Estimating Residual Life of Equipment Using Subjective Covariates

by

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Thesis presented in partial fulfilment of the requirements for the degree of Master in Industrial Engineering in the Faculty of Engineering at Stellenbosch University

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March 2015

Declaration

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Abstract

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Thesis: MEng (Industrial Engineering)

March 2015

Most industries are being forced to operate at lower costs while delivering more outputs and ensuring a safe working environment. An opportunity to achieve this for asset intensive industries lies within the complex and integrated field of Physical Asset Management (PAM). This study is specifically concerned with the maintenance subset of PAM, more specifically, the proactive maintenance strategy. A field known as prognostics emerges when combining two maintenance tactics, namely predictive and preventative maintenance.

Prognostics uses historical failure data from preventative maintenance and variable readings used in predictive maintenance to estimate asset reliability. Reliability is estimated using statistical models commonly known as reliability models or survival models. Variable readings used must describe or portray the health of the assets considered and are called covariates.

A problem that exists in the maintenance subset of PAM is concerned with the data needed for the survival models. The historical failure data is difficult to come by or non-existent in industry and the covariate data is often noisy and inaccurate. This poses a problem when wanting to make important maintenance decisions because the prognostics survival models require both the historical failure data and the covariate data. The covariate data is generally acquired by applying Condition Monitoring (CM) to assets, monitoring characteristics reflecting the asset's health. Prognostics can aid with maintenance decisions because once the equipment reliability has been estimated, it is possible to predict the time that an asset can still operate at its prescribed level of performance. This time of operation, which the asset can still operate, is more commonly known as its residual life (RL).

To overcome this problem, six of the most popular survival models found in literature, namely the Accelerated Failure Time Model (AFTM), Additive Hazards Model (AHM), Proportional Covariate Model (PCM), Proportional Hazards Model (PHM), Proportional Odds Model (POM) and the Prentice, Williams and Peterson (PWP), are considered and populated with historical failure data and the covariate data elicited from people. The people whom the data is obtained from are considered as experts in the field this study is conducted in. Also, the data is subjective because each expert has their own opinions and judgement concerning the assets in this study. The purpose of this study is, thus, to investigate whether subjective data can be used to populate survival models, therefore, allowing RL predictions of the assets considered.

A guideline consisting of five steps that aid with what system variables to consider as covariates, which people can be selected as experts and selecting the most appropriate survival model, is created and presented. Following the guideline, a case study is conducted on power transformers at an organization in South Africa.

Results from the case study reveal that the PCM is the most appropriate survival model reviewed. Using the PCM, RL predictions are made after the models are populated with subjective data and objective industry standard data. The results indicate that the subjective data yielded the same general trends but less conservative estimates when compared to industry standard data. Subjective data can, therefore, be used to populate survival models but this is inherently risky because of the less conservative results noted from this study. This study is based on a single case study, it does prove that it is possible to use the subjective data as an alternative to objective data. It is possible, however, that this characteristic does not apply for other asset types.

Uittreksel

Beraming van die Oorblywende lewe van Toerusting deur die Gebruik van Subjektiewe Kovariante

("Estimating Residual Life of Equipment Using Subjective Covariates")

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Maart 2015

Die meerderheid nywerhede word onder geweldige druk geplaas om laer bedryfskostes te handhaaf en ter selfde tyd word dit van hulle verwag om hul uitsette te vermeerder en ´n veilige werksomgewing te bied. Bate intensiewe-nywerhede het ´n geleentheid om hierdie druk te verlig deur gebruik te maak van ´n komplekse en geïntegreerde veld bekend as Fisiese Batebestuur (FB). Hierdie studie is gefokus op die instandhouding onderafdeling van FB, spesifiek die proaktiewe instandhoudingsstrategie. Twee proaktiewe instandhoudingstaktieke, naamlik voorspellende en voorkomende instandhoudingtaktieke, word saamgesmelt en vorm ´n veld bekend as prognostiek.

Prognostiek gebruik historiese falingdata van voorkomende instandhouding en veranderlike aflesings vanaf toestandmoniteering toeristing gebruik in voorspellende instandhouding om bate batroubarheid te bereken. Hierdie betroubaarheid word bereken deur gebruik te maak van statistiese modelle bekend as oorlewingsmodelle.

Een van die probleme wat voorkom in die instandhouding onderafdeling van FB het te doen met die beskikbaarheid van die data wat benodig word vir die oorlewingsmodelle. Historiese falingdata is selde beskikbaar of bestaan glad nie en die toestandsmoniteering data is dikwels onakuraat. Prognostiek word gebruik om belangrikke instandhoudingsbesluite te motiveer, dus is die

beskikbaarheid en betroubaarheid van die nodige data van belange. Om hierdie struikelblok te oorkom bestudeer hierdie studie die gebruik van subjektiewe data bekom vanaf deskundiges in prognostieke oorlewingsmodelle. Die doel van hierdie studie is dus om vas te stel of subjektiewe data gebruik kan word in prognostieke oorlewingsmodelle.

Ses oorlewingsmodelle wat gereeld voorkom in literatuur word nagesien in hierdie studie, die modelle sluit in die "Accelerated Failure Time Model" (AFTM), " Additive Hazards Model" (AHM), "Proportional Covariate Model" (PCM) , "Proportional Hazards Model" (PHM), "Proportional Odds Model" (POM) en die "Prentice Williams and Peterson" (PWP) model. Hierdie modelle word aangevul deur die subjektiewe data wat onttrek is van deskundiges in ´n sekere gebied, vir hierdie studie is die gebied krag transformators.

Met gebruik van hierdie modelle kan die betroubaarheid van die betrokke toerusting bereken word. Sodra die betroubaarheid bereken is kan die oorblywende lewe van die toerusting voorspel word. Die oorblywendelewe is die tyd wat ´n stuk toerusting nog moontlik kan werk sonder om te faal. Dit is belangrik omdat nodige instandhoudingsbesluite geneem moet word.

Hierdie studie stel ´n metode voor vir die uitvoer van die navorsing en soortgelykke studies. Die metode dui vyf stappe aan wat voorstel watter veranderlikes om te gebruik as kovariate in die oorlewingsmodelle, watter mense as deskundiges gekies kan word, en hoe om die mees toepasslikke oorlewingsmodelle te kies. Nadat hierdie metode voorgestel is word dit toegepas op krag transformators in ´n gevallestudie wat plaasgevind het in Suid Afrika.

Vir die gevallestudie is die PCM die meestoepaslikke oorlewingsmodel. Die oorblywende lewe voorspellings wat die metode opgelewer het is met die voorspellings gebaseer op die industriestandaard data vergelyk. Die resultate dui aan dat deskundiges minder konserwatiewe beramings lewer. Dus kan die subjektiewe data gebruik word in oorlewingsmodelle maar die beramings is minder konserwatief en daarom van natuur meer riskant. Hierdie studie se gevolgtrekkings is gebaseer op ´n enkele gevallestudie. Dit is dus moontlik dat die subjektiewe data dalk nie as ´n alternatief gebruik kan word met ander tipes toerusting nie.

Acknowledgements

I would like to express my sincere gratitude to the following people and organisations:

- Prof. P.J. Vlok for his valuable insight, continuous support, patience and time invested in this study.
- My parents, for their support, encouragement and infinite number of prayers.
- My friends, for their motivation and providing the entertainment to keep my spirits up.
- God Almighty, for granting me the daily strength and capabilities for this great opportunity.

Dedications

This thesis is dedicated to Fanus and Madeleine, my parents. With their love and support no challenge in life is too big to overcome.

Contents

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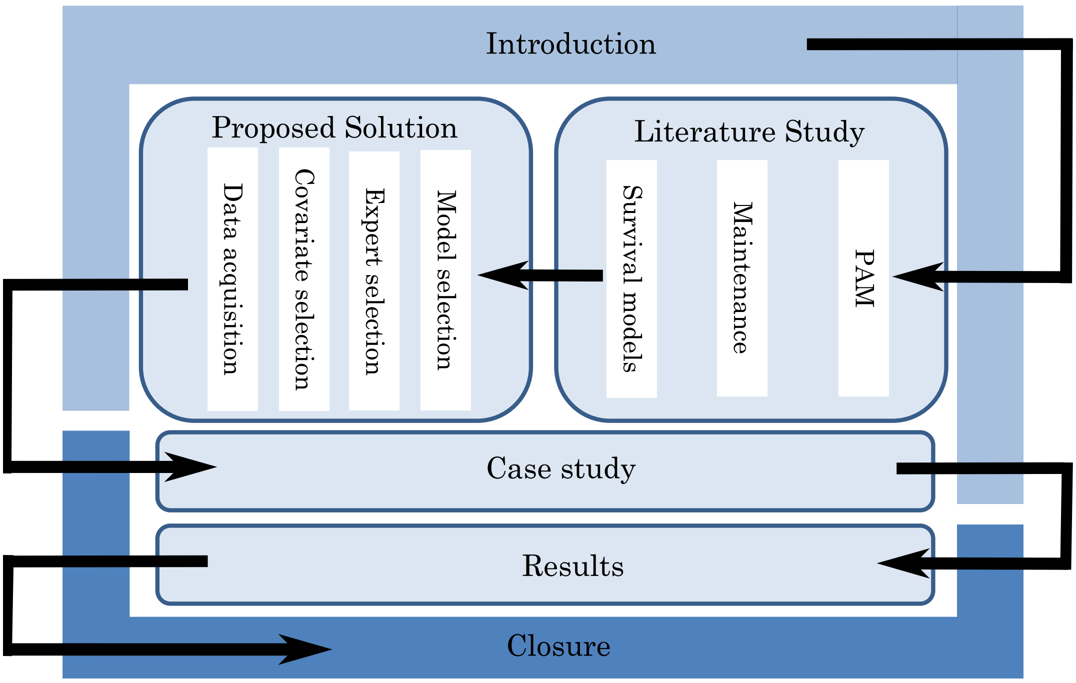
Acronyms

AFTM	Accelerated Failure Time Model
AHM	Additive Hazards Model
AM	Asset Management
BSI	British Standards Institution
CBM	Condition-based Maintenance
CDF	Cumulative Distribution Function
CM	Condition Monitoring
DP	Degree of Polymerization
EWO	Enterprise-wide Optimization
FOM	Force of Mortality
GCC	Government Competency Certificate
GOF	Goodness of Fit
HPP	Homogeneous Poison Process
HR	Hazard Ratio
IAM	Institute of Asset Management
ISO	International Standard Organization
MLE	maximum likelihood estimate
MVA	megavolt ampere
NHPP	Non-homogeneous Poisson Process
PAM	Physical Asset Management
PAS55	Publicly Available Specification 55
PCM	Proportional Covariate Model
PDF	probability density function
PHM	Proportional Hazards Model
POM	Proportional Odds Model
PwC	PricewaterhouseCoopers
PWP	Prentice, Williams and Peterson
RCM	Reliability-centred Maintenance
RL	residual life
ROCOF	Rate of Occurrence of Failure
RP	Renewal Process

Chapter 1

Introduction

This chapter aims to introduce the reader to the study conducted. It provides the core focus areas, the background information and fundamental ideas utilized in this study. The research question and objectives follow after the real world problem has been brought forth. This chapter allows the reader to place the research done in context as well as to understand where Physical Asset Management fits into the field of Industrial Engineering. The figure below illustrates how the different sections of the document relate and the work flow.



The introduction to the study is given in the first chapter, providing the motivation for this study and an overview of the background knowledge needed for this study. The second chapter describes the background knowledge in depth and explores current literature in the necessary fields. A proposed solution is provided in the third chapter, systematically laying out a methodology to conducting this study. After that, a case study is conducted and the results from this case study are discussed. The final chapter then summarizes this study and provides recommendations for future studies which could possibly improve on this study. An introduction and the background information to this study are discussed first.

1.1 Background Information

Organizations in all industries around the globe are being squeezed to churn out the same or larger profit margins while natural resources diminish and the pressure for sustainable development increase as mentioned by Gorjian *et al.* (2010*a*). Hamann (2003) describes the impact that the aforementioned has on the mining industry alone and labels this as a global shift affecting the operation of organizations. The operations and systems in the different industries have been refined and optimized to such an extent that it is now becoming increasingly difficult to find areas to stream line. Grossmann (2005) explains how this global squeeze has led to what is known as Enterprise-wide Optimization (EWO).

EWO is just as the name suggests, all facets of an organization are placed under close observation to identify areas to optimize in order to save time and money. Part of EWO is to save time and money through the application of an Asset Management (AM) system. Wassick (2009) mentions that the concept is centred around the integration of supply chain optimization, process systems engineering and operations research. This study is part of process systems engineering and makes use of operational research methods to gain insight into the failure analysis of physical assets. The entire EWO process will not be discussed in this study as the scope of this study only operates in a small section of the EWO system, namely AM. It can be debated that the single largest driving factor behind EWO is the maximization of an organization's profit.

Improved financial gain is a large driving factor for the optimization of the systems and operations but not the only one; environmental conservation is becoming increasingly important as the number of humans on earth surpassed 7 billion (Lutz, 2013). The footprint being left behind by people is becoming all too evident and irreversible (Hamann, 2003). The resource consumption and pollution is sped up by the fact that first world countries are aiding in the development of developing countries. This makes them more reliable on depleting natural resources and their emissions are increased as seen in the predictions made by Galeotti and Lanza (1999). The media fuels the process of globalization which influences the lifestyle of people worldwide and has an effect on the maintenance standards of equipment as mentioned by Campbel *et al.* (2011). The risk of safety incidents also play a large role in the pressure applied on organizations as society becomes less tolerant of occupational related injuries and fatalities.

To improve financial returns on assets, organizations are constantly developing their production systems but the risk of safety incidents on the employees encourages organizations to develop their production system while also ensuring worker safety. The modern society is ever increasing the pressure of worker safety but organizations have been dealing with this for decades as proven by Zohar (1980). From this, it can be deduced that there are three main areas in which organizations are being put under pressure, these main areas of pressure are listed as:

1. Financial pressure,
2. Environmental conservation,
3. Safety/health of employees.

A study of associated literature revealed that a sector that still has the potential to be improved or optimized is Physical Asset Management (PAM). It is a relatively new area of study that offers loads of streamlining potential. A report drafted by PricewaterhouseCoopers (PwC) reported that the South African mining industry experienced an increase of 18% in operating expenses, based on the figures of the organizations involved in the study. This report also shows that the total assets of the organizations involved in the study are comprised of more than 63% of mining and production assets (PWC, 2011). The correct and efficient management of these assets are thus of utmost importance. The listed pressure areas are all affected by the operation and management of the physical assets and, thus, PAM is a valid field to further inspect for improvement to provide relief and allow organizations to benefit financially.

1.1.1 Physical Asset Management

Asset Management (AM) can be seen as principles, concepts and processes which aid in converting the strategic plans of organizations into decisions and actions on assets in order to realize their value. The realization of value from the assets generally involves the optimization of risk opportunities, costs and performance benefits. AM systems are the interrelated and interacting elements to establish policies, objectives, strategies, plans and activities in order to maximize the value realized from the asset portfolio. The systems create a framework that helps with the control and coordination of the activities conducted in the asset portfolio. AM is, thus, a coordinated set of activities used to achieve a specified goal.

AM is said to be a disciplined approach that enables an organization to maximize the value (or minimize the liabilities) associated with their asset portfolio, which is responsible for delivering some of the organization's strategic objectives (ISO, 2014). PAM is simply the process of AM applied on physical assets. Physical assets include mobile assets (moving machines, trucks, light vehicles, etc.), plant and production machines, real estate facilities and infrastructure. There is no clear line dividing assets as physical assets or not. In an attempt to set a rough standard of what is considered to be physical assets, the British Standards Institution (BSI) and International Standard Organization (ISO) created guidelines in the documents BSi (2008) and ISO (2014). These documents are discussed later in this section.

For the purpose of this study, AM and PAM can be used interchangeably because this study will only consider physical assets. PAM is crucial for asset intensive organizations to achieve their business goals and objectives. Generally, asset intensive organizations are considered as organizations that have a heavy dependency on physical assets to create value, thus, managing these assets over their entire life cycle is of cardinal importance to achieve the desired goals. According to ISO (2014) and BSi (2008), PAM excellence is achieved by finding the balance in the conflicting factors of performance, risk and cost to achieve the optimal sustainable solution.

PAM has the potential to positively affect all three of the pressure areas. According to Hastings (2009), well managed assets allow for smooth and safe operation as well as minimizing the environmental effect. It is, therefore, worth further developing the PAM systems or methods to help in the realization of the maximum value of physical assets by organizations while relieving some of the pressure applied in the aforementioned areas. PAM is a large and complex field and has many different subsets where possible improvements can be made.

The PAM field has been around for some time but not much attention was given to it. The BSI was the first to realize the value of this field and the Publicly Available Specification 55 (PAS55) document was created in collaboration with the Institute of Asset Management (IAM) in 2004. Although this document was never recognized as a standard, it laid the foundations for the PAM field, providing the key principles and attributes of AM and suggested guidelines to follow when starting a AM system.

The ISO only recently (2014) published a suite of standards for the AM process. This came about when researchers and organizations realized the value of proper PAM systems. The suite of standards consists of three separate documents, namely ISO 55000, ISO 55001 and ISO 55002. This suite of standards provides an overview of the PAM field, the basic requirements for a PAM system and offers guidelines on the application of an AM system. The PAS55 document was used as the building blocks for the suite of standards. Both the ISO standard and the BSI document were specifically intended for physical assets.

Two other standards that are of relevance to this study are ISO 17359 and ISO 13380. Some of the data needed for this study is obtained through the Condition Monitoring (CM) process and is, thus, important to know how this process works. The international standard ISO 17359 provides general guidelines on the CM and diagnostics of machines while ISO 13380 provides more insight into the performance parameters linked to the diagnostics of machines.

According to BSi (2008), PAM system is based on four pillars that together form the foundation of the system. These four pillars support the most basic building block of a PAM system, the management of the assets themselves. The four pillars are presented on Figure 1.1.

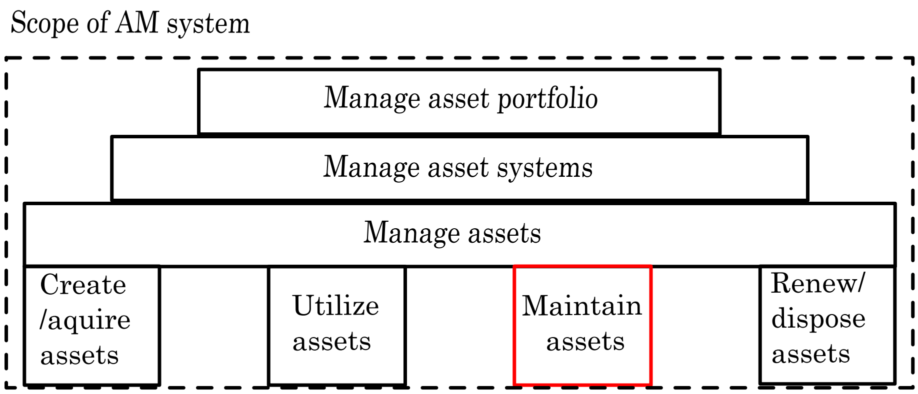


Figure 1.1: PAM pillars, adapted from BSi (2008).

This study will focus on the maintenance pillar as indicated in Figure 1.1. This study investigates whether specific data can be utilized in the maintenance pillar in order to aid in relieving pressure applied on the organization in the mentioned areas. It is clear that a lot of research has been done in the PAM environment. Even international standards have been drafted to aid and empower the processes but there are still issues involved when implementing PAM systems. The standards provide guidelines and regulations for different activities involved in the maintenance process, which is discussed next.

1.1.2 Maintenance

Maintenance encapsulates all the activities conducted on equipment which cause them to continue operating. It can be something as simple as replacing the lubricant in a machine to something as complicated as replacing a gearbox of a machine. Any activity which keeps the equipment in an operating condition or restores it to an operating condition is part of maintenance. Maintenance is a subset of PAM that affects all the mentioned pressure areas. Properly maintaining assets keeps them running more efficiently and effectively. Therefore, maintenance is an important and relatively large subset of PAM. Maintaining equipment is also a way of monitoring their reliability and Gorjian *et al.* (2010*a*) state that asset reliability is crucial for the economic pressure applied to organizations.

BSi (2008) specifies that careful consideration must be given when deciding between the trade-offs of performance, cost and risk, which are the main factors considered in PAM. The trade-off between the contradicting factors can be done by considering the different maintenance strategies and the various tactics and their application techniques. There are three different maintenance strategies that are used in different situations depending on the organization's business strategy, AM strategy and risk profile. The different maintenance strategies are:

1. Life improvement maintenance,
2. Reactive maintenance,
3. Proactive maintenance.

Each of the three maintenance strategies apply different execution tactics to deliver the optimal solution for specific situations. Proactive maintenance is the most advanced strategy but it is not always appropriate to apply. An example would be when the equipment is very cheap and not worth monitoring or they don't have significant failure consequences. The different maintenance strategies and their respective execution tactics are determined by the risk profile the organization follows and is largely dependent on the equipment cost and importance. The appropriate strategy accompanied by the correct tactics must be utilized in order to realize the maximum value from physical assets, the Chinese military strategist Sun Tzu once said:

> "Strategy without tactics is the slowest route to victory. Tactics without strategy is the noise to defeat."

The different strategies are introduced to inform the reader of the basic principles involved. The three strategies, the developed execution tactics and techniques are presented in Figure 1.2 in a hierarchical manner. These maintenance strategies will now be shortly introduced. A more in depth description is provided in Chapter 2.

1.1.2.1 Life Improvement Maintenance

Life improvement maintenance, also known as design improvement, is a strategy where failures are eliminated by identifying their root cause and then improving on the design of a physical asset. The tactic used is called design-out because the root cause is removed. This improvement then removes the initiation of the failure identified, thus, removing or reducing the possibility of it re-occurring. A system can be considered to fail as long as it can no longer operate at its prescribed level of performance, this is also known as a functional failure. A physical failure is when the asset physically breaks down, life improvement is used to eliminate or decrease both types of failure.

This maintenance strategy is very effective but it is not always possible to improve the design of a component to eliminate or reduce the failure rate. Once this improvement has been made, the equipment returns to its normal

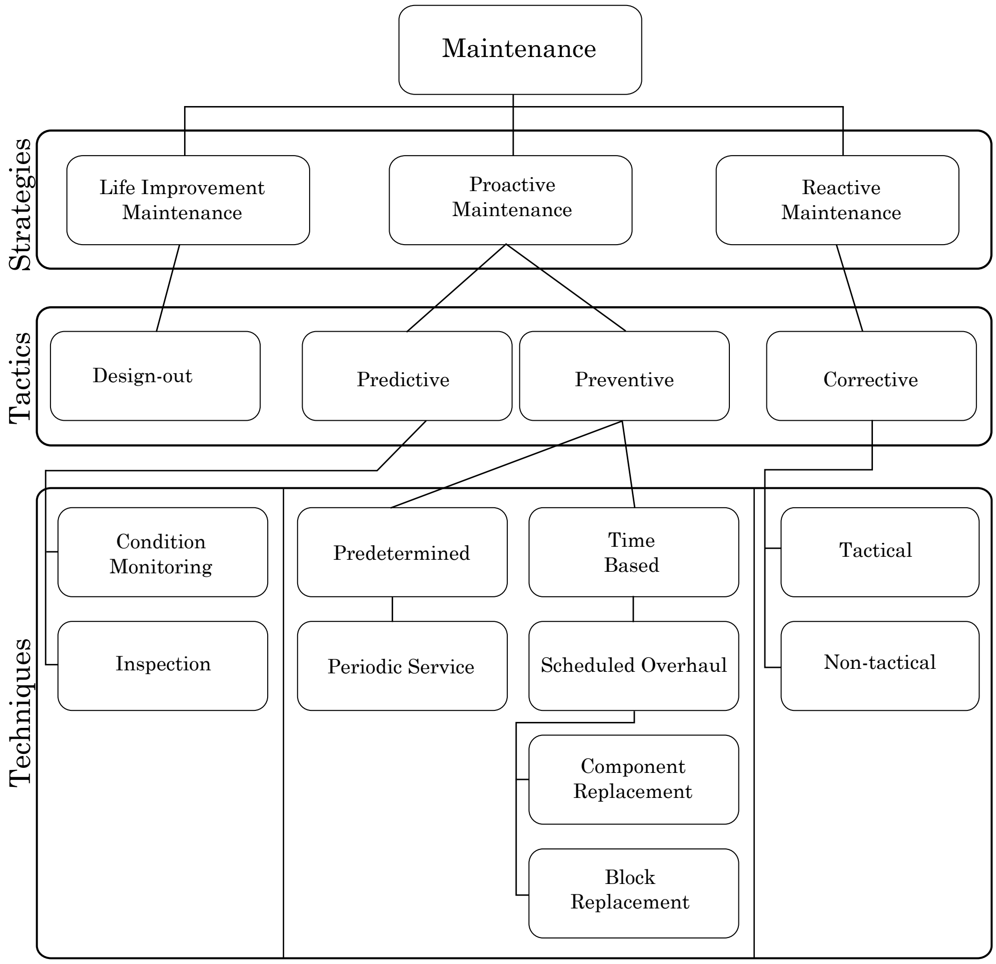


Figure 1.2: Maintenance strategies, tactics and techniques.

operation and either reactive or proactive maintenance methods are applied.

1.1.2.2 Reactive Maintenance

Reactive maintenance has one tactic known as the corrective maintenance tactic, but consists of two different application techniques. There is tactical corrective maintenance and non-tactical reactive maintenance. Tactical reactive maintenance is also known as run-to-failure, and allows the equipment to consciously run until they fail. This can be either a physical or a functional failure. This technique considers factors like the replacement cost and replacement time, which affect calculations done to determine whether it is financially favourable to apply corrective maintenance. It is important to realize that failures are allowed to occur consciously and arrangements are made prior to the failure on how to deal with the failures. Furthermore, spare parts have been ordered in advance and other strategies must also have been considered as an alternative.

The other corrective maintenance technique is to completely ignore maintenance and just have all the equipment run until they fail without any arrangements being made prior to the failures. This is known as non-tactical corrective maintenance and is not the same as tactical corrective maintenance. Unlike tactical corrective maintenance, in non-tactical corrective maintenance, no arrangements are made prior to the failure to prescribe actions to be taken in case of a failure, no spare parts are kept, no other strategies are considered and the failures are always unexpected. This technique is not considered in this study as there is no strategy or planning involved. This is simply the absence of a structured maintenance method.

1.1.2.3 Proactive Maintenance

Proactive maintenance consists of two different tactics; preventative maintenance and predictive maintenance. Preventative maintenance assigns a prescribed lifetime (the scheduled repair or replacement of components) to components irrespective of their current condition while predictive maintenance is a more advanced tactic and takes the current health of equipment into consideration.

Preventative Maintenance Tactic

The lifetime assigned in preventative maintenance can be measured in different units such as operating hours, kilometres, amount of wear, etc. The components are usually replaced while still in a good operating condition and while they still achieve their intended goals. There is, however, a probability for random failures to occur prior to the replacement time. Maintenance activities are conducted blindly, irrespective of the current condition of the piece of equipment, when the predetermined lifetime has been reached.

Decisions made in preventative maintenance make use of historical failure times of the equipment being considered. Historical failure times are the time instants at which failures (physical or functional) of the equipment considered occurred. A record is kept of the operating time between failures, thus allowing statistical models to calculate the estimated time of survival for the equipment. This process, however, does not consider the current condition that the equipment is in and once the equipment reaches its prescribed lifetime, it is replaced or the relevant maintenance actions are conducted. It should be noted that this historical failure data is often difficult to obtain in industry as Sun (2006) observes.

Predictive Maintenance Tactic

Predictive maintenance takes into consideration the current health of a system or component. A field, known as Condition Monitoring (CM), is used to obtain the characteristic data of the equipment. The data recorded to determine the state or health of equipment is referred to as CM data and can consist of a variety of different sensor recordings. The CM data is used to create degradation signals, which are analyzed to learn the characteristics of certain equipment and determine its current state or health. The data can also be used as a performance indicator for equipment. The degradation signals describing certain characteristics of a system or component are used to estimate/predict when it might fail.

CM includes the process of monitoring the current state of health of the selected systems or components with the use of special equipment and through inspections done by technicians and/or operators. The equipment used include devices such as thermal cameras, laser alignment sensors, accelerometers, viscometers, etc. These devices are used to determine when machines operate outside their prescribed or normal operating limits by recoding characteristic data. CM is, thus, a very important element of predictive maintenance.

The CM process further encompasses the analysis and interpretation of the recorded data, and not just the recording of it. This then allows for maintenance actions to be scheduled, resulting in the least amount of losses. Here, losses refer to production losses, losses regarding unplanned repair costs and any costs related to the occurrence of an injury or fatality. It can, thus, be seen that predictive maintenance require technologies and people with certain skills and knowledge to convert the CM data, the design data and operations data into useful information allowing management to make important decisions about the maintenance requirements for the assets. There is, however, a middle ground between preventative and predictive maintenance.

This middle ground is known as prognostics, referring to a field where historical failure data is used in collaboration with CM data. Prognostics is used to help accommodate for the draw-back of the preventative and predictive maintenance. The prognostics field is briefly introduced here and discussed in detail in Chapter 2.

1.1.3 Prognostics

Prognostics is an engineering discipline that makes use of survival models to estimate the reliability of equipment (Lee *et al.*, 2006). These survival models, more commonly known as a reliability models in the engineering discipline, are mathematical models which make use of both historical failure data as well as CM data to estimate asset reliability. The CM data are used as special variables known as covariates. A covariate is a variable that is possibly predictive of the outcome being investigated as Gujarati (1995) explains. It is also known as a control variable and is generally considered to be a continuous variable that is observed rather than manipulated.

Prognostics permit maintenance activities to be executed pro-actively, preventing potentially catastrophic and/or minor equipment failures. Prognostics hold the advantage of using both preventative and predictive techniques to estimate equipment reliability, thus, using both historical failure data as well as covariate values as shown in Figure 1.3.

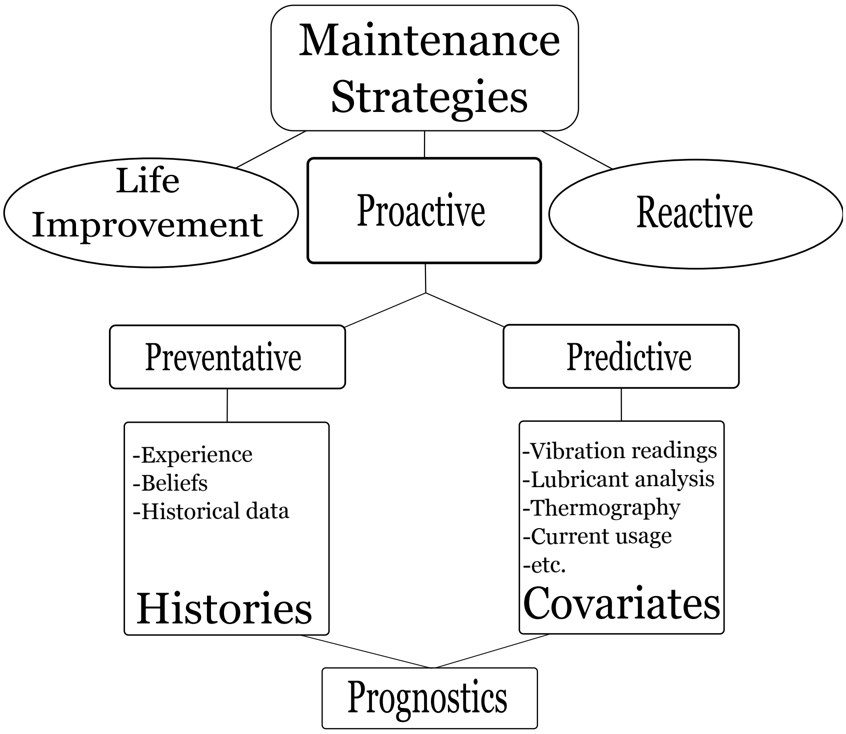


Figure 1.3: Prognostics is a combination of two maintenance tactics.

Prognostics possesses the advantage of predictive maintenance where healthy equipment is not unnecessarily replaced. It also takes into consideration the failure history of equipment, thus, allowing accurate estimates from the beginning. The modern industries consider the biggest advantage to be the drastic decrease of the unexpected asset downtime because the majority of maintenance activities are planned. Organizations benefit with favourable production gains from the planned downtime. Another advantage is that fewer maintenance activities are required because healthy equipment is not blindly replaced. Sun (2006) states that the cost of repairs for unexpected failures is much higher than that of expected failures.

Prognostics is, thus, a combination of the preventative and predictive maintenance tactics. Unfortunately, the survival models used to estimate the equipment reliability makes use of the CM data and depend strongly on the historical failure data. Without the historical failure data and covariates, the survival models are unable to estimate the equipment reliability and are, therefore, useless (Moubray, 1997; Ma, 2007; Hastings, 2009).

It can be seen that at the core of prognostics lies various survival models used to estimate equipment reliability by making use of the relevant data. These mathematical models enable prognostics to be used as a key enabler of proactive maintenance, predicting the time of failure and the residual life (RL) of equipment before the actual occurrence of the failure. The RL of an asset is the time that it has left to operate, at a desired level of performance from the current point in time, up to the time of failure as explained by Ghasemi *et al.* (2010). A brief overview of the survival models in this study is given next.

1.1.3.1 Survival Models

Survival models, as mentioned, are mathematical models which are more commonly known as reliability models in the engineering industry. Historical failure data and CM data of indicators of a system or component are required to populate the prognostics survival models. According to Wallace *et al.* (2004), survival models that represent the effect of certain indicators are called covariate models. The characteristic indicators (or CM data) are considered as covariates because they can possibly predict the outcome (the time of failure in this case) of the equipment under study.

In mechanical systems, covariates can be obtained from the CM process which provides system or component characteristic data suitable as covariates. The CM data offers completely objective data to populate the survival models, since the data is recorded by CM equipment and no human judgements or biased readings are involved in the recording of the data. The data that is required for the survival models possesses the following characteristics:

1. The time until an event (a failure or an observation) is the dependent variable.
2. The independent variables are the covariates used.
3. Some data points are recoded at an observation, thus, for some units in the study the event of interest (a failure) has not occurred.

Survival models estimate the reliability of equipment. Only after the reliability has been estimated can the RL be predicted. Considering most of these models originated from the medical discipline, especially in studying the effects of various cancers, but many were adapted and some created specifically for reliability analysis Sun (2006); Moons *et al.* (2009). These models were created with various assumptions made about the system or component under study. Ma (2007) explains that generally, the historical failure data is used to establish some baseline function. This function is then updated using the covariates. These models are meant to provide an accurate estimate of survival times for systems or subjects considered. Chapter 2 presents the survival models that were considered in this study, explaining the assumptions made as well as the advantages and drawbacks of each.

Until now the data used to populate the survival models has been completely objective data. This means that the data is not biased or subject to any human judgement. The historical failure data and the CM data is as it was recorded. These ideal data sets are discussed in the following section.

1.1.3.2 Data Sets for Survival Models

Ma (2007) explains that data sets used to populate survival models require both CM as well as historical failure data. Historical failure data consist of the failure times of the equipment considered over a certain period of time. The CM data can be considered as readings recorded from various censors, which in some way or form, reflect the performance or the health of a machine or component. Prognostics combine the data from these two fields to provide more accurate estimates of the equipment reliability, which allows better predictions to be made.

Konstantopoulos (2006) reveals that the data sets will then generally be represented in a tabular format including columns such as the event number, the time of the event, the corresponding covariate values at the time of failure and an event indicator. The covariate values are the CM data that has been recorded for the same period spanning over the failure time. The event number is simply a column counting each reading being recorded, be it at a failure or a censored occasion.

Reliability analysis generally has two possibilities of when events are recorded, either a failure or a censored case. A failure is when the system experienced a functional or physical failure and could no longer achieve its desired goal. A censored case is when the readings were recorded and are listed in the data set but the system or component has not failed yet. This includes maintenance activities which are conducted before the occurrence of a failure. A typical data set as explained will look like Table 1.1, which is slightly adapted from Jardine *et al.* (2001).

Table 1.1: Example of typical data set, adapted from Jardine *et al.* (2001).

Event #	**Operating time**	**Status**	**Covariate one**	**Covariate two**
1	52781	0	10	27
2	53048	0	470	43
3	53295	1	950	63
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

All of the data required for the prognostics survival models is objective data, not subject to judgement of any humans, though the historical failure data does not exist and the integrity of the CM data is questionable. This raised the thought of obtaining the data needed to populate the survival models from people who are considered as experts in the relevant field. To be considered as an expert, one must possess above average knowledge in the field of interest. Data obtained from the experts are considered to be subjective because each expert is free to have their own opinion on the data.

The subjective data will then encapsulate similar data as the classical objective data obtained from the CM equipment and the historical failures. The only difference will be that the objective data sets are based on the opinions and experience of people selected as experts. Currently, no evidence could be found proving that subjective data obtained from experts can be used as a viable substitute for the objective data. This study will, therefore, not attempt to develop a new survival model but rather to establish if the data used to populate current survival models can be subjective instead of objective data. The problem statement of this study, which elaborates further on the obstacles encountered when making use of prognostics is provided next.

1.2 Problem Statement

The problem that exists in the maintenance subset of PAM is concerned with the data needed for the prognostic survival models. The survival models that are used in prognostics are dependent on historical failure data as well as CM data. Sun (2006) states that it is often difficult to glean historical failure data in industry owing to poor record keeping. The CM data is generally recorded and stored but few organizations utilize the acquired data afterwards. The prognostic survival models require both the historical failure data and the recorded CM data to successfully estimate equipment reliability.

CM equipment has become cheaper and more accessible because of the advance in technology but some organizations still do not have the equipment. Therefore, cannot acquire the CM data (Mann *et al.*, 1995). The CM equipment, however, is not the biggest problem, the historical failure data does not exist in most cases and if it does exist, it is difficult to extract as mentioned by Sun *et al.* (2006).

Without the CM and failure data, prognostics cannot be made use of and the survival models driven by the data are rendered useless. The availability of the historical failure data and the integrity of the CM data are, thus, regarded as the main obstacles when considering prognostic survival models. The integrity of the CM data is questioned as a result of noisy readings or because operators or technicians have been known to alter the CM or performance readings to present their superiors with favourable reports (Mann *et al.*, 1995). This is the case in some manufacturing industries where the operators and/or technicians are under a lot of pressure to perform better.

The purpose of this study is to investigate whether subjective data obtained from people considered as experts in a specific trade, can be used to populate survival models and thereby predict the RL of equipment. The RL is important because once this is determined for any equipment, both short term as well as long term maintenance decisions can be made and financial budgets prepared. The subjective covariates will act as a substitute for the objective covariates obtained from the CM data and the failure times recorded. The data sets used in this study will, thus, be created by making use of the knowledge and experience experts have gained over time.

Considering this, the research question formulated for this study is as follows:

> "*Can subjective data obtained from experts be used as covariates to populate prognostic survival models, thus, allowing the prediction of equipment RL?*"

Following the research question, the null hypothesis can be stated as:

$\mathbf{H_0}$:
Subjective covariates obtained from experts cannot be used to populate prognostic survival models to allow the prediction of the RL of equipment.

In order to answer the research question, there are several objectives that need to be achieved. These objectives are given and elaborated on in the following section.

1.3 Research Objectives

The purpose of this study is to verify whether subjective covariates can be used in survival models as a valid alternative to objective data and thereby allowing the prediction of equipment RL. Different survival models are applicable in various situations and on different data sets. Therefore, it is necessary to select an appropriate one that is specific to the situation and the data set used. In order to ultimately allow the research question to be answered in an objective manner, the following objectives were formulated:

1. To conduct a comprehensive literature study on survival analysis and survival models.
2. To determine what CM data are suitable as covariates for the asset under study.
3. To evaluate the applicability of different survival models for this case specific study.
4. To establish a guide for selecting the experts from whom to obtain the subjective covariates.

5. To elicit subjective data from people considered as experts on the asset under study.
6. To deliver estimates of equipment reliability by making use of subjective data, allowing the RL to be predicted.
7. To validate the theory tested with a case study.

The first objective is important to this study because the background information on reliability estimation is informative on the basics of survival analysis. Acquiring the background knowledge of survival models is essential in order to select an appropriate model in an objective and reassuring manner for this particular case. The second objective is important because the correct parameters must be chosen as covariates for specific asset types. The standards published by ISO for the CM process offer valuable insights as to what characteristic parameters of specific assets to use as covariates.

The applicability of the various survival models are to be tested and compared to ensure that the most accurate results are obtained. The results will also depend on the quality of the data and this is why a guide for selecting people considered as experts is created in Chapter 3. It is important that the selected experts possess above average knowledge in the chosen field.

The final two objectives will be completed last. The other objectives are prerequisites for the last two. The RL predictions obtained by using the subjective covariates will be compared to predictions made when using industry standard data. Industry standard data is data that is accepted as a norm by the working community. The results will be validated by means of a case study conducted in South Africa. This will allow the validation of the research question.

1.4 Research Methodology

Research is a systematic process of collecting, analyzing and interpreting data to better understand a specific phenomenon. Research must originate from a question or a problem. According to Paul D. Leedy (2013), research has a clear goal and is divided into smaller more manageable problems to achieve the final objective. Two different types of research exist, namely quantitative and qualitative research.

Both quantitative and qualitative research will be used in an attempt to answer the research question of this study. This study can, therefore, be seen as a mixed method study. The qualitative procedures used in this study will enable the following questions relating to the research objectives to be answered as explained in Section 1.4.1:

1. Which survival model best fits the equipment and the specific data sets used in this study?

2. Which parameters are to be used as covariates in the survival model?
3. What effects does the chosen covariates have on the reliability of the equipment considered?
4. What criteria will be used to select experts for a specific field of interest and select the experts for this study?
5. How will the developed/proposed solution be applied in a case study?

The quantitative procedure explained in Section 1.4.2 will then allow the remaining objectives which follow to be reached:

1. Establish which of the survival models reviewed best fit the purpose of this study.
2. Make RL predictions of the equipment considered making use of the subjective data.
3. Compare the results obtained from subjective data to that of industry standard data.

Several different mixed-method designs are available but for this study a convergent design is considered. Quantitative data will be collected from CM equipment and from the selected experts. The data will be used to populate the appropriate survival model(s) to estimate the reliability of equipment. This reliability will be used to predict the RL of the equipment and the results will then be validated by a quantitative comparison to the result from normal objective data when applied to a case study. The data will be collected by qualitative research methods but still deliver quantitative results.

1.4.1 Qualitative Research

An extensive literature study must be conducted to gain more knowledge on survival analysis and to become familiar with the different survival models available. This will ensure that the most appropriate survival model is selected. The study of the different models will reveal the most favourable one to use and the reason(s) why it is preferred, listing advantages and disadvantages for all models. This will yield quantitative methods of determining the most appropriate model.

A phenomenological study will be executed by reviewing appropriate literature to determine the effects of different factors on the degradation of equipment to determine the most appropriate covariates. Available literature will act as a guide when selecting covariates, the five human senses will be considered but they will not affect the covariate selection. Although the human senses limit what can be experienced by humans, it does not prevent them from gaining knowledge of the different parameters of equipment. Parameters used as covariates do not have to be detectable by the human senses because the data will be obtained from experts based on their knowledge gained either by personal experience or research.

The humans who will be considered as experts must also be determined and a fixed method is to be laid out for selecting these experts. Relevant literature will be used to provide guidelines for what people to consider as experts in a specific field. The discretion of the one conducting the study should also be used when selecting experts because it is very seldom that the available literature will be applicable to all industries. When the survival model, the appropriate covariates and the experts are selected, a case study can be conducted.

A case study is done to validate the research question. Survey research will be made use of by providing survey type data sets to experts to obtain the data which will be used to populate the selected survival model. The data of the CM equipment and the data from the experts must be obtained simultaneously as the objective data will be used to validate the results of the subjective data. The detail on how to conduct the case study is discussed in Chapter 4. Ultimately this study is meant to deliver quantitative results but it uses mostly qualitative research methods to obtain the appropriate model and the subjective data to be used in this study.

1.4.2 Quantitative Research

In order to select a survival model which best fits the data sets used, each of the reviewed survival models are to be populated with the subjective data. Relevant mathematical tests are then conducted to determine whether they are appropriate or not. Should the models prove to be appropriate the model which can recreate the original data set the most accurately will be the final model used in the study.

The selected model is then used to estimate the reliability of the equipment considered. The reliability estimates can then be utilized to deliver RL predictions. The results delivered by the selected survival model populated with the subjective data must be compared to that of the same model populated with objective data (the industry standard data).

A correlation study can be done on the results from the data. The correlation must prove to be positive in order to accept the results of the selected model populated with the subjective data. The method of obtaining subjective data from experts and using the data in a survival model has yet to be verified in the PAM environment by existing literature. It is, therefore, important to either prove or disprove its pertinence.

1.5 Delimitations

When doing research, it is important to put boundaries in place. Up to this point the field in which this research will be conducted has been introduced and the problem that is to be solved is stated. This section states the boundaries to this study.

It is clear from the introduction thus far that this study is centred in the field of PAM, with specific focus cast upon prognostics. The study is to aid the maintenance decision making process by providing discrete values yielded from the survival models to present as reinforcing evidence in order to justify the decisions made. This study limits itself to only calculating the discrete values and validating whether the values are acceptable, not elaborating on how to present them to other functional areas in an organization. The scope of this study is:

- The applicability of this study is bounded to the PAM environment.
- This study only investigates subjective data obtained from experts in one field of occupation as a solution to the problem stated.
- The study does not attempt to develop any new survival models but rather utilizing the most popular ones found in literature.

This study is by no means intended to develop a new survival model. The study aims to verify whether subjective CM and historical failure data obtained from experts can be used to populate existing survival models. These boundaries were set and the remainder of the study is conducted with them in place.

1.6 Document Layout

This section provides an overview of the layout of this document. A short explanation of each chapter is provided, starting with the second chapter.

Chapter 2: Literature Study

The second chapter discusses the reviewed literature which aided with the formulation and execution of this study. The literature helps the reader to place where in the field of Industrial Engineering this study fits. The first topic discussed is PAM followed by the maintenance subset within the field. An introduction to the field of survival analysis is provided before reviewing the most popular survival models for reliability analysis. A final short summary of the reviewed literature is provided at the end to conclude the chapter.

Chapter 3: Proposed Solution

This chapter discusses a guide or road map to conducting this study, providing methods on how to choose the covariates, experts and the appropriate survival model. This is divided into four separate steps, each explaining the above mentioned processes in detail. This chapter equips the reader with the necessary information to plan and execute this study or one similar to it.

Chapter 4: Case Study

Conducting the case study which is used to validate the results is discussed in detail in this chapter. Thus, Chapter 4 is the application of Chapter 3 on a real life scenario. The different steps of Chapter 3 are applied in a systematic manner while documented in detail. The final survival model that is used in this study is decided upon by the end of this chapter.

Chapter 5: Results

The results of the case study in Chapter 4 are presented in this chapter. A discussion of the results is given. The subjective data is compared to objective, or industry standard data followed by a discussion on the differences and similarities. The opinions of five separate experts are compared to two different industrial standard data sets to be able to arrive at a conclusion.

Chapter 6: Closure

The final chapter provides a brief summary of the study as a whole, followed by the results obtained from the case study. It then goes on to name the limitations encountered during the conduction of the study. Final recommendations are made on how this study can be improved and on future research that can be done.

Chapter 2

Literature Study

This chapter summarizes existing knowledge and provides more background information. Another aim of this chapter is to show the relation of this study the findings of previously conducted studies. PAM is defined and one of its subsets, maintenance, is illustrated and explained. The literature is used to show how this study falls within the industrial engineering field. A large portion of this chapter elaborates on different prognostic survival models, their mathematics as well as their advantages and disadvantages. Choosing the correct survival model is very important since not all models are applicable to any data set considered.

2.1 Physical Asset Management

ISO (2014) defines an asset as: "anything that adds value or has the potential to add value". For the purpose of this study, only physical assets are considered. Before elaborating further on PAM, it must first be clarified what should be considered as physical assets. Hastings (2009) describes physical assets as items such as a plant, machinery, buildings, vehicles, pipes, wires, associated information and software systems, which are used to serve a business or organizational function. These assets are items which have a value for a period exceeding a year, thus, cash is not considered a physical asset. The scope of physical assets according to BSi (2008) is displayed in Figure 2.1. The purpose of an organization and the type of business which it conducts largely determine the types of assets which they require. The assets which organizations possess must be managed in an appropriate manner to avoid causing losses to the enterprise.

ISO (2014) defines AM as: "coordinated activities of an organization to realize the value from assets". In addition, the Asset Management Council of Australia defines AM as "the life cycle management of physical assets to achieve the stated outputs of the enterprise". Both definitions imply that financial and technical judgements are required and sound management practices must be applied throughout the life cycle of the physical assets. PAM proves to be a complex task and has many different subsets that operate within it to help

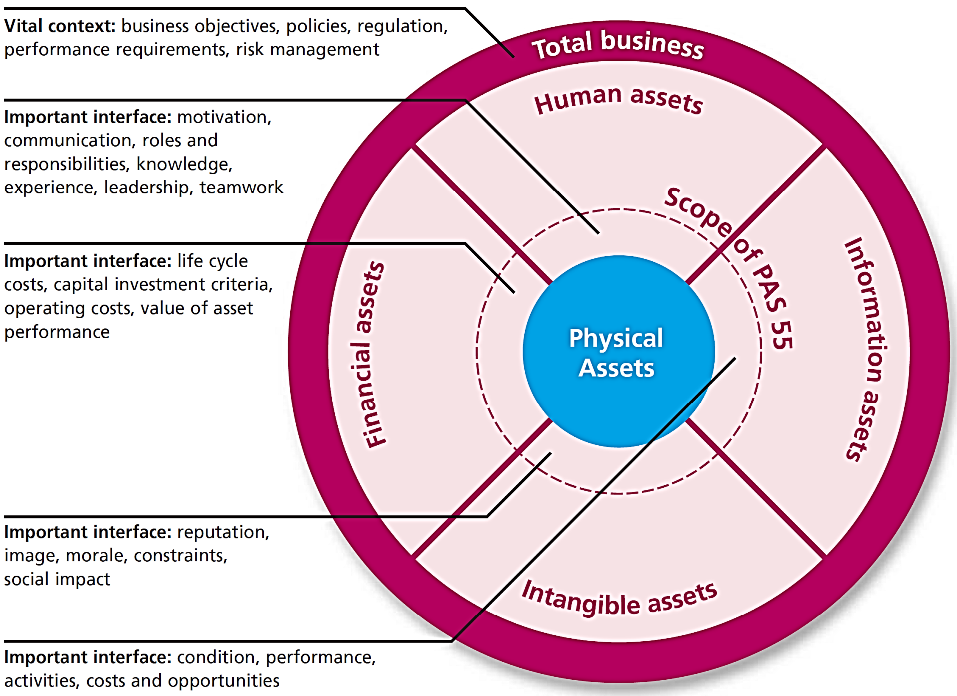


Figure 2.1: Scope of PAM (BSi, 2008).

achieve the desired outcomes. According to El-Akruti *et al.* (2013), PAM should be used to provide a holistic systems view of an organization's assets over their entire lifetime.

A PAM system is defined by ISO (2014) as a "set of interrelated and interacting elements to establish policies, objectives, strategies, plans and activities to maximize value from a portfolio of assets and asset systems in the delivery of organizational objectives over a specified period of responsibility". The PAM system is, therefore, the combination of all the AM steps and actions taken or planned in advance. The system, also serves as a framework of control and coordination. It ensures that all activities are aligned with the business objectives and AM objectives. The system further establishes an integrated and cross-functional manner to conduct all the activities.

In this study, AM and PAM can be used interchangeably since the only assets considered here will be physical assets. AM enables organizations to maximize the value from their asset, which has the responsibility of achieving the organization's strategic objectives. There are many factors influencing the formulation of an AM system. These factors emphasize how AM is fully integrated with other business functions and that it will be impossible for an AM system to operate without the consent of other functions. ISO (2014) lists the main influencing factors of an AM system formulation as:

1. The organization's vision, mission and objectives;

2. The stakeholders (internal and external, thus the customers and their expectations are considered);

3. Legal, regulatory and other absolute requirements which they must comply with;

4. Political, economic, social, technical and environmental factors that affects the organization's activities;

5. Limits which the organization has to operate within (restrictions such as financial limits, human resources and other logistical resources),

6. The organizational policy and decision making criteria (examples such as the risk evaluation, setting of priorities and the balancing of trade-offs for long term and short term objectives or goals);

7. The approach chosen to balance the short term business needs and planning cycles with the long term asset life management.

The purpose and the type of business organizations conduct have a strong bearing on what AM concepts the organization needs to develop in order to realize the organization's business objectives. According to ISO (2014) these concepts are broadly represented by the different elements in Figure 2.2. These elements are meant to allow clear interaction between the different functions, such as financial management, human resources, etc., and other elements of the organization.

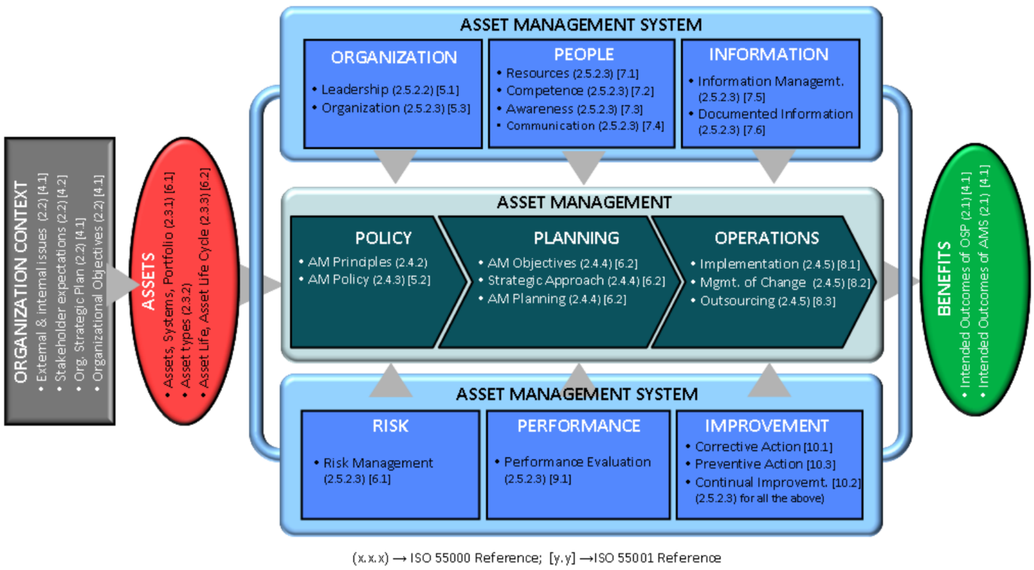


Figure 2.2: Key concepts of AM (ISO, 2014).

PAM is founded on several basic principles and the absence of any one of them is likely to result in the reduction of the value realized from the organization's assets. These principles help with the formulation of the PAM policy by top management. These principles act as foundation for a framework, which the PAM objectives and the developing plans are then built upon. ISO (2014) highlights four principles of what PAM should achieve:

1. Assets exist to provide value to the organization and its stakeholders;
2. PAM takes the organization's strategic intent and converts it into decisions and actions on the assets which are used to realize the value of assets;
3. Value realization is heavily dependent on leadership as well as a dedicated and engaged workforce;
4. Continual improvement is a requirement for effective PAM.

These principles can be achieved by following the guidelines provided by the documents PAS55 and ISO 5500x. The history and purpose of these documents and other standards which aid in the PAM process are discussed next.

2.1.1 Standards

The first documents to be discussed are those that have set the foundation for AM. Other standards that should be taken note of when conducting a study in the maintenance subset of PAM are then discussed following the AM documents' discussion.

2.1.1.1 PAS55 and ISO 5500x

The International Standard Organization (ISO) created the ISO 5500x suite of standards to set a written standard for AM. No standard for this existed until this suite of standards was published in 2014. The only other document that organizations have at their disposal as a baseline for the formation of their AM system is PAS55, a document created by the British Standards Institution (BSI) in 2008 as an attempt to create some consistency within the AM environment. The PAS55 document is not recognized as an international standard but was used by ISO as a foundation for the formulation of their suite of standards for AM.

The ISO 5500x standards can be used for the management of any assets but the suite of standards was specifically created with physical assets in mind. The ISO 5500x suite of standards have captured favourable AM practices from different industries across the world and has defined the minimum of what has to be done to ensure that organizations have an effective AM system. According to ISO (2014), the new suite of standards from ISO allows organizations that are new to the PAM environment as well as organizations with mature PAM systems the following:

1. The setting of AM policy and development of AM plans;
2. To serve as introduction to those new to management systems for assets;
3. To help the experienced with further development; implementation and continual improvement;
4. The aid of service providers in the field of PAM;
5. To aid those seeking conformation to the ISO 55001 standard and other standards;
6. The assessment of the ability to meet legal and other requirements.

To be able to achieve the listed characteristics it is clear that PAM is a cross-functional process requiring the cooperation of different departments within an organization. Since PAM is integrated with so many of functions of an organization, it quickly becomes a complex process. According to BSi (2008) delivering the best value for money in PAM is a complex process which requires that careful consideration be given when looking at the conflicting factors of performance, cost and risk through all the stages of an asset's life cycle. The next two documents discussed provide guidelines on the process of determining the equipment health, knowns as Condition Monitoring (CM).

2.1.1.2 ISO 17359 and ISO 13380

Two other standards that are of relevance to this study are ISO 17359 and ISO 13380. The CM process is important for the purpose of this study and these two international standards provide guidelines on the set-up and conduction of the CM process. Since some of the data needed for this study is obtained through the CM process, it is important to know how this process works. The international standard ISO 17359 provides general guidelines on the CM and diagnostics of machines while ISO 13380 provides more insight into the performance parameters linked to the diagnostics of machines.

The performance parameters or characteristics of the systems should be selected to display the health of the equipment considered. The survival models would generally be populated using the characteristic values as covariates as well as the historical failure data. Since this study aims to validate whether this data can be obtained from experts, the data will be subjective and needs to be validated. Thus, in order to validate the outcome of this study, the results of the subjective data must be compared to that of the objective data. These standards provide insights the characteristics to use as covariates as well as an understanding of the CM process as a whole.

PAM encompasses a spectrum of principles, concepts and processes which help to convert organizational objectives into decisions and actions on assets to aid in achieving the objectives. ISO (2014) states that there is a greater need to understand the AM field because of the pressure applied for the demand of higher output levels and better service performance to be achieved at lower costs. BSi (2008) and ISO (2014) make an effort to explain how a AM system must fit in within the context of an organization.

2.1.2 PAM in an Organization

The PAM system of an organization is required to be cross-functional and, therefore, employees need be well informed of the policies and strategies followed. Several steps should be taken to ensure this. An organization must first set the objectives which it would like to achieve, and then create a strategy of how they plan to achieve these objectives. Next, the different strategies must be considered and the appropriate tactics selected and executed. The application of this affords organizations many benefits and according to ISO (2014) these benefits have not all been discovered as yet.

Organizations with a more mature AM system are able to demonstrate enhanced levels of performance, lower costs, more productive workforces and is seen as more credible by their customers and investors as ISO (2014) and BSi (2008) state. An AM system also forces activities which have the potential to impact the driving factors (asset related risk, performance and costs) to be described and documented adequately to ensure that decisions associated with them are made in a consistent manner.

According to ISO (2014), an AM system should have several attributes in order to be successful. Firstly, it must be achievement oriented. It should be possible to develop AM plans to achieve the goals set by top management and it must be possible to determine the organizational objectives by making use of the AM system. The second attribute of an AM system is that the decisions made must be transparent, requiring that all of the data to support the decisions must be available. The system must also establish accountability and responsibility among the employees in order to provide assurance to management. The last attribute of an AM system is that it must be integrated across the organization in order to allow the use of outside resources for AM plans and activities.

The organizational strategic plan is what determines which activities are to be conducted and in what manner. Figure 2.3 shows how this ties the AM system to other functions within an organization. Buy-in of top management is of cardinal importance for implementation of PAM, since it is to be integrated across just about all of the functions of organizations. The PAM policy must be communicated to all the employees and when passed down from top management, employees are obliged to follow the policy.

Both the BSi (2008) and ISO (2014) state that the PAM function must integrate across the organization with other functions such as financial management, human resources and other management functions. Since PAM has such strong financial connotations, it requires that it be especially closely integrated

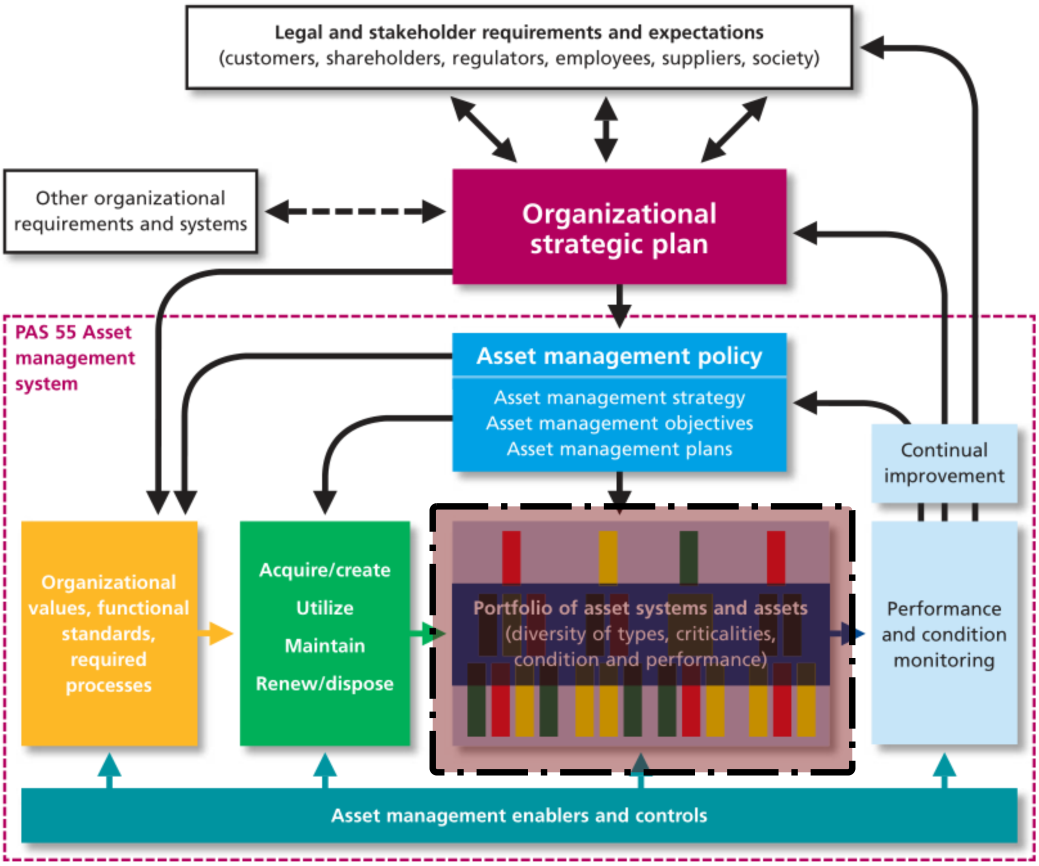


Figure 2.3: Relation of PAM to organization, adapted from BSi (2008).

with the financial management function.

A PAM system, thus, cannot be grouped under any specific functional group in an organization. Consider Figure 2.3, when focusing on the slightly shaded box, the asset portfolio, Figure 2.4 can be presented as a closer look into the asset portfolio. Here, an AM system is displayed in a hierarchical manner emphasizing that top management must be involved in an AM system.

This figure shows the relationship between the two elements in an AM system displayed in middle of the second row from the bottom of Figure 2.3. The four activities listed in the left block are presented in Figure 2.4 at the foundation of an asset portfolio. The triangles are the building blocks of the asset portfolio and represent activities through the life cycle of assets, which aid in their management. The maintenance of assets is a crucial aspect while they are in operation as emphasized in Chapter 1. This study is therefore focused specifically on this maintenance building block.

2.1.3 PAM and Maintenance

The Oxford dictionary defines maintenance as: “the act of causing to continue through repair”. Generally, a piece of equipment is considered to have failed

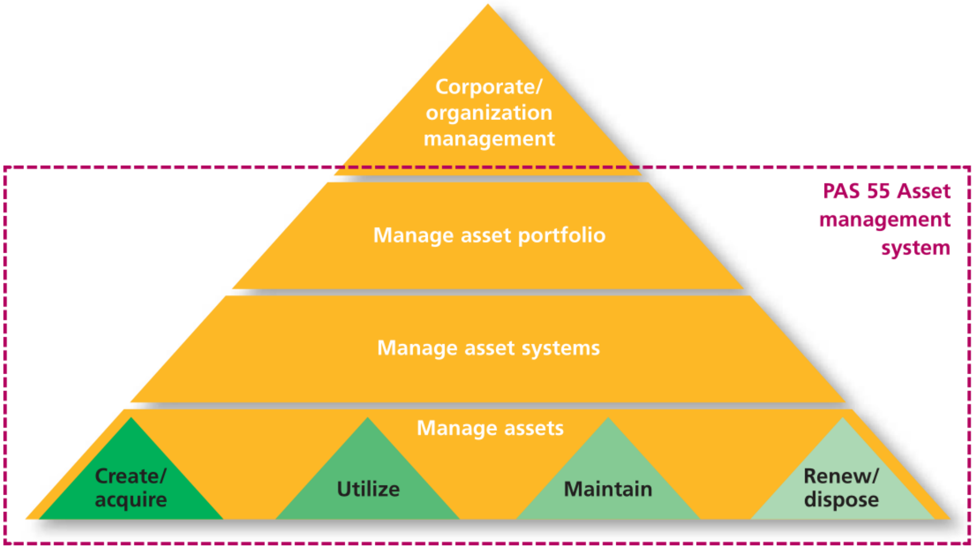


Figure 2.4: Levels of management in AM (BSi, 2008).

if it cannot meet it's functional requirements or if it physically broke down. It is clear that maintenance is the process of keeping the equipment of an organization in an operating condition until the end of its life time. In the past, there has been a misconception about maintenance and PAM.

Previously, maintenance has been used as a synonym for PAM. Maintenance is, however, not the same as PAM. Maintenance is a large subset of PAM and is important to the process but it is not equivalent to PAM. ISO (2014) and Hastings (2009) explain that PAM is much more than the maintenance of equipment, as it includes the activities listed below requiring that it be a complex and integrated system. When trying to explain that maintenance is not equivalent to PAM, the last two activities listed below can be grouped together and seen to be maintenance.

1. Determine appropriate assets to acquire or create.
2. Identify funding requirements.
3. Acquiring assets.
4. Determining how best to utilize the assets and provide logistic and maintenance support systems for the assets.
5. Adopt the optimal renewal or disposal actions of the assets.

ISO (2014) and BSi (2008) make it clear that AM is more than just keeping equipment in a working order. Maintenance has developed considerably over the past century becoming a more complex process than in the past. This evolution is as a result of a combination of the pressure being applied on organizations to produce products quicker, with greater accuracy and with the minimum costs involved as well as the evolution of physical equipment over the past century (Campbel *et al.*, 2011).

Properly maintaining an asset increases its lifespan and improves the probability of smooth and continuous operation of the asset. According to Zhou *et al.* (2007), maintenance optimization has been a popular issue over the last couple of decades and is able to offer organizations significant benefits. Some organizations only look at maintenance costs from a one-dimensional accounting point of view and, therefore, only see it as an expense. Campbel *et al.* (2011) explain that maintenance is seen as a fraction of the manufacturing cost from an accounting point of view, but that does not then consider the costs that will occur if maintenance were not conducted. Maintenance to an organization, from a business perspective, offers more benefits than realized. The correct maintenance strategy (the different strategies are discussed in the next section) will offer the potential to realize the following benefits:

1. Maximize uptime (runtime) of equipment;
2. Maximize the accuracy and quality of manufactured products;
3. Minimize the cost per unit produced;
4. Realize the lowest possible risk involved with the loss of production;
5. Reduce the safety risk to employees;
6. Ensure the lowest possible risk to negatively affecting the environment.

To achieve the organizational objectives and these benefits, the AM system must have a policy that determines the manner in which maintenance is conducted on specific equipment. The AM policy will determine which maintenance strategy must be applied to achieve the objectives. The strategies will then use certain tactics to aid with executing the plans in order to achieve the objectives.

The Oxford dictionary defines strategy as “a plan that is intended to achieve a particular purpose” (Hornby, 2010). A strategy, in the case of this study, can be a plan, method or a series of manoeuvres employed to obtain a particular purpose/goal. A tactic is then the specific method used to achieve the desired goal. One strategy can, therefore, make use of several different tactics. The strategy that an organization chooses should act as a road map in the long term while the tactics consist of different techniques of what has to be done immediately to repair/maintain assets. The maintenance process and its different strategies are discussed in the next section.

2.2 Maintenance

Any equipment used to realize value has to either be replaced or require some activity which enables it to continue operating. The first equipment that had to be maintained were purely mechanical and simple, they would generally only be fixed when they broke. This is simple reactive maintenance. Machines became more complex as technology advanced, integrating more and more electrical components into mechanical systems. The need for maintenance activities on equipment has increased with this advance not only because of the cost of the equipment but also the pressure exerted to increase production.

Manufacturing industries were put under great pressure during WWII which forced maintenance to evolve more towards a proactive strategy. Campbel *et al.* (2011) explain the evolution of maintenance strategies. Maintenance strategies developed further when operations research and reliability studies on equipment delivered results in the $1960's$ and $1970's$. The $1980's$ brought forth an important part of the evolution because computerization and the monitoring of equipment health/condition was developed and started to be used changing the proactive maintenance strategy.

Different maintenance strategies have been developed in the past and according to Campbel *et al.* (2011) they have evolved from the simple equipment used in the early $1900's$ and earlier, to accommodate the complex systems and equipment used today as explained. The three main maintenance strategies are listed as:

1. Life improvement maintenance;
2. Proactive maintenance;
3. Reactive maintenance.

The different maintenance strategies are applicable to different pieces of equipment depending on factors like the cost of failure, downtime costs, etc. Organizations must choose the appropriate maintenance strategy that suits their business plan, risk profile and the selected equipment best.

This study will focus on a proactive maintenance strategy and a combination of the predictive maintenance and the preventative maintenance tactic specifically. Methods are available to help organizations decide on what strategies to deploy, Campbel *et al.* (2011) names Reliability-centred Maintenance (RCM), which they describe as a logical process that can be conducted to do just this.

The RCM methodology can be explained by making use of an entire book, but to keep it simple a brief overview is given. According to Sun (2006) the RCM methodology has been developed over the last 30 years and was first used by the civil aviation industry. This methodology starts with significant functions and failure mode selections, and classifies the failure consequences into four groups, namely hidden failure, safety and environmental, operational and non-operational consequences.

The maintenance decisions are based on the consequences of the failures and their severity. This allows for all objects to be integrated across the organizational structure. This is only one methodology to help decide which strategy to deploy, several others exist in industry. Each of the different maintenance strategies are discussed below. It is stated when they are applicable and why they would be considered as a viable option.

2.2.1 Life Improvement Maintenance

Life improvement maintenance can be defined as: "the maintenance strategy whereby an asset is modified to improve performance, or to eliminate or reduce the consequences of failures". This method is only applicable if the performance of the component is improved and/or the effect of a failure can be minimized by changing the design of the component. It is also a once off activity and will not be part of the everyday maintenance tasks.

After the improvement, the equipment will have better reliability because the design improvement eliminates forcing functions of the root cause of a failure. Thus, the root cause of the failure must be identified in order to improve the design of the equipment. This is known as the design-out maintenance tactic.

This strategy takes a relatively long time because of the re-design of the equipment. Top management is normally also involved because this strategy requires capital investment. Generally, the system or component that is improved by the new design, returns to normal operation and is maintained as per one of the other maintenance strategies. Life improvement maintenance is generally not conducted on equipment or components indefinitely because it improves on the design of something then it returns to operation.

2.2.2 Reactive Maintenance

The earliest form of maintenance was just a run-to-failure tactic, where equipment would be repaired once they broke. Swanson (2001) refers to it as a fire-fighting approach to maintenance and states that temporary repairs are sometimes made just to get equipment in an operating condition postponing the permanent repairs. Reactive maintenance has the advantage of requiring less manpower and money to keep equipment in an operating condition. Disadvantages of this maintenance strategy that makes it unfavourable to many industries include unpredictable and fluctuating production capacity of the equipment, less accurate products, very high repair costs and the possibility of catastrophic failures.

Tactical Corrective Maintenance

Tactical reactive maintenance worked very well with simple and robust equipment, like what dominated the industries in the early $1900's$. Tactical reactive maintenance still has its place in the industry and is often used on less expensive and simple equipment. Campbel *et al.* (2011) explain how the pressure put on the manufacturing industries during WWI and WWII forced them to increase their output and run time of their machines, resulting in popularity for both preventative and planned maintenance. Careful thought is, however, needed when considering tactical reactive maintenance because if the equipment has a high repair/replacement cost or the repair/replacement time of a failure is very lengthy, it could be cheaper to apply a different maintenance strategy.

Non-tactical Corrective Maintenance

It is important to realize that tactical reactive maintenance is not identical to an absence of maintenance also known as non-tactical reactive maintenance. Tactical reactive maintenance is a strategic approach which consciously allows equipment to run until a failure occurs. Therefore, characteristics like the repair/replacement time and cost must be considered as well as the downtime costs. This requires that arrangements be made prior to a failure occurring, which may include how to deal with the failures, the keeping of spare parts in stock, etc.. It is a tactical decision made to optimize the maintenance process.

When maintenance is ignored and completely absent, no prior arrangements would have been made regarding the repair of the asset. The cost of the equipment or its repair/replacement time and costs would not have been considered when choosing not to maintain it. The absence of maintenance has no strategic or other advantage, therefore, it is not considered as a maintenance strategy in this study. It does, however, fit into the maintenance hierarchy under non-tactical reactive maintenance.

2.2.3 Proactive Maintenance

The proactive maintenance strategy aims to conduct maintenance activities before a failure occurs, therefore, requiring acting pro-actively and preventing the failure from occurring. There are two main proactive maintenance tactics are the preventative maintenance and the predictive maintenance tactics. Both tactics act pro-actively unless the failure is random and unexpected. Zhou *et al.* (2007) find that predictive maintenance is costly but has the potential to provide organizations with larger advantages than preventative maintenance. Swanson (2001) states that the proactive maintenance strategy requires more commitment in terms of training, resources and integration than the reactive maintenance but it is expected to deliver higher levels of plant and equipment performance.

2.2.3.1 Preventative Maintenance

Preventative maintenance is sometimes referred to as usage-based maintenance because the maintenance activities are scheduled according to how the much equipment has been used. According to Swanson (2001), preventative maintenance assigns a predetermined lifespan to equipment and then, blindly replaces the equipment at the specified time irrespective of its current condition.

Preventative maintenance can further be divided into two main techniques applied to achieve the preventative tactic. These two techniques are referred to as a history/time based technique and a predetermined technique. The predetermined technique involves replacing or maintaining a system or component once it has reached a specified limit. This limit is generally determined by considering how much the system or component has been used, hence the name usage-based maintenance. This limit is often measured in units like operating hours, distance travelled, amount of product moved, etc. It all depends on the function of the system or component considered.

The other technique, history/time based, considers the historical failure times of the system or component considered. These failure times are then used to create data sets to use in statistical models which estimate the reliability of the system or component at a specified time. These models can also be used to calculate the time of the next expected failure of the equipment. Thus, the historical failure times are used without any other covariates to predict the reliability of the equipment considered.

The preventative maintenance tactic often allows healthy equipment to be replaced, but the probability of failure before the scheduled replacement is also possible. Unexpected failures can be because of harsh or unexpected operating conditions or faulty equipment. Lee *et al.* (2006) describe how preventative maintenance then becomes an ineffective tactic, which squanders organizations' maintenance budget and time.

Preventative maintenance, thus, uses knowledge obtained from experience and historical failure data, to decide when to conduct maintenance activities. The immediate condition of the equipment is not considered. In industry, the historical failure data can be hard to obtain because the record keeping is often neglected and creates difficulty with the maintenance of equipment. Therefore it is desirable to know the current condition of equipment. This is not done by analyzing the historical failure times as it will become clear in the following maintenance strategy.

2.2.3.2 Predictive Maintenance

Hecht (2006) discusses how equipment have evolved from being purely mechanical to a combination of electrical and mechanical. Since electronic components have a high infant mortality rate, the failure probability curve of most equipment was altered as electronics and mechanical systems were integrated more.

The integration of electrical and mechanical components caused a change in the failure characteristics of the equipment and this, together with the pressure for increased production, brought forth the need for CM.

Monitoring the current state of equipment has become a large industry and is why the ISO created ISO 17359 (a standard for CM procedures). The process of monitoring the equipment health is known as CM. CM determines the current state of health that equipment is in through the monitoring of the equipment behaviour and the analysis of recorded diagnostic data and/or samples.

The data to be analyzed can range from the smell of the equipment to the amount of current a piece of equipment is consuming. The data is normally recorded from what is known as condition monitoring equipment. Condition monitoring equipment is the specific type of equipment used to take the readings/samples, which are then analyzed using the appropriate techniques. The current state of a machine is determined analyzing CM data and creating degradation signals. The degradation of machines and the evolution of the degradation are used to make failure predictions.

There are many CM techniques, including activities such as lubricant analysis, vibration analysis, thermography, penetrating liquids, radiography, ultrasound, the control of corrosion, etc. Ma (2007) states that when CM is considered, a continuous process that must constantly be updated and reviewed becomes an investment to organizations. If this is not implemented, it can easily end up costing the organization money while offering no or poor return because of the costs involved with the equipment and personnel.

CM has certain requirements in a modern PAM system as described in Ma (2007):

1. Predict time-to-failure (RL) and the probability of failure in the future;
2. Predict failure with greater accuracy;
3. Predict not only failure modes but also health degradation processes;
4. Predict equipment failure using a holistic view;
5. Improve equipment reliability and performance;
6. Improve data quality.

Generally, organizations would use CM to prevent failures by identifying equipment that are at risk of failure and scheduling maintenance activities to result in the smallest delay in the production schedule as emphasized by Ma (2007). The failure patterns of electrical and mechanical equipment differ and have profound effects on the maintenance strategy that will be used. Figure 2.5

displays all the failure probability curves from purely mechanical systems to the evolved electro-mechanical systems and electrical systems (Campbel *et al.*, 2011).

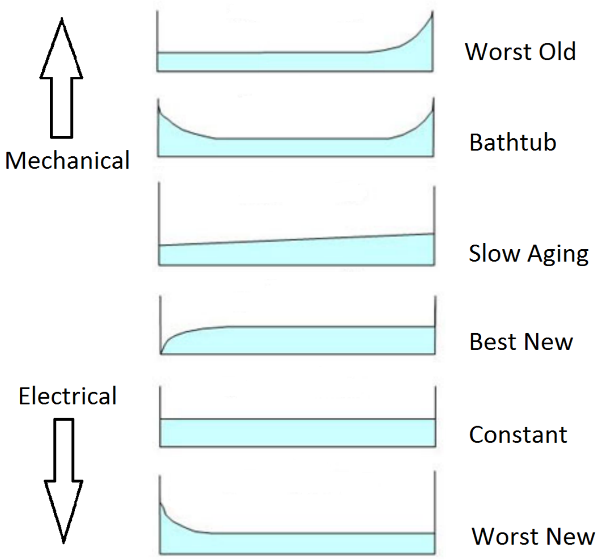


Figure 2.5: Different failure patterns.

The top three curves in the Figure 2.5 represent systems that are largely mechanical. These three failure mechanisms only account for 11% of failures in industry. The failures can be anticipated and random failures are not a common event, scheduled maintenance activities proved to be very effective for this type of equipment. Thus, preventative maintenance reigned as the preferred maintenance tactic when the equipment used had very little electronic components but Hecht (2006) mentions how equipment has evolved.

Electronic technology developed very quickly and the integration of electronic and mechanical systems became common practice and inevitable. With electronics, the probability of failure is generally higher in the beginning of its life. After that period has passed, failures occur randomly, which make it difficult to statistically predict their occurrence. These random failures require a tactic that will detect early signs of a failure to prevent them from occurring.

This is why the predictive maintenance gained popularity seeing that the tactic can detect abnormalities early on. Degradation signals recorded by CM equipment are analyzed with the aim of predicting when a failure might occur. The analysis of the data gathered reflects the current health/state of the equipment. Thus, monitoring and then acting on this data is a large part of the predictive maintenance process.

Zhou *et al.* (2007) state that predictive maintenance is also called Condition-based Maintenance (CBM), because it takes into consideration the current condition (or health) of the equipment it is applied on. Unlike preventative maintenance, components are only replaced when it is necessary, so no money is wasted on replacing healthy components. The equipment used to determine the current state/health of equipment consist of a variety of different technologies known as CM equipment. This CM equipment and the CM data are the foundation of predictive maintenance.

According to Carnero (2005), predictive maintenance can provide organizations with powerful advantages in connection to product quality, safety, equipment availability and cost reduction. These advantages include:

1. Increase in equipment availability;
2. Reduction of direct maintenance costs;
3. Lower costs in relation to spare parts and labour;
4. Information available to management to aid in decision making;
5. Improved safety;
6. Reduced energy consumption.

Predictive maintenance was developed to help drive down maintenance costs while still ensuring equipment reliability. It is costly because of the technology and training needed but worth-while. It is, therefore, usually applied on more expensive and/or critical equipment or components. Campbel *et al.* (2011) explain that the costs are driven down by understanding the past through the analysis of CM data and the tracking of the signals observed. Databases which are analyzed to determine the equipment health consists of CM data like vibration signals, lubricant analyses or thermographic readings taken from equipment. These are only some of the signals or samples tracked to better understand the failures of equipment as mentioned by Carnero (2005). Predictive maintenance does not consider the historical failure data of equipment; it makes use of the CM solely.

Reliability estimation and prediction is a relatively complicated and computationally intensive process when only considering the CM data as explained by Xu *et al.* (2008). Recent advances have allowed neural networks to be used for predicting failures by considering the CM data. This, however, requires the artificial intelligence to first be trained with previous data sets as mentioned by Mann *et al.* (1995). Solely using the CM data for failure prediction can be done for short term predictions as Lu *et al.* (2007) mention, but estimates might become excessively inaccurate for long term predictions.

When considering CM data and historical failure data, it is possible to predict when failures might occur of the equipment being monitored more accurately, than when they are considered separately. This is done by making use of a field that combines preventative and predictive maintenance tactics. This field is a middle ground between preventative and predictive maintenance. The field that considers both historical failure data as well as CM data is known as prognostics.

2.2.4 Prognostics

Prognostics is an engineering discipline utilizing both preventative and predictive maintenance techniques to conduct reliability analysis allowing the estimation of equipment reliability. Reliability analysis is more commonly known as survival analysis because of its origins from the medical discipline (Zacks, 1992). According to Singh and Mukhopadhyay (2011), survival analysis is a collection of statistical procedures used to determine the outcome variable, which is the time of an event. In the case of this study, the outcome variable is then the time at which a failure might occur.

Prognostics make use of CM data as well as historical failure data to populate survival models which estimate the reliability of monitored equipment as explained by Lee *et al.* (2006). This allows the RL of the equipment to be predicted. Predicting the RL can allow unnecessary down time, costs and possible failures to be avoided as Zhou *et al.* (2007) mention, thus, acting as an integral part of the proactive maintenance strategy. Prognostic survival models generally use the historical failure data to establish some baseline function describing the reliability of the asset under study. The CM data is then used as covariates to update the baseline function as new data is obtained. In Figure 2.6 it is mapped out, in a simplified figure, how the prognostics field fits into the proactive maintenance strategy.

The RL of equipment is important for the cost optimization of the maintenance strategy and according to Zhou *et al.* (2007) it also aids with improving the system safety. To maximize the runtime of equipment and minimize unexpected failures, the optimum replacement period must be calculated, but replacing healthy equipment or components should be avoided to save costs. In order to optimize the proactive maintenance strategy and the activities which it entails, it is necessary for the maintenance teams to conduct the activities at strategic times. The time to conduct these activities can be estimated using survival models which estimate the reliability of equipment considered.

Zhou *et al.* (2007) also mention that unscheduled maintenance actions are always more costly than scheduled maintenance actions. Thus, being able to plan in advance allows organizations to execute the maintenance actions more cheaply and plan shut downs. Estimating downtime costs can be of great benefit to the decision making of maintenance actions. Knowing the RL of equipment allows for downtime to be minimized by planning the maintenance

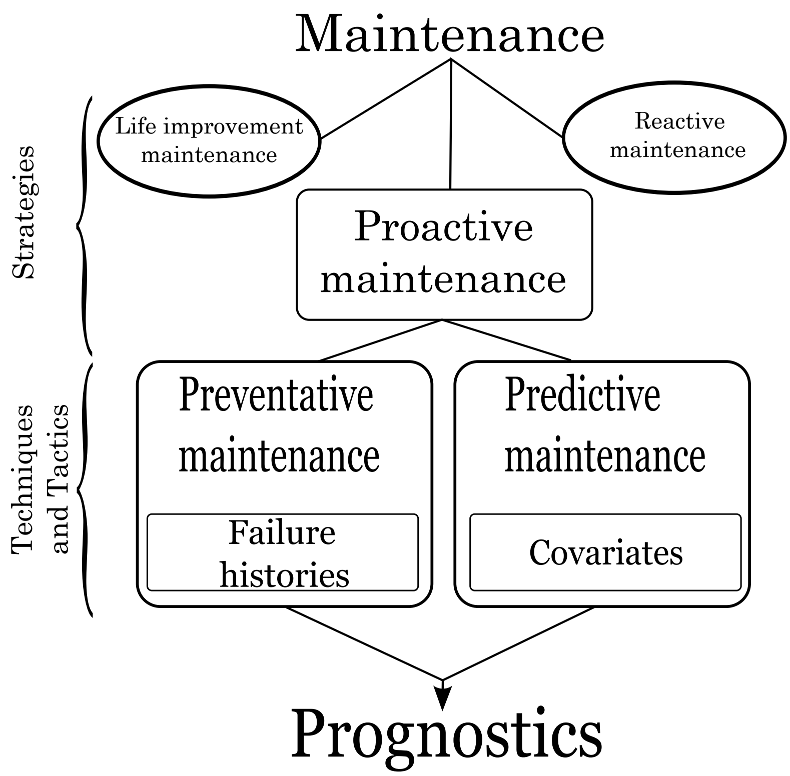


Figure 2.6: Prognostics in maintenance

actions. According to Pascual *et al.* (2008), estimating the downtime costs offer several advantages among them:

1. The measurement of the impact of equipment on system efficiency;
2. The assessment of maintenance policy effectiveness;
3. Mathematical models can be applied in the decision-making contexts (to help decide on aspects like maintenance strategies; replacement policies, spare stock levels, etc.).

Ma (2007) observes that there are a variety of survival models available, but an appropriate model must be chosen for each individual case. The next section discusses the basics of survival analysis while Section 2.4 reviews possible survival models to use in this study.

2.3 Survival Analysis

The basics of survival analysis are first presented and then the possible survival models are reviewed and their advantages and disadvantages discussed. These models can all be used to estimate the reliability of systems or components as done in the reliability analysis field of prognostics. The purpose of reviewing the different models is to become familiar with the models and weigh-up the advantages and disadvantages of each one and to allow the correct model to be selected for this study. It is also necessary to know what type of system data is being considered, the two system types are introduced.

2.3.1 Repairable and Non-repairable Systems

Pijnenburg (1991) defines a system as a collection of one or more components which are interconnected to perform specific functions. There are two system types which exist, namely repairable and non-repairable systems. When a system fails there are two possibilities, either the entire system is replaced by a new system or the old system is repaired.

A repairable system can be seen as a system that is repaired when a failure occurs. Only the components which caused the failure to occur or that failed are replaced, thus repairing the system. A non-repairable system is removed and replaced by a brand new system after the occurrence of a failure; the system is thus not repaired only replaced. There is also a difference in the data of the two system types.

The time between events of repairable systems are dependent and exhibits an underlying trend. For non-repairable systems, the time between events are independent and identically distributed (i.i.d) and hold no underlying trend. This study utilizes two different model types to model the repairable and non-repairable systems' hazard, namely the Non-homogeneous Poisson Process (NHPP) and Homogeneous Poison Process (HPP). According to Carstens (2012), the HPP is used to model non-repairable systems and NHPP for the repairable systems. With this is also the assumption of the state at which a system is after the maintenance actions are conducted. Birolini (2007) finds that the HPP is a particular case of a Renewal Process (RP) which returns a system to a "Good as New" state while the NHPP returns the repairable systems to somewhere between a "Bad as Old" and a "Good as New" state. The trend test tests the renewal assumption; when no trend is present the assumption that the data is i.i.d is confirmed and a HPP is used.

Different trend tests such as the Lewis-Robinson test, the Mann test, the generalized Anderson-Darling test or the Laplace trend test are available and any one of them can be used. The Laplace trend test is made use of in this study because of its ease and the researcher's past experience with the test. Consider a data set where T_i is the discrete event time measured in global time and r is the total number of observed events. The Laplace trend has an outcome that is calculated by;

$$U = \frac{\dfrac{\sum_{i=1}^{r-1} T_i}{r-1} - \dfrac{T_r}{2}}{T_r\sqrt{\dfrac{1}{12(r-1)}}}. \tag{2.3.1}$$

A data set is considered to have an underlying trend when $U \leq -2$ or when $U \geq 2$, no trend is present for cases where $-1 \geq U \leq 1$. The cases where $-2 \geq U \leq -1$ or $1 \geq U \leq 2$ the test is not able to confirm whether a trend is present or not and is considered a grey area. When no trend is present the data is considered to be from a non-repairable system and when a trend is present it is considered to be from a repairable system (Vlok, 2014). A system is said to have an improving reliability when $U \leq -2$ and a reliability degradation for the cases when $U \geq 2$.

It should be noted that the hazard function or also known as the Force of Mortality (FOM) is used when modelling non-repairable systems. Repairable systems, however, do not make use of the FOM because of the practical implications; instead the Rate of Occurrence of Failure (ROCOF) is used. The following section introduces the relevant functions that are needed to conduct proper survival analysis.

2.3.2 Relevant Functions and Data

There are several functions that are very important in survival analysis. The survival models are all based on these equations in some form and assumptions made for the relative models are generally based on some assumption of these functions. The first function discussed is the probability density function.

2.3.2.1 Probability Density Function and Cumulative Density Function

The probability density function (PDF) provides the probability of system or component failure at a specific time instant $P(x)$. Balakrishnan and Rao (2004) state that this function can take the shape of any one of the various parametric families later discussed. When integrating the PDF ($f(x)$) with respect to the time x, the probability of a system or component failure before time x is given. This function is known as the Cumulative Distribution Function (CDF) and indicated by $F(x)$.

$$F(x) = \int_0^x f(x)dx. \tag{2.3.2}$$

The CDF $F(x)$, or the probability of failure, is the probability of a system or component failing before the end of the time interval x. Therefore $F(x)$ is equal to $P(X \leq x)$, where X is time of failure. The survival function is dependent of the PDF, thus also the CDF.

2.3.2.2 Survival Function

All of the survival models (or more commonly known as reliability models in the engineering field) express the degree of degradation of the equipment which they are applied to in some form of the survival (or reliability) function of the equipment. The survival function is of fundamental importance to prognostics.

According to Todinov (2005), the survival function provides the probability that a system or component will operate without failure over a specified time interval $(0, x)$.

The survival function is then $R(x) = P(X > x)$. Zacks (1992) explains how the relation between reliability and the probability of failure is as presented:

$$R(x) = 1 - F(x). \tag{2.3.3}$$

This leads to another important function in survival analysis, the hazard function.

2.3.2.3 Hazard Function and Rate of Occurrence of Failure

The hazard function ($h(x)$) is also known as the instantaneous failure rate or the FOM. It would be ideal to use it for both repairable and non-repairable systems. Repairable systems, however, make use of ROCOF instead of the FOM because of the impractical implications it creates when using it with recurrent events (Vlok, 2014). According to Rinne (2014), the hazard function is often more informative about the underlying mechanism of failure than other representatives of a lifetime distribution.

A hazard function can be seen as a measure of risk, thus, the greater the hazard the greater the risk of failure. Rinne (2014) explains that the hazard rate is not a density function since it is normalized, and the integral of the hazard rate from zero to infinity will tend to infinity. The hazard rate can be explained by using the concept of conditional probability.

Have A and B be two random events, the $P(A) > 0$ and the probability of the conditional event $B|A$ (occurrence of event B given that event A has already occurred) can be given by Equation 2.3.4

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \tag{2.3.4}$$

where $A \cap B$ means that both events occur simultaneously. Therefore, the probability of event B occurring given that event A has already occurred is equal to the probability of events A and B occurring simultaneously divided by the probability of event A occurring on its own.

The hazard rate can be seen as the probability of the failure time X being within an infinitesimally small interval $\triangle$ of the current time x, thus $P(x < X \leq x + \triangle | X > x)$ where $\triangle$ tends towards zero. Simply stated, the hazard rate is the instantaneous rate of failure for assets aged x. According to Rinne

(2014), this can be represented as:

$$\begin{aligned} h(x) &= \lim_{\triangle \to 0} f(\triangle | X > x) \\ &= \lim_{\triangle \to 0} \frac{f(x+\triangle)}{R(x)} \\ &= \frac{f(x)}{R(x)}, R(x) > 0. \end{aligned} \tag{2.3.5}$$

A relation between the survival function and the PDF is thus known. The hazard rate can be also be interpreted as the rate at which failures occur per unit of time relative to the portion of the subjects included in the study which have not yet failed. The hazard measures the disposition of a system to fail at a specific age reached. This function allows deeper insight into the survival characteristics of the equipment.

The cumulative hazard function $H(x)$ is the integral of the hazard rate. The survival function of equipment can also be related to the cumulative hazard function. The relationship is presented as:

$$H(x) = -\log(R(x)). \tag{2.3.6}$$

The ROCOF of repairable systems is defined as the derivative of expected number of failures relative in terms of the time to the event. Carstens (2012) presents it as follows:

$$\text{ROCOF} = \frac{d}{dx}\text{E}[N(x)], \tag{2.3.7}$$

where $N(x)$ is the number of failures that occurred in the time interval and $\text{E}[N(x)]$ is the expected number of failures on the same time interval $(0, x]$. The format of the NHPP will determine the form of the ROCOF. According to Krivtsov (2007), in reliability analysis, the two common formats are the Log-linear NHPP and the Power Law NHPP presented below.

Log-linear NHPP

$$\begin{aligned} \text{E}[N(x_1 \to x_2)] &= \frac{1}{\alpha_1}[\exp(\alpha_0 + \alpha_1 x_2) - \exp(\alpha_0 + \alpha_1 x_1)] \\ \text{ROCOF} &= \exp(\alpha_0 + \alpha_1 x) \end{aligned} \tag{2.3.8}$$

Power Law NHPP

$$\begin{aligned} \text{E}[N(x_1 \to x_2)] &= \lambda(x_2^\delta - x_1^\delta) \\ \text{ROCOF} &= \lambda\delta x^{\delta-1} \end{aligned} \tag{2.3.9}$$

The variables of the functions are generally estimated by making use of the maximum likelihood estimate (MLE) method or by minimizing the residual between the number of failures and the expected number of failures. The next section introduces more relevant functions used to predict the equipment's residual lives.

2.3.2.4 Residual Life

The RL of equipment can be estimated once the PDF, $f(x|\mathbf{z})$, is known. The PDF can be determined by rearranging Equation 2.3.5. The hazard and the reliability functions must also be the conditional functions and not the baseline functions.

$$f(x|\mathbf{z}) = R(x|\mathbf{z})h(x|\mathbf{z}) \tag{2.3.10}$$

In the case of non-repairable systems, the expected time of failure for a system is defined by Montgomery (2013) as:

$$\mathrm{E}(X) = \int_0^\infty x \cdot f(x|\mathbf{z})dx. \tag{2.3.11}$$

The RL is can then be calculated by $\mathrm{E}(X) - x$, the RL is indicated by μ_{r+1}. The RL for repairable systems can be represented in terms of the updated estimates of the parameters of the NHPP. According to Vlok (2014), this is only for a special case; when the last observation is a recorded failure, this final failure time is indicated by T_r. The calculation will again depend on which NHPP format is chosen. The time of the next expected failure for the Log-linear NHPP is given by:

$$\mathrm{E}(T_{r+1}|t = T_r) = \frac{\log((r+1)\hat{\alpha}_1 + \exp(\hat{\alpha}_0)) - \hat{\alpha}_0}{\hat{\alpha}_1}, \tag{2.3.12}$$

where $\hat{\alpha}_0$ and $\hat{\alpha}_1$ are the estimates of the parameters obtained and r is the number of failures recorded. The time of the next expected failure for Power Law NHPP can be calculated by:

$$\mathrm{E}(T_{r+1}|t = T_r) = \left(\frac{1 + \lambda T_r^\delta}{\lambda}\right)^{1/\delta}. \tag{2.3.13}$$

The RL is then simply $\mu_{r+1} = T_{r+1} - T_r$, again λ and δ are the parameter values obtained. Once geared with these equations, consideration should be given to what data is at ones disposal and what approach is to be taken to fit some selected survival model to the data sets considered. The data required to populate survival models need to contain certain information and is generally in a specified format; this is discussed next.

2.3.2.5 Data Set and Censored Cases

The typical data set used for the purpose of survival analysis must contain three parameters. The first parameter is the discrete event time, where an event includes preventative maintenance actions, failure replacements and censored observations. Thus, an event time is the time at which an event occurred that has an effect on the survival time of the equipment considered; this time can be either global or local time. Figure 2.7 illustrates the local and global time graphically.

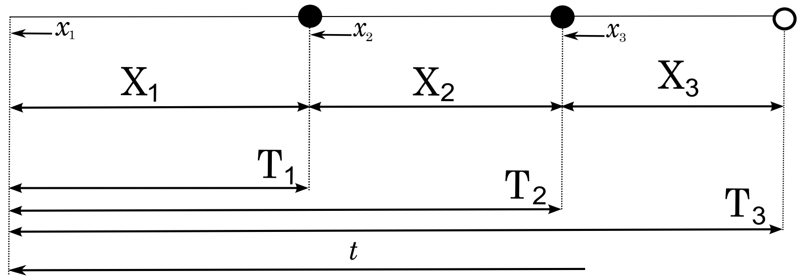


Figure 2.7: Local and global time.

The continuous local time is represented by x_i and the discrete local event time by X_i. The discrete global event times are illustrated in Figure 2.7 by T_i, while the continuous global time is given by t. Generally, the local times are provided in the data sets since it is very easy to calculate the global times from the local time. The local time can just as easily be calculated from global times, the local and global event times are related by $X_i = T_i - T_{i-1}$.

The second parameter required for survival data is the event indicator (C_i), this parameter indicates whether the recorded observation occurred at a failure or if it was a censored observation. Generally, this is a binary indicator where 1 represents a failure and 0 a censored observation. Data is considered to be censored when the event indicator is zero, therefore, not at a failure. In reliability analysis, there are generally three different types of censoring. According to Carstens (2012), these different censoring types can be listed and explained as:

1. Drop out: The equipment fails during the study, where the failure time and the starting time of operation is known, seen as an uncensored observation.

2. Termination of experiment: The equipment is still operational at the time of recording the covariates or at the end of the study; this is considered a censored observation.

3. Follow-up: The equipment was put into operation after the initiation of the study and fails before the completion of the study, also a censored observation.

Knowing whether the data set being considered has censored data is important. The likelihood function of the different models has to be altered to be able to accept the censored data. Likelihood functions are used to estimate the different parameters of the survival functions and are discussed later.

The final parameter that survival data requires is the system characteristic values (z_i), also known as the covariate values. They are the relevant CM readings which reflect the current state or health of the equipment considered.

Any number of covariates can be recorded simultaneously. A typical data set will have some sort of tabular form and include the event number as well as these three required parameters.

An example of a data set will generally be more or less in the format of Table 2.1, where the first column is the event number and the second column is the discrete local time of the events. The third column contains the values read of the first covariate. This extends over p columns where p is the number of covariates being recorded. The final column is then the event indicator, indicating whether the readings were recorded at a failure or not.

Table 2.1: Example data set format.

Event #	$\mathbf{X_i}$	$\mathbf{z_1(X)}$	...	$\mathbf{z_p(X)}$	$\mathbf{C_i}$
1	50	10	...	63	1
2	20	2	...	78	0
3	37	45	...	50	1
.	.	.	...	.	.
.	.	.	...	.	.
.	.	.	...	.	.

The covariates that are included as part of the data set are now explained further, elaborating on what readings they might consist of.

2.3.2.6 Covariates

In statistics, there are generally two main types of variables, dependent and independent. A dependent variable portrays the effect or is sometimes tested to verify if it is the effect while independent variables portray the inputs or the causes. In the field of statistics, the dependent variable is the event expected to change when the independent variable is modified as explained by Gujarati (1995).

Independent variables are also known as explanatory variables; when these variables are thought to have a possible influence on the outcome of a study they can be referred to as covariates. Therefore, in the case of this study, covariates would be considered as the CM data as well as the historical failure data, since it is expected that they have an effect on the RL of physical assets. The variables that are used as covariates can be either time dependent or time independent. When the covariates are time dependent, care must be taken when using them in the survival models since they can complicate the process of estimating the parameter values. The covariates are considered as being time dependent for the case of this study.

The CM process will provide the covariates needed for this study. Gorjian *et al.* (2010*b*) state several elements that can be used as covariates. This study will make use of subjective knowledge gained from experts, thus, the expert will be selected based on the knowledge they possess on the covariates and the selected equipment. CM systems make use of instrumentation which are permanently installed, semi-permanent as well as portable. ISO (2003) provides guidelines for which parameters should be measured for specific types of machines. Generally, covariates in the PAM environment consist of factors measured by the CM equipment as part of a proactive maintenance strategy. The factors listed below are those most commonly used but are in no way representative of all the different factors available for monitoring as ISO (2003) emphasizes:

1. Vibration readings;
2. Temperature readings;
3. Oil analysis;
4. Crack propagation and initiation;
5. Noise/Sound readings;
6. Electrical current readings.

It is important to choose the correct covariates to investigate because covariates which have no effect on the outcome are of no use. A criterion for selecting the appropriate covariates have been made for several different studies; often a correlation study is done after a certain model is conducted to identify the important covariates.

According to ISO (2002), the parameters that are to be selected as covariates are those which will indicate a fault by either increasing or decreasing their overall measured value. A guideline for selecting the covariates to include in a study is discussed in detail in Chapter 3. The covariates are considered to be time dependent and all the models are adapted accordingly. The following section describes the approaches to fitting survival models to data.

2.3.3 Fitting Models to Survival Data

When fitting survival models to the data sets, there are three approaches employed, namely parametric, semi-parametric and non-parametric. The differences in the approaches are now briefly discussed.

2.3.3.1 Parametric

According to Cox (2006), the parametric approach to fitting the survival models makes the assumption that the data considered is from a certain type of probability distribution. In the case of many survival models, this means a specific functional form of the hazard function is assumed since it depends on the PDF. Parametric methods are capable of delivering more accurate results than non-parametric and semi-parametric models but only if the assumptions made are correct.

Anderson and Keiding (2006) argue that since parametric models depends on the ability to fit the data and not as much the deep physical motivation, it is important to confirm the adequacy of the selected model. Parametric models have the important advantage of simpler methods of estimation and inference considering the likelihood function. The likelihood function is maximized in order to estimate the necessary parameter values.

2.3.3.2 Non-Parametric

Non-parametric techniques do not rely on any assumptions that data is from a specific distribution as mention by Anderson and Keiding (2006). This means that no assumptions are made about the distribution parameters of the survival data. A non-parametric approach has its main focus on estimating the regression coefficients of the selected survival model and no estimation of distribution parameters are necessary. However, not making these estimations leaves the relevant function unspecified (Cox, 1972). This approach is favoured when a small amount of data is available and when it is suspected that the data might be distributed in an unusual manner.

2.3.3.3 Semi-Parametric

The semi-parametric approach is a compromise between parametric and non-parametric models. Anderson and Keiding (2006) explain that this approach uses the rigid structure of parametric models while accessing some of the flexibility of the non-parametric models. There is no strict definition for semi-parametric models, thus, any model that is not fully parametric is considered as semi-parametric.

Survival models are developed in a manner that will cause them to be classified as non-parametric, semi-parametric or fully parametric. The models have also been adapted, so if it was originally developed as a non-parametric model, for example, another or the same researcher might have developed extensions of the model allowing it to be solved semi-parametrically or parametrically (Balakrishnan and Rao, 2004).

Different survival models are suitable for different applications. These applications and the extensions of the models are discussed in Section 2.4. This section will also provide methods for establishing when which models are customarily applied. Choosing the model most applicable in a specific case is discussed in Chapter 3. The distribution families generally used in reliability analysis are discussed in the following section.

2.3.3.4 Parametric Families

In an attempt to deliver more accurate results, assumptions are made in the parametric and semi-parametric cases about the distribution of the data. Generally, the hazard is assumed to belong to one of the distributions reviewed below. These distribution families are parametric in the sense that they have parameters that must be estimated to describe the distribution's characteristics.

Reliability analysis in the engineering field has several general parametric PDF's, referred to as distribution families (hereafter) that are commonly used to describe the failure characteristics of assets as Anderson and Keiding (2006) state. Most survival models require these models to represent the distribution of the system/component hazard. The popular PDF's of the different parametric families are listed:

1. Exponential Distribution
   $f(x) = \lambda \exp(-\lambda x) \qquad \text{for } x \leq 0,$
   with λ known as rate parameter. This distribution is used when the hazard can be assumed as constant and is why this family is often not applicable. When $\log(R(x))$ plotted versus time yield a relatively linear plot, the exponential distribution can be considered as an appropriate distribution to use. This distribution is generally used when a constant hazard for a system is desired.

2. Gamma distribution
   $f(x) = \dfrac{x^{k-1}\exp(\frac{x}{\theta})}{\theta^k \Gamma(k)} \qquad \text{for } x > 0 \text{ and } k, \theta > 0,$
   with k known as shape parameter and θ as the scale parameter. This is a more flexible distribution than most others and can be used to represent a mixture of exponential distributions.

3. Log-normal Distribution
   $f(x) = \dfrac{1}{(2\pi)^{0.5}\sigma x} \exp\left[-0.5\left(\dfrac{\log x - \mu}{\sigma}\right)^2\right] \qquad \text{for } x > 0,$
   with μ known as location parameter and σ as the scale parameter. If log of the survival times assume the shape of a normal distribution, this is the appropriate distribution to consider.

4. Log-logistic Distribution
   $f(x) = \dfrac{\delta\lambda(\lambda x)^{\delta-1}}{[1+(\lambda x)^{\delta}]^2} \qquad \text{for } x > 0 \text{ and } \delta, \lambda > 0,$
   with δ known as the shape parameter and λ as the scale parameter. If the $\log\left(\dfrac{R(x)}{1-R(x)}\right)$ versus $\log(x)$ is relatively linear this is likely the distribution that should be used.

5. Weibull Distribution
$$f(x) = \frac{\lambda}{\eta}\left(\frac{x}{\eta}\right)^{\lambda-1} \exp\left(\left(\frac{x}{\eta}\right)^{\lambda}\right) \qquad \text{for } x \leq 0,$$
with λ known as the shape parameter and η as the scale parameter. This is a very flexible distribution and has been used extensively to model physical equipment in reliability analysis. If $\log(-\log(R(x)))$ versus the $\log(x)$ is more or less linear, the Weibull distribution is likely to be the appropriate distribution to utilize.

These distribution families have all been used regularly in the field of survival analysis. They can be used to develop different parametric or semi-parametric versions of the survival models discussed.

2.4 Survival Models

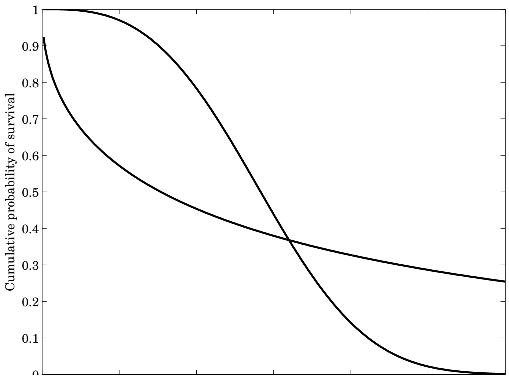
The models that were chosen to be reviewed were those that were the most common in reliability analysis literature, have been validated by preceding literature and can be used with the data sets for this study. These are the models that reappeared consistently in literature when researching survival analysis. The first model reviewed is arguably the most popular one and has been widely utilized over the past two decades.

2.4.1 Proportional Hazards Model

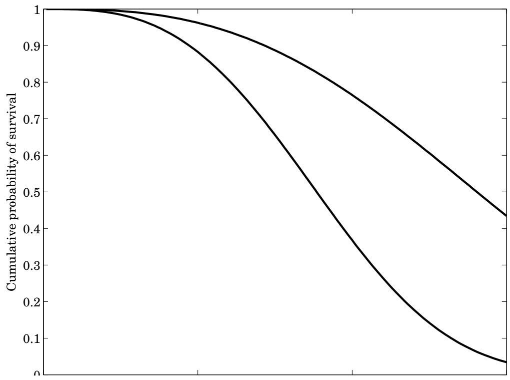
The Proportional Hazards Model (PHM) (also known as the Cox regression model) was originally developed in 1972 to analyze survival data in the medical discipline. After its success in the medical discipline, researchers adopted it for reliability analysis applications as mentioned by Carstens (2012). This model has dominated the survival analysis field for the past 20 years and has been utilized extensively in various applications. The model is meant to model non-repairable systems but extensions of the model have been developed to enable it to handle multiple events; one such model is discussed later.

This semi-parametric model is easy to use and the results are easily interpreted. The model can also be solved in a fully parametric manner should the assumptions made about the baseline hazard function be valid. The semi-parametric PHM makes no assumptions concerning the distribution of the underlying hazards function by leaving it unspecified. The parametric model specifies the distribution which the baseline hazard belongs to.

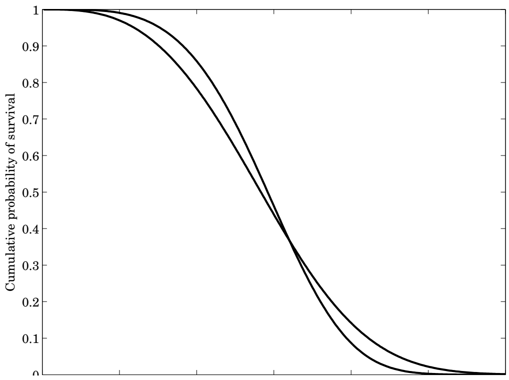
Machin *et al.* (2006) clarify the principle of the PHM as an assumption made that the hazard function of the equipment being considered change in proportion to an underlying hazard function. When the underlying hazard function and the specific hazard function do not change in proportion to one another, the PHM will yield inaccurate result and is not valid. Figure 2.8 illustrates graphically how two survival functions could look when their hazards are proportional and when not. When proportional hazard functions are plotted, the new hazard function is shifted some proportion above or below the baseline hazard function. The two figures, a) and c), of Figure 2.8 display survival functions when the hazard functions do not change in proportion to one another and the PHM would, therefore, not be applicable. The survival curves on the right represent systems where the proportional hazard assumptions are valid and the PHM would, therefore, be applicable.



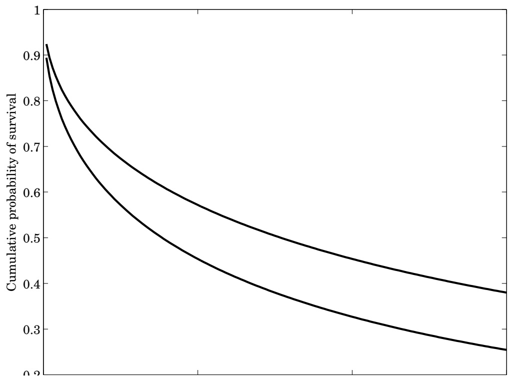
(a) Non-proportional hazard 1



(b) Proportional hazard 1



(c) Non-proportional hazard 2



(d) Proportional hazard 2

Figure 2.8: Survival curves.

The Cox model can be represented by Equation 2.4.1, where the underlying hazard function is (the general hazard function for the component or system being considered) denoted by $h_0(x)$ while the specific hazard function is denoted by $h(x|\mathbf{z}(x))$. The hazard function of a specific component/system is described by the Cox regression model. Operating conditions like temperature, vibration readings, etc. are incorporated in the model as covariates which are included as a row vector $\mathbf{z}(x)$. The corresponding parameter for each covariate is located in the column vector $\boldsymbol{\beta}$, the covariates are associated with the specific system while the parameters are unknown and describe the effect of the covariates. The semi-parametric PHM can be represented by:

$$h(x|\mathbf{z}) = h_0(x)\Psi(\mathbf{z}(x);\boldsymbol{\beta}), \tag{2.4.1}$$

where $\Psi(\mathbf{z}(x);\boldsymbol{\beta})$ is a positive functional term that incorporates the effect of the covariates. A specific component/system has a hazard function that is a multiple of the underlying hazard function, thus, the covariates act multiplicatively on the hazard function. Kumar and Klefsjö (1994) state that the baseline function is time dependent while the second element is dependent on the covariates $(\mathbf{z}(x))$. Equation 2.4.1 can be rearranged to yield the second element; this is the ratio of the particular component/system hazard to the baseline hazard, as in Equation 2.4.2

$$\Psi(\mathbf{z};\boldsymbol{\beta}) = \frac{h(x|\mathbf{z})}{h_0(x)}. \tag{2.4.2}$$

Machin *et al.* (2006) explain that even when both $h(x|\mathbf{z})$ and $h_0(x)$ change with time, their ratio can remain constant, meaning that the ratio is not time dependent and the assumption of a proportional hazard rate holds when appropriate. Liao *et al.* (2006) mention how the PHM also has the advantage of being able to accommodate time-varying covariates that conventional regression methods like linear and logistic regression do not. The functional term of the PHM $(\Psi(\mathbf{z}(x);\boldsymbol{\beta}))$ can be specified by several different functional forms, the most popular being the exponential form. For the purpose of explaining the method, the general functional form of $\Psi(\mathbf{z};\boldsymbol{\beta})$ is written in the exponential form. According to Cox (1972), this term is known as the relative risk of failure. Cox (1972) names other forms such as the logistic, inverse linear, linear and Weibull form which are all valid forms that can be used.

The PHM will have the same amount of regression parameters as covariates. The PHM for multiple covariates can therefore be written as Equation 2.4.3. The exponential form for the relative risk has been used where z_i represent the values of the covariates used and β_i the corresponding parameter values, Equation 2.4.1 then becomes

$$h(x|\mathbf{z}) = h_0(x)\exp(\beta_1 z_1 + \beta_2 z_2 + ... + \beta_k z_k). \tag{2.4.3}$$

The semi-parametric model leaves $h_0(x)$ unspecified and is one of the model's major merits when little is known about the data considered, since no assumptions of the baseline hazard function need to be made when fitting the model (Anderson and Keiding, 2006). Over the past decades, a lot of research has been done on the degradation of mechanical systems and a popular form of the baseline hazard in literature is the Weibull distribution because of its flexibility characteristics. This allows the parametric model to be implemented. The parametric model has the capability of offering more accurate results than the semi-parametric model. The baseline hazard function $h_0(x)$ is unknown but can be estimated by specifying a distribution to which it belongs in the case of a parametric model. When substituting the Weibull distribution into the PHM, Equation 2.4.3 is represented by

$$h(x|\mathbf{z}) = \frac{\lambda}{\eta}\left(\frac{x}{\eta}\right)^{\lambda-1}\exp(\boldsymbol{\beta}'\mathbf{z}), \tag{2.4.4}$$

where λ is known as the shape parameter and η as the scale parameter, it is now termed a fully parametric model. This is known as the Weibull PHM and is used by Jiang *et al.* (2010) in their study to create a general framework for making preventive maintenance decisions. Once the model to be used is decided upon, the next step is to estimate the regression parameter values of $\boldsymbol{\beta}$, and in the parametric case, the distribution parameters as well.

The distribution's parameters must be estimated together with the regression coefficients. The estimation techniques for semi-parametric and parametric models differ as explained by Carstens (2012). Cox (1972) and Machin *et al.* (2006) argue that the most suitable method for a semi-parametric model is to maximize the marginal likelihood function and the maximum likelihood estimate for parametric models. The partial likelihood function is obtained by considering the contribution that the covariates make to the hazard rate of the individual time to failure, Anderson and Keiding (2006) explain that it does not depend on the baseline hazard and is a function of $\boldsymbol{\beta}$ only. The semi-parametric model and the method of estimating the parameters is discussed next.

Semi-parametric PHM

The original Cox PHM was a semi-parametric model developed in 1972 by Cox (1972). This model then makes no assumptions about the baseline hazard. A maximum likelihood estimate cannot be made since the baseline hazard function is left unspecified; the partial likelihood developed by Cox (1975) is therefore utilized.

Consider m number of events that were observed, the failures are recorded at times x_i where $i = 1, 2, ..., k$ and $x_1 < x_2 < ... < x_k$. These are only the failure times, observations that are censored are not included here; k is, therefore, the number of recorded failures. This then still leaves $m - k$ censored observations which are then not considered here. All subjects or items still in operation prior and up to x_i form the risk set denoted by $RF(x_i)$, thus, including the equipment that have a later censored observation. The number of tied failures at time x_i is denoted by d_i; tied failures are failures which occurred on the same time x_i. The number of tied failures is normally small when compared to the number of observations in $RF(x_i)$ which is denoted by k. The partial likelihood estimate obtained from Kumar and Klefsjö (1994) is represented in Equation 2.4.5 for the Weibull PHM. This partial likelihood function will differ depending on the form given to Ψ.

$$L(\boldsymbol{\beta}) = \prod_{i=1}^{k} \frac{\exp(s_i\boldsymbol{\beta})}{\left[\sum_{m \epsilon RF(x_i)} \exp(z_m\boldsymbol{\beta})\right]^{d_i}} \tag{2.4.5}$$

The sum of the covariates of items observed to have failed at time x_i is represented by s_i, so $s_i = \sum z_{iq}$ where q is the total number of covariates. The parameter values are obtained using numerical methods in software programs such as Microsoft Excel or MATLAB. Once the regression parameters have been estimated, the hazard rate as well as the reliability can be calculated. The function *coxphfit* in MATLAB is used to calculate the coefficient;, this function also uses the method explained above. This function also returns the log-likelihood value of Equation 2.4.5 as well as discrete values for the estimated cumulative hazard (H). This then allows for the baseline reliability ($R(x|\mathbf{z})$) to be calculated as explained by Liao *et al.* (2006). The reliability is related to the hazard function by:

$$\begin{aligned} R(x|\mathbf{z}) &= \exp(-\int_0^x h(\tau|\mathbf{z})d\tau), \\ &= \exp(-H(x|\mathbf{z})). \end{aligned} \tag{2.4.6}$$

The estimated RL of the equipment can now be calculated by making use of Equation 2.3.10 and 2.3.11. The parametric models are capable of delivering more accurate estimates but as mentioned, it requires the shape of the baseline hazard to be specified.

Parametric PHM

The PHM can also be solved as a fully parametric model by specifying the baseline hazard function. Bagdonavičius and Nikulin (2004) state that the maximum likelihood estimation procedure delivers the best estimates, when estimating the regression coefficients and the distribution parameters of the baseline hazard. The maximum likelihood function for a data set with censored observations can be represented as

$$L(\boldsymbol{\beta}, \lambda, \eta) = \prod_i h(x_i, z_i(x_i)) \prod_j R(x_j, z_i(x_i)), \tag{2.4.7}$$

where i indexes failure observations and j all observations, so $j = 1, ..., m$. Substituting the hazard function and the reliability function into Equation 2.4.7 then yields

$$L(\boldsymbol{\beta}, \lambda, \eta) = \prod_i \left[\frac{\lambda}{\eta}\left(\frac{x}{\eta}\right)^{\lambda} \exp(\boldsymbol{\beta}'\mathbf{z}(x))\right] \cdot \prod_j \exp\left[-\int_0^x \exp(\boldsymbol{\beta}'\mathbf{z}(x))d\left(\left(\frac{x}{\eta}\right)^{\lambda}\right)\right], \tag{2.4.8}$$

but this form of the function demands complex calculations. Thus, the log-likelihood is used; taking the natural logarithm of Equation 2.4.8 results in

$$\begin{aligned} l(\boldsymbol{\beta}, \lambda, \eta) = k \log\left(\frac{\lambda}{\eta}\right) + \sum_i \log\left[\left(\frac{x}{\eta}\right)^{\lambda-1}\right] + \sum_i (\mathbf{z}(x_i)) - \ldots \\ \ldots - \sum_i \int_0^x \exp(\boldsymbol{\beta}'\mathbf{z}(x))d\left(\left(\frac{x}{\eta}\right)^{\lambda}\right), \end{aligned} \tag{2.4.9}$$

where k represents the number of observed failures. These calculations can be done numerically in software packages, thus, making the calculations quick and easy. The MATLAB code for the parametric PHM can be found and the specific model utilized for this study is based on the methods presented above.

The main purpose of this study is to develop a method which requires as little as possible data considering that the data needed to populate the survival models is rarely available. Kumar and Klefsjö (1994) reveal that a sample size as large as 40 observations delivers reasonable results. According to Carstens (2012), small sample sizes cause biases towards the specific covariates in the regression coefficients. This is a disadvantage because the data sets in this study will be created by experts, and will thus be a time consuming process. The data sets will be completed with surveys, therefore, it is desirable to keep the amount of data needed to a minimum.

A key advantage of the PHM is that the regression coefficients can be estimated without making any assumptions about the baseline hazard as explained in the semi-parametric model. Since this is such a popular and widely used model, it is readily available in many software programs such as SAS, R and MATLAB. There are also extensions of this model that can be used rather than the original when applicable.

This model has been used in such a wide variety of applications and has proven itself as a ground-breaking and extremely useful tool in the reliability analysis industry. The assumption of proportionality must hold to even consider using this model. The model has, however, been proven to have success with a variety of equipment.

2.4.2 Prentice, Williams and Peterson

This model is an extension of the PHM and is named after the three men who developed it and presented it for the first time (Prentice *et al.*, 1981). The Prentice, Williams and Peterson (PWP) is a generalization of the PHM to a proportional intensity function of repairable systems; this intensity is also known as the ROCOF ($\rho(t|\mathbf{z})$). This extension allows the PHM to handle cases where a single piece of equipment considered experiences multiple failures. Landers *et al.* (2001) explain that this is done by stratifying the failure data.

A PHM considers the age and the covariate values of the equipment being considered. The PWP also considers those factors but it also includes an extra characteristic, the specific stratum which the equipment is in. A stratum is dependent on the number of previous failures which the equipment has experienced, meaning that a stratum can be seen as the operating period in between each of the failures. Let $N(t)$ be the number of failures at the instant t, the stratum S at any instant is determined by $S = N(t) + 1$. Stratum one is therefore from $t = 0$ up to the time of the first failure. The PWP is presented by Equation 2.4.10 as in Prentice *et al.* (1981) and can be solved in a semi-parametric or parametric manner.

$$\rho(t|N(t),\mathbf{z}(t)) = \rho_{0S}(t - t_{S-1})\exp(\boldsymbol{\beta}'\mathbf{z}(t)) \tag{2.4.10}$$

The baseline intensity function $\rho_0(t)$ can then be chosen as either the Power Law NHPP or the Log-linear NHPP format or it is left unspecified in the semi-parametric case. The cumulative intensity function is

$$\Lambda(t) = \int_0^t \rho(t|N(t),\mathbf{z}) = \Lambda_0(t)\exp(\boldsymbol{\beta}'\mathbf{z}(t)), \tag{2.4.11}$$

where $\Lambda_0(t) = \int_0^t \rho_0(u)du$. If $\boldsymbol{\theta}$ is a vector of parameters specifying the baseline intensity, the general likelihood function is given by Lawless (1987) as

$$L(\boldsymbol{\theta},\boldsymbol{\beta}) = \prod_{i=1}^{m}\left[\prod_{j=1}^{n_i}\rho(t_{ij}|N(t),\mathbf{z})\right]\exp(-\Lambda(T_i)), \tag{2.4.12}$$

where m is the number of subjects, n the number of observed failures and T_i is the failure time of the ith subject. The baseline function is specific to each stratum created in the data. However, the semi-parametric case is not considered in this study. The parametric case is explained next.

Parametric PWP

The parametric PWP is very similar to the parametric PHM, except that the intensity function is specified according to either one of the NHPP formats and it is stratum specific. The Power Law NHPP is considered first, where the baseline intensity function is given by Jiang *et al.* (2006) as $\rho_0(t) = \lambda_0\delta t^{\delta-1}$. If λ_0 is defined as $\exp(\beta_0 z_0)$ and $z_0 = 1$, the Power Law NHPP is given by

$$\rho(t|N(t),\mathbf{z}) = \delta t^{\delta-1}\exp(\boldsymbol{\beta}'\mathbf{z}(t)), \tag{2.4.13}$$

δ is the shape parameter. The regression coefficients are estimated in a similar manner as the PHM, thus, the log likelihood function for the Power Law NHPP format is given by

$$\begin{aligned} L(\delta,\boldsymbol{\beta}) = \left(\sum_{i=1}^{m} n_i\right)\log(\delta) + (\delta-1)\sum_{i=1}^{m}\sum_{j=1}^{n_i}\log(t_{ij}) + \ldots \\ \ldots + \sum_{i=1}^{m} n_i\boldsymbol{\beta}'\mathbf{z} - \sum_{i=1}^{m} T_i^{\delta}\exp(\boldsymbol{\beta}'\mathbf{z}), \end{aligned} \tag{2.4.14}$$

where n is the number of observed failures and m is the number of subjects. The Log-linear format of the NHPP has the intensity function

$$\rho(t|N(t),\mathbf{z}) = \exp(\alpha_0 + \alpha_1 t)\exp(\boldsymbol{\beta}'\mathbf{z}), \tag{2.4.15}$$

have $z_0 = 1$ and $\exp(\alpha_0) = \exp(\beta_0)$, therefore, $\exp(\boldsymbol{\beta}'\mathbf{z}) = \exp(\beta_0 z_0 + \beta_1 z_1 + ... + \beta_p z_p)$. Equation 2.4.15 can now be written as

$$\rho(t|N(t), \mathbf{z}) = \exp(\alpha_1 t)\exp(\boldsymbol{\beta}'\mathbf{z}). \tag{2.4.16}$$

The log likelihood for the Log-linear NHPP format of the PWP is given by

$$\begin{aligned} L(\alpha_1, \boldsymbol{\beta}) = \alpha_1 \sum_{i=1}^{m}\sum_{j=1}^{n_i} t_{ij} + \sum_{i=1}^{m} n_i \boldsymbol{\beta}'\mathbf{z} - \dots \\ \dots - \frac{1}{\alpha_1}\sum_{i=1}^{m}(\exp(\alpha_1 T_i) - 1)\exp(\boldsymbol{\beta}'\mathbf{z}). \end{aligned} \tag{2.4.17}$$

Maximizing the log likelihood functions yield the estimates of the regression coefficients as well as the parameters for the NHPP baseline intensity function. This model is quite complex but very powerful and it has been proven in the field of reliability analysis.

2.4.3 Accelerated Failure Time Model

The Accelerated Failure Time Model (AFTM) is a parametric regression analysis which provides an alternative to the popular PHM (Qi, 2009). The two models differ by how they assume the covariates affect the system/component hazard. Komárek *et al.* (2005) explain that the AFTM establishes a direct relationship between the time to failure and the covariates, and not a relationship between the covariates and the hazard of a system/component like the PHM. Thus, any AFTM assumes that the covariates considered have the effect of accelerating or decelerating the survival time of equipment.

The survival function of a certain piece of equipment is represented by $R_0(x)$, which can be seen as the control group. This survival function is related to the survival function of the same equipment under the accelerated life conditions by

$$R(x) = R_0\left(\frac{x}{\theta}\right), \tag{2.4.18}$$

where θ is a function that incorporates the explanatory variables (covariates). This function then denotes the total effect of the covariates; it is known as the acceleration factor Nachlas (2005). The acceleration factor generally has the form $\theta(\mathbf{z}) = \exp(\boldsymbol{\beta}'\mathbf{z})$ where $\mathbf{z}$ denotes a vector of covariates. The hazard function of the control group h_0 is related to the equipment being considered by

$$h(x) = \frac{1}{\theta}h_0\left(\frac{x}{\theta}\right), \tag{2.4.19}$$

Anderson and Keiding (2006) then provide the survival time on a logarithmic scale as $\log T = \mu + \log\theta + \sigma\epsilon$. This equation can then be rewritten in a log-linear form as Equation 2.4.20 and is the most general form of the AFTM;

$$\log T = \mu + \boldsymbol{\beta}'\mathbf{z} + \sigma\epsilon. \tag{2.4.20}$$

Martinussen and Scheike (2006) and Komárek *et al.* (2005) explain that $\boldsymbol{\beta}'$ is a set of regression parameters and ϵ is a residual error term with an unspecified distribution in the semi-parametric model. The exponentiated value of the parameter coefficients ($\exp(\boldsymbol{\beta})$) are known as the time ratio and has a physical meaning. Carstens (2012) state that the time ratio can be used to explain the failure time characteristic of an item, where a time ratio larger than one implies that an earlier failure is more likely while a time ratio less than one implies the opposite, as illustrated in Figure 2.9. In an AFTM, the residual error (ϵ) is a random variable and its distribution must be assumed for a parametric model, where the intercept is denoted by μ and σ is the scale parameter.

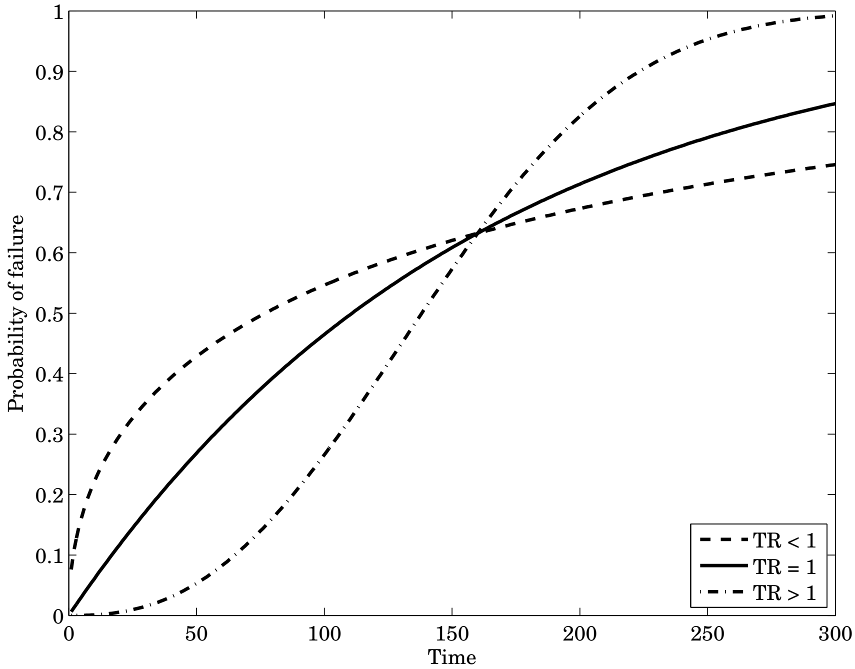


Figure 2.9: Effect of time ratio value.

The survival and hazard functions corresponding to the log-linear regression become conditional functions because the covariate vector $\mathbf{z}$ has a definite effect. They are presented as given by Zelterman and Lin (2004);

$$R_i(x_i|\mathbf{z}_i) = R_0\left(\frac{x_i}{\exp(\boldsymbol{\beta}'\mathbf{z}_i)}\right), \tag{2.4.21}$$

and

$$h_i(x_i|\mathbf{z}_i) = \frac{1}{\exp(\boldsymbol{\beta}'\mathbf{z}_i)}h_0\left(\frac{x_i}{\exp(\boldsymbol{\beta}'\mathbf{z}_i)}\right)(i = 1, ..., n). \tag{2.4.22}$$

Parametric AFTM

The semi-parametric extensions of the AFTM would estimate the regression parameters but will then not be able to predict the survival times and are, therefore, not considered here. In the non-parametric case, the regression parameters cannot be estimated and is also not covered. The survival function of T (the survival time) can be expressed by the survival function of the residual error (ϵ). Qi (2009) provides this relation as

$$R(x) = R_\epsilon \left(\frac{\log x - \mu - \boldsymbol{\beta'}\mathbf{z_i}}{\sigma} \right). \tag{2.4.23}$$

Therefore, the distribution of ϵ has a corresponding distribution of T, corresponding distributions are provided in Table 2.2. The Weibull distribution is

Table 2.2: Corresponding distributions of T and ϵ, adapted from Qi (2009).

T **distribution family**	ϵ **distribution family**
Exponential	1 Parameter, extreme value
Weibull	2 Parameter, extreme value
Log-logistic	Logistic
Log-normal	Normal
Gamma	Log-Gamma

used for the survival time T and is seen to yield an extreme value distribution for the random variable ϵ. This extreme value distribution is used to represent the limiting distributions for the minimum of a large collection of random observations; in this case, it is used to represent the random error ϵ. The survival function for ϵ can thus be given by:

$$R_\epsilon = \exp(-\exp(\epsilon)). \tag{2.4.24}$$

Therefore Equation 2.4.23 becomes:

$$R(x) = \exp \left[-\exp \left(\frac{-\mu - \boldsymbol{\beta'}\mathbf{z}_i}{\sigma} \right) x^{\frac{1}{\sigma}} \right]. \tag{2.4.25}$$

The hazard function for the Weibull AFTM can be written as

$$h_i(x_i|\mathbf{z}_i) = \frac{1}{\sigma} x^{\frac{1}{\sigma}-1} \exp \left(\frac{-\mu - \boldsymbol{\beta'}\mathbf{z}_i}{\sigma} \right). \tag{2.4.26}$$

This model is generally not presented in this form since the model assumes a direct relation between the covariates and the failure time. The AFTM written in its general form is as Equation 2.4.20 and when expanded can be written as

$$\log T = \mu + \beta_1 z_1 + ... + \beta_j z_j + \sigma\epsilon. \tag{2.4.27}$$

By exponentiating the coefficients the term $\exp(\beta_j)$ is known as the time ratio. The parametric AFTM can be fitted by using the maximum likelihood method. For a component with n observed data points, the likelihood function is presented by

$$L(\boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\sigma}) = \prod_{i=1}^{n} [f_i(x_i)]^{\delta_i} [R_i(x_i)]^{1-\delta_i}. \tag{2.4.28}$$

Qi (2009) explains the likelihood function and that it is for the ith subject at time x_i and δ_i is an event indicator, where it is one at a failure and zero otherwise. Taking the log of Equation 2.4.28 yields

$$\log L(\boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\sigma}) = \sum_{i=1}^{n} \{-\delta_i \log[\sigma x_i + \delta_i \log(f_{\epsilon_i}(y_i)) + (1-\delta_i)\log(R_{\epsilon_i}(y_i))]\}, \tag{2.4.29}$$

where $y_i = (\log(x_i) - \mu - \beta_1 z_{1i} - ... - \beta_j z_{ji})/\sigma$ and j is the number of parameters. This function then estimates the values for $j+2$ unknown parameters, $\mu, \sigma, \beta_1, ..., \beta_j$.

A disadvantage of the AFTM is that although it has been known to model electronic components and is widely utilized in the manufacturing industry, it is unfamiliar to many, especially to the medical industry. This is where most survival analysis research has traditionally been done. Another disadvantage is that even though familiar methods are used to do the parameter estimation, some complex calculations are required to derive the formulae.

One of the advantages of the AFTM is that it can be used with any of the distribution families that are appropriate for survival analysis. AFTM also relates the influence of the covariates to the failure time directly, thus, making it easier to interpret the results obtained. Another important advantage of this model is that packages in software programs such as MATLAB, R and SAS are easily accessible and very easy to implement. The next model to be reviewed is the Additive Hazard Model.

2.4.4 Additive Hazard Model

The AFTM and the PHM have multiplicative effects on their hazard functions as earlier discussed. The Additive Hazards Model (AHM), as the name suggests, has an additive effect on the system hazard. The general form of an AHM is presented by Pijnenburg (1991) as $h(x) = h_0(x) + \alpha$, where α is dependent on $\mathbf{z}$, a vector of explanatory variables, thus $\alpha(\mathbf{z})$. Aalen (1989) presented the first AHM as a non-parametric model. Semi-parametric extensions have since been developed and will be considered in this study. Since it is certain that the assumption made of the distribution of the failure characteristics is correct, the semi-parametric model will be utilized in order to obtain more accurate estimates. The hazard function for an AHM can be written as

$$h(x|\mathbf{z}_i) = h_0(x) + \alpha(\mathbf{z}_i), \tag{2.4.30}$$

where $h_0(x)$ is the baseline hazard function and $\alpha(\mathbf{z})$ can be one of many functional forms. The simplest polynomial form will be used to explain the model, which Schaubel and Wei (2007) provide as the polynomial $\alpha(\mathbf{z}) = \boldsymbol{\beta}'\mathbf{z}$. The covariate vector $\mathbf{z} = (z_1, ..., z_p)'$ has corresponding parameter values from the vector $\boldsymbol{\beta}$, where the simplest form of α is a basic linear function;

$$\alpha(\mathbf{z}) = \boldsymbol{\beta}'\mathbf{z} = \sum_{i=1}^{p} \beta_i z_i. \tag{2.4.31}$$

Gorjian *et al.* (2010*b*) ensure that the covariates have an additive effect on the hazard function, which enables it not to be zero at $x = 0$. This also allows α to be negative without the hazard function being negative. Pijnenburg (1991) provides the following information for the interpretation of $\alpha(z_i)$ from its value:

- If $\alpha(z_i) > 0$, the hazard function immediately after a failure is higher than at time of failure (worse than before).
- If $\alpha(z_i) = 0$, the hazard function is the same after the failure as at the time of failure (same as before).
- If $\alpha(z_i) < 0$, the hazard function immediately after a failure is lower than at time of failure (better than before but worse than new).

At the time of failure a maintenance action is conducted, which can be either replacing components or fixing the broken components. This action is what causes the hazard function to either change or stay constant; Figure 2.10 illustrates the properties of α.

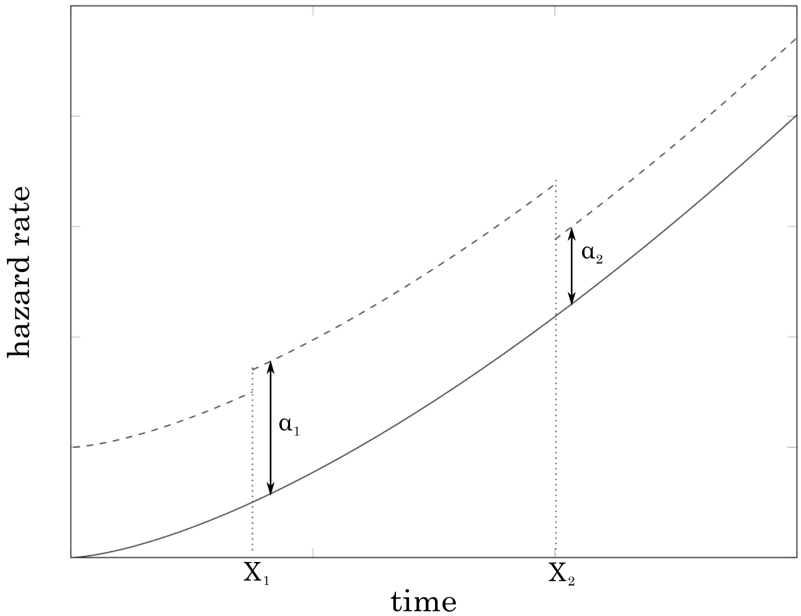


Figure 2.10: Properties of α.

Semi-parametric AHM

Lin and Ying (1994) introduce a function which mimics the partial likelihood score function's martingale feature. The solutions to the function provide estimated values for the parameter values of the parameter vector $\boldsymbol{\beta}$. Consider $i = 1, ..., n$, where n is the number of subjects and have $(X_i, C_i, \mathbf{Z}_i)$ be the failure time, censoring time and the covariate vector respectively, for the ith subject. The estimator function is

$$U(\beta) = \sum_{i=1}^{n} \int_{0}^{\infty} \left\{ \mathbf{Z}_i(x) - \bar{\mathbf{Z}}(x) \right\} \left\{ dN_i(x) - Y_i \boldsymbol{\beta} \mathbf{Z}_i(x) dx \right\}, \tag{2.4.32}$$

where

$$\bar{\mathbf{Z}}(x) = \frac{\sum_{j=1}^{n} Y_j(x) Z_j(x)}{\sum_{j=1}^{n} Y_j(x)}. \tag{2.4.33}$$

Lin and Ying (1997) utilize this estimation method and state that observations consist of n independent vectors $\{\tilde{X}_i, \delta_i, \mathbf{Z}_i(x), x \leq \tilde{X}_i\}$. The notation is such that $\tilde{X}_i = min(X_i, C_i)$ is the observation time, $\mathbf{Z}_i$ is a $p \times 1$ vector of covariates and $\delta_i = I(X_i \leq C_i)$ as the event indicator which is equal to one, if the failure time is less than or equal to the censored time.

Further, the at-risk counter $Y_i(x) = I(0 < x \leq \tilde{X}_i)$ indicates if subject i is at risk at the observation time. The observed event counter $N_i(x) = \delta_i I(\tilde{X}_i \leq x)$, $I(\cdot)$ is an indicator function. An estimate of the parameter values is obtained by setting Equation 2.4.32 equal to zero and then solving.

Pijnenburg (1991) mentions that the parameter estimation is complex and this is not a very easy to follow method, which is a drawback to this model. The AHM has rather many limitations when compared to its advantages; one such limitation is that additive hazard can lead to a negative value for the hazard function which is unrealistic. Another disadvantage is that the model cannot have failure times which are equal to zero.

Gorjian *et al.* (2010b) state that the AHM is a favourable option if the system is better than it was before a maintenance action but not as good as new. This model is also capable of depicting a hazard that is not zero at time zero, and this is generally the selling point for this model.

2.4.5 Proportional Covariate Model

Another regression technique used to predict the hazard of systems is the Proportional Covariate Model (PCM). This model is still relatively new as it was first published in 2006 and little published research on the model exist. Sun *et al.* (2006) state that it is largely based on the Cox PHM, meaning that it can be considered an extension of the Cox model. The PCM uses the covariates based on the deterioration of a system to predict the system's hazard rate. The PCM is more developed and offers several more advantages when compared to the popular PHM.

The PCM and the PHM assume proportionality of the hazard function. A PCM assumes that the covariates or a function of the covariates is proportional to the system hazard instead of using a baseline hazard function like the PHM. Certain limitations or disadvantages of the PHM that need to be avoided are listed below as mentioned by Sun (2006):

- Historical hazards estimated by using different covariates can differ,
- Fluctuations of covariates can affect hazard estimation remarkably, complicating reliability prediction,
- Sufficient historical failure data is needed to make the parameter estimations.

There are generally two categories of covariates that exist, namely environmental covariates ($Z_e(x)$) and responsive covariates ($Z_r(x)$). Environmental covariates cause characteristics of the hazard function to change while responsive covariates respond and change because of changes in the hazard function. Sun (2006) explains that most CM data are responsive covariates that reflect the deterioration of the system.

The PCM describes the relationship between the system responsive covariates and hazard with a baseline covariate function ($C(x)$) where the PHM does the same using a baseline hazard function ($h_0(x)$). According to Sun *et al.* (2006), a function of multiple covariates in PCM is expressed as:

$$\Psi(Z_r(x)) = C(x)h(x), \tag{2.4.34}$$

where $Z_r(x)$ is the covariate function and $C(x)$ is a baseline covariate function, both are normally time dependent. The hazard function is represented by $h(x)$. Since a change in the hazard function causes a change in the responsive covariates, the simplest form of the PCM is given by

$$Z_r(x) = C(x)h(x). \tag{2.4.35}$$

Here the explanatory variable is the hazard function while the covariate function is the responsive variable. The PCM and PHM have several differences and the most basic and important ones according to Sun (2006) are listed in Table 2.3. These two models are compared separately because the PCM is largely developed based on many of the assumptions made by the PHM.

The procedure of PCM is laid out in eight steps by Sun (2006) and can be summarized as follows:

1. Identify the system's failure distribution using historical failure data x_i ($i = 1, 2, ..., m_f$), m_f is the number of failure data points. The distribution that will be used in this study is the Weibull distribution and is very popular in reliability modelling because of its flexibility. The formula for

Table 2.3: PCM vs PHM

PCM	**PHM**
➢ Describes relationship between covariates and hazard using a baseline covariate function $C(x)$.	➢ Describes relationship between covariates and hazard using baseline hazard function $h_0(x)$.
➢ $C(x)$ represents rate of change of covariates when the hazard changes.	➢ $h_0(x)$ is the hazard rate of the system without the influence of the covariates.
➢ $C(x)$ is covariate dependent.	➢ $h_0(x)$ is covariate independent.
➢ Covariate value of 0 indicates hazard of system is 0.	➢ Covariate value of 0 indicates the hazard of the system change is then the baseline hazard.
➢ The hazard function estimated based on different historical covariate data is consistent.	➢ The estimated hazard function may differ when different covariate data is used to estimate.
➢ Suggests covariates (condition measurements) of a system reveals the change of equipment state of health.	➢ Suggests hazard of system affected by its conditions (covariates).

the Weibull distribution is represented in Equation 2.4.36 as given by Ushakov (2012),

$$f(x) = \frac{\lambda}{\eta}\left(\frac{x}{\eta}\right)^{\lambda-1} \exp\left(-\left(\frac{x}{\eta}\right)^{\lambda}\right). \tag{2.4.36}$$

2. Next is the initial estimation of the system's hazard function $h_{in}(x)$, generally by using a MLE. The Weibull parametric family is used in the case of this study, thus, the reliability of the system is represented by $R(x)$, where

$$R(x) = \exp\left(-\left(\frac{x}{\eta}\right)^{\lambda}\right). \tag{2.4.37}$$

The system hazard can then be written as

$$h(x) = \frac{f(x)}{R(x)} = \frac{\lambda}{\eta}\left(\frac{x}{\eta}\right)^{\lambda-1}. \tag{2.4.38}$$

To estimate the values of the shape and scale parameters, namely λ and η, the most popular method used is the MLE method. The MLE illustrates one of its advantages above other estimation methods by being able to deal with data sets including suspensions, data that was recorded when no failures occurred. The likelihood function has the form of:

$$L(x, \eta, \lambda) = \prod_{i=1}^{n} f(x) \cdot \prod_{j=1}^{r} [1 - F(x_j)], \tag{2.4.39}$$

and when taking the log of the likelihood function and writing out the equation, then the maximum of the log likelihood function shown as Equation 2.4.40 by Vlok (2014). Maximizing this will yield the appropriate values for the parameters λ and η.

$$\ln L(x, \eta, \lambda) = \sum_{i=1}^{m} [\ln \frac{\lambda}{\eta} + (\lambda - 1) \ln \frac{x_i}{\eta}] - \sum_{j=1}^{r} \left(\frac{x_j}{\eta}\right)^{\lambda} \tag{2.4.40}$$

Where m is the number of observed failures, thus excluding all suspensions, the time values used for the first term should correspond to the failure time and exclude the suspensions as well. The r is then the total amount of observed events, the time for both the first and the second term is the local time of the failure or suspension and not the global time.

3. The relationship between covariates and the hazard function is analyzed; should a covariate have a weak or poor relationship with the hazard, it should be discarded and not used again to update the hazard. This correlation analysis can be done effortlessly in software programs like MATLAB or Microsoft Excel. Covariates which have a weak relationship with the hazard of the system must not be used to update the estimate of the hazard as this will cause the hazard estimate to become inaccurate. This correlation is tested by using the data available; in this case, it will be the CM parameters obtained and the initial system hazard.

4. The baseline covariate function is the next priority. Estimating this function will require the initial system hazard ($h_{in}(x_k)$) and a set of historical covariate data ($Z_r(x_k)$). A set of discrete values is created by inserting the hazard and covariates values into $C_k = \frac{Z_r(x_k)}{h_{in}(x_k)}(k = 1, 2, ..., m_c)$.

   Using these discrete values, regression analysis can be applied to the data set $\{C_k, x_k\}$ to obtain a formula for the data set. The functions recommended for the baseline covariate function include following models where a_0, a_1, a_2, a and b are to be identified:

   a) polynomial models of different orders
      $C(x) = a_0 + a_1 x + a_2 x^2 + \cdots,$

   b) the multiplicative model
      $C(x) = ax^b,$

   c) and the exponential model
      $C(x) = a \exp(bx).$

   This is the final step to estimating the baseline covariate function but if no historical failure data is available, these first four steps are not applicable. The baseline covariate function must then be estimated in a different manner. Other options of estimating the baseline covariate function is to use historical failure data of similar equipment with more or less the same operating conditions or when no failure data can be obtained, other data such as accelerated life test data can be used.

Selecting an appropriate covariate (one with a strong relationship with the hazard of the system) and using that covariate data from the accelerated life testing, the hazard function of the system can be estimated. Sun *et al.* (2006) offer two different case studies where they applied both alternative methods. Allowing the functions to be estimated without failure data is a great improvement from the PHM which requires failure data to be used.

5. The system's hazard function must be updated by using new CM data. The data set of new data is shown as $Z_r(x_j)(j = 1, 2, ..., m_n)$ where m_n is the number of new CM data points. The updated hazard function is also characterized by using regression techniques to determine the parameter values by applying the selected technique to the discrete values $\tilde{h}_i(x)(i = 1, 2, ..., m_c, m_c + 1, ..., m_c + m_n)$ obtained by using the new CM data. To obtain these values, Equation 2.4.41 is used. It must be kept in mind that the hazard function is in the Weibull form as indicated by Equation 2.4.38.

$$\tilde{h}_i = \frac{Z_r(x_i)}{C(x_i)}(i = 1, 2, ..., m_c, m_c + 1, ..., m_c + m_n) \tag{2.4.41}$$

6. Every time new CM data is obtained both the covariate function ($C(x)$) and the estimated hazard function ($\tilde{h}(x)$) must be updated. This is done by repeating the first five steps continually as the data is obtained. When updating the hazard function in the fifth step, only the latest CM data will be used to update the function in certain cases. This can be done when the operating conditions and or environment have changed and the old data will only cause inaccuracy in the estimate.

7. The reliability function of the system should be updated using the updated hazard function. The hazard function and the reliability function are related by

$$h(x) = \frac{f(x)}{R(x)}, \tag{2.4.42}$$

where $f(x)$ is the PDF and $R(x)$ is the reliability function or the probability of survival.

8. The final step is to estimate the reliability of the system using the updated reliability function and to use this result to make maintenance decisions.

According to Sun (2006), it should be noted that if no failure data is available, the baseline covariate model can still be estimated but not by using steps one to four. The baseline covariate model can be deduced from the hazard function by using the new covariate values, using Equation 2.4.41 in step five.

Failure predictions are generally conducted in one of two ways; fault diagnosis from CM signals or statistical analysis of the past failure data of a system. The PCM utilizes both of the data types. Sun *et al.* (2006) mention that failure data is often sparse or non-existent in industry and, therefore, the PCM offers a great advantage of still being able to calculate its baseline covariate function without the use of the failure data from the specific equipment considered. Gorjian *et al.* (2010*a*) state that this is not true because Sun (2006) still used historical failure data; the author just used data from a similar piece of equipment and not the specific asset he did the study on. Failure data of a similar system or component can thus be used in the model and still yield valid results as proven by Sun (2006).

Other limitations of the PHM which PCM overcomes are that the fluctuations in the CM data has less of an influence on the model and PCM can predict time for scheduling maintenance actions where PHM only triggers an alarm when the hazard has reached its predefined limit. The PCM is robust and can handle noisy CM data as long as the noise has a mean value of zero. The model also automatically tracks the changes in the hazard of the system because of the continuous use of updated responsive covariates.

There is currently no specific method suggested to combine the covariates, the model has thus far only been used with a single covariate Z. To overcome this limitation, the PCM will be evaluated for each covariate separately and the residual life estimates obtained from each model will be averaged together to get the final estimate. The final model that is reviewed is known as the Proportional Odds Model and is popular among electrical equipment.

2.4.6 Proportional Odds Model

The Proportional Odds Model (POM) has covariates that act multiplicatively on the odds of survival for a component or system, thus, as the covariates change, the odds of failure change as well. According to Yang and Prentice (1999), the effect of the covariates diminish as time progresses. The assumption of the PHM of proportional hazards does not always hold; an alternative to this would be to consider the odds of failure and utilize the POM. Anderson and Keiding (2006) provide the function for the assumption as

$$a(x, z) = \frac{1 - R(x|z)}{R(x|z)}, \tag{2.4.43}$$

where $R(x|z)$ is the survival function of the subject considered given the covariate(s) z. This can then adopt a similar form as that of the PHM, yielding

$$a(x|z) = a_0(x)b(z) \tag{2.4.44}$$

where a baseline odds of failure function is given by $a_0(x)$, one can see that it has a similar relationship to Equation 2.4.1. This baseline odds of failure is $a_0(x) = (R(x))/1 - R(x)$, where $R(x)$ is the survival function when $z = 0$ (i.e.

without considering the covariates). The $b(z)$ term is a regression function that generally has the form of $\exp(\boldsymbol{\beta}'\mathbf{z})$.

When $\log a(x|z)$ is plotted against time for the different covariates in z, Anderson and Keiding (2006) explain that curves which lay parallel to each other will result. Each of these curves will be displaced by the amount of $\log b(z)$ from the baseline failure odds function's log curve. This can be explained with the help of Equation 2.4.45, where z_1 and z_2 are different covariate values.

$$\frac{a(x|z_1)}{a(x|z_2)} = \frac{b(z_1)}{b(z_2)} \tag{2.4.45}$$

The POM in its most common form is presented in Equation 2.4.46, from here there are several more steps before the unknown values can be estimated.

$$O(x|\mathbf{z}) = -\log \frac{1 - R(x|\mathbf{z})}{R(x|\mathbf{z})} = \log \left(\frac{1 - R(x)}{R(x)} \exp(\boldsymbol{\beta}'\mathbf{z}) \right) = G(x) + \boldsymbol{\beta}'\mathbf{z} \tag{2.4.46}$$

This function represents the odds of an event (failure) occurring in the interval $(0, x)$ provided that the vector of covariates ($\mathbf{z}$) is given. The baseline log-odds of failure is presented at time x by $G(x)$. According to Banerjee and Dey (2005), if $G(x)$ is a strictly increasing function Equation 2.4.46 can be rewritten as

$$G(x) = -\boldsymbol{\beta}'\mathbf{z} + \log(\epsilon), \tag{2.4.47}$$

where $\log(\epsilon)$ has the logistic distribution. The survival function can be presented as $R = (\exp(-G(x)))/(\exp(-G(x)) + \exp(\boldsymbol{\beta}'\mathbf{z}))$. The hazard function of the POM for the ith subject is given by

$$h(x) = -\frac{\mathrm{d}}{\mathrm{d}x} \log(R(x|\mathbf{z})) = (1 - R(x|\mathbf{z})) \left(\frac{\mathrm{d}}{\mathrm{d}t} G(x) \right). \tag{2.4.48}$$

This model is said to model the effect of covariates having a diminishing effect over time by several researchers (Carstens, 2012; Murphy *et al.*, 1997; Royston and Parmar, 2002). This is a common effect in electrical components but not in mechanical systems. Sun (2006) states that the deterioration of mechanical systems generally increases the likelihood of failure. The POM will, therefore, not be suitable for this study since the covariates that are chosen to model physical assets do not have a diminishing effect over time; in fact they have an increasing effect as time passes. Thus, this model is not considered to be a valid survival model to be used in this study and is not discussed further.

2.5 Summary of Literature

There is no shortage of literature elaborating on PAM as well as the application and improvement thereof within organizations. A lot of research is still being conducted in this field adding to fast growing body of knowledge. This study aims to add knowledge to PAM body of knowledge by obtaining results applicable in the maintenance subset of PAM. The industry is moving more towards proactive maintenance strategies in the current day and time, therefore, this study will prove to be of value in real life applications.

A large part of proactive maintenance is centred around survival analysis or better known as reliability analysis within the engineering environment. Survival models are reviewed and the models which are repeatedly found in literature are the PHM, PWP, AFTM, AHM and the POM. The PCM is included in this study because the model was designed to address the short-comings of the popular PHM. All of the various survival models are summarized in Table 2.4, highlighting the differences, similarities, strengths and weaknesses of the models.

Literature, utilising expert knowledge together with survival models exists but as seen in Zuashkiani *et al.* (2009), the study is very specific to the survival model and the data used. No literature could be found where the failure data and the CM data used as covariates in survival models are elicited from experts and compared to objective data. This study then aims to validate whether subjective data can be used as a valid alternative to objective data for populating survival models used for reliability analysis thereby allowing RL predictions of equipment.

Table 2.4: Summary of survival models.

	PHM	**PWP**	**AFTM**	**AHM**	**PCM**
General form of model:	$h(x\|\mathbf{z}) = h_0(x)\Psi(\mathbf{z}(x);\boldsymbol{\beta})$	$\rho(t\|N(t),\mathbf{z}(t)) = \rho_{0S}(t - t_{S-1})\exp(\boldsymbol{\beta}'\mathbf{z}(t))$	$\log T = \mu + \boldsymbol{\beta}'\mathbf{z} + \sigma\epsilon$	$h(x\|\mathbf{z}_i) = h_0(x)+\alpha(\mathbf{z}_i)$	$\Psi(Z_r(x)) = C(x)h(x)$
Assumption model makes:	➢Covariates have multiplicative effect on the hazard of a system.	➢Covariates have multiplicative effect on the hazard of a system.	➢Covariates have additive effect on the failure time of a system.	➢Covariates have additive effect on the hazard of a system.	➢Covariates have multiplicative effect covariate function.
Describes relationship between:	➢Covariates and hazard using a baseline hazard function $h_0(x)$.	➢Covariates and hazard using a baseline hazard function $\rho_0(t)$.	➢Covariates and time to failure.	➢Covariates and hazard using baseline hazard function $h_0(x)$.	➢Covariates and hazard using baseline covariate function $C(x)$.
Flexibility:	➢Flexible especially in semi-parametric case, no assumptions made on baseline hazard function.	➢The same as the PHM, no assumption has to be made in semi-parametric case and many distributions available for parametric case.	➢Parameters can easily be added to model but little research has been done on the effect of censored data.	➢Only applicable for cases yielding positive hazard.	➢Once trained model is very specific to the data used to train it.
Ease of use:	➢Many software toolboxes available, can make use of the model without having to understand the intricate mathematics used to develop it.	➢Similar to the PHM. These models are readily available for software programs such as MATLAB, SAS, R, etc..	➢This model is also readily available in many software programs, but if this model had to be created by the user the methods of estimating the parameters quickly become complicated.	➢This model is also available for most of the industrial software programs used for intricate mathematics, this model does however use some complex calculations for its parameter estimation.	➢None of the popular software programs were found to have a preprogrammed PCM, thus this model will have to be created from scratch when wanting to be used.

Chapter 3

Proposed Solution

This chapter aims to provide a general methodology consisting of steps for selecting relevant covariates, experts as well as the process of extracting the data from the experts. A step is also presented to ensure that an appropriate survival model is selected for the purpose of the study. A visual representation of the method and its components, which allow the RL to be predicted, is represented in Figure 3.1 as a flow diagram.

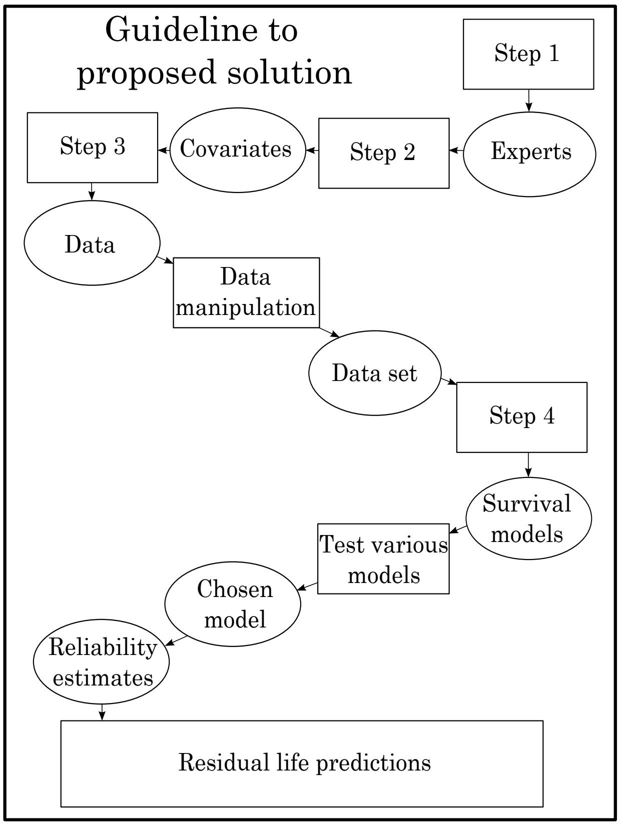


Figure 3.1: Map to solution

The problems presented with predictive and preventative maintenance are not solved by making use of prognostics because prognostics still makes use of the data from both the maintenance tactics. The availability of historical failure data and the integrity of CM data remain the prime obstacles to overcome. The prognostics field works very well when all the necessary data is available and trustworthy, thus, this chapter presents a guideline to elicit the necessary data from a panel of experts. In this way, it can be explored if subjective data can be used to populate survival models. The data will be subjective data because the opinions obtained from the experts are all subject to their own interpretation and judgement.

There is need to select experts to approach for the study, but there are also certain prerequisites to be met in order to be considered an expert. Step 1 provides a general guideline on when people can be considered experts in a certain field. Step 2, is to specify the system characteristics which are to be used as covariates in the selected survival model. The data set to use can only be obtained once the experts and the covariates are selected. Step 3 clarifies what a typical data set must contain and how it should look. The process of how the data will be extracted is also specified to ensure clarity and repeatability of this study. The last step in this guide is for selecting the correct survival model to eventually make the RL predictions. The guideline for conducting this study is now explained in detail starting with the first step, selecting appropriate experts.

3.1 Expert Selection (Step 1)

An important part of this study is setting a criterion for selecting the experts to obtain the subjective data from; this is the first step to the guide of this proposed solution. It is meant to ensure that this study is repeatable and that there is a certain standard for selecting the experts. This will decrease the probability of collecting poor data. It is the first step of the flow diagram in Figure 3.1.

An expert is a person who is seen as a reliable source of skill or technique and whose capability for judging or deciding correctly, justly or wisely is approved as authority. The Oxford dictionary defines an expert as a person with extensive knowledge or ability based on research, experience or occupation and in a particular area of study (Hornby, 2010). It can, thus, be seen that one does not have to possess professional or academic qualifications to be considered an expert, but they must be considered an expert by consensus (Hussler *et al.*, 2011). Prolonged or intensive experience can be obtained through practice, education or research and can allow a person to be considered an expert because of the knowledge he/she possesses.

The criteria for selecting experts can thus be created by considering the definitions which literature provides to consider people as such. Selecting an expert comes down to three basic elements, namely experience, education and occupation. Table 3.1 provides the criteria for when a person is considered as an expert in this specific study. It should be noted that when considering these guidelines, the selected persons are considered as experts in the relevant field of study.

Table 3.1: Criteria for selecting experts.

	Considered as an expert when:
Experience	➢ More than 10 years.
Education	➢ No formal education needed if more than 10 years experience. ➢ If $2-5$ years experience: Formal education in specific field needed, either certified university degree or relative government certificates. ➢ If $1-2$ years experience: Formal tertiary education, certified university degree in field considered. ➢ If less than 1 year of experience: Formal tertiary education, certified university degree in field being considered and specific research done on the field with publication(s) in peer reviewed sources.
Occupation	➢ If he/she is a researcher in the specific field with the necessary education and/or experience. ➢ If the person is an operator or technician with the required experience and/or training. ➢ If in a managing position the person must also meet the necessary educational and experience requirements.
Consensus	➢ Regardless of the experience and education, others in the same field consider him/her as an expert.

This table is merely a guideline on how to select experts, thus, there might be various cases when exceptions can be made. The most important criterion for a person to be considered as an expert is that he/she must be considered as such by consensus as observed by Hussler *et al.* (2011).

3.2 Covariate Selection (Step 2)

System characteristics that are to be selected as covariates are those which will indicate a fault by either increasing or decreasing their overall measured value or they reveal a change in some characteristic value. The parameters differ for different types of equipment, the cost of obtaining this data must also be considered. Data representing the industry standard gathered through extensive research is used to validate the data obtained from the experts.

According to ISO (2002), consideration must be given to the practicability of measuring the parameters as well as the operating conditions of the equipment. This will determine when and where to take the measurements required. ISO

Table 3.2: General condition monitoring parameters for several machine types, adapted from ISO (2002).

Parameter	Machine type								
	Electric motor	Steam turbine	Aero gas turbine	Industrial gas turbine	Pump	Compressor	Electric generator	RIC engine	Fan
Temperature	✓	✓	✓	✓	✓	✓	✓	✓	✓
Pressure		✓	✓	✓	✓	✓		✓	✓
Pressure (head)					✓				
Pressure ratio			✓	✓		✓			
Air flow			✓	✓		✓		✓	✓
Fuel flow			✓	✓				✓	
Fluid flow		✓			✓	✓			
Current	✓						✓		
Voltage	✓						✓		
Resistance	✓						✓		
Input power	✓				✓	✓	✓		✓
Output power	✓	✓	✓	✓			✓	✓	
Noise	✓	✓	✓	✓	✓	✓	✓	✓	✓
Vibration	✓	✓	✓	✓	✓	✓	✓	✓	✓
Acoustic techniques	✓	✓	✓	✓	✓	✓	✓	✓	✓
Oil pressure	✓	✓	✓	✓	✓	✓	✓	✓	✓
Oil consumption	✓	✓	✓	✓	✓	✓	✓	✓	✓
Oil (tribology)	✓	✓	✓	✓	✓	✓	✓	✓	✓
Torque	✓	✓		✓		✓	✓	✓	
Speed	✓	✓	✓	✓	✓	✓	✓	✓	✓
Length		✓							
Efficiency (derived)		✓	✓	✓	✓	✓		✓	

(2002) provides a table showing the parameters to be measured for specific equipment. These parameters have been studied and proven to have a relationship with the degradation of the specified type of equipment in the past. Table 3.2 is an adaptation of the table provided by ISO (2002); the (✓) symbol indicates which CM parameters are applicable for the specific machine types.

The primary reason to include covariates in a study would be if they have an association with the outcome of the study (CHMP, 2013). This association can be justified by previous studies. To have covariates that are universally applicable, the parameters that are of value in most or all types of mechanical systems must be used. Table 3.2 reveals that there are five general attributes which are applicable across all the mentioned machine types, they are shaded in the table and listed below:

1. Temperature
2. Vibration
3. Oil analysis (pressure, consumption, tribology)
4. Noise
5. Speed

These are the characteristics that are generally used as covariates in reliability analysis for mechanical systems. The standards must be used to provide guidelines as to which characteristics to consider but characteristics not mentioned may also be used. It must be ascertained that the covariates selected in the end, are capable of representing the current state or health of the assets considered. Once it has been established who will be used as experts and what covariates will be needed, the next step is to obtain the data needed.

3.3 Data (Step 3)

This section describes what data is needed, how it is obtained and what manipulation is required if any. It is appropriate to first name what data is needed and then suggest a way to ensure the necessary data is obtained.

3.3.1 Required Data

Section 2.3.2.5 mentions the three parameters which are necessary for survival analysis. Since prognostics makes use of both the historical failure data and the CM data, both are to be obtained from the selected experts. The first parameter is the historical event times (X_i) of the equipment considered. An event can be a functional or physical failure, a preventative maintenance action that was conducted or a predictive maintenance action. This leads to the second parameter required, the event indicator (C_i).

The event indicator is meant to show if the observation was recorded at a failure or not. This is usually a binary indicator with one representing a failure and zero a censored observation. The different types of censoring are discussed in Section 2.3.2.5.

The final parameter needed is a matrix of covariate values; these covariates are explained to be characteristic values of the equipment considered. These values are obtained from CM equipment. Values are needed from the initial time of operation since some models require the covariate behaviour over the entire period of operation for time-varying covariates. The final parameter can thus be a matrix that has a column for each covariate and shows the covariate behaviour over the period of the study. The progression of each covariate over time is necessary for each period of operation and for all the assets involved in the study. The final data set only contains the final covariate values (the covariate value at the time of the event) but it is also necessary to have the progression of the covariate value over time.

To summarize, the data needed to conduct reliability analysis consists of three different parameters, namely the time to the event, the event indicator and the value(s) of the selected covariate(s). This is data that should be generally available at organizations with a mature AM system but, unfortunately, it is not that simple in industry. The necessary data is rarely available, which is why this study aims to extract this data from experts and to use this subjective data in the survival models. After knowing what data needs to be obtained, the next step is to obtain the data.

3.3.2 Obtaining the Data

It is now known what data is to be obtained and from whom. The exact steps of extracting the data are now presented in a way that eases the process of collecting the data as much as possible. There are three different techniques of obtaining the required data.

First Technique

The first technique is to provide the selected experts with event times. Once the experts are given the event times, they will be required to provide the behaviour of the relevant covariates during the operating periods. This is meant to be representative of the population for the system considered. The covariate values over the operating time for each observation should also be obtained from the experts; this is important in order to compare the knowledge of the experts against the data available from research done and used in industry.

Figure 3.2 presents a table of all the data that is provided and what must be obtained from the experts. The figure also displays the behaviour of the covariates for the duration of one observation. The experts will be expected to provide this for each time period in the data set. This means that there will

#	X_i	C_i	Z_1		Z_2		Z_3	
			Start	End	Start	End	Start	End
1	200	1	?	?	?	?	?	?
2	350	1	?	?	?	?	?	?
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
12	600	1	?	?	?	?	?	?

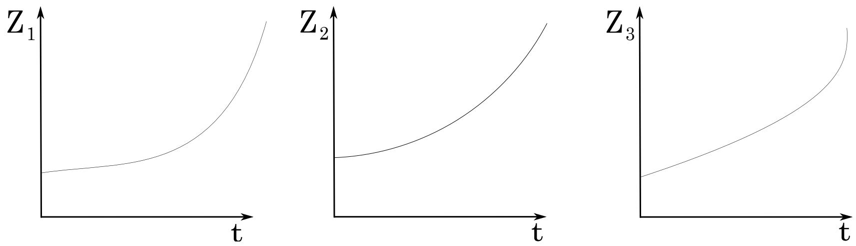


Figure 3.2: Data extracted with first technique.

be a set of covariate behaviour curves for each of the observations describing the progression of each covariate from the start until the time of observation or failure.

Second Technique

The second technique of extracting the necessary data from the experts would be to provide them with the covariate values and ask them to deliver the failure times. Here they will be provided with the starting and ending values of the characteristic covariates for certain time intervals. They are then to provide the event times as well as the progression of the covariates over the time periods which they provide.

Table 3.3: Example of data provided for second technique.

#	X_i	C_i	Z_1		Z_2		Z_3	
			Start	End	Start	End	Start	End
1	?	1	1.0	2.8	0.8	1.9	25	40
2	?	1	1.4	2.4	0.9	1.6	25	38
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
12	?	1	1.7	4.4	1.5	3.3	25	55

Table 3.3 presents the data provided before the experts deliver their opinions. Together with this, there will also be the regression of the covariates over the time period of each observation.

Third Technique

The last option is to combine the first two techniques and obtain the data through a combination the techniques. The data set will then be created by obtaining the first half of the data points with the first technique and the second half by applying the second technique. There are certain aspects which are needed for all three of the techniques as discussed next.

Collecting Data for All Techniques

With all of the different techniques it is necessary to ensure that the experts agree about the data. Consensus concerning the opinions delivered is thus important. Here, a suggestion is made to provide each expert with the same information set and have each one deliver their own opinion on that information. An attempt must then be made to ensure that the different experts agree upon their opinions before continuing. Figure 3.3 presents a flow diagram of the data acquisition process.

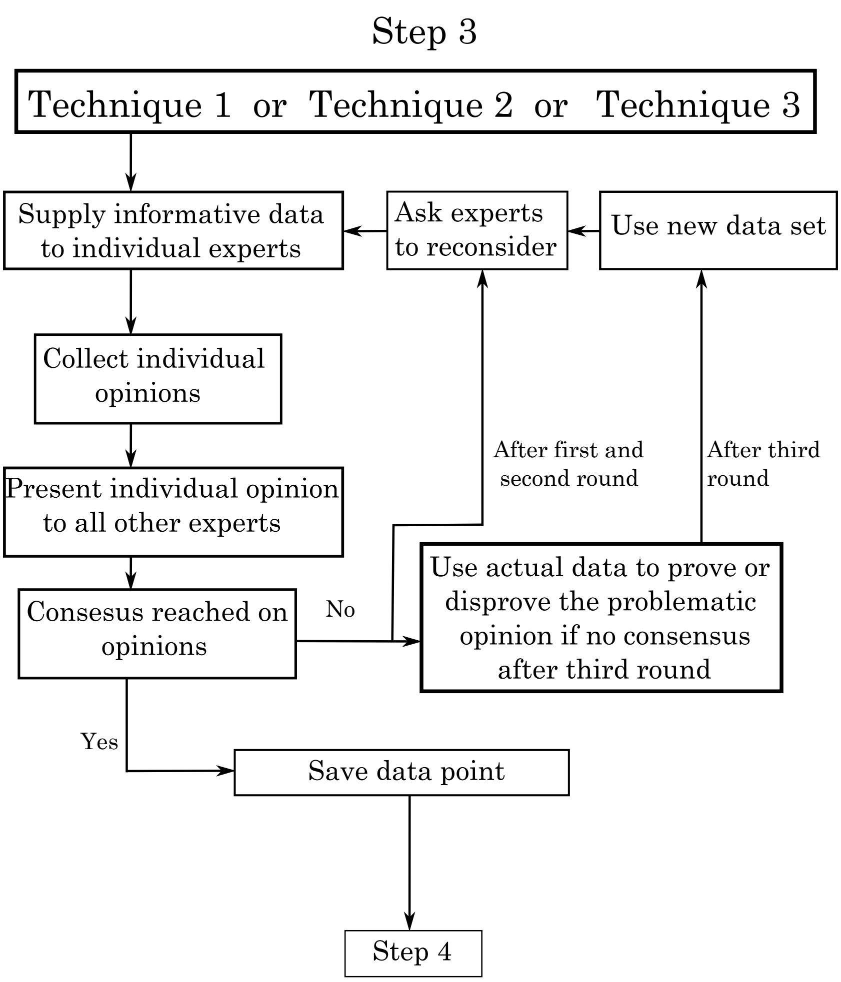


Figure 3.3: Flow diagram of data extraction process.

A single data point is thus yielded from a successful round of one of the techniques from this step. If there is no consensus after the third round, the actual data, which the data set provided to the experts is based upon, must be presented to them. The data set that is obtained at the end of this process is then to be used to populate the survival models, thus, using the Step 4 of the proposed solution to select the most appropriate survival model for the specific data set. To ensure that the data can be used in the survival models, slight manipulations might be required; this is now considered.

3.3.3 Manipulation Required

Step 3 is meant to eliminate the need for large manipulating processes of the data before feeding into the survival models. The different models might require small changes to be made depending how the models are implemented in the relevant software. Thus, small modifications might be required for some models; should this be necessary. Each model will be discussed separately in Section 3.4. The next section explains the process of selecting the appropriate survival model for the specific data set.

3.4 Model selection (Step 4)

Generally, prognostic survival models are used when enough data is available to populate them, but as mentioned in the problem statement, the availability of data in industry is problematic. Thus, existing survival models are investigated to determine which model is the most rigid while requiring the least amount of covariate values and delivering the best results. Small data sets are desirable seeing that the data sets for this study are obtained by eliciting the data from experts which is a time consuming task.

This section presents a scoring system to rate the different survival models according to their attributes as derived from the reviewed literature. Each of the survival models are reviewed and methods for determining the Goodness of Fit (GOF) for the models are presented in order to establish which models are applicable for a specific case. A dummy data set is created and used to explain the selection process. The score provided for each survival model is first presented to show the attributes of the different models.

3.4.1 Model Score Based on Literature

The reviewed literature is first considered and the chosen models' attributes and characteristics are rated. The prognostics field has numerous survival models to choose from but they were narrowed down as explained in Chapter 2. These models are each given a score based purely on the literature; this can however be the subjective opinion of the authors or case specific. The desired characteristics of a survival model are listed below. A survival model must be:

1. repeatable
2. interpretable
3. simple to estimate parameter values
4. able to deliver estimates with a relatively small data set
5. easily adaptable for different situations and equipment
6. as simple as possible to use
7. rigid to allow for noisy inputs as well as other distortions
8. proven in existing literature to be applicable in the field of reliability analysis.

A scoring system to determine the most appropriate survival model for this study is presented. If the literature reviewed specifically stated that the characteristic is an advantage or key merit of the model, it is assigned a value of 2 (✓). If nothing was explicitly stated about the characteristic but it can be deduced that the model does have the characteristic, it is assigned a blank with the value of 1, but if the literature mentioned the characteristic as a disadvantage a score of -1 (✗) is assigned. Each attribute carries equal weight; the final score for each of the models is then summed and displayed in the final column of Table 3.4. An explanation for the scores given to each model is also provided.

Table 3.4: Model attributes

Model	**Attribute**							
	Applicability in PAM	**Simplicity**	**Flexibility**	**Interpretable**	**Amount of data required**	**Repeatability**	**Rigid**	**Score**
AFTM	✓	✗	✓	✓		✓		9
AHM	✓		✗	✓				7
PHM & PWP	✓	✓	✓		✗	✗	✓	7
PCM	✓				✓	✓	✓	11

In addition to this scoring matrix of which model theoretically best fits the purpose for this study, a method to determine the GOF for each model to a data set is presented in Section 3.4.2. An explanation is now provided for each

✓and ✗ provided. The blank cells are not discussed because no evidence was found in the literature that support or act against the attribute listed.

AFTM

This model has been proven in the PAM field and delivers results which can easily be related to a physical meaning. The estimation of the parameter values become a complex process when time-dependent covariates are used. The failure time distributions are difficult to identify and time consuming to estimate. The model is, however, flexible, any parameter can be easily added and the results are not difficult to interpret.

AHM

This model has proven itself in the PAM field as Pijnenburg (1991) assures. This model is not very flexible, as it cannot model systems with zero hazard and a negative functional term can deliver results which are unrealistic. The results are relatively easy to interpret, maintenance actions can clearly be seen when the hazard function is plotted.

PHM and PWP

The PHM has been extremely successful in the PAM environment; the model can be kept simple by not making any assumptions about the baseline hazard distribution and solving semi-parametrically. The PHM is very flexible in that it allows any distribution family to be fitted to the baseline hazard when a parametric model is desired. This model is, however, sensitive to the historical failure data. The baseline hazard may also differ in functional form when calculated with different covariates, thus, affecting the repeatability. It is, however, stated that this model returns very rigid or safe results. The PWP is simply the same model only for repairable systems, thus, it has the same score.

PCM

The PCM was created and validated by making use of data from haul trucks operating in a mine, therefore, it is applicable in the PAM environment. The system hazard is able to be calculated even when no historical failure data is available for the specific equipment. Data from similar equipment can be used instead and still yield valid results. The same hazard can be obtained even when using different covariates, therefore, making the repeatability of the model easier. The model is rigid and will provide valid answers from noisy inputs provided the mean error is zero.

The model that is most fitting for a study will differ from data set to data set. It is, thus, important to first test all the relevant models with the data set being considered. The score provided by the Table 3.4 is only to make the user aware of the weak points of each model but ultimately, each model will have to be tested with every data set considered to ensure that the most appropriate model is used. The next section describes when each model is the most appropriate and a dummy data set is provided to use as an example.

3.4.2 Testing Applicability of Models

The only way to know for sure which model is the most appropriate one for a specific data set would be to apply each model to the data set and evaluate which model fits it the best. This study does not apply all possible models, only the most popular ones discovered in literature. It is, thus, possible that a model which is not reviewed in this study may have a better fit to a certain data set. It is, however, not a valid option to review all known models. The process of selecting the best model is presented in the form of a flow diagram in Figure 3.4.

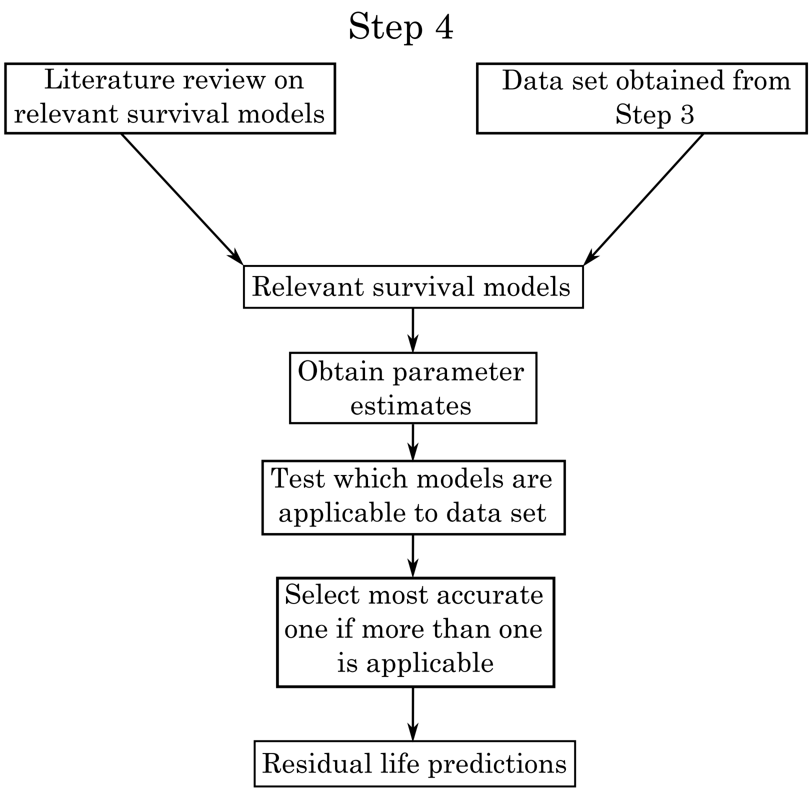


Figure 3.4: Flow diagram of Step 4.

When more than one model fits the data set, the model which yields the most accurate results should be used. Guidelines are now provided on when the reviewed models are appropriate and when not.

3.4.2.1 PHM and PWP

The PHM is the most popular and most widely used model, yet it will be useless if the data being considered does not adhere to the assumptions which the model is based on. The PWP is used when the data being considered to fit the proportionality assumption but is from a repairable system. Section 2.4.1 illustrates survival functions that indicate when the PHM is appropriate or not.

A very simple graphical method can also be used. The plot $\log(-\log(R(x)))$ vs $\log(x)$, the log of the cumulative hazard $(\log(H(x)))$ versus the log time. The result needs to be a linear plot and when the covariates are stratified, the cumulative hazard is also stratified and when plotted, the lines must be parallel to one another for the proportionality assumption to hold.

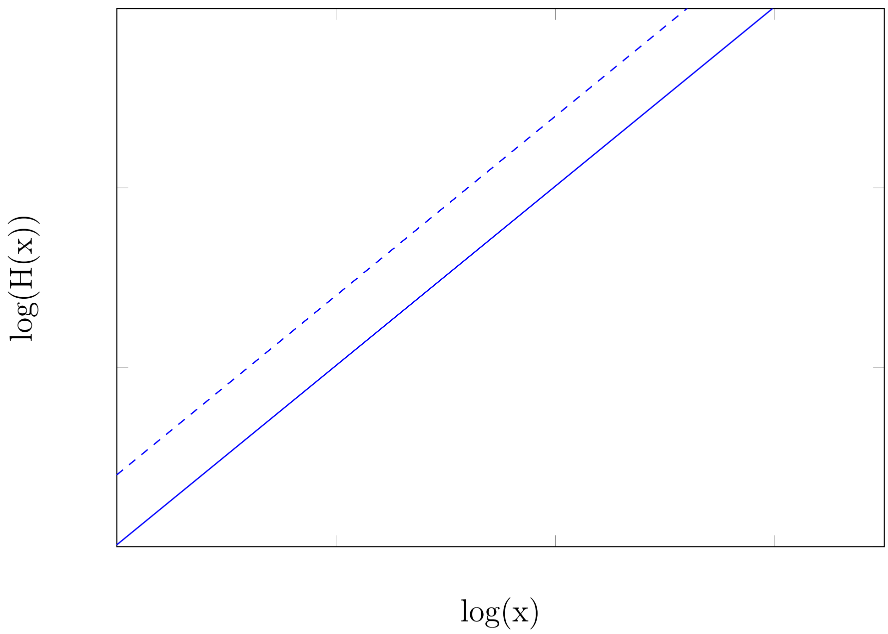


Figure 3.5: PHM goodness of fit.

A plot of the survival function with stratified covariate values can also be used as shown in Chapter 2. The survival curves of equipment are not allowed to cross in order to keep the proportionality assumption valid (Gorjian *et al.*, 2010*b*).

Another method to check the proportionality assumption is to again plot the log of the system's cumulative hazard rate, and $\log(H(x))$ multiplied by a constant determined by the estimated parameters (stratifying the cumulative hazard rate) versus time. This method is described by Kumar and Klefsjö (1994) in detail. The different strata will display whether the proportionality assumption holds or not. The vertical distance between two plotted curves need to be roughly equal for the PHM or PWP to fit the data.

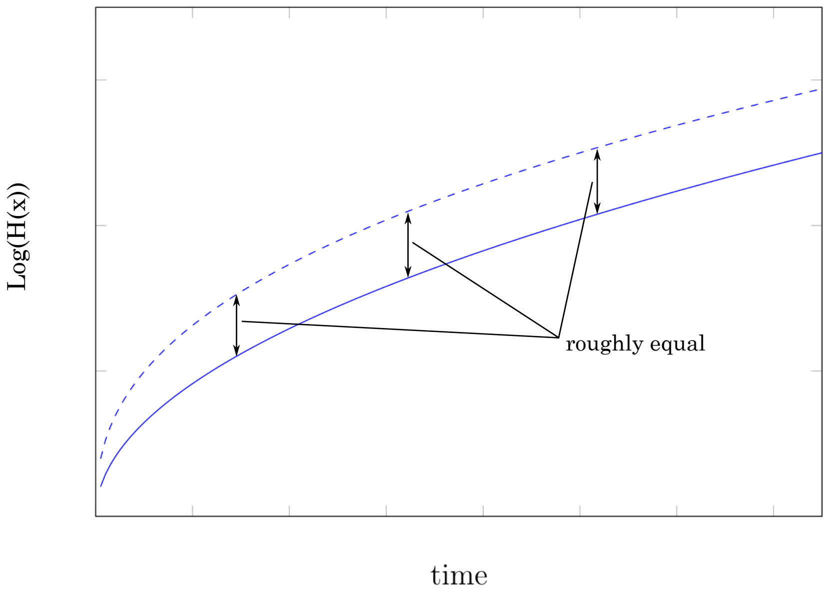


Figure 3.6: Testing proportionality assumption.

When considering a system that the proportionality assumption is valid for and the data is from a repairable system, the PWP model is used. Thus, the intensity is used instead of the hazard rate.

3.4.2.2 AFTM

The AFTM assumes that there is a direct relationship between the effect of the covariates and survival times of a system. This model can be seen as a specific case of the PHM. A simple regression test can be done to establish whether the AFTM is an appropriate model.

If a $\log(H(x))$ vs $\log(x)$ plot of different subjects in the study is linear (if the baseline hazard has the Weibull distribution) and parallel to one another, the AFTM is applicable, but because the AFTM is a special case of the PHM, the PHM will also be applicable. An AFTM will be used when this occurs because it assumes a direct relationship between the covariates and the failure time and not the covariates and the system hazard. Figure 3.7 illustrates graphically what the plot should look like.

The log of the cumulative hazard could also be plotted versus just the time, not the log time. The hazard should then also be stratified, the AFTM is said to fit the data by Kumar and Klefsjö (1994) when the horizontal distance between the curves is roughly equal. Figure 3.8 illustrates this concept.

The AFTM and the PHM will both be applicable in many cases. Should this happen, both models should be trained with all available data points. The model which can recreate the data set the most accurately should then be used.

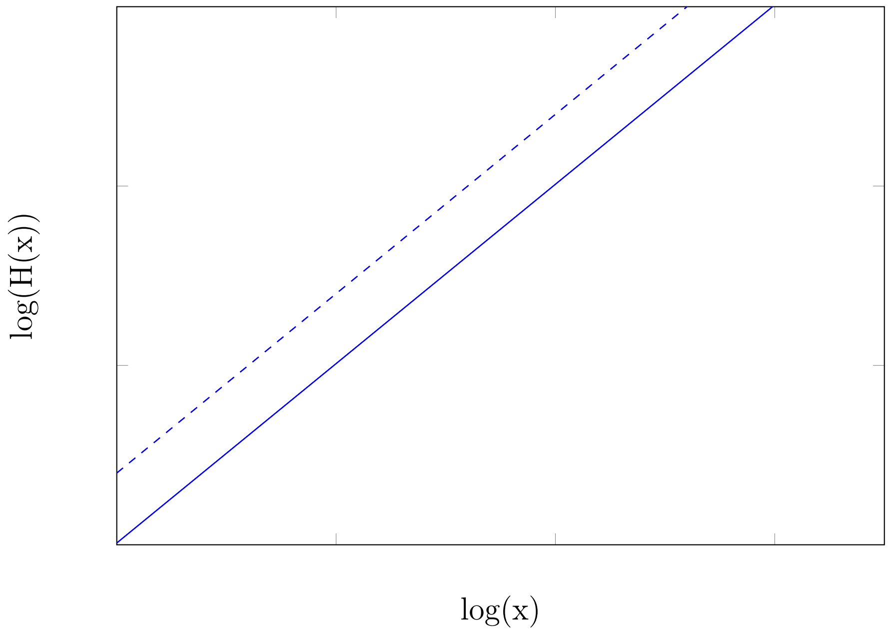


Figure 3.7: AFTM goodness of fit.

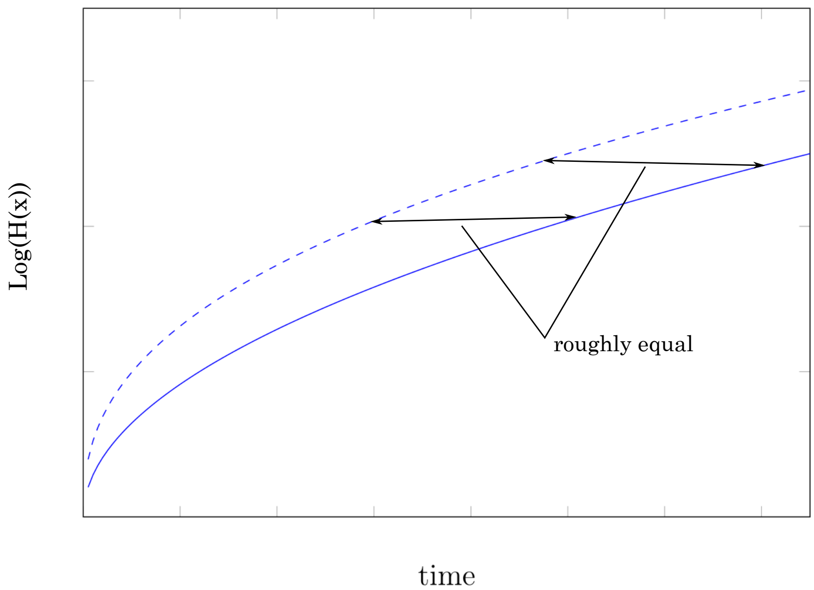


Figure 3.8: Another goodness of fit for AFTM.

3.4.2.3 AHM

This model will most likely be an option when the system does not have an initial hazard of zero. It is a key merit of this model to be able to represent a system that has non-zero hazard at time zero. Pijnenburg (1991) maintains that the AHM is often used when the proportionality assumption does not hold.

A simple example of the baseline hazard having a linear shape can be used to illustrate when an AHM is applicable and when a PHM is applicable. The set of covariate values $\mathbf{z}$ is stratified to illustrate the effect. Let $\alpha_s = (\alpha_2, ..., \alpha_p)'$, $z_s = (z_2, ..., z_p)'$ and have z_1 be a discrete variable and assign two different values ($z^\star$ and $\hat{z}$). This will result in two hazards;

$$h(x|z_i) = \begin{cases} h(x) + \alpha_s{}' z_s + \alpha_1 z^\star x & if z_i = z^\star \\ h(x) + \alpha_s{}' z_s + \alpha_1 \hat{z} x & if z_i = \hat{z}, \end{cases} \tag{3.4.1}$$

where x is the gap time (time between failures), or the time over which the curves are considered. This stratification will then indicate whether the additive assumption of the hazard is valid. This assumption can be verified if the two hazards are plotted and they are two parallel lines shifted some constant apart, as indicated in Figure 3.9 (Pijnenburg, 1991). The solid line is when $z_i = z^\star$ and the dotted line is for $z_i = \hat{z}$.

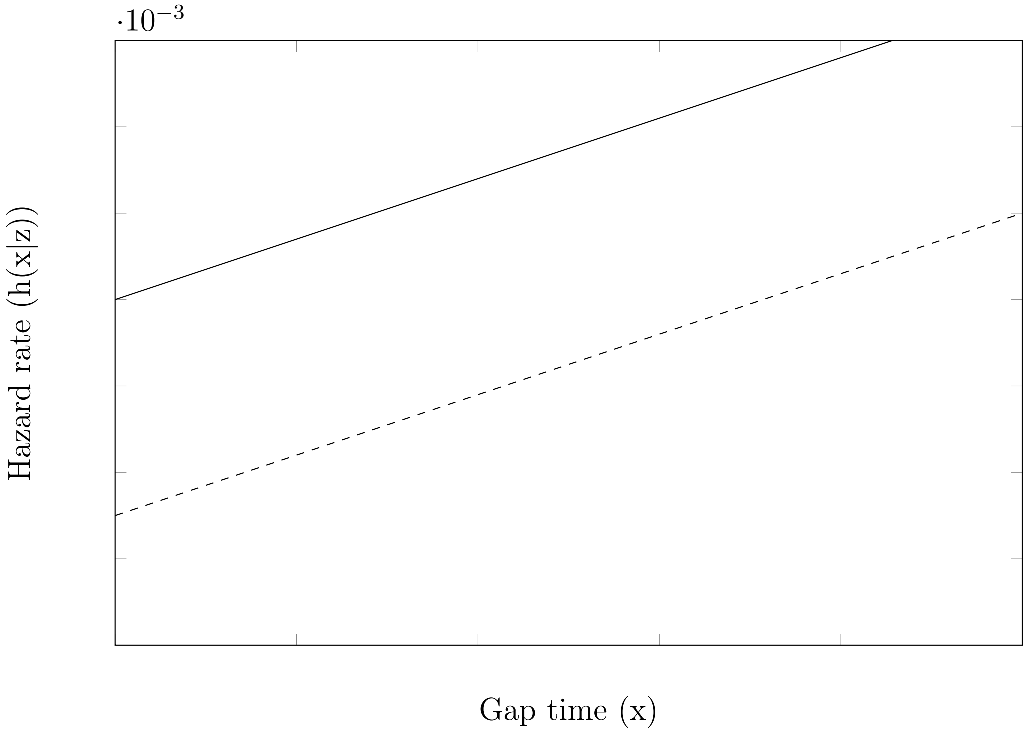


Figure 3.9: Checking additivity.

This is the case when the baseline hazard rate is assumed as linear as done in this study. One way to know for sure that the model cannot be used is if a negative hazard is returned because this is not realistic. This is also the case for a system/component that has a hazard of zero at time zero.

3.4.2.4 PCM

The PCM operates on the same assumption as the PHM, thus, if the proportionality assumption of the PHM is valid, the PCM is also appropriate (Sun, 2006). This model was developed in order to overcome the limitations of the PHM; the one limitation of the PHM is that it requires a sufficient amount of data. The PCM is, thus, favoured when a small amount of data is available. This model was developed for repairable systems but no indication is found that it could not be used for non-repairable systems. Sun (2006) states that the PHM allows for the baseline covariate function to be updated according to newly obtained CM and failure data. This then prevents error when estimating the hazard from the initial estimate of the baseline covariate function from accumulating as time progresses, illustrated in Figure 3.10.

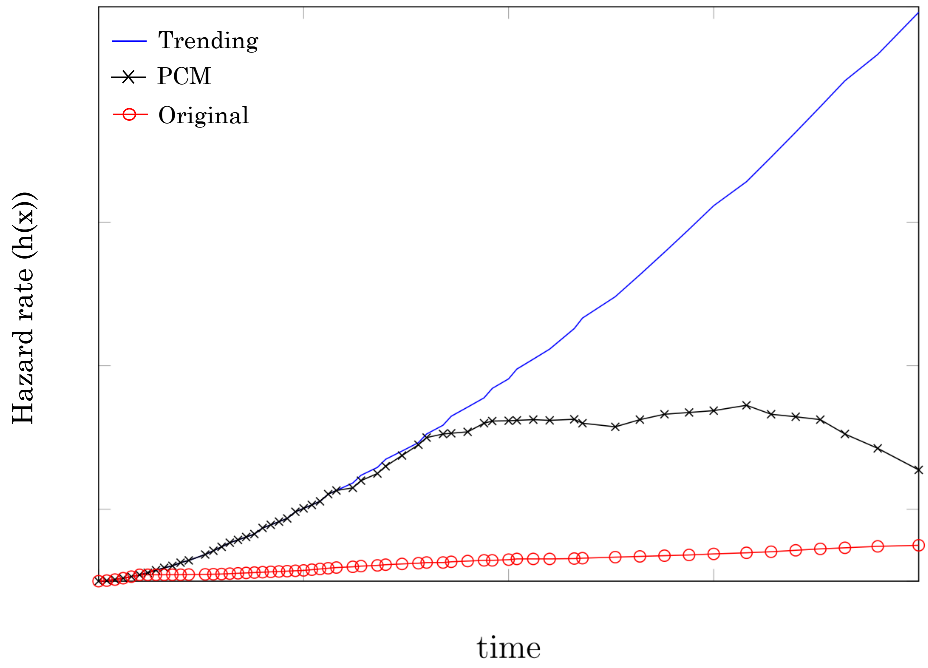


Figure 3.10: PCM hazard reduces error.

The PCM would, therefore, work very well when applied to equipment in real time. The covariate function is constantly updated with the new data received. This model will generally be applicable when the PWP and the PHM are applicable. The models should then be trained using the data set, which is then to be recreated with each model. The model which best recreates the data set will be selected to be the most appropriate survival models for the specific case.

3.4.3 Dummy Data Set

To clearly illustrate how the best survival model is to be selected, a dummy data set is created. This data set is meant to represent CM and failure data recorded from a bearing which a conveyor drum driving a belt runs on. This is purely to demonstrate how the correct model is to be selected. Three covariates (Z_1, Z_2, Z_3), represent the radial vibration, the axial vibration and the bearing temperature respectively. Figure 3.11 is a sketch of what the system typically might look like, this figure is only to demonstrate a practical application of a bearing and to annotate the forces being considered as covariates.

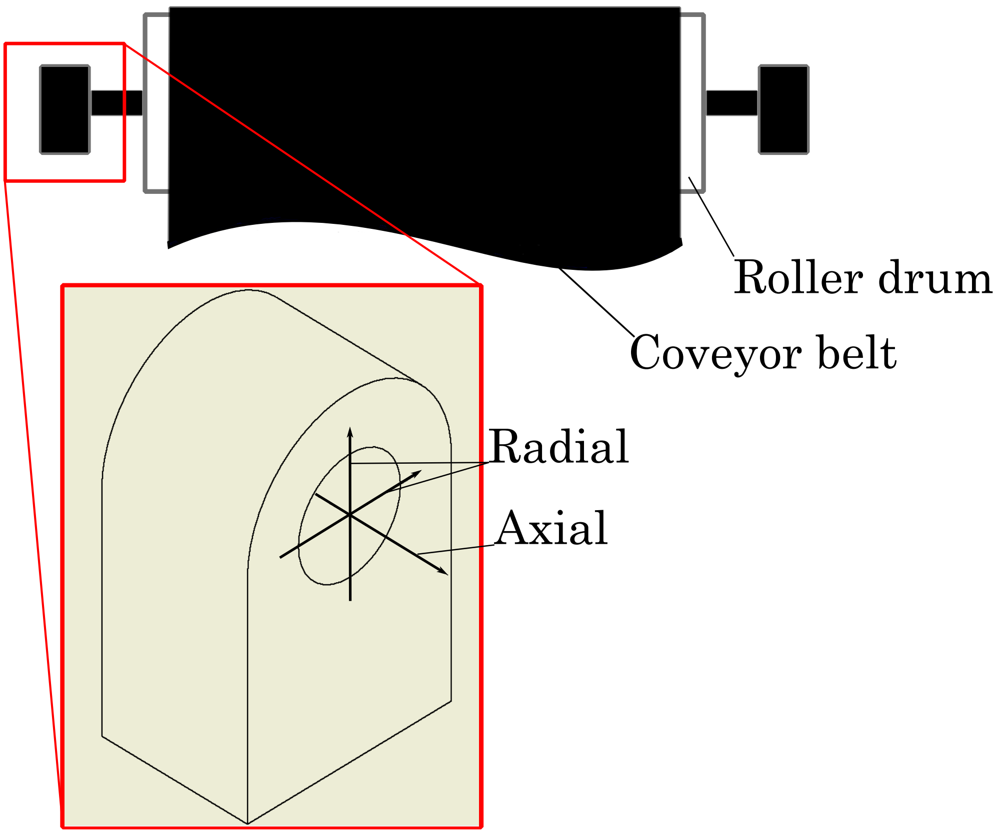


Figure 3.11: Illustration of bearing forces of roller for conveyor belt.

This data set is only created to test the various survival models. It is important to note that this is a made up data set. This data set is, however, created to be representative of an actual system because degradation characteristics of actual mechanical systems are considered to make the data as accurate as possible. The data set is shown in Table 3.5, the covariate values at the time of the failures are shown but the covariates are monitored over the entire period of operation. The values shown on Table 3.5 are all values recorded at the time of failure, the progression of the covariates values are separate.

The regression of the individual covariates up to the time of failure for the data set is also created and is used to train some of the survival models. The various survival models were populated with the dummy data set, the first eleven events were used to train the models and the last observation was then predicted. This allows a sense of how accurate the models are and if they can be used. The dummy data set is only for the purpose of demonstrating how to test the survival models. The survival model that best fits a study might differ from data set to data set.

The various methods mentioned earlier are applied to determine which models are appropriate in the case of the specific data set. Table 3.6 indicates which models are determined as being appropriate in this case, if they are appropri-

Table 3.5: Dummy data set.

Event #	**X_i (hours)**	**Z_1 (mm/s)**		**Z_2 (mm/s)**		**Z_3 (°C)**	
		Start	**End**	**Start**	**End**	**Start**	**End**
1	800.0	1.2	3.0	0.8	1.9	25	40.0
2	975.0	1.4	2.4	0.9	1.6	25	38.0
3	100.0	1.7	4.4	1.5	3.3	25	55.0
4	700.0	1.1	3.1	0.8	2.0	25	42.0
5	960.0	1.2	2.7	0.9	1.8	25	50.0
6	990.0	1.2	2.5	0.8	1.6	25	38.0
7	20.0	1.5	4.5	1.2	3.5	25	60.0
8	820.0	1.3	2.8	0.8	1.8	25	39.0
9	905.0	1.3	2.7	1.1	1.7	25	40.0
10	760.0	1.5	3.2	1.0	2.1	25	44.0
11	880.0	1.3	2.9	1.0	1.8	25	40.0
12	850.0	1.2	2.9	0.9	1.9	25	39.0

ate the results are provided. Table 3.6 provides the estimates returned by the

Table 3.6: Dummy data set results.

Model	**Residual life (Operating hours)**
PHM	851.87
PWP	N.A.
AFTM	751.94
AHM	N.A
PCM	600.78

models of the time when the covariates of the last observation was obtained. It can be seen that of the applicable models, the PHM delivered the most accurate result when predicting the last failure of the data set and would, therefore, be appropriate for the specific case. The next section introduces a specific case study that is used to validate the results for this study; it is elaborated on further in Chapter 4.

3.5 Validating Proposed Solution

In order to validate the applicability of the use of subjective covariates in survival models, a case study is done. The models used in this study have all been validated in prior studies. This study merely utilizes existing survival models to determine whether or not they can be used with subjective covariates and historical failure data. According to the proposed solution, a survival model is to be used with subjective covariates to predict the RL of physical assets. Thus, the RL estimates, which the survival model yields, must coincide with the knowledge of the experts from which the data is obtained.

The chosen survival model will be populated with the subjective data sets obtained from the individual experts and the model will be used to recreate the data set provided to them. The same survival model will then be populated by an objective data set, one from industry standard data. An attempt should then be made to recreate the same data set provided to the experts in order to give an indication as to how accurate the data obtained from them is. The final step is to compare RL predictions yielded by the model trained with the subjective data to the predictions yielded by the objective industry standard data.

Chapter 4

Case Study

This chapter covers a case study done utilising experts' knowledge on transformers for a smelting furnace in South Africa. This case study aims to evaluate the possibility of using purely subjective covariates and historical failure times to populate survival models in order to make RL predictions for physical assets. This case study seeks to apply the proposed solution and then analyzing the data acquired to obtain the results. The purpose of this chapter is

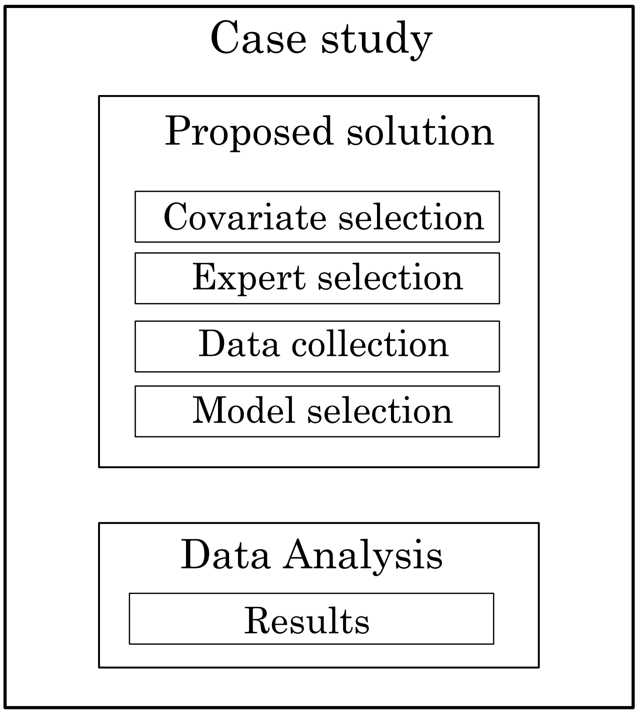


Figure 4.1: Case study steps.

application of the proposed solution from Chapter 3 to a real world problem. The steps in Figure 4.1 are systematically applied, as explained in Chapter 3, to reach a valid conclusion on the use of subjective covariates in the selected survival models.

4.1 Case Study Overview

The data for this case study was obtained from experts employed at a company that processes heavy minerals in South Africa. The desired products consist mostly of titanium dioxide slag and a high-purity pig iron, which are the products of smelting ilmenite in their electric furnaces. The temperature of the furnaces ranges from 1450 to 1700 degrees Celsius. The name of the company is withheld for confidentiality reasons. The data for this case study is based on the knowledge which the experts have gathered on the CM and failure data of the power transformers for smelting furnaces.

Separate transformers are used to power the two furnaces; both of the transformers were manufactured by ABB Power Technologies. The transformer of furnace one has a power rating of 40 megavolt ampere (MVA) and the second is slightly larger with a power rating of 50 MVA. These transformers are typically given a lifespan of 20 years by the manufacturers, but some of them have been known to operate for over 30 years. The transformers in this study are both approaching their end-of-life stage and the challenge is now to determine how long the transformers can operate within this stage before the risk of failure becomes too large.

These transformers do not come cheaply; therefore, it is necessary to manage these physical assets properly. The assets are not only expensive to replace but would also cause a substantial financial loss to the organization in the form of large production loss if they fail unexpectedly. Health and safety risks of an unexpected failure in an environment like this can also be expected if the equipment fail unexpectedly. This is, therefore, an ideal situation to use survival models because there are serious consequences and little historical data available on the specific transformers but a great deal of research has been done on transformer degradation. The knowledge of the experts can, thus, easily be compared to that of the industry standard.

4.2 Conducting Case Study

During two visits to the plant and a series of discussions, all the required data was collected. A short introduction to the transformers by the experts selected and a study of relevant literature ensured a complete understanding of the assets, their function and their maintenance. The two furnace transformers are similar except for their power ratings and their operating time. A picture of what these transformers look like can be seen in Figure 4.2, the tank seen is an oil reservoir for the cooling oil that circulates through the transformer. The transformer is too large to fit into a picture since it is inside a room which only allows a certain amount of space to distance the camera to fit the equipment in a picture.

Transformer number one is rated 40 MVA and has been in operation for 19 years, it was operating at an average of 83% loading but this has been in-

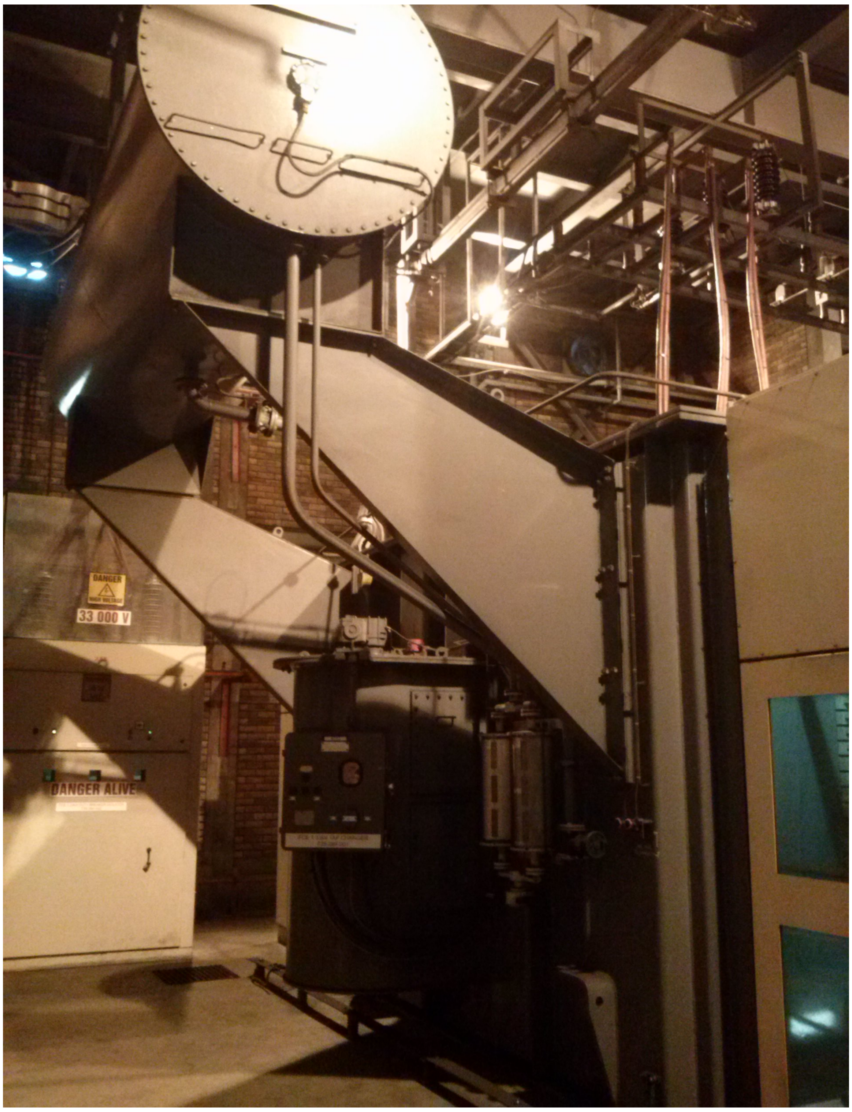


Figure 4.2: Picture of transformer number two.

creased to 96% during the last three years. The second transformer has been in operation for 15 years and is rated 50 MVA. This transformer was also operating at the ideal loading of 83% but during the last three years, that had been increased to 97%. Normal ageing of transformers is currently the problem being experienced here, thus, the one aspect to consider is the degradation of the windings' insulating paper material (Gray, 2006; Homagk *et al.*, 2008; Lundgaard *et al.*, 2004).

CM studies have enabled the loss of life to be calculated for oil filled transformers by analyzing oil samples taken from the transformers as Prevost *et al.* (2006) explain. These CM studies become necessary towards the end of the transformer's life to ensure no unexpected failures occur and to plan ahead for replacing the assets. The transformers are both operating above the ideal loading of 80% to 85%, which accelerates the degrading of the insulation material. It is, therefore, important to be able to predict when they are likely to fail.

To have the transformers operating at an ideal loading again, the organization is contemplating replacing the current transformers with ones that have a 60 MVA rating. This will be to the advantage of the organization and the new transformers can then operate under ideal conditions, thus, maximizing their life span. This venture is expected to cost the organization up R25, 517, 000.00 per new transformer. These transformers are, thus, good assets to test whether subjective covariates can be used to populate the relevant survival models. This can be assumed because the transformers have been in operation for a long time allowing the experts to become very familiar with the equipment. The real risks and financial impact of unexpected failures are also known and this will encourage the experts to provide data to the best of their knowledge.

The guidelines laid out in Chapter 3 are systematically executed to complete this case study. The execution of these steps are now presented specifically for this case study.

4.2.1 Step 1

To ensure that the data obtained is obtained from people with above average knowledge in a certain field, those being considered must possess specialized education or enough experience in the field considered or have a job title which suggests that they have more knowledge and insight than the average person. There is no set criteria for when to consider a person an expert in a certain field. However, Hussler *et al.* (2011) state that one of the most important aspects is that a person should be considered as an expert by consensus of others operating in the same field.

In relation to the three elements of experience, education and occupation to provide a guideline for selecting the experts, Table 4.1 displays the aspects of the three elements for the experts used in this study.

Table 4.1: Experts used in this study.

	Experience	**Education**	**Occupation/Job title**
Expert 1	16 years	B.Eng(electrical/electronic)	Assistant Manager, Engineering department
Expert 2	27 years	Diploma in electrical engineering, Government Competency Certificate (GCC)	Senior Electrical Engineer
Expert 3	34 years	N4 installation electrician, electrical course in all fields	Site Supervisor
Expert 4	3 years in tribology alone	Tribology course (South African Institute of Tribology), Transformer oils sampling course, Machine lubricant analyst (MLA1)	Condition Monitoring Technician
Expert 5	25 years	B.Eng(electrical/electronic), Pr.Eng, GCC	Senior Electrical Engineer

It is clear that the people used in this study possess more knowledge in the field of transformer CM and their operation than the average person, with over 100 years of combined experience between the five experts. This should not come as a surprise when looking at the experience and education that they have, and the fact that they are responsible for the safe and effective operation of multi-million rand assets.

4.2.2 Step 2

Step number two is to select the correct system characteristics to use as covariates in the survival models. The guidelines presented in Chapter 3 are a good starting point but ultimately, the covariates that will be used are system and case specific. In this case, the experts at the organization were first consulted and asked to explain what exactly the situation is and the main issues which they are experiencing with the transformers. A fair amount of literature was also reviewed to become familiar with the transformers and their degradation.

CM applied to the transformers is done by analyzing the oil circulating through the transformer to insulate and cool the windings. A discussion with the experts provided some of the most valuable information available; they were able to provide past tribology (oil analysis) reports of the transformers and explain what their greatest issues are. The tribology reports provided the values of the different characteristics discussed by Gray (2009), which included:

1. Acid content
2. Dielectric strength
3. Oil temperature
4. Moisture or water content
5. Hydrogen content
6. Nitrogen content
7. Methane content
8. Carbon monoxide content
9. Carbon dioxide content
10. Ethylene content
11. Ethane content
12. Furaldehyde or furan content.

Following a discussion with the experts, the characteristics listed below are chosen to be used as covariates, the other characteristics from the oil analysis are not considered. In this specific case, they have not had a problem or concern for the transformers, therefore, the covariates for this study are:

1. Oil temperature
2. Furan content
3. Transformer capacity loading.

The loading of a transformer is added to the list of system characteristics because it has an effect on the other characteristics considered. Furan content in the oil is used to determine an important factor, that is, the tensile strength of the insulating paper on the windings inside the transformer. Describing the tensile strength is what is known as the Degree of Polymerization (DP). Tensile strength also indicates the end-of-life criteria for the insulating paper. The DP is dependent on the furan content, and thus, will be used to explain the transformers' health. Using the DP is an industry standard criterion for the end-of-life of the insulating paper on transformer windings. Oil temperature is included because it generally has a stable range in which the transformer operates and abnormalities are quickly and easily identified.

Insulating the windings of a power transformer is kraft paper and oil within the transformer and this is why the monitoring thereof is important as Verma (2005) explains. Mineral oil is used because animal and plant oils degrade the insulting paper more rapidly when heated. The temperature of the transformers generally fluctuates around some average temperature that differs depending on factors such as where in the transformer the readings are taken and the ambient temperature. A flashpoint temperature of 140 degree Celsius for the mineral oil is convenient and allows a large operating range for the oil temperature. Flashpoint temperature is the lowest temperature at which vapours start to develop a flammable mixture.

The DP is dependent on the furan content in the oil; the samples taken are analyzed to determine the tensile strength of the insulating paper. Extensive research has been conducted on the insulating materials of power transformers yielding the various DP ranges provided in Table 4.2. Using the DP has been accepted by industry as a standard criterion to determine the insulating paper's health in power transformers (Gray, 2006; Prevost *et al.*, 2006; Patki *et al.*, 2008; Mehta and Jariwala, 2012). Table 4.2 displays the furan content, DP value and the corresponding state which the insulating paper is in. New transformers have a DP value between $1200 - 1000$. A DP value of 1000 is used in this case study.

Degradation of the insulating paper is affected by the temperature of the oil, since it degrades more rapidly at higher temperature. The loading factor of a transformer has an effect on the temperature of the oil. As expected, the closer the transformer operates to its full capacity, the higher the temperature of the circulating oil will be. The rated operational loading of a transformer is between $80-85$ percent of their rated capacity. The geographical location (the temperature), the rate at which the oil dissipates the heat from the transform-

Table 4.2: Corresponding values of furan content and DP with paper health adapted from Patki *et al.* (2008).

Furan content (ppm)	DP	State of paper
0.000-0.130	1200-700	Healthy, seen as brand new
0.131-1.060	699-450	Mild deterioration
1.061-6.600	449-250	Approaching critical condition
6.601-12.10	250-200	Extensive damage
>12.11	<200	Exceeds critical condition

ers and the loading capacity at which they operate are the ideal condition for the transformers to function. This information is as provided by the experts.

In this case study, the three covariates used are the oil temperature, the DP value and the loading of the transformers. The decision was made together with the experts to exclude the oil temperature from the covariates as it should fluctuate around an average temperature as long as no abnormalities are present or the transformers are not overloaded. These specific transformers did not experience any abnormalities causing high oil temperature or any dangerous gas level build-ups. Therefore, the decision is to use the loading of the transformer (Z_1) and the furan content, which is converted to a DP value, (Z_2) as covariates. The final decision, however, is to use only the DP because the loading factor directly affects the DP value.

4.2.3 Step 3

Experts to obtain the data from are selected and the system characteristics to be used as covariates have been identified. Next, the data must be elicited from the experts. Relevant data must be put in a format allowing the survival models to be run with ease. The data which is required is first discussed then the process of obtaining the data.

4.2.3.1 Required Data

The covariate selected is denoted by Z, for the furan content in the oil. Together with the covariate values, the times at which the readings are taken is also needed, this is denoted by X_i. An indicator to show whether the readings are recorded at a failure or not is presented as a binary variable where a 1 indicates a failure and a 0 otherwise, this indicating variable is denoted by C_i. The experts are also expected to provide their estimate of the transformers' RL given that they continue operating at the conditions at the time when the final reading is taken.

4.2.3.2 Obtaining the Data

Technique number two is used to extract the data from the experts, where experts are provided with covariate values and are expected to deliver the times when readings were recorded. Ten different scenarios are created by specifying the loading of the transformers over certain time periods and the final DP value. The loading of the transformers determines at what rate the DP decreases and therefore affects the life time of the transformers. Thus, the loading is specified and the experts must provide the progression of the DP value over time up to the time when the final reading for each scenario is taken.

All the experts are assembled in the same room and provided with Table 4.3. The operating details for each scenario are given to them and they are to provide the values for the last two columns (in years) as best as they can as well as the progression of the DP. The flow diagram provided by Figure 3.3 in Chapter 3 is followed, thus, ensuring that consensus is reached on the data extracted from the experts. Only one set of data is created here, but the experts provided their individual opinions on the progression of the covariates up to these event times. The regression of the DP by each expert is also used as data sets to compare the experts' knowledge to each other.

Table 4.3: Different scenarios provided to experts.

num	**C$_i$**	**Influencing factor (Loading)**	**Influencing factor (Oil temp °C)**	**Z (DP)**	**X$_i$ (Years)**	**Estimated RL**
1	0	0.80	48	500		
2	0	0.75	42	350		
3	0	0.80	50	700		
4	0	0.92	60	520		
5	1	0.85	66	195		
6	0	0.85	53	510		
7	0	0.95	65	400		
8	0	0.88	55	830		
9	1	0.97	67	200		
10	0	0.78	42	220		

The ideal operational loading for the transformers used in this study is $80-85$ percent and the DP value of a new transformer is taken to be 1000. Oil temperatures are also provided as a covariate to the experts to indicate that there were no abnormal events. Scenarios provided to the experts are all inspired by actual data from past research and will be explained in detail now.

Scenario number one is a transformer that started operation at 80% loading and then operated at an 85% loading for two years. The DP reading is taken to be 500, the time at which this reading was taken was given by the experts to be after 12 years since the start of operation. All of the experts agreed that this transformer can continue operation at an acceptable level of risk of failure for ten more years.

Scenario number two is a transformer that has been is in operation at a 75% loading for an unknown time. The loading stays constant from the start of operation until the DP level reaches 350 and it also continues operation at the same conditions afterwards. Considering this, the experts concluded that it has been in operation for 15 years and has eight years of operation left.

The third scenario that was given to the experts can be seen as ideal operating conditions for these transformers. Operating at a loading of 80% loading and no other abnormalities until the DP reaches a value of 700. According to the experts, this transformer has been in operation for eight years. The experts were also able to agree that if the ideal loading stayed constant for the rest of the asset's life, it would be able to continue operation for a further 13 years.

Increasing the loading above the recommended range of $80 - 85$ percent up to 92% and then continuing at this load caused the experts to decrease the total life expectancy to 18 years. The experts reached consensus that if the transformer is to operate at 92% loading from brand new (DP value of 1000) until it's DP has reached 520, it would take ten years. Thus, the RL of the transformer at that time would be eight years.

The fifth scenario is another one where the transformer operated at an ideal loading, this time 85%. This loading is constant over the entire period of operation for the transformer until the DP reading of 195 is taken which was agreed upon by the expert to be after 22 years of operation. The condition for a functional failure is when the DP value dips below 200, thus, the RL of this transformer is zero.

The next case is a transformer that's loading started at 80% and was then increased to 85%. It is not known how long the transformer operated at 80% loading but it is known that it has been operating at 85% for the last four years. The DP value at this stage was recorded to be 510. The experts then came to the conclusion that the transformer has been in operation for a total of 12 years and if it continues to operate at an 85% loading it has a RL of eleven years.

The transformer in scenario number seven started operation loaded at 80% of its rating; this loading was then increased to 95% and it is known that the transformer operated at this loading for two years. A DP reading at the time of inspection was read to be 400; the experts concluded that the time of inspection was after the transformer was in operation for 16 years. It was established that if the transformer should continue operation at 95% loading of its rating, it will operate safely for another five years, given that no unexpected events occurred.

The DP of transformers generally follows the shape of an exponential function, to test how the experts' opinions compare to the industry standard scenario number eight asks for the time of an observation for a relatively new transformer. In this case the transformer has been operating at a constant loading of 88% since the start of operation until its DP value was recorded to be 830. According to the experts this reading was taken four years after the start of operation and that the transformer has a RL of 20 years if it continues to operate under the same conditions.

The second last scenario is a transformer that started its operation with 83% loading but for the last five years, it has been operating at 97% of its rated power. A DP reading of 200 was recorded, and consensus was reached that the transformer has been in operation for a total time of 18 years. According to the judgement of the experts, the transformer can continue operation for one more year at the current condition even though it is at the critical DP level of 200.

The final scenario is a transformer which started operation at 70% loading for an unknown time, which was then increased to 78% for the last ten years of operation. The DP is recorded at 220; this led the experts to believe that the transformer has been in operation for 25 years. If the transformer continuous to operate at the loading of 78%, the experts agreed that the transformer has a RL of two years. Table 4.3 is updated to include the newly acquired data, Table 4.4 displays the updated table.

Table 4.4: Observation times obtained from experts.

num	**C**$_i$	**Influencing factor (Loading)**	**Influencing factor (Oil temp °C)**	**Z** (**DP**)	**X**$_i$ (**Years**)	**Estimated RL**
1	0	0.80	48	500	12	10
2	0	0.75	42	350	15	8
3	0	0.80	50	700	8	13
4	0	0.92	60	520	10	8
5	1	0.85	66	195	22	1
6	0	0.85	53	510	12	11
7	0	0.95	65	400	16	5
8	0	0.88	55	830	4	20
9	1	0.97	67	200	18	1
10	0	0.78	42	220	25	2

Together with this data the experts were asked to describe the progression of the DP value from the start of operation up to the time of the event for each scenario. This is done to be able to compare the industry standard, proven by multiple research outcomes, to the best knowledge of the experts. Only once this is all done can the data be put into the survival models.

4.2.4 Step 4

The data obtained is first to be tested to reveal whether there is an underlying trend or not, thus, enabling the data to be classified as repairable or non-repairable data. This will then determine which models are to be used allowing the most applicable model from the models reviewed to be used. The Laplace trend test revealed that the data set obtained from the experts has no underlying trend present ($U = -0.51$), thus, classified as a non-repairable system and the HPP can be used to model the RP. The survival models reviewed in this study that will be applied to the data set include the PHM, AFTM, AHM and the PCM.

PHM

The first model applied is the popular PHM. A parametric model can be used as the Weibull parameter assumed to model the system's failure characteristics is very flexible and has been proven applicable in reliability analysis. Utilising one of the tests from Chapter 3 to determine if the model is applicable, it is determined that the model is applicable in this case, as Figure 4.3 illustrates. The vertical distance between the two curves remain roughly equal, thus, indicating that the proportionality assumption is valid (Kumar and Klefsjö, 1994). It is certain that the PHM is applicable because by examining the reliability

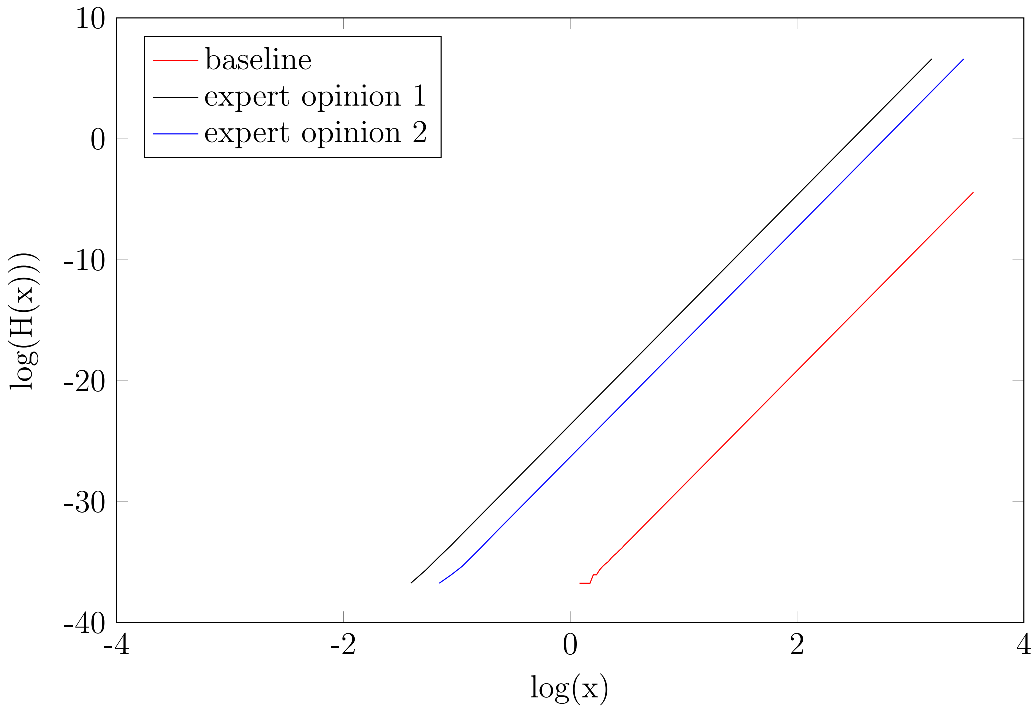


Figure 4.3: Testing proportionality assumption.

curves of separate scenarios, as shown in Figure 4.4, it can be seen that the curves meet the prescribed requirements as mentioned by Machin *et al.* (2006). These requirements are illustrated graphically in Figure 2.8.

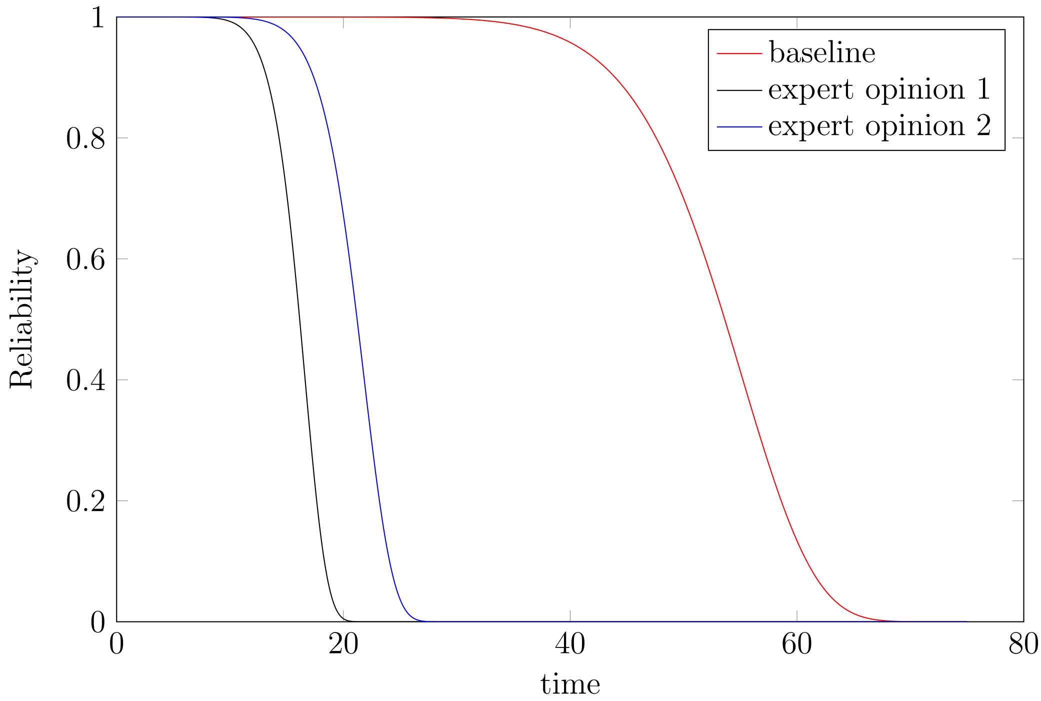


Figure 4.4: PHM reliability curves.

The parametric version of the PHM is utilized to be able to predict the RL with ease. The parameter estimation is done as explained in Chapter 2. The expert opinions are the opinions for two of the ten scenarios provided to the experts. The parameter values estimated are provided in Table 4.5, which the Weibull parameter values as well as the regression coefficient values.

Table 4.5: PHM parameter values.

Parameter	**Value**
Shape parameter (λ)	9.484
Scale parameter (η)	55.739
Regression coefficient 1 (β_1)	8.511
Regression coefficient 2 (β_2)	0.077

Using the data set obtained, the goodness of fit of the model can be tested. Feeding the covariate values of all the scenarios into the model and predicting the RL yielded the answers displayed in Table 4.6. These answers are also compared to the estimates provided by the experts, where the mean of the absolute error amounts to 2.037 years. The other models are all first applied and a comparison between the different models is only done after all the models are applied. The next model that is considered to be a valid model is the AFTM.

Table 4.6: PHM RL estimates.

Scenario	PHM estimates	Expert estimates	Error
1	9.632	10	0.368
2	8.495	8	-0.495
3	12.407	13	0.593
4	9.371	8	-1.371
5	4.793	1	-3.793
6	9.726	11	1.274
7	6.653	5	-1.653
8	16.738	20	3.262
9	3.859	1	-2.859
10	6.697	2	-4.697

AFTM

The AFTM is a common alternative to the PHM and is applied second. The assumption that the covariate has a direct effect of accelerating or decelerating the survival time is to be confirmed first. This is done as explained in Chapter 3; the *log* of the cumulative hazard is plotted against the *log* of the time of the events. Figure 4.5 illustrates the cumulative hazard for scenario number five and ten, just because their operating periods are more or less equal and thus easier to plot on the same graph. It can be seen that two lines are more or less parallel. The AFTM is, therefore, also an applicable model to use in this case.

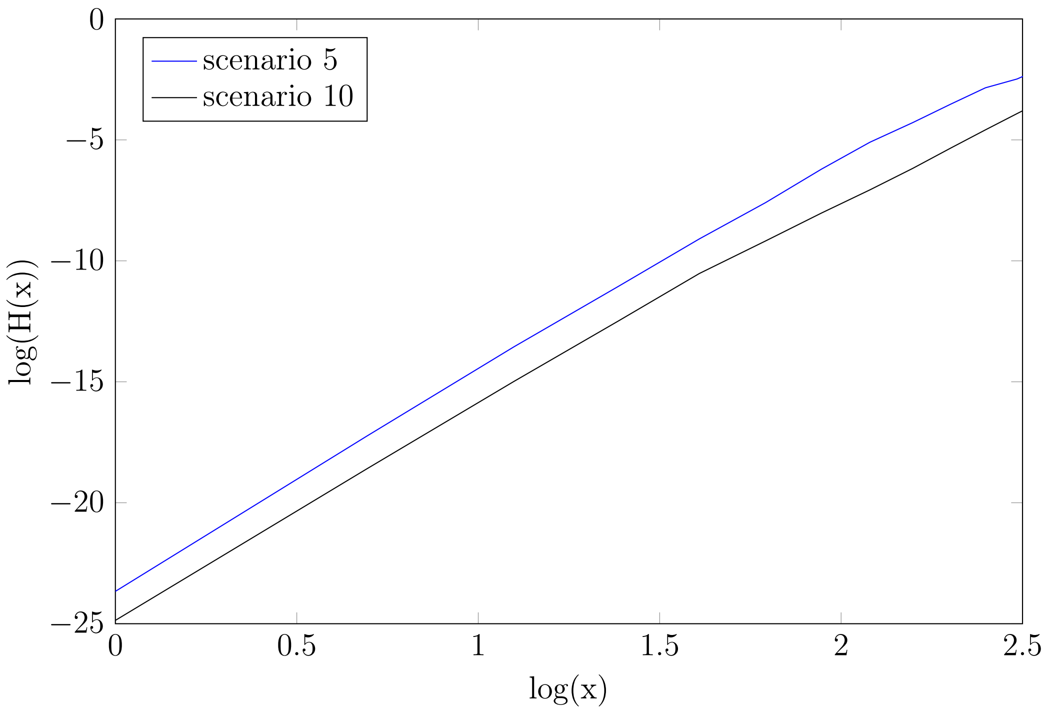


Figure 4.5: Testing AFTM assumption.

Parameter estimation for the regression coefficients and the descriptive parameter values of the AFTM returned the values as presented in Table 4.7. The distribution of the failure times is assumed to be a Weibull distribution, yielding the extreme value distribution for the error distribution as explained in Chapter 2.

Table 4.7: AFTM parameter values.

Parameter	**Value**
Intercept (μ)	4.021
Regression coefficient 1 (β_1)	-0.897
Regression coefficient 2 (β_2)	-0.001
Scale parameter (σ)	0.1054

The AFTM returns the time at which it estimates an event will occur when provided with the relevant inputs. Since the values returned are the *log* of the event times, the estimated event time is the exponent of the returned value. Once again, each of the scenarios in the data set is recreated and compared to the times provided by the experts. Table 4.8 reveal the results and the error when trying to recreate each observed event in the data set.

Table 4.8: AFTM RL estimates.

Scenario	**AFTM estimates**	**Expert estimates**	**Error**
1	6.120	10	3.880
2	6.404	8	1.596
3	7.406	13	5.594
4	6.008	8	1.992
5	0.190	1	0.810
6	5.185	11	5.815
7	1.176	5	3.824
8	8.903	20	11.097
9	1.843	1	-0.843
10	-1.846	2	3.846

Again, the average of the absolute error is calculated, and the AFTM yielded a slightly larger error than the PHM. The error is calculated to be 3.930 years, thus the model is evaluated to be less accurate than the PHM in this case. An aspect about this model that is very attractive though is how easy it is to track the individual equipment reliabilities in real time and make continuous RL predictions. The next survival model applied to the data set assumes additive effect on the hazard instead of a multiplicative effect as the first two models do.

AHM

Accordingly, the model that assumes an additive effect is known as the Additive Hazards Model. A semi-parametric AHM leaves the baseline hazard unspecified, thus, no assumptions need to be made about the baseline hazard. The model is found to not be applicable in this specific case. The AHM yields a negative cumulative hazard for the system, which is unrealistic and thus, the model is not suitable for the data set used in this case study. Figure 4.6 illustrates the cumulative hazard plot, the negative hazard can clearly be seen. The final model to test was only developed in 2006 and is meant to address

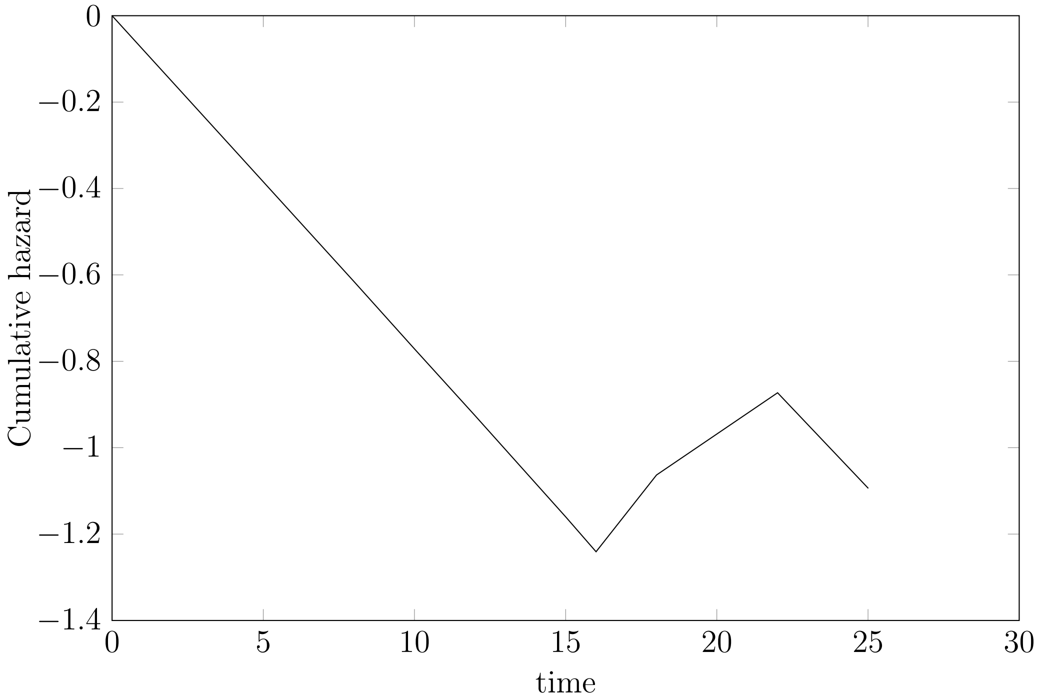


Figure 4.6: AHM cumulative hazard.

some of the limitations of the PHM.

PCM

As described by Sun (2006), the PCM assumption of the covariates having a proportional effect on the hazard of the system is valid when the proportional hazards assumption of the PHM is valid. Thus, the PCM is valid when the PHM's assumption has been proved acceptable. The PCM can be considered an applicable survival model to use in this study because the PHM's assumption has been proved acceptable, as shown in Figure 4.3.

The PCM is the only model used that is not available as a pre-programmed package or toolbox in any of the software systems. MATLAB code for the model can be found in Appendix A, based on the theoretical knowledge from Sun (2006). Parameter estimation is done where half of the data set is used for the initial estimation and the second half of the data is used to update the estimations obtained from the first half. The parameter values are updated after each recording interval, which is one year in this case.

The values of the covariate (the DP) are to be normalized to range between zero and one. This is done to reduce the sensitivity of the covariate function ($Z(x) = C(x)h(x)$) when calculating the specific covariate values at specific times. The parameter values are the mean values of all the experts used in this study.

A PHM is initially created for the covariate and the loading of the transformers (used as a covariate). A correlation test between the loading factor and the time to events revealed that the loading of transformers had a very weak correlation coefficient of 0.1. In order to prevent poor estimations, the loading is not used to estimate the parameters. The DP is the only covariate considered hereafter. The parameter estimates for the covariate as well as the hazard functions are tabulated in Table 4.9.

Table 4.9: Final PCM parameter values.

Parameter	**Value**
C(x) $= a \cdot x^b$	
Parameter 1 of covariate function (a)	$45.578 \cdot 10^9$
Parameter 2 of covariate function (b)	-7.264
h(x) $= \frac{\lambda}{\eta}\left(\frac{x}{\eta}\right)^{\lambda-1}$	
Shape parameter of Weibull function (λ)	8.201
Scale parameter of Weibull function (η)	25.695

The obtained parameter values are used to recreate the original data set. An estimate of the time and the covariate value for a specific time is provided by the experts, the model then recalculates (scales) the covariate value ($Z(x) = C(x)h(x)$) by using the parameter values obtained for the covariate function ($C(x)$) and the hazard function ($h(x)$). This scaling is necessary because of the excessively large value of the covariate function at early times together with the very small hazard in the first couple of operating years. This is because of the high reliability of transformers during these first couple of years. Figure 4.7 illustrates the characteristic curve for the covariate function, $C(x)$. Here, it can be seen how sensitive this function is to any alterations at the start of operation. The discrete points obtained from the experts are represented by the dots.

Only when zooming in closer to Figure 4.7 does it become clear just how sensitive this specific model is to an alteration in the covariate function in the first 15 years. After this the function becomes insensitive to any small alterations. Figure 4.8 illustrates the sensitivity of $C(x)$ times early in the

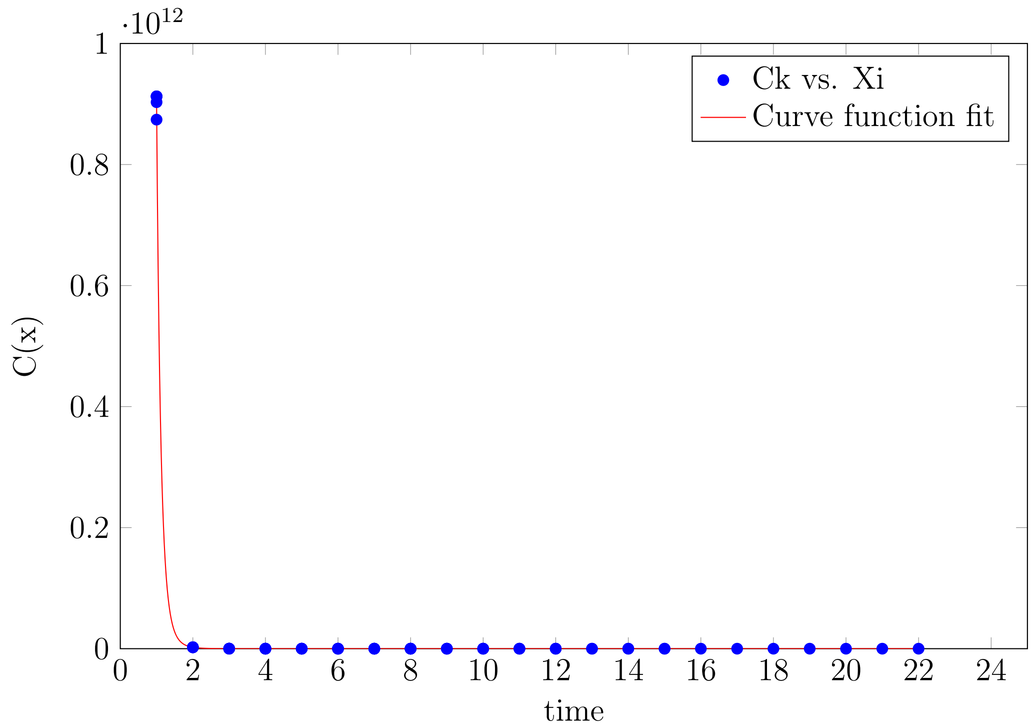


Figure 4.7: Characteristic curve of the covariate function.

life time of the transformers. The function becomes less sensitive as the time progresses and as the function slowly settles to zero.

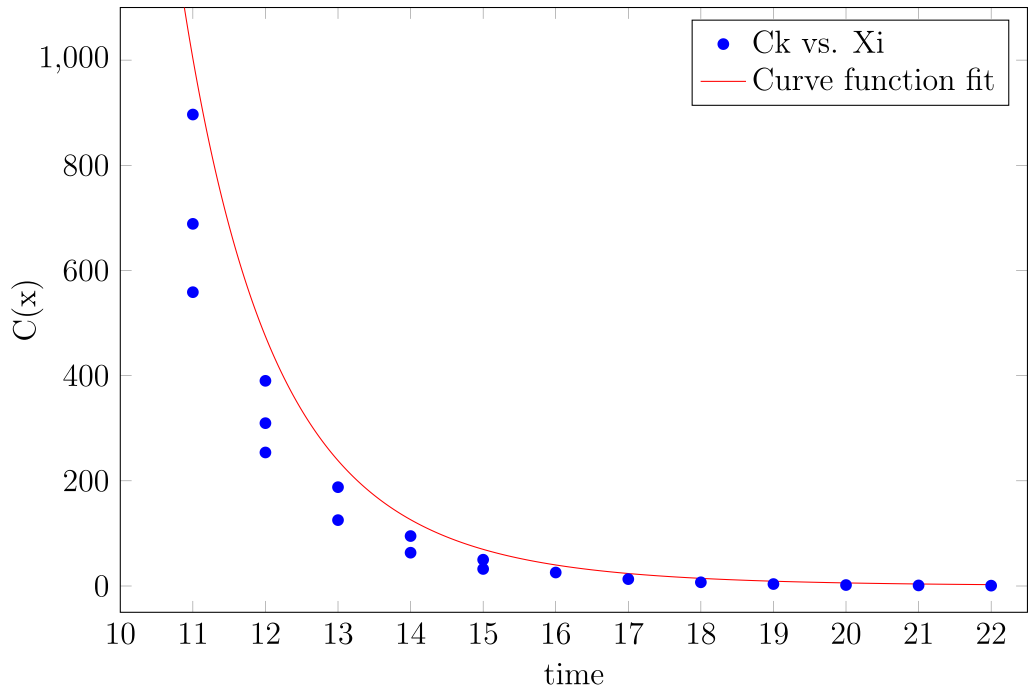


Figure 4.8: A closer look at C(x).

Using these covariate functions to scale the covariate values instead of just using the raw covariate value yield the results in Table 4.10. These are significantly more accurate but care must be taken because the model is very specific to the data used to train it.

Table 4.10: PCM RL estimates from scaled covariate values.

Scenario	**PCM estimates**	**Expert estimates**	**Error**
1	9.976	10	0.025
2	7.905	8	0.094
3	12.974	13	0.026
4	7.971	8	0.029
5	0.905	1	0.095
6	10.905	11	0.096
7	4.975	5	0.025
8	19.994	20	0.006
9	1.000	1	0
10	1.749	2	0.251

The mean error of these estimates is 0.065 years, which is the most accurate survival model, but as mentioned, the model is now heavily based on the data specific to this case. Therefore, the PCM accurately recreates the original data set but the parameter values are specific to the transformers used in this study.

In summation, of the six survival models reviewed only five are applicable for the equipment in this study. The POM is not considered because it models systems where the covariates have a diminishing effect on the reliability of the system. The covariates chosen for the transformers in this study have an accumulating degradation effect on the transformers. The five remaining models are then narrowed down further according to their applicability to the data set used. The data set for this study is determined to be that of a non-repairable system, thus, making the PWP model not applicable.

The four models finally applied to the data set include the PHM, AFTM, AHM and the PCM. The error of each model when recreating the data set is presented in Table 4.11.

Table 4.11: RL estimate errors.

Survival model	**Error (years)**
PHM	2.037
AFTM	3.930
AHM	Not applicable
PCM	0.065

The most suitable survival model for the data set in this study is the PCM. The survival model is also trained using industry standard data. Estimates obtained when using the experts' opinions and the industry standard data can now be compared. The results and the analysis of the data are discussed in the following chapter.

Chapter 5

Results

This chapter presents the results from the case study conducted in Chapter 4. The PCM is found to be the most favourable survival model for this particular case. The results of the model populated with the experts' opinions are first discussed, then the model is populated with industry standard data. A comparison is done between the results from the subjective data and that of the industry standard data. This comparison is done to validate the results yielded by the subjective data.

5.1 Expert Opinions

There are ten different operation scenarios for the transformers in the data set for this study. The data points obtained from the experts describe the scenarios in detail as the DP progresses over time. The data points specify either the time of observation or the time at which the failure was recorded as well as the RL for the subjects where the observation was not at a failure. More data points obtained from the experts include the progression of the covariate from the start of operation up to the time of the observation or failure.

The operating scenarios are given to all the experts simultaneously. They are given the details of the scenarios and are to provide the final event times for each of the ten scenarios. Also, they must reach a consensus on the values and one data set results from this. After this, each individual must provide the regression of the covariate from the start of operation up until the time of the final event. Each expert provides a data set containing 152 data points, meaning that a total of over 750 data points were obtained from the five experts. These data points are then to be compared to industry standard data of the degradation of transformers.

The initial estimates of the parameters are obtained by making use of the final event times and the MLE method. The parameters are then updated by using the regression of the covariate over time and fitting an appropriate function to the data. The new data points used for the CM data are the covariate values over the operating period for the transformer obtained from the experts. Figure 5.1 illustrates the initial reliability and hazard curves. These are the estimates calculated by using the final event times for all the scenarios. This initial estimate of the parameter values for λ and η yields 9.516 and 23.013 respectively.

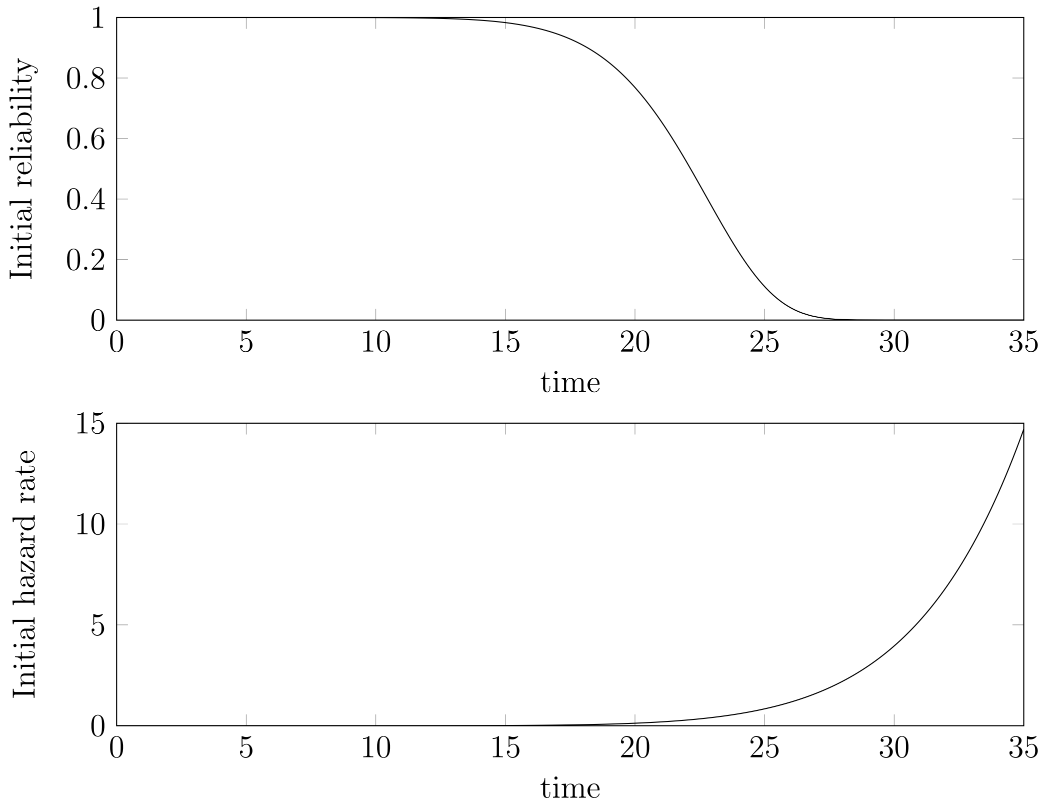


Figure 5.1: Initial reliability and hazard.

As mentioned, the parameter values are updated after the initial estimate, making use of the yearly CM data (the progression of the covariates), which is individually obtained from each expert. Therefore, there will be a different λ and a η value for each expert. The final shape parameter used is the mean value of all the λ's from the experts' data, the scale parameter is then the mean of the η's calculated. The difference between the initial reliability and the updated reliability is illustrated in Figure 5.2.

The CM data on the covariate used in this study alters the initial estimates of the Weibull parameters. The shape parameter decreases while the scale parameter increases indicating a slower degradation of the transformer. The shape parameter (λ) determines the shape of the reliability curve and also the rate at which the curve settles to zero. The shape parameter decreases from 9.516 to 8.201 when the CM data is used to update the parameters. Therefore, the reliability degrades at a slower rate than the initial estimate.

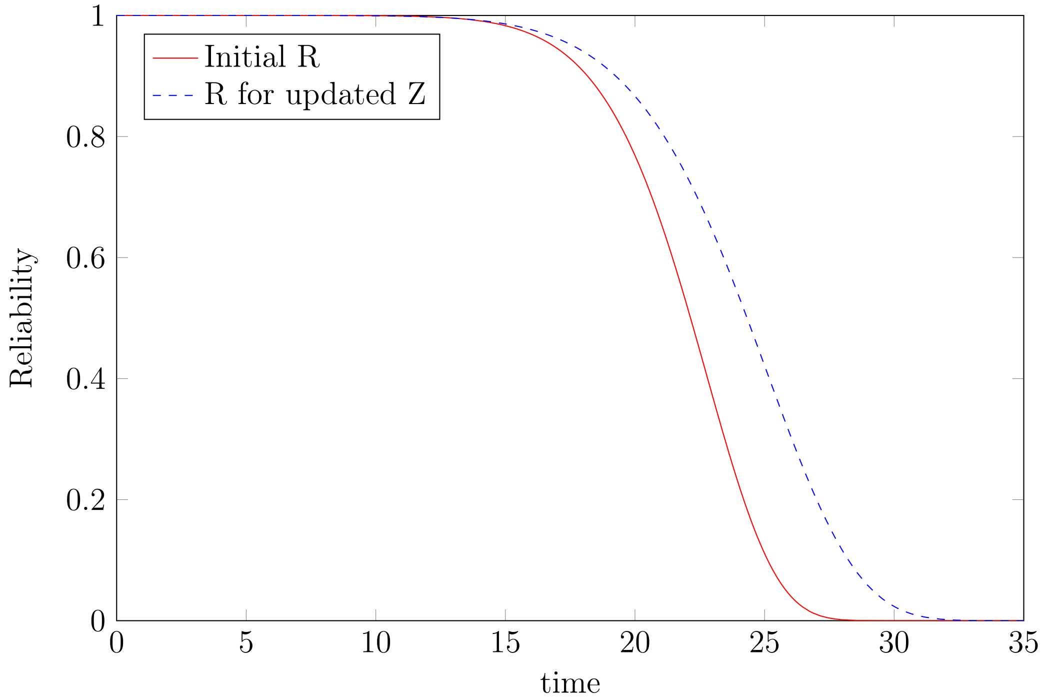


Figure 5.2: Initial reliability vs updated reliability.

The scale parameter (η) indicates when the greatest probability of failure for the system is. The scale parameter increases from 23.010 to 25.695. The reliability curve obtained after considering the updated parameters in Figure 5.2 stretches further in time than the initial curve before settling at zero. This is as a result of the increased scale parameter. The average of the parameter values obtained from all the experts is used to illustrate the difference between the initial estimates and the updated values.

The hazard rate or FOM provide the practical implications of the parameter changes more clearly; the various curves are given in Figure 5.3. The larger scale parameter delays the point at which the FOM increases while the smaller shape parameter increases the time over which the system degrades. This relates to the system degrading at a slower rate over a longer period of time. This can clearly be seen by the FOM curve resulting from the average expert opinion, as can be seen in Figure 5.3.

Since the final event times are used to calculate the initial values of λ and η the experts must all reach a consensus on the times the initial values of the parameters remain constant, for all the experts. The parameter values are however, different from expert to expert after the CM data (the progression of the DP) is used to update the Weibull parameters. This difference between the initial estimates and each expert can be seen in the reliability curves illustrated in Figure 5.4. Since the parameter values are so close to one another, the mean values of the parameters λ and η are utilized to make the RL predictions.

Figure 5.5 illustrates the difference between the FOM curves for the experts

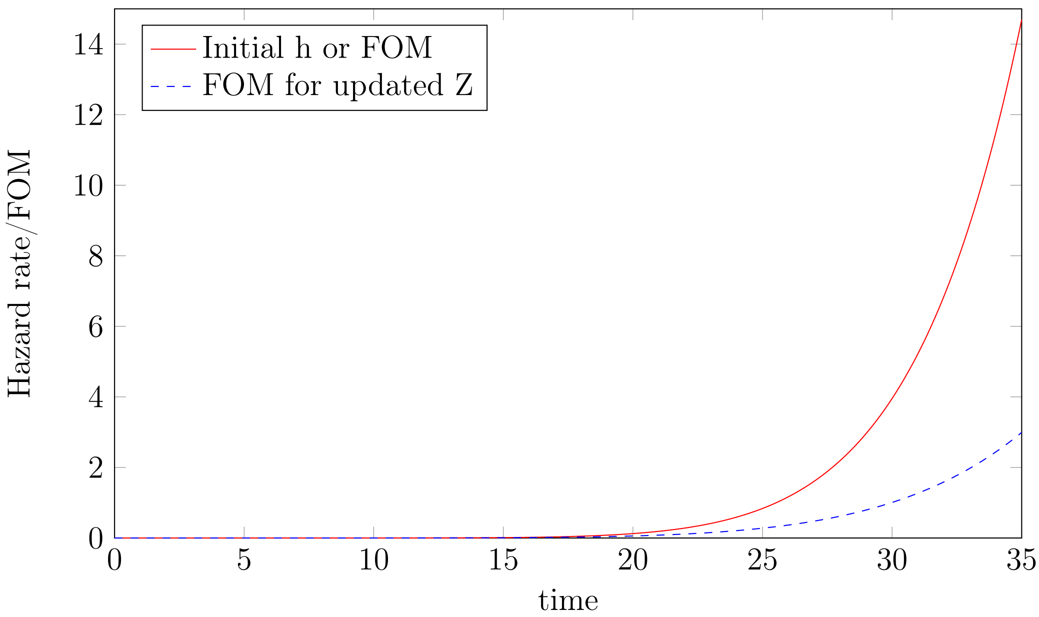


Figure 5.3: Initial hazard vs updated hazard.

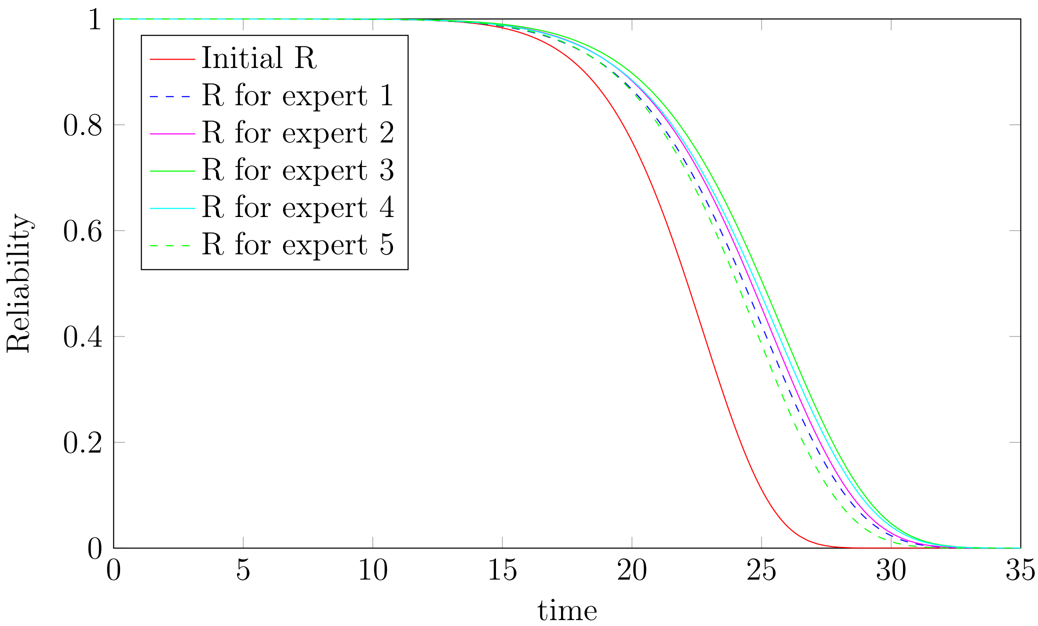


Figure 5.4: Difference in reliability between experts' opinions.

and the initial estimate. The hazard rate after the initial estimate can be seen to be higher than all of the experts' estimates. This shows that when only the failure times are used, the hazard rate of the transformers is much higher than when the covariate is also utilized. This only emphasizes the need for this study and the importance of prognostics in reliability analysis because this effect can cause healthy assets to be replaced unnecessarily.

The mean λ and η values are then used to create the final reliability and hazard curves considering the chosen covariate. Table 5.1 reveal the values calculated

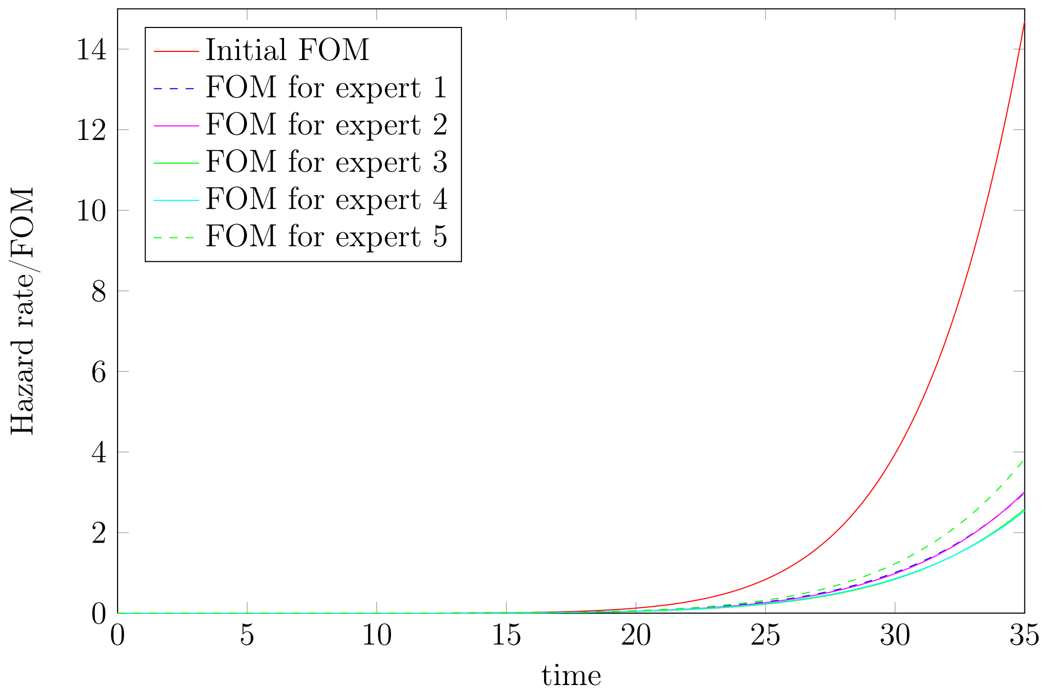


Figure 5.5: Difference in FOM between experts' opinions.

for λ and η from each expert's data set. Final parameter values used in the PCM allowing the RL predictions to be made are the mean of the updated values.

Table 5.1: Parameter values for all experts.

	Initial estimate		Updated estimate	
	λ_1	η_1	λ_2	η_2
Expert 1			8.057	25.458
Expert 2			8.274	25.732
Expert 3	9.268	23.030	8.255	26.188
Expert 4			8.040	25.959
Expert 5			8.380	25.138
Mean	9.268	23.030	8.201	25.695

To ensure that the predictions made by the subjective data are valid they must be compared to predictions yielded by industry standard data. The insulation material around the windings of the transformer coils are considered to degrade according to a set standard obtained through extensive research. This standard is discussed in the following section and is compared to the subjective data. The RL predictions obtained when using the industry standard data is also compared to the results yielded by the subjective data.

5.2 Industry Standard

As stated, the largest problem in industry with prognostic models is that the necessary data is not available. It is, therefore, no surprise that the organization in this case study did not possess any historical failure data for the transformers. The CM data that they have is also incomplete and noisy; it is not until 2011 when they started to make use of a different contracting company to do the CM that they started to build up a trustworthy data set.

This lack of data then prevented the subjective data from being validated by using objective CM data. The CM data that is needed is the degradation of the transformer represented by the regression of the DP from the start of operation up to the time of an event. The loading of the transformers over their operating period is known, which leaves the furan content or the DP to quantify the level of degradation of the transformers.

It is very seldom that two transformers in industry operate under the exact same operating conditions in the same operating environment. Therefore, no two transformers will degrade in the exact same manner, the option of using the degradation data from other transformers with the necessary data available is, therefore, ruled out. There is, however, an accepted method of calculating the DP of a transformer when partial data is available.

This method uses an ageing factor k to estimate the degradation of a transformer. The ageing factor for the transformers with their initial 83% loading can be estimated as well as the ageing factor when loaded to 96%. The ageing factor is estimated as given by Zhong (2011). Equation 5.2.1 represents the function used. The ageing factor for different loadings is then obtained by interpolating the results obtained; assuming that the ageing factor varies linearly, this assumption is justified by the results obtained by Zhong (2011).

$$k = \frac{\dfrac{1}{DP_{\text{lowest}}} - \dfrac{1}{DP_{\text{start}}}}{t_{\text{op}}} \tag{5.2.1}$$

Here, k represents the ageing factor, DP_{lowest} is the DP value at the end of the time period chosen, DP_{start} is the DP at the start of the time period and t_{op} is the duration of the operating time period in years. This enables the DP values available from the organization's CM to estimate the ageing factor of the transformers considered in this study. The ageing factor and the known operating times are now used to create the missing CM data and are considered to be industry standard data. The ageing factor values obtained are provided in Table 5.2. This linear assumption of the ageing factor is acceptable as long as the DP ranges between 1200 and 200. Zhong (2011) states that when the DP drops to below 200 this assumption is no longer valid. This is perfect for this study since any DP value below 200, is considered to be functional failure.

Table 5.2: Ageing factor for different loadings.

Loading	**k**
0.70	0.000210998
0.71	0.000214012
0.72	0.000217026
0.73	0.000220041
0.74	0.000223055
0.75	0.000226069
0.76	0.000229083
0.77	0.000232098
0.78	0.000235112
0.79	0.000238126
0.80	0.000241140
0.81	0.000244155
0.82	0.000247169
0.83	0.000250183
0.84	0.000253198
0.85	0.000256212
0.86	0.000259226
0.87	0.000262240
0.88	0.000265255
0.89	0.000268269
0.90	0.000271283
0.91	0.000274297
0.92	0.000277312
0.93	0.000280326
0.94	0.000283340
0.95	0.000286354
0.96	0.000289369
0.97	0.000292383
0.98	0.000295397
0.99	0.000298411
1.00	0.000301426

A data set is now created by rearranging Equation 5.2.1 and obtaining Equation 5.2.2 to yield the DP value at the end of an operating period with a certain loading. The same scenarios as the data sets provided to the experts are recreated. This data set is considered to be the industry standard data since the method used to calculate the degrading DP is accepted in industry as well as academic studies (Prevost *et al.*, 2006; Jarman *et al.*, 2009; Zhong, 2011).

$$DP_{\text{lowest}} = \frac{1}{kt_{\text{op}} + \dfrac{1}{DP_{\text{start}}}} \tag{5.2.2}$$

When creating the industry standard data the DP value of the transformers is calculated up to the same event time as data set created by the experts. This is labelled as "industry standard one" data . Another data set is created where the DP is calculated until the DP values have degraded to the values provided by the experts, no matter when this value occurs; this is known as the "industry standard two" data set. The parameter values yielded by the industry standard data are provided in Table 5.3.

Table 5.3: Parameter values for industry standard data.

	$\boldsymbol{\lambda}$	$\boldsymbol{\eta}$
Industry standard one	9.567	24.017
Industry standard two	8.707	22.375

The reliability and the hazard functions for the average experts' opinions and both of the industry standard data sets are compared in Figure 5.6.

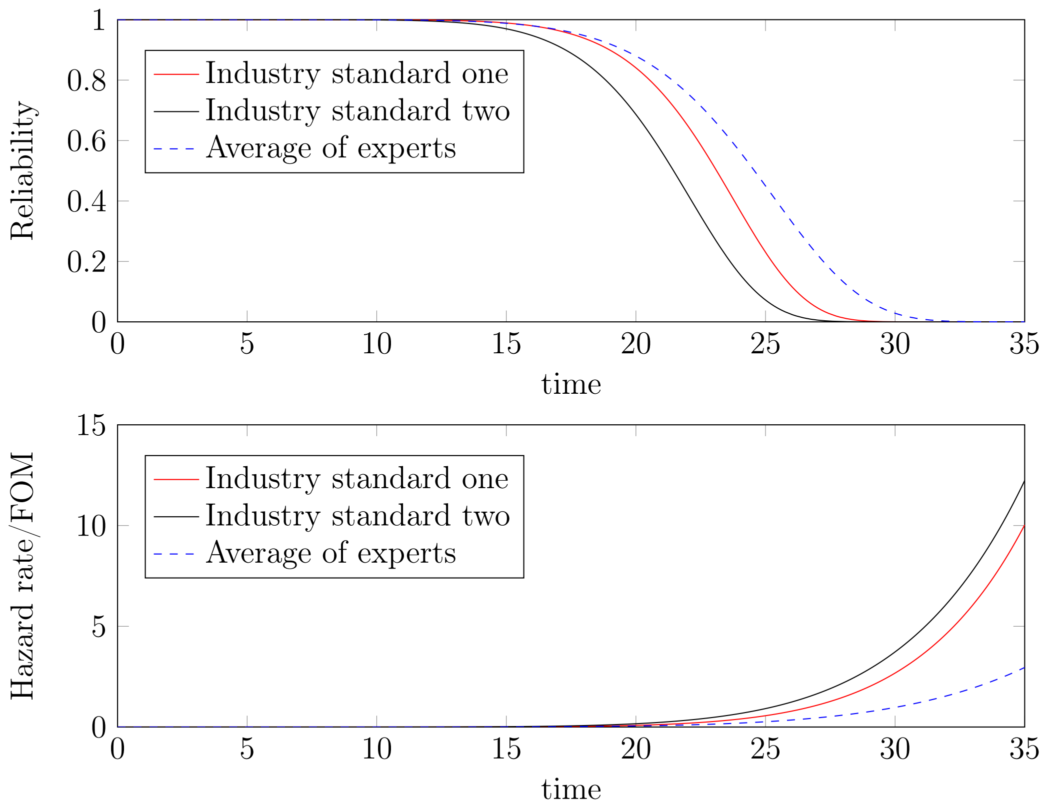


Figure 5.6: The average experts' opinions compared to industry standard reliability and hazard.

The comparison reveals a concerning but understandable characteristic of how the experts' opinions relate to the industry standard data. This characteristic can also be deduced by simply examining the shape and scale parameters of the Weibull distribution for the various data sets. One finds that the experts' opinions are less conservative than the industry standard data sets. This is understandable given that the data that is provided to the industry, especially in survival analysis, contains a safety factor. A safety factor is incorporated into a design not only to protect the users of the product but to avoid any unnecessary failures. Safety factors are necessary to include when designing anything because people in industry are continuously pushing the limits of the equipment they use.

The experts' opinions are less conservative than the industry standard data because the experts have most likely all used assets past their prescribed limits without them failing. This then causes them to subconsciously adjust their own judgements concerning the operation and health of the assets which they use. This is a bad characteristic to have especially towards the end of assets' (power transformers in this case) lives. The amount of error involved in this specific survival model increases exponentially as time progresses, as illustrated in Figure 5.7 by the FOM curves as they are extended to 80 years operation time.

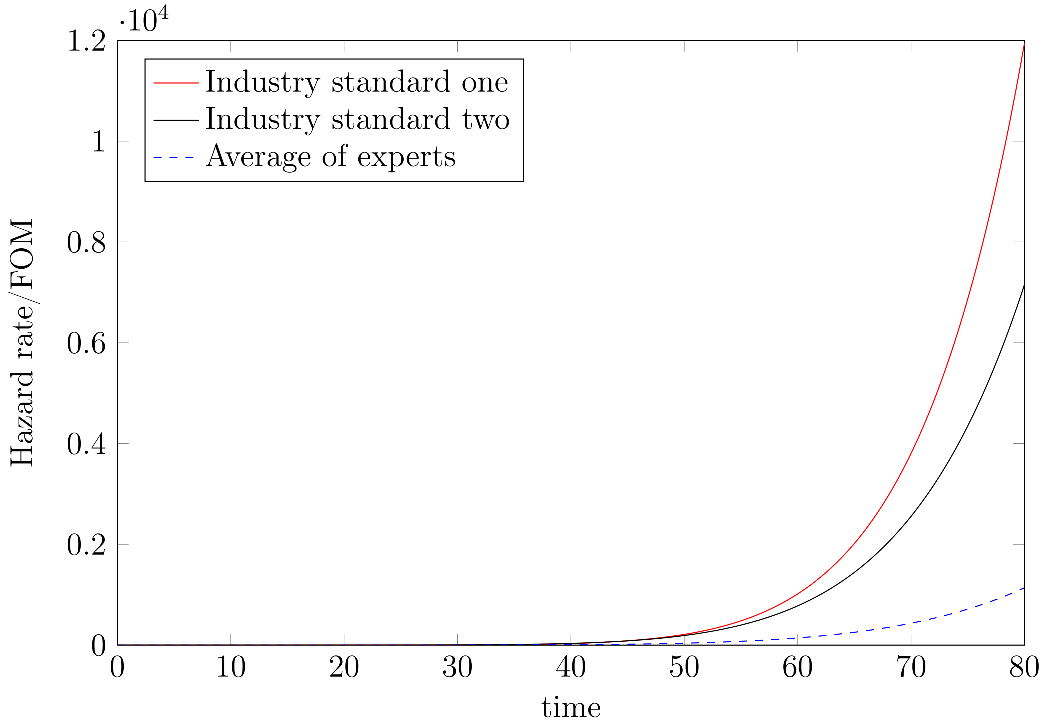


Figure 5.7: The extended FOM illustrates exponential increase in error.

The RL estimates which the experts provided can now finally be compared to the RL estimates the survival model yields when populated with the subjective data as well as the objective industry standard data. As mentioned earlier, the data set "industry standard one" uses Equation 5.2.2 to create a data set with identical operating conditions as the original data set provided to the experts. The DP is only calculated up to the same operating time and not the same level of degradation. The data set "industry standard two" also creates a data set with identical operating conditions, except that in this data set, the DP is calculated until the time when it is equal to the final DP values of the data set obtained from the experts.

The PCM is trained with all of these data sets and the parameters are calculated as explained. The RL estimates for the original data set are recreated using the parameters calculated with the data set of each expert and the industry data.

Table 5.4: RL estimates from different data sets.

Scenario	**Current operating time**	**RL from experts' data set**	**RL from industry standard one data set**	**RL from industry standard two data set**
1	12	9.976	8.555	7.139
2	15	7.905	6.492	5.000
3	8	12.974	11.625	10.273
4	10	7.971	6.824	5.652
5	22	0.905	-0.508	-2.000
6	12	10.905	9.492	8.000
7	16	4.974	3.625	2.273
8	4	19.994	18.383	16.879
9	18	1.000	-0.286	-1.500
10	25	1.749	0.172	-1.636

Inspecting the RL estimates delivered in detail shows that some of the estimates of the industry standard data yields negative values. A negative value indicates that the piece of equipment considered is past its expected lifetime. This indicates that the model expected the equipment to have already failed at the time of observation with the prescribed operating conditions.

5.3 Summary

The subjective knowledge elicited from the experts follow the same characteristics as the objective industry standard data. The shape parameter of the Weibull distribution is always greater than two, thus, indicating an increasing probability to fail. The shape parameter values for the industry standard data sets are 9.567 and 8.707 while the experts' data set shape scale is calculated to be 9.628. They all have an extremely prominent characteristic of an increasing probability of failure.

The scale parameter for the industry standard one data set is calculated as 24.017 while the industry standard two data set's value is yielded as 22.376. These are both smaller than the scale parameter calculated from the experts' knowledge which is 25.395, which then emphasizes the shorter life times in identical conditions for the industry standard data. The effect of the parameter values are illustrated graphically in Figure 5.8 by showing the RL predictions yielded by the PCM when trained with the different data sets.

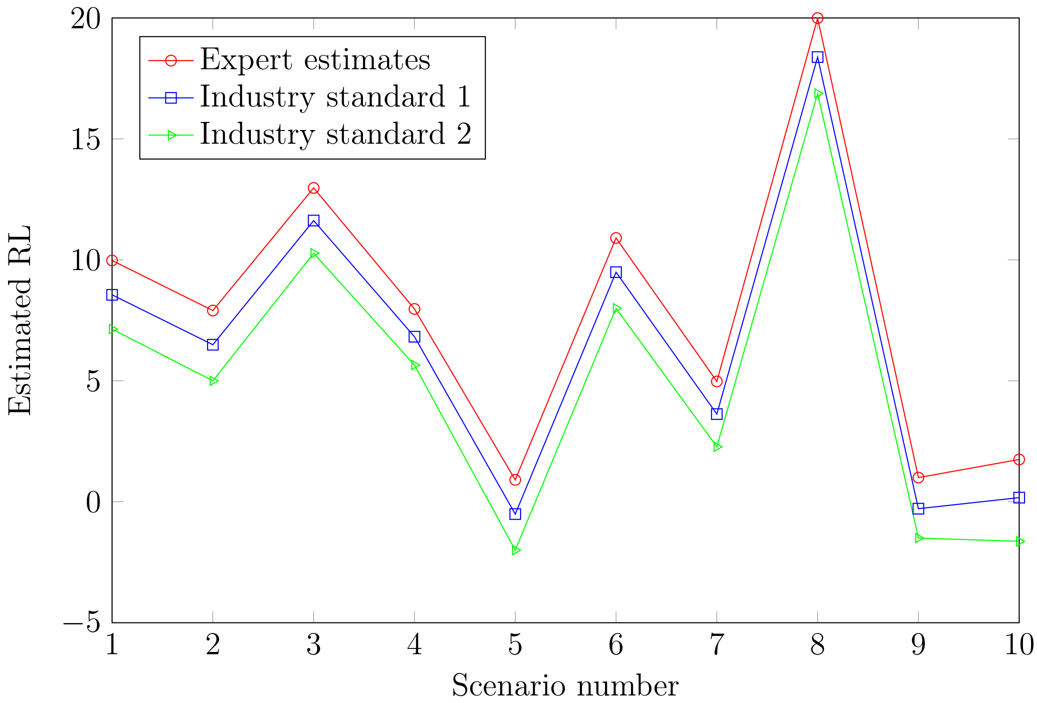


Figure 5.8: Visual illustration of RL predictions.

It is noted that the RL predictions made with the PCM trained with the experts' knowledge is less conservative than both of the industry standard data sets. This could be as a result of the experts having experienced transformers operating well beyond their prescribed operating conditions and still surviving, thus, they adapt their judgement on the operating conditions and corresponding health of the transformers to be more rigid. This can be seen in the fact that the model trained with the experts' knowledge predicts that the transformers still have a remaining useful life in all of the cases which the RL predictions are negative for the industry standard data.

Chapter 6

Closure

This final chapter aims to summarize the motive and the limitations to this study. It also provides a compact summary of the literature reviewed and the research findings. Some final recommendations are made for future research and for the improvement of this study. A brief overview of the background is given first.

6.1 Summary of study

Asset intensive industries are dependent on the assets they use to create value and are constantly improving the systems and/or methods which they use to minimize the operational downtime of their assets and the money they spend on maintaining them. The mining industry is a good example of an asset intensive industry and according to GMT (2013) they spend up to 30% of their operating costs on the maintenance of physical assets. In 2012 PricewaterhouseCoopers (PwC) conducted a study involving 39 mining companies in South Africa, this study revealed that operating expenses for the 2012 financial year alone amounted to 216 billion rand. This is only one of the asset intensive industries, with such an amount of money being spent on maintenance there is an opportunity to save large amounts of money by managing the assets correctly. The correct management of physical assets can provide an organization with multiple benefits, such as saving money and safer working environments.

This study focuses on the proactive maintenance strategy, a small subset within the Physical Asset Management (PAM) environment. Proactive maintenance is seen as a window for the opportunity to save money and increase worker safety around physical assets. A field known as prognostics combines two proactive maintenance tactics to execute reliability analysis thus allowing cost optimization and long term planning and scheduling. In order to conduct the reliability analyses, survival models are used. These survival models require data to be populated. This data includes historical failure data as well as Condition Monitoring (CM) data of the assets of interest.

The data required to populate the survival models proves to be an issue in industry. Few organizations possess historical failure data for their assets, the CM data is more readily available but it also has its issues. The CM is often very noisy or incomplete. The prognostics field requires both types of data in order to be of use when maintenance decisions are to be made. The problem in short is that the data which is required to populate the survival models is often not available at all or inaccurate. In an attempt to overcome this obstacle, an alternative to the objective data obtained from sensors and events is sought.

Subjective data obtained from people labelled as experts is considered and tested to see if it could be used as a valid alternative for populating the prognostic survival models. The research question and null hypothesis are recalled:

> "*Can subjective data obtained from experts be used as covariates to populate the prognostic survival models, thus, allowing the prediction of equipment residual life (RL)?*"

$\mathbf{H_0}$:
Subjective covariates obtained from experts cannot be used to populate prognostic survival models to allow the prediction of the RL of equipment.

To be able to answer the research question and to either reject or not reject the null hypothesis, a comprehensive literature study is necessary. An overview of PAM is provided and it is explained where maintenance fits into the PAM field. The basics of reliability analysis are given, the survival models which constantly reoccurred in the reviewed literature are explained in detail and considered when selecting an appropriate survival model for this study. These models included the Accelerated Failure Time Model (AFTM), Additive Hazards Model (AHM), Proportional Covariate Model (PCM), Proportional Hazards Model (PHM), Proportional Odds Model (POM) and the Prentice, Williams and Peterson (PWP). No literature could be found using any of these survival models solely with subjective data.

A method is provided to guide one in conducting this study. This method or road map provides guidelines on how to select the covariates and experts to be used in this study or one similar to it. The method also proposes a way to elicit the subjective data from experts and how to select the most appropriate survival models from the reviewed models. A case study is then conducted in order to obtain subjective data and to validate the results yielded.

The case study is done at a company which processes heavy minerals and has two large smelting furnaces. Each of these furnaces requires their own power transformer. The assets used in this study are the two power transformers. They are in their end-of-life stage and the company would like to have another opinion on how long they have before they have to replace them.

Experts and covariates are selected as per the criterion proposed in the method, three covariates selected are further narrowed down to only a single covariate following a discussion with the experts and a correlation test. The covariate used is the Degree of Polymerization (DP) of the insulating material inside the transformers. DP and time are, therefore, the two variables in the survival models. Data sets are created by extracting the necessary knowledge from the experts, these data sets are used to populate all the relevant survival models. The data set is then recreated by using the survival models. When recreating the data set, the most accurate model turned out to be the PCM.

A data set to compare the subjective data to is created by using an equation developed through previous research and it is assumed that this can be seen as industry standard data. Two industry standard data sets are created, one where the DP is calculated up to the same point in time as what the experts provided. The other data set calculates the DP until it reaches the same level of degradation as what the experts provided. These two data sets are then used to train the PCM and the model is again used to recreate the original data set obtained from the experts.

The Weibull parameters are calculated for each of the experts' data sets and the average parameter values calculated when considering the experts opinions are used to represent the experts' knowledge. RL predictions when considering the experts' knowledge is then compared to the RL predictions obtained from the industry standard data. The expert knowledge is proven to be less conservative than the industry standard data on all occasions. There are also cases where the RL prediction calculated by using the industry standard data is negative while the experts still think the transformer can continue operation. The negative value indicates that according to the industry standard data, the transformer should have failed already and cannot continue operation.

In conclusion, the experts' knowledge can be used as covariates to populate the survival models. The opinions which the experts delivered are less conservative than both of the industry standard data sets and, therefore, of concern because it shows that the experts always think that the transformers can continue operation for longer than what they are supposed to. This is why designing with a safety factor is so important. The research question can therefore be answered with a tentative "yes", because even though the experts' knowledge delivered results that are not far off, they insinuate that the experts are underestimating the rate of degradation of the transformers. This can be dangerous if no CM is applied to the transformers and the experts decide on how long to operate them and at which levels. When the answer to the research question is yes, it also means that the null hypothesis can be rejected.

It should be noted that only one case study was conducted in this study. This limits the applicability of the use of any subjective data in survival analysis because it is such a specific case. It does, however, indicate that it is possible to use subjective data as an alternative to objective data. Applying this study in the same industry on more case studies will reveal if the trends observed here are constant. This study is subject to several limitations, they are explicitly stated in the next section.

6.2 Limitations

It is important for any research study that the limitations discovered during the study are taken note of. Several limitations were encountered during this study, and are listed below:

- It can be argued that because the organization at which the case study was done does not possess any historical failure data of the transformers, the experts do not have any knowledge of a failure and how can they then predict when one will fail. They largely base their knowledge on the reports provided and the methods used by the contractors, which do the CM of the transformers.
- The experts first provided values as given to them by the company conducting the CM on the transformers when they were provided with scenarios the same as the current transformers. Different scenarios were then created and presented to the experts, they then seemed very unsure about their opinions on the new scenarios.
- Only one covariate is used in the final model, the experts stated that they would not even consider other covariates unless they are currently of interest. The knowledge which the experts possess might be skewed for the particular covariate used, thus, resulting in the less conservative results.
- It is not viable to include all survival models that are available and this is why only six of the most popular models were considered in this study. There is the possibility that other available models will yield more accurate results when populated with subjective data than the ones reviewed.
- The study is done on only one type of physical asset; it can be argued that this is not representative of all physical assets. The knowledge of experts in other fields might differ, and might not be less conservative than the industry standard data.

The limitations encountered in this study have now been highlighted and recommendations about future research in this particular field can be made.

6.3 Recommendations for future research

During the execution of this study, areas emerged which will allow for the improvement of this study even though all the objects were met and the null hypothesis rejected. Questions and ideas that were noted should be addressed as they have the potential to reassure the conclusion reached in this study. The questions and ideas are formulated into recommendations as provided below:

- It is not certain if the knowledge of experts is less conservative concerning the RL estimates for all types of physical assets. Therefore, it would provide valuable insight if this study could be conducted on different asset types. The results can then be compared to see if the experts' knowledge reveal the same characteristic for all of the asset types.
- This study could possibly deliver more accurate results with survival models that were not reviewed. It would, therefore, be suitable to assess the applicability of more survival models.
- In industry generally, anything somebody asks for they would have already liked to have by the time they ask, therefore, the speed at which RL estimates are made should be as quick as possible. A toolbox which combine all the different survival models and automatically runs the applicability tests and delivering real time estimates would prove favourable in industry.
- The data collection process is a tedious one, especially when asking each expert to provide a data set based on their knowledge. A way to ease and quicken this process will be of great aid when conducting a similar study.
- The results of this study are based on a single case study. Conducting another case study in the same industry would reinforce the conclusions deduced in this study.

All of the recommendations stated above are to improve research conducted. The suggested areas have the potential to not only simplify a similar study but also deliver a useful tool that can be used in industry.

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Appendices

Appendix A

Proportional Covariate Model

A.1 The PCM

The PCM was developed in 2006 by Mr. Yong Son as part of his Phd at Queensland University of Technology. No open source code for the survival model could be found so the model was programmed in Matlab. The code is based on the theoretical knowledge from Sun (2006). The model is developed to handle the specific data sets used in this study, more refining is needed to adapt the code to accept any data set in a specific format.

A.2 Matlab code

```
%Jaco Schoeman
%Proportional Covariate Model, theory based on model by Yong Son
%Written to model physical assets as part of masters thesis
%%%%
clear;
global Xi Ci;
%read data into matlab from excel sheet manipulated
data = xlsread('DataSet1.xls');
data2 = xlsread('Specialist_regressionAdriaan.xls');
data3 = xlsread('Specialist_regressionHannes.xls');
data4 = xlsread('Specialist_regressionChris.xls');
data5 = xlsread('Specialist_regressionBulent.xls');
data6 = xlsread('Specialist_regressionJan.xls');
dataIND = xlsread('Industry_standard2.xls');
num = data(:,1);
Xi = data(:,6);
Ci = data(:,2);
r = max(num);
m=Ci(1);
Ti = 1:r;
%initial starting values for parameters
beta = 1.5;
eta = sum(Xi)/r;
%assign more variable values
Ti(1)=Xi(1);
for i=2:r
   Ti(i)=Ti(i-1)+Xi(i);
   if Ci(i) == 1
       m = m+1;
   end
end
Tr = Ti(r);

%calculate Laplace trend test, see if data is repairable or non-repairable
Prompt1 = {'The Laplace trend test yields U below, click ok to continue.'};
dlg_title = 'Trend test, repairable or non repairable.';
num_line = 1;
U = ((sum(Ti)-Tr)/(r-1) - Tr/2)/(Tr*sqrt(1/(12*(r-1))));
def = {num2str(U)};
U = inputdlg(Prompt1,dlg_title,num_line,def);
U = str2double(U);
%calculate log MLE for 1st estimate with Weibull dist, step 1 & 2 of PCM
startat = [eta beta]';

fmin = @(x)likelifunc(x);
[solutions, MLE1] = fminunc(fmin,startat);
beta = solutions(2)
eta = solutions(1)
%thus initial hazard function hin() is
%hcont = @(t)(beta/eta)*(t/eta)^(beta-1);
%Step 3 is to calculate the correlation between covariates and system
%hazard

%for corellation with progression of covariates============
half = (numel(data2(:,1))/2);
```

```
halfi = (numel(dataIND(:,1))/2);
%1st set Adriaan
[values, order] = sort(data2(1:half,2));
sortedmatrix = data2(order,:);

Xi2 = sortedmatrix(:,2);
haz22 = 1:length(Xi2);

%2nd set Hannes
[valuesh, orderh] = sort(data3(1:half,2));
sortedmatrixh = data3(orderh,:);
%3rd set Chris
[valuesc, orderc] = sort(data4(1:half,2));
sortedmatrixc = data4(orderc,:);
%4th set Bulent
[valuesb, orderb] = sort(data5(1:half,2));
sortedmatrixb = data5(orderb,:);
%5th set Jan
[valuesj, orderj] = sort(data6(1:half,2));
sortedmatrixj = data6(orderj,:);
%6th set Industry standard
[valuesi, orderi] = sort(dataIND(1:halfi,2));
sortedmatrixi = dataIND(orderi,:);
Xi2i = sortedmatrixi(:,2);
haz22i = 1:length(Xi2i);

%initial hazard (FOM)
for i=1:length(Xi2)
    haz22(i) =  (beta/eta)*(Xi2(i)/eta)^(beta-1);
end

for i=1:length(Xi2i)
    haz22i(i) =  (beta/eta)*(Xi2i(i)/eta)^(beta-1);
end
correlation2 = [1:4]';
correlation2(1) = corr(Xi2,haz22');
correlation2(2) = corr(sortedmatrix(:,4),haz22');
correlation2(3) = corr(sortedmatrix(:,5)/1000,haz22');
            Prompt1 = {'Correlation of Z1:','Correlation of Z2:'};
            dlg_title = 'Correlation test for covariates and hazard.';
            num_line = 1;
            def = {num2str(correlation2(2)),num2str(correlation2(3))};
            asd = inputdlg(Prompt1,dlg_title,num_line,def);
            correlation2(2) = str2double(asd(1));
            correlation2(3) = str2double(asd(2));
%================================================
%Step 4 baseline covariate function Ck
%1st set
Z1 = sortedmatrix(:,4);
Z2 = sortedmatrix(:,5)/1000;

%2nd set
Z1h  = sortedmatrixh(:,4);
Z2h = sortedmatrixh(:,5)/1000;
%3rd set
```

```
Z1c  = sortedmatrixc(:,4);
Z2c = sortedmatrixc(:,5)/1000;
%4th set
Z1b  = sortedmatrixb(:,4);
Z2b = sortedmatrixb(:,5)/1000;
%5th set
Z1j  = sortedmatrixj(:,4);
Z2j = sortedmatrixj(:,5)/1000;
%Industry set
Z1i  = sortedmatrixi(:,4);
Z2i = sortedmatrixi(:,5)/1000;

Ck1 = 1:length(Xi2);%set size of Ck1, changed up to length(Xi2) from r
Ck2 = 1:length(Xi2);
Ck1h = 1:length(Xi2);
Ck2h = 1:length(Xi2);
Ck1c = 1:length(Xi2);
Ck2c = 1:length(Xi2);
Ck1b = 1:length(Xi2);
Ck2b = 1:length(Xi2);
Ck1j = 1:length(Xi2);
Ck2j = 1:length(Xi2);
Ck1i = 1:length(Xi2i);
Ck2i = 1:length(Xi2i);
%create discrete data sets
for i=1:length(Xi2)
    if haz22(i)==0
       Ck1(i) = NaN;
       Ck2(i) = NaN;
       Ck1h(i) = NaN;
       Ck2h(i) = NaN;
       Ck1c(i) = NaN;
       Ck2c(i) = NaN;
       Ck1b(i) = NaN;
       Ck2b(i) = NaN;
       Ck1j(i) = NaN;
       Ck2j(i) = NaN;
    else
        Ck1(i) = Z1(i)/haz22(i);
        Ck2(i) = Z2(i)/haz22(i);
        Ck1h(i) = Z1h(i)/haz22(i);
        Ck2h(i) = Z2h(i)/haz22(i);
        Ck1c(i) = Z1c(i)/haz22(i);
        Ck2c(i) = Z2c(i)/haz22(i);
        Ck1b(i) = Z1b(i)/haz22(i);
        Ck2b(i) = Z2b(i)/haz22(i);
        Ck1j(i) = Z1j(i)/haz22(i);
        Ck2j(i) = Z2j(i)/haz22(i);
    end
end

for i=1:length(Xi2i)
    if haz22i(i)==0
       Ck1i(i) = NaN;
       Ck2i(i) = NaN;
```

```
    else
        Ck1i(i) = Z1i(i)/haz22i(i);
        Ck2i(i) = Z2i(i)/haz22i(i);
    end
end
%cftool, create fit is alternative to cftool is automatic as well
[fitresult1] = createFit(Xi2,Ck1);
[fitresult2] = createFit(Xi2,Ck2);
[fitresult1h] = createFit(Xi2,Ck1h);
[fitresult2h] = createFit(Xi2,Ck2h);
[fitresult1c] = createFit(Xi2,Ck1c);
[fitresult2c] = createFit(Xi2,Ck2c);
[fitresult1b] = createFit(Xi2,Ck1b);
[fitresult2b] = createFit(Xi2,Ck2b);
[fitresult1j] = createFit(Xi2,Ck1j);
[fitresult2j] = createFit(Xi2,Ck2j);
[fitresult1i] = createFit(Xi2i,Ck1i);
[fitresult2i] = createFit(Xi2i,Ck2i);
%wait before asking values
disp('Close all Figure but the last opened on, click inside the last opened Figure to↙
continue.')
w = waitforbuttonpress;
if w == 0
    disp('Button click')
else
    disp('Key press')
end
close

Prompt1 = {'Enter the coressponding coefficient values for power function starting with↙
a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {num2str(fitresult1.a),num2str(fitresult1.b),num2str(fitresult2.a),num2str↙
(fitresult2.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
%coefficients for multiplicative functions at^b
a(1) = str2double(asd(1));
a(2) = str2double(asd(3));
b(1) = str2double(asd(2));
b(2) = str2double(asd(4));

Prompt1 = {'Enter the coressponding coefficient values for power function starting with↙
a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {num2str(fitresult1h.a),num2str(fitresult1h.b),num2str(fitresult2h.a),num2str↙
(fitresult2h.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
%coefficients for multiplicative functions at^b
ah(1) = str2double(asd(1));
ah(2) = str2double(asd(3));
bh(1) = str2double(asd(2));
bh(2) = str2double(asd(4));
```

```
Prompt1 = {'Enter the coressponding coefficient values for power function starting with
a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {num2str(fitresult1c.a),num2str(fitresult1c.b),num2str(fitresult2c.a),num2str
(fitresult2c.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
%coefficients for multiplicative functions at^b
ac(1) = str2double(asd(1));
ac(2) = str2double(asd(3));
bc(1) = str2double(asd(2));
bc(2) = str2double(asd(4));

Prompt1 = {'Enter the coressponding coefficient values for power function starting with
a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {num2str(fitresult1b.a),num2str(fitresult1b.b),num2str(fitresult2b.a),num2str
(fitresult2b.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
%coefficients for multiplicative functions at^b
ab(1) = str2double(asd(1));
ab(2) = str2double(asd(3));
bb(1) = str2double(asd(2));
bb(2) = str2double(asd(4));

Prompt1 = {'Enter the coressponding coefficient values for power function starting with
a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {num2str(fitresult1j.a),num2str(fitresult1j.b),num2str(fitresult2j.a),num2str
(fitresult2j.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
%coefficients for multiplicative functions at^b
aj(1) = str2double(asd(1));
aj(2) = str2double(asd(3));
bj(1) = str2double(asd(2));
bj(2) = str2double(asd(4));

Prompt1 = {'Enter the coressponding coefficient values for power function starting with
a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {num2str(fitresult1i.a),num2str(fitresult1i.b),num2str(fitresult2i.a),num2str
(fitresult2i.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
%coefficients for multiplicative functions has form of at^b
ai(1) = str2double(asd(1));
ai(2) = str2double(asd(3));
bi(1) = str2double(asd(2));
bi(2) = str2double(asd(4));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
betaa = beta;
etaa = eta;
```

```
Rint = @(x) exp(-(x/etaa)^betaa);
hint = @(x) (betaa/etaa)*(x/etaa)^(betaa-1);

%After initial estimate, using statistics of MLE for parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%BASELINE COVARIATE FUNCTIONS
C1 = @(t)a(1)*t^(b(1));
C2 = @(t)a(2)*t^(b(2));
C1h = @(t)ah(1)*t^(bh(1));
C2h = @(t)ah(2)*t^(bh(2));
C1c = @(t)ac(1)*t^(bc(1));
C2c = @(t)ac(2)*t^(bc(2));
C1b = @(t)ab(1)*t^(bb(1));
C2b = @(t)ab(2)*t^(bb(2));
C1j = @(t)aj(1)*t^(bj(1));
C2j = @(t)aj(2)*t^(bj(2));
C1i = @(t)ai(1)*t^(bi(1));
C2i = @(t)ai(2)*t^(bi(2));
% %Step 5 and 6 update system hazard function, get discrete htilde values to update
% %regression parameters, when new CM values obtained. Use subjective values
% %from here to update
[values, order] = sort(data2(:,2));
sortedmatrix2 = data2(order,:);
[valuesh, orderh] = sort(data3(:,2));
sortedmatrix2h = data3(orderh,:);
[valuesc, orderc] = sort(data4(:,2));
sortedmatrix2c = data4(orderc,:);
[valuesb, orderb] = sort(data5(:,2));
sortedmatrix2b = data5(orderb,:);
[valuesj, orderj] = sort(data6(:,2));
sortedmatrix2j = data6(orderj,:);
[valuesi, orderi] = sort(dataIND(:,2));
sortedmatrix2i = dataIND(orderi,:);
%update covariates
Z1up = sortedmatrix2(:,4);
Z2up = sortedmatrix2(:,5)/1000;
Z1uph = sortedmatrix2h(:,4);
Z2uph = sortedmatrix2h(:,5)/1000;
Z1upc = sortedmatrix2c(:,4);
Z2upc = sortedmatrix2c(:,5)/1000;
Z1upb = sortedmatrix2b(:,4);
Z2upb = sortedmatrix2b(:,5)/1000;
Z1upj = sortedmatrix2j(:,4);
Z2upj = sortedmatrix2j(:,5)/1000;
Z1upi = sortedmatrix2i(:,4);
Z2upi = sortedmatrix2i(:,5)/1000;

Ci = sortedmatrix2(:,3);
Xi2 = sortedmatrix2(:,2);
Xi2i = sortedmatrix2i(:,2);

r = numel(Xi2);
ri = numel(Xi2i);
m = sum(Ci(:)==1);
```

```
htilde1 = [1:r-1]';
htilde2 = [1:r-1]';
htilde1h = [1:r-1]';
htilde2h = [1:r-1]';
htilde1c = [1:r-1]';
htilde2c = [1:r-1]';
htilde1b = [1:r-1]';
htilde2b = [1:r-1]';
htilde1j = [1:r-1]';
htilde2j = [1:r-1]';
htilde1i = [1:ri-1]';
htilde2i = [1:ri-1]';

for i=1:r
      htilde1(i) = Z1up(i)/ C1(Xi2(i));
      htilde2(i) = Z2up(i)/ C2(Xi2(i));
      htilde1h(i) = Z1uph(i)/ C1h(Xi2(i));
      htilde2h(i) = Z2uph(i)/ C2h(Xi2(i));
      htilde1c(i) = Z1upc(i)/ C1c(Xi2(i));
      htilde2c(i) = Z2upc(i)/ C2c(Xi2(i));
      htilde1b(i) = Z1upb(i)/ C1b(Xi2(i));
      htilde2b(i) = Z2upb(i)/ C2b(Xi2(i));
      htilde1j(i) = Z1upj(i)/ C1j(Xi2(i));
      htilde2j(i) = Z2upj(i)/ C2j(Xi2(i));
end

for i=1:ri
      htilde1i(i) = Z1upi(i)/ C1i(Xi2i(i));
      htilde2i(i) = Z2upi(i)/ C2i(Xi2i(i));
end

hweibull = @(p,x) (p(2)./p(1)).*(x./p(1)).^(p(2)-1);
%fit using nonlinear least squares, update hazard
Coeff1 = nlinfit(Xi2,htilde1,hweibull, [eta beta]);
eta = Coeff1(1)
beta = Coeff1(2)
Coeff2 = nlinfit(Xi2,htilde2,hweibull, [eta beta]);
eta2 = Coeff2(1)
beta2 = Coeff2(2)
Coeff1h = nlinfit(Xi2,htilde1h,hweibull, [eta beta]);
etah = Coeff1h(1)
betah = Coeff1h(2)
Coeff2h = nlinfit(Xi2,htilde2h,hweibull, [eta beta]);
eta2h = Coeff2h(1)
beta2h = Coeff2h(2)
Coeff1c = nlinfit(Xi2,htilde1c,hweibull, [eta beta]);
etac = Coeff1c(1)
betac = Coeff1c(2)
Coeff2c = nlinfit(Xi2,htilde2c,hweibull, [eta beta]);
eta2c = Coeff2c(1)
beta2c = Coeff2c(2)
Coeff1b = nlinfit(Xi2,htilde1b,hweibull, [eta beta]);
etab = Coeff1b(1)
betab = Coeff1b(2)
Coeff2b = nlinfit(Xi2,htilde2b,hweibull, [eta beta]);
```

```
eta2b = Coeff2b(1)
beta2b = Coeff2b(2)
Coeff1j = nlinfit(Xi2,htilde1j,hweibull, [eta beta]);
etaj = Coeff1j(1)
betaj = Coeff1j(2)
Coeff2j = nlinfit(Xi2,htilde2j,hweibull, [eta beta]);
eta2j = Coeff2j(1)
beta2j = Coeff2j(2)
Coeff1i = nlinfit(Xi2i,htilde1i,hweibull, [eta beta]);
etai = Coeff1i(1)
betai = Coeff1i(2)
Coeff2i = nlinfit(Xi2i,htilde2i,hweibull, [eta beta]);
eta2i = Coeff2i(1)
beta2i = Coeff2i(2)

%update covariate and hazard function
haz1 =  @(t)(beta/eta)*(t/eta)^(beta-1);
haz2 =  @(t)(beta2/eta2)*(t/eta2)^(beta2-1);
haz1h =  @(t)(betah/etah)*(t/etah)^(betah-1);
haz2h =  @(t)(beta2h/eta2h)*(t/eta2h)^(beta2h-1);
haz1c =  @(t)(betac/etac)*(t/etac)^(betac-1);
haz2c =  @(t)(beta2c/eta2c)*(t/eta2c)^(beta2c-1);
haz1b =  @(t)(betab/etab)*(t/etab)^(betab-1);
haz2b =  @(t)(beta2b/eta2b)*(t/eta2b)^(beta2b-1);
haz1j =  @(t)(betaj/etaj)*(t/etaj)^(betaj-1);
haz2j =  @(t)(beta2j/eta2j)*(t/eta2j)^(beta2j-1);
haz1i =  @(t)(betai/etai)*(t/etai)^(betai-1);
haz2i =  @(t)(beta2i/eta2i)*(t/eta2i)^(beta2i-1);
%reset size of Cks
Ck1 = 1:r;
Ck2 = 1:r;
Ck1h = 1:r;
Ck2h = 1:r;
Ck1c = 1:r;
Ck2c = 1:r;
Ck1b = 1:r;
Ck2b = 1:r;
Ck1j = 1:r;
Ck2j = 1:r;
Ck1i = 1:ri;
Ck2i = 1:ri;
%Xfails = 1:r;
for i=1:r
    if i<=10%for the 10 events, all start at 0
        if haz22(i)==0
           Ck1(i)  = NaN;
           Ck2(i)  = NaN;
           Ck1h(i)  = NaN;
           Ck2h(i)  = NaN;
           Ck1c(i)  = NaN;
           Ck2c(i)  = NaN;
           Ck1b(i)  = NaN;
           Ck2b(i)  = NaN;
           Ck1j(i)  = NaN;
           Ck2j(i)  = NaN;
```

```
            Ck1i(i) = NaN;
            Ck2i(i) = NaN;
         else
            Ck1(i) = Z1up(i)/haz1(Xi2(i));
            Ck2(i) = Z2up(i)/haz2(Xi2(i));
            Ck1h(i) = Z1uph(i)/haz1h(Xi2(i));
            Ck2h(i) = Z2uph(i)/haz2h(Xi2(i));
            Ck1c(i) = Z1upc(i)/haz1c(Xi2(i));
            Ck2c(i) = Z2upc(i)/haz2c(Xi2(i));
            Ck1b(i) = Z1upb(i)/haz1b(Xi2(i));
            Ck2b(i) = Z2upb(i)/haz2b(Xi2(i));
            Ck1j(i) = Z1upj(i)/haz1j(Xi2(i));
            Ck2j(i) = Z2upj(i)/haz2j(Xi2(i));
            Ck1i(i) = Z1upi(i)/haz1i(Xi2(i));
            Ck2i(i) = Z2upi(i)/haz2i(Xi2(i));
         end
    else
        Ck1(i) = Z1up(i)/haz1(Xi2(i));
        Ck2(i) = Z2up(i)/haz2(Xi2(i));
        Ck1h(i) = Z1uph(i)/haz1h(Xi2(i));
        Ck2h(i) = Z2uph(i)/haz2h(Xi2(i));
        Ck1c(i) = Z1upc(i)/haz1c(Xi2(i));
        Ck2c(i) = Z2upc(i)/haz2c(Xi2(i));
        Ck1b(i) = Z1upb(i)/haz1b(Xi2(i));
        Ck2b(i) = Z2upb(i)/haz2b(Xi2(i));
        Ck1j(i) = Z1upj(i)/haz1j(Xi2(i));
        Ck2j(i) = Z2upj(i)/haz2j(Xi2(i));
        Ck1i(i) = Z1upi(i)/haz1i(Xi2(i));
        Ck2i(i) = Z2upi(i)/haz2i(Xi2(i));
    end
end

for i=1:ri
    if i<=10%for the 10 events, all start at 0
         if haz22i(i)==0
            Ck1i(i) = NaN;
            Ck2i(i) = NaN;
         else
            Ck1i(i) = Z1upi(i)/haz1i(Xi2i(i));
            Ck2i(i) = Z2upi(i)/haz2i(Xi2i(i));
         end
    else
        Ck1i(i) = Z1upi(i)/haz1i(Xi2i(i));
        Ck2i(i) = Z2upi(i)/haz2i(Xi2i(i));
    end
end
% cftool alternative, createfit
[fitresult1] = createFit(Xi2,Ck1);
[fitresult2] = createFit(Xi2,Ck2);
[fitresult1h] = createFit(Xi2,Ck1h);
[fitresult2h] = createFit(Xi2,Ck2h);
[fitresult1c] = createFit(Xi2,Ck1c);
[fitresult2c] = createFit(Xi2,Ck2c);
[fitresult1b] = createFit(Xi2,Ck1b);
[fitresult2b] = createFit(Xi2,Ck2b);
```

```
[fitresult1j] = createFit(Xi2,Ck1j);
[fitresult2j] = createFit(Xi2,Ck2j);
[fitresult1i] = createFit(Xi2i,Ck1i);
[fitresult2i] = createFit(Xi2i,Ck2i);
%wait before asking values
disp('Click inside the last opened Figure to continue.')
w = waitforbuttonpress;
if w == 0
    disp('Button click')
else
    disp('Key press')
end
close
%%%%%%%%%%%%%%%%%%%%%%%%%
Prompt1 = {'Enter the coressponding coefficient values starting↙
with','a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {'Start with a1',num2str(fitresult1.a),num2str(fitresult1.b),num2str(fitresult2.↙
a),num2str(fitresult2.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
a(1) = str2double(asd(2));
a(2) = str2double(asd(4));
b(1) = str2double(asd(3));
b(2) = str2double(asd(5));

Prompt1 = {'Enter the coressponding coefficient values starting↙
with','a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {'Start with a1',num2str(fitresult1h.a),num2str(fitresult1h.b),num2str↙
(fitresult2h.a),num2str(fitresult2h.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
ah(1) = str2double(asd(2));
ah(2) = str2double(asd(4));
bh(1) = str2double(asd(3));
bh(2) = str2double(asd(5));

Prompt1 = {'Enter the coressponding coefficient values starting↙
with','a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {'Start with a1',num2str(fitresult1c.a),num2str(fitresult1c.b),num2str↙
(fitresult2c.a),num2str(fitresult2c.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
ac(1) = str2double(asd(2));
ac(2) = str2double(asd(4));
bc(1) = str2double(asd(3));
bc(2) = str2double(asd(5));

Prompt1 = {'Enter the coressponding coefficient values starting↙
with','a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {'Start with a1',num2str(fitresult1b.a),num2str(fitresult1b.b),num2str↙
```

```
(fitresult2b.a),num2str(fitresult2b.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
ab(1) = str2double(asd(2));
ab(2) = str2double(asd(4));
bb(1) = str2double(asd(3));
bb(2) = str2double(asd(5));

Prompt1 = {'Enter the coressponding coefficient values starting↙
with','a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {'Start with a1',num2str(fitresult1j.a),num2str(fitresult1j.b),num2str↙
(fitresult2j.a),num2str(fitresult2j.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
aj(1) = str2double(asd(2));
aj(2) = str2double(asd(4));
bj(1) = str2double(asd(3));
bj(2) = str2double(asd(5));

Prompt1 = {'Enter the coressponding coefficient values starting↙
with','a1:','b1:','a2','b2'};
dlg_title = 'Coefficient values.';
num_line = 1;
def = {'Start with a1',num2str(fitresult1i.a),num2str(fitresult1i.b),num2str↙
(fitresult2i.a),num2str(fitresult2i.b)};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
ai(1) = str2double(asd(2));
ai(2) = str2double(asd(4));
bi(1) = str2double(asd(3));
bi(2) = str2double(asd(5));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C1 = @(t)a(1)*t^(b(1));
C2 = @(t)a(2)*t^(b(2));
C1h = @(t)ah(1)*t^(bh(1));
C2h = @(t)ah(2)*t^(bh(2));
C1c = @(t)ac(1)*t^(bc(1));
C2c = @(t)ac(2)*t^(bc(2));
C1b = @(t)ab(1)*t^(bb(1));
C2b = @(t)ab(2)*t^(bb(2));
C1j = @(t)aj(1)*t^(bj(1));
C2j = @(t)aj(2)*t^(bj(2));
C1i = @(t)ai(1)*t^(bi(1));
C2i = @(t)ai(2)*t^(bi(2));
% %step 7
R1 = @(t) exp(-1*(t/eta)^beta);
R2 = @(t) exp(-1*(t/eta2)^beta2);
R1h = @(t) exp(-1*(t/etah)^betah);
R2h = @(t) exp(-1*(t/eta2h)^beta2h);
R1c = @(t) exp(-1*(t/etac)^betac);
R2c = @(t) exp(-1*(t/eta2c)^beta2c);
R1b = @(t) exp(-1*(t/etab)^betab);
R2b = @(t) exp(-1*(t/eta2b)^beta2b);
R1j = @(t) exp(-1*(t/etaj)^betaj);
R2j = @(t) exp(-1*(t/eta2j)^beta2j);
R1i = @(t) exp(-1*(t/etai)^betai);
```

```
R2i = @(t) exp(-1*(t/eta2i)^beta2i);
% plot original R, updated Rz1 and updated Rz2
subplot(2,1,1)
fplot(Rint,[0 35],'r')
hold on
fplot(R1,[0 35],'k--')
fplot(R2,[0 35],'b--')
fplot(R2h,[0 35],'m')
fplot(R2c,[0 35],'g')
fplot(R2b,[0 35],'c')
fplot(R2j,[0 35],'g--')
xlabel('time')
ylabel('Reliability')
legend('Initial reliability', 'R for Z1', 'R for Z2', 'Location', 'NorthWest' );
subplot(2,1,2)
fplot(hint,[0 35],'r')
hold on
fplot(haz1,[0 35],'k--')
fplot(haz2,[0 35],'b--')
fplot(haz2h,[0 35],'m')
fplot(haz2c,[0 35],'g')
fplot(haz2b,[0 35],'c')
fplot(haz2j,[0 35],'g--')
xlabel('time')
ylabel('Hazard rate')
legend('Initial hazard', 'h for Z1', 'h for Z2', 'Location', 'NorthWest' );
% %Step 8 predict residual life, user given covariate values and time when
% recorded.
Prompt1 = {'Enter experts opinions, values were taken at xi↙
units','Xi:','Z1:','Z2','xi'};%,'Z3'
dlg_title = 'Expert opinions.';
num_line = 1;
%change these with each prediction, can change here or in GUI
def = {'Start with Xi','27','0.78','220','25'};
asd = inputdlg(Prompt1,dlg_title,num_line,def);
expertOP(1) = str2double(asd(2));%time of event
expertOP(2) = str2double(asd(3));%z1
expertOP(3) = str2double(asd(4))/1000;%z2
xread = str2double(asd(5));%time readings are taken
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
F1 = @(t) 1 - exp(-1*(t/eta)^beta);
F2 = @(t) 1 - exp(-1*(t/eta2)^beta2);
F1h = @(t) 1 - exp(-1*(t/etah)^betah);
F2h = @(t) 1 - exp(-1*(t/eta2h)^beta2h);
F1c = @(t) 1 - exp(-1*(t/etac)^betac);
F2c = @(t) 1 - exp(-1*(t/eta2c)^beta2c);
F1b = @(t) 1 - exp(-1*(t/etab)^betab);
F2b = @(t) 1 - exp(-1*(t/eta2b)^beta2b);
F1j = @(t) 1 - exp(-1*(t/etaj)^betaj);
F2j = @(t) 1 - exp(-1*(t/eta2j)^beta2j);

xfx1 = @(t) t*(beta/eta)*(t/eta)^(beta-1) *exp(-1*(t/eta)^beta);
xfx2 = @(t) t*(beta2/eta2)*(t/eta2)^(beta2-1) *exp(-1*(t/eta2)^beta2);
intxfx1 = xfx1(1);
intxfx2 = xfx2(1);
```

```
for i = 2:40
   intxfx1 = intxfx1 + (xfx1(i-1)+xfx1(i))/2;
   intxfx2 = intxfx2 + (xfx2(i-1)+xfx2(i))/2;
end
%results from data set, based on the data available
disp('Expected life times:')
Elife1 = intxfx1/(F1(40))      %-F1(Xstart))
Elife2 = intxfx2/(F2(40))      %-F1(Xstart))
disp('Mean expected life time:')
MElife = (Elife1+Elife2)/2  %+Elife3
% The one here closest to final reading must be used?????
disp('Residual life:')
Rlife1 = Elife1 - xread
Rlife2 = Elife2 - xread

disp('Mean residual life (MRL), the average:')
MRL = (Elife1+Elife2)/2 - xread
%time calculated using covariate values seperatly
%covariate value using time
disp('The covariate value at time expert says failure will occur:')
Cz1 = C1(expertOP(1))*haz1(expertOP(1));
Cz2 = C2(expertOP(1))*haz2(expertOP(1));
Cz1h = C1h(expertOP(1))*haz1h(expertOP(1));
Cz2h = C2h(expertOP(1))*haz2h(expertOP(1));
Cz1c = C1c(expertOP(1))*haz1c(expertOP(1));
Cz2c = C2c(expertOP(1))*haz2c(expertOP(1));
Cz1b = C1b(expertOP(1))*haz1b(expertOP(1));
Cz2b = C2b(expertOP(1))*haz2b(expertOP(1));
Cz1j = C1j(expertOP(1))*haz1j(expertOP(1));
Cz2j = C2j(expertOP(1))*haz2j(expertOP(1));
Cz1i = C1i(expertOP(1))*haz1i(expertOP(1));
Cz2i = C2i(expertOP(1))*haz2i(expertOP(1));

disp('Times when model said covariate values will be reached, values at failure time:')
%change s0 value maybe to close to expected value
betaAVG = (beta2+beta2h+beta2c+beta2b+beta2j)/5;
etaAVG = (eta2+eta2h+eta2c+eta2b+eta2j)/5;
aAVG(1) = (a(1)+ah(1)+ac(1)+ab(1)+aj(1))/5;
aAVG(2) = (a(2)+ah(2)+ac(2)+ab(2)+aj(2))/5;
bAVG(1) = (b(1)+bh(1)+bc(1)+bb(1)+bj(1))/5;
bAVG(2) = (b(2)+bh(2)+bc(2)+bb(2)+bj(2))/5;
C1AVG = @(t)aAVG(1)*t^(bAVG(1));
C2AVG = @(t)aAVG(2)*t^(bAVG(2));
haz1AVG =  @(t)(beta/eta)*(t/eta)^(beta-1);
haz2AVG =  @(t)(betaAVG/etaAVG)*(t/etaAVG)^(betaAVG-1);
RAVG = @(t) exp(-1*(t/etaAVG)^betaAVG);
Cz1AVG = C1AVG(expertOP(1))*haz1AVG(expertOP(1))
Cz2AVG = C2AVG(expertOP(1))*haz2AVG(expertOP(1))
%normalize and go past 10% to scale the Z values to each other
anorm = 0:(eta2/100):(eta2+eta2/10);
hnorm = 0:(eta2h/100):(eta2h+eta2h/10);
cnorm = 0:(eta2c/100):(eta2c+eta2c/10);
bnorm = 0:(eta2b/100):(eta2b+eta2b/10);
jnorm = 0:(eta2j/100):(eta2j+eta2j/10);
```

```
inorm = 0:(eta2i/100):(eta2i+eta2i/10);
AVGnorm = 0:(etaAVG/100):(etaAVG+etaAVG/10);
%get percentage from expertOP
percentageOL = expertOP(1)/etaAVG;
A2Itime1 = percentageOL*etai;
A2Itime2 = percentageOL*eta2i;
%Z value at times on industry scale accoding to expert
Cz1A2I = C1i(A2Itime1)*haz1i(A2Itime1);
Cz2A2I = C2i(A2Itime2)*haz2i(A2Itime2);
s0 = [20];
[X1,fval] = fsolve(@(s) myfun(a(1),b(1),Cz1,beta,eta,s),s0)
[X2,fval] = fsolve(@(s) myfun(a(2),b(2),Cz2,beta2,eta2,s),s0)
[X1h,fval] = fsolve(@(s) myfun(ah(1),bh(1),Cz1h,betah,etah,s),s0)
[X2h,fval] = fsolve(@(s) myfun(ah(2),bh(2),Cz2h,beta2h,eta2h,s),s0)
[X1c,fval] = fsolve(@(s) myfun(ac(1),bc(1),Cz1c,betac,etac,s),s0)
[X2c,fval] = fsolve(@(s) myfun(ac(2),bc(2),Cz2c,beta2c,eta2c,s),s0)
[X1b,fval] = fsolve(@(s) myfun(ab(1),bb(1),Cz1b,betab,etab,s),s0)
[X2b,fval] = fsolve(@(s) myfun(ab(2),bb(2),Cz2b,beta2b,eta2b,s),s0)
[X1j,fval] = fsolve(@(s) myfun(aj(1),bj(1),Cz1j,betaj,etaj,s),s0)
[X2j,fval] = fsolve(@(s) myfun(aj(2),bj(2),Cz2j,beta2j,eta2j,s),s0)
%   time of failure according to average experts
[X1AVG,fval] = fsolve(@(s) myfun(aAVG(1),bAVG(1),Cz1AVG,beta,eta,s),s0)
[X2AVG,fval] = fsolve(@(s) myfun(aAVG(2),bAVG(2),Cz2AVG,betaAVG,etaAVG,s),s0)
%time of failure according to industry
[X1t,fval] = fsolve(@(s) myfun(ai(1),bi(1),Cz1AVG,beta,eta,s),s0)
[X2t,fval] = fsolve(@(s) myfun(ai(2),bi(2),Cz2AVG,beta2i,eta2i,s),s0)
%time of failure on industry scale from average experts
[X1A2I,fval] = fsolve(@(s) myfun(ai(1),bi(1),Cz1A2I,beta,eta,s),s0)
[X2A2I,fval] = fsolve(@(s) myfun(ai(2),bi(2),Cz2A2I,beta2i,eta2i,s),s0)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
f1 = zeros(1001,1);
f2 = zeros(1001,1);
f2i = zeros(1001,1);
for i=1:1001
    f1(i) = (beta/eta)*(i/eta)^(beta-1) * exp(-1*(i/eta)^(beta));
    f2(i) = (betaAVG/etaAVG)*(i/etaAVG)^(betaAVG-1) * exp(-1*(i/etaAVG)^(betaAVG));
    f2i(i) = (beta2i/eta2i)*(i/eta2i)^(beta2i-1) * exp(-1*(i/eta2i)^(beta2i));
end
intxf1 = f1(1);
intf1 = f1(1);
intxf2 = f2(1);
intf2 = f2(1);
intxf2i = f2i(1);
intf2i = f2i(1);
for i=2:1001
   intxf1 = intxf1 + ((i*f1(i))+((i-1)*f1(i-1)))/2;
   intf1 = intf1 + (f1(i)+f1(i-1))/2;
   intxf2 = intxf2 + ((i*f2(i))+((i-1)*f2(i-1)))/2;
   intf2 = intf2 + (f2(i)+f2(i-1))/2;
   intxf2i = intxf2i + ((i*f2i(i))+((i-1)*f2i(i-1)))/2;
   intf2i = intf2i + (f2i(i)+f2i(i-1))/2;
```

```
end
disp('The RUL using statistic for average beta and eta values:')
% RUL1 = intxf1/intf1
% RUL2 = intxf2/intf2
% RULi = intxf2i/intf2i
%this is considering both covariates, only Z2 used for study though
RLavg = ((X1AVG+X2AVG)/2)- xread
RLA2I = ((X2A2I+X1A2I)/2)- xread
```