THE EXPERIENCES OF SECONDARY MATHEMATICS TEACHERS TEACHING MATHEMATICS THROUGH PROBLEM SOLVING

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DECLARATION

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SUMMARY

Secondary school mathematics focuses mainly on developing learners' understanding and ability to reason. Regardless of the hours of instruction, learners often fail to master basic school algorithms or to apply them correctly in mathematical situations. The assumption is that teachers still use drill and practice methods in order to teach mathematics, despite the fact that a problem-solving approach offers an efficient alternative to these methods.

In this thesis, the experiences of secondary school BEd (in-service) mathematics teachers were explored. The intention was to determine in particular what opportunities and challenges secondary mathematics teachers are faced with regarding teaching mathematics using the problem-solving approach while participating in a hybrid distance learning model offered by Rhodes University. One of the questions to be answered was "How do mathematics teachers apply a problem-solving approach in their own classrooms?"

The interpretive qualitative paradigm underpinned this research study in that the study mainly centred on the significance of participants' experiences and what meaning can be made from their experiences. As this was a case study, the focus was on four teachers (purposefully and conveniently selected among 12 teachers) in the John Taolo Gaetsewe district of the Kuruman area, Northern Cape province, South Africa, regarding how problem solving has impacted on their teaching practices as individuals. The four selected teachers were in their third and final year of the BEd (in-service) programme at the time of the study.

This study made use of a variety of data-generation techniques that included a questionnaire, semi-structured interviews and observations. The researcher analysed and reported the findings regarding the teachers' experiences using data generated from classroom observations, questionnaires and face-to-face interviews. The findings of this study indicated that teachers still facilitate mathematics lessons using a 'traditional' approach, namely 'telling and showing'. Teachers still experiences challenges that prohibit them from incorporating a problem-solving approach.

Keywords: problem solving; mathematical understanding; hybrid distance learning model; school algorithms; mathematical ideas

OPSOMMING

Hoërskoolwiskunde fokus hoofsaaklik op die ontwikkeling van leerders se begrip en redenasievermoë. Ondanks al die ure se onderrig sukkel leerders dikwels om basiese skoolalgoritmes te bemeester of om dit korrek in wiskundige situasies toe te pas. Die aanname is dat onderwysers steeds dril- en inoefeningsmetodes gebruik om wiskunde te onderrig ten spyte van die feit dat ’n probleemoplossingsbenadering ’n doeltreffende alternatief bied.

In hierdie tesis is die ervaring van BEd- (indiensopleiding)onderwysers ondersoek. Die doel was om uit te vind watter spesifieke geleenthede en uitdagings hoërskoolwiskunde-onderwysers in die oë staar tydens die onderrig van wiskunde met behulp van die probleemoplossingsmetode gedurende deelname aan ’n hibridiese leermodel wat deur Rhodes Universiteit aangebied word. Een van die vrae wat beantwoord moes word, was “Hoe pas wiskunde-onderwysers die probleemoplossingsbenadering in hulle eie klaskamers toe?”.

Die vertolkende kwalitatiewe paradigma het hierdie navorsingstudie onderlê deurdat die studie hoofsaaklik gefokus het op die beduidendheid van deelnemers se ervarings en die betekenis wat van daardie ervarings afgelei kan word. Aangesien hierdie ’n gevallestudie was, het die klem geval op vier onderwysers (doelgerig en gerieflikheidshalwe uit 12 onderwysers gekies) in die John Taolo Gaetsewe-distrik van die Kuruman-area in die Noord-Kaap, Suid-Afrika, se ervaring van hoe probleemoplossing hulle individuele onderrigpraktyke beïnvloed het. Die vier geselekteerde onderwysers was almal in hulle derde of finale jaar van die BEd- indiensopleidingsprogram ten tye van die studie.

In die studie is van ’n verskeidenheid datagenereringstegnieke gebruik gemaak, met inbegrip van ’n vraelys, semi-gestruktureerde onderhoude en waarneming. Die navorser het die bevindinge ontleed en verslag word gedoen rakende die onderwysers se ervarings deur gebruik te maak van data wat uit klaskamerwaarnemings, vraelyste en persoonlike onderhoude gegenereer is. Die bevindinge van die studie toon aan dat onderwysers steeds wiskundelesse fasiliteer deur van ’n ‘tradisionele’ benadering, naamlik ‘vertel en wys’, gebruik te maak. Onderwysers ervaar steeds uitdagings wat hulle daarvan weerhou om ’n probleemoplossingsbenadering in te sluit.

Sleutelwoorde: probleemoplossing; wiskundige begrip; hibriede afstandsleermodel; skoolalgoritmes; wiskundige idees

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CHAPTER 1

ORIENTATION TO THE STUDY

1.1 INTRODUCTION

The purpose of this study was to investigate the experiences of mathematics teachers facilitating mathematics using problem solving. This chapter describes the motivation for the study and attempts to prove its significance in relation to mathematics teaching. The aims, objectives and research questions of the study are also discussed in this chapter. In addition, the chapter provides a brief outline of the research design and the methodology followed in this study. The last section of the chapter provides an outline of the chapters of this study.

Statistics indicate that the majority of secondary mathematics teachers in South Africa are either underqualified or unqualified to teach secondary school mathematics (Department of Higher Education and Training [DHET], 2011:4). According to News Channel Africa (2013), there were 7 076 unqualified teachers and 2 642 underqualified teachers out of nearly 400 000 teachers on the Department's payroll at the time of the study. These are the teachers who either only hold a Grade 12 qualification or who completed a Grade 12 qualification but only have a one- or two-year tertiary qualification. The number of 10 000 unqualified and underqualified teachers, as a portion of the total number of around half a million, may not seem very significant, but it is still important to ensure that teachers are appropriately qualified. Therefore, in 2009, the Department of Basic Education (DBE) introduced teacher-development strategies to reduce the number of unqualified and underqualified teachers (DBE, 2011:10). One of the Department's strategies is to encourage mathematics teachers to participate in in-service university programmes.

Rhodes University (RU) is one of a number of public universities in South Africa responsible for training in-service mathematics teachers. Teachers can register for a three-year BEd (in-service) programme at RU in a project called the "Rhodes University Mathematics Education Project" (RUMEP). One of the RUMEP projects operates in the Northern Cape province in South Africa. In the Northern Cape, RUMEP is a teacher-development partnership project between Sishen Iron Ore Company Community Development Trust (SIOC-cdt) and RU.

SIOC-cdt is a mining company that had been established in August 2006 and operated in the Northern Cape and Limpopo provinces at the time of the study.

RUMEP teachers participate in face-to-face contact sessions, mainly with the same lecturer (in this case the researcher) on alternate Monday afternoons. Teachers have an opportunity to meet other lecturing staff and interact with other RU students during contact teaching blocks in Grahamstown, as interaction is considered an "important ingredient" for a successful learning experience (Beldarrain, 2006:140). The Grahamstown contact sessions only take place during school holidays. To complete the degree, teachers should attend a minimum of six weeks of contact sessions in Grahamstown. In the first two years of study, teachers meet at RU for two weeks in April and two weeks in July each year. In the third year, teachers meet during the April and September school holidays at RU. During the sessions, teachers are exposed to recent theoretical insights into mathematics education and are supported to apply these insights to the current South African context.

1.2 MOTIVATION FOR THE STUDY

This study was motivated by various factors, mainly the researcher's personal experience as a lecturer for RUMEP. It seemed as if RUMEP BEd (in-service) teachers experience obstacles to incorporating a problem-solving approach in facilitating mathematics lessons, despite being encouraged and taught through such strategies. Teachers still use drill and practice methods in order to teach mathematics. However, it is believed that a problem-solving approach to mathematics teaching offers an alternative to drill and practice methods (Marcus & Fey, 2006:59). The researcher therefore wanted to explore opportunities and challenges facing these teachers while studying at RU. The focus was on the RUMEP teachers in the John Taolo Gaetsewe district of the Kuruman area in the Northern Cape province, where the researcher was staying at the time of the study. It was convenient for the researcher to meet the teachers, as they also resided and worked in the area.

1.2.1 Aim of the study

The aim of the study was to explore the experiences of secondary school BEd (in-service) mathematics teachers in the John Taolo Gaetsewe district of the Kuruman area of the Northern Cape province regarding their teaching of mathematics through problem solving. This study assisted the researcher as a lecturer and researcher in gaining more insight into the learning of mathematics through a problem-solving approach. The insight gained into the teachers' experiences may, in future, help improve the teaching and learning of mathematics in schools and at RU.

1.2.2 Significance of the study

It is foreseen that the findings of this study may make some contribution towards the improvement of teachers' mathematics teaching skills. The study appealed as significant to the researcher as a mathematics lecturer who facilitates mathematics education modules and may potentially provide information to other lecturers at RU who participate in mathematics teacher education. In addition, the study may be helpful to individual policy makers, such as the DBE and the DHET, especially in terms of the South African DHET 2011 *Minimum requirements for teacher education qualifications*.

1.3 PROBLEM STATEMENT

This study set out to describe the experiences of mathematics teachers regarding their teaching of mathematics using a problem-solving approach. Teachers are taught during RUMEP contact sessions through problem solving and are expected to teach in their own classrooms by using a problem-solving approach. The researcher had an opportunity to observe the teachers in their own classrooms for the foregoing 24 months and observed them incorporating problem solving in their own teaching. The premise was that if teachers use innovative teaching strategies such as problem solving, there may be an improvement in the way in which mathematics is learned and ultimately a gain in the results of mathematics assessment.

It is widely understood that mathematics can play an important role in shaping how individuals deal with the various spheres of civil, private and social life (Anthony & Walshaw, 2009:147). Mathematics is considered a major branch of culture, the backbone of scientific civilization and the basis of technological, financial and insurance structures (Kline, 1978:2). At the time of the study, however, the mathematics results of many Grade 12 learners in South Africa continue to decline every year, despite the fact that good Grade 12 mathematics results open doors for learners to scarce skills careers such as engineering, medicine, biotechnology and astronomy. The National pass rate was 47,4% in 2010, while in 2011 the pass rate was 46,3% (Motshekga, 2012:4). In contrast, the Grade 12 mathematics results showed an improvement of 6,6% in 2012 and a further improvement of 5,1% in 2013, which is not considered a significant improvement by the DBE (Motshekga, 2014b: 10). Furthermore, the Grade 1 to 6 Annual National Assessment (ANA) mathematics results dropped to 27% in 2012, as compared to 30% in 2011, while the national average for Grade 9 was 13%. According to Motshekga (2014a: 31), the ANA mathematics pass percentage for Grade 1 to 6 learners was 39%, an improvement of 12% from the 2012 results, while the Grade 9 results improved by 1% in 2013. The DBE, therefore, continues to encourage mathematics teachers to participate in teacher-development programmes in order to enhance teachers' mathematics teaching skills.

As a mathematics lecturer at RUMEP, the researcher identified teachers finding it difficult to teach through a problem-solving approach during the teachers' contact sessions. In addition, the researcher observed certain teachers experiencing difficulties using a problem-solving approach in their own classrooms during the classroom support visits. Teachers still teach through a 'traditional' approach, namely 'telling and showing'. Teachers tend to always step in to show learners how to solve mathematical problems. Stacy (2005:341) states that problem solving is one of the fundamental goals of teaching mathematics and is considered to be the "heart of mathematics". In this study, the researcher wanted to determine how a group of secondary school mathematics teachers experienced teaching mathematics through a problem-solving approach while participating in a BEd (in-service) programme offered by RU via distance learning.

1.4 RESEARCH QUESTIONS

In order to realise the aim of the study, the following research questions informed and guided the research:

1.4.1 Main research question

How do secondary school mathematics teachers in the John Taolo Gaetsewe district of the Kuruman area experience the facilitation of mathematics through a problem-solving approach while participating in a BEd (in-service) programme?

1.4.2 Sub-questions

The following sub-questions were developed:

- What are the views of in-service BEd mathematics teachers in the John Taolo Gaetsewe district in Kuruman on the use of a problem-solving approach?
- How do these in-service BEd mathematics teachers apply a problem-solving approach in their own classrooms?
- How, if at all, do the RUMEP study material and activities equip these in-service mathematics teachers to facilitate mathematics through a problem-solving approach?

1.5 RESEARCH AIM

The aim of the study was to explore the experiences of BEd (in-service) secondary school mathematics teachers in the John Taolo Gaetsewe district, Kuruman, regarding teaching mathematics through a problem-solving approach.

1.6 RESEARCH OBJECTIVES

Research objectives are the steps one has to take at grassroots level within a specified time in order to attain the research aim or goal (De Vos, Strydom, Fouché & Delport, 2002:107). The research objectives of this study were the following:

- To determine the views of in-service secondary school mathematics teachers on their facilitation of mathematics through a problem-solving approach
- To observe the opportunities and challenges facing mathematics teachers who teach through a problem-solving approach
- To analyse the RUMEP study material and activities that may assist BEd (in-service) secondary school mathematics teachers to facilitate mathematics through a problem-solving approach.

1.7 RESEARCH DESIGN AND METHODOLOGY

This section represents a summary of the description of the design and methodology of the study, which follows in Chapter 3. Research design describes the plan that guides and directs all activities and processes of research. In this study, the researcher followed a multiple-case study design. According to Yin (1993:5), multiple-case studies include two or more cases within the same study. The researcher selected four teachers as particular cases operating in their natural teaching and learning contexts in order to describe and analyse their teaching and learning experiences as BEd students.

Yin (2009:13) defines case study research as an empirical inquiry that investigates a contemporary phenomenon within its real-life context. He adds that case study research is known for dealing with research questions that involve *why, how* and *what* (Yin, 2009;14). Case studies provide an intensive description and analysis of a phenomenon or social unit, such as an individual, group, institution or community (Ary, Jacobs, Razavieh & Sorensen, 2006:456). Case studies are particularly useful, because by concentrating on a single case, they point to insights that can have wider implications that may not easily be realised when dealing with a broader scope.

The greatest advantage of a case study is the possibility of depth, as it seeks to understand the individual and his or her actions within the totality of that individual's environment (Ary et al., 2006:457). Not only the present actions of an individual but his or her past, environment, emotions and thoughts can be probed. This is done by focusing on the particular to become enlightened about the general, but without generalising to wider populations. However, case studies need to be limited to the study of individuals or social units such as institutions or programmes. By means of scrutiny of the problem-solving approach in possibly promoting active teaching and learning, the researcher hoped to gain a better understanding of the phenomenon of active mathematics facilitation in the classroom within the context of an in-service BEd programme offered via a distance education model, but locally facilitated.

1.7.1 Data generation

According to Lankshear and Knobel (2005:172), data can be defined as "bits and pieces of information found in the environment" that are collected or generated in systematic ways to provide an evidential base from which to make interpretations and statements intended to advance understanding and knowledge concerning a research problem or question. However, according to David and Sutton (2004:27), for researchers using non-numerical data it is not about what is out there to collect, but rather what the researcher can generate and record. The researcher determines what counts as data depending on the questions that drive the study. Data-generation techniques in this study consisted of semi-structured interviews, questionnaires, the researcher's observations, the RUMEP study material and the researcher's reflection sheets.

1.7.1.1 Selection of participants

Selection procedures are needed if one wishes to investigate the experiences of mathematics teachers teaching by using problem solving: The experiences of teachers teaching through problem solving will differ from those of teachers who are not exposed to a problem-solving approach. The researcher purposefully selected four teachers from a group of 12 registered BEd teachers in John Taolo Gaetsewe district teaching in four different schools. There were 12 mathematics teachers registered in the BEd (in-service) programme in the John Taolo Gaetsewe district at the time of the study. At the beginning of the programme, there were 16 teachers; however, the number fell to 12 due to teachers dropping out and some of them being unable to meet the minimum requirements to proceed to the next level of the course.

The four teachers participating in this study were chosen according to their orientation towards problem solving based on their participation in contact sessions and classroom practices since the researcher observed all teachers in the foregoing 33 months. The four teachers had shown some evidence of shifting their classroom practices towards a problem-solving approach during classroom support visits. Also, the class discussions of the selected teachers during contact and workshop sessions had seemed to be more directed towards advancing problem solving. Their questions and contributions had been more aligned to a problem-solving approach. The four teachers were, therefore, likely to be better informed and 'knowledgeable' about problem solving, as they were continuously involved in learning how to use problem-solving techniques.

1.7.1.2 Questionnaire

Data were generated by means of closed-ended questions, after which the data were analysed, interpreted and reported. In this study, the researcher used a questionnaire consisting of direct statements to which the participants had to respond according to a four-point Likert-type scale. Two open-ended questions were included at the end of the questionnaire in order to obtain narrative and exploratory data. All 12 mathematics teachers registered in the BEd programme in John Taolo Gaetsewe district completed a questionnaire regarding their views on problem solving.

1.7.1.3 Semi-structured interviews

In this study, the researcher also used semi-structured interviews to generate data on a problem-solving approach regarding mathematics teaching and learning. Data were generated using the four selected BEd teachers in the John Taolo Gaetsewe district. Face-to-face interviews were used to support and strengthen data generated from the questionnaire survey and the classroom observations. The researcher used an interview schedule and electronic recording equipment with the four selected teachers in order to generate data regarding their teaching through a problem solving approach. Although an interview schedule was used in the study, there was flexibility during the interviews, as the participants were allowed to ask questions about the questions that were being asked of them, and the researcher, in support thereof, was able to follow up on some contributions made. The teachers were interviewed outside their normal teaching hours. The researcher took notes during the interviews and recorded unobtrusive responses. The participants were interviewed at their schools, which the researcher visited after having made the necessary arrangements.

1.7.1.4 Classroom observations

The researcher observed the teachers teaching in their own classrooms and completed observation sheets. The observation sheets contained observation markers detailing specific actions and strategies of problem solving. The researcher also took notes during classroom visits to support the observation sheets. A video recorder was used as a back-up to record the classroom observations and to enable the researcher to review what occurred in the classroom at a later stage. In this study, the video was mainly used to supplement the observation sheets and the field notes.

1.7.1.5 Written and other visually presented data

In this study, written data included the RUMEP study material and the researcher's workshop reflection sheets. The RUMEP study material that was prominently used by teachers for their BEd modules were considered. The researcher collected RUMEP study material used by in-service BEd mathematics teachers in the John Taolo Gaetsewe district. The inclusion of the study material in this study was useful, because the teachers mainly used the material in order to assist them in acquiring problem-solving skills. The researcher also used reflection sheets to collect data relating to his own teaching practice actions and experiences.

1.7.2 Data analysis

From the data-generation techniques used in the study, the researcher had numeric data. In this study, the researcher analysed the data using narrative data analysis. The researcher coded data by reading carefully through transcribed data, line by line, and dividing it into meaningful analytical units or segments. The researcher marked the segments of data with descriptive words or unique identifying names. The emerged data were arranged according to categories. The categories were subjected to further analysis by relating and interrelating them. The categories were used as a lens through which teachers' problem-solving experiences were discerned. Important and interesting issues arising from this analysis were cross-checked with available literature regarding the facilitation of mathematics using a problem-solving approach.

The researcher recorded data through writing during interviews and classroom observations, even though writing was sometimes found to be distractive. A digital recorder was also used to record the interviews and a video recorder was used to record the classroom observation visits. A video recorder was used in order to allow the researcher to review the classroom activities at a later stage. The main aim of the video was not to analyse the data from the video itself, but to be used as a back-up mechanism. In addition, a classroom observation sheet was used to record what transpired in the classroom during classroom visits regarding the problem-solving approach. All participants agreed with the researcher on the use of the recording instruments. The workshop reflection sheets were completed in order to record the researcher's reflections regarding the Monday afternoon workshops.

1.7.3 Delineations and limitations

This study was limited to secondary mathematics teachers in one school district who are studying for the BEd (in-service): Mathematics Education degree at Rhodes University. In order to make the study manageable, the researcher concentrated on one aspect of mathematics teaching, namely teaching mathematics through problem solving. There is no claim that the results of this study may be generalised beyond the confines of the study.

1.7.4 Assumptions

The facilitation approach in the programme is mainly based on problem solving. The teachers showed confidence in applying problem-solving strategies in their own ways during the classroom support visits. This study was based on two assumptions:

- Teachers participating in the study have some intuitive knowledge about teaching mathematics using a problem-solving approach.
- Each teacher has opportunities to apply a problem-solving approach in his or her classroom and such opportunities are therefore not restricted.

1.7.5 Trustworthiness and credibility

Multimethod strategies, namely questionnaires, interviews and observations, were used to increase the trustworthiness and credibility of the study, as recommended by McMillan and Schumacher (2001:429). The use of multimethod strategies in case study designs may compensate for any one-sidedness or distortion that may result from individual methods (Steinke, 2004:184). Cohen, Manion and Morrison (2007:141) argue that the exclusive use of one method may bias or distort the researcher’s picture of the particular “slice of reality” being investigated.

1.8 ETHICAL CONSIDERATIONS

Permission was obtained from the Northern Cape Department of Education because the study involved teachers who are employed by the Northern Cape Department of Education. Because the study also involved interviewing and observing teachers in their classrooms, the researcher obtained permission from the school principals of the schools participating in the study. The teachers were informed that their participation was important, but that their role was voluntary and that they were free to withdraw should they feel uncomfortable during the course of the study. For purposes of confidentiality and anonymity, the identities of the teachers were concealed by using alphabetical letters as pseudonyms. All participants gave their informed consent and were informed that they would remain anonymous and that the data that are to be derived from their feedback would only be used for purposes of reporting and analysis. Permission was also granted by RU’s Faculty of Education, as the teachers were enrolled under the RU education department. In addition, all ethical issues were clarified and approved by the Research Ethics Committee (Human and Social Sciences) of Stellenbosch University and adhered to by the researcher.

1.9 OUTLINE OF CHAPTERS

This study is organised into the following chapters:

Chapter 1 presents an orientation of the study. In Chapter 2, the relevant literature on the topic of this study is explored by generating a number of theoretical perspectives. Chapter 3 discusses the research design and methodology used in interpreting and describing the experiences of secondary mathematics teachers teaching through the problem-solving approach. Chapter 4 presents the empirical findings of the study and interprets and discusses the findings of the study. In Chapter 5, the researcher draws conclusions and points to a number of implications of the research.

CHAPTER 2

THEORETICAL PERSPECTIVES

2.1 INTRODUCTION

In this chapter the focus is on five aspects of this research project, which assisted in providing a theoretical perspective for the study. The first section reviews a body of literature on distance education. In the second section, mathematics education relating to teaching and learning through problem solving is reviewed. The third section outlines learners' learning processes through their mathematical activities. The fourth section explores the concepts of motivation and disposition in mathematics teaching and learning, while the fifth section focuses on reflective practices.

Much of secondary school mathematics focuses on developing learners' understanding of, and skill in using, symbolic notations to reason about and describe equations, inequalities, functions, expressions and variables (Marcus & Fey, 2006:59). Despite hours of instruction and practice, learners often fail to master basic school algorithms or to apply them correctly in mathematical situations. For many learners, the experience with algebra is a meaningless and disconnected process consisting of rules for operations with symbols that do not represent anything real or useful (Kline, 1978:161). According to Grouws (2006:129), these meaningless experiences might be as a result of the teaching they experience in learning mathematics at the secondary schooling level. Hence, Marcus and Fey (2006:59) claim that a problem-solving approach to mathematics teaching and learning offers an alternative to such experiences.

Problem solving means the process of applying mathematical knowledge and skills to new and unfamiliar situations. The purpose of encouraging learners to solve problems is that they will acquire and be able to use the process of mathematical thinking, so that they will put these processes to work whenever they are needed. Schroeder and Lester (1989:32) describe three important areas of problem solving, namely teaching *for* problem solving, teaching *about* problem solving and teaching *through* problem solving. Teaching for problem solving implies cases whereby teachers teach procedures first, after which problems related to the taught concepts are solved. Teaching for problem solving is similar to solving 'word' problems as an extension of routine computational exercises. In teaching about problem solving, learners are taught about various techniques as options when faced with a problem; these techniques can include drawing a table or a graph. Teaching through problem solving implies that problems are used to teach important mathematical concepts (Schroeder & Lester, 1989:32). Teaching through problem solving means that learners solve problems in their own ways, use mental tools already available to them and are able to learn important mathematical concepts (Schroeder & Lester, 1989:33). Teaching through problem solving entails more than simply posing the correct type of problems and then allowing learners to solve them.

2.2 DISTANCE EDUCATION AND THE RUMEP MODEL

2.2.1 Conceptualisation

Distance education emerged in response to the need to provide access to individuals who would otherwise be unable to participate in face-to-face tuition (Beldarrain, 2006:139). Distance education practices around the world use a wide range of online and audiovisual technologies to overcome the lack of direct contact between lecturers and students (Baggaley, 2008:39). However, these practices are neither universally adopted by all distance education lecturers nor even encouraged by their institutions. This is the case with the RUMEP model, where the use of online and audiovisual technologies is replaced by contact sessions in Grahamstown (10-day contact session each year), alternate Monday afternoon workshops and frequent interaction with the local facilitator (the researcher).

Distance education, describes effort where students are engaged in the learning process at a location separated from their instructor and, often from other students (Slimp, 2014:2). Distance education offers freedom from space and time constraints, increased interactivity, improved delivery of multimedia, broadened curricula and personalised learning. The term ‘distance education’ can be used interchangeably with ‘distance learning’ or ‘open learning’. The learning at RUMEP can be considered as open learning, as there is high interactivity, personalised learning and engagement in the learning situation by mathematics teachers

In South Africa, distance education formally started in 1946 with the University of South Africa (Unisa) as one of the first correspondence universities (Council on Higher Education

[CHE], 2004:8). From 1993, a number of traditionally face-to-face universities introduced distance education learning models because the then Department of Education proposed that there should be no monopoly of distance education by specific institutions (CHE, 2004:9). Also, when the colleges of education where closed or merged with universities during the early 1990s, many universities adopted distance education models to cope with the large number of students, especially in-service teacher education.

Statistics indicate that, in 2009, the headcount student enrolment in distance education was 316 349, while the headcount enrolment in full-time face-to-face contact enrolment was 521 430 (DBE, 2010:28). In 2008, 38,9% of students were enrolled in distance education, while 61,1% were enrolled in full-time face-to-face contact in higher education institutions.

2.2.2 Facilitation models in distance learning

Distance education is aimed at promoting active independent learning while reducing class seat times (Beldarrain, 2006:140). Distance education encompasses those courses that allow the student and the lecturer to be physically apart from each other during the teaching and learning processes, yet maintain communication in a variety of ways (Kaur, 1996:2). Distance education has evolved from correspondence schools to delivery mechanisms such as independent study, computer-based instruction, computer-assisted instruction, video courses, video conferencing, Web-based instruction and online learning. According to Beldarrain (2006:140), technology has played a key role in changing the dynamics of the various delivery options and models of distance education over the years, as well as the pedagogy behind distance education.

The early focus in distance education was on external studies and correspondence education (Spector, 2009:157). Both external studies and correspondence education programmes were carried on outside the confines of a university setting. At the time, correspondence models of distance education were commonly conducted by sending material back and forth through the mail. According to Spector (2009:158), such programmes are still in existence. For example, some of the Unisa programmes are still conducted using such a model, while RUMEP supplements correspondence with regular contact sessions and workshops. External studies models are commonly found in business, industry and defence contexts, where there is a desire not to take highly experienced persons away from their primary tasks to train inexperienced persons and where funds to send the inexperienced away for special training are limited (Spector, 2009:158).

Early programmes of external study drew on educational applications in a number of areas, including programmed instruction (PI) and personalised systems of instruction (PSI). PI was based on a behavioural model of learning that emphasised prompting for behaviours and responses that could then be reinforced (or not) and used to guide the student though a sequence of material. PSI evolved as an alternative to PI. PSI was specifically aimed at self-paced, mastery learning and overtly acknowledged the need for support from tutors and lecturers. These programmes are now more commonly referred to as further or continuing education programmes, most often involve a wide variety of technologies and provide students with opportunities to collaborate and experience learning not possible in text-based distance education programmes.

2.2.3 Overview of literature on study material in distance education

Distance education institutions are accustomed to writing and producing study material aimed at driving and shaping students' learning experiences in specific modules (Morgan & O'Reilly, 1999:120). Such a view is supported by Collopy (2003:288), who states that mathematics study material are considered potential vehicles for teachers learning about mathematics content, pedagogy and learners' thinking. However, these views are underscored by many universities in South Africa that use a fragmented process to develop study material (Louw & Sonnekus, 2005:15). The process is fragmented because, usually, lecturers at universities develop curricula for specific courses, write content for these curricula and pass the draft on to instructional designers who design and develop the material.

Murphy (cited in Louw & Sonnekus, 2005:15) points out that authors normally prescribe the content, as it is the authors who have content expertise in a discipline. This leads to a general agreement on what is to be learnt in the course and only then is the development of the material commissioned. In contrary, learning developers at Unisa are involved in the whole process of curricula, content and study material design (Louw & Sonnekus, 2005:15). At Unisa, learning designers have moved from being educational advisors to active participants in the learning design process. They provide input into curricula content and reach consensus with the lecturers concerned on the course-planning process. Attention is given to the development timelines and the target audience is taken into consideration, based on existing student profiles and development.

Louw and Sonnekus (2005:17) propose three phases to be followed when designing study material. The first phase is the design phase that consists of the following basic list of questions to be used and adapted where necessary to design study material:

- Who are the participants and the stakeholders in the learning environment?
- What national, provincial, global and professional aims play a role?
- What are the implied norms and values?
- What are the actions a competent student should undertake?
- What are the tools they should use?
- What are the rules?
- Who are our students?
- What is the context?
- What are the roles?
- What impact should the graduate have on the discipline, the profession and society?
- What problems do students encounter in their workplace?
- What timelines and costs are involved?

The second suggested phase is the development phase. In this phase the structure of the units is planned, layout guidelines are agreed upon, the appropriate language level for writing the material is agreed upon, the contextual resources are shared, the assessment strategies are planned and the physical material is written, reviewed and edited.

During the third phase, the production phase, the agreed-upon timeline is scheduled into concrete delivery dates and the actual layouts, printing, binding and packaging are completed. Phase four entails the delivery of the study material to the student from the warehouse in which the material is stored. The final phase in this cyclical process is the evaluation phase. This can only take place once the study material has been used by students and lecturers alike for a minimum period of one year. An evaluation instrument is used, which encompasses the students' evaluation of the material, study assignment results, throughput rates, peer reviews, focus group interviews and the appointment of quality readers who assess the general success of the product.

Study material must meet certain criteria in terms of their learning design, linguistic design, instructional devises, and visual and assessment designs (Le Roux & Le Roux, 2004:10). The design of study material is based on a learner-centred approach rather than a teacher-centred approach in order to promote problem-solving techniques. Linguistic design relate to the choice of language used in the study material. Le Roux and Le Roux (2004:13) acknowledge that dialogue is made easier by using the language that is accessible and appropriate for the students if a learning event is to be effective. Instructional devises include media and navigation tools that support and guide students through the study material. The visual design refers to the quality of the material and includes the general layout, cover design, readability of font, icons, tables, headings and subheadings.

Le Roux and Le Roux (2004:13) state that mathematics study material should include activities that respond to a problem-solving approach rather than rote learning. Course content should reflect up-to-date material in order to be relevant to the learning experience (Le Roux & Le Roux, 2004:12). A constructivist theory of learning recognises that knowledge is constructed in specific contexts and study material should therefore be connected with those contexts. Study material should offer ongoing support for pedagogy and subject matter content throughout the entire school year in order to develop new beliefs and understanding (Collopy, 2003:288). Study materials, in particular self-instructional material, not only have to convey information to the student; they also have to structure, control and manage the process by which this information is presented to and assimilated by the student (Ellington & Race, 1994:43).

2.2.4 Assessment in distance education

There are many definitions of assessment in education. Assessment is a process of gathering and analysing information using multiple and diverse sources in order to develop an understanding of what students know and understand (Huba & Freed, 2000:8). Assessment is a human activity involving interactions aimed at seeking to understand what students have achieved (Morgan & O'Reilly, 1999:13). Jaques, Gibbs and Rust (1995:14) consider assessment as a way of providing students and lecturers with feedback on how well students have learnt and what their strengths and weaknesses are.

Assessment is used as an instrument to identify which concepts are poorly understood by students and need attention, and which concepts are well understood by students. Assessment functions to measure learning and to guide and develop students' learning. Bryan and Clegg (2006:2) suggest that assessment "frames learning, creates learning activity and orientates all aspects of learning behaviour". Assessment of student learning is, therefore, key to quality teaching and learning, because classroom assessment ensures that the standards of qualifications awarded by a university are achieved (RU, 2009:2).

Current studies in the field of assessment show a shift from traditional testing practices to alternative or constructive assessment approaches that enhance quality learning and autonomy (Geyser, 2009:90). Through constructive assessment approaches, assessment becomes part of teaching and learning, while in traditional assessment practices, assessment is carried out as a separate activity. Constructive assessment forms an integral part of teaching and learning and does not become an add-on experience at the end of teaching and learning. According to Chappuis and Stiggins (2002:40), assessment that involves students in the process of teaching and learning is aimed at motivating students for learning rather than measuring students' performance for grading purposes. Morgan and O'Reilly (1999:13) state that the primary purpose of assessment is to increase students' learning and development rather than to simply rank or grade students' performance. However, one cannot grade students' performance without first assessing it. In this case, grading is a secondary activity to the primary goal of helping students to diagnose problems and improve the quality of their subsequent learning.

Assessment is classified into two categories, namely assessment *for* learning and assessment *of* learning (Morgan & O'Reilly, 1999:14). Assessment for learning (formative assessment) implies all those activities designed to motivate, enhance understanding and provide students with an indication of their progress, while assessment of learning (summative assessment) focuses on the product of learning (Morgan & O'Reilly, 1999:15). According to Morgan and O'Reilly (1999:15), most distance education institutions usually have both formative and summative components of assessment. For example, RU is one of the institutions that uses both summative and formative types of assessment, including students studying through RUMEP.

Assessments are often structured so that one assignment builds upon the next, but most distance education institutions are often criticised for inflexible pacing of assignments (Morgan & O'Reilly, 1999:23). Marks are awarded for each assignment and combined together to form a final grade. Morgan and O'Reilly (1999:16) state that formative assessment is important to distance education students because it provides some structure to learning; provides a source of ongoing dialogue between lecturers and students; encourage, motivate and build students' confidence; and provides insight for students into their progress. RUMEP imposes penalties for teachers who do not take part or perform well in their formative assessments. One of the penalties is that teachers should obtain a minimum of 40% in order to be able to write the end-of-year examinations.

2.3 MATHEMATICS TEACHING PRACTICES

2.3.1 A view of mathematics teaching in the USA and Japan

Mathematics teaching and learning evolve over time in ways that are consistent with stable beliefs and assumptions that are part of the mathematics teaching profession (Kilpatrick, 1992:1). Research indicates that, in the late 1800s, mathematics teachers believed that effective teaching involved showing learners mathematical procedures, followed by learners' use and practice of those procedures (D'Ambrosio, 2006:39). Therefore, mathematics education was primarily a process of transmission of knowledge. This means that the role of the mathematics teacher was to transmit clear information, demonstrate procedures for solving problems and explain the process of solving sample problems (Artzt, Armour-Thomas & Curcio, 2008:5). As a result, learners were expected to listen well to the teacher, remember everything the teacher told them and show that learning has occurred by applying the demonstrated procedures.

Considering American teachers, some of them believe that school mathematics is a set of procedures, because they want learners to perform a procedure in order to solve a mathematical problem (Stigler & Hiebert, 1998:2). For these teachers, practising skills and learning mathematical terms are not exciting activities. Such teachers appear to feel responsible for shaping the task into pieces that are manageable for most learners, providing all the information and assigning opportunities for practice. As a result, the majority of American teachers may occasionally interrupt the lesson to talk about non-mathematical things, trying to make the lesson interesting (Stigler & Hiebert, 1998:2). Therefore, some American teachers base their instruction on behaviourist theories of learning, as behaviourists focus on arranging the environment so that optimal interaction takes place (Schoenfeld, 1987:5).

In contrast, some Japanese teachers act as if mathematics is a set of relationships between concepts, procedures and facts (Stigler & Hiebert, 1998:2). They want learners to think about things in a new way, such as seeing new relationships between mathematical ideas. Such teachers consider mathematics as inherently interesting and they believe that learners will be interested in exploring mathematics by developing new methods for solving problems. These teachers seem less concerned about developing learners' interest in non-mathematical ways.

Stigler and Hiebert (1998:3) mention that if one believes that mathematics is a set of procedures, it would be understandable also to believe that mathematics is learned best by mastering the material incrementally piece by piece. As such, the procedures are learned by practising them many times with subsequent activities slightly more difficult than the preceding activities. Practice should be free of errors with a high level of success at every point. Teachers who believe that mathematics consists of a set of procedures argue that frustration and confusion should be minimised, because they are signs that the earlier material was not mastered (Stigler & Hiebert, 1998:2). Such teachers believe that by giving learners more activities, teaching and learning will proceed smoothly. Wertheimer (cited in Schoenfeld, 1987:3) argue that although such instruction does result in learners' 'mastering' certain procedures, knowledge acquired through drill and practice is likely to be superficial and therefore neither flexible nor useful in a range of situations.

Teachers who emphasise the mastery of a certain procedure may teach learners how to add fractions with unlike denominators, such as $\frac{3}{5}+\frac{2}{9}$ as follows. The teachers would say that the learners should first master adding fractions with like denominators, such as $\frac{1}{3}+\frac{2}{3}$; then be shown how to add simple fractions with unlike denominators , $\frac{1}{4}+\frac{1}{6}$, being warned about the common error of adding the denominators (to minimise this error), before practising the more difficult problems, such as $\frac{3}{5}+\frac{2}{9}$.

Some Japanese teachers appear to have a different set of beliefs about learning and probably would plan a different kind of lesson for adding fractions. They often choose a challenging problem to begin the lesson. Furthermore, they seem to believe that learners learn best by first struggling to solve mathematics problems, then discus how to solve the problems, and then consider the advantages and disadvantages of applying different methods and the relationships between them. Struggling and making mistakes and then seeing why they are mistakes are believed to be essential parts of the learning process for the Japanese curriculum (Stigler & Hiebert, 1998:3). For these teachers, frustration and confusion are taken to be a natural part of the process, because each person must struggle with a situation or problem first in order to make sense of the information he or she hears later.

2.3.2 A view of mathematics teaching in South Africa after 1994

Briefly considering the history of curriculum change in South Africa after 1994, one notes that Curriculum 2005 (C2005) was implemented in January 1998 with an intention of promoting learner-centred classrooms; however, it failed (Mahomed, 2004:2). C2005 was informed by the principles of outcomes-based education (OBE), as the foundation of the post-apartheid schools' curriculum (Chisholm, 2005:193). According to Jansen (1998:322), outcomes would dislocate an emphasis on content coverage, make it clear what learners should attend to and direct classroom assessment towards specified goals. However, Jansen (1998:322) is of the opinion that outcomes in fact cannot deliver what they claim. According to him, outcomes have no historical legacy because they are rooted in behavioural psychology and are derived from the competency education models associated with vocational education in the United Kingdom (Jansen, 1998:322).

However, the DBE (2009:15) states that C2005 would in essence shift teaching from a behaviourist approach, based on the idea of the teacher as transmitter of knowledge, to a constructivist learner-centred approach in terms of which the teacher becomes a facilitator of knowledge. Obviously, learners' interaction and learning experiences would depend on guidance from teachers, and teachers' key role would be to lead learners in their own discovery and understanding of mathematical concepts (Driver & Oldham, 1986:112). In particular, C2005 would offer a dialogue between learners and the curriculum where the learners interact with sources of knowledge, reconstruct knowledge and take responsibility for their own learning outcomes (Malan, 2000:26). Constructivist theory acknowledges that the teacher is not a transmitter of knowledge, but rather a facilitator and provider of experiences from which learners will learn.

Three years later, in 2000, C2005 (up to Grade 9) was reviewed and revised and the Revised National Curriculum Statement (RNCS) was named and subsequently became official policy in April 2002 (Chisholm, 2005:193). However, C2005 and the revised version RNCS were met with several challenges and were replaced by National Curriculum and Assessment Policy Statements (CAPS) with its full implementation in January 2014 up to Grade 12 (DBE, 2011:3). According to Mahomed (2004:2), these challenges included the following, among others:

- Teaching, learning and assessment: Some teachers found it difficult not to integrate teaching, learning and assessment, which was the primary requirement of C2005. Many teachers could not align assessment methods, tools and forms to learning activities and learning outcomes.
- Learner performance: Some teachers believed that learners' abilities to read, write and listen have deteriorated due to C2005.
- Not greater support from the district officials: Many teachers expressed a need for greater support from the district given the demands made on them by the new curriculum.
- Inadequate resources: About half of the textbooks at schools provided insufficient guidance for teachers. In addition, the majority of the teachers complained of comfortable classroom sizes for a number of learners.
- Teacher development: Several teachers expressed a need for more practical training that is relevant to their environmental contexts. Teachers expressed a need for training in anti-bias issues, management of diversity, accommodating learners with special educational needs, co-operative learning, lesson planning and integration across learning areas.

Moreover, Jansen (1998:323) claims that, among other reasons, C2005 had a negative impact in South African schools because it was driven by political imperatives that had little to do with the realities of classroom life; the language used was too complex, confusing and at times contradictory; and it was based on flawed assumptions about what happens in the classrooms, how classrooms are organised and what kinds of teachers exist within the system. Furthermore, Jansen (1998:323) states that the curriculum multiplied the administrative burdens placed on teachers. However, this is contradicted by Malan (2000:28), who states that C2005 forced “uncoordinated and laissez-faire educational planning, managing and teaching practices into the background and introduced strategic educational planning that was aimed at achieving results”. In essence, Malan (2000:22) notes that, as elsewhere in the world, reactions vary between curriculum commendation by its proponents and denouncement by its critics.

Kilpatrick, Swafford and Findell (2001:371) are of the opinion that the successful implementation of a mathematics curriculum is directly influenced by the proficient teaching practice of the teachers implementing the curriculum. This means that the teacher’s role is pivotal in the implementation of the curriculum. It is up to the teacher to determine how to make appropriate adaptations to accommodate the curriculum in teaching and learning within the social organisation of the individual class. However, according to Brodie, Lelliott and Davis (2001:541), teachers can create a mismatch or gap between the curriculum demands and what actually happens in the classroom. For example, in South Africa, learner-centred teaching is promoted by the new national curriculum, but instead, as in international studies, research in South Africa on the new curriculum is beginning to show that here too, teacher-centred practices are difficult to shift (Brodie et al., 2001:542).

In South Africa, many learners still do not participate fully in the learning process, as some teachers are still providing a great deal of direct instruction and are still preoccupied with content coverage (Brodie et al., 2001:542). Research conducted in South Africa found that while in general teachers are enthusiastic about the new curriculum and intend to implement learner-centred practices in their classrooms, they continue to teach in predominantly teacher-centred ways (Brodie et al., 2001:542). Obviously, where learner-centred ideas are endorsed, they do not enable learners’ commitment with key concepts in the subject area.

2.3.3 Mathematical problem solving

Problem solving has different meanings to different people and it is difficult to reach a common understanding of the concept of problem solving. Broadly, to solve a problem means finding a way where no way is known, finding a way out of difficulty, finding a way around an obstacle, or attaining a desired end that is not immediately attainable by appropriate means (Hatfield, Edwards, Bitter & Morrow, 2000:91). According to Johnson and Rising (1967:104), problem solving means "finding an appropriate response to a situation which is unique and novel to the problem solver". According to Silver (1987:40), problem solving means the application of one's knowledge to tasks that may be well structured or poorly structured, familiar or unfamiliar, simple or complex. Problem solving can also be perceived as the process of getting from givens to goals when the path is not obvious (Lesh & Doerr, 2003:31).

Not all learners in a class may view what is being taught as a problem. Orton and Frobisher (2000:25) put forward that what is a problem to one learner may be an exercise for another learner. For instance, those who have little understanding of a situation may view any mathematical idea arising from an activity associated with the situation as a problem. This means that learners who have already met a situation before and have become reasonably familiar with the different aspects of the mathematics of the activity will view their work as a repetitive exercise.

For example, in an activity that requires learners to solve for x in $2^x = 3$ using a calculator and leaving the answer in decimal form, a learner used to solving exponential equations using logarithms is likely to obtain the answer of 1,584962501. Such a learner does not see exponential equations having different bases as a problem. However, another learner who is at early stages of solving exponential equations can regard the equation $2^x = 3$ as a problem. Such a learner will employ many strategies, methods and processes to arrive at a solution. One learner may use trial and error methods, substituting x with 0,5; 1; 1,5 and 2 using a calculator and estimate the answer as lying between 1 and 2 instead of arriving at the exact solution of 1,584962501. However, a less advanced learner may say we cannot solve the equation, as the bases are not the same. Stacy (2005:342) states that successful mathematical problem solving depends on deep mathematical knowledge, reasoning abilities, communication skills, abilities to work with others and personal attributes such as persistence, organisation and confidence.

Mathematical problems consist of three types of information, namely information regarding ‘givens’, information regarding ‘operations’ and information concerning ‘goals’ (Wickelgren, 1995:10). Givens refers to the set of expressions that we accept as being present at the onset of the problem statement, operations refers to the actions one need to perform on the givens, while goals implies a terminal expression one wishes to cause to exist in the problem situation (Wickelgren, 1995:13). In the problem such as “What constant force will cause a mass of 3 kilograms to achieve a speed of 30 metres per second in 6 seconds, starting from rest?” the givens are 3 kilograms, 30 metres and 6 seconds. In the proof problems, the rule of inference that constitutes the allowable operation is the property that if $A = A'$ and $B = B'$, then $A + B = A' + B'$. For example, in the problem $2x + 7 = 19$, one can regard the goal expression as being of the form $x = -----$, where the correct number is to be found in order to fill in the blank in the goal expression.

Pollack (1969:397) cautions mathematics teachers to avoid problems that look like applications to mathematics when in actual fact they are not. He gives the following scenario as an example of such problems:

> A bee and a lump of sugar are located at different points inside a triangle. The bee wishes to reach the lump of sugar, while travelling a minimum distance, under the requirement that it must touch all three sides of the triangle before coming to the sugar. What is the shortest path?

One advantage of such problems may be that they bring a smile to a weary learner or distract him or her momentarily from a ‘dreary lesson’ and divert the imagination to a more pleasant exercise (Pollack, 1969:398). Such teachers act as if the learners’ interests must be enhanced by including problems or situations coming from outside the mathematics classroom (Stigler & Hiebert, 1998:2).

Orton and Frobisher (2000:27) mention three categories of mathematical problems, namely routine problems, environmental problems and process problems. Routine problems are problems that use knowledge and techniques already acquired by a learner in a narrow and systematic context. Environmental problems, also called 'real-world' or 'real-life' problems, are problems set in contexts that represent the practical or real world. Process problems are set in a mathematics context in contrast to real problems. These types of problems concentrate on the mathematics itself and on the mathematics thinking processes for arriving at the solution. The Pollack problem stated above can fall under the category of environmental problems, as the problem relate to real-life problems even though the problem is not purely mathematical.

2.3.4 Problem-solving frameworks

In this section, two problem-solving frameworks are described, namely that of Polya and Schoenfeld.

2.3.4.1 Polya's problem-solving framework

According to Polya (1988:5), there are four phases of problem solving. He outlines the four phases as understanding the problem, devising a plan, carrying out the plan and looking back. Proponents of Polya's framework follow a step-by-step instruction in teaching.

(a) Understanding the problem

Polya (1988:5) is of the opinion that in order to understand the problem, the verbal statement of the problem should be understood. Learners should be able to repeat the statement using their own words. To make the problem understandable, the teacher should design and pose the problem in such a way that learners are capable of determining what information is being requested (Hatfield et al., 2000:96). Learners should become involved in the problem and be able to state the parts of the problem, namely the unknowns, the data and the conditions. Modes of representation can also be used to enable the teacher to make the problem understandable and interesting to the learners. Modes of representations are described as forms for representing mathematical concepts and principles externally through the use of written or oral language, manipulatives, diagrams, calculators or computers (Artzt et al., 2008:12).

(b) Devising a plan

In this stage, the role of the teacher is to offer 'unobtrusive help' to learners. Artzt et al. (2008:1) state that the teacher is supposed to help the learners organise and formalise their ideas. Learners ought to be allowed to find strategies of solving the problem with little interference from the teacher. If the teacher interferes too much, the problem becomes his or hers and the reason for the activity is lost. The teacher can ask the learners to identify the unknown in a given equation or expression and to try to think of familiar problems having the same or similar unknowns. During this stage, the responsibility of the teacher shifts from providing information to asking questions and providing resources (Burton, 1989:20).

(c) Carrying out the plan

In this phase, the leaner should be convinced about the correctness of each step. The teacher can emphasise the difference between seeing clearly that the step is correct and proving that the step is correct. Learners should be able to apply various strategies until the problem is solved. According to Hatfield et al. (2000:96), strategies are methods by which the problem can be solved. These strategies are determined by the skill and mathematical tools that the learner has previously mastered. In this phase, learners need to be patient in order to succeed with solving the problem (Burton, 1989:21). Teachers and learners have to realise that the time for problem solving is open-ended, in the sense that the problem can be continued in the next session without feeling that a solution must be reached at the end of each session (Burton, 1989:21).

(d) Looking back

After arriving at the solution, learners should re-examine and reconsider the path that led to the required solution. Learners should be able to check each step and have a good reason to believe that their solution is correct. If the solution is not correct, another strategy should be tried. Hatfield et al. (2000:96) suggest that learners should always write the numerical answer in a full sentence. Writing numerical answers in full sentence forces learners to reflect on their answers as they translate them (Hatfield et al., 2000:96). Buschman (2004:304) supports the view of writing the solution processes into words, as such actions enhance learners' deep understanding of mathematics. Moreover, learners should be able to notice that the knowledge gained and the procedure used in arriving at the solution could be applied to some other problems. The teacher can also emphasise to the learners that no problem is completely completed, because we can always improve on our understanding of the solution.

2.3.4.2 Schoenfeld's problem-solving framework

Schoelfeld's framework is based on Gestaltism learning theory developed from a cognitive science perspective. According to him, Gestaltists believed in rich mental structures and felt that the object of instruction should be to help learners develop these structures (Schoenfeld, 1987:4). Schoenfeld (1985:12) states that any mathematical problem-solving activity is built on a foundation of basic mathematical knowledge, which is called 'resources'. Furthermore, Schoenfeld (1985:12) outlines categories such as heuristics, control and belief systems that are required when one is working on problems with mathematical content.

(a) Resources

According to Schoenfeld (1985:12), resources are tools, procedures, facts and skills potentially accessible to the problem solver. Resources imply the mathematical knowledge and procedures that the individual is bringing to bear on a particular problem. The resource stage can also be referred to as the problem solver's 'initial search stage'. In this stage, the individual must have an intuition and informal knowledge regarding the problem. The resource stage consists of four classes, namely a set of relevant facts known by the problem solver, algorithmic procedures known by the individual, routine procedures and procedural knowledge about the agreed-upon rules for working in the domain.

(b) Heuristics

Heuristics are strategies, techniques or rules of thumb for successful problem solving, and suggestions that help an individual to understand a problem better and make progress towards its solution. Larson (1983:1) also affirms that heuristics are strategies or tactics of solving problems such as mathematical problems. These strategies can include drawing figures, exploiting related problems, reformulating problems, introducing relevant notations, arguing by contradiction, working forward from available data, working backwards, and testing and verifying procedures.

(c) Control

Control implies how the individuals use the information at their disposal to solve problems. This category focuses on decisions about what to do in a problem and decisions that may 'make or break' an attempt to solve the problem. Decisions include making plans, selecting goals and sub-goals, monitoring and assessing solutions, and revising or abandoning directions when the assessments indicate that such actions should be taken.

(d) Belief systems

The category of belief systems means one's mathematical worldview; the perspective with which one approaches mathematics and mathematical tasks. This category includes the belief about the self, the topic, mathematics and the environment. One's belief about mathematics can affect how one chooses to approach a problem, which techniques will be used or avoided, and how long and how hard one will work on the problem.

From Polya and Schoenfeld's problem-solving frameworks, it follows that rather than seeking a single correct answer, learners deduce the problem, gather appropriate information, identify possible solutions, evaluate options and present conclusions. Furthermore, they contend that learners become good problem solvers by learning mathematical knowledge through the use of strategies or heuristics. The frameworks seem to be based on constructivist learning approaches that having learners construct their own solutions leads to effective learning experiences. Proponents of problem-based learning, such as Polya and Schoenfeld, believe that when learners develop frameworks for constructing their own procedures, they are integrating their conceptual knowledge with their procedural skill.

The two frameworks of teaching and learning to promote deep mathematical understanding offered the researcher an integrated view of problem solving, although Kilpatrick et al. (2001:116) argue that "no framework captures completely all aspects of expertise, competence, knowledge and facility in mathematics". However, the researcher associates the two frameworks as one of the elements necessary for anyone to teach and learn mathematics successfully.

2.3.5 Challenges of teaching through problem solving

The study discusses the experiences of in-service mathematics teachers as they move from traditional pedagogies of teacher explanation (teacher-centred) to pedagogies of learner exploration (learner-centred) of mathematical ideas within a problem-solving environment. As a result, teachers are expected to change not only what they teach but also how they teach and how they assess learners. The role of the teacher should shift to helping learners extend and expand what they already know in order to learn even more. In addition, teachers must acquire subject matter knowledge that is deeper and more flexible than that required to follow a static lesson plan or directions in a teacher's manual or textbook.

However, the development of mathematical understanding through problem solving remains a challenge for several teachers of mathematics, despite the numerous benefits of the problem-solving approach. Buschman (2004:305) acknowledges that teaching through problem solving in order to enhance understanding poses many challenges for teachers. According to Hiebert and Carpenter (1992:65), the efforts in mathematics education to promote learning with understanding have been like searching for the 'Holy Grail'. Mathematics teachers often experience some challenges in helping learners approach problems and use proper mathematical problem-solving tools (Panaoura, 2012:291). One of the challenges teachers are faced with is that their past professional development programmes did not prepare them to teach through problem solving (Buschman, 2004:306). In addition, teacher educators are unable to prepare their students to teach in a manner consistent with new ideas about learning and the nature of mathematics (Artzt et al., 2008:9).

Another challenge is that teachers must model to learners problem-solving abilities that they neither possess nor have seen demonstrated by their lecturers. One way in which teachers can learn to become successful teachers of problem solving and in which learners can learn to become competent problem solvers is to spend time in the company of others who can model appropriate problem-solving skills. Stigler and Hiebert (1998:2) consider teaching as a cultural activity. Teaching, like other cultural activities, is learned through informal participation over long periods of time. It is something one learns to do by growing up in a culture rather than by formal study. Individuals that grow up within a particular culture, for example a teaching culture, share a mental picture of what teaching is like. The last challenge for teachers of problem solving is that they may need to change some of their basic beliefs about what constitutes mathematical literacy, for example:

- Do learners demonstrate mathematical literacy when they memorise facts and computation algorithms?
- Or do learners demonstrate mathematical literacy by using mathematical information purposefully to build relationships between mathematical ideas?

2.3.6 Assessment in mathematics

Knowing classroom practice means knowing what mathematic content is to be taught and how to plan, conduct and assess lessons based on that content (Kilpatrick et al., 2001:379). In mathematics, assessment refers to the comprehensive accounting of an individual or group's functioning within mathematics or in the application of mathematics (Webb, 1992:663). As a result, classroom instruction is largely and generally organised and orchestrated around assessment. According to Webb (1992:661), mathematical assessment is carried out in order to assist plotting a national strategy that will have implications for improving mathematics education for the nation. Ziebarth (2006:178) states that one of the goals of assessment is to monitor learners' mathematical understanding, skills and problem-solving abilities so that teachers can plan and guide instruction appropriately. This means that the tasks that teachers select for assessment must convey a message to the learners about the kinds of mathematical knowledge and performance that are valued (National Council of Teachers of Mathematics [NCTM], 2000:24).

Tests are one of the quantitative tools that can be used to collect data and information for the purpose of describing an individual or group's level of performance, achievement or mathematical knowledge. However, using tests alone as quantitative assessment tools does not help to describe how learners draw relationships between different mathematical concepts and ideas. Qualitative methods such as interviews, classroom observations and teachers' opinions can be used to ascertain individual learners' knowledge of mathematics (Webb, 1993:1). The assessment of learners' mathematical understanding goes beyond assessing mathematical knowledge only to include dispositions such as learners' interests, confidence and curiosity in working with mathematical ideas (Lambdin, 1993:9).

If teachers decide to teach mathematics through problem solving, they will need to make changes in curriculum tasks, classroom norms and instructional methods (Ziebarth, 2006:177). However, Ziebarth (2006:180) states that teachers are often faced with challenges when trying to shift from traditional assessment practices to assessment that promotes problem solving. Such a premise is consistent with that of Lambdin (1993:12), who affirms that alternatives to pen-and-paper tests pose challenges to teachers. Alternatives to traditional assessment practices might involve classroom observations and individual interviews. However, classroom observations require considerable expertise to walk around a classroom as learners work in small groups, simultaneously providing guidance and making mental notes of learners' strengths, weaknesses and dispositions (Lambdin, 1993:12). Individual interviews are often time-consuming and interfere with opportunities for extended talks with learners. Despite the challenges in assessment, these changes will be possible only if learners are assessed in ways that reinforce the importance of the new teaching methods and classroom tasks.

The traditional assessment practices used to assess skill mastery alone ignores assessment that promotes deep understanding of mathematical ideas. For example, asking learners to solve the equation $y = 60 - 20x$ requires procedural recall only. To enhance understanding, the teacher may ask learners to solve the equation in more than one way or to give a real-life application that the equation can model. Another approach of designing assessment that promotes deep mathematical understanding is to begin with a context and build the assessment questions around it. Novotná, Hofmannová and Petrová (2008:24) are of the opinion that assessment is part of a teacher's decision-making process.

2.3.7 Mathematical classroom discourse

According to Walshaw and Anthony (2008:516), recent mathematics initiatives call for the development of classroom communities that take communication about mathematics as a central focus. Some of the components of the Cockcroft report (Cockcroft, 1982:71) that led to successful mathematics learning include mathematical reasoning and communication of mathematical ideas. The belief is that learners' active engagement with mathematical ideas will lead to the development of specific learner competencies. The question is what kind of quality mathematics pedagogical practices will enhance classroom mathematical discourse that can produce desirable learner competencies. The NCTM (cited in Walshaw & Anthony, 2008:517) states that some of the things that teachers might do to enhance effective classroom discourse involve observing and listening attentively to learners' explanations and ideas. In essence, making a difference through classroom discourse, teachers shift learners' cognitive attention towards making sense of their mathematical experiences, rather than limiting their focus to procedural rules (Walshaw & Anthony, 2008:522).

According to Franke and Kazemi (2001:104), teachers should listen to their learners' mathematical explanations, create strategies that evoke mathematical thinking, ask questions that elicit learners' explanations and know what to do with what they heard in order to make instructional decisions. Stigler and Hiebert (1998:2) state that mathematics teachers should elicit learners' mathematical thinking and anticipate multiple strategies for solving problems, as mathematics is not just a set of procedures or algorithms to be followed. Constructivists agree that knowledge is actively constructed by learners and not passively 'received' from the teacher. The construction process can be accomplished by learners by making connections, building mental schemata and developing new mathematics based on their prior knowledge through interactions with others (Lau, Singh & Hwa, 2009:307).

However, it is a challenge for many teachers to include classroom discourse as an integral part of an overall strategy of mathematics teaching and learning. Honouring learners' contributions is one of the pedagogical strategies used to stimulate classroom discourse. Yackel and Cobb (cited in Washaw & Anthony, 2009:523) found that classroom teachers who facilitate learner participation and elicit leaner contributions and who invite learners to listen to one another, to respect one another and themselves and to accept different viewpoints and engage in an exchange of thinking and perspectives exemplify sound pedagogical practices in mathematics.

2.4 TEACHING FOR UNDERSTANDING

It follows from Section 2.3.7 that more talk in classrooms does not necessarily enhance learner understanding (Walshaw & Anthony, 2008:522). Better understanding is dependent on particular pedagogical approaches purposefully focused on developing a discourse culture that elicits clarification and produces consensus within the classroom community. Copes and Shager (2006:195) suggest that we can first give learners a problem that they do not know how to solve, then let the mathematical ideas arise as needed to solve that problem in order to develop learners' mathematical understanding. However, if a learner is able to solve a particular problem correctly, it does not necessarily indicate that understanding of the relevant concepts is present (Cockcroft, 1982:72).

Schoenfeld (1985:12) holds the view that one understands how to think mathematically when one is resourceful, efficient and flexible in one's ability to deal with new problems in mathematics. Others state that a mathematical idea is understood if it is part of an existing internal network with stronger or more numerous connections between pieces of information (Hiebert & Carpenter, 1992:67). Such a premise is consistent with the opinion of Koehler and Grouws (1992:119) that understanding in mathematics implies making connections between ideas, facts or procedures, with a view to linking new knowledge to existing knowledge. Therefore, understanding means the ability to recognise and make use of a mathematical concept in a variety of settings, including some that are not immediately familiar (Cockcroft, 1982:68).

One way to support learners' efforts to understand is to allow mathematics to be problematic for them (Hiebert & Wearne, 2006:13). The purpose of making mathematics problematic for learners is to allow the teacher to refrain from stepping in and doing too much of the work for the learners (Hiebert & Wearne, 2006:7). According to Hiebert and Wearne (2006:6), allowing mathematics to be problematic for learners means posing problems that are just within learners' reach, allowing them to 'struggle' to find solutions and then examining the methods they have used. Understanding is enhanced by becoming curious about a specific topic, figuring out how this topic is the same as (or similar to) and different from a topic already studied, and becoming confident to handle problems about the topic, even new problems that have not been seen before (Hiebert & Wearne, 2006:3).

Kahan and Wynberg (2006:15) state that teaching mathematics through problem solving begins with the teacher identifying the mathematics that learners should learn, posing the problem to the class and making sure learners have sufficient understanding of the task, but without telling them how to solve it. Learners then explore the problem, try to make sense of it and eventually generate one or more solutions. Finally, with the teacher's guidance, the learners reflect on the problem, their work and the important mathematical ideas that have emerged. Artzt et al. (2008:15) mention that the teacher can encourage learners to reflect on what they or their classmates have asked or proposed in order to build on and extend their own understanding and solicit contributions from everyone.

2.5 LEARNERS' LEARNING PROCESSES THROUGH THEIR MATHEMATICAL ACTIVITIES

Simon, Saldanha, McClintock, Akar, Watanabe and Zembat (2010:71) argue that learners of all ages learn mathematics through (but not exclusively through) their own mathematical activities. Mathematical activities are defined as the goal-directed physical and mental actions of learners as they attempt to accomplish a particular mathematical task (Simon et al., 2010:74). Mathematical tasks are projects, problems, questions, constructions, exercises and applications in which learners engage (Chambers, 1993:18). Mathematical problems are posed in order to develop, among others, mathematical thinking and to promote mathematical understanding. Artzt et al. (2008:9) indicate that learning with understanding enhances learners' remembering strategies and assists them to relate new ideas of mathematics to what they already know and can do, to use their previous knowledge and skills to construct new meaning and to apply their learning to new contexts.

Vygotsky (cited in Lau et al., 2009:309) emphasises concept formation as a major issue in the cognitive development of a child. The process of concept formation is studied by referring to the means by which the operation is accomplished, including the use of tools, the mobilisation of the appropriate means and the means by which people learn to organise and direct their behaviours (Lau et al., 2009:309). Based on this process, Vygotsky conceptualised the idea of the zone of proximal development (ZPD). He wrote that children who by themselves are able to perform a task at a particular cognitive level, in cooperation with adults or more capable peers, will be able to perform at a higher level, and this difference between the two levels is called the child's ZPD.

The interest of this study was teacher intervention during learning and teaching through problem solving. Teacher–learner intervention is drawn based on the distance between the cognitive levels when a learner performs a task alone and in cooperation with the teacher. The assumption is that the acquisition of skills by a learner is an activity in which the readily relevant skills are combined to meet new, more complex task requirements (Lau et al., 2009:309). This activity is only successful through scaffolding by the teacher. Scaffolding is associated with teacher–learner interaction where a teacher structures tasks to facilitate learners' learning that would otherwise be beyond their reach. Scaffolding acts as a supportive tool for the learner who extends his or her skills; thereby allowing the learner to successfully accomplish a task not otherwise possible.

Manouchehri (2007:299) claims that false solutions and even unexpected results obtained by learners when solving mathematical tasks can be used to enrich their learning. According to Manouchehri (2007:299), ending learners' discussions by labelling the various solutions that learners offer as right or wrong or by giving them the correct formulas would likely close the door not only on their mathematical investigations, but also on the formation of a learning community in which members willingly explore mathematics and engage in the collaborative construction of knowledge.

2.6 MOTIVATIONS AND DISPOSITIONS IN MATHEMATICS LEARNING

Brahier (2011:4) states that mathematical learning takes place through motivations and positive dispositions. Motivation is regarded as the ‘fuel’ for mathematics learning (Bobis, Anderson, Martin & Way, 2011:32). Disposition includes attitudes towards mathematics, interest in mathematics, curiosity, perseverance, confidence in using mathematics, flexibility in exploring mathematics and attempting different strategies to solve problems, and valuing the application of mathematics (Brahier, 2011:6). Both motivation and dispositions are, therefore, related to problem solving and the process of carrying out an investigation. Learners who are motivated have a sense of self-efficacy or personal agency that enables them to succeed at a task and pursue conjectures. According to Brahier (2011:5), self-efficacy assists learners to become more willing to engage in and successfully complete a mathematical task.

When teaching learners mathematics, we do not simply help them to acquire mathematical skills and problem-solving strategies; we also attempt to develop motivation and positive dispositions towards learning mathematics that will have long-term effects on everything – from learners’ confidence to do mathematics to their career choices (Brahier, 2011:7). Each lesson in a mathematics classroom should take into account learners’ motivation level and dispositions and have as a goal the development of these affective characteristics. Artzt et al. (2008:11) suggest that the substance of motivation should be aligned with the purposes and goals of the lesson. Motivation is enhanced by using a variety of facilitation methods, a variety of resources and different assessment strategies. Visual activities (mathematics-related pictures or posters) can also be used to stimulate learners’ motivation and disposition.

2.7 REFLECTIVE PRACTICES

Reflective teaching is not a new construct in the field of higher education. Commonly, the term ‘reflection’ refers to a process of making sense of past experiences or of making sense of what has happened (Boud, Keogh & Walker, 1994:8). In education, reflection implies the ways in which teachers criticise, interrogate and evaluate the effectiveness of their teaching practice and how their teaching practice can be refined to meet the needs of the learners. In mathematics, reflective teaching practices resemble Polya's model of problem solving, namely understanding, planning and looking back (Artzt et al., 2008:4). In teaching and learning, reflection refers to the way in which teachers think about their teaching before, during and after the lesson (Gimenez, 1999:129). Keeping a teaching portfolio is one of the methods of assisting teachers to reflect on their teaching and assessment practices (Walker, 1994:63).

Griffiths and Tann (cited in Warwick 2007:6) outline a framework on how teachers can reflect on their teaching practice, which includes the following:

- Rapid reflection that is immediate and automatic: reflection-in-action
- Repair: pause for thought
- Review: re-evaluating teaching over hours and days (reflection-on-action)
- Research: a systematic reflection over a period of time, weeks or months (reflection-on-action)
- Retheorising and reformulating: long-term, clearly formulated reflection-on-action over months or years.

For effective reflection, teachers can reflect by using all of the above dimensions (Warwick 2007:7).

Reflection is based on two broad learning categories, namely the constructivist approach and the positivistic approach. The constructivist approach recognises the experiences and knowledge learners bring with them to the classroom (Huitt, 2003:2). According to the constructivist approach, new knowledge is taught based on what the learner already knows. In the constructivist approach, the learning process is centred on the learner rather than the teacher. Learning takes place by matching new knowledge against pre-existing knowledge (Thanasoulas, 2002:1). The positivistic approach considers teachers as 'experts' and learners as passive recipients of information transmitted by the teacher (Long, 2000:6).

2.8 CONCLUSION

Literature advocates for change in the way that mathematics has been taught over the years.

Obviously, such change is bound to be met by resistance from teachers who are used to the traditional methods of teaching. However, literature also provides teachers with mechanisms for dealing with these disconcerting realities. Mathematics teachers should therefore be encouraged to comply with the reform policy and teach mathematics in a more problem-orientated way.

Appreciating that the individual teacher's own beliefs about and views on teaching are a key to his or her practice, the researcher was interested in determining how the teachers incorporate the problem-solving approach into their own practice. From the literature review, it is evident that the teacher's beliefs about mathematics teaching and knowledge of mathematics content influence the way in which the teacher teaches mathematics. The teachers' beliefs lend themselves to a particular philosophy of teaching mathematics.

Chapter 3 describes the methods used to generate data on secondary school mathematics teachers' experiences with regard to the teaching of mathematics using a problem-solving approach.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

In the previous two chapters, the context of this study was discussed, relevant literature pertinent to the study was reviewed and some theoretical conclusions based on the literature review were drawn. In this chapter, the researcher explains the process followed in planning and generating data concerning the facilitation of mathematics through problem solving by secondary school mathematics teachers participating in a BEd (in-service) programme in the John Taolo Gaetsewe district, Northern Cape province, South Africa. This chapter includes an outline of the research questions, the research aim and objectives, the research design and the data-generation procedures. Delimitations and limitations, the assumptions of the study, trustworthiness and credibility, and issues of ethics are also discussed in this chapter.

3.2 THE RESEARCH QUESTIONS

Lankshear and Knobel (2005:172) argue that we cannot address problems systematically unless we have some questions to guide and structure our responses. To achieve the objectives of the study, a number of questions, as indicated in Chapter 1, were developed. The questions as stated below guided this study.

3.2.1 Main research question

How do secondary school mathematics teachers in the John Taolo Gaetsewe district of the Kuruman area experience the facilitation of mathematics through a problem-solving approach while participating in a BEd (in-service) programme?

3.2.2 Sub-questions

The study employed a deductive logic and an interpretive approach to develop the following subsidiary questions for the study:

- What are the views of in-service BEd mathematics teachers in the John Taolo Gaetsewe district in Kuruman on the use of a problem-solving approach?
- How do these in-service BEd mathematics teachers apply a problem-solving approach in their own classrooms?

- How, if at all, do the RUMEP material and activities equip these in-service mathematics teachers to facilitate mathematics through a problem-solving approach?

3.3 RESEARCH AIM

As was indicated in Chapter 1, this study aimed to analyse the experiences of BEd (in-service) secondary mathematics teachers in the John Taolo Gaetsewe district of the Kuruman area in the Northern Cape province regarding their facilitation of mathematics using a problem-solving approach.

3.4 RESEARCH OBJECTIVES

The focus of the study was on exploring the facilitation of mathematics through a problem-solving approach by secondary school mathematics teachers while participating in a three-year BEd (in-service) mathematics programme. To this end, a number of research objectives were formulated. As stated in Chapter 1, this research was based on the following research objectives:

- To determine the views of in-service secondary school mathematics teachers on their facilitation of mathematics through a problem-solving approach
- To observe the opportunities and challenges facing mathematics teachers who teach through a problem-solving approach
- To analyse the RUMEP study material and activities that may assist BEd (in-service) secondary school mathematics teachers to facilitate mathematics through a problem-solving approach.

3.5 THE INTERPRETIVE PARADIGM

According to Le Grange (2004:39), the concept of research paradigms was introduced by Kuhn during the early 1970s. Kuhn (1996:10) associated the word ‘paradigm’ with the term ‘normal science’, which means research that is based upon one or more past scientific achievements, acknowledged by a particular scientific community at a given period of time. Paradigms are frameworks that serve as maps for scientific and research communities in determining theories and methods to solve identified problems or issues (Le Grange, 2004:39). Henning, Van Rensburg and Smit (2004:16) support the view that paradigms are frameworks and state that paradigms are ‘theoretical frameworks’ or ‘philosophies of knowledge’ on which researchers can base their knowledge when conducting research.

Theoretical frameworks are therefore considered to be the 'lenses' through which the world can be viewed (Henning et al., 2004:25). Barr and Tagg (1995:14) regard paradigms as the rules, views and values that assist with the understanding of a particular issue or problem.

Adopting a particular paradigm inform researchers of a particular methodological framework to be used in conducting research (Henning et al., 2004:16). Creswell (1998:74) argues that researchers approach their studies with a certain paradigm or worldview. A paradigm serves as a scheme of thought or a frame of reference for research problems (Arjan, 1998:21). Henning et al. (2004:25) outline the following advantages of positioning a research project within a particular paradigm:

- A paradigm positions research in the discipline in which one is working. This implies that paradigms convey or channel the message embedded within a discipline. In this study, the mathematics education discipline.
- Paradigms assist one in making assumptions about the interconnectedness of things in the world.
- A paradigm provides an orientation or a 'framework' to the study. It enables the researcher to remain within the boundaries of the 'frame' in order to cover the main features of the research design.
- Positioning research within a particular paradigm leads to a certain conceptual framework. A conceptual framework implies an alignment of the key concepts of the study.
- A paradigm 'anchors' one's research within a particular literature, facilitating dialogue between the literature and the study.

An interpretive paradigm of viewing knowledge underpinned this research in that it mainly centred on the significance of participants' views and what meaning can be made from their views, based on the views of Ary et al. (2006:462). In this case, the researcher found this paradigm suitable to position his research, as the intention was to explore the experiences of teachers regarding their facilitation of mathematics through problem solving. Merriam (2009:38) states that an interpretive qualitative study "would be interested in how people interpret their experiences, how they construct their worlds and what meaning they attribute to their experiences". Moreover, Cohen, Manion and Morrison (2000:22) are of the opinion that the central endeavour in the context of the interpretive paradigm is to understand the subjective world of human experience. The central purpose of this study was to explore and understand the experiences of mathematics teachers with regard to problem solving as an approach to mathematics teaching.

The main purpose of studies using an interpretive lens is to understand the experiences or the world of others. Within this paradigm, a case study design afforded the researcher the opportunity to interpret and comprehend the experiences within set contexts (Cohen et al., 2007:85). To retain the integrity of the phenomenon being investigated, efforts were made to get 'inside' the mathematical practices of participants and to understand such practices in real classroom situations, as recommended by Cohen et al. (2000:22).

3.6 RESEARCH DESIGN

A research design describes the plan that guides and directs all activities and processes of research. According to Yin (2009:75), research designs are logical blueprints of the research process. The research design describes the procedures for conducting the study, including when, from whom and under what conditions the data will be generated (McMillan & Schumacher, 2001:30). The purpose of research design is therefore to align the research goal with the practical limitations and considerations of the research project (Mouton & Marais, 1996:32).

A multiple case study design

As the researcher's aim was to determine the teaching experiences of mathematics teachers in a limited number of school contexts within a limited timeframe, the case study was found to be a design type or genre appropriate for this study, following Cohen et al. (2007:253). Therefore, the researcher followed a multiple-case study design. According to Yin (1993:5), case study research can be based on single- or multiple-case studies. A multiple-case study consisting of four case studies (one case for each teacher) was preferred for this study. Feuerstein (1986:48) states that a case study analyses a single event, situation, person, group, institution or programme.

The researcher's choice of a case study design was further supported by Hitchcock and Hughes's (1995:317) consideration of a case study as a suitable way to investigate theories or practices in an everyday environment. In addition, Yin (2009:41) regards case study research as an empirical inquiry that investigates a contemporary phenomenon within its real-life context. This case study therefore investigated and reported the dynamics of problem solving in real-life secondary school and university tutorial situations.

The goal of case studies is to put in place an inquiry in which both educators and researchers can reflect about particular instances of educational practice (Freebody, 2004:81). Stake (cited in Freebody, 2004:82) states that case studies involve a commitment to interpretation with the aim of making principled but naturalistic generalisations. Shulman (cited in Freebody, 2004:82) outlines four attributes of an educational 'case'. A 'case' for study is available when there is:

- Intention: a plan, an itinerary or purpose that is either explicit or formal
- Chance: an intention that is interrupted by a glitch, a surprise, something unexpected
- Judgement: the exercise of judgement, when no simple answer is available in the face of the glitch
- Reflection: examination of the consequences of action taken in the light of the judgement in a way that produces the basis for a new intention.

Case studies focus on one particular instance of educational experience and attempt to gain theoretical and professional insights from a full documentation of that instance (Freebody, 2004:81). According to Freebody (2004:81), case study methodologists emphasise that teachers are always teaching some subject matter, with some particular learners, in particular places and under conditions that significantly shape teaching and learning practices. These conditions are not taken to be 'background' variables, but rather lived dimensions that are indigenous to each teaching–learning event. In this respect, case studies show a strong sense of place and time; they represent a commitment to the overwhelming significance of localised experience.

3.7 DATA-GENERATION METHODS

The data-generation methods used in this study are discussed by referring to data generation, sampling procedures and data analysis.

3.7.1 Data generation

As the researcher was seeking to explore the experiences of teachers who facilitate mathematics using a problem-solving approach, instruments were chosen to generate data that would best describe the experiences of the targeted teachers. As a result, the researcher used multiple data-generation procedures. This gave the researcher an opportunity to examine cases from several points of view (triangulation), because multiple data sources provide information in context, thereby providing rich data for analysis. According to Yin (1993:32), the important aspect of case study data generation is the use of multiple sources (interviews, observations, questionnaires) of evidence that converge on the same set of issues. Therefore, the researcher video-recorded classroom lessons, conducted face-to-face interviews and allowed the teachers to complete a questionnaire based on their problem-solving experiences. As stated before, the video recorder was only used as a back-up later when the researcher wanted to look back at what happened in the classroom.

Freebody (2004:82) states that the data that make up a case study can entail interviews, transcripts, notes and observations. However, the distinctive feature of a case study is not mainly the source of its data; rather it focuses on attempting to document the story, the moves people make in a clearly known and readily defined professional space and the consequences of those people's actions, for learning and for the ongoing conduct of the research project (Freebody, 2004:8). Therefore, data-generation procedures in this study consisted of structured questionnaires, semi-structured interviews, the researcher's observations of classroom lessons and an analysis of the RUMEP study material.

Lankshear and Knobel (2005:172) claim that data can be defined as "bits and pieces of information found in the environment" that are collected in systematic ways to provide an evidential base from which to make interpretations and statements intended to advance understanding and knowledge concerning a research problem or question. This claim is underscored by David and Sutton (2004:27), who hold the view that data should not be viewed as that which is out there to collect, but rather what the researcher 'manufactures' and records. This means that the qualitative researcher determines what counts as data depending on the questions and the objectives that drive the study.

3.7.1.1 Selection of participants

The researcher purposefully and conveniently selected four teachers from a group of 12 BEd teachers in the John Taolo Gaetsewe district teaching at four different schools. The four selected teachers participated freely and voluntarily according to agreed ethical research principles. At the time of the study there were 12 teachers participating in the BEd (in-service) mathematics education programme in the John Taolo Gaetsewe district.

According to Creswell (1994:148), the idea of purposive sampling is to purposefully select participants that will best answer the research question. Kombo and Tromp (2006:82) define purposive sampling as “a sample method, where the researcher purposely targets a group believed to be reliable for the study”. McMillan and Schumacher (2001:401) consider purposive sampling as a way of “selecting small samples of information-rich cases to study in-depth without desiring to generalize to all such cases”. The four teachers were chosen according to their ‘knowledge’ of problem solving, as the researcher had lectured and observed each teacher’s classroom lessons in the foregoing 33 months. Each teacher was observed at least once during a term and conducted at least four Monday afternoon lectures in every term. The selection was based on voluntary participation and the teachers’ willingness to share their problem-solving teaching experiences with the researcher.

According to McMillan and Schumacher (2001:37), cases are not chosen for representativeness, but a case can be selected because of its uniqueness or the case may be used to illustrate an issue. The four teachers were likely to be informative about problem solving, because the Monday afternoon workshops, the Grahamstown contact sessions and the classroom observations already exposed them to problem-solving approaches. Therefore, the four teachers were chosen according to their interest and orientation towards problem solving. Convenience played a role in the selection, because these four teachers were chosen due to their close proximity to where the researcher was staying and working at the time of the study. This enabled the researcher to return to the research participants to seek clarification, should that have proved to be necessary.

3.7.1.2 Questionnaire

A questionnaire is one of many ways information can be obtained from research participants. Questionnaires are relatively economical, present the same questions to all participants and can assist the researcher in ensuring anonymity (McMillan & Schumacher, 2001:257). According to Hofstee (2009:132), a questionnaire is a form of structured interviewing, where all respondents are asked the same question and offered the same options in answering them. In this study, all 12 teachers (including the four cases) completed a questionnaire. The questionnaire was designed and administered by the researcher. The construction of the questions was informed by the theoretical framework discussed in Chapter 2.

The questionnaire consisted of two separate sections, namely Section A and Section B (see Appendix A). Section A dealt with the individual teacher's background profile. Section B covered the teachers' experiences regarding the facilitation of mathematics using a problem-solving approach. In essence, Section B included a set of questions that requested the participants to express their views on each of the following:

- Teaching method
- Planning and preparation of lessons
- Learner communication
- Teacher questioning
- Tasks and activities
- Classroom discourse
- Monday afternoon contact sessions
- Study material.

The answers in Section B ranged from 'strongly agree' to 'strongly disagree'. The intention of this section was to allow the teachers a voice to share their impressions of some of the key features of problem solving. The researcher captured all the teacher responses in a table, as indicated in Chapter 4.

The return rate for the questionnaire was 100%, as the teachers completed the questionnaire in one venue and were required to return it immediately after completion. The questions explored areas of mathematics teaching and learning using a problem-solving approach. The questionnaire also included two open-ended questions at the end of Section B. This was to allow the teachers to express themselves in their own words, which, according to Hofstee (2009:132), helps to put the participants at ease and give them a sense of control over their answers.

The questionnaire was piloted with two teachers who were not part of the study but also participated in the same RUMEP programme in the Northern Cape province. They were from another district that was excluded from the study. The aim of piloting the questionnaire was to ensure that the questionnaire is accurate and that the participants would be able to understand each statement clearly. In particular, piloting the questionnaire served to check the clarity of the questionnaire items, instructions and layout as well as to eliminate ambiguities or difficulties in the wording. The questionnaire also enabled the researcher to check the time taken to complete it. Piloting the questionnaire therefore increased its internal validity.

The pilot participants provided the following feedback, which was incorporated into the questionnaire:

- Language editing of certain questions, for example on page 1 (a statement relating to the number of learners) was done, while questions 8, 21, 23, 24 and 36 were rephrased in order to be more concise.
- Question 37, which related to the RUMEP study material, was added.

External validity refers to the extent to which results can be generalised or used in another study in a different setting or context (McMillan & Schumacher, 2001:407). The fact that this was a case study in itself is a validity issue, because case studies cannot be generalised. A case study is an in-depth study specific to a given context. However, findings accurately documented in one study may allow for replication in other settings. Decisions that were made in this study were recorded and explained so that other researchers may be aware of the dynamic nature of the research and make similar (or different) decisions, if necessary.

All participants were informed in advance of the date and time to complete the questionnaire regarding their experiences in teaching mathematics through problem solving. Cohen et al. (2000:265) state that participants can be encouraged to complete the questionnaire; however, the decision whether to complete particular items in the questionnaire lies entirely with them. Clear instructions were provided on how to complete the questionnaire (see Appendix A for details of the questionnaire). The participants were given enough time to complete the questionnaire without interference by the researcher. The questionnaire was completed in the afternoon outside teaching hours, from 15:00 onwards. One common venue, a classroom where teachers attend the RUMEP Monday afternoon workshops, was prepared for all 12 teachers to complete the questionnaire.

A structured closed-ended questionnaire was used in this study with two open-ended questions at the end. Closed-ended questions are questions that prescribe the range of responses from which the participants may choose (Cohen et al., 2007:321). In other words, closed-ended questions promote the same frame of reference (Ary et al., 2006:422). According to Cohen et al. (2007:321), closed-ended questions directly address the research aim and are more focused and uniform than open-ended questions. Open-ended questions enable participants to write freely in their own terms, to explain and qualify their responses and to avoid the limitations of pre-set categories of responses (Cohen et al., 2007:321). The following are the two open-ended questions asked at the end of Section B (also see Appendix A):

(a) Comment on how problem solving has changed your view on mathematics teaching.

(b) Comment on how you found problem solving useful in your own mathematics classroom.

3.7.1.3 Semi-structured interviews

Semi-structured interviews with individual teachers were also used in this study to generate data. Lankshear and Knobel (2005:198) describe interviews as planned, pre-arranged interactions between two or more people, where one person is responsible for asking questions pertaining to a particular topic of formal interest and the other (or others) is (are) responsible for responding to these questions. "Interviews enable both the interviewer and the interviewee to discuss their interpretations of the world in which they live and to express how they regard situations from their own points of view" (Cohen et al., 2000:267). The researcher concluded that the interview should be regarded as a conversational partnership in which the interviewer assists a process of reflection.

In general, the interviews granted the researcher extended opportunities to explore with the participants the responses provided in the questionnaire. The researcher planned all the interview sessions and informed each participant of the date, time and place of the interview, as the interviews took place at different times and places. The researcher firstly constructed an interview guide that lists all the questions to be asked (see Appendix C); secondly, the researcher adopted the role of an interviewer and lastly, the researcher maintained the same consistent behaviour when interviewing each participant. For example, in each interview each participant was asked the same questions in the same sequence. Furthermore, in each interview the researcher was more interested in what the teachers were telling him, without holding any preconceived notions about each teacher. The researcher reminded the teachers that they have no obligation to participate in the interview. Despite the researcher's position as a lecturer, the teachers were informed that they were under no obligation to discuss anything relating to this research if they chose not to do so. The fact that the researcher was interested in learning from them was also emphasised; also that any critical comment they were to make about the research would be welcomed.

The researcher used semi-structured interviews, as they allow for some depth to be achieved by providing the opportunity on the part of the interviewer to probe and expand the participants' responses (Hitchcock & Hughes, 1995:157). The participants were encouraged to share their interpretations of questions based on an interview framework covering their teaching practices through problem solving. The main aim of the interviews was, therefore, to uncover the teachers' views on problem solving while participating in a hybrid BEd (in-service) programme. One individual interview was conducted with each teacher. Each interview took between 40 minutes and one hour. There was no need for probing teachers' responses unless greater clarity on an answer was required.

For each interview the researcher made use of an interview guide, but also gave plenty of freedom of movement in the posing of questions, follow-up questions (probes) and sequencing of questions, as recommended by Kowal and O'Connel (2004:204). The researcher used an interview schedule and digital recording equipment with the four selected teachers in order to generate data regarding their teaching experiences through problem solving. Lankshear and Knobel (2005:173) suggest that 'spoken data' should be recorded in some durable or lasting form that can be revisited as desired.

The researcher transcribed the interviews verbatim. All interviews were transcribed on the same day that they were conducted. The transcriptions reported each teacher's conversation using a pseudonym to ensure confidentiality, as per agreement. The purpose of transcription is to represent on paper as accurately as possible the 'strings' of words uttered, their acoustic form and any other non-linguistic behaviour (Kowal & O'Connel, 2004:249). Creswell

(1994:152) suggests that the researcher should also take notes during the interview in the event that the recording equipment fails and also to record unobtrusive responses. The researcher took notes of each of the interviews conducted and recorded unobtrusive responses of each teacher during the interviews. Yin (2009:157) is of the opinion that researchers should take sufficient notes to support the later analytic and compositional needs of the study.

However, interviews will not always capture everything a participant feels, thinks, believes or values about something (Lankshear & Knobel, 2005:173). The face-to-face interviews were therefore used to support, strengthen and deepen the data generated from the questionnaire survey and the classroom observations.

3.7.1.4 Classroom observations

In general, observations involve carefully planned, deliberate and systematic examinations of what is taking place, when and where everything is taking place and who is involved (Lankshear & Knobel, 2005:175). One of the advantages of using observations is that they offer the researcher an opportunity to gather 'live' data from naturally occurring situations (Cohen et al., 2007:397). This means that observations enable researchers to study behaviour as it occurs. Another advantage of observations is that they allow the researcher to collect data first hand, thereby preventing contamination of the factors standing between him or her and the object of research (Nachmias & Nachmias, 1996:206). Furthermore, observations permit researchers to validate verbal reports by comparing them with actual behaviour (Nachmias & Nachmias, 1996:207). This implies that contradictions might occur between what teachers say in interviews and their practice in the classroom. In this study, classroom observations were used in order to support and validate the data collected during the interviews.

Timing, recording and inference are some of the major parts that constitute observations (Nachmias & Nachmias, 1996:210). It is impossible to make an infinite number of observations; as a result, the researcher made a decision as to where and when is to observe, because he could not be in all the classrooms of the participating teachers at all times. For this reason, the researcher followed a time-sampling approach, which refers to the process of selecting observation units at different points in time (Nachmias & Nachmias, 1996:210). The researcher developed a time sheet specifying the time and the venues where the four teachers were to be observed while facilitating mathematics lessons in their own classrooms. Each lesson was marked as Lesson 1 (Teacher A), Lesson 2 (Teacher B), Lesson 3 (Teacher C) and Lesson 4 (Teacher D). The teachers' names were coded using alphabetical letters from A to D in order to keep their anonymity. The researcher informed all the teachers in advance of the time he would be visiting them in their classrooms.

The classroom observations lasted between 35 and 60 minutes and were digitally video-recorded, so that the researcher could review the lessons in his own time and go back to them during data analysis. The video camera was positioned on a tripod at the back and to one side of the classroom to allow panning to film the teacher and learners seated at their desks. The video camera was placed at the back of the classroom so that the learners were not intimidated by the presence of the camera. During this time, the researcher also jotted down interesting points that arose as the lesson progressed, specifically points pertaining to the problem-solving criteria, for example learner interactions and learner–teacher interactions that occurred as learners worked on the tasks and activities, the exchange that occurred during group discussions and teacher questioning.

The researcher observed all 12 teachers, including the four cases, at least once a term since the beginning of the BEd programme in 2012. The classroom observations constituted the major component of the RUMEP programme. For this study, the researcher observed each of the four cases once in addition to the normal classroom observations carried out since 2012. The purpose of the classroom observations was to provide a basis from which classroom practices of individual teachers could be discerned with respect to problem solving. After each classroom observation, the researcher and the teacher reflected on the lesson. The reflection session provided the teacher with an opportunity to respond to the following questions:

- What went well in the lesson?
- What did not go as planned in this lesson?
- What are the reasons the lesson went well?
- What are the reasons the lesson did not go well?

Based on the responses to the above questions, the researcher provided support on areas of the lesson in which the teacher needed assistance.

A classroom observation sheet (Appendix B) was used during the classroom visits. These classroom observation sheet contained observation markers detailing specific actions and strategies of problem solving. As mainly qualitative data were used in this study, the observation markers enabled the researcher to make valid qualitative judgements, based on recommendations by Yin (2011:143). The observation markers were organised under the following headings:

(a) General and classroom environment

(b) Communication and questioning

(c) Classroom discourse

(d) Teaching resources.

The items contained in the observation sheets were discussed and standardised with the study supervisors to increase the validity of the instrument. The researcher kept the observation sheets in a file after each observation session for safekeeping.

Another consideration in observation relates to the degree of inference required of the observer. Inference implies that, when the researcher observes a certain act or behaviour, he or she must process this observation and infer as to whether or not the behaviour or the act indicates a certain variable (Nachmias & Nachmias, 1996:211). During the lesson observations, the researcher was constantly aware of any action that constitutes problem-solving behaviour. Chadwick, Bahr and Albrecht (1984:75) suggest that observations must be systematic in order to assist researchers to pay particular attention to those categories of action determined by the researcher's specific objectives and questions.

Even though social science research is rooted in observations, Chadwick et al. (1984:76) outline the following challenges of using observations, which the researcher took into consideration when conducting the classroom observations:

- Observers may sometimes not see or hear what goes on or may misinterpret what is observed because only part of the situation was visible or audible to them. The researcher used a video recorder to record classroom activities so that he could later review all the actions that occurred in the classroom which he might not have seen or heard.
- Selective perception can influence what we are observing. The researcher has worked with teachers for the foregoing 24 months and during that period he had acquired professional skills that are not influenced by perceptions. Furthermore, the researcher

was a non-participant observer, detached from the activities that were taking place during the lessons.

- Senses do not operate independently from our past experiences. What we observe and the interpretations we attach to what is observed are influenced by what we have previously seen. As a mathematics education lecturer, lesson observation form a major part of the researcher's day-to-day responsibilities. This means that the researcher had gained confidence to conduct lesson observations without being influenced by his past experiences.
- The process of observation tends to influence the phenomenon that is being observed. Human beings often behave differently because they are observed. The teachers taking part in this research have been observed several times by the researcher and other RUMEP staff, therefore the researcher's presence in their classroom did not influence the outcomes of the study.

3.7.1.5 Written material

In this study, written data included the RUMEP study material. These documents were used in this study to supplement the data collected through interviews, questionnaires and lesson observations. The researcher collected RUMEP study material used by in-service BEd mathematics teachers in the Kuruman district.

3.7.2 Data analysis

According to Lankshear and Knobel (2005:266), data analysis is a process of organising pieces of information, systematically identifying their key features or relationships and interpreting them. Such a description is consistent with that of Hitchcock and Hughes (1995:139), who consider data analysis as an attempt to organise, account for and provide explanations of data so that some kind of sense can be made of them. In data analysis, the researcher summarises what he or she has seen or heard in terms of common words and phrases, as well as in terms of themes that aid the researcher's understanding and interpretation of the emerging data. The researcher was mindful of treating all of the participants' feedback with the utmost respect. The researcher considered data analysis as soon as he started generating data, as recommended by Miles and Huberman (1994:50). Therefore, the data analysis was an ongoing process.

As mentioned earlier, the study consisted of four multiple case studies where narrative and observational data were generated through different methods, namely interviews, questionnaires, observations and the RUMEP study material. However, the questionnaire was only completed in order to give the researcher a limited overview of teachers' problem-solving experiences rather than a complete questionnaire survey. The data from the questionnaire cannot be classified as the real questionnaire survey. According to Cohen et al. (2007:461), there is no single correct way of presenting and analysing qualitative data.

Content analytic procedures constituted the base of analysis in this study. Content analysis is a research method applied to written or visual material for the purpose of identifying specified characteristics of the material (Ary et al., 2006:457). According to Hitchcock and Hughes (1995:226), the aim of content analysis is to produce objective, valid, systematic and quantifiable analysis of data. The content analysis is not concerned with either circumstances of the production of the document or the motives of the producer of the document. Content analysis is concerned with the frequency or significance with which certain textual items, such as words, concepts and phrases, appear in a text (Hitchcock & Hughes, 1995:226).

3.7.2.1 Interviews and classroom observation sheets

Data generated from interviews and classroom observations were analysed by identifying categories relating to the research questions and the theory explored in Chapter 2. The researcher started by developing a general sense of the data, organising and sorting the data into segments and assigning categories. The researcher explored themes and categories in order to discover a pattern in the transcribed interviews and observation notes. In interpretive studies, data analysis involves the development of patterns and categorisation, interpreted by the researcher through his or her own disciplinary lens (Ary et al., 2006:457).

Miles and Huberman (1994:57) advise that one should go through transcripts or field notes with a pencil, marking off units that cohered because they dealt with the same topic, and then divide them into themes, categories and subcategories at different levels of analysis. In this study, the researcher adhered to this advice. The researcher used a coding system, because codes empower and speed up analysis (Miles & Huberman, 1994:65). Codes are tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study (Miles & Huberman, 1994:56). All codes were written in the right-hand margin beside the segments.

One method of creating codes is that of creating a provisional 'start list' of codes before undertaking fieldwork. Such a list is helpful, as it forces the researcher to tie research questions and conceptual interest directly to the data (Miles & Huberman, 1994:65). However, the researcher should be ready to redefine or discard codes when they look inapplicable or empirically ill-fitting to the text. The list of categories was generated from the conceptual framework and list of research questions that formed a basis for the study. The researcher kept the list on a single sheet for easy reference.

The codes were formulated using a list of categories. The five codes were identified in this study through the theoretical framework explored in Chapter 2. The codes were categorised into those that depicted teacher responses during the interview and teacher actions during the facilitation of lessons. Therefore, the initial letter was given to the first word in the code, while the last letter was given to the last word in the code. This resulted in five items, expressed in the following action statements:

1. Probing understanding

The teacher asks learners to explain and justify their responses and strategies used to arrive at a solution in order to probe understanding.

2. Sense-making

The teacher makes explicit reference to mathematical conventions, symbolisms, definitions, axioms and theorems.

3. Drive learning

The teacher uses the learners' ideas as starting points for discussion. The teacher responds to learners' contributions in order to make relevant instructional decisions.

4. Exploratory discussions

The teacher encourages argumentation between learners, including argumentation not mediated by the teacher.

5. Learner interaction

The teacher encourages and permits other learners to comment on the contribution of previous learner speakers. Clements and Battista (1990:37) suggest that learners must be encouraged to exchange points of view and to agree or disagree with one another. In this way, learners are likely to eventually agree on the truth if they debate the solution process (Clements & Battista, 1990:37).

3.7.2.2 The questionnaire

Data generated from the questionnaire were analysed and coded. Section A was coded by grouping the teacher responses in each sub-section. Each teacher response to the closed-ended questions in Section B was presented in a table, as indicated in Chapter 4. Coding of the two open-ended questions was done by copying all the responses to a particular question onto a table; the goal being to determine a small set of categories in which the responses could be sorted.

3.7.2.3 Written material

The written materials used in the data analysis were RUMEP study material and the researcher's workshop reflection sheets. The researcher analysed the study material by using the criteria suggested by Le Roux and Le Roux (2004:13) regarding the evaluation of study material.

3.7.3 Sections that constituted data analysis

The qualitative research orientation allowed the researcher to see and comment on significant issues pertaining to mathematics teachers' experiences with regard to the use of the problem-solving approach in facilitating mathematics lessons. All the data were analysed and collated in three separate sections, as discussed below:

- The individual teacher profile

The findings and discussions in this first section pertained to each teacher's personal and historical background. This created a profile for each teacher. The data were obtained through the teacher questionnaire (Appendix A: Section A). While this was a valuable research tool, the researcher also made use of material gained from interviews conducted in the course of the case study.

- Personal beliefs and experiences

The data were obtained from the questionnaire, with specific reference to Appendix A: Section B, regarding the teachers' own classroom experiences using problem solving, their experiences while being taught through problem solving and their experiences with regard to the RUMEP study material. The semi-structured interview afforded the researcher additional answers to some of the questions relating to teacher experiences and beliefs on problem solving and the RUMEP study material. The classroom observation sheets provided the researcher with even more opportunities to obtain data regarding the teachers' own classroom experiences. The reflection journal writings afforded the researcher an opportunity to reflect on his own teaching practice. In the reflection sheets the researcher indicated what seemed to go well and what seemed not to go well when presenting a particular topic. The researcher also indicated how he would facilitate the topic next time.

- Synthesis

Having considered the teachers' experiences with problem solving and the RUMEP study material, the researcher then synthesised all the data pertaining to the problem-solving approach through face-to-face interviews. Polya and Schoenfeld's problem-solving frameworks were referred to as the researcher linked data from the different experiences to discuss the findings.

3.8 DELIMITATIONS AND LIMITATIONS

Delimitations are used in order to indicate how the study was narrowed in scope, while limitations identify potential weaknesses of the study (Creswell, 1994:110). This study was limited to secondary mathematics teachers who were studying for the BEd (in-service): Mathematics Education degree at RU. In order to make the study manageable, the researcher concentrated on one aspect of mathematics teaching, namely teaching mathematics through problem solving. Even though the researcher travelled with the four teachers for a period of more than two years this study focused only one lesson observation. Furthermore, the context of curriculum demands, the school context and particular teacher's classroom, might have an impacted or constrained the teacher's use of a problem solving approach.

The results of this study were not generalised beyond the confines of the study. The purposive sampling procedure decreased the generalisability of the findings. This study was not generalisable to all areas of mathematics teaching through problem solving.

3.9 ASSUMPTIONS

The facilitation approach in the RUMEP programme is mainly based on a problem solving approach. The teachers showed confidence in applying problem-solving strategies in their own ways during the classroom support visits. This study was based on two assumptions:

- Teachers participating in the study have some intuitive knowledge about teaching mathematics through problem solving. Teachers were taught through problem solving, among other methods, and were expected to implement what they are learning at RUMEP in their own classrooms.
- Each teacher has opportunities to apply a problem-solving approach in his or her classroom and such opportunities are therefore not restricted.

3.10 TRUSTWORTHINESS AND CREDIBILITY

It was the researcher's responsibility to present as sound and impartial a report as possible. To fulfil this responsibility, the researcher took into account the notion of trustworthiness and credibility. Trustworthiness and credibility are important keys to research that makes use of qualitative data. Guba (cited in Poggenpoel, 1998:350) identifies truth value, applicability, consistency and neutrality as four important criteria applicable to the assessment of credibility and trustworthiness. The researcher applied these four criteria to assess the trustworthiness and credibility of the findings. The researcher's position as a mathematics facilitator of the teachers participating in this study was not allowed to compromise the objectivity of his role within this research study.

Multimethod strategies, namely questionnaires, interviews and observations, were used to increase the validity and credibility of the study, as recommended by McMillan and Schumacher (2001:429). The use of multimethod strategies in qualitative research is used to compensate for any one-sidedness or distortion that may result from individual methods (Steinke, 2004:184). Cohen et al. (2007:141) argue that the exclusive use of one method may bias or distort the researcher's picture of the particular 'slice of reality' being investigated.

Copies of transcripts were made available to the participants to help secure the interpretive validity of the research. This also helped to confirm the accuracy of the transcriptions.

According to Cohen et al. (2007:135), internal validity seeks to demonstrate that the explanation of a particular event, phenomenon or set of data which a piece of research provides can be sustained. It refers to the quality of the data and the 'soundness' of the reasoning that is derived from that data. The questionnaire was piloted with two teachers in order to increase its internal validity. External validity refers to the degree to which the results can be generalised to the wider population, situations or cases (Cohen et al., 2007:136). The findings of the study were also checked with RU's Mathematics Education Unit, which is familiar with the phenomenon of problem solving in the context of the RUMEP model.

3.11 ETHICAL CONSIDERATIONS

According to McMillan and Shumacher (2001:420), criteria for a research design involve not only efficient research strategies and information-rich participants, but also adherence to research ethics. Research ethics are considered to deal with beliefs about what is good or bad, wright or wrong, proper or improper (McMillan & Shumacher, 2001:196). One of the ethics conditions is that research must not result in any permanent change, damage, injury or harm to the participants (Chadwick et al., 1984:16). In addition, researchers cannot justify a course of research that will have harmful effects for the participants of that research, even in the interest of advancing scientific knowledge. Therefore, the researcher has an obligation to respect the rights, desires, needs and values of the participants (Creswell, 1994:165). Furthermore, Chadwick et al. (1984:16) state that the researcher must be aware of and respect the dignity, privacy and worth of other individuals. In this study, the researcher took cognisance of research ethics by observing, respecting and protecting the rights, dignity and privacy of all the teachers participating in the study.

An important thing to consider when conducting research is to inform participants about how the research will be conducted and how they will be involved so that they can make informed decisions about their participation. As a result, all participants gave their informed consent and were informed that they would remain anonymous and that the data that are to be derived from their feedback would only be used for purposes of reporting and analysis. In this regard, the participants completed a consent form. Informed consent means that the participants or their legal representative understands the nature of the study and the risks the participants will be exposed to and then makes a decision to participate free from force, fraud, duress, deceit or other forms of constrain or coercion (Chadwick et al., 1984:19). For purposes of confidentiality and anonymity, the identities of the teachers were indicated by means of letters of the alphabet. Before the commencement of the interviews, the participants gave the researcher permission to record the interviews.

Permission to carry out the study was obtained from the Northern Cape Department of Education, because the study involved Northern Cape teachers (see Appendix F). As the study also involved interviewing and observing teachers in their classrooms, the researcher obtained permission from the school principals of the schools participating in the study (Appendix G). The teachers were informed that their participation was important, but that their role was voluntary and that they were free to withdraw should they feel uncomfortable during the course of the study.

Permission was also granted by RUMEP, as the teachers are enrolled under the RU education department (see Appendix E). In addition, all ethical issues were clarified and approved by the Research Ethics Committee of Stellenbosch University and accepted by the researcher (Appendix D).

3.12 CONCLUSION

This chapter orientated the research within an interpretive paradigm and provided details of the choice of methodology and instruments used. The chapter also reminded the reader of the goal of the research and described the selection of participants. It was stated that the findings of the study could not be generalised due to the methodology used. Nonetheless, the study results might be beneficial to teachers and other interested individuals in teacher education to explore problem solving as one of the approaches to mathematics teaching and learning.

The chapter concluded with a discussion of the validity, trustworthiness and credibility of the data as well as the ethical considerations of the study. The findings and discussion are dealt with in the next chapter.

CHAPTER 4

FINDINGS AND DISCUSSION

4.1 INTRODUCTION

Chapter 3 outlined how the research for the empirical part of the study was conducted. In this chapter, the empirical findings of the study are presented and discussed as they occurred in the three phases outlined in Chapter 3. These findings emerged from the data generated from the questionnaire (completed by all 12 teachers who participated in the RUMEP programme) as well as classroom observations and interviews with the four teachers selected as cases for this study. Each of the four cases was selected from a different school. Data from the three methods employed were triangulated to determine the degree to which the data sets complement one another and to check for similarities or variations. The findings also include information from the RUMEP study materials.

In this chapter, the participants are indicated by alphabetical letters to protect their identity. The qualitative research data allowed the researcher to ‘see’ and comment on significant issues pertaining to how each of the teachers experienced the teaching of mathematics through problem solving. All the data were analysed and collated in three phases, as stated below in 4.2, 4.3, 4.4 and 4.5.

4.2 INDIVIDUAL TEACHER PROFILES

The findings and discussions in this first phase related to the individual teachers’ personal and historical background. The researcher’s intention in narrating each teacher’s profile was to provide an understanding as to why they may have responded in a certain way. The information regarding the teachers’ personal and historical background was obtained from the questionnaire as well as the face-to-face interviews. The four cases represented qualified and experienced secondary school mathematics teachers enrolled in the BEd RUMEP programme at the time of the study.

Teacher A

Teacher A is married and has one child and is in his early forties. Initially, mathematics was not his favourite school subject, but he eventually developed a love and passion for the subject through his half-brother. His half-brother was a learner at another school taking Mathematics as one of his subjects. Teacher A holds a two-year Diploma in Secondary Education from Uganda, majoring in mathematics and geography. He enrolled for an Advanced Certificate in Education (ACE) with the University of Johannesburg, but never completed this because the ACE programme was discontinued by the university concerned. He has completed some Department of Education courses, including on the National Curriculum Statement (NCS) and the Curriculum Assessment Policy Standards (CAPS). He was very excited to be part of this study and to study in the BEd programme offered by RU.

Teacher A started teaching in 2001. He has ever been teaching mathematics in secondary schools and the school at which he is currently teaching is the third school in his teaching career. He teaches grades 11 and 12 and his school offers schooling from Grade 8 to Grade 12. The average number of learners in his class is 35. At the end of each academic year, Teacher A is expected to 'move' with the learners to the next grade and according to him, moving with the learners is beneficial, as the learners get used to his teaching methods. Teacher A is not the only mathematics teacher at the school. The school has six mathematics teachers. He indicated that there is good co-operation among the mathematics teachers. Teacher A noted that all six teachers are always available to help each other, but that it is very rare to find that one goes to another for comments or to ask questions.

Teacher A prefers his own classroom, in which learners move to him at the end of each period instead of him following learners to another class at the end of each period. For him it is difficult to set up the class every time he moves to another classroom during the day. It is also a challenge for him to facilitate cooperative learning if he has to move to another class every time at the end of the period, as it is time-consuming to set up the classroom for group activities. Teacher A noted that his teaching approach was teacher-centred before and acknowledged that it was a challenge for him to shift to learner-centred approaches. However, he indicated that his confidence with regard to mathematics teaching has increased since he enrolled for the BEd programme in 2012.

Teacher A stated that his learners are not all well prepared for the next grades with regard to mathematics. He said learners are promoted to the next grade even if they failed mathematics in their previous grades. This is because learners are allowed to fail only one subject out of seven subjects and the subject that they normally fail is mathematics. The attitude of the school towards mathematics as a subject and towards mathematics teaching is satisfactory. However, according to Teacher A, the school is only worried that mathematics contribute to the failure rate of learners at the school. It seems Teacher A acknowledges that we live in a world that is characterised by some challenges, but he is willing to fit into that world. This is evidenced by his reporting that his confidence with regard to mathematics teaching has increased since he enrolled for the BEd programme in 2012.

Teacher B

Teacher B is married and has three children. His age falls in the age range of 41 to 50. He has 18 years' mathematics teaching experience in secondary schools. He is currently teaching Grade 10 mathematics and Grade 11 Life Sciences. The school starts from Grade 10. His Grade 10 learners are from different middle schools in the surrounding areas. Middle schools have only Grade 7, Grade 8 and Grade 9 learners who, after completing their Grade 9, move to another school starting at Grade 10. However, many of these learners are not well prepared for Grade 10 mathematics. According to Teacher B, this is because some of the learners from the middle schools are being promoted to his school because of age and not because they meet the minimum promotion requirements. In addition, they have to adjust to the secondary school environment only later, in Grade 10, as compared to their peers who are introduced to a secondary school environment as early as Grade 8.

Teacher B indicated that the wide learner age range difference in his class sometimes poses some challenges when facilitating his lessons cooperatively. One of his learners is repeating Grade 10 mathematics for the third time this year. He indicated that he follows learners at the end of the period and such arrangements do not pose any challenges to him. Teacher B thinks that the school is not happy offering mathematics as one of the subjects. At his school there are 22 classes, but only three classes are taking mathematics, that is, one class per grade. However, he indicated that the school is happy with his teaching approaches. Like Teacher A, Teacher B also indicated that his teaching approach used to be teacher-centred. However, he is currently trying to move towards learner-centred approaches, but he experiences difficulties, as he was also taught through a teacher-centred approach.

The average number of learners in his class is 40. The school has only two mathematics teachers. Teacher B confirmed that there is cooperation between him and the other mathematics teacher. He said they normally meet to discuss strategies on how to teach a certain topic. Mathematics has always being Teacher B's favourite subject. He holds a three-year Secondary Teachers' Diploma (STD) with majors in Mathematics and Biology. He attended several workshops in OBE, CAPS and NCS organised by the DBE. He once registered for an ACE programme, but did not continue with the programme due to family and job commitments. He is now excited that he is in the final stages of the BEd programme at RU and is looking forward to completing his studies in 2014. Teacher B acknowledges that we live in changing times and this is evident in his reporting on mathematics classroom practices.

Teacher C

Teacher C is married and has two children. She falls in the age range of 35 to 45. Teacher C is a qualified mathematics and natural sciences teacher at a secondary school. She holds an STD. (mathematics and physical science), an ACE (mathematics, science and technology) and a BEd Honours (natural sciences). She also attended short (one-week) OBE, CAPS and NCS workshops organised by the DBE. She is currently teaching Grade 8 mathematics and Grade 10 mathematics. Her school enrols learners from Grade 8 to Grade 12.

According to Teacher C, the majority of learners entering Grade 8 are not well prepared, because she has to re-teach some of the Grade 7 topics. The Grade 10 learners are partly prepared because some of them were taught by her while others were taught by another teacher in Grade 9. The attitude of the school towards mathematics as a subject is good. The school is happy that they have her as one of the mathematics teachers at the school and is satisfied with her teaching methods. According to her, the principal was wondering why the timetable committee cannot allocate her a Grade 12 class. She has never taught either Grade 11 or Grade 12 mathematics, and wishes that one day she may be given an opportunity to teach these classes. Teacher C also noted that her teaching was always narrative where learners will listen to her while demonstrating how to solve mathematics problems.

The average number of learners in her class is 30. There are five mathematics teachers at the school. According to her, there is little cooperation among the mathematics teachers. The teachers are always busy and never have time to meet with each other as professional mathematics teachers. According to Teacher C, mathematics teachers only meet during the moderation of learners' work.

Teacher C started teaching in 2000. She has ever being teaching mathematics in secondary schools (grades 8 and 9) and this is her third school. She believes that moving with the learners to the next grade might have a positive impact on their mathematical understanding and subsequently on learners' achievement. She owns her class (learners move to her class at the end of each period instead of her moving to the learners) and this makes it easy for her to arrange the classroom the way she likes. She was very excited to be part of the study and to be in the RUMEP programme.

She acknowledges that before studying with RUMEP she was demotivated and had no confidence in mathematics teaching and learning. "*I had no confidence in teaching mathematics, even though I am not yet there, RUMEP brought me confidence. Before joining RUMEP I thought of leaving my job".* Teacher C's confidence has increased, as is the case with Teacher A and Teacher B. According to Teacher C, on a scale of 1 to 10, where 1 is the lowest confidence and 10 is the highest, her confidence level has increased to 8 in 2014 as compared to 2 in 2012 when she was starting the RUMEP programme. As she has indicated earlier, she is now willing to teach mathematics in the higher-grade classes. The comments by Teacher C seem to indicate that she is willing to make a shift to accommodate recent reforms with regard to mathematics teaching practices.

Teacher D

Teacher D is married and has one child and his age range is also between 35 and 45. He completed an STD in 2004 with majors in mathematics and biology. He further also attended short (one-week) OBE, CAPS and NCS workshops organised by the DBE. He started teaching in 2005. Teacher D taught only in secondary schools and the school at which he is currently teaching is his second school. He teaches mathematics in Grade 11 and mathematical Literacy in grades 11 and 12.

Like Teacher B, his school starts at Grade 10 and ends at Grade 12. The school gets learners in Grade 10 from three different middle schools in the surrounding areas. According to him, the majority of the learners entering Grade 10 are not well prepared for the grade. However, the Grade 11 learners are better prepared than those in Grade 10, as they have been exposed to his school for a year and some of the learners were taught by him in the previous year in Grade 10. He does not have a problem following learners to their classrooms at the end of each period. Like the other three cases, Teacher D indicated that his teaching was teacher-centred, but he is now trying to incorporate a problem-solving approach in his teaching even though he is experiencing challenges. He claims his main challenge is time.

The average number of learners in Teacher D's class is 35. The school has five mathematics teachers. According to Teacher D, there is a high level of cooperation and support among the mathematics teachers. For example, if one teacher is absent, the other mathematics teachers are willing to teach his or her class. According to him, the mathematics teachers at his school can work as a team. If one teacher experiences a problem with a certain topic, they are able to sit and help each other.

Teacher D stated the school is happy that mathematics is one of the subjects offered in the school. Because his school is one of the Dinaledi schools, they are required by the DBE to offer mathematics as a subject. Dinaledi schools are schools that are expected to perform much better, as compared to other schools in South Africa. His school is satisfied with his mathematics teaching because last year he was teaching mathematics Grade 10 and currently he is teaching Grade 11 mathematics. Teacher D is happy to be part of the study and continuing his studies with RUMEP. Teacher D seems to be confident that learner-centred approaches are appropriate methods to follow in facilitating mathematics lessons, as he is interested in incorporating a problem-solving approach in his classrooms.

4.3 PERSONAL BELIEFS AND EXPERIENCES

In this section, the data obtained from the questionnaire, the interview and the classroom observations are presented. Firstly, the responses to the questionnaire completed by the 12 teachers registered in the RUMEP programme, including the four scrutinised cases, are presented and discussed. This is followed by an analysis of the data collected from the four selected cases. This presentation of the findings is focused on:

- The views of in-service secondary school mathematics teachers on their facilitation of mathematics through a problem-solving approach
- The classroom opportunities and challenges facing mathematics teachers teaching through a problem-solving approach
- The RUMEP study material and activities that may assist BEd (in-service) secondary school mathematics teachers to facilitate mathematics through a problem-solving approach.

The discussion provided hereafter is focused on evidence of the existence of the above aspects leading to answers to the main research question and the sub-questions.

4.3.1 Questionnaire completed by the 12 teachers

As explained in Section 3.7.2, a questionnaire was used to generate secondary data for this study. The secondary data were generated in order to complement data from the four cases. This allowed the researcher to analyse data from several points of view. The questionnaire constituted only a limited survey rather than a full questionnaire survey, whereby the teachers completed a questionnaire based on their views on the use of a problem-solving approach in the teaching and learning of mathematics. The questionnaire consisted of 37 closed-ended questions and two open-ended questions. All 12 teachers in the RUMEP programme, including the four cases, completed the questionnaire. The four cases were not separated from the other teachers for the anonymous completion of the questionnaire. The identities of the teachers were therefore not known, including those who formed the four chosen cases.

4.3.1.1 The closed-ended questions

For the purpose of this study, the data generated from the closed-ended questions were divided into seven sections, namely:

(a) Teaching method
(b) Planning and preparation of lessons
(c) Learner communication
(d) Teacher questioning
(e) Tasks and activities
(f) Classroom discourse
(g) Monday afternoon contact sessions
(h) Study material.

The eight sections listed above were identified in Chapter 2 as items that may contribute to better mathematics teaching with a focus towards problem solving for the BEd (in-service) mathematics teachers studying via a hybrid distance learning model.

(a) Teaching methods

Table 4.1 indicates how teachers responded to the issue of teaching methods. The researcher used the acronyms SA to represent 'strongly agree', A for 'agree', DA for 'disagree' and SD for 'strongly disagree'. The number in brackets represents the percentage of the number of teachers who responded to a specific question.

Table 4.1: Summary of teacher responses on teaching methods

	Item	SA	A	DA	SD	Researcher comments
1	Problem solving enhances learners' abilities to recognise what they have already learned.	4	8			It appears that more of the teachers in the RUMEP group concur with Schoenfeld's view that any mathematical problem-solving activity is built on a foundation of basic mathematical knowledge, which is called 'resources' (1985:12). Teachers seem to acknowledge that problem solving may help them to establish learners' prior knowledge.
2	Problem solving enhances learners' abilities to apply what they have already learned.	6	6			The teachers are in agreement with the notion that problem solving may better enable learners to apply what they have already learned.
3	I found a problem-solving approach appropriate in my classroom for the teaching and learning of mathematics.	6	6			The teachers gave the impression that problem solving is an appropriate method for facilitating mathematics lessons. Such a view is in line with the Cockcroft report (Cockcroft, 1982), which indicates that the major aim of teaching mathematics is the understanding of mathematics, which can be achieved through problem solving as one of the six elements of successful mathematics teaching.
4	I have to do more revision before learners can write a test if I use problem solving.	1	3	8		It appears that more teachers need not do revision before learners can write a test if they incorporate problem solving into their mathematics teaching. This is line with the meaning of problem solving, because if problem solving is consistently applied and exercised, the learners will be competent in using and applying problem-solving strategies, which means that there should be less revision necessary as they are always ready to solve a problem.
5	Using problem solving, I feel that I can complete all the prescribed work in the curriculum.		7	4	1	It appears that the teachers agree that they are able to complete the syllabus when using the problem-solving approach. The teachers indicated that they need to plan accordingly and adhere to their planning to be able to complete the curriculum on time.
6	I enjoy using the problem-solving method in my classes.	3	8	1		It appears that 67% of the 12 teachers enjoy using problem solving in their own classrooms.
7	I have confidence in using the problem-solving method in my mathematics classroom.	2	6	4		This study indicates that 33% of the teachers do not have the confidence to apply problem solving. This was evident in the four cases, as they reported finding it difficult to consistently focus their lessons on problem solving.

Table 4.1 reveals that nearly all the RUMEP teachers indicated that problem solving is one of the appropriate methods that can be used to facilitate mathematics lessons in order to learn

new mathematical skills. This is positive and praiseworthy, as it shows sensitivity for the potential of problem solving. In particular, the NCTM (2000:182) illustrates that problem solving serves as a ‘vehicle’ for learning new mathematical ideas and skills. Stacy (2005:341) states that problem solving is one of the fundamental methods of teaching mathematics and is considered to be the ‘heart of mathematics’. In essence, active learning and teaching is promoted by effectively engaging teachers and learners in work on problem solving (Marcus & Fey, 2006:55). However, at least one teacher noted strongly that problem solving is not an appropriate method for the teaching of mathematics.

(b) Planning and preparation of lessons

The questions in this section were asked in order to determine the 12 teachers’ views on the planning and the preparation of lessons if their teaching is orientated towards problem solving. Table 4.2 illustrates the number of teachers who responded to questions regarding the planning and preparation of lessons using problem solving.

Table 4.2: Summary of teacher responses regarding the planning and preparation of lessons

	Item	SA	A	DA	SD	Researcher comments
8	Problem solving improves my planning and preparation of the lessons for the next day.	3	9			All the RUMEP teachers seemed to agree that problem solving may improve their planning and preparation of lessons.
9	Problem solving enables me to plan lessons that build on learners’ existing proficiencies.	5	6	1		Most of the teachers seemed to agree that problem solving may help them plan lessons that build on learners’ prior knowledge.
10	Problem solving gives me an opportunity to facilitate planned lessons effectively.	4	7	1		Nearly all teachers are of the opinion that problem solving gives them an opportunity to facilitate lessons effectively.
11	Using problem solving, I have sufficient time to successfully finish prepared lessons as scheduled.	1	6	5		A few teachers seemed to be unable to complete prepared lessons when using problem solving.
12	I find the planning and preparation of lessons easier if I use problem solving.	1	9	1	1	Most of the teachers indicated that they found the planning and preparation of lessons easier when using problem solving.
13	Using problem solving, planning and preparation of lessons are more challenging.	5	4	3		A few teachers found the planning and preparation of lessons more challenging when using problem solving.
14	I find the planning and preparation of lessons more fulfilling if I use a problem-solving approach.	2	10			All of the teachers seemed to agree that planning and preparation of lessons are more fulfilling when using problem solving.
15	Problem solving involves less work	1	7	4		A few teachers are of the opinion that problem solving

	for the planning and preparation of my classroom lessons.					involves more work for the planning and preparation of lessons.

As illustrated in Table 4.2, most of the RUMEP teachers are of the opinion that problem solving improves their planning and preparation skills of mathematics lessons. However, this is not in line with the few teachers who find the planning and preparation of lessons difficult and challenging. Only some teachers indicated that problem solving involves more work for the planning and preparation of classroom lessons.

(c) Learner communication

In this section, the researcher wanted to determine the 12 teachers' responses with regard to learner discussions if their focus is on problem solving. Table 4.3 indicates the teachers' responses with regard to learner communication when problem solving is incorporated into the teaching of mathematics.

Table 4.3: Some indications of teachers' responses regarding learner communication

	Item	SA	A	DA	SD	Researcher comments
16	Through problem solving, learners have an ability to learn from one another.	6	6			Almost all the RUMEP teachers confirmed that problem solving provides opportunities for learners to learn from one another.
17	There is much more discussion in my classroom.	6	5	1		Nearly all the teachers confirmed that problem solving promotes more classroom discussions.
18	Learners who are usually quiet now speak more freely in the discussions.	3	8	1		Most of the teachers are of the opinion that even the quiet learners are able to take part in discussions if the problem-solving approach is used.

Table 4.3 indicates that the views of the teachers on learner communication differ. Many teachers are of the opinion that there is much discussion in their classrooms when using problem solving. In addition, nearly everyone noted that learners who are usually quiet are also motivated to take part in the class discussions.

(d) Teacher questioning

In this section, an analysis of classroom teacher questioning is presented. Table 4.4 illustrates the teachers' responses with regard to classroom questions if the facilitation of lessons is orientated towards problem solving.

Table 4.4: Some comments on teacher questioning

	Item	SA	A	DA	SD	Researcher comments
20	The questions I ask indicate a direction for learners to answer if I use the problem-solving approach.	2	9	1		Almost all of the teachers indicated that the questions they select indicate a direction for learners to answer them.
21	When using problem solving, leaners have an opportunity to ask me questions.	2	8	2		Most of the teachers agreed that learners have an opportunity to ask them questions.
22	Problem solving enables me to improve on the quality of questions that I ask my learners.	3	9			All the teachers seemed to agree that problem solving improves the quality of the questions they ask.

Table 4.4 indicates that nearly all of the RUMEP teachers are of the opinion that the questions they ask indicate a direction for learners to answer. It appears that the teachers use leading questions rather than probing questions, which lie at the ‘heart’ of problem solving. However, a few of the teachers noted that when using problem solving, learners are unable to ask them questions. All the teachers stated that problem solving enables them to improve on the quality of questions that they ask.

(e) Tasks and activities

The questions in this section were asked in order to determine the teachers’ views on the selection of tasks and activities. Table 4.5 indicates the number of teachers who responded to questions based on the tasks and activities they select for learners.

Table 4.5: Summary of teacher responses regarding tasks and activities

	Item	SA	A	DA	SD	Researcher comments
23	The tasks I select influence how learners come to make sense of mathematics.	2	10			All the teachers are of the opinion that the tasks they select make sense of mathematics. If this is true, it means that the focus of the tasks is on stimulating mathematical thinking and understanding in arriving at the solution.
24	It is difficult to select the relevant activities for the next day.		1	9	2	Almost all of the teachers indicated that it is easy to select tasks for the next day.
25	The tasks I select leave plenty of time for learners to finish them.		6	6		Half of the teachers seemed to agree that learners have sufficient time to complete given tasks. It appears to be true, because learners cannot always finish given tasks with ease.
26	The activities I provide elicit an appropriate mathematical response from the learners.		12			All teachers indicated that the tasks they select elicit appropriate mathematical responses.

All the teachers agreed that the activities they provide elicit an appropriate mathematical response from the learners. The tasks influence how learners make sense of mathematics. Also, most teachers indicated that it is easy to select relevant tasks for the next day when using problem solving. A few of the teachers noted that the tasks they select leave plenty of time for learners to finish them.

(f) Classroom discourse

Four questions in this section were asked to determine the teachers' views on classroom discussions if their teaching is problem-solving-orientated. Table 4.6 depicts the number of teachers who responded to questions regarding classroom discussions when the teaching is orientated towards problem solving.

Table 4.6: Teacher responses on classroom discourse

	Item	SA	A	DA	SD	Researcher comments
27	Problem solving arouses learners' interest in mathematics.	6	6			All the teachers seemed to confirm that problem solving arouses learners' interest in mathematics.
28	There is discussion in my classroom that supports mathematics argumentation.	3	8	1		Most of the teachers agreed that if they incorporate problem solving into their classrooms, learners' discussions are focused on mathematical argumentation.
29	Problem solving allows learners experiences of working independently.	2	7	3		Few of the teachers stated that it does not allow learners to work independently.
30	Problem solving allows learners experiences of working collaboratively to make sense of ideas.	1	11			All the teachers are of the opinion that problem solving allows learners to work collaboratively. Stacy (2005:342) states that successful mathematical problem solving depends upon, among other things, communication skills and the ability to work with others.

Almost all the RUMEP teachers seem to agree that problem solving allows learners experiences of working collaboratively and making sense of mathematical ideas. According to Gavalcova (2008:118), when learners are actively engaged, they are able to gain information for themselves, draw conclusions from this information and transform the information into knowledge. In addition, if learners are able to become involved in what they are doing, they can reach significantly high levels of mathematics (Brousseau, 1997:28).

Therefore, learning in an active way leads to engagement with a topic, thinking it through and observing connections between notions related to the topic. This kind of engagement is not promoted by just listening to or being informed about the topic. To the contrary, very few teachers indicated that problem solving does not allow learners experiences of working independently.

(g) Monday afternoon session

Table 4.7 indicates the number of teachers who responded to questions on the Monday afternoon contact sessions.

Table 4.7: Teacher responses on Monday afternoon sessions

	Item	SA	A	DA	SD	Researcher comments
31	Problem solving changed my attitude towards mathematics as a BEd student.	8	4			All the teachers are of the opinion that problem solving changed their opinions regarding mathematics teaching.
32	At the end of the class session, I am able to complete my assignments.	2	8	2		A few of the teachers indicated that they are not able to complete assignments at the end of the class.
33	The class is too noisy to maintain my interest in learning.		1	9	2	A few of the teachers are of the opinion that the class is too noisy.

Table 4.7 reveals that most teachers agreed that problem solving has changed their attitude towards mathematics as RUMEP students. However, a few of the teachers noted that at the end of the Monday afternoon session, they are unable to complete their assignments. Only very few teachers stated that the class is too noisy to maintain their interest in learning.

(h) Study material

In this section, the teachers responded to questions relating to the study material they are using in the BEd (in-service) programme offered by RUMEP. Table 4.8 illustrates the number of teachers who responded to questions regarding the RUMEP study material.

Table 4.8: Summary of teacher responses on study material

	Item	SA	A	DA	SD	Researcher comments
34	I have enough study material to support me to learn the content.	1	7	3		Most of the teachers noted that they have sufficient study material to help them learn the content.
35	I can follow the language used in the study guide during the contact sessions.	1	11			It seemed that none of the teachers have any problems with the language used in the study material.

36	The study material used in the sessions contains activities to help me learn new concepts.	1	11			All the teachers indicated that the study material helped them to learn new concepts.

All the teachers indicated that the study material used in the sessions contained activities to help them learn new concepts. Also, all the teachers indicated that they can follow the language used in the study material easily during the contact sessions. In contrast, only some of the teachers indicated that they do not have enough material in the RUMEP programme to support them to learn the content.

The following section entails an analysis and discussion of the two open-ended questions. The two open-ended questions were included in the questionnaire completed by the 12 teachers, including the four cases.

4.3.1.2 The open-ended questions

As indicated earlier, two open-ended questions were posted in the questionnaire. In this section, the open-ended questions are analysed and discussed. The two open-ended questions asked were:

(a) Comment on how problem solving has changed your view on mathematics teaching.

(b) Comment on how you found problem solving useful in your own mathematics classroom.

The researcher noticed that some of the responses to the two open-ended questions were overlapping and repeated by the participants. The participants seemed to have much the same responses to the two questions. As a result, the researcher clustered the responses that appeared the same and recorded a generalised response pointing to problem solving. Table 4.9 depicts extracts of the teachers' responses to the two open-ended questions:

Table 4.9: Sample of teacher responses to Question (a)

Teacher responses regarding how problem solving has changed their views on mathematics teaching	
Teacher responses	**Researcher comments**
I know now that learners have to give their prior knowledge. I found it much easier to explain because I worked on the learners' prior knowledge.	This response concurs with the idea that learners' existing knowledge plays an important role in problem solving. According to Schoenfeld (1985:12), the first stage in problem solving is that

	the individual must have an intuition and informal knowledge regarding the problem. Hence, learners use mental tools already available to them in order to learn new mathematical concepts (Schroeder & Lester, 1989:33). The responsibility of the teacher shifts from providing information to asking questions and providing resources (Burton, 1989:20).
Problem solving changed my views [on] mathematics teaching [. . .] it makes me aware that learners can do work independently.	The comment seems to be in line with the statement by Gavalcova (2008:118) that when learners are actively engaged they are able to gain information for themselves, draw conclusions from this information and transform the information into knowledge.
Learners are allowed to [. . .] discuss their ideas and share their opinions.	The comment cited by the teacher seems to support Yackel and Cobb's (cited in Washaw & Anthony, 2009:523) view that classroom teachers who facilitate learner participation and elicit leaner contributions and who invite leaners to listen to one another, to accept different viewpoints and to engage in an exchange of thinking and perspectives exemplify sound pedagogical practices in mathematics. It seems the teachers believe that learners learn best when they can discuss ways to solve the problems, and then consider the advantages and disadvantages of applying different methods and the relationships between them.
I used to work alone. Now they [learners] do a lot more than me. I should not talk too much in class and do everything.	The teachers appear to believe that their role is not to transmit clear information, demonstrate procedures for solving problems and explain the process of solving sample problems. The teachers' key role is to lead learners in their own discovery and understanding of mathematical concepts, as suggested by Driver and Oldham (1986:112). This is in line with constructivist theories that knowledge is actively constructed by learners and not passively 'received' from teachers (Lesh & Doerr, 2003:212).
I must allow learners to do tasks themselves, and my role is to guide them whenever they do not understand.	The literature review in Chapter 2 revealed that the teacher should provide support only when learners are stuck. Artzt et al. (2008:1) state that the teacher is supposed to help and formalise learners' ideas. Learners should be allowed to find strategies for solving the problem with little interference from the teacher. However, also for these teachers it appears that confusion and frustration are taken to be a natural part of the learning process because each person must struggle with a situation or problem first in order to make sense of the problem.
Problem solving [. . .] allows for cooperation between learners, allows learners to express themselves freely in mathematics.	For these teachers, it appears that problem solving enables learners to work effectively as individuals and with others as members of a group without too much interference from the teacher. If the teacher interferes too much, the problem becomes his/hers and the reason for the activity is lost.

As illustrated in Table 4.9, it seems as if the RUMEP teachers' views have shifted in favour of a problem-solving approach. In general, their views appear to be consistent with the opinions of the four selected cases (teachers A–D) as explored during the face-to-face interviews. However, there is a direct contradiction between what the teachers say and what happens in their actual classrooms. In the teachers' classrooms, their facilitation orientation is still mainly teacher-centred.

As noted by Brodie et al. (2001:542), the study also indicated that many learners still do not participate fully in the learning process, as teachers are still providing a great deal of direct instruction. Little is being done to incorporate problem-solving techniques. Hence, it is up to the teacher to determine how to make appropriate adaptations to accommodate the relevant teaching practices. Kilpatrick et al. (2001:371) are of the opinion that the teacher's role is pivotal in the implementation of proficient teaching practice.

Table 4.10 indicates the teachers' responses to Question (b) of the open-ended questions.

Table 4.10: Extracts of teacher responses to Question (b)

(b) Teacher responses regarding how they found problem solving useful in their own mathematics classroom	
Teacher responses	**Researcher comments**
Learners feel free to attempt any question because they are given an opportunity to do things by themselves.	It was evident in the literature review conducted that learners' active engagement with mathematical ideas will lead to the development of specific learner competencies. From the response, it appears that learners are given opportunities to engage in mathematical activities themselves.
Problem solving promotes cooperative learning in my class [. . .] Learners are encouraged to do group work and interact amongst themselves [. . .] Problem solving is useful in my classroom because learners are able to work together.	The responses suggest that learners are able to construct knowledge through interaction with others. The construction process can be accomplished by learners by making connections and developing new mathematics knowledge based on their prior knowledge (Lau et al., 2009:307). Yackel and Cobb (cited in Washaw & Anthony, 2009:523) state that teachers who facilitate learner participation and elicit leaner contributions and who invite leaners to accept different viewpoints and to engage in an exchange of thinking and perspectives exemplify sound pedagogical practices in mathematics.
The teaching of mathematics is made easier by the fact that learners are given an opportunity to say what they know than we always say what the teacher knows	The response seems to be in line with recent mathematics initiatives that call for the development of classroom communities that take communication about mathematics as a central focus (Walshaw & Anthony, 2008:516). The response seems to affirm constructivist ideas that knowledge is actively constructed by learners and not passively 'received' from the teacher (Lesh & Doerr, 2003:212).
Problem solving creates a 'room' for mathematical discussions, learners are able to discuss their findings in class and share their ideas through discussion in class.	The Cockcroft report (Cockcroft, 1982:71) states that successful mathematics learning include mathematical reasoning and communication of mathematical ideas. Indeed, making a difference through classroom communication, teachers shift leaners' cognitive attention towards making sense of their mathematical experiences, rather than limiting their focus to procedural rules (Walshaw & Anthony, 2008:522).
Problem solving in my classroom has stimulated interaction in diverse ideas and strategies	The literature review presented in Chapter 2 confirmed that learners should be allowed to find strategies for solving the problem. These strategies are considered to

amongst my learners, which has resulted in a wider range of solutions.	be the rules of thumb for successful problem solving; suggestions that help an individual to understand a problem better and make progress towards its solution (Schoenfeld, 1985:12).
Problem solving has stimulated the imagination of my learners, encouraging them to explore mathematical ideas they would normally not take into consideration. It improves learners' confidence in mathematics.	The comment seems to be in line with the views of Burton (1989:9) that the value of problem solving is to increase learners' confidence and autonomy. Leaners are expected to have confidence and passion to complete a given problem situation. In essence, when teaching learners mathematics, we do not simply help them to acquire mathematical skills and problem-solving strategies; we also attempt to develop motivation and positive dispositions towards learning mathematics that will have long-term effects on everything – from learners' confidence to do mathematics to their career choices (Brahier, 2011:7).

From the teachers' responses depicted in Table 4.10, it seems that problem solving is useful in mathematics teachers' classrooms. The views hold by the RUMEP teachers, which include the four cases, appear to be consistent with the data generated from the four cases during the face-to-face interviews. As indicated earlier, the teachers' views are the opposite of what happens in their own classrooms, as their classroom teaching methods are still heavily teacher-centred. The teacher still regards him- or herself as the main transmitter of information, while learners are expected to act as listeners.

In the following section, the views of the four selected cases on the use of a problem-solving approach are analysed and discussed.

4.3.2 The four selected teachers' views on a problem-solving approach

This section explores the four selected teachers' views on problem solving in an attempt to answer the research question: How do secondary school mathematics teachers in the John Taolo Gaetsewe district of the Kuruman area experience the facilitation of mathematics through a problem-solving approach while participating in a BEd (in-service) programme? The four cases were positive about using a problem-solving approach in their own classrooms. However, they are still experiencing difficulties in using the approach. The researcher noticed that the teaching orientation of the four selected teachers was still dominated by too much talking and demonstrations from them while the learners were attentively listening.

Five codes were identified in this study through the theoretical framework explored in Chapter 2. The codes were categorised into those that depicted teacher responses during the interview and teacher actions during the facilitation of lessons. This resulted in five items, expressed in the form of action statements. What follows are some extracts of the responses by the four cases.

4.3.2.1 Probing understanding

The teacher asks learners to explain and justify their responses and strategies used to arrive at a solution in order to probe understanding. Artzt et al. (2008:9) indicate that learning with understanding enhances learners' remembering strategies and assists them to relate new ideas of mathematics to what they already know and can do. The following question was asked in this category: What do you think about the claim that "learners should always be encouraged to justify their thinking?"

Teacher A

Teacher A stated as follows:

> *I am in full support of that claim for the reason that learners are always constructing their own meaning of the concepts. Now when they construct their own meaning, some of it is correct while some is incorrect. But then if learners justify or reason their answers it gives you an insight with regards to misconceptions, which then informs your teaching. Justification will assist you see what informs the misconceptions.*

Teacher A's statements seem to be inconsistent with what actually happened in his own classroom. Even though he agrees that learners should be given opportunities to construct their own meaning, Teacher A seems not to adhere to his convictions during lesson facilitations.

Teacher A appears to contradict constructivist ideas that knowledge is actively constructed by learners and not passively 'received' from the teacher (Lesh & Doerr, 2003:212). He will sometimes stop learners and interfere when they are working on a particular problem. He will say *"Listen, listen, before we start with anything . . . when you are looking …"*. By continuously using terms such as 'Listen', 'No', 'Before we start' and 'Look', he seems to deny to learners the opportunities to construct their own meaning. According to Manouchehri (2007:299), such actions would likely close the door not only on learners' mathematical investigations, but also on the formation of a learning community in which members willingly explore mathematics and engage in the collaborative construction of knowledge.

It seems Teacher A is telling and demonstrating to learners what to do before they can experience learning on their own. Constructivist theory states that the teacher is not a transmitter of knowledge but rather a facilitator and provider of experiences from which learners will learn (Malan, 2000:26). In addition, Stigler and Hiebert (1998:3) state that struggling and making mistakes and then seeing why learners are making mistakes is an essential part of the learning process. However, from this section of the lesson, Teacher A seems not to allow learners to learn from their mistakes.

However, the researcher also observed that Teacher A is aware that learners should be given opportunities to justify their answers (problem solving). He frequently asks learners 'why' and 'what' questions in order to probe understanding. Even though sometimes he seems to be dominating the discussion, he will give learners a chance to explore and understand the given problem. Table 4.11 depicts a sample of a conversation between Teacher A and Grade 12 learners in order to probe understanding. The problem given to the group was "Determine which investment option is better, payment start immediately. Option A: R2 000 invested for 10 years at 18% per annum compounded monthly or Option B: R2 000 for 15 years at 15% compounded monthly".

Table 4.11: Sample conversation between Teacher A and Grade 12 learners

Teacher A	Learner
You are saying Option B is the best, but I realise Option B has lower interest as compared to Option A.	Yes, mister.
Why do you take the one with lower interest rate than the one with higher interest?	Eh mister [. . .], Bank A says 18% and Bank B say 15 %. I have realised that it is the high interest rate but at the end I will have less money in my account, because here Bank A is 10 years and Bank B is 15 years.
But, why I you ending with less amount when the interest is high?	Because here Bank A is 10 years and Bank B is 15 years.
So what attracted you about this investment?	The shorter the period the lower amount at the end of investment.

Explain again.	I was saying . . . The period is 120 in Option A and the value of investment is R674 515 while on the other side [it] is high (R1 355 726,19).
What I we saying when it comes to investment? What conclusion can you deduct?	The lesser the interest the longer the period.
No, I don't agree with this.	Mister, the shorter the number of payment the less the money you get and then the more payments you give the more money you get.
Beautiful, you did a very good job.	

This part of the lesson ended by the teacher commenting: *"When it comes to investment, the longer period you invest, whatever small interest can be, you are standing to gain more . . . Thank you".* From this conversation and the teacher's last comment it appears that only the investment period has the greater influence when dealing with investments. However, the final amount received at the end of an investment does not necessarily depend on the period only, but also the interest rate plays an important role. The teacher was supposed to have given the learners sufficient opportunities to explore the problem further by linking the initial amount, the investment period and the interest rate.

Teacher B

Teacher B indicated that problem solving gives learners an opportunity to justify their answers: *"If learners just say something out of the [blue], you don't know if it's guessing or whatever, but if a learner understood something he will be able to justify it, so it's a way of showing understanding".* Teacher B gives the impression that knowledge acquired through drill and practice is likely to be superficial and therefore neither flexible nor useful in a range of situations, as suggested by Wertheimer (cited in Schoenfeld, 1987:3). Furthermore, the verbal responses of Teacher B seem to be in line with the thinking of Schroeder and Lester (1989:33) that the facilitation of mathematics using problem solving entails more than just posing the correct type of problems and then allowing learners to solve them without understanding. From the sentiments echoed by Teacher B, he seems to agree with the argument in the Cockcroft report that successful mathematics learning include mathematical reasoning and the communication of mathematical ideas (1982:71).

However, considering Table 4.12, Teacher B is dominating the lesson and learners are contributing less in the conversation. Instead, the conversation should be the other way round. Learners can be given a chance to explore their answers and solutions. A 'yes' answer from learners does not entirely imply understanding, as Teacher B claims that he stimulates learners' mathematical understanding. Teacher B can ask 'why', 'how' and 'what' questions, as was the case in some of Teacher A's lesson vignettes.

Table 4.12 represents one of the discussions that took place between Teacher B and the learners in order to probe understanding. The topic was sketching the linear graph in the Grade 10 curriculum.

Table 4.12: Sample conversation between Teacher B and Grade 10 learners

Teacher B	Learners
Now when you look at the three graphs ($y = 3x; y = 3x + 1$ and $y = 3x + 2$) what can you say about their slopes, are they the same or are they not the same?	They are the same.
What cause them to be the same? What is it that makes them to have the same slopes?	The signs, they have positive signs.
So with the positive signs they all slope in the same direction?	Yes.

Teacher B ended the conversation by reminding learners that in $y = mx + c$, the variable m represents the slope of the function. Even though the answers of the learners where not in detail and satisfactory, according to Teacher B he was following up on learners' answers to establish understanding of the concept of the slope. However, he was actually not establishing understanding. The learners were only saying *"The signs, they have positive signs"*, but he does not allow them to explore how the signs relate to the gradient or the slope.

Learners give a 'yes' answer without motivating or justifying their responses; instead, he should allow them to expand on their responses without him saying a lot on their behalf.

Teacher C

Teacher C said: *"[. . .] that is very good because learners answer or they see things differently [. . .], so I think they must be given a chance to justify their thinking because sometimes you will find that at the end they were talking the right thing"*. Teacher C seems to agree with the view of Malan (2000:26) that learners reconstruct knowledge and take responsibility for their own learning. It looks as if Teacher C wants learners to think about things in a new way, such as seeing new relationships between mathematical ideas. Her argument seems to be in line with that of Stigler and Hiebert (1998:2), who state that learners are interested in exploring mathematics by developing new methods for solving problems.

However, Teacher D does not seem to give learners a chance to justify their answers, as she claims. She asked learners a question *"What is the gradient in y = 2"*? Table 4.13 depicts some of the discussions that took place between Teacher C and the Grade 10 learners in exploring the value of the gradient in y = 2.

Table 4.13: Sample conversation between Teacher C and Grade 10 learners

Teacher C	**Learners**
May, what is the gradient?	Undefined.
[teacher laughs] Learner A, what is the gradient?	Gradient is y.
Learner B, what is the gradient?	1
Who can help them? [teacher asking the whole class] Do we have the gradient in y = 2?	No no mam.
We only have the …	The y-intercept.
That is where we are going to plot the graph at y = 2.	

From Table 4.13, Teacher C ends the conversation without finding out why learners are saying the gradient is undefined, y and 1. She ends the discussion by introducing the concept of the y-intercept instead of finding out why at least three learners gave the wrong answer. She denies the learners an opportunity to take responsibility for their own learning, as suggested by Malan (2000:26). Instead, she laughs and does not find out why these learners say the gradient is undefined, y and 1. She then asks the whole class, who respond by saying there is no gradient. Actually, in this case, the gradient is 0. It is not mathematically convincing to say there is no gradient. Also at the end of this conversation, Teacher C does not provide learners with the correct answer and establish why they are giving such responses. Her teaching does not probe understanding because she does not allow the learners to provide reasons for their responses. The researcher's opinion is that the learners did not manage to grasp the concept of the gradient in this lesson.

Teacher D

Teacher D said: "*If maybe you encourage learners to justify their thinking, then learners will have deeper understanding of mathematical concepts, because they know at the end they will have to justify their thinking".* Teacher D's comments seems to be in line with Marcus and Fey's (2006:59) claim that much of secondary school mathematics focuses on developing learners' understanding of, and skill in using, symbolic notations in order to reason about and describe equations, inequalities, functions, expressions and variables. He seems to support Driver and Oldham's (1986:112) statement that teachers' key role should be to lead learners in their own discovery and understanding of mathematical concepts. Learners who understand concepts are likely to begin to "see mathematics as a meaningful, interesting and worthwhile activity. Such learners believe that they are capable of learning and are motivated to put in the effort required to learn" (Kilpatrick et al., 2001:171).

However, from the lesson observation data, there is a mismatch between what Teacher D said and what happened in his classroom. Similar to Teacher B and Teacher C, Teacher D did not give learners opportunities to justify and explain their solutions. Table 4.14 indicates some of the Teacher D's lesson vignettes. The focus of the lesson was on analytical geometry in Grade 11. In the lesson Teacher D was discussing the previous day's homework. The learners were given a four-sided figure and had to prove in one of the questions that $PT \perp SR$ using analytical methods.

Table 4.14: Sample conversation between Teacher D and Grade 11 learners

Teacher D	**Learners**
This is what we should prove in 3.1. And now to prove that	You first find the gradient

PT ⊥ SR, what should be the approach? How do we approach this question?	of PT and the gradient of SR.
And then now after finding those gradients what should we do?	You multiply the gradients and see that they give you -1.
Ok, now let us calculate. [Teacher D calculates the gradients of both lines on the chalkboard while learners are seated and watching him demonstrating solutions]. Do you agree?	Yes.
But we should reduce $\frac{-9}{12}$.	Yes.
What is the common factor?	3
… and then now we have the two gradients … so we should say $M_{PT} \times M_{SR} = -1$ nè?	Yes.
… and then now since the products of this gradients is -1, then we can conclude by saying that PT ⊥ SR?	Yes.
You have got it?	Yes mister.

From Table 4.14, Teacher D concluded the conversation by stating that the two lines PT and SR are perpendicular because the product of their gradients equals –1. As this was a homework assignment, the learners were supposed to be given a chance to explain how they arrived at their solutions. The learners were not involved and this indicates that there is little understanding of the problem. Kahan and Wynberg (2006:15) state that teaching mathematics using problem solving requires the teacher to pose the problem to the class and to make sure that the learners have sufficient understanding of the problem, but without telling them how to solve the problem. Thereafter, the learners explore the problem, try to make sense of it and eventually generate one or more solutions. This is contrary to what Teacher D has done.

4.3.2.2 Sense making

The teacher makes explicit reference to mathematical conventions, symbolisms, definitions, axioms and theorems. The researcher used the classroom support visits and observation data to check whether the four selected teachers made reference to mathematical conventions, symbolisms, definitions, axioms and theorems during the facilitation of their lessons.

Teacher A

Teacher A showed an intention to establish ‘sense making’ of concepts in his presentation. The topic was “Comparing investments and loan options” in the Grade 12 syllabus. One of the groups, Group C, was asked to choose which loan option is the best given the following scenario: “R600 000 loan for 30 years at 15% per annum compounded monthly or R600 000 for 30 years at 18% per annum compounded monthly or R600 000 for 20 years at 18% compounded monthly”. The group was asked to present their solution on the chalkboard to the whole class. The group used financial formulas and definitions in order to arrive at an answer. Table 4.15 illustrates Group C’s solutions to the problem stated above.

Table 4.15: Summary of Group C’s solution

Option A	**Option B**	**Option C**
R600 000 for 20 years at 18% per year compounded monthly	R600 000 for 30 years at 18% per annum compounded monthly	R600 000 loan for 30 years at 15% per annum compounded monthly
R9 259,87 per month	R9 042,51 per month	R7 586,66 per month
Final payment = R2 222 368,80	Final payment = R3 255 303,60	Final payment = R2 731 197,60

The group chose Option A as the best monthly repayment loan option. For Option A, the number of payments to settle the loan is 240 months, as compared to options B and C, where the payment period is 360 months. Therefore, according to the group, the payment period in Option A is shorter than the monthly payment period in options B and C. The group also indicated that even though the monthly payment in Option A is high, the final payment at the end of the loan is less than that of options B and C. Table 4.16 illustrates some of the lesson vignettes between Teacher A and the learners.

Table 4.16: Sample conversation between Teacher A and Grade 12 learners

Teacher A	Learners
Do you understand what she did? Why did she divide by 12?	Because the interest rate is compounded monthly.
You say you take Option A?	Yes.
So you want to take the option whereby you are paying more money every month. Why?	Yes, yes sir. Because if you take the monthly payment in Option A the final total amount paid is less. I am saying you pay more money for 20 years as compared to 30 years . . .
So, from your explanations you say you multiply by . . .	Multiply by 240 payments, the money is more as compared to multiplying by 360 where the final amount is less.
Do you agree with them?	No, I want to choose Option C because I want to pay less every month.
. . . but what will happen to the final amount in Option C?	It is more. But Option A is going to suffer because he is going to pay more money every month even though at the end he will be happy. Option C will be happy but at the end he is going to suffer because is coming to pay more money at the end.
But sometimes we advise you to pay more money every month.	No. I think mister it depends on the person. You have to check your pocket.

From this conversation, the learners seem to be dominating the conversation and not the teacher. Teacher A appears to be facilitating the discussions. He is asking questions and comments from the learners in order to explore whether learners make sense of the concept of loans. These explorations of learners' answers led to worthwhile discussions and increased awareness for both teacher and learners of misinterpretations and misunderstandings of concepts, as recommended by Cockcroft (1982:72). Teacher A is, therefore, using questions and comments from learners to check whether the topic makes sense to them. In addition, the learners are able to relate the topic to everyday life situations.

Teacher B

As indicated before, Teacher B's lesson was based on sketching linear graphs in Grade 10, using the intercept method. From the classroom observation visit, the researcher observed that Teacher B could not involve learners in the discussion. Unlike Teacher A, it seems Teacher B considers himself as the only person in the classroom and the learners do not exist. Teacher B is unaware that he said *"[...] with problem solving these learners are the ones who are doing a lot in class"*. Instead, there is little evidence that Teacher B tries to establish from the learners whether particular concepts make sense to them. It appears that Teacher B finds it difficult to apply some of the problem-solving techniques, such us sense making. In essence, Panaoura (2012: 2291) states that mathematics teachers often experience some challenges in making use of proper mathematical problem-solving tools. Table 4.17 illustrates some of the lesson vignettes from Teacher B's lesson.

Table 4.17: Sample conversation between Teacher B and Grade 10 learners

Teacher B	**Learners**
Now today I want us to look at how to draw the graph using the intercepts. If we find the two intercepts that is how we are going to draw the line.	Yes sir.
So it means that if we find two points and we join those two points, they will give us a line.	[no comments from learners]
I am going to use the same one that we had: $y = 3x + 3$.	[no input from learners]
When we use the intercept method what we are going to do is like we just solve the equation.	[no remarks from learners]
Do you remember when we were doing $9 + \Delta = 15$? What is the value of Δ?	6
How did you get 6?	9 + 6 = 15, so 16 – 6 gives us Δ.
Right, because in here …… [The teacher continued dominating the discussion by finding the x-intercepts and y-intercepts and ended the lesson by presenting the graph on the Cartesian plane].	[learners sitting quietly while the teacher is talking and demonstrating]

From Table 4.17 it seems the facilitation approach employed by Teacher B is narrative and teacher-centred as opposed to learner-centred. He provides much direct instruction and the learners do not participate fully in the learning process. Teacher B only speaks to himself and does not involve the learners in the discussion. According to Franke and Kazemi (2001:104), teachers should listen to their learners' mathematical explanations, create strategies that evoke mathematical thinking, ask questions that elicit learners' explanations and know what to do with what they heard in order to make instructional decisions.

Teacher B seems not to be aware that problem solving is based on classroom discussions between the teacher and the learners. It seems that Teacher B is of the opinion that his role is to transmit clear information and demonstrate and explain procedures for solving problems. He seems to believe that learners are expected to listen well to him and to remember everything he told them. However, in the case of mathematics instruction, teachers' key role is to lead learners in their own discovery and understanding of mathematical concepts (Driver & Oldham, 1986:112). Therefore, learners' interaction and learning experiences would depend on guidance from the teacher.

Teacher C

In the case of Teacher C, the topic was determining the effect of a and q in $y = ax + q$, as outlined in the Grade 10 syllabus. The researcher wanted to also check whether Teacher C makes reference to mathematical conventions, symbolisms, definitions, axioms and theorems during the lesson facilitation. In the lesson, Teacher C divided the learners into four groups. Groups 1 and 2 investigated the effects of a in the functions $y = x$, $y = 2x$ and $y = \frac{1}{2}x$, while groups 3 and 4 investigated the effects of q in $y = x$, $y = x + 2$ and $y = x - 2$. Teacher C's facilitation techniques seem to be similar to that of Teacher B. There appears to be no evidence of sense making of concepts. Her facilitation techniques seem not to be in line with that of Teacher A. Teacher C seems not to be able to engage learners in mathematical conversations. However, he uses the correct mathematical symbols and axioms. Table 4.18 depicts some of the conversations between Teacher C and the Grade 10 learners.

Table 4.18: Sample conversation between Teacher C and Grade 10 learners

Teacher C	**Learners**
. . . OK fine. I think that we are done now. We are going to start with the tables that are doing $y = x$, $y = 2x$ and $y = \frac{1}{2}x$.	
What is the value of *a* for the first graph: $y = x$?	1
The second graph?	2
The third graph?	$\frac{1}{2}$
Now let us look at our graphs. Now look at how *a* affects the graph. Comment on *a* looking at the three graphs.	
What happens to the graphs when *a* = 1, *a* = 2 and *a* = $\frac{1}{2}$?	The graph is steep. [learners use hands to show the slope of the graph]
Which one is more steep $y = x$, it means that when *a* increases also the steepness of the graph . . .	Increases [learners complete the statement in a chorus]

The lesson continued in a similar way, whereby the teacher will elaborate and the learners provide one-word responses. Teacher C then explored the effect of the variable *q* using the same approach for three functions: $y = x$, $y = x + 2$, $y = x - 2$. From Table 4.18, it is clear that Teacher C is not applying any effort to ensure that her presentation makes sense to the learners and that learners make sense about their responses. She does not engage in a mathematical discourse. There are no ‘how’, ‘why’, ‘what’ questions, which are key to problem solving in order to establish whether the concepts under consideration make sense to learners. For the problems involving the effect of *q* the coefficient of *x* is always 1. However, using problem solving, learners can be exposed to different forms of problems to develop understanding of concepts.

Teacher D

Similar to teachers B and C, Teacher D seems not to establish whether concepts make sense to learners. Teacher D's teaching approach seems to still be teacher-centred as opposed to learner-centred. He dominates the discussions and learners make little contributions during his lessons. Table 4.19 illustrates some of the conversations between Teacher D and the Grade 11 learners. The lesson focus was proving that quadrilateral PQRS is a trapezium using analytical methods.

Table 4.19: Sample conversation between Teacher D and the Grade 11 learners

Teacher D	**Learners**
And now before we prove that, we need to know the properties of a trapezium.	Yes.
And then what we know is that a trapezium is a four-sided figure whereby one pair of opposite side is . . .	Parallel.
Are we together?	Yes sir.
… the approach here can be to find the gradients of all this four sides, nè?	Yes.
Who can find the gradient of PS?	[one of the learners writes down the solution quietly on the chalkboard]
… $\frac{1}{5}$ is that correct?	Yes.
[other three learners also go to the chalkboard and write down their solutions]	
And then now what are we saying because we have the gradients of the four lines?	Therefore, PQRS is a trapezium.
Is it a trapezium?	Yes.
Why are we saying PQRS is a trapezium?	Because two sides are parallel.
Why are we saying they are parallel?	Because they have the same gradients.
Therefore, now that only one pair of opposite sides is	Yes sir.

parallel, then we are saying this figure is a trapezium, correct?	
[end of conversation]	

From Table 4.19, it appears that Teacher D could not establish from the learners whether the concept ‘trapezium’ makes sense to them. Instead, Teacher D told the learners the properties of a trapezium. This seems not to be in line with perspectives in mathematics education that promote the development of an understanding of mathematical concepts, procedures, connections and applications through problem solving. He does not allow the learners to make their own contributions and share ideas among themselves. Current approaches to mathematics education emphasise the development of mathematical understanding through learners solving problems and sharing solutions and strategies (NCTM, 2000:46). Teacher D draws conclusions on his own. This means that mathematical procedures are being imposed on learners in ways that do not necessarily develop mathematical thinking or understanding, as suggested by Grouws (2006:129).

4.3.2.3 Drive learning

The teacher uses the learners’ ideas as starting points for discussion. The teacher responds to learners’ contributions in order to make relevant instructional decisions. In particular, learner responses drive the learning process. The establishment of learners’ prior knowledge is one of the criteria that can be used to guide the facilitation of lessons. Schoenfeld (1985:12) refers to the concept of ‘resources’ available to an individual, which consists of a set of relevant facts available to the problem solver, algorithmic procedures known by the learner, and routine procedures and procedural knowledge about the agreed-upon rules for working in the domain. In this case, one of the questions asked during the interview was: “What is your opinion regarding the statement “a major goal of mathematics instruction is to help learners develop the belief that they have the power to control their own success in mathematics”?

Teacher A

Teacher A responded: “*Obviously, my opinion would be that it is correct, but as teachers we need to create a situation whereby learners take charge of their own learning [. . .] we need to allow learners to take charge of their own learning”.* Learners’ responses should ‘drive’ the learning process. From Teacher A’s responses it appears that he is aware that any instructional decision should be based on learners' responses and abilities. He gives the impression that he is the one responsible for creating such opportunities. Teacher A's approach seems to be in line with the claim that teachers should create opportunities and strategies that evoke learners' mathematical thinking and explanations and know what to do with what they heard in order to make instructional decisions (Franke & Kazemi, 2001:104).

However, during the classroom observation carried out, Teacher A could not follow up on some of the comments the learners made during his lesson. From Table 4.16 regarding the discussion of better loan repayments options, the conversation ended without the teacher responding to one of the learner's comments that monthly repayments will depend on affordability. Instead, Teacher A maintained his opinion that he will recommend that people pay higher monthly payments so that they can pay off the loan quickly and save money on interest. However, Teacher A could in fact have followed up on the learner's response and found out why the learner said he would prefer to pay low monthly repayments, paying a higher final amount. Teacher A's actions appear to contradict the views of Stigler and Hiebert (1998: 2) that mathematics teachers should elicit learners' mathematical thinking and anticipate multiple strategies for solving problems, as mathematics is not just a set of procedures or algorithms to be followed.

Teacher B

Teacher B said: *"I must first see what they know . . . Initially, I believed that for the success of the learners it depends upon the teacher [. . .] so we were not giving them enough chance to explore on their own*". These comments from Teacher B seem to confirm that he needs to explore what learners know before he can provide his own ideas. From Teacher B's comments one can conclude that he believes learners 'drive' the facilitation process. However, from Table 4.17 it appears that the learners' responses do not guide teaching and learning. Teacher B accepts the learners' one-word responses without allowing them to explore their answers. Hence, one needs to note that the traditional way of accepting answers only is inadequate. Furthermore, he continues with the presentation without requesting the learners' responses or comments. The facilitation process in Teacher B's case looks similar to that of Teacher A. Both Teacher A and Teacher B are occasionally tempted to proceed with the lesson, ignoring learners' contributions.

Teacher B does not elicit learners' mathematical thinking and explanations, as suggested by Stigler and Hiebert (1998:2). In one of the conversations Teacher B had with the Grade 10 learners he asked "*In the equation* $y = mx + c$ *which variable represents the gradient?"* One of the learners' response was mx. But Teacher B failed to provide an explanation or ask the learner why he thinks mx represent the gradient and not the variable m. He only said m represents the slope of the function $y = mx + c$. He continued his lesson by saying *"Now today I want us to look at how to draw the graph using the intercepts"*. This is an indication that the learners' responses do not drive the flow of the lesson.

Teacher C

Teacher C said: *"Using problem solving, learners have to come up with what they know before we can tell them what to do"*. Teacher C's comments seems to be in line with Schoenfeld's (1985: 12) view that any mathematical problem-solving activity is built on a foundation of basic mathematical knowledge, which is called 'resources'. However, as is the case with teachers A and B, Teacher C seems to agree that she would build her facilitation of lessons on learner responses in order to enhance their problem-solving skills. However, Teacher C's lesson did not proceed in accordance with her claims.

From Table 4.13, three learners gave a wrong answer when asked the value of the gradient in the function y = 2. Despite these wrong answers given by the learners, Teacher C ignored the learners' misconceptions about the value of the gradient when a function is given. Instead, she proceeded with the lesson by introducing the concept of the y-intercept. She does not seem to be able to switch between her presentation in order to explore the concept of the gradient, but proceeded with the objectives of her lesson, which was to draw the graphs and then identify the effect of *a* using the graph. Yet, according to Stigler and Hiebert (1998:3), struggling and making mistakes and then seeing why learners are making mistakes are essential parts of the learning process.

Teacher D

Teacher D noted: *"Learners should be guided in their own learning to strengthen their own mathematical learning. When learners do problems themselves they will not forget easily and will have deeper understanding of concepts so that they can apply the new knowledge themselves. If I stand in front and then facilitate they might forget mathematical concepts* *easily".* From Table 4.14, Teacher D did not present his lesson in accordance with this view. He stood in front and demonstrated to the learners how to calculate the gradients of the two lines, PT and RS.

In his presentation, the learners only gave the 'Yes' answers, without exploration. He proceeded to the next stage of his presentation and never explored the learners' understanding, as he claimed to do. In the case of Teacher D, as in the case with teachers B and C, it appears that the learners' answers do not drive learning. Artzt et al. (2008: 9) mention that learning with understanding enhances learners' remembering strategies and assists learners to relate new ideas of mathematics to what they already know and can do, to use their previous knowledge and skills to construct new meaning and to apply their learning to new contexts. According to Lau et al. (2009:309), the assumption is that the acquisition of skills by a learner is an activity in which the readily relevant skills are combined to meet new, more complex task requirements.

4.3.2.4 Exploratory discussions

The teacher encourages argumentation between learners, including argumentation not mediated by the teacher. The NCTM (cited in Walshaw & Anthony, 2008:517) states that some of the things that teachers might do to enhance effective classroom discussions involves observing and listening attentively to learners' explanations and ideas. Teachers have to provide opportunities for learners to listen to one another. In essence, in order to make a difference through classroom discussions, teachers should shift learners' cognitive attention towards making sense of their mathematical experiences, rather than limiting their focus to procedural rules (Walshaw & Anthony, 2008:522).

One of the questions asked in this category was: "What do you like most or enjoy when facilitating mathematics lessons using a problem-solving approach – discussions in class?"

The following are the responses from the four selected teachers with regard to the question on exploratory discussions:

Teacher A

> *The classroom is enjoyable, you look forward to going to class because some of these learners, their discussions, they will bring up things you would never imagined and learners are giving you alternative methods of working out things that I have never talked or think about before. I realise that my learners are very more active than when I use any other teaching methods, actually I get learners to sleep when I am talking a lot, than when I use problem solving.*

This comment from Teacher A seems to reinforce that classroom discussion enhances learners' problem-solving skills. However, from Table 4.11, there is little evidence of discussions between Teacher A and the learners. The discussion is on why Option B is a better investment option as compared to Option A. The learners tried to convince the teacher why Option B appears to be better than Option A. However, the conversation ended without Teacher A sharing with the learners the significance of the period of investment and the interest rate. Also, when the discussion was about better repayment loan options, Teacher A stopped the discussion without agreeing that monthly loan repayments depends on individual affordability. The restrictions by Teacher A against learners to explore and discuss concepts are not in line with the problem-solving approach. According to Driver and Oldham (1986:112), teachers' key role is to lead learners in their own discovery and understanding of mathematical concepts.

Teacher B

Teacher B said: *"mostly is when the learners discuss among themselves [. . .] because I am able to identify the misconceptions".* Teacher B further commented: *"I have realised that problem solving has changed me because initially I used narrative methods where I was doing a lot of talking [. . .] and did not reach my learners".* These comments from Teacher B seem to suggest that more teacher talk in the classroom does not necessarily enhance learner explorations, as proposed by Walshaw and Anthony (2008:522). Obviously, Teacher B seems to agree that a teacher is not a transmitter of knowledge but rather a facilitator and provider of experiences from which learners will learn, as proposed by Malan (2000:26).

However, from Table 4.17, Teacher B seems not to allow the learners to engage in mathematical discussions. He continued his presentation without engaging the learners in how to sketch linear graphs using the intercept method. Alternatively, Teacher B could have asked the learners probing questions to stimulate their participation through discussions. In essence, Teacher B's facilitation strategies appear to be in line with Brodie et al.'s (2001: 542) assertion that in South Africa, many learners still do not participate fully in the learning process, as some teachers are still providing a great deal of direct instruction and are still preoccupied with content coverage. Teacher B endorses learner-centred approaches, but he does not enable learners' commitment with key concepts in the curriculum.

Teacher C:

Teacher C echoed: *"What I enjoy most in problem solving [is] their participation, also especially when they are in groups they are working like they are competing".* This comment from Teacher C seems to acknowledge that learner discussions are key to a problem-solving approach. She seems to believe that learners' active engagement with mathematical ideas will lead to the development of specific learner competencies. She seems to agree that learners should be allowed to engage in discussions in order to experience learning on their own.

However, from the observation notes, it emerged that Teacher C's learners only provided one-word answers without being given an opportunity to explore such answers, as illustrated in Table 4.18. She does not seem to agree with Walshaw and Anthony (2008:516), who suggest that classroom community take communication about mathematics as a central focus. In addition, the Cockcroft report (Cockcroft, 1982:71) states that successful mathematics learning includes mathematical reasoning and communication of mathematical ideas by learners. This means that honouring learners' contributions is one of the pedagogical strategies to stimulate mathematical classroom discourse.

Teacher D

Teacher D said: *"To start off, the thing that I like most in my class is when there is a discussion between myself and the learners and among the learners themselves. If the learners are just quiet and I am facilitating I don't like that kind of a class".* These comments generally indicate that Teacher D is in favour of exploratory discussions by learners. However, there appears to be a mismatch between what Teacher D advocates and what happens in his own classroom, as illustrated in Table 4.19. He demonstrates how to solve problems and the learners appear to be listening. However, the NCTM (cited in Walshaw & Anthony, 2008:517) states that some of the things that teachers might do to enhance effective classroom discussions involves observing and listening attentively to learners' explanations and ideas.

From Table 4.19, there appears to be little discussions between Teacher D and the learners and among the learners themselves. He states the properties of a trapezium without involving the learners. He continues to prove that two lines are parallel if the gradients of the two lines are equal. It seems he is showing the learners what he knows, instead of the learners showing him what they know. The Cockcroft report (1982:71) states that successful mathematics learning includes mathematical reasoning and communication of mathematical ideas by learners. The belief is that learners' active engagement with mathematical ideas will lead to the development of specific learner competencies.

4.3.2.5 Learner interaction

The teacher encourages and permits other learners to comment on the contributions of previous learner speakers. As a result, learner interaction offers a dialogue between learners and the curriculum, where learners interact with sources of knowledge, reconstruct knowledge and take responsibility for their own learning (Malan, 2000:26). However, learners' interaction and learning experiences would depend on guidance from teachers, and teachers' key role would be to lead learners in their own discovery and understanding of mathematical concepts (Driver & Oldham, 1986:112). The following are the four cases' responses with regard to learner interaction:

Teacher A

From the classroom observation notes it emerged that there is little interaction among Teacher A's learners. In Table 4.16, the conversation is only between the teacher and the learners and not among the learners themselves. Even though group work is encouraged, no efforts are made by Teacher A to ensure that the groups are effective. At end of the activity, one learner from each group is chosen as the presenter. But the presenter only talks to the solution; he or she is to the chalkboard and not to either the group members or the whole class. Teacher A's facilitation approach seems to be in contrast with the views of Yackel and Cobb (cited in Washaw & Anthony, 2009:523), who found that classroom teachers who facilitate learner participation, elicit leaner contributions and invite learners to listen to one another, to accept different viewpoints and to engage in an exchange of thinking and perspectives exemplify sound pedagogical practices in mathematics.

Teacher B

From Table 4.17, one can deduce that there is little interaction among Teacher B's learners. Also, from the classroom observation notes it emerged that Teacher B does not promote discussions with learners. The importance of discussion needs to be emphasised. In the case of Teacher B, the learners are not given opportunities to discuss the concept of graph sketching among them. Like Teacher A, Teacher B seems not to encourage conversations among learners. He seems to dominate the discussions.

Anthony and Walshaw (2009:148) state that recent mathematics initiatives focus on developing communities of practice in which learners are actively engaged with mathematics. When learners are actively engaged they are able to gain information for themselves, draw conclusions from this information and transform the information into knowledge (Gavalcova, 2008:118). Therefore, learning in an active way leads to engagement with a topic, thinking it through and observing connections between notions related to the topic.

Teacher C

From Table 4.18, it became clear that in Teacher C's lesson there is little interaction among the learners. The learners give one-word answers without explanations. The teacher then proceeds to the next stage of the presentation without allowing the learners to consolidate what they have said. From the classroom observation notes, it appears that Teacher C dominates the classroom conversation. The classroom actions of Teacher C seem to be similar to that of teachers A and B. In essence, Clements and Battista (1990:37) suggest that learners must be encouraged to exchange points of view and to agree or disagree with one another. In this way, learners are likely to eventually agree on the truth if they debate the solution process (Clements & Battista, 1990:37).

Teacher D

Considering Tables 4.14 and 4.19, Teacher D seems not to encourage learners to engage in dialogue, both with him and with one another. Artzt et al. (2008:15) mention that the teacher can encourage learners to reflect on what they or their classmates have asked or proposed in order to build on and extend their own understanding and to solicit contributions from everyone. In this case, the learners' answers are not being explored further by either the teacher or the learners themselves.

As illustrated in Table 4.19, one of the learners found the answer as '1 out of 5'. The teacher asked whether the learner is correct. The class responded by saying "Yes" and Teacher D could not allow them to reflect on and engage with the answer to see whether the learners understood. In essence, to engage in reflective actions, teachers should shift learners' cognitive attention towards making sense of their mathematical experiences, rather than limiting their focus to procedural rules (Walshaw & Anthony, 2008:522).

The next section entails an analysis and discussion of the classroom opportunities and challenges of the four selected cases who participated in the study.

4.3.3 The four teachers' classroom opportunities and challenges

The teachers indicated that problem solving presents several opportunities and challenges for them in the teaching and learning of mathematics. Buschman (2004:305) acknowledges that teaching through problem solving poses several challenges for teachers. However, the literature review presented in Chapter 2 verified that it is up to the teacher to determine how to make appropriate adaptations to accommodate the curriculum demands in teaching within the social organisation that poses several challenges.

4.3.3.1 Challenges to problem solving

In this section, the challenges experienced by the four selected teachers regarding incorporating problem solving into their teaching practices are analysed and discussed. What follows are extracts of such responses as generated by the researcher:

Teacher A

Teacher A commented: *"The biggest problem of all of it is time constraint. I struggle to finish my work these days because of problem solving. Normally problem solving takes a lot of time, sometimes in one lesson we can just complete one exercise [. . .] you end up not finishing what you have planned for".* It seems Teacher A's challenge with regard to problem solving is time, because he claims he cannot complete the prescribed curriculum in the stipulated time. However, Stacy (2005:342) states that successful mathematical problem solving depends upon deep mathematical knowledge and personal attributes such as persistence, organisation and confidence. This can imply that the preparation and organisation of lessons should always take the time factor into account.

Teacher B

The same sentiments stated by Teacher A emerged from Teacher B's comments, namely that of time management. However, Teacher B seems to agree that the time factor can be addressed by being able to manage one's presentation techniques. Teacher B said, *"Sometimes if you don't control the problem-solving approach properly, it's time consuming".* Like Teacher A, Teacher B seems to consider time as the main obstacle to problem solving. However, teachers and learners should be able to notice that the knowledge gained and the procedure used in solving a specific problem can be applied to some other problems, thereby addressing the time factor. The teacher can emphasise to learners the view that no problem is fully completed and that we can always improve our understanding of the solution (Buschman, 2004:304).

Teacher C

Teacher C stated that *"I don't see any challenges [. . .] only the types of learners that we are teaching [. . .], sometimes they expect me to give them answers instead of finding solutions for themselves."* It appears that for Teacher C, the challenge is the learners. Opportunities can be created for learners; teachers need to realise that the teacher's role is to facilitate learning and not to give them answers. According to Walshaw and Anthony (2008:522), more teacher talk in classrooms does not necessarily enhance learner explorations, but rather rote learning. A teacher is not a transmitter of knowledge, but a facilitator and provider of experiences from which learners will learn (Malan, 2000:26).

Teacher D

As is the case with teachers A and B, Teacher D seems to regard time as an obstacle to problem-oriented teaching. Teacher D said, *"Problem solving is a good method but it needs more time, a period is 40 minutes and then when you look at 40 minutes you can't do a lot".* According to the literature reviewed in Chapter 2, teachers must acquire knowledge that is deeper and more flexible than that required to follow a static lesson plan or directions in a teacher's manual or textbook.

The comments cited by the four teachers seem to indicate that time was the main challenge when trying to implement problem-solving techniques in their classrooms. Yet, in problem solving, teachers and learners need to be patient in order to succeed (Burton, 1989:21). They should realise that the time for problem solving is open-ended, in the sense that the problem can be continued in the next session without feeling that a solution should be reached at the end of each session (Burton, 1989:21). According to the literature reviewed in Chapter 2, it is evident that constructing connections between methods and problems is requiring time to explore and invent, to make mistakes, to reflect and to receive the required information at the appropriate time.

4.3.3.2 Problem-solving opportunities

In this section the problem-solving opportunities as explained by the four selected teachers are analysed and discussed. Despite the challenges mentioned above, it appears that the four teachers demonstrated a positive orientation towards problem solving. What follows are some of the comments by the four cases:

Teacher A

Teacher A stated: *"To say the truth, there are lots of benefits [. . .] from using problem solving: problem solving takes me through the mind of the kid, it shows you what this kid thinks and shows you the misconceptions that learners have about the specific concept".* He further noted: *"Problem solving takes the classroom from teacher-centred to learner-centred".* Teacher A seems to support the statement that more talk in classrooms does not necessarily enhance learner understanding (Walshaw & Anthony, 2008:522). Teacher A seems to be in line with constructivist ideas that knowledge is actively constructed by learners and not passively 'received' from the teacher (Lesh & Doerr, 2003:212). In addition, Teacher A appears to agree with Stigler and Hiebert (1998:3) that struggling and making mistakes and then seeing why learners make mistakes are essential parts of the learning process.

Teacher B

Teacher B noted that *"I see there is a good opportunity because it is easy for the teacher to identify if learners don't understand the information you are trying to give to them because with problem solving [. . .] you see whether they understood or not"*. Teacher B seems to be of the opinion that problem solving promotes understanding of mathematical concepts. It was evident in the literature conducted that mathematical problems develop, among other skills, mathematical thinking and promote mathematical understanding. Furthermore, according to Artzt et al. (2008:9), learning with understanding enhances learners' remembering strategies and assists them to relate new ideas of mathematics to what they already know and can do, to use their previous knowledge and skills to construct new meaning and to apply their learning to new contexts. In essence, understanding is necessary, as it promote learners' remembering of concepts (Hiebert & Carpenter, 1992:74).

Teacher C

Teacher C said: *"Since I started practising this approach, now I can see that even though learners do not make their best but there is an understanding when they are being taught through problem solving"*. From Teacher C's comments it seems problem solving enhances learners' understanding of mathematical concepts. Therefore, according to Teacher C, problem solving serves as a 'vehicle' for successfully learning new mathematical ideas and skills, as promoted by the NCTM (2000:182).

Teacher D

Teacher D shared his view on problem-solving advantages by saying that *"when using problem solving, learners can explore the problem further or maybe sometimes apply other methods which are not in the textbook or you did not show them which can help them to arrive at the correct answer"*. Teacher D seems to be of the opinion that problem solving enables learners to consider the advantages and disadvantages of applying different methods and the relationships between them when solving mathematical problems. Therefore, learners are able to interact with sources of knowledge, reconstruct knowledge and take responsibility for their own learning outcomes, as suggested by Malan (2000:26). He seems to be in line with Driver and Oldham (1986:112) that teachers' key role is to lead learners in their own discovery and understanding of mathematical concepts.

From these personal statements echoed by the four cases one can deduce that they are positive about incorporating problem solving into their classroom teaching practices. They appreciate the fact that through problem solving they are able to find out what learners already know before they can tell learners what to do. According to them, problem solving provides learners with opportunities to insight into and clear interpretations of mathematical concepts. For the four cases, problem solving offers opportunities for learners to develop interpersonal and communicative skills. In general, problem solving provides a learning culture that promotes critical thinking and deeper understanding.

However, from the classroom observation data discussed in Section 4.3.1, the facilitation approach of the four teachers was still narrative and lacked problem-solving attributes. Their teaching approach was predominantly based on a behaviourist teacher-centred approach as compared to constructivist learner-centred in terms of which the teacher becomes a facilitator of knowledge. They tend to show learners mathematical procedures and then ask them to use and practise those procedures. As a result, the learners are expected to listen well to the teacher, remember everything the teacher told them and show that learning has occurred by applying the demonstrated procedures. According to Stigler and Hiebert (1998:3), such teachers believe that mathematics is learned best by mastering the material incrementally piece by piece. Wertheimer (cited in Schoenfeld, 1987:3) argues that although such instruction does result in learners' 'mastering' certain procedures, knowledge acquired through drill and practice is likely to be superficial and therefore neither flexible nor useful in a range of situations.

4.4 RUMEP STUDY MATERIAL

One question relating to the RUMEP study material was asked during the interview with the four teachers constituting the selected cases. The question was "How does the RUMEP study material enhance your problem-solving skills? Please elaborate." From the interview data, all four teachers stated that they found the RUMEP study material useful in enhancing their problem-solving skills. The study revealed that the four teachers were happy with the content of the RUMEP study material. Teacher B commented *"I found them helpful; I use a lot of these hand-outs even in my class"*. Teacher A added *"obviously, the RUMEP materials has done a lot to improve the work I do in my class, for example, there is a very good trigonometric material, I had to make copies and give my learners [. . .] they give a very good introduction"*.

Teacher D expressed his views as follows: *"RUMEP materials are very simplified, when you follow their methods in class, your learners will understand easily like even when you prepare, most of the time I use them when I prepare my lessons".* Teacher C noted that *"in most cases I use RUMEP study guides because they are really helping me using this approach, they simplify things in the topics on how to teach the lesson or how to approach the lesson through problem solving".* These comments cited by the four cases reflect that the content of the study material seems to be useful to them because they can use the material in their own classrooms.

These comments seem to be consistent with the findings of Morgan and O'Reilly (1999: 120) that distance education institutions should aim at writing and producing study material aimed at driving and shaping students' learning experiences in specific modules. The four cases seem to agree that the RUMEP study material shaped their teaching through problem solving and assisted them to prepare and facilitate mathematics lessons. According to them they are able to use some of the activities in the material in their own classrooms. The teachers do not have to prepare much, for example, they can incorporate what is in the study material with the content covered in the prescribed textbook.

However, as a researcher and lecturer at RUMEP who also uses the material, the researcher found the material not sufficient to enhance problem-solving facilitation techniques. In the researcher's view the material still presents examples and then states a list of activities for learners to complete or introduces a given topic by a set of examples. According to Le Roux and Le Roux (2004:13), mathematics study material should include activities that respond to problem-solving approaches rather than rote learning. Study material must not only convey information to the learner; it also has to structure, control and manage the process by which this information is presented to and assimilated by the learner (Ellington & Race, 1994:43).

According to Collopy (2003:288), mathematics study material is considered a potential vehicle for teachers learning about mathematics pedagogy, content and learners' thinking. However, from the researcher's evaluation and tutoring, he does not view the RUMEP study material as a potential 'vehicle' for preparing in-service mathematics teachers to teach mathematics using the problem-solving approach. Even though the material is developed by the RUMEP lecturers themselves, it appears that the lectures too neglect the problem-solving criteria when compiling the study material. However, it seems the RUMEP guidelines on study material seem to be in line with Murphy's statement (cited in Louw & Sonnekus, 2005:15) that authors normally prescribe the content, as it is the authors who have content expertise in a discipline. Here too the RUMEP lecturers prescribe the content, as they are deemed experts in the discipline, but still lack problem-solving attributes. The design of study material should be based on a learner-centred approach rather than a teacher-centred approach in order to enhance problem-solving techniques.

4.5 SYNTHESIS

Polya (1988:5) and Schoenfeld's (1985:12) problem-solving frameworks were referred to as the researcher linked data from the teachers' experiences to discuss the findings. Such problem-solving frameworks include an understanding of the problem (resources available to the problem solver), devising a plan (heuristics for successful problem solving), carrying out the plan (control and belief systems) and finally looking back. The researcher used this framework to synthesise the findings from the empirical part of the study.

- Understanding of the problem and resources

Having considered the meaning of understanding, the four selected teachers agreed that understanding plays an important role in mathematics teaching. The teachers indicated that a problem-solving approach underpins deep mathematical understanding. However, from the classroom observation data, there was little evidence of learners been given opportunities to demonstrate and justify their own ideas in order to foster understanding, as illustrated in sections 4.3.1.1 (a) and (b) and 4.3.2.

To enhance understanding, learners should have the required prior knowledge regarding the problem. Even though the four cases stated the importance of establishing learners' prior knowledge, there was little evidence to show attempts by the four cases to establish learners' prior knowledge from the data generated through interviews and the classroom observation visit lists. To the researcher, the teacher's role seems of essence to help the learners develop connections between their current knowledge and new information in order to promote understanding. However, according to the four cases, the time factor remains a challenge for teaching through problem solving, yet crucial for learners in terms of learning with understanding. Teachers tend to focus on covering content rather than on developing learners' understanding (Artzt et al., 2008:4). As discussed in Section 4.3.3.1, the teachers acknowledged that it is sometimes time-consuming to allow learners to make sense of mathematics in their own way.

- Devising a plan and heuristics

From extracts of interviews and classroom observation data presented in Section 4.3.1, there was no evidence of teachers providing learners with opportunities to create strategies, suggestions and techniques that help learners understand a problem better and make progress towards its solution. It seems the four teachers continuously told their learners what to do, as depicted in Table 4.2, 4.6 and 4.10 However, from the literature reviewed in Chapter 2, in problem solving the responsibility of the teacher shifts from providing information to asking questions and providing resources (Burton, 1989:20).

From the classroom observation data presented in Section 4.3.1, there is however little evidence from the four cases of shifting from being transmitters of information to being resource providers. The learners were not given sufficient opportunities to make plans and create strategies to solve problems. Yet, it was evidenced in the literature presented in Chapter 2 that learners should be able to apply various strategies until the problem is solved. According to Artzt et al. (2008:1), the teacher is supposed to help learners organise and formalise their ideas. Learners should be allowed to find strategies of solving the problem with little interference from the teacher. If the teacher interferes too much, the problem becomes his or hers and the reason for the activity is lost. Driver and Oldham (1986:112) state that teachers' key role should be to lead learners in their own discovery and understanding of mathematical concepts.

- Carrying out the plan, control and belief system

From the classroom observation data there was limited evidence of the learners being convinced of the correctness of each step by teachers B, C and D, except in the case of Teacher A, as explored in Section 4.3.1. Table 4.16 shows how the researcher observed the learners engaging in some discussions to illustrate how they arrive at the solution. However, in general, the four teachers were observed selecting goals and sub-goals as well as providing directions, indicating actions to be taken to solve a particular problem, as discussed in Section 4.3.1. The teachers could emphasise the difference between seeing clearly that the action taken is correct and proving that the action is correct (Polya, 1988:5).

From the classroom observation data presented in Section 4.3.1, it was evident that the ‘control’ of solving problems seemed to lie entirely with the teacher and not with the learners. As presented in Chapter 2, teaching through problem solving means that learners solve problems in their own ways, use mental tools already available to them and are able to learn important mathematical concepts (Schroeder & Lester, 1989:33). In addition, if learners are able to become involved in what they are doing, then they have reached significantly high levels of mathematics (Brousseau, 1997:28). This is not the case with the four selected teachers, as they were normally dominating the discussions and not the learners, as shown, for example, in tables 4.12, 4.13 and 4.18.

- Looking back

From the classroom observation data, there was little evidence of the learners re-examining and reconsidering the ‘path’ they took to arrive at the solution. The learners were not able to check each step taken to arrive at a solution. Instead, after the leaners completed the task, they should share the process used in solving the problem. With the teacher’s guidance, learners should reflect on the problem, their work and the important mathematical ideas that have emerged.

For the four teachers who constituted the case study, it seemed to be ones who reflected on the ‘path’ taken and indicated whether the answer is correct or not as illustrated in tables 4.11, 4.13 and 4.14. However, Manouchehri (2007:299) states that ending learners’ discussions by giving them the correct formulas or answers would likely close the door not only on their mathematical investigations but also on the formation of a learning community in which members willingly explore mathematics and engage in collaborative construction of knowledge. Moreover, Schroeder and Lester (1989:33) state that the facilitation of mathematics using problem solving entails more than simply posing the correct type of problems and then allowing learners to solve them without understanding.

4.6 CONCLUSION

This chapter has covered an analysis and discussion of the findings of the study. The data generated through the interviews, classroom observations and questionnaires were analysed. By doing so, the researcher noted that there is little evidence to indicate that teachers in the RUMEP programme have made a definitive move towards a problem-solving approach. It appears that the teachers are still finding their way to implement the teaching and learning of mathematics through problem solving. From the study it was evident that there is a mismatch between what the four teachers in the case study advocate and their classroom practices. The data generated through the face-to-face interview seem to contradict the data in the classroom observation notes. From the interview data, the four selected teachers seem to be in favour of problem solving, but in the actual classroom, their teaching is mostly dominated by 'telling and showing'. According to Brodie et al. (2001:541), teachers can create a mismatch or gap between what actually happens in the classroom and curriculum demands.

The teachers in this study seem to have the knowledge of the theoretical aspects of a problem-solving approach, but the implementation aspects are still problematic. This may be attributed to some of the factors (challenges) mentioned by the participants in this study. Panaoura (2012:291) states that mathematics teachers often experience some challenges in making use of proper mathematical problem-solving tools. However, it seems the understanding and interpretation of problem solving is not the main obstacle, because the participants in this study seem to share a common understanding of problem solving, as can be seen in their responses in the interviews and the questionnaire. However, for teachers to become invested sufficiently in this process of professional development, they must first come to believe that their current practice is in some way problematic. In essence, it is very difficult for teachers to make a mind shift in their teaching orientation. It is very difficult to move out of their comfort zone.

The next chapter outlines the conclusions and implications as well as the limitations of the study.

CHAPTER 5

CONCLUSIONS AND IMPLICATIONS

5.1 INTRODUCTION

This chapter discusses the conclusions and implications of the study. Conclusions are drawn based on the empirical findings of the study and related to the literature overview presented in Chapter 2. Some implications for mathematics teaching using problem solving are also pointed out. Finally, some of the limitations encountered in this study are highlighted.

The study explored the experiences of BEd (in-service) mathematics teachers facilitating mathematics using the problem-solving approach. More specifically, the study examined the classroom opportunities and challenges facing mathematics teachers using problem solving in their classrooms. The researcher's aim was to determine how secondary school mathematics teachers experience the facilitation of mathematics using a problem-solving approach while participating in a hybrid BEd (in-service) programme offered by RUMEP. In order to investigate the teachers' experiences, data were generated through questionnaires, face-to-face interviews and classroom observations.

5.2 CONCLUSIONS

Based on the findings from the consulted literature and the empirical findings reported in Chapter 4 of this study, the following conclusions can be drawn regarding teacher experiences with problem solving.

Firstly, while the four teachers who participated in the study seemed enthusiastic about learner-centred practices and intended to implement such practices in their classrooms, they continued to teach in predominantly teacher-centred ways. The findings suggest that these teachers still facilitate mathematics lessons using a 'traditional' approach, namely 'telling and showing', despite doing a problem-solving-oriented course in mathematics education for their further studies (see Table 4.14). 'Telling and showing' practices may prohibit learners from opportunities to construct their own meaning. Furthermore, according to Wertheimer (cited in Schoenfeld, 1987:3) knowledge acquired through such practices is likely to be superficial and therefore neither flexible nor useful in a range of situations.

Secondly, the findings showed that there seems to be a mismatch between what the four selected teachers advocated and what actually happened in their classrooms. The interview findings therefore did not correspond well with the observational findings. The latter findings illustrated that the teachers regularly intervened to show learners how to solve mathematical problems and that little was done to promote problem-solving techniques, such as enhancing deep understanding (see Table 4.17).

Thirdly, the findings indicated clearly that the participating teachers are positive about incorporating a problem-solving approach into their teaching practices. They are therefore willing to move towards problem-oriented teaching. However, the four teachers still experience challenges to implement a full problem-solving approach (see Section 4.3.3.1). As explained in Chapter 2, the use of a problem-solving approach by teachers cannot be achieved overnight. It became clear that it may take some time for teachers to fully benefit from a problem-solving approach in their teaching practices.

Fourthly, the findings also showed the availability of time as a main obstacle to the implementation of a problem-solving approach. However, the teachers seem to have missed the point that the knowledge gained and the procedures used in solving a specific problem may be applied to other problems and thereby could address the time factor. This include the view that no problem is completely solved and there may always be an improved understanding of a solution (Buschman, 2004:304).

Lastly, the RUMEP (BEd in-service programme) study material currently consists of a collection of examples and activities. The results revealed that the topics in the BEd programme are mainly introduced by a set of examples followed by a list of activities. Such a design may not equip teachers fully with the necessarily skills to enhance their understanding of a problem-solving approach and accompanying facilitation techniques. Currently, the study material seems to promote rote learning instead of deep understanding of mathematical concepts. However, the four teachers seem to be satisfied with the design of the study material, which may be linked to their own difficulties in embracing a problem-solving approach.

The above conclusions seem to suggest a number of implications – not only for teaching mathematics using problem solving, but also for future research.

5.3 IMPLICATIONS

The findings and conclusions drawn in this study may have implications in at least four domains, including mathematics education, classroom practices, the (RUMEP) BEd distance education programmes and further research.

5.3.1 Implication for mathematics education

For mathematics education, the findings of this study imply that the teaching of mathematics has to include more opportunities for promoting a problem-solving approach. Mathematics education using problem solving could be carried out by teachers who were trained by university or other lecturers through such methods. Such training for mathematics education could also be presented more regularly through teacher in-service programmes. Hence, if positive attitudes and skills towards teaching mathematics via a problem-solving approach can be fully developed and extended to mathematics education programmes, improvement in the quality and success rates of mathematics learners may be the result.

5.3.2 Implications for classroom practices

Classroom practices based on learner-centred approaches as opposed to teacher-centred approaches seem to be superior in terms of learning gain. Teachers should therefore develop the ability to listen to learners' responses. This practice may allow learners to justify their answers while also motivating them to listen to one another's comments. In particular, the value of discussions among learners focusing on mathematical problems needs to be emphasised and therefore learners' answers, rather than teachers' superior knowledge, should guide instruction. Mathematics teachers' classroom practices need to be re-examined to accommodate a constructivist and problem-solving view on mathematics learning (also see Simon & Schifter, 1993).

5.3.3 Implications for the BEd distance education programmes

BEd (in-service) programmes for secondary mathematics teachers are necessary in order to allow teachers to change their roles from instructors of mathematics to facilitators of mathematical learning. This is in accordance with the stated goals of in-service programmes as well as prominent literature (see, for instance, Stacy, 2005), namely that the primary purpose of mathematics education programmes is capacitating teachers to develop competencies to assist learners with developing problem-solving skills in mathematics classrooms.

In-service programmes for teachers could therefore be created in which positive attitudes and competence towards a problem-solving approach in mathematics teaching are developed. Forms of in-service training for teachers may include locally based afternoon workshops, contact teaching blocks and classroom support visits. It may also be important to point out that teachers find traditional ‘one-shot’ workshops as of little value (also see Buschman, 2004) and such interventions may not result in the desired changes.

5.3.4 Implications for further research

This study has raised a number of issues that are critical towards the facilitation of mathematics using problem solving. Therefore, the study could potentially pave the way for further studies regarding the challenges facing mathematics teachers who attempt to incorporate problem-solving methods in their teaching. An implication for further study could be an in-depth study to establish which factors currently prevent mathematics teachers from implementing a problem-solving approach in their classrooms. Such a study would present a clearer picture of the misconceptions regarding the use of problem solving as well as why teachers are still reluctant to incorporate problem-solving approaches in their teaching.

A more in-depth study of the significance and value of fortnightly workshops, contact teaching blocks and regular classroom support visits for BEd (in-service) teachers may also render valuable information to further improve BEd in-service efforts.

5.4 LIMITATIONS

The study was limited to in-service secondary mathematics teachers in the John Taolo Gaetsewe district of the Northern Cape province. These teachers were participating in the RUMEP programme offered by Rhodes University, where the researcher was employed as a lecturer at the time of the study. The study material investigated in this study were compiled by the researcher and other RUMEP staff to train teachers. The researcher therefore interpreted the findings of the study as a RUMEP employee but also as a researcher.

In view of these potentially conflicting concerns it was a challenge for the researcher to criticise the teachers' comments as well as the material used in the role of researcher. However, the researcher 'bracketed' himself as researcher as best as he could and was aware of this possible limitation throughout the study. The researcher also cross-checked the findings of the study with other professionals who are knowledgeable in problem solving. Notwithstanding the limitations, the data generated for this study highlighted important areas to contribute to a better understanding of the experiences of BEd (in-service) secondary mathematics teachers facilitating mathematics and using (or not using) problem solving.

5.5 CONCLUSION

This study has shown how in-service secondary mathematics teachers can experience the benefits and challenges of using a problem-solving approach to their teaching. In spite of the limitations of this case study, the researcher was able to note some small changes in the attitude of teachers towards a problem-solving approach. If this positive attitude can be developed and extended to the mathematics classroom, mathematics may not be the 'dreaded' subject it is perceived to be at the moment.

However, it must also be noted that the use of a problem-solving approach by teachers is a long-term investment and cannot be achieved overnight. It may take a gradual approach to convince teachers that their present, traditional methods are less relevant and effective in relation to the needs of modern societies. To convince the majority of teachers of such a view, opportunities may be created where they successfully experience the actual methods used to enhance problem solving. Teachers also need to challenge and critically reflect on their own teaching methods more frequently.

The researcher finally contends that series of regular workshops, contact teaching blocks and classroom support visits for BEd in-service teachers, based on constructivist learning models and a problem-solving approach in mathematics teaching, may be a way to improve the quality and results of school mathematics in future. This may only materialise if universities, teachers and school authorities can synergistically muster their efforts and resources to achieve such an outcome.

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Appendix A: QUESTIONNAIRE

This questionnaire is aimed at identifying opportunities and challenges you are faced with regarding the facilitation of mathematics through a problem solving approach while participating in a B.Ed (in-service) degree offered by Rhodes University. Answer both Section A and Section B. You are requested in Section B to answer each question by marking either strongly agree, agree, disagree or strongly disagree. Please only choose ONE response per statement by marking the appropriate block with an "X".

The questionnaire is completed anonymously and will take approximately 20 minutes of your time. Thank you kindly for your cooperation.

<u>Section A</u>: Teacher Information

Your age range	Less than 30 years	31 – 40 years	41 – 50 years	Over 50 years
Your highest educational level	Teachers Diploma/ACE	Bachelor's degree in Education	B. Ed Honours in Education	Master's degree in Education
Your current post level	One	Two	Three	Four
Years in Mathematics teaching per Grade (s)	Grade 9	Grade 10	Grade 11	Grade 12
Your current Mathematics teaching Grade (s)	Grade 9	Grade 10	Grade 11	Grade 12

Average number of learners per class in your own mathematics classroom:

Section B: Experience of my Mathematics Teaching

Teaching Method					
	Item	**Strongly agree**	**Agree**	**Disagree**	**Strongly disagree**
1	Problem solving enhances learners' abilities to recognise what they already have learned.				
2	Problem solving enhances learners' abilities to apply what they already have learned.				
3	I found a problem solving approach appropriate in my classroom for the teaching and learning of mathematics.				
4	I have to do more revision before learners can write a test if I use problem solving.				
5	Using problem solving, I feel that I can complete all the prescribed work in the curriculum.				
6	I enjoy using the problem solving method in my classes.				
7	I have confidence in using the problem solving method in my mathematics classroom.				

Planning and Preparation of Lessons					
	Item	**Strongly agree**	**Agree**	**Disagree**	**Strongly disagree**
8	Problem solving improves my planning and preparation of the lessons for the next day.				

9	Problem solving enables me to plan lessons that build on learners' existing proficiencies.				
10	Problem solving gives me an opportunity to facilitate planned lessons effectively.				
11	Using problem solving, I have sufficient time to successfully finish prepared lessons as scheduled.				
12	I find the planning and preparation of lessons easier if I use problem solving.				
13	Using problem solving planning and preparation of lessons is more challenging.				
14	I find the planning and preparation of lessons more fulfilling if I use a problem solving approach.				
15	Problem solving involves less work for the planning and preparation of my classroom lessons.				

Learner Communication					
	Item	**Strongly agree**	**Agree**	**Disagree**	**Strongly disagree**
16	Through problem solving learners have an ability to learn from one another.				
17	There is much more discussion in my classroom.				

18	Learners who are usually quiet, speak now more freely in the discussions.				
19	Learners now enjoy their mathematics more than earlier.				

Teacher Questioning					
	Item	**Strongly agree**	**Agree**	**Disagree**	**Strongly disagree**
20	The questions I ask indicate a direction for learners to answer if I use a problem solving approach.				
21	When using problem solving, learners have an opportunity to ask me questions.				
22	Problem solving enables me to improve on the quality of questions that I ask my learners.				

Tasks and Activities					
	Item	**Strongly agree**	**Agree**	**Disagree**	**Strongly disagree**
23	The tasks I select influence how learners come to make sense of mathematics.				
24	It is difficult to select the relevant activities for the next day.				
25	The tasks I select leave plenty of time for learners to finish them.				
26	The activities I provide elicit an appropriate mathematical response				

	from the learners.				

Classroom discourse					
	Item	**Strongly agree**	**Agree**	**Disagree**	**Strongly disagree**
27	Problem solving arouses learners' interest to mathematics.				
28	There is discussion in my classroom that supports mathematics argumentation.				
29	Problem solving allows learners experiences of working independently.				
30	Problem solving allows learners experiences of working collaboratively to make sense of ideas.				

Monday Afternoon Contact Sessions					
	Item	**Strongly agree**	**Agree**	**Disagree**	**Strongly disagree**
31	Problem solving changed my attitude to mathematics as a B.Ed. student.				
32	At the end of the class session, I am able to complete my assignments.				
33	The class is too noisy to maintain my interest for learning.				

Study materials					
	Item	**Strongly agree**	**Agree**	**Disagree**	**Strongly disagree**
34	I have enough study materials to support me learn the content.				
35	I can follow the language used in the study guide during the contact sessions.				
36	The study material used in the sessions contains activities to help me learn new concepts.				

37 Comment on how problem solving has changed your view to mathematics teaching.

----

39 Comment on how you found problem solving useful in your own mathematics classroom.

Appendix B: CLASSROOM OBSERVATION TOOL

Grade: ---
Topic: ---
Name of teacher: ---

1. General and Classroom Environment

Items	Yes or No	Comments
Learners well organized		
Learners in groups		
General discipline		

2. Communication and Questioning

Items	Yes or No	Comments
The teacher encourages learner inquiry by asking thoughtful, open-ended questions		
Encouraging learners to ask questions of each other		
Encourage learners to ask the teacher questions		
Uses cognitive terminology such as classify, analyze, predict or create when framing questions		
Allows learner responses to:	Drive lessons	
	Shift instructional strategies	
The teacher responds to learner contributions.		
Asks learners to elaborate on initial responses		
Allows for 'wait time' after posing questions		

3. Classroom discourse

Item	Yes or No	Comments
The teacher accepts	Autonomy	

learners’	Initiative	
Presents multiple perspectives of concepts		
Inquires about learners' understandings of concepts before sharing his/her own understandings of those concepts		
Encourages learners to engage in dialogue, both with the teacher and with one another		
Helps learners list examples		
Engages learners in experiences that might bring about contradictions to their initial hypotheses		
Moves throughout the room interacting with learners		
Facilitates scaffolding to help learners perform beyond the limits of their ability		
Learners’ errors are taken into consideration and are used to gain insight into learners’ previous knowledge constructions		
Emphasizes knowledge construction as opposed to reproduction		
Considers the learners’	Previous knowledge	
	Beliefs	
Encourages cooperative learning to allow learners to see others’ viewpoints		
Encourages learner discussions		
Encourages exploration by		

learners to construct their knowledge independently		

4. Teaching Resources

Item	Yes or No	Comments
The teacher uses:	Manipulative resources	
	Interactive resources	
The teacher is text book bound		

5. General Comments:

Signature of teacher

Signature of observer

Adapted from Kilpatrick et al. (2001: 314)

Appendix C: INTERVIEW QUESTIONS

1. Personal Information: This included finding information relating the following

1.1 Teacher's marital status and number of children.

1.2 Teacher's training information: colleges and universities attended, departmental courses attended, computer courses.

1.3 Whether mathematics was the teacher's favourite subject at school.

1.4 The year the teacher started teaching mathematics in secondary school.

1.5 Previous schools and Grades where the teacher taught secondary school mathematics.

1.6 Learner composition: age range, family background

1.7 Whether the teacher moves with his or her learners to the next Grade (s).

1.8 Whether the teachers current Grades is teaching include failed learners whom he or she taught previously.

2. What do you like most or enjoy when facilitating mathematics lessons using a problem solving approach – discussions in class?

3. What do you have to say about the claim that "learners should always be encouraged to justify their thinking"?

4. Mathematics should be thought of as a collection of concepts, skills and algorithms. What is your view on this statement?

5. Are there any good opportunities for you in using problem solving to facilitate mathematics? Please elaborate.

6. What is your opinion regarding the statement that a major goal of mathematics instruction is to help learners develop the belief that they have the power to control their own success in mathematics

7. Has your personal vision of classroom practice changed or remained unchanged since your exposure to a problem solving approach? Please elaborate.

8. How has your teaching through problem solving impacted on your beliefs about the teaching and learning of mathematics?

9. What do you consider as the biggest challenge (s) in teaching mathematics using a problem solving approach?

10. Do you think it problem solving is a suitable approach to facilitate mathematics

classes? Can you explain why you say so?

11. What classroom opportunities do you find with regard to teaching and learning of mathematics through problem solving?
12. How do the RUMEP study materials enhance your problem solving skills? Elaborate
13. Do you think RUMEP prepares you adequately to facilitate mathematics lessons through problem solving? Can you explain why you say so?
14. What is the nature of the RUMEP coursework that will prepare you effectively as a teacher of secondary school mathematics teaching through problem solving?
15. Do you think problem solving added greater value to you as a RUMEP student to deep understanding of mathematics? Can you explain why you say so?
16. Are there any other comments regarding problem solving that you would like to make me aware of?

Appendix D: Ethical approval, Stellenbosch University

UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

Approval Notice

New Application

30-Sep-2013

MATLALA, Sego Jacob

Application #: DESC_Matlala2013

Title: The experiences of secondary mathematics teachers about teaching mathematics through problem solving

Dear Mr Sego MATLALA,

Your DESC approved **New Application** received on **19-Aug-2013**, was reviewed by members of the **Research Ethics Committee: Human Research (Humanities)** via Expedited review procedures on **27-Sep-2013** and was approved.

Please note the following information about your approved research proposal:

Proposal Approval Period: **27-Sep-2013 - 27-Sep-2014**

Please take note of the general Investigator Responsibilities attached to this letter. You may commence with your research after complying fully with these guidelines.

Please remember to use your proposal number **(DESC_Matlala2013)** on any documents or correspondence with the REC concerning your research proposal.

Please note that the REC has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

Also note that a progress report should be submitted to the Committee before the approval period has expired if a continuation is required. The Committee will then consider the continuation of the project for a further year (if necessary).

This committee abides by the ethical norms and principles for research, established by the Declaration of Helsinki and the Guidelines for Ethical Research: Principles Structures and Processes 2004 (Department of Health). Annually a number of projects may be selected randomly for an external audit.

National Health Research Ethics Committee (NHREC) registration number REC-050411-032.

We wish you the best as you conduct your research.

If you have any questions or need further help, please contact the REC office at 0218839027.

Included Documents:

Informed consent form

DESC form

Research proposal

Permission letter

Sincerely,

Susara Oberholzer

REC Coordinator

Research Ethics Committee: Human Research (Humanities)

Appendix E: Permission Letter, Rhodes University

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STELLENBOSCH UNIVERSITY

1 Gelda Court
17 Luipaard Street
Krugersdorp
1739
09 April 2014

The Director
RHUMEP
P O Box 94
GRAHAMSTON
6140

Dear Sir/Madam

Request for permission to conduct research study at RUMEP

I am currently studying towards an MPhil in Higher Education at Stellenbosch University. I am writing a thesis on the experiences of secondary mathematics teachers about teaching mathematics through problem solving while participating in a B.Ed. (in-service) programme offered by Rhodes University. I would like to conduct this research with the Rhodes University Mathematics Education Project (RUMEP) students in the John Taolo Gaetsewe district of the Northern Cape Province (NCP). The teachers has already shown an interest to take part in the study (see the attached consent forms). Permission to conduct the study has been granted by the Northern Cape Department of Basic Education (see attached letter).

Full anonymity and confidentiality is assured. The study, the results and any reporting of the findings of the study will not include the names of schools or teachers involved. The locations of schools, faces of learners and teachers will not be shown during any reporting. I will not interfere with the normal day to day functioning of the school where teachers are employed. I will observe the participating teachers in the classroom not more than two times over a period of two months. All other contact sessions with teachers will take place after school hours at a time that suits the participant. A copy of the complete study will be forwarded to RUMEP if you so wish.

I trust that you will grant permission for me to conduct this study using the B. Ed (in-service) NCP RUMEP students. Thank you for your understanding and cooperation in this regard.

Yours Faithfully
Sego Matlala
MPhil Student (Higher Education) Researcher
Stellenbosch University

Permission to conduct study at KP Toto High School has been approved/~~declined~~.

Signature: Director (RUMEP) Pkerlington 9|4|2014

RUMEP
P.O. BOX 94
GRAHAMSTOWN 6140
SOUTH AFRICA

Appendix F: Permission letter, Northern Cape Education Department

DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS
LEFAPHA LA THUTO
ISEBE LEZEMFUNDO

Education Building
156 Barkly Road
Homestead
KIMBERLEY 8301

Private Bag X5029
KIMBERLEY 8300
Republic of South Africa
www.ncedu.gov.za

Tel. (053) 839 6500
Fax (053) 839 6580/1

Enquiries : **A Ericksen**
Dipatlisiso :
Imibuzo :
Navrae :

Date :
Leshupelo : 04 June 2013
Umhla :
Datum :

Reference :
Tshupelo :
Isalathiso :
Verwysings :

Mr. S.J. Matlala
1 Gelda Court
17 Luiperd Street
Krugersdorp
1738

Sir,

RE: APPLICATION TO CONDUCT RESEARCH IN THE NORTHERN CAPE DEPARTMENT OF EDUCATION IN THE KURUMAN AREA OF THE JOHN TOALO GAETSEWE DISTRICT

Permission is hereby granted for Mr. Matlala to conduct research in the Northern Cape Department of Education in the Kuruman area (JTG District).

The provincial department of Education supports this research and wish you well in your future endeavours.

We would appreciate it if the findings will also be made available to the department at the end of the research.

Sincerely,

H.H. ESAU
CHIEF DIRECTOR: DISTRICT OPERATIONS

Cc: Head of Department

HIV/AIDS is everyone's concern.

Page 1 of 1

Appendix G: Permission letter, School Principals

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STELLENBOSCH UNIVERSITY

1 Gelda Court
17 Luipaard Street
Krugersdorp
1739
26 March 2014

The Principal and the SGB Chairperson
Bankhara-Bodulong High School
BANKHARA
Kuruman

Dear Sir/Madam

Request for permission to conduct research study at your school

I am currently studying towards an MPhil in Higher Education at Stellenbosch University. I am writing a thesis on the experiences of secondary mathematics teachers about teaching mathematics through problem solving while participating in a B.Ed. (in-service) programme offered by Rhodes University. I would like to conduct this research at Bankhara-Bodulong High School. One of your teachers, Ms Setae GB, who is currently participating in the Rhodes University B.Ed. programme has already shown an interest to take part in the study (see the attached consent form). Permission to conduct the study has been granted by the Northern Cape Department of Basic Education (see attached letter).

Full anonymity and confidentiality is assured. The study, the results and any reporting of the findings of the study will not include the names of schools or teachers involved. The locations of schools, faces of learners and teachers will not be shown during any reporting. I will not interfere with the normal day to day functioning of the school. I will observe the participating teacher in the classroom not more than two times over a period of two months. All other contact sessions will take place after school hours at a time that suits the participant. A copy of the complete study will be forwarded to your school if you so wish.

I trust that you will grant permission for me to conduct this study at your school. Thank you for your understanding and cooperation in this regard.

Yours Faithfully
Sego Matlala
MPhil Student (Higher Education) Researcher
Stellenbosch University

Permission to conduct study at Bankhara-Bodulong High School has been approved/declined.

Signature: Principal, Bankhara-Bodulong High School

NOFEMELA. N

DEPARTMENT OF EDUCATION
BANKHARA BODULONG HIGH SCHOOL
P.O. BOX [illegible]
20[illegible] -04- 17
Principal: ....
NORTHERN CAPE PROVINCE

Appendix I: Transcripts of part of the interview

Researcher	Transcript	Action statement
What do you have to say about the claim that "learners should always be encouraged to justify their thinking"?	*"If learners just say something out of the moon, you don't know if it's guessing or whatever, but if a learner understood something he will be able to justify it, so it's a way of showing understanding".*	**Probing understanding**
What is your opinion regarding the statement that "a major goal of mathematics instruction is to help learners develop the belief that they have the power to control their own success in mathematics"?	*"I think, let me not say I think because with problem solving this learners are the ones who are doing a lot in class and that is how you see whether they understood or what" , learners develop self-confidence and can work on their own". Initially, I believed that for the success of the learners it depends upon the teacher, but since problem solving was introduced, then that is where I realized that it is actually upon the learners, so we were not giving them enough chance to explore on their own".*	**Drive learning**
What do you like most or enjoy when facilitating mathematics lessons using problem solving approach – discussions in class?	". . . *mostly is when the learners discuss among themselves, and more especially you see when they argue that's when I enjoy it because I am able to identify the misconceptions".*	**Exploratory discussions**