Reliability Based Optimization of Concrete Structural Components

by
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ABSTRACT

Standards define target reliability levels that govern the safety of designed structures. These target levels should be around an economic optimum for the class of structure under consideration. However, society may have safety requirements in excess of that required to achieve an economic optimum. The LQI criterion can be used to determine society's willingness to invest in safety, thereby defining a minimum acceptable safety- or reliability level. This thesis determines economically optimised reliability levels for reliability class two concrete structures in South Africa, over a range of typical input parameters.

Rackwitz's (2000) approach is used here, adjusted for the South African context. The structure is described using a simple limit state function, defined as the difference between load and resistance, with resistance a function of a global safety parameter. South African construction costs, costs of increasing safety, failure costs and discount rates are used in the objective function for economic optimisation.

Life Quality Index (LQI) theory is used as a basis to derive society's willingness to pay (SWTP) for safety and the corresponding reliability level is found by applying the LQI criterion. In the South African context the derivation of SWTP presents some challenges, which is discussed.

Situations where the minimum required reliability would exceed the economically optimum reliability level are discussed.

Various reliability based cost optimization case studies are conducted covering a broad range of typical concrete design situations. From these case studies a range of target reliability indices are derived for typical concrete structural components and failure modes. Obtained values are compared to current South African target levels of reliability provided by the South African loading code and recommendations are made.

The approach used by Rackwitz (2000) is compared with results obtained from case studies and used as basis to estimate optimum reliability levels for other types of buildings.

Functions are written in MATLAB to allow replication of the study for others seeking to derive optimum reliability indices.

UITTREKSEL

Standaarde spesifiseer teiken betroubaarheidsvlakke wat die veiligheidsvlak van ontwerpte strukture bepaal. Hierdie teikenvlak moet rondom die ekonomiese optimum wees vir die klas van struktuur onder oorweging. Die samelewing verkies moontlik 'n hoër veiligheidsvlak as wat deur die ekonomiese optimum dikteer word. Die LKI (Lewens Kwaliteit Indeks) maatstaf kan gebuik word om die samelewing se bereidwilligheid om in veiligheid te belê te bepaal en sodoende 'n minimum veiligheidsvlak bepaal. Hierdie tesis bepaal die ekonomiese optimum betroubaarheidsvlak vir klas twee beton strukture in Suid-Afrika vir wisselende parameters.

Rackwitz (2000) se benadering word in hierdie studie gebruik en is aangepas vir Suid-Afrikaanse omstandighede. Die struktuur word beskryf deur 'n eenvoudige limiet staat funksie, gedefinieer as die verskil tussen die las en weerstand, met die weerstand as die funksie van 'n globale veiligheidsparameter. Suid-Afrikaanse konstruksie koste, veiligheidsvermedering koste, falingskoste en diskonteer koerse word gebruik vir optimering.

Die LKI teorie word gebruik om SBB (Samelewing Bereidheid om te Belê) vir veiligheid af te lei en die ooreenkomstige betroubaarheidsvlak word bepaal deur die LKI maatstaf toe te pas. In die afleiding hiervan vir Suid-Afrikaanse omstandighede is sekere uitdagings teegekom wat bespreek word.

Situasies waar die minimum betroubaarheidsvlak hoer is as die ekonomiese optimum word bespreek.

Verskillende betroubaarheids gebaseerde optimering gevalstudies word gedoen op tipiese beton struktuur elemente. Van hierdie gevalstudies is optimum betroubaarheidsindekse vir die tipiese beton elemente en galingsmodusie afgelei. Die betroubaarheidsindekse word vergelyk met huidige betroubaarheidsindekse soos wat voorgeskryf is in die Suid-Afrikaanse laskode (SANS10160-1(2011)).

Rackwitz (2000) se benadering word vergelyk met die resultate van die gevallestudies en word gebruik as basis om optimum betroubaarheidsvlakke vir ander tipes geboue te voorspel.

MATLAB funksies is geprogrameer om minimum en optimum betroubaarheidsindekse af te lei.

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LIST OF ABREVIATIONS

CoV:	Coefficient of Variation
D:	Deterministic Value
EU:	Eurocode
FORM:	First Order Reliability Method
GA:	Gamma Distribution
GDP:	Gross Domestic Product
GU:	Gumbel Distribution
ICAF:	Implied Cost to Avert a Fatality
ISO:	International Standards Organisation
JCSS:	Joint Committee of Structural Safety
LN:	Lognormal Distribution
LOL:	Loss of Life
LQI:	Life Quality Index
MRS:	Marginal Rate of Substitution
N:	Normal Distribution
PMC	Probabilistic Model Code
P_f:	Probability of Failure
PV_{OLAG}:	Present Value Overlapping Age Group Model
PV_{OLG}:	Present Value Overlapping Generations
PV_R:	Present Value Regular
RC:	Reliability Class
SANS:	South African National Standard
SCCR:	Society's Capacity to Commit Resources
SHC:	Societal Human Capital
SLQI:	Societal Life Quality Index
SORM:	Second Order Reliability Method
STPR:	Societal Time Preference Rate
SVSL:	Societal Value of a Statistical Life
SWTP:	Society's Willingness to Pay
VR:	Coefficient of Variance of Resistance
VS:	Coefficient of Variance of Load

WHO: World Health Organization

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Chapter 1 INTRODUCTION

1.1 Problem Statement

Risks to health and life pose a constant threat to modern day society. Engineers are often involved with projects that benefit society, but at the same time have risks associated with them. With limited resources, the engineer has to be able to mitigate the risks accordingly by asking the difficult question: How safe is safe enough?

The optimum safety could be obtained by assessing the risks through a cost optimization process. However, the consequences of structural failure is often difficult to access and consist of vastly different components such as the cost of the repair or permanent destruction of the structure, the loss of life, injuries, disabilities and the loss of economic activity.

In order to find the optimum safety, one must be able to add a monetary value to the loss of life. It is universally accepted that a human life is infinitely valuable, but resources are limited and these resources must be allocated efficiently to ensure maximum utility, thus it is necessary to determine what a society can afford to spend on safety. Ethically one cannot place a price tag on a human life, but by applying the LQI (Life Quality Index) principle, SWTP (Society's Willingness to Pay) can be derived which is what society can afford to invest in safety to save a marginal life.

There are various tables showing target reliability indices given by the PMC, SANS and ISO codes (Refer to literature review). These tables provide target reliability indices for different costs of increasing safety and failure costs. The different costs classes are typically differentiated by small, medium large costs. However, there are no clear monetary values given for these costs classes except in the PMC codes. Furthermore, these tables were created based on various assumptions that might not necessarily be true.

Thus target reliability indices must be derived for South African concrete structures in RC2 based on actual situations and by making as few assumptions as possible to ensure sufficient accuracy of results.

1.2 Aim of the Thesis

The main objective of the thesis is to use benefit/cost optimization to derive target reliability indices for concrete structures falling under reliability class 2 in South Africa. The target reliabilities will be obtained from a benefit/cost analysis of the different concrete structural components under ultimate limit state conditions. Structural failure due to non-earthquake related loads is considered.

The benefit/cost optimization does not take into account the societal requirement that investment into safety must not result in the life quality of the average individual to be negatively influenced. Thus it is theoretically possible for the optimum amount of resources to be invested in safety to result in the life quality of an average person to decrease. The LQI criterion ensures that the life quality of an average person does not decrease by setting a minimum target of safety. Optimum solutions are therefore checked with the LQI criterion to ensure that these optimum solutions are safer than the minimum safety required by society.

A study by Rackwitz (2000) derived target reliability indices based on the assumption that most structures can be accurately represented by a two variable limit state function with a lognormal distribution assumed for both the resistance and load effect. The approach used by Rackwitz (2000) is compared with results obtained from case studies and the simplified approach used by Rackwitz (2000) is adjusted to allow for accurate approximation of the complicated case studies. This allows future studies to accurately approximate the target reliability index without being too time consuming.

Along with the application, the LQI criterion will be assessed and recommendations will be made where additional research or improvements on the LQI and the benefit/cost analysis can be done. In addition to this, functions will be written in MATLAB which can be used by others seeking to derive target reliabilities for concrete structures for their nation or other nations.

1.3 Structure of the Thesis

In Chapter two an extensive literature review is conducted on the derivation and reasoning behind the LQI. A detailed explanation is made of how different theories in economics are combined to derive SWTP from the LQI. The calculation of a sustainable discount rate and how to measure society's preferences when it comes to obtaining life years or obtaining wealth is shown. The chapter also shows various forms of compensation and life saving cost estimations. The last part of the chapter deals with the derivation of the benefit/cost function and also introduces different models for mortality and consequence estimation due to structural collapse.

Chapter 3 deals with the application of the LQI and the usage of economic indicators to derive SWTP and SVSL for South Africa. A strange phenomenon of measuring of society's preferences when it comes to work is observed by conducting a comparative study for both randomly selected European and African countries. Finally the chapter also includes a study conducted as to why there exists a variable SVSL/SWTP ratio of various selected countries.

Reliability theory is covered in Chapter 4, where various methods for obtaining the probability of failure are mentioned. The methods are compared and the benefits and drawbacks of the various methods are summarised. An appropriate method, FORM (First Order Reliability Method), is chosen to compute the probability of failure for this particular study due to its efficiency. The FORM is programmed into a function written in MATLAB and the capabilities and usage of the function is covered. A comparison is done between the program VaP and the function written in MATLAB to measure its accuracy. A function is also written to find the optimum safety based on cost minimization. How the optimum point is found for a non-differentiable function is explained.

The complex relationship between optimum and minimum safety is explored in Chapter 5 by conducting parameter studies similar to those done by Rackwitz (2000). A benefit/cost function is simplified by various assumptions and using evidence from parameter studies. The effect of the two work time fraction formulations on the optimum safety is explored in this chapter. A relationship between the optimum probability of failure and the benefit/cost related parameters (Parameter K or Relative Cost of Safety) is established depending on the variances of the resistance and the load effect.

In Chapter 6 case studies representing typical failure modes on different types of slabs are conducted. The main aim of this chapter is not only to establish the target reliability indices of concrete slabs, but to see if the results from these specific limit state functions compare well with the simplified/generic approach used in Chapter 5. Comparisons are also conducted between the target reliability indices derived in this study and existing target reliability indices, which indicates that the current South African target reliability index is too low for concrete structures in RC2.

In Chapter 7 the same approach in Chapter 6 is applied, however in this particular chapter the focus is on beams. Specific limit state functions are used for typical modes of failure for reliability based optimization and the results compared to the simplified approach in Chapter 5. The results compare well with the simplified/generic approach and a conclusion is drawn that a medium variance simplified approach approximates the results the best. The results are also compared to existing target reliability indices. The current South African target reliability index for brittle failure of concrete structures in RC2 compares well with the results, but the results suggest that the target reliability should be increased. For ductile failure modes the results indicate that a definite increase is required.

Columns are investigated in Chapter 8. Short columns are considered for optimization and the approach has to be modified as a column fails due to a combination of a moment and axial force. In this chapter the probability of failure of the column is computed with a combination of an interaction diagram and a Monte Carlo simulation. The methodology and theory of the combination of these two aspects are explained and the functions used to model the failure of a column programmed in MATLAB are also mentioned. The obtained reliability indices are slightly higher than the current target reliability indices recommended by the South African codes and the results can further be accurately approximated with a medium to low variance of the simplified approach.

In Chapter 9 the simplified approach of Rackwitz (2000), which is shown in Chapters 6, 7 and 8 to be a good approximation model, is used to derive optimum reliabilities. An example with a residential building yields approximate results that took three chapters of work to derive for office structures. The sensitivity of the probability of a successful escape with the cost of increasing safety is shown.

In Chapter 10 the author makes recommendations on what the target reliability indices should be for South Africa based on the results of this study. However, if the decision maker does not agree on the assumptions regarding the probability of escape, Figure 10-1 is provided from which 50 year target reliability indices can be obtained for all the study cases considered for various probabilities of escape. Various main conclusions of the study are summarised and recommendations for future studies are provided.

The appendices contain all the MATLAB functions created and used in this study.

Chapter 2 LITERATURE REVIEW

2.1 The Derivation of the LQI

One of the key concepts in economics is that everything is exchangeable. One can exchange goods for money and money for goods. The same can be said for life years. Life years can be gained by spending money on healthcare or safety measures and can be sold in a sense that the consumer may choose not to spend this money. Rationally the consumer will spend money on life years and the size of the trade off is directly related the optimal utility obtained. This trade off is known as the marginal rate of substitution between longevity and wealth and this concept can be seen on the following graph showing the relation between the GDP per capita and the life expectancy at birth of different countries.

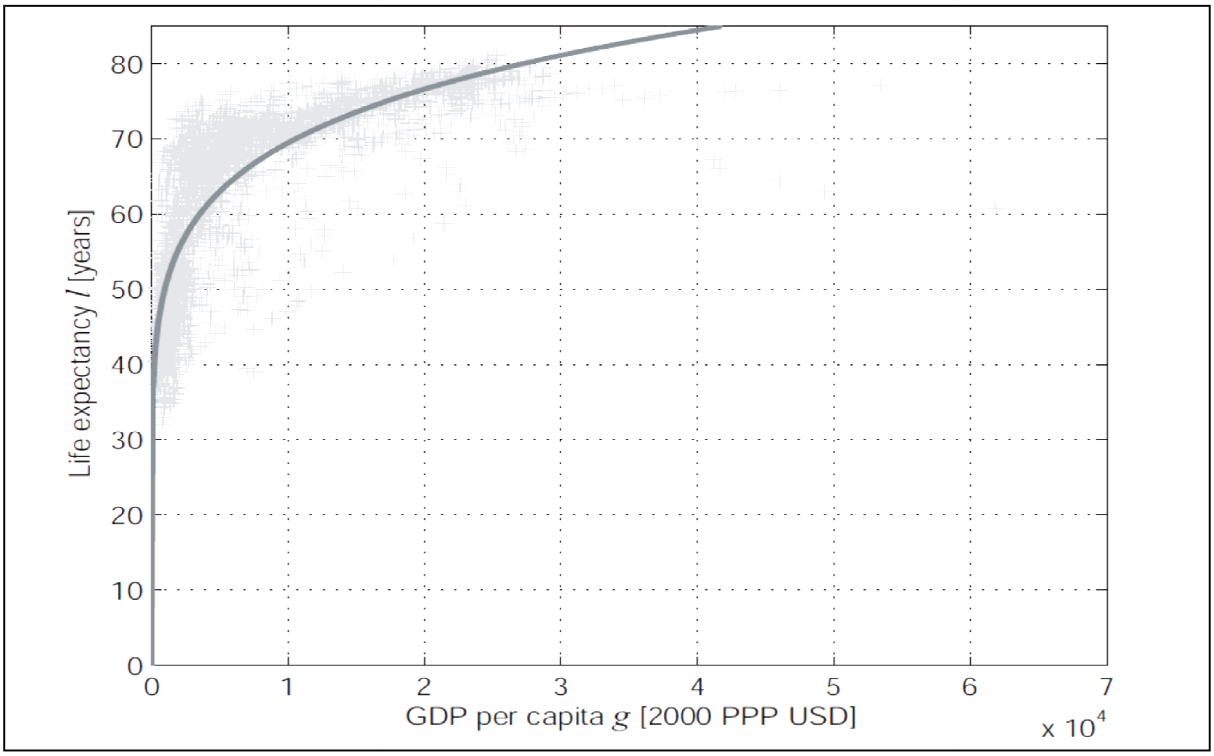

Figure 2-1: Life Expectancy and GDP per Capita of 160 countries (Kubler et al. (2005))

It is clear from the above figure that there is a correlation between the life expectancy and GDP per capita of a country.

Nathwani et al. (1997) derived the Life Quality index based on two functions namely $f_g(g)$, measuring quality of life, and $f_r(r)$, measuring the duration of life. These two functions are assumed to be mutually independent and differentiable. The utility of a society based on these functions is defined as follows:

$$L = f_g(g) f_r(r) \qquad \textbf{2-1}$$

The function is differentiated to determine the relative change caused by investment into safety:

$$\frac{dL}{L} = \left(\frac{g}{f_g(g)}\frac{df_g(g)}{dg}\right)\frac{dg}{g} + \left(\frac{r}{f_r(r)}\frac{f_r(r)}{r}\right)\frac{dr}{r} \qquad \textbf{2-2}$$

With elasticities k_g and k_r:

$$\frac{dL}{L} = k_g\frac{dg}{g} + k_r\frac{dr}{r} \qquad \textbf{2-3}$$

Nathwani et al. (1997) assumed the ratio of these elasticities is constant regardless of the actual value of g or r. This property is known as the universality requirement and implies k_r/k_g is always constant. The universality requirement allows for functions $f_g(g)$ and $f_r(r)$ to be defined by first order differentiable functions ($f_g(g) = g^r$ & $f_r(r) = ((1-w)e_0)^s$). Substituting in these functions in equation 2-1:

$$L = g^r((1-w)e_0)^s \qquad \textbf{2-4}$$

Where:

g = GDP per capita

r,s = constants to be defined

w = work time fraction of a society

e_0 = life expectancy at birth

The following figure shows indifference curves (l_{L1},l_{L2} & l_{L3}) having parameters g and l and a technology curve (lt).

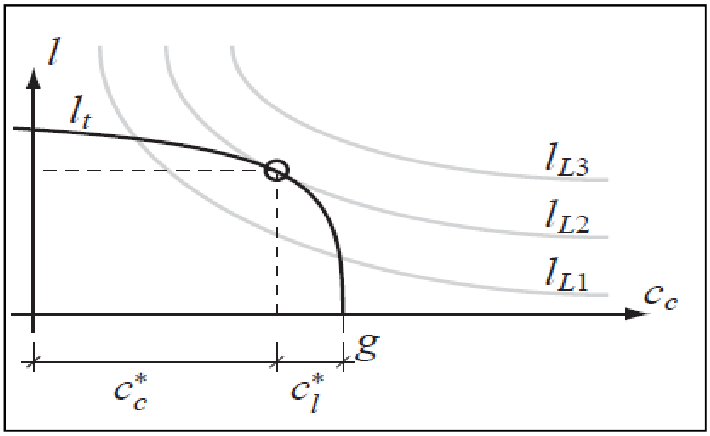

Figure 2-2: Indifference Curves having Constant Utility (Kubler (2007))

The vertical axis of the curve above is represented by the life expectancy of a society, while the horizontal axis is represented by the GDP per capita of a society. An indifference curve is a curve where society has the same amount of utility regardless of the value of l or g. In other words a society will retain the same relative quality of life for all the (g,l) points on an indifference curve. Thus equation 2-4, showing the utility of a society, is constant for all the (g,l) points on the indifference curve.

The technology curve shows the range of the possible amount of utility a society can achieve by marginally adjusting life expectancy l and g. The technology curve is dependent on the effectiveness of a society to convert an investment into safety into an increased life expectancy. The above figure is a typical example of how a technology curve would look of a typical society. As a society invests into safety the GDP decreases, but the life expectancy increases. However, investing into safety becomes less effective as the life expectancy increases. This is as a result of technology is limited as there is little doctors can do to prolong the lives of the elderly, even if increased proportions of the GDP are invested into healthcare.

If society had an increase in GDP a portion will be used to extend life expectancy and the rest will be used for reinvestments. In this particular case it is clear that the optimum utility is achieved at point C_C^* away from the origin in other words society invested C_l^* into safety. Indifference curve I_{L1} provides less utility than indifference curve I_{L2}. A conclusion can be made that the optimum utility of any society is achieved when the derivative of the technology curve and the derivative of the indifference curve are equal. (Kubler (2007)) The following figure adopted from Kubler (2007) demonstrates the optimal paths of development of a typical society through utility functions.

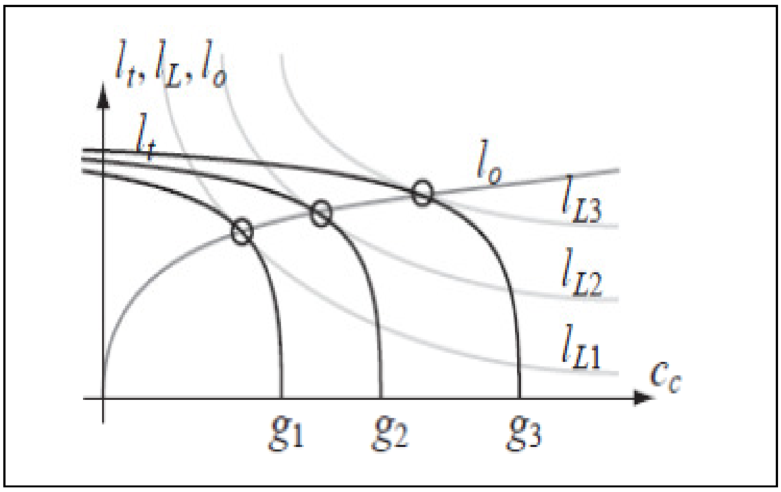

Figure 2-3: Optimum Development for a Typical Society (Kubler (2007))

In order to determine constants r and s Nathwani et al. (1997) used the work leisure optimization principle. This principle states that a person can increase his/her leisure time ((1-w) e) in two ways:

1. By increasing his/her life expectancy at birth by investing into safety.
2. By decreasing the work time fraction (w).

Nathwani et al. (1997) stated that people's choices reveal their preferences. In other words a person would just work enough to ensure that the marginal value of leisure time lost at work is equal to the marginal value of income earned, thus people optimize life expectancy (l) by adjusting the work time fraction (w). This assumption states that society is already at an optimum state when it comes to w. Remember the optimum state is where the derivative of the technology curve is equal to the derivative the indifference curve. The derivative of the indifference curve in terms of the work time fraction (w) is equal to zero as it has constant utility. Taking the derivative of equation 2-4 in terms of w, showing the utility of a society for certain (GDP per capita) g and l values, and setting it equal to the derivative of the indifference curve to solve constants r and s for a society at an optimum state:

$$\boldsymbol{r = s\frac{w}{1-w}} \qquad \textbf{2-5}$$

Kubler (2007) has shown that the LQI criterion is not dependent on the magnitude of the sum of constants r and s so for simplicity it is set equal to one:

r + s = 1

r = w

s = 1-w

$$\boldsymbol{L = g^{w}e_0^{1-w}(1-w)^{1-w} \approx g^{w}e_0^{1-w}} \qquad \textbf{2-6}$$

The term (1-w) $^{1-w}$ does not change significantly over time and between different societies and is approximately equal to 1.

The LQI can be further simplified to (Rackwitz (2005)):

$$\boldsymbol{L = \frac{g^{q}}{q}e_0} \qquad \textbf{2-7}$$

The variable q is equal to w/1-w. The reason why the equation is divided by q is for practical reasons (Rackwitz 2008 [32]). When using equation 2.6 to determine the LQI for two societies with the same g and life expectancy, the LQI is higher for the society with a higher w. This effect is cancelled out by dividing by q.

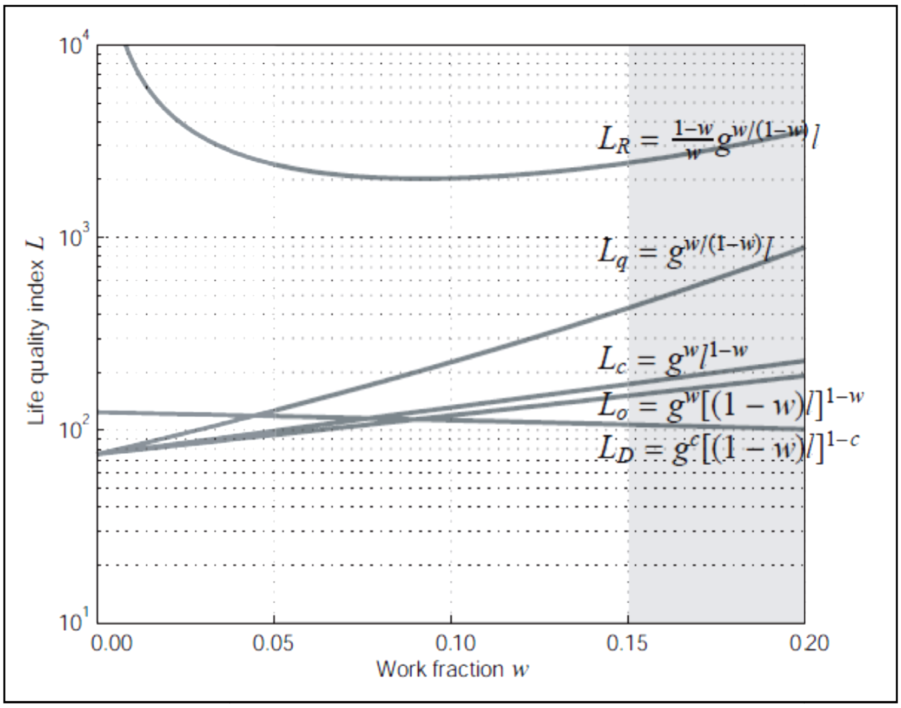

Figure 2-4: Different formulations of the LQI (Kubler 2007)

Nathwani's derivation is not the only derivation. Ditlevsen (2004) and in Ditlevsen (2003) proposed a similar looking formulation of the LQI with some minor changes. Firstly Ditlevsen takes into account the unpaid work necessary to for individual maintenance in order to produce wealth. Ditlevsen also argues that any individual needs sleep in order to produce wealth, thus the work time fraction is redefined as the time spend working divided by the total time excluding sleep (16h instead of 24h). Ditlevsen also defines two work time ratios known as w_0 and w_c. The variable w_0 corresponds to the time spent on necessary unpaid work (cleaning, cooking, education etc), while w_c corresponds to the sum of the time spent doing unpaid maintenance work and the time spent on work producing an income.

Ditlevsen (2004) proposes that the LQI formulation should look as follows:

$$\boldsymbol{L = g^r((1-w)e_o)^{1-r}} \qquad \textbf{2-8}$$

Where r is a worldwide constant and not set equal to w as was done by Nathwani et al 1997. Ditlevsen also concluded that the value r should be equal to 0.3 from two functions relating production to active time at work, thus Ditlevsen is denying the work leisure optimization principle (life quality is not optimized or even changed significantly by moderately adjusting w).

Rackwitz (2008) stated that data supports the assumptions made by Nathwani to derive the LQI. Based on this, Nathwani's formulation is chosen as the LQI formulation for this study.

2.2 The Cobb-Douglas Production Function

The formulation of the LQI above is based on the assumption the g (GDP per capita) is the product of the work time fraction (w) multiplied by the labour productivity (p). This formulation of g is relatively simple and only reflects how the GDP is produced in part.

The Cobb-Douglas production function is defined as follows (Rackwitz (2008) [32]):

$$\boldsymbol{Q = AK^{\alpha}L^{\beta}} \qquad \textbf{2-9}$$

Where Q is the output of a firm or GDP of a macroeconomic society, L is labour input, K is capital input and A is a technology constant. When α + β =1, the production will double if both variables K and L are doubled. This phenomenon is known as the return to scale property. Through mathematical manipulation, it is seen that β is the labour output to total output (wages to total GDP).

L and g are defined as follows:

$$\boldsymbol{L = N_{pop}w} \qquad \textbf{2-10}$$

$$\boldsymbol{g = \frac{Q}{N_{pop}} = A\left(\frac{K}{N_{pop}}\right)^{\alpha} w^{\beta}} \qquad \textbf{2-11}$$

From the new definition of g the final formulation of the LQI is given in 2-7 with q derived as before, but using the Cobb-Douglas production function as a definition of g:

$$\boldsymbol{q = \frac{1}{\beta}\left(\frac{w}{1-w}\right)} \qquad \textbf{2-12}$$

β is the ratio of the wages of a country divided by its total GDP. The variable q can be seen as a measure of how much value a society places on being rich (having a high GDP) or having a higher quality of life and less money (higher life expectancy). A society with a high q would rather hold on to its riches and invest less in life safety and quality where a society with a low q would rather be poorer, but have a higher life quality and longer life expectancy (Rackwitz 2008 [32]). The Cobb-Douglas production function has created a more accurate definition of q than the previous model by including β in the formulation.

The Cobb-Douglas production function has been disputed over in different economic literature, however the empirical evidence suggests that it is a good approximation of how the GDP is produced in different countries under different situations. (Rackwitz (2008) [32])

2.3 The Work Time Fraction

From the mathematical derivation of the LQI, it is assumed that society is optimizing life quality by marginally adjusting w. Searching for empirical evidence for this assumption it is clear that even though most European countries show evidence of this trend there are a few countries which form an exception from this assumption. (Rackwitz (2008) [32])

The original formulation of the work time fraction is as follows (Nathwani et al 1997):

$$W_{LWT} = \frac{life\ working\ time}{e_o} \times \frac{yearly\ working\ hours\ per\ employee}{365 \times 24} \tag{2-13}$$

Rackwitz proposed a different formulation (Rackwitz (2008)[32]):

$$W_{LF} = \frac{participating\ labour\ force}{total\ population} \times \frac{yearly\ working\ hours\ per\ employee}{365 \times 24} \times \frac{9}{8} \tag{2-14}$$

Nathwani's formulation is the work time fraction for a lifetime, while Rackwitz's formulation is a yearly work time fraction. The 9/8 factor which Rackwitz has included is due to the one hour of commuting required for eight hours of work. The two formulations will rarely be equal, as life expectancy, population growth rates and life working time changes over time.

A study was conducted by Beilenski to try and empirically verify the leisure optimization principle. A survey was conducted where couples from 16 different Western European countries were asked if they would work longer given that they will earn a higher income. These are the results in Table 2.1 (Rackwitz (2008)[32]):

Country	GDP in PPP US$	Growth rate per capita in%	Unemployment Rate in %	Part-time employment in %	Avr current weekly hours	Avr preferred weekly hours	Differ in %
Austria	26310	2.0	5.4	12.6	66.6	62.1	-6.8
Belgium	27500	2.2	8.4	14.0	65.4	62.0	-5.2
Denmark	25500	1.6	5.3	21.5	68.5	61.8	-5.5
Finland	22900	2.0	9.8	9.9	67.7	66.3	-2.1
France	24470	1.7	9.7	14.7	62.4	66.2	6.1
Germany	25010	1.9	9.9	17.1	60.8	59.6	-2.0
Greece	16900	0.9	11.3	4.3	65.1	67.3	3.4
Ireland	25470	4.0	4.1	18.3	61.8	58.3	-5.7
Italy	23400	2.1	10.4	11.8	58.0	58.9	1.5
Luxemburg	36400	3.9	2.7	7.6	58.0	55.8	-3.8
Netherlands	26170	1.8	2.6	30.4	58.3	55.9	4.1
Portugal	17000	2.9	4.3	9.3	59.1	70.8	19.7
Spain	19300	2.2	14.0	7.9	54.4	66.0	21.1
Sweden	23770	1.4	6.0	14.5	69.3	65.9	-5.0
UK	23500	2.0	5.5	23.0	66.4	58.9	-11.3
Norway	29760	2.6	3.0	20.7	66.4	66.2	-0.4

Table 2-1: Actual and Preferred Working Hours of a Couple Household (Rackwitz (2008) [32])

From Table 2-1 it is clear that there is a correlation between work preference and income (GDP per capita). In richer countries people tend to want to work less with some exceptions such as France. In poorer countries people tend to want to work more. This proves that there is to some degree a leisure optimization principle at work, however it does not confirm that the society is at an optimal state.

It can however be stated that most societies are at least close to the optimum as shown in Figure 2-5 by Rackwitz (2008). In the figure, realistic values were given to some of the random variables in the LQI formulation shown in equation 2-4 (Rackwitz (2008) [32]). It can be seen that the optimum w is in between 0.09 and 0.11. From Figure 2-6, it can be seen that the mean work time fraction of the various countries are close to the optimum. It can also be seen that the LQI does not change significantly with values of w ranging between 0.08 and 0.12 which might explain why some countries with similar g and l has a slightly different w.

Furthermore the study conducted by Rackwitz (2008) is only limited to Western European countries and there are many factors such as cultural aspects, trade unions, government subsidence and unemployment contributing to societies preferences when it comes to work (not only GDP per capita). Thus whether this assumption can be made for all societies is still questionable as shown by the exceptions in the table above.

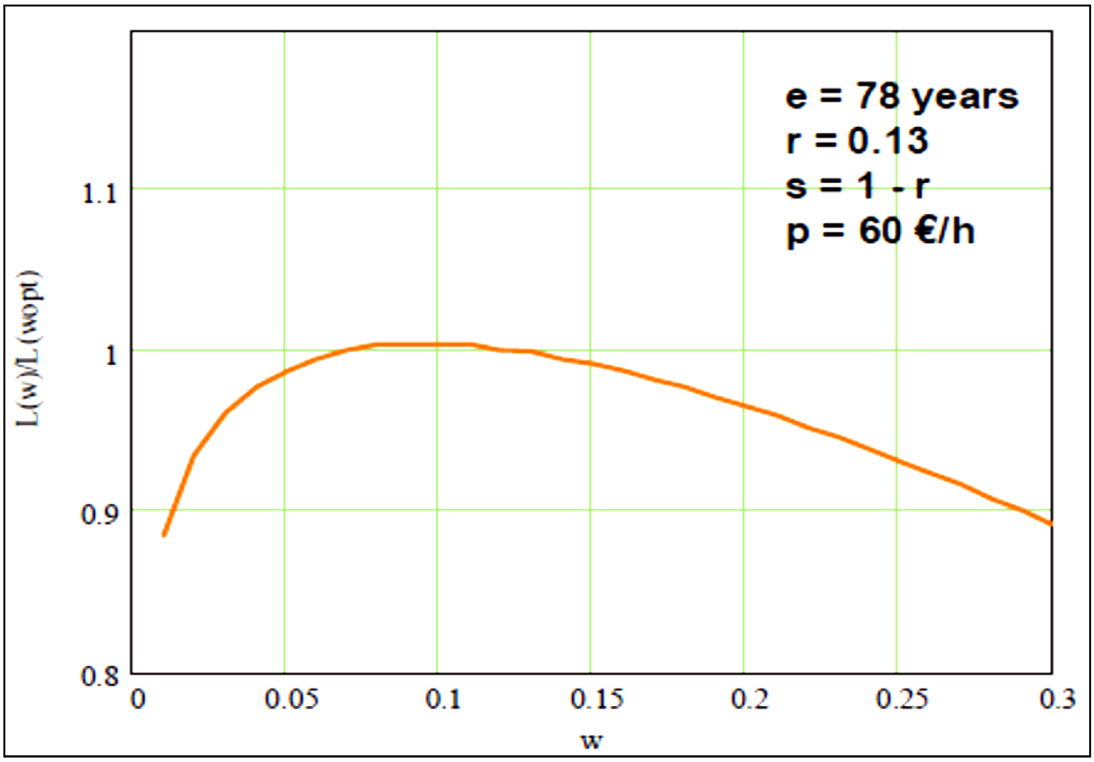

Figure 2-5: LQI for different values of w (Rackwitz (2008) [32])

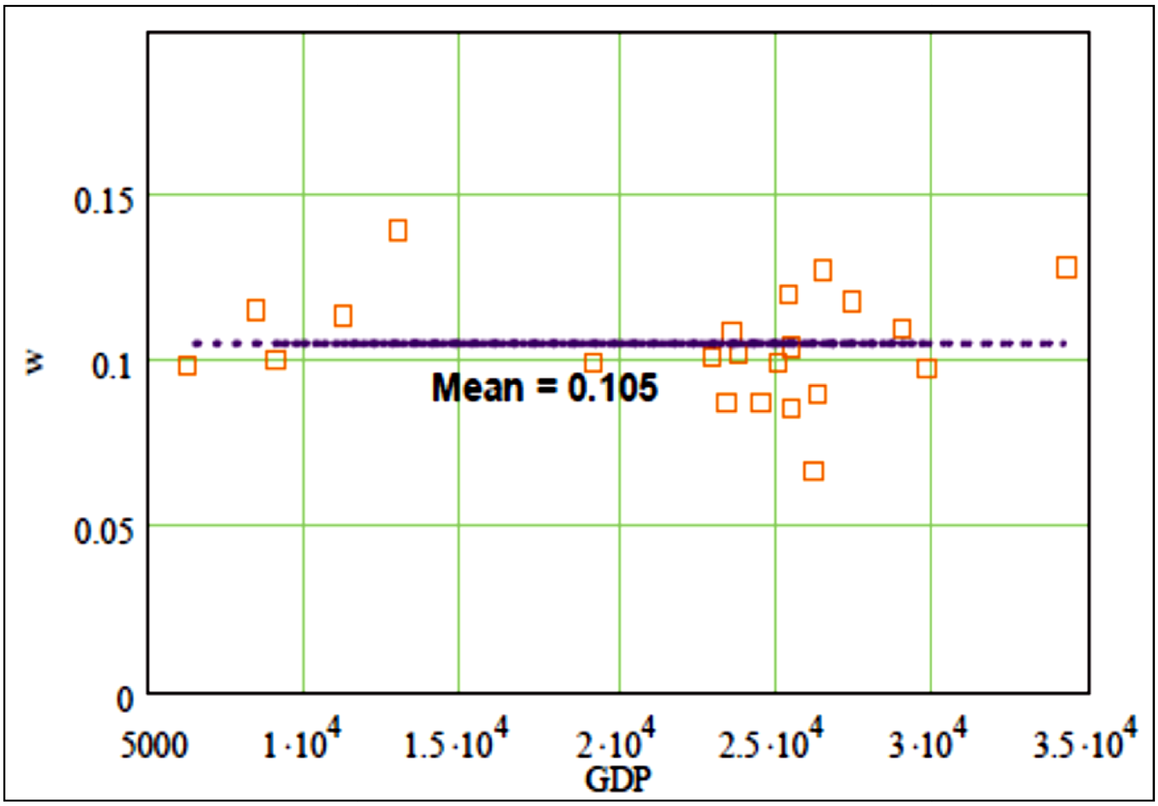

Figure 2-6: Work time fraction for different countries (equation 2.14) (Rackwitz (2008) [32])

2.4 Discount Rates

One Dollar today does not have the same value 10 years from now, in other words the value of money changes over time. It is necessary to apply the same concept when conducting a benefit/cost analysis of civil engineering infrastructures by discounting future utilities. However which rate should be used?

While the owner of the infrastructure might take the rates from the financial markets (privately owned infrastructures), obtaining an interest rate to perform a cost optimization analysis of publicly owned infrastructure is difficult as there are many different opinions on which discount rate to use.

2.4.1 Ramseyan Formulation

This section will focus on the different components that make up a sustainable discount rate from the public perspective based on the classical Ramseyan approach.

The magnitude of a discount rate must be carefully chosen or estimated. When using a high discount rate future generations will be negatively affected, but when using a low discount rate the present generation will carry too much of the financial burden. In order to understand the above statement one must think in terms of a benefit/cost optimization of a structure. The main goal of a benefit/cost optimisation is to minimize the total cost defined as the initial cost plus the failure cost multiplied by the probability of failure. If the failure costs are relatively high, the optimum solution would be to invest more into the safety of a structure. All benefit/cost analysis are conducted at the present time with present values. Suppose a structure fails 30 years from now, that failure must be discounted to a present value. Suppose a low discount rate is used, the present value of the failure will be close to the initial cost of the future failure. This will require the initial investment into the safety of the structure to be high. If the discount rate was high, less money will be invested into the safety of a structure.

Firstly the classical Ramseyan approach for a long term sustainable discount rate is as follows (Rackwitz et al. (2004)):

$$\boldsymbol{\gamma = p + \varepsilon\delta} \qquad \textbf{2-15}$$

Where p is the pure time preference rate of a society, ε is the elasticity relating to economic growth (usually equal to 0.8 or 1-q), δ is the economic growth of a society net of inflation and γ is the long term sustainable discount rate. The economic growth of a society is typically between 0.9% and 2%, but can be easily calculated by the following equation (Rackwitz (2002)):

$$\boldsymbol{r = \frac{ln\left(\frac{g2011}{g1850}\right)}{2011-1850}} \qquad \textbf{2-16}$$

Where the variable g represents the real GDP per capita for a specific year and r is calculated over the time period from 1850 to 2011.

The pure time preference rate (p) is difficult to calculate for a society as it is a psychological phenomenon where people value something less simply because it will be received in the future and not now (Bayer (2003)). This is a result partly due to the fact that humans are mortal and won't live forever, but also to impatience and economic myopia (short-sightedness). In other words in a society where people are unsure of their future, the time preference rate will be high.

Kula conducted a study where the social time preference rates were calculated for both Canada and the USA. The study was made under the assumption that a person discounts future utility simply because that person might not be alive to enjoy it.

The equation 2.17 was derived through the mathematical manipulation of a function showing present value of consumption stream (Kula (1984)):

$$\boldsymbol{P = (1+g)^e \left(\frac{1}{\pi}\right) \text{-}1} \qquad \textbf{2-17}$$

In the equation 2-17, g corresponds to the growth rate of consumption per capita, while e is the elasticity of marginal utility of consumption and π corresponds to the probability of survival by which future utility is discounted. These parameters can be calculated by the following equations:

$$\boldsymbol{log(c) = A + g(t)} \qquad \textbf{2-18}$$

$$\boldsymbol{e = \left|\frac{e_1}{e_2}\right|} \qquad \textbf{2-19}$$

Where c is the consumption per capita, A is a constant, e_1 is the income elasticity of the food demand equation and e_2 is the compensated elasticity of the food demand equation which can be obtained from economic data of a country. The results of Kula's study are given in the table below:

Pure Preference Time Rate	
United States	$(1 + 0.23)^{1.89}/(1/0.991) - 1 = 0.053$
Canada	$(1 + 0.28)^{1.56}/(1/0.992) - 1 = 0.052$

Table 2-2: P for Canada and USA (Kula (1984))

The results of this study are realistic and similar to another study conducted for the USA which yields results around 5% (Kula (1984)).

Some argue that the pure preference time rate should be zero based on intergenerational ethical grounds (Rackwitz (2008) [32]). Arrow (1995) argues that the pure preference rate should be included in the discount rate, as having a low discount rate would put too much financial strain on the present generation and proposes a discount rate of 3%. The Ramseyan model has been questioned by economic literature (Rackwitz (2008) [32]) and a different, time dependent model was proposed by Bayer/Cansier which is explained in the next section.

2.4.2 Intergenerational Discounting

Bayer (2003) proposed a model known as overlapping generation or generation adjusted discounting model. The main idea is to discount for the living generation by a discount rate, $\gamma = p + \varepsilon\delta$, while discounting with a rate $\varepsilon\delta$, for the generation not yet born.

Assuming that the consumption or loss effect (c_m) occurs at time m (time) in the future, the consumption is distributed equally among generations, a generation has a mean renewal time of L and each generation has the same preferences, the present value of this loss effect can be given by the following stationary model equation (Rackwitz et al. (2004)):

$$PV_{OLG} = \begin{cases} \sum_{j=0}^{m-1} \frac{C_m/G}{(1+\varepsilon\delta)^{m-j}(1+\rho+\varepsilon\delta)^j} + \sum_{j=m}^{L} \frac{C_m/G}{(1+\rho+\varepsilon\delta)^m} & for\ m \leq L \\ \sum_{j=0}^{L} \frac{C_m/G}{(1+\varepsilon\delta)^{m-j}(1+\rho+\varepsilon\delta)^j} & for\ m > L \end{cases} \qquad \textbf{2-20}$$

The present value can also be given by: ((Rackwitz et al. 2004)

$$PV_R = \frac{C_m}{(1+\gamma)^m} \qquad \textbf{2-21}$$

The equation above is the standard equation in economics for converting a future value (Consumption effect C_m) to a present value using compound interest (interest on interest) where γ is the discount or interest rate.

From the two equations above a time dependent discount rate can be derived (Rackwitz (2008) [32]):

$$\frac{C_m}{(1+\gamma(t))^m} - PV_{OLG}(m) = 0 \qquad \textbf{2-22}$$

Another discounting model, known as the overlapping age group model or OLAG, is where the mass age distribution function (*h(a, n)*) is included in the formulation of the present value to distribute some loss effect evenly between members of a society.(Rackwitz et al. (2004))

$$PV_{OLAG} = \begin{cases} \sum_{a=0}^{m-1} \frac{C_m h(a,n)}{(1+\varepsilon\delta)^{m-a}(1+\rho+\varepsilon\delta)^a} + \sum_{a=m}^{a_u} \frac{C_m h(a,n)}{(1+\rho+\varepsilon\delta)^m} & for\ m \leq a_u \\ \sum_{a=0}^{a_u} \frac{C_m h(a,n)}{(1+\varepsilon\delta)^{m-a}(1+\rho+\varepsilon\delta)^a} & for\ m > a_u \end{cases} \qquad \textbf{2-23}$$

The mass age distribution function can be easily obtained from a life table and will be explained in more detail later. The a in h(a,n) is the age, while n is the population growth rate and h is the percentage a specific age group forms of the total population. The variable a_u is the oldest age in the life table of a country, typically equal to 100 years.

The time dependent sustainable discount rate can be solved by equating the PV_{OLAG} to the standard equation of determining an equivalent present value from a future value as follows: (Rackwitz et al. (2004))

$$\gamma'_E(m) = \left(\frac{C_m}{PV_{OLAG}(m)}\right)^{1/m} - 1 \qquad \textbf{2-24}$$

The following figure by Rackwitz is based on equations 2-24 and 2-22, shows the discount rate for different models assuming the following values: ρ=0.03; ε=1; δ=0.02; L=40years; Using the Swiss life table for a_u and *h(a, n)*.

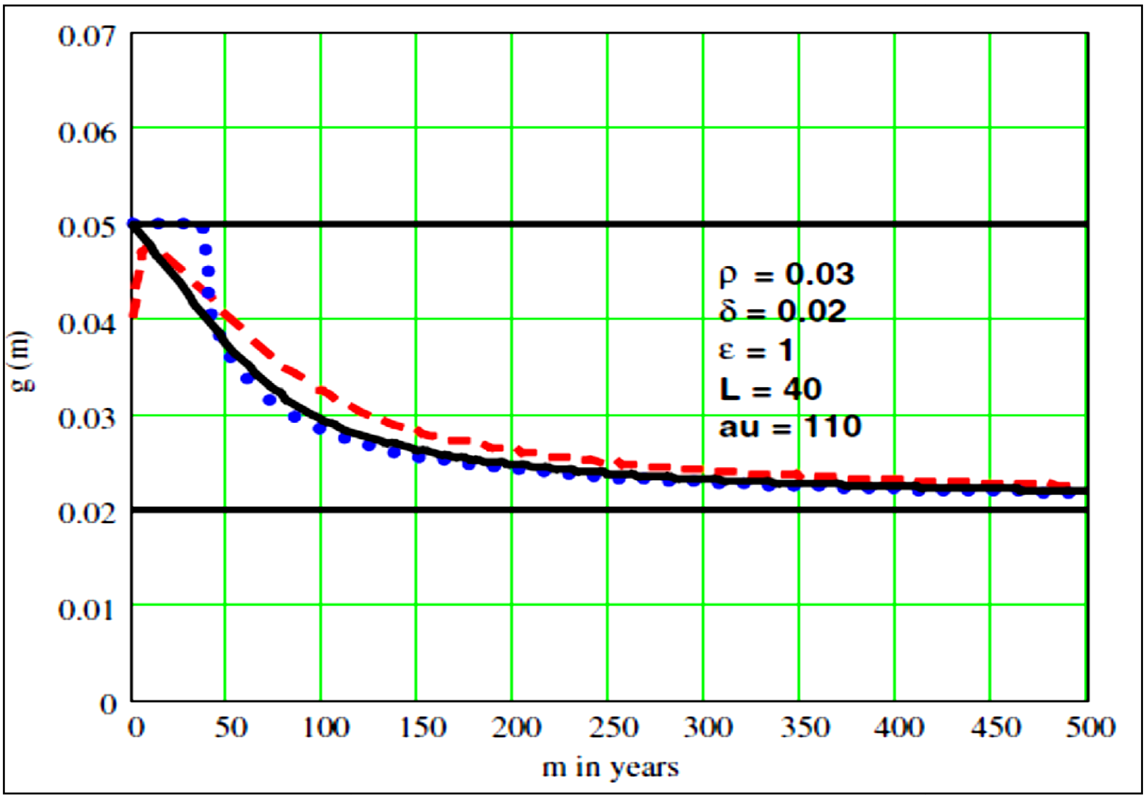

Figure 2-7: Time Dependent Discount Rates using different Models (Rackwitz et al. (2004))

The solid line is represents the OLAG model, the striped line represents the OLAG model excluding the age mass distribution function and the dotted line represents the OLG model. In the above figure, m denotes the expected useful lifetime of the project or facility. When the facility has a long expected lifetime a low discount rate is used, because the majority of people using this facility are part of the future, unborn generation. The burden is thus not transferred from one generation to the next generation. This view is a much more ethically defensible than that of the classical Ramseyan view where a constant discount rate is proposed.

It is also clear that there is little difference between the different models and that these models can be closely approximated by the following function:

$$\gamma(m) \approx \varepsilon\delta + \rho exp[-am] \qquad \textbf{2-25}$$

Where a is a constant typically equal to 0.013. (Rackwitz (2008) [32])

2.5 Derivation of Different Monetary Compensations & Life saving Costs

2.5.1 Life Saving Costs

From the different formulations of LQI, an acceptability criterion based on society's preferences can be derived. The LQI formulation assuming 1/q = K and $\beta(1 - w)^2/w = 1$ (Nathwani et al.(2009)) is as follows(Nathwani defined E as life expectancy and G is defined as GDP per capita):

$$\boldsymbol{L = E^K G} \qquad \textbf{2-26}$$

From this formulation, Nathwani et al. (2009) assumed that some risk related project causes a small change in LQI as stated by the derivative:

$$\boldsymbol{\frac{dL}{L} = K\frac{dE}{E} + \frac{dG}{G}} \qquad \textbf{2-27}$$

For the risk related project, the change in LQI must be larger than or equal to zero in order for the project to be justified. From this SCCR can be derived.

$$\boldsymbol{-\frac{dG}{dE} \geq \frac{KG}{E} = SCCR} \qquad \textbf{2-28}$$

In this case the units of SCCR (Societal Capacity to Commit Resources) are defined as money/year/life year/person saved. This acceptability criterion is flexible, but it is difficult to use. Investment into some life saving intervention will produce a change in mortality rate. From the change in mortality rate a change in remaining life expectancy for each age group can be calculated from the actuarial life table of the society as follows:

$$\mathbf{e}(\boldsymbol{a}) = \int_{\boldsymbol{a}}^{\boldsymbol{a_u}} \boldsymbol{exp}\left[-\int_{\boldsymbol{a}}^{\boldsymbol{t}} \mu[\boldsymbol{\tau}]\boldsymbol{d\tau}\right]\boldsymbol{dt} \qquad \textbf{2-29}$$

Where μ(a) is the age dependent mortality rate, a is the age group in the life table, a_u is the oldest age group in the life table and t is the time in years.

The project is acceptable if:

$$\boldsymbol{SCCR} \times \boldsymbol{\Delta E} \times \boldsymbol{E_A} - \boldsymbol{C} > 0 \qquad \textbf{2-30}$$

Where ΔE is the change in life expectancy, E_A is the age specific remaining life expectancy and C is the investment in to safety. Note that this is if only one age group is exposed to a risk. If the risk is uniformly distributed over all age groups, the first part of the acceptability criterion (SCCRxΔExE_A) is calculated for each age group and summed over the age distribution (Age averaged). It should also be taken into account that money has a changing buying power over time and the above criterion still has to be discounted which further complicates the procedure.

There exists a slightly different acceptability criterion based on the same formulation of the LQI (q=1/k). This acceptability criterion is derived in the same way as Nathwani's formulation by assuming some investment into safety as a minimum requirement results in a constant LQI. By taking the derivative of the LQI formulation the acceptability criterion can be derived as follows: (Rackwitz (2008) [32])

$$dL = \frac{\partial L}{\partial l} dl + \frac{\partial L}{\partial g} dg = 0 \qquad \textbf{2-31}$$

Inserting equation 2.4

$$-dg \leq \frac{g}{q}\frac{de}{e} \qquad \textbf{2-32}$$

Age averaging the de/e term leads to:

$$\mathbf{E}\left[\frac{de}{e}\right] = \int_0^{a_u} \left[\frac{de(a)}{e(a)}\right] h(a, n) \qquad \textbf{2-33}$$

Where h(a, n) can be defined by:

$$h(a, n) = \frac{exp[-na]\int_0^a exp\mu(t)dt}{\int_0^{a_u} exp[-na]\int_0^a exp\mu(t)dtda} \qquad \textbf{2-34}$$

Where n is the population growth rate usually taken as an average over the last few years and μ is the mortality rate. Assuming that a change in life expectancy due to an infinitesimal change in mortality is defined as follows (discounted by an exponential function): (Rackwitz (2008) [32])

$$\mathbf{E}\left[\frac{de(a,\rho,\delta)}{e(a,p,\delta)}\right] \approx \mathbf{E}\left[\frac{\frac{d}{dx}e(a,\rho,\delta,x)|_{x=0}\,x}{e(a,p,\delta)}\right] = C_x(\rho,\delta)x \qquad \textbf{2-35}$$

Where C_x is the demographic constant, which is dependent on type of mortality reduction (x), discount rate and the age distribution of the population. The future life years are discounted by an exponential discounting model.

Based on the above, for small reductions in mortality, Societies Willingness to Pay (SWTP) to save a life can be given by: (Rackwitz (2008) [31])

$$SWTP = \frac{g}{q} C_x(a, n, \rho, \delta) \qquad \textbf{2-36}$$

In this case, SWTP has a unit of money/prevented fatality and is based on the same concepts as SCCR. In the case of SCCR the de/e term not simplified further by elegant mathematical manipulations.

There are three different mortality reduction schemes. One of these schemes is the delta mortality reduction scheme, where the change in mortality is not age dependent and constantly distributed over all age groups. A practical example of this situation would be a collapse of a structure or building where the probability of dying is equally distributed among all age groups. (Rackwitz (2008) [31])

The change in mortality can be given by the following function:

$$\mu_{a,\Delta} = \mu_a + \Delta \tag{2-37}$$

The delta demographic constant is given by:

$$C_\Delta = \int_0^{a_u} \frac{\int_a^{a_u} (t-a)exp\left[-\left(\int_a^t (\mu(\tau)+\rho(\tau))d\tau + \delta(t-a)\right)\right]dt}{\int_a^{a_u} exp\left[-\left(\int_a^t (\mu(\tau)+\rho(\tau))d\tau + \delta(t-a)\right)\right]dt} h(a,n)da \tag{2-38}$$

For this study it is assumed that ρ (pure preference time rate) is the same for all age groups and thus not dependent on age (a).

The following scheme is the Pi mortality scheme, where the change in mortality is age depended. An example of this situation is investing into health care. Older people are more depended on healthcare than younger people. Thus investing into health care will have a higher influence on the mortality of older people. The following function shows the age depended change in mortality: (Rackwitz (2008) [31])

$$\mu_{a,\pi} = \mu_a(1+\pi) \tag{2-39}$$

The last scheme is known as the alpha mortality scheme, where there exists a change in mortality only for a certain age group or age groups. The change in mortality is given by the following equation: (Rackwitz (2008) [31])

$$\mu_{a,\alpha} \begin{cases} = \mu_a + \alpha & \text{for } a > 60 \\ = \mu_a & \text{for } a \leq 60 \end{cases} \tag{2-40}$$

Another model used to determine what society can afford to spend on saving a life is known as ICAF (Implied Cost to Avert a Fatality). The ICAF is formulated as follows: (Rackwitz (2002))

$$\Delta g = g\left[1 - \left(1 + \frac{\Delta e}{e}\right)^{1-\frac{1}{w}}\right] \tag{2-41}$$

When applying the above formulation to technical facilities, one should set e to Δe and multiply Δg by e, because the above formulation is a yearly cost and undiscounted. By applying these changes, one can derive the amount of money a society can spend on averting a fatality or (ICAF). The above formulation is derived by clever mathematical manipulations of one of the older versions of the LQI and has a unit of money/prevented fatality/year. (Rackwitz (2002))

SCCR has a unit of money/year/life year/person saved and is highly flexible, but difficult to use as it must be discounted and age-averaged. The ICAF is similar to SCCR as it is a yearly cost and undiscounted, therefore for this study SWTP will be used as it is discounted, age averaged and easier to apply than the other two life saving costs. From these life saving costs the minimum safety can be determined as they are based on the LQI criterion.

2.5.2 Life Compensation Costs

It is noted that one must not confuse SHC (Societal Human Capital) with SCCR, SWTP or ICAF. SHC is the amount of money a society can afford to compensate for a lost life where the other formulations are the amount of money a society can afford to invest to save a life. SHC can be understood as the possible earnings lost and can be calculated as follows: (Rackwitz (2008) [31])

$$\boldsymbol{SHC = g \int_0^{a_u} l(a)h(a,n)} \qquad \textbf{2-42}$$

Another formulation of a compensation cost is the SVSL or societal value of a statistical life. The formulation is as follows:

$$\boldsymbol{SVSL = \frac{g}{q} e(a,\rho,\delta,n)} \qquad \textbf{2-43}$$

SVSL multiplied by the change in mortality will be equal to SWTP and can thus be understood as the societal monetary amount to reduce a risk by unit mortality. For this particular study SWTP and SVSL will be used as the derivations of these two formulations are clearly shown in the literature. In addition to this, Nathwani derived a utility function known as SLQI, Societal Life Quality Index. In another independent study conducted by Shepard and Zeckhauser the SVSL was derived and the two formulations SLQI and SVSL are identical when the SLQI is divided by a marginal utility. SVSL also assumes that the risk is distributed equally between all age groups, which is typically the case for structural failure. Therefore for this particular study SVSL will be used as the compensation cost. (Rackwitz (2005))

It is further important to determine which part of the GDP is available to invest into safety. This is still a widely disputed topic as there are various opinions on which part of the GDP is available for risk reduction. The entire GDP should be seen as an upper limit. About 20% of the GDP is used by the government while 10% is used for re-investment. This leaves about 70% available for private consumption and for investment in to safety. It is recommended that a lower bound of the GDP available for safety measures should be about 60%. (Rackwitz (2008) [31])

2.6 Reliability Based Optimization of Technical Facilities

The optimal target reliability of a structural member can be determined by applying a benefit/cost analysis of the technical facility. The following equation is a basic formulation of this process: (Rackwitz (2000))

$$\boldsymbol{Z(p) = B(p) - C(p) - D(p)} \qquad \textbf{2-44}$$

It is assumed that the variables above can all be represented in monetary terms by a vector p of safety related parameters. B(p) is the benefit of the existing structure, C(p) is the cost associated with the construction of the structure and D(p) is cost associated with the failure of the structure. Another assumption is that the parameters above are differentiable in terms of p. In the view of sustainability, one has to at least consider four different replacement strategies: (Rackwitz et al. (2004))

- The facility is given up after service or failure
- The facility is systematically replaced after failure
- The facility is renewed after deterioration
- The facility is renewed due to obsolesce

Another classification is made between facilities that fail upon completion or never (time invariant) and facilities that fail at a random point in time due to extreme loading (time variant). It is also important to optimize the structural member from the public's point of view or from the owner's point of view as the two parties have different economical interests. For this particular study, structures will be optimized from the public's perspective. (Rackwitz et al. (2004)) The following figure shows the optimization process in terms of safety parameter p: (Rackwitz (2000))

For systematic reconstruction after failure, negligibly short reconstruction times and a single mode of failure the appropriate renewal model can be given as follows:

$$\boldsymbol{Z(p) = \frac{b}{\gamma} - C(p) - (C(p) + H_m + H_F)h^*(\gamma, p)} \qquad \textbf{2-45}$$

Where b is the benefit per time unit or benefit rate, H_m is the other failure costs (environmental, injuries, economical & equipment loss), H_F is the compensation cost (Number of fatalities x SVSL), the term, $h^*(y,p)$, is the Laplace transform of the renewal intensity or the failure rate and γ is the long term sustainable discount rate. The cost of the structural member can be defined by: (Rackwitz (2008) [31])

$$C(p) = C_0 + C_1 p \qquad \text{2-46}$$

Where C_1 is the cost associated with the most cost effective parameter to increase the reliability of the structural component. (Cost of increasing safety)

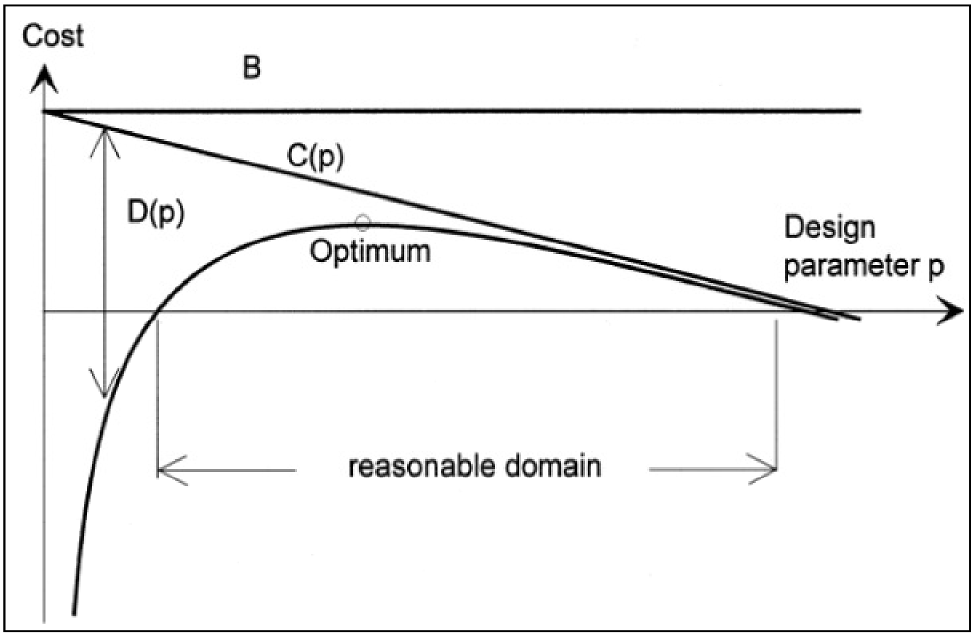

Figure 2-8: Benefit/Cost Optimization Process (Rackwitz (2000))

Assuming that some extreme event has a Poison occurrence rate of λ the failure rate can be defined by: (Rackwitz (2008) [31])

$$h^*(\gamma, p) = \frac{\lambda P_f(p)}{\gamma} \qquad \text{2-47}$$

The 1/λ can be understood as the time period a live load changes significantly. For example the furniture in an office building is changed every 5-years, therefore λ is equal to 0.2 in units of 1/year. (Holický (2009))

Structures become obsolete after a period of time because they no longer fulfil their original requirements. For the case of obsolescence Rackwitz proposed the following equation: (Rackwitz (2000))

$$A(p) = (C(p) + A)\frac{\omega}{\gamma} \qquad \text{2-48}$$

Where A is the cost of demolition and ω is the obsolescence rate typically equal to 2%. The cost benefit function can finally be defined as follows: (Rackwitz (2000))

$$Z(p) = \frac{b}{\gamma} - C_0 - C_1 p - (C_0 + C_1 p + A)\frac{\omega}{\gamma} - (C_0 + C_1 p + H_m + H_F)\frac{\lambda P_f(p)}{\gamma} \tag{2-49}$$

The optimum safety is achieved when the function Z (p) is maximised. However, the following criterion derived from the LQI has to be satisfied: (Rackwitz (2008) [31])

$$\frac{d}{dp}\left(C_0 + C_1 p + (C(p) + A)\frac{\omega}{\gamma}\right) \geq -\frac{g}{q} C_x N_f \frac{d}{dp}\left(\frac{\lambda P_f(p)}{\gamma}\right) \tag{2-50}$$

The criterion above is derived by taking the derivative of the optimisation function in terms of safety parameter p and setting it equal to zero. The left hand side can be understood as the change investment into safety of the structure. The $\frac{g}{q}C_x$ part of the right hand side of the equation is equal to SWTP. The right hand side can be understood as the change in risk to human life, remember that risk is defined as probability of failure times the cost of the failure. It is also important to note that SWTP is the minimum amount of money a society is willing to invest into saving a life. Thus the change in investment into the structure must, as a minimum requirement, be equal to or larger than the change in the reduction of a risk to human life as required by society (LQI criterion).

As long as the optimum safety is safer than the above criterion requires, the LQI criterion is not active. When the LQI criterion is active the safety parameter p is determined from the inequality above. This ensures that the structural member is safe enough from the perspective of the public. When the LQI criterion is not active, the target reliability is obtained from the optimization process where:

$$\frac{dZ(p)}{dp} = 0 \tag{2-51}$$

The probability of failure is determined from some limit state function for the mode of failure under consideration. Rackwitz (2000) recommended that the optimization process contain the cost of serviceability failure of a structure denoted by U(p). In order to simplify the approach, serviceability failure will be excluded from the cost/benefit analysis for this study. Further reasons are given in Chapter 5.

The following figure shows the results of a parameter study conducted by Rackwitz where the LQI criterion (minimum safety requirement) can be seen as P_{Lim} of the cost/benefit analysis. Rackwitz (2008) also demonstrates that the optimum (indicated by a dot) varies little with two different discount rates indicated by a solid line and a dashed line on the figure. (Rackwitz et al. (2004))

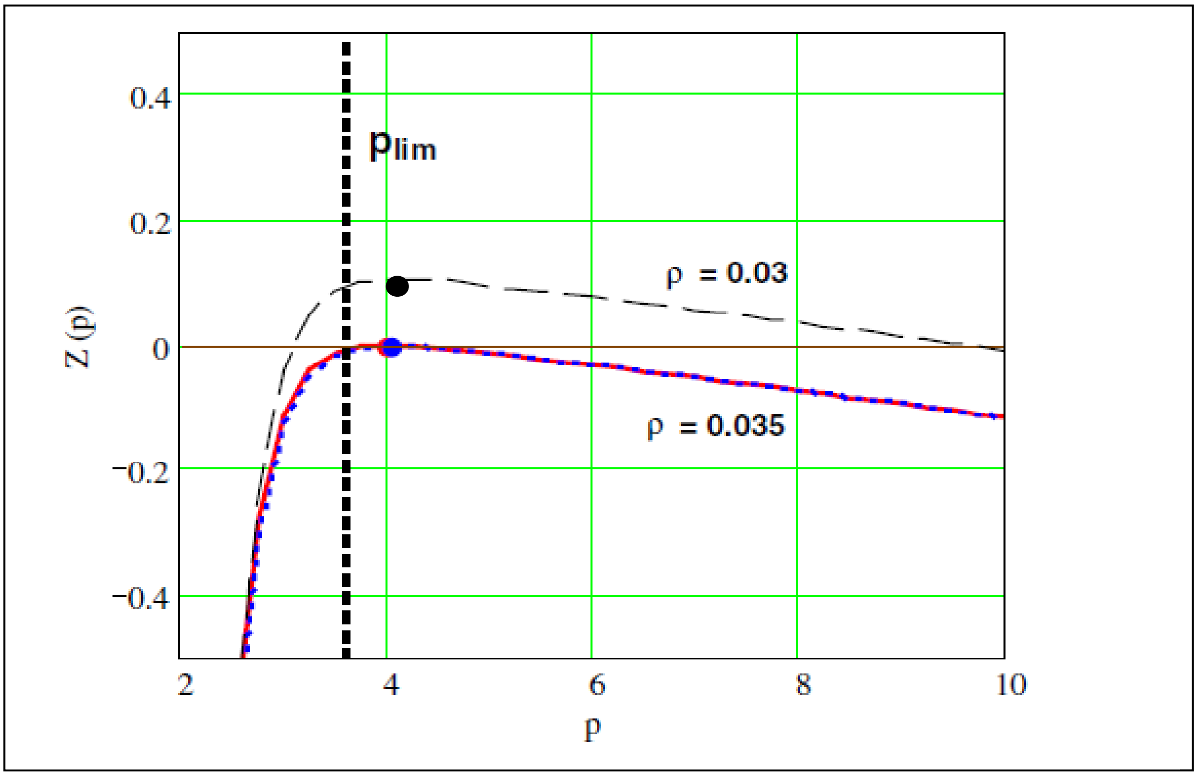

Figure 2-9: Parameter Study of Benefit/Cost Optimization Process (Rackwitz et al. (2008))

2.7 Estimation of Failure Consequences for Civil Engineering Infrastructure

A crucial component of a benefit/cost analysis to optimize structures is the estimation of failure consequences. A parameter study conducted by Rackwitz (2008) showed that the cost of increasing safety and the cost of failure consequences have a significant effect on the optimal structural safety. Two models for estimating fatalities due to structural collapse will be discussed in this section. A combination of the two models is used in the reliability optimization of this study.

2.7.1 Fatality Estimation by Lentz et al. (2004)

The basic equation determining the number of lost lives given failure ($N_{LOL|F}$) is as follows: (Lentz et al. (2004))

$$N_{LOL|F} = N_{par}\big(1 - P(Q)\big).k \qquad \textbf{2-52}$$

Where N_{par} is the population exposed to the failure, P(Q) is the probability of escape and k is the probability of dying. It should also be noted that the occupational population of a structure depends heavily on the function of the structure (Lentz et al. (2004)).The probability of a successful escape given some warning (W) can be calculated as follows:

$$\boldsymbol{P(Q) = P(W).P(Q|W)} \qquad \textbf{2-53}$$

Where the probabilities can be calculated as follows: (Lentz et al. (2004))

$$\boldsymbol{P(W) = P(W_0)P(W_{prc})P(W_{dc})} \qquad \textbf{2-54}$$

$$\boldsymbol{P(Q|W) = P\{T_w - T_Q < 0\}} \qquad \textbf{2-55}$$

Where $P(W_0)$ is the probability of a warning, $P(W_{prc})$ is the probability of perceiving the warning and $P(W_{dc})$ is the probability of attempting to escape. The probability of escaping given a warning is equal to the probability that the time required to escape (T_w) is shorter than the time it takes for the failure to occur (T_Q). (Lentz et al. (2004))

The probability of death is determined empirically by looking at previous disasters such as floods, fires and earthquakes. The model above can be applied to vastly different risk related fatality estimations such as flooding and fires and is thus highly flexible. There is however little information available for fatalities estimation in a building collapse due to a random failure of a structural component.

2.7.2 Fatality Estimation by Coburn et al. (1992)

The next model is similar, but specializes more in fatality estimation due to collapse of structures as a result of earthquakes. Coburn et al. proposed a fatality estimation model due structural failure depending on five different components. The formulation is as follows: (Coburn et al. (1992))

$$\boldsymbol{K_{sb} = D5_b[M1_b \times M2_b \times M3_b \times (M4_b + M5_b)]} \qquad \textbf{2-56}$$

Where $D5_b$ is the amount of structures collapsed due to the earthquake. In this particular study only one structure will be considered at a time. The other components are defined as follows: (Coburn et al. (1992))

$M1_b$= Population of the building

$M2_b$= Occupancy of building during time of earthquake

$M3_b$= Probability of being trapped during collapse

$M4_b$=Immediate casualties

$M5_b$=Percentage of casualties long after the collapse

This model uses information based on empirical evidence all over the world. However, this model is focussed on the fatality estimation due to general failure of structures as a result of earthquakes. For this particular study the failure consequences must be determined for the case where one of the structural components fails due to overloading. Another problem with the above model is it does not have a classification system between the type of failure (brittle or ductile).

This model does however provide empirical information on fatality estimation for trapped victims. The following tables show the immediate casualties after collapse and the percentage of casualties of the remainder of the trapped victims after the structural failure. (Coburn et al. (1992))

M4 Estimated Injury Distributions at Collapse (% of M3)		
Triage Injury Category	**Masonry**	**RC**
1) Dead or Unsaveable	20	40
2) Life threatening cases needing immediate medical attention	30	10
3) Injury requiring hospital treatment	30	40
4) Light injury not necessitating hospitalization	20	10

Table 2-3: Immediate Casualty Estimation of Trapped Victims (Coburn et al. (1992))

M5 (as % of M3-M4) Living victims trapped in collapsed buildings that subsequently die		
Situation	**Masonry**	**RC**
Community incapacitated by high casualty rate:	95	-
Community capable of organising rescue activities:	60	90
Community + emergency squads after 12 hours	50	80
Community + emergency squads + SAR experts after 36 Hours	45	70

Table 2-4: Post Event Casualty Estimation of Living Trapped Victims (Coburn et al. (1992))

In addition to the empirical data provided on probability of death of trapped victims, Coburn et al. (1992) also provides information on the probability of being trapped in RC structures 3-5 stories in height. There are two average probabilities of being trapped, 50% for a Top-Down collapse and 75% for a Bottom-Up collapse. These two collapses are shown in the following figure adopted from Coburn et al. (1992):

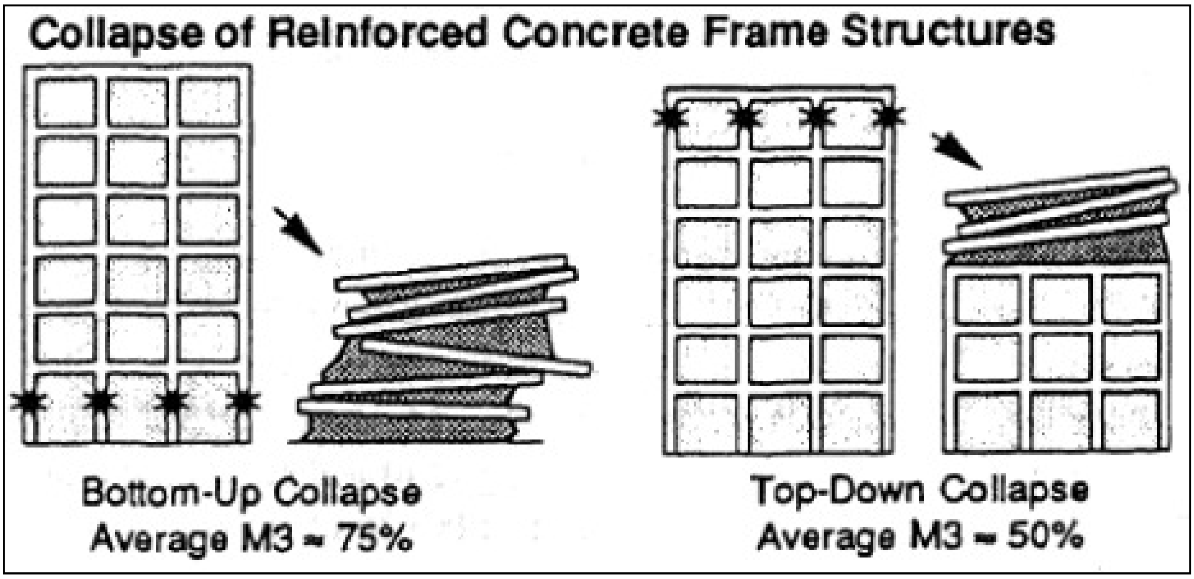

Figure 2-10: Different Type of Collapses of RC Frames (Coburn et al. (1992))

Even though the data above is based on earthquake data, the information above can be used as a basis to estimate the probability of escape for this study.

2.7.3 Estimating Area of Collapse

Estimating the area of collapse as a result of random failure of a structural component is not clearly defined in literature. However annex B.4 of the SANS 10160 Part 1 (2010) specifies that there should be some measures taken to ensure that localized failure is limited and that the structure as a whole still remains stable. For structures in RC2 effective horizontal ties should be provided, or effective anchorage of suspended floors to walls should be provided to limit damage to the structure as a whole. To restrict damage to structures in RC3 when localized failure occurs, two strategies can be adopted.

- Horizontal ties can be provided as for RC2 structures.
- "Checking the building to ensure that upon the notional removal of each supporting column and each beam supporting a column, or any nominal section of load bearing wall as defined in B.7 (one structural element at a time in each storey of the building), the building remains stable and that any local damage does not exceed a certain limit"

SANS 10160 B.4 recommends a limit of 15% of the floor, or 100m^2, whichever is smaller. For this study an assumption is made that concrete structures in RC2 are provided with horizontal ties which will limit localized failure to the smallest of 100m^2 or 15% of the floor area. Therefore in this study it is assumed that the failure of a structural element will not result in the entire structure or portions larger than 15% of the floor area collapsing.

The following figure adopted from SANS 10160 Part 1 (2010) shows the limiting of damage as a result of localized failure where a) is the plan view and b) is the side view:

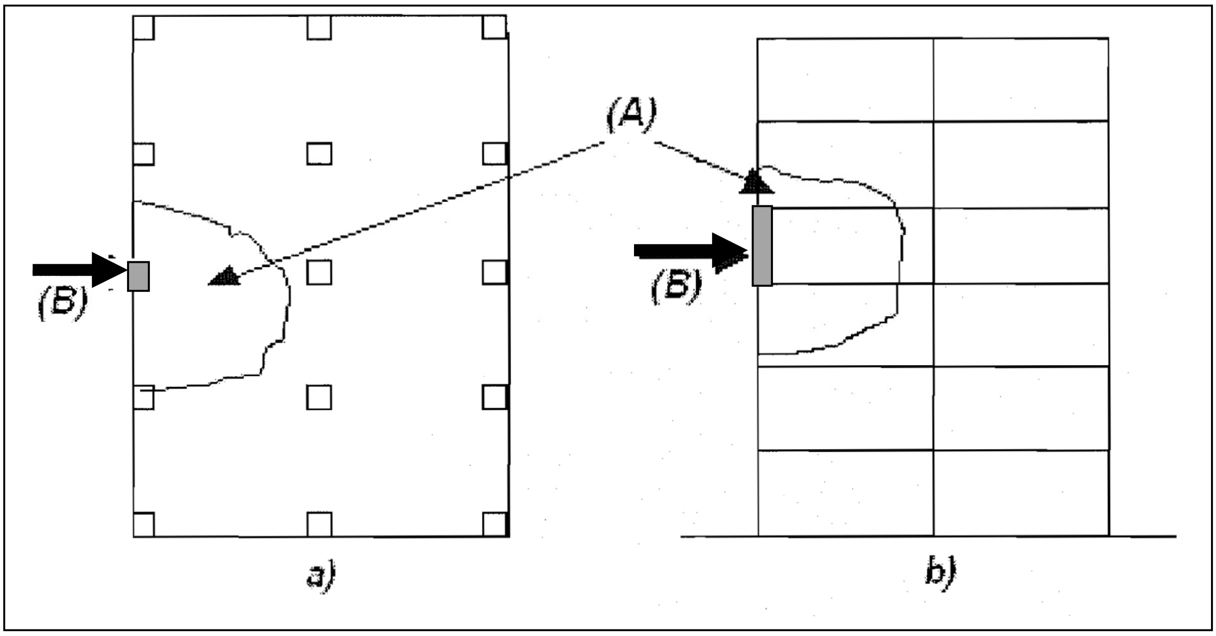

Figure 2-11: A) Localized Failure Damage Limit B) Column Removed (SANS 10160 Part 1 (2010))

2.7.4 Estimating Other Losses Due to Structural Failure

There are methods to estimate other losses due to structural failure such as economical losses and losses due to injuries, but these are often difficult to assess as they are highly case specific. In a study conducted by Kanda et al. (1997) various components of structural failure costs were defined and estimated based on earthquake data in Japan as shown in the following table.

It can be seen that as the importance of the structure increases, the loss of life component starts dominating the other cost components. The other failure costs (non-LOL (Loss of Life) costs) are approximately in the same order of magnitude of construction costs for office buildings which is the type of structure that is considered for this study.

Type of Loss	House	Apartments	Small Shops	Office Buildings	Hospitals	Nuclear Power Plant	Fire Stations
Damage to structure	0.3	0.3	0.4	0.25	0.25	0.15	0.3
Damage to Contents	0.4	0.2	0.4	0.5	0.5	0.1	0.1
Damage to non-structural components	0.2	0.2	0.1	0.3	0.3	0.2	0.2
Damage to Equipment	0.1	0.1	0.1	0.3	0.5	10	0.2
Economical (Functional)	0.1	0.1	0.2	0.2	10	2	10
Injuries	0.2	0.5	0.2	0.5	2	100	0.1
Fatalities	0.1	0.2	0.1	5	20	2000	5
Psychological Damage	1	0.2	0.2	0.1	0.1	0.1	0.1

Table 2-5: Normalized Loss Estimations (Kanda et al. (1997))

In a study conducted by Alfredo et al. (2001) to optimize the life cycle cost for a RC structure under seismic loading, a model was provided to determine the economical or functional losses of a structure. This model is known as an Input-Output or I-O model and is made up of two rounds of losses. The first round of economic losses is based on the functional loss of the structure, for example production in a factory is slowed or stopped for a period of time until the factory is repaired. The first round is formulated as follows:

$$C_{B1} = \sum_{i=1}^{n} \varepsilon_i Y_i^P \left(\frac{t_{loss}}{t_{IO}}\right) v_i \qquad \textbf{2-57}$$

Where ε_i is the economic output per unit total output of production sector i in the I-O table; Y_i^P is total output of sector i without any disaster; t_{loss} is the loss of function measured in time; t_{IO} is the time interval of the I-O model; and v_i the participation factor of sector i.

The second round loss is as a result of the output of other industries being dependent on the output of the damaged structure. The formulation is as follows:

$$C_{B2} = \sum_{i=1}^{n} \varepsilon_i \left(Y_i^* - Y_i^D\right) \qquad \textbf{2-58}$$

Where Y_1^* is the difference between the normal production of sector i and the production after the disaster of sector i; and Y_1^D is new production level.

The model above is highly case specific and time consuming to calculate the economic losses of a structure, thus for this study an estimation of the economic and other losses is made based on Table 2-5 and other calculated failure costs. Furthermore the dominating failure cost is the cost of lost lives, thus the focus of this study is to determine this cost accurately. Coburn et al (1992) also concluded from previous earthquake data that there are very few injuries relative to fatalities after a concrete structure collapses. In other words most trapped victims do not survive. Therefore the injuries component should not have a significant impact and can be estimated along with the other costs for this study.

2.8 Current Target Reliability Indices Recommended by Various Codes

Currently there are various different recommendations for target reliability indices by different codes. The PMC (Probabilistic Model Code) recommends one year target reliability indices for structures shown in Table 2-6. These target reliability indices are based on the consequence of failure and the cost of increasing the safety of the structure. The classification for the consequence classes are differentiated by a ratio ρ defined by the sum of construction costs and failure cost over construction cost as follows: (PMC Part 1 (JCSS, 2001))

- For a ρ less than 2 a structure is classified as a minor failure consequence class (Agricultural Buildings)
- For a ρ between 2 and 5 a structure is classified as a moderate failure consequence class (office buildings, industrial buildings, apartment buildings)
- For a ρ between 5 and 10 the structure is classified as a large failure consequence class (bridges, theatres, hospitals, high rise buildings)

The relative cost of safety is classified as medium depending on the following aspects: (PMC Part 1 (JCSS, 2001))

- Medium variabilities of the total loads and resistances ($0.1<V<0.3$)
- Relative costs of safety measure
- Normal design life and normal obsolesce rate composed of construction costs of the order of 3%

1	2	3	4
Relative Cost of Safety Measure	**Minor Consequences of Failure**	**Moderate Consequences of Failure**	**Large Consequences of Failure**
Large(A)	β=3.1	β=3.3	β=3.7
Medium(B)	β=3.7	β=4.2	β=4.4
Small(C)	β=4.2	β=4.4	β=4.7

Table 2-6: Tentative Target Reliability Indices Related to One Year Reference Period (PMC Part 1 (JCSS, 2001))

The following table, adopted from Roberts and Marshall, shows the target reliability indices recommended by ISO 2394.

Relative Costs of Increasing Safety	Consequences of Failure			
	Small	Some	Moderate	Great
High	0	1.5	2.3	3.1
Moderate	1.3	2.3	3.1	3.8
Low	2.3	3.1	3.8	4.3

Table 2-7: Target Reliability Indices Related to Life-time Reliability Index (Robberts et al. (2010))

The current target reliability classes as recommended by the South African code: (SANS10160-1: (2011))

1	2	3	4
Reliability Class	**Function of Facility, Probability or Consequence of Failure**	**Examples**	**Minimum Level of Reliability β_t**
RC1	Low loss of human life, economic, social or small or negligible for environmental consequences	Agricultural buildings	2.5
RC2	Moderate loss of human life, economic, social or considerable for environmental consequences	Residential and office buildings	3
RC3	High loss of human life, extremely high for economic, social or environmental consequences	Grandstands and concert halls	3.5
RC4	Post-disaster function or consequences beyond the boundaries of the facility	Hospitals, communication centres and rescue centres	4

Table 2-8: Target Reliability Indices Related to a Life-time Reference Period (SANS10160- Part 1 (2010))

It is clear from Table 2-8 that South African reliability classes lacks a numerical orientated classification system, as for example the reliability classification table recommended by the PMC. Furthermore the ISO does also not provide clear guidance as what is meant by small/moderate/great consequences nor by low/moderate/high relative costs of increasing safety. The ISO also used a basic model to calculate the probability of failure. A three random variable limit state function is assumed to represent the failure of structures accurately. A lognormal distribution is used for the resistance of the structure, a normal distribution is used for the dead load and a Gumbel distribution is used for the live load. Due to the computationally high requirements of reliability based optimization the target reliability indices are usually derived by roughly estimating costs and simplifying limit state functions.

Chapter 3 CALCULATING SWTP AND SVSL FOR SOUTH AFRICA

3.1. Introduction

The theory behind the LQI (Life Quality Index) and the different formulations of the LQI has been extensively covered in the literature review. The LQI must now be applied to calculate SWTP (Society's Willingness to Pay) and SVSL (Societal Value of a Statistical Life) for South Africa. The main focus of this chapter will be to show how this is done.

In order to determine SWTP and SVSL for South Africa a few parameters must be determined or obtained from statistical data. These parameters are the discount rate, the GDP per capita, the work time fraction, the wages component of GDP and the age dependent mortality rate of South Africa. These will be determined in the first section of this chapter.

The second section of this chapter will deal with the steps taken to calculate the societal time preference rate (p) based on the method used in Kula's study. This time preference rate will be combined with the economic growth rate of South Africa along with Bayer's and Cansier's model to derive discount rate for South African concrete structures in RC2.

The third section will deal with the calculation of a work time fraction for South Africa. This yielded an unexpectedly low value which prompted further investigation in the form of a comparative study for randomly selected European and African countries. The results of the comparative study are used to explain the unexpected low work time fraction value.

The fourth section will deal with the rest of the parameters as well as the functions written in MATLAB to calculate SWTP and SVSL for South Africa. A parameter study is conducted to determine which parameters cause a relatively large or small difference between SWTP and SVSL. The formulation of SWTP and SVSL is studied and after the narrowing down the possible parameters causing this phenomenon to three possible parameters, a multi-national comparative study is conducted.

At the end of this chapter a summary will be given of all the conclusions drawn from this chapter with regards to the derivation of SWTP and SVSL for South Africa.

3.2. Discount Rate for South Africa

Applying equation 2.13, the economic growth rate was calculated for South Africa using GDP per capita from 1960 to 2011 minus the average inflation calculated between the years 1960 and 2011. The data was obtained from the World Bank.

It is clear from the table below that the economic growth rate for South Africa is relatively high compared with the recommendations made by Rackwitz which is typically between 1.2% and 1.9% for industrialized countries between 1975 and 1998 and only 0.9% for African Countries. (Rackwitz (2002)) The reason for the rapid growth is the fact that South Africa has produced consistently high growth rates in the last 20 years.

Economic Growth Rate	
i	1.52%
Time Preference rate	
p(demand)	2.29%
p(system)	2.30%

Table 3-1: Economic Growth Rate and Time Preference Rate for South Africa

The time preference rate is difficult to calculate as it is a physiological phenomenon where people discount something simply because it will be received in the future and not now. By using equation 2.17 the time preference rate can be determined for South Africa.

The first parameter which must be calculated is the growth rate of consumption of South Africa which is determined by relating the natural logarithm of the consumption per capita to a time dependent straight line graph. The following figure shows the natural logarithm of consumption per capita for South Africa from 1960 to 2010 being fitted by a straight line graph. The result provides a solution for g and A in equation 2.17. The data was obtained from Indexmundi. It is interesting to note that the consumption growth is slow during the period of 1980-1990.

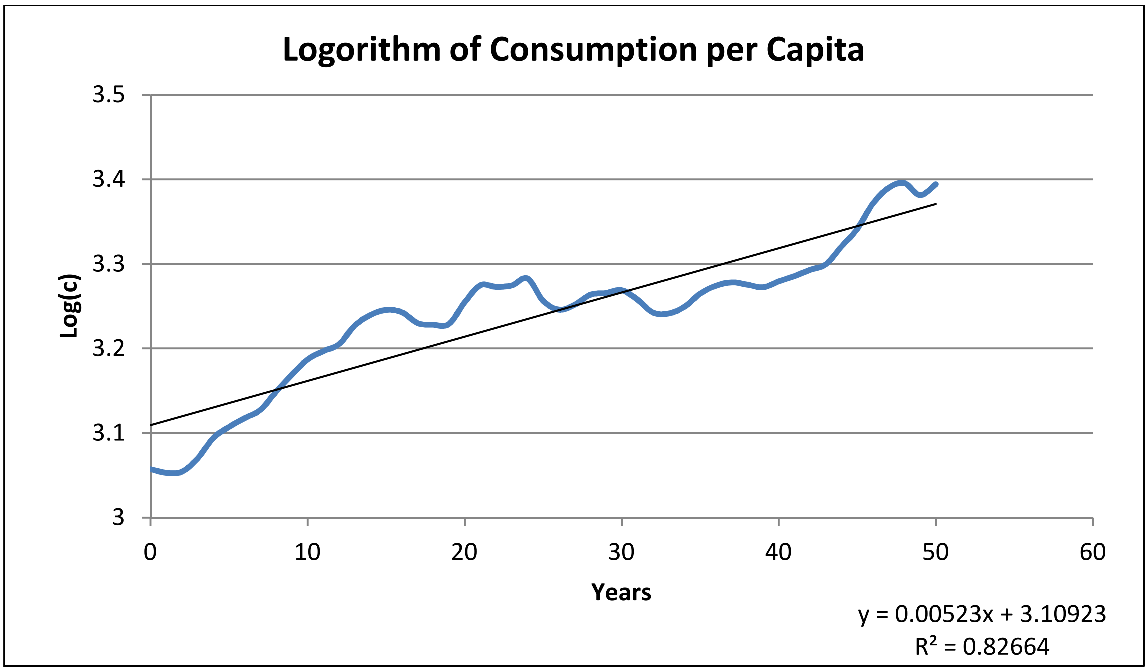

Figure 3-1: Natural Logarithm of Consumption per Capita versus Time

The income elasticity and the compensated elasticity for a food demand equation must be obtained. In the study conducted by Dunne et al. (2005), these elasticities were calculated using two food demand equations, namely the dynamic demand equation and the ideal system demand equation. Two societal time preference rates based on each of the results of the two different food demand equations were calculated as shown in Table 3.1. The following table shows the elasticities and the parameter e calculated from these elasticities for both food demand equations.

Elasticities	
Dynamic e_1	0.6
Dynamic e_2	-0.8
Ideal System e_1	1
Ideal System e_2	-1.3
Dynamic e	0.75
Ideal System e	0.77

Table 3-2: Equation and System Food Demand Elasticities (Dunne et al. 2005)

Finally the probability of survival is obtained by using the mean of data, obtained from Statistics South Africa, showing the probability of the average South African surviving over the last 11 years. The following table contains this data.

Year	Death Rate	PI
2000	14.69	98.53%
2001	16.77	98.32%
2002	18.86	98.11%
2003	18.42	98.16%
2004	20.54	97.95%
2005	21.32	97.87%
2006	22	97.80%
2007	22.45	97.76%
2008	16.94	98.31%
2009	16.99	98.30%
2010	16.99	98.30%
2011	17.09	98.29%
	Mean	**98.14%**

Table 3-3: Yearly Probability of a South African Being Alive in SA (Statistics South Africa)

The STPR for the two food demand equations are identical in Table 3.1 and a conclusion can be made that the STPR can be calculated accurately from these two different types of food demand equations. A time dependent discount rate can be obtained by using the approximation equation 2-25 for inter-generational discounting. Remember that the living generation discounts by $\rho + \varepsilon\delta$ while the generation not yet born discounts by only $\varepsilon\delta$. The following figure shows the two time dependent discount rates derived from the two different food demand equations. As mentioned before the results are identical. (A value of 0.013 was assumed for variable a in equation 2-25)

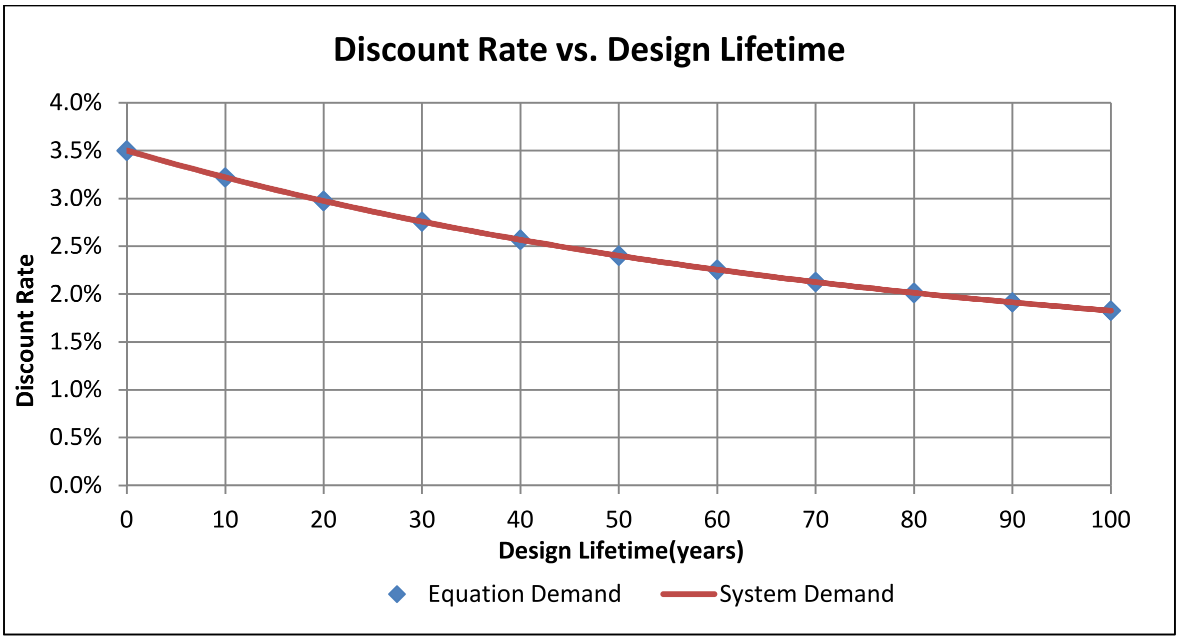

Figure 3-2: The Time Dependent Discount Rate for South Africa

Most structures are designed for a period of 50 years thus a discount rate of 2.4% is the discount rate that will be used for a benefit/cost analysis of South African structures. The discount rate is low compared to discount rates used in literature by Rackwitz ((2008) [32]). This is due to the low p calculated.

3.2. Work Time Fraction for South Africa

There are two main ways to calculate the work time fraction for a society as shown by Rackwitz and Nathwani. In this particular study it was decided to use Rackwitz's formulation. The reason for this is Nathwani's formulation uses the lifetime working years, which changes from generation to generation and might not reflect the current society's preferences. Rackwitz's formulation compensates for this problem by using the yearly work time thus representing the preferences of the current working generation. Rackwitz's formulation has the benefit of having the unemployed included in the labour force who want to work, but can't find work. The advantage of this is that Rackwitz's formulation is measuring a society's preferences accurately by including the unemployed.

A study conducted by Budlender et al. (2000) determined what South Africans do with their time. The study was conducted with three rounds namely February, June and October of the year 2000. The study covered nine provinces and covered all settlement types, namely formal urban, informal urban, commercial farms and rural settlements. The results were obtained by using 24-hour diaries updated every half an hour. The results of the study revealed that employed women spend an average of 19% of their 24-hour day working while men spend an average of 24.5% working. (Budlender et al. (2000))

The number of women making up the labour force and the number of men making up the labour force were used to calculate a weighted average of the amount of time an average employed South African spend working which came to 22.01%. Converting this to the amount of yearly hours spend working it amounts to 1928 hours a year. This corresponds well with the data compiled by Rackwitz where countries like the USA and Australia in the year 1996 had yearly working hours of 1950 and 1875. (Rackwitz (2008) [32])

The work time fraction for South Africa using Rackwitz's formulation was calculated as 0.086. This value is quite low compared to the mean of 0.109 for a multi-national study conducted by Rackwitz. Rackwitz concluded in his study that there are many factors contributing to the work time fraction, but the main factor is the GDP: The higher the GDP the smaller the work/time fraction of a country becomes. (Rackwitz (2008) [32])

South Africa has a lower GDP than the average and it was therefore expected that the w calculated for South Africa would be larger than average w; i.e. that society will prefer to work longer to increase wealth. However the fact that South Africa has a low GDP per capita does not reflect in its work/time fraction. Due to this unexpected result and because SVSL and SWTP are extremely sensitive to slight changes in the parameter w, it was decided to explore this aspect further. The following tables make a comparison between the top African countries and some European countries in terms of the work time fraction, computed according to Rackwitz's formulation (equation 2-14)

Constant yearly working hours of 1600 for European countries (Rackwitz (2008) calculated yearly working hours of 1400 for the Netherlands and 1800 for Finland) and a total of 2000 hours for African countries were assumed to simplify calculations as there is no data available for African countries in Rackwitz's study.

Country	(1) GDP Per Capita (PPP) US$	(2) Population(a)	(3) Labour force (b)	(4) b/a x 100	(5) w(1600ywh)
Belgium	37800	10787790	4834126	44.81%	0.092
Denmark	36600	5592738	2952665	52.79%	0.108
Finland	35400	5402627	2692717	49.84%	0.102
France	33100	63457780	29087950	45.84%	0.094
Germany	35700	81990840	42283030	51.57%	0.106
Greece	29600	11418880	5347508	46.83%	0.096
Ireland	38300	4579498	2179318	47.59%	0.098
Italy	30500	60964140	25390590	41.65%	0.086
Luxembourg	82600	523362	246823	47.16%	0.097
Netherlands	40300	16714230	8917892	53.36%	0.110
Portugal	23000	10699330	5653983	52.84%	0.109
Spain	29400	46771600	23512970	50.27%	0.103
Sweden	39100	9495392	5043060	53.11%	0.109
United Kingdom	34800	62798100	32093060	51.11%	0.105
Mean	37586	27942593	13588264	49.20%	0.101
Standard dev.	13777			3.61%	0.007

1) Obtained from OECD (2010) in US$
2) Obtained from Labourista
3) Obtained from Labourista
4) Labour force as % of Total Population
5) Work time Fraction assuming 1600 yearly working hours

Table 3-4: Work time fraction for European countries

From these Tables (3-4 & 3-5) it can be confirmed that South Africa's work time fraction is not the only exception from the general rule that a low GDP per capita should result in a high work time fraction. Even though the yearly working hours for an average African forming part of the labour force is 25% more than his/her European counterpart, the European countries have a higher work time fraction.

Country	(1) GDP Per Capita (PPP) US$	(2) Population(a)	(3) Labour force (b)	(4) b/a x 100	(5) w(2000ywh)
Algeria	7300	36485830	11727130	32.14%	0.083
Angola	8200	20162520	7611537	37.75%	0.097
Egypt	6200	83958370	28396610	33.82%	0.087
Libya	14000	6469497	2359688	36.47%	0.094
Morocco	4800	32598540	11754700	36.06%	0.093
Nigeria	2500	166629400	53095240	31.86%	0.082
South Africa	10700	50738260	18752530	36.96%	0.095
Sudan	2300	45722080	14891270	32.57%	0.084
Tunisia	9400	10704950	3939954	36.80%	0.095
Mean	7150	50385494	16947629	34.94%	0.090
Standard dev.	4074			2.32%	0.006

1) Obtained from OECD[36] (2010) in US$

2) Obtained from Labourista

3) Obtained from Labourista

4) Labour force as % of Total Population

5) Work time Fraction assuming 2000 yearly working hours

Table 3-5: Work time fraction for African Countries

Another clear difference between African and European countries is the percentage that the labour force forms of the total population. African countries have a mean of only 34.94% compared to 49.2% for European countries.

The significant difference between the African and European work time fraction is because of the difference of the population distributions of the two continents. In other words, the percentage of the population over the age of 15 is higher in European countries than in African countries. For example, 87% of the population in Germany is over 15 while only 57% of the population is over 15 in Nigeria. This gives the European countries a larger labour force relative to the total population than African countries.

Statistics South Africa conducted survey to determine why the labour force was so low. The not economically active population was split up into different groups. 41.5% of the not economically active group consisted of students, 15.7% of this group consisted of those who are discouraged work seekers, 11.5% of this group were ill or disabled and could not work and about 8.2% of this group were either too young or too old to work. (Statistics South Africa) It can be concluded from the results of the study above that the main reason for the low labour force in South Africa is because of the high percentage of the population still studying. This is also shown in the population distribution of South Africa where a large proportion of the population is younger than 21. (Refer to Figure 3-5)

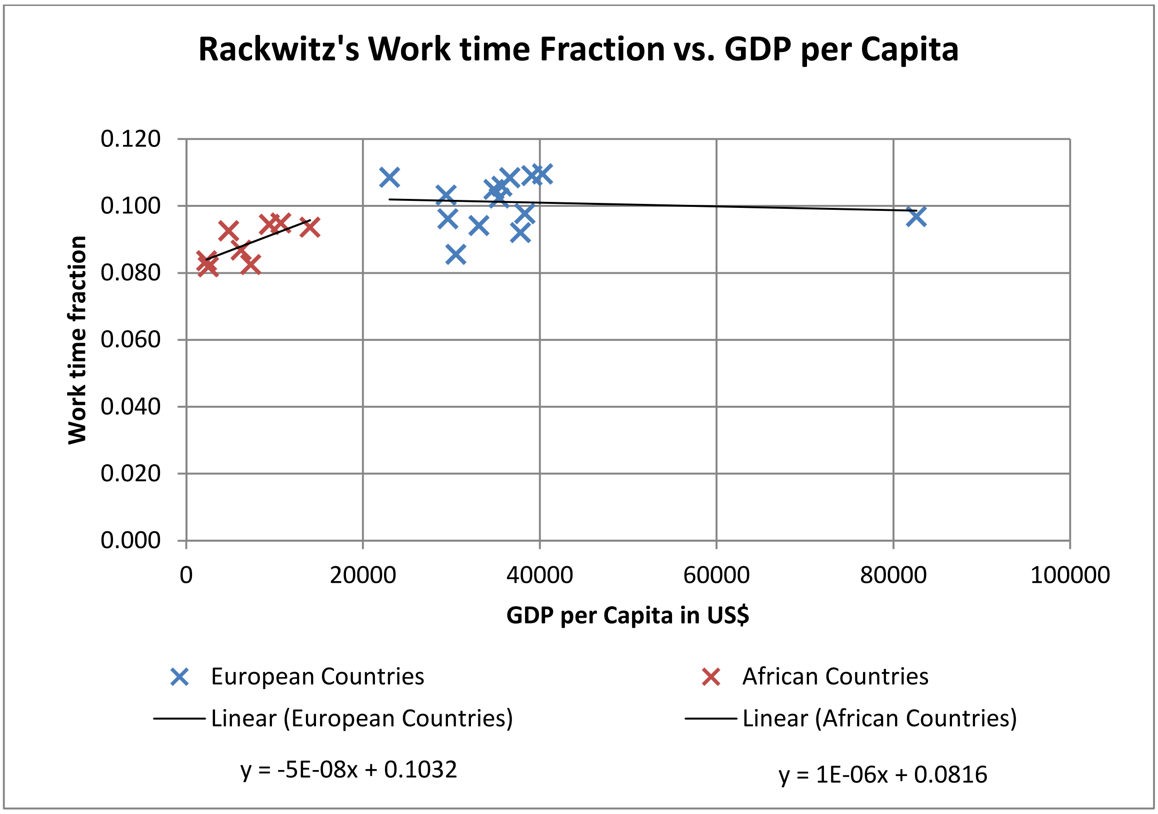

Figure 3-3: Work time fraction of African & European Countries

Rackwitz states that the work time fraction depends further on subsidy from government. (Rackwitz (2008) [32]) A high subsidy could decrease the work time fraction. All South African subsidies are shown in the following table.

Type of Grant	Value in Rand	Value in 2012 US$
State old-age grant	1140	128
State old-age grant, over 75's	1160	130
War veterans grant	1160	130
Disability grant	1140	128
Foster care grant	740	83
Care dependency grant	1140	128
Child support grant	265	30

Table 3-6: South African Government Subsidies (Budget Summary (2012))

It is clear from Table-3-6 that these government subsidies are small, so South Africans should be motivated to find work to live comfortably. Rackwitz (2008) also states that cultural aspects has an effect on the work time fraction, but this aspect will not be explored further as it is difficult to quantify and obtain relevant data.

Given the low GDP per Capita of African countries, the work time fraction should be higher than European countries, but yet it is not. Even though fairly high assumed yearly working hours was assigned to the African countries their work time fractions are low. This was found to be due to the small labour force as a percentage of the total population, due in turn to a population distribution that has many young people (younger than 21 years). The increase in work time fraction with for African countries was initially expected, but after careful consideration it was found that the labour force as a percentage of the total population increases with GDP, creating a trend. This is probably because countries with a higher GDP per capita can afford to invest in better healthcare, thus increasing life expectancy which causes a higher labour force percentage of total population.

In fact on Figure 3-3 based on the tables another strange trend is observed. Increasing GDP per capita for African countries increases the work time fraction while the opposite is observed for European countries. In the following figure it is also noted how the labour force of African Countries, as a percentage of total population, increases with increasing GDP per capita. This would explain the relationship of w and GDP per capita observed for African Countries.

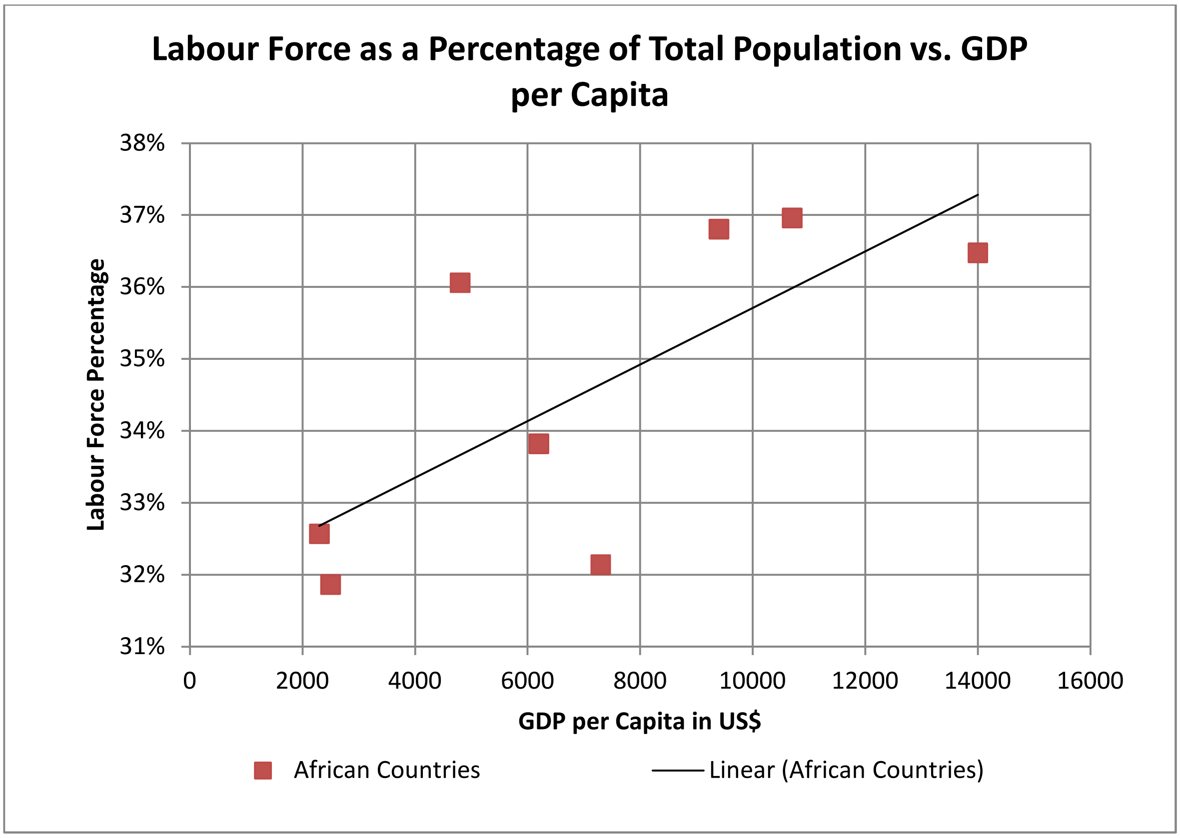

Figure 3-4: Labour Force of African Countries Dependency on GDP per Capita

Referring back to the general formulation of SWTP and SVSL, a low work time fraction results in higher life compensation and saving costs. These increased costs means more resources are invested into safety and therefore the life expectancy is increased. From this the work time fraction is now increased, because of an increased proportion of the population distribution being older than 21 as a result of the higher life expectancy. A higher w can now be achieved by working fewer hours. Finally the society will reach a stable state, where the life expectancy is at acceptable level and the work time fraction is close to the mean of countries in this state. (i.e. most European countries) Basically from the results above it can be concluded that the calculated low work time fraction is correct. Therefore, a low work time fraction requires more resources to be invested into safety which is true for South Africa because of its low life expectancy.

In the next section the sensitivity of parameter w with regards to SVSL and SWTP is explored by using two work time fractions. These different life saving and life compensation costs will be used in Chapter 5 to determine the sensitivity of the target safety due to different work time fractions. This is due to the fact that data for average yearly working hours are not necessarily available to all countries. Thus an assumption is necessary to determine the work time fraction and the assumed w might result in a target safety to be far from the actual target safety due to a lower or higher life compensation and life saving costs.

3.3. Final Calculations for Determining SWTP & SVSL

Due to the complex nature of the formulation of SWTP and SVSL, it was decided to program a function in MATLAB that will be able to calculate demographic constant and age averaged and discounted life expectancy. The program requires a matrix containing unitary age specific mortality rates and a discount rate.

Another function was needed as most life tables do not have unitary, age specific mortality rates. This function is named *LTConverter* and is based on the work of Fischer et al. (2012). The function requires an input of the population growth rate, which is the mean population growth over a period of 10 years or so, as well as a life table. The first column of the life table is the starting age of the bin, the second column is the size of the bin and the third column is the mortality rate. LTConverter does not only convert a life table into a unitary age specific mortality rate life table, but it also returns the stable population distribution obtained by using equation 2-34. The integrals are solved by assuming that the area under the curve can be approximated by a sum of rectangular bars having a width of a year. LTConverter returns the results in a matrix containing 100 by 4 entries. The actual code, input and output data of the function LTConverter can be found in Appendix A. All the life table and population growth rate data was obtained from Statistics South Africa.

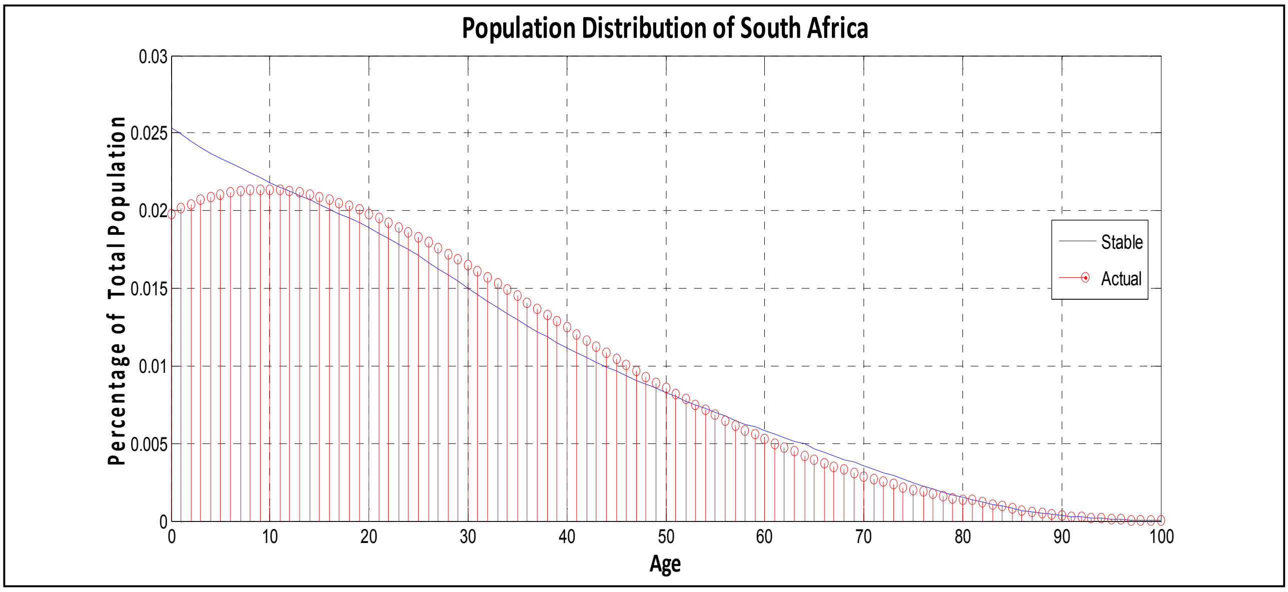

Figure 3-5: Stable and Actual Population Distributions of South Africa

The above figure shows the actual distribution of South Africa, obtained by fitting a polynomial trend line to data taken from Statistics South Africa, as well as the stable population distribution calculated by *LTConverter*.

The results of *LTConverter* are used by a MATLAB function named *DeltaConstantCalc* which is based on the work of Fischer et al. (2012). *DeltaConstantCalc* also uses the actual population distribution obtained by trend line fitting data from Statistics South Africa as well as a time dependent discount rate and the population distribution obtained from the stable population assumption.

Equation 2-38 and the age averaged, discounted life expectancy in equation 2-33 is solved in *DeltaConstantCalc*. Integration is done by assuming that the area under the curve can be accurately approximated by using rectangular bars each with a one year width and thus effectively using a Riemann sum. *DeltaConstantCalc* returns a demographic constant, based on the delta mortality regime, for the stable population distribution and the actual population. Only the aged averaged, discounted life expectancy for the actual population is used to calculate the delta constant. The actual population distribution is a more accurate representation of reality, but it can be seen in the following table that the demographic constants are both similar in magnitude.

To see the code, output and input of *DeltaConstantCalc* refer to Appendix B. The following table shows the final calculations of SWTP and SVSL for South Africa. In the table, the part of the GDP available for SWTP and SVSL was determined by looking at the budget available to the department of treasury for South Africa of 2012. Rackwitz stated that about 10% of the GDP is needed for reinvestment back into the economy to keep it growing. This leaves 90% of the GDP available for SVSL and SWTP. (Rackwitz (2008) [32]) The department of treasury of South Africa obtained revenue of about R830 billion which is about 28% of the GDP. The amount available for investment into safety is thus 62% of the total GDP. (Budget summary (2012))

Social Indicators for South Africa		
GDP	R 3 000 000 000 000	
Part of GDP Available for Safety	R 1 860 000 000 000	
Population	50586757	
Part of GDP per Capita	R 36 769	
Wages	R 1 346 726 000 000	
β	0.45	
Labour Force	17 482 000	
Employment	13 118 000	
Average Hours Worked	1928	
Source of w	Calculated	Approximated by Rackwitz (Rackwitz (2000))
Work Time Fraction(w)	0.086	0.125
Q	0.21	0.32
CΔ Stable population assumption	17.84	N.A.
CΔ Actual population	17.55	17.95
SWTP	R 3 078 640	R 2 093 057
Age averaged & discounted Life Expectancy	21.39	22.54
SVSL	R 3 752 257	R 2 628 273

Table 3-7: Social Indicators for South Africa

The factor β used in calculating SVSL and SWTP, is determined by taking the total wages paid out over the total GDP. The variable q is determined from the variable w and the factor β. All other data not specified was obtained from Statistics South Africa. A work time fraction approximated by Rackwitz is used in the third column of Table 3-7, while a work time fraction based on the calculations of this thesis is used in the second column of Table 3-7. Furthermore, various economists concluded from multinational studies that the q should be approximately equal to 0.2 according to Rackwitz (2008). This is a further confirmation of the fact that the low w is correct for South Africa.

It can be seen from the table that SWTP and SVSL for the two work time fractions differ by about 50%. This is a display of how sensitive SWTP and SVSL are to the parameter w. The effect of the two life saving and compensation costs on target reliability is explored in Chapter 5.

3.4. Parameter Study of SWTP & SVSL

The following table shows some of the results of a study where various SWTP and SVSL values were derived by Rackwitz for various countries. During an analysis of the table it was noted that the ratio between SVSL and SWTP for some countries were particularly small in comparison to the same ratio for other countries. The optimum safety can be less than the minimum required safety derived from the LQI if SVSL is less than SWTP according to Fischer et al. (2012).

Country	Ratio (SVSL/SWTP)	GDP per Capita in US$	Economic Growth Rate	Life Expectancy	Population Growth Rate
Japan	1.100	26460	2.7%	80	0.17%
Germany	1.130	25010	1.9%	78	0.72%
Poland	1.143	9030	1.6%	73	-0.03%
Switzerland	1.143	29000	1.9%	79	0.27%
Sweden	1.158	23770	1.9%	79	0.02%
Austria	1.167	26310	1.3%	77	0.24%
Canada	1.174	27330	2.0%	78	0.99%
USA	1.179	34260	1.8%	77	0.90%
France	1.182	24470	1.9%	78	0.37%
Finland	1.188	22900	1.8%	77	0.16%
Denmark	1.190	25500	1.8%	77	0.30%
Bulgaria	1.200	6200	1.3%	70	-1.14%
Ireland	1.200	25470	1.5%	76	1.12%
Netherlands	1.200	29760	2.1%	78	0.49%
UK	1.217	23500	1.3%	78	0.23%
Australia	1.250	25370	1.2%	78	0.99%
Czech Rep.	1.286	12900	1.5%	73	-0.07%
Russia	1.333	8377	1.2%	66	-0.35%

Table 3-8: Ratio between SVSL & SWTP for Selected Countries (Rackwitz (2005))

The ratio being smaller for some countries meant that SVSL could be less than SWTP under certain conditions. Therefore, selected countries are chosen to determine which variable or combination of variables will cause this to happen.

It was noted from the equations formulating the SWTP and SVSL that the ratio between the magnitude of the two formulations is dependent on two variables, namely the age averaged discounted life expectancy and the demographic constant. The ratio between the age averaged life expectancy and the demographic constant is identical to the ratio of the SVSL and SWTP. The previous table shows the results of Rackwitz's study and is organized from a the smallest ratio between SVSL and SWTP to the largest ratio.

The age averaged discounted life expectancy and the demographic constant are two complex formulations, each depending on the same three parameters, namely the age dependent mortality rate, the population growth rate and the sustainable discount rate. A study of a few selected countries is conducted to explore the effect of these parameters on the age averaged discounted life expectancy. Two countries with a relatively high difference between SWTP and SVSL were selected as well as a country having a relatively low difference between SWTP and SVSL. Therefore a few countries were selected such as Japan, South Africa and Australia in order to investigate this aspect further. South Africa has the highest ratio between SVSL and SWTP, while Australia has a lower ratio and Japan has the lowest ratio. In order to stay consistent, the same data was used as in the study conducted by Rackwitz. The following table shows the relative difference for the three countries in descending order.

Country	**Relative Difference SVSL/SWTP**
South Africa	1.256
Australia	1.250
Japan	1.100

Table 3-9: Ratio between SVSL & SWTP for Study Case (Rackwitz (2005))

The three parameters are plotted for each country to identify which parameters can cause the difference between the various SVSL/SWTP ratios. Figure 3-6 shows the different age dependent mortality rates between the different countries plotted on a log scale with base 10. All age dependent mortality rate data was obtained from WHO (World Health Organisation). It is clear that South Africa has the highest age dependent mortality rates while Japan and Australia have almost identical age dependent mortality rates.

The age dependent population distribution (Figure 3-7) was obtained from the MATLAB function *LTConverter* and by using the Stable population assumption. The population growth rates were obtained from the World Bank. It is clear that South Africa and Australia both have a similar age dependent population distribution.

Age averaged life expectancies and demographic constants for the three different countries are calculated and plotted against a changing sustainable discount rate in Figure 3-8. From the figure a conclusion is made that the relative difference is not at all dependent on the age dependent population distribution, but the relative difference is extremely sensitive and dependent on the sustainable discount rate and slightly dependent on age dependent mortality rate. The SWTP and SVSL become identical for all countries at around a discount rate of 7%. It is also clear that at a discount rate of 1% South Africa has the lowest ratio between SVSL and SWTP while this ratio is identical for both Japan and Australia. A conclusion can be made that at low discount rates the age dependent mortality rate has a significant effect on the relative difference between SWTP and SVSL.

The fact that a discount rate has an effect on the relative difference between SWTP and SVSL can be confirmed by the Table 3-9. Countries with a high economic growth rate have a lower SVSL/SWTP ratio. Rackwitz rounded off the SWTP and SVSL to the nearest 100 000 US$, which explains some of the minor inconsistencies. The inconsistencies can also result from the fact that Rackwitz does not specify the time preference rate used for the derivation of SWTP and SVSL. The only information given is the economic growth rate, which is only a component of a sustainable discount rate.

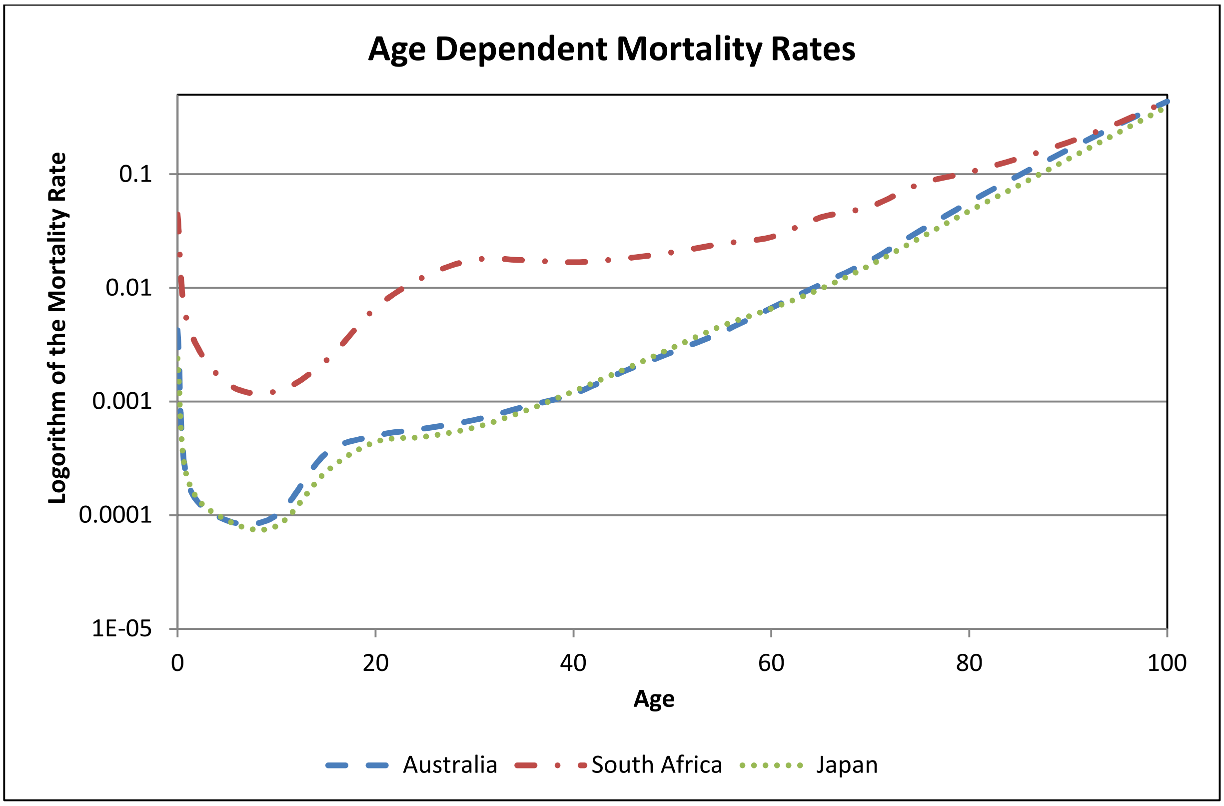

Figure 3-6: Age Dependent Mortality Rates for Selected Countries

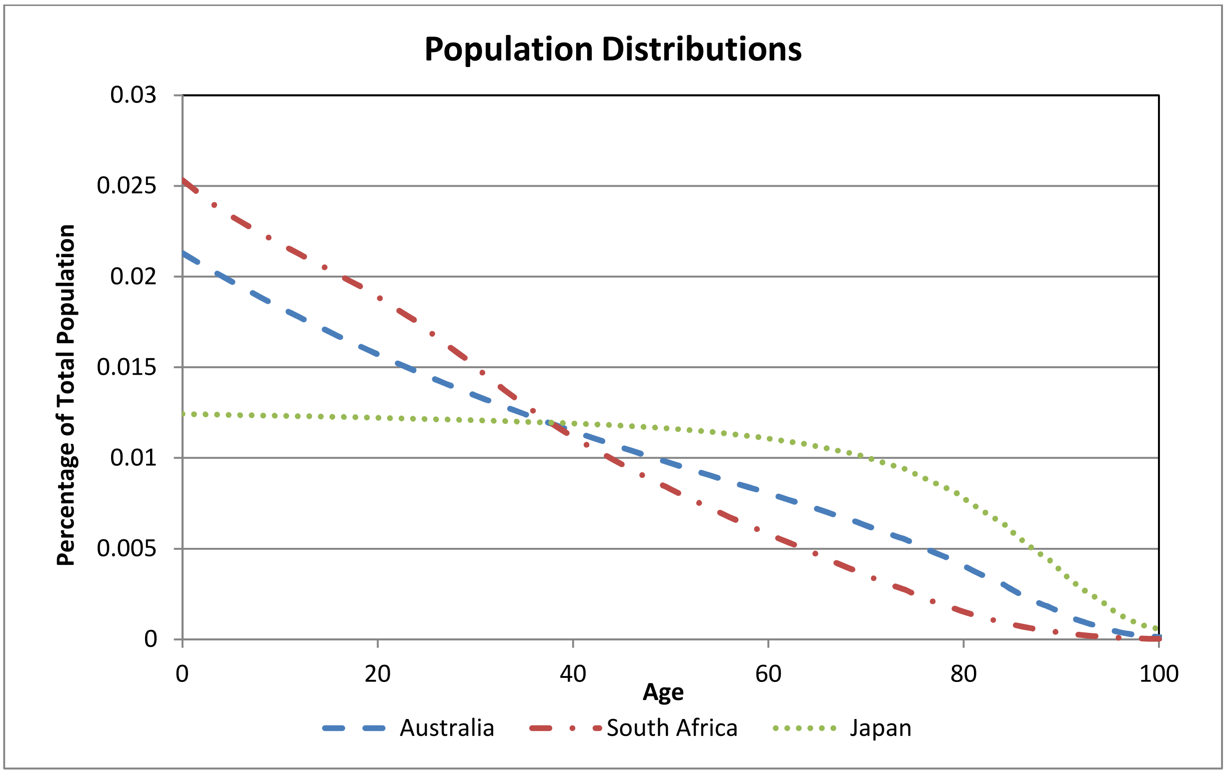

Figure 3-7: Population Distributions for Selected Countries

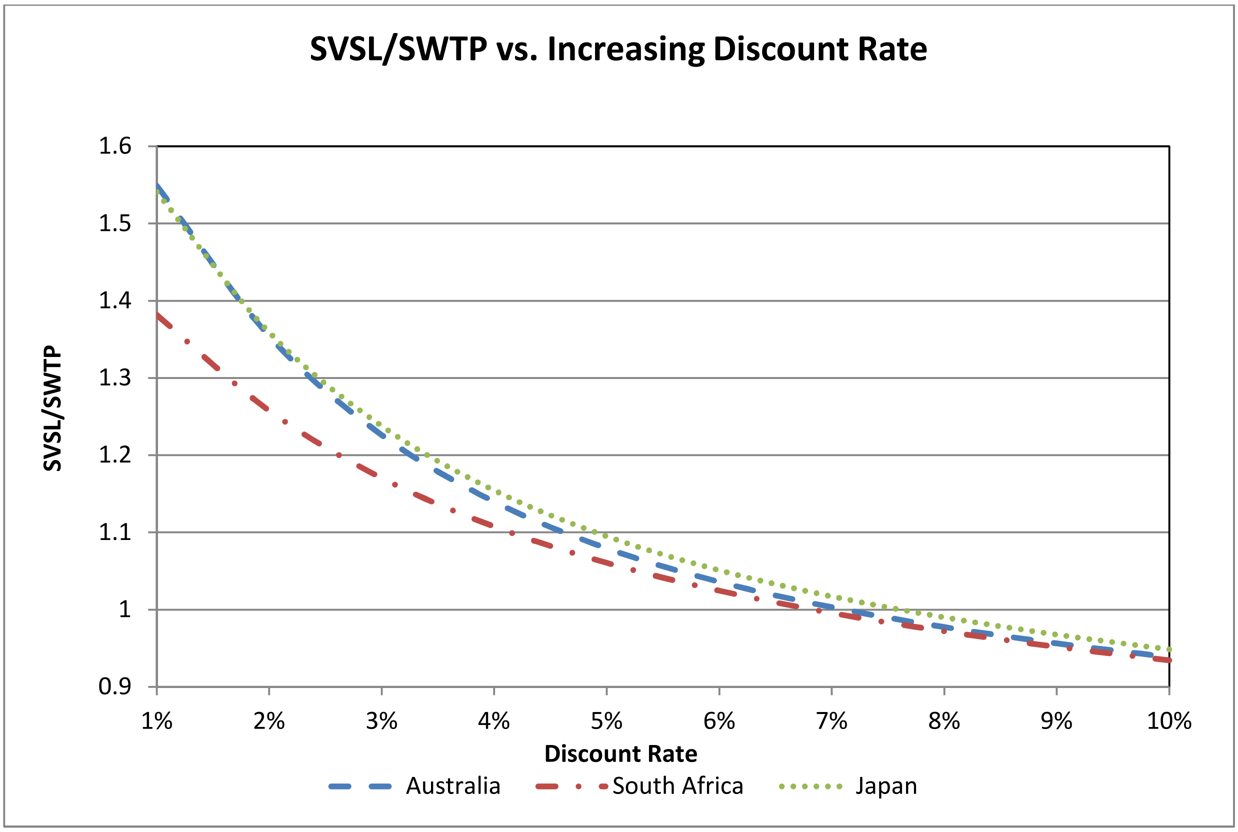

Figure 3-8: The Effect of Increasing Discount Rate on SVSL/SWTP for Selected Countries

3.5. Conclusions

With regards to the discount rate, two different food demand equations were used to determine the compensated and the uncompensated elasticities. By using these elasticities and equation 2-19 the elasticities of marginal utility can be calculated from which two pure preference time rates can be further obtained from equation 2-17. The fact that the two food demand equations yields identical discount rates gives some degree of confirmation of the accuracy of Kula's work.

Rackwitz (2008) stated that with increasing GDP per Capita a country's work time fraction should, as a general trend with some exceptions, decrease. However during the multi-national study the opposite was observed for African countries. The fact that European countries have higher work time fractions than African countries is as a result of high life expectancy and the population distribution. It was also interesting to see that there is a correlation between GDP per capita and labour force percentage of the total population for African countries which could be as a result of the output (GDP) of African countries being heavily dependent on labour input. This statement requires further investigation to confirm or disprove.

SWTP and SVSL for South Africa were calculated for two work time fractions. This was done to see how sensitive these life saving and life compensation costs are by changing w. The two life saving and life compensation costs will be used in the following chapter to measure the minimum and optimum safeties dependency on w. Yearly working hours is data that will not necessarily be available to all countries, thus a work time fraction value will have to be assumed. If the effect of the work time fraction is low the assumption made for w in future studies will not be critical to the accuracy of the results. It was found that with Rackwitz's approximated work time fraction the monetary value of SWTP and SVSL decreased by about 30%.

A study conducted to determine why some countries have a higher difference between SWTP and SVSL was conducted. It was found that the ratio SVSL/SWTP is mostly influenced by the discount rate. At low discount rates the age dependent mortality rate also influences the ratio to some extent. Furthermore, it is also interesting to note that at a discount rate of approximately 7% SWTP and SVSL was equal to one for all three countries. Thus at a discount rate of 7% or more the minimum safety will be more than the optimum safety.

Chapter 4 THEORY AND PROGRAMMING OF THE FORM AND OPTIMIZATION PROCESS

4.1. Introduction

Various methods can be used to for determine the probability of failure of structural components and systems. This chapter focus on both the theory and programming of the optimization process and the FORM (First Order Reliability Method). The first part of this chapter covers the theory behind the FORM and briefly discuss the benefits and drawbacks of some other methods used for determining the probability of failure. Justification is given as to why the FORM was chosen above the other methods.

A small section showing the theory behind computation of a stationary out-crossing rate where more than one live load with its own occurrence rate is involved is shown. This method is used to calculate the jump rate used in the objective function for cost optimization.

The programming structure, limitations and capabilities of a function written in MATLAB which can perform the FORM are covered in the following section. A comparative study comparing the accuracy of the written function in MATLAB with commercially available software, VaP is also shown.

The programming structure and the methodology of a function written in MATLAB calculating the optimum point during the benefit/cost optimization process are covered in the following section along with a graphical demonstration.

4.2. The Theory of the FORM

The safety margin (G) of a structural member can be simply represented by the difference between two variables, namely the resistance (R) and the load effect (L). The member fails when the load effect is larger than the resistance of the structural member, or in other words when the safety margin is equal to or less than zero. (Holický (2009))

$$\boldsymbol{G = R - E} \qquad \textbf{4-1}$$

The fact that the resistance and the load effect are not deterministic parameters, but have some degree of variance in reality means that they can be accurately represented by statistical distributions. When this is done there exist a multitude of possible combinations of values for R and E. Thus the probability of the failure can be calculated for the structural member under consideration. The following figure shows the load effect and resistance of a structural member with assigned statistical distributions. The probability of failure of the structural member is equal to the integral of the product of the probability density function of load effect E and the cumulative probability function of resistance R (Holický (2009)).

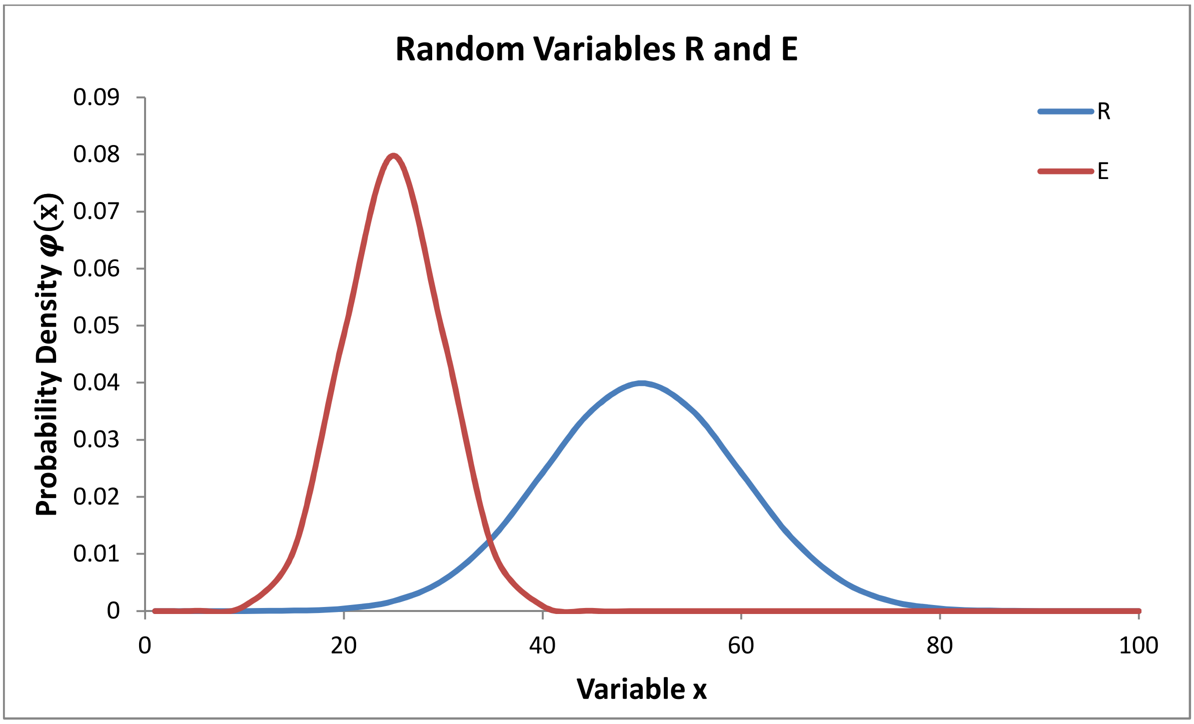

Figure 4-1: Probability Density Functions for Random Variables R & E

There exist various different methods to compute the probability of failure of a structural member: (Holický (2009))

- Exact analytical integration
- Numerical integration
- Approximate analytical methods (FORM,SORM)
- Simulation methods;
- Or by a combination of these methods

Exact analytical integration can only be applied in exceptional academic cases, and is not suitable to determine the probability of failure for this study. Numerical integration is a strong candidate as an alternative method as it can be applied to various different situations. However it is not practical as structural reliability problems generally have a high dimensionality. Simulation methods are also extremely popular and attractive because of their simplicity and transparency. However they require a large number of iterations which requires a large amount of time and computer memory. For example 10^7 realisations are required to be able to accurately simulate a situation where the probability of failure of a structural member is in the order of 10^{-5}. Approximate analytical methods are extremely accurate and require little time and computing power relative to the other methods mentioned. These methods can be programmed into a function that can be applied to various different situations (Holický (2009)). Due to the fact that the probability of failure has to be computed iteratively during the optimization process the FORM is the method of choice for this study as a result of its accuracy and relatively low computational requirements.

The FORM is an iterative method where the design point is estimated along the failure surface. The design point is the point where the failure surface is the closest to the mean of the joint probability density function. The accuracy of the FORM is determined by the number of iterations it completes. The FORM method can be summarised by the following steps adopted from (Holický (2009)):

1) The limit state function G(x)=0 is formulated and theoretical models of basic variables $\mathbf{X} = \{\mathbf{X_1}, \mathbf{X_2}, \ldots \mathbf{X_n}\}$ are specified.
2) The initial assessment of the design point is made by using the mean values of n-1 basic variables and the last one is determined from the limit state function $\mathbf{G(x^*)=0}$
3) At the design point equivalent normal distributions are found for all variables using the following equations and assumptions:

   Assumptions:

   Equal distribution functions:

$$\boldsymbol{\phi_X(x^*) = \phi_u\left(\frac{x^* - \mu_x^e}{\sigma_x^e}\right)} \qquad \textbf{4-2}$$

   Equal probability increments

$$\boldsymbol{\varphi_X(x^*) = \frac{1}{\sigma_x^e}\varphi_u\left(\frac{x^* - \mu_x^e}{\sigma_x^e}\right)} \qquad \textbf{4-3}$$

   The equivalent normal distributions are obtained by assuming other statistical distributions can be accurately approximated by a lognormal distribution. The transformation process is as follows:

$$\boldsymbol{c = \left[\left(\sqrt{\alpha_x^2 + 4} + \alpha_x\right)^{1/3} - \left(\sqrt{\alpha_x^2 + 4} - \alpha_x\right)^{1/3}\right] 2^{-1/3}} \qquad \textbf{4-4}$$

$$u = \frac{x - \mu_x}{\sigma_x} \qquad \text{4-5}$$

$$u' = \frac{\ln\left(\left|\frac{1}{c} + u\right|\right) + \ln\left(|c|\sqrt{1 + c^2}\right)}{\sqrt{\ln(1 + c^2)}} sign(\alpha_x) \qquad \text{4-6}$$

$$\varphi_{LN,U}(u) = \frac{\varphi(u')}{\sigma_x\left(\frac{1}{c} + u\right)\sqrt{\ln(1 + c^2)}} \qquad \text{4-7}$$

$$\sigma_N = \frac{\varphi_N\left(\phi_N^{-1}\left(\phi_N(u', 0, 1)\right), 0, 1\right)}{\varphi_{LN,U}(u)} \qquad \text{4-8}$$

$$\mu_N = \mu_x - \sigma_x \phi_N^{-1}\left(\phi_N(u', 0, 1)\right) \qquad \text{4-9}$$

$$u_N = \frac{x - \mu_u}{\sigma_u} \qquad \text{4-10}$$

In equation 4.9 the equivalent normal distribution mean is calculated and the equivalent normal distribution standard deviation is calculated in equation 4.8.

4) Partial derivatives dented as vector D of the limit state function in respect of the standardized variables **U = {U$_1$, U$_{2,}$ ....U$_n$}** are evaluated at the design point

$$D = \begin{Bmatrix} D_1 \\ \vdots \\ D_n \end{Bmatrix} \qquad \text{4-11}$$

Where:

$$D_i = \frac{\partial G}{\partial X_i} \sigma_{Xi}^e \qquad \text{4-12}$$

5) The reliability index β is estimated as follows:

$$\beta = -\frac{\{D\}^T\{u^*\}}{\sqrt{\{D\}^T\{D\}}} \qquad \text{4-13}$$

6) Sensitivity factors are determined as:

$$\alpha = -\frac{\{D\}^T}{\sqrt{\{D\}^T\{D\}}} \qquad \text{4-14}$$

7) A new design point is determined for n-1 standardized variables:

$$u_i^* = \alpha_i \beta_i \qquad \text{4-15}$$

$$\boldsymbol{x}_i^* = \mu_{Xi}^e - \boldsymbol{u}_i^* \boldsymbol{\sigma}_{Xi}^e \qquad \textbf{4-16}$$

Steps three to seven are repeated until satisfactory results are achieved. A result is seen as satisfactory when the result is accurate enough i.e. when the difference between two target reliability indices calculated at consecutive iterations is sufficiently small.

4.3. Out-crossing Rates for Stationary Random Processes

Assuming some load has an extreme occurrence rate of λ and is modelled by a rectangular wave process, Rackwitz (2000) showed that the failure rate can be defined as shown in equation 2-47. However, a structural reliability problem usually has multiple different types of loads i.e. wind load, short term live load and long term live load. Each of these loads has their own λ. This section briefly shows the work of Kuschel et al. (2000) demonstrating how a failure rate can be computed for a reliability problem with more than one live load each having its own λ (occurrence rate).

A stationary out-crossing rate can be calculated by taking the product of the jump rate and the probability that a component of the rectangular wave jumps from the safe domain (F) to the unsafe domain (F_u), summed up over all n_s components of the rectangular wave renewal process. (Kuschel et al. (2000)) Where p is the safety parameter explained in the literature review:

$$\boldsymbol{V}^+(\boldsymbol{F}(\boldsymbol{p})) = \sum_{i=1}^{ns} \boldsymbol{\lambda}_i \boldsymbol{P}\left(\{\boldsymbol{S}_i^- \, \boldsymbol{\varepsilon} \, \boldsymbol{F}_u\} \cap \{\boldsymbol{S}_i^+ \boldsymbol{\varepsilon} \boldsymbol{F}\}\right) \qquad \textbf{4-17}$$

It is assumed that S_i changes its position from a random value to a new random value after the jump. An additional assumption is made that appropriate probability transformations are done so that the out-crossing rate can be computed by:

$$\boldsymbol{V}^+(\boldsymbol{F}(\boldsymbol{p})) = \boldsymbol{\phi}(-\boldsymbol{\beta}_p) \sum_{i=1}^{ns} \boldsymbol{\lambda}_i \left(\mathbf{1} - \frac{\boldsymbol{\phi}_2(-\boldsymbol{\beta}_p, -\boldsymbol{\beta}_p, \boldsymbol{\rho})}{\boldsymbol{\phi}(-\boldsymbol{\beta}_p)}\right) \qquad \textbf{4-18}$$

Where λ is the jump rate and Φ_2 is the two-dimensional normal integral with correlation coefficient ρ. For high reliability indices (β), the two-dimensional normal integral becomes small compared to the one dimensional normal integral and can be ignored. High reliabilities (1-year reference period) are worked with in this study, thus the out-crossing rate or yearly failure rate is computed as follows:

$$\boldsymbol{V}^+(\boldsymbol{F}(\boldsymbol{p})) \approx \boldsymbol{\phi}(-\boldsymbol{\beta}_p) \sum_{i=1}^{ns} \boldsymbol{\lambda}_i \qquad \textbf{4-19}$$

From the above equation two things are observed: The out-crossing rate is approximately equal to the failure rate and the failure rate can be computed by summing the different occurrence rates.

4.4. The Programming and the Testing of the FORM Function

A function written in MATLAB to perform the FORM is called *form* and requires a symbolic array called *stat*, a matrix called *data* and a function (y) to return the probability of failure. The variable stat is a symbolic row array, containing the various symbols denoting a certain type of statistical distribution. The form function is programmed to handle the following statistical distributions:

- Normal distribution (N)
- Two parameter lognormal distribution (LN)
- Gumbel distribution (GU)
- Gamma distribution (GA)

The *form* function can also handle parameters that are assumed to have zero variance so they don't have a mean or standard deviation. These are assumed to be deterministic and are denoted by the symbol D in the vector *stat*.

A statistical distribution has a mean, standard deviation and skewness. The main function of the matrix *data* is to give this information to the function *form*. The first column of matrix *data* contains the mean, the second column contains the standard deviation and the third column contains the skewness of the statistical distribution.

The function *y* is the limit state function that defines a structural failure when the function is smaller than zero. In order for the form function to be able to use the limit state function in the calculating of the probability of failure the limit state function must follow these rules:

1. All parameters must be written as X1,X2 ...X20
2. There may not be more than 20 different parameters
3. The first parameter (X1) may never be deterministic

It is further also important that the number of the row in *stat* where the type of distribution is defined is equal to the row number in *data* where the statistical properties of that statistical distribution are defined. It is also important that matrix *data* must be 20 rows long even if there are less than 20 random variables. (Other rows must be assigned zeros)

If the *form* does not converge after 20 iterations an error message is returned. The solution is assumed to be sufficiently accurate if the difference between the reliability indices is less than 0.001. There are also various other error messages programmed in the *form* function to increase the user friendliness.

The *form* function was tested with a simple and typical engineering problem. A reinforced concrete beam failing in tension is used as an example to compare the function *form* with commercially available software VaP. VaP can use both the SORM and FORM and direct Monte Carlo simulation if required. For this exercise only the FORM capability of VaP is used. The following equation shows the limit state function of the failure under consideration:

$$\boldsymbol{G} = \boldsymbol{X1} \times (\boldsymbol{0.9}) \times (\boldsymbol{X2} - \boldsymbol{X3}) \times \boldsymbol{X4} - \boldsymbol{X5} \qquad \textbf{4-20}$$

The following table shows the different parameters and their statistical properties:

Parameter	Description	Statistical Distribution	Mean	Standard Deviation	Skewness	Units
X1	Yield Strength of Reinforcement	LN	510000000	30000000	0.18	Pa
X2	Depth of Beam	N	0.5	0.007	0	m
X3	Concrete Cover	GA	0.04	0.015	0.75	m
X4	Area of Reinforcement	D	0.0015	0	0	m^2
X5	Applied Moment	GU	150000	30000	1.14	N.m

Table 4-1: Statistical Parameters for Reinforced Concrete Beam

The reason for the 0.9 included in the limit state function is an assumption that the lever-arm of the beam is 90% of the effective depth of the beam. This is just to simplify the limit state function and it must be noted that the aim of this exercise is not to model the reliability of a beam accurately, but to test the function *form*. The following table shows the results of the two reliability indices obtained for different areas of reinforcement.

Area of Reinforcement (mm^2)	β (form)	β (VaP)
1500	3.22	3.22
1600	3.46	3.46
1700	3.67	3.68
1800	3.86	3.89
1900	4.05	4.09

Table 4-2: Reliability Indices for VaP and the MATLAB function form

It can be seen that for both the MATLAB function form and the software VaP, the reliability indices increases as the reinforcement area is increased as expected. The differences between VaP and the form function are insignificantly small. The reason for the differences can be as a result of the fact that an assumption is made during the transformation process that Gumbel and Gamma distributions can be well approximated by lognormal distributions in the MATLAB function *form*.

Remember that the FORM is an iterative method approximating the probability of failure along a failure plane. The FORM always initially underestimates the probability of failure and incrementally the probability of failure is increased as it approaches the design point closest to the origin. (Refer to following figure) (The shorter the length of β the larger the probability of failure)

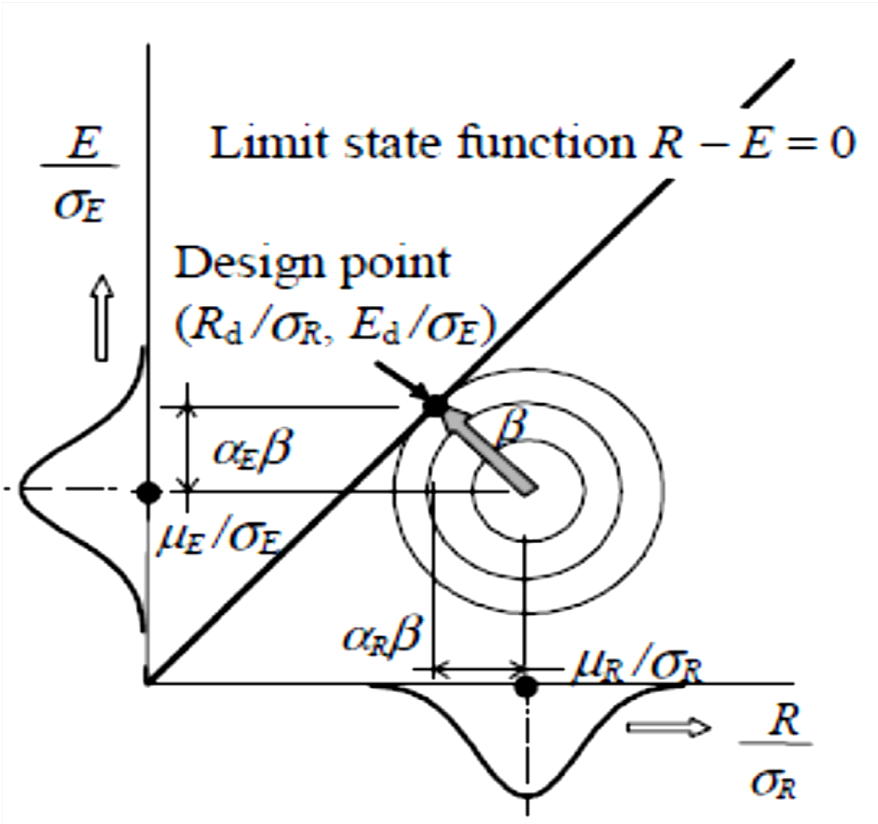

Figure 4-2: Transformed Variables (R & E) & Failure Plane (Straight Line) (Holický (2009))

Thus the difference in reliability indices between VaP and *form* can be as a result of different acceptable levels of accuracy. Referring to Table 4-2, it is noted that the reliability indices of both programmes are identical at a high probability of failure. This is due to the fact that the design point closest to the origin can be accurately approximated with relatively few iterations and a relatively low accuracy level for a high probability of failure. However as the probability of failure of the structural member decreases (in this case a beam), the design point closest to the origin requires more iterations and a higher level of accuracy. The reliability indices given by VaP is larger for the higher reinforcement areas, thus giving a smaller probability of failure than *form* and a longer distance between the origin and design point than the reliability indices obtained by *form*. It is thus possible that VaP accepts a higher difference than 0.001 between consecutive reliability indices as an acceptable level of accuracy. Thus *form* could actually be more accurate than VaP.

4.5. The Programming of the Optimization Function

The optimum safety of a structural member is found by taking the derivative of the benefit/cost function (equation 2-48) and setting it equal to zero. However the benefit/cost function cannot be differentiated due to the complex formulation of the probability of failure of the structural component under consideration. Therefore an alternative method must be used to find the optimum solution.

It was decided to use a simple approximate searching method to find the optimum. The starting point of the method is firstly to define a range over which the optimum solution can possibly be. This is done by defining a reasonable range for the safety parameter for the specific mode of failure under consideration.

The method then breaks up the range into ten equally spaced points for the safety parameter and assigns these points to the benefit/cost function. The size of the steps between these points is known as the rough step size. After this is done, the next step is to find the smallest of these ten points (initial optimum point).

The rough step is then reduced to a 30th of its size and a finer step size can now be used to estimate the optimum point more accurately. The method tests whether the optimum solution lies to the left or to the right of the initial estimated optimum safety point. This is done by taking a small step to the left of the initial optimum point and seeing whether the benefit/cost function is increasing or decreasing. If the benefit/cost function is increasing the optimum solution is towards the left of the initial optimum point and the optimum solution is estimated by finer steps moving to the left until the estimated optimum point is found.

For illustration purposes a simple cost function was optimized in terms of a safety parameter. In this case the optimum solution is where the total costs are the lowest and only 5 rough steps and 20 finer steps were allowed for demonstration purposes. The probability of failure was obtained from a very basic limit state function of two random variables, each assumed to be best described by a lognormal distribution. The following table and graph shows the results of the optimization process explained above.

Safety Parameter (p)	Probability of Failure	Total Cost
2.00	6.70E-03	2941.486
2.60	3.27E-04	324.526
3.20	1.68E-05	238.145
3.80	9.72E-07	274.852
3.17	1.94E-05	237.082
3.20	1.68E-05	238.145
3.17	1.94E-05	237.082
3.14	2.25E-05	236.196
3.11	2.60E-05	235.514
3.08	3.02E-05	235.070
3.05	3.49E-05	234.903
3.02	4.05E-05	235.057
Initial rough estimation of optimum		
Finer estimation of optimum		
Optimum solution for safety parameter p		

Table 4-3: Optimization Process

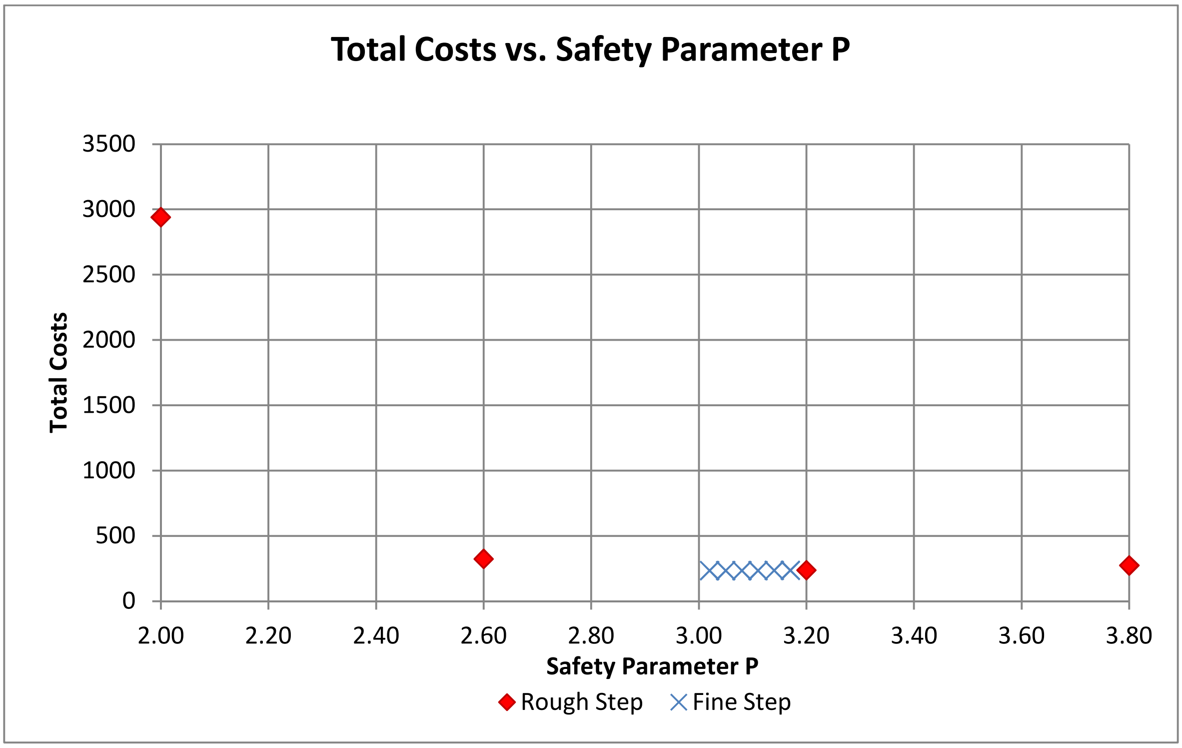

Figure 4-3: Optimization Process

The process explained above is programmed into a MATLAB function *Generic_Optimization* and the table and graph was obtained by using this function. The function uses various if statements and while loops to determine the optimum solution. *Generic_Optimization* repetitively calls the *form* function from which the probability of failure is obtained and when the optimum safety is determined the function *Generic_Optimization* returns the optimum reliability index. The function *Generic_Optimization* was adjusted to enable it to return the data seen in the table above, but as mentioned earlier its main purpose is to return an optimum reliability index given certain parameters. Refer to Appendix D to see the code and an input example for the function *Generic_Optimization*.

It can be seen from the results that the optimum solution estimated by *Generic_Optimization* is sufficiently accurate. The smaller steps can however be reduced if higher accuracy is required. The accuracy also depends on the size of the range the user chooses. If the user wants a very accurate solution, he/she can simply reduce the size of the range where the solution might exist after an initial optimum solution was estimated by *Generic_Optimization* over an initial large range.

4.5. Conclusions

From all the various methods, the FORM is the most appropriate method for estimating the probability of failure for this study due to its numerical efficiency. It is accurate and uses little computing power. Even though there are slight differences between the commercially available software VaP and the function *form*, they can be deemed as insignificantly small for this particular study.

The optimization function is also shown to have an acceptable level of accuracy. Even though the optimum point is not an exact solution, the difference between the costs of the finer step size next to the optimum estimated solution is less than 0.107. Considering that the costs are in the order of magnitude of 200 to 3000, the estimated optimum solution is sufficiently accurate.

Chapter 5 GENERIC OPTIMIZATION OF STRUCTURES

5.1. Introduction

The target reliability indices recommended by the SANS 2394 (2003) in Table 2-7 and by (PMC Part 1 (JCSS, 2001)) Table 2-6 are based on a benefit/cost optimization of structures. These optimization studies were conducted by the JCSS and the ISO. In the benefit/cost optimization study conducted by the ISO, the limit state function was simplified to a three random variable limit state function and assumed to be an accurate representation of most structures. The resistance is modelled by a lognormal distribution, the live load by a Gumbel distribution and the dead load by a normal distribution. An approach similar to the JCSS and ISO studies, conducted by Rackwitz (2000), used a simplified limit state function to derive target reliability indices.

The main aim of this chapter is to replicate the study conducted by Rackwitz (2000) explained in the following section. Various parameters will be varied between reasonable bounds in order to measure their respective effects on the optimum reliability indices. The results of these parameter studies will be used to simplify the benefit/cost optimization process which will then be used for case studies conducted in Chapters 6, 7 and 8. In Chapter 9 the results of the simplified approach in this chapter will be compared to the results of the case studies.

5.2. The Study Conducted by Rackwitz

Firstly the benefit/cost optimization function used by Rackwitz (2000):

$$Z(p) = \frac{b}{\gamma} - C_0 - C_1 p - (C_0 + C_1 p + A)\frac{\omega}{\gamma} - (C_0 + C_1 p + H_m + H_F)\frac{\lambda P_f(p)}{\gamma} - \mathrm{U}\left(\frac{\lambda P_f(p/a)}{\gamma}\right) \qquad \textbf{5-1}$$

Refer to section 2.6 for definition and discussion of the terms in equation 5-1. In this case, U is the cost of serviceability failure and p is the central safety factor explained later. In the above function obsolescence, systematic reconstruction upon failure and serviceability failure are taken into account by Rackwitz (2000). Fatigue costs and maintenance costs were not taken into account. Rackwitz then normalized all the costs by defining them in terms of C_0 (Construction costs of the structure excluding C_1). The following costs and parameters were assumed by Rackwitz to be an accurate representation of a typical structure:

$\lambda = 1$ $a=1.5$

$b = 0.07$ $\omega = 0.02$

$U/C_0 = A/C_0 = 0.2$ $C_1/C_0 = 0.03$

$H/C_0 = 3$

The limit state function for ultimate failure assumed by Rackwitz to be an accurate representation of all structures is as follows (Rackwitz (2000)):

$$\boldsymbol{G = R - E} \qquad \textbf{5-2}$$

The following table shows the statistical properties of the two random variables.

Parameter	Description	Statistical Distribution	Mean	Standard Deviation
R	Resistance	LN	1*p	VR *p
E	Load Effect	LN	1	VS

Table 5-1: Statistical Properties of R & E (Rackwitz (2000))

The central safety factor p is defined as the mean value of the resistance divided by the mean of the load. The central safety factor is increased throughout the optimization process until the optimum point is found. It can be seen in Table 5-1 that as the central safety factor is increased the mean of the resistance is increased as well. The standard variance is determined by using a fixed value for VR and VS. Rackwitz (2000) does not clearly state what values of VR and VS is used to derive target reliability indices. Unless otherwise stated, a VR of 0.2 and a VS of 0.2 is used in this chapter.

The serviceability probability of failure was obtained by using the limit state function shown in equation 5-2, however the mean of the resistance is now further divided by a variable (a) assumed to be equal to 1.5. The following table shows the results of Rackwitz's (2000) study:

Ultimate Limit State			
Expected Failure Consequences			
Relative Effort To Achieve Reliability	**Insignificant**	**Normal**	**Large**
High	2.3	3.1	3.7
Medium	3.1	3.7	4.3
Low	3.7	4.3	4.7
Serviceability Limit State			
High	1.3		
Medium	1.7		
Low	2.3		

Table 5-2: Target Reliability Indices According to Rackwitz's Numerical Study (Rackwitz (2000))

The results of Table 5-2 were obtained by varying some of the assumed costs. Rackwitz (2000) does not however clearly state what is meant by normal or any of the other costs defined by words. The reliability indices above are based on a one year reference period. Comparing Table 5-2 to Tables 2-6 and 2-7 it is clear that the ISO table recommends higher target reliability indices and that the target reliability indices recommended by the PMC table are slightly higher. The following figure shows the various costs components of the benefit/cost functions as a function of the central safety factor obtained by Rackwitz (2000):

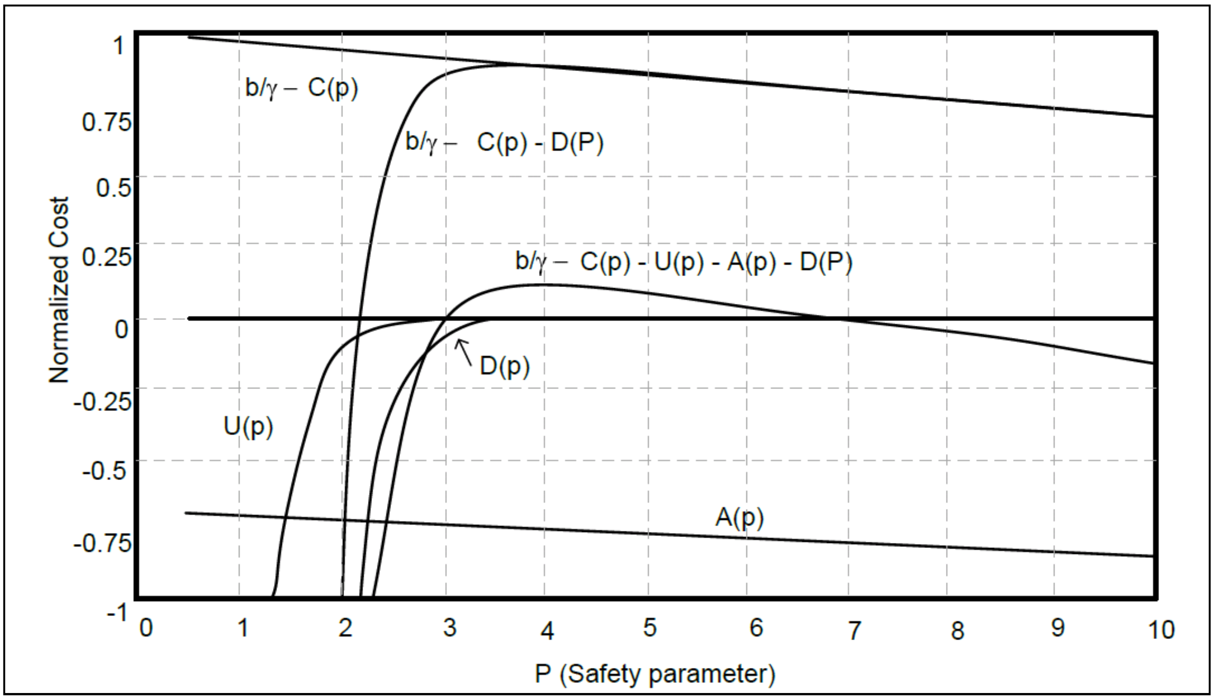

Figure 5-1: The Reliability Optimization of Safety Parameter P (Rackwitz (2000))

Figure 5-1 was obtained by using the values for the various parameters specified on top of page 63. From this figure the various effects of different costs can be seen on the optimum solution. It is also an indication of how sensitive Rackwitz assumed the various parameters to be in terms of increasing the central safety factor (p). Increasing the central safety factor is a simple approximation of a designer increasing the structural resistance of a structure. It can be seen that the effect of serviceability failure (U(p)) can be neglected as it has an insignificantly small effect on the optimum solution. Furthermore, little information has been found in literature to estimate the losses associated with serviceability failure. Thus serviceability failure is excluded for this study due to its insignificant effect on the optimum solution of p. (Refer to literature review for definition of costs A(p), C(p) & D(p)) .

5.3. Modification of the Benefit/Cost Function

The benefit/cost function (equation 5-1) excluding the serviceability costs is used in the cost optimization carried out in this chapter. Due to the fact that various parameters in this function are difficult to compute, the benefit/cost analysis function was simplified through realistic assumptions. An assumption is made that the benefit society obtains from the construction of a building is always more than the costs. If this was not true society would not be building new structures as the benefit obtained would be less than the costs, but this is obviously not true. However, this assumption is only realistic for typical structures and cannot be made for specialized structures such as stadiums and high rise (buildings falling outside RC2). In these cases a case specific benefit/cost analysis must be conducted. Due to the assumption above the benefit/cost function is now as follows:

$$\boldsymbol{Z(p) = -C_0 - C_1 p - (C_0 + C_1 p + A)\frac{\omega}{\gamma} - (C_0 + C_1 p + H_m + H_F)\frac{\lambda P_f(p)}{\gamma}} \qquad \textbf{5-3}$$

It is also assumed that the demolition costs are independent of safety parameter p. This can be justified by the fact that the demolition costs does not change significantly with safety parameter p (area of reinforcement) for concrete structures falling under reliability class 2 and this is also observed on Figure 5-1 showing various cost components as functions of the safety parameter p assumed by Rackwitz. A(p) does not change significantly with increasing p.

The optimum of the cost function is only determined by variables dependent on the central safety factor or the safety parameter (p). Therefore the cost function above can be simplified further by excluding the costs C_0 from the left hand side of the benefit cost function. However, the cost of consequences of failure is dependent on the cost $C_{0.}$

With all the information above, a reliability based optimization parameter study is done in the rest of the chapter based on the following cost function:

$$\boldsymbol{Z(p) = -C_1 p - (C_1 p)\frac{\omega}{\gamma} - (C_0 + H_m + C_1 p + H_F)\frac{\lambda P_f(p)}{\gamma}} \qquad \textbf{5-4}$$

The optimization process is simplified further by normalising all the different cost components assigning a unit of Rand per collapsed floor area to the different costs components. The number of fatalities is also taken as per collapsed floor area. In this chapter the C_0 component is assumed to be R1150/m^2. It is also assumed that the majority of structures have a reference period of 50 years, therefore the sustainable discount rate of 2.4% derived in Chapter 3 is used as well as an obsolescence rate of 2%. All of the costs in this chapter are roughly estimated. In the following chapters the various costs components will be determined by careful analysis of data obtained in the literature review. The main aim of this chapter is to establish the effect of certain cost components on the optimum and minimum safety, thus the exact value of the costs estimated in this chapter is not really important. The assumed costs must only be in approximately the same order of magnitude as the actual costs.

5.4 Difference in Reliability Indices between the two Work Time Fractions

In Chapter 3 a significant difference was found for different SWTP and SVSL values based on the two work time fractions. The calculated work time fraction suggested that South Africans prefer working less than most counties in Europe. This was found to be a particularly low work preference for South Africa's GDP per capita, as people in poorer countries tend to prefer more work than people in richer countries according to Rackwitz (2008). A much higher preference for work for South Africa was estimated by Rackwitz (2000) resulting in significantly lower life saving and life compensation costs.

In this section a sensitivity study is conducted to see to what extend the different magnitudes of life compensation costs effect the optimum reliability indices. Figure 5-2 shows the different optimum reliability indices resulting from the cost optimization process based on two life compensation costs obtained from the two different work time fractions.

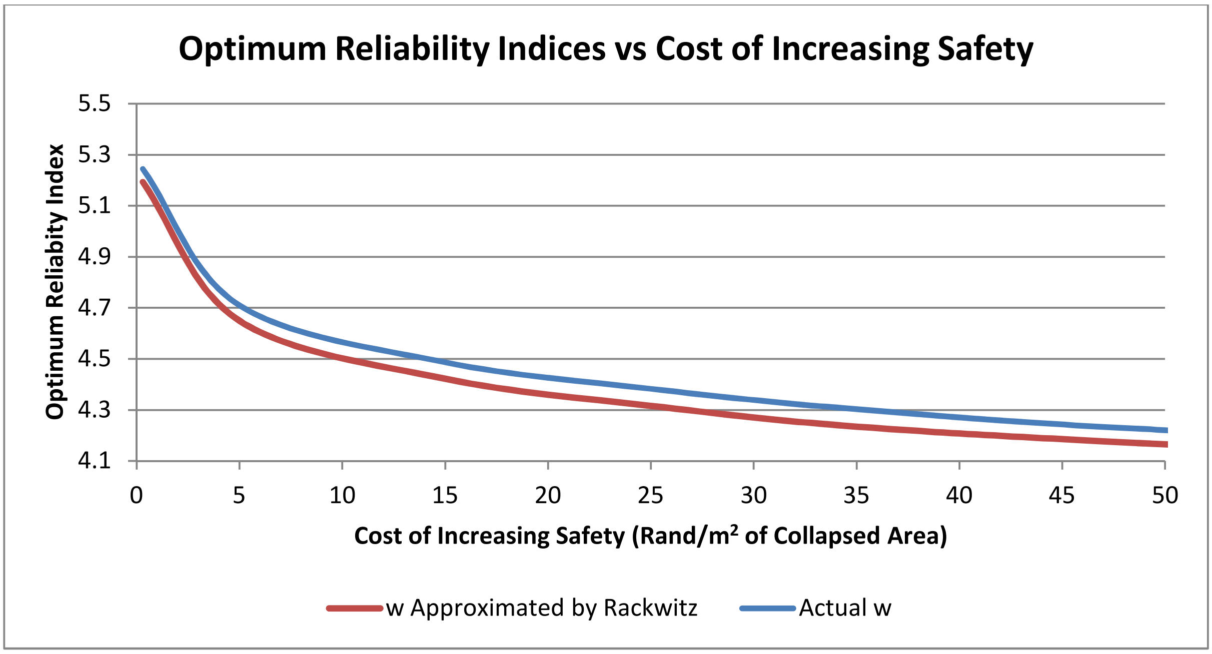

Figure 5-2: Difference in Optimum Reliability Indices for two Work Time Fractions (N=0.01)

It is clear from the Figure 5-2 that even though the difference between the two SVSL's is more than a million rand, it has a fairly small influence on the optimum reliability index subsequently calculated. The following figure shows a similar comparison when the assumed number of fatalities is taken as $0.1/m^2$ instead of $0.01/m^2$.

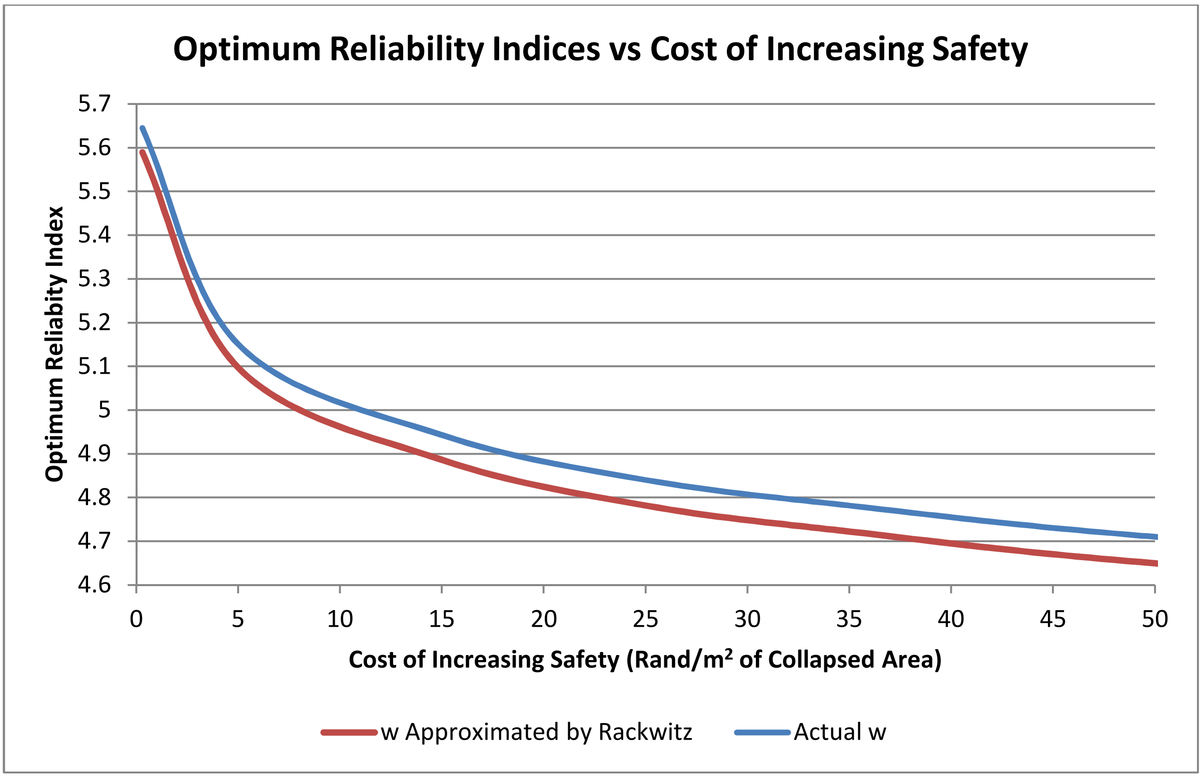

Figure 5-3: Difference in Optimum Reliability Indices for two Work Time Fractions (N=0.1)

It can be concluded from the two above figures that increasing the number of fatalities per collapsed area increases the target reliability indices. However the difference between the reliability indices for the two life compensation costs (work time fractions) is still the same in both figures. Therefore as long as two different life compensation costs do not differ by a factor of more than 2, the optimum safeties obtained from these two costs will differ insignificantly if the number of fatalities is the same for both cases. The work time fraction has a small effect on the optimum safety from the above results which means that it is not necessary to determine the exact value for w.

It is also noted that when the cost of increasing safety approaches zero, the target safety exponentially increases, as would be expected.

5.5 Effect of Various Costs of Failure on Optimum Reliability Indices

The sensitivity of the target reliability to different magnitudes of costs of increasing safety and work time fractions has been shown in the previous section. However, the focus will now shift to the effect of various components of failure costs on target reliability. Firstly the value of SVSL was varied between reasonable bounds in order to observe the impact a different life compensation cost can have on the optimum safety. The cost of increasing safety was assumed to be equal to R10/m^2, the number of fatalities was kept constant as 0.035/m^2, the C_0 cost was assumed to be R1150/m^2 and the H_m component was assumed to be zero. Figure 5-4 shows the results of the study:

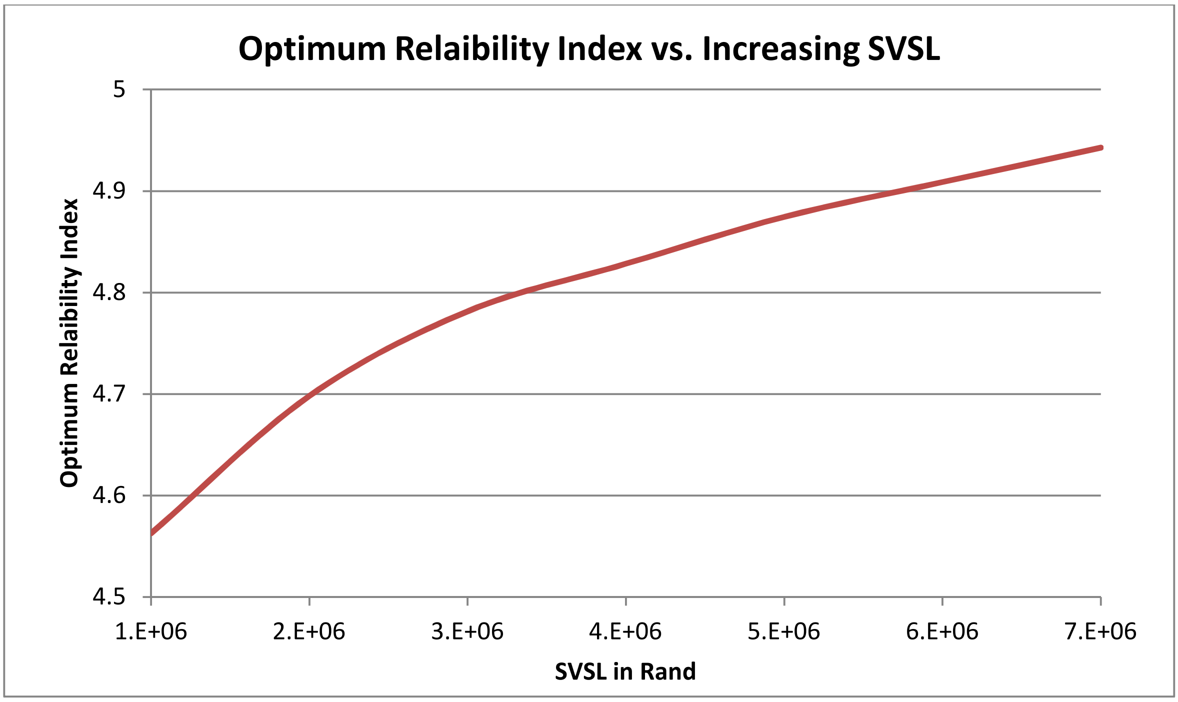

Figure 5-4: Target Reliability Indices vs. Changing Life Compensation Cost (SVSL)

In figure 5-4 it can be seen that if the compensation cost is doubled the reliability index only increases by approximately 0.1. Thus same observation is made as in the previous section, if the compensation costs do not differ by more than a factor of two the target reliability indices will differ insignificantly.

The effect of the number of fatalities per collapsed floor area on target reliability index is explored in the next section. The number of fatalities is varied between reasonable bounds and the same magnitudes are assumed for various costs as in Figure 5-4. However, the SVSL value is equal to R3 752 257 and is based on the calculated w for South Africa. Figure 5-5 shows the results:

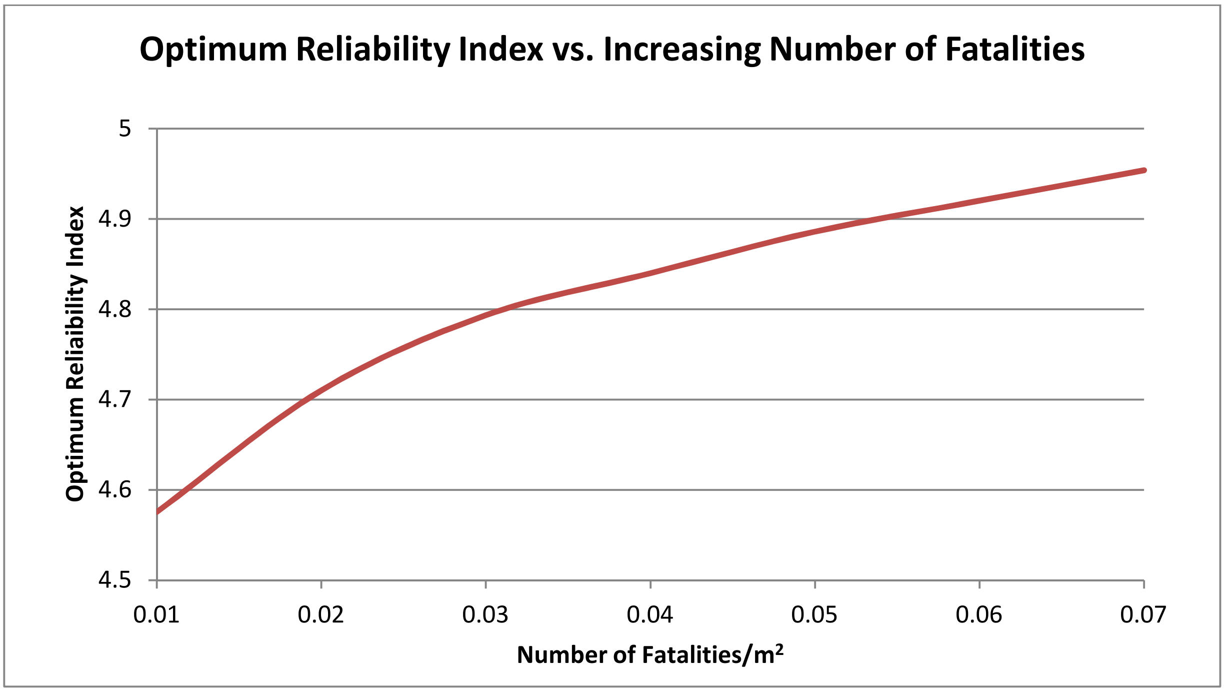

Figure 5-5: Optimum Reliability Indices vs. Changing Number of Fatalities (N)

The target reliability index has approximately the same sensitivity towards changing SVSL than it does towards changing the number of fatalities.

The other failure costs, in units of Rand/m^2, are varied between reasonable bounds to measure the sensitivity of target reliability to these costs. These costs are C_0, which is the construction costs of the structure and H_m (refer to literature review section 2-6). Figure 5-6 shows the results:

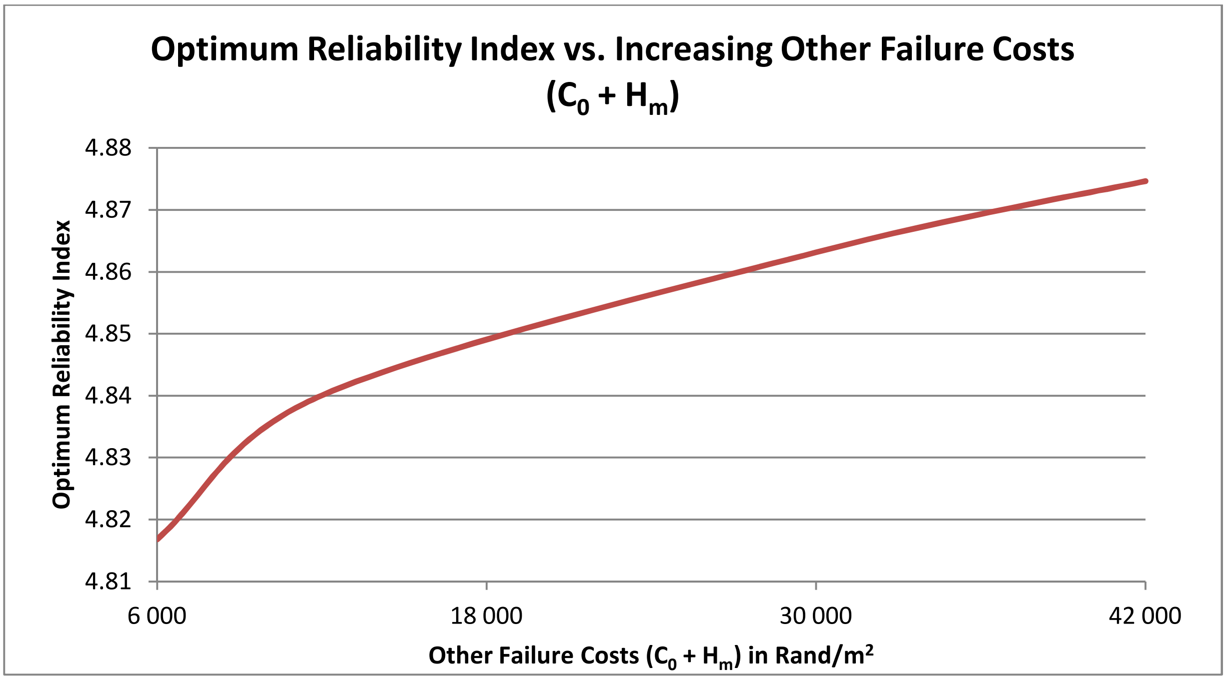

Figure 5-6: Optimum Reliability Indices vs. Changing Other Failure Costs

From Figure 5-6 it is clear that the target reliability index is not influenced greatly by the other failure costs. This is due to the fact the life compensation failure cost dominates these other costs. This aspect was also observed in Table 2-5 in the literature review.

5.6 The Relationship between Optimum Probability of Failure & Optimization Parameters

A study conducted by Fischer et al. (2012) which partly replicated the study conducted by Rackwitz (2000), determined that the minimum probability (derived from the LQI criterion) can be well approximated by a straight line graph relating the benefit/cost parameters and the yearly failure rate provided that the variance of the random variables is small. The focus of this section is whether this can also be done for the optimum safety determined from the modified cost function and the basic two random variables limit state function. The following figure shows the results of the study conducted by Fischer et al. (2012):

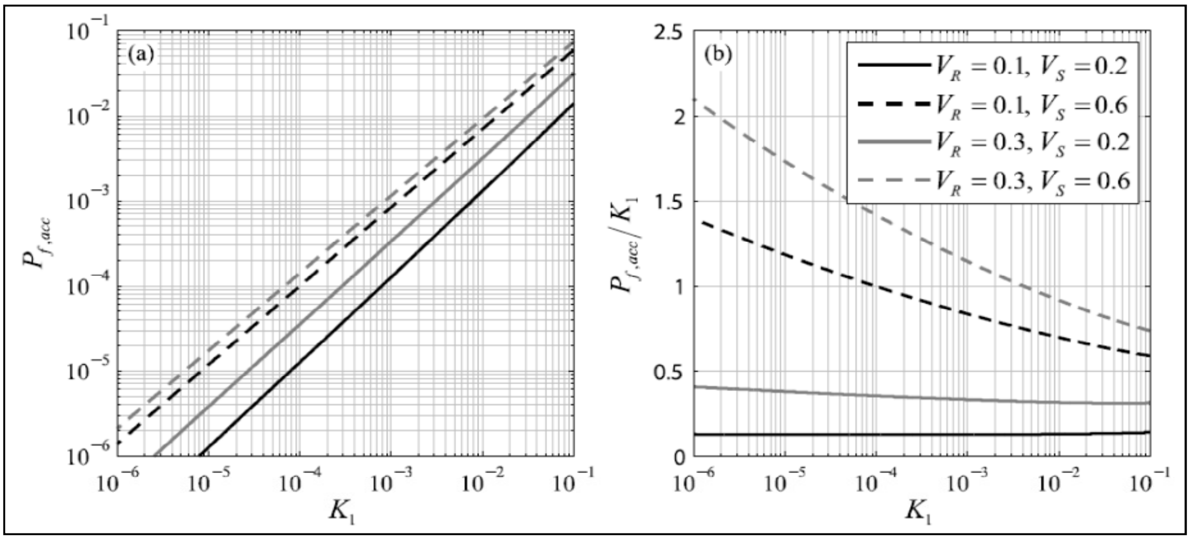

Figure 5-7: Effect of Variance on the Relationship between Minimum P_f and K_1 (Fischer et al. (2012))

The parameter K_1 is the cost of increasing safety plus the cost of obsolescence divided by the costs of failure, while the minimum probability of failure is the probability of failure derived from the LQI criterion. It can be seen that with decreasing K, the minimum probability of failure is also decreasing. For low variances the relationship between the minimum probability of failure and parameter K_1 is almost linear, but for high variances the relationship becomes non-linear according to Fischer (2012).

It was decided to replicate the study above to try and verify whether or not a similar relationship can be established between the optimum probability of failure and relative cost of safety.

The parameter K or relative cost of safety is defined as follows (K is different from K_1, as K_1 based on LQI criterion and K is based on optimization):

$$K = \frac{C_1(\omega+\gamma)}{(C_0+H_m+C_1+H_F)\lambda} \tag{5-5}$$

In the following figure the optimum probability of failure and relative cost of safety are plotted against each other for different variances. A log scale is used for both the probability of failure axis and the relative cost of safety axis.

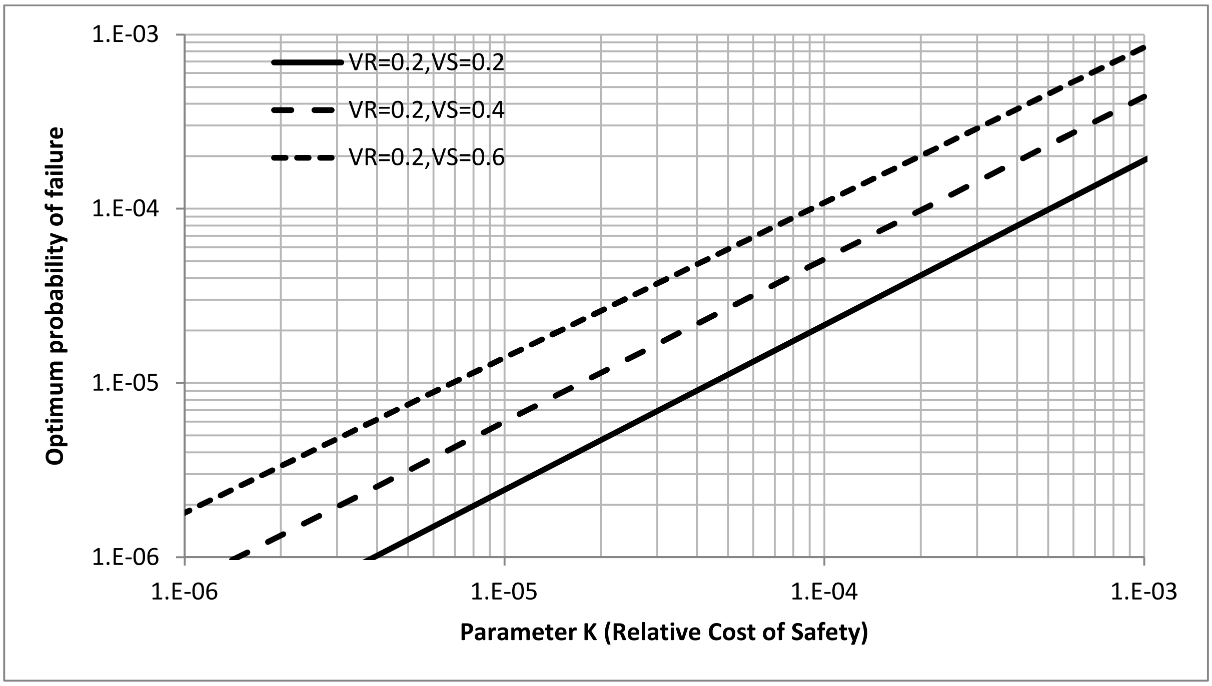

Figure 5-8: Effect of Variance on the Relationship between Optimum Probability of Failure and K

It is clear from Figure 5-8 that a similar relationship is observed for optimum probability of failure than the relationship between minimum probability of failure and the relative cost of safety as determined by Fisher et al. (2012). This result implies that the optimum probability of failure can be reasonably approximated without having to conduct a reliability optimization analysis for any typical structure falling under RC2. If the position of the optimum solution of the realistic limit state function relative to the lines shown in Figure 5-8 does not differ by varying K, the above statement is correct. The difficulty is to determine the variance of the resistance and load effect for the failure mode under consideration. The two above aspects will be explored by conducting various case studies for various elements and various limit sate functions in the following chapters.

5.7 Difference between Optimum Safety and Minimum Safety

In this chapter the focus has been on obtaining the optimum safety of a structural member. However the question now arises whether this optimum safety is always larger than the minimum safety derived from the LQI?

Studies conducted by Rackwitz (2002) suggested that for most cases the optimum safety will be more than the minimum safety required from a societal point of view. The focus in this section is to see if changes in some parameters cause the optimum safety to be less than the minimum safety.

Updating equation 2-50 with the assumptions made in section 5-3, results in the following LQI criterion:

$$C_1\left(1+\frac{\omega}{\gamma}\right) \geq -\frac{d}{dp}\left((\mathrm{SWTP} \times N_F)\frac{\lambda P_f(p)}{\gamma}\right) \qquad \textbf{5-6}$$

The optimum reliability is computed for different assumed number of fatalities per collapsed area and cost of increasing safety. The same is done for the minimum reliability by using the MATLAB function *Generic_Min* and Figure 5-9 compares the results.

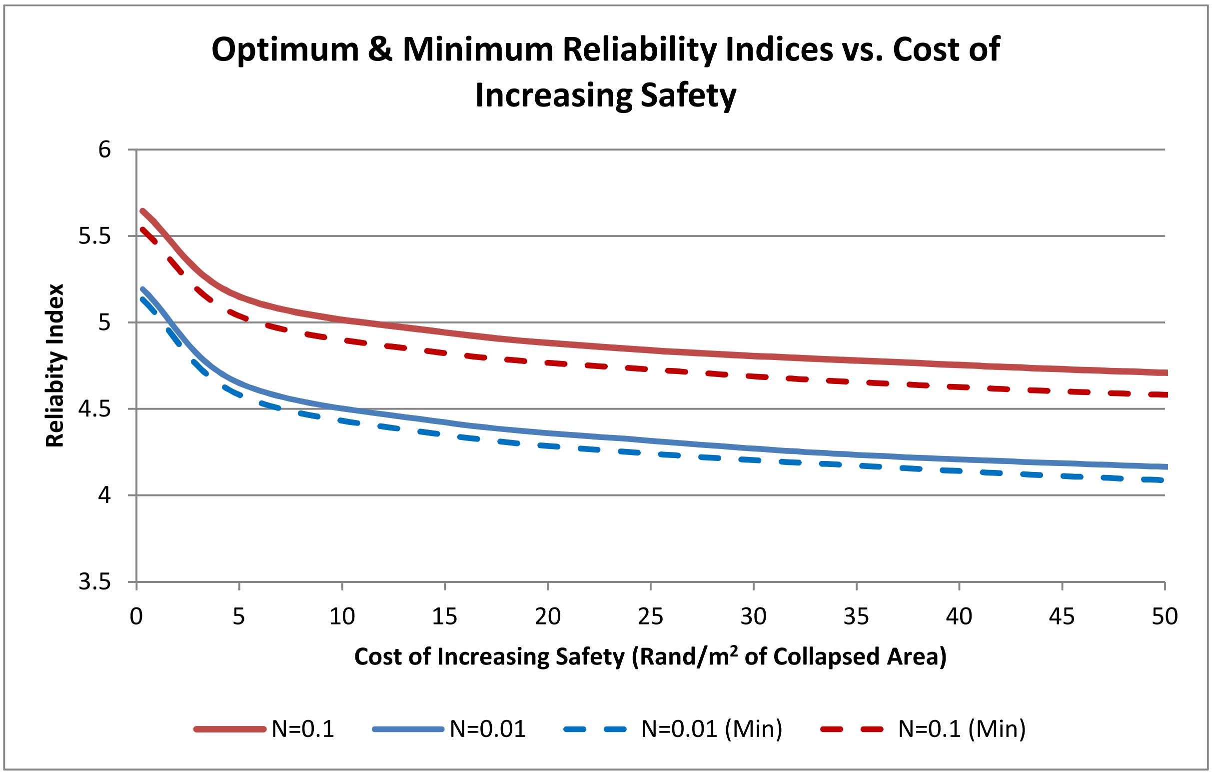

Figure 5-9: Optimum & Minimum Target Reliability Indices

It can be seen that the optimum safety is always larger than the minimum safety for different cost of increasing safety and fatalities per collapsed floor area. It is however interesting to note that with increasing number of fatalities the difference between the optimum and minimum safety increases.

The study conducted by Rackwitz (2002) concluded that the minimum safety will be more than the optimum safety if the optimization is done for privately owned infrastructure with a high discount rate. For the following study the costs of increasing safety is assumed as a constant value of R35/m^2 and the interest rate was varied between 2% and 7%.

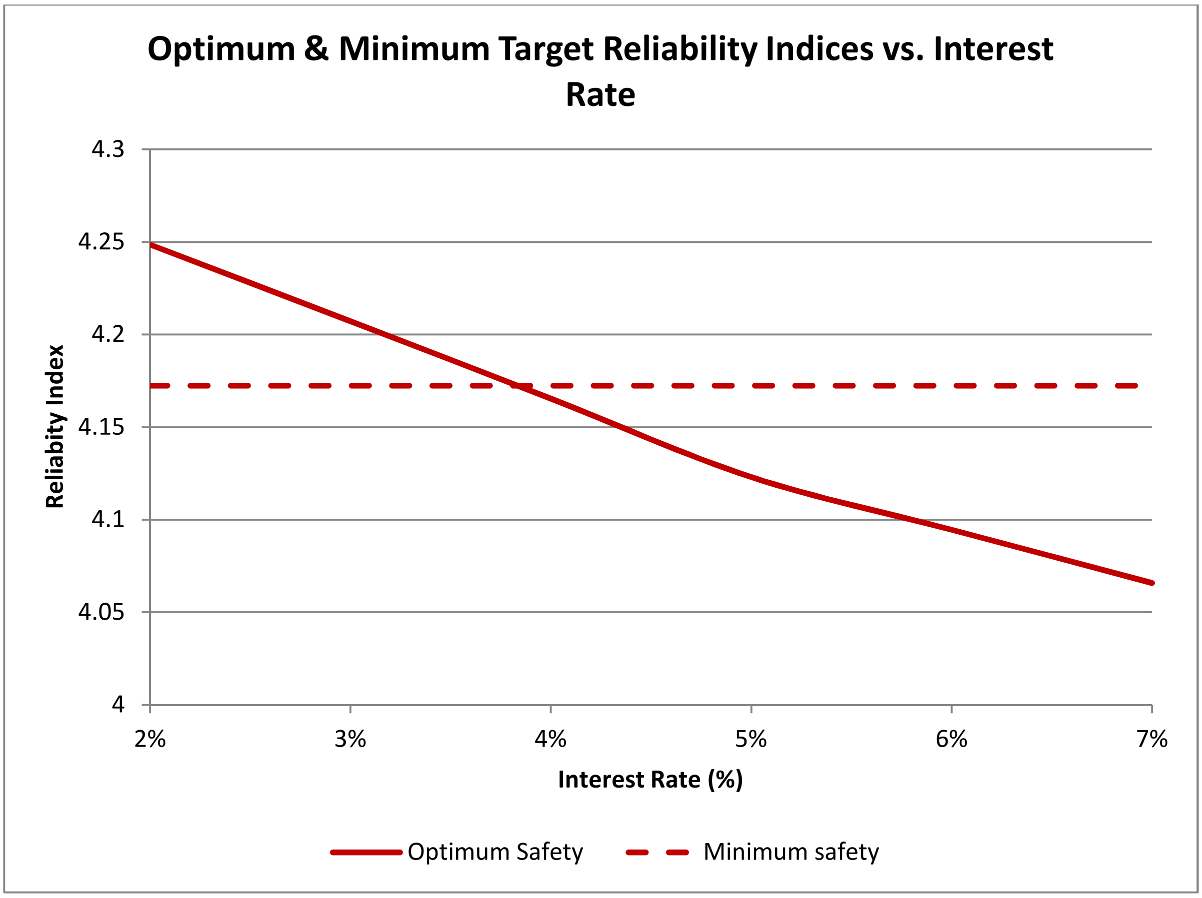

Figure 5-10: Optimum & Minimum Target Reliability Indices vs. Changing Interest Rate

At a discount rate of 4% the optimum safety is less than the minimum safety derived from the LQI. Therefore the discount rate chosen by the owner will determine whether or not the optimum or minimum safety is set as the target safety.

The following figure shows the results of increasing the coefficient of variance for the resistance variable for both optimum and minimum probability of failures. The cost of increasing safety is assumed to be equal to R35/m^2 and the number of fatalities is assumed to be equal to 0.01/m^2.

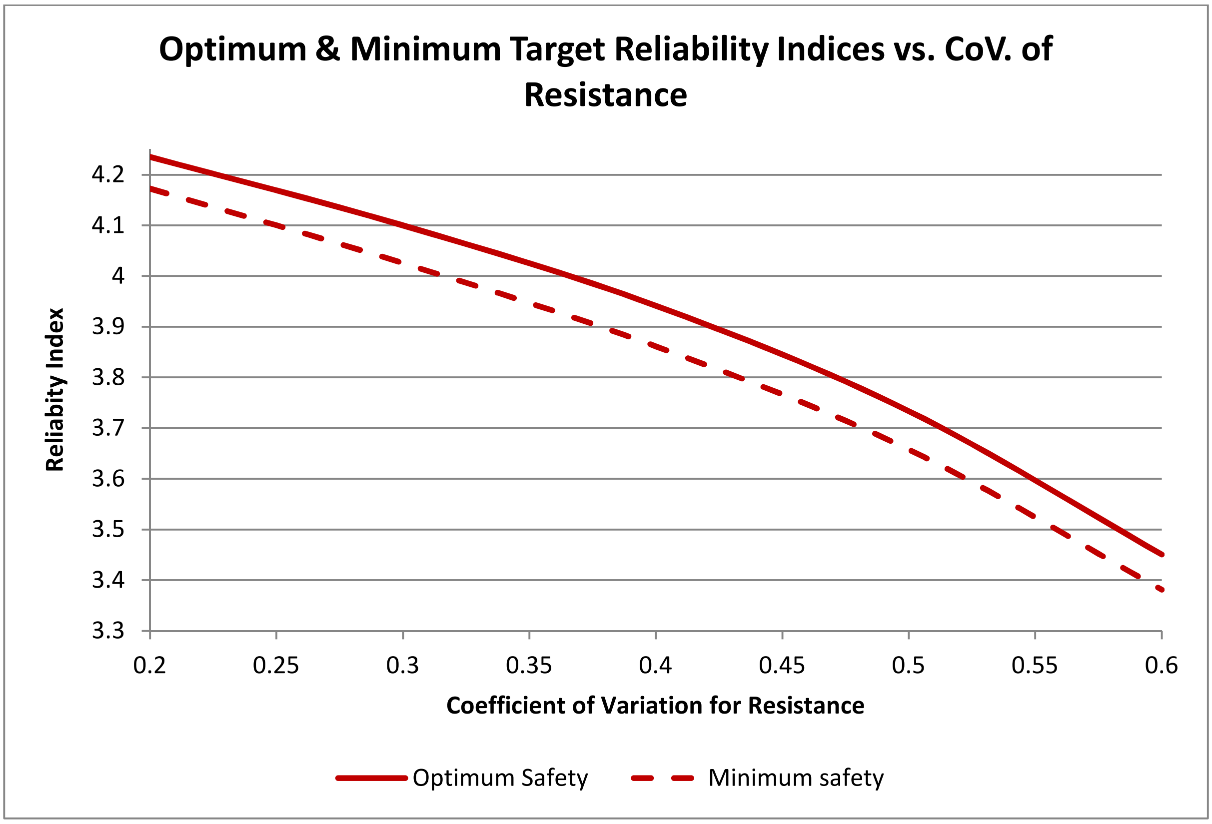

Figure 5-11: Optimum & Minimum Target Reliability Indices vs. Changing CoV.

With increasing variance the optimum and minimum reliability indices decreases. This is as a result of the fact that increasing the variance decreases the cost effectiveness of increasing a safety parameter. In other words it is more costly to obtain the same safety for a structural member with a high variance than a structural member with a low variance. The coefficient of variation has no effect on the difference between optimum safety and minimum safety as seen in the above figure.

For the purpose of this thesis, a discount rate of 2.4% is used thus the optimum reliability index will be more than the minimum reliability index required by society.

Another study conducted by Fischer et al. (2012) showed mathematically that if SVSL is larger than SWTP the optimum safety will be larger than the minimum safety given that the reliability optimization is done from the perspective of the public.

5.8 Conclusions

The two work time fractions used to determine different SVSL's and SWTP's has a relatively insignificant effect on the optimum safety. A conclusion is therefore made that if two life compensation costs have the same order of magnitude and does not differ by a factor of more than 2, the results will differ insignificantly. It was also clear that the other failure costs (C_0 + H_m) have an insignificant impact on the optimum safety, while the cost of increasing safety, the assumed number of fatalities and coefficients of variance of load and resistance have a significant impact on optimum safety.

This section has also shown that it is unnecessary to check whether or not the optimum safety is more than the minimum safety derived from LQI for typical concrete structures in RC2. The parameter influencing this switch is the discount rate. The LQI minimum safety level may govern for high discount rates. The discount rate is not more than 2.4% for this particular study, thus the optimum safety will be more than the minimum safety.

The relationship established between the cost relative cost of safety and the optimum probability of failure allows for an approximation of the optimum safety for various forms of concrete structures falling under RC2. The difficulty comes in determining the optimum probability of failure or the implicit variance of load and resistance for more detailed limit state functions describing different element types and structural configurations with various different types of distributions for random variables as well as to establish realistic areas of collapse and corresponding fatalities for these. In Chapters 6, 7 and 8 various case studies are considered and the relationship between the cost relative cost of safety (parameter K) and the optimum probability of failure is determined by comparing the results to Figure 5-8. From this comparison it will be determined if the simplified approach can be used to approximate the optimum solution of structural elements with complex limit state functions.

Chapter 6 OPTIMIZATION OF CONCRETE SLABS

6.1. Introduction

Floors form an essential part of the structure as their main function is to provide a flat usable surface. In the first part of this chapter the different types of slabs are shown and the different modes of failures are briefly introduced.

The optimization is done for various locations of failures as each location of failure has a unique combination of costs associated with it. This causes different optimum reliability indices derived from the different locations of failure by using the optimization process shown in Chapter 5. During the optimization process the length dimensions of the slab is varied as it has an effect on the costs associated with the location of failure and the statistical properties of the reliability problem. For this particular study it is assumed that the cost of serviceability failure is negligible and that maintenance, fatigue and inspection costs are independent of safety parameter p. Other assumptions stated in Chapter 5 are included. Furthermore, an assumption is made that in all the case studies the structure is braced in both horizontal directions.

The optimum reliabilities for specific cases of slab failure are compared to optimum reliabilities obtained from the generic formulations of Chapter 5. All the target reliability indices are averaged, as a code does not specify a specific target reliability index for a specific location of failure.

6.2. Types of Slabs

Besides the purpose of slabs providing a flat surface, the structural function of slabs are to transfer uniformly distributed loads to the beams or columns supporting them. A slab supported by beams on two edges is known as a one-way spanning slab where the load is transferred in one direction to the beams. A two-way spanning slab is where the slab is supported by beams on four edges and the loads are transferred in two perpendicular directions. A slab can also be supported by only using columns and no beams. A slab supported by only columns is known as a flat slab. The support conditions of a slab are important as it determines the distribution of forces through the slab and thus determines the structural performance of the slab. (Robberts et al. (2010))

The three types of slabs can be further altered by the designer. One-way spanning slabs can be altered by a creating a ribbed slab to reduce the weight of the slab. This alteration can also be applied to two-way spanning slabs and flat slabs in two directions creating a waffle slab. A flat slab can be further altered by creating drop panels at the slab-column connection to increase the load carrying capacity of the connection between the slab and column. The following figure shows the different types of slabs: (Robberts et al. (2010))

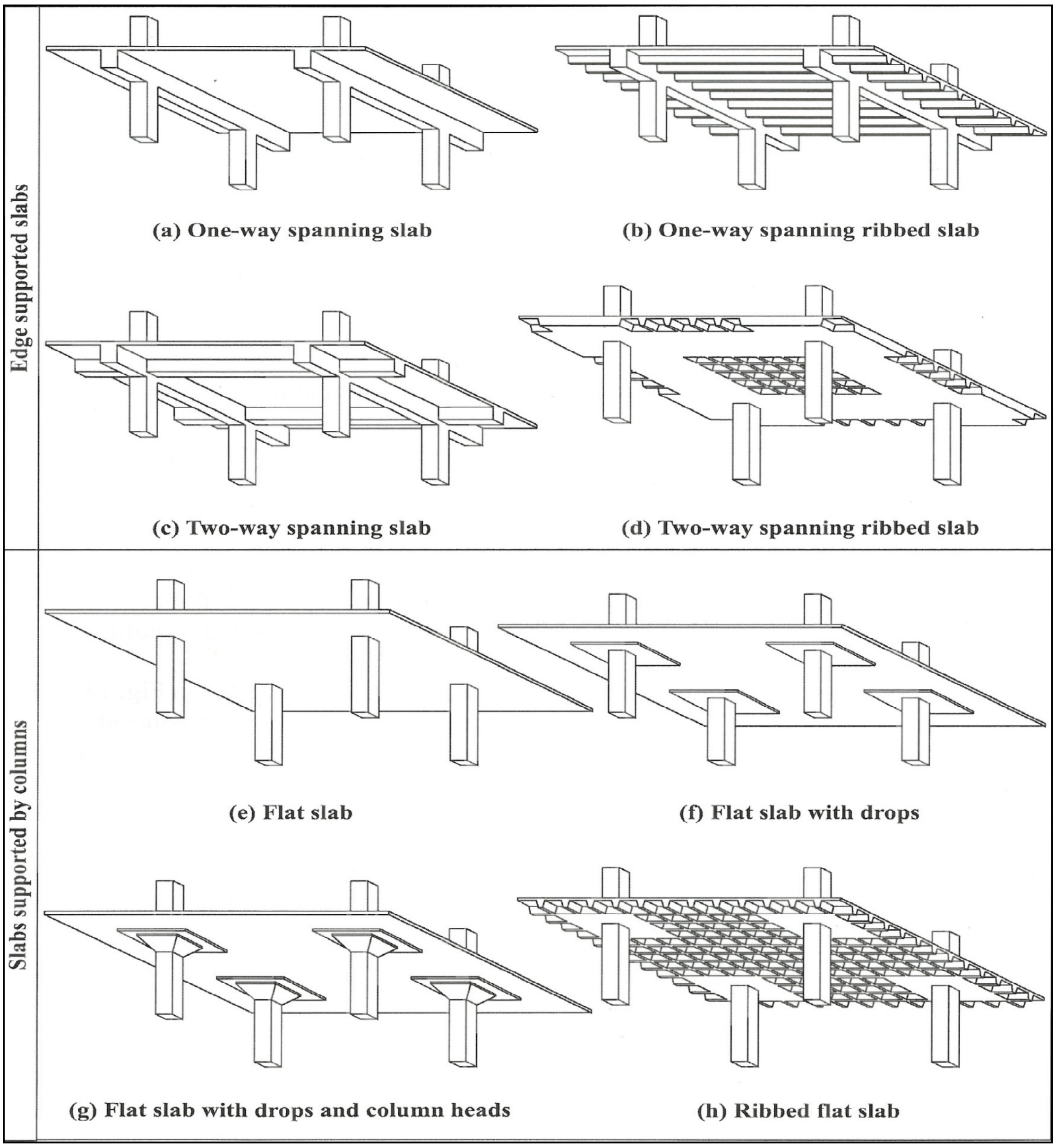

Figure 6-1: Different Types of Slabs (Robberts et al. (2010))

For this study only non-ribbed slabs are considered. Therefore only one-way, two-way and flat slabs are considered for optimization.

6.3. Estimation of Various Parameters

Some of the variables, such as the number of fatalities due to structural collapse and the area of collapsed floor, have to be assumed due to lack of data. Even though models for fatality estimation exist, like those shown in the literature review, data on how people react in the sense whether they perceive the warning signs and whether they are able to react in time to a structural element failing has not been found. There is however data available on how people respond to structural failure due to earthquakes, but this kind of data cannot be applied to this particular study as the focus of this study is on structures failing due to non-earthquake related overloading. In earthquakes the large, sudden deflections of the entire structure gives occupants different warning signs than local structural failure which gives a small vertical displacement changing over time. Therefore the number of people escaping is a parameter where assumptions are made based on structural robustness and other factors and is shown in Table 6-1. A structure is considered as robust if there are various alternative load paths.

Table 6-1 shows the different locations of flexural failure that are considered for this study and the assumptions regarding the probability of escape (P(Q)). The probability of escape depends on the robustness of the structural member and is assigned a best case scenario and a worst case scenario to cover a reasonable range based on Figure 2-10. In this figure the probability of being trapped, based on earthquake data, when a top-bottom collapse occurs is 50%. For this study top-bottom collapses are assumed. Thus the probability of escape is assumed in the same order of magnitude as 50% even though the warning signs differ for earthquakes and localized structural failure. Basically it is assumed that the probabilities of escape will be similar in magnitude for both cases. A feasible range is assigned to the parameters mentioned above. Due to the high uncertainty of the assumed values an upper and a lower bound optimum reliability index will be computed for the structural member for the failure considered. The level of uncertainty of a parameter will determine how wide a range is considered.

The population at risk is obtained by personal judgement. It is assumed that in a typical office building the population is 1 person per 10m^2. An assumption is made that each person in the office has a personal office space of about 4m^2, however the office building requires stairs and hallways which increases the average floor area per person which was estimated as 10m^2.

Slab Type	Location	Mode of Failure	Best Case P(Q)	Worst Case P(Q)	Population at Risk
One-way	Middle	Flexure	0.65	0.2	$0.1/m^2$
One-way	Support	Flexure	0.65	0.2	$0.1/m^2$
Two-way	Middle Strip two different panels	Flexure	0.65	0.3	$0.1/m^2$
Two-way	Edge Strip Two Different Panels	Flexure	0.65	0.3	$0.1/m^2$
Two-way	Simply Supported at Middle	Flexure	0.65	0.3	$0.1/m^2$
Flat Slab	Middle Strip	Flexure	0.5	0.2	$0.1/m^2$
Flat Slab	Edge Strip	Flexure	0.5	0.2	$0.1/m^2$

Table 6-1: Locations of Flexural Failures for Optimization Study

The number of fatalities due to structural failure is determined by using equation 2-52. The relative cost of safety is determined by using the two Tables (2-3 & 2-4) and assuming that the community, emergency squads and SAR experts are all involved in the rescue process. Due to the fact the failure is ductile, an assumption is made that there is a relatively high probability of escape for the best case scenario. As a result of the fact that the probability of escape is estimated, a high possible range is used by assuming a relatively low worst case.

The area of reinforcement is defined as the parameter of safety that is increased during the analysis. The optimization process is done by using a safety parameter known as p (refer to Chapter 5). The unit of area reinforcement equivalent to a safety factor of 1 is determined by substituting the mean values in the limit state function and solving the amount of area reinforcement that will cause G=0. The volume of steel reinforcement is calculated by using simplified curtailment rules for slabs taken from SANS 0100-01(2000) and multiplying the reinforcement lengths by the area of reinforcement that is equivalent to the safety factor (p) being equal to 1. The cost of reinforcement is R9000/ton, including fixing costs, taken from Robberts et al (2010) and the density of steel of $7850kg/m^3$ is used for this study. All other live saving and life compensation costs are based on 2011 data, therefore the cost of R9000 is increased to R9500/ton due to inflation. From this the cost of increasing safety is obtained, which has a unit of Rand per collapsed floor area.

The concrete cover is determined by assuming that Y16 reinforcement bars are used. And that the distance between the centre of the reinforcement and the outside of the slab is equal to 33mm.

The loading model is described by a rectangular wave process with jump rate λ. The live load has two components, because of the time related component of the optimization process explained in section 4.3. The optimization calculates an optimum failure rate in units of probability of failure per year. Thus a long term load with an intermittent load or short term load is used to model the live load. The long term load is as a result of equipment and furniture, while the short term load is as a result of a large gathering of people or furniture moved to one room for remodelling of the building. The sustained long term load has an occurrence rate of 0.2/year while a short term load has an occurrence rate of 1/year (Probabilistic Model Code part 2(JCSS, 2001)). The long term load is modelled by a gamma distribution and a Gumbel distribution is used to model the short term load. The standard deviation is calculated by using the following equation adopted from the Probabilistic Model part 2(JCCS, 2001):

$$\boldsymbol{\sigma^2 = \sigma_v^2 + \sigma_u^2 \frac{A_0}{A} k} \qquad \textbf{6-1}$$

σ= Standard deviation of the live load

σ_v= Standard deviation of the overall load intensity

σ_u= Standard deviation of the random field describing space variation of the load

A_0= The reference area

A= The loaded area under consideration for the reliability problem

K= The influence factor

The influence factor is determined to be equal to 2 from the PMC and in this particular case A was always smaller than A_0 so that the ratio of A_0 and A is always taken as equal to one as required by the PMC. In the study cases where A is not smaller than A_0 the ratio in equation 6-1 is calculated. The standard deviation for the short term imposed load is calculated by using the second part of equation 6-1.

Furthermore, an assumption is made that the C_0 cost component is R8000/m^2. The area of the collapsed slab is estimated based on the requirements of SANS 10160 Part 1 (2010) shown in Figure 2-11 that limits disproportionate damage of a localized element failure to the entire structure by providing horizontal ties. In addition to the above, it is assumed that the slab supporting the collapsed slab is damaged beyond repair and needs to be replaced, but has not collapsed on the slab below. Table 2-5 shows that the cost of replacing lost structural building components and non-structural building components is about 50% of the initial cost of the structure per floor area. The other losses are 1.6 times the C_0 cost according to Table 2-5 and calculated to be approximately R12000/m^2. Even though this is a rough estimation Chapter 5 has shown that the C_o and H_m have little effect on the optimum solution.

Parameter	Type of Statistical Distribution	Description	Mean	Standard Deviation	Skewness	λ	Units
θ_R	Lognormal	Model uncertainty factor for resistance	1.20	0.18	0.45	NA	NA
fy	Lognormal	Yield Strength of Reinforcement	510000000	30000000	0.18	NA	Pa
A_s	Deterministic	Area of reinforcement	0.00013	NA	NA	NA	m^2
h_s	Normal	Height of slab	0.15	0.01	0	NA	m
d'	Gamma	Cover of concrete + 16/2	0.033	0.005	0.30	NA	m
θ_E	Lognormal	Model uncertainty factor for load	1.00	0.20	0.60	NA	NA
Q_L	Gamma	Long-term load	500	900	3.60	0.2	Pa
Q_s	Gumbel	Short-term load	200	565.69	1.14	1	Pa
γ	Normal	Density of Concrete	24000	960	0	NA	N/m^3
l2	Deterministic	Length y	5.00	NA	NA	NA	m
f_{cu}	Lognormal	Strength of Concrete	39062500	7031250	0.54	NA	Pa
l1	NA	Length x	5.00	NA	NA	NA	m
C1	NA	Cost of Increasing Safety	8.09	NA	NA	NA	$Rand/m^2$
w	NA	Obsolescence rate	0.02	NA	NA	NA	NA
y	NA	Discount rate	0.024	NA	NA	NA	NA
P(Q)(Best Case)	NA	Probability of Escape	0.65	NA	NA	NA	NA
P(Q)(Worst Case)	NA	Probability of Escape	0.2	NA	NA	NA	NA
Npar	NA	Exposed Population	0.10	NA	NA	NA	$People/m^2$
k	NA	Probability of Dying	0.82	NA	NA	NA	NA
N_f(Best Case)	NA	Number of Fatalities	0.029	NA	NA	NA	$People/m^2$
N_f(Worst Case)	NA	Number of Fatalities	0.066	NA	NA	NA	$People/m^2$
SVSL	NA	Compensation Cost	3.752 mil	NA	NA	NA	Rand
$H_m + C_0$	NA	Other Losses and C_0	12000+8000	NA	NA	NA	$Rand/m^2$

Table 6-2: Parameters Used for Optimization Case study

The mean, standard deviation and statistical distribution type of the random variables are obtained from the probabilistic model code provided by the Probabilistic Model Code (Part1, Part 2 & Part 3)(JCSS, 2001). Table 6.2 shows the statistical and other properties of all the various parameters used in this chapter.

6.4. Optimization of One-way Spanning Slabs

This section focuses on obtaining a range of possible optimum reliability indices for one-way spanning slabs. Figure 6-2 shows the example of the slab under consideration. It is assumed that the slab is 150mm thick. It is further assumed that a concrete strength of 25MPa is used and that the primary function of the structure is to serve as an office building. During the analysis the length of the slab (l2) is varied. The thickness to effective length ratio is kept constant throughout the optimization process to normalize the results.

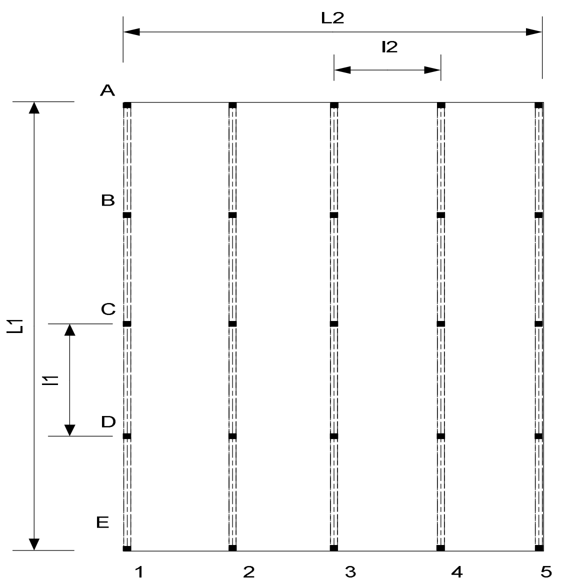

Figure 6-2: Plan View of One-way Spanning Slab

The moments are calculated from the following table adopted from the SANS 0100-1(2000):

Position	Factor
At outer support	$0Fl_{eff}$
Near middle end of span	$0.086Fl_{eff}$
At first interior support	$-0.086Fl_{eff}$
At middle of interior spans	$0.063Fl_{eff}$
At interior supports	$-0.063Fl_{eff}$

Table 6-3: Factors to Determine Moments (SANS 0100-01 (2000) Table13)

The slab spanning between supports A1, A2, E1 and E2 is optimized by assuming flexural tension failure at mid-span. The following table shows the various parameters used for the optimization process. The l1 is assumed to have a constant value of 5m.

The limit state function for a slab in flexure is as follows:

$$G = \theta_R\, f_y A_s \times \left(h_s - d' - \frac{0.75 f_y A_s}{f_{cu}}\right) - 0.086 \times \theta_E(\, Q_L + Q_s + \gamma \times h_s) l2^2 \qquad \textbf{6-2}$$

The cost function to be optimized is the same as equation 5-4. Table 6-4 and Figure 6-3 shows the optimum reliability indices obtained for different lengths of the slab. The results are compared with medium costs of increasing safety and medium consequences from various tables recommending target reliability indices obtained from the PMC, EN and ISO. The SANS target reliability index is obtained from recommendations made for structures in RC2.

L2	Cost of Increasing Safety (R/m^2)	β Best Case	β Worst Case
5	8.49	4.50	4.63
8	18.49	4.43	4.57
10	27.43	4.40	4.54

Table 6-4: Results for One-way Spanning Slab Optimization Assuming Tension Failure at Mid-span

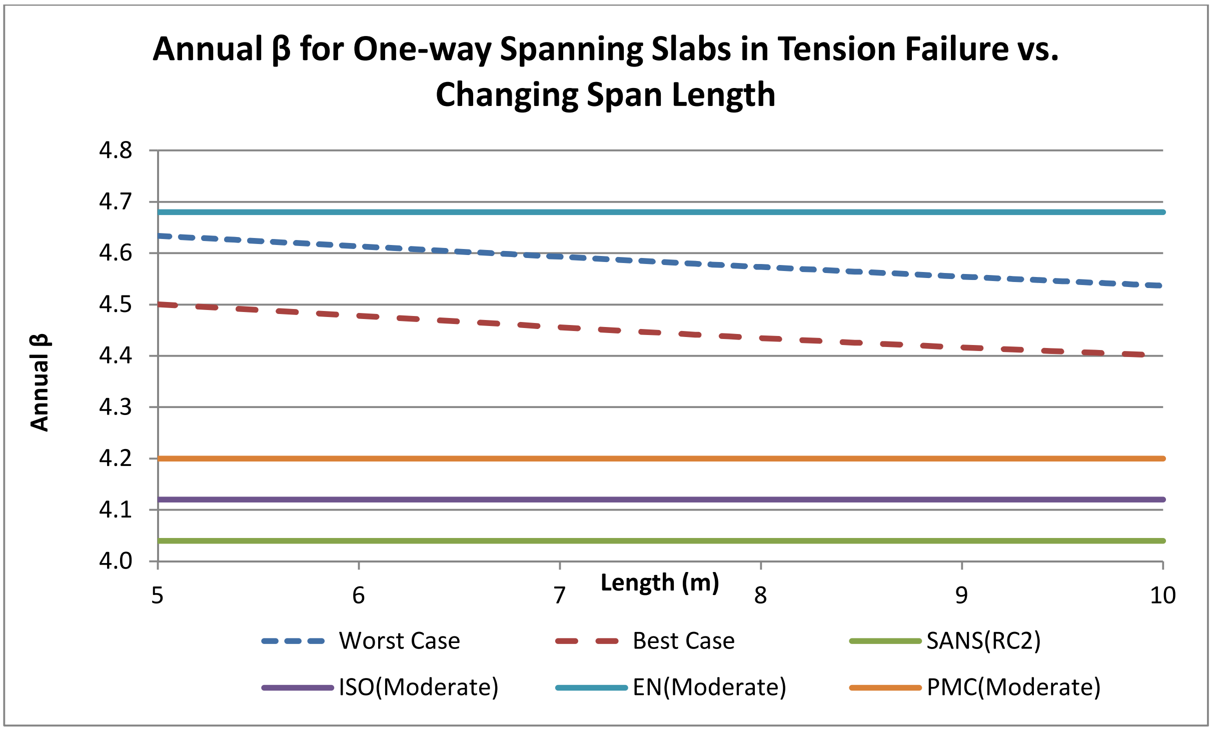

Figure 6-3: Results for One-way Spanning Slab Optimization Assuming Tension Failure at Mid-span

An observation is made that the relative cost of increasing safety increases as the span length of the slab increases. This causes the slab to be less cost effective to increase the safety for larger spans and in turn results in lower optimum reliability indices.

It is also interesting to note that from the different target reliability indices proposed by different codes the target reliability indices proposed by the Eurocode is the closest to the results obtained for this particular study case.

Even though the worst case scenario has a probability of escape (P(Q)) 0.45 higher than the best case scenario, the difference in target reliability indices is only 0.14.

The same reliability based cost optimization procedure is followed for tension failure at the support of the slab as a result of an internal moment. The limit state function and random variables are mostly the same. However, in this case the cost of increasing safety is significantly less because the length of reinforcement is significantly shorter at the support of the slab than the length of steel reinforcement required at mid-span of the slab to resist flexural tension.

The slab spanning between supports A1, A2, E1 and E2 is optimized in terms of the flexural tension failure along support line 2 (Area of l2 by five times l1 is assumed to collapse). Another difference between this particular analysis and the previous analysis is the standard deviation of the cover of concrete is higher for top steel according the JCSS Probabilistic Model Code part 3 (2001). The following table shows the results.

l2	Cost of Increasing Safety (R/m^2)	β Best Case	β Worst Case
5	2.42	4.73	4.85
8	5.25	4.67	4.80
10	7.79	4.63	4.76

Table 6-5: Results for One-way Spanning Slab Optimization Assuming Tension Failure at Support

It is clear from the results of the study that a decreased relative cost to increase safety has resulted in higher optimum reliability indices. As observed before the relative cost of increasing safety increases slightly with increasing span length. The following figure shows the results.

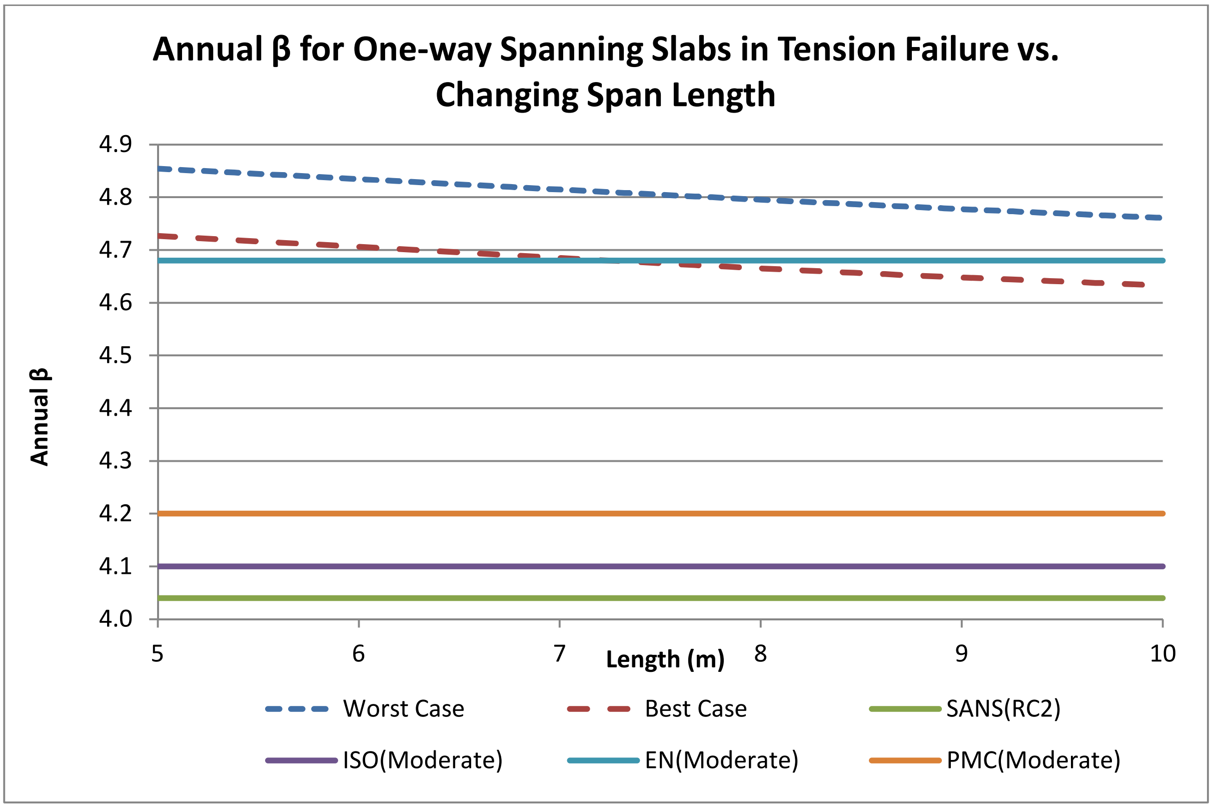

Figure 6-4: Results for One-way Spanning Slab Optimization Assuming Tension Failure at Support

The target reliability indices are higher than target reliability indices recommended by the SANS10160, ISO2394 & JCSS Probabilistic Model Code Part 1 listed in section 2.8, but compares well with target reliability indices set in the EuroCode (EN 1990).

6.4. Optimization of Two-way Spanning Slabs

The main difference between the different types of slabs is not only the geometrical properties, but also the support conditions and thus the distribution of forces through the slab. The following table shows the different cases and case specific coefficients used to calculate the moments in both simply supported and edge restrained two-way spanning slabs. (SANS 10100-1 (2000))

ℓ_y / ℓ_x	α_{sx}	α_{sy}
1.0	0.045	0.045
1.1	0.061	0.038
1.2	0.071	0.031
1.3	0.080	0.027
1.4	0.087	0.023
1.5	0.092	0.020
1.6	0.097	0.017
1.7	0.100	0.015
1.8	0.102	0.016
1.9	0.103	0.016
2.0	0.104	0.016
2.5	0.108	0.016
3.0	0.111	0.017

Table 6-6: Bending Moment Coefficient for Simply Supported Slabs (Table 14 SANS 0100 (2000))

The following figure shows the various specific slab panel cases for a slab that is cast monolithically with its supporting beams. (Robberts et al. (2010))

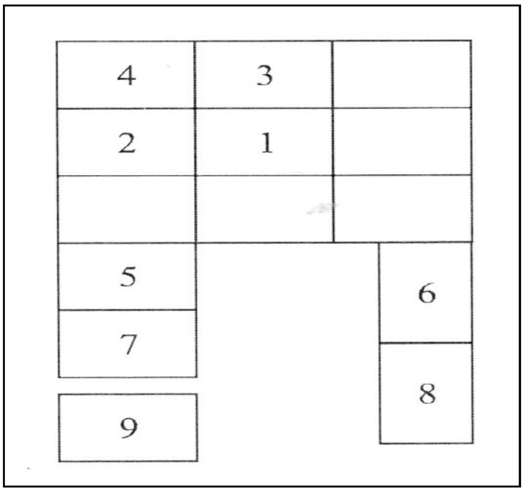

Figure 6-5: Support Conditions for Slab Panels (Robberts et al. (2010))

The following table shows the moment coefficients for slabs that are cast monolithically with their supporting beams. (SANS 0100 (2000))

Type of panel and moments considered	Short span coefficients β_{sx} for ℓ_y/ℓ_x								Long span coefficients β_{sy} for all ℓ_y/ℓ_x
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
1. Interior panel									
Negative moment at continuous edge	0.031	0.037	0.042	0.046	0.050	0.053	0.059	0.063	0.032
Positive moment at midspan	0.024	0.028	0.032	0.036	0.039	0.041	0.045	0.049	0.024
2. One short edge discontinuous									
Negative moment at continuous edge	0.039	0.044	0.048	0.052	0.055	0.058	0.063	0.067	0.037
Positive moment at midspan	0.029	0.033	0.036	0.039	0.041	0.043	0.047	0.050	0.028
3. One long edge discontinuous									
Negative moment at continuous edge	0.039	0.049	0.056	0.062	0.068	0.073	0.082	0.089	0.037
Positive moment at midspan	0.030	0.036	0.042	0.047	0.051	0.055	0.062	0.067	0.028
4. Two adjacent edges discontinuous									
Negative moment at continuous edge	0.047	0.056	0.063	0.069	0.074	0.078	0.087	0.092	0.045
Positive moment at midspan	0.036	0.042	0.047	0.051	0.055	0.059	0.065	0.070	0.034
5. Two short edges discontinuous									
Negative moment at continuous edge	0.046	0.050	0.054	0.057	0.060	0.062	0.067	0.070	-
Positive moment at midspan	0.034	0.038	0.040	0.043	0.045	0.045	0.047	0.053	0.034
6. Two long edges discontinuous									
Negative moment at continuous edge	-	-	-	-	-	-	-	-	0.045
Positive moment at midspan	0.034	0.046	0.056	0.065	0.072	0.078	0.091	0.100	0.034
7. Three edges discontinuous (One long edge continuous)									
Negative moment at continuous edge	0.057	0.065	0.071	0.076	0.080	0.084	0.092	0.098	-
Positive moment at midspan	0.043	0.048	0.053	0.057	0.060	0.063	0.069	0.074	0.044
8. Three edges discontinuous (One short edge continuous)									
Negative moment at continuous edge	-	-	-	-	-	-	-	-	0.058
Positive moment at midspan	0.042	0.054	0.063	0.071	0.078	0.084	0.096	0.105	0.044
9. Four edges discontinuous									
Positive moment at midspan	0.055	0.065	0.074	0.081	0.087	0.092	0.103	0.111	0.056

Table 6-7: Bending Moment Coefficient for Slabs with Restrained Edges (Table 15 SANS 0100 (2000))

Due to the fact that a two-way spanning slab is more robust than a one-way spanning slab, as a result of alternate load paths, the probability of escape is increased for the worst case scenario to 0.3. The following figure shows the plan view of the two-way spanning slab under consideration:

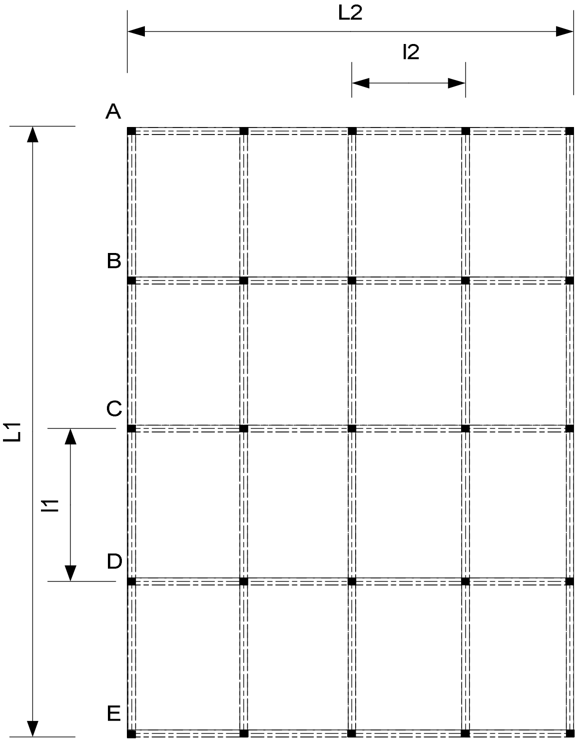

Figure 6-6: Plan View of Two-way Spanning Slab

Most of the variables are as defined in Table 6-2, however the amount of steel that results in a safety factor of 1 is significantly lower as the bending moment coefficient is significantly less in this case than in the case of one-way spanning slabs. This results in a significantly lower relative cost of increasing safety. The magnitude of the moment coefficient is dependent on the ratio of the long span to the short span. So the limit sate function is as follows:

$$G = \theta_R\, f_y\, A_s \times \left(h_s - d' - \frac{0.75 f_y A_s}{f_{cu}}\right) - \alpha(l2/l1) \times \theta_E(\,Q_L + Q_s + \gamma \times h_s)l1^2 \qquad \text{6-3}$$

Where α is the moment coefficient depended on the ratio of the dimensions of the slab.

L2	Cost of Increasing Safety (R/m^2)	β Best Case	β Worst Case
5	4.62	4.63	4.74
8	8.44	4.60	4.71
10	8.59	4.64	4.75

Table 6-8: Results for a Simply Supported Two-way Slab

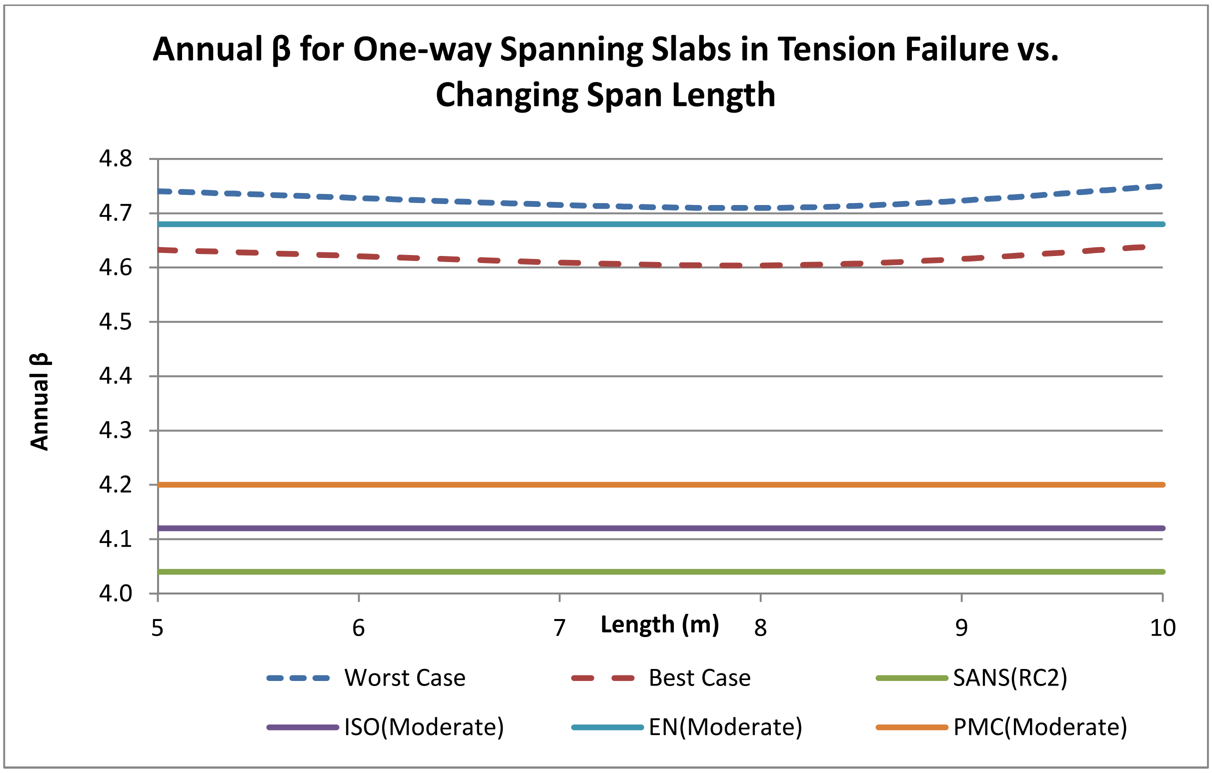

Figure 6-7: Results for a Simply Supported Two-way Slab

In this case, the bending moment coefficient increases as the ratio of the long span over the short span of the slab increases. However, the optimum reliability index remains approximately constant for different span lengths. This is due to the fact that the cost of increasing safety is less sensitive to a changing span length in this case, but the coefficient of variance of the combination of the dead load and live load decreases with a significant amount due to changing span lengths as in the previous cases. Thus the overall effort to achieve a higher level of reliability remains the same over all spans which results in the target reliability indices to remain almost constant.

The probability of escape was increased by 10% for the worst case due to the high robustness of this particular study case. The difference between the optimum reliability indices has decreased by only 0.03. This is an indication of the sensitivity of the target reliability index to changes in the probability of a successful escape. Unless the probability of escape is changed by 30% or more the optimum reliability index will not differ more than 0.1.

The optimization of a two-way spanning slab with restrained edges for mid-span tension failure is now considered. The limit state function is as follows:

$$G = \theta_R\, f_y\, A_s \times \left(h_s - d' - \frac{0.75 f_y A_s}{f_{cu}}\right) - \beta(l12/l1) \times \theta_E(\, Q_L + Q_s + \gamma \times h_s) l1^2 \qquad \textbf{6-4}$$

In the above formulation β is the bending moment coefficient obtained from Table 6-7. All other parameters are kept the same as the previous study case, except the cost of increasing safety, as shown in the following table.

l2	Cost of Increasing Safety (R/m^2)	β Best Case	β Worst Case
5	2.93	4.72	4.82
8	4.37	4.73	4.83
10	4.62	4.75	4.86

Table 6-9: Results for a Partially Restrained Two-way Slab for Flexural Failure at Mid-span

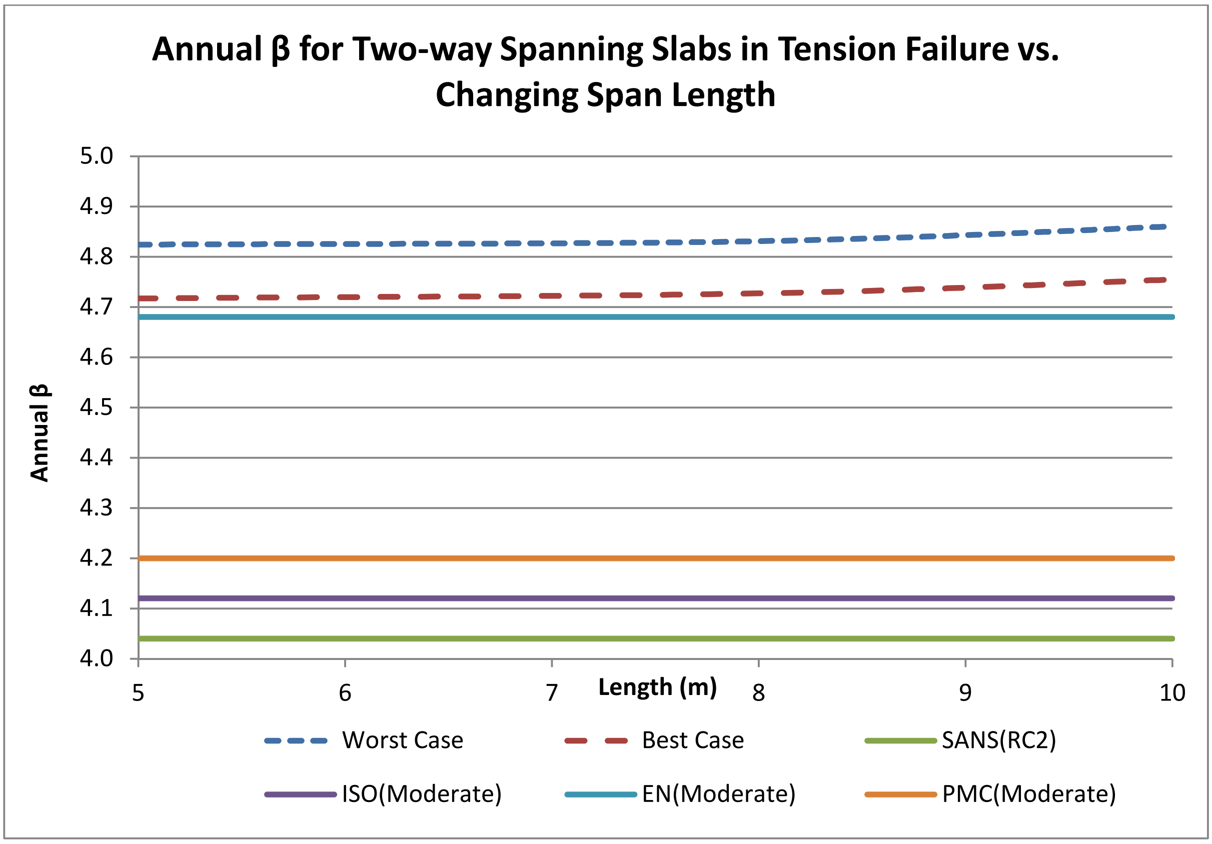

Figure 6-8: Results for a Partially Restrained Two-way Slab for Flexural Failure at Mid-span

For this particular case study the optimum reliability index does not change significantly with changing span length. Even though the cost of increasing safety increases the target reliability index is constant.

The following particular case study considers tension failure for slabs at the supports best described by panel 4. Most of the parameters are kept the same except for the relative cost of increasing safety and the standard deviation of the concrete cover.

The following table and graph shows the results of the optimization study:

l2	Cost of Increasing Safety (R/m^2)	β Best Case	β Worst Case
5	1.13	4.87	4.97
8	1.69	4.88	4.98
10	1.77	4.91	5.01

Table 6-10: Results for a Partially Restrained Two-way Slab for Flexural Failure at the Support

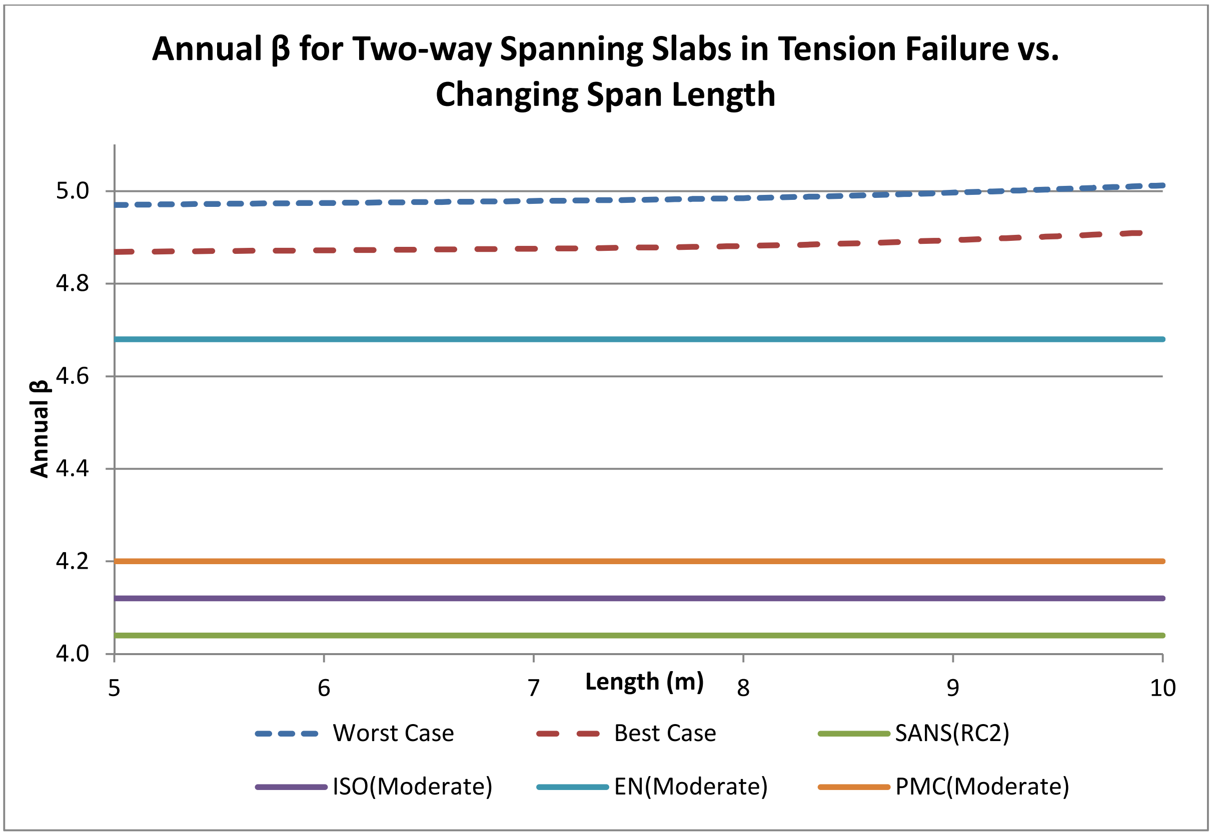

Figure 6-9: Results for a Partially Restrained Two-way Slab for Flexural Failure at the Support

In this case the relative cost of increasing safety is significantly smaller than the other case studies considered which results in relatively high optimum reliability indices. The optimum reliability indices increase slightly with changing span length indicating the low effect span length has on optimum reliability index in this particular case. This is due to the low sensitivity of the cost of increasing safety for a changing span length and the fact that the combined variance of the dead load and live load significantly decreases when the span lengths are increased. The overall effort to achieve a higher reliability is the same for all the span lengths considered above.

A general conclusion can be made that the cost of increasing safety is relatively cheap for flexural failure at the supports of slabs.

6.5. Optimization of Flat Slabs

Flat slabs are only supported by columns. However, a slab with a ring beam is also considered as a flat slab as the behaviour is identical. (Robberts et al. (2010))

Both flexural tension failures at the support and at mid-span are once again considered for optimization case studies, but punching shear is not considered as the contribution of concrete and tension reinforcement to the shear capacity is an equation derived empirically. This equation contributes unrealistic amount of shear capacity for small sections such as a slab and is therefore not considered further.

Concrete strength is assumed to be 25MPa and slab thickness is considered to be 150mm thick. The following figure shows the flat slab considered for optimization studies:

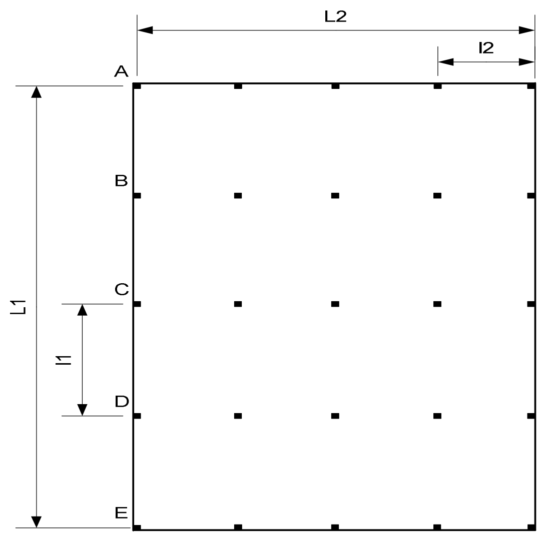

Figure 6-10: Flat Slab Considered for Optimization

The first case study considered is flexural tension failure at the middle of the column strip of the first span from the edge. For flat slabs only the strip under consideration is assumed to collapse. (i.e. column strip, middle strip)

There are various obvious differences between the parameters of flat slabs and the other slabs. Firstly the relative cost of increasing safety is significantly higher due to the increased magnitude of the moments distributed through the slab. Due to the fact that there are less alternative load paths, the probability of escape for both the worst case and best case scenarios are decreased by 10% and 15%. As a result the number of fatalities increases for both cases.

The limit state function is as follows:

$$G = \theta_R\, f_y\, A_s \times \left(h_s - d' - \frac{0.75 f_y A_s}{b f_{cu}}\right) - 0.55 \times 0.083 \times \theta_E(\,Q_L + Q_s + \gamma \times h_s)l2^2 l1 \qquad \text{6-5}$$

The following table shows how the forces can be calculated in a flat slab:

Position	Moment	Shear	Total column moment
Outer support: Column	- 0.04Fl	0.45F	0.04Fl
Wall	- 0.02Fl	0.4F	-
Near centre end of span	+ 0.083Fl	-	-
First internal support	- 0.063Fl	0.6F	0.022Fl
Centre of interior span	+ 0.071Fl	-	-
Interior support	- 0.055Fl	0.5F	0.022Fl

Table 6-11: Ultimate Bending Moments & Shear in Flat Slabs (SANS 0100 Table 16 (2000))

The following table shows the results of the optimization study.

l2	Cost of Increasing Safety (R/m^2)	β Best Case	β Worst Case
5	8.69	4.48	4.57
8	18.98	4.44	4.53
10	28.23	4.40	4.50

Table 6-12: Results for a Flat Slab for Flexural Tension Failure at Mid-span on the Column Strip

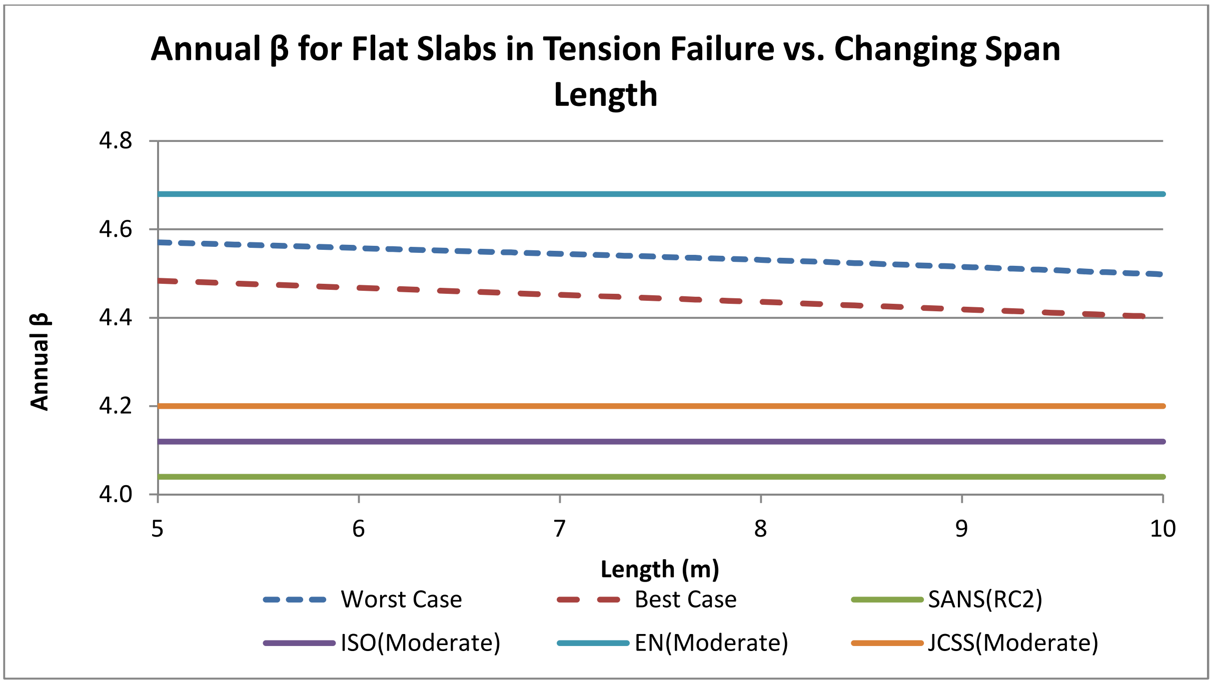

Figure 6-11: Results for a Flat Slab for Flexural Tension Failure at Mid-span on the Column Strip

It is clear from the results that the optimum reliability indices are relatively low compared to the results of the other case studies. The main reason for this relatively low set of optimum reliability indices is as a result of the high magnitude of forces in the flat slab. As a consequence the relative cost of increasing safety is significantly larger than for the other case studies which results in lower target reliability indices.

The same mode of failure is considered above, except the focus is on the middle strip of the slab. For this case the limit state function is as follows:

$$G = \theta_R \, f_y \, A_s \times \left(h_s - d' - \frac{0.75 f_y A_s}{b f_{cu}}\right) - 0.45 \times 0.083 \times \theta_E(\, Q_L + Q_s + \gamma \times h_s) l2^2 l1 \qquad \textbf{6-6}$$

The results of the study are given in the following table.

l2	Cost of Increasing Safety (R/m^2)	β Best Case	β Worst Case
5	7.09	4.54	4.63
8	15.46	4.49	4.59
10	22.97	4.46	4.55

Table 6-13: Results for a Flat Slab for Flexural Tension Failure at Mid-span on the Middle Strip

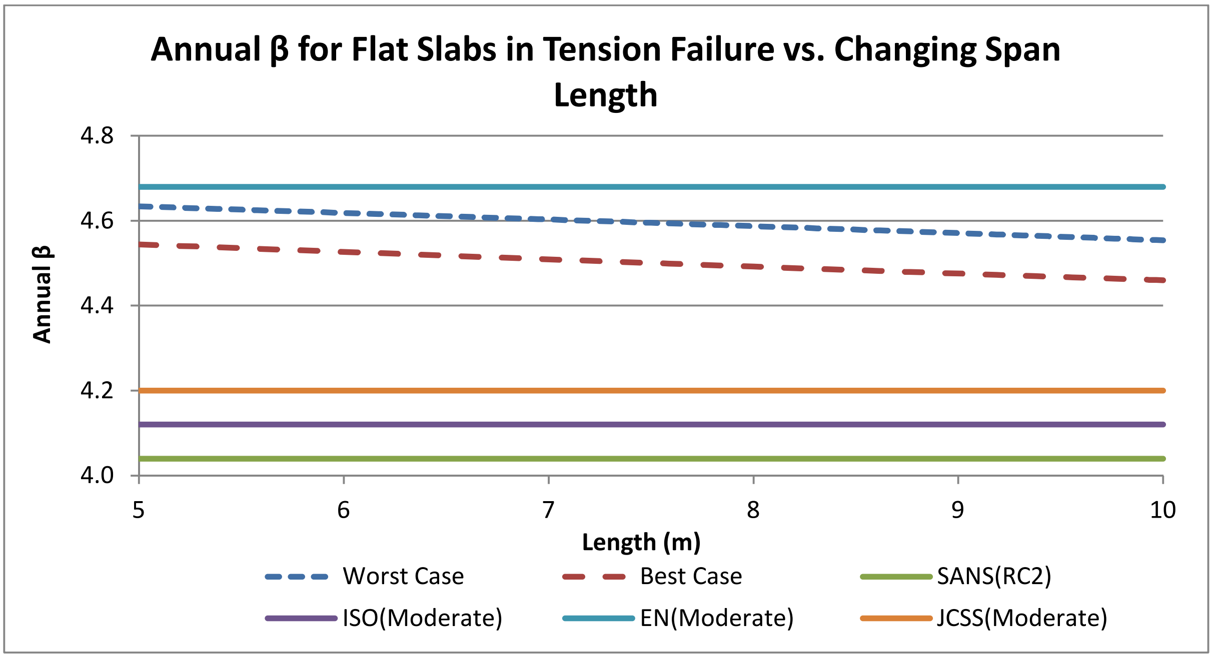

Figure 6-12: Results for a Flat Slab for Flexural Tension Failure at Mid-span on the Middle Strip

The smaller magnitude of forces is distributed in the middle strip of the flat slab at mid-span resulting in lower relative cost for increasing safety. The optimum reliability indices for this case are thus higher than the previous case study.

Flexural tension failure is considered next for both middle and column strips at the support of the flat slab. The standard deviation is higher for the top steel than for the bottom steel and the cost of increasing safety is significantly lower than in the previous case studies.

The following function shows the limit state function for flexural tension failure at the support of a flat slab in the column strip.

$$G = \theta_R\, f_y\, A_s \times \left(h_s - d' - \frac{0.75 f_y A_s}{b f_{cu}}\right) - 0.75 \times 0.063 \times \theta_E(\,Q_L + Q_s + \gamma \times h_s)l2^2 l1 \qquad \textbf{6-7}$$

The following table shows the results of the optimization case study.

l2	Cost of Increasing Safety (R/m^2)	β Best Case	β Worst Case
5	2.56	4.65	4.72
8	5.59	4.64	4.72
10	8.31	4.61	4.70

Table 6-14: Results for a Flat Slab for Flexural Tension Failure at Support on the Column Strip

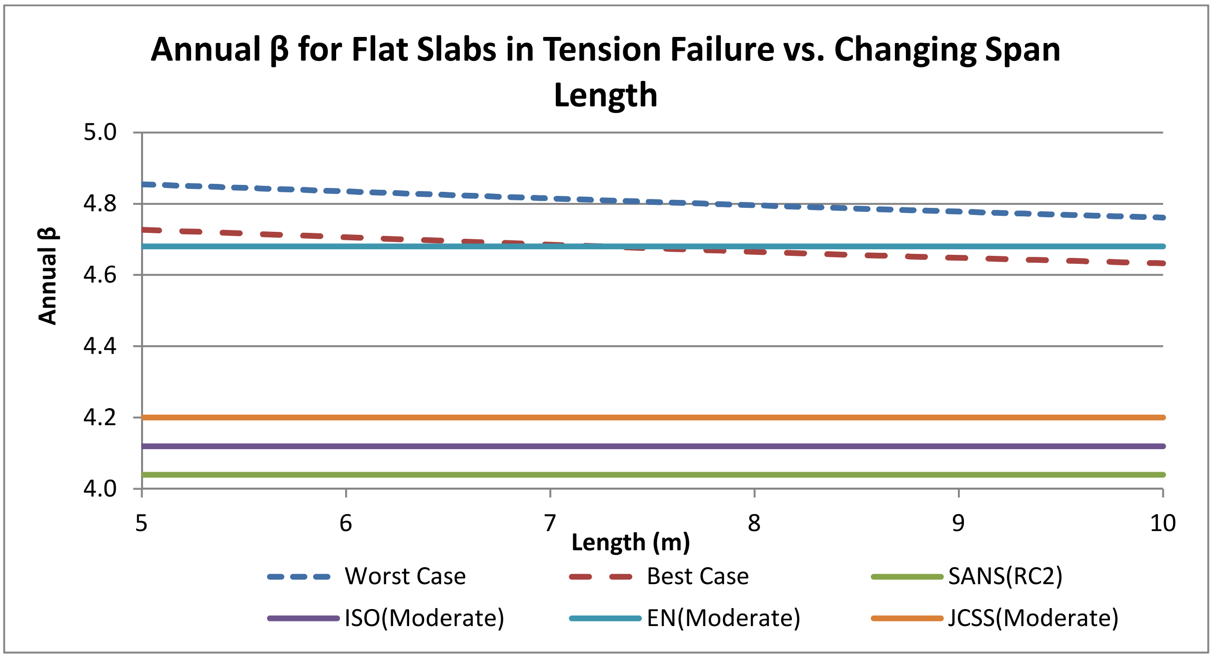

Figure 6-13: Results for a Flat Slab for Flexural Tension Failure at Support on the Column Strip

Due to relatively low costs of increasing safety compare to flexural failure at mid-span of the slab, the optimum reliability indices are significantly higher.

Failure at the support is once again considered except this time the middle strip is considered. The limit state function is now as follows:

$$G = \theta_R\, f_y\, A_s \times \left(h_s - d' - \frac{0.75 f_y A_s}{b f_{cu}}\right) - 0.25 \times 0.063 \times \theta_E(\, Q_L + Q_s + \gamma \times h_s) l2^2 l1 \qquad \textbf{6-8}$$

The results of the case study are shown in the following table.

l2	Cost of Increasing Safety (R/m^2)	β_t Best Case	β_t Worst Case
5	0.84	4.90	4.99
8	1.83	4.90	5.00E
10	2.72	4.87	4.96

Table 6-15: Results for a Flat Slab for Flexural Tension Failure at Support on the Middle Strip

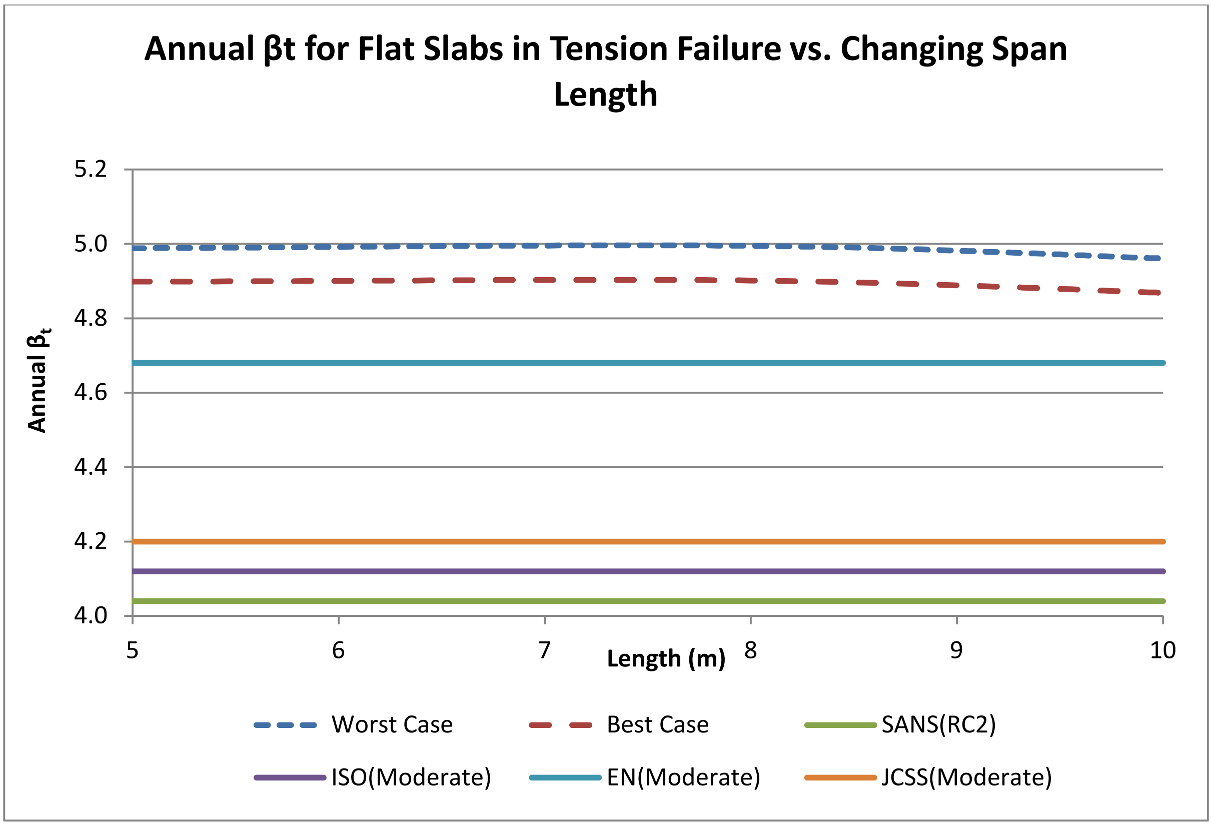

Figure 6-14: Results for a Flat Slab for Flexural Tension Failure at Support on the Middle Strip

The cost of increasing safety is significantly less at the support than at mid-span, thus higher target reliability indices are obtained. The cost of increasing safety is less at the support for the middle strip compared to the column strip resulting in higher optimum reliability indices.

6.6. Conclusions

The following graph shows all of the study cases conducted in this chapter in terms of optimum probability of failure and relative cost of safety for ductile modes of failure:

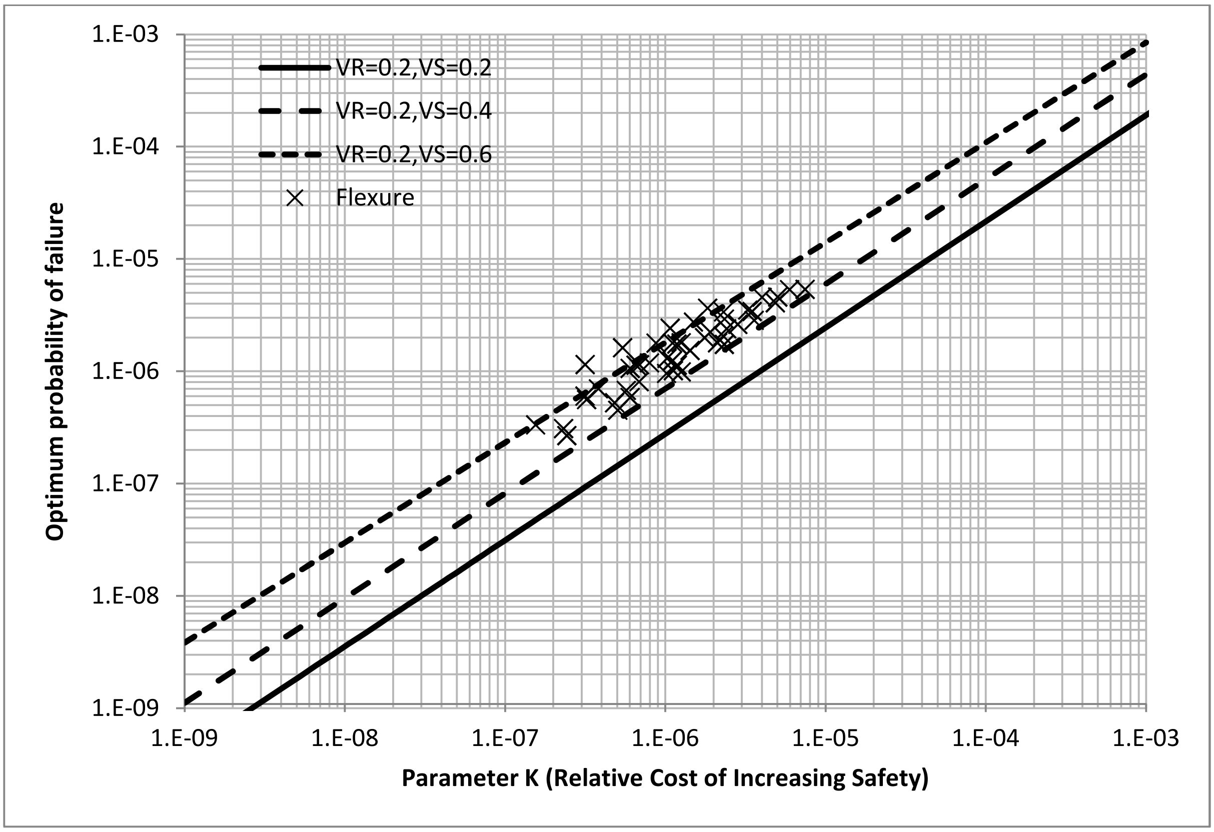

Figure 6-15: Optimum Probability of Failure vs. Increasing Relative cost of safety for Flexure

It is observed that the results from slabs in flexure compare well with the generic trend lines previously generated. Note that changes in span length imply changes in the variance of resistance and loads. Typically the cost of increasing safety increases as the span of the slab is increased. The effective coefficient of variance of the slab decreases with increasing length for both the resistance and load effect. For flexure in slabs a high to medium variance of the simplified lognormal approach seems to be an accurate approximation. Refer to Figure 6-16 where this is more clearly shown by only considering the one-way spanning slab case studies. The points closer to the medium variance generic approach line are the longer span lengths of the case studies and the lower K-value points are related to the situation when failure at the support is considered in Figure 6-16.

It is assumed in Figures 6-15 and 6-16 that the variance of the resistance does not change significantly with increasing span length.

A general observation during the case studies was that the cost of increasing safety at the supports of all types of slabs is relatively cheap resulting in a relatively higher set of optimum reliability indices. Increasing the safety of flat slabs was the most expensive while it was the least expensive for two-way spanning slabs. Table 6-16 shows the mean optimum reliability indices obtained for slabs of various lengths for flexure based on a 50 year reference period.

Slabs failing flexure				
Parameter	**Mean**	**Range**	**Mean (Best Case)**	**Mean (Worst Case)**
β target	3.71	3.42 - 4.16	3.66	3.78

Table 6-16: Target Reliability Indices based on a 50-year Reference Period

The results of the study cases suggest that the target reliability indices set by the current South African codes are too low. The optimum reliabilities calculated in this chapter for ductile failures compares well with the current target reliabilities set by the EuroCode for RC2 structures which is 3.8. Furthermore, there is relatively little difference between the best and worst case scenarios.

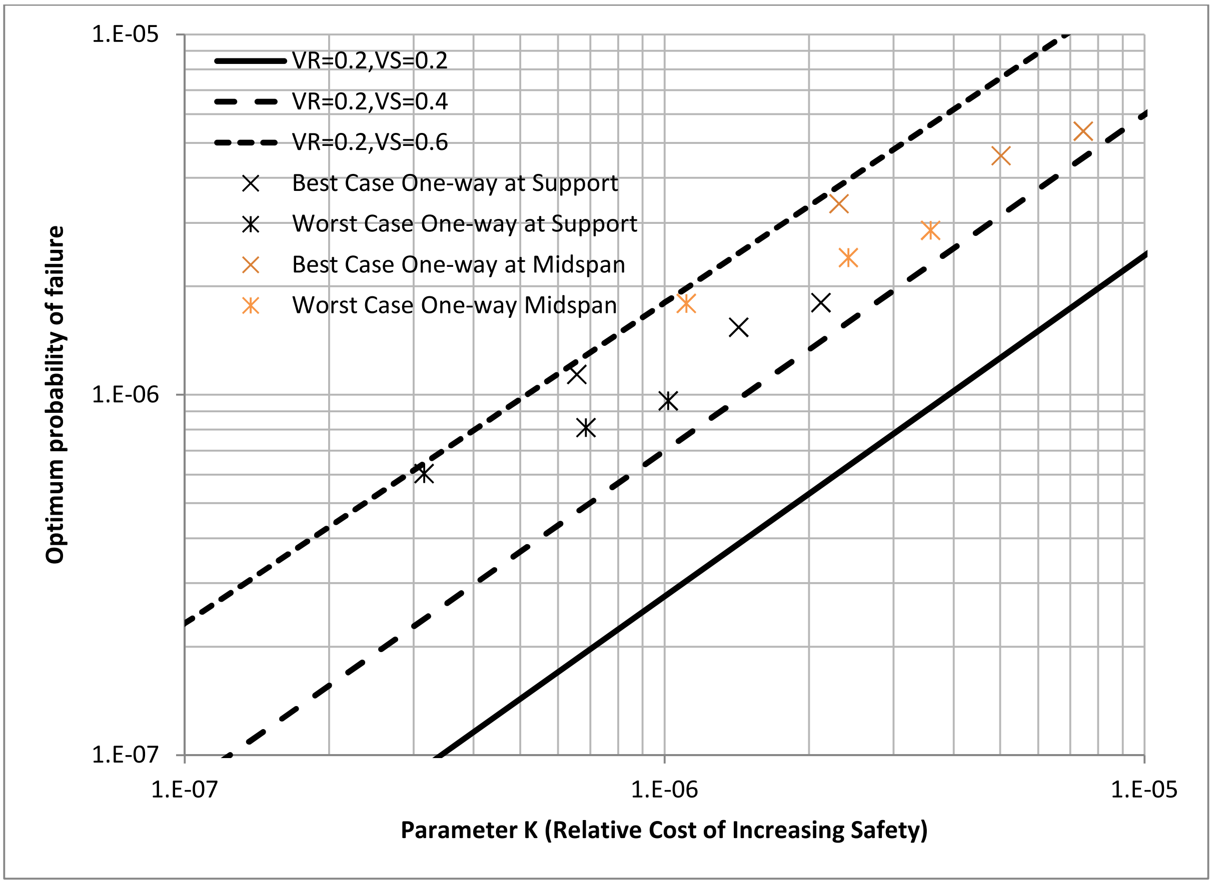

Figure 6-16: Optimum Probability of Failure vs. Increasing Relative cost of safety for One-way Slabs

Chapter 7 OPTIMIZATION OF CONCRETE BEAMS

7.1. Introduction

The main function of beams is to assist the slabs in transferring the loads to the columns. As seen in the previous chapter, the beams greatly reduce the moments in the slab as they provide additional support. The focus of this chapter is to obtain the target reliability indices for various modes of failures of beams. The depth to length ratios of all structural members are kept constant for different span lengths to normalize the results.

Serviceability costs are assumed to be insignificantly small and maintenance, inspection and fatigue costs are assumed to be independent of safety parameter p. It is assumed that the behaviour of T-beams and L-beams are similar to rectangular beams. This is true if the compression block falls within only the flange or web, which is usually the case for typical dimensions and reinforcement ratios. Thus for this particular study only rectangular beams are considered for optimization. Furthermore, the structure is assumed to be braced in both the horizontal directions. Various different failure modes are considered as detailed in the following table.

Beam Type	Location	Mode of Failure	Best Case P(Q)	Worst Case P(Q)	Population at Risk
Rectangular	Mid-span	Flexure(tension)	0.85	0.6	$0.1/m^2$
Rectangular	Support	Flexure (tension)	0.85	0.6	$0.1/m^2$
Rectangular	Support	Shear	0.5	0.35	$0.1/m^2$

Table 7-1: Different Study Cases and Assumptions Regarding Probability of Escape

The probability of escape is higher for beams than slabs, as the main cause of injuries and fatalities is as a consequence of a floor or slab collapsing. Assuming that the slab still has a ductile behaviour and has some structural resistance after the beam fails, will provide the occupants with additional escape time. Furthermore, the relatively larger deflections of both the slab and beam failing together will provide the occupants with clearer warnings than a floor failing alone. However for brittle failure mode of beams the warning time and the warning signs are less and thus a smaller probability of escape is assumed.

The optimum reliabilities for specific cases of beam failure are compared to optimum reliabilities obtained from the generic formulations of Chapter 5. All the target reliability indices are averaged, as a design code does not specify a specific target reliability index for a specific location of failure.

7.2. Flexural Failure

Two cases are considered in this section, which is a beam failing in tension at mid-span and at the support respectively. The most critical beam is chosen for optimization, which is beam 1-2 on support line B shown in the following figure:

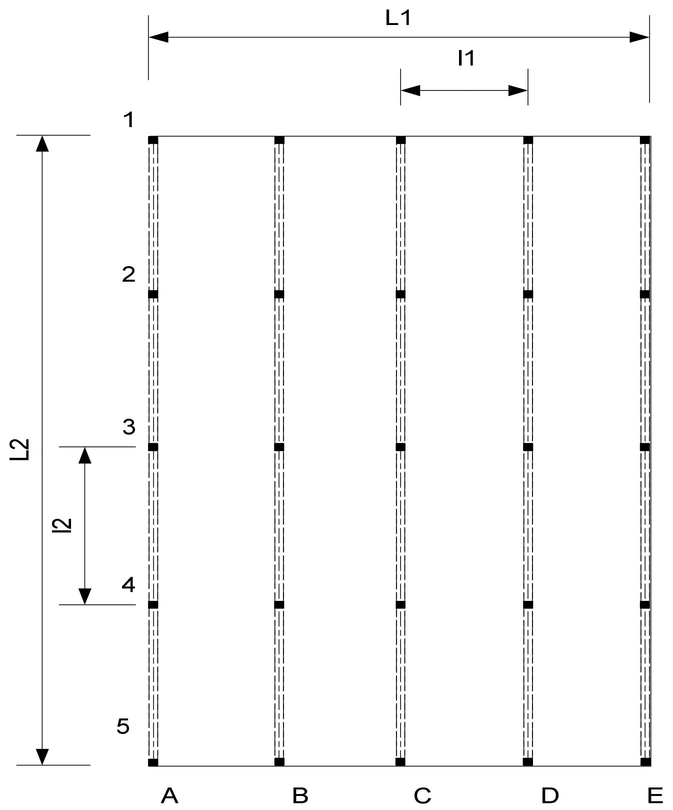

Figure 7-1: Plan View of Structure for Beam Case Studies

The internal bending moments in the beam is calculated by using the following figure adopted from Southern African Institute of Steel Construction (2010).

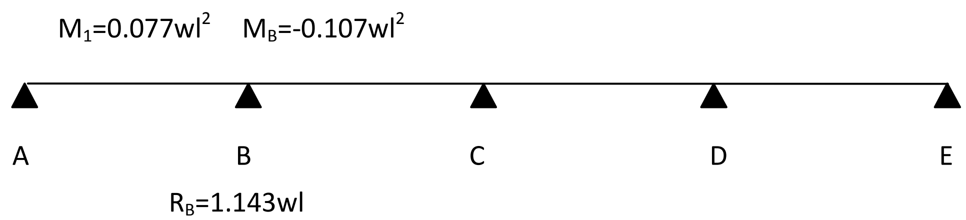

Figure 7-2: Distribution of Forces in a Beam

The following table shows the various different statistical parameters used for the optimization study:

Parameter	Type of Statistical Distribution	Description	Mean	Standard Deviation	Skewness	λ	Units
θ_R	Lognormal	Model uncertainty factor for resistance	1.20	0.18	0.45	NA	NA
fy	Lognormal	Yield Strength of Reinforcement	510000000	30000000	0.18	NA	Pa
A_s	Deterministic	Area of reinforcement	0.000606	NA	NA	NA	m^2
h_b	Normal	Height of beam	1	0.01	0	NA	m
d'	Gamma	Cover of concrete	0.033	0.005	0.30	NA	m
θ_E	Lognormal	Model uncertainty factor for load	1.00	0.20	0.60	NA	NA
Q_L	Gamma	Long-term load	500.00	614.82	2.46	0.2	Pa
Q_s	Gumbel	Short-term load	200.00	357.77	1.14	1	Pa
γ	Normal	Density of Concrete	24000	960	0	NA	N/m^3
l2	Deterministic	Length y	10	NA	NA	NA	m
f_{cu}	Lognormal	Strength of Concrete	39062500	7031250	0.54	NA	Pa
b	Normal	Width of beam	0.2	0.01	0	NA	m
h_s	Normal	Height of Slab	0.3	0.01	0	NA	m

Table 7-2: Statistical Parameters Used for Optimization of Beams

By using the SANS 0100 (2000) and Figure 7-2, the limit state function for tension failure is derived as follows:

$$G = R - E \qquad \textbf{7-1}$$

$$R = \theta_R f_y A_s \times \left(h_b - d' - 0.75\frac{f_y A_s}{b f_{cu}}\right) \qquad \textbf{7-2}$$

$$E = \theta_E(5.715(h_s\gamma + Q_L + Q_s) + b h_s \gamma) \times 0.077 l_2^2 \qquad \textbf{7-3}$$

The 5.715 factor comes from the fact that the l1 span is kept equal to 5m and multiplied by the reaction coefficient of the distributed load on support line B. A few additional parameters are included in the limit state function such as the height and width of the beam under consideration. The beam height over span ratio of the beam is kept as a constant equal to 0.1 to normalize the results. The influence factor, K, of 2 is used as flexural tension failure is the mode of failure under consideration. (PMC Part 2 (JCSS, 2001)) (refer to equation 6-1)

All economic parameters are kept the same as the previous chapter such as the discount and obsolescence rates. However, economic parameters such as the cost of increasing safety and the total compensation cost due to the number of fatalities are different from the previous case studies. The following table shows the other parameters used for the optimization case study.

Parameter	Type of Statistical Distribution	Description	Mean	Standard Deviation	Skewness	λ	Units
L1	NA	Length x	5.00	NA	NA	NA	m
C1	NA	Cost of Increasing Safety	8.32	NA	NA	NA	Rand/m^2
w	NA	Obsolescence rate	0.02	NA	NA	NA	NA
y	NA	Discount rate	0.024	NA	NA	NA	NA
P(Q)(Best Case)	NA	Probability of Escape	0.85	NA	NA	NA	NA
P(Q)(Worst Case)	NA	Probability of Escape	0.6	NA	NA	NA	NA
N_{par}	NA	Exposed Population	0.10	NA	NA	NA	People/m^2
k	NA	Probability of Dying	0.82	NA	NA	NA	NA
N_f(Best Case)	NA	Number of Fatalities	0.012	NA	NA	NA	People/m^2
N_f(Worst Case)	NA	Number of Fatalities	0.033	NA	NA	NA	People/m^2
SVSL	NA	Compensation Cost	3.752 mil	NA	NA	NA	Rand
$H_m + C_0$	NA	Other Losses and C_0	R12000 + R8000	NA	NA	NA	Rand/m^2

Table 7-3: Other Parameters Used for Optimization of Beams

The cost of increasing safety is calculated in two major steps. Firstly, by substituting all the mean values of the random variables into the limit state function and solving the area of reinforcement that causes the limit state function to be equal to zero. Secondly, this area of reinforcement is multiplied by lengths in the beam based on distribution of reinforcement in a typical beam as shown by the simplified curtailment rules given in SANS 0100 (2000). The above process is based on the safety parameter (p) concept explained in Chapter 5 and is done so that a unit of safety is defined in the same way for all case studies.

As seen in the following table there are two sets of costs of increasing safety known as the best case scenario and the worst case scenario. There is some degree of uncertainty regarding the size of the floor area that will collapse when a beam fails. Thus it was decided to create a best and worst case costs of increasing safety scenarios to cover this uncertainty. The higher cost of increasing safety or best case scenario is when a floor area of l1 by l2 is assumed to collapse. However for the worst case scenario a floor area of two times l1 by l2 is assumed to collapse. The following table shows the results of the case study:

L2	Cost of Increasing Safety (R/m^2) Best Case	Cost of Increasing Safety (R/m^2) Worst Case	β Best Case	β Worst Case
5	2.29	1.15	4.63	4.91
8	5.44	2.72	4.64	4.92
10	8.32	4.16	4.64	4.91

Table 7-4: Results of Flexural Tension Failure at Mid-span of Beam

The following figure shows the results of the reliability based optimization study:

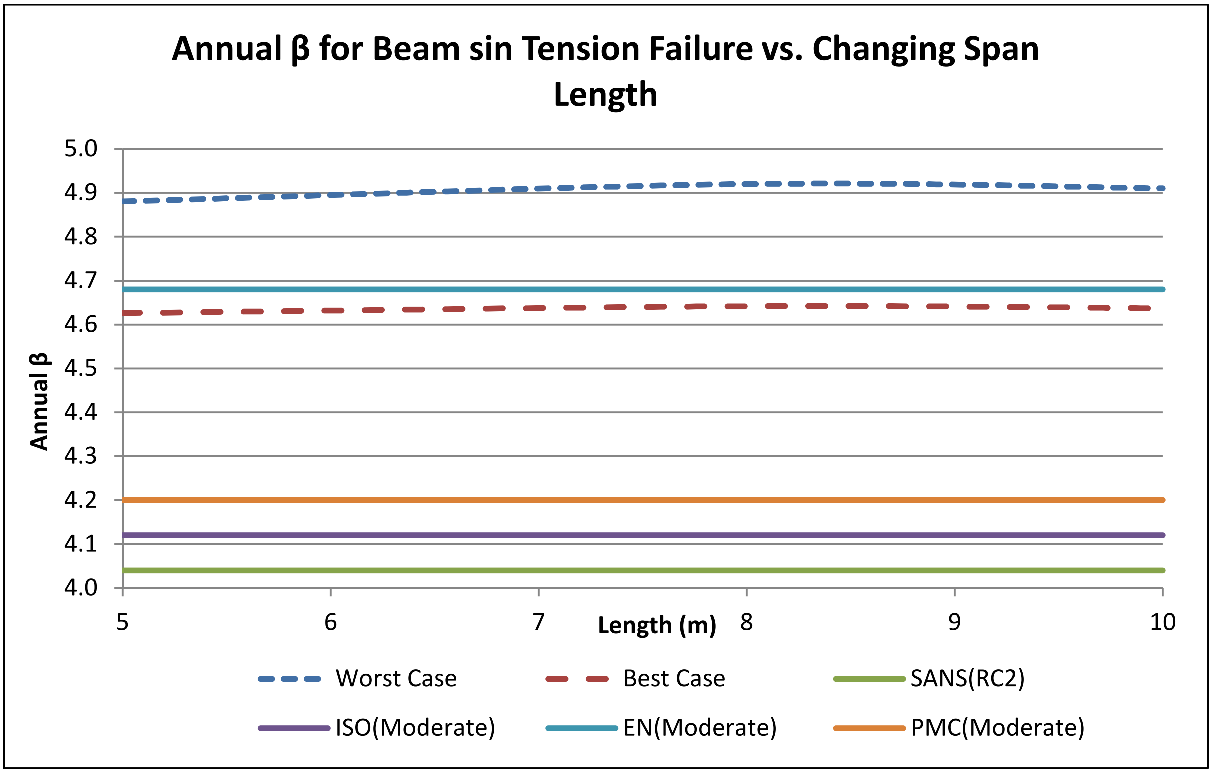

Figure 7-3: Results of Optimization of a Beam at Mid-span in Flexural Tension

Even though the cost of increasing safety increases as the span length changes, a slight increase in target reliability indices is observed for increasing span lengths. This is an indication that the coefficient of variance is sensitive to increases in span lengths. The coefficient of variance for various random variables decreases with changing span lengths resulting in the beam being less costly to increase the safety and thus approximately the same value target reliability indices are calculated for different costs of increasing safety. Comparing the results to existing target reliability indices indicates that the EuroCode is the closest to the results obtained as observed with the previous case studies.

Flexural tension failure at the support is considered for optimization. The main difference between the mid-span case and this one is the limit state function and one of the random variables. The standard deviation of the sum of the concrete cover and half the radius of the reinforcement is increased to 10mm according to the Probabilistic Model Code, Part 3 (JCSS, 2001). The magnitude of the bending moment is higher at the support than at the mid-span according to Figure 7-2. The load effect of the limit state function is as follows:

$$\boldsymbol{E = \theta_E(5.715(h_s\gamma + Q_L + Q_s) + bh_s\gamma) \times 0.107 l_2^2} \qquad \textbf{7-4}$$

The following table shows the results of the optimization study:

L2	Cost of Increasing Safety (R/m^2) Best Case	Cost of Increasing Safety (R/m^2) Worst Case	β Best Case	β Worst Case
5	1.19	0.6	4.67	4.94
8	2.84	1.42	4.71	4.98
10	4.36	2.33	4.70	4.97

Table 7-5: Results of Flexural Tension Failure at Support of Beam

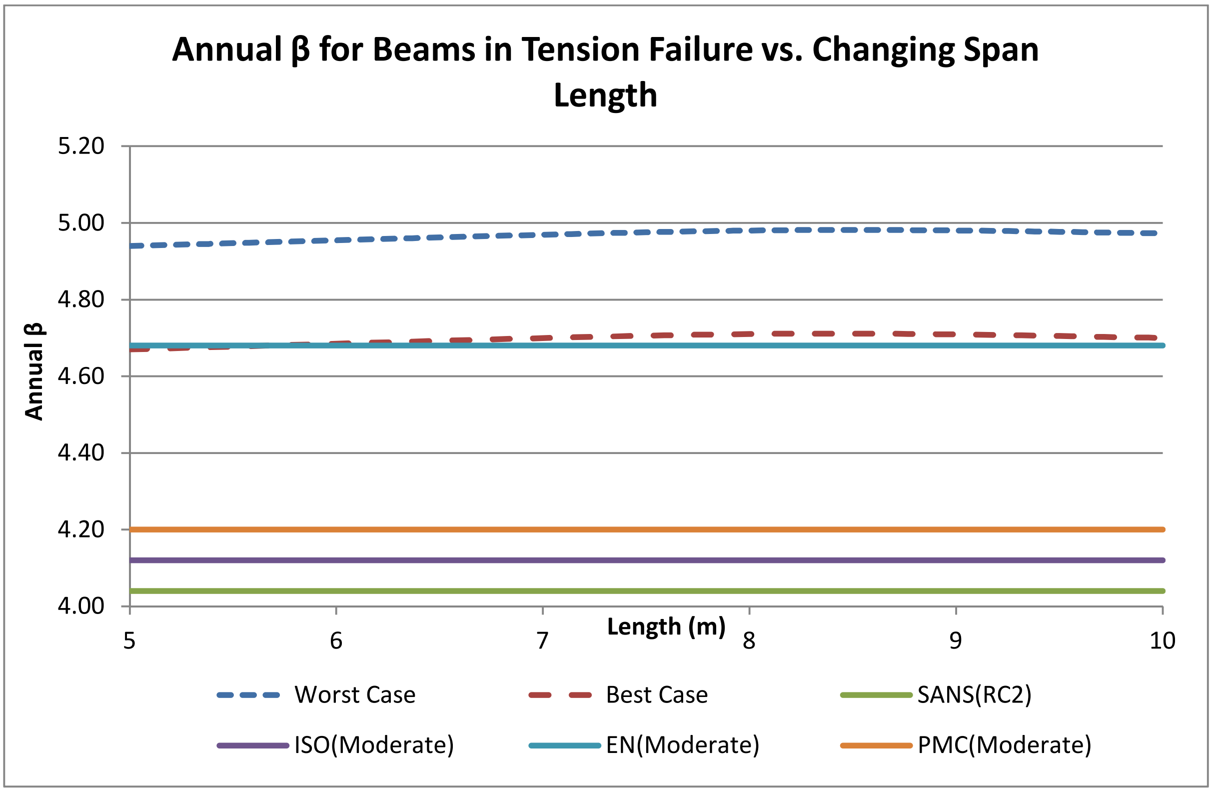

Figure 7-4: Results of Optimization of a Beam at Support in Flexural Tension

The same observation is made for this case study as the previous case study. Even though the cost of increasing safety increases with increasing span length the target reliability indices increases slightly. The above observation is as a result of the decreasing coefficient of variation of some of the random variables with increasing span length. These random variables where the CoV changes with an increased span length are the height of the beam, height of the slab, dead load and the live load. The cost of increasing safety is less than the previous case study, thus higher target reliability indices are calculated. The relatively lower cost of increasing safety is as a result of the short length of reinforcement at the support required by the simplified curtailment rules as given by SANS 0100 (2000).

7.3. Shear Failure

The last study case considered for beams is shear failure at the first internal support of the beam. Shear failure is considered as a brittle mode of failure. By using the design methods recommended by SANS (2000) and excluding all material factors the limit state function is derived and is as follows:

$$\boldsymbol{G = R - E} \qquad \textbf{7-5}$$

$$\boldsymbol{R = \theta_R \left(\frac{f_{yv}A_{sv}(h_b - d')}{s_v} + (0.95\left(\frac{f_{cu}}{25*10^6}\right)^{1/3}\left(\frac{0.4}{h_b - d'}\right)^{1/4}) \times 10^6 \times b(h_b - d')\right)} \qquad \textbf{7-6}$$

$$\boldsymbol{E = \theta_E(5.715(h_s\gamma + Q_s + Q_L) + bh_s\gamma) \times 0.61l_2} \qquad \textbf{7-7}$$

The 0.95 factor comes from the fact that 2% reinforcement is assumed for the beam. Furthermore, the spacing of the links is assumed to be a deterministic variable and to always be equal to 200mm. The influence factor, k, is equal to 1.4 according to the Probabilistic Model Code (JCSS, 2001) (refer to equation 6.1).

The cost of increasing safety is calculated by first solving the area of the shear links that will cause the limit state function to be zero. However the contribution of the concrete and reinforcement to the shear resistance resulted in the area of the shear links to be extremely small. Therefore, the contributing resistance of the concrete and flexural reinforcement was removed to calculate the area of the links that will result in the limit state function to be equal to zero. This area of shear links was used along with the link spacing and the simplified curtailment rules to calculate the cost of increasing safety. For the best case scenario an area of l2 by l1 is assumed to collapse, while for the worst case scenario an area of two times l1 X l2 is assumed to collapse.

The beam depth was assumed to be 400mm and the length to depth ratio was kept constant for both the beam and the slab. The following table and figure shows the results of the optimization process:

L2	Cost of Increasing Safety (R/m^2) Best Case	Cost of Increasing Safety (R/m^2) Worst Case	β Best Case	β Worst Case
5	4.07	2.04	4.71	4.87
8	5.28	2.64	4.81	4.96
10	6.14	3.07	4.85	5.02

Table 7-6: Results of Shear Failure in Beam

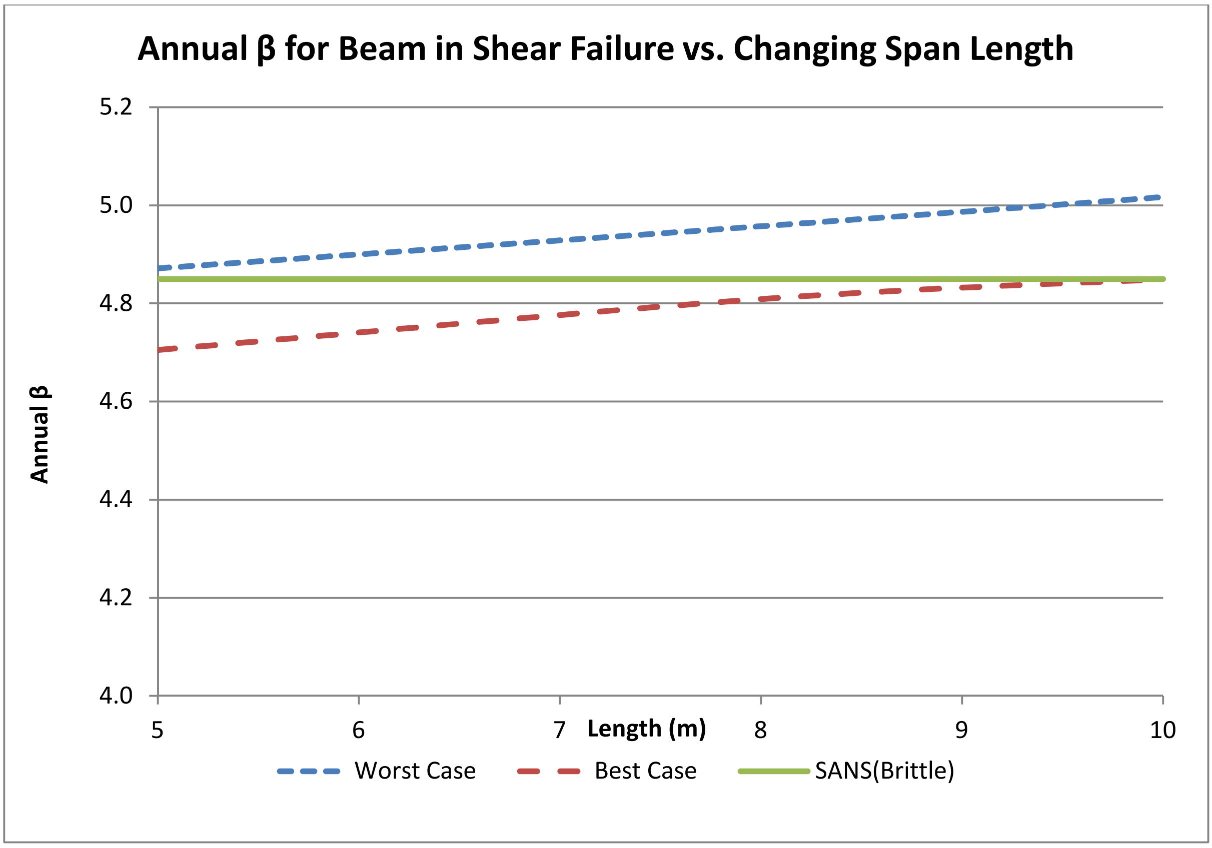

Figure 7-5: Results of Optimization of Shear failure in Beam

Even though the costs of increasing safety increases with changing span length, the optimum reliability index increases. This is as a result of the decreasing coefficient of variation of most of the random variables as the span length is increased. The results compares well with current target reliability index set by the South African codes for brittle failure.

7.4. Conclusions

The following graph shows the optimum probability of failure plotted against the variable K for all the flexural failures considered in this study. Shear failure are not included as it is brittle a mode of failure.

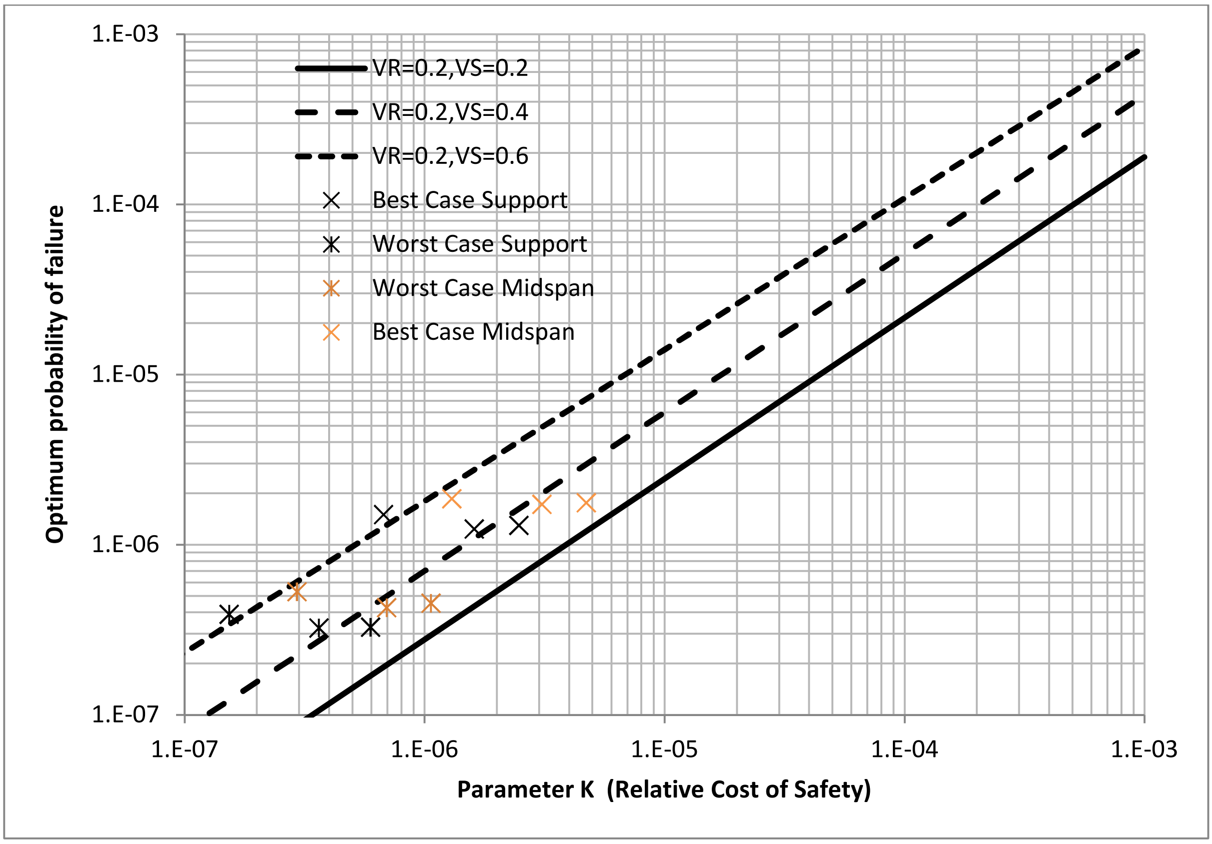

Figure 7-6: Optimum Probability of Failure vs. Increasing Relative cost of safety for Flexural Failure

As the span length is increased, the relative cost of safety increases. However the optimum probability of failure stays relatively constant due to the reduction in variance of the random variables that is clearly observed on the above figure. For flexural failure at the support of the beam, the K values are smaller than those for flexural failure at mid-span. It is also clear that for flexural failure at the support the variance of the random variables is lower than at the mid-span of the beam. This is as a result of the high moment coefficient for flexural tension failure resulting in the load effect to have a larger influence on the overall variance of the random variables. From Figure 7-6 a conclusion is made that medium variance simplified generic approach (VR=0.2, Vs=0.4) best approximates flexural tension failure in beams.

The following graph shows the optimum probability of failure plotted against relative cost of safety for all the brittle mode of failure (Shear) considered for beams.

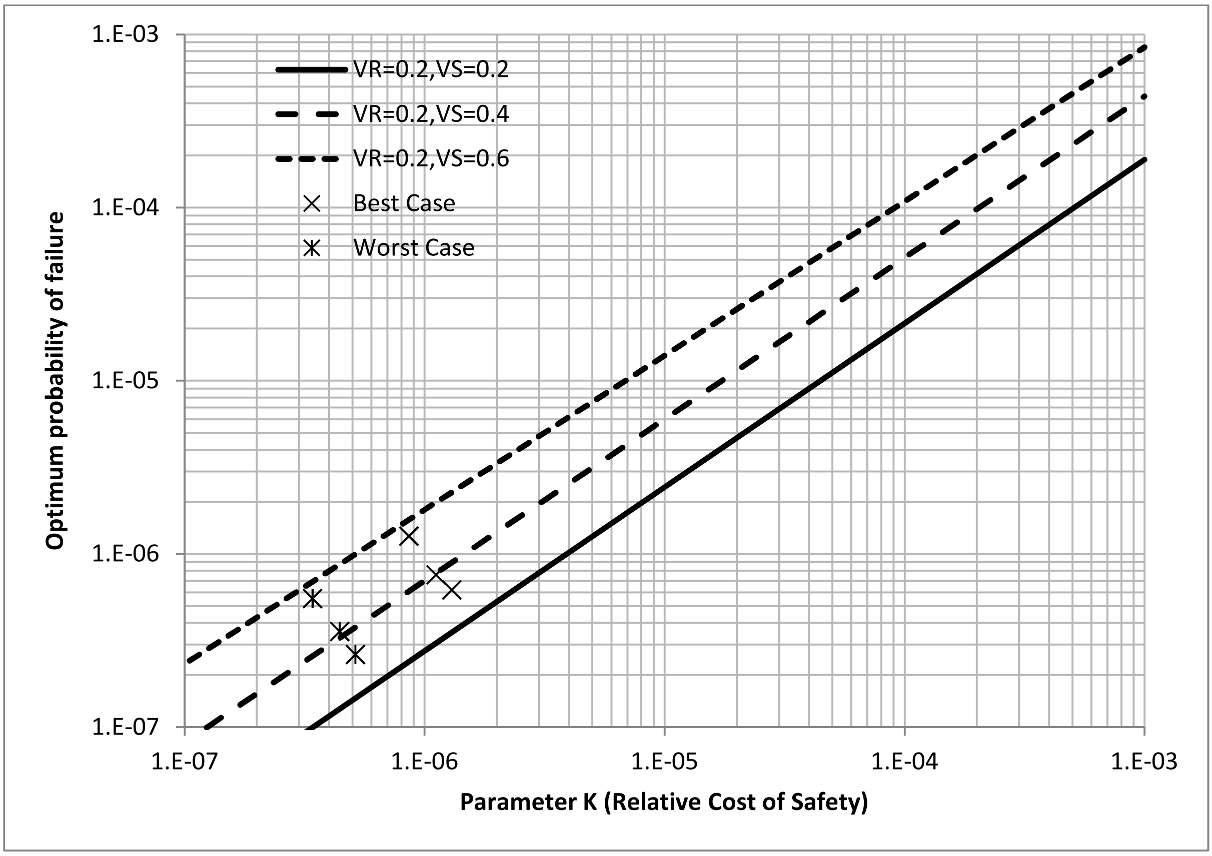

Figure 7-7: Optimum Probability of Failure vs. Increasing Relative Cost of Safety for Brittle Failure

The same property of decreasing variance with increasing span length is observed on the above figure for brittle failure. It is however clear that the relationship between optimum probability of failure and relative cost of safety can be best approximated by a simplified generic approach with a slightly lower variance than medium variance, as the mean of the results are located around medium variances. The following table shows the fifty year reference period target reliability indices for both brittle and ductile failure modes.

Beams Ductile Failure				
Parameter	**Mean**	**Range**	**Mean (Best Case)**	**Mean (Worst Case)**
β target	3.85	3.69 - 4.11	3.74	4.06

Table 7-7: Summary of Ductile Failure in Beams

The values in Table 7-9 suggest that the target reliability index of 3 set for ductile failure for structures in RC2 is far too low, but it compares well with the target reliability index set by the EuroCode of 3.8.

Beams Brittle Failure				
Parameter	Mean	Range	Mean (Best Case)	Mean (Worst Case)
β target	3.95	3.79 - 4.16	3.88	4.07

Table 7-8: Summary of Brittle Failure in Beams

For brittle failures the optimum reliability index is comparable to the target of 4 set by the South African loading code (SANS 10160-1).

Chapter 8 OPTIMIZATION OF CONCRETE COLUMNS

8.1. Introduction

Columns are vertical structural members that transfer the load from the superstructure to the foundation of a structure. Generally columns are designed to resist axial loads, but there are moments transferred from the beams to the columns. Additional moments can also arise in the column due to large deflections. If the column is slender large deflections will occur, if however the column is short small deflections will occur resulting in little or no additional moments.

Serviceability failure will not be considered during the optimization process and the dimensions of the column will be increased with increasing span length. Bi-axial bending and single bending is considered for the failure of columns. The limit state function along with an interaction diagram is used to define failure. The structure is assumed to be horizontally braced, so only vertical loads are considered. The case study considered for this chapter is as follows:

Column Type	Mode of Failure	Best Case P(Q)	Worst Case P(Q)	Population at Risk
Rectangular, Supporting One-way Slab	Brittle (Moment+ Axial Load)	0.5	0.2	0.1/m^2

Table 8-1: Description of Case Study and Assumptions Regarding Probability of Escape

Due to the fact that all the losses are caused by a slab failing, the probability of escape is relatively high considering the column undergoes brittle failure. The slab and beam is assumed to provide some ductile structural resistance after the failure of a column, thus providing the occupants with clear warning signs and some time to escape. A reasonable range is assigned to the probability of escape to ensure that the uncertainty of this parameter is covered.

The optimum reliabilities for specific cases of column failure are compared to optimum reliabilities obtained from the generic formulations of Chapter 5. All the target reliability indices are averaged, as a code does not specify a specific target reliability index for a specific location of failure.

8.2. Determining the Probability of Failure of a Column

8.2.1. Interaction Diagram & Column Resistance

At failure a column can have various different stress distributions with a varying location of the neutral axis depending on the magnitude of the bending moment relative to its axial force. Correspondingly, the reinforcement may be fully plastic or elastic and in tension or compression. The following figure shows a typical interaction diagram. Various points are shown on the figure (a,b,c....) that indicates various different states of stress.

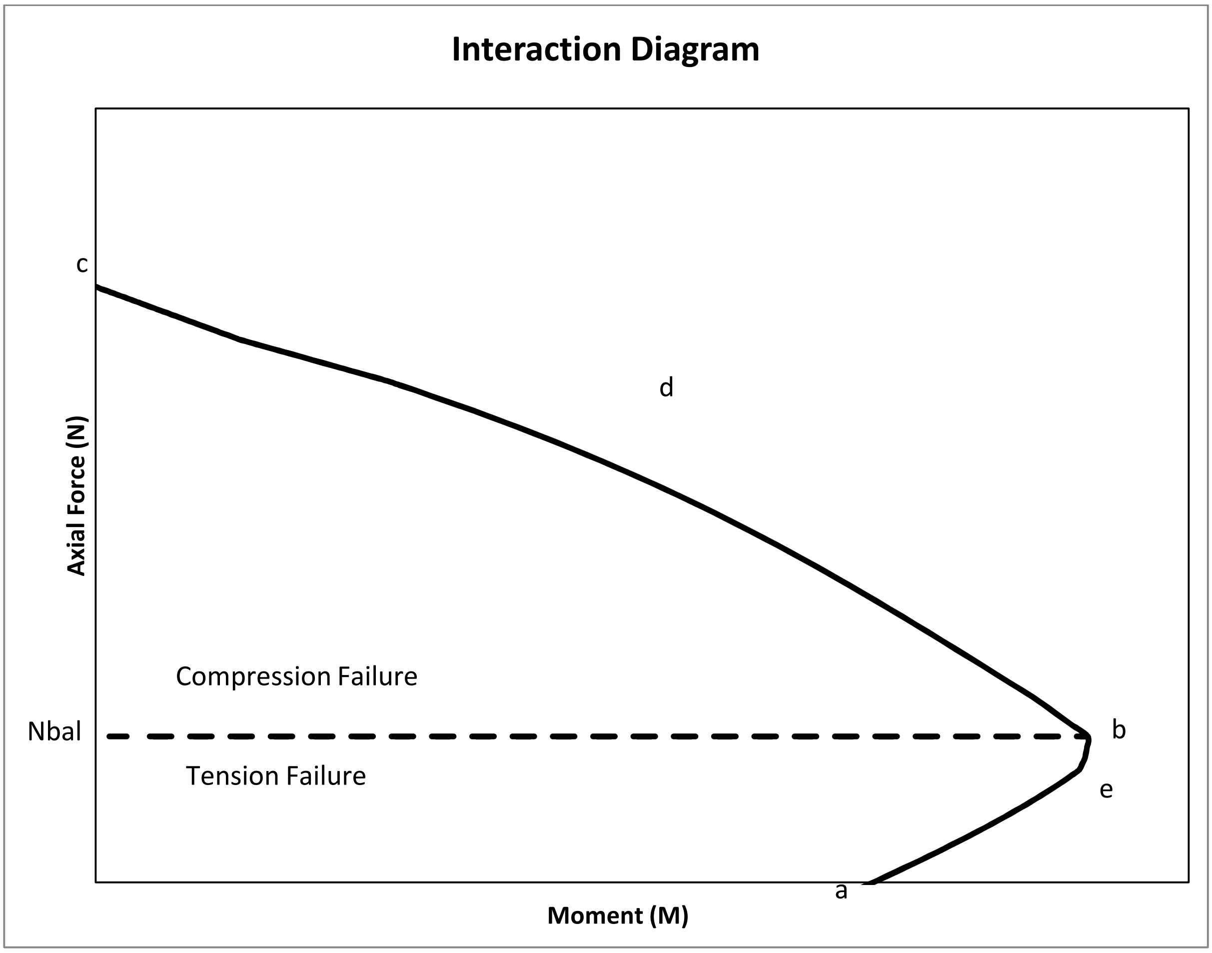

Figure 8-1: Interaction Diagram of a Column Showing Various States of Stress (Robberts et al (2010))

When the depth of the compression block is small enough and the moment is large relative to the axial load the column fails in tension. Generally, however the axial force is significantly larger than this limit and the failure is then brittle. The points a to e is explained in the following sections obtained from Robberts et al (2010).

Pure Bending (a)

For pure bending (N=0) the columns behaviour is identical to that of a beam. If normal ranges of reinforcement are used the tension reinforcement yields when the ultimate strain is reached in the concrete. Between points a and b the moment capacity of the column actually increases with increasing axial load.

Balance Point (b)

The Balance point is where both the concrete reaches its ultimate strain and the reinforcement yields at the same time. If the combinations of M and N fall under the balance point the failure is ductile and when it is above the balance point it is brittle. Unfortunately the failure mode of a column cannot be controlled with the amount of reinforcement as with beams and will typically have a brittle failure as the axial force is usually significantly larger than the moment.

Pure axial compression (c)

For this particular case the moment is zero and the entire section is in compression. The tension reinforcement contributes to the overall compression strength as it is in compression.

Zero strain in the tension reinforcement (d)

Moving from the balance point to point c, the tension reinforcement moves from yielding in tension to zero strain and then in compression. The neutral axis falls outside of the section moving from point d towards c.

Yielding of Compression Reinforcement (e)

At this point the compression reinforcement changes from elastic behaviour to plastic behaviour as it starts yielding. It can be seen on Figure 8.1 there is a definite change in slope.

From the above points it can be seen that the stress distribution of the column is highly variable depending on the relative magnitudes of the axial load and moment applied to the column. A further complication is the fact that steel reinforcement has three possible different stress states as mentioned earlier and the three states are:

- Yielding in tension ($F_{st} = f_y \times A_s$)

- Linear elastic (Fst = E x ε x As)
- Yielding in compression (Fst = Fyc x As)

The interaction diagram is needed to determine if the reinforcement is in tension or in compression to define the stress state of the steel in the limit state function. The interaction diagram is also needed to establish the exact depth of the compression block. There are two functions which in combination determine if a column fails. R1 is the axial resistance of the column, while R2 is the moment resistance of a column.

Another mode of failure is buckling, but this mode is highly unlikely. According to Robberts et al (2010) if the slenderness ratio (l/h) is kept smaller than 20 the column will fail in crushing due to the axial load before it buckles.

A column is a system reliability problem where the area of reinforcement has to overcome both a moment and an axial load. As a result the depth of the neutral axis will depend on the ratio of the axial load and the moment which are modelled by various random variables. Thus the probability of failure of a column is calculated by using a Monte Carlo simulation described in section 8.2.3. The following functions show the two interacting limit state function of a column.

$$\boldsymbol{G_1 = R_1 - E_1} \quad \textbf{8-1}$$

$$\boldsymbol{G_2 = R_2 - E_2} \quad \textbf{8-2}$$

$$\boldsymbol{R_1 = \theta_R\left(0.67f_{cu}b_c0.9x + {A_sf_{yc}}/{2} + {A_sf_y}/{2}\right)} \quad \textbf{8-3}$$

$$\boldsymbol{R_2 = \theta_R\left(0.67f_{cu}b_c0.9x\left(\frac{h_c}{2} - \frac{0.9x}{2}\right) + {A_sf_{yc}}/{2}\left(\frac{h_c}{2} - d'\right) + {A_sf_y}/{2}\left(\frac{h_c}{2} - d'\right)\right)} \quad \textbf{8-4}$$

Where the load effect E_1 is the axial load and load effect E_2 is the moment applied to the column. The formulation of these load effects are shown in the next section.

8.2.2. Forces Acting on Column

The axial force of a column is easy to calculate as the reaction coefficient of the simplified beam introduced in Chapter 7 can be used to calculate the axial load. The moment in a column is however less straight forward to calculate. Robberts et al. (2010) proposes that the ratio of the column with half of the stiffness of the surrounding columns be used along with the unbalanced fixed end moment to determine the moment transferred to the column.

The SANS 0100 (2000) states if a column is supporting a symmetrical arrangements of beams only the axial load and the moments generated from the eccentricities and deflections have to be considered. This is due to the fact that the unbalanced fixed end moments are insignificantly small when the beams are in a symmetrical arrangement.

The axial force in a column is calculated as follows:

$$\boldsymbol{E_1 = \theta_E\Big(\big(5.715\big(h_s\gamma n + (Q_L + Q_s)(n-1)\big) + h_b b_b \gamma n\big) l_1 1.143\Big)}$$

$$\boldsymbol{+\theta_E(\gamma h_c b_c) n 3.5} \qquad \textbf{8-5}$$

Where n is the number of storeys, for this study only n=4 is considered. All other parameters are defined in the following section.

The initial column size is assumed to be 250mmx250mmx3500mm and is short around both axis. The eccentricity of the x axis is calculated as 0.0125m and as 0.0125m for the y axis. Therefore the two moments considered is summarised as follows:

- Strong Axis (Nxe_{min})
- Weak Axis (Nxe_{min})

The PMC Part 3 (JCSS, 2001) states that eccentricities can be modelled by random variables having normal distributions, therefore these two eccentricities will be modelled is random variables during the reliability based optimization process.

The formulation below is conservative, transforming a bi-axial bending situation to a uni-axial bending situation with an increased uni-axial bending moment. This is done in order to simplify the model, otherwise a three dimensional interaction diagram is required.

The uni-axial moment in the column is calculated as follows:

$$E_2 = \theta_E e_y E1 + \beta_b \frac{b_c}{h_c} \theta_E e_x E1 \qquad \textbf{8-6}$$

Where E_1 is the axial load, β_p is a factor dependent on the size of the axial load relative to the axial resistance of the concrete in the column alone. A conservative value of 0.7 is assumed for β_p for this study. All other variables are defined in the following section.

8.2.3. Computing Probability of Failure with Monte Carlo Simulation

A Monte Carlo simulation is where a random numbers are generated from a statistical distribution with certain parameters (mean, skewness). A limit state function defines failure. For each random variable a random realisation is generated. A set of such realisations are generated at each iteration and substituted into the limit state function. The required number of iterations to achieve an acceptable level of accuracy is determined by the approximate probability of failure of the structural member under consideration. (Refer to section 4.2) The number of failures are summed and divided by the number of iterations to compute the probability of failure.

The probability of failure of a column as computed using a Monte Carlo approach. Firstly a realisation is generated for each random variable. From this an axial load realisation is computed by substituting the generated random input realisations into the axial load equation. The depth of the neutral axis and corresponding moment capacity that would lead to a limit state, given this axial load realisation, is computed. Figure 8-2 indicates this process.

This neutral axis depth, together with the random realisations for the input variables are used to compute the moment capacity (MR). This is compared to a random realisation of the applied moment (MU) to evaluate whether or not failure will occur. If the moment load is higher than the moment resistance of the column, the column fails. This is done repeatedly and the total number of failures divided by the number of repetitions yields the probability of failure.

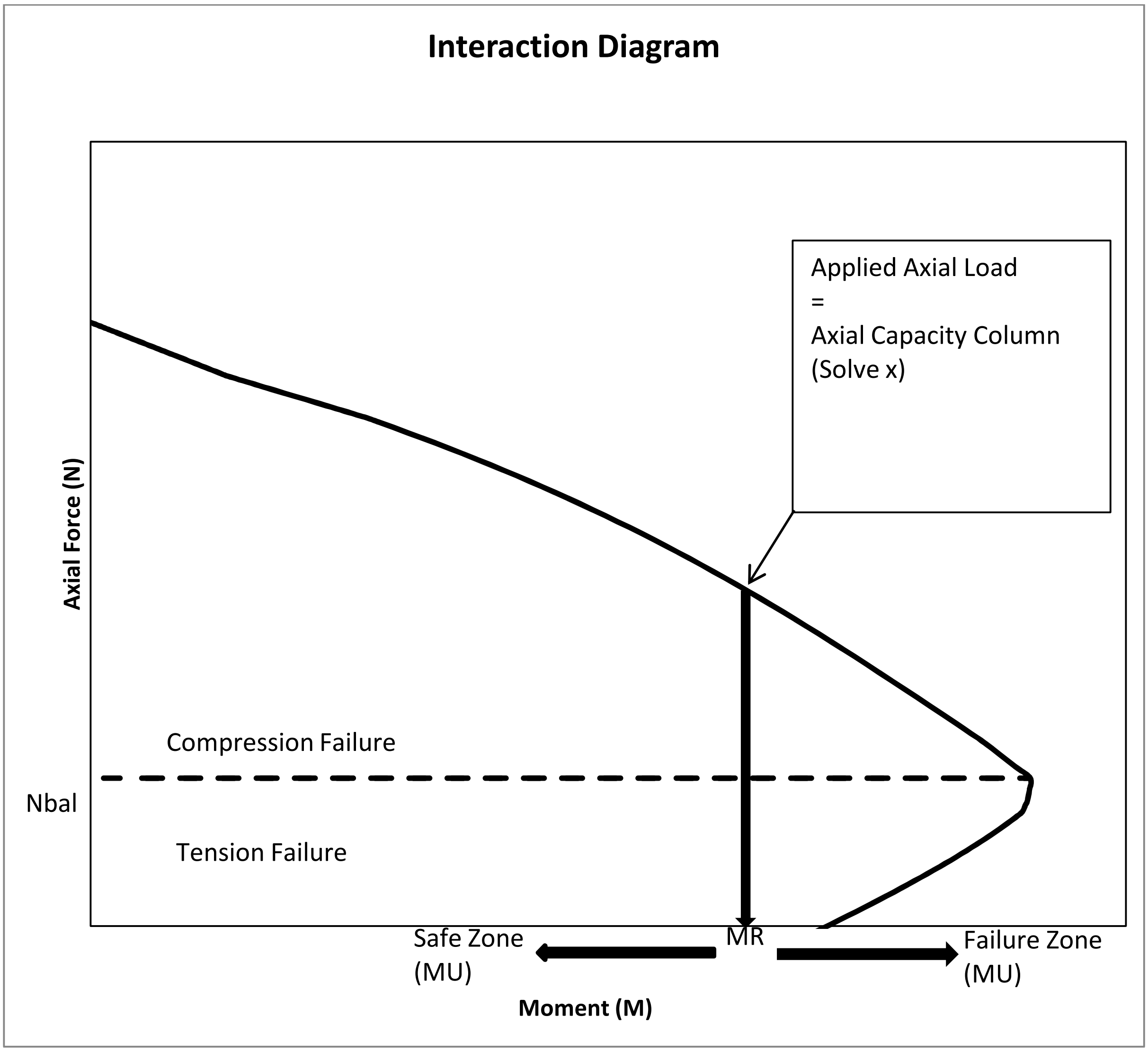

Figure 8-2: Calculating Probability of Failure of a Column Using an Interaction Diagram

All of these steps are programmed into various functions in MATLAB. The main function is called *MonteCarloC* and it calls various other functions which are used to model the resistance and load effects. The various functions are:(Refer to Appendices G-L for the code)

- *Generic_OptimizationC*
- *MonteCarloC (10^9 iterations)*
- *resistance2*
- *loadaxial*
- *MRC*
- *loadmoment*

8.3. Testing of the Monte Carlo Function

A Monte Carlo function was programmed in MATLAB that is used primarily as a comparative test of accuracy. This function is based on the same principles as the function that specializes in the Monte Carlo simulation of a column, but has a different resistance model. The same example of a basic limit state function using a beam, as conducted in Chapter 4, is used so that the results can be compared to VaP and the other function *form*. The function is called *MonteCarlo1*. The various functions used for this exercise are as follows. (Refer to Appendix M-O for the code)

- MonteCarlo1
- resistance1
- load1

The following table shows the results of VaP (FORM), *form* (FORM) and *MonteCarlo1* (Monte Carlo Simulation 10^7 trials).

Area of Reinforcement (mm^2)	β (form)	β (VaP)	β (MonteCarlo1)
1500	3.22	3.22	3.22
1600	3.46	3.46	3.44
1700	3.67	3.68	3.66
1800	3.86	3.89	3.87
1900	4.05	4.09	4.06

Table 8-2: Reliability Indices Obtained from Various Sources

The results compare well to both the FORM of VaP and *form* in MATLAB. There are slight differences, but these differences can be deemed as insignificantly small. The above test shows that the principles of the Monte Carlo simulation programmed in MATLAB works. Unfortunately it is impossible to verify the accuracy of the Monte Carlo simulation programmed for the column as it requires a built in model of an interaction diagram which VaP does not have.

8.4. Case Study and Results

A 4-storey structure with one-way spanning slabs is chosen as a case study. The following figure shows the plan view of a floor in the structure.

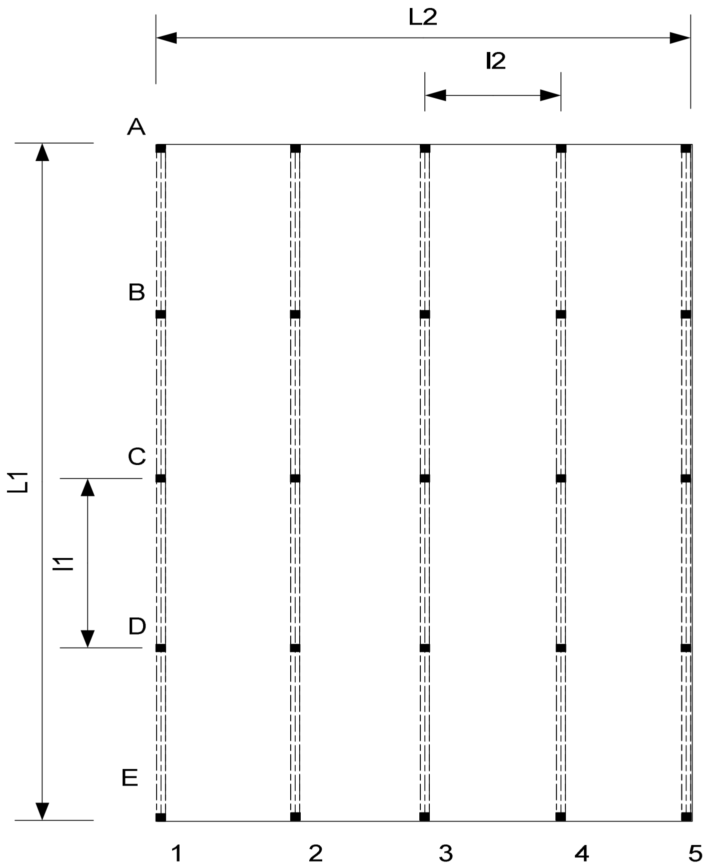

Figure 8-3: Plan View of a Typical Floor of the Case Study

The height of the columns is chosen as 3.5m while the initial length of the spans between supports is 5m. The span in the l1 direction is increased systematically throughout the reliability based optimization of the column. The column at point B2 on the first floor is chosen for optimization.

The cost of increasing safety is calculated by assuming that the actual depth of the neutral axis is equal to the height of the column. It was further observed on the interaction diagram created in Robberts et al. (2010) (Figure 8-4) that if the moment is calculated via an eccentricity the neutral axis depth is approximately equal to the height of the section. ($x \approx h$) Since this is the case for this case study this assumption should be approximately be true for this case study. The following interaction diagram adopted from Robberts et al. (2010) shows this property:

The following table shows the random variables used for this study:

Parameter	Type of Statistical Distribution	Description	Mean	Standard Deviation	Skewness	λ	Units
θ_R (R1)	Lognormal	Model uncertainty factor for resistance	1.2	0.18	0.45	NA	NA
θ_R (R2)	Lognormal	Model uncertainty factor for resistance	1.2	0.18	0.45	NA	NA
f_y	Deterministic	Stress in Tension Reinforcement	Variable	NA	NA	NA	Pa
A_s	Deterministic	Area of Reinforcement	0.000607	NA	NA	NA	m^2
h_b	Normal	Height of Beam	0.8	0.01	0	NA	m
d'	Gamma	Cover of concrete	0.05	0.01	0.33	NA	m
θ_E (E1)	Lognormal	Model Uncertainty Factor for Load	1	0.05	0.15	NA	NA
θ_E (E2)	Lognormal	Model Uncertainty Factor for Load	1	0.1	0.3	NA	NA
Q_L	Gamma	Long-term load	500	614.82	2.46	0.2	Pa
Q_s	Gumbel	Short-term load	200	357.77	1.14	1	Pa
γ	Normal	Density of Concrete	24000	960	0	NA	N/m^3
l_1	Deterministic	Length y	10	NA	NA	NA	m
f_{cu}	Lognormal	Strength of Concrete	39062500	7000000	0.54	NA	Pa
b_b	Normal	Width of Beam	0.2	0.01	0	NA	m
h_s	Normal	Height of Slab	0.3	0.01	0	NA	m
b_c	Normal	Width of Column	0.2	0.01	0	NA	m
h_c	Normal	Height of Column	0.5	0.01	0	NA	m
f_{yc}	Deterministic	Stress in Compression Reinforcement	Variable	NA	NA	NA	Pa
e_x	Normal	Eccentricity Strong Axis	0.02	0.004	NA	NA	m
e_y	Normal	Eccentricity Weak Axis	0.01	0.004	NA	NA	m

Table 8-3: Statistical Parameters Used for Optimization of Columns

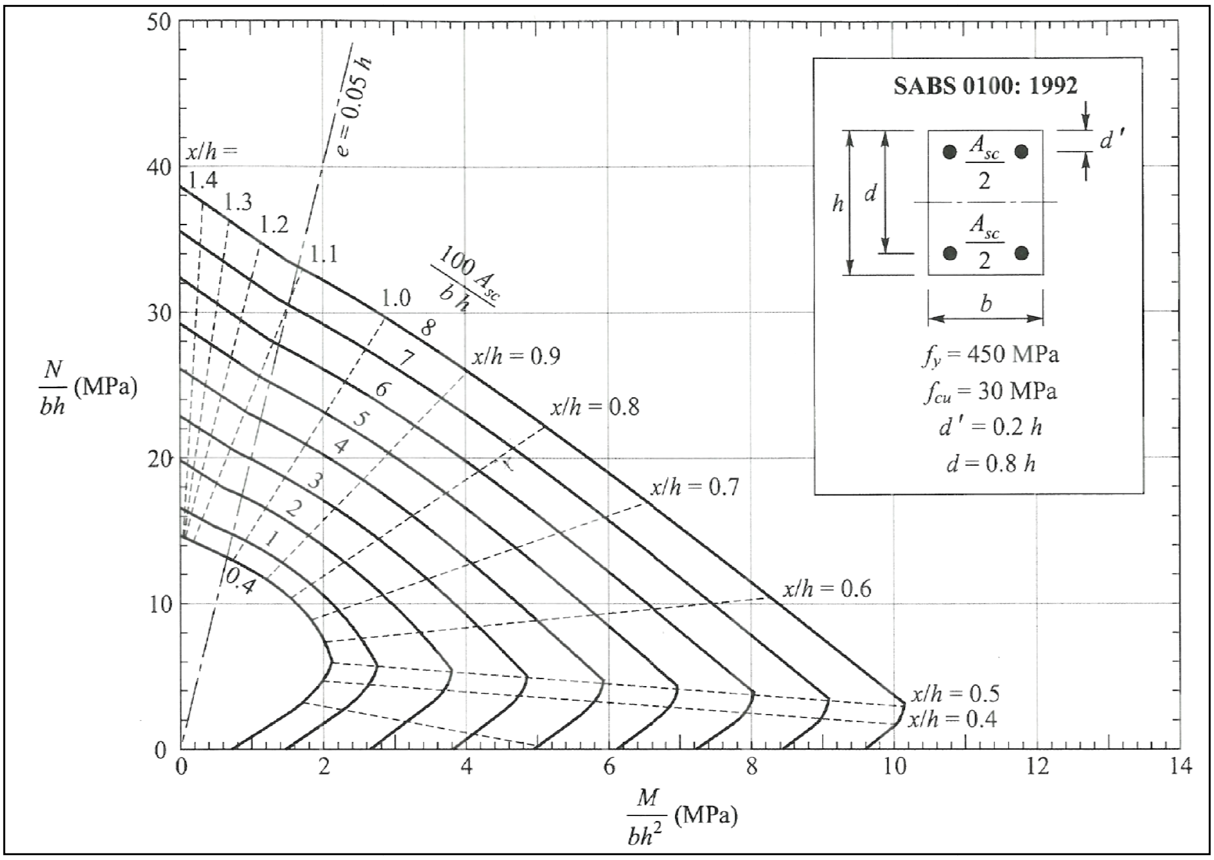

Figure 8-4: Interaction Diagram Indicating Relative Position of Neutral Axis

From this property an assumption is made that as long as the moments are calculated from eccentricities the neutral axis depth is approximately equal to the height of the column. The compression reinforcement will thus have a stress of 401 MPa and the tension reinforcement will thus have a stress of 126 MPa. These values are used along with the moment force related limit state function to calculate a cost of increasing safety by using the safety parameter concept. Using the simplified curtailment rules as shown by SANS 0100 (2000) the cost of increasing safety can be calculated by using the cost of reinforcement as R9500/ton. The R9500/ton includes fixing costs. The cost of increasing safety is defined in terms of Rand per collapsed area. For this case study two areas of collapse are defined. For the best case scenario an area of l1 by l2 is assumed and for the worst case scenario an area of 2xl2 by 2xl1 is assumed to collapse.

The fact that a Monte Carlo simulation is used to calculate the probability of failure the function Generic_Optimization must be adjusted to allow for the new computation of probability of failure. Generic_OptimizationC is created to compute the probability of failure and can be seen in appendix G. All other parameters used for the study are shown in the following table.

Parameter	Type of Statistical Distribution	Description	Mean	Standard Deviation	Skewness	λ	Units
n	NA	Number of Storeys	4	NA	NA	NA	NA
l2	NA	Length x	5	NA	NA	NA	m
C1	NA	Cost of Increasing Safety	3	NA	NA	NA	Rand/m^2
w	NA	Obsolescence rate	0.02	NA	NA	NA	NA
y	NA	Discount rate	0.024	NA	NA	NA	NA
P(Q)(Best Case)	NA	Probability of Escape	0.5	NA	NA	NA	NA
P(Q)(Worst Case)	NA	Probability of Escape	0.3	NA	NA	NA	NA
Npar	NA	Exposed Population	0.1	NA	NA	NA	People/m^2
k	NA	Probability of Dying	0.82	NA	NA	NA	NA
Nf(Best Case)	NA	Number of Fatalities	0.041	NA	NA	NA	People/m^2
Nf(Worst Case)	NA	Number of Fatalities	0.057	NA	NA	NA	People/m^2
SVSL	NA	Compensation Cost	3.752 mil	NA	NA	NA	Rand
$H_m + C_0$	NA	Other Losses and C_0	R12000 + R8000	NA	NA	NA	Rand/m^2

Table 8-4: Other Parameters Used for Optimization of Columns

During the analysis the L/d ratio's of the beam and slabs are kept constant. However this will be unrealistic to keep the height of the column over the length of the one span constant as it would result in unrealistic dimensions for a column. Thus three column heights, 250mm, 300mm and 350mm are considered as l1 is increased. All other parameters in the Table 8-3 were obtained from the three parts of the PMC codes.

The following table shows the results of the case study:

L2	Cost of Increasing Safety (R/m^2) Best Case	Cost of Increasing Safety (R/m^2) Worst Case	β Best Case	β Worst Case
5	4.80	1.20	4.85	5.17
8	4.69	1.17	4.87	5.22
10	4.65	1.16	4.95	5.27

Table 8-5: Results of Case study

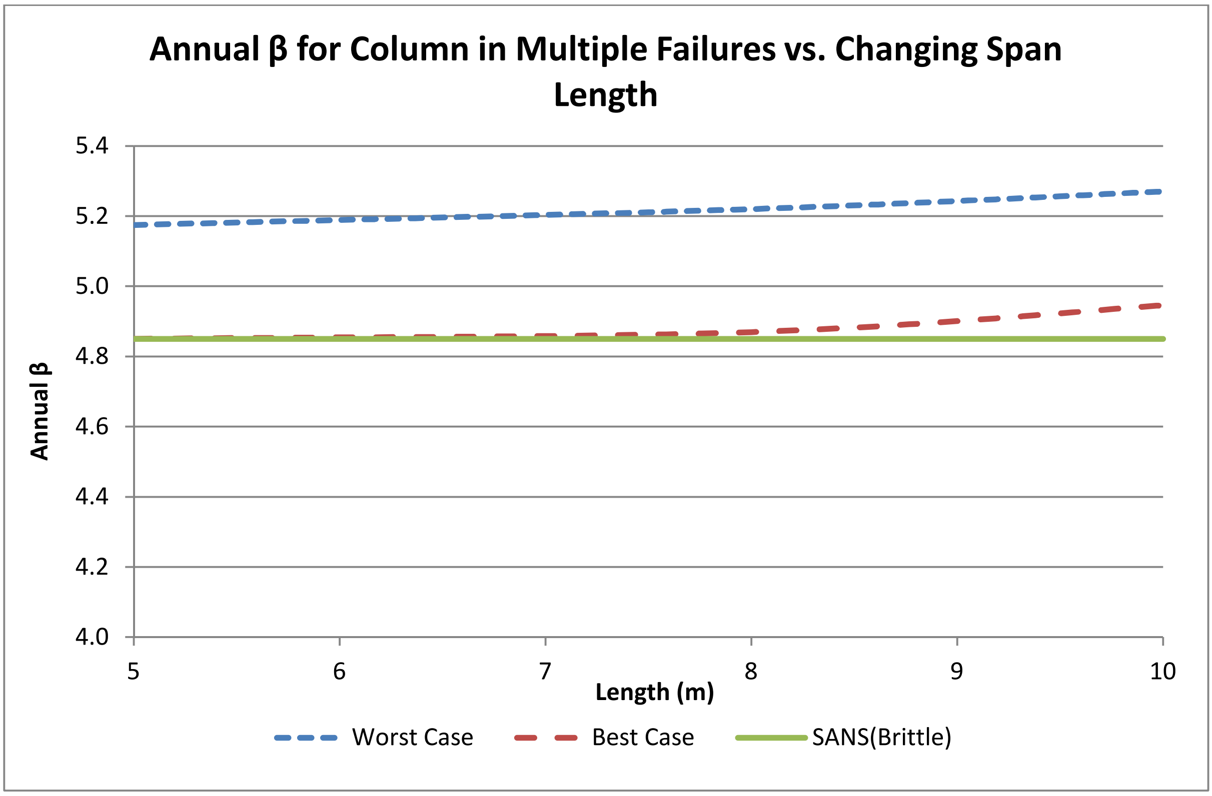

Figure 8-5: Results of Case Study

As the span length is increased, two components of the optimization process changes. The cost of increasing safety (R/m^2) decreases, which was not the case in the previous case studies. This is due to the fact that the cost of increasing safety is normalized by the collapsed area. If the span length is increased the amount of reinforcement that is equivalent to a safety parameter of one increases as well. However, as the span length is increased the area of collapse also increases, thus causing a smaller relative cost of increasing safety.

The other component that is depended on increasing span length is the CoV of some of the statistical parameters. The dead load and live load are in combination affected significantly with an increase in span length, while the resistance are not influenced significantly by increasing span length. The magnitude of the dead load relative to the live load increases significantly reducing overall variance of the load effect and the CoV of the live load decreases with increasing span length by using equation 6-1.

The above occurrence is also an explanation of why an increased target reliability index is observed for a fairly insignificant decreasing cost of increasing safety as shown in Table 8-4. The target reliability indices are close, but once again slightly higher than the target reliability index set by the South African codes.

8.5. Conclusions

The following figure shows the optimum probability of failure plotted against relative cost of safety for the column case study.

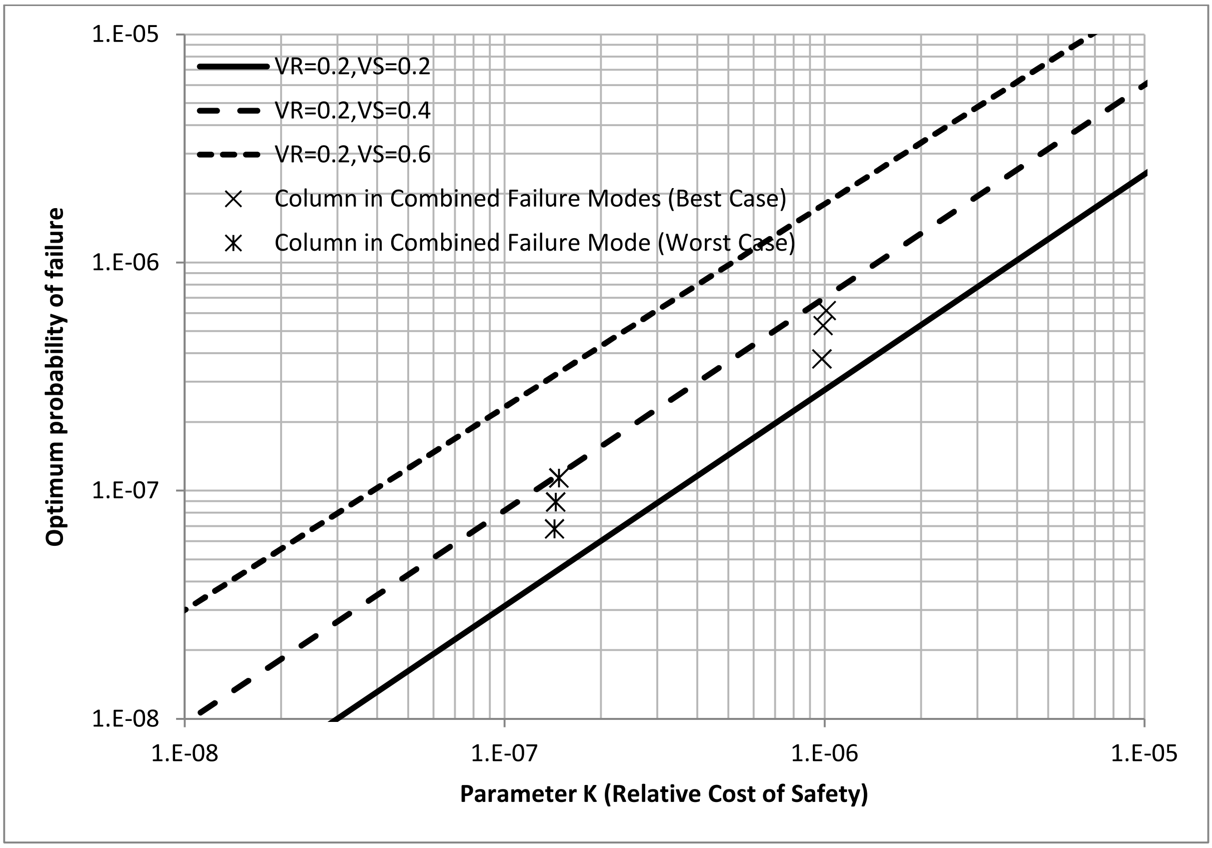

Figure 8-6: Optimum Probability of Failure of Columns vs. Relative cost of safety

From the above figure it is clear that a medium to low variance of the generic approach will best approximate the relationship between the optimum probability of failure and the relative cost of safety. For this particular study, the cost of increasing safety was the lowest for columns out of all the case studies and the highest optimum reliabilities were thus obtained. Columns also had the lowest overall CoV from all the elements, due to a larger dead load relative to live load contributing to the total load than in other element case studies.

The following table summarises the results of this study showing the target reliability indices based on a 50 year reference period:

Columns Brittle Failure				
Parameter	**Mean**	**Range**	**Mean (Best Case)**	**Mean (Worst Case)**
β target	4.18	3.96 - 4.54	4.02	4.41

Table 8-6: Summary of Results

The above results indicate that the current target reliability index of 4 is too low for brittle failure. All means were calculated by obtaining the average of the target probability of failures and converting the average probability of failure to reliability index.

Chapter 9 COMPARING THE RESULTS OF THE CASE STUDIES WITH THE SIMPLIFIED APPROACH

9.1. Introduction

The target reliability indices in the ISO 2394 (1998), or also known as SANS 2394 (2003), are determined by assuming lognormal or Weibull distributions for resistance, Gaussian distribution for the dead load and a Gumbel distribution for the live load. This is a very basic reliability model and is similar to that of Rackwitz's (2000) simplified approach. The problem with these models is that they are assumed to be an accurate representation of most structures which is not necessarily true.

The main aim of the case studies was to derive target reliability indices based on actual situations modelled as accurately as possible. The secondary purpose of the case studies is to gather data on the relationship between optimum probability of failure and relative cost of safety.

The data obtained in the case studies is used in this particular chapter to find which variance of the simplified approach used by Rackwitz (2000) best represents the results obtained. This allows for a more accurate approximation of the target reliability index than what is achieved by the current simplified approaches used by ISO.

9.2. Combination of Results with Simplified Approach

The average K and average P_{fopt} is determined for each of the elements (slab, beam and column) and also for the type of failure (Ductile or brittle). From these results the simplified approach will be used to approximate the relationship of P_{fopt} and K for each element and type of failure. The simplified approach allow for an accurate approximation of the various cases described above as the variance is not estimated, but based on the case studies and can be used to determine the optimum probability of failure graphically in future studies. The following figure shows the average K value and average P_{fopt} points for all the ductile failures. The average value is taken, as an conclusion is made from the results of the case studies that the optimum safety does not change significantly with increasing span length.

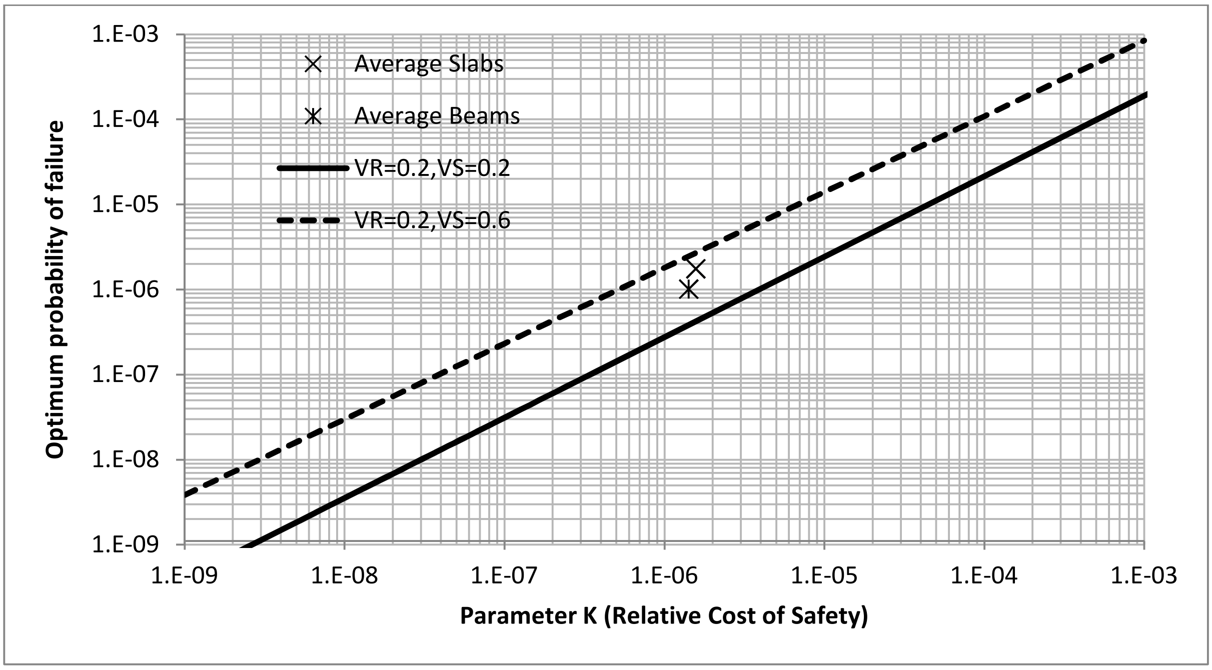

Figure 9-1: Optimum Probability of Failure & Relative cost of safety for Ductile Failures

From the above figure it can be seen that the average P_{fopt} and relative cost of safety relationship for slabs corresponds well with a high variance, while a medium variance corresponds well for the P_{fopt}/K relationship for beams. The same process above is used to determine the variance of the simplified approach best approximating the P_{fopt}/K relationship for brittle failures of certain concrete elements shown in the following figure:

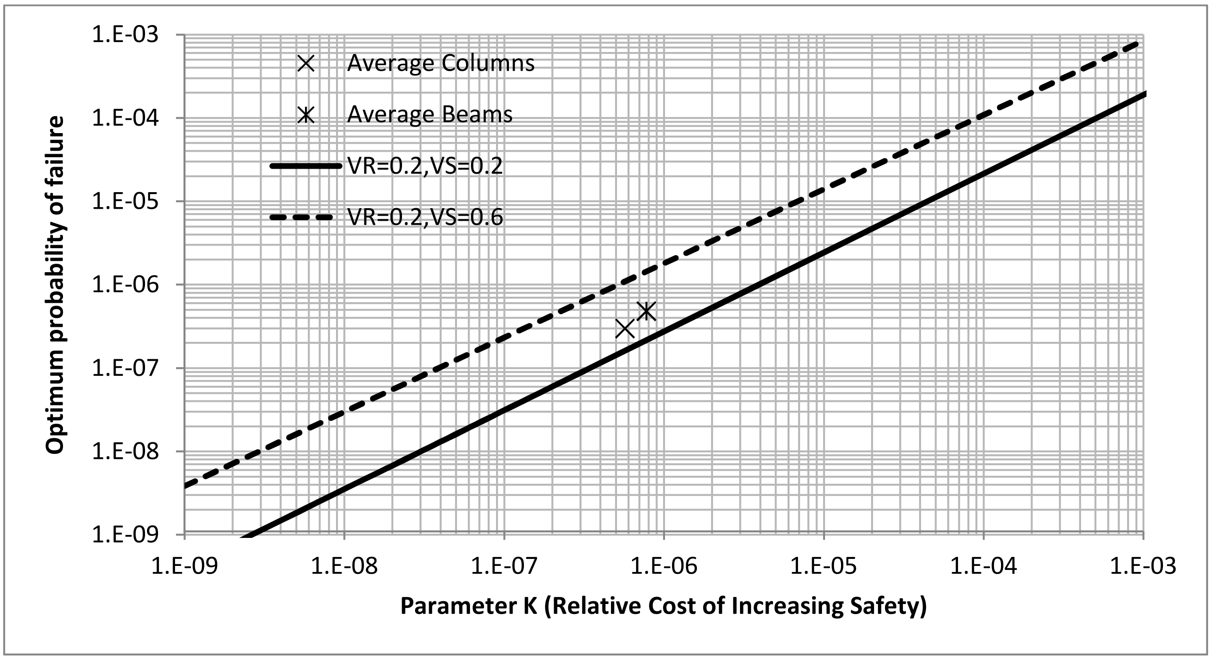

Figure 9-2: Optimum Probability of Failure & Relative cost of safety for Brittle Failures

These two above figures can be used in future studies to approximate the optimum probability of failure graphically, for the specific cases, without having to conduct a reliability based optimization. The following section will demonstrate the application of these figures by using an example. The results of the example will also contribute to the overall results of this thesis as the example will focus on residential buildings. Compared to office buildings, residential buildings will have smaller consequences of structural failure due to a lower occupancy density and smaller costs due to functional losses resulting in lower optimum reliability indices. Refer to Table 2-5.

9.3. Example of the Application of the Simplified Approach

Suppose the target reliability index for a concrete residential building has to be determined and the case study is ductile failure of a slab. For this specific situation a medium to high variance can be used to determine the optimum probability of failure and the first step is to calculate relative cost of safety defined as:

$$\boldsymbol{K} = \frac{\boldsymbol{C_1}(\boldsymbol{\omega}+\boldsymbol{\gamma})}{(\boldsymbol{C_0}+\boldsymbol{C_1}+\boldsymbol{H_F}+\boldsymbol{H_m})\boldsymbol{\lambda}} \qquad \textbf{9-1}$$

An assumption is made that the cost of the entire structure amounts to R8000 per m^2 as in the case studies. When the slab collapses, it is assumed that the slab below supports the collapsed slab do to the requirement SANS 10160 Part 1 (2010) shown in Figure 2-10 that limits the damage of a localized element failure to the entire structure by providing horizontal ties. Furthermore, it is assumed that the slab supporting the collapsed slab is damaged beyond repair and needs to be replaced, but has not collapsed on the slab below. Table 2-5 shows that the cost of replacing lost structural building components and non-structural building components is about 50% of the initial cost of the structure per floor area. Therefore C_0 is R8000 per m^2 collapsed area.

The cost of fatalities relative to other costs is small according to Table 2-5. This could be because the earthquake case studies on which the table is based the earthquakes occurred when occupancy levels were low. It is definitely in part due to the fact that the occupancy density of residential buildings is significantly lower than office buildings. The high injury cost relative to fatality cost could be as a result that the collapsed residential buildings on which the data is based on were mainly constructed out of masonry. Coburn et al (1992) showed that collapsed masonry buildings have a larger proportion of injuries to fatalities than collapsed concrete structures. (Refer to Tables 2-3 & 2-4)

The occupancy density or exposed population is assumed to be 1 person per $35m^2$ while the probability of escape will be kept as a variable to graphically demonstrate the sensitivity of this parameter.

The renewal rate (λ) is taken as 1/7 for the long term load and 1 for the short term load according to the (PMC Part 2 (JCSS, 2001)). The coefficient of variance of the live loads of residential buildings is slightly less than office buildings, but this difference is insignificantly small.

Finally the cost of increasing safety will be less for residential buildings than office buildings, as the mean of the residential live load is less than that of office buildings. The cost of increasing safety for this example is calculated by solving the area of reinforcement resulting in E(G)=0 for a case study having a cost of increasing safety close to the average cost of increasing safety of all the case studies. The average cost of increasing safety for all ductile slab failure case studies was R8.50 per square meter of collapsed area. The cost of increasing safety for residential buildings is calculated as R8.20 per meter square of collapsed area for medium span lengths (8m long span & 5m short span). The insignificantly small difference is as a result of the mean of the dead load being significantly larger than the mean of the live load.

The following figure was obtained by varying parameter P(Q) and assuming additional failure costs (H_m) is estimated as R12000 based on Table 2-5.

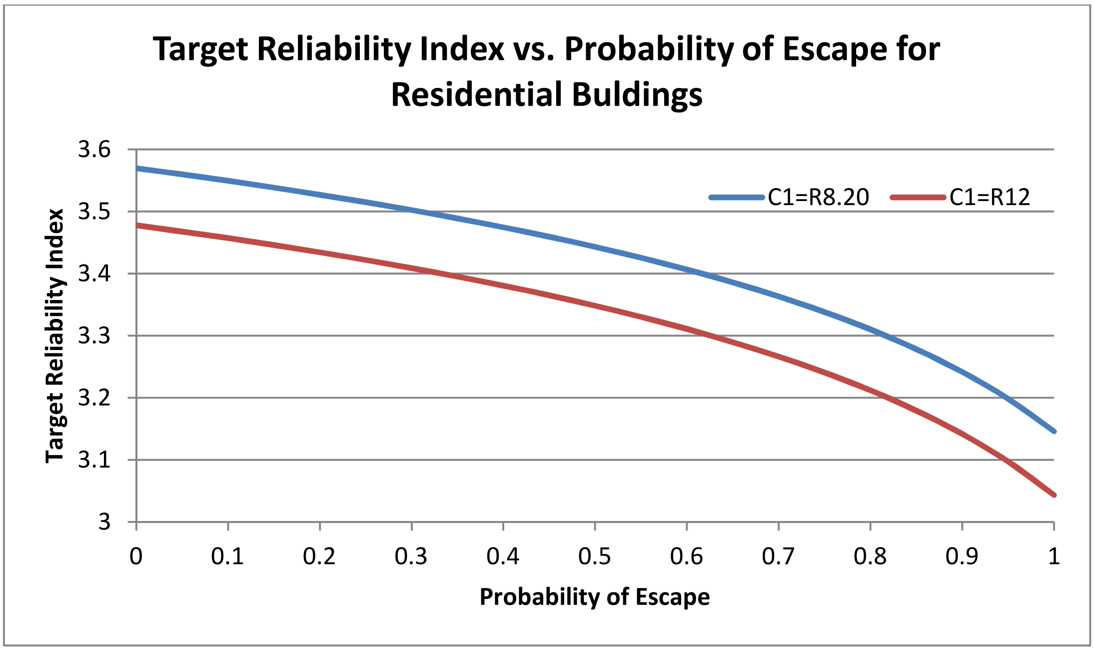

Figure 9-3: Effect of Probability of Escape on 50 Year Reference Period Target Reliability Index

In the above figure the effect of two parameters can be seen. Firstly, there are two lines representing two different costs of increasing safety. The calculated cost is increased with about 50% to R12/m^2 and the result is a difference in target reliability indices of less than 0.1.

Secondly, the probability of escape is varied between 0 and 1, and it is interesting to note that if there are zero fatalities the target reliability index is still larger than 3 as recommended by the current South African code for ductile failures of concrete structures in RC2.

For ductile failures in slabs a probability of escape of 0.5 is assumed as reasonable. Thus for ductile slabs residential buildings a target reliability index of 3.44 deemed as the optimum safety. This is about 0.31 less than the target reliability index for slabs derived in the office building case studies. The following table gives a summary of the target reliability index obtained, in the same manner as the above example, for different elements having different failure modes in residential buildings:

Situation	P(Q)	$C_1(R/m^2)$	K	β_{target}	Pfopt
Ductile Failure of Slabs	0.6	8.2	5.72E-06	3.41	6.57E-06
Ductile Failure of Beams	0.8	3	3.13E-06	3.67	2.43E-06
Brittle Failure of Beams	0.5	3.9	2.35E-06	3.81	1.40E-06
Brittle Failure of Columns	0.3	2.1	9.92E-07	4.02	5.70E-07

Table 9-1: Summary of Results for Residential Buildings by Proposed Approximation Method

The cost of increasing safety was obtained from the average cost of increasing safety for the office case studies as the change of live load from the office building has little impact on this particular cost. From Table 9-1 it is clear that target reliability indices for residential buildings are slightly smaller than those derived from the case studies for office buildings.

9.4. Conclusions

The work in this particular chapter has shown that the complicated optimization process can be approximated by Rackwitz's (2000) simplified approach accurately approximating the P_{fopt}/K relationship which can further be simplified by obtaining the optimum safety graphically. The two main advantages this approach is relative accuracy and the fact that it does not require the repetitive computation of a probability of failure to obtain the optimum safety if the optimum safety is obtained graphically.

The applicability and the simplicity of the simplified approach used by Rackwitz (2000) is demonstrated by acquiring target reliability indices in one basic example of a residential building that took three chapters of work to derive for office buildings. In this example the effect of certain parameters, cost of increasing safety and probability of escape, on the target reliability index was explored. From these results the sensitivity of optimum safety with regards to the cost of increasing safety and probability of escape is shown.

The simplified approach used by Rackwitz (2000) is simplified by graphical approximation. The advantages are as follows:

- No Probability of failure computation required.
- Less time needed for a result.

The disadvantages are:

- These are case specific (A benefit/cost function with other cost components (fatigue and serviceability failure) dependent on the safety parameter p will not necessarily work as relative cost of safety is dependent on the benefit/cost function).

This graphical approximation can be used to conduct other reliability based optimization studies for concrete structures which greatly reduce the complexity and the effort required from the user to acquire results. However there are some certain requirements:

- SVSL>SWTP.
- All cost in units of Money/m^2 of collapsed area.
- Cost of increasing safety determined the same way as done in this study.
- If the user knows the P_{fopt}/K relationship of the situation, an appropriate graphical approximation can be done
- Approximations derived from these case studies may only be used for systematic reconstruction policy and the assumption that failure does not occur upon reconstruction.
- Fatigue, maintenance and demolition costs are assumed to be independent of safety parameter p, while serviceability failure is assumed to be insignificantly small.
- Can be used for various reference periods (Obsolescence rate and discount rate modified for required life time of structure).
- P_{fopt} obtained from graphical approximation is not an exact solution, only an approximation.

Chapter 10 CONCLUSIONS & RECOMENDATIONS

10.1. Main Findings

Life Saving and Compensation Costs

There are various life compensation costs and life saving costs as shown by the literature review. The target reliability indices obtained from the reliability optimization process using two life saving costs will be almost identical as long as these life saving costs do not differ by a factor of two or more.

The general assumption that the work time fraction is in large determined by the country's GDP per capita has been disproved in this study. The assumption above is based on the assumption that people in poorer countries generally want to obtain more wealth so they are willing to work harder and that people in richer countries want to spend more time enjoying life. Even though the second assumption is true, the work time fraction was found to be lower in poorer African countries than in richer European countries. This is as a result of the work time fraction actually being largely dependent on the population distribution of a country, which in turn is to a large degree dependent on the life expectancy of a country.

The work time fraction was shown to have an insignificant effect on the optimum safety, but does increase the SVSL life compensation cost by a significant amount. Thus work time fractions between the ranges of 0.08-0.012 should result in approximately the same target reliability index obtained from the reliability based optimization process.

In this study it was also shown that the LQI criterion check is seldomly necessary for reliability based optimization situations. If the life saving cost is more than the life compensation cost the LQI criterion must always be checked. However this situation will only arise if a long term sustainable discount rate of more than 7% is used to derive SVSL and SWTP.

The pure time preference rate is determined based on Kula's formulation (1984) and identical results were obtained from using the elasticities obtained from two different food demand equations. The pure time preference rate was low for South Africa compared to those of USA and Canada determined by Kula. This resulted in a low long term sustainable discount rate for South Africa.

Target Safety

Based on the results of the various case studies and the final simplified example conducted in Chapter 9 the author recommends the following values for ultimate limit states failure for South African RC2 structures:

Failure Mode	Target Reliability Index
Ductile Failures	≈3.7
Brittle Failures	≈4.1

Table 10-1: Recommended Target Reliability Indices for RC2 Structures

It is the personal opinion of the author that the probability of escape is low for the worst case scenarios thus the values in Table 10-1 are significantly less than the means derived in this study for office buildings. Figure 10-1 shows various target reliability indices against changing probability of escape for various structural components and for different types of structures.

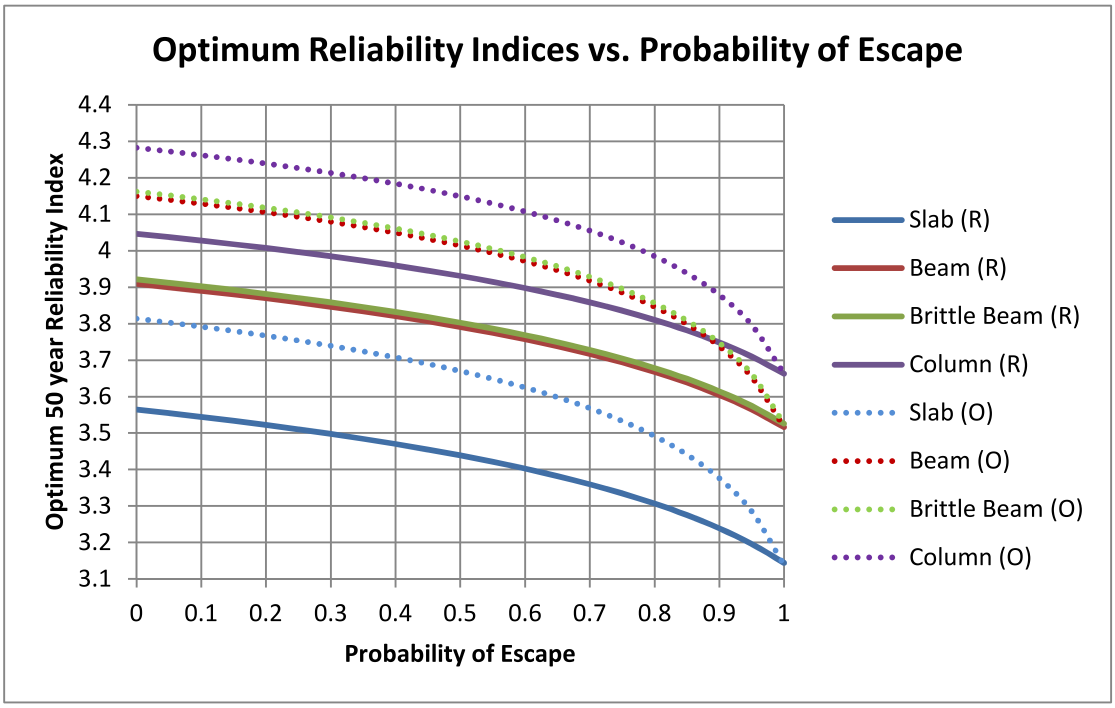

Figure 10-1: Effect of Probability of Escape on 50 Year Reference Period Target Reliability Index for all Cases

Figure 10-1 was created using the simplified approach graphically and using the average cost of increasing safety obtained from the case studies. The population density of an office is assumed as 1 person 10m^2, while a population density of 1 person per 35m^2 is assumed for a residential building. If the reader disagrees with the author's recommended target reliability indices he/she may use the above figure to choose reliabilities with the appropriate probability of escape. (R=Residence, O=Office)

From the table the recommended safety level for brittle failures compares well with the current target reliability index of β = 4.0 recommended by the current South African codes. However, the recommended target reliability index for ductile failure is significantly higher than the current South African target of 3. This is not because of the fact that the human fatalities consequences are grossly overestimated in this study and can this can be confirmed by referring to Figures 9-3 and 10-1. Even if there are zero fatalities after structural failure, the target reliability index for a residential structure is still 3.15 which is still higher than the current specified target reliability index of β = 3.0 for ductile failures.

From the various case studies a conclusion is drawn that the costs of increasing safety or relative effort to increase safety, is low for concrete structures. This result in high target reliability indices obtained even if there are zero fatalities. In this study, comparisons were drawn with target reliability index tables based on medium costs and medium consequences. However since the cost of increasing safety is low for concrete structures, comparisons must be with tabulated target safety levels with a low cost of increasing safety. The recommended target reliability index is slightly higher than the β = 3.5 recommended by the PMC (refer to Table 2-6) and slightly less than the value of β = 3.8 recommended by the ISO 2394 (refer to Table 2-7) for ductile failures.

Furthermore, this study has clearly defined costs of increasing safety in numerical values based on actual situations and determined methods of obtaining normalized costs of increasing safety. Where in other studies, such as that conducted by Rackwitz (2000), the costs of increasing safety were assumed. It was also interesting to note that for slabs the highest cost of increasing safety is observed while the lowest is observed for columns.

The differences between these recommended values in Table 10-1 are due to the fact that both the tables are based on simplified approaches assumed to be an accurate approximation of typical structures. Another reason for different target safeties is due to the fact that in only one of the tables some guidelines are given for the classification of large or small costs of increasing safety and consequences. Furthermore, due to lack of information and various methodologies of estimating consequences, the consequences in these studies are estimated using some data from other disasters (floods, fires and earthquakes) and personal judgement. This leads to some degree of either overestimating or under estimating the consequences and will lead to different levels of safety. For example the consequences in this study are about two to four times of the medium consequences in the PMC. (Refer to Section 2.8)

Confirmation of the Simplified Approach

The results of the case studies showed that the simplified approach used by Rackwitz (2000) is a sufficiently accurate approximation. The results of the case studies were also used to determine which kind of variances of the simplified approach, used by Rackwitz (2000), best represents certain types of element failure. Ductile failures in slabs can be best approximated by medium to high variances, ductile failures in beams can be best approximated by medium variances, while brittle failures in both columns and beams can best approximated by medium to low variances.

10.2. Recommendations for Future Studies

The broad range of possible applications of benefit/cost functions has been shown literature. However, while good data/statistical models are available for most of the variables required for reliability based optimization, the consequences of a localized structural failure have been estimated in this study due to lack of data. Furthermore, various models exist to calculate the other losses due to structural failure, but they are highly case specific and not practically applicable in this study. Thus the estimation of consequences based on earthquake data and other studies might result in an overestimation or underestimation of consequences. The following areas of research are recommended:

- Compile data for fatality estimation of structural failure due to failures other than earthquakes
- Determining people's reaction to localized structural failure in concrete structures in terms of probability of escape (perceiving warning signs, reacting in time)
- Robustness studies: Will other components of the structure fail as well if one of the components is removed or fails?
- Deflection of the failing structural components and time until components fail for both ductile and brittle failure.
- A study determining typical other (non-fatality related) losses in concrete buildings in RC2.
- Simplification of existing models of other (non-fatality related) losses estimation.
- System reliability based optimization instead of component reliability based optimization.

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APPENDIX A: LTConverter

MATLAB Code

```
function [ LifeTable ] = LTConverter( Data,n )
%Converts a life table matrix (nx3) with bins larger than one using age at
%beginning of bin, size of bin,
%mortality rate of bin & growth rate of population

%n = 0.01214;
%growth rate factor
NumberOfBins = size(Data,1);
%determine number of bins
LifeTableSize = Data(NumberOfBins,1)+Data(NumberOfBins,2);
%determine required size of life table with one year bins
LifeTable = zeros(LifeTableSize,4);
%set previous life table calculated to zero
counter = 0;
%set counter to zero

%converting life tables to uniform tables
for i=1: NumberOfBins
    for j=1: Data(i,2)
        d = counter+1;
        LifeTable (d,2) = Data(i,3);
        LifeTable (d,1) =counter;
         counter=counter+1;
    end
end

for k=1:(LifeTableSize)
    LifeTable (k,3) = exp(-n*k)*exp(-sum(LifeTable(1:k,2)));
end

Div = sum(LifeTable(1:LifeTableSize,3));

%percentage of population at age a stable
for l=1:(LifeTableSize)
    LifeTable(l,4) = LifeTable(l,3)/Div;
end

end
```

Input of LTConverter

(1) Age at Beginning of Bin	(2) Size of Bin	(3) Mortality rate
0	1	0.04447
1	4	0.00495
5	5	0.00145
10	5	0.00123
15	5	0.00229
20	5	0.00667
25	5	0.01264
30	5	0.01774
35	5	0.01757
40	5	0.01684
45	5	0.01814
50	5	0.02061
55	5	0.02449
60	5	0.0281
65	5	0.04184
70	5	0.05219
75	5	0.08234
80	5	0.10452
85	5	0.13798
90	5	0.19177
95	6	0.2824
1), 2) & 3) obtained from Statistics South Africa		

LTConverter Input

Output of LTConverter

Age	Mortality rate	Denominator of Eq 2.34	Stable Population as % of Total Population	Empirical Population as % of Total Population	Age	Mortality rate	Denominator of Eq 2.34	Stable Population as % of Total Population	Empirical Population as % of Total Population
0	0.04447	0.944962533	0.025351462	0.0198	50	0.02061	0.308242028	0.008269519	0.00855
1	0.00495	0.928950337	0.024921887	0.020141595	51	0.02061	0.298310616	0.008003079	0.008188363
2	0.00495	0.913209465	0.024499591	0.020437549	52	0.02061	0.288699189	0.007745224	0.007833693
3	0.00495	0.897735318	0.02408445	0.020689598	53	0.02061	0.279397439	0.007495677	0.007486334
4	0.00495	0.882523378	0.023676344	0.020899447	54	0.02061	0.270395386	0.007254169	0.007146599
5	0.00145	0.870611013	0.023356759	0.021068775	55	0.02449	0.26067001	0.006993257	0.006814775
6	0.00145	0.858859442	0.023041488	0.021199233	56	0.02449	0.25129443	0.006741729	0.006491121
7	0.00145	0.847266495	0.022730472	0.021292443	57	0.02449	0.242256063	0.006499248	0.006175867
8	0.00145	0.83583003	0.022423654	0.021350001	58	0.02449	0.233542782	0.006265488	0.005869217
9	0.00145	0.824547935	0.022120978	0.021373473	59	0.02449	0.225142893	0.006040135	0.005571345
10	0.00123	0.813597099	0.021827189	0.0213644	60	0.0281	0.216263004	0.005801906	0.0052824
11	0.00123	0.802791701	0.021537302	0.021324292	61	0.0281	0.207733349	0.005573072	0.0050025
12	0.00123	0.792129809	0.021251265	0.021254634	62	0.0281	0.199540112	0.005353264	0.004731738
13	0.00123	0.781609519	0.020969026	0.021156881	63	0.0281	0.191670026	0.005142125	0.004470177
14	0.00123	0.771228948	0.020690536	0.021032461	64	0.0281	0.184110345	0.004939314	0.004217853
15	0.00229	0.760180024	0.020394116	0.020882775	65	0.04184	0.174435541	0.004679758	0.003974775
16	0.00229	0.749289391	0.020101942	0.020709194	66	0.04184	0.165269137	0.004433842	0.003740922
17	0.00229	0.738554782	0.019813953	0.020513064	67	0.04184	0.156584418	0.004200848	0.003516248
18	0.00229	0.72797396	0.019530091	0.0202957	68	0.04184	0.148356072	0.003980098	0.003300676
19	0.00229	0.717544724	0.019250295	0.020058392	69	0.04184	0.140560117	0.003770948	0.003094104
20	0.00667	0.704173855	0.018891582	0.0198024	70	0.05219	0.131802591	0.003536001	0.0028964
21	0.00667	0.691052142	0.018539552	0.019528958	71	0.05219	0.123590697	0.003315692	0.002707406
22	0.00667	0.67817494	0.018194083	0.019239271	72	0.05219	0.115890442	0.00310911	0.002526935
23	0.00667	0.665537696	0.01785505	0.018934516	73	0.05219	0.108669947	0.002915398	0.002354772
24	0.00667	0.653135936	0.017522336	0.018615844	74	0.05219	0.101899321	0.002733756	0.002190676
25	0.01264	0.63715011	0.017093468	0.018284375	75	0.08234	0.092712683	0.002487297	0.002034375
26	0.01264	0.621555545	0.016675097	0.017941204	76	0.08234	0.084354257	0.002263057	0.001885572
27	0.01264	0.606342664	0.016266966	0.017587396	77	0.08234	0.076749378	0.002059033	0.00174394
28	0.01264	0.591502127	0.015868824	0.017223991	78	0.08234	0.069830109	0.001873403	0.001609127
29	0.01264	0.577024819	0.015480426	0.016851998	79	0.08234	0.063534641	0.001704508	0.00148075
30	0.01774	0.560038359	0.015024713	0.0164724	80	0.10452	0.056538695	0.001516821	0.0013584
31	0.01774	0.543551946	0.014582416	0.016086152	81	0.10452	0.05031309	0.0013498	0.0013498
32	0.01774	0.527550861	0.014153139	0.01569418	82	0.10452	0.044773	0.00120117	0.00120117
33	0.01774	0.512020815	0.013736498	0.015297384	83	0.10452	0.039842942	0.001068907	0.001068907
34	0.01774	0.496947943	0.013332123	0.014896634	84	0.10452	0.035455744	0.000951207	0.000951207
35	0.01757	0.482400788	0.012941852	0.014492775	85	0.13798	0.03051338	0.000818613	0.000818613
36	0.01757	0.468279471	0.012563005	0.014086621	86	0.13798	0.026259958	0.000704502	0.000704502
37	0.01757	0.454571528	0.012195248	0.013678961	87	0.13798	0.022599443	0.000606298	0.000606298
38	0.01757	0.441264857	0.011838257	0.013270554	88	0.13798	0.019449187	0.000521783	0.000521783
39	0.01757	0.428347713	0.011491716	0.012862132	89	0.13798	0.016738062	0.000449049	0.000449049
40	0.01684	0.416112343	0.011163465	0.0124544	90	0.19177	0.013650488	0.000366215	0.000366215
41	0.01684	0.404226465	0.010844591	0.012048033	91	0.19177	0.011132461	0.000298662	0.000298662
42	0.01684	0.392680097	0.010534825	0.011643681	92	0.19177	0.00907892	0.000243569	0.000243569
43	0.01684	0.381463541	0.010233907	0.011241963	93	0.19177	0.007404184	0.00019864	0.00019864
44	0.01684	0.370567376	0.009941584	0.010843473	94	0.19177	0.006038377	0.000161998	0.000161998
45	0.01814	0.359514776	0.009645065	0.010448775	95	0.2824	0.004497831	0.000120668	0.000120668
46	0.01814	0.348791834	0.00935739	0.010058407	96	0.2824	0.003350318	8.99E-05	8.99E-05
47	0.01814	0.338388715	0.009078295	0.009672878	97	0.2824	0.002495565	6.70E-05	6.70E-05
48	0.01814	0.328295881	0.008807524	0.009292669	98	0.2824	0.001858882	4.99E-05	4.99E-05
49	0.01814	0.318504078	0.00854483	0.008918235	99	0.2824	0.001384633	3.71E-05	3.71E-05
					100	0.2824	0.001031377	2.77E-05	2.77E-05

Output of LTConverter

APPENDIX B:*DeltaConstantCalc*

MATLAB Code

```
function [DeltaConstantS,DeltaConstantE,e] = DeltaConstantCalc(Data,g)
% Calculates Delta Constant and age averaged and discounted
% life expectancy using a lifetable containing:
%age-column 1
%mortality column 2
%numinator in calculating percentage of the population which are a years
%old-column 3
%percentage of population which are a years old (stable) column 4
%percentage of population which are a years old (actual) column 5
%& a discount rate
NumberOfBins = size(Data,1);

DeltaConstantS = 0;
DeltaConstantE = 0;
results  = zeros(NumberOfBins,5);
e=0;



%integration of exp term in terms of age,i, and lifeyears discounted,j,
for i=1:NumberOfBins
    for j=i:NumberOfBins
        results(i,1) = results(i,1) + (j-i+1)*exp(-(sum(Data(i:j,2))+(j-
i+1)*g));
        %Calculate de
        results (i,2) = results(i,2) + exp(-(sum(Data(i:j,2))+(j-i+1)*g));
        %Calculate d
    end
end

results(:,5) =(results(:,1)./results(:,2));

DeltaConstantS = dot(results(:,5),Data(:,4));
DeltaConstantE = dot(results(:,5),Data(:,5));
e = dot(results(:,2),Data(:,5));

end
```

Input of *DeltaConstantCalc*

Age	Mortality rate	Denominator of Eq 2.34	Stable Population as % of Total Population	Empirical Population as % of Total Population	Age	Mortality rate	Denominator of Eq 2.34	Stable Population as % of Total Population	Empirical Population as % of Total Population
0	0.04447	0.944962533	0.025351462	0.0198	50	0.02061	0.308242028	0.008269519	0.00855
1	0.00495	0.928950337	0.024921887	0.020141595	51	0.02061	0.298310616	0.008003079	0.008188363
2	0.00495	0.913209465	0.024499591	0.020437549	52	0.02061	0.288699189	0.007745224	0.007833693
3	0.00495	0.897735318	0.02408445	0.020689598	53	0.02061	0.279397439	0.007495677	0.007486334
4	0.00495	0.882523378	0.023676344	0.020899447	54	0.02061	0.270395386	0.007254169	0.007146599
5	0.00145	0.870611013	0.023356759	0.021068775	55	0.02449	0.26067001	0.006993257	0.006814775
6	0.00145	0.858859442	0.023041488	0.021199233	56	0.02449	0.25129443	0.006741729	0.006491121
7	0.00145	0.847266495	0.022730472	0.021292443	57	0.02449	0.242256063	0.006499248	0.006175867
8	0.00145	0.83583003	0.022423654	0.021350001	58	0.02449	0.233542782	0.006265488	0.005869217
9	0.00145	0.824547935	0.022120978	0.021373473	59	0.02449	0.225142893	0.006040135	0.005571345
10	0.00123	0.813597099	0.021827189	0.0213644	60	0.0281	0.216263004	0.005801906	0.0052824
11	0.00123	0.802791701	0.021537302	0.021324292	61	0.0281	0.207733349	0.005573072	0.0050025
12	0.00123	0.792129809	0.021251265	0.021254634	62	0.0281	0.199540112	0.005353264	0.004731738
13	0.00123	0.781609519	0.020969026	0.021156881	63	0.0281	0.191670026	0.005142125	0.004470177
14	0.00123	0.771228948	0.020690536	0.021032461	64	0.0281	0.184110345	0.004939314	0.004217853
15	0.00229	0.760180024	0.020394116	0.020882775	65	0.04184	0.174435541	0.004679758	0.003974775
16	0.00229	0.749289391	0.020101942	0.020709194	66	0.04184	0.165269137	0.004433842	0.003740922
17	0.00229	0.738554782	0.019813953	0.020513064	67	0.04184	0.156584418	0.004200848	0.003516248
18	0.00229	0.72797396	0.019530091	0.0202957	68	0.04184	0.148356072	0.003980098	0.003300676
19	0.00229	0.717544724	0.019250295	0.020058392	69	0.04184	0.140560117	0.003770948	0.003094104
20	0.00667	0.704173855	0.018891582	0.0198024	70	0.05219	0.131802591	0.003536001	0.0028964
21	0.00667	0.691052142	0.018539552	0.019528958	71	0.05219	0.123590697	0.003315692	0.002707406
22	0.00667	0.67817494	0.018194083	0.019239271	72	0.05219	0.115890442	0.00310911	0.002526935
23	0.00667	0.665537696	0.01785505	0.018934516	73	0.05219	0.108669947	0.002915398	0.002354772
24	0.00667	0.653135936	0.017522336	0.018615844	74	0.05219	0.101899321	0.002733756	0.002190676
25	0.01264	0.63715011	0.017093468	0.018284375	75	0.08234	0.092712683	0.002487297	0.002034375
26	0.01264	0.621555545	0.016675097	0.017941204	76	0.08234	0.084354257	0.002263057	0.001885572
27	0.01264	0.606342664	0.016266966	0.017587396	77	0.08234	0.076749378	0.002059033	0.00174394
28	0.01264	0.591502127	0.015868824	0.017223991	78	0.08234	0.069830109	0.001873403	0.001609127
29	0.01264	0.577024819	0.015480426	0.016851998	79	0.08234	0.063534641	0.001704508	0.00148075
30	0.01774	0.560038359	0.015024713	0.0164724	80	0.10452	0.056538695	0.001516821	0.0013584
31	0.01774	0.543551946	0.014582416	0.016086152	81	0.10452	0.05031309	0.0013498	0.0013498
32	0.01774	0.527550861	0.014153139	0.01569418	82	0.10452	0.044773	0.00120117	0.00120117
33	0.01774	0.512020815	0.013736498	0.015297384	83	0.10452	0.039842942	0.001068907	0.001068907
34	0.01774	0.496947943	0.013332123	0.014896634	84	0.10452	0.035455744	0.000951207	0.000951207
35	0.01757	0.482400788	0.012941852	0.014492775	85	0.13798	0.03051338	0.000818613	0.000818613
36	0.01757	0.468279471	0.012563005	0.014086621	86	0.13798	0.026259958	0.000704502	0.000704502
37	0.01757	0.454571528	0.012195248	0.013678961	87	0.13798	0.022599443	0.000606298	0.000606298
38	0.01757	0.441264857	0.011838257	0.013270554	88	0.13798	0.019449187	0.000521783	0.000521783
39	0.01757	0.428347713	0.011491716	0.012862132	89	0.13798	0.016738062	0.000449049	0.000449049
40	0.01684	0.416112343	0.011163465	0.0124544	90	0.19177	0.013650488	0.000366215	0.000366215
41	0.01684	0.404226465	0.010844591	0.012048033	91	0.19177	0.011132461	0.000298662	0.000298662
42	0.01684	0.392680097	0.010534825	0.011643681	92	0.19177	0.00907892	0.000243569	0.000243569
43	0.01684	0.381463541	0.010233907	0.011241963	93	0.19177	0.007404184	0.00019864	0.00019864
44	0.01684	0.370567376	0.009941584	0.010843473	94	0.19177	0.006038377	0.000161998	0.000161998
45	0.01814	0.359514776	0.009645065	0.010448775	95	0.2824	0.004497831	0.000120668	0.000120668
46	0.01814	0.348791834	0.00935739	0.010058407	96	0.2824	0.003350318	8.99E-05	8.99E-05
47	0.01814	0.338388715	0.009078295	0.009672878	97	0.2824	0.002495565	6.70E-05	6.70E-05
48	0.01814	0.328295881	0.008807524	0.009292669	98	0.2824	0.001858882	4.99E-05	4.99E-05
49	0.01814	0.318504078	0.00854483	0.008918235	99	0.2824	0.001384633	3.71E-05	3.71E-05
					100	0.2824	0.001031377	2.77E-05	2.77E-05

***DeltaConstantCalc* Input**

Output of *DeltaConstantCalc*

Demographic constant stable =15.5676

Demographic constant empirical=15.4918

Age averaged, discounted life expectancy = 17.7110

APPENDIX C:FORM

```
function [pf ] = form( stat,data,y)
%FORM-(First Order Reliability Method)
% This function imports Matrices "stat", "data", performance function "y"
%  and returns  probability of failure "pf" through the
% use of the first order reliability method.
%
%INPUT
% data(numerical matrix)not larger than 20 rows:
%
    %-Column 1 mean of variable
    %-Column 2 standard deviation of variable
    %-Column 3 skewness of variable
%

%stat
    %-Row 1 statistical distribution of variable as follows:
%
    % 'N'   Normal Distribution
    % 'LN'  Lognormal Distribution(2 parameter): Skewness = 3*sigma/mu
    % 'GU'  Gumbel Distribution:Skewness = 1.14
    % 'GA'  Gamma Distribution:Skewness = 2*sigma/mu also sigma+->mu/7.5
    % 'D'   Determinate variable std Dev & skewness = 0
    % or smaller than zero

format longEng;
results2 =(zeros(21,7));
BT=zeros(20,1);
B=0;
b1=0;
i=1;
s=1;
syms  e N LN GU GA D X1 X2 X3 X4 X5 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X15
X16 X17 X18 X19 X20;


%INPUT EXAMPLE
% syms LN GU N y X1 X2 X3
%data = double( [1500 ,100,0.2;
%        800,100,1.14;
%        1.5,0.15,0;

%        zeros(17,3)]);
%Data must be a 20x3 matrix
%
%stat = ([LN;GU;N;]);
 %Stat can be any size but not larger than 20 columns
%
%y=X1-X2*X3;
%Performance function, each variable must be represented by an X
%
%symbols =(sym ('X',[3,1]));
%matrix containing symbols representing random variables
%
%form(stat,data,y,symbols)
% calling function
```

```
%End of Example



num = size(stat);
%number of variables

solution = solve(y,X1);
%solution of performance function ensuring design point remains on failure
plane



 results(1,1) = diff(y,X1);
results(2,1) = diff(y,X2);
results(3,1) = diff(y,X3);
results(4,1) = diff(y,X4);
results(5,1) = diff(y,X5);
results(6,1) = diff(y,X6);
results(7,1) = diff(y,X7);
results(8,1) = diff(y,X8);
results(9,1) = diff(y,X9);
results(10,1) = diff(y,X10);
results(11,1) = diff(y,X11);
results(12,1) = diff(y,X12);
results(13,1) = diff(y,X13);
results(14,1) = diff(y,X14);
results(15,1) = diff(y,X15);
results(16,1) = diff(y,X16);
results(17,1) = diff(y,X17);
results(18,1) = diff(y,X18);
results(19,1) = diff(y,X19);
results(20,1) = diff(y,X20);
%Partial derivatives of performance function

X1 = data(1,1);
X2 = data(2,1);
X3 = data(3,1);
X4 = data(4,1);
X5 = data(5,1);
X6 = data(6,1);
X7 = data(7,1);
X8 = data(8,1);
X9 = data(9,1);
X10 = data(10,1);
X11 = data(11,1);
X12 = data(12,1);
X13 = data(13,1);
X14 = data(14,1);
X15 = data(15,1);
X16 = data(16,1);
X17 = data(17,1);
X18 = data(18,1);
X19 = data(19,1);
X20 = data(20,1);
%Assign mean values to variables
```

```
results2 (1,1) = subs(solution);
%calculate value of variable X1 from g=0 when X2-XN are assigned mean
values

 for j=2:num
     results2(j,1) = data(j,1);
 end
%assign mean values to a matrix showing numerical results of FORM

while (i<20)
%for loop completing number of iterations
 for j=1:num
%for loop transforming statistical distributions into a space of
standardized
%normal variables
     if ((stat(j)==N))
         %Normal Distribution

         if((data(j,3)==0))
         %Checking for incorrect skewness

        results2 (j,2) =
(normpdf((norminv((normcdf(results2(j,1),data(j,1),data(j,2))),0,1)),0,1))/
normpdf(results2(j,1),data(j,1),data(j,2));
        %Equivalent standard deviation

        results2 (j,3) =results2(j,1)-
results2(j,2)*(norminv((normcdf(results2(j,1),data(j,1),data(j,2))),0,1));
        %Equivalent mean

        results2 (j,4) = (results2(j,1)-results2(j,3))/results2(j,2);
         %Equivalent Standardized Mean
         else
             e=1;
         end


     elseif ((stat(j)==LN))
         %Lognormal Distribution
            if(abs(data(j,3)-(3*data(j,2))/data(j,1))<0.1)
                %Checking for incorrect skewness
                c = ((sqrt(data(j,3)^2+4)+data(j,3))^(1/3)-
(sqrt(data(j,3)^2+4)-data(j,3))^(1/3))*2^(-1/3);
                %Coefficient c

                u = (results2(j,1)-data(j,1))/data(j,2);
                %Standardized varible u

                u1 =
(log(abs(u+1/c))+log(abs(c)*sqrt(1+c^2)))/((sign(data(j,3))*sqrt(log(1+c^2)
)));
                %Modified Standardized Variable u

                 LNpdf =
(normpdf(u1,0,1))/(data(j,2)*(abs(u+(1/c))*sqrt(log(1+c^2))));
                %Standardized Lognormal PDF

                LNcdf = normcdf(u1,0,1);
```

```
                %Standardized Lognormal CDF

                results2 (j,2) = (normpdf((norminv(LNcdf,0,1)),0,1))/LNpdf;
                %Equivalent standard deviation

                results2 (j,3) = results2(j,1)-
results2(j,2)*(norminv(LNcdf,0,1));
                %Equivalent  mean

                results2 (j,4) = (results2(j,1)-
results2(j,3))/results2(j,2);
                %Equivalent Standardized Mean
           else
             e=2;
           end

     elseif ((stat(j)==GU))
         %Gumbel Distribution
         if(data(j,3)==1.14)
            %Cheking for incorrect skewness
                c = ((sqrt(data(j,3)^2+4)+data(j,3))^(1/3)-
(sqrt(data(j,3)^2+4)-data(j,3))^(1/3))*2^(-1/3);
                %Coefficient c

                u = (results2(j,1)-data(j,1))/data(j,2);
                %Standardized variable u

                u1 =
(log(abs(u+1/c))+log(abs(c)*sqrt(1+c^2)))/((sign(data(j,3))*sqrt(log(1+c^2)
)));
                %Modified Standardized Variable u

                GUpdf =
(normpdf(u1,0,1))/(data(j,2)*(abs(u+(1/c))*sqrt(log(1+c^2))));
                %Standardized Gumbel PDF

                GUcdf = normcdf(u1,0,1);
                %Standardized Gumbel CDF

                results2 (j,2) = (normpdf((norminv(GUcdf,0,1)),0,1))/GUpdf;
                %Equivalent standard deviation

                results2 (j,3) = results2(j,1)-
results2(j,2)*(norminv(GUcdf,0,1));
                %Equivalent  mean

                results2 (j,4) = (results2(j,1)-
results2(j,3))/results2(j,2);
                %Equivalent Standardized Mean

         else
             e=3;
         end

     elseif ((stat(j)==GA))
         %Gamma Distribution
         if(abs(data(j,3)-(2*data(j,2))/data(j,1))<0.1)
            %Checking for incorrect skewness
```

```
                c = ((sqrt(data(j,3)^2+4)+data(j,3))^(1/3)-
(sqrt(data(j,3)^2+4)-data(j,3))^(1/3))*2^(-1/3);
                %Coefficient c

                u = (results2(j,1)-data(j,1))/data(j,2);
                %Standardized variable u

                u1 =
(log(abs(u+1/c))+log(abs(c)*sqrt(1+c^2)))/((sign(data(j,3))*sqrt(log(1+c^2)
)));
                %Modified Standardized Variable u

                GApdf =
(normpdf(u1,0,1))/(data(j,2)*(abs(u+(1/c))*sqrt(log(1+c^2))));
                %Standardized Gamma PDF

                GAcdf = normcdf(u1,0,1);
                %Standardized Gamma CDF

                results2 (j,2) = (normpdf((norminv(GAcdf,0,1)),0,1))/GApdf;
                %Equivalent standard deviation

                results2 (j,3) = results2(j,1)-
results2(j,2)*(norminv(GAcdf,0,1));
                %Equivalent  mean

                results2 (j,4) = (results2(j,1)-
results2(j,3))/results2(j,2);
                %Equivalent Standardized Mean

        else
            e=4;
        end
    elseif(stat(j)==D)
        %Determinate variable
        if(and(data(j,2),data(j,3))==0)
            %Check if skewness and std Dev is zero
                results2(j,2)=0;

                results2(j,3)=data(j,1);

                results2(j,4)=data(j,1);
        else
             e=5;
        end


    else
    %If Distribution is not recognized
       e=0;
    end

 for k=1:num
 %Calculating partial derivative vector D
 results2(k,5) = subs(results(k,1)*results2(k,2));

 end
```

```
 end

%Calculating Reliability Index Beta
BT(i,1) = subs((-
transpose(results2(1:num,5))*results2(1:num,4))/(((transpose((results2(1:nu
m,5)))*(results2(1:num,5))))^0.5))

 for j=1:num
 %Calculating Sensitivity factors

results2(j,6)=subs(transpose(results2(j,5))/(((transpose((results2(1:num,5)
))*(results2(1:num,5))))^0.5));
 end
 for j=2:num
 %Calculating new design point
      results2(j,1)=results2(j,3)-results2(j,6)*BT(i,1)*results2(j,2);
 end

%Assigning values of new design point to symbols
X2 = results2(2,1);
X3 = results2(3,1);
X4 = results2(4,1);
X5 = results2(5,1);
X6 = results2(6,1);
X7 = results2(7,1);
X8 = results2(8,1);
X9 = results2(9,1);
X10 = results2(10,1);
X11 = results2(11,1);
X12 = results2(12,1);
X13 = results2(13,1);
X14 = results2(14,1);
X15 = results2(15,1);
X16 = results2(16,1);
X17 = results2(17,1);
X18 = results2(18,1);
X19 = results2(19,1);
X20 = results2(20,1);

results2 (1,1) = subs(solution);
%Ensuring design point remains on failure plane

X1 = results2(1,1);
%Assigning values of new design point to X1

    if (i>1)
        if (abs(BT(i-1,1)-BT(i,1))<0.001)
            B=BT(i,1);
            b1=1;
            i=20;
        end
    end
i=i+1;
 end
%error messages
if( e==0)
    pf = 'Error Statistical distribution not Recognized';

elseif(e==1)
    pf = 'Error Normal Distribution has a non-zero Skewness';
```

```
elseif(e==2)
    pf = 'Error 2 parameter Lognormal Distribution Skewness not equal to
3*sigma/mu';
elseif(e==3)
    pf = 'Error Gumbel Distribution Skewness not equal to 1.14';
elseif(e==4)
    pf = 'Error Gamma Distribution Skewness not equal to 2*sigma/mu';
elseif(e==5)
    pf='Error Determinate variable must have a Std. Deviation and Skewness
equal to zero';
elseif(e==6)
    pf='Error Three Parameter Lognormal Disrtibution must have non-zero
skewness';
elseif(b1==0)
    pf='Error Could Not Converge';
else

    pf=normcdf(-B,0,1);
end


end
```

APPENDIX D: Generic_Optimization

```
function  [Beta] =
Generic_Optimization(p0,p1,c1,c0,hm,SVSL,Nf,w,y,stat,data,G,lamla)
%
%This function performs a benefit/cost analysis and returns the optimum
%target reliability via a reliability index Beta.
%This function calls the form function witch returns the probability of
%failure.
%
%INPUT PARAMETERS
%
%p0      = starting value of safety factor p
%p1      = end value of safety factor p
%c1      = cost of safety measures
%c0      = cost of failed components
%hm      = Other losses
%SVSL    = Societal Value of a Statistical Life
%N       = Number of fatalities
%w       = Obsolescence rate
%y       = Discount Rate
%stat    = Vector of Statistical Distributions (Refer to form)
%data    = Matrix of Statistical Data(Refer to form)
%G       = Performance function(Refer to form)

%lamla   = Jump Rate

%Input Example
%
%syms X1 X2 G LN LN p;

%format longEng;
%p0=2;
%p1=5;
%c1=50;
%c0=8000;
%hm=12000;
%SWTP=2093057;
%N=0.01;
%w=0.02;
%y=0.045;
%SVSL=2628273;
%stat = [LN;LN];
%data = [(1*p),0.2*p,3*0.2*p/p;
%         1,0.2,0.6;
%         zeros(18,3)];
%G = X1-X2;

%lamla=1;
format longEng;

i=1;
j=1;
k=1;
l=0;
m=0;
%counters measuring number of iterarions
Beta=0;
```

```
results = zeros(40,3);
%define size of matrix containing results for efficientcy

dz1=(p1-p0)/10;
dz2=dz1/30;
%step size of seeking function
p=p0;
while (p<p1)&& Beta==0

    a=subs(data);
    pf=form(stat,a,G);
    %calculating probability of failure

    results(i,1)=p;
    results(i,2)=pf;
    results(i,3) = c1*p+(c1*p)*(w/y) + (c1*p + Nf*SVSL + c0+hm)*pf*lamla/y;
        if(Beta==0)
            if((i-j)==1)


                if(results(j,3)-results(i,3)<0)
                k=i+1;
                results(k,1)=results(j,1)-dz2;
                p=results(k,1);
                b=subs(data);
                pf=form(stat,b,G);
                results(k,2)=pf;
                results(k,3)= c1*p+(c1*p)*(w/y) + (c1*p + Nf*SVSL +
c0+hm)*pf*lamla/y;

                    if(results(k,3)-results(j,3)<0)
                        p=results(j,1);
                        l=k+1;
                        m=k+1;
                        while p>results(j-1,1)&& Beta==0
                            c=subs(data);
                            pf=form(stat,c,G);
                            results(l,1)=p;
                            results(l,2)=pf;
                            results(l,3)=c1*p+(c1*p)*(w/y) + (c1*p +
Nf*SVSL + c0+hm)*pf*lamla/y;
                              if(Beta==0)
                                  if((l-m)==1)
                                      if(results(l,3)-results(m,3)>0)
                                          Zmaxpf= results(m,2);
                                          Beta=-norminv(Zmaxpf,0,1);

                                      end
                                      m=m+1;
                                  end
                              end
                             l=l+1;
                             p=p-dz2;

                        end
                    else
                        p=results(j,1);
                        l=k+1;
                        m=k+1;
```

```
                            while p<results(i,1)&& Beta==0
                               c=subs(data);
                               pf=form(stat,c,G);
                               results(l,1)=p;
                               results(l,2)=pf;
                               results(l,3)=c1*p+(c1*p)*(w/y) + (c1*p +
Nf*SVSL + c0+hm)*pf*lamla/y;
                                 if(Beta==0)
                                     if((l-m)==1)
                                         if(results(l,3)-results(m,3)>0)
                                             Zmaxpf= results(m,2);
                                             Beta=-norminv(Zmaxpf,0,1);

                                         end
                                         m=m+1;
                                     end
                                 end
                                l=l+1;
                                p=p+dz2;

                           end

                       end
                   end

               j=j+1;
               end
           end
       i=i+1;
       p=p+dz1;
end


%A while loop finding the optimum results, if the optimum result is out of
%the range selected by p, error message will provide user with following
%statement.
if(Beta==0)
    Beta = 'Increase range, optimum is out of bounds';
end
results
end
```

APPENDIX E: StartArea

```
function [ a ] = StartArea( data,y,i)
%Returns starting Area of Reinforcement so that limit state function is
%zero when mean values are substituted in
%Input Variables
%data=3 x 20 matrix containing random and deterministic variables
%y=Limit State Function
%i=row number of variable that represents the area of reinforcement
format longEng;
syms  X1 X2 X3 X4 X5 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X15 X16 X17 X18 X19
X20;
if(i==1)
solution = solve(y,X1);
end

if(i==2)
solution = solve(y,X2);
end

if(i==3)
solution = solve(y,X3);
end

if(i==4)
solution = solve(y,X4);
end

if(i==5)
solution = solve(y,X5);
end

if(i==6)
solution = solve(y,X6);
end

if(i==7)
solution = solve(y,X7);
end

if(i==8)
solution = solve(y,X8);
end

if(i==9)
solution = solve(y,X9);
end

if(i==10)
solution = solve(y,X10);
end

if(i==11)
solution = solve(y,X11);
end
```

```
if(i==12)
solution = solve(y,X12);
end

if(i==13)
solution = solve(y,X13);
end

if(i==14)
solution = solve(y,X14);
end

if(i==15)
solution = solve(y,X15);
end

if(i==16)
solution = solve(y,X16);
end

if(i==17)
solution = solve(y,X17);
end

if(i==18)
solution = solve(y,X18);
end

if(i==19)
solution = solve(y,X19);
end

if(i==20)
solution = solve(y,X20);
end

X1 = data(1,1);
X2 = data(2,1);
X3 = data(3,1);
X4 = data(4,1);
X5 = data(5,1);
X6 = data(6,1);
X7 = data(7,1);
X8 = data(8,1);
X9 = data(9,1);
X10 = data(10,1);
X11 = data(11,1);
X12 = data(12,1);
X13 = data(13,1);
X14 = data(14,1);
X15 = data(15,1);
X16 = data(16,1);
X17 = data(17,1);
X18 = data(18,1);
X19 = data(19,1);
X20 = data(20,1);

a= subs(solution);
end
```

APPENDIX F: Generic_Min

```
function [BetaMin] = Generic_Min( p0,p1,c1,SWTP,N,w,y,stat,data,G,lamla )
%
%This function performs a benefit/cost analysis and returns the optimum
%target reliability via a reliability index Beta.
%This function calls the form function witch returns the probability of
%failure.
%
%INPUT PARAMETERS
%
%p0       = starting value of safety factor p
%p1       = end value of safety factor p
%c1       = cost of safety measures
%SWTP     = Society's Willingnes to Pay
%N        = Number of Fatalities
%y        = Discount Rate
%stat     = Vector of Statistical Distributions (Refer to form)
%data     = Matrix of Statistical Data(Refer to form)
%G        = Performance function(Refer to form)

%lamla    = Jump Rate

%Input Example
%
%syms X1 X2 G LN LN p;

%format longEng;

%p0=2;
%p1=5;
%c1=50;
%SWTP=2093057;
%N=0.01;
%w=0.02;
%y=0.022;

%stat = [LN;LN];
%data = [(1*p),0.2*p,3*0.2*p/p;
%         1,0.2,0.6;
%         zeros(18,3)];
%G = X1-X2;

%lamla=1;
format longEng;

dz1=(p1-p0)/10;
dz2=dz1/30;
BetaMin=0;
results=zeros(70,4);

j=1;
i=1;
k=1;
l=1;
p=p0;
```

```
while p<p1 && BetaMin==0
    results(i,1)=p;
    a=subs(data);
    results(i,2)=form(stat,a,G);
    results(i,3)=SWTP*N*results(i,2)*lamla/y;

    if(i-j==1)

        results(i,4)=(results(j,3)-results(i,3))/dz1;

        if(results(i,4)<(c1*(1+y/w)))

           p=results(j-1,1);
           k=i+1;
           l=i+1;
           while p<results(i,1)&&BetaMin==0
               results(k,1)=p;
               c=subs(data);
               results(k,2)=form(stat,c,G);
               results(k,3)=SWTP*N*results(k,2)*lamla/y;

               if(k-l==1)
                   results(k,4)=(results(l,3)-results(k,3))/dz2;

                   if(results(k,4)< (c1*(1+y/w)))
                      results(k+1,1)=results(k,1)-(dz2/2);
                      p=results(k+1,1);
                      d=subs(data);
                      results(k+1,2)=form(stat,d,G);
                      pfMin=results(k+1,2);
                      BetaMin=-norminv(pfMin,0,1);

                   end
                   l=l+1;
               end
               k=k+1;
               p=p+dz2;
           end
        end

        j=j+1;
    end
    i=i+1;
    p=p+dz1;
end

if(BetaMin==0)
    BetaMin = 'Increase range, optimum is out of bounds';
end


end
```

APPENDIX G: Generic_OptimizationC

```
function  [Beta] =
Generic_OptimizationC(p0,p1,c1,SVSL,Nf,w,y,data,lamla,Hm,C0)
%
%This function performs a benefit/cost analysis and returns the optimum
%target reliability via a reliability index Beta.
%This function calls the MonteCarloC function witch returns the number of
%failures.
%
%INPUT PARAMETERS
%
%p0      = starting value of safety factor p
%p1      = end value of safety factor p
%c1      = cost of safety measures
%SVSL    = Societal Value of a Statistical Life
%N       = Number of fatalities
%w       = Obsolescence rate
%y       = Discount Rate
%stat    = Vector of Statistical Distributions (Refer to form)
%data    = Matrix of Statistical Data(Refer to form)
%Hm      = Other failure costs
%C0      = Construction cost of structure

%lamla   = Jump Rate

%Input Example
%
%syms  p;

%format longEng;
%p0=2;
%p1=5;
%c1=50;
%SWTP=2093057;
%N=0.01;
%w=0.02;
%y=0.045;
%SVSL=2628273;

%data = [(1*p),0.2*p,3*0.2*p/p;
%        1,0.2,0.6;
%        zeros(18,3)];

%lamla=1;
format longEng;

i=1;
j=1;
k=1;
l=0;
m=0;
%counters measuring number of iterarions
Beta=0;
nf=0;
results = zeros(25,3);
%define size of matrix containing results for efficientcy
```

```
dz1=(p1-p0)/10;
dz2=dz1/3;
%step size of seeking function
p=p0;
while (p<p1)&& Beta==0

    a=subs(data);
    nf=0;
    for u=1:1:100
         count=MonteCarloC(a);
         nf=nf+count;
         u
    end

    pf=nf/(10^9)
    %calculating probability of failure

    results(i,1)=p;
    results(i,2)=pf;
    results(i,3) = c1*p+(c1*p)*(w/y) + (c1*p + Nf*SVSL+ Hm +C0)*pf*lamla/y
        if(Beta==0)
            if((i-j)==1)


                if(results(j,3)-results(i,3)<0)
                k=i+1;
                results(k,1)=results(j,1)-dz2;
                p=results(k,1);
                b=subs(data);
                nf=0;
                for u=1:1:100
                count=MonteCarloC(b);
                nf=nf+count;
                u
                end

                pf=nf/(10^9)
                results(k,2)=pf;
                results(k,3)= c1*p+(c1*p)*(w/y) + (c1*p + Nf*SVSL + Hm
+C0)*pf*lamla/y

                    if(results(k,3)-results(j,3)<0)
                        p=results(j,1);
                        l=k+1;
                        m=k+1;
                        while p>results(j-1,1)&& Beta==0
                            c=subs(data);
                            nf=0;
                            for u=1:1:100
                            count=MonteCarloC(c);
                            nf=nf+count;
                            u
                            end

                            pf=nf/(10^9)
                            results(l,1)=p;
                            results(l,2)=pf;
```

```
                                        results(l,3)=c1*p+(c1*p)*(w/y) + (c1*p +
Nf*SVSL+Hm+C0)*pf*lamla/y
                                          if(Beta==0)
                                              if((l-m)==1)
                                                  if(results(l,3)-results(m,3)>0)
                                                      Zmaxpf= results(m,2);
                                                      Beta=-norminv(Zmaxpf,0,1);

                                                  end
                                                  m=m+1;
                                              end
                                          end
                                         l=l+1;
                                         p=p-dz2;

                                    end
                                else
                                    p=results(j,1);
                                    l=k+1;
                                    m=k+1;

                                     while p<results(i,1)&& Beta==0
                                        c=subs(data);
                                        nf=0;
                                         for u=1:1:100
                                        count=MonteCarloC(c);
                                        nf=nf+count;
                                        u
                                         end

                                        pf=nf/(10^9)
                                        results(l,1)=p;
                                        results(l,2)=pf;
                                        results(l,3)=c1*p+(c1*p)*(w/y) + (c1*p +
Nf*SVSL+ Hm +C0)*pf*lamla/y
                                          if(Beta==0)
                                              if((l-m)==1)
                                                  if(results(l,3)-results(m,3)>0)
                                                      Zmaxpf= results(m,2);
                                                      Beta=-norminv(Zmaxpf,0,1);

                                                  end
                                                  m=m+1;
                                              end
                                          end
                                         l=l+1;
                                         p=p+dz2;

                                    end

                                end
                            end

                        j=j+1;
                        end
                    end
    i=i+1;
    p=p+dz1;
end
```

```
%A while loop finding the optimum results, if the optimum result is out of
%the range selected by p, error message will provide user with following
%statement.
if(Beta==0)
    Beta = 'Increase range, optimum is out of bounds';
end
results
end
```

APPENDIX H: MonteCarloC

```
function [nf] = MonteCarloC( data )
%Calculates the probability of failure of a column using montecarlo
%simulation.
%Input is a matrix data containing mean, std. dev. and skewness
%of random parameters

n=10^7;
pf=0;

results=zeros(21,2);

%Obtaining location and shape parameters from statistical variables

%Variable 1 is lognormal
results(1,2)=sqrt(log(((data(1,2))^2)/((data(1,1))^2)+1));
results(1,1)=log(data(1,1))-results(1,2)^2/2;

%Variable 2 is determistic
results(2,1)=data(2,1);

%Variable 3 is normal
results(3,1)=data(3,1);
results(3,2)=data(3,2);

%Variable 4 is gamma
results(4,1)=(2/data(4,3))^2;
results(4,2)=data(4,1)/results(4,1);

%Variable 5 is lognormal
results(5,2)=sqrt(log(((data(5,2))^2)/((data(5,1))^2)+1));
results(5,1)=log(data(5,1))-results(5,2)^2/2;

%Variable 6 is gamma
results(6,1)=(2/data(6,3))^2;
results(6,2)=data(6,1)/results(6,1);

%Variable 7 is gumbel
results(7,2)=data(7,2)*sqrt(6)/pi;
results(7,1)=data(7,1)-0.5772*results(7,2);

%Variable 8 is normal
results(8,1)=data(8,1);
results(8,2)=data(8,2);

%Variable 9 is determistic
results(9,1)=data(9,1);

%Variable 10 is lognormal
results(10,2)=sqrt(log(((data(10,2))^2)/((data(10,1))^2)+1));
results(10,1)=log(data(10,1))-results(10,2)^2/2;

%Variables 11-14 are Normal
results(11,1)=data(11,1);
```

```
results(11,2)=data(11,2);

results(12,1)=data(12,1);
results(12,2)=data(12,2);

results(13,1)=data(13,1);
results(13,2)=data(13,2);

results(14,1)=data(14,1);
results(14,2)=data(14,2);


%Variables 15-16 are Normal
results(15,1)=data(15,1);
results(15,2)=data(15,2);

results(16,1)=data(16,1);
results(16,2)=data(16,2);


%Variables 17-18 are lognormal

results(17,2)=sqrt(log(((data(17,2))^2)/((data(17,1))^2)+1));
results(17,1)=log(data(17,1))-results(17,2)^2/2;

results(18,2)=sqrt(log(((data(18,2))^2)/((data(18,1))^2)+1));
results(18,1)=log(data(18,1))-results(18,2)^2/2;

%Neutral axis depth and Compression/Tension Steel Stress are determined in
%MRC
[r1,r2]=resistance2(lognrnd(results(1,1),results(1,2),n,1),...
results(2,1),...
lognrnd(results(10,1),results(10,2),n,1),...
normrnd(results(13,1),results(13,2),n,1));
R1=transpose(r1);
R2=transpose(r2);

e1=Loadaxial(normrnd(results(3,1),results(3,2),1,n),...
   lognrnd(results(5,1),results(5,2),1,n),...
   gamrnd(results(6,1),results(6,2),1,n),...
   -evrnd(-results(7,1),results(7,2),1,n),...
   normrnd(results(8,1),results(8,2),1,n),...
   results(9,1),...
   normrnd(results(11,1),results(11,2),1,n),...
   normrnd(results(12,1),results(12,2),1,n),...
   normrnd(results(13,1),results(13,2),1,n),...
   normrnd(results(14,1),results(14,2),1,n));

MR=MRC(R1,R2,e1,n,lognrnd(results(1,1),results(1,2),1,n),...
    gamrnd(results(4,1),results(4,2),1,n),...
    normrnd(results(14,1),results(14,2),1,n),...
    lognrnd(results(17,1),results(17,2),1,n));

MU=Loadmoment (e1,lognrnd(results(5,1),results(5,2),1,n),...
    normrnd(results(13,1),results(13,2),1,n),...
normrnd(results(14,1),results(14,2),1,n),...
   normrnd(results(15,1),results(15,2),1,n),...
  normrnd(results(16,1),results(16,2),1,n),...
```

```
  lognrnd(results(18,1),results(18,2),1,n));

G=MR-MU;


nf = sum(G < 0);





%lognrnd(results(1,1),results(1,2),1,n),...
%results(2,1),...
%normrnd(results(3,1),results(3,2),1,n),...
%gamrnd(results(4,1),results(4,2),1,n),...
%lognrnd(results(5,1),results(5,2),1,n),...
%gamrnd(results(6,1),results(6,2),1,n),...
%-evrnd(-results(7,1),results(7,2),1,n),...
%normrnd(results(8,1),results(8,2),1,n),...
%results(9,1),...
%lognrnd(results(10,1),results(10,2),1,n),...
%normrnd(results(11,1),results(11,2),1,n),...
%normrnd(results(12,1),results(12,2),1,n),...
%normrnd(results(13,1),results(13,2),1,n),..
%normrnd(results(14,1),results(14,2),1,n),...
%normrnd(results(15,1),results(15,2),1,n),...
%normrnd(results(16,1),results(16,2),1,n),...
%normrnd(results(17,1),results(17,2),1,n),...
%lognrnd(results(18,1),results(18,2),1,n),...
%lognrnd(results(19,1),results(19,2),1,n));
end
```

APPENDIX I: resistance2

```
function [ R1,R2] = resistance2( X1,X2,X10,X13)
%This function calculates the resistance of some of the parameters of a
%column in axial compression excluding steel stresses and neutral axis
%depth

R1=X1.*0.67.*X10.*X13;
R2=X1.*X2/2;


end
```

APPENDIX J: Loadaxial

```
function [ e1 ] = Loadaxial( X3,X5,X6,X7,X8,X9,X11,X12,X13,X14 )
%This function calculates the axial load

e1= X5.*(5.175.*(X12.*X8.*4+(X6+X7).*3))*X9.*1.143...
    + X5.*(X11.*X3.*X8.*4).*X9.*1.143...
    +X5.*X13.*X14.*14;
end
```

APPENDIX K: Loadmoment

```
function [ Mu ] = Loadmoment( e1,X5,X13,X14,X15,X16,X18)
%This function returns the moment acting on a column due to an eccentriciy
%of the axial load. Also converts bi-axial bending to uni-axial
%bending state
e2=(e1./X5).*X18;
Mu=X16.*e2+0.7.*(X14./X13).*X15.*e2;

end
```

APPENDIX L: MRC

```
function [MR] = MRC( R1,R2,e1,n,X1,X4,X14,X17)
%This function calculates the depth of the neutral axis, stress state of
%the compression/tension reinforcement and the moment resistance of a
%column.
MR=zeros(1,n);


for i=1:n
    z=1;
    N=1;
    while (N>0&&z>=0)
          x=z*X14(1,i);
             N=1;




             es1=-0.0035*(X14(1,i)-X4(1,i)-x)/x;

             es2=-0.0035*(X4(1,i)-x)/x;

             if(es1<=(-510*10^6/(200*10^9)))
                 fy=-510*10^6;

             elseif(es1>(-510*10^6/(200*10^9))&& es1<(430*10^6/(200*10^9)))
                 fy=es1*200*10^9;
             elseif(es1>=(430*10^6/(200*10^9)))
                  fyc=430*10^6;
             end

             if(es2>=(430*10^6/(200*10^9)))
                 fyc=430*10^6;
             elseif(es2<(430*10^6/200*10^9)&& es2>(-510*10^6/(200*10^9)))
                 fyc=es2*200*10^9;
             elseif(es2>(-510*10^6/(200*10^9))&& es1<(430*10^6/(200*10^9)))
                 fy=es2*200*10^9;
             end

            if(x<X14(1,i))
                N=R1(1,i)*x+fyc*R2(1,i)+R2(1,i)*fy-e1(1,i);
            else
                N=R1(1,i)*X14(1,i)  + fyc*R2(1,i)+R2(1,i)*fy-e1(1,i);
            end


        z=z-0.01;
    end


    if(x<X14(1,i))
    MR(1,i)=(R1(1,i)*0.9*x*(X14(1,i)-x)/2 +fyc*R2(1,i)*(X14(1,i)/2-
X4(1,i))-R2(1,i)*fy*(X14(1,i)/2-X4(1,i)))*X17(1,i)/X1(1,i);
    elseif(x<X4(1,i))
```

```
    MR(1,i)=(R1(1,i)*0.9*x*(X14(1,i)-x)/2 -fyc*R2(1,i)*(X14(1,i)/2-
X4(1,i))-R2(1,i)*fy*(X14(1,i)/2-X4(1,i)))*X17(1,i)/X1(1,i);

    elseif(x>=X14(1,i))
        MR(1,i)=(fyc*R2(1,i)*(X14(1,i)/2-X4(1,i))-R2(1,i)*fy*(X14(1,i)/2-
X4(1,i)))*X17(1,i)/X1(1,i);
    end
end

end
```

APPENDIX M: MonteCarlo1

```
function [pf] = MonteCarlo1(data)
%This function calculates the probability of failure of example
%   Detailed explanation goes here

a1=0;
a2=0;
a3=0;
a4=0;
a5=0;
a6=0;
a7=0;
a8=0;
a9=0;



n=10^7;
pf=0;

%Obtaining location and shape parameters from statistical variables

%Variable 1 is 2-parameter lognormal
a2=sqrt(log(((data(1,2))^2)/((data(1,1))^2)+1));
a1=log(data(1,1))-a2^2/2;
%Variable 2 is Normal
a3=data(2,1);
a4=data(2,2);

%Variable 3 is Gamma
a5=(2/data(3,3))^2;
a6=data(3,1)/a5;

%Variable 4 is deterministic
a7=data(4,1);



%Variable 5 is Gumbel
a9=data(5,2)*sqrt(6)/pi;
a8=data(5,1)-0.5772*a9;

R=resistance1(lognrnd(a1,a2,1,n),normrnd(a3,a4,1,n),gamrnd(a5,a6,1,n),a7);
E=load1(-evrnd(-a8,a9,1,n));
G=R-E;

count = sum(G < 0);

pf=count/n;


end
```

APPENDIX N: resistance1

```
function [ r ] = resistance1( X1,X2,X3,X4 )
%Calculates the resistance
%
r=X1.*0.9.*(X2-X3).*X4;
end
```

APPENDIX O: load1

```
function [e] = load1(X5)
%UNTITLED5 Summary of this function goes here
%   Detailed explanation goes here
e=0;
e=X5;
end
```