677
	40	0.0072	0	0	0	0	1	−0.0013	−0.0604	−0.0294	−0.0063	0.0000	0.0030
	41	0.0652	0	1	0	0	0	−0.0922	0.0000	−0.1375	−0.1281	−0.0562	0.0639
	42	0.0565	0	1	0	0	0	−0.1194	0.0000	−0.0069	−0.1200	−0.0591	0.0554
	43	0.1889	0	0	1	0	0	−0.1410	−0.1619	0.0000	−0.1317	−0.1374	0.1520
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0337	0.0000	−0.0403	−0.0589	0.0314
	2	0.0709	0	1	0	0	0	−0.0644	0.0000	−0.0408	−0.0543	−0.0455	0.0448
	3	0.0856	0	1	0	0	0	−0.0881	0.0000	−0.0998	−0.1125	−0.1062	0.0802
	4	0.1619	0	1	0	0	0	−0.0886	0.0000	−0.0846	−0.1018	−0.1415	0.1157
	5	0.0504	1	0	0	0	0	0.0000	−0.0594	−0.0153	−0.0200	−0.0278	0.0439
	6	0.0414	0	0	0	0	1	−0.0805	−0.0176	−0.0710	−0.0708	0.0000	0.0394
	7	0.0221	0	0	0	1	0	−0.0302	−0.0436	−0.0093	0.0000	−0.0263	0.0215
	8	−0.0239	0	0	0	0	1	−0.0694	−0.0650	−0.0410	−0.0321	0.0000	−0.0257
	9	0.0578	0	0.2510	0.7490	0	0	−0.0302	0.0000	0.0000	−0.0145	−0.0675	0.0532
	10	0.0670	0	0.5038	0	0.4962	0	−0.0060	0.0000	−0.0238	0.0000	−0.0611	0.0602

Table B.21: Optimisation: Standard deviation as risk, US consumer sentiment as indicator and a moderate investment

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
							Aggressive Investment						
Upward	1	0.0435	0	0	1	0	0	−0.0558	−0.0536	0.0000	−0.0537	−0.0802	0.0424
	2	0.0877	0	0	1	0	0	−0.0454	−0.0773	0.0000	−0.0319	−0.0981	0.0748
	3	0.0359	0	0	0	1	0	−0.0230	−0.0949	−0.0048	0.0000	−0.0104	0.0339
	4	0.0354	1	0	0	0	0	0.0000	−0.0533	−0.0698	−0.0452	−0.0297	0.0323
	5	−0.0063	1	0	0	0	0	0.0000	−0.0246	−0.0123	−0.0354	−0.0187	−0.0106
	6	0.0441	0	0	1	0	0	−0.0644	−0.0665	0.0000	−0.0195	−0.0364	0.0415
	7	−0.0368	0	0	1	0	0	−0.1168	−0.1308	0.0000	−0.0698	−0.0031	−0.0413
	8	−0.0112	0	0	0	0	1	−0.0381	−0.0041	−0.0133	−0.0458	0.0000	−0.0113
	9	0.0651	0	0	1	0	0	−0.2039	−0.1338	0.0000	−0.0560	−0.0138	0.0581
	10	0.1377	0	0	1	0	0	−0.0210	−0.0053	0.0000	−0.0335	−0.0837	0.1254
	11	0.1403	0	0	1	0	0	−0.1801	−0.1163	0.0000	−0.0310	−0.0852	0.1260
	12	0.1746	0	1	0	0	0	−0.0125	0.0000	−0.1324	−0.0526	−0.1646	0.1525
	13	−0.0516	0	0	0.7606	0.2394	0	−0.0908	−0.0811	0.0000	0.0000	−0.0207	−0.0559
	14	−0.0849	0	0	1	0	0	−0.1041	−0.0538	0.0000	−0.0347	−0.0150	−0.0999
	15	0.1367	0	1	0	0	0	−0.0658	0.0000	−0.1551	−0.1563	−0.2188	0.1287
	16	−0.1394	0	0	0	0	1	−0.1832	−0.1080	−0.3377	−0.2104	0.0000	−0.1576
	17	0.1762	0	1	0	0	0	−0.1187	0.0000	−0.1804	−0.1869	−0.1630	0.1685
	18	0.0096	0	0.3807	0	0.4194	0.1999	−0.1145	0.0000	−0.0113	0.0000	0.0000	0.0075
	19	0.0045	0	0	0	0	1	−0.0865	−0.0924	−0.1069	−0.0504	0.0000	0.0028
	20	0.1104	0	0	0.2370	0.7630	0	−0.0980	−0.0286	0.0000	0.0000	−0.0503	0.1001
	21	0.0732	0	1	0	0	0	−0.0294	0.0000	−0.0633	−0.0571	−0.0344	0.0622
	22	0.1716	1	0	0	0	0	0.0000	−0.0824	−0.0990	−0.0697	−0.1008	0.1624
	23	0.1357	0	1	0	0	0	−0.0329	0.0000	−0.0819	−0.0606	−0.0212	0.0384
	24	−0.0038	0	0	0	0	1	−0.0814	−0.0415	−0.0325	−0.0515	0.0000	−0.0051
	25	0.1209	1	0	0	0	0	0.0000	−0.0688	−0.0274	−0.0410	−0.1568	0.1130
	26	0.1685	1	0	0	0	0	0.0000	−0.0656	−0.1452	−0.1626	−0.1336	0.1528
	27	0.0363	0	0.7741	0	0	0.2259	−0.0653	0.0000	−0.0852	−0.0224	0.0000	0.0218
	28	−0.0056	0.4156	0.5844	0	0	0	0.0000	0.0000	−0.0906	−0.0764	−0.0014	−0.0059
	29	0.0993	0	0	0	0.9932	0.0068	−0.1160	−0.0349	−0.0242	0.0000	0.0000	0.0913
	30	0.0948	0	0	0	1	0	−0.0472	−0.0580	−0.0514	0.0000	−0.1354	0.0828
	31	0.1204	0	0	1	0	0	−0.0242	−0.0419	0.0000	−0.0070	−0.0931	0.0937
	32	0.0210	0.5563	0	0	0.4437	0	0.0000	−0.0529	−0.0061	0.0000	−0.0183	0.0120
	33	0.0159	0	0	0	1	0	−0.2069	−0.1174	−0.0674	0.0000	−0.0489	0.0157
	34	0.0220	0.6115	0	0	0	0.3885	0.0000	−0.0309	−0.0084	−0.0110	0.0000	0.0158
	35	0.0099	0	0	0	0	1	−0.1108	−0.0983	−0.0783	−0.1360	0.0000	0.0085
	36	0.1027	1	0	0	0	0	0.0000	−0.0251	−0.1701	−0.1585	−0.0621	0.0937
	37	0.0726	0	0	0	1	0	−0.0806	−0.0501	−0.0307	0.0000	−0.0107	0.0718
	38	0.0427	0	1	0	0	0	−0.0227	0.0000	−0.0177	−0.0248	−0.0055	0.0317
	39	0.1144	0	1	0	0	0	−0.0162	0.0000	−0.0407	−0.0648	−0.0734	0.0911
	40	0.0086	0	0	0	0	1	−0.0062	−0.0679	−0.0335	−0.0101	0.0000	0.0058
	41	0.0656	0	1	0	0	0	−0.0929	0.0000	−0.1391	−0.1298	−0.0565	0.0647
	42	0.0569	0	1	0	0	0	−0.1186	0.0000	−0.0069	−0.1181	−0.0597	0.0561
	43	0.2012	0	0	1	0	0	−0.1723	−0.1756	0.0000	−0.1522	−0.1565	0.1766
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0589	0.0314
	2	0.0796	0	1	0	0	0	−0.0870	0.0000	−0.0537	−0.0675	−0.0633	0.0622
	3	0.0874	0	1	0	0	0	−0.0893	0.0000	−0.1037	−0.1177	−0.1114	0.0838
	4	0.1773	0	1	0	0	0	−0.1219	0.0000	−0.1062	−0.1249	−0.1777	0.1465
	5	0.0526	1	0	0	0	0	0.0000	−0.0640	−0.0157	−0.0218	−0.0302	0.0483
	6	0.0421	0	0	0	0	1	−0.0835	−0.0203	−0.0738	−0.0732	0.0000	0.0407
	7	0.0223	0	0	0	1	0	−0.0303	−0.0441	−0.0094	0.0000	−0.0270	0.0219
	8	−0.0233	0	0	0	0	1	−0.0672	−0.0643	−0.0399	−0.0312	0.0000	−0.0245
	9	0.0601	0	0.8094	0.1906	0	0	−0.0301	0.0000	0.0000	−0.0147	−0.0677	0.0533
	10	0.0703	0	0.1996	0	0.8004	0	−0.0059	0.0000	−0.0245	0.0000	−0.0617	0.0609

Table B.22: Optimisation: Standard deviation as risk, US consumer sentiment as indicator and an aggressive investment

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Upward	1	0.0835	0	1	0	0	0	−0.0867	0.0000	−0.0952	−0.1063	−0.1001	0.0760
	2	−0.0246	0	0	0	0	1	−0.0719	−0.0659	−0.0424	−0.0331	0.0000	−0.0272
	3	0.0650	0	0.6741	0	0.3259	0	−0.0062	0.0000	−0.0280	0.0000	−0.0604	0.0594
	4	0.0854	0	0.6780	0	0.3220	0	−0.0283	0.0000	−0.0276	0.0000	−0.0149	0.0477
	5	0.1112	0	1	0	0	0	−0.0140	0.0000	−0.0791	−0.0679	−0.0893	0.1012
	6	0.0811	0	0	1	0	0	−0.0580	−0.0892	0.0000	−0.0201	−0.0923	0.0666
	7	0.0029	0	0	0	1	0	−0.0388	−0.0270	−0.0232	0.0000	−0.0178	0.0026
	8	−0.0122	0	0	0	0	1	−0.0304	−0.0149	−0.0101	−0.0219	0.0000	−0.0130
	9	−0.0068	0	0	0	0	1	−0.0414	−0.0211	−0.0314	−0.0348	0.0000	−0.0069
	10	0.0423	0	0	1	0	0	−0.0490	−0.0505	0.0000	−0.0495	−0.0761	0.0398
	11	0.1292	0	0.3150	0.6850	0	0	−0.0173	0.0000	0.0000	−0.0309	−0.0799	0.1206
	12	0.0085	0	0.6547	0	0.2555	0.0898	−0.1161	0.0000	−0.0106	0.0000	0.0000	0.0074
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0336	0.0000	−0.0403	−0.0590	0.0313
	2	0.0607	0	1	0	0	0	−0.0380	0.0000	−0.0257	−0.0388	−0.0247	0.0245
	3	0.1439	0	1	0	0	0	−0.0498	0.0000	−0.0595	−0.0750	−0.0993	0.0797
	4	0.0144	0	1	0	0	0	−0.0555	0.0000	−0.0079	−0.0264	−0.0311	0.0120
	5	0.1613	0	1	0	0	0	−0.0734	0.0000	−0.0783	−0.0764	−0.1307	0.1342
	6	0.0469	0	0.8155	0.1124	0.0721	0	−0.0730	0.0000	0.0000	0.0000	−0.0154	0.0342
Moderate Investment													
Upward	1	0.0856	0	1	0	0	0	−0.0881	0.0000	−0.0998	−0.1125	−0.1062	0.0802
	2	−0.0239	0	0	0	0	1	−0.0694	−0.0650	−0.0410	−0.0321	0.0000	−0.0257
	3	0.0670	0	0.5038	0	0.4962	0	−0.0060	0.0000	−0.0288	0.0000	−0.0611	0.0602
	4	0.0985	0	1	0	0	0	−0.0330	0.0000	−0.0332	−0.0044	−0.0205	0.0607
	5	0.1140	0	1	0	0	0	−0.0138	0.0000	−0.0841	−0.0717	−0.0932	0.1068
	6	0.0852	0	0	1	0	0	−0.0639	−0.1024	0.0000	−0.0231	−0.1032	0.0747
	7	0.0030	0	0	0	1	0	−0.0392	−0.0271	−0.0235	0.0000	−0.0182	0.0028
	8	−0.0120	0	0	0	0	1	−0.0305	−0.0136	−0.0104	−0.0220	0.0000	−0.0125
	9	−0.0068	0	0	0	0	1	−0.0412	−0.0211	−0.0313	−0.0346	0.0000	−0.0069
	10	0.0429	0	0	1	0	0	−0.0527	−0.0521	0.0000	−0.0517	−0.0783	0.0412
	11	0.1324	0	0.1612	0.8388	0	0	−0.0174	0.0000	0.0000	−0.0311	−0.0804	0.1211
	12	0.0089	0	0.5563	0	0.3144	0.1293	−0.1152	0.0000	−0.0110	0.0000	0.0000	0.0075
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0337	0.0000	−0.0403	−0.0589	0.0314
	2	0.0709	0	1	0	0	0	−0.0644	0.0000	−0.0408	−0.0543	−0.0455	0.0448
	3	0.1619	0	1	0	0	0	−0.0886	0.0000	−0.0846	−0.1018	−0.1415	0.1157
	4	0.0151	0	1	0	0	0	−0.0570	0.0000	−0.0090	−0.0277	−0.0326	0.0133
	5	0.1689	0	1	0	0	0	−0.0824	0.0000	−0.0855	−0.0830	−0.1442	0.1494
	6	0.0517	0	0.7763	0.2237	0	0	−0.0724	0.0000	0.0000	−0.0012	−0.0172	0.0351
Aggressive Investment													
Upward	1	0.0874	0	1	0	0	0	−0.0893	0.0000	−0.1037	−0.1177	−0.1114	0.0838
	2	−0.0233	0	0	0	0	1	−0.0672	−0.0643	−0.0399	−0.0312	0.0000	−0.0245
	3	0.0703	0	0.1996	0	0.8004	0	−0.0059	0.0000	−0.0294	0.0000	−0.0617	0.0609
	4	0.1111	0	1	0	0	0	−0.0481	0.0000	−0.0473	−0.0166	−0.0335	0.0859
	5	0.1164	0	1	0	0	0	−0.0136	0.0000	−0.0884	−0.0751	−0.0966	0.1116
	6	0.0886	0	0	1	0	0	−0.0689	−0.1138	0.0000	−0.0256	−0.1126	0.0817
	7	0.0030	0	0	0	1	0	−0.0395	−0.0272	−0.0238	0.0000	−0.0185	0.0029
	8	−0.0118	0	0	0	0	1	−0.0306	−0.0126	−0.0106	−0.0222	0.0000	−0.0122
	9	−0.0067	0	0	0	0	1	−0.0410	−0.0210	−0.0312	−0.0344	0.0000	−0.0068
	10	0.0435	0	0	1	0	0	−0.0558	−0.0536	0.0000	−0.0537	−0.0802	0.0424
	11	0.1377	0	0	1	0	0	−0.0210	−0.0053	0.0000	−0.0335	−0.0837	0.1254
	12	0.0096	0	0.3807	0	0.4194	0.1999	−0.1145	0.0000	−0.0113	0.0000	0.0000	0.0075
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0589	0.0314
	2	0.0796	0	1	0	0	0	−0.0870	0.0000	−0.0537	−0.0675	−0.0633	0.0622
	3	0.1773	0	1	0	0	0	−0.1219	0.0000	−0.1062	−0.1249	−0.1777	0.1465
	4	0.0156	0	1	0	0	0	−0.0583	0.0000	−0.0099	−0.0287	−0.0339	0.0145
	5	0.1754	0	1	0	0	0	−0.0901	0.0000	−0.0916	−0.0886	−0.1557	0.1624
	6	0.0597	0	0.6444	0.3556	0	0	−0.0719	0.0000	0.0000	−0.0031	−0.0195	0.0364

Table B.23: Optimisation: Standard deviation as risk and rate of change of US consumer sentiment as indicator

		Solution					Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
1	0.0237	1	0	0	0	0	0.0000	−0.0040	−0.0276	−0.0304	−0.0317	0.0222
2	0.0310	0	1	0	0	0	−0.0170	0.0000	−0.0346	−0.0295	−0.0305	0.0288
3	−0.0021	0	0	1	0	0	−0.0276	−0.0118	0.0000	−0.0093	−0.0014	−0.0022
4	−0.0028	0	1	0	0	0	−0.0068	0.0000	−0.0196	−0.0120	−0.0035	−0.0047
5	0.0006	0	1	0	0	0	−0.0196	0.0000	−0.0168	−0.0052	−0.0078	0.0005
6	0.0149	0	0	1	0	0	−0.0223	−0.0019	0.0000	−0.0049	−0.0262	0.0143
7	0.0375	0	1	0	0	0	−0.0189	0.0000	−0.0245	−0.0026	−0.0606	0.0362
8	0.0040	0	0	1	0	0	−0.0104	−0.0308	0.0000	−0.0023	−0.0013	0.0040
9	0.0309	0	0	1	0	0	−0.0223	−0.0197	0.0000	−0.0061	−0.0389	0.0298
10	0.0244	0	0	1	0	0	−0.0322	−0.0219	0.0000	−0.0159	−0.0265	0.0235
11	0.0292	0	0	1	0	0	−0.0336	−0.0058	0.0000	−0.0065	−0.0332	0.0280
12	0.0001	0	0	1	0	0	−0.0039	−0.0079	0.0000	−0.0043	−0.0035	0.0001
13	0.0329	0.2093	0.7907	0	0	0	0.0000	0.0000	−0.0152	−0.0264	−0.0341	0.0322
14	0.0170	0	0	0	0	1	−0.0021	−0.0061	−0.0069	−0.0090	0.0000	0.0165
15	0.0100	0	1	0	0	0	−0.0403	0.0000	−0.0183	−0.0127	−0.0181	0.0097
16	−0.0022	0	0	0	0	1	−0.0173	−0.0299	−0.0108	−0.0079	0.0000	−0.0022
17	−0.0066	0	0	0	0	1	−0.0432	−0.0357	−0.0379	−0.0298	0.0000	−0.0066
18	−0.0067	0	0	0	0	1	−0.0601	−0.0224	−0.0275	−0.0276	0.0000	−0.0068
19	0.0385	0	1	0	0	0	−0.0043	0.0000	−0.0231	−0.0181	−0.0313	0.0328
20	−0.0028	0	0	1	0	0	−0.0236	−0.0310	0.0000	−0.0127	−0.0118	−0.0029
21	−0.0110	0	0	1	0	0	−0.0256	−0.0084	0.0000	−0.0026	−0.0021	−0.0111
22	0.0138	1	0	0	0	0	0.0000	−0.0166	−0.0212	−0.0072	−0.0223	0.0134
23	0.0273	0	1	0	0	0	−0.0397	0.0000	−0.0024	−0.0056	−0.0247	0.0221
24	0.0054	0	1	0	0	0	−0.0110	0.0000	−0.0065	−0.0051	−0.0038	0.0052
25	0.0000	0	0	0	0	1	−0.0258	−0.0161	−0.0051	−0.0035	0.0000	−0.0001
26	0.0184	1	0	0	0	0	0.0000	−0.0157	−0.0074	−0.0160	−0.0149	0.0179
27	0.0620	1	0	0	0	0	0.0000	−0.0749	−0.0322	−0.0490	−0.0659	0.0572
28	0.0025	0	0	1	0	0	−0.0380	−0.0004	0.0000	−0.0095	−0.0059	0.0024
29	0.0256	0	1	0	0	0	−0.0115	0.0000	−0.0189	−0.0215	−0.0308	0.0254
30	0.0327	0	1	0	0	0	−0.0332	0.0000	−0.0135	−0.0155	−0.0285	0.0319
31	0.0337	0	0	1	0	0	−0.0164	−0.0060	0.0000	−0.0106	−0.0281	0.0288
32	0.0084	0	0	1	0	0	−0.0283	−0.0064	0.0000	−0.0041	−0.0131	0.0083
33	−0.0091	0	0	0	0	1	−0.0058	−0.0185	−0.0120	−0.0004	0.0000	−0.0093
34	0.0057	1	0	0	0	0	0.0000	−0.0004	−0.0186	−0.0008	−0.0092	0.0056
35	0.0067	0	0	1	0	0	−0.0012	−0.0120	0.0000	−0.0041	−0.0149	0.0066
36	−0.0055	0	0	1	0	0	−0.0114	−0.0225	0.0000	−0.0095	−0.0001	−0.0059
37	0.0105	1	0	0	0	0	0.0000	−0.0302	−0.0180	−0.0182	−0.0153	0.0103
38	0.0166	1	0	0	0	0	0.0000	−0.0093	−0.0061	−0.0109	−0.0167	0.0164
39	−0.0055	0	0	0	0	1	−0.0178	−0.0069	−0.0029	−0.0010	0.0000	−0.0056
40	0.0072	0	0	0	1	0	−0.0028	−0.0159	−0.0047	0.0000	−0.0173	0.0071
41	0.0203	0	1	0	0	0	−0.0185	0.0000	−0.0255	−0.0202	−0.0251	0.0198
42	0.0048	0	0	0	0	1	−0.0065	−0.0196	−0.0082	−0.0037	0.0000	0.0048
43	−0.0327	0	0	0	0	1.000000278	−0.0309	−0.0301	−0.0309	−0.0087	0.0000	−0.0327
44	0.0095	1	0	0	0	0	0.0000	−0.0850	−0.0690	−0.0470	−0.0146	0.0032
45	0.0036	0	1	0	0	0	−0.1283	0.0000	−0.2577	−0.1830	−0.0378	0.0022
46	0.1103	1	0	0	0	0	0.0000	−0.0812	−0.1980	−0.1451	−0.1259	0.0969
47	0.0696	1	0	0	0	0	0.0000	−0.0514	−0.1390	−0.1291	−0.0516	0.0677
48	0.1026	0	0	1	0	0	−0.1151	−0.0609	0.0000	−0.0651	−0.0860	0.0881
49	0.0691	0	0	1	0	0	−0.0203	−0.0626	0.0000	−0.0151	−0.0323	0.0384
50	0.0376	1	0	0	0	0	0.0000	−0.1018	−0.1249	−0.0695	−0.0349	0.0351
51	0.0555	0	0	1	0	0	−0.0366	−0.0566	0.0000	−0.0508	−0.0557	0.0499
52	−0.0081	0	0	0	1	0	−0.0206	−0.0094	−0.0220	0.0000	−0.0091	−0.0081

Table B.24: Optimisation: Downside risk, upward shock in Bond yields as indicator and a conservative investment

		Solution					Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
						Conservative Investment						
1	−0.0076	0	0	0	0	1	−0.0326	−0.0071	−0.0145	−0.0061	0.0000	−0.0077
2	0.0117	1	0	0	0	0	0.0000	−0.0135	−0.0148	−0.0166	−0.0196	0.0098
3	−0.0083	0	0	0	0	1	−0.0194	−0.0539	−0.0220	−0.0201	0.0000	−0.0084
4	0.0232	0	0	1	0	0	−0.0326	−0.0247	0.0000	−0.0008	−0.0198	0.0225
5	0.0426	1	0	0	0	0	0.0000	−0.0298	−0.0187	−0.0356	−0.0461	0.0372
6	0.0354	0	1	0	0	0	−0.0149	0.0000	−0.0240	−0.0287	−0.0255	0.0330
7	−0.0007	0	0	0	1	0	−0.0096	−0.0175	−0.0014	0.0000	−0.0127	−0.0007
8	0.0223	1	0	0	0	0	0.0000	−0.0187	−0.0176	−0.0113	−0.0264	0.0220
9	0.0438	1	0	0	0	0	0.0000	−0.0377	−0.0313	−0.0319	−0.0368	0.0392
10	0.0190	0.0262	0.9738	0	0	0	0.0000	0.0000	−0.0230	−0.0012	−0.0186	0.0180
11	0.0616	0	1	0	0	0	−0.0554	0.0000	−0.0142	−0.0046	−0.0474	0.0549
12	−0.0056	0.3117	0	0	0.6883	0	0.0000	−0.0089	−0.0048	0.0000	−0.0060	−0.0058
13	0.0191	0	0	0	1	0	−0.0115	−0.0250	−0.0172	0.0000	−0.0050	0.0179
14	0.0663	1	0	0	0	0	0.0000	−0.0579	−0.0510	−0.0409	−0.0437	0.0559
15	0.0349	1	0	0	0	0	0.0000	−0.0171	−0.0097	−0.0170	−0.0058	0.0343
16	0.0436	0	1	0	0	0	−0.0305	0.0000	−0.0219	−0.0131	−0.0295	0.0417
17	0.0426	0	0	0	0	1	−0.0086	−0.0102	−0.0114	−0.0052	0.0000	0.0408
18	0.0437	1	0	0	0	0	0.0000	−0.0179	−0.0215	−0.0135	−0.0245	0.0364
19	0.0062	0	0	0	0	1	−0.0502	−0.0392	−0.0293	−0.0641	0.0000	0.0062
20	0.0209	0	1	0	0	0	−0.0610	0.0000	−0.0093	−0.0197	−0.0114	0.0199
21	0.0093	0	0	0	0	1	−0.0298	−0.0151	−0.0098	−0.0036	0.0000	0.0090
22	0.0659	0	1	0	0	0	−0.0014	0.0000	−0.0421	−0.0237	−0.0582	0.0597
23	−0.0019	0	1	0	0	0	−0.0655	0.0000	−0.0180	−0.0227	−0.0073	−0.0023
24	0.0218	1	0	0	0	0	0.0000	−0.0239	−0.0226	−0.0102	−0.0199	0.0160
25	0.0047	0	1	0	0	0	−0.0227	0.0000	−0.0034	−0.0132	−0.0036	0.0044
26	0.0608	1	0	0	0	0	0.0000	−0.0234	−0.0470	−0.0448	−0.0590	0.0597
27	0.0467	0	1	0	0	0	−0.0229	0.0000	−0.0190	−0.0121	−0.0323	0.0393
28	0.0592	0	1	0	0	0	−0.0440	0.0000	−0.0350	−0.0351	−0.0506	0.0557
29	0.0277	1	0	0	0	0	0.0000	−0.0719	−0.0614	−0.0564	−0.0231	0.0262
30	−0.0093	0	0	0	0	1	−0.0437	−0.0313	−0.0286	−0.0240	0.0000	−0.0094
31	0.0120	0	0	0	1	0	−0.0226	−0.0042	−0.0072	0.0000	−0.1188	0.0114
32	0.0271	0	1	0	0	0	−0.0134	0.0000	−0.0308	−0.0284	−0.0345	0.0260
33	0.0522	0	1	0	0	0	−0.0659	0.0000	−0.0090	−0.0181	−0.0390	0.0471
34	0.0625	0	1	0	0	0	−0.0110	0.0000	−0.0142	−0.0058	−0.0417	0.0535
35	0.0330	0	0	0	1	0	−0.0744	−0.0577	−0.0106	0.0000	−0.0182	0.0325
36	0.0511	1	0	0	0	0	0.0000	−0.0369	−0.0012	−0.0181	−0.0410	0.0501
37	0.0087	0	0	0	0	1.000001	−0.0045	−0.0345	−0.0370	−0.0151	0.0000	0.0086
38	0.0211	0	1	0	0	0	−0.0472	0.0000	−0.0280	−0.0277	−0.0194	0.0208
39	0.0438	1	0	0	0	0	0.0000	−0.0364	−0.0336	−0.0391	−0.0377	0.0408
40	0.0234	0	0	0	1	0	−0.0268	−0.0082	−0.0012	0.0000	−0.0209	0.0225
41	−0.0114	0	0	0	0	1	−0.0488	−0.0794	−0.0085	−0.0123	0.0000	−0.0117
42	−0.0012	0	0	0.1021	0	0.8979	−0.0306	−0.0680	0.0000	−0.0262	0.0000	−0.0015
43	0.0554	0	0	1	0	0	−0.0450	−0.0680	0.0000	−0.0182	−0.0342	0.0515
44	0.1198	1	0	0	0	0	0.0000	−0.0490	−0.0612	−0.0598	−0.0833	0.1018
45	0.0325	0	1	0	0	0	−0.0266	0.0000	−0.0226	−0.0200	−0.0223	0.0322
46	0.0302	0	0	1	0	0	−0.0588	−0.0418	0.0000	−0.0208	−0.0205	0.0296
47	0.0401	0	0	1	0	0	−0.0342	−0.0356	0.0000	−0.0224	−0.0351	0.0376
48	0.0829	0	0	1	0	0	−0.0919	−0.0924	0.0000	−0.0383	−0.0444	0.0718
49	0.0013	0	0	0	0.6945	0.3055	−0.0206	−0.0758	−0.0578	0.0000	0.0000	0.0007
50	0.0623	0	1	0	0	0	−0.0214	0.0000	−0.0147	−0.0264	−0.0560	0.0610
51	0.0042	0	0	0	1	0	−0.0254	−0.0302	−0.0041	0.0000	−0.0123	0.0041
52	0.0571	0	0	1	0	0	−0.0111	−0.0302	0.0000	−0.0334	−0.0415	0.0501
53	0.0644	1	0	0	0	0	0.0000	−0.0312	−0.0424	−0.0079	−0.0555	0.0607
54	0.0418	0.9587	0.0413	0	0	0	0.0000	0.0000	−0.0470	−0.0279	−0.0504	0.0417
55	0.1319	0	1	0	0	0	−0.0818	0.0000	−0.0959	−0.0921	−0.1186	0.1099
56	0.0272	1	0	0	0	0	0.0000	−0.0408	−0.0344	−0.0084	−0.0336	0.0248
57	0.0240	1	0	0	0	0	0.0000	−0.0164	−0.0468	−0.0210	−0.0412	0.0201
58	0.0024	0	0	0	1	0	−0.0447	−0.0175	−0.0332	0.0000	−0.0068	0.0022
59	0.0100	0	0	1	0	0	−0.0408	−0.0254	0.0000	−0.0010	−0.0136	0.0092

Table B.25: Optimisation: Downside risk, downward shocks in Bond yields as indicator and a conservative investment

		Solution					Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
60	0.0212	0	0	1	0	0	−0.0435	−0.0328	0.0000	−0.0166	−0.0229	0.0208
61	0.0257	0	0	0	1	0	−0.0340	−0.0796	−0.0097	0.0000	−0.0202	0.0248
62	0.0736	0	1	0	0	0	−0.0326	0.0000	−0.0602	−0.0326	−0.0777	0.0689
63	0.0221	0	1	0	0	0	−0.0049	0.0000	−0.0147	−0.0020	−0.0236	0.0211
64	0.0637	0	0	1	0	0	−0.0240	−0.0322	0.0000	−0.0265	−0.0729	0.0602
65	0.0594	1	0	0	0	0	0.0000	−0.0199	−0.0245	−0.0519	−0.0545	0.0547
66	0.0529	0	0	0	1	0	−0.0845	−0.0453	−0.0185	0.0000	−0.0384	0.0512
67	0.0137	0	0	1	0	0	−0.0028	−0.0177	0.0000	−0.0071	−0.0067	0.0134
68	0.0433	0	1	0	0	0	−0.0171	0.0000	−0.0390	−0.0317	−0.0644	0.0375
69	0.0869	0	0	1	0	0	−0.0950	−0.0916	0.0000	−0.0228	−0.0711	0.0859
70	0.0436	0	1	0	0	0	−0.0630	0.0000	−0.0294	−0.0327	−0.0903	0.0420
71	0.0414	0	0	0	0	1	−0.0831	−0.0853	−0.1106	−0.0887	0.0000	0.0390
72	0.0248	0	0	1	0	0	−0.0262	−0.0285	0.0000	−0.0009	−0.0228	0.0189
73	0.0106	0	0	1	0	0	−0.0017	−0.0388	0.0000	−0.0137	−0.0428	0.0096
74	0.0033	0	0	0	0	1	−0.0087	−0.0368	−0.0597	−0.0082	0.0000	0.0027
75	−0.0001	0	0	0	0	1	−0.0964	−0.1163	−0.0312	−0.0064	0.0000	−0.0006
76	0.0227	0	1	0	0	0	−0.0313	0.0000	−0.0565	−0.0243	−0.0151	0.0159
77	0.0108	0	0	0	0	1	−0.0447	−0.0837	−0.0105	−0.0072	0.0000	0.0105
78	0.0231	1	0	0	0	0	0.0000	−0.0064	−0.0188	−0.0176	−0.0286	0.0121
79	0.0655	0	0	1	0	0	−0.0549	−0.0160	0.0000	−0.0510	−0.0306	0.0594
80	0.0144	0	0	0	0	1	−0.0128	−0.0590	−0.0475	−0.0568	0.0000	0.0136
81	0.0050	0	0	0	0	1	−0.0863	−0.0657	−0.0821	−0.1011	0.0000	0.0050
82	0.0301	1	0	0	0	0	0.0000	−0.0419	−0.0105	−0.0185	−0.0013	0.0292
83	0.0157	0	0	0	0	1	−0.0153	−0.0196	−0.0119	−0.0144	0.0000	0.0156
84	0.0765	1	0	0	0	0	0.0000	−0.0406	−0.0647	−0.0499	−0.0939	0.0703
85	0.0130	0	1	0	0	0	−0.0476	0.0000	−0.0141	−0.0110	−0.0108	0.0127
86	−0.0015	0	0	0	0	1	−0.0190	−0.0028	−0.0115	−0.0200	0.0000	−0.0020
87	0.0258	0	0	0	0	1	−0.0119	−0.0160	−0.0046	−0.0163	0.0000	0.0248
88	0.0535	0	1	0	0	0	−0.0046	0.0000	−0.0421	−0.0354	−0.0604	0.0505
89	0.0445	1	0	0	0	0	0.0000	−0.0445	−0.0235	−0.0393	−0.0338	0.0420
90	0.0390	0	1	0	0	0	−0.0248	0.0000	−0.0235	−0.0312	−0.0500	0.0387
91	0.0653	0	1	0	0	0	−0.0487	0.0000	−0.0413	−0.0390	−0.0633	0.0604
92	0.0240	1	0	0	0	0	0.0000	−0.0333	−0.0337	−0.0324	−0.0269	0.0236
93	−0.0009	0	0	0	1	0	−0.0238	−0.0157	−0.0106	0.0000	−0.0232	−0.0010
94	0.0407	0	0	0	0	1	−0.1129	−0.0759	−0.0652	−0.0556	0.0000	0.0379
95	0.0561	1	0	0	0	0	0.0000	−0.0368	−0.0508	−0.0436	−0.0500	0.0514
96	0.0023	0.6294	0	0.3706	0	0	0.0000	−0.0293	0.0000	−0.0149	−0.0148	0.0018
97	0.0172	0	1	0	0	0	−0.0242	0.0000	−0.0531	−0.0750	−0.0209	0.0170
98	0.0244	1	0	0	0	0	0.0000	−0.0185	−0.0316	−0.0181	−0.0022	0.0236
99	0.0020	0	0	0	0	1	−0.0164	−0.0468	−0.0412	−0.0134	0.0000	0.0018
100	0.0658	0.9465	0	0.0535	0	0	0.0000	−0.0328	0.0000	−0.0269	−0.0471	0.0541
101	−0.0075	0	0	0	0	1	−0.0002	−0.0301	−0.0062	−0.0350	0.0000	−0.0076
102	0.0076	1	0	0	0	0	0.0000	−0.0627	−0.0093	−0.0114	−0.0128	0.0064
103	0.0016	0	1	0	0	0	−0.0214	0.0000	−0.0141	−0.0524	−0.0003	0.0014
104	0.0101	0	1	0	0	0	−0.0677	0.0000	−0.0160	−0.0092	−0.0005	0.0086
105	0.0863	1	0	0	0	0	0.0000	−0.1026	−0.0437	−0.0653	−0.0726	0.0766
106	0.0329	0	0	1	0	0	−0.0720	−0.0599	0.0000	−0.0384	−0.0136	0.0326
107	0.0442	0	0	1	0	0	−0.0177	−0.0380	0.0000	−0.0420	−0.0410	0.0436
108	0.0343	0	1	0	0	0	−0.0302	0.0000	−0.0316	−0.0135	−0.0362	0.0340

Table B.26: Optimisation: Downside risk, downward shock in Bond yields as indicator and a conservative investment (continue)

	Solution						Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment												
1	0.0241	1	0	0	0	0	0.0000	−0.0045	−0.0285	−0.0314	−0.0328	0.0230
2	0.0316	0	1	0	0	0	−0.0177	0.0000	−0.0358	−0.0307	−0.0318	0.0300
3	−0.0021	0	0	1	0	0	−0.0277	−0.0118	0.0000	−0.0093	−0.0014	−0.0022
4	−0.0023	0	1	0	0	0	−0.0075	0.0000	−0.0206	−0.0129	−0.0046	−0.0037
5	0.0006	0	1	0	0	0	−0.0196	0.0000	−0.0168	−0.0052	−0.0078	0.0006
6	0.0151	0	0	1	0	0	−0.0222	−0.0018	0.0000	−0.0050	−0.0267	0.0146
7	0.0379	0	1	0	0	0	−0.0194	0.0000	−0.0251	−0.0026	−0.0619	0.0369
8	0.0040	0	0	1	0	0	−0.0103	−0.0308	0.0000	−0.0023	−0.0013	0.0040
9	0.0312	0	0	1	0	0	−0.0223	−0.0199	0.0000	−0.0062	−0.0397	0.0304
10	0.0247	0	0	1	0	0	−0.0326	−0.0222	0.0000	−0.0162	−0.0271	0.0240
11	0.0295	0	0	1	0	0	−0.0342	−0.0058	0.0000	−0.0066	−0.0340	0.0287
12	0.0001	0	0	1	0	0	−0.0039	−0.0079	0.0000	−0.0043	−0.0035	0.0001
13	0.0332	0.5653	0.4347	0	0	0	0.0000	0.0000	−0.0152	−0.0264	−0.0341	0.0323
14	0.0171	0	0	0	0	1	−0.0021	−0.0062	−0.0072	−0.0091	0.0000	0.0167
15	0.0101	0	1	0	0	0	−0.0408	0.0000	−0.0186	−0.0129	−0.0184	0.0099
16	−0.0022	0	0	0	0	1	−0.0174	−0.0298	−0.0108	−0.0079	0.0000	−0.0022
17	−0.0065	0	0	0	0	1	−0.0431	−0.0356	−0.0378	−0.0296	0.0000	−0.0066
18	−0.0066	0	0	0	0	1	−0.0597	−0.0223	−0.0274	−0.0275	0.0000	−0.0067
19	0.0401	0	1	0	0	0	−0.0041	0.0000	−0.0247	−0.0193	−0.0342	0.0360
20	−0.0028	0	0	1	0	0	−0.0235	−0.0309	0.0000	−0.0126	−0.0118	−0.0029
21	−0.0110	0	0	1	0	0	−0.0256	−0.0083	0.0000	−0.0026	−0.0021	−0.0110
22	0.0140	1	0	0	0	0	0.0000	−0.0167	−0.0216	−0.0074	−0.0226	0.0136
23	0.0287	0	1	0	0	0	−0.0426	0.0000	−0.0037	−0.0069	−0.0276	0.0250
24	0.0054	0	1	0	0	0	−0.0111	0.0000	−0.0067	−0.0052	−0.0039	0.0053
25	0.0000	0	0	0	0	1	−0.0258	−0.0161	−0.0051	−0.0036	0.0000	−0.0001
26	0.0185	1	0	0	0	0	0.0000	−0.0160	−0.0076	−0.0161	−0.0151	0.0182
27	0.0633	1	0	0	0	0	0.0000	−0.0786	−0.0334	−0.0510	−0.0688	0.0599
28	0.0025	0	0	1	0	0	−0.0380	−0.0004	0.0000	−0.0095	−0.0060	0.0025
29	0.0257	0	1	0	0	0	−0.0117	0.0000	−0.0190	−0.0215	−0.0310	0.0255
30	0.0329	0	1	0	0	0	−0.0337	0.0000	−0.0136	−0.0156	−0.0289	0.0323
31	0.0351	0	0	1	0	0	−0.0186	−0.0080	0.0000	−0.0121	−0.0308	0.0316
32	0.0084	0	0	1	0	0	−0.0283	−0.0064	0.0000	−0.0041	−0.0132	0.0084
33	−0.0090	0	0	0	0	1	−0.0057	−0.0185	−0.0120	−0.0005	0.0000	−0.0092
34	0.0057	1	0	0	0	0	0.0000	−0.0004	−0.0187	−0.0008	−0.0092	0.0056
35	0.0068	0	0	1	0	0	−0.0012	−0.0120	0.0000	−0.0041	−0.0151	0.0067
36	−0.0053	0	0	1	0	0	−0.0114	−0.0223	0.0000	−0.0095	−0.0002	−0.0057
37	0.0105	1	0	0	0	0	0.0000	−0.0304	−0.0180	−0.0183	−0.0154	0.0104
38	0.0166	1	0	0	0	0	0.0000	−0.0094	−0.0061	−0.0109	−0.0168	0.0165
39	−0.0055	0	0	0	0	1	−0.0177	−0.0069	−0.0029	−0.0010	0.0000	−0.0056
40	0.0073	0	0	0	1	0	−0.0027	−0.0161	−0.0048	0.0000	−0.0175	0.0072
41	0.0205	0	1	0	0	0	−0.0185	0.0000	−0.0258	−0.0204	−0.0255	0.0201
42	0.0048	0	0	0	0	1	−0.0065	−0.0197	−0.0082	−0.0037	0.0000	0.0048
43	−0.0327	0	0	0	0	1	−0.0308	−0.0300	−0.0309	−0.0086	0.0000	−0.0327
44	0.0113	1	0	0	0	0	0.0000	−0.0891	−0.0727	−0.0506	−0.0182	0.0067
45	0.0040	0	1	0	0	0	−0.1266	0.0000	−0.2536	−0.1806	−0.0379	0.0030
46	0.1140	1	0	0	0	0	0.0000	−0.0915	−0.2136	−0.1553	−0.1332	0.1044
47	0.0701	1	0	0	0	0	0.0000	−0.0546	−0.1425	−0.1318	−0.0526	0.0688
48	0.1067	0	0	1	0	0	−0.1246	−0.0662	0.0000	−0.0718	−0.0930	0.0962
49	0.0777	0	0	1	0	0	−0.0381	−0.0726	0.0000	−0.0174	−0.0463	0.0556
50	0.0382	1	0	0	0	0	0.0000	−0.1038	−0.1294	−0.0719	−0.0366	0.0365
51	0.0571	0	0	1	0	0	−0.0393	−0.0593	0.0000	−0.0533	−0.0580	0.0530
52	−0.0081	0	0	0	1	0	−0.0207	−0.0093	−0.0220	0.0000	−0.0091	−0.0081

Table B.27: Optimisation: Downside risk, upward shock in Bond yields as indicator and a moderate investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
						Moderate Investment						
1	–0.0076	0	0	0	0	1	–0.0325	–0.0071	–0.0145	–0.0061	0.0000	–0.0077
2	0.0122	1	0	0	0	0	0.0000	–0.0147	–0.0159	–0.0178	–0.0209	0.0108
3	–0.0083	0	0	0	0	1	–0.0194	–0.0538	–0.0219	–0.0200	0.0000	–0.0083
4	0.0234	0	0	1	0	0	–0.0334	–0.0254	0.0000	–0.0010	–0.0202	0.0229
5	0.0441	1	0	0	0	0	0.0000	–0.0313	–0.0205	–0.0380	–0.0495	0.0402
6	0.0360	0	1	0	0	0	–0.0159	0.0000	–0.0251	–0.0300	–0.0267	0.0343
7	–0.0007	0	0	0	1	0	–0.0096	–0.0175	–0.0014	0.0000	–0.0128	–0.0007
8	0.0224	1	0	0	0	0	0.0000	–0.0189	–0.0177	–0.0114	–0.0266	0.0222
9	0.0452	1	0	0	0	0	0.0000	–0.0396	–0.0337	–0.0343	–0.0394	0.0418
10	0.0193	0.1205	0.8795	0	0	0	0.0000	0.0000	–0.0232	–0.0016	–0.0190	0.0185
11	0.0635	0	1	0	0	0	–0.0592	0.0000	–0.0148	–0.0050	–0.0504	0.0587
12	–0.0056	0.4732	0	0	0.5268	0	0.0000	–0.0088	–0.0048	0.0000	–0.0060	–0.0058
13	0.0194	0	0	0	1	0	–0.0115	–0.0255	–0.0178	0.0000	–0.0053	0.0186
14	0.0692	1	0	0	0	0	0.0000	–0.0632	–0.0552	–0.0447	–0.0481	0.0617
15	0.0350	1	0	0	0	0	0.0000	–0.0172	–0.0096	–0.0172	–0.0058	0.0346
16	0.0442	0	1	0	0	0	–0.0318	0.0000	–0.0225	–0.0134	–0.0301	0.0428
17	0.0431	0	0	0	0	1	–0.0090	–0.0106	–0.0117	–0.0055	0.0000	0.0418
18	0.0457	1	0	0	0	0	0.0000	–0.0199	–0.0233	–0.0160	–0.0277	0.0405
19	0.0063	0	0	0	0	1	–0.0501	–0.0393	–0.0294	–0.0643	0.0000	0.0062
20	0.0212	0	1	0	0	0	–0.0625	0.0000	–0.0097	–0.0203	–0.0118	0.0205
21	0.0094	0	0	0	0	1	–0.0301	–0.0153	–0.0100	–0.0036	0.0000	0.0092
22	0.0676	0	1	0	0	0	–0.0012	0.0000	–0.0435	–0.0250	–0.0612	0.0632
23	–0.0018	0	1	0	0	0	–0.0662	0.0000	–0.0185	–0.0231	–0.0077	–0.0021
24	0.0235	1	0	0	0	0	0.0000	–0.0255	–0.0250	–0.0124	–0.0235	0.0193
25	0.0048	0	1	0	0	0	–0.0226	0.0000	–0.0035	–0.0132	–0.0037	0.0046
26	0.0611	1	0	0	0	0	0.0000	–0.0232	–0.0473	–0.0452	–0.0596	0.0603
27	0.0488	0	1	0	0	0	–0.0276	0.0000	–0.0217	–0.0145	–0.0361	0.0435
28	0.0601	0	1	0	0	0	–0.0461	0.0000	–0.0360	–0.0363	–0.0524	0.0576
29	0.0281	1	0	0	0	0	0.0000	–0.0734	–0.0630	–0.0578	–0.0239	0.0270
30	–0.0092	0	0	0	0	1	–0.0437	–0.0311	–0.0285	–0.0239	0.0000	–0.0093
31	0.0122	0	0	0	1	0	–0.0226	–0.0041	–0.0073	0.0000	–0.1191	0.0117
32	0.0275	0	1	0	0	0	–0.0136	0.0000	–0.0316	–0.0291	–0.0354	0.0266
33	0.0536	0	1	0	0	0	–0.0695	0.0000	–0.0087	–0.0192	–0.0414	0.0500
34	0.0651	0	1	0	0	0	–0.0118	0.0000	–0.0154	–0.0064	–0.0455	0.0586
35	0.0332	0	0	0	1	0	–0.0751	–0.0579	–0.0106	0.0000	–0.0183	0.0328
36	0.0513	1	0	0	0	0	0.0000	–0.0374	–0.0016	–0.0182	–0.0414	0.0506
37	0.0087	0	0	0	0	1	–0.0045	–0.0347	–0.0371	–0.0152	0.0000	0.0087
38	0.0212	0	1	0	0	0	–0.0473	0.0000	–0.0281	–0.0278	–0.0196	0.0210
39	0.0446	1	0	0	0	0	0.0000	–0.0381	–0.0350	–0.0406	–0.0394	0.0425
40	0.0237	0	0	0	1	0	–0.0268	–0.0085	–0.0014	0.0000	–0.0212	0.0230
41	–0.0114	0	0	0	0	1	–0.0483	–0.0784	–0.0085	–0.0123	0.0000	–0.0115
42	–0.0011	0	0	0.1759	0	0.8241	–0.0305	–0.0680	0.0000	–0.0262	0.0000	–0.0015
43	0.0565	0	0	1	0	0	–0.0465	–0.0709	0.0000	–0.0193	–0.0354	0.0537
44	0.1248	1	0	0	0	0	0.0000	–0.0507	–0.0658	–0.0645	–0.0917	0.1119
45	0.0326	0	1	0	0	0	–0.0269	0.0000	–0.0228	–0.0203	–0.0223	0.0323
46	0.0304	0	0	1	0	0	–0.0599	–0.0421	0.0000	–0.0211	–0.0207	0.0299
47	0.0408	0	0	1	0	0	–0.0356	–0.0369	0.0000	–0.0232	–0.0365	0.0390
48	0.0860	0	0	1	0	0	–0.0982	–0.0993	0.0000	–0.0420	–0.0488	0.0780
49	0.0015	0	0	0	0.8915	0.1085	–0.0205	–0.0758	–0.0577	0.0000	0.0000	0.0008
50	0.0627	0	1	0	0	0	–0.0220	0.0000	–0.0154	–0.0269	–0.0570	0.0618
51	0.0043	0	0	0	1	0	–0.0252	–0.0301	–0.0041	0.0000	–0.0123	0.0042
52	0.0591	0	0	1	0	0	–0.0130	–0.0341	0.0000	–0.0360	–0.0444	0.0540
53	0.0655	1	0	0	0	0	0.0000	–0.0328	–0.0433	–0.0075	–0.0575	0.0628
54	0.0419	0.7929	0.2071	0	0	0	0.0000	0.0000	–0.0471	–0.0280	–0.0504	0.0417
55	0.1380	0	1	0	0	0	–0.0896	0.0000	–0.1072	–0.1023	–0.1311	0.1222
56	0.0279	1	0	0	0	0	0.0000	–0.0436	–0.0363	–0.0098	–0.0353	0.0261
57	0.0251	1	0	0	0	0	0.0000	–0.0179	–0.0498	–0.0224	–0.0438	0.0223
58	0.0024	0	0	0	1	0	–0.0448	–0.0176	–0.0333	0.0000	–0.0069	0.0023
59	0.0102	0	0	1	0	0	–0.0414	–0.0254	0.0000	–0.0012	–0.0142	0.0097

Table B.28: Optimisation: Downside risk, downward shock in Bond yields as indicator and a moderate investment

		Solution					Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment												
60	0.0214	0	0	1	0	0	−0.0439	−0.0332	0.0000	−0.0168	−0.0231	0.0210
61	0.0260	0	0	0	1	0	−0.0345	−0.0804	−0.0099	0.0000	−0.0206	0.0253
62	0.0749	0	1	0	0	0	−0.0326	0.0000	−0.0632	−0.0346	−0.0808	0.0715
63	0.0224	0	1	0	0	0	−0.0051	0.0000	−0.0148	−0.0019	−0.0239	0.0217
64	0.0647	0	0	1	0	0	−0.0244	−0.0329	0.0000	−0.0272	−0.0745	0.0621
65	0.0608	1	0	0	0	0	0.0000	−0.0216	−0.0266	−0.0542	−0.0571	0.0574
66	0.0534	0	0	0	1	0	−0.0866	−0.0463	−0.0193	0.0000	−0.0392	0.0522
67	0.0138	0	0	1	0	0	−0.0027	−0.0177	0.0000	−0.0071	−0.0067	0.0136
68	0.0449	0	1	0	0	0	−0.0185	0.0000	−0.0416	−0.0342	−0.0687	0.0407
69	0.0872	0	0	1	0	0	−0.0959	−0.0929	0.0000	−0.0230	−0.0718	0.0864
70	0.0441	0	1	0	0	0	−0.0639	0.0000	−0.0307	−0.0340	−0.0920	0.0429
71	0.0421	0	0	0	0	1	−0.0848	−0.0879	−0.1133	−0.0909	0.0000	0.0404
72	0.0265	0	0	1	0	0	−0.0284	−0.0318	0.0000	−0.0012	−0.0265	0.0222
73	0.0109	0	0	1	0	0	−0.0017	−0.0389	0.0000	−0.0137	−0.0429	0.0102
74	0.0035	0	0	0	0	1	−0.0085	−0.0377	−0.0600	−0.0082	0.0000	0.0030
75	0.0000	0	0	0	0	1	−0.0975	−0.1176	−0.0314	−0.0064	0.0000	−0.0003
76	0.0246	0	1	0	0	0	−0.0332	0.0000	−0.0621	−0.0283	−0.0190	0.0197
77	0.0109	0	0	0	0	1	−0.0453	−0.0844	−0.0107	−0.0074	0.0000	0.0107
78	0.0262	1	0	0	0	0	0.0000	−0.0060	−0.0237	−0.0222	−0.0375	0.0183
79	0.0672	0	0	1	0	0	−0.0591	−0.0160	0.0000	−0.0534	−0.0324	0.0628
80	0.0146	0	0	0	0	1	−0.0131	−0.0606	−0.0487	−0.0584	0.0000	0.0140
81	0.0050	0	0	0	0	1	−0.0867	−0.0660	−0.0825	−0.1016	0.0000	0.0050
82	0.0304	1	0	0	0	0	0.0000	−0.0431	−0.0109	−0.0191	−0.0012	0.0297
83	0.0158	0	0	0	0	1	−0.0155	−0.0197	−0.0119	−0.0143	0.0000	0.0157
84	0.0783	1	0	0	0	0	0.0000	−0.0442	−0.0670	−0.0525	−0.0976	0.0738
85	0.0131	0	1	0	0	0	−0.0479	0.0000	−0.0142	−0.0111	−0.0110	0.0129
86	−0.0014	0	0	0	0	1	−0.0194	−0.0033	−0.0118	−0.0204	0.0000	−0.0017
87	0.0261	0	0	0	0	1	−0.0119	−0.0162	−0.0049	−0.0167	0.0000	0.0254
88	0.0543	0	1	0	0	0	−0.0038	0.0000	−0.0431	−0.0366	−0.0623	0.0522
89	0.0453	1	0	0	0	0	0.0000	−0.0461	−0.0248	−0.0409	−0.0346	0.0434
90	0.0391	0	1	0	0	0	−0.0247	0.0000	−0.0236	−0.0314	−0.0500	0.0389
91	0.0667	0	1	0	0	0	−0.0506	0.0000	−0.0435	−0.0409	−0.0661	0.0631
92	0.0241	1	0	0	0	0	0.0000	−0.0336	−0.0341	−0.0327	−0.0272	0.0238
93	−0.0009	0	0	0	1	0	−0.0239	−0.0157	−0.0106	0.0000	−0.0233	−0.0010
94	0.0414	0	0	0	0	1	−0.1170	−0.0794	−0.0672	−0.0579	0.0000	0.0395
95	0.0574	1	0	0	0	0	0.0000	−0.0381	−0.0535	−0.0465	−0.0522	0.0540
96	0.0025	0.9302	0	0.0698	0	0	0.0000	−0.0292	0.0000	−0.0148	−0.0149	0.0019
97	0.0173	0	1	0	0	0	−0.0244	0.0000	−0.0534	−0.0754	−0.0211	0.0171
98	0.0246	1	0	0	0	0	0.0000	−0.0182	−0.0307	−0.0176	−0.0027	0.0240
99	0.0020	0	0	0	0	1	−0.0166	−0.0471	−0.0415	−0.0136	0.0000	0.0019
100	0.0693	0.3563	0	0.6437	0	0	0.0000	−0.0346	0.0000	−0.0295	−0.0518	0.0593
101	−0.0075	0	0	0	0	1	−0.0001	−0.0301	−0.0062	−0.0349	0.0000	−0.0075
102	0.0079	1	0	0	0	0	0.0000	−0.0638	−0.0096	−0.0115	−0.0133	0.0070
103	0.0016	0	1	0	0	0	−0.0214	0.0000	−0.0142	−0.0523	−0.0004	0.0015
104	0.0105	0	1	0	0	0	−0.0671	0.0000	−0.0165	−0.0100	−0.0016	0.0094
105	0.0890	1	0	0	0	0	0.0000	−0.1093	−0.0469	−0.0693	−0.0780	0.0821
106	0.0330	0	0	1	0	0	−0.0731	−0.0606	0.0000	−0.0390	−0.0138	0.0328
107	0.0444	0	0	1	0	0	−0.0178	−0.0382	0.0000	−0.0422	−0.0413	0.0439
108	0.0343	0	1	0	0	0	−0.0303	0.0000	−0.0317	−0.0135	−0.0363	0.0342

Table B.29: Optimisation: Downside risk, downward shock in Bond yields as indicator and a moderate investment (continue)

	Solution						Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
1	0.0245	1	0	0	0	0	0.0000	−0.0049	−0.0293	−0.0322	−0.0337	0.0238
2	0.0321	0	1	0	0	0	−0.0182	0.0000	−0.0368	−0.0317	−0.0329	0.0311
3	−0.0021	0	0	1	0	0	−0.0277	−0.0118	0.0000	−0.0094	−0.0015	−0.0021
4	−0.0018	0	1	0	0	0	−0.0081	0.0000	−0.0214	−0.0136	−0.0056	−0.0027
5	0.0006	0	1	0	0	0	−0.0196	0.0000	−0.0168	−0.0052	−0.0078	0.0006
6	0.0152	0	0	1	0	0	−0.0222	−0.0017	0.0000	−0.0051	−0.0271	0.0149
7	0.0382	0	1	0	0	0	−0.0198	0.0000	−0.0256	−0.0025	−0.0630	0.0376
8	0.0041	0	0	1	0	0	−0.0103	−0.0308	0.0000	−0.0023	−0.0013	0.0040
9	0.0314	0	0	1	0	0	−0.0223	−0.0201	0.0000	−0.0062	−0.0404	0.0309
10	0.0249	0	0	1	0	0	−0.0330	−0.0225	0.0000	−0.0165	−0.0277	0.0245
11	0.0298	0	0	1	0	0	−0.0348	−0.0058	0.0000	−0.0067	−0.0346	0.0292
12	0.0001	0	0	1	0	0	−0.0040	−0.0079	0.0000	−0.0043	−0.0035	0.0001
13	0.0336	1	0	0	0	0	0.0000	−0.0001	−0.0153	−0.0266	−0.0343	0.0325
14	0.0172	0	0	0	0	1	−0.0021	−0.0063	−0.0074	−0.0093	0.0000	0.0170
15	0.0102	0	1	0	0	0	−0.0412	0.0000	−0.0188	−0.0131	−0.0186	0.0100
16	−0.0022	0	0	0	0	1	−0.0174	−0.0298	−0.0108	−0.0079	0.0000	−0.0022
17	−0.0065	0	0	0	0	1	−0.0431	−0.0355	−0.0376	−0.0295	0.0000	−0.0066
18	−0.0066	0	0	0	0	1	−0.0593	−0.0222	−0.0273	−0.0274	0.0000	−0.0067
19	0.0415	0	1	0	0	0	−0.0039	0.0000	−0.0260	−0.0203	−0.0367	0.0388
20	−0.0028	0	0	1	0	0	−0.0233	−0.0309	0.0000	−0.0126	−0.0119	−0.0028
21	−0.0109	0	0	1	0	0	−0.0255	−0.0083	0.0000	−0.0026	−0.0021	−0.0110
22	0.0141	1	0	0	0	0	0.0000	−0.0168	−0.0220	−0.0076	−0.0230	0.0138
23	0.0299	0	1	0	0	0	−0.0451	0.0000	−0.0049	−0.0080	−0.0301	0.0275
24	0.0054	0	1	0	0	0	−0.0111	0.0000	−0.0068	−0.0053	−0.0040	0.0054
25	0.0000	0	0	0	0	1	−0.0259	−0.0162	−0.0051	−0.0036	0.0000	0.0000
26	0.0187	1	0	0	0	0	0.0000	−0.0162	−0.0077	−0.0163	−0.0153	0.0184
27	0.0645	1	0	0	0	0	0.0000	−0.0818	−0.0345	−0.0528	−0.0714	0.0622
28	0.0025	0	0	1	0	0	−0.0380	−0.0005	0.0000	−0.0096	−0.0060	0.0025
29	0.0257	0	1	0	0	0	−0.0118	0.0000	−0.0191	−0.0216	−0.0311	0.0256
30	0.0331	0	1	0	0	0	−0.0342	0.0000	−0.0138	−0.0157	−0.0292	0.0327
31	0.0362	0	0	1	0	0	−0.0204	−0.0098	0.0000	−0.0135	−0.0332	0.0339
32	0.0085	0	0	1	0	0	−0.0283	−0.0064	0.0000	−0.0040	−0.0133	0.0084
33	−0.0090	0	0	0	0	1	−0.0057	−0.0184	−0.0120	−0.0005	0.0000	−0.0091
34	0.0057	1	0	0	0	0	0.0000	−0.0005	−0.0188	−0.0009	−0.0093	0.0057
35	0.0068	0	0	1	0	0	−0.0012	−0.0120	0.0000	−0.0041	−0.0152	0.0068
36	−0.0052	0	0	1	0	0	−0.0113	−0.0220	0.0000	−0.0095	−0.0004	−0.0054
37	0.0106	1	0	0	0	0	0.0000	−0.0305	−0.0180	−0.0184	−0.0155	0.0105
38	0.0167	1	0	0	0	0	0.0000	−0.0094	−0.0061	−0.0110	−0.0169	0.0166
39	−0.0055	0	0	0	0	1	−0.0176	−0.0068	−0.0029	−0.0010	0.0000	−0.0055
40	0.0073	0	0	0	1	0	−0.0027	−0.0162	−0.0048	0.0000	−0.0177	0.0072
41	0.0206	0	1	0	0	0	−0.0185	0.0000	−0.0261	−0.0206	−0.0258	0.0203
42	0.0049	0	0	0	0	1	−0.0065	−0.0197	−0.0083	−0.0037	0.0000	0.0048
43	−0.0327	0	0	0	0	1	−0.0306	−0.0300	−0.0309	−0.0086	0.0000	−0.0327
44	0.0128	1	0	0	0	0	0.0000	−0.0926	−0.0759	−0.0536	−0.0212	0.0097
45	0.0043	0	1	0	0	0	−0.1252	0.0000	−0.2501	−0.1785	−0.0380	0.0037
46	0.1172	1	0	0	0	0	0.0000	−0.1003	−0.2269	−0.1640	−0.1394	0.1108
47	0.0705	1	0	0	0	0	0.0000	−0.0573	−0.1455	−0.1341	−0.0535	0.0696
48	0.1102	0	0	1	0	0	−0.1328	−0.0708	0.0000	−0.0775	−0.0990	0.1032
49	0.0850	0	0	1	0	0	−0.0533	−0.0810	0.0000	−0.0193	−0.0583	0.0703
50	0.0388	1	0	0	0	0	0.0000	−0.1056	−0.1332	−0.0740	−0.0381	0.0377
51	0.0585	0	0	1	0	0	−0.0417	−0.0615	0.0000	−0.0554	−0.0600	0.0558
52	−0.0081	0	0	0	1	0	−0.0207	−0.0092	−0.0221	0.0000	−0.0092	−0.0081

Table B.30: Optimisation: Downside risk, upward shock in Bond yields as indicator and an aggressive investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
1	−0.0076	0	0	0	0	1	−0.0325	−0.0071	−0.0145	−0.0061	0.0000	−0.0076
2	0.0127	1	0	0	0	0	0.0000	−0.0158	−0.0169	−0.0188	−0.0220	0.0118
3	−0.0083	0	0	0	0	1	−0.0193	−0.0537	−0.0218	−0.0199	0.0000	−0.0083
4	0.0235	0	0	1	0	0	−0.0341	−0.0260	0.0000	−0.0011	−0.0204	0.0232
5	0.0454	1	0	0	0	0	0.0000	−0.0326	−0.0220	−0.0401	−0.0525	0.0428
6	0.0366	0	1	0	0	0	−0.0168	0.0000	−0.0261	−0.0311	−0.0276	0.0355
7	−0.0007	0	0	0	1	0	−0.0096	−0.0176	−0.0014	0.0000	−0.0128	−0.0007
8	0.0225	1	0	0	0	0	0.0000	−0.0191	−0.0177	−0.0115	−0.0268	0.0223
9	0.0463	1	0	0	0	0	0.0000	−0.0412	−0.0357	−0.0363	−0.0416	0.0440
10	0.0196	0.2888	0.7112	0	0	0	0.0000	0.0000	−0.0235	−0.0020	−0.0194	0.0190
11	0.0651	0	1	0	0	0	−0.0624	0.0000	−0.0153	−0.0053	−0.0530	0.0619
12	−0.0054	0.7616	0	0	0.2384	0	0.0000	−0.0088	−0.0048	0.0000	−0.0060	−0.0058
13	0.0197	0	0	0	1	0	−0.0116	−0.0259	−0.0183	0.0000	−0.0056	0.0191
14	0.0717	1	0	0	0	0	0.0000	−0.0677	−0.0588	−0.0479	−0.0518	0.0667
15	0.0351	1	0	0	0	0	0.0000	−0.0172	−0.0095	−0.0172	−0.0059	0.0349
16	0.0446	0	1	0	0	0	−0.0329	0.0000	−0.0230	−0.0137	−0.0306	0.0437
17	0.0435	0	0	0	0	1	−0.0093	−0.0111	−0.0119	−0.0058	0.0000	0.0427
18	0.0474	1	0	0	0	0	0.0000	−0.0216	−0.0249	−0.0181	−0.0304	0.0440
19	0.0063	0	0	0	0	1	−0.0500	−0.0395	−0.0295	−0.0644	0.0000	0.0062
20	0.0215	0	1	0	0	0	−0.0637	0.0000	−0.0100	−0.0208	−0.0120	0.0210
21	0.0095	0	0	0	0	1	−0.0303	−0.0154	−0.0101	−0.0037	0.0000	0.0093
22	0.0691	0	1	0	0	0	−0.0011	0.0000	−0.0447	−0.0262	−0.0637	0.0662
23	−0.0017	0	1	0	0	0	−0.0667	0.0000	−0.0190	−0.0234	−0.0079	−0.0019
24	0.0249	1	0	0	0	0	0.0000	−0.0270	−0.0270	−0.0143	−0.0265	0.0221
25	0.0048	0	1	0	0	0	−0.0224	0.0000	−0.0036	−0.0133	−0.0039	0.0047
26	0.0614	1	0	0	0	0	0.0000	−0.0230	−0.0476	−0.0454	−0.0601	0.0609
27	0.0506	0	1	0	0	0	−0.0316	0.0000	−0.0241	−0.0166	−0.0394	0.0470
28	0.0610	0	1	0	0	0	−0.0478	0.0000	−0.0368	−0.0374	−0.0540	0.0593
29	0.0284	1	0	0	0	0	0.0000	−0.0748	−0.0644	−0.0590	−0.0246	0.0277
30	−0.0092	0	0	0	0	1	−0.0437	−0.0310	−0.0285	−0.0239	0.0000	−0.0093
31	0.0123	0	0	0	1	0	−0.0225	−0.0039	−0.0074	0.0000	−0.1193	0.0120
32	0.0278	0	1	0	0	0	−0.0138	0.0000	−0.0322	−0.0297	−0.0362	0.0272
33	0.0548	0	1	0	0	0	−0.0725	0.0000	−0.0085	−0.0201	−0.0435	0.0524
34	0.0672	0	1	0	0	0	−0.0125	0.0000	−0.0166	−0.0069	−0.0487	0.0629
35	0.0333	0	0	0	1	0	−0.0757	−0.0581	−0.0107	0.0000	−0.0183	0.0331
36	0.0516	1	0	0	0	0	0.0000	−0.0379	−0.0019	−0.0183	−0.0417	0.0511
37	0.0087	0	0	0	0	1	−0.0046	−0.0348	−0.0372	−0.0152	0.0000	0.0087
38	0.0213	0	1	0	0	0	−0.0475	0.0000	−0.0282	−0.0279	−0.0197	0.0211
39	0.0453	1	0	0	0	0	0.0000	−0.0396	−0.0363	−0.0418	−0.0409	0.0439
40	0.0239	0	0	0	1	0	−0.0267	−0.0086	−0.0015	0.0000	−0.0215	0.0235
41	−0.0113	0	0	0	0	1	−0.0479	−0.0776	−0.0085	−0.0123	0.0000	−0.0114
42	−0.0009	0	0	0.3075	0	0.6925	−0.0305	−0.0680	0.0000	−0.0262	0.0000	−0.0015
43	0.0575	0	0	1	0	0	−0.0478	−0.0734	0.0000	−0.0202	−0.0364	0.0556
44	0.1292	1	0	0	0	0	0.0000	−0.0522	−0.0698	−0.0685	−0.0988	0.1205
45	0.0326	0	1	0	0	0	−0.0271	0.0000	−0.0231	−0.0204	−0.0223	0.0325
46	0.0306	0	0	1	0	0	−0.0608	−0.0424	0.0000	−0.0214	−0.0209	0.0303
47	0.0414	0	0	1	0	0	−0.0368	−0.0380	0.0000	−0.0239	−0.0378	0.0402
48	0.0887	0	0	1	0	0	−0.1035	−0.1053	0.0000	−0.0453	−0.0527	0.0834
49	0.0018	0	0	0	1	0	−0.0203	−0.0752	−0.0570	0.0000	−0.0004	0.0012
50	0.0630	0	1	0	0	0	−0.0224	0.0000	−0.0159	−0.0273	−0.0579	0.0624
51	0.0043	0	0	0	1	0	−0.0251	−0.0300	−0.0041	0.0000	−0.0123	0.0043
52	0.0608	0	0	1	0	0	−0.0147	−0.0374	0.0000	−0.0383	−0.0468	0.0574
53	0.0664	1	0	0	0	0	0.0000	−0.0342	−0.0441	−0.0071	−0.0592	0.0646
54	0.0420	0.4967	0.5033	0	0	0	0.0000	0.0000	−0.0471	−0.0280	−0.0504	0.0417
55	0.1433	0	1	0	0	0	−0.0962	0.0000	−0.1169	−0.1111	−0.1418	0.1328
56	0.0284	1	0	0	0	0	0.0000	−0.0459	−0.0380	−0.0110	−0.0369	0.0273
57	0.0260	1	0	0	0	0	0.0000	−0.0192	−0.0523	−0.0236	−0.0460	0.0241
58	0.0025	0	0	0	1	0	−0.0449	−0.0176	−0.0334	0.0000	−0.0070	0.0024
59	0.0104	0	0	1	0	0	−0.0420	−0.0254	0.0000	−0.0013	−0.0147	0.0100

Table B.31: Optimisation: Downside risk, downward shock in Bond yields as indicator and an aggressive investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
60	0.0215	0	0	1	0	0	−0.0442	−0.0335	0.0000	−0.0169	−0.0233	0.0213
61	0.0262	0	0	0	1	0	−0.0349	−0.0811	−0.0100	0.0000	−0.0209	0.0258
62	0.0760	0	1	0	0	0	−0.0326	0.0000	−0.0657	−0.0364	−0.0834	0.0737
63	0.0226	0	1	0	0	0	−0.0053	0.0000	−0.0148	−0.0018	−0.0242	0.0221
64	0.0655	0	0	1	0	0	−0.0248	−0.0335	0.0000	−0.0279	−0.0759	0.0638
65	0.0619	1	0	0	0	0	0.0000	−0.0231	−0.0285	−0.0562	−0.0593	0.0596
66	0.0538	0	0	0	1	0	−0.0883	−0.0471	−0.0201	0.0000	−0.0399	0.0530
67	0.0139	0	0	1	0	0	−0.0026	−0.0178	0.0000	−0.0071	−0.0067	0.0137
68	0.0463	0	1	0	0	0	−0.0198	0.0000	−0.0438	−0.0364	−0.0724	0.0435
69	0.0874	0	0	1	0	0	−0.0967	−0.0939	0.0000	−0.0231	−0.0724	0.0869
70	0.0444	0	1	0	0	0	−0.0648	0.0000	−0.0318	−0.0350	−0.0934	0.0437
71	0.0427	0	0	0	0	1	−0.0862	−0.0900	−0.1156	−0.0929	0.0000	0.0415
72	0.0279	0	0	1	0	0	−0.0302	−0.0346	0.0000	−0.0014	−0.0298	0.0251
73	0.0111	0	0	1	0	0	−0.0017	−0.0390	0.0000	−0.0137	−0.0429	0.0106
74	0.0036	0	0	0	0	1	−0.0083	−0.0384	−0.0603	−0.0082	0.0000	0.0033
75	0.0002	0	0	0	0	1	−0.0984	−0.1187	−0.0316	−0.0064	0.0000	−0.0001
76	0.0262	0	1	0	0	0	−0.0349	0.0000	−0.0668	−0.0317	−0.0224	0.0230
77	0.0110	0	0	0	0	1	−0.0458	−0.0850	−0.0109	−0.0075	0.0000	0.0108
78	0.0288	1	0	0	0	0	0.0000	−0.0057	−0.0279	−0.0261	−0.0451	0.0235
79	0.0687	0	0	1	0	0	−0.0626	−0.0161	0.0000	−0.0555	−0.0340	0.0658
80	0.0148	0	0	0	0	1	−0.0135	−0.0620	−0.0497	−0.0598	0.0000	0.0144
81	0.0050	0	0	0	0	1	−0.0870	−0.0662	−0.0828	−0.1020	0.0000	0.0050
82	0.0306	1	0	0	0	0	0.0000	−0.0441	−0.0113	−0.0197	−0.0012	0.0301
83	0.0158	0	0	0	0	1	−0.0156	−0.0198	−0.0119	−0.0143	0.0000	0.0157
84	0.0798	1	0	0	0	0	0.0000	−0.0472	−0.0690	−0.0547	−0.1007	0.0768
85	0.0131	0	1	0	0	0	−0.0482	0.0000	−0.0143	−0.0112	−0.0111	0.0130
86	−0.0013	0	0	0	0	1	−0.0198	−0.0038	−0.0120	−0.0207	0.0000	−0.0015
87	0.0263	0	0	0	0	1	−0.0118	−0.0163	−0.0051	−0.0170	0.0000	0.0258
88	0.0551	0	1	0	0	0	−0.0032	0.0000	−0.0440	−0.0376	−0.0639	0.0536
89	0.0459	1	0	0	0	0	0.0000	−0.0476	−0.0259	−0.0424	−0.0354	0.0446
90	0.0392	0	1	0	0	0	−0.0246	0.0000	−0.0237	−0.0315	−0.0501	0.0390
91	0.0679	0	1	0	0	0	−0.0522	0.0000	−0.0454	−0.0425	−0.0684	0.0655
92	0.0242	1	0	0	0	0	0.0000	−0.0338	−0.0344	−0.0330	−0.0275	0.0240
93	−0.0009	0	0	0	1	0	−0.0239	−0.0157	−0.0107	0.0000	−0.0234	−0.0009
94	0.0421	0	0	0	0	1	−0.1205	−0.0823	−0.0689	−0.0598	0.0000	0.0408
95	0.0585	1	0	0	0	0	0.0000	−0.0393	−0.0559	−0.0490	−0.0541	0.0563
96	0.0027	1	0	0	0	0	0.0000	−0.0288	−0.0003	−0.0149	−0.0152	0.0023
97	0.0173	0	1	0	0	0	−0.0245	0.0000	−0.0537	−0.0756	−0.0212	0.0172
98	0.0248	1	0	0	0	0	0.0000	−0.0179	−0.0298	−0.0172	−0.0031	0.0244
99	0.0021	0	0	0	0	1	−0.0169	−0.0475	−0.0418	−0.0138	0.0000	0.0020
100	0.0730	0	0	1	0	0	−0.0007	−0.0374	0.0000	−0.0326	−0.0575	0.0655
101	−0.0075	0	0	0	0	1	0.0000	−0.0301	−0.0062	−0.0349	0.0000	−0.0075
102	0.0082	1	0	0	0	0	0.0000	−0.0648	−0.0099	−0.0115	−0.0137	0.0076
103	0.0017	0	1	0	0	0	−0.0214	0.0000	−0.0142	−0.0522	−0.0005	0.0016
104	0.0109	0	1	0	0	0	−0.0665	0.0000	−0.0169	−0.0107	−0.0025	0.0102
105	0.0914	1	0	0	0	0	0.0000	−0.1151	−0.0497	−0.0727	−0.0827	0.0867
106	0.0331	0	0	1	0	0	−0.0741	−0.0612	0.0000	−0.0394	−0.0140	0.0329
107	0.0446	0	0	1	0	0	−0.0180	−0.0384	0.0000	−0.0424	−0.0416	0.0443
108	0.0344	0	1	0	0	0	−0.0304	0.0000	−0.0318	−0.0134	−0.0364	0.0343

Table B.32: Optimisation: Downside risk, downward shock in Bond yields as indicator and an aggressive investment (continue)

	Solution						Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
1	0.0068	0	0	1	0	0	−0.0248	−0.0092	0.0000	−0.0092	−0.0135	0.0065
2	−0.0007	0	0	0	1	0	−0.0096	−0.0175	−0.0014	0.0000	−0.0127	−0.0007
3	−0.0056	0.3117	0	0	0.6883	0	0.0000	−0.0089	−0.0048	0.0000	−0.0060	−0.0058
4	0.0209	0	1	0	0	0	−0.0610	0.0000	−0.0093	−0.0197	−0.0114	0.0199
5	0.0659	0	1	0	0	0	−0.0014	0.0000	−0.0421	−0.0237	−0.0582	0.0597
6	0.0047	0	1	0	0	0	−0.0227	0.0000	−0.0034	−0.0132	−0.0036	0.0044
7	0.0277	1	0	0	0	0	0.0000	−0.0719	−0.0614	−0.0564	−0.0231	0.0262
8	0.0087	0	0	0	0	1	−0.0045	−0.0345	−0.0370	−0.0151	0.0000	0.0086
9	0.0211	0	1	0	0	0	−0.0472	0.0000	−0.0280	−0.0277	−0.0194	0.0208
10	0.0363	1	0	0	0	0	0.0000	−0.0207	−0.0225	−0.0254	−0.0159	0.0322
11	−0.0058	0	0	0	0	1	−0.0257	−0.0186	−0.0290	−0.0305	0.0000	−0.0059
12	0.0121	0	0	1	0	0	−0.0136	−0.0179	0.0000	−0.0106	−0.0188	0.0119
13	0.0250	0	1	0	0	0	−0.0282	0.0000	−0.0354	−0.0248	−0.0297	0.0240
14	0.0303	0	1	0	0	0	−0.0065	0.0000	−0.0271	−0.0234	−0.0206	0.0286
15	0.0237	1	0	0	0	0	0.0000	−0.0040	−0.0276	−0.0304	−0.0317	0.0222
16	−0.0028	0	1	0	0	0	−0.0068	0.0000	−0.0196	−0.0120	−0.0035	−0.0047
17	0.0487	1	0	0	0	0	0.0000	−0.0312	−0.0024	−0.0207	−0.0501	0.0438
18	0.0250	0	1	0	0	0	−0.0214	0.0000	−0.0482	−0.0456	−0.0215	0.0236
19	0.0126	0	1	0	0	0	−0.0212	0.0000	−0.0163	−0.0219	−0.0211	0.0111
20	0.0316	0	1	0	0	0	−0.0358	0.0000	−0.0363	−0.0313	−0.0375	0.0297
21	0.0317	0	1	0	0	0	−0.0047	0.0000	−0.0354	−0.0358	−0.0432	0.0315
22	0.0038	0	0	1	0	0	−0.0058	−0.0120	0.0000	−0.0097	−0.0183	0.0038
23	0.0449	1	0	0	0	0	0.0000	−0.0339	−0.0554	−0.0501	−0.0443	0.0428
24	0.0005	1	0	0	0	0	0.0000	−0.0238	−0.0384	−0.0222	−0.0161	−0.0002
25	0.0092	1	0	0	0	0	0.0000	−0.0245	−0.0047	−0.0074	−0.0112	0.0085
26	0.0007	0	0	0	0	1	−0.0211	−0.0265	−0.0243	−0.0255	0.0000	0.0007
27	0.0161	0	1	0	0	0	−0.0166	0.0000	−0.0313	−0.0163	−0.0206	0.0155
28	0.0228	1	0	0	0	0	0.0000	−0.0462	−0.0093	−0.0365	−0.0201	0.0219
29	0.0023	0	0	0	1	0	−0.0158	−0.0140	−0.0001	0.0000	−0.0165	0.0022
30	0.0223	0	0	1	0	0	−0.0902	−0.0394	0.0000	−0.0222	−0.0248	0.0190
31	−0.0174	0	0	0	0	1	−0.0811	−0.0640	−0.0732	−0.0604	0.0000	−0.0179
32	0.0218	0	0	1	0	0	−0.0565	−0.0248	0.0000	−0.0450	−0.0167	0.0204
33	0.0272	1	0	0	0	0	0.0000	−0.0408	−0.0344	−0.0084	−0.0336	0.0248
34	0.0100	0	0	1	0	0	−0.0408	−0.0254	0.0000	−0.0010	−0.0136	0.0092
35	−0.0386	0	0	0	0	1	−0.0195	−0.0638	−0.0115	−0.0264	0.0000	−0.0396
36	−0.0326	0	0	0	0	1	−0.0415	−0.0837	−0.0482	−0.0525	0.0000	−0.0338
37	0.1697	1	0	0	0	0	0.0000	−0.0807	−0.1704	−0.1489	−0.1417	0.1267
38	0.0619	0	1	0	0	0	−0.0536	0.0000	−0.0504	−0.0431	−0.0672	0.0611
39	−0.0281	1	0	0	0	0	0.0000	−0.0992	−0.0386	−0.0251	−0.0070	−0.0282
40	−0.0327	0	0	0	0	1	−0.0309	−0.0301	−0.0309	−0.0087	0.0000	−0.0327
41	0.0095	1	0	0	0	0	0.0000	−0.0850	−0.0690	−0.0470	−0.0146	0.0032
42	0.0036	0	1	0	0	0	−0.1283	0.0000	−0.2577	−0.1830	−0.0378	0.0022
43	0.0696	1	0	0	0	0	0.0000	−0.0514	−0.1390	−0.1291	−0.0516	0.0677
44	0.0376	1	0	0	0	0	0.0000	−0.1018	−0.1249	−0.0695	−0.0349	0.0351
45	0.0072	1	0	0	0	0	0.0000	−0.0158	−0.1003	−0.0534	−0.0121	0.0062
46	−0.0090	0	0	0	0	1	−0.0493	−0.0478	−0.1050	−0.0595	0.0000	−0.0092
47	0.0518	1	0	0	0	0	0.0000	−0.0537	−0.1426	−0.0739	−0.0623	0.0490
48	−0.0046	0	0	0	0	1	−0.0189	−0.0507	−0.0203	−0.0117	0.0000	−0.0050
49	0.0222	0	0	0	0	1	−0.0066	−0.0273	−0.0281	−0.0351	0.0000	0.0214
50	0.0678	0	1	0	0	0	−0.0147	0.0000	−0.0946	−0.0752	−0.0169	0.0632
51	0.0172	0	0	0	1	0	−0.0111	−0.0047	−0.0158	0.0000	−0.0299	0.0169
52	0.0414	0	0	0	0	1	−0.0831	−0.0853	−0.1106	−0.0887	0.0000	0.0390
53	0.0227	0	1	0	0	0	−0.0313	0.0000	−0.0565	−0.0243	−0.0151	0.0159
54	0.0144	0	0	0	0	1	−0.0128	−0.0590	−0.0475	−0.0568	0.0000	0.0136
55	0.0178	0	0	1	0	0	−0.0391	−0.0492	0.0000	−0.0308	−0.0279	−0.0034
56	0.0696	1	0	0	0	0	0.0000	−0.0291	−0.1085	−0.0776	−0.0486	0.0599
57	0.0541	1	0	0	0	0	0.0000	−0.0136	−0.0273	−0.0372	−0.0177	0.0510
58	0.0172	0	1	0	0	0	−0.0242	0.0000	−0.0531	−0.0750	−0.0209	0.0170

Table B.33: Optimisation: Downside risk, upward shocks in rate of change of bond yields as indicator and a conservative investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
1	0.0567	1	0	0	0	0	0.0000	−0.0541	−0.0566	−0.0597	−0.0386	0.0539
2	−0.0016	0	0	0	1	0	−0.0206	−0.0112	−0.0044	0.0000	−0.0034	−0.0017
3	−0.0076	0	0	0	0	1	−0.0326	−0.0071	−0.0145	−0.0061	0.0000	−0.0077
4	−0.0083	0	0	0	0	1	−0.0194	−0.0539	−0.0220	−0.0201	0.0000	−0.0084
5	0.0438	1	0	0	0	0	0.0000	−0.0377	−0.0313	−0.0319	−0.0368	0.0392
6	0.0190	0.0262	0.9738	0	0	0	0.0000	0.0000	−0.0230	−0.0012	−0.0186	0.0180
7	0.0191	0	0	0	1	0	−0.0115	−0.0250	−0.0172	0.0000	−0.0050	0.0179
8	0.0663	1	0	0	0	0	0.0000	−0.0579	−0.0510	−0.0409	−0.0437	0.0559
9	0.0426	0	0	0	0	1	−0.0086	−0.0102	−0.0114	−0.0052	0.0000	0.0408
10	0.0218	1	0	0	0	0	0.0000	−0.0239	−0.0226	−0.0102	−0.0199	0.0160
11	0.0522	0	1	0	0	0	−0.0659	0.0000	−0.0090	−0.0181	−0.0390	0.0471
12	0.0625	0	1	0	0	0	−0.0110	0.0000	−0.0142	−0.0058	−0.0417	0.0535
13	0.0911	0	1	0	0	0	−0.0457	0.0000	−0.0723	−0.0780	−0.0701	0.0822
14	0.0149	0	0	1	0	0	−0.0223	−0.0019	0.0000	−0.0049	−0.0262	0.0143
15	0.0113	0	0	0	1	0	−0.0075	−0.0065	−0.0089	0.0000	−0.0105	0.0111
16	0.0146	0	0	0	0	1	−0.0238	−0.0223	−0.0119	−0.0073	0.0000	0.0140
17	0.0101	0	0	1	0	0	−0.0401	−0.0362	0.0000	−0.0095	−0.0145	0.0096
18	0.0500	1	0	0	0	0	0.0000	−0.0633	−0.0082	−0.0332	−0.0296	0.0457
19	0.0724	1	0	0	0	0	0.0000	−0.0354	−0.0380	−0.0401	−0.0628	0.0620
20	0.0279	0	0	0	1	0	−0.0167	−0.0196	−0.0051	0.0000	−0.0392	0.0269
21	0.0272	0	0	0	0	1	−0.0364	−0.0295	−0.0143	−0.0150	0.0000	0.0268
22	0.0257	0	0	1	0	0	−0.0264	−0.0161	0.0000	−0.0089	−0.0162	0.0251
23	0.0266	0	1	0	0	0	−0.0296	0.0000	−0.0089	−0.0066	−0.0386	0.0243
24	0.0245	0	0	0	0	1	−0.0257	−0.0569	−0.0127	−0.0240	0.0000	0.0238
25	0.0327	0	0	1	0	0	−0.0160	−0.0250	0.0000	−0.0053	−0.0253	0.0318
26	0.0162	0	0	0	1	0	−0.0336	−0.0098	−0.0125	0.0000	−0.0126	0.0159
27	0.0246	0	0	1	0	0	−0.0380	−0.0192	0.0000	−0.0211	−0.0364	0.0244
28	0.0571	0	0	1	0	0	−0.0111	−0.0302	0.0000	−0.0334	−0.0415	0.0501
29	0.0418	0.9698	0.0302	0	0	0	0.0000	0.0000	−0.0471	−0.0279	−0.0504	0.0417
30	−0.0438	0	0	0	0	1	−0.0202	−0.0398	−0.0238	−0.0129	0.0000	−0.0455
31	0.1103	1	0	0	0	0	0.0000	−0.0812	−0.1980	−0.1451	−0.1259	0.0969
32	0.1026	0	0	1	0	0	−0.1151	−0.0609	0.0000	−0.0651	−0.0860	0.0881
33	0.0691	0	0	1	0	0	−0.0203	−0.0626	0.0000	−0.0151	−0.0323	0.0384
34	0.0555	0	0	1	0	0	−0.0366	−0.0566	0.0000	−0.0508	−0.0557	0.0499
35	0.1903	0	0	1	0	0	−0.1501	−0.0436	0.0000	−0.0661	−0.1629	0.1606
36	0.0488	0	0	1	0	0	−0.0408	−0.0322	0.0000	−0.0040	−0.0148	0.0460
37	0.0099	0	0	0.2129	0.7871	0	−0.0087	−0.0242	0.0000	0.0000	−0.0534	0.0096
38	0.0566	0	0	1	0	0	−0.0672	−0.0533	0.0000	−0.0293	−0.0229	0.0541
39	0.0799	0	0	1	0	0	−0.0366	−0.0467	0.0000	−0.0160	−0.0603	0.0521
40	0.0584	0	0	1	0	0	−0.0451	−0.0490	0.0000	−0.0245	−0.0553	0.0503
41	0.0305	0	0	1	0	0	−0.0269	−0.0538	0.0000	−0.0338	−0.0038	0.0294
42	0.0242	0	1	0	0	0	−0.0149	0.0000	−0.0212	−0.0144	−0.0313	0.0235
43	0.0674	1	0	0	0	0	0.0000	−0.0329	−0.0718	−0.0677	−0.0477	0.0605
44	0.0638	0	1	0	0	0	−0.0442	0.0000	−0.0207	−0.0123	−0.0599	0.0574
45	0.0246	1	0	0	0	0	0.0000	−0.0489	−0.0433	−0.0390	−0.0378	0.0224
46	0.0445	0	0	0	0	1	−0.0655	−0.0406	−0.0306	−0.0266	0.0000	0.0411
47	0.0869	0	0	1	0	0	−0.0950	−0.0916	0.0000	−0.0228	−0.0711	0.0859
48	0.0248	0	0	1	0	0	−0.0262	−0.0285	0.0000	−0.0009	−0.0228	0.0189
49	0.0531	0	0	1	0	0	−0.0372	−0.0615	0.0000	−0.0031	−0.0473	0.0510
50	0.0157	0	0	0	0	1	−0.0153	−0.0196	−0.0119	−0.0144	0.0000	0.0156
51	0.0658	0.9465	0	0.0535	0	0	0.0000	−0.0328	0.0000	−0.0269	−0.0471	0.0541
52	0.0101	0	1	0	0	0	−0.0677	0.0000	−0.0160	−0.0092	−0.0005	0.0086

Table B.34: Optimisation: Downside risk, downward shocks in rate of change of bond yields as indicator and a conservative investment

	Solution						Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment												
1	0.0069	0	0	1	0	0	−0.0247	−0.0092	0.0000	−0.0093	−0.0137	0.0067
2	−0.0007	0	0	0	1	0	−0.0096	−0.0175	−0.0014	0.0000	−0.0128	−0.0007
3	−0.0056	0.4732	0	0	0.5268	0	0.0000	−0.0088	−0.0048	0.0000	−0.0060	−0.0058
4	0.0212	0	1	0	0	0	−0.0625	0.0000	−0.0097	−0.0203	−0.0118	0.0205
5	0.0676	0	1	0	0	0	−0.0012	0.0000	−0.0435	−0.0250	−0.0612	0.0632
6	0.0048	0	1	0	0	0	−0.0226	0.0000	−0.0035	−0.0132	−0.0037	0.0046
7	0.0281	1	0	0	0	0	0.0000	−0.0734	−0.0630	−0.0578	−0.0239	0.0270
8	0.0087	0	0	0	0	1	−0.0045	−0.0347	−0.0371	−0.0152	0.0000	0.0087
9	0.0212	0	1	0	0	0	−0.0473	0.0000	−0.0281	−0.0278	−0.0196	0.0210
10	0.0375	1	0	0	0	0	0.0000	−0.0222	−0.0242	−0.0273	−0.0178	0.0345
11	−0.0058	0	0	0	0	1	−0.0257	−0.0187	−0.0289	−0.0304	0.0000	−0.0058
12	0.0122	0	0	1	0	0	−0.0138	−0.0181	0.0000	−0.0107	−0.0190	0.0120
13	0.0253	0	1	0	0	0	−0.0284	0.0000	−0.0362	−0.0254	−0.0305	0.0246
14	0.0308	0	1	0	0	0	−0.0070	0.0000	−0.0280	−0.0242	−0.0212	0.0295
15	0.0241	1	0	0	0	0	0.0000	−0.0045	−0.0285	−0.0314	−0.0328	0.0230
16	−0.0023	0	1	0	0	0	−0.0075	0.0000	−0.0206	−0.0129	−0.0046	−0.0037
17	0.0501	1	0	0	0	0	0.0000	−0.0320	−0.0019	−0.0217	−0.0530	0.0465
18	0.0253	0	1	0	0	0	−0.0216	0.0000	−0.0492	−0.0465	−0.0223	0.0243
19	0.0130	0	1	0	0	0	−0.0222	0.0000	−0.0170	−0.0228	−0.0221	0.0119
20	0.0321	0	1	0	0	0	−0.0369	0.0000	−0.0375	−0.0326	−0.0387	0.0308
21	0.0317	0	1	0	0	0	−0.0047	0.0000	−0.0355	−0.0359	−0.0433	0.0316
22	0.0038	0	0	1	0	0	−0.0057	−0.0120	0.0000	−0.0098	−0.0183	0.0038
23	0.0455	1	0	0	0	0	0.0000	−0.0345	−0.0575	−0.0518	−0.0454	0.0440
24	0.0007	1	0	0	0	0	0.0000	−0.0243	−0.0391	−0.0226	−0.0167	0.0002
25	0.0094	1	0	0	0	0	0.0000	−0.0250	−0.0049	−0.0075	−0.0116	0.0089
26	0.0007	0	0	0	0	1	−0.0211	−0.0265	−0.0243	−0.0255	0.0000	0.0007
27	0.0163	0	1	0	0	0	−0.0170	0.0000	−0.0319	−0.0167	−0.0210	0.0159
28	0.0231	1	0	0	0	0	0.0000	−0.0472	−0.0096	−0.0373	−0.0204	0.0224
29	0.0023	0	0	0	1	0	−0.0158	−0.0140	−0.0001	0.0000	−0.0166	0.0023
30	0.0232	0	0	1	0	0	−0.0903	−0.0403	0.0000	−0.0228	−0.0261	0.0208
31	−0.0172	0	0	0	0	1	−0.0798	−0.0627	−0.0715	−0.0591	0.0000	−0.0176
32	0.0222	0	0	1	0	0	−0.0558	−0.0243	0.0000	−0.0445	−0.0174	0.0212
33	0.0279	1	0	0	0	0	0.0000	−0.0436	−0.0363	−0.0098	−0.0353	0.0261
34	0.0102	0	0	1	0	0	−0.0414	−0.0254	0.0000	−0.0012	−0.0142	0.0097
35	−0.0383	0	0	0	0	1	−0.0194	−0.0625	−0.0113	−0.0259	0.0000	−0.0390
36	−0.0323	0	0	0	0	1	−0.0406	−0.0815	−0.0472	−0.0512	0.0000	−0.0331
37	0.1818	1	0	0	0	0	0.0000	−0.0907	−0.1994	−0.1720	−0.1675	0.1508
38	0.0621	0	1	0	0	0	−0.0549	0.0000	−0.0499	−0.0431	−0.0677	0.0615
39	−0.0280	1	0	0	0	0	0.0000	−0.0987	−0.0383	−0.0249	−0.0069	−0.0281
40	−0.0327	0	0	0	0	1	−0.0308	−0.0300	−0.0309	−0.0086	0.0000	−0.0327
41	0.0113	1	0	0	0	0	0.0000	−0.0891	−0.0727	−0.0506	−0.0182	0.0067
42	0.0040	0	1	0	0	0	−0.1266	0.0000	−0.2536	−0.1806	−0.0379	0.0030
43	0.0701	1	0	0	0	0	0.0000	−0.0546	−0.1425	−0.1318	−0.0526	0.0688
44	0.0382	1	0	0	0	0	0.0000	−0.1038	−0.1294	−0.0719	−0.0366	0.0365
45	0.0075	1	0	0	0	0	0.0000	−0.0163	−0.1022	−0.0544	−0.0127	0.0068
46	−0.0090	0	0	0	0	1	−0.0490	−0.0477	−0.1046	−0.0593	0.0000	−0.0091
47	0.0526	1	0	0	0	0	0.0000	−0.0563	−0.1474	−0.0768	−0.0645	0.0506
48	−0.0045	0	0	0	0	1	−0.0194	−0.0511	−0.0210	−0.0121	0.0000	−0.0048
49	0.0224	0	0	0	0	1	−0.0068	−0.0275	−0.0283	−0.0356	0.0000	0.0218
50	0.0691	0	1	0	0	0	−0.0141	0.0000	−0.0982	−0.0781	−0.0155	0.0658
51	0.0172	0	0	0	1	0	−0.0112	−0.0047	−0.0159	0.0000	−0.0300	0.0171
52	0.0421	0	0	0	0	1	−0.0848	−0.0879	−0.1133	−0.0909	0.0000	0.0404
53	0.0246	0	1	0	0	0	−0.0332	0.0000	−0.0621	−0.0283	−0.0190	0.0197
54	0.0146	0	0	0	0	1	−0.0131	−0.0606	−0.0487	−0.0584	0.0000	0.0140
55	0.0237	0	0	1	0	0	−0.0482	−0.0535	0.0000	−0.0314	−0.0371	0.0084
56	0.0723	1	0	0	0	0	0.0000	−0.0279	−0.1165	−0.0847	−0.0537	0.0653
57	0.0550	1	0	0	0	0	0.0000	−0.0139	−0.0280	−0.0379	−0.0192	0.0528
58	0.0173	0	1	0	0	0	−0.0244	0.0000	−0.0534	−0.0754	−0.0211	0.0171

Table B.35: Optimisation: Downside risk, upward shocks in rate of change of bond yields as indicator and a moderate investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment												
1	0.0575	1	0	0	0	0	0.0000	−0.0554	−0.0582	−0.0616	−0.0398	0.0555
2	−0.0016	0	0	0	1	0	−0.0202	−0.0112	−0.0045	0.0000	−0.0034	−0.0017
3	−0.0076	0	0	0	0	1	−0.0325	−0.0071	−0.0145	−0.0061	0.0000	−0.0077
4	−0.0083	0	0	0	0	1	−0.0194	−0.0538	−0.0219	−0.0200	0.0000	−0.0083
5	0.0452	1	0	0	0	0	0.0000	−0.0396	−0.0337	−0.0343	−0.0394	0.0418
6	0.0193	0.1205	0.8795	0	0	0	0.0000	0.0000	−0.0232	−0.0016	−0.0190	0.0185
7	0.0194	0	0	0	1	0	−0.0115	−0.0255	−0.0178	0.0000	−0.0053	0.0186
8	0.0692	1	0	0	0	0	0.0000	−0.0632	−0.0552	−0.0447	−0.0481	0.0617
9	0.0431	0	0	0	0	1	−0.0090	−0.0106	−0.0117	−0.0055	0.0000	0.0418
10	0.0235	1	0	0	0	0	0.0000	−0.0255	−0.0250	−0.0124	−0.0235	0.0193
11	0.0536	0	1	0	0	0	−0.0695	0.0000	−0.0087	−0.0192	−0.0414	0.0500
12	0.0651	0	1	0	0	0	−0.0118	0.0000	−0.0154	−0.0064	−0.0455	0.0586
13	0.0935	0	1	0	0	0	−0.0481	0.0000	−0.0770	−0.0828	−0.0739	0.0872
14	0.0151	0	0	1	0	0	−0.0222	−0.0018	0.0000	−0.0050	−0.0267	0.0146
15	0.0114	0	0	0	1	0	−0.0076	−0.0066	−0.0089	0.0000	−0.0106	0.0112
16	0.0148	0	0	0	0	1	−0.0244	−0.0228	−0.0122	−0.0075	0.0000	0.0143
17	0.0102	0	0	1	0	0	−0.0408	−0.0367	0.0000	−0.0095	−0.0148	0.0098
18	0.0512	1	0	0	0	0	0.0000	−0.0673	−0.0088	−0.0354	−0.0315	0.0481
19	0.0753	1	0	0	0	0	0.0000	−0.0376	−0.0417	−0.0432	−0.0688	0.0678
20	0.0282	0	0	0	1	0	−0.0169	−0.0201	−0.0052	0.0000	−0.0401	0.0275
21	0.0273	0	0	0	0	1	−0.0371	−0.0299	−0.0144	−0.0151	0.0000	0.0270
22	0.0258	0	0	1	0	0	−0.0267	−0.0164	0.0000	−0.0090	−0.0164	0.0254
23	0.0273	0	1	0	0	0	−0.0309	0.0000	−0.0094	−0.0072	−0.0404	0.0256
24	0.0247	0	0	0	0	1	−0.0263	−0.0581	−0.0130	−0.0244	0.0000	0.0242
25	0.0329	0	0	1	0	0	−0.0164	−0.0255	0.0000	−0.0053	−0.0257	0.0323
26	0.0162	0	0	0	1	0	−0.0340	−0.0100	−0.0125	0.0000	−0.0127	0.0160
27	0.0247	0	0	1	0	0	−0.0383	−0.0194	0.0000	−0.0212	−0.0367	0.0245
28	0.0591	0	0	1	0	0	−0.0130	−0.0341	0.0000	−0.0360	−0.0444	0.0540
29	0.0419	0.7929	0.2071	0	0	0	0.0000	0.0000	−0.0471	−0.0280	−0.0504	0.0417
30	−0.0433	0	0	0	0	1	−0.0194	−0.0386	−0.0235	−0.0127	0.0000	−0.0445
31	0.1140	1	0	0	0	0	0.0000	−0.0915	−0.2136	−0.1553	−0.1332	0.1044
32	0.1067	0	0	1	0	0	−0.1246	−0.0662	0.0000	−0.0718	−0.0930	0.0962
33	0.0777	0	0	1	0	0	−0.0381	−0.0726	0.0000	−0.0174	−0.0463	0.0556
34	0.0571	0	0	1	0	0	−0.0393	−0.0593	0.0000	−0.0533	−0.0580	0.0530
35	0.1986	0	0	1	0	0	−0.1659	−0.0468	0.0000	−0.0752	−0.1799	0.1772
36	0.0496	0	0	1	0	0	−0.0419	−0.0330	0.0000	−0.0042	−0.0157	0.0476
37	0.0100	0	0	0.5023	0.4977	0	−0.0087	−0.0242	0.0000	0.0000	−0.0534	0.0097
38	0.0573	0	0	1	0	0	−0.0693	−0.0551	0.0000	−0.0300	−0.0235	0.0555
39	0.0877	0	0	1	0	0	−0.0473	−0.0583	0.0000	−0.0217	−0.0729	0.0677
40	0.0607	0	0	1	0	0	−0.0490	−0.0521	0.0000	−0.0260	−0.0597	0.0548
41	0.0308	0	0	1	0	0	−0.0272	−0.0544	0.0000	−0.0344	−0.0043	0.0300
42	0.0245	0	1	0	0	0	−0.0153	0.0000	−0.0216	−0.0146	−0.0318	0.0239
43	0.0693	1	0	0	0	0	0.0000	−0.0358	−0.0768	−0.0721	−0.0511	0.0644
44	0.0656	0	1	0	0	0	−0.0456	0.0000	−0.0214	−0.0124	−0.0640	0.0610
45	0.0252	1	0	0	0	0	0.0000	−0.0510	−0.0455	−0.0410	−0.0394	0.0236
46	0.0455	0	0	0	0	1	−0.0680	−0.0428	−0.0314	−0.0278	0.0000	0.0430
47	0.0872	0	0	1	0	0	−0.0959	−0.0929	0.0000	−0.0230	−0.0718	0.0864
48	0.0265	0	0	1	0	0	−0.0284	−0.0318	0.0000	−0.0012	−0.0265	0.0222
49	0.0538	0	0	1	0	0	−0.0386	−0.0633	0.0000	−0.0033	−0.0488	0.0522
50	0.0158	0	0	0	0	1	−0.0155	−0.0197	−0.0119	−0.0143	0.0000	0.0157
51	0.0693	0.3563	0	0.6437	0	0	0.0000	−0.0346	0.0000	−0.0295	−0.0518	0.0593
52	0.0105	0	1	0	0	0	−0.0671	0.0000	−0.0165	−0.0100	−0.0016	0.0094

Table B.36: Optimisation: Downside risk, downward shocks in rate of change of bond yields as indicator and a moderate investment

		Solution					Reduced Gradient					
Upward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
1	0.0070	0	0	1	0	0	−0.0246	−0.0093	0.0000	−0.0094	−0.0139	0.0068
2	−0.0007	0	0	0	1	0	−0.0096	−0.0176	−0.0014	0.0000	−0.0128	−0.0007
3	−0.0054	0.7616	0	0	0.2384	0	0.0000	−0.0088	−0.0048	0.0000	−0.0060	−0.0058
4	0.0215	0	1	0	0	0	−0.0637	0.0000	−0.0100	−0.0208	−0.0120	0.0210
5	0.0691	0	1	0	0	0	−0.0011	0.0000	−0.0447	−0.0262	−0.0637	0.0662
6	0.0048	0	1	0	0	0	−0.0224	0.0000	−0.0036	−0.0133	−0.0039	0.0047
7	0.0284	1	0	0	0	0	0.0000	−0.0748	−0.0644	−0.0590	−0.0246	0.0277
8	0.0087	0	0	0	0	1	−0.0046	−0.0348	−0.0372	−0.0152	0.0000	0.0087
9	0.0213	0	1	0	0	0	−0.0475	0.0000	−0.0282	−0.0279	−0.0197	0.0211
10	0.0385	1	0	0	0	0	0.0000	−0.0235	−0.0257	−0.0289	−0.0194	0.0365
11	−0.0057	0	0	0	0	1	−0.0257	−0.0189	−0.0288	−0.0304	0.0000	−0.0058
12	0.0122	0	0	1	0	0	−0.0140	−0.0182	0.0000	−0.0107	−0.0191	0.0121
13	0.0256	0	1	0	0	0	−0.0286	0.0000	−0.0368	−0.0258	−0.0312	0.0251
14	0.0312	0	1	0	0	0	−0.0075	0.0000	−0.0288	−0.0248	−0.0217	0.0303
15	0.0245	1	0	0	0	0	0.0000	−0.0049	−0.0293	−0.0322	−0.0337	0.0238
16	−0.0018	0	1	0	0	0	−0.0081	0.0000	−0.0214	−0.0136	−0.0056	−0.0027
17	0.0513	1	0	0	0	0	0.0000	−0.0327	−0.0015	−0.0226	−0.0556	0.0489
18	0.0257	0	1	0	0	0	−0.0217	0.0000	−0.0501	−0.0472	−0.0229	0.0250
19	0.0134	0	1	0	0	0	−0.0230	0.0000	−0.0177	−0.0236	−0.0228	0.0127
20	0.0326	0	1	0	0	0	−0.0377	0.0000	−0.0386	−0.0337	−0.0398	0.0317
21	0.0317	0	1	0	0	0	−0.0046	0.0000	−0.0356	−0.0360	−0.0434	0.0317
22	0.0038	0	0	1	0	0	−0.0057	−0.0121	0.0000	−0.0098	−0.0183	0.0038
23	0.0460	1	0	0	0	0	0.0000	−0.0350	−0.0593	−0.0532	−0.0464	0.0450
24	0.0009	1	0	0	0	0	0.0000	−0.0248	−0.0396	−0.0228	−0.0172	0.0005
25	0.0095	1	0	0	0	0	0.0000	−0.0253	−0.0051	−0.0077	−0.0120	0.0092
26	0.0007	0	0	0	0	1	−0.0211	−0.0264	−0.0243	−0.0255	0.0000	0.0007
27	0.0164	0	1	0	0	0	−0.0173	0.0000	−0.0324	−0.0170	−0.0214	0.0161
28	0.0233	1	0	0	0	0	0.0000	−0.0482	−0.0098	−0.0379	−0.0207	0.0229
29	0.0023	0	0	0	1	0	−0.0158	−0.0141	−0.0001	0.0000	−0.0166	0.0023
30	0.0240	0	0	1	0	0	−0.0903	−0.0410	0.0000	−0.0233	−0.0272	0.0224
31	−0.0171	0	0	0	0	1	−0.0787	−0.0616	−0.0701	−0.0579	0.0000	−0.0173
32	0.0226	0	0	1	0	0	−0.0551	−0.0239	0.0000	−0.0441	−0.0180	0.0219
33	0.0284	1	0	0	0	0	0.0000	−0.0459	−0.0380	−0.0110	−0.0369	0.0273
34	0.0104	0	0	1	0	0	−0.0420	−0.0254	0.0000	−0.0013	−0.0147	0.0100
35	−0.0381	0	0	0	0	1	−0.0192	−0.0614	−0.0110	−0.0255	0.0000	−0.0386
36	−0.0320	0	0	0	0	1	−0.0398	−0.0796	−0.0463	−0.0502	0.0000	−0.0326
37	0.1921	1	0	0	0	0	0.0000	−0.0994	−0.2242	−0.1918	−0.1896	0.1714
38	0.0623	0	1	0	0	0	−0.0560	0.0000	−0.0494	−0.0431	−0.0681	0.0619
39	−0.0280	1	0	0	0	0	0.0000	−0.0983	−0.0380	−0.0247	−0.0067	−0.0281
40	−0.0327	0	0	0	0	1	−0.0306	−0.0300	−0.0309	−0.0086	0.0000	−0.0327
41	0.0128	1	0	0	0	0	0.0000	−0.0926	−0.0759	−0.0536	−0.0212	0.0097
42	0.0043	0	1	0	0	0	−0.1252	0.0000	−0.2501	−0.1785	−0.0380	0.0037
43	0.0705	1	0	0	0	0	0.0000	−0.0573	−0.1455	−0.1341	−0.0535	0.0696
44	0.0388	1	0	0	0	0	0.0000	−0.1056	−0.1332	−0.0740	−0.0381	0.0377
45	0.0078	1	0	0	0	0	0.0000	−0.0167	−0.1038	−0.0553	−0.0131	0.0073
46	−0.0089	0	0	0	0	1	−0.0488	−0.0476	−0.1043	−0.0592	0.0000	−0.0090
47	0.0532	1	0	0	0	0	0.0000	−0.0585	−0.1515	−0.0794	−0.0663	0.0519
48	−0.0044	0	0	0	0	1	−0.0197	−0.0515	−0.0215	−0.0124	0.0000	−0.0046
49	0.0226	0	0	0	0	1	−0.0071	−0.0276	−0.0286	−0.0360	0.0000	0.0222
50	0.0702	0	1	0	0	0	−0.0135	0.0000	−0.1013	−0.0805	−0.0142	0.0680
51	0.0173	0	0	0	1	0	−0.0112	−0.0046	−0.0160	0.0000	−0.0301	0.0172
52	0.0427	0	0	0	0	1	−0.0862	−0.0900	−0.1156	−0.0929	0.0000	0.0415
53	0.0262	0	1	0	0	0	−0.0349	0.0000	−0.0668	−0.0317	−0.0224	0.0230
54	0.0148	0	0	0	0	1	−0.0135	−0.0620	−0.0497	−0.0598	0.0000	0.0144
55	0.0288	0	0	1	0	0	−0.0560	−0.0572	0.0000	−0.0320	−0.0450	0.0186
56	0.0747	1	0	0	0	0	0.0000	−0.0268	−0.1233	−0.0907	−0.0581	0.0700
57	0.0558	1	0	0	0	0	0.0000	−0.0141	−0.0286	−0.0385	−0.0205	0.0543
58	0.0173	0	1	0	0	0	−0.0245	0.0000	−0.0537	−0.0756	−0.0212	0.0172

Table B.37: Optimisation: Downside risk, upward shocks in rate of change of bond yields as indicator and an aggressive investment

	Solution						Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment												
1	0.0582	1	0	0	0	0	0.0000	−0.0566	−0.0596	−0.0631	−0.0407	0.0569
2	−0.0016	0	0	0	1	0	−0.0200	−0.0112	−0.0045	0.0000	−0.0034	−0.0016
3	−0.0076	0	0	0	0	1	−0.0325	−0.0071	−0.0145	−0.0061	0.0000	−0.0076
4	−0.0083	0	0	0	0	1	−0.0193	−0.0537	−0.0218	−0.0199	0.0000	−0.0083
5	0.0463	1	0	0	0	0	0.0000	−0.0412	−0.0357	−0.0363	−0.0416	0.0440
6	0.0196	0.2888	0.7112	0	0	0	0.0000	0.0000	−0.0235	−0.0020	−0.0194	0.0190
7	0.0197	0	0	0	1	0	−0.0116	−0.0259	−0.0183	0.0000	−0.0056	0.0191
8	0.0717	1	0	0	0	0	0.0000	−0.0677	−0.0588	−0.0479	−0.0518	0.0667
9	0.0435	0	0	0	0	1	−0.0093	−0.0111	−0.0119	−0.0058	0.0000	0.0427
10	0.0249	1	0	0	0	0	0.0000	−0.0270	−0.0270	−0.0143	−0.0265	0.0221
11	0.0548	0	1	0	0	0	−0.0725	0.0000	−0.0085	−0.0201	−0.0435	0.0524
12	0.0672	0	1	0	0	0	−0.0125	0.0000	−0.0166	−0.0069	−0.0487	0.0629
13	0.0957	0	1	0	0	0	−0.0503	0.0000	−0.0810	−0.0870	−0.0772	0.0914
14	0.0152	0	0	1	0	0	−0.0222	−0.0017	0.0000	−0.0051	−0.0271	0.0149
15	0.0115	0	0	0	1	0	−0.0077	−0.0067	−0.0090	0.0000	−0.0107	0.0113
16	0.0149	0	0	0	0	1	−0.0249	−0.0232	−0.0125	−0.0077	0.0000	0.0146
17	0.0103	0	0	1	0	0	−0.0414	−0.0371	0.0000	−0.0096	−0.0151	0.0101
18	0.0523	1	0	0	0	0	0.0000	−0.0707	−0.0093	−0.0373	−0.0332	0.0502
19	0.0778	1	0	0	0	0	0.0000	−0.0394	−0.0449	−0.0460	−0.0739	0.0728
20	0.0285	0	0	0	1	0	−0.0172	−0.0206	−0.0054	0.0000	−0.0409	0.0280
21	0.0275	0	0	0	0	1	−0.0376	−0.0302	−0.0144	−0.0152	0.0000	0.0272
22	0.0260	0	0	1	0	0	−0.0270	−0.0166	0.0000	−0.0092	−0.0166	0.0257
23	0.0278	0	1	0	0	0	−0.0320	0.0000	−0.0098	−0.0078	−0.0419	0.0267
24	0.0248	0	0	0	0	1	−0.0267	−0.0590	−0.0133	−0.0248	0.0000	0.0245
25	0.0331	0	0	1	0	0	−0.0167	−0.0258	0.0000	−0.0052	−0.0261	0.0327
26	0.0163	0	0	0	1	0	−0.0342	−0.0101	−0.0126	0.0000	−0.0128	0.0162
27	0.0248	0	0	1	0	0	−0.0386	−0.0195	0.0000	−0.0213	−0.0369	0.0246
28	0.0608	0	0	1	0	0	−0.0147	−0.0374	0.0000	−0.0383	−0.0468	0.0574
29	0.0420	0.4967	0.5033	0	0	0	0.0000	0.0000	−0.0471	−0.0280	−0.0504	0.0417
30	−0.0429	0	0	0	0	1	−0.0186	−0.0376	−0.0232	−0.0124	0.0000	−0.0437
31	0.1172	1	0	0	0	0	0.0000	−0.1003	−0.2269	−0.1640	−0.1394	0.1108
32	0.1102	0	0	1	0	0	−0.1328	−0.0708	0.0000	−0.0775	−0.0990	0.1032
33	0.0850	0	0	1	0	0	−0.0533	−0.0810	0.0000	−0.0193	−0.0583	0.0703
34	0.0585	0	0	1	0	0	−0.0417	−0.0615	0.0000	−0.0554	−0.0600	0.0558
35	0.2057	0	0	1	0	0	−0.1794	−0.0495	0.0000	−0.0831	−0.1945	0.1915
36	0.0503	0	0	1	0	0	−0.0428	−0.0337	0.0000	−0.0043	−0.0164	0.0489
37	0.0102	0	0	1	0	0	−0.0087	−0.0242	0.0000	0.0000	−0.0534	0.0097
38	0.0579	0	0	1	0	0	−0.0710	−0.0567	0.0000	−0.0306	−0.0239	0.0567
39	0.0944	0	0	1	0	0	−0.0565	−0.0683	0.0000	−0.0266	−0.0837	0.0810
40	0.0626	0	0	1	0	0	−0.0524	−0.0548	0.0000	−0.0273	−0.0634	0.0587
41	0.0311	0	0	1	0	0	−0.0275	−0.0550	0.0000	−0.0349	−0.0047	0.0305
42	0.0246	0	1	0	0	0	−0.0156	0.0000	−0.0220	−0.0148	−0.0322	0.0243
43	0.0710	1	0	0	0	0	0.0000	−0.0384	−0.0811	−0.0758	−0.0540	0.0677
44	0.0671	0	1	0	0	0	−0.0469	0.0000	−0.0220	−0.0124	−0.0675	0.0641
45	0.0257	1	0	0	0	0	0.0000	−0.0528	−0.0474	−0.0427	−0.0407	0.0247
46	0.0463	0	0	0	0	1	−0.0702	−0.0446	−0.0321	−0.0288	0.0000	0.0447
47	0.0874	0	0	1	0	0	−0.0967	−0.0939	0.0000	−0.0231	−0.0724	0.0869
48	0.0279	0	0	1	0	0	−0.0302	−0.0346	0.0000	−0.0014	−0.0298	0.0251
49	0.0543	0	0	1	0	0	−0.0397	−0.0649	0.0000	−0.0036	−0.0502	0.0532
50	0.0158	0	0	0	0	1	−0.0156	−0.0198	−0.0119	−0.0143	0.0000	0.0157
51	0.0730	0	0	1	0	0	−0.0007	−0.0374	0.0000	−0.0326	−0.0575	0.0655
52	0.0109	0	1	0	0	0	−0.0665	0.0000	−0.0169	−0.0107	−0.0025	0.0102

Table B.38: Optimisation: Downside risk, downward shocks in rate of change of bond yields as indicator and an aggressive investment

Shock	Window	Objective Value	Solution CI09	Solution CI10	Solution CI24	Solution CI27	Solution CI70	Reduced Gradient CI09	Reduced Gradient CI10	Reduced Gradient CI24	Reduced Gradient CI27	Reduced Gradient CI70	Lagrange Multiplier
Conservative Investment													
Bull	1	0.1446	0	1	0	0	0	−0.1372	0.0000	−0.1010	−0.1130	−0.1215	0.1159
	2	0.1895	0	1	0	0	0	−0.0876	0.0000	−0.1033	−0.1191	−0.1544	0.1234
	3	0.0929	0	0	0	1	0	−0.0050	−0.0063	−0.0320	0.0000	−0.0785	0.0841
	4	0.1830	0	1	0	0	0	−0.0695	0.0000	−0.0685	−0.0293	−0.0520	0.1309
	5	0.1389	1	0	0	0	0	0.0000	−0.0726	−0.0442	−0.0323	−0.2019	0.1026
	6	0.2029	0	1	0	0	0	−0.1453	0.0000	−0.1144	−0.1237	−0.1774	0.1819
	7	0.0506	0	0	1	0	0	−0.0465	−0.0689	0.0000	−0.0245	−0.0009	0.0493
	8	0.0852	0	0	1	0	0	−0.0129	−0.0321	0.0000	−0.0389	−0.0891	0.0775
	9	0.0868	0	0	1	0	0	−0.0132	−0.0828	0.0000	−0.0281	−0.0958	0.0835
	10	0.1008	0	0.9046	0.0954	0	0	−0.1143	0.0000	0.0000	−0.0522	−0.0359	0.0857
	11	0.0536	0	0	1	0	0	−0.0073	−0.0571	0.0000	−0.0273	−0.0444	0.0484
	12	0.0532	0.0517	0	0	0.9483	0	0.0000	−0.0464	−0.0199	0.0000	−0.0247	0.0503
	13	0.2818	0	0	1	0	0	−0.2257	−0.1871	0.0000	−0.1041	−0.1616	0.2309
	14	0.1980	0	1	0	0	0	−0.0292	0.0000	−0.0738	−0.0681	−0.1412	0.1439
	15	0.1300	1	0	0	0	0	0.0000	−0.0212	−0.0461	−0.0050	−0.0332	0.0603
	16	0.2554	0	0	1	0	0	−0.0875	−0.4121	0.0000	−0.6575	−0.1458	0.1580
	17	0.1439	0	1	0	0	0	−0.0027	0.0000	−0.0586	−0.0749	−0.0904	0.1050
Bear	1	−0.0268	0	0	0	0	1	−0.0010	−0.0320	−0.0379	−0.0298	0.0000	−0.0277
	2	0.0044	0	0	1	0	0	−0.0639	−0.0902	0.0000	−0.0071	−0.0227	0.0044
	3	−0.0241	0	0	0	0	1	−0.0712	−0.1407	−0.0775	−0.0613	0.0000	−0.0246
	4	0.0360	1	0	0	0	0	0.0000	−0.0880	−0.0581	−0.0578	−0.0523	0.0287
	5	−0.0218	0	1	0	0	0	−0.0542	0.0000	−0.0323	−0.0525	−0.0534	−0.0230
	6	0.0303	1	0	0	0	0	0.0000	−0.0629	−0.0746	−0.0679	−0.0457	0.0281
	7	−0.0407	0	0	0	1	0	−0.0202	−0.0298	−0.0160	0.0000	−0.0322	−0.0433
	8	−0.0246	0	0	0	0	1	−0.1025	−0.2094	−0.0203	−0.0803	0.0000	−0.0251
	9	−0.0708	0	1	0	0	0	−0.1390	0.0000	−0.4104	−0.2675	−0.0035	−0.1248
	10	−0.0253	0	0	0	0	1	−0.1399	−0.0739	−0.0494	−0.0546	0.0000	−0.0272
	11	−0.0332	0	0	0	0	1	−0.0573	−0.0509	−0.0686	−0.0138	0.0000	−0.0345
	12	0.0158	0	0	0	0	1	−0.2029	−0.1546	−0.1433	−0.0314	0.0000	0.0156
	13	−0.0051	0	0	0	0	1	−0.0772	−0.0639	−0.0279	−0.0877	0.0000	−0.0052
Moderate Investment													
Bull	1	0.1526	0	1	0	0	0	−0.1591	0.0000	−0.1189	−0.1308	−0.1386	0.1320
	2	0.2080	0	1	0	0	0	−0.1221	0.0000	−0.1305	−0.1501	−0.1984	0.1604
	3	0.0954	0	0	0	1	0	−0.0047	−0.0068	−0.0342	0.0000	−0.0828	0.0890
	4	0.1976	0	1	0	0	0	−0.0822	0.0000	−0.0817	−0.0395	−0.0636	0.1601
	5	0.1491	1	0	0	0	0	0.0000	−0.0904	−0.0517	−0.0410	−0.2204	0.1229
	6	0.2087	0	1	0	0	0	−0.1555	0.0000	−0.1202	−0.1297	−0.1878	0.1937
	7	0.0509	0	0	1	0	0	−0.0473	−0.0695	0.0000	−0.0248	−0.0006	0.0500
	8	0.0873	0	0	1	0	0	−0.0127	−0.0339	0.0000	−0.0399	−0.0924	0.0818
	9	0.0877	0	0	1	0	0	−0.0131	−0.0845	0.0000	−0.0281	−0.0983	0.0853
	10	0.1056	0	1	0	0	0	−0.1245	0.0000	−0.0055	−0.0580	−0.0422	0.0930
	11	0.0551	0	0	1	0	0	−0.0075	−0.0607	0.0000	−0.0284	−0.0477	0.0513
	12	0.0540	0	0	0	1	0	−0.0007	−0.0480	−0.0213	0.0000	−0.0255	0.0518
	13	0.2961	0	0	1	0	0	−0.2385	−0.2003	0.0000	−0.1144	−0.1800	0.2594
	14	0.2131	0	1	0	0	0	−0.0248	0.0000	−0.0991	−0.0788	−0.1753	0.1742
	15	0.1495	1	0	0	0	0	0.0000	−0.0417	−0.0664	−0.0190	−0.0576	0.0993
	16	0.2826	0	0	1	0	0	−0.0980	−0.4274	0.0000	−0.6532	−0.2144	0.2125
	17	0.1547	0	1	0	0	0	−0.0051	0.0000	−0.0713	−0.0906	−0.1138	0.1268
Bear	1	−0.0266	0	0	0	0	1	−0.0013	−0.0317	−0.0371	−0.0295	0.0000	−0.0272
	2	0.0044	0	0	1	0	0	−0.0638	−0.0901	0.0000	−0.0071	−0.0227	0.0044
	3	−0.0240	0	0	0	0	1	−0.0697	−0.1384	−0.0761	−0.0601	0.0000	−0.0243
	4	0.0380	1	0	0	0	0	0.0000	−0.0939	−0.0645	−0.0644	−0.0580	0.0328
	5	−0.0215	0	1	0	0	0	−0.0542	0.0000	−0.0329	−0.0525	−0.0525	−0.0223
	6	0.0309	1	0	0	0	0	0.0000	−0.0646	−0.0771	−0.0702	−0.0471	0.0293
	7	−0.0400	0	0	0	1	0	−0.0197	−0.0296	−0.0156	0.0000	−0.0319	−0.0419
	8	−0.0245	0	0	0	0	1	−0.1014	−0.2073	−0.0197	−0.0794	0.0000	−0.0248
	9	−0.0557	0	1	0	0	0	−0.1609	0.0000	−0.4313	−0.2904	−0.0309	−0.0945
	10	−0.0247	0	0	0	0	1	−0.1376	−0.0727	−0.0478	−0.0542	0.0000	−0.0261
	11	−0.0329	0	0	0	0	1	−0.0575	−0.0517	−0.0688	−0.0143	0.0000	−0.0338
	12	0.0158	0	0	0	0	1	−0.2045	−0.1557	−0.1440	−0.0316	0.0000	0.0157
	13	−0.0051	0	0	0	0	1	−0.0769	−0.0646	−0.0281	−0.0879	0.0000	−0.0052
Aggressive Investment													
Bull	1	0.1595	0	1	0	0	0	−0.1780	0.0000	−0.1342	−0.1461	−0.1533	0.1457
	2	0.2239	0	1	0	0	0	−0.1516	0.0000	−0.1538	−0.1766	−0.2362	0.1922
	3	0.0975	0	0	0	1	0	−0.0045	−0.0072	−0.0362	0.0000	−0.0865	0.0933
	4	0.2101	0	1	0	0	0	−0.0930	0.0000	−0.0930	−0.0482	−0.0736	0.1851
	5	0.1578	1	0	0	0	0	0.0000	−0.1056	−0.0582	−0.0484	−0.2363	0.1404
	6	0.2138	0	1	0	0	0	−0.1643	0.0000	−0.1253	−0.1349	−0.1968	0.2037
	7	0.0512	0	0	1	0	0	−0.0479	−0.0700	0.0000	−0.0250	−0.0003	0.0506
	8	0.0891	0	0	1	0	0	−0.0125	−0.0354	0.0000	−0.0408	−0.0953	0.0855
	9	0.0885	0	0	1	0	0	−0.0130	−0.0859	0.0000	−0.0281	−0.1003	0.0869
	10	0.1099	0	1	0	0	0	−0.1362	0.0000	−0.0118	−0.0648	−0.0494	0.1014
	11	0.0563	0	0	1	0	0	−0.0077	−0.0638	0.0000	−0.0294	−0.0505	0.0538
	12	0.0547	0	0	0	1	0	−0.0015	−0.0497	−0.0228	0.0000	−0.0261	0.0533
	13	0.3083	0	0	1	0	0	−0.2495	−0.2116	0.0000	−0.1233	−0.1957	0.2839
	14	0.2261	0	1	0	0	0	−0.0210	0.0000	−0.1208	−0.0880	−0.2046	0.2001
	15	0.1662	1	0	0	0	0	0.0000	−0.0592	−0.0839	−0.0310	−0.0786	0.1327
	16	0.3060	0	0	1	0	0	−0.1069	−0.4405	0.0000	−0.6495	−0.2731	0.2592
	17	0.1641	0	1	0	0	0	−0.0071	0.0000	−0.0821	−0.1041	−0.1339	0.1454
Bear	1	−0.0263	0	0	0	0	1	−0.0016	−0.0315	−0.0363	−0.0292	0.0000	−0.0268
	2	0.0044	0	0	1	0	0	−0.0638	−0.0901	0.0000	−0.0071	−0.0227	0.0044
	3	−0.0238	0	0	0	0	1	−0.0685	−0.1365	−0.0749	−0.0591	0.0000	−0.0241
	4	0.0398	1	0	0	0	0	0.0000	−0.0990	−0.0700	−0.0700	−0.0628	0.0363
	5	−0.0212	0	1	0	0	0	−0.0543	0.0000	−0.0334	−0.0524	−0.0517	−0.0218
	6	0.0314	1	0	0	0	0	0.0000	−0.0660	−0.0794	−0.0722	−0.0483	0.0304
	7	−0.0394	0	0	0	1	0	−0.0194	−0.0295	−0.0152	0.0000	−0.0316	−0.0406
	8	−0.0244	0	0	0	0	1	−0.1003	−0.2056	−0.0192	−0.0786	0.0000	−0.0246
	9	−0.0427	0	1	0	0	0	−0.1797	0.0000	−0.4492	−0.3099	−0.0544	−0.0686
	10	−0.0243	0	0	0	0	1	−0.1356	−0.0717	−0.0466	−0.0539	0.0000	−0.0252
	11	−0.0326	0	0	0	0	1	−0.0578	−0.0525	−0.0690	−0.0147	0.0000	−0.0332
	12	0.0159	0	0	0	0	1	−0.2058	−0.1567	−0.1446	−0.0317	0.0000	0.0158
	13	−0.0050	0	0	0	0	1	−0.0767	−0.0651	−0.0282	−0.0881	0.0000	−0.0051

Table B.39: Optimisation: Downside risk and bull and bear markets (Definition 14) as indicator

			Solution					Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Bull	1	0.1821	0	1	0	0	0	−0.1353	0.0000	−0.1208	−0.1306	−0.1815	0.1516
	2	0.1830	0	1	0	0	0	−0.0695	0.0000	−0.0685	−0.0293	−0.0520	0.1309
	3	0.1614	0	0	1	0	0	−0.0482	−0.1174	0.0000	−0.0157	−0.1034	0.1374
	4	0.1755	0	1	0	0	0	−0.0996	0.0000	−0.0933	−0.0904	−0.1569	0.1626
	5	0.1138	0	0	1	0	0	−0.1184	−0.0792	0.0000	−0.0511	−0.0367	0.0569
	6	0.2130	1	0	0	0	0	0.0000	−0.0072	−0.0621	−0.0435	−0.1452	0.1419
	7	0.2166	0	0	0.5698	0.4302	0	−0.1494	−0.0365	0.0000	0.0000	−0.0054	0.0771
	8	0.1356	0	0	1	0	0	−0.0745	−0.0818	0.0000	−0.0175	−0.0637	0.0695
	9	0.1300	1	0	0	0	0	0.0000	−0.0212	−0.0461	−0.0050	−0.0332	0.0603
	10	0.1746	1	0	0	0	0	0.0000	−0.0449	−0.0420	−0.0099	−0.1602	0.1497
Bear	1	−0.0241	0	0	0	0	1	−0.0712	−0.1407	−0.0775	−0.0613	0.0000	−0.0246
	2	−0.0264	0	0	1	0	0	−0.0900	−0.2066	0.0000	−0.0697	−0.0015	−0.0344
	3	−0.1013	0	0	0	0	1	−0.2659	−0.1194	−0.0176	−0.0769	0.0000	−0.1133
	4	−0.1771	0	0.5581	0	0	0.4419	−0.1498	0.0000	−0.4593	−0.3067	0.0000	−0.2097
	5	−0.0556	0	1	0	0	0	−0.1902	0.0000	−0.1885	−0.0924	−0.0203	−0.0672
	6	0.2135	0.0897	0.9103	0	0	0	0.0000	0.0000	−0.1235	−0.1246	−0.0995	0.1265
	7	0.0159	0	0	0	0	1	−0.0913	−0.1438	−0.0453	−0.1712	0.0000	0.0144
Moderate Investment													
Bull	1	0.1906	0	1	0	0	0	−0.1539	0.0000	−0.1336	−0.1444	−0.2015	0.1687
	2	0.1976	0	1	0	0	0	−0.0822	0.0000	−0.0817	−0.0395	−0.0636	0.1601
	3	0.1681	0	0	1	0	0	−0.0532	−0.1248	0.0000	−0.0139	−0.1119	0.1509
	4	0.1791	0	1	0	0	0	−0.1012	0.0000	−0.0963	−0.0931	−0.1630	0.1698
	5	0.1298	0	0	1	0	0	−0.1301	−0.0814	0.0000	−0.0580	−0.0558	0.0888
	6	0.2329	1	0	0	0	0	0.0000	−0.0075	−0.0854	−0.0572	−0.1883	0.1817
	7	0.2671	0	0	1	0	0	−0.1713	−0.0777	0.0000	−0.0162	−0.0411	0.1214
	8	0.1541	0	0	1	0	0	−0.1075	−0.1157	0.0000	−0.0320	−0.0787	0.1065
	9	0.1495	1	0	0	0	0	0.0000	−0.0417	−0.0664	−0.0190	−0.0576	0.0993
	10	0.1816	1	0	0	0	0	0.0000	−0.0447	−0.0376	−0.0086	−0.1756	0.1637
Bear	1	−0.0240	0	0	0	0	1	−0.0697	−0.1384	−0.0761	−0.0601	0.0000	−0.0243
	2	−0.0242	0	0	1	0	0	−0.0927	−0.2100	0.0000	−0.0725	−0.0061	−0.0299
	3	−0.0980	0	0	0	0	1	−0.2475	−0.1119	−0.0140	−0.0707	0.0000	−0.1066
	4	−0.1673	0	0.6816	0	0	0.3184	−0.1385	0.0000	−0.4006	−0.2720	0.0000	−0.1945
	5	−0.0523	0	1	0	0	0	−0.1803	0.0000	−0.1810	−0.0894	−0.0231	−0.0607
	6	0.2474	0.6444	0.3556	0	0	0	0.0000	0.0000	−0.1237	−0.1249	−0.0996	0.1265
	7	0.0164	0	0	0	0	1	−0.0895	−0.1411	−0.0451	−0.1690	0.0000	0.0153
Aggressive Investment													
Bull	1	0.1979	0	1	0	0	0	−0.1699	0.0000	−0.1445	−0.1561	−0.2185	0.1833
	2	0.2101	0	1	0	0	0	−0.0930	0.0000	−0.0930	−0.0482	−0.0736	0.1851
	3	0.1739	0	0	1	0	0	−0.0575	−0.1311	0.0000	−0.0124	−0.1192	0.1624
	4	0.1822	0	1	0	0	0	−0.1027	0.0000	−0.0988	−0.0953	−0.1683	0.1760
	5	0.1434	0	0	1	0	0	−0.1402	−0.0833	0.0000	−0.0639	−0.0722	0.1161
	6	0.2499	1	0	0	0	0	0.0000	−0.0078	−0.1053	−0.0690	−0.2252	0.2158
	7	0.3157	0	0	1	0	0	−0.2211	−0.1689	0.0000	−0.0547	−0.1231	0.2185
	8	0.1699	0	0	1	0	0	−0.1358	−0.1447	0.0000	−0.0444	−0.0915	0.1382
	9	0.1662	1	0	0	0	0	0.0000	−0.0592	−0.0839	−0.0310	−0.0786	0.1327
	10	0.1876	1	0	0	0	0	0.0000	−0.0446	−0.0339	−0.0076	−0.1888	0.1756
Bear	1	−0.0238	0	0	0	0	1	−0.0685	−0.1365	−0.0749	−0.0591	0.0000	−0.0241
	2	−0.0222	0	0	1	0	0	−0.0950	−0.2130	0.0000	−0.0749	−0.0100	−0.0261
	3	−0.0951	0	0	0	0	1	−0.2317	−0.1055	−0.0109	−0.0653	0.0000	−0.1008
	4	−0.1570	0	0.9021	0	0	0.0979	−0.1289	0.0000	−0.3503	−0.2423	0.0000	−0.1814
	5	−0.0495	0	1	0	0	0	−0.1718	0.0000	−0.1746	−0.0868	−0.0255	−0.0551
	6	0.2995	1	0	0	0	0	0.0000	−0.0261	−0.1739	−0.1856	−0.1550	0.1874
	7	0.0167	0	0	0	0	1	−0.0879	−0.1389	−0.0449	−0.1672	0.0000	0.0160

Table B.40: Optimisation: Downside risk and bull and bear markets (Definition 15) as indicator

		Solution					Reduced Gradient					
Downward Shock	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment												
1	0.0333	0	0	1	0	0	−0.0306	−0.0094	0.0000	−0.0274	−0.0332	0.0316
2	0.0314	0	0	1	0	0	−0.0036	−0.0337	0.0000	−0.0402	−0.0588	0.0314
3	0.0784	0	1	0	0	0	−0.0843	0.0000	−0.0535	−0.0667	−0.0607	0.0599
4	0.0879	0	1	0	0	0	−0.0891	0.0000	−0.1047	−0.1187	−0.1122	0.0848
5	0.1712	0	1	0	0	0	−0.1108	0.0000	−0.0972	−0.1140	−0.1651	0.1344
6	0.0518	1	0	0	0	0	0.0000	−0.0606	−0.0156	−0.0205	−0.0290	0.0466
7	0.0417	0	0	0	0	1	−0.0825	−0.0192	−0.0730	−0.0725	0.0000	0.0401
8	0.0224	0	0	0	1	0	−0.0309	−0.0455	−0.0097	0.0000	−0.0271	0.0220
9	0.1323	0	1	0	0	0	−0.0625	0.0000	−0.1404	−0.1495	−0.2176	0.1198
10	−0.1382	0	0	0	0	1	−0.1670	−0.1145	−0.3562	−0.2243	0.0000	−0.1551
11	0.1743	0	1	0	0	0	−0.1196	0.0000	−0.1726	−0.1792	−0.1626	0.1648
12	0.0589	0	0.6686	0.3315	0	0	−0.0679	0.0000	0.0000	−0.0022	−0.0184	0.0375
13	0.0100	0	0.2806	0	0.5199	0.1995	−0.1216	0.0000	−0.0089	0.0000	0.0000	0.0078
14	0.0044	0	0	0	0	1	−0.0872	−0.0928	−0.1077	−0.0494	0.0000	0.0025
15	0.1116	0	0	0.5670	0.4330	0	−0.0964	−0.0297	0.0000	0.0000	−0.0494	0.0992
16	0.0674	0	1	0	0	0	−0.0252	0.0000	−0.0502	−0.0459	−0.0220	0.0505
17	0.1672	1	0	0	0	0	0.0000	−0.0795	−0.0932	−0.0718	−0.1014	0.1537
18	0.1257	0	1	0	0	0	−0.0253	0.0000	−0.0589	−0.0358	−0.0053	0.0185
19	−0.0043	0	0	0	0	1	−0.0841	−0.0446	−0.0334	−0.0523	0.0000	−0.0061
20	0.1203	1	0	0	0	0	0.0000	−0.0737	−0.0280	−0.0405	−0.1522	0.1118
21	0.1678	1	0	0	0	0	0.0000	−0.0647	−0.1431	−0.1607	−0.1401	0.1514
Moderate Investment												
1	0.0338	0	0	1	0	0	−0.0316	−0.0098	0.0000	−0.0281	−0.0342	0.0325
2	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0588	0.0314
3	0.0836	0	1	0	0	0	−0.0977	0.0000	−0.0608	−0.0743	−0.0714	0.0703
4	0.0887	0	1	0	0	0	−0.0898	0.0000	−0.1066	−0.1214	−0.1149	0.0865
5	0.1816	0	1	0	0	0	−0.1325	0.0000	−0.1118	−0.1299	−0.1889	0.1550
6	0.0532	1	0	0	0	0	0.0000	−0.0641	−0.0159	−0.0219	−0.0307	0.0495
7	0.0422	0	0	0	0	1	−0.0845	−0.0210	−0.0748	−0.0740	0.0000	0.0410
8	0.0225	0	0	0	1	0	−0.0309	−0.0454	−0.0097	0.0000	−0.0274	0.0222
9	0.1357	0	1	0	0	0	−0.0633	0.0000	−0.1486	−0.1538	−0.2187	0.1268
10	−0.1334	0	0	0	0	1	−0.1544	−0.1060	−0.3239	−0.2046	0.0000	−0.1456
11	0.1770	0	1	0	0	0	−0.1225	0.0000	−0.1804	−0.1881	−0.1656	0.1701
12	0.0672	0	0.5268	0.4732	0	0	−0.0686	0.0000	0.0000	−0.0031	−0.0195	0.0381
13	0.0109	0	0.0486	0	0.6819	0.2696	−0.1209	0.0000	−0.0092	0.0000	0.0000	0.0078
14	0.0049	0	0	0	0	1	−0.0879	−0.0940	−0.1068	−0.0499	0.0000	0.0035
15	0.1154	0	0	0.7626	0.2374	0	−0.1017	−0.0307	0.0000	0.0000	−0.0555	0.1048
16	0.0721	0	1	0	0	0	−0.0275	0.0000	−0.0634	−0.0546	−0.0314	0.0600
17	0.1710	1	0	0	0	0	0.0000	−0.0831	−0.0983	−0.0721	−0.1027	0.1613
18	0.1557	0.0019	0.9981	0	0	0	0.0000	0.0000	−0.1121	−0.0993	−0.0563	0.0784
19	−0.0038	0	0	0	0	1	−0.0810	−0.0431	−0.0318	−0.0514	0.0000	−0.0051
20	0.1227	1	0	0	0	0	0.0000	−0.0768	−0.0293	−0.0434	−0.1588	0.1165
21	0.1724	1	0	0	0	0	0.0000	−0.0706	−0.1575	−0.1738	−0.1432	0.1606
Aggressive Investment												
1	0.0342	0	0	1	0	0	−0.0324	−0.0101	0.0000	−0.0286	−0.0351	0.0334
2	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0588	0.0314
3	0.0881	0	1	0	0	0	−0.1092	0.0000	−0.0670	−0.0809	−0.0805	0.0792
4	0.0895	0	1	0	0	0	−0.0905	0.0000	−0.1082	−0.1237	−0.1173	0.0880
5	0.1904	0	1	0	0	0	−0.1511	0.0000	−0.1243	−0.1436	−0.2092	0.1727
6	0.0545	1	0	0	0	0	0.0000	−0.0671	−0.0161	−0.0230	−0.0321	0.0520
7	0.0426	0	0	0	0	1	−0.0862	−0.0225	−0.0763	−0.0754	0.0000	0.0418
8	0.0226	0	0	0	1	0	−0.0308	−0.0452	−0.0097	0.0000	−0.0278	0.0224
9	0.1387	0	1	0	0	0	−0.0641	0.0000	−0.1556	−0.1574	−0.2196	0.1328
10	−0.1294	0	0	0	0	1	−0.1436	−0.0988	−0.2961	−0.1876	0.0000	−0.1375
11	0.1793	0	1	0	0	0	−0.1250	0.0000	−0.1872	−0.1958	−0.1680	0.1747
12	0.0815	0	0.2736	0.7264	0	0	−0.0692	0.0000	0.0000	−0.0040	−0.0205	0.0386
13	0.0121	0	0	0	0.6828	0.3172	−0.1168	−0.0022	−0.0088	0.0000	0.0000	0.0097
14	0.0053	0	0	0	0	1	−0.0886	−0.0951	−0.1060	−0.0503	0.0000	0.0044
15	0.1194	0	0	1	0	0	−0.1071	−0.0335	0.0000	−0.0011	−0.0615	0.1105
16	0.0762	0	1	0	0	0	−0.0295	0.0000	−0.0747	−0.0621	−0.0395	0.0681
17	0.1742	1	0	0	0	0	0.0000	−0.0862	−0.1027	−0.0724	−0.1039	0.1677
18	0.1885	0.6484	0.3516	0	0	0	0.0000	0.0000	−0.1362	−0.1248	−0.0756	0.1018
19	−0.0034	0	0	0	0	1	−0.0784	−0.0419	−0.0304	−0.0506	0.0000	−0.0043
20	0.1247	1	0	0	0	0	0.0000	−0.0794	−0.0304	−0.0458	−0.1644	0.1206
21	0.1764	1	0	0	0	0	0.0000	−0.0757	−0.1698	−0.1850	−0.1458	0.1685

Table B.41: Optimisation: Downside risk and SA business confidence as indicator

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Upward	1	0.0417	0	0	0	0	1	−0.0825	−0.0192	−0.0730	−0.0725	0.0000	0.0401
	2	0.0824	0	1	0	0	0	−0.0704	0.0000	−0.0474	−0.0890	−0.0328	0.0733
	3	0.0541	1	0	0	0	0	0.0000	−0.0928	−0.0148	−0.0207	−0.0486	0.0514
	4	0.0399	0	0	0	0	1	−0.0624	−0.0255	−0.0196	−0.0155	0.0000	0.0375
	5	−0.0118	0	0	0	0	1	−0.0306	−0.0126	−0.0106	−0.0222	0.0000	−0.0122
	6	−0.0218	0	1	0	0	0	−0.0542	0.0000	−0.0323	−0.0525	−0.0534	−0.0230
	7	0.0772	0	0.7354	0.2646	0	0	−0.1097	0.0000	0.0000	−0.0264	−0.0044	0.0642
	8	0.0450	0	0	1	0	0	−0.0660	−0.0679	0.0000	−0.0213	−0.0382	0.0433
	9	0.1399	0	0	1	0	0	−0.0281	−0.0094	0.0000	−0.0364	−0.0884	0.1298
	10	0.1418	0	0	1	0	0	−0.1918	−0.1166	0.0000	−0.0313	−0.0869	0.1290
	11	0.1203	1	0	0	0	0	0.0000	−0.0737	−0.0280	−0.0405	−0.1522	0.1118
	12	0.1678	1	0	0	0	0	0.0000	−0.0647	−0.1431	−0.1607	−0.1401	0.1514
	13	0.0363	0	1	0	0	0	−0.0653	0.0000	−0.0779	−0.0148	−0.0017	0.0165
	14	−0.0058	0.5037	0.4963	0	0	0	0.0000	0.0000	−0.0907	−0.0766	−0.0008	−0.0061
	15	0.0995	0	0	0	1	0	−0.1074	−0.0306	−0.0263	0.0000	−0.0018	0.0916
	16	0.0216	0.4696	0	0	0.5304	0	0.0000	−0.0542	−0.0047	0.0000	−0.0181	0.0129
	17	0.0723	0	0	0	1	0	−0.0810	−0.0504	−0.0307	0.0000	−0.0121	0.0712
Downward	1	−0.0067	0	0	0	0	1	−0.0409	−0.0207	−0.0308	−0.0340	0.0000	−0.0068
	2	−0.0042	0	0	1	0	0	−0.0145	−0.0317	0.0000	−0.0105	−0.0430	−0.0083
	3	−0.0777	0	0	1	0	0	−0.0969	−0.0558	0.0000	−0.0336	−0.0212	−0.0855
	4	−0.1382	0	0	0	0	1	−0.1670	−0.1145	−0.3562	−0.2243	0.0000	−0.1551
	5	0.0094	0	0	0	0	1	−0.1122	−0.1000	−0.0765	−0.1359	0.0000	0.0075
	6	0.0984	1	0	0	0	0	0.0000	−0.0156	−0.1597	−0.1459	−0.0535	0.0850
	7	0.0659	0	1	0	0	0	−0.0923	0.0000	−0.1399	−0.1317	−0.0572	0.0653
Moderate Investment													
Upward	1	0.0422	0	0	0	0	1	−0.0845	−0.0210	−0.0748	−0.0740	0.0000	0.0410
	2	0.0850	0	1	0	0	0	−0.0741	0.0000	−0.0496	−0.0937	−0.0344	0.0784
	3	0.0548	1	0	0	0	0	0.0000	−0.0948	−0.0155	−0.0216	−0.0498	0.0529
	4	0.0406	0	0	0	0	1	−0.0638	−0.0259	−0.0201	−0.0158	0.0000	0.0389
	5	−0.0117	0	0	0	0	1	−0.0306	−0.0120	−0.0107	−0.0223	0.0000	−0.0120
	6	−0.0215	0	1	0	0	0	−0.0542	0.0000	−0.0329	−0.0525	−0.0525	−0.0223
	7	0.0808	0	0.6756	0.3244	0	0	−0.1184	0.0000	0.0000	−0.0288	−0.0067	0.0714
	8	0.0455	0	0	1	0	0	−0.0680	−0.0717	0.0000	−0.0220	−0.0397	0.0442
	9	0.1427	0	0	1	0	0	−0.0322	−0.0176	0.0000	−0.0396	−0.0923	0.1355
	10	0.1454	0	0	1	0	0	−0.1927	−0.1199	0.0000	−0.0351	−0.0938	0.1362
	11	0.1227	1	0	0	0	0	0.0000	−0.0768	−0.0293	−0.0434	−0.1588	0.1165
	12	0.1724	1	0	0	0	0	0.0000	−0.0706	−0.1575	−0.1738	−0.1432	0.1606
	13	0.0418	0	1	0	0	0	−0.0680	0.0000	−0.0939	−0.0292	−0.0078	0.0276
	14	−0.0057	0.4689	0.5311	0	0	0	0.0000	0.0000	−0.0900	−0.0760	−0.0012	−0.0060
	15	0.1017	0	0	0	1	0	−0.1131	−0.0344	−0.0258	0.0000	−0.0028	0.0960
	16	0.0241	0.3540	0	0	0.6460	0	0.0000	−0.0581	−0.0039	0.0000	−0.0267	0.0175
	17	0.0726	0	0	0	1	0	−0.0809	−0.0501	−0.0310	0.0000	−0.0111	0.0718
Downward	1	−0.0067	0	0	0	0	1	−0.0408	−0.0208	−0.0308	−0.0340	0.0000	−0.0067
	2	−0.0031	0	0	1	0	0	−0.0150	−0.0337	0.0000	−0.0116	−0.0452	−0.0060
	3	−0.0755	0	0	1	0	0	−0.0950	−0.0564	0.0000	−0.0331	−0.0222	−0.0812
	4	−0.1334	0	0	0	0	1	−0.1544	−0.1060	−0.3239	−0.2046	0.0000	−0.1456
	5	0.0100	0	0	0	0	1	−0.1115	−0.0982	−0.0768	−0.1347	0.0000	0.0086
	6	0.1021	1	0	0	0	0	0.0000	−0.0242	−0.1702	−0.1547	−0.0611	0.0925
	7	0.0660	0	1	0	0	0	−0.0928	0.0000	−0.1406	−0.1321	−0.0571	0.0656
Aggressive Investment													
Upward	1	0.0426	0	0	0	0	1	−0.0862	−0.0225	−0.0763	−0.0754	0.0000	0.0418
	2	0.0871	0	1	0	0	0	−0.0773	0.0000	−0.0514	−0.0977	−0.0357	0.0828
	3	0.0555	1	0	0	0	0	0.0000	−0.0965	−0.0161	−0.0224	−0.0508	0.0542
	4	0.0411	0	0	0	0	1	−0.0650	−0.0262	−0.0206	−0.0159	0.0000	0.0400
	5	−0.0116	0	0	0	0	1	−0.0307	−0.0115	−0.0108	−0.0223	0.0000	−0.0118
	6	−0.0212	0	1	0	0	0	−0.0543	0.0000	−0.0334	−0.0524	−0.0517	−0.0218
	7	0.0840	0	0.5688	0.4312	0	0	−0.1259	0.0000	0.0000	−0.0308	−0.0087	0.0775
	8	0.0459	0	0	1	0	0	−0.0697	−0.0749	0.0000	−0.0227	−0.0410	0.0451
	9	0.1451	0	0	1	0	0	−0.0357	−0.0247	0.0000	−0.0423	−0.0957	0.1403
	10	0.1485	0	0	1	0	0	−0.1935	−0.1227	0.0000	−0.0383	−0.0997	0.1423
	11	0.1247	1	0	0	0	0	0.0000	−0.0794	−0.0304	−0.0458	−0.1644	0.1206
	12	0.1764	1	0	0	0	0	0.0000	−0.0757	−0.1698	−0.1850	−0.1458	0.1685
	13	0.0466	0	1	0	0	0	−0.0704	0.0000	−0.1077	−0.0415	−0.0130	0.0371
	14	−0.0056	0.4068	0.5932	0	0	0	0.0000	0.0000	−0.0894	−0.0755	−0.0015	−0.0058
	15	0.1036	0	0	0	1	0	−0.1181	−0.0377	−0.0253	0.0000	−0.0037	0.0998
	16	0.0264	0.1475	0	0	0.8525	0	0.0000	−0.0615	−0.0032	0.0000	−0.0341	0.0214
	17	0.0728	0	0	0	1	0	−0.0808	−0.0499	−0.0313	0.0000	−0.0101	0.0723
Downward	1	−0.0067	0	0	0	0	1	−0.0407	−0.0208	−0.0309	−0.0340	0.0000	−0.0067
	2	−0.0021	0	0	1	0	0	−0.0154	−0.0354	0.0000	−0.0126	−0.0470	−0.0041
	3	−0.0737	0	0	1	0	0	−0.0933	−0.0569	0.0000	−0.0328	−0.0230	−0.0774
	4	−0.1294	0	0	0	0	1	−0.1436	−0.0988	−0.2961	−0.1876	0.0000	−0.1375
	5	0.0104	0	0	0	0	1	−0.1109	−0.0967	−0.0771	−0.1337	0.0000	0.0095
	6	0.1053	1	0	0	0	0	0.0000	−0.0317	−0.1792	−0.1622	−0.0675	0.0989
	7	0.0662	0	1	0	0	0	−0.0932	0.0000	−0.1412	−0.1325	−0.0571	0.0659

Table B.42: Optimisation: Downside risk and rate of change in SA business confidence as indicator

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Upward	1	0.0434	0	0	1	0	0	−0.0554	−0.0533	0.0000	−0.0534	−0.0797	0.0421
	2	0.0846	0	0	1	0	0	−0.0453	−0.0730	0.0000	−0.0272	−0.0914	0.0685
	3	0.0362	0	0	0	1	0	−0.0234	−0.0954	−0.0055	0.0000	−0.0110	0.0345
	4	0.0342	1	0	0	0	0	0.0000	−0.0493	−0.0661	−0.0417	−0.0271	0.0299
	5	−0.0044	1	0	0	0	0	0.0000	−0.0247	−0.0136	−0.0352	−0.0210	−0.0069
	6	0.0450	0	0	1	0	0	−0.0660	−0.0679	0.0000	−0.0213	−0.0382	0.0433
	7	−0.0369	0	0	1	0	0	−0.1158	−0.1311	0.0000	−0.0698	−0.0028	−0.0414
	8	−0.0112	0	0	0	0	1	−0.0381	−0.0038	−0.0133	−0.0458	0.0000	−0.0114
	9	0.0674	0	0	1	0	0	−0.2141	−0.1423	0.0000	−0.0621	−0.0175	0.0626
	10	0.1399	0	0	1	0	0	−0.0281	−0.0094	0.0000	−0.0364	−0.0884	0.1298
	11	0.1418	0	0	1	0	0	−0.1918	−0.1166	0.0000	−0.0313	−0.0869	0.1290
	12	0.1727	0	1	0	0	0	−0.0186	0.0000	−0.1285	−0.0498	−0.1603	0.1486
	13	−0.0522	0	0	1	0	0	−0.0972	−0.0839	0.0000	−0.0021	−0.0223	−0.0569
	14	−0.0777	0	0	1	0	0	−0.0969	−0.0558	0.0000	−0.0336	−0.0212	−0.0855
	15	0.1323	0	1	0	0	0	−0.0625	0.0000	−0.1404	−0.1495	−0.2176	0.1198
	16	−0.1382	0	0	0	0	1	−0.1670	−0.1145	−0.3562	−0.2243	0.0000	−0.1551
	17	0.1743	0	1	0	0	0	−0.1196	0.0000	−0.1726	−0.1792	−0.1626	0.1648
	18	0.0100	0	0.2806	0	0.5199	0.1995	−0.1216	0.0000	−0.0089	0.0000	0.0000	0.0078
	19	0.0044	0	0	0	0	1	−0.0872	−0.0928	−0.1077	−0.0494	0.0000	0.0025
	20	0.1116	0	0	0.5670	0.4330	0	−0.0964	−0.0297	0.0000	0.0000	−0.0494	0.0992
	21	0.0674	0	1	0	0	0	−0.0252	0.0000	−0.0502	−0.0459	−0.0220	0.0505
	22	0.1672	1	0	0	0	0	0.0000	−0.0795	−0.0932	−0.0718	−0.1014	0.1537
	23	0.1257	0	1	0	0	0	−0.0253	0.0000	−0.0589	−0.0358	−0.0053	0.0185
	24	−0.0043	0	0	0	0	1	−0.0841	−0.0446	−0.0334	−0.0523	0.0000	−0.0061
	25	0.1203	1	0	0	0	0	0.0000	−0.0737	−0.0280	−0.0405	−0.1522	0.1118
	26	0.1678	1	0	0	0	0	0.0000	−0.0647	−0.1431	−0.1607	−0.1401	0.1514
	27	0.0363	0	1	0	0	0	−0.0653	0.0000	−0.0779	−0.0148	−0.0017	0.0165
	28	−0.0058	0.5037	0.4963	0	0	0	0.0000	0.0000	−0.0907	−0.0766	−0.0008	−0.0061
	29	0.0995	0	0	0	1	0	−0.1074	−0.0306	−0.0263	0.0000	−0.0018	0.0916
	30	0.0938	0	0	0	1	0	−0.0411	−0.0555	−0.0509	0.0000	−0.1332	0.0807
	31	0.1150	0	0	0.8765	0.1235	0	−0.0213	−0.0451	0.0000	0.0000	−0.0834	0.0861
	32	0.0216	0.4696	0	0	0.5304	0	0.0000	−0.0542	−0.0047	0.0000	−0.0181	0.0129
	33	0.0159	0	0	0	1	0	−0.2093	−0.1197	−0.0681	0.0000	−0.0484	0.0155
	34	0.0216	0.5497	0	0	0	0.4503	0.0000	−0.0322	−0.0050	−0.0102	0.0000	0.0159
	35	0.0094	0	0	0	0	1	−0.1122	−0.1000	−0.0765	−0.1359	0.0000	0.0075
	36	0.0984	1	0	0	0	0	0.0000	−0.0156	−0.1597	−0.1459	−0.0535	0.0850
	37	0.0723	0	0	0	1	0	−0.0810	−0.0504	−0.0307	0.0000	−0.0121	0.0712
	38	0.0450	0	1	0	0	0	−0.0304	0.0000	−0.0213	−0.0286	−0.0102	0.0363
	39	0.1097	0	1	0	0	0	−0.0153	0.0000	−0.0330	−0.0582	−0.0634	0.0817
	40	0.0078	0	0	0	0	1	−0.0038	−0.0655	−0.0315	−0.0082	0.0000	0.0043
	41	0.0659	0	1	0	0	0	−0.0923	0.0000	−0.1399	−0.1317	−0.0572	0.0653
	42	0.0566	0	1	0	0	0	−0.1189	0.0000	−0.0071	−0.1199	−0.0593	0.0557
	43	0.1979	0	0	1	0	0	−0.1620	−0.1810	0.0000	−0.1461	−0.1527	0.1700
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0337	0.0000	−0.0402	−0.0588	0.0314
	2	0.0784	0	1	0	0	0	−0.0843	0.0000	−0.0535	−0.0667	−0.0607	0.0599
	3	0.0879	0	1	0	0	0	−0.0891	0.0000	−0.1047	−0.1187	−0.1122	0.0848
	4	0.1712	0	1	0	0	0	−0.1108	0.0000	−0.0972	−0.1140	−0.1651	0.1344
	5	0.0518	1	0	0	0	0	0.0000	−0.0606	−0.0156	−0.0205	−0.0290	0.0466
	6	0.0417	0	0	0	0	1	−0.0825	−0.0192	−0.0730	−0.0725	0.0000	0.0401
	7	0.0224	0	0	0	1	0	−0.0309	−0.0455	−0.0097	0.0000	−0.0271	0.0220
	8	−0.0233	0	0	0	0	1	−0.0690	−0.0638	−0.0402	−0.0317	0.0000	−0.0246
	9	0.0579	0	0.3344	0.6656	0	0	−0.0295	0.0000	0.0000	−0.0136	−0.0675	0.0528
	10	0.0693	0	0.2040	0	0.7960	0	−0.0059	0.0000	−0.0229	0.0000	−0.0591	0.0590

Table B.43: Optimisation: Downside risk, US consumer sentiment as indicator and a conservative investment

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Moderate Investment													
Upward	1	0.0438	0	0	1	0	0	−0.0572	−0.0542	0.0000	−0.0546	−0.0809	0.0428
	2	0.0891	0	0	1	0	0	−0.0494	−0.0813	0.0000	−0.0312	−0.1011	0.0775
	3	0.0367	0	0	0	1	0	−0.0238	−0.0976	−0.0052	0.0000	−0.0111	0.0355
	4	0.0354	1	0	0	0	0	0.0000	−0.0519	−0.0693	−0.0448	−0.0300	0.0323
	5	−0.0037	1	0	0	0	0	0.0000	−0.0251	−0.0143	−0.0358	−0.0220	−0.0055
	6	0.0455	0	0	1	0	0	−0.0680	−0.0717	0.0000	−0.0220	−0.0397	0.0442
	7	−0.0356	0	0	1	0	0	−0.1227	−0.1385	0.0000	−0.0742	−0.0070	−0.0388
	8	−0.0112	0	0	0	0	1	−0.0383	−0.0041	−0.0134	−0.0460	0.0000	−0.0113
	9	0.0687	0	0	1	0	0	−0.2189	−0.1444	0.0000	−0.0629	−0.0190	0.0653
	10	0.1427	0	0	1	0	0	−0.0322	−0.0176	0.0000	−0.0396	−0.0923	0.1355
	11	0.1454	0	0	1	0	0	−0.1927	−0.1199	0.0000	−0.0351	−0.0938	0.1362
	12	0.1794	0	1	0	0	0	−0.0085	0.0000	−0.1414	−0.0569	−0.1770	0.1621
	13	−0.0509	0	0	1	0	0	−0.0929	−0.0813	0.0000	−0.0013	−0.0224	−0.0543
	14	−0.0755	0	0	1	0	0	−0.0950	−0.0564	0.0000	−0.0331	−0.0222	−0.0812
	15	0.1357	0	1	0	0	0	−0.0633	0.0000	−0.1486	−0.1538	−0.2187	0.1268
	16	−0.1334	0	0	0	0	1	−0.1544	−0.1060	−0.3239	−0.2046	0.0000	−0.1456
	17	0.1770	0	1	0	0	0	−0.1225	0.0000	−0.1804	−0.1881	−0.1656	0.1701
	18	0.0109	0	0.0486	0	0.6819	0.2696	−0.1209	0.0000	−0.0092	0.0000	0.0000	0.0078
	19	0.0049	0	0	0	0	1	−0.0879	−0.0940	−0.1068	−0.0499	0.0000	0.0035
	20	0.1154	0	0	0.7626	0.2374	0	−0.1017	−0.0307	0.0000	0.0000	−0.0555	0.1048
	21	0.0721	0	1	0	0	0	−0.0275	0.0000	−0.0634	−0.0546	−0.0314	0.0600
	22	0.1710	1	0	0	0	0	0.0000	−0.0831	−0.0983	−0.0721	−0.1027	0.1613
	23	0.1557	0.0019	0.9981	0	0	0	0.0000	0.0000	−0.1121	−0.0993	−0.0563	0.0784
	24	−0.0038	0	0	0	0	1	−0.0810	−0.0431	−0.0318	−0.0514	0.0000	−0.0051
	25	0.1227	1	0	0	0	0	0.0000	−0.0768	−0.0293	−0.0434	−0.1588	0.1165
	26	0.1724	1	0	0	0	0	0.0000	−0.0706	−0.1575	−0.1738	−0.1432	0.1606
	27	0.0418	0	1	0	0	0	−0.0680	0.0000	−0.0939	−0.0292	−0.0078	0.0276
	28	−0.0057	0.4689	0.5311	0	0	0	0.0000	0.0000	−0.0900	−0.0760	−0.0012	−0.0060
	29	0.1017	0	0	0	1	0	−0.1131	−0.0344	−0.0258	0.0000	−0.0028	0.0960
	30	0.0975	0	0	0	1	0	−0.0453	−0.0587	−0.0558	0.0000	−0.1420	0.0881
	31	0.1239	0	0	1	0	0	−0.0269	−0.0491	0.0000	−0.0059	−0.0987	0.1008
	32	0.0241	0.3540	0	0	0.6460	0	0.0000	−0.0581	−0.0039	0.0000	−0.0267	0.0175
	33	0.0160	0	0	0	1	0	−0.2065	−0.1180	−0.0669	0.0000	−0.0484	0.0157
	34	0.0236	0.7284	0	0	0	0.2716	0.0000	−0.0336	−0.0049	−0.0112	0.0000	0.0171
	35	0.0100	0	0	0	0	1	−0.1115	−0.0982	−0.0768	−0.1347	0.0000	0.0086
	36	0.1021	1	0	0	0	0	0.0000	−0.0242	−0.1702	−0.1547	−0.0611	0.0925
	37	0.0726	0	0	0	1	0	−0.0809	−0.0501	−0.0310	0.0000	−0.0111	0.0718
	38	0.0474	0	1	0	0	0	−0.0322	0.0000	−0.0254	−0.0316	−0.0163	0.0412
	39	0.1175	0	1	0	0	0	−0.0200	0.0000	−0.0433	−0.0704	−0.0799	0.0974
	40	0.0088	0	0	0	0	1	−0.0073	−0.0704	−0.0344	−0.0109	0.0000	0.0063
	41	0.0660	0	1	0	0	0	−0.0928	0.0000	−0.1406	−0.1321	−0.0571	0.0656
	42	0.0569	0	1	0	0	0	−0.1184	0.0000	−0.0070	−0.1183	−0.0597	0.0562
	43	0.2057	0	0	1	0	0	−0.1824	−0.1872	0.0000	−0.1593	−0.1645	0.1856
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0588	0.0314
	2	0.0836	0	1	0	0	0	−0.0977	0.0000	−0.0608	−0.0743	−0.0714	0.0703
	3	0.0887	0	1	0	0	0	−0.0898	0.0000	−0.1066	−0.1214	−0.1149	0.0865
	4	0.1816	0	1	0	0	0	−0.1325	0.0000	−0.1118	−0.1299	−0.1889	0.1550
	5	0.0532	1	0	0	0	0	0.0000	−0.0641	−0.0159	−0.0219	−0.0307	0.0495
	6	0.0422	0	0	0	0	1	−0.0845	−0.0210	−0.0748	−0.0740	0.0000	0.0410
	7	0.0225	0	0	0	1	0	−0.0309	−0.0454	−0.0097	0.0000	−0.0274	0.0222
	8	−0.0230	0	0	0	0	1	−0.0673	−0.0635	−0.0394	−0.0311	0.0000	−0.0239
	9	0.0600	0	0.8466	0.1534	0	0	−0.0295	0.0000	0.0000	−0.0136	−0.0674	0.0528
	10	0.0730	0	0	0	1	0	−0.0057	−0.0025	−0.0245	0.0000	−0.0625	0.0625

Table B.44: Optimisation: Downside risk, US consumer sentiment as indicator and a moderate investment

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Aggressive Investment													
Upward	1	0.0441	0	0	1	0	0	−0.0589	−0.0549	0.0000	−0.0556	−0.0819	0.0434
	2	0.0929	0	0	1	0	0	−0.0529	−0.0885	0.0000	−0.0347	−0.1095	0.0852
	3	0.0371	0	0	0	1	0	−0.0242	−0.0995	−0.0050	0.0000	−0.0112	0.0363
	4	0.0365	1	0	0	0	0	0.0000	−0.0541	−0.0721	−0.0474	−0.0325	0.0344
	5	−0.0031	1	0	0	0	0	0.0000	−0.0255	−0.0149	−0.0363	−0.0229	−0.0043
	6	0.0459	0	0	1	0	0	−0.0697	−0.0749	0.0000	−0.0227	−0.0410	0.0451
	7	−0.0345	0	0	1	0	0	−0.1286	−0.1449	0.0000	−0.0780	−0.0105	−0.0367
	8	−0.0112	0	0	0	0	1	−0.0385	−0.0043	−0.0135	−0.0461	0.0000	−0.0112
	9	0.0699	0	0	1	0	0	−0.2231	−0.1462	0.0000	−0.0635	−0.0204	0.0676
	10	0.1451	0	0	1	0	0	−0.0357	−0.0247	0.0000	−0.0423	−0.0957	0.1403
	11	0.1485	0	0	1	0	0	−0.1935	−0.1227	0.0000	−0.0383	−0.0997	0.1423
	12	0.1852	0.0111	0.9889	0	0	0	0.0000	0.0000	−0.1522	−0.0629	−0.1911	0.1735
	13	−0.0498	0	0	1	0	0	−0.0891	−0.0791	0.0000	−0.0007	−0.0225	−0.0520
	14	−0.0737	0	0	1	0	0	−0.0933	−0.0569	0.0000	−0.0328	−0.0230	−0.0774
	15	0.1387	0	1	0	0	0	−0.0641	0.0000	−0.1556	−0.1574	−0.2196	0.1328
	16	−0.1294	0	0	0	0	1	−0.1436	−0.0988	−0.2961	−0.1876	0.0000	−0.1375
	17	0.1793	0	1	0	0	0	−0.1250	0.0000	−0.1872	−0.1958	−0.1680	0.1747
	18	0.0121	0	0	0	0.6828	0.3172	−0.1168	−0.0022	−0.0088	0.0000	0.0000	0.0097
	19	0.0053	0	0	0	0	1	−0.0886	−0.0951	−0.1060	−0.0503	0.0000	0.0044
	20	0.1194	0	0	1	0	0	−0.1071	−0.0335	0.0000	−0.0011	−0.0615	0.1105
	21	0.0762	0	1	0	0	0	−0.0295	0.0000	−0.0747	−0.0621	−0.0395	0.0681
	22	0.1742	1	0	0	0	0	0.0000	−0.0862	−0.1027	−0.0724	−0.1039	0.1677
	23	0.1885	0.6484	0.3516	0	0	0	0.0000	0.0000	−0.1362	−0.1248	−0.0756	0.1018
	24	−0.0034	0	0	0	0	1	−0.0784	−0.0419	−0.0304	−0.0506	0.0000	−0.0043
	25	0.1247	1	0	0	0	0	0.0000	−0.0794	−0.0304	−0.0458	−0.1644	0.1206
	26	0.1764	1	0	0	0	0	0.0000	−0.0757	−0.1698	−0.1850	−0.1458	0.1685
	27	0.0466	0	1	0	0	0	−0.0704	0.0000	−0.1077	−0.0415	−0.0130	0.0371
	28	−0.0056	0.4068	0.5932	0	0	0	0.0000	0.0000	−0.0894	−0.0755	−0.0015	−0.0058
	29	0.1036	0	0	0	1	0	−0.1181	−0.0377	−0.0253	0.0000	−0.0037	0.0998
	30	0.1006	0	0	0	1	0	−0.0489	−0.0615	−0.0600	0.0000	−0.1496	0.0943
	31	0.1316	0	0	1	0	0	−0.0326	−0.0532	0.0000	−0.0120	−0.1146	0.1162
	32	0.0264	0.1475	0	0	0.8525	0	0.0000	−0.0615	−0.0032	0.0000	−0.0341	0.0214
	33	0.0160	0	0	0	1	0	−0.2041	−0.1165	−0.0659	0.0000	−0.0485	0.0159
	34	0.0265	1	0	0	0	0	0.0000	−0.0352	−0.0053	−0.0128	−0.0008	0.0189
	35	0.0104	0	0	0	0	1	−0.1109	−0.0967	−0.0771	−0.1337	0.0000	0.0095
	36	0.1053	1	0	0	0	0	0.0000	−0.0317	−0.1792	−0.1622	−0.0675	0.0989
	37	0.0728	0	0	0	1	0	−0.0808	−0.0499	−0.0313	0.0000	−0.0101	0.0723
	38	0.0495	0	1	0	0	0	−0.0336	0.0000	−0.0289	−0.0341	−0.0215	0.0453
	39	0.1242	0	1	0	0	0	−0.0241	0.0000	−0.0521	−0.0809	−0.0941	0.1108
	40	0.0097	0	0	0	0	1	−0.0102	−0.0745	−0.0368	−0.0132	0.0000	0.0080
	41	0.0662	0	1	0	0	0	−0.0932	0.0000	−0.1412	−0.1325	−0.0571	0.0659
	42	0.0571	0	1	0	0	0	−0.1179	0.0000	−0.0070	−0.1169	−0.0601	0.0567
	43	0.2124	0	0	1	0	0	−0.1999	−0.1924	0.0000	−0.1707	−0.1746	0.1990
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0588	0.0314
	2	0.0881	0	1	0	0	0	−0.1092	0.0000	−0.0670	−0.0809	−0.0805	0.0792
	3	0.0895	0	1	0	0	0	−0.0905	0.0000	−0.1082	−0.1237	−0.1173	0.0880
	4	0.1904	0	1	0	0	0	−0.1511	0.0000	−0.1243	−0.1436	−0.2092	0.1727
	5	0.0545	1	0	0	0	0	0.0000	−0.0671	−0.0161	−0.0230	−0.0321	0.0520
	6	0.0426	0	0	0	0	1	−0.0862	−0.0225	−0.0763	−0.0754	0.0000	0.0418
	7	0.0226	0	0	0	1	0	−0.0308	−0.0452	−0.0097	0.0000	−0.0278	0.0224
	8	−0.0226	0	0	0	0	1	−0.0659	−0.0633	−0.0388	−0.0306	0.0000	−0.0233
	9	0.0627	0	1	0	0	0	−0.0315	0.0000	−0.0022	−0.0172	−0.0725	0.0571
	10	0.0765	0	0	0	1	0	−0.0054	−0.0081	−0.0273	0.0000	−0.0693	0.0695

Table B.45: Optimisation: Downside risk, US consumer sentiment as indicator and an aggressive investment

		Solution						Reduced Gradient					
Shock	Window	Objective Value	CI09	CI10	CI24	CI27	CI70	CI09	CI10	CI24	CI27	CI70	Lagrange Multiplier
Conservative Investment													
Upward	1	0.0879	0	1	0	0	0	−0.0891	0.0000	−0.1047	−0.1187	−0.1122	0.0848
	2	−0.0233	0	0	0	0	1	−0.0690	−0.0638	−0.0402	−0.0317	0.0000	−0.0246
	3	0.0693	0	0.2040	0	0.7960	0	−0.0059	0.0000	−0.0279	0.0000	−0.0591	0.0590
	4	0.1023	0	1	0	0	0	−0.0294	0.0000	−0.0323	−0.0039	−0.0202	0.0684
	5	0.1172	0	1	0	0	0	−0.0226	0.0000	−0.0929	−0.0806	−0.0985	0.1132
	6	0.0875	0	0	1	0	0	−0.0667	−0.1105	0.0000	−0.0241	−0.1099	0.0793
	7	0.0030	0	0	0	1	0	−0.0393	−0.0271	−0.0236	0.0000	−0.0184	0.0028
	8	−0.0118	0	0	0	0	1	−0.0306	−0.0126	−0.0106	−0.0222	0.0000	−0.0122
	9	−0.0067	0	0	0	0	1	−0.0409	−0.0207	−0.0308	−0.0340	0.0000	−0.0068
	10	0.0434	0	0	1	0	0	−0.0554	−0.0533	0.0000	−0.0534	−0.0797	0.0421
	11	0.1399	0	0	1	0	0	−0.0281	−0.0094	0.0000	−0.0364	−0.0884	0.1298
	12	0.0100	0	0.2806	0	0.5199	0.1995	−0.1216	0.0000	−0.0089	0.0000	0.0000	0.0078
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0337	0.0000	−0.0402	−0.0588	0.0314
	2	0.0784	0	1	0	0	0	−0.0843	0.0000	−0.0535	−0.0667	−0.0607	0.0599
	3	0.1712	0	1	0	0	0	−0.1108	0.0000	−0.0972	−0.1140	−0.1651	0.1344
	4	0.0156	0	1	0	0	0	−0.0581	0.0000	−0.0097	−0.0287	−0.0338	0.0144
	5	0.1755	0	1	0	0	0	−0.0996	0.0000	−0.0933	−0.0904	−0.1569	0.1626
	6	0.0589	0	0.6686	0.3315	0	0	−0.0679	0.0000	0.0000	−0.0022	−0.0184	0.0375
Moderate Investment													
Upward	1	0.0887	0	1	0	0	0	−0.0898	0.0000	−0.1066	−0.1214	−0.1149	0.0865
	2	−0.0230	0	0	0	0	1	−0.0673	−0.0635	−0.0394	−0.0311	0.0000	−0.0239
	3	0.0730	0	0	0	1	0	−0.0057	−0.0025	−0.0295	0.0000	−0.0625	0.0625
	4	0.1118	0	1	0	0	0	−0.0432	0.0000	−0.0445	−0.0143	−0.0312	0.0874
	5	0.1184	0	1	0	0	0	−0.0200	0.0000	−0.0940	−0.0809	−0.0999	0.1155
	6	0.0897	0	0	1	0	0	−0.0701	−0.1178	0.0000	−0.0259	−0.1159	0.0839
	7	0.0030	0	0	0	1	0	−0.0395	−0.0272	−0.0238	0.0000	−0.0186	0.0029
	8	−0.0117	0	0	0	0	1	−0.0306	−0.0120	−0.0107	−0.0223	0.0000	−0.0120
	9	−0.0067	0	0	0	0	1	−0.0408	−0.0208	−0.0308	−0.0340	0.0000	−0.0067
	10	0.0438	0	0	1	0	0	−0.0572	−0.0542	0.0000	−0.0546	−0.0809	0.0428
	11	0.1427	0	0	1	0	0	−0.0322	−0.0176	0.0000	−0.0396	−0.0923	0.1355
	12	0.0109	0	0.0486	0	0.6819	0.2696	−0.1209	0.0000	−0.0092	0.0000	0.0000	0.0078
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0588	0.0314
	2	0.0836	0	1	0	0	0	−0.0977	0.0000	−0.0608	−0.0743	−0.0714	0.0703
	3	0.1816	0	1	0	0	0	−0.1325	0.0000	−0.1118	−0.1299	−0.1889	0.1550
	4	0.0160	0	1	0	0	0	−0.0588	0.0000	−0.0103	−0.0293	−0.0345	0.0151
	5	0.1791	0	1	0	0	0	−0.1012	0.0000	−0.0963	−0.0931	−0.1630	0.1698
	6	0.0672	0	0.5268	0.4732	0	0	−0.0686	0.0000	0.0000	−0.0031	−0.0195	0.0381
Aggressive Investment													
Upward	1	0.0895	0	1	0	0	0	−0.0905	0.0000	−0.1082	−0.1237	−0.1173	0.0880
	2	−0.0226	0	0	0	0	1	−0.0659	−0.0633	−0.0388	−0.0306	0.0000	−0.0233
	3	0.0765	0	0	0	1	0	−0.0054	−0.0081	−0.0323	0.0000	−0.0693	0.0695
	4	0.1200	0	1	0	0	0	−0.0550	0.0000	−0.0549	−0.0232	−0.0406	0.1037
	5	0.1193	0	1	0	0	0	−0.0177	0.0000	−0.0950	−0.0812	−0.1011	0.1174
	6	0.0917	0	0	1	0	0	−0.0731	−0.1240	0.0000	−0.0275	−0.1210	0.0878
	7	0.0031	0	0	0	1	0	−0.0398	−0.0273	−0.0240	0.0000	−0.0188	0.0030
	8	−0.0116	0	0	0	0	1	−0.0307	−0.0115	−0.0108	−0.0223	0.0000	−0.0118
	9	−0.0067	0	0	0	0	1	−0.0407	−0.0208	−0.0309	−0.0340	0.0000	−0.0067
	10	0.0441	0	0	1	0	0	−0.0589	−0.0549	0.0000	−0.0556	−0.0819	0.0434
	11	0.1451	0	0	1	0	0	−0.0357	−0.0247	0.0000	−0.0423	−0.0957	0.1403
	12	0.0121	0	0	0	0.6828	0.3172	−0.1168	−0.0022	−0.0088	0.0000	0.0000	0.0097
Downward	1	0.0314	0	0	1	0	0	−0.0036	−0.0338	0.0000	−0.0402	−0.0588	0.0314
	2	0.0881	0	1	0	0	0	−0.1092	0.0000	−0.0670	−0.0809	−0.0805	0.0792
	3	0.1904	0	1	0	0	0	−0.1511	0.0000	−0.1243	−0.1436	−0.2092	0.1727
	4	0.0162	0	1	0	0	0	−0.0595	0.0000	−0.0108	−0.0298	−0.0352	0.0157
	5	0.1822	0	1	0	0	0	−0.1027	0.0000	−0.0988	−0.0953	−0.1683	0.1760
	6	0.0815	0	0.2736	0.7264	0	0	−0.0692	0.0000	0.0000	−0.0040	−0.0205	0.0386

Table B.46: Optimisation: Downside risk and rate of change of US consumer sentiment as indicator

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