A RE-ASSESSMENT OF WAVE RUN UP FORMULAE

by

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Declaration

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ABSTRACT

Over the last few decades, wave run up prediction has gained the interest of numerous researchers and every newly-published paper has aimed to predict wave run up with greater accuracy. Wave run up is defined as the vertical elevation reached by a wave’s, front water edge as it runs up a beach, measured relative to the still water line. Wave run up is dependent on the incidental wave height, the wave period, the beach slope and the wave steepness. The majority of publications incorporate all of these factors, but some do not, which has led to numerous debates.

The goal of this study is to do a re-assessment of previously published wave run up formulae, to obtain a more informed understanding about wave run up and the available predictive empirical formulae. The study also seeks to evaluate the Mather, Stretch & Garland (2011) formula. The method for undertaking this objective comprised a physical model test series with 10 regular wave conditions on a constant slope, being 1/24, performed with an impermeable floor. Also, a beach study in the field was done on Long Beach, Noordhoek, where run up measurements were taken for 30 minute intervals, resulting in five test conditions.

A numerical model was employed in conjunction with the beach study to determine the local offshore wave parameters transformed from a deep water wave rider. This information was used to correlate the run up measurements with known wave parameters.

Firstly, the physical model assessment was performed to provide a proper foundation for run up understanding. Plotting empirical normalised run up values \( \frac{R}{H_0} \) versus the Iribarren number for different formulae, a grouping was achieved with upper and lower boundaries. The physical model results plotted on the lower end of this grouping, resulted in prediction differences of more than 10\%. These differences may have been caused by the unevenness of the physical model slope or the fact that only one slope had been tested. Despite this, the results fell within a band of wave run up formulae located on the lower end of this grouping.

An assessment of the beach measurements in the field gave a better correlation than the physical model results when compared to normalised predicted wave run up formulae. These measurements also plotted on the lower end of the grouping, resulting in prediction differences of less than 10\% for some empirical formulae.

When comparing these empirical predictions to one another, the results demonstrate that the formulae comparing best with the beach measurements were Holman (1986) and Stockdon, Holman, Howd, & Sallenger Jr. (2006). Extreme over predictions were found by Mase & Iwagaki (1984), Hedges & Mase (2004) and Douglass (1992). Nielsen & Hanslow (1991) only compared best with the beach
measurements and De la Pena, Sanchez Gonzalez, Diaz-Sanchez, & Martin Huescar (2012) only compared best to the physical model results.

This study supports the formula proposed by Mather, Stretch, & Garland (2011). Applying their formula to the measured results presented a $C$ constant of 3.3 for the physical model and 8.6 for the beach results. Both values are within the range prescribed by the authors.

Further research minimized the array of possible ‘$C$’ values by correlating this coefficient to Iribarren numbers. ‘$C$’ values between 3.0–5.0 is prescribed for low Iribarren conditions (0.25-0.4) and values between 7.0–10 for higher Iribarren conditions are 0.75-0.8. However, this formula is still open for operator errors whereby the ‘$C$’ value has a big influence in the final result. The best formulae to use, from results within this thesis, is proposed by Holman (1986) and Stockdon et.al (2006). These formulae are not open to operator errors and uses the significant wave height, deep water wave length and the beach face slope to calculate the wave run up.
OPSOMMING

Gedurende die afgelope paar dekades, het golf-oploop voorspellings die aandag van talle navorsers gelok en elke nuwe geskrewe voorlegging het gepoog om meer akkurate golf-oploop voorspellings te verwesenlik. Golf-oploop kan definieer word as die vertikale elevasie bereik deur ‘n golf se voorwaterkant soos dit op die strand uitrol, gemeet relatief vanaf die stilwaterlyn. Golf-oploop is afhanklik van die invalsgolfhoogte, die golfperiode, die strandhelling en die golfsteilheid. Die oorgrote mederheid publikasies uit die literatuur inkorporeer al hierdie faktore, maar sommige nie, wat groot debatvoering tot gevolg het.

Die doel met hierdie studie is om vorige gepubliseerde golf-oploop formules te re-evalueer, om ‘n meer ingeligte begrip van golf-oploop en beskikbare voorspellende formules te verkry. Die studie poog terselfdertyd ook om golf-opvolg tendense, uniek aan Suid Afrikaanse strande te evalueer deur die huidige formule wat tans hier gebruik word, te assesseer. Om hierdie doelwit te bereik, is gebruik gemaak van ‘n fisiese model toetsreeks bestaande uit 10 reëlmatige golfstoestande op ‘n konstante ondeurlaatbare strandhelling van 1/24. ’n Veldstudie was ook uitgevoer op Langstrand, Noordhoek, waar golf-oploopmetings met 30 minute tussenposes uitgevoer is, vir vyf toets-toestande.

Tesame met die veldstudie, is ‘n numeriese model aangewend om die gemete diepsee data nader ann die strand wat bestudeer is te transformeer. Hierdie inligting is benodig om ‘n verband tussen tussenlop-metings en bekende golf parameters te bepaal.

Eerstens is die fisiese model assessering uitgevoer om ‘n behoorlike basis vir die begrip van golf-oploop in die veld te verkry. Deur die emperiese, genormaliseerde oploop waardes ($R_2/H_0$) vir verkeie formules teenoor die Iribarren getal te plot, is ‘n groepering met hoër en laer grense gevind. Daar is gevind dat die fisiese modelwaardes op die laer grens plot, en het verskille met die emperiese waardes van meer as 10% getoon. Hierdie verskille is moontlik veroorsaak as gevolg van ‘n oneweredige fisiese model strandhelling of deur die feit dat slegs een helling getoets is. Ten spyte hiervan, het die model oploop waardes binne die bestek van golf-oploop formules geval.

Assessering van die veldmetings het ‘n beter korrelasie as die fisiese modelresultate getoon, tydens vergelykings met genormaliseerde golf-oploop formules van die emperiese formules. Die oploop waardes van hierdie metings het ook geplot aan die laer grens van die groepering, met verskille van minder as 10% vir die meeste gevalle van die emperiese formules.

Wanneer hierdie emperiese voorspellings vergelyk word, is gevind dat die formules wat die beste ooreenstem met die fisiese model, die van Holman (1986) en Stockdon, Howd, & Sallenger Jr. (2006)

Hierdie studie ondersteun die formule voorgestel deur Mather, Stretch, & Garland (2011). Deur hul formules op die gemete bevindings toe te pas, is ‘n C konstante van 3.3 vir die fisiese model resultate, en 8.0 vir die stranduitslae bepaal. Beide waardes lê binne die grense wat deur die auteurs voorgestel is.

Verdere navorsing het getoon dat moontlike waardes vir die ‘C’ konstante tussen 3.0 en 5.0 moet wees vir Iribarren waardes van tussen 0.25 en 0.4. Vir hoër Iribarren waardes, 0.75-0.8, moet die ‘C’ konstante tussen 7.0 en 10 wees; dog is die formule steeds oop vir operateur foute. Die hoofbevindinge van die tesis is gevind dat die beste golf-oploop formules, om tans te gebruik, die van Holman (1986) en Stockdon et.al (2006) is. Hierdie formules kan glad nie beinvloed word deur operateurs foute nie en maak gebruik van die invals golfhoogte, die golfperiode en die strandhelling om die golf-oploop te bepaal.
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LIST OF SYMBOLS AND ABBREVIATIONS

\( \alpha \) Beach slope in degrees

\( \beta_f \) Beach face slope

\( \beta_{tr} \) Near shore beach slope

\( \xi_0 \) Iribarren number

\( \xi_{m-1.0} \) Iribarren number based on mean spectral wave period

\( \rho \) Square correlation coefficient

\( \gamma_b \) Wave breaking index

A Shape parameter

C Dimensionless coefficient used in the formula by A Mather

d Water depth

\( d_b \) Breaking wave water depth

\( d_{50} \) Mean diameter of sand

\( F_i \) Inertial forces

\( F_g \) Gravitational forces

h Closure depth

\( H_I \) Incident wave height

\( H_R \) Reflected wave height

\( H_s \) Significant wave height

\( H_b \) Breaking wave height

\( H_{max} \) Maximum wave height

\( H_{m0} \) Spectral significant wave height

\( H_0 \) Deep water significant wave height

\( H_{0}^{(rms)} \) Root mean squared deep water wave height

\( H_{tr} \) Transitional wave height

\( H_{1/10} \) Average of the highest 10% wave heights

\( H_1, H_2 \) Scaling parameters

k Shape parameter

\( K_r \) Bulk reflection factor

LLD Land Levelling Datum

L Wave length

\( L_c \) Critical wave length

\( L_0 \) Deep water wave length

\( L_{m-1,0} \) Spectral mean wave length

MHW Mean high water

MLW Mean low water

MSL Mean Sea Level
MWL  Mean Water Level
N_L  Froude scale factor
n_s  Wave set up
n_s,max  Maximum wave set up
n_b  Wave set down
p  Porosity factor
Q_p  Peakedness factor
r  Roughness factor
R  Wave run up
R_ave  Average wave run up
R_high  Wave run up for high tide
R_low  Wave run up for low tide
R_max  Maximum wave run up
R_min  Minimum wave run up
R_Mid  Wave run up for mid tide
R_2wm(rms)  Vertical scale of wave run up
R_2  Wave run up exceeded by 2% of the record
s_0  Wave steepness
SWL  Still Water Level
S_s  Maximum wave setup
S_inc  Incident swash
S_IG  Infragravity swash
T_p  Peak wave period
T_m & T_m0.2  Mean wave period
T_m-1.0  Spectral mean wave period
x_h  Horizontal distance to closure depth
x_wm  Horizontal run up limit
z_wm  Maximum waterline elevation achieved by an individual wave
z_100  Highest elevation which is transgressed by 100% of the waves
CHAPTER 1: INTRODUCTION

1.1 Background

Wave run up, also known as beach run up, can be described as the maximum water surface excursion measured vertically upwards from the still water level (SWL), which is caused by a turbulent layer of water rushing up the beach when a wave has broken onshore (Burcharth & Hughes 2011). The SWL is the water surface elevation when all wave actions have ceased and the sea is seen as flat (SWL-Still Water Level n.d). Run up levels are mainly dependent on the incident wave height and steepness, the interaction between the incident wave with the proceeding reflective wave, the slope of the beach above and below the SWL line, the surface roughness of the slope and also the permeability thereof (Burcharth & Hughes 2011). Wave run up is normally defined by two parameters, namely \( R_{\text{max}} \) and \( R_2 \). \( R_{\text{max}} \) is the maximum run up at any specific time and \( R_2 \) represents a run up value for waves in an event exceeded by 2% of all the run up records within the event. The 2% run up value is more commonly used in the coastal engineering field.

In Figure 1.1, \( R \) represents the vertical excursion of the water surface i.e. wave run up; SWL represents the still water level and \( \beta_f \) is the beach face slope located in the foreshore, between mean low water (MLW) and mean high water (MHW). Wave run up plays a vital role in beach dynamics because it is the main element in promoting or causing beach/bluff erosion and cross-shore sediment transport.
Wave run up can be broken down into two components: wave setup and swash. Both these components are added to one another to produce a run-up value. Wave setup can be described as the elevated water level above the SWL, measured at the location where the SWL intersects the beach. Swash is the water level fluctuation around this wave setup elevation (Smith 2003). In Figure 1.2, the zero of the y-axis represents the SWL for a given situation. The dashed, mean water line (MWL) indicates the ‘setup’ height over time and the parameter $S_s$ indicates the maximum value of setup. Note, the maximum value of ‘setup’ is measured from the SWL to the point where the MWL intersects the slope (Hedges & Mase 2004). ‘Swash’ is the term used for all the peaks and troughs above and below this setup line. Adding together setup and swash, produces the total wave run up ($R$). As indicated from the figure, it can be seen that both ‘setup’ and ‘swash’ can vary significantly over time for irregular waves. In the literature review, both ‘setup’ and ‘swash’ will be defined in greater detail.

![Figure 1.2 - Definition Sketch and Wave Record Showing Setup and Swash (Hedges & Mase 2004)](image)

Run up is of utmost importance to coastal engineers, land planners and environmental managers (Mather, Stretch & Garland 2011). It provides them with a line from which they can estimate suitable
set back lines from for the development of private or municipal projects. These lines can be designed on a determined manner for a 1 in 10 year storm or even a 1 in 50 year storm.

Numerous studies about wave run up have mainly focused on solid structures with steep slopes such as dykes and breakwaters, with very little focus on beaches. Some have tried to apply these procedures of solid structures on sandy beaches, but some complications always arise. These problems can be any one of the following as mentioned by (Douglass 1992):

- Sandy beaches tend to have much flatter slopes than solid structures.
- Beaches can move freely in response to wave action.
- A beach slope is not constant across the whole profile.
- Sandbars located offshore can have an effect on wave energy.

In the last decade, much interest has been directed at wave run up calculations, especially for irregular waves. This started with regular wave run up, which was investigated by Granthem (1953), Saville (1958), Savage (1959) and Hunt (1959) for which they accurately defined maximum wave run up values from physical model tests. After regular waves, engineers investigated irregular wave run up given the irregular character of natural sea states. Wassing (1957), Battjies (1974b), Holman (1986), Nielsen & Hanslow (1991) and De la Pena et al (2012) all published research documents on irregular waves with all of them publishing new formulae or updated previous research with different views about the wave run up process.

### 1.2 Problem statement

One of the latest publications for a wave run up formula in South Africa, was published by Mather et al (2011). Their model took a different approach than the average wave run up formulae by using a modification of the beach face slope $\beta_f$ and not using the Iribarren number, $\zeta_0$, for calculations. Reasons would be described at a later stage in this document. Instead, they predicted wave run up using the distance offshore $x_h$ to a certain prescribed water depth $h$. The distance $x_h$ was measured from the point where the SWL intersects the beach. For $h$, the depth of closure was used, to estimate the near shore slope $S$ in Equation 1.1. The closure depth is described as the seaward limit where minimal or no sediment movement occurs, leading to limited beach profile fluctuations (Dean, Kriebel & Walton 2008). All of the above parameters are used to calculate maximum run up, which is written as follows:

$$\frac{R}{H_0} = C S^{\frac{2}{3}}$$

**EQUATION 1.1 - INITIAL RUN UP FORMULAE (MATHER ET AL 2011)**
Where $H_0$ is the deep-water significant wave height, $C$ is a dimensionless coefficient and $S = h/x_h$. 

Mather et al. (2011) mentioned that their model reduces some uncertainties with the prediction for wave run up on natural beaches compared to other previous models because the beach slope is a very dynamic entity which can change numerous times within one storm event. This formula was proposed for use as a first estimate for predicting wave run up (Andrew Mather, Personal Communication, May 29 2014). The study was done by collecting run up data from a single storm event from a large number of beaches with varying slopes, Figure 1.4.

Their study relied on visible debris signs that were left on the beach, caused by a storm event, for the measuring of the maximum beach run up achieved. This maximum run up was occasionally taken as the vegetation line on the beach/dune and is thus not the true maximum level. With a dune present, the run up would be measured at the toe of the dune, thus the wave could be stopped by the dune and not
reach its full excursion as shown in Figure 1.5.

The motivation behind this thesis was to validate Equation 1.1 by comparing it against other formulae, which will be discussed further in the literature review. Concerns around this formulae were: (1) the formula does not take into account the wave period or beach slope; (2) the measuring techniques used for wave run up are doubtful (3) it includes a dimensionless constant, which can be chosen by the user and varies between 3 and 10.

![Figure 1.5 - Storm Wave Damage in KZN (Mather et al. 2011)](https://scholar.sun.ac.za)

1.3 Proposed solution

The thesis was set out to include two different types of tests. The first type will be a physical model test in a 2D flume where a series of regular wave runs would be performed. These wave runs should have different wave heights and periods leading to a comprehensive data set. Results from the physical model would be compared to some published formulae, which are discussed in Chapter 2, to validate the empirical formulae. The second type would be beach measurements where wave run up measurements will be recorded on Long Beach, Noordhoek. These measurements would then be associated to an offshore wave height, which will be provided by a numerical model. From the above two solutions, the student expects to confirm the validity of Equation 1.1.

1.4 Chapter overview

This thesis consists of seven chapters in total, including the present chapter, which is the Introduction. Chapter 2, the Literature Review, introduces the reader to coastal engineering terms and defines the fundamentals of propagating waves from deep water to the shoreline. The Literature Review also discusses the chronological evolution of wave run up research including most of the published formulae to predict wave run up.

The physical model testing is described in Chapter 3. Within this chapter the model setup is explained and the test procedure is provided. The results provide run up values for 10 tests taken from video imagery, with the agreeing wave characteristics using regular waves.
The beach study performed at Long Beach, Noordhoek, is described in Chapter 4. The site conditions are explained including a general scope of the survey and measuring technique implemented. The wave run up results is provided for five different events, each being 30 min long, starting at low tide and ending at high tide.

Chapter 5 describes the numerical model implemented to provide the necessary wave characteristics of waves being propagated from deep water to just in front of Long Beach. Two models were used, a regional and a local model. The bathymetry data used was provided from public organisations, and the significant wave height and wave period was taken from an offshore wave rider.

Chapter 6 examines the results from Chapters 3 - 5 and provides an interpretation and discussion on these results. Conclusions are drawn from each type of tests, providing the best and worst fit equations. A joint comparison is also considered and values for the constant ‘C’ (Equation 1.1) is provided.

The concluding chapter of this report, Chapter 7, provides the conclusion for this thesis report and gives general recommendations regarding any future research that can be made.
CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter briefly describes wave regions and propagation effects of waves, travelling from deep water to shallow water conditions, including wave set up and set down. A review is also done on the majority of published wave run up formulae, given in chronologic sequence. To conclude, a review on relevant numerical models was also done.

2.2 Wave regions and propagation effects

When a group of waves propagate from the deep water to shallow water, they travel through three different zones before they reach the shoreline. These are deep -, transitional - and shallow water regions. For each of the regions above, a wave is known to act differently and thus different formulae exist to determine essential variables. It is thus vital to classify a wave within a specific region before calculations can be made. The boundaries between the different zones are described by a ratio of water depth over wavelength (d/L) known as the relative water depth. When d/L > 0.5, a deep water region exists; when d/L < 0.05, a shallow water region exists and when 0.05 < d/L < 0.5 it can be classified as a transitional region.

One essential variable to identify for each region is when a wave would start to break. For deep water, the breaking wave height is solely dependent on the deep water wave length, \( H_b = 0.142 L_0 \). Once the wave enters the transitional zone, the breaking height is a function of the water depth and can be calculated by iteration with \( (H_b/L) = 0.142 \tanh (2\pi d/L) \). For the shallow water zone, the wave length and period do not have an influence anymore, thus the breaking wave height is only dependent on the water depth, \( H_b/d = 0.78 \). The ratio of \( H_b/d \) is also better known as the breaker index (\( \gamma_b \)).

Within these regions, processes exist which influences the characteristics of waves as it travels through them. These processes can be one or a combination of the following: refraction, shoaling, diffraction, wave breaking and dissipation due to friction (Zeki & Linwood 2008). The first three processes cause waves to diverge or converge in a certain direction and are mainly influenced by the bathymetry of the site. They are called propagation effects.
The first propagation effect, refraction, is caused primarily when the wave undergoes a change in water depth. If a wave is defined by a water column, stretching from the surface to the sea bed, with a known width, the particle velocities and displacements within the column can be calculated very accurately. For a deep water region, wave particles travel in a circular motion all the way through the water column and the particles at the bottom of the water column, never touch the sea bed. When the same wave travels into the transitional region, the particle motion changes from circular to elliptical, which leads to the wave slowing down. Within this water column, the top particles travel in an elliptical pattern but the bottom particles tend to oscillate linearly, Figure 2.1.

Once the wave enters this transitional zone, the bottom of the wave interacts with the seabed and the wave attributes would begin to change because the water column is shallower. This will result in a change in wave celerity, wave length and the direction of travel. This directional change is called refraction. Refraction is when a wave attempts to orientate itself to be parallel to the contours of the bathymetry.

The second propagation effect is ‘wave shoaling’. When the wave speed, length and the water depth changes as the wave nears the shoreline, the wave height will increase inside of the shallow water region because the energy per unit area should always be constant within a wave. This process is
known as ‘wave shoaling’ (Dalrymple & Fearing n.d). When the crest of the wave becomes too steep, it would tend to fall over and cause wave breaking. A wave can break in multiple ways, but this will be explained further in section 2.3. Figure 2.2 provides a good representation of the propagation effects on waves travelling from deep water into shallow water.

The last propagation effect is wave diffraction. This is a process where the direction of a wave is influenced by a surface piercing obstacle (Dalrymple & Fearing n.d) or a slit in an obstacle like an island, headland or harbour mouth. On the lee side of this obstacle, the water would normally be calmer than on the seaside, but the waves would curve around the corners of these obstacles. For a slit in a wall, the wave would enter and radiate sideways through it in a half-circular pattern.

2.3 Estimating wave parameters within the surf zone

Once a wave enters the surf zone, the largest waves within the energy spectrum would tend to break first as the water depth decreases. This is the result of depth-induced wave breaking. When this happens, the shape of the wave height distribution changes because the largest waves have been eliminated from the wave record. Shoaling, refraction and diffraction could also affect the wave height distribution. For deep-water cases, a simple Rayleigh distribution can be used with constant relationships to calculate $H_s$ or $H_{1/10}$. However, when the largest wave breaks in the surf zone, the whole distribution is altered and another model should be applied to find $H_s$ or $H_{1/10}$.

Battjies & Groenendijk (2000) compiled a study to find a wave height distribution for shallow foreshores, which was analysed and parameterised with data from physical modelling tests. They found that a Composite Weibull Distribution consisting of two independent Weibull distributions, best matched their physical model records plotted on an exceedance plot, Figure 2.3. These two distributions delivered two independent equations, which were calibrated and validated with the
physical model results. The point where the two distributions meet, located between the 40% and 50% exceedance values (Figure 2.3), is known as the ‘transitional wave height point’ ($H_{tr}$), which denotes the first wave breaking through depth induced breaking.

The equations used to best fit the data are provided in Equation 2.1. Equation 2.1 classifies the wave height for each of the two slopes in Figure 2.3. The distribution line on the left side of $H_{tr}$, fits the Rayleigh distribution precisely, with a shape parameter of $k = 2$. The distribution to the right of $H_{tr}$, has a shape parameter of $k = 3.6$ and differs from the Rayleigh distribution. In the paper presented by (Battjies & Groenendijk 2000) a table is provided which can be used predict $H_{0.1\%}$, $H_{1\%}$, $H_{2\%}$, $H_{10\%}$ and $H_{1/3}$. The input variables, used for the predicted shallow water wave heights, are calculated with Equations 2.2 and 2.3.

$P(H) = P(H < H) =$

\[
\begin{align*}
1 - \exp\left(\frac{-H}{H_1}^2\right) & \quad \text{for } H < H_{tr} \\
1 - \exp\left(\frac{-H}{H_2}^{3.6}\right) & \quad \text{for } H \geq H_{tr}
\end{align*}
\]

Equation 2.1 - Composite Weibull distribution formulae (CIRIA, CUR & CETMEF 2007)

$H_1, H_2 = $ Scaling parameters

$H_{tr} = (0.35 + 5.8\tan \alpha) d$

Equation 2.2 - Transitional wave height (CIRIA, CUR & CETMEF 2007)

$\alpha = $ Slope angle

$d = $ Water depth where wave height is calculated
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\[ H_{RMS} = \left[ 0.6725 + 0.2025 \left( \frac{H_{m0}}{d} \right) \right] H_{m0} \]

EQUATION 2.3 - ROOT MEAN SQUARED WAVE HEIGHT (CIRIA, CUR & CETMEF 2007)

2.4 Wave setup and set down

When waves break onshore, the mean water level changes with respect to the SWL by means of wave setup and wave set down. The CEM defines wave set up as the super elevation of the water level at the point where the SWL meets the beach (Smith 2003). Wave set up originates by a change in radiation stresses in the surf zone, which can be best described by (Longuet-Higgins & Stewart 1964) in the following quote:

“It is well known that surface waves possess momentum which is directed parallel to the direction of propagation and is proportional to the square of the wave amplitude. Now if a wave train is reflected from an obstacle, its momentum must be reversed. Conservation of momentum then requires that there be a force exerted on the obstacle, equal to the rate of change of a wave momentum. This force is the manifestation of the radiation stress.”

When deep-water waves approaches a beach with a sloping gradient, the waves would start to shorten, become steeper and end up breaking. Even though the wave has broken, it will continue to propagate up the beach with decreasing amplitude. This change in radiation stress causes the change in the mean surface water profile, MWL, (Longuet-Higgins & Stewart 1964). At the point where the crest of a wave is at its highest, the mean water surface decreases to a minimum and this is identified as wave set down, \( \eta_b \). The water surface increases hereafter up to the wave setup point, \( \eta_s \), and up to the maximum wave setup elevation, \( \eta_{s,max} \). Figure 2.4 provides a good representation of wave setup and set down in regards to the SWL. The formula for mean wave setup (\( \bar{\eta}_s \)) and mean wave set down (\( \bar{\eta}_b \)) are provided with Equation 2.4.

FIGURE 2.4 - WAVE SET UP AND SET DOWN
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\[ \bar{n}_s = \bar{n}_b + \left[ \frac{1}{1 + \frac{8}{3y_b^2}} \right] * H_b \]

\[ \bar{n}_b = -\frac{1}{16} \gamma_b^2 d_b \]

EQUATION 2.4 - MEAN WAVE SET UP AND SET DOWN (SMITH 2003)

\[ \gamma_b = \text{Breaker index} \]

\[ h_b = \text{Breaking wave height} \]

\[ d_b = \text{Water depth where wave breaks} \]

2.5 Iribarren number

This factious number, also known as the surf similarity parameter (Battjies 1974), is very important in the field of coastal engineering because it provides an idea of the form in which a wave breaks. The Iribarren number is defined as follows:

\[ \zeta_0 = \frac{\tan \alpha}{\sqrt{s_0}} \]

EQUATION 2.5 - IRIBARREN NUMBER (BATTJIES 1974)

Where \( \alpha \) is the angle of the beach where the wave breaking occurs on and \( s_0 \) is the wave steepness. The wave steepness can be defined as the ratio between the wave height and the wavelength, \( s_0 = H_0/L_0 \). Both the wave height and wave period are generally used for deep-water conditions. \( H_0 \) is sometimes substituted with \( H_b \), which is the breaking wave height at the toe of a slope. The wave steepness can tell us a lot about the wave's history and its characteristics. A steepness of \( s_0 \approx 0.01 \) provides an indication of a typical swell sea state, and a steepness between \( s_0 \approx 0.04-0.06 \) indicates a typical wind driven sea (EurOtop 2007).

The ratio of the beach slope over the wave steepness adheres to a certain type of wave breaking. These are classified as surging, collapsing, plunging or spilling. They are listed with respect to an increased wave steepness or decreasing slope angle (Battjies 1974). For a surging wave, \( \zeta_0 > 3.0 \), it is assumed that the wave has limited breaking, but it mostly just flows up and down the slope. For \( \zeta_0 = 3.0 \), the wave is classified as collapsing. Collapsing waves forms in the changeover stage from surging to plunging waves. These waves have an almost vertical front and cause a greater wave run up and run down than the other wave types (EurOtop 2007). For plunging waves, the Iribarren number is normally between 0.5 and 3.0. Any value smaller than 0.5, would cause wave to be a spilling wave. Figure 2.5 provides a visual description of the four different wave types.
2.6 Wave run up formulae

Hunt (1959) investigated one of the first formulae on wave run up. The formula estimates that wave run up is directly proportional to the Iribarren number, roughness factor and porosity factor. Several formulae were proposed for different slope styles, i.e. continuous and composite slopes. The formula prescribed for continuous slopes, is as follows:

\[
\frac{R}{H_0} = 1.84 \cdot \zeta_0 \cdot (r) \cdot (p)
\]

**EQUATION 2.6 - HUNT'S EQUATION (HUNT 1959)**

\[(r) = \text{Roughness factor}\]

\[(p) = \text{Porosity factor}\]

When working in a laboratory, with a continuous, impermeable and smooth surface, the roughness and porosity factor will become 1.0. For beach face slopes at 1/10 and 1/30, the product of the roughness and porosity factor is 0.91 and 0.77 respectively, given a sand grain size of 0.2mm. This product will decrease as the sand size increases, or when the slope decreases.
Mase & Iwagaki (1984) were one of the first researchers to investigate the run up of irregular waves on gentle slopes in a research facility. Their objectives were to examine the characteristics that could influence the run up on beaches. Tests were performed in a 2D wave flume with beach slopes of 1/5, 1/10, 1/20 and 1/30. Run up measurements were measured with a wave probe that was built into a groove, cut into the beach face. The irregular waves were simulated with a Pierson-Moskovitz spectra. The run up results were analysed according to the crest method and not the zero up cross method, because this leads to a reduction in run up peaks (Mase & Iwagaki 1984). A zero up cross method only accepts a run up value if it is higher than the mean run up level and the crest method accepts every wave as a run up wave (Mase & Iwagaki 1984). The following equation was proposed from the physical model results:

$$\frac{R_2}{H_0} = 1.86\zeta_0^{0.71}$$

EQUATION 2.7 - WAVE RUN UP FORMULA (MASE & IWAGAKI 1984)

When comparing the number of run up waves to the number of incident waves, it is apparent that the ratio decreases as the beach slope becomes milder and the wave steepness larger. The ratio is almost linearly dependant on the Iribarren number. This is because when a wave comes to the shoreline, it cannot run up if the backrush of the previous wave is larger and when a wave is overtaken and captured by a larger wave before it reaches the maximum run up elevation (Mase & Iwagaki 1984). When wave groupings were analysed, the maximum run up heights showed 10% increase associated with a higher wave grouping. For significant and mean wave run up, no increase was found.

Holman & Sallenger (1982) compiled a study to find relationships between swash, setup and maximum wave run up. Their report, which is titled: Setup and Swash on a Natural Beach, defined total wave run up as the sum of setup and half the swash height generated. They performed 154 wave run up tests, where data was gathered by analysing time lapse photography recorded with video cameras at +13m MSL. The analyses were done with a computer aided digitization scheme designed by Holman & Guza (1984).

Their tests were done on a beach that was classified as moderately steep with a beach foreshore slope of 1/10. The significant wave heights were recorded from an offshore buoy, which measured waves with heights of between 0.4 and 4.0 m. One of Holman & Sallenger (1982) objectives was to use their field data to parameterise a linear relationship between the normalised wave run up and the Iribarren number.
With compiling the first plot of setup ($\eta_{\text{setup}}$, Figure 2.4) against incident wave height, they found no correlation between the two as the data was well scattered. The scatter was reduced by plotting the non-dimensional setup ($\eta_{\text{setup}} / H_0$) against the surf similarity parameter and this delivered far better results. These results were improved again by plotting the results in different tidal band plots, i.e. low-, mid- and high tide. For both the high- and mid-tide, a positive relationship was found between non-dimensional setup and the surf similarity number, but for the low tides, no trend was evident. This was perhaps caused by an offshore sand bar which influenced the results. The results also showed that when the surf similarity number increases, the non-dimensional setup data tend to scatter more, thus the only respectable correlation was found between surf similarity numbers of 0 and 1.5. The same methodology was used to draw up plots for swash heights. Holman & Sallenger (1982) found that swash data showed no difference with a change in tide, thus concluding that the foreshore slope was the determining factor for the swash dynamics.

For the maximum run up comparison, the non-dimensional wave run up ($R_{\text{run up}} / H_0$) was plotted against the surf similarity parameter. A direct comparison was found between the two variables with acceptable linear trends for mid- and high tides. When the data was plotted for high- and mid-tides, the data conformed to a more linear relationship, but delivered poor grouping results. Once the Iribarren number passed 1.5, the data scatter increased. Plotted for low tide, the data did not conform, but the data grouping improved significantly. The following relationships were found at the end of the research:

$$\frac{R_{\text{High}}}{H_0} = 0.86 \xi_0 + 0.2$$
$$\frac{R_{\text{Mid}}}{H_0} = 1.06 \xi_0 + 0.25$$
$$\frac{R_{\text{Low}}}{H_0} = 0.32 \xi_0 + 0.79$$

**EQUATION 2.8 - NORMALISED RUN UP FOR HIGH-, MID- AND LOW TIDES (HOLMAN & SALLenger 1982)**

$$R_{\text{High}} = \text{Wave run up for high tide}$$
$$R_{\text{Mid}} = \text{Wave run up for mid tide}$$
$$R_{\text{Low}} = \text{Wave run up for low tide}$$

Thus, Equations 2.8 should probably be used for cases where the surf similarity parameter is 1.5 or smaller. Holman & Sallenger (1982) explained that the possible difference between their data and Hunt’s (1959) could have been caused by the time-lapsed photography technique of defining the swash rundown line. It is user-interpreted and can easily affect the outcome. For plunging and collapsing breakers, this was very difficult to define. Some concerns about this study were that
Holman & Sallenger (1982) only had two sets of full beach survey data for the entire three week test period and all of the above results where plotted on these values. They admitted that this was a limiting factor. They found that the set up component of run up was partially influenced by the full beach slope and not swash, which was influenced by the beach face slope.

Four years later, Holman (1986) published a second paper titled: *Extreme value statistics for wave run-up on a natural beach*. By using the same data used by Sallenger and himself, his objective was to extend the analysis of their data with extreme value statistics. With this research paper, Holman (1986) published a new wave run up formulae compiled on a statistical method resulting in a 2% run up value.

Holman (1986) did not use all the data used by Holman & Sallenger (1982), but utilised only 149 data runs with videos of 35 minutes in length. Incident wave heights were measured between 0.4 and 4.0 m, at a water depth of 20m, with periods in the range of 6 to 16s. Holman (1986) used the same technique as Holman & Sallenger (1982) to plot the 2% run up level in non-dimensionless terms \( \frac{R_2}{H_0} \) against the Iribarren number. The reason for the non-dimensional plot variable is that for the same significant wave height, the Iribarren number could differ, thus the run up statistics were normalized to the non-dimensional value.

The same problems arose when the plots were drawn up, where high Iribarren numbers led to a large scatter of the values, but for low Iribarren numbers, the data conformed to the Hunt formula, *Equation 2.6*. This was because lower Iribarren numbers were generated by two storm events and forced the beach to become more linear. The 2% run up formula, compiled from statistical analysis and regression coefficients, proposed by (Holman 1986) is given below. It is almost similar to Equation 2.8, of Holman & Sallenger (1982), for wave run up measured at high tide.

\[
\frac{R_2}{H_0} = 0.83\zeta_0 + 0.2
\]

*EQUATION 2.9 - HOLMAN’S WAVE RUN UP (HOLMAN 1986)*

Holman (1986) found that when the data was plotted, more scatter was found when the setup variable is included in the calculations. When the Iribarren number exceeded a value of 1.5, the data set tended to scatter significantly because the shoreline-elevation time series were dominated by incident wave frequencies. Another conclusion that Holman made is that for an erodible beach under a storm attack, the beach slope cannot be measured a-priori but should be estimated with good knowledge and experience (Holman 1986).
Nielsen & Hanslow (1991) investigated wave run up on six different sandy beaches on the east coast of Australia. They collected data with a deep water root mean squared wave height ($H_{0\text{rms}}$) range of between 0.53 and 3.76m and significant wave periods between of 6.4s to 11.5s. All of the above data were obtained from an offshore wave buoy in 80m water depth. For this study, they used the beach face slope which stayed linear and was easily measured under storm conditions.

Their method of measuring wave run up in the field was different from the previously known techniques. Instead of using video imagery or resistance wires, they used a visual method of counting the number of waves running up past stakes planted in the beach with known elevations. Each stake had a water line elevation ($z_{wm}$). They did not state how the elevation of each stake was measured. If it is measured at the bottom of the stake, where it intersects with the beach, their water line elevations can change over a period of a few minutes because of littoral transport. Their test time was set at 20 minute intervals. Initial data indicated that for a wide range of sandy beaches, the Rayleigh distribution provides a decent explanation for the distribution of $z_{wm}$. This accounts only for beach face slopes, $\beta_f$, in the range of 0.026 to 0.19. Vertical scaling for run up, or wave run up, which is used in the final run up equation, is defined as follows with root mean squared values:

$$R_{zwm(rms)} = (Z_{wm} - Z_{100})_{rms}$$

EQUATION 2. 10 - VERTICAL SCALE OF RUN UP (NIELSEN & HANSLOW 1991)

$Z_{wm} = \text{Maximum water line elevation achieved from an individual wave}$

$Z_{100} = \text{The highest point elevation which is transgressed by 100% of the waves}$
For steep beaches, Nielsen & Hanslow (1991) produced the following formula for $R_{zwms}$:

$$R_{zwms} \approx 0.6\sqrt{H_{rms}L_0} \cdot \tan \beta_f \quad \text{tan}\beta_f > 0.1$$

EQUATION 2.11 - RUN UP FOR STEEP BEACHES (NIELSEN & HANSLOW 1991)

And for flatter beaches:

$$R_{zwms} = 0.05\sqrt{H_{rms}L_0} \quad \text{tan}\beta_f \leq 0.1$$

EQUATION 2.12 - RUN UP FOR FLAT BEACHES (NIELSEN & HANSLOW 1991)

$H_{rms} = $ Deep water root mean squared wave height

$L_0 = $ Deep water wave length

$\tan \beta_f = $ Beach face slope measured in the foreshore

The authors found that for steep beaches, the beach slope will have an effect on the $L_{zwms}$ value and that the $z_{100}$ value should be taken as the SWL. For flat beaches, the beach slope does not influence $L_{zwms}$, but $z_{100}$ will be lower than SWL. This is because on flat beaches waves normally transform into bores, broken waves, seaward of the $z_{100}$ location (Nielsen & Hanslow 1991). These bores unite by absorbing one another and results in less bores reaching the shoreline and surpassing the $z_{100}$ point. Finally, the maximum run up was found to be:

$$R_2 = 1.98R_{zwms}$$

EQUATION 2.13 - 2% RUN UP (NIELSEN & HANSLOW 1991)

One interesting finding that Nielsen and Hanslow (1991) made was, that for steep beaches, the water that infiltrated the beach surface, after run up, drained very efficiently. This produced a mean water surface elevation far below the maximum run up point. This is because steep beaches are more coarse and porous causing quicker drainage.

Douglass (1992) took a new approach than the previous authors and argued that normalised wave run up is directly equal to the Iribarren number, but the beach face slope term should be omitted from the equation because it is a dependant variable, which responds to wave conditions. The beach face slope is also very difficult to estimate a-priori, which was mentioned by Holman (1986). Douglass (1992) wanted to prove that by removing the beach face slope, the run up levels could still be predicted accurately. Douglass (1992) used data obtained by Holman & Sallenger (1982) to plot the ratio of $R_{max}/H_0$ against the beach face angle, $\alpha$. This showed that these relationships are completely independent of one another, thus the $(\tan \alpha)$ can be omitted from the Iribarren number and be replaced by a constant, $C$, shown in Equation 2.14.
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\[ \frac{R}{H_s} = \frac{C}{\sqrt{\frac{H_m}{L_0}}} \]

**EQUATION 2.14 - PROVISIONAL WAVE RUN UP FORMULA (DOUGLASS 1992)**

The constant value C was estimated as 0.12 based on Holman & Sallenger’s (1982) dataset. Douglass (1992) mentioned that his model should be applied to more data sets because the value for the coefficient C was based on just one set of data from the same beach.

The final formula is as follows:

\[ \frac{R}{H_0} = \frac{0.12}{\sqrt{\frac{H_0}{L_0}}} \]

**EQUATION 2.15 - FINAL WAVE RUN UP FORMULA (DOUGLASS 1992)**

Some years later, Hedges & Mase (2004) tried to modify Hunt’s formula, *Equation 2.1*, to give more weight to the setup parameter. They argued that when the beach becomes completely flat, and applying the equation proposed by Hunt (1959), the run up equals zero. But this is technically incorrect because the incoming waves would still cause wave setup. Thus, run up would have a minimal limiting value when \( \tan \alpha \) from Hunt’s equation tends to zero. But run up would never be zero. The improved relationship that they found is as follows:

\[ \frac{R_2}{H_0} = 1.49 \xi_0 + 0.34 \]

**EQUATION 2.16 - WAVE RUN UP FORMULA (HEDGES & MASE 2004)**

They found that the use of their equation increases the prediction of wave run up by 77%, through comparing it to the physical model results published by Mase and Iwagaki (1984). When the Iribarren number is below 1.5, both equations are equally good, but when the number increases above 1.5, the Mase and Iwagaki’s (1984) formula underpredicts the run up. The dimensionless significant run up was also found to have a maximum asymptote of 2.7 to which the grouping tends.

Thereafter, Stockdon, Holman, Howd, & Sallenger Jr. (2006) performed a study to empirically parameterize extreme wave run up, taken as the 2% exceedance value, for natural beaches with a wide range of conditions. Ten field experiments were analysed and used for this study. Their objective was to improve the work done by Holman (1986). These authors defined wave run up through two diverse processes: ‘total swash excursion’ and ‘time averaged wave set up’. Each of the above processes was parameterised on its own and the basic 2% run up formula found by this study is given in *Equation 2.17*. 
This formula was tested extensively on 10 field experiments and provided a 0.94 linear regression when $R_2$ was plotted against $(\eta_s + \text{Swash}/2)$. The coefficient of correlation was 1.1, which is the slope of the best-fit line to the data sets. Stockdon et al (2006) decided that they would plot their regression graphs with the dimensional parameterisation $R_2$ rather than using the non-dimensional ratio. According to their findings, this could introduce large errors in regression statistics.

$$R_2 = 1.1(\eta_{s,max} + \frac{S}{2})$$

EQUATION 2.17 - BASIC RUN UP EQUATION (STOCKDON ET AL 2006)

Where,

$$S = \text{Swash} = \sqrt{(S_{inc})^2 + (S_{IG})^2}$$

$$\eta_{s,max} = \text{Maximum setup elevation above SWL}$$

$S_{inc}$ represents the swash heights of all incident-frequency-band-waves ($f_0 > 0.05$ Hz) and $S_{IG}$ the swash heights for all infragravity-frequency-band-waves ($f_0 < 0.05$ Hz). The above two swash excursions were modelled independently from one another. For incident swash levels, the beach face slope, offshore wave height and offshore wave length were used which resulted in a square-correlation ($\rho$) of 0.44. The square correlation was attained from plotting the left side of the equation against the right side, applicable to Equations 2.18 and 2.19. A low value means the data are more scattered. For the infragravity frequency bands, the correlation was identical, but by removing the beach face slope, the correlation of the data improved significantly to $\rho = 0.65$. This means that waves with periods of longer than 20 seconds are not influenced by the beach face slope at all. These equations are defined below:

$$S_{inc} = 0.75\sqrt{H_0 \cdot L_0} \cdot \beta_f$$

$$S_{IG} = \sqrt{H_0} \cdot L_0 \cdot 0.004$$

EQUATION 2.18 - SWASH HEIGHTS FOR INCIDENT AND INFRAGRAVITY WAVES (STOCKDON ET AL 2006)

Investigation was done on two different wave heights, $H_0$ and $H_b$, to find a proper choice for the total swash excursion equations. When the swash level was parameterised, using the deep water wave height ($H_0$) the regression achieved was 0.46. When the breaking wave height ($H_b$) was used, it showed no improvement and the regression stayed at 0.46. Thus, Stockdon et al (2006) concluded that any of the two wave heights could be used for wave run up.

They also investigated the effect of different slopes used in the swash parameterisation, the surf zone slope ($\beta_{sz}$) or the beach face slope ($\beta_f$). The surf zone slope is defined as the slope between the position of maximum setup and the location of wave breaking (Stockdon et al 2006). The beach face
slope is defined as the average slope around maximum setup, over a region of plus/minus two times the standard deviation of the continuous water level (Stockdon et al 2006). $\beta_{sz}$ was used in the swash formulation on the entire data set and gave a very low regression value of only 0.01 compared to the 0.68 achieved by using $B_c$. They concluded that the beach face slope had a bigger influence on the swash process than the surf zone slope, thus the surf zone slope should not be used.

Wave set up (Equation 2.19) was best parameterised using the beach face slope, offshore wave height and wave length. The formula provided an overall correlation of 0.48. When the setup data were grouped in tidal bands, it was found that for high and mid tides, the correlation increases to 0.52, but that for low tides, it decreases significantly to 0.29. This supports Holman and Sallenger’s (1982) findings that the bathymetry plays a larger role in setup when the sea is at low tide. Even by altering the beach slope parameter between the beach face and surf zone slope, no improvement in correlation was found. Stockdon et al (2006) also experimented with the use of the whole surf zone slope in the formula but this did not prove to have an impact on the results. In addition, for waves with Iribarren numbers of 0.3 or smaller, the setup was best formulated by only using offshore wave conditions and not the beach slope in the model. This gave a regression value of 0.68.

The final model for 2% wave run up was proposed as follows:

$$R_2 = 1.1 \left[ 0.35 \beta_f \sqrt{H_0L_0} + 0.5 \sqrt{H_0L_0(0.563\beta_f^2 + 0.004)} \right], \quad 0.1 < \zeta_0 < 2.2$$

EQUATION 2. 20 - WAVE RUN UP (STOCKDON ET AL 2006)
the run up values with an average of 17cm.

Physical model tests were performed by Roberts, Wang, & Kraus (2007) in a ‘SUPERTANK’ flume to find the upper limits of beach run up, in response to a change in water level. A total of 30 runs were performed, which included erosion and accretion wave conditions, with regular and irregular seas. Erosion waves depleted the beach of sand above the mean water line and accretion waves built the beach up above the mean water line. Tests were performed on a movable, sandy sea bed, which was surveyed before and after every test condition. The slope of the beach was constructed with the Dean (1977) equilibrium beach profile equation, and it was only built once. All subsequent tests were done on the changed profile from the previous test. Figure 2.11 shows how the beach profile of a selected test changed over time.

![Figure 2.8 - Beach Profile Change (Roberts, Wang, & Kraus 2007)](image)

It was found that the beach similarity parameter does not influence the wave run up at all, and they proposed a direct relationship between beach run up and the significant breaking wave height, Equation 2.21. Initial comparisons showed that maximum run up equals to 0.94 times the breaking wave height, but to be more conservative, the value was increased to 1.0. Tests were only done on wave heights between 0.4 and 1.2m. The equation was published as follows:

\[
R_{\text{max}} = 1.0 \ H_b
\]

\[
R_{\text{max}} = \text{Maximum beach run up}
\]

\[
H_b = \text{Breaking wave height}
\]

EQUATION 2.21 - WAVE RUN UP (ROBERTS ET AL 2007)
Mather et al (2011) proposed a new beach run up formula that was derived from common beach attributes, with interconnected relationships to one another. This relationship involved the beach sand grain size \( (d_{50}) \). They state that wave run up is directly proportional to the beach face slope, \( \beta_f \), and this slope is proportional to the sand grain size of \( d_{50} \). Thus, \( \beta_f \) is a function of \( d_{50} \). In turn, the offshore equilibrium beach profile, published by Dean (1977), provides an estimation of the sea bed profile (Equation 2.22). This equation is governed by the shape parameter, \( A \), which is also directly proportional to the sediment fall velocity and the sediment grain size, \( d_{50} \). These two relationships suggest that the beach equilibrium profile and the beach face slope are connected by a common factor, \( d_{50} \).

\[
h = Ax^{2/3}
\]

**EQUATION 2. 22 - DEAN’S EQUILIBRIUM BEACH PROFILE (DEAN 1977)**

With this finding, the authors suggested that the run up can be predicted to a specified point on the seabed and then correlated with the offshore beach profile, to deliver the run up value that would occur on the beach, Equation 2.23.

\[
\frac{R}{H_0} \sim \left(\frac{x_h}{h}\right)^p
\]

**EQUATION 2. 23 - FORMULA DERIVED FROM \( d_{50} \) RELATIONSHIP (MATHER ET AL 2011)**

\( p = \) shape parameter  
\( x_h = \) Distance offshore to closure depth, measured from SWL intersecting beach  
\( h = \) Closure depth

To test this theory, they compared normalised maximum run up \( (R_{\text{max}}/H_0) \) against the offshore distance \( (x_h) \) to the -15m depth contour, taken as the depth of closure. This could be done because if \( (h) \) in Equation 2.23 is a constant it will result in only one variable being present on the right hand side of the equation, being \( x_h \). The run up information, gathered from a KwaZulu Natal (KZN) storm event, was plotted and is shown in Figure 2.9. It was found that Equation 2.24 best fits the plotted values with a shape parameter \( (p) \) of 2/3.
The run up value ($R_u$), in Figure 2.9, was taken from visible debris lines on the beach or scour points located at the vegetation or dune edges. The normalised beach slope, Figure 2.10, is defined from the closure depth to the location where the SWL meets the beach. The final formula proposed is as follows:

$$\frac{R}{H_0} = C \left(\frac{h}{x_R}\right)^{\frac{2}{3}}$$

EQUATION 2.24 - WAVE RUNUP (MATHER ET AL 2011)

Where,

- $R = Maximum \ run \ up \ level \ above \ still \ water \ line$
- $C = Dimensionless \ coefficient$
- $x_R = Distance \ offshore \ to \ closure \ depth$
- $h = Closure \ depth \ (i.e. \ 15 \ m \ water \ depth)$
Values for the dimensionless constant $C$ were categorised for three different coastline types: (1) open coastline, (2) large embayment and (3) small embayment. The difference between the large and small embayment is that the large embayment has a distance of 40 km between its headlands and the small embayment a distance of 3 km. The following table was drawn up to provide a range for the model coefficient $C$ (Mather et al 2011).

**TABLE 2.1 - $C$ CONSTANT VALUES (MATHER ET AL 2011)**

<table>
<thead>
<tr>
<th>Coastline Type</th>
<th>Upper Bound</th>
<th>Median</th>
<th>Lower Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open coast</td>
<td>10</td>
<td>7.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Large embayment</td>
<td>10</td>
<td>5.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Small embayment</td>
<td>10</td>
<td>4.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

From the table above, it can be concluded that for all coastline types the upper bound $C$ value is identical. Thus, when predicting the most extreme wave run up, the coastline type does not have an influence in the calculation. The same accounts for the lower bounds. The only difference is seen in the median values for the three different coastline types.

This model was tested against the extreme storm event on the KZN coastline that happened in 2007. **Figure 2.11.** The storm conditions resulted in a $H_0$ of around 8.5m. Comparisons between the measured run up lines and the result from **Equation 2.24** were made. The results showed that the model predicted wave run up, within a horizontal distance of 1m, for 52% of the 1000 sample points.
On average, the horizontal position was over-predicted by 1.6m. The reason behind the over-prediction was that the beach scarp used for indicating the maximum run up location, may have moved further inland because of a longer storm duration.

A revision study, performed by De la Pena et al (2012) was done on most of the previous mentioned run up formula, which are currently being used by coastal engineers. Their objective was to compare physical model results against these chosen equations and thereafter propose a new run up formula. The physical model tests were done in a 2D flume with a movable bed. The flume was sectioned in half, with one side housing a sand diameter of 0.7mm and the other 0.12mm. All measurements were done with capacitive probes and three different beach face slopes were used: 1/20, 1/30 and 1/50, which was a continuous slope. The significant wave heights ranged from 0.5m – 4m and peak periods of between 4 and 14 seconds. The first attribute that the authors investigated was the effect of the sand grain size on the run up measurements. They found that a change in the grain size does not affect the run up measurements at all. The result shown in Figure 2.12 compares the run up measurement between sand grain sizes of 0.7mm and 0.12mm.

Thereafter, they compared measured readings to published formulae. These formulae were first
broken into two distinctive groups. The first group grouped the 2% exceedance wave run up as a function of the Iribarren number \( R_2 (\zeta_0) \) and the second group as a function of the wave height and wave length \( R_2(H_0, L_0) \). Comparisons were compiled per wave run up group. For the first group, *Figure 2.13*, Hunt (1959) is the minimal limiting factor and Mase (1984) and Hedges & Mase (2004) are the maximum limits. Holman (1986) formula provided the best fit for the physical model results. For the results of the second group, *Figure 2.14*, the maximum limit came from Nielsen and Hanslow (1991). There was not a clear limiting equation and data were more scattered for this case because of the lack of a beach slope parameter in these equations.

![Figure 2.13 - Wave Run Up Comparisons Group 1 (De La Pena et al. 2012)](image)
Furthermore, De la Pena et al (2012) published two new equations for 2% exceeded wave run up, which they found fit their physical model data far better than the previously published formulae, *Equation 2.25*. The results increased significantly in comparison to *Figure 2.13 and 2.14*, using the same data, and the comparison is given in *Figure 2.15*. In *Figure 2.15*, the normalised wave run up is plotted against the product of the beach slope parameter and Iribarren number.

\[ \frac{R_2}{H_{mo}} = 4\beta_f 0.3\zeta_0 \]  
\[ R_2 = 4\beta_f 1.3 \sqrt{H_0L_0} \]

*Equation 2.25 - Wave run for two different cases (De la Pena et al 2012)*
Researchers at HR Wallingford also tried to design a maximum run up formula for beaches on the South East coast of England comprising a sand and shingle mixture (Polidoro, Dombusch & Pullen 2013). When they reviewed some of the formulae published by previous authors, like Stockdon et.al (2006), they observed a large scatter for high run up values (Figure 2.16). The filled black dots represent predicted wave run up using a formulae from Powell (1990) and the empty dots are predicted wave run up from Stockdon et al (2006).

Three hundred and eighty wave run up elevation points were used from a single site (Worthing), measured by different features i.e. the border between wet and dry sand, the line between smooth and disturbed beach sand, the line of sea weed washed ashore or a berm feature on the beach. Run up measurements were taken directly after a wave event or within 24h after such an event had occurred. Wave data and tidal data acquired from offshore wave and tidal gauges were split into 30 min intervals for simplification.
Polidoro et al (2013) argued that when a sea state comprises of wind and swell waves, spectral wave parameters should be used to define the maximum wave run up because the sea state would include two peak frequencies with a possibility of even two peak wave directions (Polidoro et al 2013). For the correct energy spectral shape to be defined, Polidoro et al. (2013) used power spectral density files to calculate the peakedness parameter ($Q_p$) to use in the formula. This factor was directly proportional to the spectral shape parameter ($\gamma$) which varied from 3.3 for a JONSWAP, 1.0 for Pierson-Moskowitz and 7.0 for short fetches sea states. In using the information above, as well as a strong belief that set up is influenced directly by the Iribarren number, they published the following formula:

$$R_{u2\%} = 1.04 H_{m0} \left(\frac{T_{m-1,0}}{T_{m0,2}}\right)^{0.5} \xi_{m-1,0}^{0.5} \exp(-Q_p)^{0.5} + (0.095 H_{m0}^{0.5} L_{m-1,0}^{0.5})$$

**EQUATION 2. 26 - 2\% EXCEEDANCE RUN UP FORMULA (POLIDORO ET AL 2013)**

- $H_{m0} = \text{Spectral wave height}$
- $T_{m-1,0} = \text{Spectral mean wave period}$
- $T_{m0,2} = \text{Mean wave period}$
- $\xi_{m-1,0} = \text{Iribarren number based on mean spectral wave period}$
- $L_{m-1,0} = \text{Spectral mean wave length}$
When the measured values were plotted against Equation 2.26, Figure 2.17, a big improvement is seen for higher values of run up. Comparing Figure 2.16 against Figure 2.17, a clear difference is seen for run up values of above four meters.

![Figure 2.17 - Predicted Wave Run Up Against Measured Run Up, Improvement Against Figure 2.12 (Polidoro et al. 2013)](image)

### 2.7 Numerical models

There are a number of numerical models in the field of coastal engineering that can assist one when analysing the wave processes inside the surf zone. There are two types of numerical models available. The one applies time-phase-resolving-analysis (frequency dispersion models) and the other applies phase-averaging-analysis (non-frequency dispersion models). Phase resolving means that for a wave field, the phase for each component is retained to resolve the displacement of the sea surface with Boussinesq equations, i.e. the surface profile is given as an output for time intervals. For phase averaging models, the sea surface is described by means of a spectral energy density function and the output is given for an instance and is not time dependant. The reasons for two very different models are based on the application of each and the time constraints associated with a project. One would use a phase averaging model to get a quick result that is less demanding and has a large area. For smaller areas like harbours where wave motions are of importance, a phase resolving model would be better suited because a smaller time step and grid size can be used to resolve time domain wave information.

Because wave run up occurs in the surf zone and within a small area compared to the bigger ocean, phase resolving models should be preferably used. They can provide detailed wave heights along the length of the surf zone, the water level and the run up height at any given time step. *Figure 2.18* is a
good representation of the output given by such a model.

One such numerical model is MIKE21-BW developed by DHI in Denmark. This is a Boussinesq wave model working in 1D or 2D space coordinates. The model uses flux-formulation with improved frequency dispersion characteristics to solve the Boussinesq type equations (DHI (a) 2007). These enhanced equations simplify the propagation of nonlinear waves from deep to shallow water. For wave run up modelling, a 1D model would suffice. This 1D model uses a different numerical interpolation technique than the 2D model. This technique makes use of a three step Taylor-Galerkin scheme compared to a 2D time-centred implicit scheme. Possible types of outputs for these two models are deterministic parameters, phase averaged parameters, wave disturbance parameters, hot start parameters and moving shoreline parameters. The one parameter of interest is the moving shoreline output because this provides the horizontal run up and vertical run up. (DHI (a) 2007)

Another model is SWASH (Simulating WAves till SHore) developed by TU Delft. This is a free surface, terrain following wave flow model specifically written to analyse the coastal region up to the shoreline using a phase averaging energy balance. This is a general-purpose numerical model for the simulation free surface, non-hydrostatic flows in one, two or three dimensions. It is governed by the nonlinear shallow water equations and includes non-hydrostatic pressures. SWASH provides a good basis for simulating, “wave transformation in both surf and swash zones due to nonlinear wave-wave interactions, interaction of waves with currents, interaction of waves with structures, wave damping due to vegetation, wave breaking and run up at the shoreline.” (The SWASH Team 2014)

SWASH was built from the well-known SWAN code and thus has the same numerical stability and
robustness as SWAN. It also delivers accurate results in an acceptable turnaround time. The aim was to design a model which could model surface waves and shallow water flows with a wide range of time and space scales in complex environments, for example, waves approaching a beach or waves penetrating into a harbour. This model also includes Coriolis and meteorological forces for the inclusion of tidal waves and storm surges (The SWASH Team 2014).

SWASH is based on an explicit, second order finite difference method whereby mass and momentum are strictly conserved and thus it is not a Boussinesq-type wave model. It can be run in depth-averaged mode or multi-layered mode where the water column is divided into different layers of known thickness. For the multi-layer approach, a typical Boussinesq model uses one layer and increases the number of derivatives to improve its frequency dispersion, whereas SWASH just increases the number of layers (The SWASH Team 2014). The model includes the following physical phenomena (only a handful are mentioned): wave propagation, shoaling, refraction, wave breaking, run up and run down, moving shoreline and wave current interaction. Possible outputs from a SWASH model are: water surface elevation, significant wave height, wave induced setup and maximum horizontal run up. (The SWASH Team 2014). Figure 2.19 is a representation of a 2D wave run up model which provides run up values with respect to time.

![Figure 2.19 - SWASH Representation for 2D Run Up (Luger 2013)]
CHAPTER 3: PHYSICAL MODEL TEST

3.1 Introduction
The physical model test was performed in a 2D glass flume located at the University of Stellenbosch. The model comprised of a simple straight concrete slope which represented a beach with a 1/24 beach face slope. The test series included 10 regular wave tests with wave heights ranging between 1.5 and 4.1m and wave periods of between 10s and 13s. Wave run up was measured with a video recording and analysed afterwards by the student.

3.2 Test facility
The physical model tests were performed in the Hydraulic Laboratory, Civil Engineering Department, at Stellenbosch University. The laboratory has four glass flumes, ranging in widths from 0.6m to 1.0m and in length from 20m to 40m. There is also one large concrete flume with a width of 2m and a length of 60m. Both the concrete flume and glass flume has a built in wave paddle, to reproduce real sea conditions. The student used the glass flume with dimensions of 1.0m x 1.2m x 40m to study the wave run up on a simple slope.

The 2D glass flume has a wave paddle installed at one end. The paddle is a stainless steel piston-type paddle, which moves in a longitudinal direction, to generate regular or irregular sea states. It is a single paddle setup with dimensions of 1.0m x 1.2m. The wave paddle contains two fixed dynamic
wave absorption probes that use an absorption technique to compensate against the waves that are reflected from the structures down the channel back towards the paddle. The maximum model wave height that the paddle can produce is 0.4 m wave at 0.8m water depth.

The wave’s characteristics were measured with four wave probes, each being 750 mm long, placed at specified distances along the channel. These probes are all resistance wave probes, which are more sensitive to a change in environment. Every probe measured the wave height by sending a voltage signal, with a specified frequency, from the probe-box to the wave-probe. This provided the wave height and wave period measurements in prototype scale, at the end of each wave test. The accompanying wave data acquisition solution provided the wave characteristics with the zero up crossing technique as well as providing some statistical wave height outputs, i.e. $H_s$, $H_{1/10}$ and $H_{\text{max}}$.

3.3 Model set-up

The author used an already built concrete slope, available from a previous study, which represented a typical averaged beach face slope, from the coast of South Africa for his tests. This slope was calculated by averaging the measurements from five different locations, stretching from Saldanha to Richards Bay (Schoonees 2014). The averaged slope derived to 1/28, measured between -1m MSL and +1m MSL. The different slope values are provided in Table 3.1.

<table>
<thead>
<tr>
<th>Location</th>
<th>Beach face Slope (-1m MSL to +1m MSL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saldanha</td>
<td>1 : 11.5</td>
</tr>
<tr>
<td>False Bay</td>
<td>1 : 16.5</td>
</tr>
<tr>
<td>Glentana</td>
<td>1 : 32</td>
</tr>
<tr>
<td>Table Bay</td>
<td>1 : 41.5</td>
</tr>
<tr>
<td>Richards Bay</td>
<td>1 : 42</td>
</tr>
</tbody>
</table>

Before any tests were conducted, the upper concrete slope was measured again and the upper section displayed a gradient of 1/24. This was acceptable with the available resources. This survey exposed the skewness and non-uniformity of the slope. It was found that the slope changes four times over the 10 metre floor length. The first incline (1/2.5) is the transitional slope taking the wave from deep water directly into transitional water. Thereafter, the slope flattens out to 1/21 and then it becomes completely flat at 1/119. After this, the area of importance had a slope of 1/24 where the waves were expected to break and run up. Figure 3.2 presents a sectional view of the 2D glass flume from the side.
3.4 Run up measurement technique

Different experimental techniques were used to measure the wave run up. The initial idea was to use bathymetry lines that were made up by connecting nails with nylon string, with the same elevations, to one another. The second idea was laying a probe directly on the floor to measure the wave run up, which was first tried by Mase & Iwagaki (1984). The final idea was to record the run up with a video camera and to analyse the video footage after every completed test. After failed attempts with the first two techniques, the last technique was used. A 2m by 40mm white plastic-paint strip was painted on the floor and elevation lines were drawn on top, at 1mm elevation increments. The video camera was positioned at the side, focussing on the marked strip. The maximum run up elevation that could be measured, was 25 mm above SWL, model scale.

3.5 Probe setup

Four probes were used for all tests. They were spaced from one another at a prescribed distance, as recommended by the method of (Mansard & Funke 1980). This method only provided formulae for the spacing of three probes, but the employed data acquisition software required four probes for the
reflection calculations. Thus, the fourth probe was placed three metres from the first probe along the flume. The first three probes were placed at a distance stated from one another, in order to easily measure all the wave periods within the test schedule. The flume was then filled with municipal water to a depth of 360 mm, which was the maximum water level for all tests. When the water level was increased above 360 mm, the wave run up would have spilled past the edge of the built slope, thus it was kept to 360 mm.

![FIGURE 3. 4 - PROBE SPACINGS USED](image1)

![FIGURE 3. 5 - IMAGE OF FIRST THREE RESISTANCES WAVE PROBES](image2)
3.6 Model scale

The scaling used for the physical model was the Froude criterion of similitude. This similitude assumes that the inertial forces ($F_i$) and the gravitational forces ($F_g$) dominate in a physical model. All the other forces like viscous, surface tension, elastic compression and pressure forces are accepted as being close to zero. Scale laws in the Froude similitude are related to the Froude number which is equal to $F_i / F_g$ (Hughes 1995). Equating the prototype Froude number with the model Froude number, the model scale is attained. This model scale is better identified as $N_L$. Froude similitude describes all of the necessary scaling parameters in terms of this model scale, $N_L$, refer to Table 3.2.

Several model scales ranging from 1:50 to 1:100 were experimented with. After testing all the waves in the test schedule, it was found that the best suited scale was 1:100. The large scale eliminated early wave breaking, which was a big concern for all the other model scales. One limitation in the physical model was the fixed concrete floor that could not be altered. The flume could only be filled to a certain water level which directly influenced the maximum wave height that could be generated. Applying the scale of 1:100 to the Froude similitude, the following table provides all the necessary values with which the model dimension must be multiplied.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prototype Value / Model Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>100</td>
</tr>
<tr>
<td>Time</td>
<td>$\sqrt{100} = 10$</td>
</tr>
</tbody>
</table>

3.7 Test procedure

At the start of every test sequence, the wavemaker was programmed to generate regular waves for a period of three minutes. This was done since the water in the flume becomes stagnant overnight or if left untouched for a few days. When stratification happens, the water temperature differs from the top layer to the bottom layer; as a result, the probes cannot be calibrated. Being resistance probes, sensitivity to temperature played a big role. Thus, by producing waves, the water is adequately mixed for calibration. After the three minute mixing procedure, the physical model was left alone for the water level to settle and any energy to dissipate, from long or short waves.

After the water level had become completely still, verified by a visual inspection as well as inspecting the absorption probe readings from the wave box, the probes were calibrated. The probes were calibrated twice a day to compromise for any change in temperature within the laboratory. The aim of the probe calibration process was to acquire a linear relationship between different still water depths.
The process of calibration was performed as follows: The probes were moved upwards and downwards, with a reading being taken at each height, for a total of three different elevations. These three values were plotted on a graph and the regression coefficient was verified. If the regression for all four probes was higher than 0.998, the calibration was saved and testing could commence.

After the calibration was completed, the wave maker was then activated, running the specific wave condition with the associated gain setting. These wave files were programmed beforehand as a run file, i.e. the wave maker could be started but it would stop automatically after a set time. As waves were generated, measurements were taken by the probes and a video recording was started to measure the run up. The test schedule was performed twice - first to calibrate the physical model and then to measure the run up values for every test condition. For the model calibration process, the wave reflection was measured and wave software provided a bulk reflection coefficient ($K_r$) for every test condition. This coefficient had an influence on the deviation of the incident wave height ($H_i$) and the gain setting used with the wavemaker.

A method designed by (Mansard & Funke 1980) was applied to distinguish the two different wave heights as only the incident wave height is important for the run up measurements. Mansard & Funke (1980) found a relationship linking the incident wave height ($H_i$) and reflected wave height ($H_R$) to the spectral significant wave height, $H_{m0}$. The relationship is represented as follows:

$$H_{m0} = \sqrt{H_i^2 + H_R^2}$$

**EQUATION 3.1 - SIGNIFICANT WAVE HEIGHT COMPOSITION (MANSARD & FUNKE 1980)**

This equation can be rewritten by introducing a relationship between the incident wave height, the reflected wave height and the bulk reflection coefficient. The reflected wave height equals the incident wave height multiplied by the bulk reflection coefficient given by Equation 3.2. Combining Equation 3.1 and 3.2, results in the final relationship between the incident wave height, the deep-water significant wave height and the bulk reflection coefficient, Equation 3.3.

$$H_r = H_i * K_r$$

**EQUATION 3.2 - RELATIONSHIP BETWEEN $H_r$ AND $H_i$ (MANSARD & FUNKE 1980)**

$$\therefore H_i = H_{m0} / \sqrt{1 + K_r^2}$$

**EQUATION 3.3 - INCIDENT WAVE HEIGHT FORMULA (MANSARD & FUNKE 1980)**
3.8 Test schedule and wave conditions

The test schedule consisted of 10 different wave conditions, all being regular waves, with different combinations of wave heights and wave periods. When testing started, it was found that four wave conditions (#4, #12, #13 and #14) delivered incompatible results because of early wave breaking and measuring capabilities of the wave probes, and these tests were rejected. The full test schedule is provided in Table 3.3. To simplify the run up measurements, it was decided to only perform the tests with regular waves.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Wave Height [m]</th>
<th>Wave Period [s]</th>
<th>Wave Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>10</td>
<td>Regular</td>
</tr>
<tr>
<td>2</td>
<td>2.1</td>
<td>11</td>
<td>Regular</td>
</tr>
<tr>
<td>3</td>
<td>2.2</td>
<td>12</td>
<td>Regular</td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
<td>10</td>
<td>Regular</td>
</tr>
<tr>
<td>6</td>
<td>2.8</td>
<td>12</td>
<td>Regular</td>
</tr>
<tr>
<td>7</td>
<td>3.0</td>
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<td>Regular</td>
</tr>
<tr>
<td>8</td>
<td>3.4</td>
<td>11</td>
<td>Regular</td>
</tr>
<tr>
<td>9</td>
<td>3.7</td>
<td>13</td>
<td>Regular</td>
</tr>
<tr>
<td>10</td>
<td>3.9</td>
<td>11</td>
<td>Regular</td>
</tr>
<tr>
<td>11</td>
<td>4.1</td>
<td>12</td>
<td>Regular</td>
</tr>
</tbody>
</table>

3.9 Test duration

The test duration was identical for every test because all tests were regular wave tests, thus it could be assumed that every wave generated by the wave maker was constant in height and period. Waves were produced for 10 minutes without any measuring, allowing all initial flume oscillations to dissipate. After this fixed time, the wave probes were initialised to measure the wave heights and periods for a duration of three minutes. Within this three minutes of data acquisition, a video sample was recorded for the run up measurements. Refer to Figure 3.6.
3.10 Data acquisition

The data acquisition comprised of three different tasks. The first task was acquiring the significant wave height and peak wave period; secondly, the bulk reflection coefficient was calculated to produce the incident wave height and thirdly the wave run up value was taken from the video recording. The wave heights and periods were recorded for 180 seconds, as mentioned with Figure 3.6.

3.11 Sensitivity tests

A sensitivity analysis was done to assess the influence of the wave period on run up measurements. Test 11 was selected from Table 3.2 for this analysis. The wave height was kept constant at 4.1m and the run up was measured for wave periods of 10s, 11s, 12s and 13s. The results are provided in section 3.12 and will be further discussed in Chapter 6.

3.12 Results

The total number of physical model experiments performed tallied 45 tests which include the wave calibration tests (23), wave run up tests (19) and sensitivity tests (3).

3.12.1 Wave run up results

<table>
<thead>
<tr>
<th>Test Name</th>
<th>R01_003</th>
<th>R02_001</th>
<th>R03_002</th>
<th>R05_002</th>
<th>R06_001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave Height [m]</td>
<td>1.5</td>
<td>2.1</td>
<td>2.2</td>
<td>2.5</td>
<td>2.8</td>
</tr>
<tr>
<td>Wave Period [s]</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Water depth at wave paddle [m]</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Gain Parameter</td>
<td>1.02</td>
<td>0.955</td>
<td>0.98</td>
<td>0.99</td>
<td>0.95</td>
</tr>
<tr>
<td>$H_{m0}$ probes [m]</td>
<td>1.61</td>
<td>2.18</td>
<td>2.37</td>
<td>2.65</td>
<td>2.99</td>
</tr>
<tr>
<td>$K_r$</td>
<td>0.36</td>
<td>0.26</td>
<td>0.39</td>
<td>0.35</td>
<td>0.36</td>
</tr>
<tr>
<td>$H_{Incident}$ [m]</td>
<td>1.52</td>
<td>2.11</td>
<td>2.20</td>
<td>2.50</td>
<td>2.81</td>
</tr>
</tbody>
</table>
TABLE 3.5 - WAVE RUN UP RESULTS 2/2

<table>
<thead>
<tr>
<th>Test Name</th>
<th>R07_001</th>
<th>R08_001</th>
<th>R09_002</th>
<th>R10_001</th>
<th>R11_001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave Height [m]</td>
<td>3</td>
<td>3.4</td>
<td>3.7</td>
<td>3.9</td>
<td>4.1</td>
</tr>
<tr>
<td>Wave Period [s]</td>
<td>11</td>
<td>11</td>
<td>13</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Water depth at wave paddle [m]</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Gain Parameter</td>
<td>0.93</td>
<td>0.91</td>
<td>0.9</td>
<td>0.92</td>
<td>0.96</td>
</tr>
<tr>
<td>$H_{m0}$ probes [m]</td>
<td>3.17</td>
<td>3.51</td>
<td>3.89</td>
<td>4.09</td>
<td>4.35</td>
</tr>
<tr>
<td>$K_r$</td>
<td>0.303</td>
<td>0.27</td>
<td>0.32</td>
<td>0.255</td>
<td>0.372</td>
</tr>
<tr>
<td>$H_{incident}$ [m]</td>
<td>3.03</td>
<td>3.38</td>
<td>3.70</td>
<td>3.96</td>
<td>4.08</td>
</tr>
<tr>
<td>Wave run up [m]</td>
<td>1.2</td>
<td>1.5</td>
<td>1.7</td>
<td>1.5</td>
<td>1.7</td>
</tr>
</tbody>
</table>

3.12.2 Sensitivity results

TABLE 3.6 - SENSITIVITY TEST RESULTS

<table>
<thead>
<tr>
<th>Test Name</th>
<th>S01</th>
<th>S02</th>
<th>S03</th>
<th>S04</th>
</tr>
</thead>
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<tr>
<td>Wave Height [m]</td>
<td>4.1</td>
<td>4.1</td>
<td>4.1</td>
<td>4.1</td>
</tr>
<tr>
<td>Wave Period [s]</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>Water depth at wave paddle [m]</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Gain Parameter</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$H_{m0}$ probes [m]</td>
<td>4.49</td>
<td>4.61</td>
<td>4.35</td>
<td>-</td>
</tr>
<tr>
<td>$K_r$</td>
<td>0.372</td>
<td>0.372</td>
<td>0.372</td>
<td>-</td>
</tr>
<tr>
<td>$H_{incident}$ [m]</td>
<td>4.21</td>
<td>4.32</td>
<td>4.08</td>
<td>-</td>
</tr>
<tr>
<td>Wave run up [m]</td>
<td>1.45</td>
<td>1.6</td>
<td>1.7</td>
<td>-</td>
</tr>
</tbody>
</table>
CHAPTER 4: BEACH TESTS

4.1 Introduction

The author visited the location of Long beach, Noordhoek, South Africa, to acquire run up field data to use for this study. This site was chosen because of the offshore CSIR wave buoy which is located south west of Long Beach. Figure 4.1 illustrates where Long Beach is located in South Africa, and includes an enlarged aerial photograph of the area.

![Figure 4.1 - Long Beach's location in South Africa (Google Earth 2014)](image_url)

The authors objective for the beach test’s was to measure real time run up values as a wave breaks on the beach, which would then be correlated to the offshore wave height that was measured on that day by the CSIR wave rider. If the run up value and the significant wave height is known, the results can be plotted and compared to the published run up formulae. Measurements were taken on the weekend of 13th and 14th of September 2014. Data for the 13th was discarded due to poor planning and a faulty GPS receiver. This chapter describes the technique that was used to measure the run up on Long Beach.

4.2 Site details

Noordhoek beach is an 8km stretch of unspoiled beach on the Cape Peninsula, facing west. The northernmost point has huge boulders scattered around, has an estuary opening and is the start of
Chapman’s Peak Drive. The southernmost point is Kommetjie Lighthouse, which is a famous spot among the Capetonian surfers (refer to Figure 4.3). Behind the beach, one can find a National Park, which houses protected wetlands that vary in size throughout the seasons. The width of the beach is more or less dependent on the wetlands behind it. In the wet season, when there is a large run off from the surrounding mountains, the width of the beach is close to 100m. In the summer season, the width increases to over 350m in some areas. Refer to Figure 4.3 and 4.4.
4.3 Site survey

A location was chosen on the beach by doing a visual inspection to decide where the best section of beach would be for run up measurements. The criteria were: the area should be close to the public area, for safety reasons, there should be no rock outcrops offshore of the location that could interfere with the wave heights and lastly, the location should be free of rip currents. Once the area was located (as shown in Figure 4.4), it was marked out to a length of 50 m (along the beach) and a width (dimension towards the sea) of 42 m. The total area spanned 2100 m$^2$. This area was divided into grid points, for easier surveying, where five meter spacing’s were used for areas with minimal undulations and a denser two metre spacing was used for areas that have large undulations i.e. cusps near the berm of the beach. Figure 4.5 shows the layout of the surveyed points. The points in the left of Figure 4.5 represent the area below the berm, going into the sea. The dense area in the middle represents the berm and to the right of this is the top of the beach furthest away from the sea.

The survey was started two hours before the estimated low tide. On foot, the student conducted a survey in lines running along the beach, from the highest point (the location of the base station) to the lowest possible point that could be reached in the surf zone without getting the instruments wet from the wave action. The survey apparatus used was a Trimble TSC3 GPS that was set up in VRS mode (real time corrections made to land level data) working in the WG19 spatial coordinate. The accuracy was set to 30mm for the vertical coordinate and 50mm for the horizontal coordinate. The Trimble GPS could be set more accurately but this, in turn, would increase the time to store a reading. A total of 214 surveyed points were taken for the 2100 m$^2$ area. These points were exported and analysed with Surfer Demo 12 which is a contouring surface mapping application released by Golden Software.
A 3D contour map output given by Surfer Demo 12 is shown in Figure 4.7. The beach-face slope-section was taken through the centre of Figure 4.7, to provide a visual of the beach face slope. This visual is given in Figure 4.8, on the following page.
4.4 Site preparation

After the beach survey was completed, run up stakes were hit into the sand to provide visual markers for the wave run up measurements. These stakes were 20mm hollow PVC electrical conduits cut at 800mm lengths. A length of 800mm was chosen to allow for a penetration of 500mm into the sand leaving 300mm above the sand. The initial plan was to hit the markers into the sand at set z-elevation differences between every marker, measured at the top of the stake. This method failed because of the incorrect GPS readings. An improvised method was implemented where the PVC stakes were hit into the sand, at three paces from one another, all to the same depth. This was made possible by creating a indicator line, 300mm from the top of the stake, and hitting the stake into the sand until this marker was just not visible. Figure 4.9 shows two pictures of the PVC stakes being hit into the ground at three paces from one another.
The reason for utilising the stakes was to have visual markers of known elevations above land level datum. This was done by putting the GPS instrument on the top of every stake and taking a measurement. The data was stored and analysed afterwards. Because every stake was individually marked, the tests were conducted by using the marker identities, and were correlated afterwards with the measured elevations.

4.5 Wave run up tests

The plan was to test from low tide to high tide because at low tide more beach area was exposed for surveying and putting out the stakes. Thus, the first test started at 12:30 which was close to the predicted low tide. A total of five tests were performed at 30 min lengths. The procedure for measuring the wave run-up entailed walking to the point to where the wave had run up to, and then taking the measurement with the GPS device. The measurement included the position (x,y) as well as the elevation of the run up (z). The author tried to measure as many run up points as possible within the allocated test time. It is well known that the run up points would differ from one location on the beach to another, thus the author only measured the points where the water line intersected the stake-line. This was done by assuming that the stake-line represented a unit width of the beach, and the run up was measured on this unit width. Figure 4.10 to 4.12 shows some images including the measurement of the run up points by the author.

![Figure 4.10 - Test with notice board and camera](image)
4.6 Visual observations of the run up process on the beach face slope

Spending 8 hours on the beach, for each of the two days, the student spotted three practical effects that causes high wave run up points:

- Wave run up can be broken down into wave setup and swash, as discussed in the literature review. Setup is the momentary increase of the water level above the SWL and swash is the water uprush above this water level. One effect spotted, was regarding setup and swash. When the broken wave propagated up the beach, the water level increased as the front of the wave moved higher up the beach slope (*Figure 4.13 (A)*). It increased only to a certain elevation before subsiding again (*Figure 4.13 (B)*). When this maximum setup elevation was reached, the wave front continued to propagate up the beach, even while the water level...
started going down(Figure 4.13 (C)). Thus, the maximum set up and maximum swash was found to not always occur at the same time.

- The second effect originated from evaluating wave patterns against maximum run up patterns. One cause for maximum run up was found to originate from sets of wave approaching the shore and breaking shortly after one another. This led to a build-up of wave setup on top of one another, which caused the swash of the last wave, to reach a maximum run up elevation. Figure 4.14 shows such an event between 10 and 13 minutes.
• The third effect has to do with the uprush and down rush interaction between wave fronts. Before a maximum run up value has happened, the preceding run up should have been a minimum, *Figure 4.15*. This minimum run up is produced when the down rush of a preceding wave is larger than the uprush of the minimum run up wave, causing a low run up value. The wave following this wave, then achieves a high run up value, because the preceding wave with a minimum run up has a very small down rush. And this small down rush does not significantly interfere with the following waves uprush, causing a maximum run up elevation.
4.7 Beach test results

The following process was performed from information gathered by the author, to find the run up value for every measurement taken. The process is explained by the following bullets:

- The GPS measurements were exported from the Trimble TSC to a comma-separated values file (.csv). This file included the wave run up id, Y-coordinate, X-coordinate and the Z elevation. The Z elevation gave measurements relative to the Land Levelling Datum (LLD).
- Tide information was received from SANHO (Figure 5.7) for the day on which testing took place. This information was broken down into 30 min sections and then evenly distributed for each test, between the start- and end time of every test. This was done because the tide information is conveyed in 1 minute intervals and the wave run up record is not time dependant. The tide information was correlated to LLD, where a constant factor of 0.843m was deducted from the tide measurements (Feun 2014). This correlated value represents the SWL at the given beach.
- The wave run up can now be calculated as the correlated tide value subtracted from the measured run up elevation. The formula was implemented for every measurement and the answers were tabled and sorted from large too small.
- From this descending wave run up record the following values was retrieved by applying an cumulative probability analysis and general assessment: \( R_{\text{max}} \), \( R_{2\%} \), \( R_{\text{min}} \) and \( R_{\text{ave}} \). These values are provided in Table 4.1

Figure 4.16 illustrates the beach face slopes at the location on Long Beach where the measurements were taken. Five slopes were plotted, each representing a 10m width of beach. The maximum elevation of the beach was at +2.5m LLD and the slope was measured down to just below 0.00 LLD. The average slope was 0.0833 or 1/12, which was double the steepness compared to the physical model bathymetry slope.

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Wave runup T1</th>
<th>Wave runup T2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>30 min</td>
<td>30min</td>
</tr>
<tr>
<td>Total run up points</td>
<td>52</td>
<td>81</td>
</tr>
<tr>
<td>Maximum run up</td>
<td>1.113m</td>
<td>1.442m</td>
</tr>
<tr>
<td>Minimum run up</td>
<td>0.121m</td>
<td>0.010m</td>
</tr>
<tr>
<td>Mean run up</td>
<td>0.590m</td>
<td>0.638m</td>
</tr>
<tr>
<td>2% Run up</td>
<td>1.113m</td>
<td>1.400m</td>
</tr>
<tr>
<td>Test start time</td>
<td>12:36:00</td>
<td>13:40:00</td>
</tr>
<tr>
<td>Test end time</td>
<td>13:06:00</td>
<td>14:10:00</td>
</tr>
</tbody>
</table>
## Table 4.2 - Beach Run Up Results (2/2)

<table>
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<tr>
<th>Test Name</th>
<th>Wave runup T3</th>
<th>Wave runup T4</th>
<th>Wave runup T5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>30 min</td>
<td>30 min</td>
<td>30 min</td>
</tr>
<tr>
<td>Total run up points</td>
<td>78</td>
<td>84</td>
<td>90</td>
</tr>
<tr>
<td>Maximum run up</td>
<td>1.778m</td>
<td>1.606m</td>
<td>1.730m</td>
</tr>
<tr>
<td>Minimum run up</td>
<td>0.190m</td>
<td>0.162m</td>
<td>0.116m</td>
</tr>
<tr>
<td>Mean run up</td>
<td>0.748m</td>
<td>0.802m</td>
<td>0.761m</td>
</tr>
<tr>
<td>2% Run up</td>
<td>1.696m</td>
<td>1.523m</td>
<td>1.512m</td>
</tr>
<tr>
<td>Test start time</td>
<td>14:30:00</td>
<td>15:30:00</td>
<td>16:15:00</td>
</tr>
<tr>
<td>Test end time</td>
<td>15:00:00</td>
<td>16:00:00</td>
<td>16:45:00</td>
</tr>
</tbody>
</table>
Combination of beach slopes measured at 5 locations

FIGURE 4.16 - MULTIPLE CROSS SECTIONS OF SURVEYED BEACH FACE SLOPES (A) AND LOCATIONS (B)
CHAPTER 5: NUMERICAL MODELLING

5.1 Introduction
Two numerical models, a regional- and a local grid model, were utilised on the coastal waters of Long Beach, located just west of Noordhoek in the Western Cape of South Africa. The regional grid model transferred waves from an offshore wave rider (Cape Point wave rider located at 70 water depth) to a depth of -15.9m, in front of Long Beach. The local grid model then used the output from the regional grid model and modelled the waves from -15.9m to -1.0m. These numerical models form a vital part in the beach tests results as discussed in Chapter 4, as the model provides the $H_0$ and $T_p$ values, which are to be used with the results obtained from the beach measurements, to investigate wave run up relationships.

5.2 Numerical model implemented
It was decided to make use of a Mike21 Spectral waves FM numerical model, (DHI (b) 2007). This is a spectral wind-wave model, which can be implemented on a regional scale, while still delivering accurate results. Wind and swell generated waves are allowed to grow, decay and transform from the original boundary to the user’s defined area of interest, delivering a variety of output parameters (DHI (b) 2007). Section 5.3 will describe the model set-up in more detail. Both the regional- and local grid models used the same set up as described in section 5.3.

5.3 Model Setup

5.3.1 Domain
The model domains were set up in three phases. With the first phase, the bathymetry of the coastal area was imported into the model. Bathymetry information was received from the CSIR (Rossouw 2014) to use for this study. The information received comprised the whole Cape Peninsula coastal region, but only the necessary area was selected. Figure 5.1 (A) shows the soundings received from the CSIR, and the area used in shown in Figure 5.1(B).
For the second phase, this bathymetry was trimmed to a local domain area where the necessary boundary conditions were applied. The local area selected was large enough around Long Beach to allow waves to propagate around the necessary rock formation at Kommetjie, just south of Long Beach. The initial plan was to do the placement of the boundaries at the wave rider that was situated at -70m water depth, but problems arose for modelling waves coming from the south. The propagating area was too narrow and short, thus the boundaries were moved further away from shore and into deeper water. After a quick test, it was found to be acceptable to move the boundaries away from the wave rider, further offshore, without a significant loss in wave characteristics.

The last phase involved applying a triangular mesh onto the domain. Because the area was broken down into three individual domains, each domain had its own mesh properties. The first section was located furthest offshore and had a mesh element area of 1,000,000 m². The second section was in shallower water and had a denser element area of 250,000 m². The last section was closest to the beach and had a very dense element area of 10,000 m². The mesh area provides an estimation of the grid that is used inside of the mesh, i.e. 10,000 m² means a grid size of around 100 by 100 metres. A denser mesh provides results that are more accurate, but in turn increases the computational time.

*Figure 5.2*, on the following page, shows a contour map for the chosen domain area. *Figures 5.3* and *5.4* displays how this domain area was meshed, showing the three different mesh sizes.
For the local grid model, the student received soundings from the South African Navy Hydrographic Office (further referred to as SANHO). These soundings were more detailed and included values shallower than -15.9 m, which was the minimum depth from the CSIR soundings. The domain for the local grid model included only one mesh element area.
Figure 5.5 displays the mesh used for the local grid model and Figure 5.6 displays the contour map of the domain.

5.3.2 Time integration

The simulation was run for the 14th of September 2014, from 12:00am to 17:00 the evening to match the fieldwork. The time steps used by the model were 30 min intervals, resulting in 24 time steps in total. Because the fully spectral computational formulation takes significantly longer than the directionally decoupled parametric formulation, the simulation was only run for the interested time-period and not for the whole day.

5.3.3 Basic equations

The model used the fully spectral formulation instead of the directionally decoupled parametric formulation as this simulates the directional-frequency wave action spectrum (DHI (b) 2007) and introduces white capping into the numerical model which can influence the final result. The fully spectral formulation uses a wave action conservation equation where the directional-frequency wave action spectrum is the dependant variable (DHI (b) 2007). When comparing the two spectral formulation results, at a randomly chosen point in the domain, the fully spectral formulation delivered slightly larger results.

The time formulation was set to a quasi-stationary formulation that allows “time to be removed as an independent variable and a steady state solution is calculated at each time step.” (DHI (b) 2007)

5.3.4 Spectral discretization

The minimum frequency was set at 0.0704 Hz with a frequency ratio of 1.1. This was left at the
default value prescribed by Mike21. The total discrete frequencies were 24 in this case. The frequency spectrum ranged from 0.0704 to 0.0901 Hz. The discretization rose was resolved into 36 discrete directions.

5.3.5 Solution technique

The model was run on a quasi-stationary technique. The geographical space discretization was set to a low order, fast algorithm. This discretization makes use of an unstructured mesh, using the cell-centred finite volume method (DHI (b) 2007). This is also a steady state solution implementing the Newton-Raphson iteration method. This was highly recommended by DHI (DHI (b) 2007). The number of iterations allowed was set to 500 with a tolerance of 1e-005. The relaxation factor was left at default which, was 0.1. The smaller the relaxation value, the greater the model’s convergence will be.

5.3.6 Water level conditions

The water level conditions were specified for the model. The data format was set to vary in time but constant in the domain. Water level readings were attained from (SANHO) for the 14th of September 2014. Every minute, a reading was measured with a water level gauge at the port of Simon’s Town and recorded. Figure 5.7 provides the tidal information as received from SANHO. The soft start option was not used, thus the value was set to 0 seconds.

![Tide Data Graph](image-url)  
**FIGURE 5.7 - REGIONAL- AND LOCAL GRID WATER LEVEL READINGS - SIMONS TOWN (SANHO 2014)**
5.3.7 Other parameters

**TABLE 5.1 - NUMERICAL MODEL PARAMETERS**

<table>
<thead>
<tr>
<th>Selections</th>
<th>Option</th>
<th>Selected</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
</tr>
<tr>
<td>Energy transfer</td>
<td>Quadruplet wave interaction</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>Triad Wave interaction</td>
<td>Yes</td>
</tr>
<tr>
<td>Wave breaking</td>
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</tr>
<tr>
<td></td>
<td>Alpha</td>
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<tr>
<td>Bottom Friction</td>
<td>Roughness coefficient</td>
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</tr>
<tr>
<td>White Capping</td>
<td>Cdis</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>DELTAdis</td>
<td>0.5</td>
</tr>
</tbody>
</table>

5.3.8 Boundary conditions

The model domain was specified to include four boundary conditions. The table below lists each boundary name as well as the boundary type selected.

**TABLE 5.2 - REGIONAL- AND LOCAL GRID BOUNDARY CONDITIONS FOR NUMERICAL MODEL**

<table>
<thead>
<tr>
<th>Boundary name</th>
<th>Boundary Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>North (Red)</td>
<td>Lateral boundary</td>
</tr>
<tr>
<td>East (Yellow)</td>
<td>Land boundary</td>
</tr>
<tr>
<td>South (Blue)</td>
<td>Wave parameters (version 1)</td>
</tr>
<tr>
<td>West (Green)</td>
<td>Wave parameters (version 1)</td>
</tr>
</tbody>
</table>

The average wave direction for the western half of the Cape Peninsula is normally from the South West, thus waves were simulated on the western and southern boundaries. The same wave file was used on both boundaries. Each boundary was provided a wave file of version 1 specification, which was set up to be varying in time but constant along the boundary line. This wave data was received from the CSIR a week after the beach tests were performed. *Figures 5.8 and 5.9* displays the domain areas with the associated boundary colours.
For version 1 - wave parameters’ type, a wave direction index should be assigned. A value of 100 was used, as prescribed in Table 6.1 in the Mike21 SW User Manual (DHI (b) 2007). A directional index larger than 10, meant the waves are more swell driven waves. No soft start was used and the interpolation type was selected as linear, in time. The wave heights for the regional grid model ranged between 2.6m and 3.1m, and peak periods between 11.1s and 14.2s. The wave direction was between 230 and 250 degrees. For the local grid model, the wave heights ranged between 1.5m and 2.1m, peak periods between 11.8s and 14.2s, with a wave direction between 249 and 257 degrees.

5.3.9 Outputs

The regional grid model was set up to deliver two output files, with both providing the Hs, Tp, and the mean wave direction. The first output was formatted as a point series, located in the near shore of Long Beach and comprising a line of points on the -15.0 m bathymetry contour. This contour was the closest surveyed point on the regional grid domain. Figure 5.10 shows the point series output.

Two output files were set up for the local grid model. The first being an area output file, of the whole local grid domain. The second being a point series file, Figure 5.11, which included points at 100m distance intervals, measured from the beach to the sea. Only six points were used. This provided information with regard to wave transformation in the surf zone.
5.4 Model results

The only results needed from the numerical model were the wave height and wave period on the beachfront, just outside of the surf zone. This was because all the formulae as described in Chapter 2.6 used wave heights and periods located outside of the surf zone between -10m and -80m water depth.

From the local grid model, the normalised beach slope was calculated, as prescribed by Mather et.al (2011) where the distance offshore was measured, to the -15.0 m contour line, from where the mean tide level intersects the beach. The mean tide level was selected because the SWL will always change over time. Figure 5.12 displays the normalised beach slope with the SANHO soundings. Both these slopes are drawn on the same graph with the blue line representing the detailed near shore slope from the SANHO soundings, measured relative to chart datum. The red line represents the normalised slope as defined by Mather et al (2011), which provides a slope of 0.030. The normalised slope starts at +0.98 m, which is the difference between the mean level and chart datum.

From Figure 5.12 it can be noted that the slope defined by Mather et al (2011) has two possibilities because of the offshore sandbar located just in front of 600 meters, providing two locations to a -15.0 m contour. Using a different slope will influence the final result by some means. This study used the first location for the -15m contour for its Mather slope.
The wave height and wave period data are given in Table 5.3. This information was taken from a point in -15.0m water depth, in front of the location where the beach measurements were performed. The numerical model was set to deliver the output data in 30 min intervals.

### TABLE 5.3 - WAVE DATA INFORMATION AT -15.0M, TRANSFERRED FROM MEASUREMENTS AT -70M

<table>
<thead>
<tr>
<th>Time</th>
<th>$H_0$ [m]</th>
<th>$T_p$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:00</td>
<td>1.5</td>
<td>12.6</td>
</tr>
<tr>
<td>12:30</td>
<td>1.75</td>
<td>13.2</td>
</tr>
<tr>
<td>13:00</td>
<td>2.0</td>
<td>14.2</td>
</tr>
<tr>
<td>13:30</td>
<td>1.9</td>
<td>14.2</td>
</tr>
<tr>
<td>14:00</td>
<td>1.7</td>
<td>13.2</td>
</tr>
<tr>
<td>14:30</td>
<td>1.8</td>
<td>13.2</td>
</tr>
<tr>
<td>15:00</td>
<td>1.9</td>
<td>13.1</td>
</tr>
<tr>
<td>15:30</td>
<td>1.8</td>
<td>12.9</td>
</tr>
<tr>
<td>16:00</td>
<td>1.7</td>
<td>12.6</td>
</tr>
<tr>
<td>16:30</td>
<td>1.7</td>
<td>12.9</td>
</tr>
<tr>
<td>17:00</td>
<td>1.8</td>
<td>13.2</td>
</tr>
</tbody>
</table>

With the local grid wave model, the significant wave heights were also measured at 100m increments measured from chart datum. This information is given in Figure 5.13. It can be perceived that the significant wave height decreases as the wave moves closer to the shoreline. This is in line with
theory because as the water depth becomes shallower, the waves at the higher end of the spectra would break first, leading to a reduction in the significant wave height.

Figure 5.14 represents a snapshot view of the wave conditions at 13:00 on the 14th of September 2014. The arrows symbolize vectors pointing in the direction that the significant wave height propagates. The box within the figure represents Long Beach. When following the wave direction, it is clear that it bends around the south headland and refracts to become near perpendicular to the shoreline.
CHAPTER 6: ANALYSIS AND DISCUSSION

6.1  Introduction

In this analysis and discussion chapter, the results from the authors physical model tests will first be shown and described briefly. Thereafter, these results will be compared to nine published wave run up formulae and analysed in detail. The second section of this chapter focuses on the numerical modelling, field measurements, and their comparison to the formulae. In the last section, this study proposes how the Iribarren number could be correlated to the C constant evident in Mather et al (2011) formulae.

6.2  Physical model study

6.2.1  Run up comparison with literature

The results for the 10 official physical model tests are shown in Table 6.1. In the first column the test number is shown. The second column lists the incident wave height that was gathered from the data acquisition software. The wave period, column three, was also derived from the data acquisition software. The wave length was calculated for each test by using the standardised, transitional water, wave length formula which is dependent on the wave period. The slope stayed constant in the physical model at 1/24 as seen in column five.

The wave steepness parameter, $s_0$, was taken as $H_i / L_0$. The Iribarren number was then calculated using Equation 2.5 that incorporates the wave steepness and beach slope parameter. Column nine, $R_{\text{model (Measured)}}$, provides the model scale run up height that was taken from the video analysis for every test and the values are expressed in mm (model scale). These values were then scaled up to prototype and the run up was expressed in meters. The last column normalises the run up by dividing this with the incident wave height. This is a common practice with research about wave run up because this is plotted against the Iribarren number which is also dimensionless.
The run up measurements show an increase in value with an increase in wave height from the test schedule as would be expected. For test five and test ten, the run up values were lower compared to the test preceding each of the above two, even though their wave heights were higher. Whereas each test’s wave height was higher, the wave periods, compared to the proceeding tests, were lower. This brings us to the conclusion that wave run up is dependent on the wave height, but also dependent on the wave period. This argument can be backed up by also comparing test six and seven. For test six, the wave height is 2.8 m and the period is 12 sec that results in a 1.2 m run up value. Comparing this result to test seven with a wave height of 3.0 m but a period of 11 sec, the run up value is precisely identical at 1.2 m.

The slope of the physical model beach was constructed from concrete and was 1/24. This slope is close to a 1/20 slope which is regarded as a reflective beach from previous research papers. This was confirmed by the high bulk reflection coefficients that were produced by the reflection analysis. The reflection values ranged from 0.25 to 0.38 for the above tests. Figure 6.1 provides a summary of the three different beach face slopes used for the formulae comparisons relative to LLD. The green line represent the true beach face slope, The blue line, the physical model slope, and the red line the normalised beach slope as defined by Mather et al (2011), starting at mean sea level, +0.98 m CD.
The physical model results will now be compared to the following run up formulae as described in Chapter 2.6. It should be noted that the physical model comprised of regular waves and not irregular waves for which the following equations are designed. Thus, it is not a very accurate comparison. The run up values provided by the physical model can be assumed similar to a R_{\text{mean}} value, and not R_{2\%} as the empirical formulae provides. The ratio between the R_{2\%} and R_{\text{mean}} is similar to that of H_{2\%} and H_{\text{mean}}.

The following table recaps the run up formulae chosen, listed in chronological order.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Hunt 1959)</td>
<td>$\frac{R}{H_0} = 1.84 \cdot \zeta_0 \cdot (r) \cdot (p)$</td>
</tr>
<tr>
<td>(Mase &amp; Iwagaki 1984)</td>
<td>$\frac{R_2}{H_0} = 1.86 \zeta_0^{0.71}$</td>
</tr>
<tr>
<td>(Holman 1986)</td>
<td>$\frac{R_2}{H_0} = 0.83 \zeta_0 + 0.2$</td>
</tr>
<tr>
<td>(Nielsen &amp; Hanslow 1991)</td>
<td>$R_2 = 0.099 \sqrt{\frac{H_0}{1.42}} L_0$</td>
</tr>
<tr>
<td>(Douglass 1992)</td>
<td>$\frac{R_2}{H_0} = 0.12 \frac{H_0}{\zeta_0}$</td>
</tr>
<tr>
<td>(Hedges &amp; Mase 2004)</td>
<td>$\frac{R_2}{H_0} = 1.49 \zeta_0 + 0.34$</td>
</tr>
<tr>
<td>(Stockdon et al, 2006)</td>
<td>$R_2 = 1.1 \left[ 0.35\beta_l \sqrt{H_0 L_0} + 0.5 \sqrt{H_0 L_0 (0.563 \beta_f^2 + 0.004)} \right]$</td>
</tr>
<tr>
<td>(Mather et al, 2011)</td>
<td>$\frac{R_2}{H_0} = C \left( \frac{b}{x_0} \right)^2$</td>
</tr>
<tr>
<td>(De la Pena et al, 2012)</td>
<td>$\frac{R_2}{H_0} = 4 \beta_f^{0.3} \zeta_0$ &amp; $R_2 = 4 \beta_f^{1.3} \sqrt{H_0 L_0}$</td>
</tr>
</tbody>
</table>

The following figures, *Figures 6.2 – 6.9*, compare the author’s run up measurements to the formulae results listed in *Table 6.2*. These results were calculated by using the same H_0 and T, as used in the physical model, and given in *Table 6.1*. The red line in each figure represents a one-to-one ratio, meaning that the run up measurement was exactly the same between the author and the given formula, thus if a data point falls below this line, the formula over-predicts the run up and points above this line will result in an under-prediction. The test results are plotted in order, i.e. the point furthest to the left represents the first physical model test and the point furthest to the right the last test, test number 11. Note, test 4 was left out.
A RE-ASSESSMENT OF WAVE RUN UP FORMULAE

FIGURE 6.2 - RUN UP MEASUREMENTS COMPARED TO (HUNT 1959)

FIGURE 6.3 - RUN UP MEASUREMENTS COMPARED TO (MASE & IWAGAKI 1984)

FIGURE 6.4 - RUN UP MEASUREMENTS COMPARED TO (HOLMAN 1986)

FIGURE 6.5 - RUN UP MEASUREMENTS COMPARED TO (NIELSEN & HANSLOW 1991)
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**Figure 6.6 - Run Up Measurements Compared to (Douglas 1992)**

**Figure 6.7 - Run Up Measurements Compared to (Hedges & Mase 2004)**

**Figure 6.8 - Run Up Measurements Compared to (Stockdon et al. 2006)**

**Figure 6.9 - Run Up Measurements Compared to (De la Pena et al. 2012)**
From the figures above it can be seen that the author’s run up measurements correlate best with the formulae published by Holman (1986), Stockdon et al (2006) and De la Pena et al (2012). Looking at the results for De la Pena et al (2012), it shows that for the first half of the tests schedule (test 1 – 7), the data has a poor fit against the equation, with a run up prediction difference of above 30%. But when the wave heights increases to above 3.0m, the data fit is far better and this prediction difference falls to below 7% with a minimum value of 5% achieved. The overall prediction difference for De la Pena et al (2012) is at 29%. The formula to which the measurements fits second best to, was published by Holman (1986). The same scenario is found when comparing the measurements to the calculated run up values. For waves heights lower than 3.0m the correlation fit is quite poor, with an difference percentage more than 20%. When comparing the tests, with wave heights larger than 3.0m, the difference percentage decreases to below 6% and a minimum difference of 3% is achieved. The overall prediction difference was calculated at 20% for the Holman (1986) formula. The publication to which the author’s data shows the best fit, is that of Stockdon et al (2006). For the first six tests, the formula overpredicts the wave run up with an average difference of 22%, and for the last four test the run up is underpredicted by 9%. This summarises to a overall difference of only 10%.

Comparing the measurement to the equation published by Hunt (1959), the majority of the run up is overpredicted. This overprediction decreases in percentage as the wave height increases over the test period. The overall overprediction difference was 54%. When the measurements were compared to Mase & Iwagaki (1984), Nielsen and Hanslow (1991), Douglass (1992) and Hedges and Mase (2004), the correlation was also very poor. All of the above four equations overpredict the run up by some margin. The averaged difference percentages associated to these equations are all over 50% with a maximum individual difference of 254% for test 1 by Douglass (1992). For test 1, the student measured a run up value of 0.5m, compared to a calculated value of 1.77m according to Douglass (1992) [144% over prediction]. This could be supported by the fact that the formula proposed by Douglass (1992) does not include the beach slope and is only dependant on the wave height and period. The same is applicable to the formula proposed by Nielsen and Hanslow (1991) [66% over prediction]. When looking at the results of Mase & Iwagaki (1984) and Hedges and Mase (2004), their results are similar because both of them used the same physical model test information, and the latter’s equation was an improvement of the first. The results for Hedges and Mase (2004) are slightly better [111% over prediction] than for Mase & Iwagaki (1984) [115% over prediction]. De la Pena et al (2012) also mentioned that for their paper found that Mase & Iwagaki (1984) caused a maximum limit, which over estimated their results as well. Thus it seems acceptable that an over-prediction of this magnitude is possible.
Lastly, the physical model test results were directly compared to the formula of Mather et al (2011), to examine what the dimensionless constant \((C)\) would be. Note: the equation given by Mather et al (2011) was formulated with large wave heights, \(H_0 = 8.0\) m, and they found when \(C\) is 7.5, the equation fits their data the best. The physical model only included wave heights of up to 4.3 m. Using the physical model slope of 1/24 and a dimensionless constant of 3.3, the following graph, Figure 6.10 was achieved. It can be seen that the points fit the measured results exceptionally well with an average difference of 0.8%. The \(C\) value of 3.3 is on the lower bound of Mather et al (2011) specifications. Referring to Figure 2.9, the value of 3.3 is acceptable with a \(R/H_0\) of between 0.3 and 0.45; to an offshore distance of 360 m for a 1/24 slope. Using a value any higher than 3.3, will result in an overprediction of the run up which is evident in Mather et al (2011) report, where 75% of the Cape values were overpredicted and 55% of the KZN values, by using a constant coefficient of 7.5. Importing 7.5 into these results will deliver a run up overprediction of 130%, which is also plotted on Figure 6.10 (Green triangles in Figure 6.10).

![Model results vs Mather et al. (2011)](image)

**FIGURE 6.10 - RUN UP COMPARISON BETWEEN MEASURED RESULTS AND MATHER ET AL. (2011) PREDICTIONS**

When the normalised run up measurements \((R/H_{inc})\) are plotted against the Irribarren number, Figure 6.11, the results from the empirical formulae form two distinctive groups. The top group consists of Douglass (1992), Mase & Iwagaki (1984) and Hedges and Mase (2004). This is expected because all three of them over-predicted the measured run up measurements. The lower group consists of Holman (1986), Stockdon et al (2006), De la Pena et al (2012) and Mather et al (2011).

The physical model run up results fall within this group, but at the lower end. This is as expected, because the physical model results are plotted in terms of \(R_{mean}/H_{incipient}\). If a correction factor is applied to the data, the physical model results will plot around the top of all the empirical values. In Figure 6.11 it is also evident that the normalised mean run up values are decreasing in magnitude as the Irribarren number increases, against the empirical formulae that increases. This could be caused by sidewall reflections inside of the wave flume which was triggered by the uneven model slope causing some uneven breaking.
6.2.2 Sensitivity analysis

TABLE 6.3 - SENSITIVITY TEST RESULTS

<table>
<thead>
<tr>
<th>Test Name</th>
<th>S01</th>
<th>S02</th>
<th>S03</th>
<th>S04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave Height [m]</td>
<td>4.1</td>
<td>4.1</td>
<td>4.1</td>
<td>4.1</td>
</tr>
<tr>
<td>Wave Period [s]</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>Gain Parameter</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$H_{\text{m0}}$ probes [m]</td>
<td>4.49</td>
<td>4.61</td>
<td>4.35</td>
<td>-</td>
</tr>
<tr>
<td>$H_{\text{incident}}$ [m]</td>
<td>4.21</td>
<td>4.32</td>
<td>4.08</td>
<td>-</td>
</tr>
<tr>
<td>Wave run up [m]</td>
<td>1.45</td>
<td>1.6</td>
<td>1.7</td>
<td>-</td>
</tr>
</tbody>
</table>

*For test S04, the waves displayed double breaking and thus the test was discarded and no run up was measured*

The sensitivity results entailed keeping the wave height constant in the test schedule, but increasing the wave period. When observing the sensitivity results, there is a clear trend between the peak wave periods and the wave run up achieved. For S01, the $H_{\text{incident}}$ was 4.21m; $T_p$ was 10sec and the run up was 1.45m. For S02, the $H_{\text{incident}}$ was 4.32m that was greater compared to S01; $T_p$ was 11sec and the run up was 1.6m. The larger measured run up could be as the result of the increase in incident wave height or an increase in wave period. For S03, the $H_{\text{incident}}$ was 4.08m that was less compared to S02; $T_p$ was 12sec and the run up was 1.7m. This shows that the wave period is just as important as the wave height for the prediction of wave run up. The wave height dropped by 0.24m between test S02 and S03, but the run up in turn increased.
6.3 Numerical model and beach test results

As with the physical model results, the beach run up measurements was also compared to the formulae listed in Table 6.2, for each of the five beach tests. Table 6.4 provides a summary of the numerical model results and the coinciding beach measurements. The wave heights as chosen from the wave record (Table 5.3) were selected 30min before a test has begun, or the closest 30min time interval calculated. ‘Wave runup T1’ started at 12:36:00, thus the wave parameters associated to these results were taken at 12:00:00 from the wave record. A wave with a period of 13-14s, travels at a wave celerity of around 11.5 m/s. Thus for a wave to cover the 18 km distance between the wave rider and the beach, it would take about 26 minutes. For smaller peak periods, the time periods would increase.

TABLE 6.4 - BEACH TEST RUN UP RESULTS

<table>
<thead>
<tr>
<th>Test#</th>
<th>(H_{\text{incident}})</th>
<th>(T_p)</th>
<th>Wave length</th>
<th>Beach face slope</th>
<th>Slope Mather</th>
<th>Wave Steepness</th>
<th>Iribarren</th>
<th>Run up</th>
<th>(R_s/H_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>m</td>
<td>s</td>
<td>m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
<td>12.61</td>
<td>146.88</td>
<td>0.0812</td>
<td>0.0300</td>
<td>0.0102</td>
<td>0.804</td>
<td>1.113</td>
<td>0.742</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>14.2</td>
<td>167.93</td>
<td>0.0812</td>
<td>0.0300</td>
<td>0.0119</td>
<td>0.744</td>
<td>1.421</td>
<td>0.711</td>
</tr>
<tr>
<td>3</td>
<td>1.7</td>
<td>13.5</td>
<td>158</td>
<td>0.0812</td>
<td>0.0300</td>
<td>0.0108</td>
<td>0.783</td>
<td>1.778</td>
<td>1.046</td>
</tr>
<tr>
<td>4</td>
<td>1.9</td>
<td>13.14</td>
<td>154</td>
<td>0.0812</td>
<td>0.0300</td>
<td>0.0123</td>
<td>0.731</td>
<td>1.565</td>
<td>0.824</td>
</tr>
<tr>
<td>5</td>
<td>1.7</td>
<td>12.61</td>
<td>146.88</td>
<td>0.0812</td>
<td>0.0300</td>
<td>0.0116</td>
<td>0.755</td>
<td>1.621</td>
<td>0.954</td>
</tr>
</tbody>
</table>

Because the entire test series was measured in one day, the range between the wave heights and periods are not vast. The difference between the highest and lowest significant wave heights is 0.493m. In Table 6.4, two slopes are provided. The beach face slope originated from the beach survey taken and the Mather et al (2011) normalised slope. The beach face slope was used for all calculations. The Iribarren numbers compared to the physical model tests are significantly larger, which is caused directly by the beach face slope being two times steeper, Figure 6.1, thus the Iribarren numbers are also double those of the physical model tests. The normalised run up values were also larger in comparison with the physical model results. Further research into De la Pena et al (2012) report, justified high ratios with steep slopes. In their report, physical model testing on a 1/20 beach face slope gave ratios up to 0.8 for \(R_s/H_0\). This study’s measurements were taken on a beach with a face slope of 1/12, validating these high ratios.

Figure 6.12 – 6.16 presents the wave run up record plotted on a wave index axis. It should be noted that the run up is not plotted against time. The wave run up index only represents the measurement number associated to a run up elevation, i.e. point 15 represents the fifteenth measurement taken for that test. The symbol T in the title represents the word test, i.e. T1 means test one.
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FIGURE 6.12 - WAVE RUN UP RESULTS FOR T1

FIGURE 6.13 - WAVE RUN UP RESULTS FOR T2

FIGURE 6.14 - WAVE RUN UP RESULTS FOR T3

FIGURE 6.15 - WAVE RUN UP RESULTS FOR T4
From the figures above, it is evident that the SWL line (represented by the red dotted lines) increases from Figure 6.12 (SWL = -0.01m) to Figure 6.16 (SWL = 0.5m). This is since the first test started at low tide and the last test ended near high tide. The run up points show one run up event, and is not time dependant. The points are also biased towards higher run up values, because it was not always possible to measure the minimum run up values in the field.

Comparing these \( \frac{R_2}{H_0} \) beach measurements against the formulae published in Table 6.2, Figure 6.17, the formulae fitting the measurements the best are Holman (1986), Nielsen and Hanslow (1991) and Stockdon et al (2006). Stockdon et al (2006) underpredicts the run up by 11% and Nielsen and Hanslow (1991) by 7%. Holman (1986) overpredicts the measurements by 1%, which is taken as the average difference from the five events, which is also the best result from all the empirical formulae.

The formulae that presented large prediction difference were those of Mase and Iwagaki (1984), Hedges and Mase (2004) and De la Pena et al (2012). The over-prediction differences are 83%, 76% and 72% respectively. This over-prediction was also evident in the physical model analysis for Mase and Iwagaki (1984) and Hedges and Mase (2004). A reason why the results for De la Pena et al (2012) was so high was because their formula gives high weight to the beach face slope, thus when
the beach slope increase marginally, the run up value increases significantly. Douglass (1992) formula predictions positioned between these maximum and minimum regions with an difference of 35%. The reason for the 35% difference by Douglass (1992) could be caused by the lack of a beach slope parameter.

The beach measurements falls again in the lower region of Figure 6.17, but the measurements are more scattered than the physical model’s. The scatter could be decreased if information from more tests are included in the analysis from different wave conditions.

Comparing the measurements with the formulae published by Mather et al (2011), a value of the constant $C$ set at 8.6 delivered a 1% prediction difference, Figure 6.18. In reference to Figure 2.9, the value of 8.6 is acceptable with a $R/H_0$ of between 0.7 and 1.0; for an offshore distance of 500m. Also plotted on Figure 6.18 are the predicted run up measurements for the constant value ($C$) equal to 7.5, which has an prediction difference of 13% from the measured value.

![Figure 6.18 - Run comparison between measured and Mather et al. (2011) for $C=8.6$](image-url)
FIGURE 6.19 - NORMALISED RUN UP VS IRIBARREN INCLUDING POSSIBLE RANGES OF C VALUES
6.4 Joint comparison

By joining the theoretical normalised run up graphs (Figure 6.11 and Figure 6.17) for both the physical and beach measurements, Figure 6.19 was produced, shown on the previous page. The values on the left represent the physical model measurements and the values on the right, the beach measurements. For the physical model results, the measurements were increased by a factor of 2.2, which is equal to the ratio of $H_{2%}/H_{\text{mean}}$ (CIRIA, CUR & CETMEF 2007). With both data sets, the true measurements are plotted with the bright red colour squares for identification.

Because the Iribarren number differs by a factor of two between the different data sets, normalised run up measurements could not be compared directly to one another. Thus, an extrapolation technique was implemented where predicted run up values from the same equation were linked to one another, across the two data sets. The dashed lines represent the extrapolation of an individual formula between the physical model tests and beach tests. The top two lines follow the Mase & Iwagaki (1984) formula very accurately, which is also the case for the lower lines, which represents the equation from Hunt (1959), Stockdon et al (2011) and Holman (1986).

The joint comparison was made to compare the empirical formulae over a large span of Iribarren numbers and to find the best overall run up prediction formula. With the beach measurements not particularly valid in this comparison, because of the regular wave scenario, the best formula should be found from the beach study values. These measurements correlates best with three formulae as mentioned in Chapter 6.3: Holman (1986), Stockdon et al (2006) and Nielsen and Hanslow (1991).

From Figure 6.19 there can be seen that all the formulae that have a linear relationship, are written as a first degree function of the Iribarren number, except the formula from Stockdon et al (2011), which is far more complex in nature, but its results appear to be similar to the first degree function.

The formula by Nielsen and Hanslow (1991) predicts high normalised run up values for low Iribarren numbers compared against low run up values for higher Iribarren numbers. This could be because their formula does not take into account the beach face slope, when the slope is less than 0.1, which is the case for both the physical model and beach tests. Where Nielsen and Hanslow (1991) showed a significant loss in normalised run up, De la Pena et al (2012) had a significant increase in normalised run up. This jump is because their formulae gives more weight to the beach face slope than any other formulae.
6.5 The dilemma around the “C” constant in Mather’s equation

It was found that the constant coefficient, $C$, varied from 3.3, for the physical model results, to 8.6 for the beach results. The ratio of $R_2/H_0$ was also higher for the beach tests. The key attribute that could cause this increase can only be that the beach face slope is double that of the model slope. This initiated a thought that the constant can be correlated to the Iribarren number. The Iribarren number is in turn dependent on the beach face slope and wave steepness. If the wave steepness is the same for the physical and beach results, the only variable that could influence the Iribarren number is then, the beach face slope.

Plotting this theory on a normalised run up vs Iribarren graph, Figure 6.19, the idea is visualised. On the lower end of the Iribarren axis, the physical model predictions are plotted having a gentle slope of $1/24$ and on the higher end, the beach measurements are plotted as having a slope of $1/12$. On top of both data groups, a vertical line is plotted representing the predicted normalised run up measurements for associated C values, for a wave steepness of 0.012. These C values ascend by a factor one, from the bottom to the top, and start at $C=2$ for the physical model tests and $C = 6$ for the beach tests.

For the left-hand data set, an acceptable value for C would be between 3.0 and 5.0 for Iribarren numbers between 0.25 and 0.4. Using a C value between these ranges, would provide a similar run up prediction as Holman (1986) and Stockdon et al (1991). On the right-hand data set, the acceptable range for C would be between 7.0 and 10 for Iribarren numbers between 0.75 and 0.8. Using these C values, run up predictions would be similar to Holman (1986), Stockdon et al (1991) and Nielsen and Hanslow (1991). This presents a way of linking the beach slope and wave period to the C constant in the formulae because a common point of debate is that this formula does not include these two parameters.
CHAPTER 7: CONCLUSION AND RECOMMENDATIONS

To predict wave run up accurately under all possible wave conditions is still a massive challenge in the coastal engineering field because of the interaction between various processes inside the surf zone. But as this idea has challenged many minds, we have come really close from where it all started a few decades ago. The study was set out to do a re-assessment of previously published wave run up formulae and to provide the reader with a better understanding of wave run up and how the results of one formula compares to another. The study also sought to evaluate run up for various coastal types in South Africa by assessing the formulae published by Mather, Stretch & Garland (2011).

The method for undertaking these objectives consisted of (1) performing a physical model test with a constant slope and impermeable floor running regular waves, and (2) compiling a numerical analysis to propagate waves from an offshore location to Long Beach and performing run up measurements at this location.

The physical model with a constant slope indicated that wave run up is dependent on the wave height, but also dependent on the peak wave period associated with this wave wave record. The physical model results fell within a band of theoretical predicted run up measurements, using identical wave parameters, among the likes of Holman (1986), Stockdon et al (2006) and De la Pena (2012). Prediction differences larger than 10% were found for all the formulae, compared to the physical models results, which could have been caused by comparing regular waves to irregular wave run up and the unevenness in the concrete model slope.

An assessment of wave run up measurements on Long Beach resulted in a clearer understanding of exactly how a maximum run up elevation is achieved by the down-rush and uprush wave interactions between preceding waves, as well as the build-up between a group of waves breaking shortly after another. A theoretical comparison showed some scatter between the beach measurements and formulae predictions, compared to the physical model results. These measurements correlated well to the likes of Holman (1986), Stockdon et al (2006) and Nielsen & Hanslow (1991).
It was found that the formulae proposed by Mase & Iwagaki (1984), Hedges & Mase (2004) and Douglass (1992) always over-estimated the wave run up for both of the tests. The formulae proposed by Holman (1986) and Stockdon et al (2006) delivered reasonably accurate run up estimation compared to the physical and beach measurement results. The other two formulae by Nielsen & Hanslow (1991) and De la Pena et al (2012) compared well against one test, but not for both tests.

The formulae proposed by Mather et al (2011) were applied on both assessments and delivered a constant value, C, equal to 3.3 for the physical model results and 8.6 for the beach results. The value of 3.3, found from the physical model comparison is very low, but it should be taken into account that the test provided mean run up and not $R_{25}$. Both results were found to be within the range of the author’s predictions when considering the near shore slope on which these measurements were measured. This led to a hypothesis around correlating the Iribarren number to the unknown constant ‘C’, which in turn, introduces the beach face slope and wave period into this equation. For low Iribarren numbers between 0.25 and 0.4, an accurate prediction for the ‘C’ constant in the Mather et al (2011) formula should be between 3.0 and 5.0. For higher Iribarren numbers, 0.75 -0.8, an accurate ‘C’ prediction should be between 7.0 and 10.

Fitting the two normalised run up plots onto one graph resulted a split distribution of data between the physical model and beach measurements. The physical model’s Iribarren values were between 0.3 - 0.4 and the beach measurements Iribarren numbers started at 0.75. However, by extrapolating from the one group to the other, the trend lines fitted flawlessly for the upper and lower bound formulae. This graph showed that the formula published by Holman (1986) and Stockdon et al (2006) predicts the wave run up the best, based on the beach run up data.

Future research into validating or disregarding this hypothesis can be done by acquiring more beach data points for Iribarren numbers between 0.4 and 0.75 or by running the wave schedule on different slopes in the physical model with irregular waves. A larger collection of test results (beach and physical) could, in the end, lower the prediction differences, as found with this report.

With a multitude of wave run up prediction models, this study sought to provide the reader with a better understanding of the origins of the named formulae, how they compare relative to one another and which one is the best at present. Focus was also directed on the formula proposed by Mather et al (2011) and this report found that the formulae can predict wave run accurately, hence choosing the correct constant factor $C$. This formula is still open to operator errors, which can lead to higher or lower than expected wave run up predictions. The formula by Holman (1986) and Stockdon et al (2006) is the best formula to use at present, which is not open to operator errors.
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