Novel implementation of a phase-only spatial light modulator for laser beam shaping

by

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March 2016
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Abstract

Novel implementation of a phase-only spatial light modulator for laser beam shaping

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Dissertation: PhDPhys

December 2015

The phase-only spatial light modulator (SLM) has revolutionized the field of laser beam shaping. In this thesis we describe in detail the considerations necessary to build a “digital laser” which incorporates an SLM into a laser cavity as an intracavity element to dynamically generate a wide variety of custom beams. We then present a theoretical analysis of the healing of petal-like (or Laguerre-Gaussian) beams using rotational considerations, and using digital laser technology to demonstrate this healing experimentally. We extend our investigation into self-healing beams with the theoretical derivation of a new type of Bessel-like beams, which retains a concentric ring structure on propagation, and which self-heal axially. These beams are generated using an SLM, and self-healing is demonstrated experimentally.
Uittreksel

Nuwe implementering van ’n fase-alleen ruimtelike ligmodulator vir laserbundelvorming

(“Novel implementation of a phase-only spatial light modulator for laser beam shaping”)

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Desember 2015

Die fase-alleen ruimtelike ligmodulator (RLM) het die veld van laserbundelvorming revolusionêr verander. In hierdie tesis beskryf ons sorgvuldig wat die nodige oorwegings is om ’n “digitale laser” te bou, wat ’n RLM in die laserholte as in ’n intra-holte element bevat, sodat dit ’n wye verskeidenheid van laserbundel vorms dinamies kan genereer. Voorts verskaf ons ’n teoretiese analyse van die korreksie van blom-patroon (of Laguerre-Gauss) bundels, met die hulp van rotasie beginsels, en ons gebruik die digitale laser tegnologie om hierdie korreksie eksperimenteel te demonstreer. Ons brei ons ondersoek na self-korregerende bundels uit met die teoretiese afleiding van ’n nuwe soort Bessel-tipe bundel wat ’n konsentriese ringstruktuur behou tydens voortplanting en wat aksiaal self-korregerend is. Hierdie bundels is met behulp van ’n RLM gegenereer en self-korreksie is eksperimenteel gede- monstreer.
Acknowledgements

I am thankful to my supervisors Dr Andrew Forbes and Dr Igor Litvin for their patience, encouragement and excellent academic guidance. I am also grateful to the National Laser Centre which facilitated and sponsored my post-graduate studies. Thanks to my colleagues Dr Angela Dudley, Dr Darryl Naidoo and Ms Thandeka Mhlanga, who assisted with discussions and in the lab. Thanks in particular to Dr Darryl Naidoo also for the considered and very helpful proof-reading of my manuscript. Thanks also to Prof. Ian Underwood of the University of Edinburgh for his interest in my work and assistance with SLM design and behaviour, and to Prof. Helen Gleeson OBE of the University of Leeds for assistance with liquid crystal material properties. Thanks especially to my family, Herman, Paul and Disa, for their love and encouragement, even if they do call me “Dr Monkey”. Thanks also to friends and family who offered support and sustenance, a list which includes Margaret Haines, Adrian & Bonnie Haines, Joyce Burger & Johan Pretorius, and Sandra & Glen McGavigan.
Dedications

To my family: Herman, Paul & Disa
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Nomenclature

List of symbols

\[
e = 2.718
\]
\[
\pi = 3.142
\]
\[
c = 3 \times 10^8 \text{ ms}^{-1}
\]
\[
\lambda \quad \text{wavelength} \quad [ \text{m} ]
\]
\[
L \quad \text{resonator length} \quad [ \text{m} ]
\]
\[
R_1, R_2 \quad \text{mirror radius} \quad [ \text{m} ]
\]
\[
g_1, g_2 \quad \text{resonator stability parameters} \quad [ \text{ } ]
\]
\[
I \quad \text{intensity} \quad [ \text{Wm}^{-2} ]
\]
\[
w \quad 1/e^2 \text{ beam radius} \quad [ \text{m} ]
\]
\[
w_0 \quad \text{Gaussian beam waist} \quad [ \text{m} ]
\]
\[
w_1, w_2 \quad \text{beam radius on mirror} \quad [ \text{m} ]
\]
\[
r \quad \text{radial coordinate} \quad [ \text{m} ]
\]
\[
z \quad \text{axial coordinate} \quad [ \text{m} ]
\]
\[
z_R \quad \text{Rayleigh range} \quad [ \text{m} ]
\]
\[
C \quad \text{curvature} \quad [ \text{m}^{-1} ]
\]
\[
\theta \quad \text{divergence} \quad [ \text{radians} ]
\]
\[
N_F \quad \text{Fresnel number} \quad [ \text{ } ]
\]
\[
a \quad \text{aperture radius} \quad [ \text{m} ]
\]
\[
M^2 \quad \text{quality factor} \quad [ \text{ } ]
\]
NOMENCLATURE

$H$ Hermite polynomial ........................................ [ ]

$n, m$ order in the x, y directions ........................... [ ]

$L^{[\ell]}_{p}$ Laguerre polynomial ................................. [ ]

$p$ radial mode index ........................................... [ ]

$\ell$ azimuthal mode index ..................................... [ ]

$\alpha$ prism rotation angle .................................... [ radians ]

$N$ number of petals ........................................... [ ]

$n_e, n_o$ extraordinary, ordinary refractive index ........ [ ]

$\beta$ birefringence ........................................... [ ]

$R$ reflectivity ................................................... [ ]

$f$ focal length .................................................. [ m ]

Acronyms/Abbreviations

DOE diffractive optical element

SLM spatial light modulator

TEM transverse electromagnetic

HG Hermite-Gaussian

LG Laguerre-Gaussian

BG Bessel-Gaussian

LCD liquid crystal display

CMOS complementary metal oxide semiconductor

OASLM optically activated SLM

EASLM electrically activated SLM

ITO indium tin oxide

LC liquid crystal
ACRONYMS/ABBREVIATIONS

TN-LC  twisted nematic liquid crystal
PA-LC  parallel aligned liquid crystal
OC    output coupler
BW    Brewster window
DLP   diode laser pump
BET   beam-expanding telescope
AOM   acousto-optic modulator
Chapter 1

Introduction

Laser beam shaping is a dynamic and vibrant field of study which deals with the selection and manipulation of laser modes and with the modification of existing beams to create new patterns with particular phase and intensity properties. The earliest beam-shaping methods were aimed at simply achieving a Gaussian beam profile, which is the preferred output beam for many industrial materials processing applications like cutting and welding [2] because it has a low divergence and can be focussed to a very small spot, and achieved by resonator designs which limit the transverse extent of the beam while extracting maximum energy [3]. The study of the modes which form in laser resonators has led to amplitude and phase masking techniques and gain shaping techniques which allow the selection of particular chosen transverse modes with their characteristic phase and intensity distributions [4, 5, 6]. Phase modulation masks were used to create custom output intensity profiles inside the resonator [7], and using a holographic technique to shape Gaussian beams to form custom intensity profiles outside the resonator [8]. (See Section 1.3 for more on beam-shaping techniques.)

Diffractive optical elements (DOEs) and phase-only spatial light modulators (SLMs) are two common methods of phase modulation for laser beam shaping. A DOE has a phase pattern etched onto a glass substrate, and is tailored for a specific laser configuration and output pattern. DOEs have the disadvantage that a master DOE is expensive to manufacture, although it can be used to make many inexpensive copies, such as the type commonly distributed with laser pointers. A phase-only SLM allows digitally generated phase patterns to be displayed on a pixelated liquid crystal display panel controlled by a computer, in
order to dynamically generate custom phase patterns. The invention of this device has revolutionised the field of holographic laser beam shaping, with new beams with specific properties being continuously discovered [9; 10; 11; 12; 13; 14]. The work covered in this thesis highlights the role of the phase-only SLM in laser beam shaping, first as an intracavity phase modulating device to dynamically generate a wide variety of custom beams (see Chapter 2), and in the generation of new beams with self-healing properties (see Chapters 3 and 4).

The development of the phase-only SLM device started when a new material (cholesteryl benzoate) with a mesophase between the liquid and solid state (at a certain temperature range) was discovered by an Austrian botanist, Friedrich Reinitzer in 1888. The following year Otto Lehmann, a German Professor of Physics, studied the material and found that it had a double refraction effect characteristic of crystals, and called it a "liquid crystal". The material remained a scientific curiosity with very little research into the material properties until 1962 when Richard Williams discovered interesting electro-optical characteristics. This subsequently led to the development by George H. Heilmeier of the first LCD screens, which were first used in 1972 [15]. By 1975 the dynamic switching of nematic LCs (see Chapter 1.2) in an electric field was well understood [16]. LC SLMs were developed in the later 1980s, emerging from the development of the LC television (TV) [17] which was developed in the late 1980s and early 1990s [18; 19; 20]. They were used first for amplitude modulation [21], and then for phase modulation [22], and for cross-coupled amplitude and phase modulation [23]. LC SLMs were low-cost, proof-of-concept devices, with typically 100 × 100 pixels, a pixel size of 100µm and a frame rate of 20kHz [24]. Two types of SLM devices had emerged: the optically activated (OA) SLM [25; 26], and the electrically activated (EA) SLM. The development of both occurred concurrently, but the OA SLM suffered from the drawback of slow switching speeds, and the development of EA SLMs was favoured. This preference accelerated with the integration of the SLM with a silicon chip [27], know as LCOS (liquid crystal on silicon) technology. Electrical activation was simpler and more efficient than optical activation, and the resolution quickly increased to 256 × 256 pixels [28], and to 1024 × 768 pixels by 2004 [29]. At around this time, with the implementation of new liquid crystal material [30], phase-mostly SLMs became truly phase-only. This is discussed in more detail in Chapter 2.2. Modern phase-only LCOS
SLM devices can have up to $1920 \times 1200$ pixels with pixel size $4\mu m$, and run at 60 Hz [31].

The first application of LC SLMs was 3D holography [32; 33; 34; 35]. Soon after, computer-generated holograms were created and used for rudimentary beam shaping of Fresnel lenses [18; 36; 37] and for static [38] and dynamic [39] mirror tilt. It was shown that a phase-only SLM can produce a large set of Zernike polynomials [40], and this was demonstrated in real-time [41], making them suitable for adaptive optical wave-front correction [30] in applications such as medical imaging [42] and terrestrial telescopic systems to correct for atmospheric aberration [43; 44]. The biggest impact of these devices, however, has been in the field of laser beam shaping. Phase-only SLMs made possible the creation of a wide range of novel laser beams, most of which cannot be created in any other way [45; 46; 47; 48; 10; 11]. Novel beams created by SLMs form the basis of the field of laser tweezing [49; 50; 51], have been used in the study of atmospheric aberrations [52], and have been used for commercial applications such as laser marking [53] and micro-machining [54].

This thesis presents new examples of how phase-only SLMs are advancing the field of laser beam shaping. The first example is of a new invention, the “digital laser” [55; 56; 57; 58; 59; 60], which arose out of a need to fine-tune the design of intracavity diffractive optical elements. This work uncovered subtle properties of SLMs that become dominant when used in an intracavity configuration, and led to a device which is able to dynamically generate a wide range of laser modes and beam shapes. The second and third examples use an SLM to generate beams with different regeneration properties, the mechanisms of which are presented along with experimental results [61; 62].

1.1 Basic laser theory

1.1.1 Laser resonators

An optical resonator is a series of optical components that allows laser light to circulate. The simplest and most common configuration is the Fabry-Perot resonator, which consists of two spherical mirrors: a fully reflective back reflector and a partially reflective output coupler.

A laser resonator can be categorised as either stable or unstable. If a ray launched
CHAPTER 1. INTRODUCTION

parallel to the resonator axis tends to remain inside it after multiple round trips then the resonator is stable. Conversely, in an unstable resonator a ray will tend to leave after a few round trips [63]. Using the ray-transfer matrix of a resonator and the self-consistency requirement for stability gives the condition for stability [64]:

\[ 0 \leq g_1 g_2 \leq 1, \quad (1.1.1) \]

where mirror 1 has radius \( R_1 \), mirror 2 has radius \( R_2 \), \( L \) is the resonator length, and \( g_1 = 1 - \frac{L}{R_1} \) and \( g_2 = 1 - \frac{L}{R_2} \).

This is illustrated in Fig. 1.2, where the resonator configurations yielding stable resonators are shown in coloured areas, and unstable resonators in white areas.

1.1.2 Gaussian beams

A geometrical or ray transfer approach is useful to quantify the degree of stability of a resonator but does not predict the intensity distribution of a laser beam. Light in a resonator is more accurately regarded as the electromagnetic field that is a solution of the one-dimensional wave equation. One important solution is the Gaussian intensity distribution, which while not the only solution, is the most fundamental and the most commonly selected in commercial lasers. The Gaussian intensity distribution is illustrated in Fig. 1.3 and has the form:

\[ I(r) = I_0 \exp \left( -\frac{2r^2}{w^2} \right), \quad (1.1.2) \]
1.1. BASIC LASER THEORY

Figure 1.2: Plot of the stability function as a function of $g_1$ (x-axis) and $g_2$ (y-axis), with stability in the coloured areas.

Figure 1.3: A Gaussian beam profile showing the beam radius $w$.

where $I_0$ is the peak intensity of the beam. $w$ is the size of the laser beam and is defined as the radius at which the beam intensity falls to $1/e^2$ (13.5 percent) of its peak value (see Fig. 1.3).

At some point along the axis of propagation (usually denoted $z = 0$) the beam has the smallest transverse extent, known as the waist, which is also the point at which the wave front is planar.

Diffraction causes light to spread transversely and causes the wave-fronts to acquire
CHAPTER 1. INTRODUCTION

Figure 1.4: Propagation of a Gaussian laser beam.

Curvature as they propagate (see Fig. [1.4]) according to:

\[ w(z) = w_0 \left[ 1 + \left( \frac{z}{z_R} \right)^2 \right]^{\frac{1}{2}} \]  

(1.1.3)

and

\[ R(z) = z \left[ 1 + \left( \frac{z_R}{z} \right)^2 \right], \]  

(1.1.4)

where \( z \) is the distance propagated from the plane with flat wave-front, \( w_0 \) is the \( 1/e^2 \) radius of the beam waist, \( z_R = \pi w_0^2 / \lambda \) is the Rayleigh range, \( w(z) \) is the \( 1/e^2 \) beam radius at \( z \), and \( R(z) \) is the wave-front radius of curvature at \( z \).

If the waist is at \( z = 0 \) (where \( R(z) \) is infinite), then as the beam propagates \( R(z) \) passes through a minimum at some \( z = z_R \), and increases again toward infinity with \( z \). Simultaneously, the \( 1/e^2 \) intensity contours asymptotically approach a cone of angular radius:

\[ \theta = \frac{\lambda}{\pi w_0}. \]  

(1.1.5)

This is called the half-angle divergence of the Gaussian beam and is a measure of the divergence or transverse spread of the beam with distance.

Referring back to Fig. [1.1] and applying the steady-state condition that the radius of the phase front must be static at any arbitrary plane, and that it must match the radius of curvature of the mirrors, yields expressions for the Gaussian beam emerging from the resonator. The waist size is given by:

\[ w_0^4 = \left( \frac{\lambda}{\pi} \right) \frac{L(R_1 - L)(R_2 - L)(R_1 + R_2 - L)}{(R_1 + R_2 - 2L)^2} \]  

(1.1.6)

and is located at

\[ z_1 = \frac{L(R_2 - L)}{R_1 + R_2 - 2L} \text{ and } z_2 = \frac{L(R_1 - L)}{R_1 + R_2 - 2L} \]  

(1.1.7)
1.1. BASIC LASER THEORY

Gaussian propagation in the resonator gives the spot sizes on the mirrors:

$$w_1^4 = \left( \frac{\lambda R_1}{\pi} \right)^2 \frac{L(R_2 - L)}{(R_1 - L)(R_1 + R_2 - L)} \quad \text{and} \quad w_2^4 = \left( \frac{\lambda R_2}{\pi} \right)^2 \frac{L(R_1 - L)}{(R_2 - L)(R_1 + R_2 - L)}$$

(1.1.8)

A real resonator will always contain a limiting aperture of radius $a$, which might be the smallest dimension of the laser gain medium, a cavity mirror, or an intracavity iris. The Fresnel number is defined as:

$$N_F = \frac{a^2}{\lambda L}$$

(1.1.9)

where $a$ is the limiting aperture radius, $\lambda$ is the laser wavelength, and $L$ is the resonator length.

The significance of the Fresnel number is that it defines the transverse extent available to the laser beam, which limits the number of higher-order modes which can be supported in a resonator. This is discussed in more detail in Section [1.1.4].

1.1.3 Beam quality

Many laser applications require a high-quality beam, in other words a beam with a defined cross-section that does not diverge too quickly. The Second Moment beam propagation ratio $M^2$ is a common and widely-used parameter which summarizes the beam quality in one number [65]. According to this definition, the $M^2$ of a Gaussian beam is 1, and greater than one for all other beams.

1.1.4 Laser modes

The paraxial wave equation for the Fabry-Perot resonator can be solved by a number of complete and orthogonal sets of polynomials, which obey the orthogonality relationship [66]:

$$\int_{-\infty}^{\infty} a(x) \Psi_m(x) \Psi_n(x) dx = \delta_{mn},$$

(1.1.10)

where $\Psi_n(x)$ is the set of polynomials, $a(x)$ is a weighting function, and $\delta_{mn}$ is the Kronecker delta.

The two most common basis sets for mathematically describing the intensity distribution in a resonator are the Hermite and Laguerre polynomials, which describe the family
of Hermite-Gauss (HG) and Laguerre-Gauss (LG) laser modes, respectively. In a resonator with no apertures there are infinitely many eigenmodes, and these are referred to as transverse electromagnetic (TEM) resonator modes. The HG modes tend to form in resonators with rectangular symmetry, while LG modes tend to form in resonators with circular symmetry.

One important property of laser modes is that the intensity distribution is identical at any arbitrary plane along the optical axis inside (and outside) the resonator.

1.1.4.1 Hermite-Gaussian modes

One set of eigenmodes has the form of Hermite-Gaussian (HG) functions \([67, 68]\) in rectangular coordinates and are denoted by TEM HG\(_{nm}\), where \(n\) is the order in the \(x\)-direction, \(m\) is the order in the \(y\)-direction, and \(w\) is the beam radius of the associated TEM HG\(_{00}\) or Gaussian mode. These modes have an intensity distribution with the form \([69]\):

\[
u_{nm}(x, y, z) = \frac{1}{w(\zeta)} H_m \left[ \sqrt{2} \frac{x}{w(\zeta)} \right] H_n \left[ \sqrt{2} \frac{y}{w(\zeta)} \right] \exp \left[ ikz - \frac{\rho^2}{2w_0^2(1 + i\zeta)} - i\Psi_{m,n} \right],
\]

(1.1.11)

where \(k = \frac{2\pi}{\lambda}\) is the wave number, \(w(z)\) is the beam size at longitudinal position \(z\), \(w_0\) is the beam waist, \(z_R\) is the Rayleigh range, \(\zeta = z/z_R\), \(\Psi_{m,n} = (m + n + 1) \arctan(\zeta)\), and \(H_m, H_n\) are the Hermite polynomials are found using \([70, 71]\):

\[
H_n(z) = (-1)^n \exp(z^2) \frac{d^n}{dz^n} \exp(-z^2),
\]

(1.1.12)

and the first few Hermite polynomials are given by:

\[
\begin{align*}
H_0(z) &= 1 \\
H_1(z) &= 2z \\
H_2(z) &= 4z^2 - 2 \\
H_3(z) &= 8z^3 - 12z.
\end{align*}
\]

The intensity distribution of the Hermite-Gaussian modes is given by:
1.1. BASIC LASER THEORY

\[ I_{n,m}(x,y,z) = |u_{n,m}(x,y,z)|^2 \]  

(1.1.13)

<table>
<thead>
<tr>
<th>n/m</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<tbody>
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<td><img src="image17" alt="Plot" /></td>
<td><img src="image18" alt="Plot" /></td>
</tr>
</tbody>
</table>

**Figure 1.5:** An array of plots of the TEM HG intensity distributions where the mode indices correspond to the horizontal, \( n \) and vertical, \( m \) index, respectively.

Fig. 1.5 shows transverse mode patterns for TEM HG modes of various orders.

**Figure 1.6:** The cross-section of a HG\(_{00}\) (blue), HG\(_{01}\) (purple) and HG\(_{02}\) (yellow) mode, all with the same \( w \) in 1.1.11.

Fig. 1.6 illustrates an important property of higher-order modes which is that the transverse extent of the modes increases with order. The spot sizes of higher-order modes in rectangular coordinates can be approximated by:

\[ w_n = w\sqrt{n+1} \] and \[ w_m = w\sqrt{m+1}, \]  

(1.1.14)
where \( w \) is the spot size of the corresponding TEM HG\(_{00} \) mode and \( m \) and \( n \) are the orders of the \( x \)- and \( y \)-modes respectively.

For a Hermite-Gaussian mode, if \( w_{nm} \) and \( w_{00} \) are the waists of a high-order and fundamental beam respectively, then \( w_{nm}(z) = Mw_{00}(z) \) and

\[
M_x^2 = 2n + 1 \\
M_y^2 = 2m + 1
\]

### 1.1.4.2 Laguerre-Gaussian modes

Another common set of eigenmodes can be expressed in cylindrical coordinates using Laguerre functions. The fields of the Laguerre-Gaussian (LG) beams \([72]\) are given by:

\[
u_{\ell}^p(r, \phi, z) = \sqrt{\frac{2p!}{\pi(p + |\ell|)!}} \frac{1}{w(z)} \exp \left[ -\frac{r^2}{w^2(z)} - \frac{ikr^2}{2R(z)} \right] L_{\ell}^p \left[ \frac{2r^2}{w^2(z)} \right] \\
\times \left[ \frac{\sqrt{2}r}{w(z)} \right]^{|\ell|} \exp[i\ell\phi] \exp \left[ -i(2p + |\ell| + 1) \arctan \left( \frac{z}{z_R} \right) \right]
\]

where \( w(z) \) is the beam radius at longitudinal position \( z \), \( p \) and \( \ell \) are the radial and azimuthal indices, \( z_R \) is the Rayleigh range, and \( L_{\ell}^p \) are the Laguerre polynomials, which are the solutions of the differential equation:

\[
\frac{d^2L_{\ell}^p}{dx^2} + (\ell + 1 - x) \frac{dL_{\ell}^p}{dx} + pL_{\ell}^p = 0,
\]

where the first few LG polynomials are given by:

\[
L_0^0(x) = 1 \\
L_1^1(x) = \ell + 1 - x \\
L_2^2(x) = \frac{1}{2}(\ell + 1)(\ell + 2) - (\ell + 2)x + = \frac{1}{2}x^2.
\]

The intensity distribution of Laguerre-Gauss beams is given by:

\[
I_{\ell}^p(r, \phi, z) = \left| \nu_{\ell}^p(r, \phi, z) \right|^2
\]
1.1. BASIC LASER THEORY

![Figure 1.7: An array of plots of the LG intensity distributions where the mode indices correspond to the radial, \( p \) and azimuthal, \( \ell \) index, respectively.](image)

Fig. 1.7 shows transverse mode patterns for LG modes of various orders. Notice that the transverse extent of each mode increases with increasing order.

For a Laguerre-Gaussian mode defined by Eqn. 1.1.16 [73], the beam quality is found to be:

\[
M^2 = 2p + \ell + 1.
\] (1.1.19)

1.1.5 Mode discrimination

In the absence of an intracavity aperture and any other obstructing elements, a Fabry-Perot resonator will have a fundamental mode with beam radius \( w_1 \) given by Eq. 1.1.8 on mirror 1 say, where \( w_1 \geq w_2 \). The same resonator could, in theory, also produce the entire set of Hermite-Gaussian (or Laguerre-Gaussian) modes as described by Eq. 1.1.11. As can be seen in Fig. 1.6 however, for a resonator with the same length \( L \) and mirror radii \( R_1 \) and \( R_2 \), that higher order modes are physically wider than lower order modes. Fig. 1.8 shows the transmission of several TEM orders as a function of the limiting aperture radius \( a \) on mirror 1. It can be seen that an aperture with \( a = 2w \) confers a 10% loss on the TEM\(_{22} \) mode, but insignificant losses to the TEM\(_{00} \) and TEM\(_{11} \) modes. An aperture with \( a = 1.5w \) however confers a 13% loss on the TEM\(_{11} \) mode, and insignificant losses to the TEM\(_{00} \) modes. It is clear that in general a smaller aperture will cut off a higher percentage of a higher order mode than a lower order mode.

A laser mode will only oscillate in a resonator if the gain available to it is greater than
1.1.6 Modal decomposition

It has been noted [65] that measuring the intensity profile of a laser beam is not a reliable method of identifying the modal composition of the beam. For example, a beam with an apparent Gaussian profile can be the sum of a number of non-Gaussian modal components, which will propagate very differently to a Gaussian beam. It is often necessary therefore to identify the components of a beam in terms of some complete, orthogonal polynomial set (see Eq. 1.1.10). The first method for the decomposition of transverse modes was devised in 1982, before phase-plate synthesis was possible [74]. Thereafter, methods devised for
1.2. INTRODUCTION TO SLM TECHNOLOGY

phase plates became easier and more convenient using a phase-only SLM. The optical inner product technique \cite{75, 76} is one convenient method of performing modal decomposition. Using the basis set $\exp[i\ell \phi]$ as an example, a field $u$ can be expressed in terms of these harmonics:

A beam which emerges from a Fabry-Perot resonator can be described by a linear combination of eigenmodes:

$$U(r) = \sum_{n=1}^{\infty} a_n \Psi_n(r),$$

where $\Psi_n(r)$ are eigenmodes, $a_n$ are weighting coefficients, and $U(r)$ is the resulting output field. The coefficients $a_n$ can be determined using an inner product, given by:

$$a_n = \langle U, \Psi_n \rangle = \int \int_R U(r) \Psi_n^*(r) \, d^2r,$$  

where the $^*$ represents the complex conjugate. An arbitrary paraxial optical beam can be completely decomposed into the basis elements once the respective correlation coefficients have been determined. The weightings are optically determined by sampling the resultant field ($u(x,y) = U(x,y)\Psi_n^*(x,y)$) in the Fourier plane where the corresponding Fourier transformation is expressed as:

$$U_1(k_x,k_y) = \mathcal{F}\{u(x,y)\} = \int \int U(x,y)\Psi_n^*(x,y) \exp[-i(k_x x + k_y y)] \, dx \, dy.$$  

The weightings $a_n$ can be found using an inner product, by measuring the on-axis intensity of the field in the Fourier plane by setting the propagation vectors in (1.1.22) to zero ($k_x = k_y = 0$) to get:

$$I(0,0) = |U_1(0,0)|^2 = \left| \int \int U(x,y)\Psi_n^*(x,y) \, dx \, dy \right|^2 = |a_n|^2.$$

1.2 Introduction to SLM technology

A spatial light modulator (SLM) refers to a device which spatially modulates coherent light \cite{77}. They make use of liquid crystals, and are dynamically controlled by computers. There are two types of SLMs:
• those which modulate the *intensity* of light, and are commonly used in computer-controlled projectors, and

• those which modulate *phase* (or phase and intensity simultaneously), and are used to modify the wave-front of laser beams in applications like laser tweezing, wave-front correction, and data processing.

Liquid crystals are used to modulate both intensity and phase. They are transparent rod-shaped molecules which align similarly to crystals but are free to slide across each other similarly to liquids. In the nematic phase, as used in SLMs, molecules are *positioned* randomly, but can be *aligned* by an applied electric field. Birefringence is another important property of liquid crystals, meaning that they have a different refractive index perpendicular to the optical axis \( (n_o) \) from parallel to it \( (n_e) \), see Fig. 1.9. The property of birefringence is denoted:

\[
\beta = n_e - n_o
\]  

(1.2.1)

These molecules align themselves along an electric field, and therefore the optical axis can be rotated by modulating an electric field. In this way light passing through a liquid crystal layer can be slowed by between \( [n_o - 1]c \) and \( [n_e - 1]c \) (where \( c \) is the speed of light), resulting in a phase delay [78].

![Figure 1.9: Schematic of liquid crystal molecule, showing the origin of birefringence \( \beta \).](image-url)
There are two types of liquid crystals in use in SLMs, twisted nematic liquid crystals (TN-LCs) and parallel aligned liquid crystals (PA-LCs). The difference between these two types is shown in Fig. 1.10.

![Figure 1.10: Two examples of LC alignment schemes. (a) twisted nematic, (b) parallel aligned.](image)

When a TN-LC layer is trapped between two sheets of glass, with no applied electric field, the molecules align to be parallel with the glass surfaces and with the optical axis of the crystals creating a twist or spiral through 90° from the top surface to the bottom surface. An applied electric field aligns the molecules to be perpendicular to the glass surfaces.

In a PA-LC layer however the molecules align parallel to the glass surfaces, and all point in the same direction. An applied electric field rotates all the molecules, keeping them parallel to each other, but perpendicular to the glass surfaces.

Both TN-LCs and PA-LCs are used in SLMs, with TN-LCs in older devices and PA-LCs in newer devices.

There are two basic types of SLMs [79]:

- an optically addressed (OA) SLM which uses incoherent light to map spatial modulation, and
- an electrically addressed (EA) SLM, which uses electrical signals to map spatial modulation.
1.2.1 Optically Addressed SLM (OASLM)

The basic OASLM (also known as a “light valve”) system is shown in Fig. 1.11.

![Typical OASLM layout](image)

**Figure 1.11:** Typical OASLM layout, showing that the phase pattern is written to the detector of the OASLM with “write light”, and is imparted onto the coherent “read light” by the modulator [1].

The OASLM works as follows: Incoherent light (or ‘write light’) is used as a signal, and a desired phase pattern is imaged onto the detector as a grey-scale image. The intensity of the write light is detected by a photo-detector and is converted to an electrical charge distribution. This charge distribution aligns the liquid crystal molecules in regions of higher intensity, which changes the phase of the coherent light (or ‘read light’) in these regions.

OASLMs are capable of forming large high-resolution holograms, and since they are not pixelated they avoid the two-dimensional grating effect found in EASLMs [80]. However, these devices suffer several disadvantages:

- It is difficult to keep the contrast and sensitivity across the device constant;
- The device is relatively insensitive to write light, and has a low contrast ratio of only 20:1;
- They tend to retain the written image;
- The liquid crystal material used in the device degrades.
Table 1.1: Comparison of specifications of some commercially-available OASLMs.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Resolution (lp/mm)</th>
<th>Active area (mm)</th>
<th>Reflectivity (%)</th>
<th>Switch frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telecom Bretagne</td>
<td>100</td>
<td>35 × 45</td>
<td>&gt; 85</td>
<td>2 − 3 × 10⁻³</td>
</tr>
<tr>
<td>Vavilov State Optical Institute</td>
<td>100</td>
<td>diam 30 − 45</td>
<td>unknown</td>
<td>100</td>
</tr>
<tr>
<td>University of Cambridge</td>
<td>825</td>
<td>16 × 21</td>
<td>unknown</td>
<td>0.1 − 10</td>
</tr>
</tbody>
</table>

For these reasons OASLMs are regarded as being experimental devices, and are therefore very expensive. Some specifications of commercially-available OASLMs are shown in Table

1.2.2 Electrically Addressed SLM (EASLM)

The basic EASLM system [81] is shown in Fig. 1.12. They combine liquid crystal technology with existing complementary metal oxide semiconductor (CMOS) technology, which has been used for many years for video display applications. The silicon back plane of the CMOS (Fig. 1.12(a)) consists of electronic circuitry under pixel arrays (b), and forms the substrate of the device. The pixels are reflective aluminium deposited on the silicon backplane. A cell consisting of LC material (c) trapped between two alignment layers (d) is placed on top of the reflective pixel layer. An indium tin oxide (ITO) layer (e) forms a transmissive electrode, and this layer is protected by a glass substrate (f). The circuitry in the CMOS allows a voltage to be applied to each pixel, which is used to alter the refractive index of the liquid crystal layer and thereby changing the phase delay of incident light (g) which is reflected and modulated (h) by the cell. The SLM device is attached to driver electronics, and controlled by a computer as an additional display. The required phase is plotted to a bitmap image, the phase screen, the resolution of which matches the resolution of the SLM, and the required phase of each pixel mapped to 255 grey levels. This phase screen is displayed on the SLM device.

While SLMs can be used for a wide range of incident light wavelengths, it is important to note that the topmost glass substrate (Fig. 1.12(f)) is anti-reflection coated for a specific wavelength band, which must be matched to the incident light. Also, because of the
Table 1.2: Comparison of specifications of some commercially-available EASLMs.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Resolution (lp/mm)</th>
<th>Active area (mm)</th>
<th>Reflectivity (%)</th>
<th>Switch frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HoloEye</td>
<td>1920 × 1080</td>
<td>15.4 × 8.64</td>
<td>75</td>
<td>60</td>
</tr>
<tr>
<td>Hamamatsu</td>
<td>792 × 600</td>
<td>16 × 12</td>
<td>98</td>
<td>60</td>
</tr>
<tr>
<td>Boulder NLS</td>
<td>512 × 512</td>
<td>7.68 × 7.68</td>
<td>80 – 95</td>
<td>60</td>
</tr>
</tbody>
</table>

The birefringent nature of LCs, SLMs only work as phase modulators for polarized light; the specific polarization direction is specified by the manufacturer.

Figure 1.12: Structure of an EASLM, showing a liquid crystal layer (c) sandwiched between two alignment layers (d) on an aluminium pixel array (b) which is mounted on a CMOS chip (a). A voltage between the CMOS chip and indium tin oxide layer (e) controls the birefringence of the liquid crystals in each pixel, which changes the phase of the incident light (g) to obtain modulated, reflected light (h) [1].

The most common experimental setup in order to use an SLM for beam-shaping is shown in Fig. 1.13. A polarized laser beam must be used, which is orientated to match the SLM polarization direction. A beam expanding telescope (BET) is often necessary to match the beam size to the size of the SLM. A collimated beam with flat wave-front is reflected off the SLM displaying the required phase screen, thereby acquiring the required phase modulation. Since an SLM is a repetitive structure, several orders of diffraction will be present, separated by a characteristic angle. If, as is often the case, the far-field pattern of the modulated beam is required, then an iris is used to block off unwanted orders and only
allow the selected order through to be captured on a camera placed in the Fourier plane of a lens.

![Figure 1.13: Common experimental setup of an SLM to generate custom beams. Laser light is polarized and then expanded onto the SLM. One diffraction order is selected using an iris at the focal plane of a lens and recorded on a camera (CAM).](image)

### 1.3 Custom modes and laser beam shaping

Soon after the invention of the laser in 1960 [82], distinctive intensity patterns in the beams or modes were modelled using round-trip loss considerations [83]. By 1962 scientists had inserted a circular aperture into a resonator to select for the lowest-order mode, the Gaussian beam [3; 84], and by 1972 the first-order $TEM_{01}$ was selected for as the output beam [85]. Hermite-Gaussian beams were reasonably easy to obtain, but Laguerre-Gauss beams proved to be more difficult. They were first preferentially selected in 1990 using a pump-shaping technique [86].

Several techniques have been devised for selecting higher-order modes. The first and simplest was to insert fine metal wires near one of the end mirrors coinciding with node lines which give high loss to all but the the desired mode, and was demonstrated for both HG modes [87] and LG modes [88]. Wires work by introducing scattering losses, but heat up and become inefficient. An alternative is to use non-absorbing phase elements, which
introduce losses by interference and diffraction. A phase plate with a $\pi$ phase shift line acts as a loss line inside a resonator, but with higher mode discrimination [6]. A $\pi$ phase shift is used so that there is a $2\pi$ phase shift after one round trip through the resonator, resulting in no net phase shift. This concept can be expanded to multiple lines or rings to select low-order HG or LG modes [89; 90]. The discovery that LG modes have wave-fronts with $n\pi$ spiral phase shifts [91] can be exploited to create $LG_n^0$ modes by inserting a $2\pi$ spiral phase plate into the resonator [92]. Different spiral beams have also been created by inserting a Dove prism into a ring resonator in order to rotate the internal beam [93].

Pump-shaping provides regions of high and low gain inside a resonator and can thus function in a similar way to inserted regions of loss. The pump intensity profile can be modulated by, for example, diffraction from an aperture [94], or by a phase-plate [95].

It is generally true that beam-shaping methods can be applied to any wavelength of laser light, although some are more suited to longer wavelengths due to challenges in manufacturing processes. For example, custom-shaped or aspherical mirrors have been used to shape the output beam [96], but this method is limited to lasers with longer wavelengths like CO$_2$ lasers because of relatively large feature size. Deformable mirrors are active aspherical mirrors, able to respond to changes in wave-front using a wave-front sensor in a closed loop. These have been used first to improve beam quality [97], for the formation of a super-Gaussian output beam [98], for aberration correction [99], and to dynamically switch between various low- and high-order output modes [100].

Modulating the phase inside a resonator allows the formation of new and custom intensity distributions. One common method is using diffractive optical elements (DOEs), which are etched using an electron beam using a multi-stage mask process. Intracavity DOEs have been used for intracavity beam-shaping to generate, for example, a square flattop beam [7]. Holographic elements have been recorded in photo-polymer sheets, with an SLM to control the phase of a signal beam, and used as intracavity elements to generate super-Gaussian output beams from an Nd:YAG laser [101]. Phase modulation has also been provided by an intracavity optically-activated SLM, and used to produce circular and square super-Gaussian beams [102]. However this design has the disadvantage of requiring a complicated optical imaging system to address the OA SLM.

Until the work which follows in Chapter 2, an electrically-activated (EA) SLM had
1.3. CUSTOM MODES AND LASER BEAM SHAPING

never before been used as a phase modulating element inside a laser resonator. The design
consideration necessary to achieve this are discussed.
1.4 Outline

This thesis deals with the use of SLMs in the field of novel laser beams. It is structured as follows.

An EA-SLM was used for the first time as an intracavity optical element in order to achieve real-time intracavity beam shaping. The details of how this was achieved is explained in Chapter 2.

An SLM was used to generate Laguerre-Gauss beams, which have self-healing properties. The radial flow of energy which results in self-healing is explained, with experimental verification, in Chapter 3.

An SLM was used to generate completely new beams, belonging to a class of Bessel-like beams, which also have self-healing properties resulting from a longitudinal energy flow. This is explained, with experimental verification, in Chapter 4.

Finally, this thesis is concluded in Chapter 5 with a summary of our contributions to the field of beam shaping using an SLM, and a discussion of future work within this field.
Chapter 2

SLM for intracavity beam shaping

In this chapter we outline the steps necessary to create a laser with an intra-cavity spatial light modulator (SLM) for transverse mode control. We employ a commercial SLM as the back reflector in an otherwise conventional diode-pumped solid state laser. We show that the geometry of the liquid crystal (LC) arrangement strongly influences the operation regime of the laser, from nominally amplitude-only mode control for twisted nematic LCs to nominally phase-only mode control for parallel-aligned LCs. We demonstrate both operating regimes experimentally and discuss the potential advantages of and improvements to this new technology.
2.1 Introduction

Good beam quality associated with lower-order modes is a fundamental requirement for industrial applications like cutting and welding that require a tightly focused beam. Applications such as paint stripping, penetration laser drilling and thin-film welding, however, require a flat-top beam profile, while high-volume parallel processes require a single beam to be split into an array of beams. The required beam shape for a particular application may be created by a range of techniques [8].

For example, a simple amplitude filter may be used to produce a Gaussian beam, but at the expense of power. A more efficient method of manipulating the intensity distribution of a given beam is using phase plates, but these are static, custom components, and their performance deteriorates with any variation in size of the initial beam [103]. Deformable mirrors were originally developed to correct for atmospheric disturbance in telescopes, but have proved useful for beam shaping applications, and have been used for producing circular and rectangular flat-top intensity profiles. They have the drawback however that the number of mirror elements is limited, and so the feature size of the beams produced by deformable mirrors is therefore limited [99; 104]. A more common approach today is to use liquid crystal displays in the form of spatial light modulators (SLMs) to dynamically mimic both amplitude and phase transformations. These devices are easily programmed by simply displaying the required phase, represented by a bitmap image, on the high-resolution SLM screen [54].

For the most part the aforementioned techniques are used to modify an existing beam outside a resonator, but it is possible to reduce the number of optical elements and increase the efficiency of a system by putting the modulating device inside the resonator. Intracavity amplitude filters, phase plates and deformable mirrors have all been used to modify the output beam [105; 106]. An intracavity optically addressed SLM has also been used to manipulate the beam intensity profile [101], but required a complex intracavity imaging system to create a phase screen. (Intracavity mode selection and custom modes are discussed in more detail in Section 1.3). More recently we have demonstrated the on-demand creation of modes with an intracavity electrically addressed SLM [55; 57; 58; 60]. The unique advantages of using an intracavity electrically-addressed SLM are the ability to create a very wide range of free-space beams, and the ability to do so dynamically.
2.1. INTRODUCTION

In this chapter we outline the necessary steps to construct a laser incorporating an intracavity electrically addressed SLM for transverse mode selection. We outline the design considerations, advantages and disadvantages of this approach, and provide a detailed performance evaluation of the SLM and laser. This work can be a useful reference for others wishing to build such devices.

2.1.1 Liquid crystal considerations

In a typical SLM cell a thin layer of liquid crystal (LC) material is sandwiched between transparent electrodes. The LC materials used in SLMs are birefringent, with two refractive indices that depend on the direction of the molecular axis. The LCs are also electro-active, and align according to the applied electric field. The most important configurations are twisted nematic (TN), and parallel aligned (PA) cells. In a twisted cell, the orientation of the molecules differs by typically 90° between the top and the bottom of the LC cell and rotates helically between (see 1.10 (left)). In PA cells, the alignment layers are parallel, so the LC molecules are oriented in the same plane (see 1.10 (right)).

The birefringence of the liquid crystals causes a change in the polarization of monochromatic, polarized incident light, leading to a modulation of phase and/or amplitude. For TN cells the situation is complex, and a $2 \times 2$ Jones matrix is used to model the change in polarization of light passing through a TN LC cell, the eigenvectors of which correspond to elliptically polarized waves that propagate through the system without a change in the polarization state, and are subject only to phase modulation [22] [107] [108]. A small degree of amplitude modulation is introduced by a polarizer behind the SLM, leading to the mode of operation which is often referred to as phase-mostly operation [109]. Fig. 2.1 is derived from reference [109], and shows that varying the incident polarizer angle can at best reduce the amplitude modulation of a field passing through a TN-LC cell, but never eliminate it completely.

In PA cells, light polarized linearly parallel to the extraordinary axis of the LC material is retarded as a function of the birefringence. Therefore, these cells are true phase-only modulators of linearly polarized light, with no amplitude modulation.

It is impossible to specify the chemical composition of the liquid crystal used in commercial SLMs, since this is always commercially confidential. It is usually a mixture, en-
Figure 2.1: Theoretically calculated curves show the transmission through a TN-LC layer for twist angle 90°, for incident polarizer angles 0°; 5°; 10° (solid line), 15° and 20°. The solid line shows the configuration with the most constant transmission [109].

gineered by a chemical company (Merck, for example), and suitable for the application. There will however be several key features that are common for this application, and the mixture will be optimised for them [110]. Firstly, both TN and PA LCs have a nematic phase, so specifying this is an obvious starting point. Secondly, a birefringence of around 0.2 is fairly typical; 5CB is just one well-known material with this property. Thirdly, LC mixtures need to have little temperature dependence at and around room temperature, so are chosen to undergo a transition to an isotropic liquid at temperatures greater than 60°C, and often as high as 100°C. It is for this reason that LC cannot be used at high powers. Fourthly, the response time (typically 10 ms) depends on the viscosity, elastic constants, device thickness and applied voltage. Also, a compromise must be found between a small cell gap which gives a faster response, and a thicker gap which is slower but gives a wider phase modulation range.

### 2.2 SLM characterisation and design considerations

In standard operation, each pixel on an SLM is addressed by a pixel on a grey-scale bitmap. For each pixel a grey scale level between 0 and 255 corresponds to a phase change of between 0° and 360° being imparted on the reflected beam by the corresponding pixel on the SLM.
2.2. SLM CHARACTERISATION AND DESIGN CONSIDERATIONS

Table 2.1: Comparison of typical specifications of the older type of SLM using TN-LCs and the newer type using PA-LCs.

<table>
<thead>
<tr>
<th>SLM type</th>
<th>Resolution (pixels)</th>
<th>Area (mm)</th>
<th>LC</th>
<th>Reflectivity</th>
<th>Damage Threshold (W/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TN-LC</td>
<td>1920 × 1080</td>
<td>15.36 × 8.64</td>
<td>TN</td>
<td>∼ 60%</td>
<td>2</td>
</tr>
<tr>
<td>PA-LC</td>
<td>792 × 600</td>
<td>16 × 12</td>
<td>PA</td>
<td>&gt; 90%</td>
<td>15</td>
</tr>
</tbody>
</table>

As will be seen in the discussion that follows, the most significant differences in performance of SLMs used as an intracavity component were a result of the type of liquid crystal used in the SLM. We consider here the two most common liquid crystal geometries: twisted-nematic liquid crystals (TN-LC), and parallel-aligned liquid crystals (PA-LC). Table (2.1) shows a comparison of typical specifications.

For most LCD applications a high resolution is regarded as being desirable. For SLMs used inside a resonator, however, the lower resolution of the PA-LC SLM presented no limitations.

Both types of SLM require linear vertically polarized light to perform optimally as phase screens, and behave as plane mirrors for light polarized perpendicular to this axis. It is therefore necessary to ensure vertical polarization in all experiments, and in the design of the SLM resonator. In addition, the possibility that either SLM would depolarize the beam was also considered. Experiments confirm however that there is no depolarization of the incident light on a single reflection off the SLMs for any phase.

Since the intracavity power is typically one order of magnitude higher than the extracavity power, one of the primary considerations is to prevent damage to the SLM. Depending on the expected intracavity power density it could be necessary to expand the beam in order to decrease the power density. Any clipping of the beam by the edges of the SLM active area will result in distortion of the desired mode. Using the second moment definition of beam radius, as a starting point the expected beam radius should be designed to be between 1/4 and 1/6 of the shorter dimension of the SLM active area. For example, the SLM with active area 15.36 × 8.64 mm would be illuminated by a spot radius no larger than 2.16 mm. An intracavity telescope is typically used to achieve this.

The zeroth-order reflectivity of the TN-LC SLM and PA-LC SLM were specified to be 60% and > 90% respectively, with no specification given as to the variation in reflectivity.
with phase. The reflectivity of each SLM as a function of phase \( R(\theta) \) was measured experimentally by reflecting a 1.064 \( \mu \text{m} \) Nd:YAG laser beam of constant power off the SLM and recording the reflected power. The grey level or phase shift of a uniform screen was increased in steps from 0 to 255 grey shades, or from 0° to 360°. The mean reflectivity of the TN-LC SLM over 360° was measured to be 51% for vertical (correct) polarization and 64% for horizontal (incorrect) polarization. The mean reflectivity of the PA-LC SLM over 360° was measured to be 91% for vertical (correct) polarization and 93% for horizontal (incorrect) polarization. One result of this is that a laser with this intracavity component will tend to produce radiation with polarization which is incorrect for the SLM, and that another polarization-selecting component needs to be included to ensure the correct polarization. Typically a Brewster window is used inside a cavity to select one polarization direction.

In conventional use, the beam reflected off an SLM is at a small angle from the incident beam, and a phase grating superimposed on the desired phase serves to separate the diffracted light away from the undiffracted light. Since it replaces a mirror, an SLM inside a resonator must be aligned perpendicular to the optical axis, with the reflected beam returning along the path of the incident beam, with the diffracted and undiffracted beams coaxial. Since a laser preferentially amplifies the mode with lowest loss, it tends to amplify the mode which is selected for by the SLM screen, and suppress any other modes, including that containing the undiffracted beam. In a similar way, as it is a pixelated device an SLM has the property of any periodic structure in that a small fraction of incident light is diffracted into higher orders. Fortunately these higher orders are diffracted away from the optical axis and lost, and only the lowest order containing the selected mode is amplified.

The average reflectivity of an SLM acts as a loss in the cavity, which can be compensated for with higher gain, typically by increasing the pump power. Far more important for our application is the variation in reflectivity with phase. The percentage variation in reflectivity was measured at 9.5% and 0.75% for the TN-LC SLM and PA-LC SLM respectively, as in Figure 2.2.

The explanation for this variation in reflectivity can be found in reference [109], which explains that in TN-LC modulators with no field applied, the liquid crystal molecules are aligned in a 90° spiral between the front and back of the layer. When an electric field is applied across the layer, the liquid crystal molecules become tilted and cause an ellipticity
Figure 2.2: Measured reflectivities of TN-LC SLM and PA-LC SLM as a function of phase, for vertical polarization.

In the polarization state. In TN-LC SLMs, the transmitted intensity is determined by the LC properties of twist angle and birefringence, as well as the orientation angles of internal polarizers which are used to cut out the non-linear component. The result is an unavoidable amplitude modulation that can be reduced but not eliminated altogether (compare Fig. 2.1 to Fig. 2.2 (TN-LC curve)), and therefore TN-LC SLMs are referred to in the literature as “phase-only” or phase-mostly SLMs [109; 111; 112].

Remember however that SLMs are designed to be used as single reflectors, and in typical applications this residual amplitude modulation is negligible. Inside a laser resonator, however, it will be shown to have a determining effect on the laser performance. In PA-LC devices, the liquid crystal molecules are aligned in parallel, not in spirals, when no electric field is applied. When an electric field is applied the molecules are tilted in the direction of the substrates. When the polarization direction of the incident light is parallel to the axis of the liquid crystal molecules only the refractive index along the optic axis is changed, the light is not depolarized, and in theory phase-only modulation can be achieved [30; 113; 114]. In practice a small residual amplitude modulation can be measured, caused by reflection changes at the optical boundary and diffraction due to index changes of the LC material [115].

If \( R_1(\theta) \) is the reflectivity of the SLM, and \( R_2 \) is the reflectivity of the output coupler, then after \( n \) round trips through the laser cavity, a unit of intensity will have intensity \( I(\theta, n) \).
given by

\[ I(\theta, n) = (R_1(\theta)R_2)^n. \]  

(2.2.1)

This simple amplification model reveals that this small variation in SLM amplitude modulation, when amplified through many round trips in a resonator, is sufficient to cause a higher lasing threshold at some phase values than at others. The model uses the amplitude modulation data from Figure 2.2 and is used to generate the graphs in Figure 2.3 which predict relative intensity as a function of phase after 0, 5, 10 and 20 round trips for the TN-LC SLM and the PA-LC SLM resonators respectively.

Figure 2.3: Plots of normalized relative intensity as a function of phase after \( n = 0, 5, 10 \) and 20 round-trip reflections of a beam with stable mode off an SLM in a resonator, using measured intensity modulation data for (a) a TN-LC SLM and (b) a PA-LC SLM.

Figure 2.3 (a) suggests that when a TN-LC SLM is used as an intracavity element, the variation in amplitude modulation which accompanies the desired phase modulation
will be significant, and cautions that amplitude modulation effects could swamp the phase
modulating effect. Figure 2.3 (b) shows that any small residual amplitude modulation in
a PA-LC SLM will have a far smaller effect on laser output, which will allow phase-only
behaviour to dominate.

2.3 SLM laser description

Our experimental approach, illustrated schematically in Figure 2.4, was to proceed in a
step-by-step manner from a known, conventional resonator, through a series of equivalent
resonators, ending with our goal configuration, and ascertaining equivalence in terms of
the output beam at each stage. The conventional resonator (Figure 2.4 (a)) contains two
reflective mirrors $M_1$ and $M_2$ and comprises a stable cavity. Next a lens $L_2$ (focal length $f = R$) combined with a flat mirror replaces the back reflector $M_2$ (radius $R$), see Figure 2.4(b). Then, with the lens still in place, a reflective SLM without phase modulation (effectively
just a flat mirror) replaces the flat mirror, see Figure 2.4(c). Lastly the lens $L_2$ is removed,
and curvature equivalent to this lens is displayed on the SLM (Figure 2.4(d)).

The prototype SLM laser was constructed as shown in Figure 2.6. It employed a 1%
doped Nd:YAG crystal rod with dimension of 30 mm (length) $\times$ 4 mm (diameter) as gain
medium, which was end-pumped with a 75 W Jenoptik (JOLD 75 CPXF 2P W) multimode
fibre-coupled laser diode (see Fig. 2.5). A $4 \times$ Galilean beam-expanding telescope (BET)
was used to increase the spot size on the SLM to 2 mm, in order to optimally fill the SLM
while reducing the power density on it. The resonator was folded to facilitate the pump
scheme, as well as to exclude the pump power from the leg containing the SLM.

Both the output coupler and the SLM were flat, and the resonator was marginally stable
due to a small degree of thermal lensing. The output coupler reflectivity was 95%. In
order to facilitate the alignment of the SLM, as well as to characterize the resonator without
the SLM, the resonator included a flat 60% mirror immediately in front of the SLM on a
flip-up mount (FUM). Since it was necessary to work at low laser powers in order to avoid
damaging the SLM, two techniques were used to facilitate alignment. The SLM displaying
a uniform grey-level screen was inserted once the resonator was lasing with the 60% back
reflector. With the current set just above threshold, the SLM alignment was adjusted until
a jump in output power was observed. At this position the SLM was contributing to the
reflectivity of the back reflector, and was perfectly aligned. The 60% FUM could now be moved out of the optical path. The second technique centred the SLM on the optical axis: The SLM with a uniform grey screen was aligned to give a good Gaussian. The phase screen was changed to one with a centred horizontal strip (as in Fig. 2.7 (b) bottom), and the SLM was adjusted vertically until the Gaussian beam split into a Hermite-Gauss beam $(n = 1, m = 0)$, as in Fig. 2.7 (b) top, thereby centring the screen along the vertical axis. This was then repeated for the horizontal axis. The nominal length of the cavity was 390 mm but was determined to have an effective length of 373 mm to compensate for the small thermal lensing due to pump absorption in the crystal as well as the refractive index of the crystal. The effective length was used in calculations of the beam sizes.

Figure 2.4: (a) A simple, conventional resonator comprising two reflective mirrors, (b) Mirror $M_2$ replaced with a flat mirror and lens $L_2$, (c) Flat mirror $M_2$ replaced with blank-phase SLM, (d) Phase on SLM to simulate lens $L_2$. 
2.3. SLM LASER DESCRIPTION

The laser included a BK7 Brewster window orientated at 56.4° to ensure that the beam was correctly polarized for the SLM. Filters separated the pump beam from the required beam, which was recorded using a Photon USBamPro beam profiling system.

Two models, each with a different SLM, were constructed successively in order to evaluate the performance differences between the two systems. The TN-LC model included a 4x BET, but this was not included in the PA-LC model due to a higher SLM damage threshold.

Figure 2.5: Graph of the 75 W Jenoptik (JOLD 75 CPXF 2P W) multimode fibre-coupled laser diode output power as a function of current.

Figure 2.6: Diagram of the SLM diode-pumped Nd:YAG laser. The Nd:YAG crystal was end-pumped by an 808 nm diode laser pump (DLP). The resonator has a spatial light modulator (SLM) as back-reflector, and a 95% flat output coupler (OC). The resonator also contains a 60% flip-up mirror (FUM) in front of the SLM, a Brewster window (BW) and a 4x beam-expanding telescope (BET).
CHAPTER 2. SLM FOR INTRACAVITY BEAM SHAPING

2.4 Experimental results

The first prototype of a laser with an intracavity SLM included the TN-LC SLM. The initial equivalence of two configurations was tested: the resonator with a flat 60% reflectivity mirror (flip-up mount up) as back reflector, and the same resonator with the TN-LC SLM with uniform phase as back reflector (flip-up mount down). Both produced a Gaussian beam with radius 0.26 mm on the output mirror. Note however that the beam size on the SLM was many times bigger than this, and typically hundreds of pixels of the holograms will be used.

A lens phase pattern on the SLM, which is equivalent to a curved back reflector, resulted in the cessation of lasing instead of the expected change in beam size. Similarly, a linearly varying phase pattern also stopped lasing, instead of producing the expected misalignment effects. This indicated that when used in an intracavity configuration, the phase modulation effects of a TN-LC SLM are swamped by amplitude modulation effects and that it behaves primarily as an amplitude modulator. The bitmaps shown in Figure 2.7 (bottom row) were generated in order to use this effect to select the laser mode in the manner analogous to the use of intracavity wires [88]. They consist of a geometric shape with a uniform grey level corresponding to the value for which minimum power output was obtained (grey level 85, phase 120°) superimposed on a uniform background with grey level corresponding to the maximum power output (grey level 225, phase 316°). The resulting laser beams are shown in Figure 2.7 (top row), and confirm that the features behave as localized regions of loss: in Figure 2.7 (a) a uniform grey bitmap generates a Gaussian beam; in Figure 2.7 (b) a central horizontal strip forces the laser into a Hermite-Gauss beam ($n = 1, m = 0$); in Figure 2.7 (c) a pattern of intersecting strips at 45° to each other generates an 8-petal patterned beam; and in Figure 2.7 (d) a spot forces it into a donut-shaped beam.

The beam patterns were measured in the near- and far-fields for several output patterns (see Figure 2.8). In each case the intensity distribution pattern in the near-field was the same as that of the far-field, which showed that these laser modes are also free-space modes and invariant on propagation.

Although the beams appeared to comprise of pure transverse modes, an azimuthal modal decomposition was performed on the 6-petal output beam using the optical inner product technique [75]. To do this, the petal field $U(x, y)$ is decomposed into a set of angular
Figure 2.7: Beam patterns produced by the laser containing the intracavity TN-LC SLM are shown in the top row, with the corresponding bitmaps below each. (a) is a Gaussian beam; (b) is a Hermite-Gauss beam \((n = 1, m = 0)\); (c) is an 8-petal patterned beam, and (d) is a donut beam.

Figure 2.8: Examples of laser modes produced by the laser. In each case (a - h) the near-field pattern is shown, with the far-field pattern inset. Notice that the near-field beam pattern matches the far-field pattern. The modes can be identified as: (a) Gaussian; (b) Hermite-Gauss \((n = 0, m = 1)\); (c) Hermite-Gauss \((n = 1, m = 0)\); (d) donut mode; (e) Hermite-Gauss \((n = 0, m = 2)\); (f) Laguerre-Gauss \((p = 0, l = \pm 2)\); (g) Laguerre-Gauss \((p = 0, l = \pm 3)\); (h) Laguerre-Gauss \((p = 0, l = \pm 4)\).

harmonics, \(\exp[i\ell\phi(x, y)]\), where \(\ell\) is referred to as the topological charge and can take any integer value, and \(\phi\) is the azimuthal coordinate. These harmonics are orthogonal over the azimuthal plane, but do not form a complete basis set since they have no radial dependence. The first five (both positive and negative) are shown in Fig. 2.9 The unknown field \(U(x, y)\) is directed onto a second (external) SLM displaying the conjugate phase of each angular harmonic in turn, \(\Psi_n^*(x, y) = \exp[-in\phi(x, y)]\). The resulting field, \(u(x, y) = U(x, y)\Psi_n^*(x, y)\), is Fourier transformed using a thin lens positioned a focal length from the SLM. The intensity measured on the optical axis gives the relative weightings by Eq. 1.1.23.
The modal decomposition confirmed that the six-petal beam consists of Laguerre-Gauss modes \( (p = 0, l = \pm 3) \) with purity measured at greater than 90%. (see Figure 2.10). The reason for the high mode purity is that features on the SLM bitmaps serve as localized regions of loss along the nodal lines of a particular transverse mode, which results in a loss differential which selects for that mode and against all others.

In order to avoid thermal effects the SLM was mounted onto a heat sink, and the power incident on the SLM surface was limited to the manufacturer’s specified damage threshold. Priority was also given to minimising thermal distortion of the laser crystal and other optical components, so most work was done just far enough above threshold to avoid flickering, with the output power not exceeding 200 mW. A weak thermal lens of about 25 m in the laser crystal caused the cavity to be stable even in a flat-flat configuration, and an unvarying mode was obtained from the system for periods in excess of an hour.
2.4. EXPERIMENTAL RESULTS

The second prototype used the PA-LC SLM as an intracavity component. To test whether the SLM screen serves as a phase modulator, an experiment was performed to determine whether a resonator containing an intracavity SLM displaying digital holograms of curved mirrors with radius of curvature \( R \) does indeed produce beams equivalent to the identical conventional resonator, as required in Figure 2.4(a) and (d). The beam waist \( w_0 \) on the output coupler was measured for a number of hologram curvatures as well as for two curved conventional mirrors and compared to the analytical expression \[116\]:

\[
w_0^2 = \left(\frac{\lambda}{\pi}\right) \sqrt{L(R-L)}
\]  

(2.4.1)

where \( L \) is the effective length of the resonator and \( \lambda \) is the laser wavelength.

Figure 2.11 shows these beam size changes with hologram curvature.

The results in reference \[55\] confirmed that the amplitude modulation effects of an intracavity PA-LC SLM screen are negligible, and that it does indeed behave as a phase modulator. The losses due to the SLM are higher than for physical mirrors (the threshold when the resonator contained the SLM was 27.5 W compared to 11.3 W with physical mirrors), but are easily overcome by increasing the pump power.

The beams labelled (a) to (d) in Figure 2.12 were produced using the corresponding digital holograms shown below each beam, and can be identified as (a) a circular flat-top
beam; (b) an Airy beam; (c) a Laguerre-Gauss beam \((p = 1, l = \pm 2)\); and (d) a Laguerre-Gauss beam \((p = 1, l = 0)\).

Figure 2.12: Beam patterns produced by the second prototype containing the intracavity PA-LC SLM, identified as (a) a circular flat-top beam; (b) an Airy beam; (c) a Laguerre-Gauss beam \((p = 1, l = \pm 2)\); and (d) a Laguerre-Gauss beam \((p = 1, l = 0)\). The corresponding digital holograms are shown below each beam. Detail of the insert of digital hologram (c) is shown below to illustrate the use of a complex amplitude modulation technique (here using a checker-board pattern) to modulate amplitude in addition to phase.

The digital holograms required to produce beams (a), (b) and (d) in Figure 2.12 contain only phase features, but the flexibility of the device was increased by incorporating a localized checker-board pattern to make use of a complex amplitude modulation technique \[117\]. This technique, used to produce beam (c) in Figure 2.12 and shown in more detail below the beams, demonstrates that almost any required output beam can be achieved using digital holograms containing a combination of phase- and amplitude-modulation patterns.

2.5 Conclusion

While it is well understood how to use a phase-only (or phase-mostly) SLM in a conventional, single reflection configuration, the effects of subtle properties of an SLM become apparent when it is used as an intracavity component. For example, the average reflectivity over all phase is an unwanted loss in conventional use, but in intracavity use can be read-
2.5. CONCLUSION

ily compensated for by increasing pump power and consequently has a negligible effect. Conversely, the variation in reflectivity is negligible in conventional use, but is amplified in intracavity use until amplitude modulation is obtained from an SLM which is nominally a phase modulator. In terms of mode selection, there is a significant advantage to placing an SLM inside a resonator, namely that laser resonators are very effective filters, with the ability to amplify only the mode with the lowest loss and to suppress any others. This results in pure output modes, free of any unwanted superimposed modes.

A virtually infinite set of free-space output beams can be produced by this device by the judicious combination of phase- and amplitude-modulation techniques in digital holograms displayed on the SLM screen, limited only by the resolution of the SLM. In addition, changing a digital hologram on the SLM screen requires no realignment, and the output beam can be cycled at the SLM refresh rate.

The biggest limitation of this device is on the output power, which is imposed by the damage threshold of the SLM. Further experiments are planned to amplify shaped beams to higher power levels.
Chapter 3

Angular self-reconstruction of petal-like beams

The self-reconstruction of superpositions of Laguerre-Gaussian beams has been observed experimentally, but the results appear anomalous and without a means to predict under what conditions this take place. In this chapter we offer a simple equation for predicting the self-reconstruction distance of superpositions of Laguerre-Gaussian beams, which we confirm by numerical propagation as well as by experiment. We explain that the self-reconstruction process is not guaranteed and predict its dependence on the obstacle location and obstacle size.
CHAPTER 3. ANGULAR SELF-RECONSTRUCTION OF PETAL-LIKE BEAMS

3.1 Introduction

Laguerre-Gaussian (LG) modes were known to be solutions to the paraxial wave equation since the 1960s [63], but theoretical work on the subject predominated because reliable experiments were difficult to carry out [118]. Hermite-Gaussian (HG) modes could be created by the insertion of wires into a resonator, but intracavity LG modes were more difficult to generate [72]. The prediction that Laguerre-Gaussian beams carried orbital angular momentum [91], however, stimulated attempts to generate these beams. The first LG beams were produced external to the laser cavity by passing a Gaussian beam through a $2\pi$ spiral phase filter [119; 120], through a hologram [121; 122], and through a pair of cylindrical lenses [123; 124; 125; 126]. Digital holograms have become the most convenient method of producing LG beams since the development of the phase-only SLM into a common laboratory device [127; 128; 129; 130; 131; 132].

One of the major fields of application of LG modes is optical trapping, where a beam is engineered to trap and hold microscopic particles. Single and multiple metallic particles were manipulated in a controlled fashion [133; 127; 134], and LG beams can be used to sort particles by size [135]. Doughnut modes can be used to trap ultra-cold atomic particles in the dark axial region of the beam [136; 137; 138]. Many of the applications of LG beams result from their carrying orbital angular momentum, and they are used to both trap and to rotate particles by transferring their optical orbital angular momentum [139; 133]. They are used to study optical vortices [140] for applications like quantum information transfer [141]. Other applications include the improvement of confocal microscope performance [142], in LIDAR installations [133], and in extremely sensitive laser interferometers for use in future gravitational wave detectors [144].

3.2 Introduction to petal-like beams

It is possible to form higher-order modes inside a resonator, so that a high-order mode emerges from the laser. Pure LG modes $LG_+ \text{ or } LG_-$, also known as doughnut modes, are difficult to produce inside a resonator, with resonators tending to produce both $LG_+$ and $LG_-$ beams, which superpose coherently to form petal-shaped beams. Petal modes have been produced by a ring resonator incorporating a Dove prism [93], by shaping the gain
profile by shaping the pump beam \cite{86,94,145,95} (useful when there is no space for intracavity optics), by inserting a spiral phase element into the resonator \cite{146,120,92} \cite{90}, and using an amplitude mask \cite{147}. These modes are also spontaneously formed in Porro prism resonators.

It was the study of petal-like output beams of Porro prism lasers \cite{148}, and the subsequent capacity to generate petal-like beams using the “digital laser” detailed in Section 2 which allowed the study of the healing or self-reconstruction of these beams on propagation.

### 3.2.1 Porro prism lasers

The Porro prism laser configuration has been widely used for over 30 years in commercial and military applications, where their inherent ruggedness makes them ideally suited to applications where a laser beam is required at a large distance from the source, and where the source is not mounted on a stable platform. In typical field use the conditions these resonators are subject to could include shock and large temperature variations, and will experience some degree of optical misalignment. Porro resonators have been extensively used in long-range military beam applications like range finders and laser designators (\cite{139}, \cite{150}, \cite{151}, \cite{152}), as well as in exotic laser systems such as the Mars Observer Laser Altimeter \cite{153}, and the CALIOP lidar system \cite{154}.

Porro (or roof) prisms are right angled prisms which use the principle of total internal reflection to reflect incoming light. As shown in Fig. 3.1 when the prism is aligned (when the hypotenuse or input face is vertical), an incoming ray enters through this face, and is reflected off the two 45° faces in turn, exiting the hypotenuse face parallel to the input ray. When the prism is misaligned by being rotated through angle $\beta$ in the plane of the page the exit ray remains parallel to the input ray, and is only offset by a small amount $\delta$.

In contrast, mirrors have the well-known property that they reflect an incident ray at an angle equal to the incident angle \cite{155}. As a consequence, any tilt or misalignment of a laser mirror will result in a deflection of the reflected ray away from the optical axis, and it will tend to “walk off” the mirrors and cause the cessation of lasing. Replacing the laser mirror in a flat-flat Fabry-Perot cavity with crossed Porro prisms makes the resonator insensitive to misalignment in the sense that the tilt of either Porro prism results in just a small reduction in the active volume of the laser gain medium with a corresponding small drop in output.
CHAPTER 3. ANGULAR SELF-RECONSTRUCTION OF PETAL-LIKE BEAMS

Figure 3.1: Sketch of a Porro or roof prism, showing a correctly aligned prism on the left, and a misaligned prism with vertical offset $\delta$ corresponding to an angular offset $\beta$ on the right.

Figure 3.2: Schematic diagram of a Porro prism resonator, showing the following optical elements: (a, h) Porro prisms, (b, g) lenses, (c) polarizing beam cube, (d) quarter-wave plate, (e) Q-switch, (f) Nd:YAG rod.

Fig. 3.2 shows an example of a Porro prism resonator. Gain is provided by a pumped laser crystal. The end mirrors are replaced by “crossed” Porro prisms, so that the apexes are at 90° to each other. With this configuration any misalignment in one direction is compensated for by one prism and any misalignment in the orthogonal direction is compensated for by the other prism, thus making the resonator insensitive to misalignment. In a conventional Fabry-Perot resonator the stability of the resonator is determined by the radius of curvature on the mirrors. In a Porro resonator however the Porro prisms do not contribute any focusing power and so intracavity lenses may be included to determine the stability. In a traditional mirror resonator the laser beam is coupled out through a partially transmitting output coupling mirror. In a Porro prism resonator, with both resonator mirrors replaced by roof prisms, output coupling is realized by polarization techniques using a polarizer.
3.2. Modes from Porro prism lasers

Despite the ubiquitous nature of Porro prism lasers in the field, for a long time the output modes from such lasers were not fully understood. Beams with either radially-symmetric lobed (or “petal”) patterns, or flattened doughnut patterns are reported to be characteristic of Porro prism lasers [157; 158; 159]. An early paper [160] which considers the theoretical properties of prism resonators, mentions the bevel at the apex of each prism as a possible explanation for sectors of the beam to oscillate independently, but does not develop this idea into an explanation for experimentally observed petal patterns. A physical optics model which treats the Porro prisms as perfect mirrors [161] predicts that Hermite-Gauss modes can be expected from Porro prism resonators, which is clearly in opposition to experimentally observed fields. Despite this, this remained the preferred model [162; 166; 163] until recently.

However it must be recognised that Porro prisms differ from end mirrors in two important respects:

- the field which falls on a mirror is reflected off directly, but the field which falls onto a Porro prism undergoes a reflection across the apex before being reflected away from the prism, and

- for a mirror it is only necessary to consider the diffraction losses from the limiting aperture, but for a Porro prism it is necessary to consider losses from the limiting aperture as well as from the (small) bevel at the apex of the prism.

We then developed a new approach [164; 148] which included the loss from the apex of the prism as a loss screen as well as the inversion of the field about the prism apex on every pass. This approach predicted the formation of ‘petals’, where the number of petals $N$ can be calculated for discrete values of the Porro angle $\alpha$, defined as the angle between the two loss lines indicated in red on Fig. 3.2 as viewed along the optical axis:

$$N = \frac{j2\pi}{\alpha}$$  \hspace{1cm} (3.2.1)

for some integer $j$ such that $N$ is also an integer.
CHAPTER 3. ANGULAR SELF-RECONSTRUCTION OF PETAL-LIKE BEAMS

This approach was confirmed in a numerical simulation as well as experimentally. Fig. 3.3 shows examples of the beams produced by varying the Porro angle $\alpha$. Note that petals are only formed for integer values of $N$ in Fig. 3.3(a)-(c), but not for Fig. 3.3(d).

![Figure 3.3: Output of the numerical model of Porro prism laser showing examples of beams produced with (a) Porro angle $\alpha = \frac{\pi}{2}$, (b) $\alpha = \frac{\pi}{3}$, (c) $\alpha = \frac{\pi}{4}$, (d) $\alpha = 0.7625$.](image)

This numerical model was then used [165, 166, 167, 168] to investigate the effect of varying the stability parameter $g_1g_2$ (see Eq. 1.1.1) by varying intracavity lenses, to explore the temporal development of modes, and to see the effect of increasing the Fresnel number $N_F$ (see Eq. 1.1.9) by increasing the clear aperture available to the mode.

![Figure 3.4: Output of the numerical model of Porro prism laser showing examples of beams produced with (a) Porro angle $\alpha = 0.174, N_F = 0.371$, (b) $\alpha = 0.523, N_F = 0.428$, (c) $\alpha = 0.523, N_F = 0.269$, (d) $\alpha = 0.523, N_F = 0.306$.](image)

Fig. 3.4 shows some example of beams with varying $N_F$ value and Porro angles $\alpha$. It is apparent that beams formed with large $N_F$ values are higher-order modes of the petal modes, and strongly resemble the recently reported kaleidoscope modes [169, 170, 171] (after the patterns formed in a kaleidoscope), and show an increasing complexity with Fresnel number.

Subsequent work [75] analysed “petal” modes experimentally and confirmed that they are a coherent superposition of Laguerre-Gauss modes of zero radial order but opposite
3.3. RECONSTRUCTION OF LAGUERRE-GAUSS BEAMS

azimuthal order, and are therefore the lowest-order or fundamental modes of a Porro prism laser. It follows that the kaleidoscope modes formed in the numerical model of this laser are Laguerre-Gauss modes with higher radial order and opposite azimuthal order, which are possible given sufficient transverse extent to oscillate.

3.3 Reconstruction of Laguerre-Gauss beams

That some optical fields may self-heal, or self-reconstruct, is now well-known, having first been discovered and studied in the context of Bessel modes \[172\] and their superpositions \[174\]. In such cases the self-healing is understood as the interference of plane waves, travelling on a cone, that bypass the obstacle. The reconstruction distance in this instance is determined from geometric arguments.

More recently it has been recognized that there are other classes of optical fields that self-reconstruct \[175\], and interestingly also the Laguerre-Gaussian (LG) modes \[176\]. It was shown experimentally that such LG modes do, at least in some instances, reconstruct after an obstacle, but at present there is no means to predict under what conditions or to what extent the reconstruction will take place. In this chapter we offer a simple concept for the reconstruction of superpositions of LG beams based on the handedness of the modes and the rotation of the Poynting vector (which gives the rate of energy transfer per unit area). While these concepts are not new, we apply them for the first time to derive, from geometrical principles, an expression for the angular self-reconstruction distance after an obstacle. We show that reconstruction is not guaranteed and is influenced by the distance between the obstacle and the waist plane of the LG modes interacting with it. We confirm the model both numerically and experimentally.

We consider the propagation properties of a superposition of two azimuthal LG modes, so-called petal modes \[148\] or optical Ferris wheels \[177\], of opposite helicity and direction of the Pointing vector. The electric field for such a superposition may be written in the general form:

\[
u(r, \phi) = A(r)[\exp(i\ell \phi) + \exp(-i\ell \phi)], \tag{3.3.1}\]

where \((r, \phi)\) are the co-ordinates, \(A(r)\) is a general radial enveloping function and \(\ell\) is the
azimuthal index. Each mode rotates during propagation by an amount given by \([178, 179]\) 
\[\theta = \arctan(z/z_R),\]
exponentially independent of the azimuthal index \(\ell\), and where the propagation distance \((z)\) from the waist plane is normalized to the Rayleigh range, \(z_R\). This rotation effect has been observed experimentally \([178]\). In Fig. 3.5(a) we show a numerical simulation of the propagation of two obstructed LG beams with opposite helicity and thus differing direction of the orbital angular momentum vector. We notice that the obstructed areas of these beams effectively rotate in opposite directions. Intuitively it appears that regions of obstructed light from one mode eventually overlap with regions of unobstructed light from the other mode. Since this is true for each component of the superposition, such an obstructed area in the initial plane will be self-reconstructed on propagation. It is clear that this “angular self-reconstruction” distance will depend on the angular size of obstruction.

To derive a simple expression for this self-reconstruction distance, we recall that the maximum rotation angle of the Poynting vector for any vortex beam is \(\pi/2\) \([178]\), thus limiting the maximum angular size of the obstruction for angular self-healing. This maximum rotation is based on propagation from the waist plane through the Rayleigh range. In the case of an obstruction that is not at the waist plane, the maximum rotation angle will decrease to \(\theta = \arctan((z + z_I)/z_R) - \arctan(z_I/z_R)\) where the distance to the waist plane is \(z_I\). Let’s assume that total reconstruction is achieved when the angular rotation of each component of the field exceeds the angular extent of the obstruction: \(\theta > \theta_I\). An example of an obstructed beam is shown in Fig. 3.5(b), with an angular extent of \(\theta_I\). We have placed the obstacle at a distance of \(r_p = w_g \sqrt{I/2}\) from the beam centre since the peak intensity of azimuthal LG beams (and their superpositions) is found on a ring of this radius (where \(w_g\) is the Gaussian beam size), but it is only the angular extent that matters. Following this argument, the angular self-reconstruction distance, \(z_{min}\) can easily be found to be

\[z_{min} = z_R \tan(\theta_I + \arctan(z_I/z_R) - z_I).\] (3.3.2)

We see that the reconstruction distance depends on the Rayleigh range. The initial position of the obstacle influences the reconstruction process significantly, namely, that if the obstacle is placed a at distance equal to the Rayleigh range then angular self-healing will fully reconstruct the initial field only if the angular obstruction is less than \(\pi/4\). In Fig. 3.5(c) we have represented the dependence of the self-reconstruction distance on the
initial position \((z_I)\) for the different angular sizes of the obstruction.

![Figure 3.5](image.png)

**Figure 3.5:** (a) Schematic representation of the rotation of the shadow region in an obstructed LG beam with different sign of the angular momentum. (b) A schematic for the derivation of the self-reconstruction distance \(z_r\), and (c) the dependence of the reconstruction distance on initial position \((z_I)\) and angular size \((\theta_I)\) of the obstacle. (d) The dependence of the maximum angular size obstruction \(\theta(z_I)_{max}\) on the initial position of the obstacle \(z_I\) for the different Rayleigh range of the beam.

From Eq. 3.3.2 we can find the maximum angular size for the obstruction that can be reconstructed:

\[
\theta(z_I)_{max} = \frac{1}{2} \left( \pi - 2 \arctan \left( \frac{z_I}{z_R} \right) \right)
\]  

(3.3.3)

We see that the maximum angular size of the obstruction decreases with the distance to the waist plane and drops twice (to \(\pi/4\)) at the Rayleigh range distance (see Fig. 3.5(d)).

Experimental verification was carried out using an intra-cavity generation technique for such petal beams [164, 75] but implemented with a digital laser setup [55].

The laser output was a superposition mode, shown in Fig. 3.6, of equal weightings of two azimuthal modes. The infrared laser beam (1064 nm wavelength) was relay imaged to a waist plane with beam waist radius \(w \approx 300\mu m\). An obstacle consisting of a metal bead
of diameter \( d = 200\mu \text{m} \) fixed to a thin fused silica plate was located in this plane \((z_I = 0)\) to overlap with the one of the petal structures, as illustrated in Fig. 3.5(b), for an angular obstruction angle of \( \theta_I = 27^\circ \). The bead diameter \( d \) and radial position \( r_p \) was chosen to best overlap with and obscure a single petal at plane \( z_I \). These two values determine the angular obstruction angle \( \theta_I = d \times r_p \) at plane \( z_I \). Using Eq. 3.3.2 we predicted a self-reconstruction distance of \( z_{min} \approx 140\text{mm} \). We numerically propagate the obstructed field and show the impact of the obstruction on each LG mode individually as well as the superposition, shown in Fig. 3.7 (a) and (b) respectively, as a function of distance.

In Fig. 3.7(b) we present a simulation and an experimental verification of the self-reconstruction, with the two in excellent agreement. The results also confirm the analytical expression of Eq. 3.3.2. We see in Fig. 3.7(b) that the petal which was obstructed in the waist plane of the beam will have reconstructed completely by \( z = 140 \text{ mm} \), as predicted.

### 3.4 Conclusion

In this work we have presented an intuitive argument for the self-reconstruction of petal-like beams, and derived a simple analytical equation for the self-reconstruction distance. Our analysis explains previous anomalous observations [176] of why some superpositions appeared to self-heal, while others did not: we note that the self-reconstruction distance is independent of the azimuthal orders in the superposition, but depends on the distance to the waist plane of the petal-like beam. Indeed, there are conditions where it is not possible to self-heal, for example, the maximum obstruction size drops by a factor of two to \( \pi/4 \).
3.4. CONCLUSION

Figure 3.7: (a) The simulation of the free space propagation of obstructed $LG_{04}$ and $LG_{0-4}$ beams. (b) The simulation and corresponding experimental verification of the reconstruction of the superposition beam ($LG_{04}$ and $LG_{0-4}$).

when placed at a Rayleigh length from the waist, decreasing further thereafter. Our new results, summarized in Eqs. 3.3.2 and 3.3.3 allow these properties to be calculated for any superposition.
Chapter 4

Self-healing of Bessel-like beams

Bessel beams have been extensively studied, but to date have been created over a finite region inside the laboratory. Recently Bessel-like beams with longitudinally dependent cone angles have been introduced allowing for a potentially infinite quasi non-diffracting propagation region. Here we show that such beams can self-heal. Moreover, in contrast to Bessel beams where the self-healing distance is constant, here the self-healing distance is dependent on where the obstruction is placed in the field, with the distance increasing as the Bessel-like beam propagates farther. We outline the theoretical concept for this self-healing and confirm it experimentally.
4.1 Introduction

4.1.1 Introduction to Bessel beams

Bessel beams [180] are solutions to the Helmholtz equation, with field:

\[ u(r, \phi, z) = A_0 \exp(ik_z z)J_n(k_r r) \exp(\pm in\phi), \]  

(4.1.1)

where \( J_n \) is an \( n \)-th-order Bessel function, \( k_z \) and \( k_r \) are the longitudinal and radial wave-vectors, with \( k = \sqrt{k_z^2 + k_r^2} = 2\pi \lambda \) (\( \lambda \) being the wavelength of the electromagnetic radiation making up the Bessel beam) and \( r, \phi \) and \( z \) are the radial, azimuthal and longitudinal components respectively.

Bessel beams (B) are commonly created by passing a Gaussian beam (G) through an axicon or conical lens (A), as shown in Fig. 4.1. The axicon bends the incident beam so that the waves travel along a cone. The opening angle of the cone is given by:

\[ \theta = (n - 1)\gamma, \]  

(4.1.2)

where \( n \) is the refractive index of the axicon material, and \( \gamma \) is the opening angle of the axicon. For a beam generated by an axicon the maximum propagation distance \( z_{\text{max}} \) is given by:

\[ z_{\text{max}} = \frac{w_0}{\theta}, \]  

(4.1.3)

Figure 4.1: A Bessel beam (B) formed by passing a Gaussian beam (G) through an axicon (A).
4.1. INTRODUCTION

Figure 4.2: Examples of Bessel beams generated by plotting Eq. 4.1.1 with \( n = 0, 1 \) and 2. Notice that the beam of order 0 has a central peak but that higher orders have a central null.

Fig. 4.2 shows the patterns of light in Bessel beams with orders 0, 1 and 2 respectively.

One interesting property of Bessel beams is that they self-heal. An obstruction placed in the beam will form a shadow region for a distance \( z_{\text{min}} \), after which the intersection of conical plane waves will cause the beam pattern to reform. The distance \( z_{\text{min}} \) also shown in Fig. 4.1 is given by:

\[
z_{\text{min}} = \frac{a}{2\theta},
\]

where \( a \) is the extent of the obstruction.

4.1.2 The ray approximation of light waves

The geometrical or ray approximation of light waves is an approach which can be used when the wavelength of light is much smaller than the feature sizes of the optical system under consideration [181, 182]. Consider a field \( E \) propagating from some initial plane \( i \) to a screen \( s \). The field at \( s \) can be calculated using the Fresnel diffraction integral in cylindrical coordinates:

\[
E(r_s) = A \int_s^\infty rE(r_i) \exp \left\{ \frac{k}{2z} \left[ r_i^2 - r_i r_s \right] \right\} dr,
\]

where \( k = \frac{\pi}{\lambda} \), \( r \) is the cylinder radius, \( z \) is the cylinder length, and \( A \) is some constant term.

Since \( k \to \infty \) as \( \lambda \to 0 \), the phase term is a rapidly oscillating function. The stationary phase approximation states that the integral of a rapidly varying function will be 0 everywhere except where the function is constant (and the derivative is 0),
CHAPTER 4. SELF-HEALING OF BESSEL-LIKE BEAMS

\[
\frac{d}{dr_i} \left( E(r_i) + \frac{r_i^2}{2z} - \frac{r_i r_s}{z} \right) = 0 \quad (4.1.6)
\]

Now:

\[
E'_r(r_i) + \frac{r_i}{z} - \frac{r_s}{z} = 0 \quad (4.1.7)
\]

Eq. 4.1.7 provides the link between the wave theory of light and the geometrical or ray theory of light, and is valid for short wavelengths.

To illustrate the equivalence of these theories, Eq. 4.1.7 can be applied to the case of a lens, which has a quadratic phase term:

\[
f(r_i) = -\frac{r_i^2}{2f}, \quad \text{so} \quad f'(r_i) = -\frac{r_i}{f} \quad (4.1.8)
\]

Applying Eq. 4.1.7 gives:

\[
- \frac{r_i}{f} + \frac{r_i}{2f} - \frac{r_s}{2f} = 0 \quad (4.1.9)
\]

\[
- \frac{r_i}{2f} - \frac{r_s}{2f} = 0 \quad (4.1.10)
\]

thus \(r_i = -r_s\) as expected from geometrical optics.

4.2 Introduction to self-healing Bessel-like beams

Self-healing is a property that is usually associated with Bessel beams (BBs) [183, 184, 180, 185, 186, 187], and describes the ability of the field to reform in amplitude after some distance beyond an obstruction. It is usually explained through a simple concept of rays: since the Bessel beam,

\[
u(r) \propto J(k\theta r),
\]

where \(u(r)\) is the field of the Bessel beam, \(J\) is a Bessel function and \(k = 2\pi/\lambda\) is the wave number of the incident light, may be seen as the interference of waves travelling on a
cone of angle, \( \theta \), some waves may bypass the obstruction and hence interfere to create the original beam again \[188\].

Experimentally such self-healing was first demonstrated with zero-order BBs \[189; 172\], and later with BBs carrying orbital angular momentum \[190\]. More recently the concept of self-healing has been extended to other classes of optical fields, such as Airy beams \[191\], scaled propagation invariant beams \[192\] and rotating fields \[174; 175\], as well as to the angular domain \[61\] and beyond classical light to quantum states \[188\]. Self-healing of BBs has been a useful tool in a variety of applications ranging from communication \[193\], atmospheric studies \[194; 195\] microscopy \[196; 197; 198\] and optical trapping and tweezing \[199; 200; 201; 202\]. Despite its many experimental demonstrations, it remains a topical field theoretically \[203; 204\].

A new class of BB was recently introduced where the intensity profile of the beam remains shape-invariant during propagation \[205; 206\]. This is in stark contrast to conventional BBs where the near-field is a Bessel function but the far field is an annular ring. In keeping with the literature we refer to such beams as Bessel-like beams (BLBs), which have a propagation-invariant Bessel-function intensity profile for a long propagation distance. These BLBs are engineered such that their cone angle is not constant but rather a function of propagation distance, \( \theta(z) \). Based on this property we can assume that these beams will have self-healing properties similarly to Bessel beams but with a self-healing distance that is dependent on where the obstruction is placed in the field. Such behaviour has not been observed previously.

In this chapter we study the self-healing properties of BLBs both theoretically and experimentally. We find that the self-reconstruction properties are similar to Bessel beams but that the self-reconstruction distance depends on the distance between the initial field (at the SLM plane) and the obstruction. This property is a result of the longitudinal dependence of the cone angle. This behaviour is a unique property of these beams in contrast with Bessel beams, where the self-reconstruction distance is constant.

### 4.3 Theoretical approach

Consider the case where a BLB is created by a single phase-only element of the form
\[ \phi(r) = \exp[i k (ar^n + br^m)], \quad (4.3.1) \]

where \( k \) is the wave number of the incident light, and \( a, b, n \) and \( m \) are design parameters. If the clear aperture of the entrance optic is \( r_I \), then the parameter set that gives rise to a BLB is given by \( \text{[206]} \)

\[ b = -a \left( \frac{n}{m} \right) r_I^{n-m}. \quad (4.3.2) \]

Note that the phase terms in Eq. 4.3.1 can be viewed as optical aberrations. The physical interpretation of \( n \) and \( m \) is that a power of 1 gives the linear phase term equivalent to an axicon, a power of 2 gives the quadratic phase term equivalent to a lens, a power of 3 gives a cubic phase term, and so on. \( a \) and \( b \) are weighting terms that necessarily obey the relationship in Eq. 4.3.2 in order to produce a long-range Bessel-like beam with reconstruction properties.

Fig. 4.3 shows the effect that changing the \( a \) parameter (with \( b \) calculated according to Eq. 4.3.2) has on the resulting BLB. A small \( a \)-value concentrates the energy in the first couple of rings, but a larger \( a \)-value distributes the energy into a wider area.

![Figure 4.3: A sequence of BLBs showing the effects of the parameter \( a \) can be seen. (a) \( a = 0.0001 \), (b) \( a = 0.001 \), (c) \( a = 0.05 \), (d) \( a = 0.01 \) (\( n = 1, m = 2 \)).](image)

Now our BLB at any transverse plane can be written as the superposition of conical waves where the angle of arrival of the conical waves, \( \theta(z) \), is identical and decreases with distance. The cone angles can be calculated from the stationary phase approximation to the diffraction equation, where rays from the source plane are mapped to new transverse positions at some distance \( z \) away.

From Eq. 4.1.7 and Eq. 4.3.1 we can find the mapping of rays from the initial plane \( r_i \) to some screen plane \( r_s \), a distance \( z \) away, given by
4.3. THEORETICAL APPROACH

\[ an_{i}^{n-1} + bmr_{i}^{m-1} + \frac{r_{i}}{z} - \frac{r_{s}}{z} = 0. \]  

(4.3.3)

Figure 4.4: A longitudinal cross-section of the intensity distribution of a BLB illustrating the derivation of the self-reconstruction distance for BLBs. An obstruction with radius \( r_{0} \) is located at \( z \) on the optical axis OC at position AB. Self-reconstruction occurs in the zone with length \( z_{r} \).

Now consider the case where the central part of the beam is obscured by an obstruction with half-width \( r_{0} \) which self-reconstructs after distance \( z_{r} \). We need to solve the following two simultaneous equations for \( z_{r} \):

\[ an_{i}^{n-1} + bmr_{i}^{m-1} + \frac{r_{i}}{z} - \frac{r_{0}}{z} = 0 \]  

(4.3.4a)

\[ an_{i}^{n-1} + bmr_{i}^{m-1} + \frac{r_{i}}{z + z_{r}} = 0 \]  

(4.3.4b)

which describe the propagation of ray AC (see Fig. 4.4) from the initial plane \( r_{i} \) to (Eq. 4.3.4a) the obstruction plane at \( z \), and to (Eq. 4.3.4b) the reconstruction plane at distance \( z_{r} \) beyond \( z \) where AC intersects with the optical axis OC. These equations can equally be written in terms of the cone angle at some distance \( z \):

\[ \theta + an(r_{f})^{n-m}(z\theta)^{m-1} + an[-r_{f}^{n-m}(-r_{f} + 2z\theta)^{m-1} + (-r_{f} + 2z\theta)^{n-1} - an(z\theta)^{n-1}] = 0. \]  

(4.3.5)

This is solved for \( \theta \) (which will be a function of \( z \)). We provide the cone angles for some example hologram parameters in Table 4.1.
### Table 4.1: The cone angle, $\theta(z)$, of BLBs for three example cases: $n = 1, m = 2$ (an axicon-lens doublet), $n = 2, m = 3$ (an aberrated lens) and $n = 1, m = 3$ (an aberrated axicon).

<table>
<thead>
<tr>
<th>Case</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n=1; m=2$</td>
<td>$\theta(z) = \frac{ar_I}{az-r_I}$</td>
</tr>
<tr>
<td>$n=2; m=3$</td>
<td>$\theta(z) = \frac{r_I}{12az^2} (1 + 10az \pm F(a,z))$</td>
</tr>
<tr>
<td>$n=1; m=3$</td>
<td>$\theta(z) = \frac{r_I}{6az^2} (r_I + 4az \pm G(a,z))$</td>
</tr>
</tbody>
</table>

$F(a,z) = \sqrt{1 + 20az + 4a^2z^2}$

$G(a,z) = \sqrt{(r_I)^2 + 8ar_Iz + 4a^2z^2}$

To predict the reconstruction distance $z_r$ we make use of a simple geometric argument, but which is consistent with a full diffraction analysis. Consider the scenario depicted in Fig. 4.4 where an obstruction of radius $r_0$ is placed in the path of the BLB at some distance $z$ from the source. From simple trigonometric arguments we can see that the shadow distance, $z_r$, is given by the solution to

$$z_r = \frac{r_0}{\theta(z + z_r)}.$$  \hspace{1cm} (4.3.6)

By way of example, consider the axicon-lens doublet in Table 4.1 ($n = 1, m = 2$) from which we find

$$\theta(z) = \frac{ar_I}{az-r_I},$$  \hspace{1cm} (4.3.7)

Substituting into Eq. 4.3.6 and solving for $z_r$ we find

$$z_r = \frac{r_0}{\theta(z + z_r)} = \frac{r_0}{\theta(z + z_r)}$$

and thus

$$z_r(z) = \frac{r_0(az-r_I)}{a(r_I-r_0)}.$$  \hspace{1cm} (4.3.9)

The same approach is followed for any parameter combination of the BLB, and example expressions are provided for various parameters sets in Table 4.2.
4.3. THEORETICAL APPROACH

Table 4.2: The self-reconstruction distance $z_r$ for example values of $n$ and $m$ of the transformation system.

<table>
<thead>
<tr>
<th>$n$=1; $m$=2</th>
<th>$n$=2; $m$=3</th>
<th>$n$=1; $m$=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_r(z) = \frac{r_0(a_0-c_1)}{a(r_1-c_0)}$</td>
<td>$z_r(z) = \frac{4ar_0z^2}{r_1 - 4ar_0z + 2ar_1z \pm r_1 \sqrt{\frac{8a^2r_0z^2}{r_1^2} + (1 + 2az)^2}}$</td>
<td>$z_r(z) = \frac{-2ar_0z^2}{2ar_0z \pm r_1 \left( \pm 1 \mp \sqrt{\frac{4a^2z^2}{r_1^2} + (az - r_0)} \right)}$</td>
</tr>
<tr>
<td>(+) for $a &gt; 0$ and (-) for $a &lt; 0$</td>
<td>(upper sign) for $a &gt; 0$ (lower sign) for $a &lt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

The phase screen used to generate the BLB used in reconstruction experiments is shown in Fig. 4.5. This beam was well suited to the experiments because the energy is spread out into a large area (relative to the obstructions used), with a fine ring structure.

![Phase screen](image)

Figure 4.5: Phase screen generated using the approach outlined in this section, and used for reconstruction experiments following. $n = 1, m = 2, a = 0.05$

We note from Fig. 4.6 that the self-reconstruction distance depends on the distance between the initial plane and the obstruction. This behaviour is a unique property of BLBs in contrast to BBs where the self-reconstruction distance is constant. The representation of BLBs as the interference of two diverging conical-like waves helps to explain the nature of the self-reconstruction property which is similar to BBs [173]. For BBs the self-reconstruction distance is constant as a result of the constant cone angle, in contrast with BLBs where the self-reconstruction distance increases with distance as a result of the cone angle decreasing with distance, as shown in Fig. 4.4.
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Figure 4.6: Dependence of self-reconstruction distance $z_r$ (for certain values of $n$ and $m$ of the transformation system) on distance to obstruction $z$ (see Fig. 4.4) for the following parameters of initial field and system: $w = 2$ mm, $r_0 = w/3$, $r_1 = 3w$, $a = 3 \times 10^{-3}$; (black) $n = 1; m = 2$; (red) $n = 2, m = 3$; (blue) $n = 1, m = 3$.

4.4 Experimental results

Figure 4.7: The experimental setup consists of an expanded HeNe beam reflected off the phase screen displayed on an SLM, creating a BLB with $n = 1, m = 2$. An obstruction OBST (either a bead or thin wire) was positioned at a distance of $z$ from the phase screen. A 4-f imaging system transfers the object plane OBJ to the image plane IMG on the camera sensor CAM at several axial positions $z_I$.

For experimental verification a Gaussian beam from a HeNe laser was expanded by a 4× beam-expanding telescope (BET) to a beam radius $w = 1.7$ mm and reflected off a HoloEye PLUTO spatial light modulator (SLM). A phase screen was generated for a BLB with $n = 1, m = 2$ (equivalent to an axicon-lens doublet, and chosen to allow comparison with previous work, for example [207]) with $a = 0.05$ and $r_I = 2$ mm. An obstruction was placed at $z = 240$ mm from the SLM, and a series of transverse planes at $z_I$ from the
obstruction plane were imaged using a 4-f system onto an image plane coincident with the sensor of a Spiricon LBA-USB-L130 camera and recorded. The experimental setup is shown in Fig. 4.7.

![Figure 4.8: Unobstructed BLB](image)

**Figure 4.8:** (a) Unobstructed BLB \((n = 1, m = 2, w = 1.7\, \text{mm}, a = 0.05)\) at the obstruction plane \((z_I = 0)\), (b) BLB at the same plane but obstructed by a centred 400\(\mu\text{m}\) bead, (c) unobstructed BLB at \(z_I = 110\, \text{mm}\), and (d) obstructed BLB at \(z_I = 110\, \text{mm}\).

We first verified that reconstruction does indeed occur. The obstruction used consisted of a bead at the obstruction plane \((z_I = 0)\) and centred on the BLB. We imaged this plane both without [see Fig. 4.8(a)] and with [see Fig. 4.8(b)] the bead. Notice the dark area at the centre of the beam in Fig. 4.8(b). We then moved our imaging system and camera a distance of \(z_I = 110\, \text{mm}\) away, and imaged the beam at this point. Fig. 4.8(c) shows the unobstructed beam at \(z_I = 110\, \text{mm}\), and Fig. 4.8(d) shows the obstructed beam at the same plane. Notice that the ring structure of the beam has been completely reconstructed. We notice however that the intensity of the outer region of the obstructed beam (Fig. 4.8(d)) is lower than in the unobstructed beam (Fig. 4.8(c)).

Fig. 4.9 is a zoomed-in view of the obstructed area, for incomplete reconstruction. A diffraction pattern is evident in the central region, which merges with the BLB pattern.

In order to study how well other obstruction configurations would reconstruct we followed the method of the previous experiment, now using both a bead with diameter 400\(\mu\text{m}\)
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Figure 4.9: (a) unobstructed BLB, (b) obstructed BLB, (c) partial reconstruction showing diffraction.

and a wire with diameter $180 \mu m$, in both centred and off-centred positions relative to the BLB, which was the same as generated previously, i.e. $n = 1, m = 2, w = 1.7 \text{ mm}$, $a = 0.05$ and $r_l = 2 \text{ mm}$. When in their off-centre positions the bead and the wire were respectively $839 \mu m$ and $526 \mu m$ from the centre of the BLB. For each obstruction configuration the beam was imaged and recorded every $10 \text{ mm}$. Fig. 4.10 shows beam reconstruction for: (a) off-centre wire, (b) off-centre bead, (c) centred wire, and (d) centred bead. In each case the beam is shown at axial positions $z_I = 0 \text{ mm}, 30 \text{ mm}, 60 \text{ mm}$ and $90 \text{ mm}$ from the obstruction. Full reconstruction is found at $z = 70 \text{ mm}$ for the bead, and at $z = 27 \text{ mm}$ for the wire.

Our approach to calculating the BLB shadow after the obstruction is based on the conical wave approach. By considering the projection of the obstruction in space which results from the two travelling conical waves which produce the BLBs [173] we are able to predict the movement of the shadow region of the obstructed area with beam propagation. The approach of projecting the obstruction boundaries rather than the field itself results in the fast and accurate prediction of the field after an obstruction. We successfully predict the reconstruction properties of a BLB after obstructions in both the central region of beam and off-centre by calculating the boundaries of the various projected regions (see Fig. 4.10). The projection results in the creation of two zones defined by a single conical wave, with the boundaries of these zones moving farther apart at a rate of $\delta = 2z \tan[\theta(z)]$, where $\theta(z)$ is the cone angle at the obstruction position (see Table 4.1) and $z$ is the longitudinal position of obstruction.

For each position $z_I$ of each experiment the shadow pattern predicted by the simulation is shown as an inset. It is clear that the shadow pattern is characteristic of the shape of
obstruction, as well as the position of the obstruction in the beam. Fig. 4.10 reveals good agreement between the theory and the experimental results.

![Figure 4.10: Beams shown at increasing distances z_l from the obstruction of four reconstruction experiments: (a) off-centre wire, (b) off-centre bead, (c) centred wire, and (d) centred bead. In each case the calculated shadow pattern is shown (inset) for n = 1, m = 2, w = 1.7 mm, and α = 0.05 and r_l = 2 mm.](image)

In our third experiment we investigated the dependence of reconstruction distance z_r on the distance z of the obstruction from the initial plane at the SLM. A wire with diameter 0.7 mm was placed off-centre in the same BLB as generated previously (i.e. n = 1, m = 2, w = 1.7 mm, α = 0.05 and r_l = 2 mm), first (a) at z = 248 mm, and then (b) at z = 748 mm. A camera captured the beam at a distance z_l after the obstruction.
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Figure 4.11: The self-reconstruction of the same beam with a wire obstruction placed off-centre at (a) 248 mm, and (b) 748 mm. (c) shows that the beam is partially reconstructed 100 mm after (a), contrasted with (d) which shows very little reconstruction 100 mm after (b). (e) shows complete reconstruction of the obscured area at 200 mm after (a), but (f) shows that complete reconstruction is only evident at 400 mm after (b).

Referring to Fig. 4.11, we observed that in both cases reconstruction is incomplete after 100 mm, but for (a) reconstruction is complete at 200 mm, whereas for (b) reconstruction was only complete after 400 mm.

4.5 Conclusion

Here we demonstrate the self-healing property of Bessel-like beams. We outline theoretically and confirm experimentally that the shadow region is dependent on where the obstruction is placed in the field, with the self-healing distance increasing with distance from the source plane.
Chapter 5

Conclusion and future work

5.1 Conclusion

We have demonstrated new ways in which SLMs can be used to generate new types of beams with special properties, both intra- and extra-cavity.

In Chapter 2, we have shown that properties like the variation in reflectivity of an EA SLM, which is insignificant when the SLM is used in the standard extra-cavity configuration, become significant and even dominant in an intracavity configuration, leading to a “phase-only” SLM behaving as an intensity modulating element. This effect can be countered by the selection of an EA SLM which uses PA LCs rather than TN LCs. This discovery confirmed that a true phase-only SLM could be used as a phase modulator in a resonator, allowing a wide variety of modes and output beams to be generated without manufacturing custom DOEs or even realigning the resonator.

In Chapter 3, we use the “digital laser” to generate “petal” modes, which we confirm to be comprised of a superposition of $LG_{0+n}$ and $LG_{0-n}$ modes. We demonstrate experimentally that if an obstruction in the path of the beam is not too large then the rotation of the fields causes the shadow of the obstruction to be healed after some distance.

In Chapter 4, we use an SLM in a standard extra-cavity configuration to generate a new class of beam: the Bessel-like beams, which consist of concentric rings that extend from the creation plane to infinity. We show that, like the petal beams, these beams can self-heal, but that the self-healing mechanism differs. In the case of BLBs the mechanism relies on diffraction to fill in the cone of shadow behind the obstruction and along the optical axis. We
show too that for this class of beams that the self-healing distance increases with distance from the creation plane.

5.2 Future work

While the digital laser will find immediate application in laser tweezing and particle micro-manipulation, the low damage threshold of modern SLMs has proven to be the biggest obstacle to its use in applications such as rapid prototyping and laser marking. In order to address this, we propose an experiment to determine the feasibility of amplifying the digital laser output. The effects of gain modulation and gain saturation would need to be established before this could be demonstrated.

Another way of making the output beam more useful for commercial applications would be to pulse the output, creating a beam with low average power but high peak power by Q-switching the output.

Modern SLMs can be switched at 60 Hz. This rate could be increased by using an intracavity acousto-optic modulator (AOM) to switch the direction of the beam to $n$ distinct areas of the SLM. Different phase screens could be displayed in these areas, allowing the switching speed to be increased $n$-fold.

One important application of self-healing beams is in data transfer. A beam used to transmit data over a distance in free space might be obstructed by small objects (blown debris, for example) in its path, but self-heal as it propagates along the optical axis. To be of practical use the number of obstacle particles and particle size which would still permit error-free signal transfer should be investigated, and data transfer should be demonstrated with these beams.

The possibility of devising beams that self-heal both radially and axially should be investigated, and the self-healing distance determined for this new class of beams.
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