INVESTIGATING TEACHING AND LEARNING OF GRADE 9 ALGEBRA THROUGH *EXCEL* SPREADSHEETS: A MIXED-METHODS CASE STUDY FOR LESOTHO

‘M’AMOSA M.E. NTŠOHI

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DECLARATION

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously, in its entirety or in part, submitted it at any university for a degree.

Signature………… ………………

Date………………………………….

DEDICATIONS

This thesis is dedicated to my husband, Tau, who has been with me throughout this study, from its inception through the turbulences I experienced, always encouraging me to sail on to the end. He shouldered all family responsibilities without hesitation, with unflagging support and took great care of the children and me.

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ABSTRACT

The teaching and learning of algebra in the middle school grades in Lesotho is dominated by the mechanistic approach where learners are drilled on procedures for solving certain types of problems in algebra, without making any connection to the experience learners had with arithmetic. This is one of the sources of learners' difficulties in mathematics. Research indicates that use of spreadsheets such as *Excel* has a potential of bridging the gap between arithmetic and algebra and thus enhancing the teaching and learning of algebra, making it meaningful to the learners.

The study sought to answer the question: *How do Grade 9 learners in Lesotho experience teaching and learning of algebra through Excel spreadsheets?* The research commenced with a literature review that was followed by the empirical study. The theories of *instrumental genesis* and *instrumental orchestration* were identified as the framework for the investigation. *Instrumental genesis* is the process in which learners develop facility with the artifact as they use it towards achieving lesson objectives; technical (conceptual, mechanical) and personal (attitudes, behavior and preferred learning styles) aspects of learners' experiences were identified. *Instrumental orchestration* is the steering of learners' instrumental genesis by the teacher and the manner in which this process is carried out, depends on the teacher's Technological Pedagogical Content Knowledge (TPCK).

The research was a multi-case study following a mixed-methods approach, where both qualitative and quantitative methods were used. The empirical study was conducted in two schools in Lesotho. In each school, fifteen learners volunteered participation. The investigation was done through classroom teaching by me as the researcher. The focus was on what challenges learners encountered and how they benefited from their "spreadsheets algebra" learning experience. Data were collected through classroom observations where field notes were recorded and an observation schedule was used by the researcher and the Assistant Observer respectively. A questionnaire was also administered to all learner participants after the whole teaching period. Six learners, representative of high, medium and low performances in class, were also interviewed with a goal of finding out their experiences. The Assistant Observer was

also interviewed to reduce the bias that may result from to the researcher studying her own practice

The study found that learners experiences with learning algebra through spreadsheets, comprised of both challenges and benefits. The challenges encountered by learners could be classified into those that were school-based and those that were instruction-based. The school-based challenges related to inadequate physical structures and lack of well-functioning equipment in the computer laboratories. Instruction-based challenges encountered by learners were both technical and personal. Technical challenges related to the physical manipulation of the artifact and the lack of understanding of concepts involved, where the spreadsheets meet the algebra. Personal challenges related to learners' attitude and behavior towards use of spreadsheets for algebra teaching and learning. While school authorities could address some of the challenges, it was found that both the teacher and learners could initiate strategies that could be used to overcome the instruction-based challenges. Teaching strategies such as "*technical-demo"*, "*explain-the-screen*", "*discuss-the–screen*", "*link-screen-board*" and *"spot-and-show*" (building on learners' responses), and the general organization of classroom environment were helpful in orchestrating algebra learning within the spreadsheets. It was also found that use of spreadsheets had both cognitive and affective values for the learners.

Even though use of spreadsheets may benefit both teachers and learners in algebra teaching and learning, implementation of the practice would require critical considerations in terms of teacher preparation and infra-structural improvements in the schools.

OPSOMMING

Die onderrig en leer van algebra in die middelbare skoolgrade in Lesotho word oorheers deur die meganiese benadering waarvolgens leerders gedril word in prosedures om oplossings vir sekere tipe algebraprobleme te vind en die ervaring wat leerders in rekenkunde opgedoen het, nie daarmee in verband gebring word nie. Dit is een van die oorsake waarom leerders met wiskunde sukkel. Navorsing toon dat die gebruik van sigblaaie soos *Excel* moontlik die gaping tussen rekenkunde en algebra kan oorbrug, en dat die onderrig en leer van algebra daardeur kan verbeter, wat dit sinvol vir leerders sal maak.

Die studie was daarop gemik om ’n antwoord op die volgende vraag te vind: *Hoe ervaar graad 9-leerders in Lesotho die onderrig en leer van algebra deur middel van Excel-sigblaaie?* Die navorsing het met ’n literatuuroorsig begin en is deur ’n empiriese studie opgevolg. Die teorieë *instrumentele genese* en *instrumentele orkestrasie* is uitgewys as die raamwerk vir die ondersoek. *Instrumentele genese* is die proses waarvolgens leerders bedrewenheid in die produk ontwikkel namate hulle dit gebruik om lesdoelstellings te bereik; tegniese (konseptuele, meganiese) en persoonlike (ingesteldheid, gedrag en voorkeurleerstyle) aspekte van leerders se ervarings is geïdentifiseer. *Instrumentele orkestrasie* is die stuur van leerders se instrumentele genese deur die onderwyser; en die wyse waarop hierdie proses uitgevoer word, hang van die onderwyser se Tegnologiese Pedagogiese Inhoudskennis (TPCK) af.

Die navorsing het ’n meervoudige gevallestudie gebruik en ’n gemengde metodebenadering is gevolg, terwyl beide kwalitatiewe en kwantitatiewe metodes gebruik is. Die empiriese studie is in twee skole in Lesotho uitgevoer. Vyftien leerlinge uit elke skool het vrywillig deelgeneem. Die ondersoek is by wyse van klaskameronderrig deur my as die navorser gedoen. Daar is gefokus op die uitdagings wat leerders teëgekom het en hoe hulle by die “sigbladalgebra”-leerervaring gebaat het. Data is aan die hand van klaskamerwaarnemings versamel, waar veldnotas afgeneem is en ’n waarnemingskedule onderskeidelik deur die navorser en die assistentwaarnemer gebruik is. ’n Vraelys is na die volle onderrigtydperk by al die leerderdeelnemers afgeneem. Onderhoude is gevoer met ses leerders, wat hoë, medium en lae prestasies in die klaskamer verteenwoordig, met die doel om hulle ervarings te bekom. ’n

Onderhoud is ook met die Assistentwaarnemer gevoer om vooroordeel, deurdat die navorser moontlik haar eie praktyk kon bestudeer, te verminder.

Die studie het bevind dat leerders se ervarings met die leer van algebra deur middel van sigblaaie uitdagings sowel as voordele inhou. Die uitdagings wat leerders teëgekom het, kan onderskeidelik as skoolgebaseerde uitdagings en onderrig-gebaseerde uitdagings geklassifiseer word. Die skoolgebaseerde uitdagings hou verband met onvoldoende fisiese strukture en 'n gebrek aan behoorlik funksionerende toerusting in die rekenaarlaboratoriums. Die onderrig-gebaseerde uitdagings vir leerders is beide tegnies en persoonlik. Tegniese uitdagings hou verband met die fisiese manipulering van die produk en 'n gebrek aan begrip ten opsigte van die betrokke konsepte, naamlik waar die verband tussen die sigblaaie en algebra bestaan. Persoonlike uitdagings hou verband met leerders se ingesteldheid en gedrag rakende die gebruik van sigblaaie by die onderrig en leer van algebra. Alhoewel die skoolowerhede bepaalde uitdagings kan aanspreek, is daar bevind dat onderwysers sowel as leerders strategieë kan aanvoor om onderrig-gebaseerde uitdagings te oorkom. Onderrigstrategieë soos "tegniese demo", "verduidelik die skerm", "bespreek die skerm", "koppel-skerm-bord" en "vind-en-wys" (wat op leerders se reaksie voortbou), asook die algemene organisasie van die klaskameromgewing, dra tot die orkestrasie van algebraonderrig met die sigblaaie by. Daar is ook bevind dat die gebruik van sigblaaie kognitiewe sowel as affektiewe waarde vir die leerders inhou.

Selfs al sou onderwysers en leerders voordeel uit die gebruik van sigblaaie in die onderrig en leer van algebra kon trek, sou die implementering van die praktyk kritiese oorwegings moet geniet ten opsigte van onderwyservoorbereiding en infrastruktuurverbetering by die skole.

LIST OF ABBREVIATIONS

AO-Assistant Observer

CAS-Computer Algebra Systems

ECOL-Examination Council of Lesotho

IAS-Instrumented Action Schemes

ICT-Information and Communication Technologies

LJC-Lesotho Junior Certificate

MOET-Ministry of Education and Training

NCDC-National Curriculum Development Centre

NCTM-National Council for Teachers of Mathematics

NRC-National Research Council

RME-Realistic Mathematics Education

TPCK-Technological Pedagogical Content Knowledge

CONTENTS

CHAPTER 1

INTRODUCTION & ORIENTATION OF THE STUDY

1.1 Introduction

This chapter provides introduction and orientation of the study. It starts by giving background and the context of the inquiry. It also provides a justification and motivation to the study. The Statement of the Problem and the research question is defined, as well as the corresponding sub-questions. Then a brief outline of the design and methodology which are followed in the study are given. The Scope and Limitations of the Study are also discussed. The importance of this study to the field of Mathematics Education is highlighted in the chapter.

1.2 Background and Context of Inquiry

Mathematics is one of the subjects that are given high status in national curricula. This is in realization of its usefulness to humankind. Mathematics is used in homes, sports and more importantly in industry, as well as in business for solving practical problems in various situations. It provides a powerful and precise means of communication to learners, academics, developers and designers amongst others. The UK Curriculum document, Mathematics for Ages 5 to16, cited by (Pimm, 1995, p. 10) offers the following explanation about Mathematics:

> In the broadest sense, mathematics provides a means for organizing, communicating and manipulating information. The language of mathematics consists of diagrams and symbols, with associated conventions and theorems.

One of the goals of mathematics teaching is to enhance acquisition and application of mathematical knowledge and skills to solve problems and understand our environment. In problem solving, competency in algebra is a useful requirement for generalization, representation, analysis and identification of effective solution procedures.

Algebra is that branch of school mathematics that deals with symbolizing and generalizing relationships and mathematical structures, and operating within those structures (Kieran, 1992, p.

391). Due to its nature and role, algebra is seen as abstract arithmetic (Kieran, 1988, p. 91; Usiskin, 1988, p. 11) a language of mathematics, (Lee, 1996, p. 87)or a tool for studying functional relationships and mathematical modeling (Usiskin, 1988, p. 13; French, 2002, p. 3). Teaching of algebra in schools is geared towards helping learners develop these three different understandings of algebra. Learners should be helped to appreciate a need for symbolism in generalizing mathematical ideas and in expressing functional relationships between variables in their everyday life contexts. They should be able to make different representations of the same situation for better comprehension of the phenomenon under study. Research reports, (MacGregor, 1990; Kieran, 1992; English & Halford, 1995; Linchevski & Herscovics, 1996; French, 2002) however, indicate that algebra is one branch of mathematics that poses great difficulties for most learners, despite its importance within mathematics, across the school curricula and beyond. Miles (1999, P. 46) also indicated that “there is a possibility that some students who have learning problems in Mathematics have difficulty in Algebra, not because their difficulty is mathematics-based but because it is language–based, thus delaying their assimilation and comprehension of mathematics instruction”. Some of the learners’ difficulties emanate from the nature of the subject while others result from the way algebra is introduced and taught in schools.

In Lesotho, traditionally learners are introduced to algebra through the “mechanistic approach” (Wubbels, Korthagen, & Broekman, 1997, pp. 1-2). In this approach, algebra is introduced by way of defining letters as representing unknown value(s) or as representing any number, in the case of expressions. They are then introduced to the rules that govern operations on these objects. They engage in substitution, simplifying, factorizing, expanding brackets and ultimately solving equations (MOET, 2002). This approach is evident in many textbooks used in schools, not only in Lesotho, and it seems to provide learners with only limited understanding of algebra. Arcavi notes “even those students who manage to handle the algebraic techniques successfully, often fail to see algebra as a tool for understanding, expressing and communicating generalization, for revealing structure, and for establishing connections and formulating mathematical arguments” (Arcavi, 1994, p. 24). Reports from the Examinations Council of Lesotho (ECOL) that include 1992-2012 indicate that learners’ difficulties in mathematics examinations are mostly in the domain of algebra. Despite many workshops directed at making teachers aware of learners' difficulties in mathematics, problems pertaining to learners' facility with algebra still exist.

The mechanistic approach to algebra does not offer learners opportunity to make sense of the domain. There is poor link established between the arithmetic that is learned in their primary mathematics and the mathematics they study at Grade 9. Learners are not well introduced to generalizing, which is one of the central activities in algebra (Kieran, 2004b, pp. 22-23), and mathematics as a whole (Mason, 1996, p. 65). In fact, Mason (1996, P. 65) notes "generalization is the heartbeat of mathematics,…If teachers are not aware of its presence and are not in the habit of getting students to work at expressing their own generalizations, then mathematical thinking is not taking place". *Excel* spreadsheets, though initially developed to solve problems in accounting, have been identified as a computer program that can be useful in introducing learners to algebra (Tabach, Hershkowitz, & Arcavi, 2008, p. 50; Tabach & Friedlander, 2004, p. 428; Berdnarz, Kieran, & Lee, 1996; Heugl, Barzel, & Furukawa, 1997; Yerushalmy, 2005).

One of the aims of mathematics education is to produce citizens who are able to live meaningfully in the rapidly growing technological world. It is imperative, therefore, that the teaching and learning of mathematics should involve use of available technologies, to enhance acquisition of concepts and application of the learned concepts to solve contextual mathematical problems that learners are likely to encounter in their real life experiences. The Government of Lesotho, through the Ministry of Education and Training (MOET), is currently engaged in projects that are targeted at equipping teachers, both in primary and high schools, with skills that would enable them to integrate use of technology in their teaching. The establishment of the School Technology Innovation Centre (STIC) at the Lesotho College of Education was the MOET initiative, in partnership with public, private and civil society, to "support and help determine the Information and Communication Technologies (ICT) in education model for national deployment" (MOET, 2002, p. 2). Through this initiative, several public schools in the country have been provided with computer laboratories, while in some cases, private school proprietors found ways of securing such facilities for their schools. With promises that use of technologies such as computers (particularly *Excel* spreadsheets) for the teaching and learning of mathematics, can improve learner performance, I found it compelling to investigate what Grade

9 learners' experiences would be like, when they were taught algebra through the spreadsheets. Pierce, Stacey, and Barkatsas(2007, p. 286) urge

> With substantial investment in providing information technology to assist in teaching and learning mathematics, it is important to monitor students' reactions and decide how best to use both forms of technology, the mathematics analysis tools and the real world interfaces (Pierce, Stacey, & Barkatsas, 2007, p. 286).

The study sought to identify possible challenges that may be encountered and the strategies that teachers and learners may use to overcome the challenges. This was also driven by the observation that studies related to algebra teaching in spreadsheets environment have been conducted in contexts different from that of Lesotho.

Context of study

The study was conducted in two high schools, Retha and Palo (pseudo-names) in Lesotho**.** Lesotho is approximately 30,000 km^2 and a population of about 1.8 million. Maseru is the capital city and the administration centre of the country. It has the largest number and highest density of the population, with the largest number of schools and institutions of higher learning. According to the Human Development Report (2011), Lesotho is among the "Low Human Development" countries with a Human Development Index value at 0.450, positioning it at 160 out of the 187 countries. Approximately 43.4 % of the population is living below poverty line (below $1.25 per person per day); while 11.1% live in severe poverty. The Gross National Income per capita in 2005 is $1 664. (Human Development Report, 2011, pp. 1-5).

Access to technology in the form of telephones and cell phones and computers is still limited in Lesotho. There are only two telecommunication services providers and these have reached only 39% market penetration, which is below average for Africa. In June 2010, Lesotho recorded only 4% of its population as internet users. (Retrieved on 18 August 2012 from www.Mbendi.com/indy/cotl/af/le/p0005.htm). Not many households in Lesotho have computers, this means only a few learners have the opportunity to interact with this form of ICT on a regular basis.

The process of integrating technology in the curriculum is still new in Lesotho, only few schools have computers. Only forty-four (44) of the 215 (20.5%) registered secondary and high schools offer Computer Studies as an examinable subject at Junior Certificate (JC) level. In some of the schools that have acquired computers, teachers are still to be trained on the use of the technology for teaching. This study will therefore lay foundation for effective use of ICT in general and of spreadsheets in particular in algebra teaching and learning in Lesotho schools. Details of the schools used in the study are provided in Chapter 3.

The two schools chosen for the study are located within the Maseru city centre. They have adequate infrastructure in the form of administrative buildings as well as classrooms. Most learners came from areas within a radius of 5km from the Maseru City centre, there were however those who came from areas as far as 30km away. All learners in the two schools were day-scholars, that is, they came to school from their homes on daily basis. They did not dwell in the school premises. The majority of the learners depended on public transport (taxis) to-and-from school. These learners came from families with different economic backgrounds and the school fees were affordable by most parents.

1.3 Rationale of the Inquiry

The purpose of this study was to investigate classroom events during the teaching and learning of Grade 9 algebra through spreadsheets in Lesotho. The study sought to find out how Grade 9 learners experienced learning algebra through use of spreadsheets; paying particular attention to the kind of problems that were encountered, how such difficulties were overcome and what benefits were derived from the practice. Learners' interactions with the technology, the algebra, and the collaboration among pairs or groups of learners as well as teacher's intervention strategies in facilitating effective construction and reconstruction of algebraic concepts were the major focal points.

1.3.1 Why Grade 9 algebra?

In Lesotho, learners encounter formal algebra learning when they enter junior secondary school (MOET, 2002, pp. 2-8). This is when they are around 13 to 14 years of age. Earlier research studies (Kieran, 1989; MacGregor, 1990; Linchevski & Herscovics, 1996) indicate that difficulties that most people have with mathematics, emanate from their inability to make sense of algebra, which begins from their early experiences with algebra.

The study that the author undertook towards a Masters' Degree focused on problems that Grade 9 algebra learners encounter in understanding linear equations. Learners' difficulties were identified and critically analyzed. The study established that there was no smooth transition in cognition for learners from arithmetic into algebra, and that the manner in which algebra was taught also contributed to learners' failure to appreciate the power of algebra. The major problems that learners had included understanding of the concept of variable, interpretation of terms and expressions, basic laws of algebra and the structure of equations (Ntšohi, 2005, p. 40). Research reports (MacGregor, 1990; Kieran, 1992; English & Halford, 1995; Linchevski & Herscovics, 1996) also confirm these as areas that pose difficulties for most learners.

Learners had different interpretations for the concept of variable, leading to their inability to handle algebra (Kuchemann, 1981, p. 104). There are symbols in algebra that learners have met in their arithmetic learning; however, the same familiarity became a source of difficulty due to interpretations that learners attached to those symbols. For example, in arithmetic, learners interpret the equal sign as implying activity or command to perform an operation instead of understanding it as a symbol for expressing equality between quantities. There are properties of numbers upon which algebraic understanding can build from as early as primary school (Carpenter, Franke, & Levi, 2003, p. 2). It was found that there was need to develop smooth link between arithmetic and algebra. The ultimate goal is to make algebra learning a meaningful and painless exercise.

1.3.2 Why *Excel* Spreadsheets?

Research studies on the integration of technology in mathematics teaching and learning (Waits & Demana, 2000; Peressini & Knuth, 2005; Jonassen & Reeves, 2001) point to benefits that may be derived from such endeavors. Proper use of technology in the teaching of mathematics enhances learning. It offers learners opportunity to inquire, test their hypothesis and perform some functions related to their learning more quickly than they would otherwise do without the technology. Advocates of the technology integration in mathematics classrooms however warn that technology should only be used if it enhances cognitive development of the learners in the subject domain (Jonassen & Reeves, 2001, p. 716). Spreadsheets even though initially designed to solve problems of accountants, have been found to have potentialities, affordances as well as constraints that may be useful in enhancing learning of algebra.

Spreadsheets have the potential to bridge the gap between arithmetic and algebra as "they allow students to handle, observe and generate large sets of numerical instances" (Tabach et al 2008, p. 50). With their "dragging" power, spreadsheets can enable learners to extend any given sequences of numbers, slowly realize need for generalization, and finally appreciate the power of symbolism (Tabach & Friedlander, 2004, p. 428). The concept of variable as a placeholder for a general number may be developed through spreadsheets. In working with spreadsheets, learners may be challenged to make extensions to problems and investigate what happens when certain variations are made.

The mediated use of spreadsheets like *Excel*, can potentially offer opportunities for learners to develop algebraic skills such as generalization, modeling and shifting between different forms of representations (graphical, numerical and symbolization) (Berdnarz et al, 1996; Heuglet al, 1997; Yerushalmy, 2005; Tabach et al, 2008). Through use of spreadsheets, different learners are able to analyze problem situations, to conjecture critique and justify their solutions, proving correctness of their solutions and interpretation of results (Tabach et al, 2008, p. 49). Spreadsheets can thus be seen as tools that may be used to enable learners to generalize, mathematize and communicate mathematical ideas.

Another advantage of spreadsheets is that they are readily available as part of every computer. In comparison with other computer programs that would have to be purchased, there are no extra costs incurred with regard to accessing spreadsheets. It was on the bases of the above that I decided to investigate the teaching and learning of Grade 9 algebra through spreadsheets in Lesotho. The ultimate goal was trying to find out how learners would react to the use of this technology for algebra teaching and learning, and perhaps establish conditions under which effective teaching and learning of some parts of the algebra content through spreadsheets may be achieved.

1.4 Statement of the Problem

With the introduction of ICT for education, there has been controversy with regard to effectiveness of these technologies. Research indicates that use of computers in education, particularly in mathematics, has shown positive results (Heid, 1988; Palmitter, 1991; Dunham, 1992; Bokhove & Drijvers, 2012, p. 205). Advocates for integration of technology in mathematics teaching and learning, such as Waits and Demana (2000, pp.56-57), point out that use of technology (such as graphing calculators, spreadsheets, etc) may relieve learners of some technical or manipulation part of the mathematical activity, and allow them to concentrate on conceptual development. However, there are challenges associated with these technologies. According to Drijvers and Gravemeijer (2005, p. 164), "the idea that we can separate techniques from conceptual understanding and that leaving the first to the technological tool would enable us to concentrate on the latter has been shown to be inadequate and naïve." Lagrange (2003, p. 271) notes, "Technical difficulties in the use of CAS replaced the usual difficulties students encountered in pencil-and-paper calculations. Easier calculation did not automatically enhance students' reflections and understanding". Drijvers (2000, p. 189) identified five obstacles that learners encountered as they learned algebra through Computer Algebra Systems (CAS). Examples of these obstacles include the differences between the algebraic representations provided by CAS and the formal algebra notation, and the implicit ways in which the CAS performs calculations. These obstacles were defined as the barriers that prevented learners from using mathematical schemes they had in solving mathematical problems (Drijvers, 2000, p. 189).

Tabach and Friedlander (2008, p. 27) also pointed out that even though the use of spreadsheets may enhance initiating learners into algebra, potential sources of difficulties still exist. Learning algebra in a technologically (spreadsheets)-based environment would involve development of schemes in which technical and conceptual aspects co-emerge and evolve (Drijvers & Gravemeijer, 2005, p. 168). Considering the co-emergence of these aspects, it thus became imperative that a study was carried out to identify the kind of obstacles that could be encountered and the benefits that could be derived from the teaching and learning of Grade 9 algebra in Lesotho, through spreadsheets. The problems that were encountered were of different types: those related to algebra learning within spreadsheets, as learners grow in their *instrumental genesis* (Hoyles, Noss, & Kent, 2004, p. 310) and those that resulted from learners' perceptions and attitudes (Pierce & Stacey, 2004, p. 71).

In this study, it was also found important to identify the role teachers need to play in teaching algebra through spreadsheets in order to enhance learning. The teacher's involvement ranged from organization of the environment, *the didactical configurations*, planning of how the algebraic concepts would be developed within the spreadsheets environment, *the exploration modes*, and the nature of ad hoc actions that the teacher took within the classroom in order to address learners' needs, *the didactical performances*. This external steering of learners' instrumental genesis by the teacher is called *Instrumental Orchestration* (Trouche, 2004., p. 296).

This study was therefore intended to address the following research question:
How do Grade 9 learners in Lesotho experience the teaching and learning of algebra through Excel spreadsheets?
The sub-questions addressing this question were:

1) What challenges do Grade 9 learners in Lesotho encounter during teaching and learning of algebra through spreadsheets?
2) How do Grade 9 learners in Lesotho overcome the identified challenges to enable effective teaching and learning of Grade 9 algebra through spreadsheets?

3) How do Grade 9 learners in Lesotho perceive the teaching and learning of algebra through Excel spreadsheets?

Perceptions as implied in sub-question 3 refer to the opinions, feelings or beliefs that learners may have developed about the teaching and learning of algebra through spreadsheets. Earlier research indicates that some of the problems encountered by learners in their learning of mathematics through technology emanated from their perceptions and attitudes towards the subject domain and the technology (Pierce & Stacey, 2004, p. 71; McCulloch, Campbell & Hedges, 2009, p. 901).

1.5 Research Methodology and Design

A research design may be described in simple terms as the path taken to connect the empirical study to its initial research questions and finally to the answers or conclusions arrived at the end of the study (Yin, 2009, p. 26). It may also be seen as a "blue print" of the study with the main purpose of helping to avoid the situation in which the evidence does not address the initial research questions (Yin 2009, p. 27).

For this investigation, the research method is a multiple case study, intended to explore *how Grade 9 learners in Lesotho would experience the teaching and algebra through Excel spreadsheets.* A case study approach was carried out because the researcher wanted to obtain an in-depth understanding of the phenomena explored. The study was a multi-case study design in which two similar groups of Grade 9 learners were used in the investigation. The study falls within a mixed methods research paradigm (Johnson & Christensen, 2008, p. 52), where both qualitative and quantitative approaches were used. The mixed-methods paradigm was used primarily for two purposes, confirmation or complementarity (Small, 2011, p. 63). *Confirmation* involved verification of results obtained from the qualitative data with those obtained from the quantitative data, while *complementarity* was concerned with availability of findings from one type of data to compensate for the weaknesses of those obtained from another (Small, 2011, p. 64).

In this particular study the design is denoted by

$$\text{QUAL} \rightarrow \text{quant} \rightarrow \text{QUAL.}$$

This indicates that the methods occur in a sequence, with greater dominance of the qualitative methods while the quantitative methods followed (Creswell & Plano Clark, 2011, p. 110).

Qualitative data was collected through classroom observations and semi-structured in-person interviews. The focus here, was on identifying challenges encountered, strategies used to overcome the challenges, and the benefits derived from teaching and learning of algebra through spreadsheets. A video- audio recorder was used to capture the interviews and the activities in most lessons, while field notes were used to record important activities that took place in the all lessons in the classrooms. A questionnaire was administered to collect quantitative data, which was focused at determining the difficulties and benefits from learners' point of view, their perceptions, attitudes and feelings (Johnson & Christensen, 2004, p. 170) towards the algebra learning experience they had.

The researcher, who also played the role of the teacher, and the Assistant Observer (AO) both, collected the data. The details of how the study was conducted are presented in Chapter 3.

1.6 Scope

Use of spreadsheets in the teaching of algebra requires consideration of:

- The algebraic content that needs to be taught;
- The potentials and affordances (clues for possible functions) that the spreadsheets provide to enhance learning of the targeted algebraic concepts;
- The challenges that may be brought about by use of spreadsheets in the teaching and learning of algebra;
- Learners' perceptions and attitudes towards use of the spreadsheets as an instrument for algebra learning; and

- The teacher's role in enhancing effective teaching and learning within the spreadsheets environment.

Earlier research studies have already been conducted investigating the affordances provided by spreadsheets for the teaching of algebra. These identified some topics in algebra that are most suitable for teaching and learning through spreadsheets. This study was not directed in that area.

The focus of this study was to investigate how Grade 9 learners experienced teaching and learning of algebra through *Excel* spreadsheets, paying attention to the problems encountered and benefits derived, and identifying the strategies that were helpful in addressing the problems. The targeted learning areas were:

a) The concept of variable;
b) Recognizing and expressing relationships;
c) Exploring graphs of linear functions;
d) Simplifying algebraic expressions; and
e) Solving problems involving linear equations.

Learners' problems in relation to these areas and the role of spreadsheets in enhancing teaching and learning of these concepts were studied through review of related literature. Instructional materials, in the form of worksheets, focused on these areas were developed and used in the teaching.

1.7 Limitations

Limitations as used here refer to the conditions that hindered the smooth conduct of the study.

1.7.1 Time for the lessons

The study took place in the first two months (August and September) of the second session of the school calendar. It was, therefore, difficult to find the slot in the school timetables where all learners participating in the study would be available. The procedure used to select learners into the study also contributed to the timetable problem. Learners volunteered participation in the study and this ended up with participants coming from different streams of Grade 9 in the schools. Eventually the only available times were at the end of the school day, which was ideally for one hour only, twice a week. These were on days when learners were free to do individual study or go home.

These times were also not practically one-hour sessions, as learners also spent a lot of time moving from their different classrooms to the computer laboratories. Sometimes, teachers would still be in their classes, so learners could not walkout and leave them behind. In some cases, some of the learners would not available for the lessons because they would have attended a public lecture by a guest invited into the school or they would have gone for sporting activities. Even though the participants knew they were not expected to attend such activities by the school authorities, some still did.

1.7.2 **Involving teachers in the schools**

It was my expectation that I would be able to get at least one teacher from each school to participate in the study as an observer. This was, not possible. All the mathematics teachers in the two schools indicated that they would have liked to observe my lessons but they had a lot of work, in the form of preparations for their lessons and marking of learners work. I ended up having only one observer, the AO, who was also not available for all lessons due to work commitments.

1.8 Significance of the Study

The results of the study can potentially add to the body of research in the field directed at improving the teaching and learning of algebra at junior secondary school level. Earlier research studies conducted point to the positive role played by use of computer programs for enhancing mathematics learning. Identifying the problems and benefits that Grade 9 learners in Lesotho had when they were taught algebra through spreadsheets would be important to teachers, as the knowledge would help them be aware of the conditions under which the software can function as a cognitive tool for learning of algebra. Even though the study was conducted in Lesotho, the findings would be applicable to other countries with similar economic and historical background and beyond context of a developing country.

The effectiveness of use of spreadsheets as cognitive tools depends largely on the facility of the learners with the software, their attitude towards use of technology for learning mathematics as well as their confidence in their mathematical abilities in the domain (Hennessy, Ruthven, & Brindley, 2005, p. 162; Reed, Drijvers, & Kirschner, 2010, p. 2). It also depends on the teacher's knowledge of the potentialities, affordances and constraints inherent within the software with regard to the content to be taught and the strategies applied to orchestrate learners through the process of *instrumental genesis.* When planned well, teaching of algebra through spreadsheets can reduce the amount of time spent on the topics taught. There would be a shift of focus on the concepts that need to be emphasized at that level. The focus would only be on those aspects that require and develop cognitive thinking skills instead of mechanistic manipulations. The results of this study could therefore signal to mathematics teacher-educators, of the need to review their programmes in order to meet the demands and explore possibilities offered by technology for teaching in the 21st Century. The worksheets used in this study and their corresponding activities may be adopted and used for algebra teaching and learning in the classrooms.

1.9 Overview and Organization of the Report

This report is sub-divided into six (6) chapters as suggested by Creswell and Plano Clark (2011, pp. 258-259).

Chapter 1 provides an introduction and orientation of the study. It highlights the background to the study and gives a brief outline of the design and methodology followed. The scope and limitations of the study are briefly discussed here. The importance of this study to the field of Mathematics Education is also provided in this chapter.

Chapter 2 discusses literature related to the study. It discusses algebra from different perspectives, and pays particular attention to what comprises schools algebra. Grade 9 algebra as outlined in the Lesotho curriculum is described. A discussion of the curriculum as implemented in the schools and the nature of problems that learners and teachers encounter in algebra classrooms are provided. Reference is made to studies conducted by researchers such as Kuchemann (1981), Kieran (1989, 2004a, 2004b), Tabach and Friedlander (2004), Drijvers and Gravemeijer (2004), Trouche, (2004, 2005), Ntšohi (2005).

The chapter further discusses the role of technology in mathematics education. The potentials and affordances provided by use of spreadsheets in enhancing effective teaching and learning of beginning algebra is the focus of the discussion. A detailed account of what learning algebra through spreadsheets may involve with regard to demands put on the learner and the teacher is provided, focusing mainly on the processes of instrumental genesis and orchestration. This chapter provides a basis upon which the study is founded. It provides that, if this is the algebra to be taught, and research indicates problems with current practices in the teaching of the same algebra, then what other alternative approaches can be explored, and what problems are likely to be encountered in its execution? How such difficulties could be overcome is also important.

*Chapter 3*discusses how the empirical study was conducted. As the study involved use of a mixture of a number of data collection techniques and instruments, it was important that these were tried out first before the main study was conducted. A discussion of the pilot study and its impact on the conduct of main study is provided in the first part of this chapter.

The second part of the chapter discusses the design and methodology of the main study in detail. A description of this multi-case study and the associated mixed methods approach is given. The chapter outlines the different phases of the study and their corresponding data collection methods. How issues of validity and reliability were addressed in the study also forms part of the discussions in the chapter.

Chapter 4 presents findings from the qualitative part of the study. It reports on findings from classroom observations, and interviews conducted with both the learners and the Assistant Observer. The processes involved include exploration, analysis and interpretation of the data.

Chapter 5 presents findings from the quantitative part of the study. Data was collected by means of a questionnaire that was administered to learners who had remained in the study to the end, in both schools. Findings from both sets of data were then compared using statistical methods.

Chapter 6 provides a discussion on the findings from the qualitative and quantitative parts of the study. The findings are merged and compared, as collaboration and corroboration is sought. Conclusions are drawn and recommendations are made concerning the study.

1.10 Conclusion

The chapter provided the introduction and orientation of the study. It gave a brief discussion on the rationale and justification for the inquiry. The research question and its sub-questions were defined in this chapter. The chapter also highlighted on the design and methodology followed in conducting the research. It also provided the scope and limitations of the study. This chapter also highlighted on the significance of the study in mathematics education and finally outlined an overview of the whole report.

CHAPTER 2

THE TEACHING AND LEARNING OF ALGEBRA IN LESOTHO SCHOOLS-THE ROLE OF SPREADSHEETS

2.1 Introduction

This chapter discusses the literature related to the study and consists of two sections. The first section of the chapter examines different perspectives on what constitutes algebra, algebraic thinking and reasoning, and how these could be developed through the algebra curriculum offered to Grade 9 learners, as outlined by the National Curriculum Development Centre (NCDC) of the Government of Lesotho. The discussion covers the algebra content knowledge and the targeted learning outcomes (abilities that characterize competencies in algebraic knowledge and skills). The teaching approach to be followed in the mathematics classrooms is also discussed in detail. The researcher followed this approach in conducting the lessons for the study. A detailed account of learning obstacles pupils encounter in algebra is provided. School algebra as conceptualized by Kieran (2004a, pp. 24-26), is discussed as a basis upon which the teaching and learning activities were designed.

The second part of the chapter is focused on the teaching and learning of Grade 9 algebra through spreadsheets. It discusses the general role of technology in the form of computers in the teaching and learning of school mathematics, with particular reference to concepts found in school algebra. This section further pays particular attention to the affordances and "enablement" provided by spreadsheets in the case of the identified algebraic concepts. It highlights the constraints imposed when spreadsheets are used in the teaching and learning of algebra. The discussion also looks at how the complexities of using the *Excel* spreadsheets for teaching and learning of concepts found in the school algebra at the Grade 9 level may be managed.

2.2. Algebra and Algebraic Thinking

2.2.1 Defining Algebra

There is no agreement on a definition of algebra. In the mathematics education literature, we see different conceptions of algebra. Algebra can be conceptualized as generalized or abstract arithmetic (Kieran, 1988, p. 91, Usiskin, 1988, p. 11), a language of mathematics (Lee, 1996, p. 87) or a tool for studying functional relationships and mathematical modeling (Usiskin, 1988, p. 13; French, 2002, p. 3). These different conceptions of algebra are based on the different roles that algebra plays in mathematical situations.

2.2.1.1 Algebra: generalized or abstract arithmetic

This aspect of algebra is attributed to its symbolic form. Letters are used in many ways, in different contexts. They are used as variables, as unknowns, as parameters, etcetera, and it is this "use of letters to represent numbers which is a characteristic feature of algebra" (Hart in Usiskin, 1988, p. 9). The use of symbols helps mathematicians to abstract from specifics (arithmetic) to generality (algebra). Generalization involves realization of an arithmetic pattern, expression of the pattern in verbal terms, and finally expressing it in the symbolic language. The resulting mathematical objects can then be manipulated by application of mathematical operators, without any reference to where the objects originated. This is where the power of algebra can be realized.

While still acknowledging algebra as a *symbolic system*, Wheeler (1996, p. 319) also sees algebra as a *calculus,* since one of its main uses is computation of numerical solutions to problems. In the latter, algebra is now described not only by form, but also by how it is used and actions that can be performed on its objects (terms, expressions, statements, etc.). Saunders Mac Lane and Garret Birkhoff, in Usiskin (1988, p. 8) describe algebra as follows:

> Algebra starts as the art of manipulating sums, products, and powers of numbers. The rules for these manipulations hold for all numbers, so the manipulations may be carried out with letters standing for

> numbers. It then appears that the same rules hold for various different sorts of numbers… and that the rules even apply to things …which are not numbers at all. An algebraic system, as we will study it, is thus a set of all elements of any sort on which functions such as addition and multiplication operate, provided only that these operations satisfy certain basic rules. (Usiskin, 1988, p. 8).

The nature of algebra as portrayed here contributes to the strand of algebra that is described as 'the study of structures and systems abstracted from computations and relations, including those arising from arithmetic (algebra as generalized arithmetic) and in quantitative reasoning" (Kaput, 2008, p. 11).

2.2.1.2 Algebra: the language of mathematics

Unknowns, formulas, generalized patterns, placeholders and relationships (Usiskin, 1997, p. 346) characterize algebra as a language of mathematics. Algebra is described as a language, since it can be analyzed in terms of properties of ordinary language such as syntax (the organization and transformation of symbols), semantics (level of meaning) and pragmatics , which is about the relation between the signs and their user (Drouhard & Teppo, 2004, p. 231). *Metalinguistic awareness,* a characteristic associated with ordinary language, applies very well to the description of proficiency with algebra (MacGregor & Price, 1999, p. 451). McGregor and Price illustrate this as follows:

> Metalinguistic awareness enables the language user to reflect on the structural and functional features of text as an object, to make choices about how to communicate information, and to manipulate perceived units of language. Because analyzing structure and making choices about representation, and manipulating expressions are intrinsic to mathematics, and particularly to algebra, it seems that metalinguistic awareness in ordinary language has an equivalent in algebraic language. (MacGregor & Price, 1999, p. 451)

Algebra is indeed a language: it is used to represent and communicate mathematical ideas. This representation can be in the form of tables, equations, and graphs. The same representations also serve as a means of communicating the mathematical information. These algebraic representations aid better understanding of the phenomenon under consideration.

For example, Figure 2.1 shows information about heights of seedlings of a certain plant recorded at different times during their growth. In deciding on the appropriate time for spraying plants against insecticides, it may be necessary to consider their height above the soil.

No. of days	HEIGHT (cm)
0	15
5	40
10	65
15	90
20	115
25	140
30	165
35	190
40	215

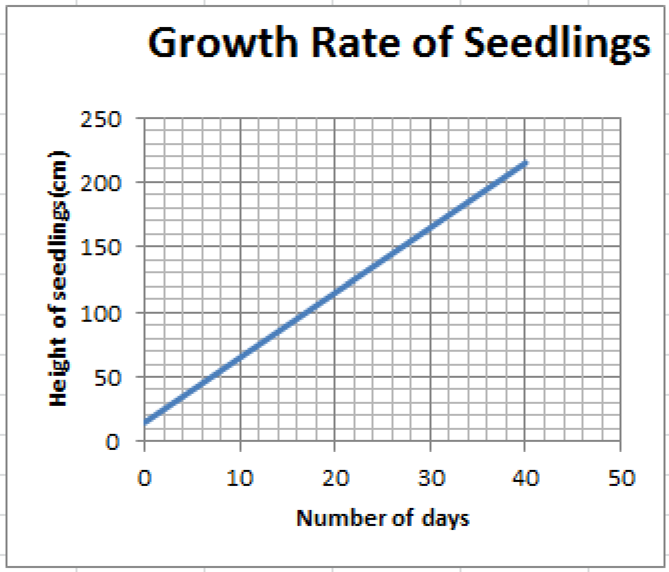


Figure 2.1: Height of seedlings (cm) per number of days

Displaying information in different ways can enhance interpretation and understanding, as well as making appropriate decisions about the situation at hand.When we verbalise algebraic expressions, formulas and functions, it confirms algebra as a language, and this verbalisation is a confirmation that algebra is not limited to symbols only. It is therefore important that learners be helped to develop metalinguistic awareness in algebra.

2.2.1.3 Algebra: A tool for the study of functions

Algebra is also viewed as a tool to study functional relationships and mathematical modeling, in fact, (Charbonneau, 1996, p. 35) suggests "algebra is foremost a means to manipulate relations". These relations can be between numbers or between magnitudes or between numbers and magnitudes. The power of algebra lies in its use of the economical and consistent symbolic system to express relationships, which can then be used to formulate arguments concerned with prediction, problem solving, explanation and proof (French, 2002, p.3). Once relationships are represented in symbolic form they can be easily transformed (manipulated) and thus complex situations can be analyzed and better understood (Wheeler, 1996, p. 319; French, 2002, p. 3). It is on the basis of these manipulative actions that Wheeler (1996, p. 319) further suggests that algebra could be described as *action*, as it involves actions such as collecting like terms, factoring, expanding, solving equations, summing sequences, drawing graphs, etc.

It is worth noting that these views about algebra have evolved over time, from the initial limited view where algebra was only recognized as the process of solving only a certain class of problems (Mason, 1996, p. 73).Vance (1998, p.382) notes that “algebra is more than a set of rules for manipulating symbols: it is a *way of thinking*.” We now look more closely at what characterizes algebraic thinking.

2.2.2 Algebraic Thinking

Love in Kieran and Yerushalmy (2004, p. 101) suggests algebra “concerns itself with those modes of thought that are essentially algebraic, for example, handling the as yet unknown, inverting and reversing operations, seeing the general in the particular; being aware of these processes, and in control of them, is what it means to think algebraically”. This means that algebraic thought is in operation whenever a pattern or some commonality is observed and an attempt is made to express it in verbal language or in symbolic form. Learners are involved in algebraic thinking when they determine an unknown in a mathematical context, when they can recognize and apply inverse operations (for example, addition versus subtraction) to solve problems.

Algebraic thinking, therefore, can be developed as early as in primary school, since it “involves the development of mathematical reasoning within an algebraic frame of mind by building meaning for the symbols and operations of algebra in terms of arithmetic” (Kieran &

Chalouh, 1993, p. 179). As young learners solve arithmetic problems, recognize relationships between operations, recognize patterns and express these verbally, they are engaged in algebraic thinking. Chambers (1994, p. 85) notes "algebraic thinking involves the construction and representation of patterns and regularities, deliberate generalization, and most important, active exploration and conjecture".

Kieran (2004b, p. 140) suggests an approach that would include the following aspects, in order to help learners to develop an algebraic way of thinking:

1. A focus on relations and not merely on the calculation of a numerical answer;
2. A focus on operations as well as their inverses, and on the related idea of doing/undoing;
3. A focus on both representing and solving a problem rather than on merely solving it; and
4. A focus on both numbers and letters, rather than on numbers alone. This includes:

   i) working with letters that may at times be unknowns, variables, or parameters;
   ii) accepting unclosed literal expressions as responses; and
   iii) comparing expressions for equivalence based on properties rather than on numerical evaluation;

5. A refocusing of the meaning of the equal sign. Kieran (2004b, p. 140)

Algebraic thinking may also be described in terms of problem solving in the context of mathematics. Herbert and Brown (1997, p. 340) assert:

> Algebraic thinking is using mathematical symbols and tools to analyze different situations by (1) extracting information from the situation... (2) representing that information mathematically in words, diagrams, tables, graphs, and equations; and (3) interpreting and applying mathematical findings, such as solving for unknowns, testing conjectures, and identifying functional relationships. (Herbert & Brown, 1997, p. 340).

Fig 2.1 shows a diagrammatical representation of the process of algebraic thinking as problem solving in the context of mathematics.

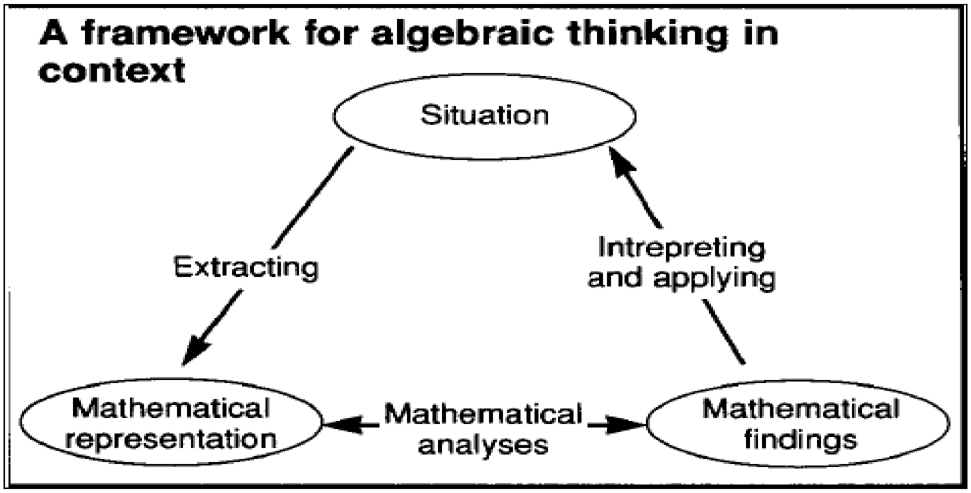


Figure 2.2: Algebraic Thinking Herbert & Brown (1997, p. 340)

According to Kriegler, algebraic thinking is composed of two parts, namely: development of mathematical thinking tools; and study of fundamental algebraic ideas (Kriegler, 2007, p. 2). On one hand, the mathematical thinking tools are those analytical thinking skills involved in problem solving, representation and inductive and deductive reasoning. The fundamental algebraic ideas, on the other hand, include those concepts that may be categorized under algebra as abstract/generalized arithmetic, as a language and as a tool for studying functions and mathematical modeling (Kriegler, 2007, p. 2). This view of algebraic thinking is closely related to Herbert & Brown's description cited earlier. This reveals that there is a close association between mathematical thinking and algebraic thinking.

2.3 Grade 9 Algebra in Lesotho Schools: The Curriculum

The Grade 9 Algebra curriculum discussed here refers to the mathematics content, the learning outcomes (objectives), the related pedagogical processes as well as assessment standards in the domain of algebra education at junior secondary level as found in Lesotho schools. There are two categories of curriculum that are discussed: *the intended* (written) curriculum and *the enacted* (implemented) curriculum (Hoadley & Jansen, 2010, p. 45). The intended curriculum is the curriculum, as set by national curriculum developers, which is intended to be implemented in the schools. The taught curriculum, on the other hand, is the curriculum as interpreted and implemented by teachers in the classrooms. The discussion will thus focus on both categories of curriculum since there seems to be a difference (gap) between curriculum prescribed for teaching and what is actually implemented and consequently learned in the classroom (Hoadley & Jansen, 2010, p. 45)

2.3.1 The intended Lesotho Junior Certificate algebra curriculum

According to the MOET (2002, p. 1) curriculum document, the main themes that are stressed in the teaching of mathematics are classified as:

- Knowledge and skills
- Applications and problem solving;
- Appreciation of the environment.

Table 2.1 shows the different topics (*knowledge and skills*) and corresponding end-of-level outcomes/objectives as adopted from MOET curriculum document for Lesotho Junior Certificate (LJC) Mathematics (2002, pp. 2-8):

Content	End of level objectives
Types of numbers and their sequences	• Identify properties of sets of numbers - Find the rule for a sequence - Fill in missing numbers in a sequence
Algebraic representation and formulae	• Simplify algebraic expressions - Use letters for numbers - Evaluate expressions by substitution - Differentiate between, variables, terms, coefficients and expressions; like and unlike terms - Expand and factorize algebraic expressions - Change subject of formulae
Coordinates, graphs, relations and function notation	• Draw and label linear graphs - Draw and label x- and y-axis - Plot points on a Cartesian plane - Evaluate given mappings - Draw graphs of straight lines - Solve equations using graphs - determine equations of straight lines • Evaluate a function - Recognize a function as a special relation - Use function notation
Solutions of equations and inequalities	• Solve linear equations - Solve equation by inspection, then by algorithm - Formulate an equation from a statement - Find algebraic and graphical solutions to linear inequalities(with one variable)
Indices	• express numbers in index form - Perform basic operations(+, -, x, ÷) on indices - Express numbers in standard form

Table 2.1:LJC Algebra Curriculum

On elaborating on *application and problem-solving*, and *appreciation of environment*, the document indicates that teaching approaches should have the following characteristics:

- teaching should be pupil-centered (i.e. more activities for pupils)
- application of concepts should be emphasized (this means that when developing a concept one should start with examples in the immediate environment of pupils i.e. from known to the unknown)
- understanding of mathematical concepts is encouraged as opposed to memorization of a collection of formulae
- learners' experiences and interests should be taken into account in the development of concepts
- hands-on activities are mostly encouraged to aid recall, understanding and a better attitude towards the subject;

- pupils should be made aware of the need to study a given topic by the application of the topic in their everyday life.

(MOET, 2002, p. 1).

The assessment objectives relating to algebra as listed in the document, state that candidates will be tested on their ability to:

- Recognize the appropriate mathematical procedures for a given situation;
- Interpret, use, and present information in written, graphical, diagrammatic and tabular forms;
- Recognize patterns and structures in a variety of situations in order to form and justify generalizations;
- Understand and use mathematical language and symbols to present mathematical arguments in a logical and clear fashion;
- Apply and interpret mathematics in a variety of situations, including daily life;
- Formulate problems into mathematical terms (and solve them).

(MOET 2000, p.2)

From the LJC curriculum, it can be deduced that the teaching and learning of school algebra should leave learners with a residue of what is called algebraic competency (Crawford, 2001, p. 192). According to Crawford (2001, p. 192), algebraic competency may be defined as:

- ability to think in symbolic language, to understand algebra as generalized arithmetic and to understand algebra as a study of mathematical structures;
- ability to understand equality and equations of algebra and to apply these within real-world problem-solving settings;
- ability to understand relationships of quantities through patterns, defining functions and applying mathematical modeling.

In the next section, I will look at how algebra (school algebra) is conceptualized by Kieran and thus make connection between Kieran's conceptualization with the kind of activities embedded in the Lesotho curriculum.

2.3.2 School algebra as seen by Kieran

In this section, we take a closer look at the LJC algebra curriculum content and compare it with Kieran's conceptualization of school algebra. Analysis of the algebraic content in the LJC mathematics curriculum indicates similarity to what Kieran (2004a, p. 24) outlines as core activities in school algebra.

Kieran (2004a, p. 24)identifies three types of core activities in algebra teaching and learning. These are: generational, transformational and global/meta-level. The focus of *generational activities* is "representation (and interpretation) of situations, properties, patterns, and relations" (Kieran, 2004a, p. 25). Examples here include formulating expressions and equations for situations. The activities in this category offer opportunities for making algebra learning meaningful.

Transformational activities are described as those that are rule-based. These include algebraic manipulation processes such as collecting like terms, factorizing, expanding brackets, substitutions, simplifying algebraic expressions through addition or multiplication, exponentiation with polynomials, solving equations etc. (Kieran, 2004a, p. 26). The main concern here is to lead learners into transforming the original given expression into its equivalent forms (Kieran, 2004b, p. 142). This class of activities has dominated and has been the main focus in the teaching and learning of school algebra. Traditionally algebraic manipulations such as those involved in solving linear equations have been taught without any meanings attached, with no connections to learners' life experiences. Learners are drilled on applying the rules and procedures for solving such equations. (Chazan, 1999, p. 124; Ntšohi, 2005, pp. 38-39).

The third type of activities, the *global/meta-level activities,* consists of "activities for which algebra is used as a tool but which are not exclusive to algebra" (Kieran, 2004a, p. 24). Examples of such activities include problem-solving, predicting, modeling, generalizing and justifying, etc. These are activities that may be carried out without using any algebra at all, they suggest more general mathematical processes and activity (Kieran, 2004b, p. 142).

The Lesotho algebra curriculum for Grade 9 reflects these three categories of core activities in school algebra, as seen by Kieran, as elaborated by the end of level objectives, in Table 2.1. Learners engage in those rule-based activities of simplifying, expanding and factorizing expressions. They seek for patterns and generalize them verbally and eventually using the symbolic mathematical language. They solve contextualized mathematical problems involving relations, which require formulation of equations, modeling and interpretation of those graphical representations.

The above descriptions about the nature of algebra and algebraic thinking encompass the objects and activities that learners use and engage in when learning algebra. Even though learners in Lesotho only begin to use letters in the manner reflected here, at around 15 years of age, they would have long experienced and engaged in algebraic thinking in their early grades and in their everyday encounters in life. Ideally, there seems to be continuity in algebraic thinking from primary school arithmetic and what learners encounter in their algebra lessons. The question is "why does algebra learning then become so problematic and perhaps impossible" for some learners? This question leads us to the discussion on what actually goes on in the teaching of algebra in Lesotho schools and what problems learners do encounter in their algebra learning.

This study thus seeks to establish how learners would experience spreadsheets-based generational activities intended to enhance a deep understanding of the concept of variable and hence, structure in algebraic expressions. It was hoped that in working through such activities, learners would be able to express relationships and represent those relationships in multiple ways (numerical/symbolic, tabular, and graphical). Algebra was taught in such a way that learners were given opportunity to make meaning of the expressions or mathematical statements, connecting them to real-life contexts from where they may arise. Activities (involving use of spreadsheets) geared towards developing the concept of equivalence in algebraic expressions, were designed. These spreadsheets activities were designed such that they would make these algebraic manipulations and transformations meaningful and would lead to solution of linear equations. Learners engaged in activities that involved problem-solving, modeling, making conjectures and justifying solutions for the given problem situations.

2.3.3 The Implemented Curriculum

From the author's experience as a learner and a teacher, at secondary school level, traditionally learners are introduced to algebra by way of defining letters as representing unknown value(s) or as representing any number, in the case of expressions. They are then introduced to the rules that govern operations on these objects. They engage in activities that involve use of letters as representing variables, substitution, simplifying, factorizing expressions, expanding brackets and ultimately solving equations, (MOET, 2002). The teaching is characterized by the teacher introducing the concept to be learned by way of a definition. This is then followed by demonstrations of how problems are solved, showing the learners the procedures to be followed leading to the correct answers.

The teacher may then bring up a question with the same structure, and ask learners to work it out on the chalkboard. After discussing the solution, learners would then be given an assignment with a set of drill exercises. This approach is usually referred to as "mechanistic" (Wubbels et al, 1997, pp. 1-2) as it concerns drilling learners on how to manipulate symbols, and learners may master these manipulations without attaching any meaning behind the operations. Learners are denied opportunity to realize and appreciate need for algebra. They do not appreciate the power of symbolism and its beauty as truths emerge because of generalizations on observed patterns. Learners fail to appreciate algebra as a language, as a way of expressing and communicating ideas. The approach leads to the perception that learning algebra or mathematics in general, is all about rules and procedures that remain meaningless to such a viewer. This approach really needs to change.

The approach is evident in the way many textbooks are written for learners. It has provided learners with only a limited understanding of algebra, as revealed in the mathematics examinations reports by ECOL. Despite the many workshops directed at making teachers aware of learners' problems in mathematics, the problems still exist.

2.3.4 Problems learners encounter in learning algebra

The previous discussions only elicit what ought to be taught in the classrooms. What is taught and what is learned is usually different, and in most cases, it is only a fraction of the stipulated subject matter. Why is this the case? The teaching-learning process involves interaction between the teacher, the subject matter and the learner, within an environment that involves use of the teaching/learning aids. There can be countless factors that may be attributed to why all that is taught is not learned. For the purpose of this study, we will only look at those problems/obstacles that relate to the nature of the subject matter (algebra), and the methods used in the teaching of algebra.

Learning obstacles are the hiccups or hurdles that get into the learners' way, preventing or delaying learners from understanding the targeted algebraic concepts. These obstacles lead to misconceptions that learners develop with regard to such concepts. Misconceptions can form serious barriers that may impede pupils' learning and understanding of new concepts. From the constructivist perspective, however, "misconceptions are crucially important to learning and teaching, because misconceptions form part of a pupils' conceptual structure that will interact with new concepts, and influence new learning, mostly in a negative way, because misconceptions generate errors" (Olivier, 1989). Olivier further notes that errors and misconceptions should be seen as the natural result of children's effort to construct their own knowledge, and that these misconceptions are intelligent constructions based on correct or incomplete (but not wrong) previous knowledge. Teachers should be able to diagnose the constructed misconceptions and help learners construct correct understandings of concepts.

From the latter perspective, it becomes apparent that the role of the teacher is to identify the type of thinking in the learners that results in the errors displayed in order to overcome the underlying misconceptions Errors are signposts that direct teachers to pupils' problem areas. According to Heiman, Narode, Slomianko and Lochhead (1987, p.18) "misconceptions can even be used as instructional aids; when a teacher presents a misconception that is troublesome for pupils, s/he forces them to test their conceptions against the paradox and against their common sense." Through listening to what pupils say in reasoning and justifying their solutions, and watching them carefully as they execute problem solving, the teacher can discover the misconceptions that pupils may have. Sharing the misconceptions with the pupils and other teachers can improve pupils' understanding of concepts as well as instructional activities designed by the teachers.

2.3.4.1 Learners problems related to the nature of algebra.

French (2002, p. 2) notes "the essence of algebra is that it uses an economical and consistent symbol system to represent expressions and relationships which are then used in formulating arguments concerned with prediction, problem solving, explanation and proof". Despite its usefulness in the field of mathematics and across the different disciplines within school curricula, algebra creates serious problems to learners even in the later stages of their mathematics learning. In many countries, learners encounter algebra in the form of symbols around the age of fifteen (15), when they enter secondary education. Lesotho is no exception in this regard. According to Kieran (1989, p. 39), difficulties that learners have with algebra are centered on:

i. the meaning of letters,
ii. the shift to a set of conventions different from those used in arithmetic, and
iii. recognition and use of algebraic structure.

Other researchers (Berger & Wilde, 1987; MacGregor, 1990; Kieran, 1992; English & Halford, 1995, p. 241) indicate that learners also experience difficulties when solving mathematical word problems, which require iv) formulation of algebraic expressions and equations.

i) The meaning of letters

From the study conducted by Kuchemann (1981, p. 104), learners go through six stages in acquiring the concept of variable. The stages are: a) Letter evaluated, b) Letter not used, c) Letter used as an object, d) Letter used as a specific unknown, e) Letter as a generalized number and f) Letter used as a variable. In their first stage, learners avoid operating on a specific unknown and as such simply assign a numerical value to the unknown from the outset. They may recall any number or the number fact about the expressed relationship. In the last stage, they now see the letter as representing a range of unspecified values and understand that a relationship exists between such sets of values.

Understanding the use of letters is fundamental for the study of algebra. There are three uses of letters, which are common in school algebra. These are:

- Letter as a specific unknown: this is found in equations such as x + 2 = 5, 2x +3 = 3x –2, etc. The value of x is a specific number that can be determined;
- Letter as a general number: here the letter is used as a generalizer, regarded as representing or at least being able to take several values rather than just one value. The letter is used in statements that are true for all numbers, for example, a + b =b + a , and
- Letter as a variable: In this case, the letter is used to represent a range of unspecified values, usually in functional relationships where a change in one variable determines change in the other. For example, y = 2*x* +5, C = 2πr, etcetera.

Lack of understanding of the different roles of letters in algebra by learners, hinders their (learners) ability to generalize situations and express them algebraically (Ntšohi, 2005, p. 90). An example of learners difficulties in understanding use of letters, is their inability to evaluate $a + b + 2$, given that $a + b = 43$ (French, 2002, p.15).

ii) The shift to a set of conventions different from those used in arithmetic

Research studies such as that conducted by Carpenter et al(2003,p.2) indicate that some of learners' problems in algebra originate from conflicts between arithmetic language and the technical algebraic language. While in arithmetic 3 + ⅓ = 3⅓, in algebra $a + b \neq ab$; also $pq = qp$ is true in algebra but 42≠24 in arithmetic. Learners need to be helped to make a smooth transition from arithmetic into algebra.

iii) Recognition and use of algebraic structure

Algebraic structure may be described in terms of *surface structure* and *systemic structure*. According to Kieran (1992, p. 397), surface structure refers to the arrangement of different terms and operations that go to make up an algebra (or arithmetic) expression. Nickson (2000, p. 11) on the other hand defines systemic structure as the "properties of operations within an algebraic expression and relationships between the terms of the expression that come from within the mathematical system". *Structure of an equation* incorporates the systemic structure and the relationship of equality (Nickson, 2000, p. 11).

Learners exhibit confusion when dealing with algebraic manipulations involving simplifying expressions and/or solving equations. When confronted with a problem whose solution results in an expression as the answer, learners would opt to give a numerical solution. For example the solution "$3x + 4$" would be simplified to "7 or $7x$". This indicates that learners have trouble in understanding algebraic structure. Again, in simplifying an expression such as $\frac{3m}{2} + \frac{m}{5}$ *to* $\frac{3m}{7}$ indicate learners' inability to understand the role of letters as well as their failure to relate what they learned in arithmetic to algebra (Ntšohi, 2005, p. 55). Learners still encounter the same difficulties regarding the equal sign. Some learners still interpret the equal sign as a command to carry out an operation, to the extent that the following solution was seen in learners' work:

$$7 + 9 = x + 10$$

$$= 16 + 10$$

$$= 26\ .$$

Solutions such as $7 + 9 = x + 10$

$\underline{-9\ =\ \ \ \ -9}$

$$7 = x + 1$$

$\underline{-1 =\ \ \ \ \ -1}$

$6 = x, \quad x = 6.$

indicate that learners just apply the learnt algorithms to get to the correct solutions, without any comprehension of the equation statement. (Ntšohi, 2005, pp. 57-58).

iv) Formulation of algebraic expressions and equations

Another problem area identified was that of translating mathematical statements from ordinary language to the symbolic algebraic language (Ntšohi, 2005, p. 33). This seems to be an on-going problem that was identified in earlier research studies (Berger & Wilde, 1987, p. 23; MacGregor, 1990; Kieran, 1992; English & Halford, 1995, p. 241; Setati & Adler, 2001, p. 247)). Berger and Wilde (1987, p.23) affirm, "algebra word problems have been a source of consternation to generations of students." The most commonly cited problem is the *student-professor* problem. In the problem it is stated that "there are six times as many students as professors in this university" and in translating this problem into symbolic from many students came up with $6s = p$ as the equation representing the relationship between number of students and professors. An explanation to this type of error is that "students rely on direct syntactic approach to solving these problems, that is, they use a phrase-by-phrase translation of the problem into variables and equations" (English & Halford, 1995, p.241).

According to the Lesotho Education Policy, all children are taught all subjects, except English, in their mother tongue, from their first day up to end of their third year in formal school (MOET, 2004, p. 6). This late start in learning English contributes to their lack of fluency in the language even at Grade 9level. The only exceptions are those learners who went to the privately run English medium primary schools, as the schools are usually called, where the medium of communication and instruction is English all the way. When they cannot comprehend the mathematical problem statement, learners find it very difficult to translate the relationship between variables in the problem from ordinary language to the symbolic mathematical language.

It is clear from the above discussions that lack of understanding on any of the above-mentioned algebraic concepts and skills prevents learners from "doing mathematics". Doing mathematics includes investigating, formalizing patterns and regularities, generalizing, representing mathematical situations in multiple ways, and solving mathematical problem situations.

2.3.4.2 Learners' problems due to teaching methods

As indicated earlier, some of the problems that learners have with regard to understanding algebra can be attributed to the way algebra is taught. Teachers' beliefs about their work greatly influence the manner in which they teach. Beliefs that teachers hold are so instrumental in shaping their instructional decisions and actions that they need not be underestimated (Nathan & Koedinger, 2000, p. 210). Their perceptions about algebra, and regarding learners' difficulties in learning/understanding school algebra, largely determine the way they (teachers) plan and implement their instructional activities. It was found that "Some teachers still hold a formalistic view of mathematics; as such believe that teaching mathematics is about drilling pupils on procedures (and rules) on working out solutions to linear equations"(Ntšohi, 2005, p.90).

In Lesotho, the teaching of algebra is characterized by the mechanistic approach. *See Section 2.3.3.* Mason (2007, p.914-915) noted that

> Where learners are thrown straight into manipulation of symbols as 'if they were numbers', the whole process becomes mysterious, the purpose unclear, and the practice merely routine; motivation and performance suffer, and interest in mathematics itself declines. Mason (2007, pp. 914-915).

Another contributing factor to learner's inability to handle algebra is that some teachers are not aware of the importance of linking algebra learning to pupils' previous experiences with arithmetic. The consequence of this is that teachers miss out leading learners from known to the unknown, thus creating a gap, "a cognitive gap" (Linchevski & Herscovics, 1996, pp. 40-41) between arithmetic and algebra.

Some teachers also hold a view that discussion, as a strategy of teaching, consumes a lot of time and allowing discussions may even cause confusion to slow learners. This is not in line with what most pupils prefer. According to the study conducted by Ntšohi, learners indicated that they preferred learning mathematics through discussions. They mentioned that they got motivated to learn more mathematics when their ideas were responded to in a positive manner and were allowed to use their own methods in solving problems. (Ntšohi, 2005, p. 90).

2.3.5 Effective teaching and learning of algebra: according to NCDC

According to the LJC curriculum document, effective teaching of mathematics should be learner-centered and must give learners opportunity to engage in hands-on activities. Teaching should be geared towards enhancing learning. (MOET,2002, p.1).A closer look at the teaching approach recommended by the MOET, the LJC mathematics curriculum document, reveals similar teaching approaches to those followed in other countries, for example, Realistic Mathematics Education (RME) in the Netherlands (Van den Heuvel-Panhuizen, 1996, pp. 9-13) and the USA, (*see* National Council for Teachers of Mathematics (2000, pp. 11-26). Having recognized these similarities in the curricula, the main principles in the teaching of algebra in Lesotho will be discussed within the frameworks of RME and NCTM.

The teaching approaches recommended by NCTM, RME and MOET are founded on constructivism as a theory of learning. Constructivism is the most widely accepted theory of learning and it posits that children construct their own knowledge based on their prior knowledge, and emphasizes that learning requires active participation by learners (Fry, Ketteridge, & Marshall, 2009, pp. 9-10). Individual learners construct their own knowledge, through social interactions with each other, their teacher and the subject matter. Such knowledge, according to Jones,

> "…is the dynamic product of the work of individuals operating in the communities, not a solid body of immutable facts and procedures independent of mathematicians. In this view learning is considered more as a matter of meaning-making and of constructing one's own knowledge than of memorizing mathematical results and absorbing facts from the teacher's mind or the textbook; teaching is the facilitation of knowledge construction and not delivery of information"(Jones, 1997, p. 145).

Constructivism rejects the idea that learners are blank slates upon which teachers write new knowledge. Instead, it recognizes that learners come to a learning situation with some already perceived ideas about the concept to be learned. Some of these ideas are ad hoc and unstable, while others are deeply rooted and well developed. The role of teachers is, therefore, to ascertain the nature of these understandings and model them in such a way that they form a basis upon which new knowledge can be constructed. Experiences of the learner create mental networks and these existing networks influence the relationships that are constructed, thereby helping to reshape the new networks that are formed (Hiebert & Carpenter, 1992, p. 70). Learning is thus regarded as a process of making connections between existing schemas and new structures of knowledge through social interactions.

From the LJC document, learning is reflected both as a process and as an outcome (S*eeSection*2.3.1). Kostova & Atasoy (2008, p. 51) note that, learning, as a process, can be seen as a conscious endeavor a learner undertakes to accomplish personal educational necessities, interests and aims for purposes of effective adaptation and integration in social life. In addition, Demkanin, Kibble, Lavonen, Mas and Turko (2008, p.8), suggest that learning should be seen as a cumulative process in which learners are "aided in noticing how a new concept or skill is related to other already familiar concepts or network of concepts and skills". This construction of knowledge may be regarded as a process whereby learners combine earlier knowledge with new topics being learned, and thereby tailor information structures that they can abstract and assimilate (Demkanin, Kibble, Lavonen, Mas, & Turko, 2008, p. 8). This therefore means that the teacher should consider learners' experiences and interests in the development of new knowledge; teaching must lead learners from known to unknown concepts. As an outcome, learning is represented by the desired outcomes acquired by the learner through study, practice or information enhancing learning (Kostova & Atasoy, 2008, p. 51). Learning may, thus, be defined as knowledge acquired through experience, in the manner described above.

Within the framework provided above, the main principles underlying algebra instruction are i) use of meaningful contexts, ii) development of problem solving skills and iii) interaction and guidance. These are discussed in detail in the next sections.

2.3.5.1 Use of meaningful contexts in teaching and learning of algebra

Meaningful contexts, according to the curriculum documents, are those situations that are familiar to the learners, and relate to their environment (Van den Heuvel-Panhuizen, 1996, p.13). It must be noted, however, that "real contexts" does not necessarily refer to real life situations. What is important is that the contexts "can be organized mathematically and students can place themselves within them" (Van den Heuvel-Panhuizen, 1996, p. 13). In other words, students should be able to visualize the situation and understand the problem and "must image themselves in the situation", in this way, the situation becomes real to the learners' world. When learners work within situations that are familiar to them, they tend to develop understandings of what they are engaged in. Concepts that are learned without any link to learners' reality and experiences, are easily forgotten and learners will not be able to apply them (Freudenthal, 1973). Through algebra, learners would be able to model the situation at hand, representing it in different ways and thus accessing better interpretations and analysis of the problem. This would then make the solution process much easier.

Contexts also offer learners opportunity to appreciate usefulness of algebra. When learners realize usefulness of what they are learning, through *application*, they tend to develop interest in it. Students tend to learn better that which they consider important and useful. It is important here note that application of mathematical concepts is not recognized at the end, but it is in fact the source for learning mathematics (Van den Heuvel-Panhuizen, 2000, p. 5). Instead of beginning with abstract ideas that learners will only apply later, rich mathematical contexts are provided to engage learners in mathematising the situation and hence discovering the embedded mathematical concepts. Freudenthal (1973), as cited in Van den Heuvel-Panhuizen (1996, p. 10), suggests that mathematics must be linked to children's reality and be relevant to society in order to be considered important.

Another advantage of using real life contexts in the learning of algebra is that they provide opportunity for making *connections* with different branches of mathematics and across different disciplines within the school curricula (NCTM, 2000, pp. 64-66). This may be interpreted as the *inter-twinement* of topics within the field of mathematics and across other school subject (Van den Heuvel-Panhuizen, 2000, p. 7). Algebra as a language of mathematics can be applied in different branches of mathematics and thus making it possible for learners to view mathematics as a meaningful connected discipline. Gattegno (1970, p. 26) pointed out that "algebra is present in all mathematics because it is an attribute of the functioning mind". Bell (1996, p. 167) further elaborates

> I do not see algebra as an identifiable course, separate from other branches of mathematics, but as appearing throughout the mathematics course, its symbolism, concepts, and methods being used wherever appropriate in the other fields—in expressing arithmetic generalizations, solving geometric problems, denoting unknown elements, solving equations and establishing relations in trigonometry, as formulas for statistics and mensuration, and so on. Bell (1996, p. 167).

Making connections within mathematical topics is very important in strengthening conceptual understandings. For example, formulating and solving linear equations can be done in geometry, statistics, mensuration, ratio and proportion, and so on. This would even lead learners into appreciating the power of algebraic symbolism. Subjects such as geography, accounting, and science, provide contexts where algebra learning may be made meaningful and worthwhile to learners. In this case, algebra would thus be seen as a tool for studying relationships in contexts outside "mathematics".

The aim of the researcher is that teaching of algebra should be done in context. Learners should be able to develop appreciation for the power of algebra through its application in contexts that are meaningful and interesting to them. There is a very close relationship between the role of contexts and development of problem solving skills in the learners.

2.3.5.2 Development of problem solving skills

According to the curriculum document, mathematics should be learned through problematic contexts, to develop problem-solving skills in the learners. Helping learners to become better problem solvers is an important aim in algebra teaching (French, 2002, p. 5). While to some people problem solving in mathematics may be thought of as just about getting answers for questions in an exercise at the end of a topic, to mathematics educators, it is much more than that. For the purposes of this study, we will consider two perspectives of problem solving namely: a) problem-solving as a skill and b) problem solving as a teaching approach. We thus view problem solving not only as a goal of learning algebra but as a means of learning algebra as well (NCTM, 2000, p.52).

i) Problem solving as a goal of algebra learning

Problem solving as defined by Bell (1996, p.167) is a mathematical activity that involves "exploring problems in an open way, extending and developing them in the search for more results and more general ones". When confronted with a realistic problematic situation, learners bring their mathematical tools to unravel the problem. They discover and organize the mathematics through mathematising horizontally and vertically. Learners, with as little guidance as possible, are given opportunity to "aspire, to climb, and to dive to heights and depths as steep and as deep as they can reach and afford" (Freudenthal, 1991, p. 47) according to demands of each particular mathematical problem situation. In this way learners go through what Freudenthal (1991, p.46) termed *guided reinvention*. When mathematics is learned in this way, the valuable knowledge and abilities so acquired are there to stay, they will be more easily retained and transferred; and mathematics learning becomes enjoyable as learners discover (reinvent) knowledge and the "aha"(eureka!!) effect is felt.

The experience that learners must go through in problem-solving, demands thoughtful planning and a lot of creativity on the part of the teacher, regarding the nature of problems to be solved. A mathematical problem is one that has no immediate solution. It requires mathematical knowledge and skills to solve. According to this description, the problems that learners engage with should be those that provide for exploration thus investigation, reflective enquiry and discovery come into the picture. They should require from learners, answers to "what if" type of questions, thus offering learners opportunity to make conjectures, make extensions and analyze their own solution procedures. Learners should be able to generalize about the problem situation they are confronted with. They should even be able to make predictions based on data gathered during the solution process.

The problem solving process involves four stages as identified by Polya (1973). These are sequentially arranged as: Understanding the problem, devising a plan or deciding on an approach for tackling a problem, carrying out the plan, and looking back (at the problem, the answer, what has been done to get there and interpreting the solution in terms of the question asked). This means that the problem solver needs to understand the problem clearly; this may involve representing it in different ways, identifying different variables and establishing the existing relationships between variables. The problem solver would then devise and implement a strategy for solving the problem. Having found the solution, s/he then reflects on the solution and checks if it makes sense; that is, interpreting it in terms of the original problem situation. The question is now "how can these problem solving skills be developed in the learners?" This then brings us to the discussion on problem solving as an approach to teaching of algebra.

ii) Problem solving as a means to algebra learning

Problem solving skills cannot be taught to learners directly. They should be acquired through real experience of solving problems. Hiebert, Carpenter, Fennema, Fuson, Human, Olivier &Wearne (1996, p. 12) argue that mathematics instruction should be geared towards students "problematizing the subject". According to them, the subject matter should be made more problematic, that is

> allowing students to wonder why things are, to inquire, to search for solutions and to resolve incongruities. It means that both curriculum and instruction should begin with problems, dilemmas, and questions for students (Hiebert et al, 1996, p. 12).

Learners should be given opportunity to grapple with the mathematics and discover the solutions themselves. The result of this is that learners will invent their own knowledge and gain better understandings of the subject matter. The problems posed should offer learners opportunity to acquire the symbolic language and be able to represent information in different ways. They should be able to conjecture about situations, critique and justify their solutions. Thus through logical reasoning, they would be able to make decisions about the problem at hand.

The teaching here is aimed at enhancing problem solving skills in the learners. Use of models is also very vital in problem solving. Learners should be able to analyze the problem confronting them, identify the variables, establish the relationships between the different variables, and tackle the problem. They should be able to use the different representations for situations. A mathematical problem situation may be represented algebraically, be it graphically or in the form of equations. Mathematical models serve as a bridge between the informal contexts, related mathematics and a more formal (abstract) mathematics (Van den Heuvel-Panhuizen, 2000, p. 5). As these forms can be easily manipulated, a solution may be sought using these representations and the results then interpreted in terms of the original problem situation. This would therefore increase learners' understanding of the problem and help them make better decisions about the situation confronting them.

Indeed, contextualized mathematical problems are ideal for enhancing problem-solving skills. They offer learners opportunity to move from one level of understanding to another, from informal solution procedures to more advanced, formal mathematical understandings. Learners first develop solution strategies (representations) that are closely related to the context and later on, the context can be treated more generally thus acquiring the character of a model that can then be used to solve problems of a similar nature. In this way, the model shifts from being only of a particular situation to being a model for a certain class of situations. Through reflection on activities at the lower level, the learner is able to move to the next level of understanding (van den Heuvel-Panhuizen, 2000, p. 6).

This approach has many implications on the part of the teacher, with regard to their role in teaching through problem solving. These concern mainly the nature and organization of problems, and the classroom environment and culture.

a) Nature and organization of problems

Teaching through problems is very demanding on the part of the teacher. It demands that the teacher is very conversant with the curriculum and knows his/her learners' mathematical abilities very well. The teacher needs to very resourceful and creative in designing the problems. This needs a lot of time and commitment.

Teaching through problem solving requires that the teacher has the learners' interests at heart and is determined to see them gain mathematical understandings. The teacher should therefore have a collection of problems from where learners would choose. The problems should cater for a wide range of abilities so that at the end of each session everyone would have been successful. This has motivational value, as learners would always look forward to the next session with all eagerness. The problems should also have multiple paths to solutions, thus allowing learners to use their existing knowledge and ideas to get to the solution. In executing their solution strategies, learners expand on their knowledge and grow in their understanding as they interact with the problem and listen to other students' ideas as well (Van de Walle, 2004, p. 38).

Problems that learners engage with should take into consideration the mathematical knowledge that learners have acquired but should still challenge their thinking. As has been discussed earlier, they should be those that are appealing to the learners. They should be related to the learners' experiences and be in situations that learners can imagine and understand. The problems should provoke learners' cognitive abilities, engage them in mathematical "habits of mind" (Cuoco, Goldenberg, & Mark, 1996, pp. 378-383) and develop their mathematical power. Cuoco et al (1996, pp.3-8) suggests that, learners should be pattern sniffers, experimenters, tinkerers, describers, inventers, visualizers, conjecturers, and guessers. The problems they tackle should be focused on the mathematical concepts that learners are to learn. This means that in teaching algebra, the problems chosen should be based on the current algebraic knowledge and skills that learners have and be aimed at enhancing acquisition of knowledge that is more advanced and skills. Problems that they engage in should require them to use algebraic reasoning to justify their procedures and thus solutions.

Teaching through problem solving also means confronting learners with different problems with varying levels of complexity. This is important, as the problems would be representative of problem situations that learners encounter in real life (Van den Heuvel-Panhuizen, 1996, p.13). Problems that learners encounter in real life are not all solvable; some require a certain

level of maturity while some may just need more time before a solution could be realized. Problems of this nature thus develop in the learners, perseverance towards problem solving.
I now turn to another feature of the recommended teaching approach, interaction and guidance.

2.3.5.3 Interaction and guidance principle in teaching and learning of algebra

The discussion in this section is based on Vygotsky's theory of "Zone of Proximal Development' (ZPD) (Fry, Ketteridge, & Marshall, 2009, p. 21). In his theory, Vygotsky emphasizes the role of social and cultural context in enhancing learning. He suggests that learners can be moved up their learning ladder more quickly through guidance and support by an elderly person (Fry et al, 2009, p. 21). The discussion therefore outlines the role played by the teacher, in facilitating learning by steering interactions between learners, teacher and the subject matter.

Embedded within the principle of *interaction and guidance* is the idea that the classroom environment should offer learners opportunity to exchange ideas and share their experiences as they interact with the subject matter, they should be able to share their strategies and reinventions with each other (van den Heuvel-Panhuizen, 2000, p. 9), under the guidance of the teacher. According to NCTM (1991, p. 3), a mathematics classroom should be seen a community of learners, characterized by discourse. "Discourse refers to the ways of representing, thinking, talking, and agreeing and disagreeing that teachers and students use to engage in [mathematical tasks]" (NCTM 1991, p. 20).

Setati (2005, p. 449) identifies four types of discourses that are common in mathematics classrooms, *procedural*, *contextual*, *conceptual* and *regulatory discourses.* Procedural discourses in an algebra lesson, concerns taking learners systematically through steps followed in carrying out an algebraic process. An example of this would be showing learners how to solve a linear equation through algebraic manipulations. *Contextual discourses* here, would surface when learners are helped to understand the context within which the relationship between the variables represented in the equation is established- this does not include explanations about the relationship between the variables. The latter would be left to the learners to formulate.

i) Conceptual discourses

In an algebra lesson, conceptual discourses concerns leading learners into understanding reasons behind performing specific operations in solving the problem (Setati, 2005, p. 449). When learners explain their own ideas, evaluate others' methods of solving problems and pose questions for others to explore, they can develop deeper understandings of algebra (NCTM 1991, p. 21). *Self-explanation* is a metacognitive strategy that facilitates knowledge construction (Aleven & Koedinger, 2002, pp. 149-150) as such needs to be supported by creating a classroom atmosphere conducive to learning. the teacher is seen as a facilitator of learning, guiding learners through different levels of understanding, (Van den Heuvel-Panhuizen, 2000, p. 9). The teacher may create a classroom culture where learners at the end of every problem solving session, they would answer questions:

- how did you solve the problem?
- why did you solve it that way? and
- why do you think your solution is correct or makes sense? Van de Walle (2004, p. 52).

The teacher may also promote discussions by asking learners to comment on answers suggested by another learner. This may be through whole-class or small group discussions. When learners are provided with opportunity to talk about mathematics, with the teacher or other learners, they verbalize their thinking, justify their solution procedures, ask for clarifications, etc. This promotes their construction and reconstruction of their own knowledge and understandings. This also helps the teacher to connect the learner's own language with the specialized algebraic language.

The teacher may also ask learners to clarify and justify their ideas in writing. S/he should listen to learners' ideas carefully, monitor their ways of doing things, e.g. their solution strategies, and where necessary, intervene by probing instead of giving hints. Through this type of discourse learners are provided with opportunity to make their thinking public and through others' questions and ideas, learners negotiate meaning, modify, and refine their conceptual understandings.

Effectiveness of this type of discourse in promoting learning, however, would be achieved when learners are taught to respect one another's ideas and hence engage in constructive criticism. Fuson, de la Cruz, Smith, Cicero, Hudson, Ron & Steeby (2000, p. 199) suggest a "separate knowing" strategy where "all listeners can separate an idea from the person having the idea, so an idea can be respectfully modified or improved, or errors fixed without diminishing its originator". They further note that "debugging errors helps everyone learn more deeply; everyone including the teacher makes errors in math; [so there should be] no laughing or making fun about errors; errors deserve respectful and sensitive help from everyone" (Fuson, et al., 2000, p. 199)

It is important to note that even though teachers may try to encourage all learners to participate in whole class discussions, some learners may still be uncomfortable and prefer engaging in mall group discussions. Small group discussions thus provide more opportunity for participation of learners. They relieve the teacher's workload in that there would be more learner-learner interaction as opposed to teacher–learner interactions in whole-class discussions. In small group discussions, learners acquire team-building skills. They practice tolerance, negotiation and collaboration within one another in solving problems. (Retrieved on 18th, April 2013, from www.umuc.edu/ctl/upload/smgroups.pdf).

ii) Regulatory discourses

These type of discourses are not concerned directly with development of concepts, but rather with motivating, mobilizing and encouraging learners to focus on their learning, thus regulating learners' behavior towards achievement of learning objectives Setati (2005, p. 449). Positive reinforcement and motivation for work well done is very important in learning. The teacher should therefore be conscious about reinforcing those who were able to provide convincing solutions and even to those who did not get to the solution. Nobody should be made to feel like a looser. Being able to provide learners with problems with varying levels of difficulty and allowing them to choose problems to solve would provide motivation to learners as they succeed in solving problems. They would be encouraged to do even more.

Learners should learn to persevere, in problem solving. At first, they should be encouraged to work individually and then discuss their work in small groups. They should also be asked not to help one another, as this would deny them opportunity to enjoy getting the solutions by themselves. They should know that putting a problem aside when the solution is not realized, gives them time to rethink of other strategies to use and rework on it. The teacher should also know the right time to provide intervention so that learners, much as they must be given time to grapple with problems, they should not be left to toil on a problem until they get frustrated and discouraged.

In Lesotho, mathematics is taught in a second language (English), which is the language of teaching and learning. This can have a negative impact on the assimilation and understanding of concepts, as communication is not smooth. It is the responsibility of the teacher to help learners to develop ways of talking about mathematics and understanding what is being taught. Language should be viewed as a tool for thinking and communication (Setati, 2005, p. 448). Code switching occurs when a person uses two or more languages interchangeably during a conversation and it is a technique that teachers could use to use in multi-lingual mathematics classrooms to improve communication. Switching to learners’ main language facilitates their thinking process as they access information. Code switching can be used when an emphasis is made, for ease and efficiency of expression, to interject in a conversation. Teachers should be careful, however, on the extent to which code switching is allowed in the classroom. Excessive use of this technique can slow down learners’ process of developing fluency in the Language of Teaching and Learning (LoTL). (Setati, 2002, pp. 13-16).

From the above discussions, it becomes very clear that problems based in meaningful contexts provide a basis for developing problem solving, representation, connections, and reasoning about mathematics. On the other hand, Kriegler (2007, p.3) suggests that problem solving, representation and reasoning should be contexts within which the fundamental ideas of algebra develop. It needs to be emphasized that successful learning also requires from the learner, "a conscious approach to cognitive tasks, studiousness, curiosity, tolerance, self-criticism, realistic attitude, open-mindedness, active learning and organization" (Kostova & Atasoy, 2008, p. 52).

The purpose of the above discussions was to provide background information on what constitutes effective teaching and learning of school algebra, as seen through the LJC curriculum. This was done by first defining algebra and finding out what and how algebra is taught in schools, particularly at Grade 9 level. The discussions provided a detailed account of what algebra is expected to be taught and what should actually happen in the classrooms with regard to algebra teaching and learning. The types of problems learners should encounter as they learn algebra were also highlighted.

This study was aimed at exploring experiences that Grade 9 learners might have when they are taught algebra through use of spreadsheets. The next section is focused on the role that spreadsheets may play in the teaching and learning of algebra.

2.4 A Possible Way Forward: Teaching and Learning of Grade 9 Algebra Through Use of Spreadsheets

This section first provides a general overview on the role of technology (computers and calculators) in the teaching and learning of mathematics. Some reference is made on perceptions that some teachers have regarding use of these technologies in mathematics classrooms. Particular attention, in this section, is paid to the use of spreadsheets, specifically *Excel*, in the teaching and learning of Grade 9 algebra. A description of what spreadsheets are and what affordances they have to enable acquisition of algebraic concepts, and some potential sources of difficulties caused by the use of spreadsheets in the learning of algebra is also provided. The teacher's role in the algebra classroom within spreadsheets environment is also discussed. A theoretical framework is developed and a Hypothetical Learning

Trajectory (Simon, 1995, p. 136) outlining the learning objectives and the sequence of activities for the study is provided in the last section of the chapter.

2.4.1 The role of technology (computers and calculators) in mathematics education

There are some contradicting ideas concerning use of technology in the form of computers and calculators in mathematics classrooms. Amongst mathematics educators, including teachers, there are those who still believe that these forms of technology will do more harm than good to pupils' mathematics learning, particularly regarding development of arithmetic skills. On the use of calculators, these critics express fears such as pupils "won't learn the basics", " calculators make students lazy", "students will be overly dependent on calculators"(Van de Walle, 2004, pp. 104-105). On the use of computers in mathematics classrooms, the concerns were on the likely misuse by the learners and that the technology could very much distract pupils from their intended learning activities.

The fears about the use of these technologies may be attributed to ignorance regarding how and when these technologies are used in the teaching–learning situation. For example, there are certain activities with a calculator that may be carried out to develop mental computation and estimation skills, which are the "basics" that the critics talk about (Van de Walle, 2004, p. 104). Perhaps it also needs mentioning that irrespective of the medium of instruction, mastery of computation skills and development of number sense will remain important goals in mathematics teaching, and thus may never be compromised.

Waits and Demana (2000, p. 57) argue out that as long as mathematics is still viewed as a "bag of tricks and rules to memorize for computing or solving something", then people will not be able to appreciate that appropriate use of technology can enhance mathematics teaching and learning. Instead of viewing calculators as promoting laziness in learners, they should be appreciated as tools that relieve learners from the tedious work of paper-and-pencil computations, enabling them to focus on the important aspects of learning mathematics such as problem solving. Waits and Demana (2000, p. 56) note, "calculators reduce the drudgery of applying arithmetic and algebraic procedures when those procedures are not the focus of the lesson".

The above-cited fears and others not mentioned here necessitated a lot of research with regard to use of technologies in mathematics classrooms. Based on such research, which advocates for use of these technologies in mathematics teaching and learning, many benefits that may be derived from the integration are shown. The National Research Council (NRC) (2001) report indicates that calculators can be used for developing mathematical concepts such as numeration and computation even from as early as from Grade 4. They could be used very successfully in drill exercises and thus enhance problem-solving skills. Waits and Demana (2000, p. 56) further affirm, "we have learned that calculators cause changes in the way we teach and in the way students learn". They indicate that use of calculators in mathematics classrooms enhance better understanding of mathematical concepts, they develop positive attitudes in the learners towards mathematics, and help learners visualize multiple representations of mathematical situations.

Computers may be viewed as management, communication, evaluation, motivational as well as cognitive tools (Peressini & Knuth, 2005, p. 278). Computers can help both teachers and learners to organize and do their work more efficiently. Teachers can use computers for administrative purposes, such as organizing their records, preparing teaching/ learning activities and modifying multiple aspects of their classroom responsibilities more effectively. Teachers can access more information on their subject content through the internet. They can also communicate within and across schools and share ideas about mathematics teaching. The interconnections in the computer laboratory can also enable the teacher to have access to individual learners work on screen to monitor their progress.

Computers may also function as cognitive tools. Cognitive tools, according to Jonassen and Reeves, (2001, p. 693) "refer to technologies, tangible and intangible, that enhance the cognitive powers of human beings during thinking, problem solving and learning". Examples of such tools include written language, mathematical notation, computers, etc. As cognitive tools, computers like calculators, provide learners with new and more dynamic ways of exploring and engaging with mathematics. When using cognitive tools and interactive learning environments learners engage in knowledge construction. Due to the dynamic nature of computer-based instruction, learners become active participants, focused on their own learning.

The cognitive tools activate complex learning strategies as well as critical and analytical thinking. They extend the mind, reorganize mental functioning and engage learners in high level generative processing of information. (Jonassen & Reeves, 2001, p. 697). In using the cognitive tools, learners' problem solving skills are enhanced and they (learners) function as designers; they analyze the situation at hand, access information, interpret and organize their own knowledge and represent what they know to others. Jonassen and Reeves (2001, p 695) further assert, "some of the best thinking results when students try to represent what they know". In order to be able to represent what they know, learners think in meaningful ways to use the capabilities and features of the cognitive tools.

Cognitive tools support reflective thinking (Jonassen & Reeves, 2001, p.697). As learners engage in problem solving, they think about the problem situation, reflect on what they know, make inferences about the situation, determine implications and reason about it. Cognitive tool thus support reflective thinking when they enable learners to construct new knowledge by adding new representations, modify old ones and compare the two. Knuth & Hartmann (2005, pp.151-163) also demonstrate, with several examples, how computers may be used to foster students' mathematical understandings and intuitions. It is imperative therefore that these tools are made accessible to learners within the context of learning to support the learning process (Jonassen & Reeves, 2001, p. 696). It is worth-noting that even though these technologies have the potential to impact positively on learning, their effect depends on the learners' levels of engagement with the tasks afforded by these tools. (Jonassen & Reeves, 2001, p. 696).

Clement and Sarama warn that "all children can benefit from high quality use of technology and all [can be] harmed by inappropriate use" (Clement & Sahara, 2005, p. 60). This statement calls for caution on educators about care that should be taken in their efforts to integrate these technologies in their mathematics classrooms. Careful planning of lessons is very vital as that may help direct the lesson towards the desired goals. Oldknow and Taylor (2000, p. 181) add that in planning for teaching using ICT, much as the objectives of the lesson must be very clear to the teacher, it is important that the teacher identifies ways in which the selected ICT will be used to meet the targeted teaching and learning objectives.

Teaching-learning tasks should be structured such that they only focus on relevant aspects and maximize use of time and resources. Waits and Demana (2000, pp. 58-59) also urge that appropriate use of technology in mathematics teaching and learning should be that of a balanced approach, whereby both technology and pencil-and–paper techniques are essential. They further demonstrate that engaging students in tasks that involve the following strategies could achieve this balance:

1. Solve problems using paper and pencil and then support the results using technology
2. Solve problems using technology and then confirm the results using paper-and pencil techniques
3. Solve problems for which they choose whether it is appropriate to use paper and pencil, technology or a combination of both (Waits & Demana, 2000, p. 59).

The teacher should also consider possible impact of the ICT use on the organization and conduct of the lesson and devise appropriate management strategies. He should be able to ask key questions and identify opportunities for teacher intervention in order to stimulate and direct pupils' learning. Chazan (1999, p. 122), further elicits that "technology-by virtue of the ways in which it restructures activity and provides new opportunities for analysis and solution of problems can shape the thinking of teachers and curriculum developers, in addition to supporting the learning of students".

Assessing and monitoring learners' progress in an ICT environment should also form part of the teacher's concern. There has to be set criteria to ensure that judgments about pupils' attainment and progress in mathematics are not masked because ICT has been used. Priority should be given to the quality of learning reflected by pupils' work not just the manner in which it is presented. Different methods of assessment, formative, diagnostic and summative assessment should be used to monitor learners' progress and evaluate their achievement.

In summary, technology should only be used if it enhances cognitive development of the learners, thus at this point I need to look critically into the affordances that spreadsheets provide with regard to how they may function as cognitive tools in the context of algebra learning.

2.4.2 Teaching and learning of algebra in a spreadsheets environment

Algebra is one branch of school mathematics that causes many difficulties that learners have with the subject. This state of affairs triggered a lot of research into the field(Kieran, 1988, 1989; 2004; Usiskin, 1988, 1997; Lee, 1996; Charbonneau, 1996, wheeler, 1996, Herbert & Brown, 1997;French, 2002; Kieran & Yerushalmy, 2004; Kriegler, 2007), trying to identify the nature and sources of learners' difficulties, and ways to combat those with the advancements in technology. Further research studies are still being carried out to seek strategies that may be more appropriate in teaching this "problematic'' area such that learners are able to progress in their mathematics related careers without any difficulty.

Computer programs and software packages have been designed to address both teachers' and learners' problems regarding algebra instruction. Some of these packages need to be purchased while others may be downloaded free from the internet. The implication here is that there has to be some form of funding in order that teachers and learners could access these teaching-learning tools. With continuing research, mathematics educators have found out that spreadsheets, though initially developed to address issues in Accounting (Forman & Steen, 2000, p. 144), can be very useful in the mathematics classrooms as a medium through which algebraic concepts could be developed. Unlike other software packages, spreadsheets are readily available as they come as part of every computer. Thus, this makes them more accessible and hence more appropriate for the classroom use.

2.4.2.1 Potentials and affordances provided by spreadsheets for teaching and learning of Grade 9 algebra

This section outlines the potential use of spreadsheets for the teaching and learning of school algebra. This is achieved by first providing an overview of how spreadsheets may be function as cognitive tool for teaching and learning of school algebra. This is followed by a description of how spreadsheets may be suitable for use to enhance learning of some concepts (concept of variable; simplifying expressions; solving equations, graphs) within the Grade 9 algebra curriculum.

Arganbright (1984, p.185) gives the following description of a spreadsheets program:

> A spreadsheets program uses a large matrix whose rows are identified by positive integers and columns by letters. Locations, or cells, in the matrix are identified by row and column. Each cell can contain a descriptive label, a number, or an algebraic expression that refers to other cells in the spreadsheets. The program calculates values for the expressions using the values of the cells to which the expressions refer and displays the evaluated spreadsheets on the computer screen. (Arganbright, 1984, p. 185).

Figure 2.3is an illustration of the features of spreadsheets.

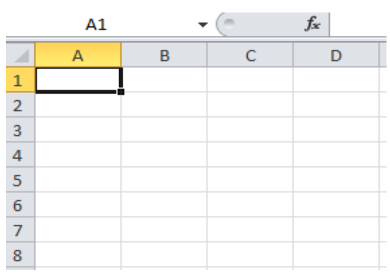


Figure 2.3 Part of *Excel* Spreadsheetsss

Spreadsheets, as cognitive tools, may be used for amplifying and reorganizing mental functioning. Spreadsheets programs were mainly used to create budgets and manage accounts. They affected ways of dealing with accounting process in handling large sets of data. They can be used to perform various mathematical calculations more quickly and accurately. Spreadsheets can also be used to store and modify data quite easily. When data used in a calculation is altered, the rest of the data resulting from the calculation is automatically modified. Spreadsheets can also be used to perform multiple calculations and allow the user to handle large sets of numerical data. They can also convert numerical data that reflects relationships, into charts and graphs.

It is on the bases of these affordances that spreadsheets were then investigated for potential use in the teaching of school algebra. Research studies (Rojano, 1996; Berdnarz, et al, 1996; Heugl, et al, 1997; Yerushalmy, 2005; Tabach et al, 2008) indicate that spreadsheets like *Excel*, offer opportunities for learners to develop algebraic skills such as generalization, modeling and shifting between different forms of representations (graphical, numerical and symbolization).

Spreadsheets have the potential to bridge the gap between arithmetic and algebra. They provide learners with opportunity to progress from their arithmetical intuitive methods to more algebraic ones (Haspekian, 2005, p. 114). Use of spreadsheets involves rule making, as such it requires learners to be able to formulate rules connecting data presented in the cells. The rules represent some form of modeling of the mathematical situation dealt with. Identifying values and developing rules that show relationship between them enhances understanding of algorithms that are used to compare them. In this way, learners would be working on numerical representations while at the same time working on generalizations and patterns. (Jonassen & Reeves, 2001, p. 712).

Spreadsheets also function as cognitive tools in that they may be seen as learners' intellectual partners sharing the cognitive burden of performing long calculations. Learners are left with cognitive processing of information while the technology does that which it performs best. They "allow students to handle, observe and generate large sets of numerical instances" (Tabachet al, 2008, p. 50) offering them opportunity to conjecture, compare and identify patterns and relationships in data.

Chazan (1999, p. 123) noted that the capacity of this technology to carry out many calculations rapidly supports the transition from examination of single cases towards the examination of multiple cases at once. With their "dragging" power, spreadsheets can enable learners to extend any given sequences of numbers, slowly realize need for generalization, and finally appreciate the power of symbolism (Tabach & Friedlander, 2004, p. 428).

Spreadsheets can thus be seen as tools that "allow children to appreciate the need for an algebra-like notation and provide new ways for children to be introduced to it" Ainley in (Haspekian, 2005, p. 114). From analysis of the learners' work in the Anglo/Mexican Project, Rojano (1996, p.144) also affirms, "spreadsheets environment supported pupils in moving from thinking with the specific to the general, both in terms of the unknown and of the mathematical relationships expressed in the problem". Spreadsheets may thus be seen as tools that may enable learners to generalize, mathematize and communicate mathematical ideas.

The relationship that has been identified and expressed using spreadsheets notation may then be modeled mathematically, using rules to describe relationships in the model. Spreadsheets offer opportunities for learners to model the same situation in multiple ways, at the same time, all on the same display. The models can be symbolic (equation), tabular and graphical. Multiple representations afforded by the spreadsheets help learners to get deeper understanding of the situation at hand. Constructing spreadsheets representation of situations requires abstract reasoning by the user, which is consistent with one of the goals of using cognitive tools. (Jonassen & Reeves, 2001, p. 716).

Spreadsheets may function as cognitive tools supporting problem solving. In the spreadsheets environment, learners engage in more pupil-centered experimental learning where they explore mathematical ideas through supposing, testing, correcting their suppositions, testing again, applying results, proving their results, etc. Working in spreadsheets offers learners opportunity to construct their own conceptualization of the organization of the content domain. In this way the spreadsheets do not make the work easier, but requires from the learner higher thinking skills, processing the information at hand, generating thoughts that would otherwise not be possible without the technology. (Jonassen & Reeves, 2001, p. 697). The immediate feedback that learners get while working on a formula allows them to experiment, conjecture, realize, and correct their errors (Haspekian, 2005, p. 114). Problem solving within spreadsheets reveals all steps taken, showing the progression of calculations as they are carried out. This models all the logic implied by the calculations and making these obvious to the learners, helps them understand the interrelationships and the procedures involved.

Spreadsheets may also be seen as tools that support reflective thinking. They provide for learners' flexibility to quickly rearrange information and re-engage with the activities from fresh perspectives. Here learners can test and reflect on their output. In this way, spreadsheets allow and foster risk-taking and experimentation in an attempt to make sense of a situation at hand (Calder, 2010, p. 2). Heugl et al indicate that problem solving in the age of CAS involves modeling, operating and interpreting versus the earlier approaches to problem solving which viewed it as involving problem formulation and looking for solutions, with calculations as the main activity (Heugl et al, 1997, pp. 37-38).

The next section provides a discussion of how spreadsheets may be used to enhance some basic algebraic concepts and mathematical skills at Grade 9 level.

i) Concept of variable and algebra notation

The concept of variable as a placeholder for a general number may be more visible to learners through spreadsheets. A cell in spreadsheets can be viewed as a container into which numbers may be placed, and a cell address or cell reference describes the physical location of the cell. This cell address may also be used to refer to the particular number in a cell or any number that may be entered into that cell. According to Haspekian (2005, p. 121), the cell argument A1, highlighted in Figure 2.4, may be seen as:

- An abstract, general reference: it represents the variable (indeed, the formula does refer to it, making it play the role of variable);
- A particular concrete reference: it is here a number (in case nothing is edited there, some spreadsheets attribute the value 0);
- A geographic reference (it is a spatial address on the sheet);
- A material reference (it is a compartment of the grid; some pupils can see it as a box). (Haspekian, 2005, p. 121).

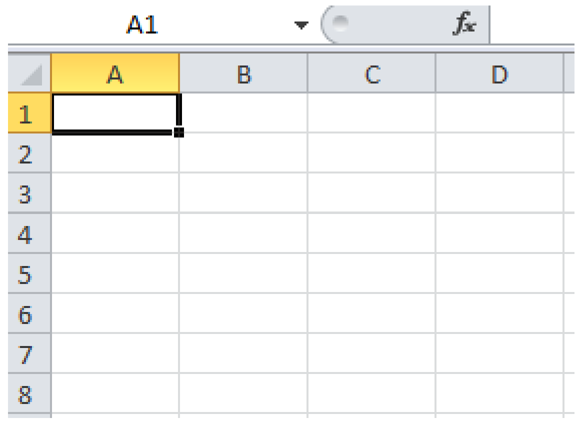


Figure 2.4. Cell argument A1 in Spreadsheetsss

The ambiguity with which a cell reference is used, “offers a strong visual image of the cell as a number container whose contents may be changed” (Bills, Ainley, & Wilson, 2006, p. 42). In this way, cell addresses are concrete representations of variables. The “fill down” command on a column using a formula again supports the idea of “variable as a range of numbers in functional relationships” (Bills et al, 2006, p. 42). On clicking on the individual values that have been generated by filling down, learners may realize that each one of them was produced by substituting entries in adjacent cells in place of the cell reference used in the formula, hence the concept of a “variable column” also emerges (Haspekian, 2003, p. 6). Using a letter to name the variable column and consequently asking them to write a formula for the column with generated values, learners may be able to form a link from the spreadsheets notation to the formal algebraic symbolism and thus appreciate the role of the letters in this case(Ainley, Bills & Wilson, 2005, p. 190).

ii) Simplifying expressions

The process of simplifying expressions through expansion of brackets is also one of the problem areas in algebra learning. Learners tend to give, for example, $2x + 3$ as a simplified form of $2(x + 3)$. When sequences are generated from already formulated expressions in a spreadsheets environment, learners may be led into identifying those that yield similar results and thus appreciate how the distributive property is applied. The enablement that spreadsheets permit working with actual values can be used to develop the concept of equivalent expressions, involving factorization and expansion of brackets. (Tabach &

Friedlander, 2008, p. 29). The two expressions may also be treated as functions, say, f(x) = 2x + 3, f(x) = 2x + 6 and f(x) = 2(x+ 3), and comparing the graphs of the three drawn on the same axes would help learners recognize the differences and similarities.

iii) Solving equations (linear and quadratic)

Solutions of equations may also be done by using tables of values generated through the “dragging” or “fill down” command of spreadsheets. Solving equations in this case may be viewed as identifying the value of the variable for which the output for the expression is a given number. This case may also be seen as comparing two functions, where the task is to identify the value of the independent variable for which the two functions or expressions are equal.

For example, Figure 2.5 shows how spreadsheets may be used in solving for x in $2x + 3 = 3x$.

	A	B	C
1	x	y= 2x +3	g= 3x
2	1	5	3
3	2	7	6
4	3	9	9
5	4	11	12
6	5	13	15
7	6	15	18
8	7	17	21
9	8	19	24
10	9	21	27
11	10	23	30
12	11	25	33
13			

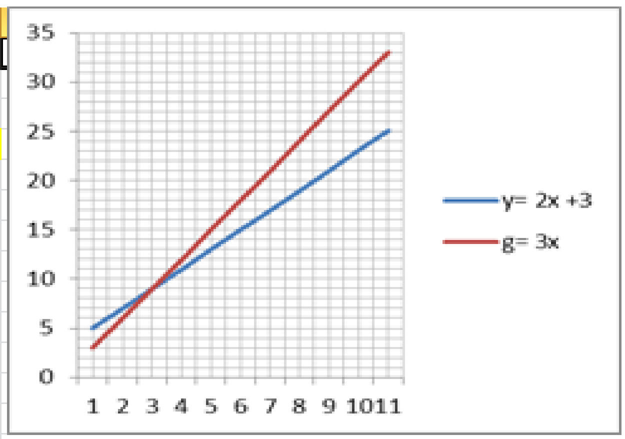


Figure 2.5: Solving Linear Equations in Spreadsheets

Solution of equations may also be done through graphs. In this case, an equation is viewed as representation two expressions that are connected together by an equal sign. Creating separate graphs, on the same pair of axes, for functions formulated from each expression, and identifying the point of intersection, provide the solution for the original linear equation. This is illustrated in Figure 2.5.

iv) Multiple representations

Three main functions served by use of multiple representations in the teaching and learning of mathematics have been identified. Multiple representations may be used to complement each other, that is, one representation may complement information displayed in another form or they may support complementary processes. Using multiple representations for this purpose means that one form of representation would not be sufficient to carry or reveal all information about a situation. Information displayed in these cases may bear redundant features as well as those that are unique to each form, or each may encode different aspects of the domain. (Ainsworth, 1999, p. 137).

Another function of multiple representations is seen when one form of representation may also be used to constrain possible (mis)interpretations from use of the other. In this case, a familiar form of representation is used to help learners make correct interpretations of the less familiar form. The familiar form is used to support learners' cognitive processes as they extend or construct and reconstruct meaning and understanding on the unfamiliar form. (Ainsworth, 1999, p. 139).

Multiple representations can also be used to help learners construct deeper understandings of a situation represented (Ainsworth, 1999, p.134). Kaput (1989) in Ainsworth (1999, p. 141) notes "the cognitive linking of representations creates a whole that is more than the sum of its parts…it enables us to "see" complex ideas in a new way and apply them more effectively". This includes situations where multiple representations may be introduced simultaneously to learners so that they (learners) may be able to establish relationships between them. Integration of information from different representations of a situation permits learners to gain deeper insight into the situation that would rather be more difficult with one representation. (Ainsworth & Van Labeke, 2004, p. 250).

Figure 2.6 shows the different functions for which multiple representations may serve, as viewed by Ainsworth (1999, p.134).

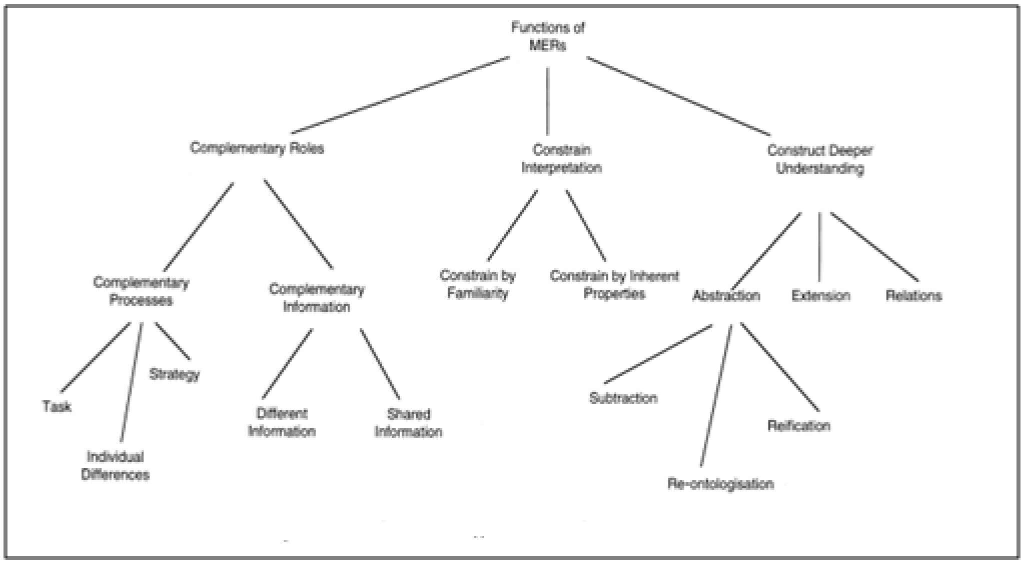


Figure 2.6: A Functional Taxonomy of Multiple representations

The multiple representations (graphical, tabular and symbolic) that spreadsheets permit on the same screen, helps learners to make connections between all data displayed. Even though there are suggestions that translations between different forms of representations are difficult (Ainsworth, 1999, p.132), the design of spreadsheets allows for automatic translation, "dyna-linking". When learners can see the immediate change on a graph, as a result of altering values in a table, this can help them establish the connection between the two forms of representation of the same relationship (Calder, 2010, p. 2). These multiple representations enable the learners to visualize those aspects of a problem that may not be apparent in one form and thus provide for better interpretations and understanding of the mathematical situation at hand.

Within spreadsheets environment, learners can choose to use any of the forms of representation depending on their preference or may use them in parallel as may be determined by the needs of the problem tackled. Depending on their levels of experience and expertise with varying representations, an appropriate combination of representations leaves each learner with freedom to select and exploit that which they feel most comfortable with. (Ainsworth, 1999, p. 136).

Multiple representations enable learners to conjecture critique and justify their solutions. They enable learners to prove correctness or reasonableness of their responses to problem situations (Tabach et al, 2008, p. 49). According to constructivism, best learning results from thinking in meaningful, mindful ways. On working in spreadsheets, learners may represent what they know in multiple ways and this requires them to think mindfully about what the spreadsheets can afford them. Empowering learners to design and produce representations of what they know and share it with each other is a powerful learning experience. (Jonassen & Reeves, 2001, p. 695). In addition, using more than one form of representation is more likely to have a motivational effect on the learners.

v) Exploring properties of graphs

The fact that spreadsheets also allow for specifying a formula for performing a calculation is very important in constructing tables as well as graphs for a range of values for any given relationship. Spreadsheets promote experimentation with data. Learners may be challenged to make extensions to problems and investigate what happens when certain variations are made. This would thus make it possible for learners to explore behavior of graphs of linear equations, under varying values for the gradient and y-intercept. Learners would therefore be able to generalize about the properties of graphs of linear functions. According to Tabach and Friedlander (2008, p. 28), learners' investigations of variation processes can be considered as generational and global /meta-level mathematical activities.

Having reviewed the potentialities that spreadsheets may provide for the teaching and learning of Grade 9 algebra, I now get into the discussion of what teaching and learning algebra through spreadsheets requires, paying particular attention to the kind of cognition involved.

2.4.2.2 Challenges associated with spreadsheets-enhanced instruction

Even though spreadsheets seem to provide opportunity for smooth transition from arithmetic into algebra and also in helping learners in breaking down problems in real life contexts into bits of manageable related pieces of data and thus getting deeper insights into the problem, the complexity involved in the whole process may not be underestimated. Classroom dynamics in a technologically assisted environment may be described in terms of the processes of *instrumental genesis* and *instrumental orchestration,* in which the key players are the learners and the teacher respectively..

i) Instrumental genesis

Learning occurs when individual learners construct their own knowledge through meaning making through social interactions with each other, their teacher and the subject matter situated in their environment. *Instrumental genesis* is described as "concerning mutual transformation of the learner and the *artifact* in the course of constructing knowledge with the technology" (Hoyles et al, 2004, p. 310). The artifact, as used here, may be defined as a "bare tool", the spreadsheets, which is the material or abstract object, that is available to the user to sustain a kind of activity, but which may remain meaningless to him/her as long as he does not know what kind of tasks it can support and in which ways (Drijvers & Gravemeijer, 2005, p. 165). This transformation evolves in two processes: *instrumentalisation* and *instrumentation.*

Instrumentalisation is a process in which the subject shapes the artifact for specific uses (Hoyles et al, 2004, p.313). According to Haspekian, this is a process whereby "various potentialities of the artifact are progressively discovered, or possibly transformed in personal ways" (Haspekian, 2005, p.118). Trouche suggests that this process can progress in different stages: a stage of *discovery and selection* of the functions relevant to the task at hand, a stage of *personalization* in which one fits the artifact to one's hand; and a stage of *transformation* of the artifact, sometimes in directions unplanned by the designer (Trouche, 2004, p. 293).

For example, transformation of the artifact may refer to modification of the task bar, creation of keyboard shortcuts, storage of game programs, automatic execution of some tasks, etc. This is a process whereby the technological tool is transformed into a learning instrument and is a complex process that does not necessarily lead to better mathematical understandings (Guin & Trouche, 1999, p. 195).

The second process, *instrumentation*, is one in which the subject is shaped by actions with the artifact, the spreadsheets (Hoyles et al, 2004, p.313). According to Tabach et al, *instrumentation* "describes the diversity of strategies to solve the same task with the same tools, within the classroom"(Tabach et al, 2008, p. 51). Trouche suggests, "it is a process through which the constraints and potentialities of the artifact shape the subject" (Trouche, 2005, p. 148). In other words, through interaction with it, the spreadsheets shape the thinking of the user. This process also goes through two phases: the explosion and purification phases.

The first phase is the *explosion phase* and is characterized by "oscillation, zapping or over verification" (Trouche, 2005, p.143). The *oscillation phenomenon* involves learners moving between several strategies and techniques, while *zapping* may describe actions such as quickly changing the graph window without having time to analyze each of the representations obtained. *Over verification* refers to the learner's actions in carrying out multiple checks using all means provided by the spreadsheets. This is where learner's work appears as though they were looking for equilibrium between their former paper–and-pencil techniques and strategies linked to use of spreadsheets and various other new potentialities opened up by use of the artifact and the evolution of classroom knowledge. (Trouche, 2005, p.148).

With more interaction with the spreadsheets, the learner then enters the second phase, the *purification phase*. Here learner's strategies and techniques of using the artifact tend to stabilize. The learner is now more confident in using the artifact even though they may not be aware of other uses of some commands. For example, they can only delete using the "delete" command not realizing they could also use the "backspace", depending on the position of the cursor. In addition to this, Verillon and Rabardel (1995, pp. 84-85) indicate that

> A machine or a technical system does not immediately constitute a tool for the subject….it becomes so when the subject has been able to appropriate it for himself - has been able to subordinate it as a means to his ends- and , in this respect, has integrated it with his activity. (Verillon & Rabardel, 1995, pp. 84-85).

In this way, the spreadsheets would only become a cognitive instrument for the learning of algebra, when the learners themselves are aware of what they can do with it, and view it as a valuable and useful instrument, a means to their algebra learning. The instrument here is a mixed entity, part of which is the artifact and then the cognitive *schemes* that make it an instrument.

A cognitive scheme may be defined as the "invariant organization of behavior for a given class of situations" composed of goals, and anticipations, the rules of action, gathering of information, control-taking and the *operative invariants* (Vergnaud in Trouche (2004, p.286). It may thus be regarded as having three functions, the *pragmatic*, where it allows the learner to do something, *heuristic*, in the sense that it allows him/her to anticipate and plan his/her actions and, the *epistemic* functions where the learner is allowed to understand what she/he is doing (Trouche, 2004, p.286). In this context, a cognitive scheme may thus be viewed as a mental organization of knowledge and skills that a learner engages or uses when resolving a problem situation that is intended to lead him/her to discovering knowledge using the spreadsheets. Schemes for carrying out a particular task with an artifact are classified as *utilization schemes.* Two types of utilization schemes are distinguished, the *usage schemes* and the *instrumented action schemes.* (Drijvers & Gravemeijer, 2005, p. 167)

Drijvers and Gravemeijer defined a *usage scheme* is a basic elementary scheme that is directly related to the use of the artifact (Drijvers & Gravemeijer, 2005, p. 167). For example, in a spreadsheets environment, a whole set of numbers may be deleted by highlighting the whole block and then deleting them all at once. A novice learner on the other hand would be deleting each of the numbers one by one. *Instrumented Action schemes* refer to those focused at producing certain kinds of transformations on the objects of study, for example, using spreadsheets on a set of values to produce a graph. Instrumented action schemes may thus be described as coherent and meaningful mental schemes built up from the elementary usage schemes through instrumental genesis. (Drijvers & Gravemeijer, 2005, p. 167). Such a scheme involves both technical as well as conceptual knowledge and skills. It is worth noting at this point that sometimes the distinction between these schemes is not so easy to make, it is merely dependent on the level of operation of the learner. What may be regarded as the instrumented action scheme at a time, may later on become the usage scheme from which a higher order scheme may developed. (Drijvers & Gravemeijer, 2005, p. 168).

The situation where an instrument in this case, spreadsheets, is used to enhance algebra learning is a triad that may be represented as shown in Figure 2.7. This model clearly reveals the complexity involved. There is interaction between technical and conceptual aspects. The *instrumented technique* "concerns the external visible and manifest part of the instrumented action scheme" (Drijvers & Gravemeijer, 2005, p.169), it is the technical part of the scheme. The conceptual part consists of the mathematical concepts that "are the students' mental images of the techniques that they carry out and of the mathematical objects and the procedural knowledge that play a role in this technique" (Drijvers, 2003, p. 8). While technical actions may be seen and observed, it is impossible to assess and observe cognitive schemes directly. It is the visibility of the *instrumented techniques* that makes them a gateway to assessment and monitoring of instrumental genesis.

While in some instances, the technological tool may be viewed as "mediators between the subject and the object of his actions, due to their generative or amplifying character which these technologies confer on the subject's action, the appropriation of such objects at school age raises new questions concerning the relation between learning functioning and development" (Verillon & Rabardel, 1995, p. 80). Learning of algebra within spreadsheets may also bring additional problems to the teaching-learning situation.

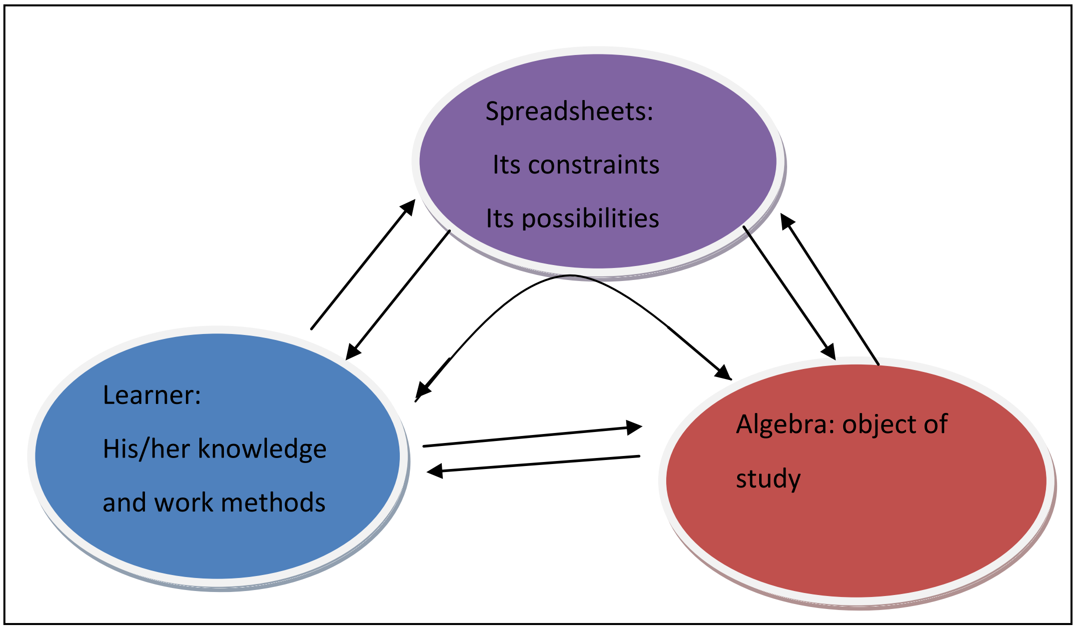


Figure 2.7:The Instrumented Activity Situation (IAS) Model(Modified)(Verillon et al, 1995: 85)

Learning obstacles in the spreadsheets environment are barriers provided by use of spreadsheets, which prevent learners from utilizing the mathematical schemes that they have in mind to solve mathematical problems (Drijvers, 2000, p. 189). These obstacles may be two-fold: those related to the technical knowledge (technical challenges) and those related to personal aspects of the learner (personal challenges).

a) Technical challenges

Learning of algebra in a spreadsheets environment requires use of traditional mathematical knowledge, constant interplay between algebraic knowledge and knowledge about the spreadsheets, and knowledge related to the computer hardware (Pierce & Stacey, 2004, p. 61). These forms of knowledge can be visualized as existing on a continuum, where on one extreme is knowledge about the machine only and on the other is knowledge about algebra only and the interplay between these forms of knowledge lies in-between. The substantial body of knowledge involving the algebra and the machine is what is termed *Technical knowledge* (Pierce & Stacey, 2004, p. 62). Technical challenges within spreadsheets-based instruction may, therefore, be described as those that are brought about by the constraints imposed by the artifact. Trouche distinguishes between three types of constraints:

1. Internal constraints intrinsically linked to the hardware;
2. Command constraints linked to the existence and to the form(i.e. syntax) of various commands
3. Organization constraints (i.e., linked to the organization of the keyboard and more generally, of the interface between artifact and the user.(Trouche, 2004, p 290)

Internal constraints refer to those related to the knowledge about what the spreadsheets can do, about the internal representation of objects and their calculation processing. Spreadsheets do not reveal to the user how calculations are performed and how graphs are drawn- only the results are visible. C*ommand constraints* are related to knowledge about the available commands and their associated syntax. *Organizational constraints* relate to access and organization of the available keys and commands.

The level of individual learner's technical knowledge determines the effectiveness with which s/he learns the targeted algebraic concepts. Hoyleset al, (2004, p. 311) further warn that use of computers in mathematics classrooms adds new complexities which not recognized may lead to rejection of mathematical meanings and discourse fostered by computer use, due to the inevitable discrepancy between this and the official mathematical discourse of the classroom. For example, according to Wilson et al (2005b, p. 322), the spreadsheets language, though related to the usual algebraic language, may pose difficulties in learners' efforts to use spreadsheets in solving algebraic problems. The translation from the spreadsheets formula to the formal algebraic language may not be so obvious to all learners (Wilson et al, 2005b, p.322). Spreadsheets do not reveal to learners how certain operations are done as opposed to their pencil-and-paper techniques. It may also be noted that the technical "fill down" command, may not be easily interpreted algebraically.

Any limitations on the conceptual or technical knowhow regarding the use of the spreadsheets or on the relevant content knowledge required for the acquisition of the targeted algebraic concepts will create problems on the implementation of the scheme. Instrumental genesis involves evolution and development of usage and instrumented action schemes. The rate at which learners progress through instrumental genesis, depends on their (learners) relationship with both the mathematics and the involved technologies (Defouad in Trouche, 2005, p. 201).

The amount of time available for learners to interact with the technology and the learners' confidence and competence in mathematics determine the pace at which they progress through instrumental genesis. This is determined by the availability of the machines and the way in which the teacher takes the artifact into account (Hoyles et al, 2004, p. 311).

b) Personal challenges

*Attitudes***.** When learners are introduced to the technology, they begin to develop ideas as to what the tool can and should do. These ideas are primarily based on their experiences and beliefs, conversations with peers and or their perceived nature of the classroom activities they are about to perform (Tabach et al, 2008, p. 51). When such experiences are negative then learners may lose interest in using the technology. When the experience is positive then learners would show high level of engagement with the technology. The individual learner's attitudes, confidence levels, cognitive and emotional styles and social identities, can influence their participation in the use of technology (Hennessy, Ruthven, & Brindley, 2005, p. 162; Reed, Drijvers, & Kirschner, 2010, p. 2).

Reed et al (2010, p. 3) note that learners' attitude towards the subject domain and the technological tool (spreadsheets in this case), and the kind of behaviors they undertake when using such tools appear to moderate what they learn and how they learn it. They suggest that learners who have a positive attitude towards mathematics and the spreadsheets would tend to persevere in learning through the technology, eventually overcome their initial challenges, and even go further to explore more mathematics and what the spreadsheets can offer. Pierce and Stacey (2004, p. 71) refer to the latter as judicious use of spreadsheets by learners. This happens when they have developed facility and confidence in using the tool and have developed appreciation of what the tool can do.

Learners who have a negative attitude towards the mathematics domain studied and the technology, tend to despair very easily and resort to withdrawal behaviors. (Pierce & Stacey, 2004, p. 71). They tend to avoid use of the technology that further leads to lack of success in using it to gain better understanding of the learnt concepts. It was also observed that students, who just focus on the mechanical manipulation of the technological tool, lose sight of the meaning of the underlying mathematical concepts, while those who show reflective behavior are able to connect the technical and the conceptual parts of the mathematical activity (Vom Hofe, 2001, p. 1170). Barkatsas, Gialamas, & Kasimatis (2009, p. 9), further indicate, "the two factors that seem to be associated with the development of a positive attitude to learning mathematics with computers are mathematics confidence and affective engagement". In addition to these factors, Forgasz (2002, p. 369) also notes that

> Compared to males, females are generally reported to be less positive about computers, like them less, perceive them as less useful, fear them more, feel more helpless around them, view themselves as having less aptitude with them, and show less interest in learning about and using computers. (Forgasz, 2002, p. 369).

The implication here is that, female learners would usually show lower levels of confidence, hence lower performance in tasks that involve use of computers than their male counterparts.

Mureșan, Catalano and Bocoș (2010, p.70) indicate that use of technology in the form of computers in a lesson may have a negative impact on the learning process. Technology is increasingly becoming part of our lives and there is a general eagerness for people, particularly the youth, to be up-to-date with the developments thereof in order to live as part of this rapidly changing and fascinating world. Mureșan et al noted that

> The curiosity, challenge, the excitement when encountering novelty, the specific attraction of the cyberspace, the desire to inform, the model and social behavior of the people of the same age are all motivational factors which bring primary school students into virtual reality, which they explore and try to learn its mechanisms, so that they can become part of this fascinating world (Mureșan et al, 2010, p. 70).

While excitement may be good for individuals, to inspire them to learn and explore, however, this may also prevent learners from concentrating in the lessons and make them to be just too eager to manipulate the technology. Even though this was directed to children at the primary school, secondary school learners, with limited exposure to the technology, may experience the same effect as well.

Learning Styles. People prefer different learning styles. According to Hu, Hui, Clark, & Tam, (2007, p. 1101) a learning style may be viewed as "characteristic behaviors of a learner that can serve as a relatively stable indicator of how he or she perceives, interacts with, and responds to a learning environment". Kolb, Rubin and Osland's(1990) learning style model, represented in Figure 2.8, distinguishes between four distinct learning styles, namely: *Accommodating, Diverging, Assimilating and Converging.* These learning styles are based on the four-stage learning cycle that individuals go through in the process of acquiring knowledge.

According to the model, the four stages in the learning cycle begin with C*oncrete Experiences (feeling),* where a new experience of a situation is encountered, or a there is reinterpretation of existing experience. The next stage involves *Reflective Observations (watching)* of new experience, where any inconsistencies between experience and understanding are particularly important. The third stage is that of *Abstract Conceptualization* (*thinking*) in which reflection gives rise to a new idea, or a modification of an existing abstract concept. In the last stage, *Active Implementation (doing),* the learner applies the learned concepts to the world around him/her to see what results. (Retrieved on 15 April 2013 from http://www.simplypsychology.org/learning-kolb.html)

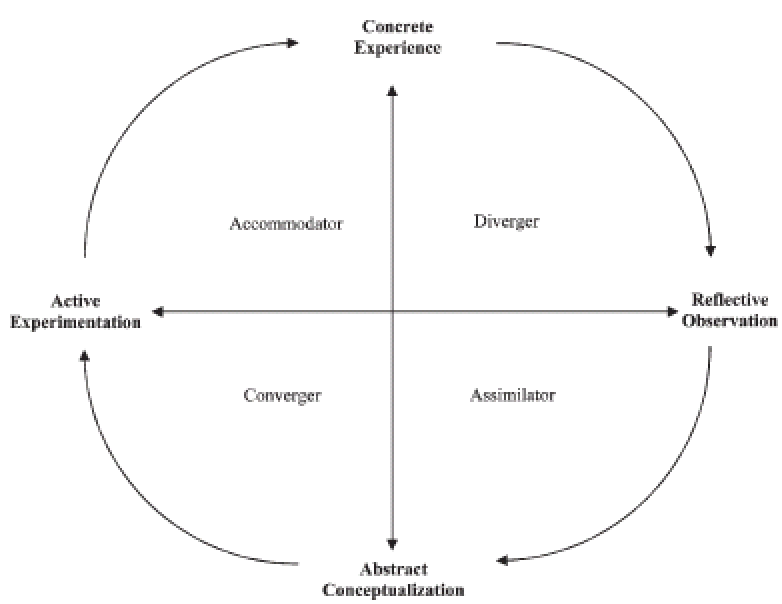


Figure 2.8: Kolb et al.'s Learning Styles Model

McLeod (2010) posits that, in any learning situation, including technology-enhanced learning situations, individuals first decide on whether they want to do, watch, think or feel and that the learning style that is going to be adopted will be a combination of any two of the choices made. Table 2.2 shows the different learning styles and the combination of learning choices from which they arise.

	Doing (Active Experimentation-AE)	Watching (Reflective Observation-RO)
Feeling (Concrete Experience-CE)	**Accommodating** (CE/AE)	**Diverging** (CE/RO)
Thinking (Abstract Conceptualization-AC)	**Converging** (AC/AE	**Assimilating** (AC/RO)

Table 2.2: Kolb's Learning Styles

(*Retrieved* from http://www.simplypsychology.org/learning-kolb.html)

Diverging learners prefer concrete experiences and reflective observation. They prefer watching than to doing, and perform best when in situations that require generation of ideas, hence like working in groups. *Assimilating* learners are sensitive, and prefer watching (reflective observation) and thinking (abstract conceptualization). They also require clear and concise explanations of things, and prefer reading and having time to work through tasks, organizing information in a clear logical manner. *Converging* learners prefer thinking(abstract conceptualization) and doing (active experimentation). They use learning to find solutions and prefer technical tasks to social/personal ones. *Accommodating* learners prefer doing (active experimentation) and feeling (concrete experiences). They tend to be hands-on, value intuition than logic and enjoy new challenges and experiences.

McLeod (2010) further suggests " everybody responds to and needs the stimulus of all types of learning styles, to one extent or another-it's the matter of emphasis that fits best with the given situation and a person's learning style preferences". It therefore becomes the teacher's responsibility to plan, design and conduct instructional activities within a spreadsheets-enhanced environment, such that all learners, with their different learning styles, are catered for. Learning opportunities should provide for identification and strengthening of the individual's lesser-preferred learning styles. Montgomery and Groat (1998, p. 7) note that learning styles are not immutable; they can be modified over time and for different classroom contexts. The teacher should also monitor behaviors and attitudes exhibited by learners during instruction with spreadsheets and provide the necessary support. The purpose of this research was to investigate what challenges learners encountered in their algebra learning through spreadsheets and the strategies that were used to overcome the identified challenges.

ii) Instrumental Orchestration

Orchestration here refers to the external steering of learners' process of instrumental genesis in order to enhance their learning of mathematics (Trouche 2004, p.296). Teachers are the key players in this orchestration process. The purpose is to help learners to develop effective relationship with the technology to reach their mathematical learning goals. The word "effective" is used here to mean, "helping learners engage with, develop and articulate understandings of mathematical procedures, structures and relationships through the technology and according time and status to this process" (Hoyles et al, 2004, p.311). Understanding the role that teachers are to play in the spreadsheets-based learning environment would throw light on the nature of the mathematical knowledge involved, its relationship with the planned official algebra curriculum and help clarify how the use of spreadsheets in algebra teaching and learning may be facilitated. Drijvers, Doorman, Boon, Reed and Gravemeijer (2010, p. 2), distinguish three elements within instrumental orchestration. These are didactical configuration, an exploration mode and a didactical performance.

Didactical configuration is defined as "an arrangement of artifacts in the environment… a configuration of the teaching setting and the artifacts involved in it" (Drijvers et al, 2010, p. 215). *An exploitation* mode refers to the decisions the teacher makes concerning how s/he is going to use the didactical configuration towards attaining teaching objectives. This includes a plan on how the lesson would be introduced and developed, the anticipated role the spreadsheets would play in enhancing the learning, paying attention to the kind of knowledge and skills the learner would acquire. It is the responsibility of the teacher to determine when and how the spreadsheets are used to support learning. All learners need to be provided time to work on their own, at the same time they also need to be given opportunity to share ideas and learn from their peers. (Anthony & Walshaw, 2009, p. 151).

A didactical performance refers to the ad hoc actions the teacher carries in the classroom. This includes the teacher's reactions towards learners' needs, unexpected aspect of a mathematical task or of the spreadsheets, and/or decisions concerning the type of question s/he asks at a particular time to lead learners into discovering knowledge. (Drijvers et al, 2010, p. 215). Providing for whole class discussions can be beneficial as learners may get a broader forum where they may clarify their interpretations of the situation at hand and may be able to solve challenging questions when a solution is not initially available. (Anthony & Walshaw, 2009, p. 151). Instrumental orchestration may thus be seen as consisting of teacher's repertoire of teaching techniques and the flexibility with which s/he reacts to unexpected aspects that emerge while executing his teaching plan.

Drijvers et al (2010, p. 219) identified six types of orchestrations that may be carried out during instruction in a technology-enhanced classroom. These are "technical demo", "explain-the-screen", "discuss-the-screen", "link-screen-board", "spot-and-show" and, "Sherpa-at-work". *Technical demo* orchestrations are demonstrations of tool techniques and in the case of spreadsheets, these would involve showing learners how to carry out certain commands, for example, how to highlight, or create a graph. *Explain-the -screen* orchestrations involve the teacher explaining what may be displayed on the computer screen. This may be explanations for the whole class or a smaller group of learners. During the *link-screen-board* orchestrations, the teacher establishes links between the technology (spreadsheets) representations and those encountered in the usual pencil-and-paper environment. (Drijvers et al, 2010, p. 219).

Discuss-the-screen orchestrations are those during which the teacher conducts a discussion on what is displayed on the computer screen. In s*pot-and-show* orchestrations, the teacher identifies some interesting learner's work during preparation and brings it to the attention of the rest of the class for discussion. The *Sherpa-at-work* orchestration involves a particular student (Sherpa-student) whose work is displayed on the larger screen, through an overhead projector, so that the rest of the students can follow what is happening in the lesson. The Sherpa-student is purposefully chosen by the teacher as someone who has control over the technology and able to carry out actions as instructed by the teacher. Compared to the first three types discussed earlier these types of orchestrations are more interactive as learners participate actively during their execution. (Drijvers et al, 2010, p. 219).

The interventions, in the form of discourses, provided by the teacher in the in a technology-assisted classroom, may be classified into two categories, depending on the purpose they serve (Abboud-Blanchard, 2009, p. 1881). *Procedural help* refers to situations where the teacher modifies the activities for the prescribed task. This corresponds to the indications or clarifications that the teacher gives to learners before or during their work. *Constructive help* involves adding something between the strict activity of the learner and the expected knowledge construction that could result from the activity. (Abboud-Blanchard, 2009, p. 1881). Teachers' discourse has been identified as the main mode through which intervention is provided in the classroom.

Analysis of the teacher's discourse provides information as to how s/he contributes in modifying learners' activities. Teacher's discourse plays a significant role in the mediation of cognitive development. Two functions of discourse can be distinguished, specifying the manner in which the mathematics teacher intervenes in the learners' work. These are *cognitive functions* and *functions of enrolment. Cognitive functions* are linked to the realization of the tasks and to the mathematics content. These include functions such as introducing the tasks, dividing the task into realizable sub-tasks, assessment, justification and structuring. Through discourse, the teacher can foster the use and understanding of the conventional mathematical language. (Anthony & Walshaw, 2009, p. 153). Re-voicing, repeating, rephrasing or expanding on what learners have said, are useful techniques in promoting understanding. Anthony & Walshaw (2009, p. 153) further pointed out that a teacher may use re-voicing

i) to highlight ideas that have come directly from students;
ii) to help development of students' understandings implicit in those ideas,
iii) to negotiate meaning with their students, and

iv) to add new ideas, or move discussion in another direction. (Anthony & Walshaw, 2009, p. 153).

The teacher could also make connections between paper-and-pencil and the spreadsheets environment and thus linking the spreadsheets algebra to the official algebra outside the micro-world. Highlighting the constraints and the limitations of the spreadsheets to the learners may also support their instrumental genesis. (Guin & Trouche, 1999, p. 224). *Functions of enrolment* are not related to the task, but can have impact on the realization of the task. These include functions such as enhancing learners' engagement in the task, mobilizing their attention and encouraging them to move on. (Abboud-Blanchard, 2009, p. 1881). Creating a classroom atmosphere where all learners feel comfortable and free ask questions, offers every learner opportunity to get involved in the lesson activities (Anthony & Walshaw, 2009, p.150).

The success of the instrumental orchestration process depends on the teacher's mathematical content knowledge, experiences with and views on mathematics education, and the role of technology in the teaching and learning of the targeted mathematical concepts (Mishra & Koehler, 2006, p. 1025). According to Mishra and Koehler (2006, p. 1025), the interplay between the subject content knowledge(C), the pedagogical knowledge (P), and the knowledge about the involved technology (T) is termed Technological Pedagogical Content Knowledge (TPCK). They describe TPCK as

> "the basis of good teaching with technology and requires an understanding of representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and develop new epistemologies or strengthen the old ones" (Mishra & Koehler, 2006, p. 1029).

Figure 2.9 illustrates Mishra and Koehler's model for their TPCK approach in which the three types of knowledge are blended.

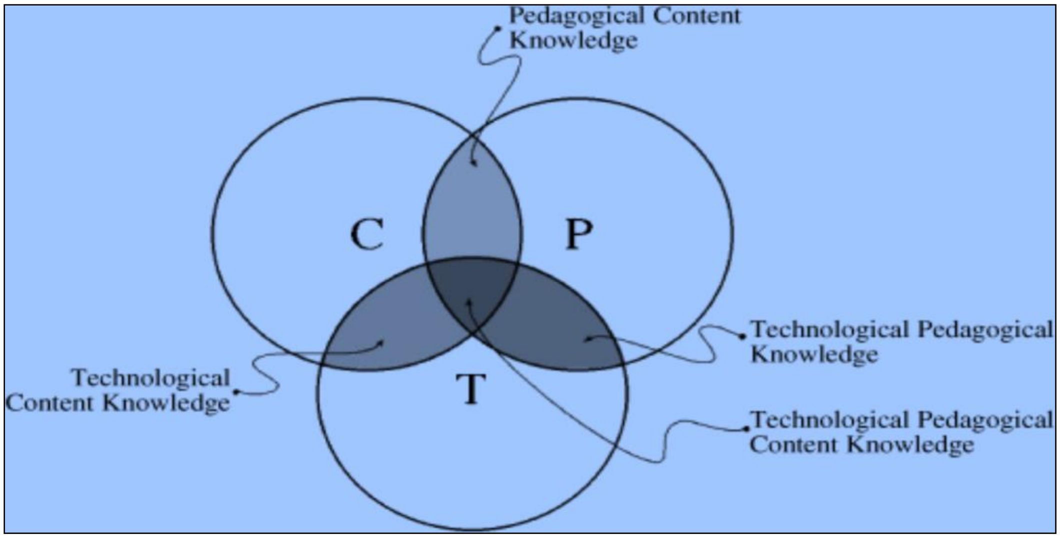


Figure 2.9: Technological Pedagogical Content Knowledge (TPCK) (Mishra & Koehler, 2006: 1025)

In the case of using spreadsheets to teach algebra, this TPCK includes knowledge about the potentialities, affordances, enablement provided as well as constraints that may be imposed by the artifact (T), and how spreadsheets may be used to enhance learning of those algebraic concepts that are usually problematic to learners. What is required of the software and computational tools, is to be pedagogical instruments for learning the mathematical knowledge and values which were defined in the past, mostly before these tools existed and to "help in the fight against inadequate teaching practices: practices too much oriented towards pure lecturing or the procedural learning of mathematical skills" (Artigue, 2002, p. 245). It therefore becomes important that issues relating to computational instrumentation, relationships between technical and conceptual learning, and relationship between paper-and–pencil and instrumented techniques are dealt with in a more sensible way.

Before any technological tool is used in the classroom for the teaching and learning purposes, it is important that its potentialities and limitations be assessed, in order to make informed decisions with regard to its use. The decisions can be on whether to use the technology, based on its potentialities with regard to enhancing acquisition of targeted mathematical concepts, and more importantly, on how the constraints that are associated with the use of the instrument may be managed, in order to attain the teaching-learning goals.

The discussion in the next section provides a general framework that was followed in the study.

2.4.3 Towards a theoretical framework for the study

The purpose of this study was to investigate how learners experienced the teaching and learning of algebra within a spreadsheets environment. The three sub-questions that study addressed were:

- What challenges were encountered during the teaching and learning of Grade 9 algebra through *Excel* spreadsheets in Lesotho?
- How did Grade 9 learners overcome the identified challenges to enable effective learning algebra through *Excel* spreadsheets in Lesotho?
- How did Grade 9 learners in Lesotho perceive the teaching and learning of algebra through *Excel* spreadsheets?

These research questions suggest that the focus of this study was on the events taking place in the classroom as Grade 9 learners were taught algebra through spreadsheets. This involved identification of challenges encountered and the actions that learners took in order to progress in their instrumental genesis, as well as strategies that the teacher used in orchestrating algebra learning within the spreadsheets environment. The processes of instrumental genesis and instrumental orchestration were thus identified as frameworks for this investigation.

Instrumental genesis is a developmental process that takes place in the mind of the leaner as they interact with the artifact, in this case spreadsheets, in the learning of algebra (Trouche, 2005, p. 148). The challenges encountered during instruction were interpreted, taking into consideration the constraints imposed by the artifact. Research indicates that "learners' attitudes color their experiences in learning situations to which those attitudes apply" (Reed et al, 2010, p. 2). Learners' experiences of the teaching and learning of algebra through spreadsheets were gathered from their views, beliefs and attitudes that they (learners) developed and displayed while participating in the study. The study, therefore, investigated what learners liked, appreciated or found useful and what they disliked about the teaching and learning process. These were based on the affordances and constraints imposed by spreadsheets as well as the organization and conduct of the teaching and learning, as experienced during the study. Other challenges that were brought about by external factors such the context or environment within which the teaching and learning took place, were also considered.

Instrumental orchestration (Trouche, 2004, p.296) involves the actions that the teacher took to help learners get more familiar with the operations with the artifact, spreadsheets, in order to enhance learning of the targeted algebraic concepts. Instrumental orchestration included organization of the learning environment *(didactical configurations*), the plan of how the targeted concepts were developed in the spreadsheets environment (*exploitation mode)*, exploiting the affordances and managing the constraints provided by use of spreadsheets, and the kind of ad hoc actions the teacher carried out in the classroom to address learners needs *(didactical performance)* (Drijvers et al, 2010, p. 215). The study sought to find out what type of organization and didactical performances (teacher-initiated strategies) were helpful to the learners in attaining their mathematical goals.

The following section outlines the mathematical content and the associated learning objectives/outcomes that were covered during the study.

2.4.4 The Hypothetical Learning Trajectory

A Hypothetical Learning Trajectory (HLT) was developed for the teaching-learning session. Simon (1995, p. 136) describes an HTL as follows:

> The hypothetical learning trajectory is made up of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning process– a prediction of how the students' thinking and understanding will evolve in the context of the learning activities. (Simon, 1995, p. 136)

It comprised of activities that were intended to lead learners through the following cognitive developmental shifts:

a. identify and express, in algebraic terms, patterns and functional relationships found in mathematical real-life situations**;**
b. demonstrate understanding of the different roles that letters take in algebraic expressions**;**

c. develop the concept of equivalence in algebraic expressions;
d. translate between different forms of representations of mathematical information;
e. manipulate algebraic objects in solving linear equations emerging from mathematical situations in real life contexts; and
f. use algebra to solve mathematical problems situated in real life contexts

2.4.5 The matrix for teaching

The main concepts, the targeted learning objectives and a brief description of the activities that were used in developing the algebra concepts, are provided in Table 2.3. The title of the worksheet and the number of lessons budgeted for each worksheet is also indicated.

CONTENT	OBJECTIVES Learners should be able to:	ACTIVITIES	No. of lessons spent	Worksheets used
Generalization of Relationships	• Identify and express patterns verbally and algebraically.	-identifying and expressing patterns verbally. -formulating spreadsheets formulas for patterns; -generating sequences using spreadsheets formulas; -Moving from verbal to symbolic expressions; -shifting from spreadsheets formulas to formal algebraic notation; -Moving between verbal and symbolic expressions.	5 1 3	**a) Worksheet 1 :** Formulas on a Spreadsheets b)**Worksheet 1A**: Angles **c)Worksheet 2** Formulating expressions for functional linear relations
Algebraic Manipulations	• Simplify algebraic expressions • Solve linear equations	-Identifying equivalent expressions -Expanding and factorizing expressions -Equations: change of subject -Solving linear equations	4 **3**	a) **Worksheet 5:** Simplifying expressions and Equivalence b**) Worksheet 5A:** simplifying expressions
Modeling mathematical situations	• Use algebra to solve problems that arise in real life contexts	-Formulating algebraic expressions/equations for situations -solving formulated equations -interpreting solution in terms of the solved problem	2 4 2	**Worksheet 3:** creating graphs of linear functions; **Worksheet 4:** exploring properties of graphs; **Worksheet 6:**; Solving equations from real life contexts

Table 2.3: Matrix for Teaching

2.5 Conclusion

This chapter discussed the nature of school algebra and provided an outline of the Grade 9 algebra curriculum in Lesotho. Algebra may be seen through three lenses: as generalized or abstract arithmetic**,** a language of mathematics, and a tool for the study of functions. The power of algebra lies in its use of the economical and consistent symbolic system to express relationships, which can then be used to formulate arguments concerned with prediction, problem-solving, explanation and proof (French, 2002, p. 3).

The algebra curriculum in Lesotho offers learners opportunity to interact with algebra in such a manner that they acquire appropriate knowledge and develop necessary skills to attain proficiency in the domain. The curriculum also engages learners in such activities as categorized in Kieran's framework on conceptualization of the school algebra. A distinction was drawn between the intended and the implemented grade 9 algebra curriculum. Problems that learners encounter when learning algebra in Lesotho were highlighted, and possible sources were discussed. Some of the sources of learners' problems were associated with the nature of the subject while others were attributed to the teaching methods/strategies implemented in the classrooms.

The main principles presented in the teaching approach, as described in the NCDC curriculum document, were discussed and found to be characteristic of the RME, the Dutch approach to mathematics teaching and learning, and those expressed in the USA, NCTM standards. The approach advocates that algebra teaching should be in the contexts of meaningful real life situations, which are of interest to learners. By the word "real", it is meant that the contexts should be those that learners are familiar with or at least those learners can be able to imagine. The contexts should also help learners make connections between topics within mathematics and across the school curriculum. Problem solving is also seen as central to teaching and learning of algebra. Problems that learners engage in should allow them to mathematize both horizontally and vertically.

It was also established that technological tools, in the form of computers, provide learners with new and more dynamic ways of exploring and engaging with mathematics. An in depth discussion of the role played by computers and calculators in mathematics education was done. It was revealed that technology, in the form of spreadsheets, has a potential of mediating the learning of algebraic concepts, particularly in early stages. The use of spreadsheets has the potential to bridge the gap between arithmetic and algebra learning. They can help enhance understanding of the problematic concepts such as role of letters in algebra, concept of equivalence of algebraic expressions and solving equations. Through multiple representations of situations, spreadsheets enhance better analysis, interpretation, and hence understanding of problem situations.

Before adopting use of spreadsheets as a complementary approach to address the problems or obstacles in algebra learning in Lesotho, it was found important that an investigation be carried out to identify the possible obstacles that may be encountered when teaching algebra through spreadsheets. Literature Review indicates that there are certain complications involved with the learning of algebra through technology, spreadsheets. *Instrumental genesis* is a process whereby learners develop skills of using an artifact in such a manner that they exploit its potentials towards attaining their mathematical goals. *Instrumental orchestration* is a process whereby a teacher steers the learner's process of instrumental genesis. This includes all forms of interventions provided by the teacher to enhance effective learning through the artifact. This study investigated what happened when Grade 9 learners in Lesotho were taught algebra through spreadsheets. The focal points of the study were determining the challenges that learners experienced, and the role played by the teacher and the learners to achieve the targeted teaching/learning outcomes. The quality of teachers' TPCK was found to be a determinant factor on the type of interventions provided by the teacher in the technologically-assisted (spreadsheets) learning environment. The Hypothetical Learning Trajectory with accompanying worksheets used in the teaching and learning of the identified algebraic concepts was thus developed. The next chapter provides a discussion of how the research study was conducted.

CHAPTER 3

THE RESEARCH DESIGN & METHOLOGY

3.1 Introduction

This chapter provides a detailed account of how the study was conducted. The study commenced with a pilot, which was then followed by the main study. As the study involved use of a mix of a number of data collection techniques and instruments, it was important that these be tried out first before the main study was conducted.

3.2 The Pilot Study

After permission was granted to carry out the study by the Ministry of Education and Training of the Kingdom of Lesotho and the School Authorities, and the Ethical Clearance Certificate received from the University, the researcher went out to conduct the pilot study. The piloting was carried out in a school other than the ones used for the main study. This section provides justification for the pilot study and discusses how the study was conducted. The implications of the findings to the main study and the necessary steps taken are also discussed.

Pilot studies are important as they may provide advance warning about whether the main study will succeed or not. The pilot study was helpful as it provided with data necessary for the planning of the main study. Potential practical problems in following the research protocol were detected through the pilot study. Such problems included local politics within the study site that might have affected the research process. Moreover, the proposed approach for recruiting participants into the study was also assessed through the pilot study. The pilot study was carried out to assess the appropriateness of the instructional materials in the form of worksheets, particularly with regard to the level of the language used; and to determine the appropriateness and applicability of the data collection protocol. However, it needs to be pointed out that the success of a pilot study does not necessarily imply success of the main project. (Van Taijlingen & Hundley, 2002, pp. 33-34).

All teaching was conducted in the computer laboratory of the pilot school. This part of the study only covered a portion of the Grade 9 algebra content that would be taught during the main study (de Vos, 2002, p. 338). The areas of algebra covered during the pilot study included the concept variable, identifying relationships and expressing generalizations. Recruitment procedure and all the data collection instruments (the questionnaire, observation schedule and one-on-one interview schedule, focus group interviews) were pilot-tested. Upon completion of each worksheet, focus groups were conducted. Use of the video-recorder was also practiced during the pilot study.

The pilot study offered the opportunity to test the instruments on the subjects with similar characteristics as those in the main study (de Vos, 2002, p. 337). This was important for "refining the wording, ordering, layout, filtering, and so on, and in helping to prune" the instruments (Hoinville in de Vos 2002, p.216). Thus, constructs such as internal validity of the questionnaire and observation schedule were improved based on the results of the study. The language used in the instructional materials was also adjusted based on findings from the pilot study. The pilot study also gave the researcher an idea of how much time was needed to complete each worksheet. The time constraints experienced, during the pilot, led to the decision to have the questions used for focus group interviews inserted at the end of the learners' worksheets, hence there were no more focus group interviews in the main study.

3.3 The Main Study

This section discusses the path taken to collect information that addresses the purpose of the whole study. A description of the design or plan, sometimes called the blue print of the study, is provided. The section also provides a detailed account of the contexts in which the study was carried out. The key players and their roles in each phase of the study are also mentioned. The instruments and methods used to collect the data, as well as the procedures followed are also discussed here. Finally, an outline of how the data was dealt with is presented. This consists of strategies for data organization, presentation, analysis and interpretation.

3.3.1 Research Design

A research design may be described, in simple terms, as the path taken to connect the empirical study to its initial research questions and finally to the answers or conclusions arrived at the end of the study (Yin, 2009, p. 26). It is a plan that guides the investigator in determining which data to collect, and how to analyze and interpret data in a logical manner. The main purpose of a research design, according to Yin (2009, p, 27), "is to avoid a situation in which the evidence does not address the initial research questions".

This research was a multi-case study, intended to address an exploration, research question: *How do Grade 9 learners in Lesotho experience teaching and learning of algebra through spreadsheets?* According to Creswell(1998 , p. 61), a case study may be regarded as an exploration or in-depth analysis of a "bounded system" over a period. This bounded system could be a single or a multiple case. A case study research is defined as "an empirical inquiry that investigates a contemporary phenomenon within its real-life context" (Yin, 2009, p. 18). It was on the bases of these that the researcher decided to follow a case study design for this research project.

A case study may also be described as particularistic, descriptive and heuristic(Merriam, 1998, p. 30).This case study is particularistic in the sense that it is focused, and takes a holistic view of teaching and learning of algebra through spreadsheets.. It is descriptive as it provides a "thick" (complete, lateral) description of the processes under investigation. This description is presented using prose and literary techniques to elicit images through quotes, samples and artifacts. This case study may be seen as heuristic as it illuminates understanding of the phenomenon being studied and may bring about discovery of new meaning or widening conceptions about the teaching and learning of grade 9 algebra through spreadsheets. (Merriam, 1998, p. 30). It provides explanation to reasons for a problem, the background of a situation, about what happened and why? Case studies are therefore suitable for investigating practical problems and issues such as those found in educational settings. This particular first-person research study is designed by instances where teaching of grade 9 algebra through spreadsheets is done and examined how it works from inside.

A case study provides a detailed account of one or more cases (Johnson & Christensen, 2008, p. 49). This particular study was of a multiple-case design in which two groups of Grade 9 learners in similar classroom settings would be used in the investigation. A multiple-case design was used because the researcher believed she could get deeper insights into the research topic. The multiple-case in this study was selected so that it would predict similar results that are lateral replication (Yin, 2009, p. 54). There are three advantages advanced for using multiple- case studies:

- they provide for comparison of cases for similarities and differences;
- one can effectively test a theory by observing results of multiple cases; and
- that the researcher is more likely to generalize the results from multiple cases than from a single case. (Johnson & Christensen, 2008, p. 408).

This therefore means that a multiple-case study may be viewed as a means to validate the findings of the study. Figure 3.1 is an illustration of the path followed in conducting a multiple–case study.

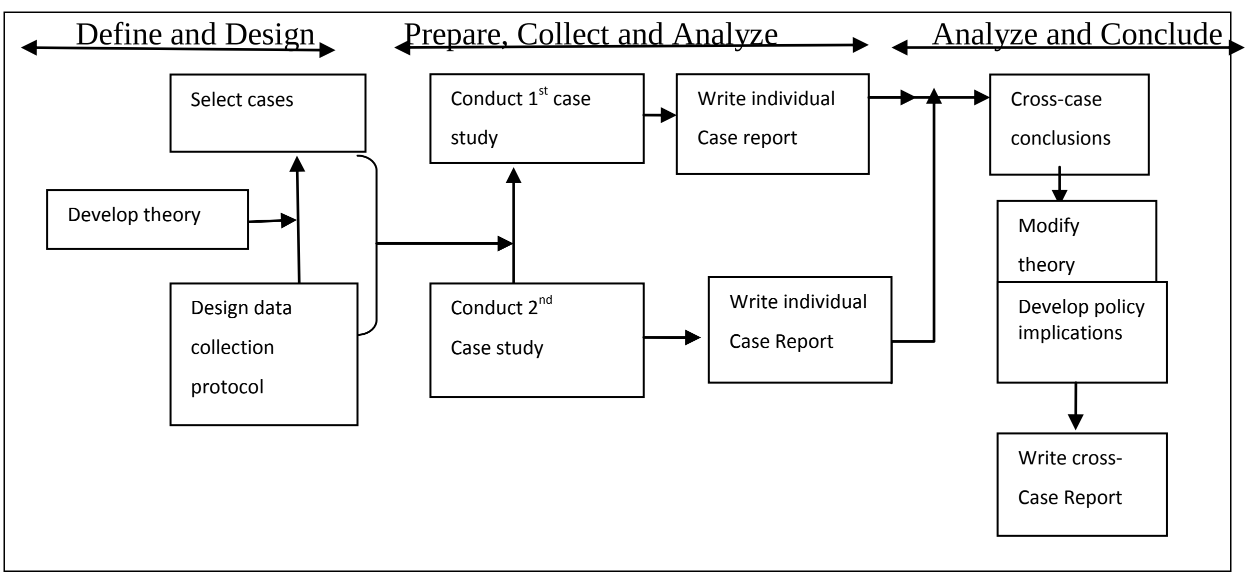


Figure 3. 1: Path followed in conducting a multi-case study (Yin, 2009, p. 57)

This illustration indicates that a case study research process begins with theory development, which would then be followed by case selection and data collection procedures. For this particular study, the process begun with a literature review that led to formulation of a theoretical framework for the study. A Hypothetical Learning Trajectory (HLT), *(See Section 2.4.4)*, was then designed and was followed by case selection and data collection procedures. An HLT is a plan of teaching, showing the topics to be covered, the targeted learning objectives as well as the related sequence of activities. The sequencing of topics was done in hierarchical order from simple to more demanding, taking into consideration the relatedness of the areas. This is treated in more detail under *Section 3.3.3.2*. The data was then collected from each case, each of the two cases was treated as a whole study, and evidence was sought regarding the phenomenon being investigated. Observations were made and reports were compiled for the individual cases. The last stage was characterized by comparison of the findings from the two cases leading to drawing of conclusions for the whole study. A cross-case report was then compiled.

During the conduct of the study, particularly during the teaching phase at each site, the researcher had her eyes wide open and antennas up, as this was the area where important discovery occurred. It needs mentioning that in conducting a multiple case study, there could be a situation where one of the cases under study does not suit the original design and thus requires the researcher to pay attention to the initial propositions. This could lead to a "redesign" and selection of alternative cases or change in the data collection protocol. This latter situation did not happen in this particular study. The same groups of learners were used to the end of the study.

3.3.2 Methodology

This study falls within a mixed methods research paradigm, using both qualitative and quantitative research methods (Johnson & Christensen, 2008, p. 52). Mixed methods researchers such as Greene, Caracelli & Graham, (1989) and Creswell & Plano Clark (2011) advance many reasons why mixed methods may be used in a research project. The mixed-methods paradigm was used primarily for two purposes, confirmation and complementarity

(Small, 2011, p. 63). Confirmation involved verification of results obtained from the qualitative data by those obtained from the quantitative data. For example, interviews were conducted to get in-depth understanding of learners' responses to the some of the items in questionnaire. This was an attempt to ensure that the findings were not dependent on the method of data collection.

Mixed methods were used to bring about *complementarity*, which "seeks elaboration, enhancement, illustration and clarification of the results from one method with the results from the other"(Creswell & Plano Clark, 2011, p. 62; Small, 2011, p. 64). This means that in combining qualitative and quantitative methods, the strengths of one method were used to overcome the weaknesses of the other. This approach added insights and enhanced better understanding that might have, otherwise, been missed when a single method was used. Use of mixed-methods provided completeness and opportunity for instrument development (Creswell & Plano Clark, 2011, pp. 62-63). Completeness, according to Bryman (2006, p. 106) refers to the notion that a "researcher can bring together a more comprehensive account" of the issue under investigation. The results from the questionnaire were also helpful in developing some question items for the interviews, hence the aspect of instrument development. In this way, the methods complemented one another. (Johnson & Christensen, 2008, p. 444)

Johnson & Onwuegbuzie (2004, p. 15) further argues that

> Today's research world is becoming increasingly interdisciplinary, complex, and dynamic, therefore, many researchers need to complement one method with another, and all researchers need a solid understanding of multiple methods used by other scholars to facilitate communication, to promote collaboration, and to provide superior research.(Johnson & Onwuegbuzie, 2004, p. 15)

It is believed that employing mixed methods approach offers researchers opportunity to mix and match design components in the best possible way to address their research questions more fully. The *Fundamental Principle of Mixed Methods* requires the researcher to "strategically mix or combine qualitative and quantitative approaches to produce an overall design with complementary strengths and non-overlapping weaknesses" (Johnson & Christensen, 2008, p. 443). The qualitative and quantitative parts of the study may be conducted concurrently (in parallel) or sequentially. This study was a sequential exploratory study in which classroom observations were conducted first, these were followed by the administration of the questionnaire, and lastly the interviews were conducted. The manner in which the events took place in the study, thus defines the design as the sequential mixed methods design (Creswell & Plano Clark, 2011, p. 242). The study is thus denoted as

$$\text{QUAL} \rightarrow \text{Quant} \rightarrow \text{QUAL}$$

This notation indicates that the methods occurred in a sequence, but with greater dominance of the QUALitative methods and the quantitative methods followed (Creswell & Plano Clark, 2011, p. 109). Two categories of mixed methods designs are distinguished, the *Fixed* and the *Emergent* mixed methods (Creswell & Plano Clark, 2011, p. 54). Fixed mixed methods designs are those in which the use of quantitative and qualitative methods is predetermined and planned at the beginning of the research process and the procedures are implemented as planned. Emergent mixed methods designs, on the other hand, are those mixed methods studies in which use of mixed methods arise due to issues that emerge during the course of the study. It has been noted, however, that a lot of mixed-methods studies fall in-between the two categories, in the sense that a researcher may plan to have a qualitative phase first followed by a quantitative method, but find that procedures in the second phase will be determined, to some extent, by results of the first phase (Creswell & Plano Clark, 2011, p. 54). It is therefore evident that this particular study is a combination of both fixed and emergent designs.

Mixed methods were therefore used to provide stronger evidence for conclusions or claims that were arrived at through convergence, corroboration and correspondence of findings. Both qualitative and quantitative methods and strategies were necessary for collecting, analyzing and interpreting data from different and/or similar sources. This was done in order to reduce the bias from observing my own influence in the teaching/learning process and for validation of claims that I made about the results of the study. The intention here was to explore and gain a deeper understanding of learners' difficulties in learning algebra through spreadsheets.

3.3.2.1 Sampling

The schools.

Purposive sampling (Johnson & Christensen, 2008, p. 239) was used for selecting the schools whose learners participated in this study since not all schools in Lesotho are currently teaching computer skills at junior secondary level. Due to financial constraints, the two high schools, Retha and Palo (pseudonyms used for ethical considerations) chosen for the study are located within the Maseru city center, which is urban and hence easily accessible. They had adequate infrastructure in the form of administrative buildings as well as classrooms. Most learners came from areas within a radius of 5km from the Maseru city center, there were however those who came from areas as far as 30km away. All learners in the two schools were day-scholars, that is, they came to school from their homes on daily basis. They did not dwell in the school premises. The majority of the learners depended on public transport (taxis) to-and-from school. These learners came from families with different economic backgrounds and the school fees were affordable by most parents.

Retha High School

There were two computer laboratories in this school, one for the junior classes and the other for senior classes. The computer laboratory for the juniors was quite small; there was not enough space between the workstations and in between the rows, to allow free movement during the lessons. There were three teachers employed to teach Computer Studies in the whole school, although none of them held a teaching certificate.

Retha High school usually showed high performance in Junior Certificate external examinations. For example, in the 2010 and 2011 JC examinations, the pass rate was 93% (ECOL, 2010, p. 48; ECOL, 2011, p. 36), as a result, learners enrolled in the school may be influenced and come with a positive mindset that they have to do well in their academic work and maintain the good image of the school.

Palo High School

The school had fair infrastructure, and had only one computer laboratory, for both the junior and senior classes. The computer laboratory was also small but bigger compared to that in Retha High School, and there was enough space between the workstations and in between the rows to allow free movement during the lessons. Only one teacher was employed to teach Computer Studies and, like in Retha High School, did not hold a teaching certificate.

In this school, performance of learners in Junior Certificate external examinations was usually low. In the years 2010 and 2011, JC external examinations the pass rates were 47% and 35% respectively (ECOL, 2010, p. 44; 2011, p. 32). Learners enrolled in this school have to work very hard to improve the long standing (poor performance) history of the school.

The Learners.

In this study, the population was all Grade 9 learners who were taught computer skills, in Lesotho secondary/high schools. The schools followed the same curriculum for Computer Studies, as designed by the NCDC, and all learners had already been taught some basic skill in *Excel* spreadsheets in their first year of secondary school and were still following the course over a three-year period. The research study was conducted during the second half of the school calendar, after a period of one-and-half ($1\frac{1}{2}$) years of learners' exposure to the Computer Studies curriculum. Learners had also begun some algebra lessons in their normal mathematics class. The traditional algebra topics covered include using letters for numbers, simplifying algebraic expressions, and solving linear equations. Since each group of learners was a mix of learners from different Grade 9 classes, they had differing experiences in their algebra learning. Two methods were used to draw a representative sample for the study.

Convenience sampling of participants was done as learners were invited to participate in the study (Johnson & Christensen, 2008, p. 238). Learners were first informed of the purpose of the study and how it was going to be conducted. The briefing highlighted on the nature of the activities, the times and days when the lessons would be conducted and what was expected of them (learners), as was done in the Pilot study *(See section 3.1).* In both cases, the group of volunteers was much more than the desired number, hence *systematic random sampling* was used. This is a type of probabilistic sampling method that involves selection of every *k*th unit from population into the sample, where $k = \frac{population\ \ size}{sample\, size}$ (Babbie & Mouton, 2011, p. 648). Each sample group had fifteen (15) learners. This was done for classroom management purposes as well as regulation of the amount of data that would be handled. Sample size for case studies need not be large. A case may be an individual, a particular group of people, an institution, et cetera (Babbie & Mouton, 2011, p. 281; de Vos, 2002, p. 275).

3.3.2.2 Phase 1: Qualitative

The first phase of the investigation took a form of an exploration where learners in the two groups were taught algebra in a spreadsheets-assisted environment. All teaching was done in the computer laboratories

In order to gain an in-depth understanding and first-hand information on learners' behaviors and attitudes as they interacted with algebra within the spreadsheets environment, this study was conducted within a classroom setting. The unit of analysis was each teaching episode. The author played the dual role of a teacher-researcher. Unlike in other case studies in education where a researcher is an outsider, getting into a classroom, positioning him/herself near a student, observing the kind of interactions that go on, listening to what students say and observing the teacher's moves, and in some cases failing to understand the local language, peculiar to the class, the situation was different here. The teacher-researcher role provided opportunities to get better insights and understandings about the phenomenon studied through first-hand experience of the teacher. It was also binding for the researcher to assume the dual role (teacher –researcher) since the teaching of algebra through spreadsheets is not yet in practice in Lesotho schools.

Here, the researcher created the situation that she studied. Based on what the LJC mathematics curriculum points as pillars of effective teaching and learning of algebra and on the understandings of the affordances that spreadsheets can offer towards teaching and learning of Grade 9 algebra, the teacher designed a hypothetical learning trajectory for the study. She carried out the teaching process to examine, from inside, what it takes to help learners grasp the targeted algebraic concepts within the spreadsheets environment. Works such as this “can be seen as a means to legitimate perspectives of practice in the construction of knowledge about teaching. First-person research enters a teacher’s voice and perspective into a discourse of scholarship” (Ball, 2000, p. 375). As with Lampert in (Ball, 2000, p. 376), the research reported here is not towards making conclusions about how to teach Grade 9 algebra through spreadsheets, but is rather intended to outline an approach that is worth trying, based on the actual classroom experimentation. The interest here is in insight, discovery and interpretation of learners’ difficulties in the context of spreadsheets-assisted algebra instruction (Merriam, 1998 , p. 28).

i) Participants

The participants here are the teacher-researcher, the Assistant Observer and all the Grade 9 learners who had been selected into the study. As the researcher was in a way studying her own practice, it was necessary that an Assistant Observer (AO) was involved in the study to minimize bias on the findings. The AO was a mathematics educator and was purposively chosen to participate in the study because of her familiarity with classroom dynamics pertaining to effective teaching and learning of algebra and had some knowledge on the use of spreadsheets though not in relation to algebra teaching and learning.

ii) The lessons

During the course of the teaching, the researcher implemented the plan of teaching as reflected in the HLT. (*See section 2.4.4.1).* The teaching of algebra within the spreadsheets environment was in line with the teaching approach as recommended in the NCDC curriculum document. The following guidelines were followed in the development of the activities in a spreadsheets-based teaching-learning environment:

a) Introduce spreadsheets in context
b) Address worth-while mathematical tasks through realistic problems within the spreadsheets;
c) Take advantage of the spreadsheets;

d) Connect mathematics topics;

e) Incorporate multiple representations; and

f) Involve use of spreadsheets as well as pencil-and-paper techniques.

The worksheets for both groups were designed by the researcher, with some ideas obtained from review of related literature and input from experts in the field (including supervisor).(Gürbüz, Çatlioğlu, Birgin, & Erdem, 2010, p.1057). Introducing spreadsheets in context, as used here, refers to familiarizing learners with the spreadsheets and its language through activities, as seen with Worksheet 1. Worthwhile mathematical tasks are those that are based on sound and significant mathematics, knowledge of students' understandings, interests and experiences and knowledge of a range of ways that different learners acquire mathematical concepts (NCTM, 1991, p. 25). This also refers to the algebra content, as stipulated in the syllabus, that learners be led to acquire through spreadsheets. The objectives of the lessons listed at the beginning of each worksheet.

The worksheets comprised the activities that fall within Kieran's categorization: generational, transformational and global/meta-level activities. According to (Kieran, 2004a, p.25), the focus of *generational activities* is "representation (and interpretation) of situations, properties, patterns, and relations". Examples here include formulating expressions and equations for situations. The activities in this category offer opportunities for making algebra learning meaningful. *Transformational activities* are described as those that are rule-based. These include algebraic manipulation processes such as collecting like terms, factorizing, expanding brackets, substitutions, simplifying algebraic expressions through addition or multiplication, exponentiation with polynomials, solving equations etc. (Kieran, 2004a, p. 26). The main concern here was to lead learners into transforming the original given expression into its equivalent forms. The third types of activities, the global/meta-level activities are the "activities for which algebra is used as a tool but which are not exclusive to algebra" (Kieran, 2004a, p. 24). Examples of such activities include problem-solving, predicting, modeling, generalizing and justifying, etc. (*See section 2.3.2)*

The instruction provided here was geared towards helping learners understand and appreciate algebra as: abstract or generalized arithmetic (Kieran, 1988, p. 91); the language of mathematics (Lee, 1996, p. 87; MacGregor & Price, 1999, p. 451; Usiskin, 1997, p. 346; Drouhard & Teppo, 2004, p. 231) and a tool for study of functions and mathematical modeling (French, 2002, p. 3; Kriegler, 2007, p. 2; Herbert & Brown, 1997, p. 340; Charbonneau, 1996, p. 35).

The activities in the learners' worksheets were problem-based, learner-centered and formulated within contexts that were realistic to learners. Realistic contexts were those that are familiar to learners or those in which learners could be able to situate themselves. They were designed to lead learners into discovering new knowledge beginning from known to unknown. Problems that learners solved also provided connections between mathematical topics. For example, learners were led to acquire concepts in algebra through problems situated in statistics. In the activities, there was provision for learners to explain and justify their thinking regarding the problem situations they were confronted with. Through the activities, learners were led to realize that mathematical ideas could be represented symbolically, through equations, graphically and in tabular form.

In the study the teaching of algebra was done taking advantage of what the spreadsheets afforded. Taking advantage of spreadsheets refers to use of spreadsheets to achieve algebra-learning objectives. This involved taking into consideration the opportunities, affordances and enablements provided by spreadsheets as well as possible constraints that spreadsheets imposed and planning tasks such that all these would be exploited to enhance learning of the targeted concepts. The spreadsheets environment also provided for extensions of problem situations and allowed learners to conjecture about the situations at hand. It is suggested that "the use of technology in mathematics teaching is not for the purpose of teaching about technology, but for the purpose of enhancing mathematics teaching and learning with technology(Garofalo, Drier, Harper, Timmerman, & Shockey, 2000),retrieved on August 7, 2012 from http://www.citejournal.org/vol1/iss1/currentissues/mathematics/article1.htm

Spreadsheets were used in the lessons because they can potentially function as a cognitive tool, enhancing learning of the targeted algebraic concepts. (*See section 2.4.2.1.* The groups were taught algebra with spreadsheets to complement the traditional pencil-and-paper instruction. Learners were led through different stages of algebraic understandings by active engagement in classroom activities in the form of worksheets. (*See Appendix A).*

iii) The design of classroom activities

The activities used in the study were in the form of worksheets. There were six (6) worksheets in all.

Worksheet 1: Introduction- Formulas in spreadsheets

Specific objectives for this worksheet were:

- Expressing generalizations in own words
- Creating spreadsheets formulas for generating sequences

Here learners were led into recognizing patterns of numbers in contexts they are familiar with. There were three activities in this worksheet. In the first activity, learners were to create the *Calendar for June 2009*. They had to establish the relationship between dates of consecutive days in a week, and for consecutive Mondays or Tuesdays, verbalize the relationship and then create spreadsheets formulas that they use to create the calendar for the month. The second activity involved generating the *Pascal's Triangle*. They were confronted with identifying patterns and creating the spreadsheets formulas to extend the pattern, now *Pascal Square*, in spreadsheets. The third activity was on generating the 20- by-20 *Multiplication Table.* The purpose of these activities was to give learners opportunity to familiarize themselves with the spreadsheets notation, tapping on their prior experience with arithmetic. They generalized the patterns, created spreadsheets formulas for the patterns and extended the sequences using spreadsheets. Here learners exploited spreadsheets features such as cell reference and "drag command" to develop the concept of variable and algebra notation.

Worksheet 1A: Angles: Generalizing Patterns

Unlike the other worksheets where instructions came on hardcopy, the tasks in this case as well as instructions were uploaded on each learner's computer. There were several activities in this worksheet. In *Growing Angles*, a circle is divided into two sectors, blue (b) and pink (p). Each sector angle is given and on clicking in one sector, the corresponding angle increases, while the other decreases. Learners were to identify the relationship between the pairs of angles. The next activity was based on the first one, here the learners solve an equation $b + p = 360^0$, given either b or p. In the third activity, the circle represented a Pizza that was being shared equally among a number of children. Learners were to determine the relationship between the number of children and the angle size of each piece of pizza they got. The fourth activity involved solving the equation, $p \times a = 360^{0.}$

Here learners were to realize the different roles of letters in algebraic expressions and equations. They were introduced to solving equations, using their own methods. Connections were made between different topics in mathematics through contexts that are realistic to learners. Learners were left to discover knowledge. The tasks were designed such that they provided learners with immediate feedback and thus gave them opportunity to reflect immediately and seek alternative solutions. The other activities were left for learners to do at their own time.

Worksheet 2: Formulating Expressions for Functional Linear Relations

Specific objectives for this worksheet were:

- Identify patterns of numbers arising in real life contexts;
- Identify dependent and independent variables in functional relationships;
- Express relationships between sets/pairs of numbers in own words;
- Express relationships between sets/pairs of numbers in algebraic form.

There were four activities in this worksheet. The first one was on a *Restaurant Furniture* layout. Learners are required to construct a table of values showing *Number of Tables* in one column and the corresponding *Number of Chairs* in the other. They were led into identifying and expressing the relationship between the number of chairs to the number of tables in the set up in their own words, finally in algebraic form. Activity 2 consisted of more examples of relations between pairs of numbers and learners were asked to use letters of their own choices to express them. The third activity was in a form of *assignment*, which was intended for assessing achievement of the set objectives. An additional activity also helped learners into formulating algebraic expressions / equations for contextualized exploration problems.

Worksheet 3: Drawing Graphs of Linear Functions

Specific objectives for this worksheet were:

- write relationships for linear functions in the form $y = mx + c$, where m = gradient and c = y-intercept;
- represent linear functions in the form of a graph;
- determine the gradient of a linear graph;
- determine the gradient of a linear function given two points that lie on the line;
- realize that the gradient of a horizontal line is zero;
- realize that the gradient of a vertical line is undefined.
- interpret gradient as rate of change;
- relate the coordinates of y-intercept to the value of c in $y = mx + c;$ and
- solve problems involving linear functions.

The worksheet comprised of a series of instructions that when followed by the learner would lead them to visualizing a cell reference as a placeholder for a general number. Learners were then introduced to the general formula for the equation of a straight line and were helped to discover meanings for y-intercept and gradient. Through an assignment, achievement of stated objectives was assessed.

Worksheet 4: Exploring properties of graphs

By the end of the session, learners were expected to be able to:

- realize that the gradient of a horizontal line is zero;
- realize that the value of the gradient determines the steepness of the graph of the function;
- recognize graphs of relations with similar gradients;
- relationship between gradients of parallel and perpendicular lines;
- relate the symbolic, tabular and graphical representations of linear functional relationships; and

- solve equations using spreadsheets (tabular and graphical representations).

Here learners were given opportunity to explore properties of linear graphs by varying values of the *m* and *c* in the general equation $y = mx + c$. They also compared different graphs and noted relationships (equations for parallel and perpendicular graphs). Learners were then introduced to idea of solving linear equation through graphing and checking answers through algebraic manipulations.

Worksheet 5: *Simplifying Expressions*

By the end of the session, learners were expected to be able to:

- Identify equivalent expressions using spreadsheets tables; and
- Use the distributive property to simplify expressions.

The three activities in the worksheet were intended to lead learners into identifying equivalent expressions through comparing tables of values created by application of the distributive property. A corresponding assessment exercise was given where learners were asked to determine expressions equivalent to the given ones.

Worksheet 5A: Mappings: Equivalent Expressions

In this activity, two mapping machines were used to formulate different functions f and g, which were obtained through varying the operator in one or the input in the other. Symbolic, tabular and graphical representations of the two functions were shown on the screen. A change of value on the operator or input caused change in all the forms of representation. Learners were instructed to make the variations and observe when f and g were equivalent and explain their observations. Through this activity, learners were expected to be able to realize that:

- same information may be represented in different forms, (symbolic, graphical, and tabular).
- that a factor outside the brackets multiplies every term inside the brackets;
- graphs of equivalent expressions overlap; and that
- the order of operations (multiplication and addition) is important.

Due to time constraints, the other activities in the same worksheet were left to learners to explore in their own time.

Worksheet 6. Equivalence; solving equations

By the end of the lesson, learners were expected to be able to:

- formulate expressions for situations arising in real life situations;
- use *Excel* spreadsheets to construct a table of values for formulated expressions;
- draw graphs of the linear functions; and
- solve problems using tabular and/or graphical representations.

This worksheet served as an assessment tool. Learners were confronted with real life problem situations whose solutions required formulation of equations as representations of the situations, solving those problems through graphing, using pencil and paper or using spreadsheets.

Table 3.1 shows the dates and content covered for each of the two schools that participated in the study.

Worksheet	TOPIC	Dates For Lessons	
		Retha HS, Tue, Thurs & Sat	Palo HS, Wed, Fri & Sat
1	*Introduction- Formulas in spreadsheets*	9th-18th Aug	10th-19th Aug
1A	*Angles: Generalizing Patterns*	20th Aug	20thAug
2	*Formulating Expressions for Functional Linear Relations*	23rd-27th Aug	24th-27th Aug
3	*Drawing Graphs of Linear Functions*	30th Aug-1st Sept	31st Aug-2nd Sept
4	*Exploring properties of graphs*	3rd -10 Sept	3rd-10Sept
5	*Simplifying Expressions*	13th-20thSept	14th– 21Sept
5A	*Mappings: equivalent expression*	22nd-27th Sept	23rd-28thSept
6	*Equivalence; solving equations*	29th Sept-1st Oct	No more classes

Table 3.1: Dates for Lessons in Schools

In conducting the lessons, the teacher's role was that of a facilitator of learning, giving learners opportunity to grapple with the problems, and only providing interventions by way of probing and calling for whole class discussions whenever necessary. Each learner was afforded own workstation even though they could still discuss problems with their group members. There were three learners per group. A competition kind of atmosphere was created and any group or individual that would finish any given task or part thereof was given chance to explain to the rest of the class how he went about solving the problem. (*See section 2.3.5(iii)).*

Monitoring the learning process involved observing the behavior of learners as they interact with the subject matter and *Excel* spreadsheets as a medium of instruction. Here the focus was on the learners' progress as they went through *instrumental genesis*. Attention was paid to the interactions of learners with the technology as they worked through the tasks in the worksheets. As mentioned earlier the schemes learners used were determined by the learners' facility with the artifact, spreadsheets, being aware what the spreadsheets can do and knowing how to use the spreadsheets to attain their learning objectives. The researcher-teacher also engaged in reflective thinking about her own teaching practice, how she steered the *orchestration* process. *Instrumental orchestration* as indicated earlier consists of teacher's repertoire of teaching techniques within the spreadsheets environment and the flexibility with which s/he reacts to unexpected aspects that emerge while executing her teaching plan. The reflective practice here involved identifying the nature of those strategies and techniques that seemed to help learners to overcome their' difficulties. She was able to identify strengths and weaknesses in her planning of the activities. She paid attention to those teacher- and learner-initiated pedagogical strategies that enabled and enhanced effective teaching and learning of algebra through spreadsheets. (*See section 4.2.2)*

At the end of the lessons the researcher also reflected on the classroom events and based on the notes she took in class, identified the points where learners had difficulties and reflect on the nature of interventions she provided. At that stage, the researcher compared notes with those obtained by the assistant observer through the observation schedule. All lessons took place in the computer laboratory of the schools. The two groups used the same worksheets.

The teaching sessions were of 80-minutes (double-period) duration and they ran over a period of eight weeks, during the first part of the third quarter of the school calendar.

iv) Data collection

a) Classroom observations

As the study was investigating learners' experiences (challenges, moments of excitement) when they were taught algebra through spreadsheets, it was found necessary that a teaching-learning environment be organized so that real information may be gathered from actual classroom experiences. Observation is defined as the act of watching behavioral patterns of people in certain situations to gather information about a phenomenon of interest (Johnson & Christensen, 2008, p. 211). Advantages of observations as suggested by (Babbie & Mouton, 2011, p. 295) are that they provide opportunity for the participants (observer and subjects) to get familiar with one another. Observations allows for those aspects that were previously unnoticed or ignored to be clearly visible and recognized. It is also noted that the subjects' actions are more telling than what they may express verbally and observing them would be more valuable to the researcher. (Babbie& Mouton, 2011, p. 295). In this case, the researcher-teacher's interest was on the behavioral patterns exhibited by learners as they interacted with the algebra in the spreadsheets environment. It has also been noted that learners' attitudes, confidence levels, cognitive and emotional styles and social identities, could influence their participation in the use of technology (Hennessy et al, 2005, p. 162; Reed et al, 2010, p. 2) . Learners had to be carefully assisted (instrumental orchestration) as they progressed through the "instrumental genesis" processes. Video-recordings, field notes and a prepared observation schedule were used to collect the relevant data in the classrooms.

Video recording: The purpose of using a video recorder was to capture all the events taking place in the classroom. However, it was difficult to get learners discourse during the course of their learning, particularly when they were discussing amongst themselves. As soon as the camera was brought nearer to them, they whispered to each other. It was decided that the video camera was then used mainly to capture teacher interventions as well as those aspects of learners' work, which would be useful to the teacher-researcher. These recordings were very useful during analysis of the classroom events as they were illuminating and made behavioral patterns of the learners more visible. They provided useful starting points for further investigation. It is worth mentioning that not all lessons could be recorded as the equipment was borrowed and hence not available at all times.

Field Notes: These are notes taken by a researcher during an observation. The purpose of these notes is to capture details about events, people involved, and their interactions, in relation to the issue being investigated (de Vos, 2002, p. 285). In this particular study, the researcher was also the teacher. She engaged in observing learners' behavior as they worked through the tasks in spreadsheets. The focus was on the learners' problems arising from use of the technology, how learners overcome such difficulties, and which of the strategies she used seemed to be helpful to the learners. In a way the researcher was investigating her own teaching. When conducting these open observations, the teacher-researcher used blank sheets of paper to record the key events of a lesson (Babbie & Mouton, 2011, p. 294). The recordings were made using personal form of shorthand. Formative assessments in the form of classroom and homework assignments, and classroom discourse were used during the course of the teaching/learning to monitor cognitive growth as well as determining the learning obstacles learners experienced in the algebra lessons. Learners' behavior during the classroom interactions was observed and noted.

The researcher's field notes were edited immediately after the lesson when the events were still fresh. If a lot of time elapses before the notes are edited, this might result in making the data less accurate (de Vos, 2002, p. 285). The researcher would reflect on the events in the classroom and analyzed learners' activities and classroom environment to identify indicators for existence of learning difficulties and the learning shifts that may have occurred and the means that supported them. The video-recordings proved very helpful in capturing the conduct of the teacher-researcher, portraying her teaching and intervention strategies as implemented. Viewing these at the end of the lesson helped the researcher to fill up the field notes. *Learners' work* also provided evidence about their performance on the given tasks. These consisted of learners' responses and solutions to the tasks in the worksheets. Some of their productions in spreadsheets were also assessed. These enriched and provided more information for the researcher's field notes. Analysis of learning obstacles led to modifications on the conduct of the next lessons.

Observation schedule : (See Appendix B). This was used by the Assistant Observer (AO) to observe learners' behavior as exhibited during their interaction with the algebra in the spreadsheets environment. Like with an observational protocol (Creswell & Plano Clark, 2011, p. 178), where the observer records descriptions of events and processes observed, the observation schedule consisted of a set of guiding questions on the aspects of classroom interactions that were considered important for the study. It also provided space for the user to write down responses to the questions. The researcher was in a way researching her own practice, and in order to reduce possibilities of bias in the data collected, (Koshy, 2005, p. 99), she decided to include the AO. The AO was a mathematics educator who has some knowledge about spreadsheets, though not in relation to its use for the teaching and learning of algebra. The AO paid particular attention to aspects that related to the learners' difficulties as they went through the process of *instrumental genesis*, monitoring their facility with the technology, spreadsheets-algebra and ultimately their understanding of the targeted algebraic concepts. These pieces of data were accessed through assessment of learners' answers/ responses to items in the worksheets, listening to classroom discourse - the discussions they had amongst themselves and with the teacher.

The AO was also confronted with a set of questions relating to strong and weak points in the lessons. The observer was expected to give his/her opinion of learner experiences, whether use of spreadsheets helped or hindered learners' understanding of algebra and provide reasons for the stand she took. Sources of learners' difficulties were also sought. The observer also had to indicate the kind of strategies the teacher-researcher used and say which of those strategies worked for the learners. The observations on the teacher-researcher's interaction with the learners were intended to establish the necessary conditions under which effective learning of Grade 9 algebra in the spreadsheets environment can be realized. Opportunity to make suggestions on how the lessons could be improved was also provided for.

a) Open-ended questions.

Classroom activities were organized in the form of worksheets. *See Appendix A.* At the end of each of six main worksheets, there was a list of open-ended questions that learners had to answer (the other two worksheets were extensions, hence the responses were covered in the main worksheet). The following is a list of questions that learners answered :

1. What challenges did you encounter in working through this worksheet?
2. What do you think were the sources of your problems?
3. What do you think could have been done to help you?
4. What have you learned and understood in this session?
5. What do you think helped you to learn successfully? ("successfully" was used here because there was evidence that learners had successfully grasped some of the concepts taught).

The purpose of the questions was to get the learners' impressions on their algebra learning experience within spreadsheets. Learners were to reflect on the challenges they encountered, and the sources or causes of their difficulties. They had to explain how they came out of their difficulties; paying attention to what they did and what role their teacher played in helping them overcome their difficulties. Learners were also provided opportunity to say what they thought should have been done to help them.

Learners' responses to these questions were transcribed and compiled into one document. These were then analyzed and coded using ATLAS.ti. This was done separately for each school. Learners were assigned numbered letters as names to distinguish them. For example, *R13* and *P4* have been used to name learners from Retha and Palo High schools respectively. The main themes identified here were based on the research sub-questions while some sub-themes (sub-headings) emerged from the data. This phase was then followed by the second phase of the investigation.

3.3.2.3 Phase 2: Quantitative

i) Participants

The participants in this phase were all Grade 9 learners who participated and remained in the study to the end.

ii) Data Collection: Questionnaires

Group questionnaires (See Appendix E) were administered to learners and this took place at the end of the teaching session. (de Vos, 2002, p. 174). The aim here was to determine learners' perceptions, attitudes and feelings (Johnson & Christensen, 2004, p. 170) towards the algebra learning experience they had during the teaching session. The intention was to assess learners' overall impression about learning algebra through spreadsheets. The purpose was to find out what they found challenging and how they handled the challenges. Learners were required to rate, on a Lickert Scale, their achievements related to their technical knowledge and skills they attained during the course of the study. They also rated their experience of the teaching approach used in conducting the lessons. The questionnaire consisted of selected-response in Section A and scaled-response questions in Section B (Gillham, 2000, p. 39).

3.3.2.4 Phase 3: Qualitative

i) Participants

For both schools, Retha and Palo High School, participants in this phase were learners selected from the groups participating in the study. They were selected based on performance in the lesson activities, as seen by the teacher-researcher. These were representative of the high, mediocre and least achievers. One learner was chosen for each category, making a total of three learners in each case (school). It is worth mentioning that the selection of learners was restricted, as the number of learners in each school had decreased from the initial number of fifteen (15) to ten (10) and seven (7), by the end of the study, for Retha and Palo High School respectively.

The AO was also interviewed on her impressions regarding her experience of the teaching and learning of algebra through spreadsheets, from an observer point of view.

ii) Data collection

a) Interviews: Learners and the Assistant Observer

Semi-structured in-person interviews (See Appendices C & D) were conducted at the end of the teaching period, with three learners from each school and the Assistant Observer. Table 3.2 shows the dates for the AO and learner interviews. The purpose of learner interviews was to make a follow up on the learners' responses to the questionnaire items. The interviews were conducted to seek further clarifications and get deeper insights into the algebraic understandings and learning difficulties and successes as experienced by the participants. Interviews provided the researcher opportunity to explore and gather data in a more relaxed context.

Interviewee	**Name of School**	**Date**	**Day**
AO	N/A	01/10/2011	Saturday
R1	Retha H.S	07/10/2011	Friday
R2	Retha H.S	07/10/2011	Friday
R3	Retha H.S	07/10/2011	Friday
P1	Palo H.S	08/10/2011	Saturday
P2	Palo H.S	08/10/2011	Saturday
P3	Palo H.S	08/10/2011	Saturday

Table 3.2: Dates for Interviews

This type of questioning allowed participants to express themselves freely. The semi-structured interviews also permitted the researcher to steer the discussions through fruitful paths. (Koshy, 2005, p. 92) These were conducted by the researcher and were audio-recorded. These were one-to-one interviews that included open-ended questions. The researcher prepared an interview schedule, containing logically sequenced questions addressing the areas of concern in the learners' experiences in their algebra learning through spreadsheets. The questions were arranged according to degree of challenge, from simple to more challenging and from broad to more specific (Koshy, 2005, p. 93).

The Assistant Observer was interviewed to get in-depth understanding and interpretation of her observations regarding the learner experiences in learning algebra through spreadsheets and on the project as a whole.

We now turn to the discussion on how the issues of validity were addressed in this study.

3.3.3 Validity and reliability

3.3.3.1 Validity

Mixed methods research involves the mixing of both qualitative and quantitative methods, approaches and concepts into a single research study (Johnson & Onwuegbuzie, 2004, p. 17; Creswell & Plano Clark, 2007, p. 5). As this is a new research paradigm, it is mentioned that the issue of validity in mixed methods is not yet clearly defined (Onwuegbuzie & Johnson, 2006, p. 48; Creswell & Plano Clark, 2011, p. 238). Validity, as defined by Onwuegbuzie & Johnson (2006, p. 48), it

> is not about singular truths, and it certainly is not limited to quantitative measurement; rather by validity we mean that a research study, its parts, the conclusions drawn, and the applications based on it can be of high or low quality. (Onwuegbuzie & Johnson, 2006, p. 48)

Creswell & Plano Clark (2011, p. 239), further elaborates on this definition and define validity in mixed methods as

> employing strategies that address potential issues in data collection, data analysis and the interpretations that might compromise the merging or connecting of the quantitative and qualitative strands of the study and the conclusions drawn from the combination. (Creswell & Plano Clark, 2011, p. 239).

In this study, particular attention was paid to the data collection procedures, how each related to others in order to minimize threats to validity, as recommended by Creswell & Plano Clark (2011, p. 239). The open-ended questions at the end of learners' worksheets were designed to follow up on the findings drawn during the lessons. They were meant to get more information from the learners in order to complement the Assistant Observer's and the teacher-researcher's field notes. The number of participants in the interviews (3 per group)(qualitative) was smaller than that used in answering the questionnaire (quantitative), and the learners interviewed were selected from the same group that participated in answering the questionnaire (Creswell & Plano Clark, 2011, p. 242). This therefore means that the interview questions (developed from major themes) were largely based on the findings from the questionnaire. The interviews were conducted to complement data obtained from the classroom observations and questionnaire.

Direct observations were conducted because the researcher was out there organizing the case, observing what was happening and using personal judgments about what was being investigated. The fact that the researcher was also observing her own teaching and observations were based on her personal judgment brought in risk of biasness on the results of the study (Koshy, 2005, p. 99). In order to minimize this risk, the AO, knowledgeable in the field, was engaged.

The issue of validity was also addressed by prolonged process of data collection on the sites. Using the many worksheets with different tasks over a period of eight weeks (24 lessons) was enough to expose learners' problems as they went through the *instrumentation* and *instrumentalisation* processes.

One of the main purposes of using mixed methods was to provide stronger evidence for conclusions or claims that were arrived at through convergence, corroboration and correspondence of findings, a property also sought by triangulation (Johnson & Christensen, 2008, p. 444; Greene et al, 1989.; Bryman, 2006, p. 105; Creswell & Plano Clark, 2011, p. 62). Use of the various methods of data collection was important for validating the data collected. The study progressed in three phases as illustrated earlier; the different samples at each phase were drawn from the same population, making the data comparable. (Creswell & Plano Clark, 2011, p. 240).

3.3.3.2 Reliability

Reliability of a research instrument is a measure of the extent to which the instrument yields the same results on repeated trials. For this study, the questionnaire was tested for internal consistency. The Cronbach alpha coefficient is the most popular index of reliability used to determine the extent to which scaled items in a questionnaire measure a given construct. The formula used for the coefficient is:

$$\alpha = \frac{n}{n-1}\left(1 - \frac{\sum s_i^{\,2}}{s_T^{\,2}}\right),$$

Where n = number of items; $s_i^{\,2}$ is the variance of the *i*th item and $s_T^{\,2}$ is the variance of the total score formed by summing all the items (Bland & Altman, 1997, p. 572). The values of the coefficient range between 0 and 1, and the acceptable values are between 0.7 and 0.9. (Tavakol & Dennick, 2011, p. 53). Tavalok and Dennick further note that the lower values may indicate that the items are poorly related, that the questionnaire is one-dimensional or may be due to a fewer number of items for the construct measured. In order to attain acceptable levels of reliability some items may be revised or deleted from the questionnaire. In the case where there were few items, additional ones may be formulated. (Tavakol & Dennick, 2011, p. 54). The researcher believed that the questionnaire was three-dimensional, the list of items that were formulated to investigate each of the three constructs (attitudes towards spreadsheets, preferred learning styles, and technical knowledge and skills) were also tested for internal correlation.

The multiple-case design also provides for generalization of results. Although generalization was not the main purpose for this case studies, these generalization, as against those made from surveys, were analytic conclusions arising from the two cases, as with experiments, and thus more powerful than those coming from single case study designs (Yin, 2009, p. 61).

3.3.4 Strategy for data presentation and analysis

A case study database was compiled for the study. This is a collection of all evidence collected during the study. It consists of all data, electronic and non-electronic, and all other evidence that has been collected by the researcher. Data analysis, according to (de Vos, 2002, p. 339) is the process of "bringing order, structure and meaning to the mass of collected data". The entire set of data generated during the experiment was systematically analyzed while simultaneously documenting the grounds for particular assertions and inferences. Audio tapes were studied and transcriptions of interviews were done. The videotapes were also studied and information about the study was noted.

In mixed methods research, data may be analyzed differently in different designs. Creswell & Plano Clark (2011, p. 218) suggest the following model, which was adopted with modifications, in analyzing data for this exploratory design:

1) Collect the qualitative data;
2) Analyze the qualitative data qualitatively using analytic approaches best suited to the qualitative research question;
3) Design the qualitative strand based on the qualitative results;
4) Develop and pilot test the new instrument;
5) Collect the quantitative data;
6) Analyze the quantitative data quantitatively using analytic approaches best suited to the quantitative and mixed methods research questions;
7) Interpret how the connected results answer the qualitative, quantitative, and mixed methods questions.

(Creswell & Plano Clark, 2011, p. 218)

It is on the bases of this approach that qualitative and quantitative data are presented and analyzed separately in Chapters 4 and 5 respectively. Qualitative methods were used to analyze data from observations and interviews, while statistical numerical methods were used to analyze data collected from questionnaires. Some categories, guided by literature, were utilized to organize the information. Analysis of the quantitative data was done individually for each case, based on the model suggested by (Yin, 2009, p. 57). (See *section 3.3.1)*. Integration of the qualitative and quantitative data took place during the analysis. The findings, both quantitative and qualitative, were integrated during interpretation, and conclusions were drawn about the research. (Johnson & Christensen, 2004, p. 447).

When analyzing quantitative and qualitative data within the mixed methods framework, the researcher carried out the following processes: a) data reduction, b) data display, c) data transformation, d) data correlation, e) data consolidation, f) data comparison, and g) data integration (Onwuegbuzie & Leech, 2006, pp. 490-491; Onwuegbuzie & Teddlie, 2003). Figure 3.2 shows the steps in mixed methods data analysis process.

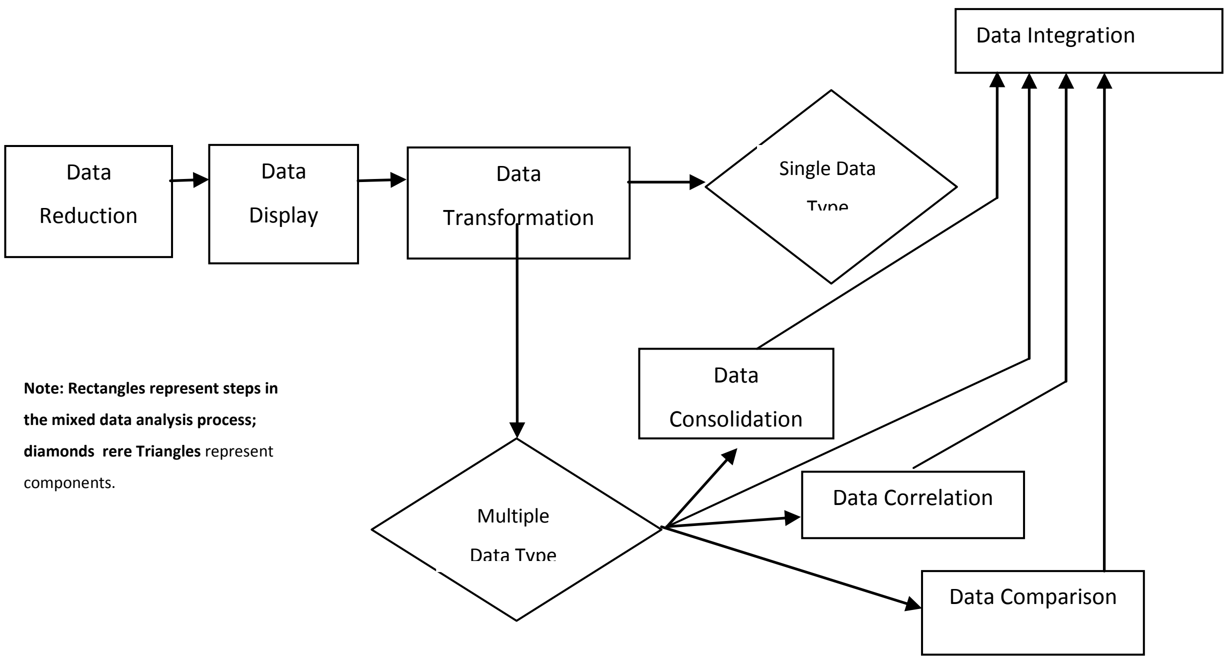


Figure 3. 2: The mixed methods Data Analysis Process (Onwuegbuzie & Leech, 2006, p. 492)

Data reduction involved reducing the number of dimensions in data by identifying and classifying it into themes, in the case of qualitative data or via descriptive methods in the case of quantitative data. According to Braun and Clarke, “a theme captures something important about the data in relation to the research question, and represents some level of patterned response or meaning within the data set” (Braun & Clarke, 2006, p. 10). It is worth noting, however, that the researcher’s judgment is necessary to determine what counts as a theme and what is not. A theme is not determined by the frequency of its prevalence within the data set, but by whether it captures something important with regard to the overall research question" (Braun & Clarke, 2006, p.10)

Thematic analysis is defined as "a method for identifying, analyzing and reporting patterns (themes) within data" (Braun & Clarke, 2006, p. 6). Thematic analysis of data may be *inductive or theoretically* driven. In *An inductive thematic approach*, on one hand, the themes are strongly linked to the data themselves, they emerge from the data, and are not necessarily driven by the theoretical interest of the researcher in the topic. On the other hand, a *theoretically driven thematic analysis,* which was followed in this study, is driven by the researcher's theoretical or analytic interest in the area under study. The coding of themes is focused on those aspects as have been mentioned or implicated in the previous studies or literature in the area, as was the situation in the current study (Braun & Clarke, 2006, p. 12).

Thematic analysis in this study was done at the latent level. At this level, the identification of themes does not stop at surface (semantic or explicit) meaning of the data, but goes beyond and "starts to identify or examine the underlying ideas, assumptions and conceptualizations- and ideologies- that are theorized as shaping or informing the semantic content of the data" (Braun & Clarke, 2006, p. 13). As Braun and Clarke further affirm, "for latent thematic analysis, the development of themes themselves involves interpretive work, and the analysis that is produced is not just description, but is already theorized" (Braun & Clarke, 2006, p. 13). It is therefore imperative that in order for theoretically driven thematic analysis to be done successfully, thorough engagement with literature relevant to study be done.

The software, ATLAS.ti, was used to aid the process of data analysis. Data was then displayed or represented in the form of statements, tables and figures. This included discussion of the evidence of the themes, supported with actual quotations from the data. Quotations conveying the different themes or sub-themes were obtained from varying sources of data to provide multiple perspectives from individuals in the study. (Creswell & Plano Clark, 2011, p. 209). Interrelationships between themes and sub-themes were established.

During the transformation stage, quantitative data are converted into narrative data for analysis using quantitative methods and qualitative data are similarly converted to numerical data (coding) that can be statistically analyzed. Data correlation involves making correlation between quantitative and qualitative data. This could mean cross-classifying different data types, such as transforming qualitative data into categorical variables and examining their relationships with quantitative variables. In this particular study, however, no transformation of data occurred, qualitative data was analyzed using qualitatively and quantitative data was analyzed using analytic techniques best suited to the approach, as indicated earlier, and the results were then merged in order to draw conclusions for the whole study.

Table 3.3 shows phases of thematic analysis and corresponding descriptions of what happens at each phase.

PHASE	DESCRIPTION OF PROCESS
1. Familiarization with data	Transcribing data, reading the data, noting down initial ideas
2. Generating initial codes	Coding interesting features of the data in a systematic fashion across entire data set, collating data relevant to each code
3. Searching for themes	Collating codes into potential themes, gathering all data relevant to each potential theme.
4. Reviewing themes	Checking if the themes work in relation to the coded extracts(level 1) and the entire data set (level 2), generating a thematic "map" of the analysis
5. Defining and Naming Themes	Ongoing analysis to refine the specifics of each theme, and the overall story the analysis tells; generating clear definitions and names for each theme.
6. Producing the report	The final opportunity for analysis. Selecting of vivid, compelling extract examples, final analysis of selected extracts, relating back of the analysis to the research question and literature, producing a scholarly report of the analysis.

Table 3.3: Phases of Thematic Analysis (Braun & Clarke, 2006, p.:35)

Results from the data collected through the questionnaire were interpreted in terms of the themes used for the qualitative data. One-way*Analysis of variance (ANOVA)* was then carried out to determine the statistical significance of differences between means of responses from the two schools. This is a statistical technique that can be used with two or more groups and uses the *F*-test statistic (Urdan, 2005, p. 101). Urdan further explains that "The F-value is the statistic used to indicate the average amount of difference between group means relative to the average amount of variance within each group"(Urdan, 2005, p. 114). The *Mann-Whitney U-test (U)* was also used to determine whether there was any significant difference between the groups. This is a nonparametric inferential statistic used to determine whether two uncorrelated groups differ significantly. This statistical test was used because the used data represented ordinal level scale and the sample sizes were small. Non-parametric tests are a class of tests that do not hold the assumptions of normality. (MacFarland, 1998)

In the last phase, findings from the qualitative and quantitative data were then triangulated, and collaboration and corroboration were sought. The exploratory nature of this study indicates that the aim was not to prove any proposition or generalization, but to seek better understanding of how learners experienced learning algebra in the spreadsheets environment, paying special attention to the type of obstacles they encountered and how they were be assisted, through instrumental orchestration, to overcome their problems. Here the researcher compared the results with the initial research problem and questions in the study and determined how these were answered. The results of the study were also compared with prior explanations from past research studies. In addition to these, the researcher also brought into picture, personal experiences and drew assessment of meanings of the findings.

3.4 Conclusion

This chapter provided an outline of how the study was conducted. The research commenced with a pilot study whose main purpose was to assess the appropriateness of the selection procedure for the participants, usability of the instructional materials and the questionnaire, in terms of language accessibility by the learners, and the appropriateness and applicability of the data collection protocol. The necessary modifications were done on some instruments and in the manner in which the study was conducted and the main study followed.

The main study progressed in three phases, the classroom observations, the administration of the questionnaire and the interviews. The design of classroom activities was provided. Data from the two schools, Retha and Palo High school was collected and analyzed through qualitative and quantitative methods. Triangulation of findings was done; comparisons were made to determine the differences, similarities, collaboration and correspondence between the findings from the different sources of data. Conclusions were drawn and recommendations were made concerning the study. The next chapter presents findings from the qualitative data.

CHAPTER 4

FINDINGS: QUALITATIVE DATA

4.1 Introduction

The previous chapter outlined how the empirical study was conducted. This chapter presents findings from the qualitative phase of the empirical study. The qualitative data was collected through classroom observations, learners' responses to questions at the end of each worksheet, and in-person interviews. The findings provided in this chapter are from the sets of data from the two schools and are presented simultaneously for the two schools. This was done on realizing that there were overlaps in the data from the schools studied.

This presentation of findings is focused on:

1) The challenges that were encountered in the teaching and learning of grade 9 algebra through spreadsheets;
2) The strategies that enabled effective teaching and learning of Grade 9 algebra through spreadsheets, and
3) Learners' attitudes and perceptions with regard to the teaching and learning of algebra through spreadsheets.

The discussion provided hereafter is focused on evidence of existence of the above aspects leading to answers to the main research question. The data in the form of extracts from each of these sources is provided and categorized according to the research sub-question they addressed. Letters R and P have been used to distinguish between data excerpts for learners from Retha and Palo High schools respectively.

4.2. Findings

4.2.1 Challenges encountered in the teaching and learning of Grade 9 algebra through spreadsheets.

From a critical analysis of the challenges encountered during the teaching and learning of Grade 9 algebra through spreadsheets, two categories of challenges emerged: those that were school-based and those that were instruction-based.

4.2.1.1 School-based challenges

The challenges in this category are related to the physical space and the technological equipment required for effective teaching and learning of algebra through spreadsheets.

i) Lack of adequate facilities

Laboratory space: In one of the schools, the computer laboratory was very small and this created many problems for the learners and the teacher. The space between the furniture did not permit free movement for the teacher when attending to individual learners' needs. This restricted the teacher in the choice and execution of intervention techniques and strategies she used during the lessons. Some learners preferred asking questions and being assisted individually or better in smaller groups, so the kind of set up in the laboratory was not favorable to their preferred mode of learning. Such learners withdrew from the discussions. Most of the time the teacher had to use whole-class discussions, wherein a learner would have to ask a question aloud and the rest of the class called to discuss the raised issue. Even though whole-class discussions played a significant role in helping learners in constructing and reconstructing their knowledge, sometimes these caused unwelcomed disruptions to what some learners were busy with at a time.

Lack of adequate equipment: The equipment here refers to the computers that learners used. Some of these were very old and took a lot of time to open. This forced some of the learners to be moving from one workstation to another, which was inconvenient as expressed by one of the learners at Retha High school,

> *R11: All computers should be working well. .....I hated moving up and down changing computers*.

This was not only frustrating to the learners but to the teacher as well. The same situation was also experienced at Palo High school. Learners had to change from working on one computer to another because the computers were old and would therefore pick up different problems at different times. This problem led to learners having to re-do work they had already done and the teacher not being able to proceed with the lesson as planned, as some learners would then be left behind.

The AO felt that it was important to have an overhead projector in the computer laboratories, as this would have made the teachers' work easier. This observation applied to both schools. Some explanations and illustrations that the teacher provided could have been even clearer and presented more quickly with the help of an overhead projector. The teacher spent a considerable amount of time in some cases trying to make a pictorial representation of spreadsheets on the board, when the focus was not teaching about the spreadsheets but on concepts in spreadsheets.

> *Use of overhead [data] projector could be more helpful to the teacher in demonstrating certain aspects on spreadsheets (Retha H.S)*
>
> *Use of an overhead [data projector] would enable the teacher to do demonstrations much easier than on the board with pen or doing same thing for each and every group*. *(Palo HS.)*

The AO also felt that monitoring learners' work would also be made easier with interconnections between computers in the laboratory. *(See Section 2.4.1).* Some learners had a tendency to engage in off-task activities once the teacher was not near. In this way they lost track of what was happening in class; they remained behind and were not able to answer the teacher's questions and neither were they able to contribute to classroom discussions. As mentioned earlier, (*See Section 2.4.1*), great care has to be taken when using technologies such as computers in for learning purposes. The teachers have to make sure that they have management strategies in place. Salomon et al in Jonassen and Reeves (2001, p. 696) notes that it is only when learners engage mindfully with computer tools that success in learning with the tools may be realized. The AO suggested:

If it was possible to have all students' workstations connected to the teachers' then monitoring their use of the computer. (Palo H.S)

ii) Lack of maintenance of facilities

Supply of electricity. In some instances, there were disruptions of the lessons due to electricity cuts, which were caused by use of faulty equipment. These also led to learners losing their work particularly because most of them would not have been able to save it. (Bingimlas, 2009, p. 241) affirms that lack of technical support provided by the school greatly affects ICT integration in the schools. Sicilia (2005, p. 43) noted, "technical barriers impede the smooth delivery of the lesson or the natural flow of the classroom activity". Even if teachers would appreciate the usefulness of spreadsheets in enhancing the teaching of algebra, they may be discouraged from implementing its usage due to fear of failure of technology in their lessons (Bingimlas, 2009, p. 241).

4.2.1.2 Instruction-based challenges

The challenges in this category are those that were encountered by learners during instruction. These are classified into three groups namely: technical, personal and those that are language related.

i) Technical challenges

The challenges discussed here are categorized into mechanical and conceptual. Mechanical challenges are those that are related to physical manipulation of the technology and the conceptual ones are those that relate to the associated algebra (mathematics) content. The challenges can be attributed to the constraints imposed by the use of the artifact.

a) Mechanical challenges.

The following are the mechanical challenges that were experienced by learners.

Mechanical manipulation of the computer. During the lessons, particularly at the beginning of the project, learners displayed a lot of incompetence on the use of the computer and of *Excel* spreadsheets in particular. Even though this group of learners was expected to have had some basic exposure on the use of the computer, almost all of them were still struggling with basics such as opening the spreadsheets, saving their work, accessing their previous work, use of undo, backspace and delete keys. This was also observed by the AO. Learners also had problems with handling the mouse particularly when applying the "dragging command" or when highlighting/selecting data.

> *They lacked basic computer skills and spreadsheets in particular* (Palo *H.S.)*
>
> *Handling the mouse to drag*. (Retha H.S).
>
> *They are too slow in operating the mouse and remain behind*. (Retha H.S).
>
> *Learners had a problem handling the mouse. They spend a lot of time trying to select and drag* (Palo H.S)
>
> *They were not aware they could open new books in the same window. When starting a new task they reopened Excel spreadsheets anew*(Palo H.S.)

Even though one can still perform a lot of functions or commands on a computer without using a mouse, lack of basic computer skills confined learners to using the mouse even when they found it difficult to handle. Their *usage schemes* (Drijvers & Gravemeijer, 2005, p. 167) were still very limited. During the interview, the AO indicated that learners had inadequate computer skills.

> *At the beginning, they lacked the necessary computer skill.*
>
> *There was also a problem with a "dragging command" in filling the rows and columns*.

Learners also confessed that they lacked the mentioned skills.

> *P4:… Dragging down was a problem to me.*
>
> *R13: I think the sources are caused by my typing speed and my slow work with a lot of time when typing*.

R13's confession about his/her inability to do work within reasonable time could be due to organizational constraints imposed by the computer. *Organizational constraints* are those related to learners' inability to operate the hardware of the computer (Trouche, 2004, p. 290).

(*See section 2.4.2.2.).* Lack of knowledge about how different keys were organized on the keyboard led to situations where learners could not type quickly. This lack of the necessary skills hindered learners' progress in learning the intended mathematics. They spent a lot of time on Worksheet 1 as they were being familiarized with the use of the *Excel* spreadsheets.

Accessing available commands. This was more evident when learners had to create graphs in spreadsheets. Worksheets 3 and 4 engaged learners in working with graphs, creating graphs and exploring their properties in spreadsheets. In Worksheet 3, Learners were introduced to the general formula for the equation of a straight line and were guided to discover meanings for intercept and gradient. They started by creating the graph of y =2x + 3 in spreadsheets. Learners indicated that creating a graph was a problem to them.

> *R1: I think the sources is I don't know how to insert graph using spreadsheets*
>
> *P9: The source of my problem is how to make a graph on the computer*
>
> *R4: I encountered dozens of hectic and mind teasing texas[tasks] like selecting the right graph and following instructions.*
>
> *R5: drawing graphs of linear functions and graphs for seedlings*
>
> *R5: entering the formulas, inserting the vertical lines and the horizontal lines to the graphs; inserting the minor units, major units, the maximum and minimum units.*

Learners' inability to create a graph could be because they had never done it before; even in their computer lessons or that their instrumented action schemes were not yet developed in that regard. For example, when they were first introduced to the process, they said they had no idea about graphs in spreadsheets. The teacher then gave them steps they should follow in creating a graph. They were able to do it with the teacher, but when they had to do it for the different situations that they came across; learners complained that there were too many steps. Even during interviews, learners could still remember the difficulties they experienced.

> *R3: The problem was creating the graph, the steps after highlighting the table, madam. The steps to take in order to get the correct graph.*

From her classroom observations and interview response, the AO also confirmed learners' difficulties with creating graphs

Learners found it difficult to follow steps in getting the graph, particularly where adjusting scales were necessary. (Retha *H*.S)

Creating the graph was problematic. Too many steps, adjusting scales was more serious. (Palo H.S)

Command constraints (Trouche, 2004, p. 290) were experienced when learners did not know what the technology could offer and they did not know how to get certain commands and, more importantly, they were unable to make decisions and justify why they needed to do certain operations. This usually hindered their use of the software to attain their algebra learning goals.

b) Conceptual Challenges

The conceptual challenges that were encountered are discussed in relation to the different algebraic concepts that were taught during the study.

In Worksheet 1, learners were engaged in creating spreadsheets formulas and using those formulas to extend the sequences of numbers. The purpose of tasks in this worksheet was to familiarize learners with spreadsheets notation, the knowledge that would later on help them to translate mathematical relationships from ordinary language to the formal algebraic notation. The intended knowledge was to be building on their knowledge about number behaviors, number sense, which they should have acquired in their arithmetic lessons in the primary school. The following areas of learners' difficulty were identified:

Lack of necessary background knowledge of mathematical concepts. This was evident in learners' inability to carry out tasks in Worksheets 1 and 2, which involved identifying number patterns. In response to the open-ended questions at the end of the worksheets, learners confessed that they had problems in determining missing numbers in sequences, as they could not establish the relationships between numbers.

P10: identifying patterns and sequences. The formulas used.

P7: the challenge is how to find relationship between numbers

R9: trying to find out how the sequences are build up ….

R2: I met a challenge whereby I was to fill in the next numbers of the rows and columns

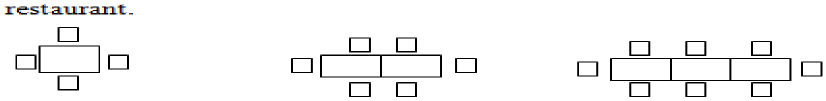
These statements indicate that learners lacked number sense and were not familiar with the behavior of numbers. From the AO's perspective learners in both schools showed lack of sufficient arithmetic skills to enhance development of algebraic concepts. Initially learners had difficulty in filling up the Pascal's triangle due to their inability to relate numbers in sequences in worksheet 1.

> *I think the main problem here was identifying relationships between consecutive numbers in a sequence*. (Retha H.S).
>
> *Pascal's triangle was the most difficult to recognize* (Palo H.S)

With more engagement with finding the "rules" for the sequences, they became more comfortable with the concept. Their next challenge was on determining the association between sets of numbers. In Worksheet 2, learners were led through tasks that were intended to strengthen their understanding on the role of letters in expressing generalizations. This also involved identifying relationships between sets of numbers in functional relationships. Figure 4.1 shows one of the tasks that learners had to do:

1. Expressing relationships

The following is representation of different arrangements of tables and chairs in Mpho's restaurant.



chair □
table ▭

a) Use the Excel spreadsheets to construct a table with columns for *number of tables* and *number of chairs*.

b) Generate the values for the columns and determine the number of people that may be accommodated by 14 tables.

c) What formula did you use to generate the table of values?

d) Explain the meaning of the formula in your own words.

e) We can also relate the number of chairs to the number of tables available. Calculate the number of chairs that would be required for
   i) 50 tables.
   ii) 540 tables.

f) Describe, in your own words, how you can calculate the number of chairs given the number of tables.

Figure 4.1: A Task on Expressing Relationship Between Sets of Values

Although they could answer all questions correctly, learners had difficulties in the last part; they found it difficult to express the relationship between sets of numbers, the number of tables and the number of chairs. They preferred to use the relationship between terms in sequences in each column, which they used in filling up the table of values for a) and b). They also took advantage of the enablement of spreadsheets (dragging) to get number of chairs for 50, and 540 tables instead of calculating the values. Examples of their responses to this question were "*the table consists of 4 chairs, but if a table is added, then 2 chairs are added" [R9]* or *"the tables increase with 1 and chairs with 2" [R10].*

A mention of this type of problems came up in learners' confessions:

> *R10:…when I had to write the relationship between people accommodated and the tables…*
>
> *R14: when we had to write relationship between people accommodated and the tables.*

When they had to extend the pattern, the table of values, using spreadsheets formulas, they resorted to using the "rules" for the numbers in each column. The AO's comments on learners' performance in Worksheets 1 and 2 also confirmed poor arithmetic skills as a source of their difficulties.

> *Learners' difficulties lie on not knowing number patterns and not being able to identify relationships between sets of numbers. They lack number sense. Getting relationship between numbers in a sequence is much easier to them, so they rather use this to fill up columns in the tables.* (AO-Retha H.S)
>
> *The main challenge in this worksheet is making connections between sets of values* (AO-Palo H.S)
>
> *They found writing relationships between pairs of numbers difficult* (AO-Palo HS)

During interviews, which were held at the end of the whole teaching period, learners could still remember that they had difficulty with number patterns.

> ***I:****…Can you just give me examples of difficulties you encountered?*
>
> *P3:The relationships between numbers in Pascal's triangle*

I = Interviewer, P3= learner 3 from Palo H.S.

In an interview with the AO, she further pointed out that learners were not able to generalize about patterns and attributed that to poor arithmetic skills.

> *Poor arithmetic skills. I think if they do not have knowledge about number patterns and number behaviors then it will be difficult for them to identify the relationships in the sequences.*

Learners' lack of underlying mathematical concepts was also observed when they were working through the activity that was intended to lead them to discovering the *distributive* and *commutative properties* in algebra. The AO confirmed that

Learners could not see 2A + 2B and 2B + 2A as two ways of writing the same thing hence were frustrated by not getting the second formula or expression. (Retha H.S)

The AO's comments on learners' performance in this exercise, again, pointed to poor arithmetic skills as the main source of learners' difficulties.

The main problem is that of lack of necessary background knowledge (number relationships) (Palo H.S)

This was due to learners' lack of numbers sense. They are not aware of number behaviors. (Palo H.S.)

Learners also indicated that they encountered challenges when they had to choose or adjust scales for their graphs. Although this may be taken to be lack of knowledge concerning use of the software, the difficulties are likely to be associated with the arithmetic skills involved in choosing selecting appropriate scales. Figure 4.2 shows one learner's (R7) part of the solution to Question 1 of Worksheet 6. The graph had not been formatted, (the axes, the scales, the gridlines) and as such, it would be very difficult to read information from it. Hence the comment:

R6: The challenge in working through this worksheet is when finding distance and time when Lele overtake Thato

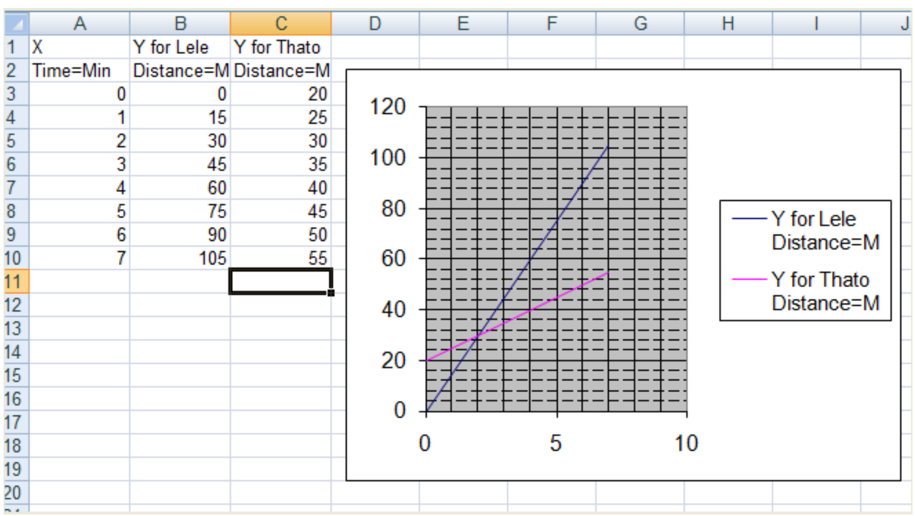


Fig. 4.2: Part of R7's Solution to Worksheet 6: Question 1

Learners indicated that they had problems in choosing appropriate scales for their graphs.

R6: expanding the graph

R5:*…inserting the minor units, major units, the maximum and minimum units*

It is important that arithmetic be taught in the manner that it develops algebraic thinking in the learners, thus paving way to their learning of algebra. Algebraic thinking can be developed as early as in primary school. As young learners solve arithmetic problems, recognize relationships between operations, recognize patterns and express these verbally, they are engaged in algebraic thinking. (*See section 2.2.2*). Some of the problems that learners indicated, were aspects of the targeted content to be learned which by the end of the tasks they were able to grasp. Lack of necessary mathematical background for developing the intended algebraic content within spreadsheets was bound to bring about the challenges encountered (Pierce & Stacey, 2004, p. 61).

Syntax in spreadsheets notation. Another area of difficulty for the learners was that of using the spreadsheets notation. Since the aim of these initial tasks was to develop this skill, this was expected. Learners were even introduced to absolute referencing which was completely new to them. The AO noted

> *It was difficult for them to use the spreadsheets notation, if ever there was a formula to write about a pattern, they were not able to do that*.

She also pointed out that learners displayed lack of understanding of the entries in the spreadsheets cells. For example, at one instance A2, is used to refer to the location on the spreadsheets, at another it is used to refer to the contents within the cell itself (Haspekian, 2005, p. 121). Although the ambiguity within which a cell-reference is used provides an enablement for the development of the concept of variable (Bills et al, 2006, p. 42), it also became a source of confusing to learners.(*See section 2.4.2.1)* This was observed during their engagement with tasks in Worksheet 1, where they were required to explain the meanings of the spreadsheets formulas that they used to generate the sequences. This exercise was intended to help learners to realize that entries within cells could be connected to those in the neighboring cells by means of formulas. The AO explained that

> *understanding the meaning of the entries in the cells, for example if there was a 3 in a cell, then they did not understand the meaning of the entries, this project helped them to understand these things.*

It is also important to note that the strict syntax followed in writing the spreadsheets formulas also brought challenges to the learners as they engaged in creating formulas. The following statements are some of the learners' responses to the open-ended questions in the worksheets.

> *R7: I didn't know how to use the $A2*B$1 but madam explain it to us and we used it for writing multiplication table*
>
> *P9: my problem is that it is extremely hard to find the correct formula in the spreadsheets*

R3: I met the challenge of forgetting how is the formula used

R15: it was difficult to find the formula for Pascal's triangle

P8: multiplication, Pascal triangle and square, creating a calendar and entering formulas.

P1: Sometimes not easy to get the formulas to give to the computer.

R2: Challenges I met are not knowing the formulas even though I struggled and found them

The AO had also observed that learners had difficulty in formulating spreadsheets formulas.

Creating formulas for the patterns was very difficult as a result. (Palo H.S)

They syntax followed was not mastered. They usually forgot the equal sign at the beginning. (Palo H.S)

They were not aware of the absolute referencing. (Palo H.S.)

This was evident during the early sessions of learners' encounters with learning algebra through spreadsheets. In worksheet 2, learners were led into identifying and expressing the relationship between the number of chairs to the number of tables in "*Mpho's restaurant"* in their own words, and finally in ordinary algebraic form. Activity 2 consisted of more examples of relations between pairs of numbers and learners were asked to use letters of their own choices to express them.

P1: I knew the relationship well but I did not know how to guide the computer to give other figure

It is evident from the following interview excerpts that even when learners could recognize the relationships, they had difficulty in formulating spreadsheets formulas to enable them get the next numbers in the sequences.

I: *What was problematic there?*

P1: Finding the formulas

Learners in both schools encountered this common problem.

I: you said what was really difficult?

R2: When using the formulas.

I: The formulas were a bit problematic!

R2: Yes, madam.

N.B. I=interviewer and R= learner from Retha High school

Learners even suggested that algebra should not be taught through spreadsheets because the formulas became a source of problems.

R2: ....I don't think algebra should be taught by spreadsheets because sometimes when it comes to formulas it is difficult to use them…

From the interviews, it also surfaced that learners were not very particular with the strict syntax in the spreadsheets formulas. When asked what spreadsheets formula they would use to generate a table of values for the graph of $y = 2x + 3$, all the learners interviewed left out the equal sign in their formulations(*command constraint*), even though the rest of the formula was correct. They only included the equal sign after a probe, as illustrated in the following conversation:

I*: if you had to draw the graph of y =2x +3 using spreadsheets, can you remember the spreadsheets formula you would enter, suppose this is cell…*

R1: A1

I: yes suppose it is that A1, what would be the formula you would enter in that cell which you are going to drag to get all other values in that column?

R1: It would be A1×2 +3.

I: just like that?

R1: yes madam!

I: Don't you think you have left something that is very important?

R1: Oh. The equal sign. It would be = A1×2 +3.

N.B. I=interviewer and R= learner from Retha High School

I observed that when working on Worksheet 1, learners were not familiar with absolute referencing which they could use to create formulas for extending the multiplication table, the Pascal's triangle and the calendar more quickly. Even after working through several exercises, learners still had difficulties with creating spreadsheets formulas.

R3: Sometimes when you highlight after entering the formula and you draw the graph, may be the formula you entered was wrong then you get the wrong graph

R9: I did not calculate /enter the correct values on the table of values, so my graph was very different from the one that the teacher expected

In Worksheet 3 and 4, learners indicated that due to their inability to get the right formulas, they ended up getting the wrong graphs. Because it was not visible to the learners how the calculation processing was done by the spreadsheets (*internal constraint)*, learners could only see from their final product (graph) that the formula they used was wrong. Learning algebra in a spreadsheets environment requires use of traditional mathematical knowledge, a constant interplay between algebraic knowledge and the knowledge about the spreadsheets and knowledge related to the computer hardware (Pierce & Stacey, 2004, p. 61). When any of these components is lacking, problems are likely to emerge.

Implicit processing of data. As has been indicated above, the fact that spreadsheets data processing is not visible to learners, this created a hiccup on learners' understanding of some concepts. For instance, dragging has both mechanical and conceptual dimensions. The mechanical dimension involves the learners' ability to handle the mouse properly in selecting and dragging (which is about *usage schemes*). The conceptual dimension involves that which is being dragged, that is, the contents of the cell from where dragging begins. It was also found that even for those who knew how to drag; there was still lack of understanding of the underlying meaning to the application of the command. Statements such as

> *R9: I couldn't get how possible it is (if it is) to not enter a formula in each and every cell*
>
> *R7:When I supposed to enter the formula for multiplication, it gives me the correct answer but when I drag to the place I want, it writes the answer that I don't want to.*

These statements indicate that these particular learners did not clearly understand how the "dragging" command operates. This reveals that even though the learner knew the kind of results s/he was expecting, s/he did not know the correct formula to enter at the beginning.

The AO also observed the learners' difficulty with the dragging command.

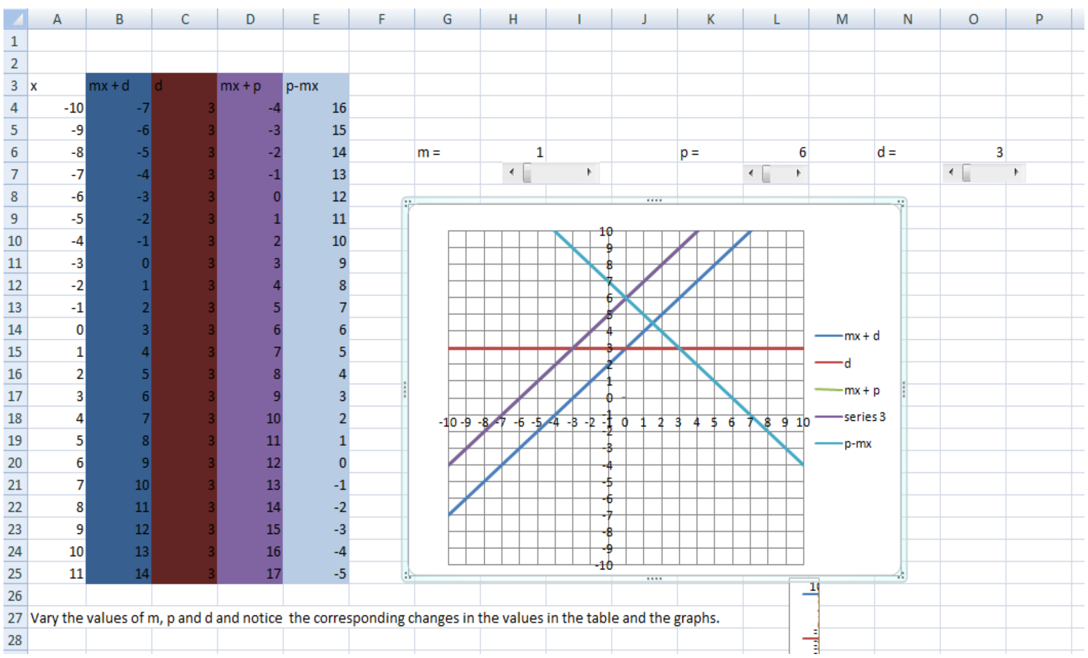
> *Also, they did not understand the meaning of the "drag command* (Retha H.S).

When asked what they thought the dragging command instructed the computer to do, most learners responded as follows:

> *R3: …this fill down command tell the computer to give more answers, it tells us that the computer should … it means that we should highlight down so that we can get more answers*

This challenge is due to the *internal constraints* imposed by the use of the spreadsheets. It is not explicit to the learners how the processing of data is done by the spreadsheets, otherwise learners would immediately see even before they dragged that their formula was wrong.

Confusing multiple representations. While this dynamic environment and multiple representations afforded by spreadsheets helped some learners to acquire deeper understandings of the targeted algebraic concepts (Calder, 2010., p. 2); they became a source of other learners' difficulties. In Worksheet 4, learners were provided with four (4) different linear graphs drawn on the same pair of axes. The corresponding tables of values were also visible on the screen. By varying the parameters *m* and *p*, through clicking on the scroll-bars, learners would observe the effects on the graphs and the corresponding tables of values. The purpose here was to lead learners into being able to compare positions and equations of graphs with same gradients. By reducing the value of *m* to until they got to zero by clicking on the scroll-bar, they were expected to observe and deduce that the gradient of a horizontal line is zero. They were also expected to realize that the graph of a line whose equation is $y = d$ is horizontal passing through the y-intercept, *d*. Learners found relating all the changes in the equations, tables and graphs, all happening at the same time, challenging and confusing. Figure 4.3 shows the representation as seen by learners on the screen.



x	mx + d	d	mx + p	p-mx
-10	-7	3	-4	16
-9	-6	3	-3	15
-8	-5	3	-2	14
-7	-4	3	-1	13
-6	-3	3	0	12
-5	-2	3	1	11
-4	-1	3	2	10
-3	0	3	3	9
-2	1	3	4	8
-1	2	3	5	7
0	3	3	6	6
1	4	3	7	5
2	5	3	8	4
3	6	3	9	3
4	7	3	10	2
5	8	3	11	1
6	9	3	12	0
7	10	3	13	-1
8	11	3	14	-2
9	12	3	15	-3
10	13	3	16	-4
11	14	3	17	-5

Fig. 4.3: Exploring Properties of Graph 2

From the AO's perspective, the exercise was also found challenging to learners.

> *Learners found it rather challenging to observe what was happening in the table of values and the graphs at the same time when the values in the scroll bars were changed. This involved more than one graph and more than one column in the table* (Retha H.S)
>
> *Being able to realize the effect of changing the parameters on the values in the table and the corresponding graphs was a bit challenging. With one click all changes happened at the same time* (Palo H.S)

Learners indicated that they were confused by the graphs; they were not able to see what was happening.

> *P9: I encountered the challenge of comparing the properties of graphs*
>
> *P10: I was not able to see what was going on*
>
> *P9: The graphs confused me.*
>
> *R12: I met the challenges of finding the differences of the graphs and my group*

Some of learners' challenges here could be due to the *internal constraints* of the spreadsheets. The dynamic nature of the spreadsheets environment, where changes in one parameter caused changes in the table of values and on the graph, all these happened too quickly for some learners to follow. While these affordances and enablement of spreadsheets were helpful for linking the different representations of the same relationship and for comparison of different graphs, they also provided a challenge to learners especially when more than one graph was involved. This was observed in learners in both schools. Verillon and Rabardel (1995, p. 80) cautioned, " due to the generative or amplifying character which these technologies confer on the subject's actions, the appropriation of such objects at school age raises new questions concerning the relation between learning functioning and development". It was important therefore that the teacher was aware of the possibilities that spreadsheets open to and plan the activities that take advantage of the constraints. Another set of problems that learners encountered were categorized as *personal*.

ii) Personal Challenges

These problems that related to learners' attitudes and behavior, affect their performance in learning algebra through spreadsheets. (*See section 2.4.2.2).*

a) Lack of motivation and focus

When learners lack motivation for their learning, they tend to lose focus on the events in the classroom. When asked what kind of challenges they encountered during the learning, some learners mentioned lack of concentration in classroom activities.

R7: First lack of concentration…

R10: paying little attention to what I am doing.

According to the AO, some of the learners seemed to lack commitment in the work they were supposed to be doing. This situation was mostly evident at Palo High School. Several factors may be attributed to this kind of behavior, but the main one could be that learners in this school are generally less motivated to learn. There is no urge for them to do better; they just become part of the long-standing history of poor performance in external examinations by learners in this school. (*See section 3.3.2.1*). Wirth and Perkins (2008) note that struggling learners "seldom identify specific actions needed to accomplish a task, and when they do, they tend to procrastinate". The observations and confessions made here show that some of the learners were challenged by the classroom activities involving use of spreadsheets.

The AO noted that some of the learners were not committed to learning.

Some displayed lack of commitment towards learning. (Palo HS)

Learners were also not focused on their work. (Palo HS)

During an interview, one learner indicated that s/he was not prepared to try out problems.

I: Why didn't you try first and see if it's wrong and then change.

P1: I was afraid that if I enter a wrong formula and then I would have to start all over again

I=Interviewer

Wirth and Perkins (2008), however, warn that in some cases average learners may be "quite talented but lack organization and motivation, … rarely willing to devote time necessary to develop deeper understanding".

When learners lack confidence in their abilities to meet the demands of the tasks lying ahead of them, they usually engage in non-goal-oriented behaviors (Pierce & Stacey, 2004, p. 71). They show lack of interest, disengage from the activities, and lose track of the happenings in the class. *(See section 2.4.3.1(ii.).*One common behavior observed with learners in this category was that of *zapping* (Trouche, 2005, p. 143) aimlessly between different windows, pretending to be on task. One learner indicated that the difficulties that she had in class were due to her lack of participation in the activities.

> *P10: Not asking questions where I did not understand and less discussions with the group members.*

Wirth and Perkins (2008) suggest that struggling learners are "solitary, seldom requesting, even rejecting offers of assistance from legitimate resources". The behavior projected in P10's confession is characteristic to struggling learners and the call for concern is their reaction to the challenges they encountered.

It thus remained the teacher's responsibility to monitor learners' progress and help them to develop effective relationship with the technology. The teacher ought to encourage them as they worked through the activities, "helping them to engage with, develop and articulate understandings of mathematical procedures, structures and relationships through the technology" (Hoyles et al, 2004, p. 311).

b) Distractive effect of the technology

As mentioned earlier, most households in Lesotho do not own computers. The majority of the learners in the study only had access to this technology during their normal computer lessons. This situation led to increased levels of excitement when they got opportunities to explore the mechanisms of these technologies, and learned to use the computers to become part of this "fascinating world" (Muresan et al, 2010, p. 70). In stating some of the challenges they came across in their learning of algebra through spreadsheets, learners indicated that:

> *P12:….I don't listen to what the teacher is teaching and at that time the computers are on, I don't listen I am doing something on the computer.*
>
> *P8: The source of my problem is that every time I lose concentration because I will always look at the computer without listening what you are saying and it is extremely hard for me to ignore it because I like it.*

This state of affairs calls for the teachers to pay particular attention to indicative behavior and help learners to overcome their anxiety. Some learners had a tendency to engage in off-task activities once the teacher was not near. In a way, they lost track of what was happening in the class, they remained behind and were not able to answer the teacher's questions and neither were they able to contribute to classroom discussions. As mentioned earlier, (*See section 2.4.1*), great care has to be taken when using technologies such as computers in for learning purposes.

Teachers have to make sure that they deploy appropriate management strategies to enhance learners' classroom engagement.

c) Clash between learning styles and teaching approach

Some learners encountered difficulty in adjusting to the manner in which the teaching was conducted. Because they were accustomed to the usual classroom practice in which the teacher did more of the talking, drilling them with procedures for solving problems, learners were bored when they were confronted with problems and were left to figure out the solutions on their own.

> *R2: It was sometimes boring because I am used to talking to the teacher, not solving problems on my own.*

The manner in which the teaching-learning process is conducted in the classroom, usually induces certain behavioral patterns and expectations from the learners. For example, in lessons where problem solving is used as the approach to teaching, learners adapt to that and are always ready to engage in problem-solving activities. They get used to discovering knowledge themselves. For learners in Palo High School, problem solving was not a preferred approach; whenever they encountered difficulties, they expected the teacher to provide them with a solution. This situation was confirmed by the AO, based on her classroom observations.

> *Learners were expecting the teacher to give them the correct formula*. (Palo HS).

One of the aims of teaching mathematics through problem solving is to develop in them perseverance towards solving problems. Learning through spreadsheets offered them opportunity to try out their solution procedures, reflect on them and see if they made sense and if not try alternative procedures. The problems that learners engaged in were of differing levels of difficulty and some were solvable within a short time while others required a considerable amount of time. When asked about the kind of problems they were solving in class, learners expressed their opinions in the following manner:

> *P1: I liked the easy questions*
>
> *R2: but some of them I did not like where I encountered a problem and I got stuck and did not know what to do.*

During the solution process, the teacher's role was that of a technical assistant, facilitator of learning and guiding learners when they experienced challenges. This guidance was, in some cases, in a form of questions that were sequenced strategically to lead learners into discovering knowledge. *(See section 2.3.5.2. (iv).* Some learners were not happy with this approach. When asked whether he liked the probing questions that the teacher asked, one learner stated

> *R3: Yes some of them, some people get sad when they ask a question and the teacher may be returns another question to you, instead of answering him or her*.

Learners were used to the teaching where they were provided with answers whenever they encountered difficulties, they felt discouraged when that was not happening. Learners were not familiar with this kind of approach to learning; it took them time to adjust as will be seen in the coming sections. Much as teaching should be directed to match learners' preferred learning styles, it is also important to note that learning styles can also be modified over time (Montgomery & Groat, 1998, p. 7).

d) Lack of freedom in the class

In some situations, learners indicated that they were held back by the fact that they were not free with the new teacher; they were not yet familiar with her. The consequence of this was that they were not free to ask questions where they did not understand. .

> *P8: I was afraid because I met with people especially teachers who I was not familiar with but as time went, I became familiar and comfortable with them*.
>
> *R10:... Not asking when having a problem. Being fearful when I have to ask a question.*

Learning algebra within spreadsheets created a very interactive environment. This required from participants, a warm relationship to foster collaboration and cooperation as construction and re-construction of new knowledge took place. For some learners, this kind of atmosphere was lacking. The responsibility of the teacher was to develop good rapport with learners, creating an atmosphere that was conducive to learning, where learners would be free to ask questions and share ideas (Van den Heuvel-Panhuizen, 2000, p. 9). In order for effective learning to take place, the learners and the teacher need to understand the roles they need to play and be willing to act accordingly. The role of a teacher should always be that of a facilitator, while the learner should take a conscious effort and engage in activities that are intended to help him/her to acquire the targeted knowledge. Learners should feel free to ask and answer questions in class. *See section 2.3.5.1.*

e) Lack of cooperation and collaboration within groups.

Learners were divided into groups of three, even though each one of them had individual computer to use. They were supposed to work individually and only consult each other when they encountered difficulties. I observed that in other groups, some members were not cooperative. Two members would engage in some chat that was not related to the task, leaving only one member to do the work and when I came nearer to them, they would immediately pretend to be on task and disturb the one who had been working all along. The statements below show that learners were not engaging with other members in their groups.

R6: I could have asked for help or work with my partners

R8:… apart from that I should cooperate with my group-mates

R7:…Third, my group members were not concentrating; I was just working alone, two of them

This challenge points to need for clarification of roles and proper mindset for learners whenever group activities are involved. It is worth mentioning also that it is the responsibility of the teacher to monitor and encourage learners' involvement in the teaching-learning process. In some of the statements learners indicated that they (learners) were aware of their own practices that hindered their progress in learning, as such they needed support to be able to overcome them.

One of the purposes of discourse in the classroom is that of enrolment, which includes enhancing learner engagement in the learning tasks. *(See Section 2.4.3.4).*

It was also revealed from the data that some of learners' difficulties emanated from the language used in the classrooms. The next section provides findings in this regard.

iii) Language related challenges

Another set of problems identified related to the language used in the classroom. One of the problems in the learning of algebra or mathematics in general, is that of learning in a second language. (Setati & Adler, 2001, p. 247; Ntšohi, 2005, p. 33). During problem solving in a spreadsheets environment, learners were required to analyze the problems confronting them, identify the variables and establish the relationships between the different variables and then tackle the problems using spreadsheets. The kind of problems used in this study were context-based (word problems), situated in contexts that were "real" to learners. (*See Section 2.3.5.1.).* It has been found that these kinds of problems also brought with them certain challenges to learners (Berger & Wilde, 1987, p. 23; MacGregor, 1990; Kieran, 1992; English & Halford, 1995, p. 241; Ntšohi, 2005). Berger & Wilde(1987, p. 23) affirm, "algebra word problems have been a source of consternation to generations of students." The terminology that was sometimes technical or new to learners became a source of learners' difficulties.

During the teaching period, learners complained that they could not understand the tasks due to the language problem.

> *R6: I did not understand the meaning of independent value and dependent values*
>
> *R4: The challenges I encountered are of understanding the meening [meaning] of the words flatter and steeper*
>
> *R2: I was not able to answer some questions because of language used here. It is difficult English and I couldn't understand some of the questions.*
>
> *R3: Sometimes I do not understand what we are asked to do, and I do things without asking;*
>
> *R5: In some, I did not understand what the question was saying*
>
> ***P6:*** *I did not understand question. The English was difficult.*
>
> *R7: understanding the English in the question*
>
> ***P2:*** *May be not understanding the question.*
>
> *R8:... When not understanding the questions and what it requires.*

From their interviews, learners also confessed that sometimes it was difficult for them to comprehend what was required in the given tasks.

> *R1: In some, I did not understand what the question was saying.*
>
> ***P3:*** *Sometimes madam, the English that was written in the worksheets was difficult. We could not understand that question, if she used simpler English, we could understand easily.*

Indeed lack of proficiency in English, which was the LoTL, affected the learning process negatively (Setati & Adler, 2001, p. 247) *(See section 2.3.4.1(iv)).*

While working on Worksheet 2, for example, learners were required to explain, in their own words, the meaning of the spreadsheets formula (=2*A + 2). This was the formula that they used to get the number of people who may be accommodated for any given number of tables. Those who could write the relationship in words: "*We multiply the number of tables by 2 and add 2 to get the number of people accommodated",* were able to translate that to $y = 2x + 2$ at a later stage. Some learners found it difficult to verbalize the relationship. When learners were unable to verbalize their thinking or their understanding of a particular situation, then it was not surprising that they were not able to translate relationships from ordinary language to the symbolic mathematical language.

> *R4:It was difficult when translating to symbols from ordinary language.*
>
> *R12: Not being able to write the relationship between the number of people and the tables, expressing relationships between the numbers in letters*
>
> *R6: It was difficult for me to express relationship between the variables for sequence 2.*
>
> *P13: The sources is that I did not know the relationship and how to express it.*

In other cases, learners were confronted with a difficulty when they had to express themselves. This was evident in Worksheet 3 where they were exploring properties of graphs. (*See Fig 4.2*.) One learner indicated that s/he was able to see what was going on (*epistemic function of the cognitive scheme)*, she engaged in negotiating meaning, attempting to express it in her own words, but the language became a barrier.

> *R2: I could see what was going on with the graph but I couldn't explain what was going on with the graph*

Algebra is the language used to represent mathematical information and data. When its structure is not understood then difficulties are manifested (Kieran, 1989, p. 39; Ntšohi, 2005, p. 55) *(See section 2.3.4.).* Problems situations that learners were confronted with in Worksheet 6, were realistic problems that required learners to be able to identify variables, determine the relationship between those variables and eventually express those in the symbolic mathematical language. For example, Question 2 read as follows:

> A video shop offers two plans for renting videos. In Plan A, you pay a subscription fee of M66.00 per year and pay M5.00 for every video hired. In Plan B, there is no subscription fee, but only pay M8.00 for every video hired.
>
> Let x = the number of video rentals.
>
> a) Make a table of values for the two plans
>
> b) Formulate an expression for the amount paid for video rentals according to each plan.
>
> Plan A: ____
>
> Plan B____
>
> c) Draw graphs showing the cost for the rentals according to each plan.

d) For how many video rentals will: i) payment in both plans be equal? ______________

ii) Plan A be cheaper? _____________________

It is from the formulated algebraic expressions and equations that learners made translations into the spreadsheets formulas in order to work out solutions to the original problems using spreadsheets. Learners indicated that one of their challenges was that of translating mathematical statements from ordinary language to the symbolic the mathematical language.

> *I: …what basically was the problem with that worksheet?*
>
> *R2: Drawing the graph for the given questions the information given.*
>
> I*: So at which state did you find it difficult to do, was it on translating the information into the symbols or moving from the graph to the table or what exactly?*
>
> *R2: Translating the information to the table to the graph.*
>
> I*: If the table were there, would you be able to draw the graph from the table?*
>
> *R2: Yes, madam.*
>
> I = interviewer, R2 = Learner number 2 from Retha H.S.
>
> *R1: when we were doing with the last worksheet, when we were required to change from ordinary language to mathematical language madam*

As has been indicated earlier mathematics, and algebra in particular, should be taught in context, as problems that learners are prepared for are in real life contexts. It is important that teachers take particular care when helping learners acquire skills involved in solving this type of problems.

The AO also observed that learners' progress through the activities was hindered by lack of proficiency in the LoLT.

> *Learners seem to have difficulty with understanding the statements expressing the problem. But once the statement is explained they are able to solve the problem* (Retha H.S).
>
> *They found it difficult to ask questions, they could not express themselves clearly and were not free to speak aloud.* (Palo H.S).
>
> *They also had problem with understanding the long explanations provided in the worksheet. English was the problem* (Palo HS)

In Lesotho, learners at the primary school level are only taught in English when they enter Grade 4, and this becomes the language of instruction thereafter *(See section 2.3.4.1)*. This late start in learning English leads to learners' struggle with the language, even in later years of their secondary and tertiary education, for some. This has some implications on the role played by the teacher during instruction to second-language learners. In this situation, learners were confronted with learning algebra as a language of mathematics, they were learning how to express ideas in mathematical terms through transition from ordinary English(LoLT) to the spreadsheets then to formal algebraic language and vice versa. This created serious problems to the learners.

4.2.1.3 Summary

From the data, similar categories of challenges were identified. Figure 4.4 shows the classification of the challenges as obtained from the data.

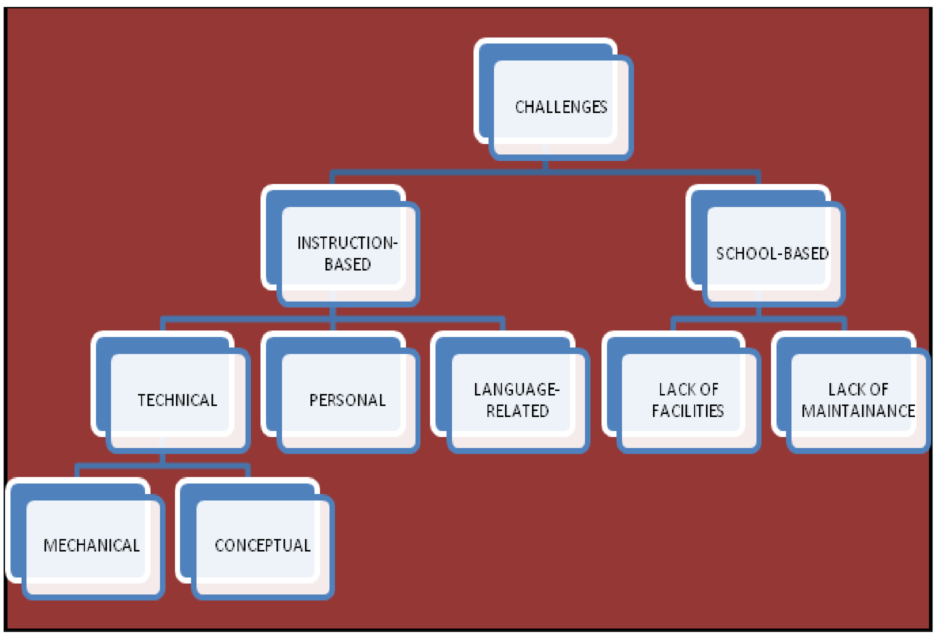


Fig. 4.4: Challenges in Algebra Instruction in Schools

Two categories of challenges were obtained. These were the school-based challenges and the instruction-based challenges. From the study, there were challenges related to the facilities in the schools. In both schools, the computers used were very old and some took a lot of time to open. This caused a lot of inconvenience on the progress of the lessons, as learners had to shift from one computer to another, not being able to access their previous work.

Another challenge observed by the researcher was that of lack of maintenance of facilities in the schools, which lead to electricity cuts, thus hindering the smooth running of the lessons. The computer laboratories were also very small. This made it almost impossible for the teacher to move around the class monitoring learners' progress and attending to their individual needs.

The instruction-based challenges are those related to the events occurring in the classroom. These were categorized as technical, personal and language related challenges. *Technical challenges* are those related to the *mechanical knowledge* about the artifact as well as the *conceptual knowledge* in algebra. (Pierce & Stacey, 2004, p. 61). Some of these challenges were brought about by constraints imposed by the use of the artifact. Three types of constraints were identified. These were organizational constraints, which related to access and organization of available commands on the artifact; the command constraints that related to the knowledge about the commands available on the artifact and the syntax associated with them and the internal constraints, which related to the knowledge the learner had about what the artifact could do. *(See section 2.4.3.1).* It was observed however that with time, the learners' facility with the spreadsheets increased. This is discussed in detail under "enabling strategies'' and "benefits derived" in the next sections.

In some cases, conceptual challenges were mainly due to learners' lack of necessary background knowledge upon which new knowledge may build. Learners' poor arithmetic skills greatly hindered their progress in acquiring the targeted algebraic concepts. (Linchevski & Herscovics, 1996, pp. 40-41).

Undesirable learners' attitudes and behaviors constitute what is referred to as *personal challenges.* Behaviors such as withdrawal from participation in the classroom activities, lack of concentration and focus were seen among some learners. The AO and the learners themselves reported these. These were due to, among other reasons, clash between the teaching approach used in the project and the normal classroom practice that learners were used to. Some learners did not like discovering knowledge on their own since they were used to the teacher "showing them how to do it" and theirs had always been to practice the skill. In other cases, learners were not comfortable with group-work; they preferred working on their own, where they could focus more on what they were doing.

For other learners, their source of distraction was the exposure to the computer itself, they had very little experience with the machine and the only interaction they had with the machine was during computer lessons, this was not enough, so they were overcome by anxiety. Learners who were not confident of their abilities with the computer and the algebra withdrew their participation in the lesson activities and resorted to some coping behaviors such as oscillating between windows without any purpose. These findings are consistent with those in earlier studies, which indicated that the individual's attitudes, confidence levels, cognitive, and emotional styles and social identities, could influence their participation in the use of technology (Hennessy et al, 2005, p. 162; Reed et al, 2010, p. 2).

The *language-related* challenges are those that were brought about by instruction in a second language. It was observed that learners' encounter with new vocabulary challenged them. Learners were not able to follow the instructions in the worksheets because of some technical terms that were used in some of the texts. Transitions between ordinary language, spreadsheets notation and the formal algebraic language provided challenges to learners. Translating mathematical relationships from ordinary language to symbolic language was a problem to them. This seems to be an on-going problem that was identified in earlier research studies (Berger & Wilde, 1987, p. 23; MacGregor M. E., 1990; Kieran 1992; English & Halford, 1995, p. 241; Ntšohi, 2005, p. 33)

4.2.2. Pedagogical strategies that enabled effective teaching and learning of algebra through spreadsheets.

In the previous section, we discussed the problems that Grade 9 learners encountered in their algebra learning in the spreadsheets environment. It is worth noting that most challenges that the learners mentioned were overcome with more interactions with the technology, the mathematics involved and the teacher. In this section, we now look at those strategies that enabled the teacher and the learners to overcome problems that they encountered during the teaching and learning of algebra through spreadsheets. Some of the strategies discussed here, are those that learners found helpful or believed could have helped them to overcome their difficulties.

Data collected for this section was led by the questions:

1. What helped learners to learn successfully?
2. What do you think could have been done to help them learn successfully?

It was found that both the teacher and the learner played critical roles in facilitating effective learning. The strategies are thus discussed under two headings: teacher–initiated strategies and learner-initiated strategies.

4.2.2.1 Teacher-initiated strategies

The quality of the TPCK a teacher possesses determines the manner in which s/he conducts teaching in a technologically-assisted environment. While planning to teach in algebra through spreadsheets, the teacher had to consider the *didactical configuration*, the *exploitation mode* as well as his/her *didactical performances* within the classroom.

i) Didactical configurations.

This refers to the organization of the classroom space with the purpose of enhancing teaching and learning. This may involve allocation of available equipment to the learners and arrangement of how the learners would work. The following strategies and techniques were found helpful in enhancing learning of algebra through spreadsheets:

a) Organization of Machines

Individual learners had their own workstations from where they worked. This arrangement gave learners opportunity to have hands-on experiences with manipulation of the artifact. Unlike in situations where learners would have only one computer to work from, where only one of them would be manipulating while others are spectators, this set-up promoted active engagement of learners both cognitively and mechanically.

> *P2: Madam I liked working in groups and working alone. Working alone helps you to see how much you understand. It makes you aware of yourself, because you are working on your own computer.*
>
> *P1: I think it was better that each one had their own computer….A chance to do our own thing*

Learners indicated that they liked working as individuals within a group setting. During interviews, learners indicated how individual work was important to their learning.

> *I***:** *Ok while still on groups, suppose you were given one computer for the whole group, which would you prefer one computer for the whole group or each one having their own computer?*
>
> *R1:Each one having a computer because even after explaining if you are not doing the task yourself it would not be easy to tackle the task even when you are alone. But if you are using your own computer when the group members are not around you will be able to tackle the task alone.*

When individual learners are given opportunity to manipulate the artifact then their rates of progression in instrumental genesis become accelerated.

b) Organization of Learners

While learners indicated that in working alone they were able to assess their own progress, they also pointed out that group discussions and peer collaborations played a very important role in learning in enhancing their construction and reconstruction of knowledge. Learners were organized in groups of three to facilitate sharing of ideas and experiences.

> *P3: well madam, my opinion is that when we were doing this we had different minds madam. And some could understand so we would have information.*
>
> *P2: It was helpful because when you do not understand others; your colleagues will help you understand the questions.*
>
> *P1: I like working in groups particularly working in groups of three, because unlike when in groups of two the other one will have their opinion and the other one will have their own opinion and we will never end but if in threes then the majority will rule.*
>
> *R1: Yes because sometimes it happened that one of the members did not understand but when we discussed then the task it was easy for the members to understand.*

Through discussions, connections between networks of knowledge schemes were either strengthened or broken and new ones established. *(See Section 2.3.5.1).* Learners appreciated that through discussions tasks or concepts that were difficult for individuals became easy to perform or understand.Hoyles et al, (2004, p. 318) notes that collaborative learning encourages a shift in relationships between teacher and learner, but enhances task-based interactions between learners and learners, and between learners and teachers. When this collaboration happens in spreadsheets environment, the spreadsheets may “not only be seen as a cognitive tool but also as a genuine mediator of social interaction through which shared expression can be constructed”

(Hoyles, et al, 2004, p. 318). The AO added that the classroom organization provided learners with opportunity for independent learning.

Learners are given autonomy over their work (Retha HS)

Learners did more independent work and only consulted each other when they got stuck (Palo H.S)

This helped the learners to engage in reflective, critical and analytical thinking which were important in problem solving.

ii) Exploitation mode

This refers to decisions that the teacher made concerning use of the didactical configurations, and the role of spreadsheets in developing the targeted algebraic concepts (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010, p. 215). The teacher prepared a hypothetical learning trajectory, in which the sequence of topics and the corresponding teaching objectives and timed activities were outlined (Simon, 1995, p. 23).

a) Use of worksheets.

This was seen as another strategy that helped learners to be focused on their learning, going systematically through the process of acquiring the targeted algebraic concepts. The length of worksheets was also another aspect that had motivational effect on the part of learners. Learners also liked the idea of working with worksheets. They indicated that in using worksheets, they were able to learn the topics faster as they were able to focus on one thing at a time.

R2: I think it is easier when you are using worksheets because it helps students to learn faster because with worksheets you are able to do one thing at a time

R1: Yes but here madam, when you are using worksheets you are more focused madam and you learn the topic more faster.

P3: I like it when you gave us worksheets madam, you did not tell us everything that was there madam, some of them we were thinking for ourselves.

R5: In worksheets are useful because you have instructions that you follow one by one and you have time to think and you can still go back and look at the instruction but in class the teacher talks once she doesn't talk much and you must remember what he said. But here the instructions are there.

The AO also appreciated the idea of using worksheets and noted that it made learners concentrate on the task. She reckoned

> *This idea of worksheet is helpful in engaging learners, focusing their attention to the business of the lesson*. (Palo HS)

Through the worksheets, learners were introduced to new concepts gradually, as they engaged in tasks that were broken into realizable pieces. The AO also commended use of shorter worksheets. These were favored because they gave learners a sense of accomplishment, after completing a task within a short period, thus have motivational value. Worksheets engaged individual learners on task and thus promoted effective learning.

b) Use of real life contexts.

Learners also indicated that they liked the kind of problems they dealt with, problems related to real life situations as that made them understand better what they were being taught. When learners can situate themselves within the context of the problem they are dealing with, they are more likely to be motivated into engaging with the solution process (Van den Heuvel-Panhuizen, 1996, p. 13). They would appreciate the usefulness and applications of the mathematical content involved. (See section 2.3.5 (ii)a). Use of contexts in teaching makes learners appreciate and understand things they encounter in their everyday life.

> *R1: I prefer the problems that are related to real life because they help us understand what we are taught madam.*
>
> *P3: Yes madam. And even something that the person cannot be aware of sometimes she can be aware of it when it is now appearing in the activities or in the test because he or she could know that this thing had happened and I know this thing.*

The following part of conversation illustrates how learners felt about problems based in real life contexts:

> *P2: We were asked to find the kilometers and the hours the plane was moving.*
>
> *I: Oh the question on the plane moving between two towns?*
>
> *P2: Yes madam. I was not aware of those things the Km and the hours, I can now see them on cars and now I understand what it is all about.*

The AO believed that learners were able to learn effectively because the tasks that they engaged in were based on contexts with which learners were familiar. For example, the tasks for developing skills for generalizing were based on number patterns and patterns arising in realistic contexts. The functional relationships that learners studied were also on realistic situations; these helped learners to appreciate the graphical representations and were able to answer questions based on them because they were meaningful to them. Use of angles in a pie chart helped learners to generalize the relationship for the sum of angles in a circle.

They were also about things learners know and meet, there was enough background upon which the activities were based and as such, they were meaningful.

Use of contexts familiar to learners… Contexts say to learners-mathematics is real and applicable (Retha H.S)

The AO also indicated that it was important that the problems that learners were at their level. It was easy for them to situate themselves in them. The tasks were appealing to the learners; they were about things that interest them, for example, the tasks on sharing the Pizza in Worksheet 1A.

Yes, the tasks were valuable. They were at their level.

I liked the tasks. They were simple and learners could easily understand them (Palo HS)

c) Use of examples.

Learners also believed that use of examples by the teacher enabled them to gain understanding of concepts taught. They even went further to suggest that use of simple examples of things they meet in their everyday life experiences helped them understand better.

P5: Making more examples.

P8: The examples that my teacher gave to me helped me to learn successfully

Working with more and different examples of a situation provided learners with opportunity to get more clarification of the concepts involved.

d) *Use of teaching Aids.*

The teacher also used teaching aids to help learners develop some of the concepts. For example in helping learners create spreadsheets formulas for relationships, a calendar and multiplication table were used. The AO saw these as a helpful strategy as the teaching aid was also something with which learners were familiar.

The teacher foresaw that and took advantage of what they were familiar with, like the multiplication table to develop expressing relationships between sets of numbers. (Palo HS)

I liked the introduction on the Calendar. It was quite realistic. Learners could relate with it. (Palo HS)

Use of teaching aids in the form of calendar (Retha H.S)

e) Time allocated for activities.

As the learning was hands-on and experimental in nature, sufficient time had to be allocated so that learners would complete their tasks and hence attain the targeted learning objectives. *(See Section 2.3.4.2 (iv))* The more time they got to work on the technology, the more they improved their technical skills. Time was a contributory factor to the learners' process of instrumental genesis. Learners also confirmed that the time that was provided for their activities was sufficient and helped them to learn successfully.

R9: The teacher's supervision, the discussions with my group members and ***the time provided***

Hoyles et al, (2004, p. 311) notes that the effectiveness of the instrumental orchestration process is determined, among other factors, by the time and the status the teacher accords the artifact. *(See section 2.4.3.2.(ii)).* Learners further suggested that being allowed to interact with artifact during spare time would afford them the opportunity to explore the learned skills and concepts.

P7: I think if we could be allowed during our spare time to use the computers alone without teachers next to us, we could be able to work out numbers without the help of the teachers

iii) Didactical performances

The ad hoc actions that the teacher carries during the teaching-learning process, attending to the needs of the learners constitute his/her didactical performances (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010, p. 215). (*See Section 2.4.3.4). Discourse* was the main form of intervention the teacher used to enhance learners' process of instrumental genesis. The strategies used are therefore categorized according to the functions served by the discourses within spreadsheets environment.

a) Procedural and conceptual discourses.

Procedural discourses are those discourses that the teacher use to lead learners in being able to manipulate the artifact to carry out some commands, while conceptual discourses are those discourses associated with helping learners to develop conceptual understandings of what is being taught (Setati, 2005, p. 449). In this study, four types of orchestrations were found in this category, namely: "*Technical demos*", *"Discuss-the-screen*", "*Explain-the-screen*", and the "*Link-screen-board*" (Drijvers et al, 2010, p. 219).

- "*Technical-demo*" *orchestrations*. These were the actions that were carried out by the teacher, showing the learners how to operate the computer. They include cases where the teacher demonstrated to individual learners where to access certain commands, for example, in creating graphs.

The AO had indicated that learners lacked some skills, computer and mathematical, which hindered their progress in learning the targeted algebraic concepts, as outlined in the worksheets, using spreadsheets. When asked how the teacher addressed that problem, the AO noted that sometimes the teacher had to help learners out of their struggles with computer manipulations by way of demonstrating to individuals.

> *Sometimes she would show them what they should do on their computers* (Palo HS)

Learners also confirmed that the demonstrations were helpful to them.

> *R6: It was difficult because we knew nothing about drawing graphs. And we asked our teacher because all of us in our group didn't know how to do it and we asked our teacher and it was easier when she showed it to us and we started from the beginning so that we did it ourselves and we saw that we could do it.*

She pointed out that the learners were helped to acquire mechanical skills as they used them.

> *She somehow taught the skills along; they learned them as they required them.*

- *"Explain-the-Screen" orchestrations.* These were discourses that the teacher engaged in when explaining outputs displayed on the screens of learners' computers. The teacher would use the display on one learner's computer screen as reference while other learners are watching their own as explanations were provided. For example, explaining solution of equations by graphical method. Use of a calendar in Worksheet1 was helpful in clarifying to learners about creating spreadsheets formulas. In another instance, the teacher had to make an attempt to draw a spreadsheets on the board to aid explanations of what learners saw on their screens. With the availability of a projector, the explanations could have been done more effectively. The AO suggested that some explanations could have been made clearer through use of screen projectors.

> *The explanations about the meaning conveyed by point of intersection of the graphs* (Palo HS)
>
> *Explanations* (Palo HS)

Other incidents in this category included the systematic instructions that the teacher provided to learners in carrying out certain commands that were not familiar to them, for example, creating a graph, naming a column, etc. When learners carried out the instructions systematically as given by the teacher, they were able to obtain the desirable results. The AO gave examples of such cases:

> *On how to create the graph, the teacher had to give instructions slowly and repeatedly* (Palo HS)

Learners would then be given opportunity to apply what they learned.

- "*Discuss–the-Screen*" *orchestrations***.** These involved discussions of outputs as displayed on the computer screens.

The AO suggested that when concepts, are developed in a meaningful way, making sense to the learner then they (learners) are more likely to understand them. For example, concepts such as gradient and *y*-intercept were developed more successfully through sense-making (S*ee section 2.3.5.1*) using spreadsheets.

> *Concepts of gradient and intercepts were developed through sense-making* (Retha H.S)

In worksheet 4, learners were given opportunity to explore properties of linear graphs by varying values of the *m* and *c* in the general equation $y = mx + c$. By varying the values of m and p on the scroll bars, they would be able to recognize the effect on the position and steepness of the graph. Fig.4.4 shows an activity that learners carried out in exploring properties of graphs.

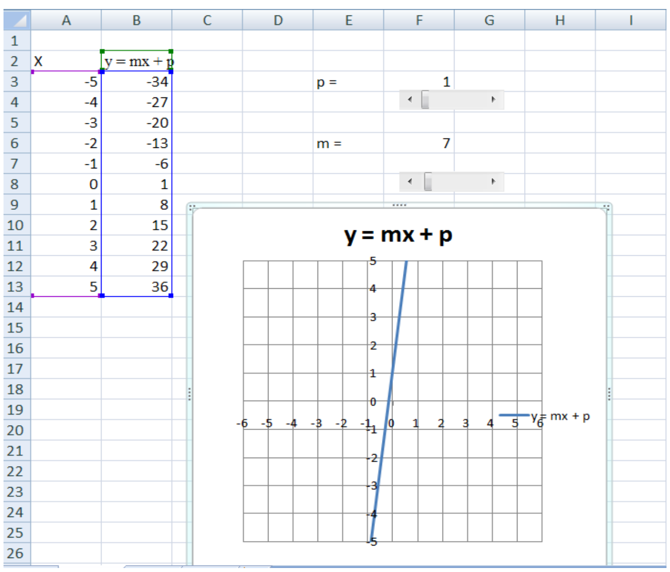


Fig. 4.5: Exploring Properties of graphs 1

This is one of the activities that learners enjoyed most. The activity was based on the dynamic nature of the spreadsheets environment. They were able to see the effects of varying parameters because everything was just there on the screen. They were able to explain that "*when the gradient increases, the steepness of the increases*" and that *"the gradient determines the steepness of the graph"*.

The screen discussions were conducted at small-group and whole-class levels. The AO noted that the manner in which the teacher conducts the discussions had great impact on the success of the learning-teaching process. Attending to learners in their small groups before conducting a whole class discussion, is another strategy that helps in developing confidence in the learners. This gave them opportunity to debate in a more focused, less threatening and controlled forum before opening to a larger group (retrieved on 18 April 2013, from www.umuc.edu/ctl/upload/smgroups.pdf).Giving learners a chance to explain and justify their responses to any given task helps them to construct and reconstruct conceptions of new knowledge.

> *Attending to individual groups, using questioning and probing to lead them to the solutions* (Retha HS)
>
> *Interactions-group discussions, and whole -class discussions*. (Retha H.S)

Learners indicated that through discussion the teacher was able to evoke their thinking through questioning and probing. Learners were able to correct mistakes such as the one displayed in Figure 4.5 (dates beyond 31) through probing during screen discussions.

	A	B	C	D	E	F	G
1	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
2		1	2	3	4	5	6
3	7	8	9	10	11	12	13
4	14	15	16	17	18	19	20
5	21	22	23	24	25	26	27
6	28	29	30	31	32	33	34

Fig.4.6: P3's Part of Work in Creating a Calender

The AO asserted that discussion played a very important role in the learning process. Giving learners opportunity to share ideas helps those who may have difficulties to learn from others.

> *I mean we all learn from others. If one understands something better and you allow him or her to share the idea with others, then these others can grab the idea and perform better.*

Learners appreciated this as they indicated that it helped them to discover knowledge. The following interview reveals learners' expectations during discussions.

> *I: Now was your teacher aware of the problems you encountered?*
>
> *P1: Yes, she was but we had to figure it out ourselves. But if all of us did not know the answer, we would ask the teacher*
>
> *I: And what did she do. Did she give you solutions?*
>
> *P1: No, she would not just give us right away. She would ask us what if you did this and this or that, then we would get the answer.*
>
> *I: So she would ask you questions ….*
>
> *P1: That would lead us to the answers.*
>
> *I: Did you like that or you would have loved it if the teacher gave you the answer straight away?*
>
> *P1: I liked it because it teaches me much about that we should not always ask and be told I should try out first for myself.*

The AO also confirmed the usefulness of probing questions in developing learners' understanding of concepts.

> *She would probe them trying to help them connect the situation with what they had learned* (Palo H.S).

The AO further indicated that, building on learners constructions encouraged them to see that their ideas were valued and gave them confidence about what they had learned and understood. This also emphasized to them that they could learn from one another and therefore needed to develop positive attitudes towards teamwork.

> *Asking learners to reiterate the steps one after the other until the graph was produced. Probing to justify the next step* (Retha HS)
>
> *She asked learners to instruct one learner to do it.* (Palo HS)

Discussions in the algebra lessons provided learners opportunity to make their thinking public and through others' questions and ideas, they negotiated meaning, modified and refined their conceptual understandings. *See section 2.3.5.2. (iv).*

- **"***Link-screen-board***"** *orchestrations***.** Even though the teaching and learning of algebra was done to support the paper-and-pencil instruction, the kinds of interventions discussed here refer to those situations where, results from spreadsheets were verified through pencil-and-paper algebraic methods. The AO indicated that this form of orchestrations were useful in

Demonstrating how to check solution by algebraic manipulations. (Palo HS)

Sometimes the latter were used to explain certain concepts. For example, reading the gradient from the graph was illustrated using the grid on the chalkboard. The AO also observed these strategies to be working positively for learners in both schools.

Explaining how to read the gradient and y-intercept from the graph and how to obtain the gradient from coordinates by calculations. (Palo HS)

Explanations/illustrations on the board (Retha H.S)

b) Contextual discourses.

These refer to those discourses that do not relate directly to enhancing understanding of learned concepts, but may influence learners' engagement with the tasks intended to lead to knowledge acquisition (Setati, 2005, p. 449).

- *Facilitating understanding of tasks*. The tasks that learners engaged with involved problem solving within real-life contexts. Learning occurred as learners grapple with problems(Hiebert et al., 1997, p. 12), and the first stage in problem solving involved interpretation of the question, that is, understanding what the problem required (Polya, 1973). (*See section 2.3.5.2. (ii)a*). It was to the learners' advantage that they are left to figure out what the question required before the teacher intervened. When learners failed to comprehend the problem statements, they were encouraged to read the questions aloud and that was helpful to them, as noted by the AO.

Encouraging learners to read the questions aloud (Retha HS*)*

Oueini, Bahous, and Nabhani(2008, p. 153) suggest that the read-aloud strategy enhances learners' development of vocabulary, reading and comprehension and that this strategy can be used with second language learners of all ages

The AO and the learners again indicated that when the teacher provided explanations of what tasks and questions require learners to do, that was also helpful for the learning process. Learners also felt that using simple words that they could understand helped them to understand the given tasks ("questions") before they could start working on them.

P1: Questions should be explained before we do anything

The teacher had to provide explanations for the tasks (Palo HS)

She also provided explanations to words learners found difficult (Palo HS)

P13: I have learned successfully in the help of discussion with my classmates and the teacher herself by explaining the questions in words that I understand very well and making some simple examples which we can meet in daily life.

While learners should be given opportunity to acquire vocabulary in the language of instruction, new terminology must be explained. Some learners also indicated that during exposition, it was important that the teacher went slowly over the explanations and or illustrations so that they could follow.

R12: You should teach us slowly…

- *Code switching.* As a way of addressing the language problem, learners also felt that code switching could help them learn successfully, particularly when learning in a second language, as suggested by P11. Van de Walle notes that "when children recognize that their teacher is willing to allow them time to process mathematical concepts in their own language and develop ways of expressing in the English language; they are more likely to invest their effort and extend themselves in the mathematics process" (Van de Walle, 2004, p. 97). Studies such as those conducted by (Setati & Adler, 2001)and(Jegede, 2011) suggest that code switching should be seen as a valuable tool for fostering conceptual, procedural, contextual as well as regulatory discourses in multilingual mathematics classrooms.

P11: I think I can be helped by talking English and explain them in Sesotho. So that I can ask if there is anything I don't understand.

c) Regulatory discourses

These discourses are associated with regulating learners' behavior in order to attain maximum active engagement in the learning tasks. The following orchestration techniques were found in this category.

Monitoring and evaluation of learners' progress. Learners indicated that they were able to learn successfully because they were being monitored ("supervised") as they worked through their activities.

R9: The teacher's supervision...

R4: Yes because every time in the class there was a camera and she was moving around to see that we were doing the right thing.

Indeed monitoring learning in the classroom was beneficial to both the teacher and the learners. It provided the teacher with feedback on how learners are progressing and helped her to identify those who were struggling and those who were faring smoothly through the process. The following part of an interview shows that the teacher was aware of events in the classroom.

> *I: Was the teacher aware of the learners' problems?*
>
> *R:Yes, the teacher was aware of that.*

The AO further confirmed this by indicating some of her reactions to learners' difficulties.

> *That is why in most cases she intervened, tried to discuss. There were these whole class discussions, between the teacher and the whole group. The teacher also encouraged peer discussions among members of a group. There was also use of the chalkboard to elaborate and clarify some concepts.*

Monitoring learners' progress also helped in maintaining order and discipline in the classroom. Learners became focused on the learning activities. Due to the distractive effect of computers, the AO proposed that an arrangement where learners' computers are connected to the teachers' would help the teacher to see what each learner is engaged with at any particular time. *(See Section 2.4.1)*

> *If it was possible to have all students' workstations connected to the teachers' then monitoring their use of the computer. (Palo H.S)*

Learners also acknowledged importance of monitoring and assessment of their progress in learning. They also felt that the teacher should assess their understanding in the course of learning.

> *R12:… and you must ask us questions so that we could hear whether we are wrong or right*
>
> *R10: I think we should be given a quiz at the end so that we can be able to see if we understand*

These statements also indicate the importance of feedback in the learning process. In other worksheets like Worksheet 1A and 5A, the activities were designed such that learners got immediate feedback on the work they were doing. The activities also provided the quiz-like evaluation exercises that the learners wanted. Figure 4.6 shows one of the activities in Worksheet 1A in which learners were led into formulating generalizations about patterns.

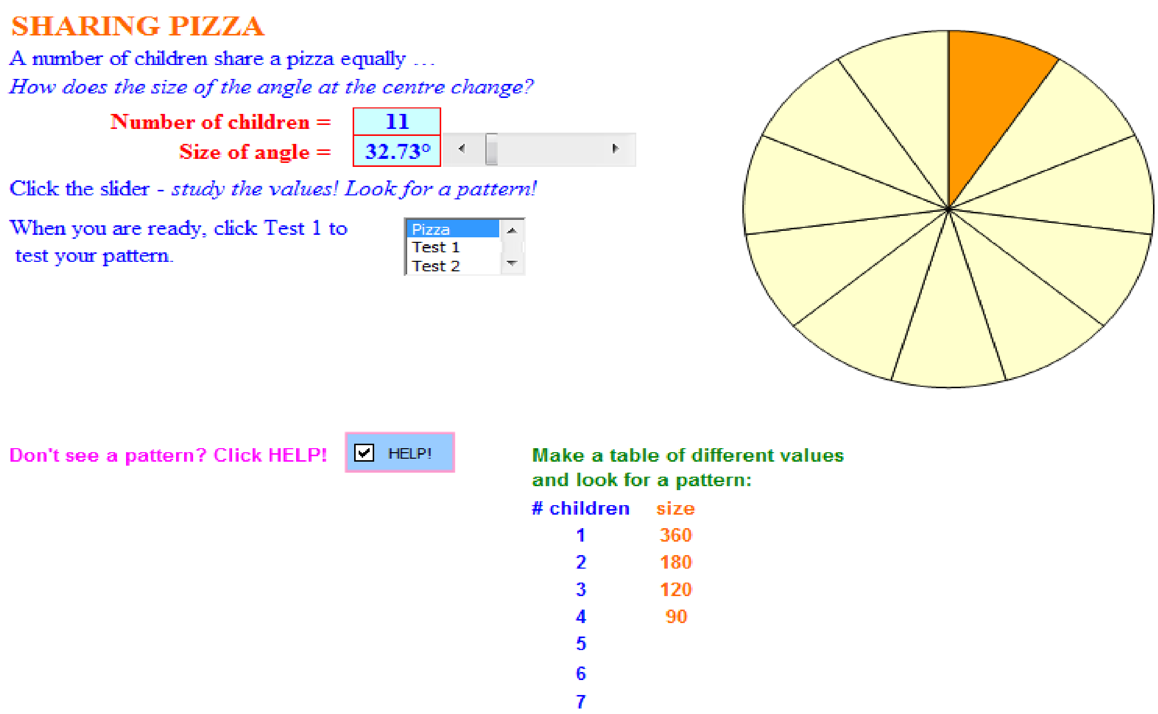


Fig. 4.7: Task on Generalizing Patterns

(designed by Alwyn Olivier, Stellenbosch University)

Provision of immediate positive feedback was necessary as it had motivational effect; it encouraged learners to pursue their journey in learning even further. However, if the feedback were negative, that is, showing that learners lacked understanding on the concept being learned, it would also evoke reflective thinking on the part of the learners. This would recall learners to review their solution procedures and justify their thinking. In this way, they would engage in constructive knowledge acquisition.

Learners also appreciated that their teacher was easy to approach and willing to help them.

> R3: *That she helped us, that she come to us, and speak politely to us not I such way that a person can say the teacher is not happy or stressed, not in such a manner that a person cannot like.*

Creating a warm but work-oriented climate within classroom was vital for effective learning.
From the above discussion, it is evident that in order for effective teaching and learning to be achieved in a technologically assisted environment, the teacher's TPCK (Mishra & Koehler, 2006, p. 1025) is very critical. The teacher's content knowledge, knowledge about the potentialities and affordances provided by spreadsheets as well as the possible constraints that

may be imposed by the artifact were very critical in shaping the teachers' orchestrations within the classroom.

4.2.2.2 Learner-initiated Strategies

In order for learners to be able to attain their learning goals within the spreadsheets –assisted instruction, they need to engage fully in the learning activities and have positive attitudes towards the learning processes involved.

i) Engagement with lesson activities.

During the learning process, learners engaged in practical, hands-on activities that enhanced their cognition and development of targeted algebraic concepts. Even though the activities were designed to help them acquire knowledge, the extent to which learners developed understanding of targeted concepts was dependent on their level engagement in these activities. Most learners confirmed that they were able to learn the targeted algebraic concepts through different forms of participation in the learning activities

- *Following questions/instructions:* In response to the question, "what could have helped you to learn successfully", learners indicated that being able to follow instructions and understanding what was required in the questions had been helpful to them. This involved reading and re-reading the problem in an attempt to make sense out of it. It was only when the problem was understood that the problem-solver would devise a plan to solve it. *(See Section 2.3.5.2. (ii)a).*

> *R9:The best was also to read questions carefully and …*

Other learners confessed as follows:

> *R5: I could have read the question over and over till I can say it in my own language*

When learners verbalized their thinking, this promoted their construction and reconstruction of their own knowledge and understandings. Being able to translate what they were doing and expressing it in own words or even own language reflected understanding of concepts studied. Indeed learners were able to develop their technical knowledge relating to algebra learning and ultimately attained the learning goals in the Grade 9 algebra curriculum. They were able to express the relationships between sets of values (which they expressed as "*variables*") using letters. Self-explanation is a metacognitive strategy that facilitates knowledge construction (Aleven & Koedinger, 2002, p. 145) as such needs to be supported by creating an atmosphere conducive to teaching and learning. (*See Section 2.3.5.2(iv)*). In the research conducted by (Ainsworth & Loizou, 2003, p. 678), it was found that students who produced more self-explanations in multiple representation contexts performed better than those who produced less.

The following are examples that illustrate their abilities in expressing relationships between values in the columns in the multiplication table for multiples of 2, 3 and 5: "$b = 2a$"; "$b = 3a$" and "$b = 5a$". They had also learned how to translate spreadsheets formulas into ordinary algebraic notation. Examples of such cases include a situation where they had identified the relationship between number of tables and number of chairs in Mpho's Restaurant. They first expressed relationship verbally and then into spreadsheets formula "=2*A + 2" and then to ordinary algebraic form, $y = 2x + 2$.

- *Trying and testing:* The AO indicated that another strategy that learners used to enhance their own learning was that of experimenting with their own ideas. Spreadsheets allowed learners to explore patterns and relationships and to form and test their thinking. Trying and testing out ideas helped learners in reshaping thinking and modifying solution procedures and strategies.

> *Trying out and testing their thinking* (Retha HS)
>
> *They also tested what they came up with* (Palo HS)

When they realized their solutions were not correct, learners indicated that they would retrace their steps and check where they might have gone wrong. Once that was picked up, learners would try an alternative route until they got the desirable outcomes. This is evident from the following statement:

> *R3: In such cases we just look at our first step that we did when we do the graph and then we can see where is our problems and then we check our formulas whether they are correct or wrong and when we see that they are wrong, then we start another formula and then create another graph with another formula.*
>
> *P3: Before I could ask, I had to think about the question and the answer*

The AO pointed out that when confronted with a problem, learners would attempt to solve it before they could seek help.

> *They only ask the teacher after attempts were made* (Palo H.S)
>
> *They would try, talk with their neighbors, then ask the teacher. (Palo HS)*

Learners constructed their own knowledge within the existing social and dynamic spreadsheets environment.

- *Practice of learned skills*: With more interactions with the artifact, learners' facility with the spreadsheets was enhanced and their *instrumental genesis* accelerated. As mentioned earlier, learning, as a process, may be defined as acquisition of knowledge and skills through practice and study (Kostova & Atasoy, 2008, p. 51). In order for learning to take place there must be active participation of the learner, s/he must engage in activities that would help promote understanding of learned concepts. This means that the practice that the learners here talk of indeed helped them to gain understanding of the learned concepts.Figure4.7 shows one of the learner's (R1) work where she went over to explore her own graphs, and formatting the plot space on her own.

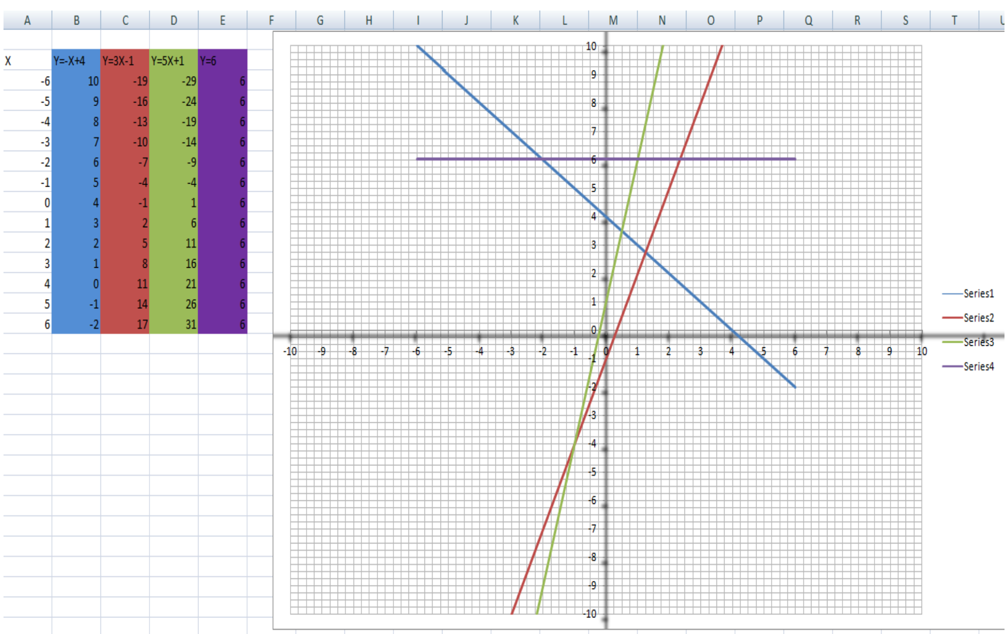


Fig. 4.8: R1's Work on Exploring Properties of Graphs 2

The learner confidently mentioned, "*It was difficult at first but now it is easy because I now know how to do it*". Learners indicate that if afforded more time to interact with the artifact they would gain competence and be able to solve problems in mathematics without much assistance from the teacher.

> *R9: I could have practiced more, by making different examples of what I have been taught*
>
> *P3: I should have practiced the speed and I should have concentrated*.
>
> *P11: I think trying has helped me to learn successfully because it is better than doing nothing at the moment*

One of the goals of learning is that learners should be able to translate knowledge from one context to another. Being able to give examples of what has been learned engaged learners in reflective thinking and negotiation of meaning. Differentiating examples from non-examples promoted better understanding of the concepts being learned. When learners were able to give such examples, it showed that they were able to transfer the learned concepts to different situations. This involved critical thinking about the situation at hand.

ii) Developing positive attitudes and behavior

This related to the attitudes and behavior that learners identified as important for effective learning in a spreadsheets based environment.

- *Class attendance:* Learners attitudes and behaviors towards learning in a computer-assisted environment, greatly influences their success in using the computer tool to attain their educational goals. Positive attitudes lead to goal-oriented behaviors. *(See section 2.4.3.1(ii).* This therefore supports the learner's believe that taking initiative in learning helped them to learn successfully. Learners pointed out that it was important that they attended the classes regularly, as inability to do so was a source of the problems that they encountered.

> *R13:By attending classes correctly*
>
> *P10: Attending class and listening in class*
>
> *P11: The sources of my problem is that the first day my teacher was teaching us about this topic I was upsent[absent] now it was difficult to learn some as my classmates have more knowledge than I have.*

- *Participation in Discussions:* Having passion for learning, cooperation and willingness to listen to others' opinion was very helpful to the learners. When learners are motivated towards their learning, they become open about their discoveries and challenges. The following statements reveal what learners saw as helpful attitudes and behavioral patterns required for successful learning within the spreadsheets environment.

> *R9: enjoying what I was doing with full dedication and love, discussing with my group members and using the teachers word helped me to learn successfully*.
>
> *P13: A lot has helped me to learn successfully today, like concentrating in class enjoying the day.*
>
> *R1: Concentrating and participating in my group*
>
> *P10: Discussion with the group members and asking questions*
>
> *P8: I have to see by myself that I must do my work quickly and save it and to ask question.*
>
> *P12: I have to stop doing what I am not expected to do in the lesson so that I can concentrate to what the teacher is teaching me.*

Learning as a process, requires a conscious effort of the learner to accomplish personal educational necessities, interests and goals. (*See section 2.3.5).* Acknowledging ones' limitations and moving on to ask for assistance, from either colleagues or teacher, helped the learners in achieving the goals of learning.

> *P4: What I think can be done is to ask for help and don't be lazy to think, and to work in groups so that we can talk about one another's problem*
>
> *P4: to ask for help from my colleagues, and to ask help from my teacher and concentrating to what the teacher is saying.*
>
> *R9: Asking question where I did not understand and following instructions and working together with my group or partners*

4.2.2.3 Summary

From the above discussion it is evident that both the teacher and the learner have significant roles to play to achieve effective teaching and learning in the classroom. The strategies that were helpful in addressing the earlier mentioned challenges were thus found to be those initiated by the teacher and those that learners used. Figure 4.8 shows the different strategies and their relationships as found from the data.

Teacher-initiated strategies include all planning, implementation and actions that the teacher took to orchestrate learning in the spreadsheets-assisted learning environment. Since the teacher was aware of the limitations and constraints that were brought about by the use of the artifact, the planning and implementation of the plan in the classroom were conducted in a way that assisted learners in overcoming their difficulties. *Didactical configurations* include organization of the learning environment, which comprised assigning the machines to learners, allocating each learner a workstation and grouping of learners in threes. This kind of arrangement and organization was appreciated by both learners and the AO, as it afforded each learner opportunity to have personal interaction with the algebra within spreadsheets at the same time giving them opportunity to discuss in smaller groups where they could talk in the language they understood.

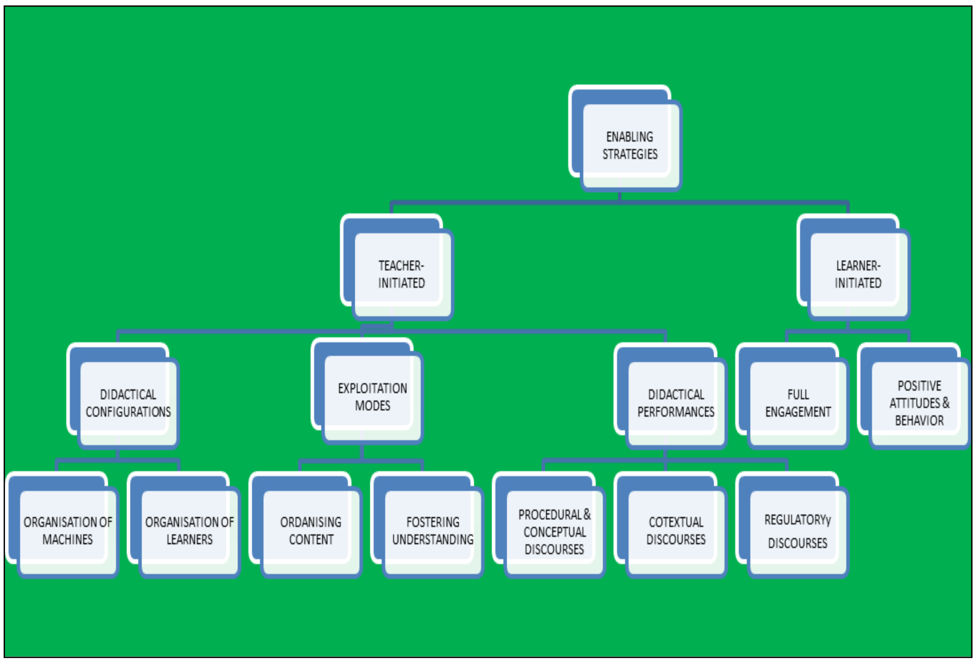


Fig. 4.9: Enabling Pedagogical Strategies for Algebra Instruction in Spreadsheets

The *Exploitation mode* involves the plan of how the targeted concepts were developed and this includes the design of the tasks. These engaged learners in problem-solving within realistic contexts. Even though at the beginning learners were not happy with "finding things on their own", they later on appreciated the approach; they liked being left to discover solutions on their own. Learners and the AO indicated that, realistic contexts made them (learners) aware of some aspects in their environment and that made them appreciate the usefulness of algebra (mathematics). Learners indicated that realistic contexts made their learning interesting. The Assistant observer also appreciated that learners' tasks were designed such that learners would focus on the targeted concepts, taking advantage of the some constraints imposed by the artifact, for example, exploring properties of graphs. Use of the worksheets in this case provided guidance, to learners, into a systematic discovery of knowledge. These were also useful in keeping learners focused on their work. The learners and the AO also appreciated this. Provision of self-evaluation activities, such as one in Fig. 4.6, supported the investigative inquiry by the learners, testing their ideas and seeing if they were wrong. This kind of planning is consistent with what constitutes effective teaching within spreadsheets environment as seen by (Jonassen & Reeves, 2001, pp. 693-697). *(See section 2.4.1).*

Didactical performances refer to the ad hoc actions that the teacher carried out during the teaching-learning process, attending to the needs of the learners. Both the AO and the learners agreed that the interventions that the teacher-researcher provided were helpful to the learners. The main forms of interventions provided by the teacher were through discourses. These included procedural and constructive discourses (Abboud-Blanchard, 2009, p. 1881), (which comprise conceptual and contextual discourses), as well as regulatory discourses (Setati, 2005, p. 449). Examples of the procedural and constructive discourses within a spreadsheets-environment in this study, included “explain-the-screen”, “discuss-the-screen”, “technical-demos” and “link-screen-board” orchestrations (Drijvers, et al, 2010, p. 219). Techniques such as probing, questioning, providing examples, making illustrations and demonstrations, promoting and holding discussions, and all other forms of help, constructive and procedural help, that the teacher offered in this study, fall under this category. Regulatory or enrolment (Abboud-Blanchard, 2009, p. 1881) discourses are those that were associated with regulating learners’ behavior in order to get them back on task.

Hoyles, Noss, & Kent (2004, p. 311) note that the effectiveness of the instrumental orchestration process is determined, among other factors, by the time and the status the teacher accords the artifact. During the lessons, all teaching was based on the spreadsheets, except in those cases whereby the results or solutions and explanations were verified through pencil-and-paper. With more interactions with the artifact, learners’ facility with the spreadsheets was enhanced and their *instrumental genesis* accelerated. In addressing the language problem, code switching was another strategy that was employed while teaching in a second language, a suggestion that is consistent with earlier studies.

Learner-initiated strategies involved full participation (active participation) and development of positive attitudes and behavior, as advocated for by. (Kostova & Atasoy, 2008, p. 52). Learners indicated that through discussions and asking questions, cooperating and working together as partners with their peers, they were able to learn successfully. Discourse in algebra lessons provided learners opportunity to make their thinking public and through others' questions and ideas, they negotiated meaning and modified and refined their conceptual understandings. Being able to give examples of what had been learned, engaged learners in reflective thinking and negotiation of meaning; differentiating examples from non-examples promoted better understanding of the concepts being learned. Learners reported that by carefully reading the questions and translating them into own words or language helped to get even better understanding of the problems they were confronted with.

4.2.3. Learners' Perceptions regarding use of spreadsheets for teaching and learning algebra through spreadsheets.

From data collected in this study, learners indicated that use of spreadsheets affected teaching and learning of algebra both positively and negatively. This section will, however, only deal with the benefits (positive impact) derived through use of spreadsheets, as the negative aspects have already been captured in earlier sections, under challenges *(See section 4.2.1).*Opinions that the participants developed regarding use of spreadsheets for teaching and learning of algebra, compared to the usual pencil-and–paper algebra instruction, were also investigated.

4.2.3.1 Benefits derived from use of spreadsheets.

Even though there were challenges that learners encountered during the teaching-learning sessions, there are benefits that they derived from the exercise. Just as with the challenges that learners encountered, the benefits derived are categorized into technical (conceptual and mechanical) and personal (related to attitudes and behavior).

i) Technical Benefits

Learners indicated that they appreciated the experience in spreadsheets as that helped them to learn better.

> *P1: Well, I liked it because it gave me a lot of knowledge about computers interacting with mathematics.*
>
> *R6: What I can say is that the learning experience that we have had with spreadsheets has been very helpful to us as learners at grade 9 it is useful because we can learn better*.

Knowledge in this section relates to learners' growth in the process of instrumental genesis. The learners' mechanical facility with the artifact promoted their ability to use it technically; hence, the two are discussed together. Learners seemed to have moved from the initial, *discovery and selection* to the second phase, *personalization*, of the instrumentalisation process. The following were excerpts of learners' confessions:

> *I have learned how to fill the Pascal triangle and multiplication table using formula. I have learned how to fill the calendar not entering cell by cell*
>
> *I have learned the formulas and how to use them*
>
> *Learned that the fastest way of calculating is by using or entering a formula*

The potentialities of the artifact had begun *shaping* the learners; they (learners) had come to appreciate the value of spreadsheets formulas in computation. Their interaction with the technology has had impact on their perception about learning mathematics. They could then appreciate the application of their knowledge in real life situations.

> *P3: The calendar and graphs. Madam, madam, I didn't know how people make calendars, and I am now able to make a calendar with spreadsheets.*

(*See Figure 4.6*.

The assertion that one could insert the graph and change scales reflects some positive advancement in the use of the artifact. They have been able to discover and select the keys and menus relevant to what they were learning at a time. Again, one could see evidence of the learners' utilization schemes and instrumented actions schemes developed.

> *I have learned how to insert the graph and to change scale*
>
> *R1: I didn't know the graphs, but I learned them*.

Learners had gained some stability and confidence in the use of spreadsheets, they had moved through the *explosion* to the *purification* level of *instrumentation* process. They pointed out that in working with spreadsheets; they were able to do tasks more quickly and accurately. The following conversations reveal learners views in this regard:

> Interviewer*: so which parts of your learning experience did you like?*
>
> *R1: I liked when we were entering the formulas in the spreadsheets and when we were drawing graphs with the spreadsheets. The calculations with spreadsheets are easy when entering the formula and when plotting the points. While the point we are having when we doing them in class on our own it becomes difficult and the person takes a long time.*
>
> *I: What is it that you found very satisfying when drawing the graph with spreadsheets rather than when you draw with pen and paper?*
>
> *R1: It's because once you draw the graph with* spreadsheets*, when you have already got the table, you don't have to think of where to plot the points, when you have highlighted the table when you draw the graph it's going to draw the line for you.*
>
> *P4: eh madam, because when a person knows the formula he/she will not calculate with a pen ,she or he will just give the computer instruction and the computer will respond.*
>
> *I: so you don't want to do the calculations yourself?*
>
> *P4: yes madam. (Laughs).*

Learners mentioned that they liked it when spreadsheets did calculations and created graphs for them. They said that these were more accurate and produced more quickly than those they would get with pencil-and-paper. These revelations are consistent with the benefits that could be derived from use of spreadsheets for algebra teaching learning as mentioned in the literature (see (Chazan, 1999, p. 123). Learners' cognitive schemes allowed them to create graphs (pragmatic function).

Learners indicated that they had learned the intended concepts. They had also understood the relationship between the algebraic expressions and spreadsheets formulas, and were able to translate from one form of representation (tabular) to the other (graphical). Learners' competencies in the mentioned areas were evident in their responses to tasks in the worksheets and highlighted in *Section 4.3.2*. The following statements reflect some of the concepts that learners picked out:

R10: I have learned how to write the relationship of variables using letters. That we can change computer formulas into mathematical [formal] algebra

R1: I have learned that the gradient of a horizontal line is zero. The steepness of a graph depends on the value of the gradient. I learned that graphs of the same gradient are parallel

R7:How to solve the equations, read the graph and how to get the equations and the gradient without calculating

P2: I have learned how to make a graph. I was taught with a ruler how to make those columns and now I can see the x-axis, the y-axis and now I can see where that graph comes from. That time when we were taught with paper-and-pencil, I did not understand now I understand

R1: When the number of y-intercept is changed, the line will cut the y-axis at that point that I have selected. And the gradient determines the slope of the line.

Learners benefitted from the affordances provided by spreadsheets. According to the AO, use of spreadsheets was beneficial to the learners for developing the concept of variable, in generalizing.

I think spreadsheets helped learners to conceptualize the meaning of variable, through generation of long lists /tables (Retha HS)

Connection between spreadsheets formulas and the algebraic notation is very close, so learners were able to make generalizations using letters.

Generating long lists of numbers with spreadsheets helped them to check their formulas. (Palo HS)

*Learners were able to check their solutions quickly and reformulate their generalizations (*Palo HS)

The AO's observations are consistent with views shared by earlier researchers such as Rojano (1996, p. 144); Jonassen & Reeves (2001, p. 718); Tabach & Friedlander (2004, p. 428); Haspekian (2005, p. 114) and Tabach et al (2008, p. 50). The AO further noted that, use of spreadsheets helped learners in linking different representation of relationships and in learning properties of graphs.

Spreadsheets provided a quicker and accurate way of drawing the graph. Because the focus was not to teach how to draw graph with pencil-and-paper, then they could concentrate on understanding the concept of gradient. Getting coordinates with cursor was helpful in making learners see they are same points as in the table

I liked the use of scroll bars. This helped the learners to see the effect of changing the value of the gradient on the steepness of the graph and in demonstrating the meaning of y-intercept. It made this clearer.

The following excerpt confirms what the AO observed:

R1: When the number of y-intercept is changed, the line will cut the y-axis at that point that I have selected. And the gradient determines the slope of the line.

Through exploitation of spreadsheets affordances and enablements, learners were able to develop the concept of gradient of a straight line. They were able to determine the equations of straight lines from their graphs by just reading the gradient (reading "rise" over "run" and the y-intercept, not through calculations. The instrumented action schemes in a dynamic environment permitted learners to vary the parameters (*pragmatic function*) and observe the corresponding effects on the graph, thus enhancing learners' understanding of the phenomena studied (*epistemic function).*

Besides the conceptual aspects of their learning experience, learners indicated that they learned a lot during the period, their computer skills improved and the lessons helped them to understand the earlier learned concepts better.

I: Something like what? What are the advantages of doing algebra with spreadsheets?

R: Madam, the advantages madam, sometimes we are just given questions in a computer test, and some other questions needs what you know about the computer. So when we just learn algebra in class you will not get the experience with the computer.

I: Ok, now I get you. Now this exercise required that you have some certain basic computer skills, right from worksheet 1 up to worksheet 6. Did you have all the necessary skills?

R1: Not all madam. Some I learned them as we were working through the worksheets.

Growth in terms of learners use of the computer and of spreadsheets, in particular, was also observed by the AO. The AO attested to learners' increased knowledge in mechanical manipulation of the technology.

AO: Not only were they gaining computer skills they were also understanding algebra as well

The following parts of conversations depict some of the skills learned,

P11: I have learned how to delete quickly

I: How would you draw that graph from the table of values, tell me the exact steps on how to draw the graph from the table?

R2: There were graphs on the menu and we had to choose which one to use so we chose the line graph, and insert and after that we had to format it.

I: Format which parts?

R2: The numbering, the scales.

I: Okay. And sometimes were all the vertical lines or horizontal lines there?

R2:Sometimes you may see that they were too small and we had to enlarge them but sometimes they were not enough.

The extracts reflect the interrelationship and coexistence between the processes of *instrumentalisation* and *instrumentation* during *instrumental genesis*. (*See section 2.4.2.2).*

The teaching and learning of school algebra should leave learners with a residue of what could be termed algebraic competency. *(See section 2.3.1*). This competency is defined in terms of attainment of the targeted learning objectives as outlined in the LJC curriculum. Learners should be able to recognize mathematical patterns and structures in a variety of situations and make generalizations using mathematical symbolism. They should also be able to model mathematical situations in the form of tables, graphs and equations. Through learning algebra, they should be able to apply and interpret mathematics in a variety of problem situations they may encounter in their life. (MOET, 2002, pp. 2-8). From the learners' responses to tasks in the worksheets, particularly Worksheets 2, 3, 4, and 6 (for the few who were able to get to worksheet 6), they were then able to generalize, model mathematical situations and answer questions based on those models.

ii) Personal Benefits

a) Development of positive attitudes and behaviors.

One learner indicated that he was happy that he was then more knowledgeable compared to other people (presumably his peers in the school) in creating graphs with spreadsheets. As mentioned earlier, use of technology has that motivational aspect in the learners that they were then enabled to become part of this exciting virtual world (Muresan et al, 2010, p. 70). This is illustrated in the following utterances:

R: What I can say, what I have learned most and I like most is when drawing graphs because most people do not know how to draw graphs, yes madam, through the spreadsheets.

P2: Madam, they were attracted into this particular project. They were many madam, just the word computer madam, we came we were attracted.

Learning algebra through spreadsheets promoted positive attitudes towards mathematics (Peressini & Knuth, 2005, p. 278; Waits & Demana, 2000, p. 56). The following statements by learners verify this view:

R: Most of the students hate maths, sometimes they hate the teacher and so do not listen in class, but students like computers they even pass their computers [course] so if they learn maths through computers they will like it and they will enjoy and that will help them pass maths

P3: ... I think it is going to be easier to understand Mathematics because most students enjoy sitting in front of the computer no matter what they are doing on a computer.

I: So you think learning Mathematics using computers is being enjoyed by most students?

R: Yes, madam.

Learners indicated that learning algebra with spreadsheets was fun and that, besides learning the content, they also learned that trying out things promoted their understanding of concepts.

P1: I liked it because it teaches me much about that we should not always ask and be told I should try out first for myself.

R2: It was a very good experience and surprising too

R2: I liked graphing and charts because it was fun doing it on computer and I gained passion for them.

AO: They were very excited about the whole thing. It was hands-on and that made them to be happy; and promoted positive attitude; it was challenging and normally it is exciting for learners to be challenged. It was exposing them to the unknown making them realize what they knew but was not aware of, so such things help in exciting them.

Perseverance is another attribute that learners that learners develop through problem solving. By learning through problem solving in a spreadsheets-assisted environment, learners acquired this trait and as a result, they were able to achieve their learning goals.

P2: I think not begging down on what I have been attempting to do and not give up has assisted me to learn successfully.

R5: Patience-if I was not patient I would not be successful in doing what I was doing.

Learners' acknowledgement that cooperation played an important role in enhancing learning was another achievement brought about by their experience of learning algebra through spreadsheets.

P5: cooperation with my group members.

Spreadsheets can thus be seen as a tool for social construction of algebraic knowledge and skills.

4.2.3.2 Spreadsheets-assisted learning compared with traditional pencil and paper methods

Learners were asked to compare their experience of the teaching and learning of algebra through spreadsheets and the traditional pencil and paper instruction. They appreciated the role that was played by spreadsheets in enhancing learning, and that the two approaches should be used together.

> *R1: …I would like to advice all teachers to use spreadsheets when teaching algebra because it makes it easier for students to understand better than when we were using just pen-and-paper.*
>
> *R1:Yes, it can also be taught through pen-and paper but it is easier when we are using spreadsheets madam.*

They felt they still needed the skills involved in performing algebraic manipulations.

> *R: Yes madam. And I think algebra should be taught through both spreadsheets and pencil–and-paper because we still need to be able to solve equations using pencil-and-paper.*

Learners also acknowledged that they had different abilities and preferences and that pencil-and paper instruction could still benefit those students who have difficulties in using the computers/ spreadsheets.

> Interviewer: Oh*! I see. Having learned algebra in class where you did not use any spreadsheets and having had this experienced this learning algebra through spreadsheets, what would you suggest for the teaching of algebra in the schools in Lesotho? Do you think children should be taught through spreadsheets alone or pencil and paper alone or through both spreadsheets and pencil-and-paper?*
>
> *P2: I think they should be taught in both because some of us do not understand computers and some of us do not understand when we write down on paper and do our own thinking madam. But some of us can do it better through worksheets through computers so they should be taught in both, madam.*

Some learners had mixed feelings regarding this issue due to challenges they were confronted with and the benefits they derived from use of spreadsheets.

> *R2: It was a very good experience surprising too but I don't think algebra should be taught by spreadsheets because sometimes when it comes to formulas it is difficult to use them but it is more easier to address them on the paper. But it is good though when it comes to drawing graphs because when using the paper you make silly mistakes.*

The AO also expressed the same views as learners. Even though she acknowledged that spreadsheets helped learners to understand algebraic concepts, she urged that spreadsheets should be used with the usual pencil-and-paper instruction.

The views expressed here, are consistent with (Waits & Demana, 2000, pp. 58-59) who argued that the appropriate use of technology in mathematics should be that of a balanced approached, whereby both technology and pencil-and-paper techniques are essential. In the study, spreadsheets were used to complement the normal pencil-and-paper instruction commonly used in the schools. This approach was chosen taking into consideration that the examinations that learners were going to write at the end of their junior and senior secondary education were based on pencil-and-paper instruction.

4.2.3.3 Summary

In this section, views of participants in the study, regarding use of spreadsheets for the teaching and learning of algebra were discussed. Benefits that were derived from use of spreadsheets for algebra teaching were found to be both technical and personal. Technical benefits include growth of learners in terms knowledge and skills pertaining to algebra as well as use of the technology (spreadsheets and the computer). At the start of the project, learners displayed lack of most of the basic computer skills that they were expected to have acquired from their computer lessons. Through their interaction with the technology in learning algebra, their computer skills improved and hence were able to use the technology to learn the targeted algebraic concepts.

Through spreadsheets, learners were able to develop the concept of variable and thus able to express mathematical relationships in algebraic form. They confessed, and were observed to have acquired the targeted algebraic knowledge and skills. Some of the algebra taught during the study was already covered in some of the classes that some of the learners came from. According to learners from such classes, learning through spreadsheets helped them to understand those concepts even better. Learners were thus able to benefit from the affordances and enablements provided by spreadsheets.

Learners also confessed that use of spreadsheets also changed their behavior in learning and attitudes towards mathematics. They learned that cooperation, patience and perseverance were essential characteristics for successful learning. It was also found that all opinions pointed to use of spreadsheets to complement the usual pencil-and-paper instruction for algebra.

4.3. Conclusion

This chapter presented findings from the qualitative part of the study. Learners from both schools expressed similar views about their experiences with algebra learning in a spreadsheets environment. The findings indicated that use of spreadsheets for the teaching and learning of Grade 9 algebra through spreadsheets might bring with it some challenges and opportunities to the learner, the teacher, and education authorities.

The challenges that confronted the learners were technical (relating to the mechanical knowledge about the spreadsheets and the associated conceptual knowledge in algebra), personal (related to the learner behavior and attitudes towards learning in spreadsheets), and related to the language of instruction. As much as learners could still gain some computer skills as part of their algebra learning experience within spreadsheets, they needed to have basic computer skills in order to learn algebra effectively within a spreadsheets environment.

The teacher's TPCK determined the kind of orchestrations that s/he provided in a spreadsheets-enhanced learning environment. The challenges to the teacher included knowledge about spreadsheets, its potentialities, affordances and enablements as well as the associated constraints that such use may impose and how to manage them in order to attain the goals of algebra learning. Another challenge concerned the teachers' knowledge about the dynamics of teaching in a technologically based environment and of providing instruction to second-language learners. The teachers' attitude towards teaching in such an environment determined to a larger extent the effort s/he invested in planning for and conducting the lessons in a manner that would benefit the learners.

Use of spreadsheets for instruction in algebra did not only bring positive results in as far as algebra learning was concerned, it also resulted in development of positive attitudes and behaviors towards algebra learning and group dynamics for the learners. Learning algebra through spreadsheets offered learners an enjoyable experience that left them with a feeling that all learners in schools should be exposed to such an experience in their algebra lessons. Learners appreciated use of spreadsheets as an approach to complement the usual pencil-and-paper instruction.

Teaching and learning algebra through spreadsheets requires computer laboratories that are equipped with adequate well-functioning equipment as well as well trained teachers. Even if teachers were passionate about integrating technology such as spreadsheets in their teaching of mathematics, lack of necessary support from school administrators would frustrate their efforts in that direction. The challenge is for the school authorities to provide for adequate facilities and infrastructure, if use of spreadsheets for the teaching and learning of algebra were to be effectively implemented. The next chapter will provide a presentation of findings from the quantitative part of the study.

CHAPTER 5

THE FINDINGS: QUANTITATIVE DATA

5.1 Introduction

The study followed a mixed–methods approach to data collection, using both qualitative and quantitative methods. The previous chapter provided a report on the findings from the qualitative part of the empirical study. This chapter provides findings from the quantitative part of the study where data was collected through administration of a questionnaire. *See Appendix E*.

5.2 Findings

Findings of the study are presented in two sections: Section A is focused on biographical information of the learners while Section B provides analysis of data obtained through the questionnaire, with the aim of addressing the main research question "*How did Grade 9 Learners in Lesotho experience the teaching and learning of algebra through spreadsheets*"?

5.2.1 Section A: Personal and General Information

A questionnaire was administered at the end of the teaching period. Table 5.1 shows biographic information of learners who participated, and remained in the study, throughout the entire period. The results show that ten out of the original fifteen (10/15) learners (66.7%) remained to the end of the study at Retha High School. This is different from what was found at Palo High School where only seven of the fifteen (7/15) learners (46.7%) had remained to the end. The grades in the Primary School Leaving Examinations (PSLE) range from 1 (one) which is the highest to F which is the lowest and in fact a fail. From the table, it can be realized that ninety percent (90%) of the learners from Retha had obtained the highest grades, ("one" or "two"), while 86.7% from Palo HS had obtained these grades; none of the learners had failed the mathematics examination. Earlier research (Barkatsas et al, 2009, p. 9), indicated that mathematics confidence and affective engagement in mathematics learning activities based within computer environment contribute towards development of positive attitudes towards learning mathematics through computers. As mentioned earlier, mathematics performance for learners from Palo high school were always lower than those from Retha High school. This condition may have contributed to higher drop-out rate of learners from Palo High school.

		SCHOOLS			
		RETHA HS		PALO HS	
		Frequency	%	Frequency	%
Gender	**M**	4	40	7	100
	F	6	60	0	0
Total		**10**	**100**	**7**	**100**
Age of respondents	14	7	70	1	14.3
	15	2	20	3	42.8
	16	1	10	2	28.5
	16+	0	0	1	14.3
Total		**10**	**100**	**7**	**100**
Maths Grade (PSLE)	1	5	50	4	57.1
	2	4	40	2	28.5
	3	1	10	1	14.3
	F	**0**	**0**	**0**	**0**
Total		**10**	**100**	**7**	**100**

Table 5. 1: Personal Information for Respondents from Both Schools

It was also found that of the learners from Retha, forty percent (40%) and sixty percent (60%) were male and female respectively. From Palo HS, all the remaining learners were boys; all girls dropped out. Forgasz also noted that female students are reported to be less positive about learning mathematics using computers; this could be a reason why all girls from Palo High school dropped out of the study (Forgasz, 2002, p. 369). It was also realized that most learners (90%) from Retha HS were 15 years of age or less, against 57.1% from Palo HS. Only one learner (6%) was more than 16 years of age. The age of learners (around 15 years) in this study was consistent with the usual age of learners at Grade 9 in most countries.

5.2.2 Section B: Learner experiences in learning algebra through spreadsheets

The twenty-four (24) items in the questionnaire were categorized into three areas that they addressed. These areas were based on the factors that influence learners' performance when learning algebra through spreadsheets. These were: attitudes towards algebra and spreadsheets, engagement in the classroom activities, and their technical knowledge and skills required for learning algebra within spreadsheets environment.

The questionnaire was assessed for internal consistency reliability using the alpha coefficient. The results from the test for all items in the questionnaire indicated the alpha coefficient of 0.74. (*See Appendix F)*. Items designed to investigate each of the three constructs were also tested for internal consistency reliability. Only those items for investigating learners' attitudes towards spreadsheets and algebra were found to have the acceptable value of $\alpha \approx 0.7$. The coefficient for the measuring learners' levels of engagement and their knowledge and skills were far below the acceptable levels. The possible explanation here could be that there were very few items per each of the two categories (Tavakol & Dennick, 2011, p. 54). Instead of increasing the number of items in these categories, it was decided that the total number of items in the whole questionnaire be kept at twenty-four (24); considering the age of participants in the study, too many questions may have led to inaccurate responses or low response rate (Leung, 2001, p. 189). Learners' responses to items in the two categories were analyzed and compared one-by-one for the two schools involved in the study.

Learners had to rate their opinions on a 5-point Lickert scale, ranging from strongly disagree to strongly agree. Learner's responses to items in these areas were analyzed separately for each school and the results compared (Yin, 2009, p. 57) as illustrated in tables and graphs.

5.2.2.1 Learners' attitudes towards algebra and spreadsheets

Research indicates that learners' attitudes towards mathematical domain and use of technology affect their effective use of the artifact towards achieving mathematical learning goals. The following is a list of items on the questionnaire which were formulated to investigate learners' attitude towards learning algebra within spreadsheets environment.

1. Spreadsheets helped me to recognize patterns in sequences easily
2. I enjoyed learning algebra using spreadsheets
3. Spreadsheets helped me to understand algebra
7. I could solve algebra problems more quickly using spreadsheets
9. I enjoyed creating graphs in spreadsheets
10. The activities in spreadsheets helped me to make sense of algebra
11. I like drawing graphs in pencil and paper than in spreadsheets
13. Doing computations using spreadsheets helped me to solve mathematics problems successfully.
15. I feel more confident in doing mathematics after using spreadsheets.
18. I prefer to perform calculations using pencil and paper than in Excel spreadsheets.
19. Solving real life problems in spreadsheets makes algebra learning interesting.
21. I learned algebra better through pencil-and-paper than through spreadsheets.
24. Knowledge of algebra can help me solve problems in other.

Table 5.1 shows the distribution of learners' responses to items regarding their attitudes towards algebra and spreadsheets

Items in this category were tested for internal consistency using the Cronbach's alpha coefficient. The negatively phrased items (highlighted) were scored in reverse. The results indicated the value of alpha as 0.71, which is an acceptable value for reliability.

	SCHOOLS											
	RETHA HS						PALO HS					
	Frequency per response						**Frequency per response**					
Item\Response	1	2	3	4	5	Mean	1	2	3	4	5	Mean
1	0	0	0	7	3	**4.3**	0	0	1	2	4	**4.4**
2	0	0	0	2	8	**4.8**	1	0	2	1	3	**3.7**
3	0	0	0	8	2	**4.2**	0	1	1	1	4	**4.1**
7	0	0	0	7	3	**4.3**	0	0	1	3	3	**4.3**
9	0	0	1	3	6	**4.5**	0	1	1	3	2	**3.9**
10	0	0	1	6	3	**4.2**	0	2	0	1	4	**4.0**
11	7	2	0	1	0	**1.5**	2	0	2	1	2	**3.1**
13	0	0	0	4	6	**4.6**	1	0	0	2	4	**4.1**
15	0	0	1	5	4	**4.3**	0	0	1	1	5	**4.6**
18	4	4	0	2	0	**2.0**	2	0	1	4	0	**3.0**
19	0	0	3	4	3	**4.0**	1	0	0	1	5	**4.3**
21	4	6	0	0	0	**1.6**	1	0	0	1	5	**4.3**
24	0	0	1	6	3	**4.2**	1	1	1	2	2	**3.4**

Table 5. 2: Distribution of Learners' Ratings of their Attitudes Towards Algebra and Spreadsheets.

The means (M), standard deviation (SD) and *standard error of the means (SEM)* of responses from the two schools were then calculated. The SEM is an estimate of the amount by which an obtained mean may be expected to differ by chance from the true mean. It is an indication of how well the mean of a sample estimates the mean of a population. The results are displayed in Table 5.3.

SCHOOL	**N**	**Mean**	**Standard Deviation**	**Standard Error of Means**
RETHA HS	10	3.73	0.26	0.08
PALO HS	7	3.95	0.37	0.14
TOTAL	17	3.82	0.32	0.08

Table 5. 3: Descriptive Statistics for Responses on Attitudes towards Algebra and spreadsheets

The results show that the mean for the learners' responses about their attitudes towards algebra and spreadsheets differ in the two schools. The mean for Retha HS was approximately 3.73 with standard error of 0.084, while for Palo, the mean and standard error were 3.95 and 0.14 respectively. The standard errors were calculated at 95% confidence level. The Least Square (LS) means were also calculated and then plotted. The LS means are used when inferential comparisons need to be made. Figure 5.1 shows the plot of the LS means for the schools.

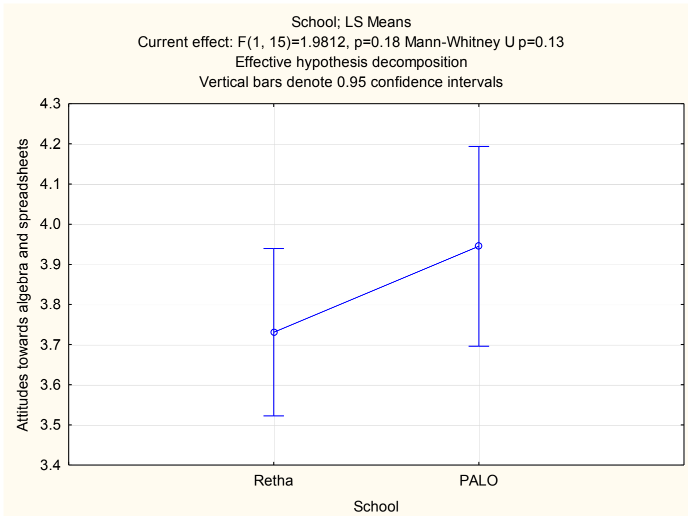


Figure 5. 1: The LS Means for Retha and Palo HS

Analysis of the plots also indicates the values for Retha being lower than those for Palo HS. The task was then to establish whether the difference between the means was significant or not.

When comparing means from two groups in an experiment, we want to find out whether the two means truly differ from one another, or if the difference in the means of the groups is simply due to random chance. The test was done on the two hypotheses:

The Null hypothesis, H_0: There is no difference in opinions relating to attitudes towards algebra and spreadsheets between learners from Retha and Palo High schools.

The Alternative Hypothesis, H_1: There is a significant difference in the opinions relating to attitudes towards algebra and spreadsheets between learners from Retha and Palo High schools.

The statistical "rule of thumb" is that- "whenever a statistical test returns a probability value (or "*p*-value") equal to or less than 0.05, we reject the null hypothesis that our results fit the distribution we are testing" (downloaded on 12 June 2013, from basics-cap-online.pdf). This means that a *p*-value equal to or less than 5% (0.05) indicates that the opinions expressed in the two groups are significantly different from one another.

Results indicate $F(1, 15) = 1.9812$, $p = 0.18$, and the Mann-Whitney U $p = 0.13$. This indicates that the opinions expressed by the learners from the two schools regarding their attitudes towards algebra and spreadsheets were the same. That is, there was no significant difference between the opinions expressed by these groups of learners.

All the statements, in this section, were positive about learners' attitudes towards learning through spreadsheets, except three (11, 18, and 21, highlighted in yellow) which were negatively phrased in this regard. Learners were to rate how much they agreed with the statements. In both schools, learners indicated that they enjoyed the experience, they agreed that spreadsheets helped them to understand algebra better. Learners agreed that spreadsheets helped them to recognize patterns in sequences easily (item 1) and that they could solve algebra problems more quickly using spreadsheets (item 7). They indicated that they enjoyed creating graphs using spreadsheets (item 9). Learners had developed positive attitude and confidence towards learning mathematics as a result of learning through spreadsheets (item 15). They agreed that solving realistic problems in spreadsheets made algebra learning interesting (item 19). They indicated that they enjoyed learning algebra through spreadsheets (item 2). These findings were consistent with those obtained through qualitative methods (interviews and classroom observations). These further confirmed that spreadsheets can serve as cognitive and motivational tools for the learning of algebra (Tabach & Friedlander, 2004; Rojano, 1996; Jonassen & Reeves, 2001; Haspekian, 2005,etc.). Learners benefitted from the affordances provided by spreadsheets with the help of the strategies that were initiated by their teacher. (*See section4.2.2.1*)

The responses to item 13 indicate that 57.1% of the learners from Palo HS did not agree that doing calculations in spreadsheets helped them solve problems successfully, while 42.9% agreed with the statement. A possible explanation to this distribution could be that, while still appreciating the power of spreadsheets in doing computations quickly and accurately, those learners may have realized that solving algebraic problems in spreadsheets is not only about doing computations right, but it also requires much more than that. Lack of understanding of what the problems required, and inadequate technical skills were some of the challenges that learners encountered in this study. *(See Section 4.2.1.2).*The feelings expressed by learners from Palo high school, may also come as a result of their low mathematical background. As was observed, these learners struggled a lot with worksheet 1, which was basically on number sense and their ability to recognize relationships between numbers.

The other three statements (11, 18 and 21) were on learners' preferences between using paper-and-pencil and spreadsheets for doing computations, drawing /creating graphs and general algebra learning. The mean scores for statements 11, 18 and 21 indicate that learners from Retha HS are opposed to being in favor of using pencil-and-paper than to spreadsheets. When scored in reverse, the mean values for these items become consistent with the positive views towards spreadsheets expressed by learners in this category. Learners indicated that they preferred using spreadsheets than pencil-and-paper. They had developed positive attitude towards learning algebra through spreadsheets than just pencil and paper. These findings confirm what was found from literature search. For example, (Jonassen & Reeves, 2001, p. 697). Again learners from Retha showed better performance in Worksheet 1 than those from Palo, hence were less challenged by doing mathematics in spreadsheets. The responses here show that students with low mathematics background struggle with using spreadsheets than those with higher mathematics skills and thus develop negative attitudes towards using spreadsheets to solve mathematical problems.

Figures 5.2 and 5.3 show the distribution of learners' responses to the items regarding their attitudes towards algebra and spreadsheets, for each of the two schools, Retha and Palo HS, respectively. From the bar chart for Retha HS, it can be seen that the high frequencies are for agreeing with all the other statements, except for 11, 18, and 21, where responses are mostly on the negative, due to the way they were formulated.

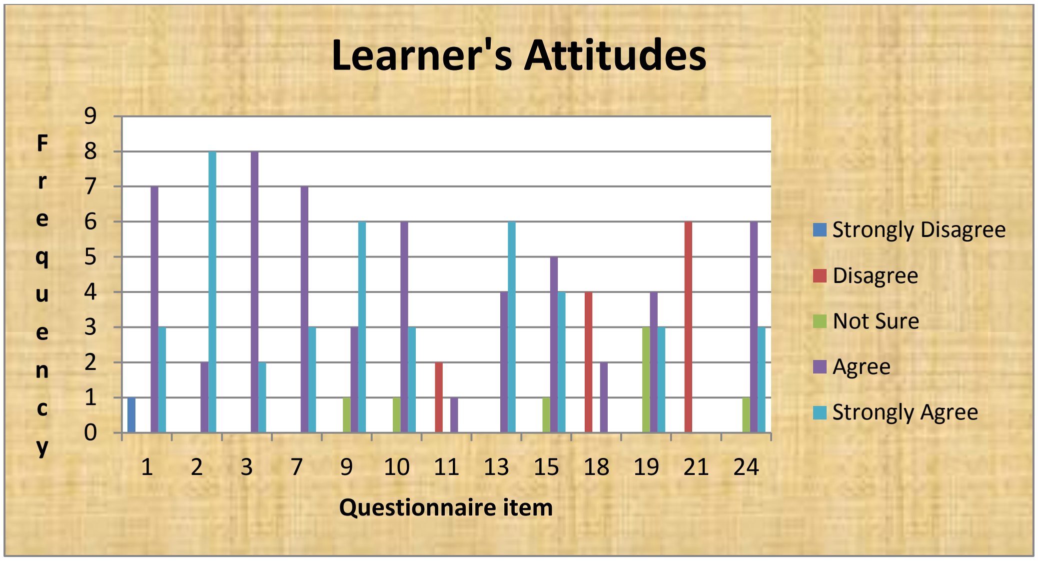


Figure 5. 2: Distribution of Ratings on Learners' Attitudes at Retha HS

Eventhough the general impression of opinions expresed by learners from both schools may be considered the same, responses from Palo HS on items 11, 18 and 21, show different trends from what was expressed by learners at Retha: they are towards the positive side, learners are in agreement with being happier with use of pencil-and-paper in their algebra learning than with spreadsheets. There are possible explanations to these kind of responses:one possibility could be that, for this group,learning algebra through pencil-and-paper was more ideal than through use of spreadsheets, as many of them were observed to be greatly challenged by working on the computer, they lacked the required technical knowledge and skills.Another possibility could be that learners interpreted use of spreadsheets as implying doing away with paper-and-pencil, for which learners still expressed appreciation and value. The implication to theopinions expressed thus calls for a balanced approach, use of spreadsheets together with pencil-and-paper, for teaching and learning of algebra (Waits & Demana, 2000, pp. 58-59).(*See Section 4.2.3.2*). Although these kind of responses were not surprising, there is also a possibility that the negative phrasing of the items may have also caused some confusion to this group of learners (Sauro, 2011, downloaded on 20 June 2013 from www.measuringusability.com/positive-negative.php).

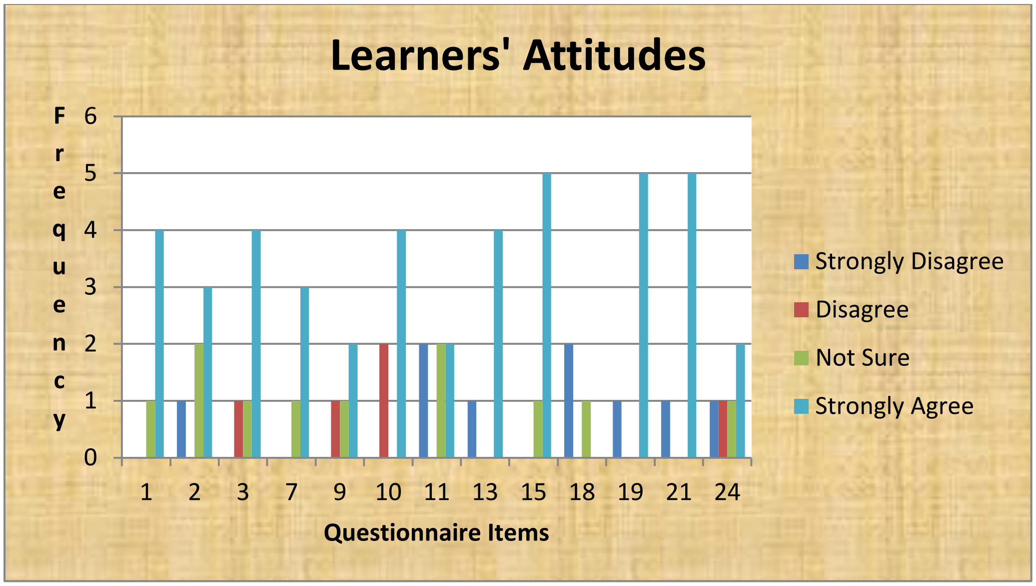


Figure 5. 3: Distribution of Ratings on Learners' Attitudes at Palo HS

Earlier studies (Peressini & Knuth, 2005, p. 278; Waits & Demana, 2000, p. 56) indicate that use of technology is expected to bring about motivation towards learning, and there is evidence in this study that learners enjoyed learning algebra through spreadsheets. If use of spreadsheets imposed more obstacles that completely impede algebra learning, there is no doubt learners would not have enjoyed the experience.

5.2.2.2 Learners' engagement in lesson activities

The following is a list of questionnaire items that were designed to assess learners' views on the given modes of engagement in the learning activities:

6. I actively participated in the algebra lessons
8. I like working alone than in a group
12. I learn better, when I work and discuss problems with my peers
14. I was happy to work, talk and discuss ideas with my peers in the algebra classroom

Table 5.4 shows the distribution of responses on Learners' engagement during the lesson activities.

	SCHOOLS											
	RETHA HS						**PALO HS**					
Item\Response	1	2	3	4	5	**Mean**	1	2	3	4	5	**Mean**
6	0	0	0	6	4	**4.4**	0	1	1	1	4	**4.1**
8	0	1	1	4	4	**4.1**	1	3	1	1	1	**2.7**
12	0	0	1	3	6	**4.5**	1	1	0	1	4	**3.9**
14	0	0	0	1	9	**4.9**	0	1	2	2	2	**3.7**

Table 5. 4: Distribution of Learners' responses on engagement during the lessons

Items in this category were tested for internal consistency, and the Cronbach's coefficient was found to be unsatisfactory, thus comparison for the two schools was done for each individual item. The smaller number of items in this group is very likely to have contributed to the low value for alpha. More items were not added to the list to avoid a lengthy questionnaire. Means of responses for individual items were calculated and compared. The hypotheses tested were of the form:

H_0: There is no difference between the opinions expressed by learners from both schools concerning the issue reflected in the given item.

H_1: There is a significant difference between the opinions expressed by learners from both schools concerning the issue reflected in the given item.

As with the items on " learners' attitudes" in *Section 5.2.2.1*, the tests were done within 95%confidence interval, that is for p = 0.05 and the Null hypothesis is rejected for the values of p less or equal to 0.05.

The means for item 6 for both schools indicate that learners agreed that they were actively involved in the lesson activities, with values of 4.4 and 4.1 for learners from Retha and Palo HS respectively. Statistical analyses of the means using Fisher test indicated F (1, 15) = 0.36297, with p = 0.56 and calculating the Mann-Whitney Up yielded p= 1.00, within the 95% confidence interval. Since $p > 0.05$ then we accepted the Null hypothesis that there was no difference in the opinions expressed by learners from both schools concerning their participation in the lesson activities. The difference between their levels of engagement is not statistically significant. This finding is in agreement with what advocates of technology integration in mathematics education posit-use of technology promote active participation of learners (Peressini & Knuth, 2005, p. 278; Jonassen & Reeves, 2001, p. 696).

In item 8, learners were to rate their preference for working alone against group work. Means for their responses were 1.90 and 3.29 for Retha and Palo HS respectively. Applying Fisher test on the means indicated F (1, 15) = 5.8342, $p = 0.03$ and the calculated Mann-Whitney U $p = 0.06$. These results suggested that we rejected the Null hypothesis and accepted the alternative hypothesis that, there was a significant difference in the opinions expressed by learners from the two schools regarding individual work. Learners from Retha HS did not favor individual work while those from Palo were much happier working alone than in groups. This finding is not surprising as learners prefer different learning styles and teaching in a technology-assisted environment has to provide opportunities that cater for different learning styles (Kolb et al, 1990; Hu, et al., 2007, p. 1101; McLeod, 2010). This also confirms what some of the learners indicated during interviews, that they preferred working alone as that gave them opportunity to test their own understanding. (*See Section4.2.2.2).*It is important to remember that students who are struggling with their studies usually want to remain in isolation, hiding their inabilities to peers and even tend to reject assistance from reliable sources (Wirth & Perkins, 2008).

In item 12, learners were to rate their learning because of working and discussing problems with their peers. Means for their responses were 4.50 and 3.86 for Retha and Palo HS respectively. Application of the Fisher test on the means indicate F (1, 15) = 1.1952, $p = 0.29$ and the Mann-Whitney U-test gave $p = 0.70$.Since $p>0.05$, we accepted the Null hypothesis that there was no difference in the opinions expressed by learners from the two schools regarding value of discussion in a spreadsheets-enhanced learning environment. All learners from both schools benefited, learned better, from discussions with their peers. Research indicates that learner-interactions in mathematics classroom are very important in fostering understanding. When learners are provided opportunity to share ideas, they are able to communicate in a language that they best understand and hence engage in continuous construction of new knowledge (NCTM, 1991, p. 21; Van den Heuvel-Panhuizen, 2000, p. 9). From the qualitative data, learners also confessed that discussions with their peers helped them to learn successfully. (*See Section 4.2.2.2*).

Item 14 was also about effects of discourses in the classroom. Learners were to rate their appreciation for working, talking and discussing ideas in the algebra classroom. Means for their responses were 4.90 and 3.71 for Retha and Palo HS respectively. Analyses of the means using Fisher test indicated F (1, 15) = 10.426, $p < 0.01$ and the p-value from the Mann-Whitney U-test was $p = 0.03$. These results suggested that we rejected the Null hypothesis and accepted the alternative hypothesis that there was a significant difference in the opinions expressed by learners from the two schools regarding their engagement in classroom talking. Learners from Retha HS were happy to talk, work and discuss ideas in class, while those from Palo were less happy to engage in such discourses. During classroom observations, I observed that learners at Retha HS were more interactive than those at Palo HS. There were more arguments (work-related), and even more progress in the work covered by learners from Retha than PaloHS-learners at Retha HS were able to reach Worksheet 6, while very few from Palo were able to get to Worksheet 6.While some learners may have been comfortable to disclose their strengths and weaknesses, others may felt uncomfortable to do so, as seen with item 8 (Wirth & Perkins, 2008).

From learners' responses to items in this section, it can be realized that use of spreadsheets encouraged active involvement of learners in the learning of algebra. Some learners preferred working individually while others enjoyed group work. Classroom activities should cater for different learning styles in order to maximize success of the teaching learning process. Discussions with peers played an important role in the development of understanding of learned concepts. These also need to be guided discussions driven towards attainment of learning objectives.

5.2.2.3 Learner’s opinions on their technical knowledge and skills

The following is a list of items from the questionnaire that were formulated to assess learners competency level in technical knowledge and skills (TKS) involved in learning algebra through spreadsheets:

4. I can develop formulas for use in spreadsheets.
5. I can use the fill down command correctly
16. I can always choose a suitable scale for the graph I draw.
17. I can interpret the results I get from the computer.
20. I can translate expressions in ordinary language to spreadsheets notation.
22. I can translate expressions in spreadsheets notation to ordinary algebra language.
23. When different graphs are drawn in spreadsheets, they can be easily compared.

Table 5.4 shows the distribution of learners’ responses on each questionnaire items pertaining to technical knowledge and skills.

	SCHOOLS											
	RETHA HS						PALO HS					
Item\Response	**1**	**2**	**3**	**4**	**5**	**Mean**	**1**	**2**	**3**	**4**	**5**	**Mean**
4	0	0	2	6	2	**4.0**	0	0	1	1	5	**4.6**
5	0	1	1	2	6	**4.3**	0	0	0	1	6	**4.9**
16	0	0	3	5	2	**3.9**	1	1	2	1	2	**3.3**
17	0	1	2	7	0	**3.6**	1	0	0	5	1	**3.7**
20	0	0	4	4	2	**3.8**	1	0	1	0	5	**4.1**
22	0	2	4	4	0	**3.2**	3	0	0	4	0	**2.7**
23	1	2	2	3	2	**3.3**	4	1	1	1	0	**1.9**

Table 5. 5: Distribution of learners' responses to Technical Knowledge and skills

As with the earlier categories of items, items in this category were also tested for internal consistency, and the Cronbach’s coefficient was found to be unsatisfactory, thus comparison of results for the two schools was done for each individual item. Means of responses for individual items were calculated and compared using the Fisher test. The Mann-Whitney U *p*-values were also calculated for each item. Table 5.5 shows the results-the p-values for the Fisher and the Mann-Whitney test and the corresponding decision with regard to the null hypotheses.

Learners from both schools indicated that they had acquired technical knowledge and skills involved in learning algebra through spreadsheets. For example, in item 4, learners were to rate their competency in developing spreadsheets formulas for given mathematical relationships

Item	Means		*p*-values		Decision on H_0
	Retha	Palo	Fisher test	Mann-Whitney	
4	4.0	4.6	0.13	0.13	Accept
5	4.3	4.9	0.21	0.35	Accept
16	3.9	3.3	0.28	0.43	Accept
17	3.6	3.7	0.81	0.49	Accept
20	3.8	4.1	0.56	0.26	Accept
22	3.2	2.7	0.42	0.81	Accept
23	3.3	1.9	0.04	0.05	Reject

Table 5. 6: Analysis of Responses on Technical Knowledge and skills

Means for their responses were 4.00 and 4.57 for Retha and Palo HS respectively. Application of the Fisher test indicated $F(1, 15) = 2.6144$, p =0.13 and calculation of the Mann-Whitney U-test yielded $p = 0.13$. Since the value of $p > 0.05$, then I accepted the Null hypothesis that learners in both schools were able to develop formulas for use in spreadsheets.

For items 5, 16, 17, and 20, results indicate that learners had learned how to choose suitable scales for their graphs, they were able to interpret the results they obtained from the computer and that they could translate expressions in ordinary language to spreadsheets notation. This was evident in learners' work and their confessions that they had learned and acquired these skills through continued work on the activities. *(See sections 4.2.3.1).* Learners were less competent to make translations from spreadsheets notation to formal algebra notation (item 22).This was also observed during the lessons, that it was only those learners who were able to verbalize and express relationships in their own words, that were then able to translate from spreadsheets notation to the formal algebraic language. (*See section 4.2.1.2(iii)).*

Learners had different experiences with regard to comparing many graphs drawn on the same pair of axes. Analysis of responses to item 23, indicated that, while those from Retha still had some difficulties with the exercise, it was even more difficult for those from Palo HS. This revelation is consistent with what learners mentioned as one of the technical challenges they encountered. *(See section 4.2.1.2, (i))*. Some of the affordances provided by spreadsheets may become sources of difficulties for learners; the multiple representations provided by spreadsheets were a source of difficulties for learners in this study. (Ainsworth, 1999, p. 132).

From their responses in this section, it appears that learning algebra through spreadsheets provided learners opportunity to acquire technical (mechanical and conceptual) knowledge and skills. They learned how to create formulas for use in spreadsheets, how to use the "drag" command, create graphs and format them, and make translations within ordinary language, spreadsheets and formal algebra notation. Embedded in the statements in this section are problem solving skills such as generalization, multiple representation of information (modeling.), and interpretation of solutions to address the original questions. Learners indicated that they were able to solve problems in algebra using spreadsheets. The degree of competence in the acquired knowledge and skills differed from learner to learner in the two schools, with those from Retha being higher.

The next section provides a summary of the findings from learners' responses to the questionnaire.

5.3 Summary

The purpose of the using the questionnaire was to collect data that would complement the data obtained through qualitative methods, in order to address the main research question. From the analysis of data, it can be realized again that generally, there was no significant difference between views expressed by learners from the two schools. The two cases used in the study served as means of validating and possibly generalizing results of the study (Yin, 2009, p. 61). Findings from the learners' responses indicated that learners experienced some challenges and successes in their learning of algebra through spreadsheets. This is discussed in detail in the next sections.

5.3.1 Challenges encountered by learners while learning algebra through spreadsheets.

The challenges encountered by learners were technical, personal and language related.

5.3.1.1 Technical challenges

Learners' responses indicated that even though they learned the algebra content that was offered and the associated computer skills, they were not confident with some of the selected aspects on knowledge and skills related to algebra learning within spreadsheets. Learners encountered challenges in developing spreadsheets formulas for given situations. They lacked the relevant arithmetic skills upon which algebraic concepts (particularly generalizing) could build. They also had difficulties with choosing scales for their graphs; they lacked number sense. Learners who had lower mathematics background struggled more with using spreadsheets.

Some of the affordances provided by spreadsheets became sources of difficulties for learners. The multiple representations provided by spreadsheets were a source of learners' difficulties when they had to observe the impact of varying parameters on the symbolic, graphical and tabular forms of same relationship. This became even worse when different relationships were presented on one pair of axes. Learners also had problems interpreting the results they got from the spreadsheets. These findings confirm what was confessed by learners and was also observed during qualitative phases of the study. *(See Section 4.2.1.2)*

5.3.1.2 Personal challenges

Even though learners were taught together as a group, they still had their individual preferences. Some learners were not comfortable with group work. They still preferred to have time to learn and explore the algebraic ideas on their own. They needed time to reflect on their constructions and assess their own understanding of the learned algebraic concepts. It is important that teachers monitor learners' progress as they work individually, so that the teaching strategy would not be used as an opportunity to hide inadequacies, but as an opportunity to reflect and reconstruct own understandings of taught concepts.

5.3.1.3 Language related challenges

Learners indicated that translating from ordinary language to spreadsheets notation was more difficult than moving from spreadsheets notation to formal algebraic notation. While it was expected that the similarity between spreadsheets notation would advantage learners in translating in between the two forms, it appeared that this was not the case for some learners. This kind of difficulty was more evident in those leaners who were not able to verbalize the involved relationships in their own words. This perhaps may suggest that there is a link between fluency in language of instruction (English) and mathematical thinking, which was beyond the scope of this study and may need to be researched.

5.3.2 Pedagogical strategies that enabled teaching and learning of algebra through spreadsheets

From learners' responses, it could be realized that individual and group work helped learners to enjoy learning algebra through spreadsheets. Learners were able to acquire algebraic concepts better through individual engagement, group and whole-class discussions. Teaching algebra in a spreadsheets-based environment through tasks organized in the form of worksheets encouraged learners to be actively engaged in their learning as individuals , while the group arrangement and whole-class discussions facilitated by the teacher gave learners opportunity to share their ideas and help each other to gain understanding of targeted concepts.

5.3.3 Learners' perception of their learning of algebra through spreadsheets

In both schools, learners indicated that they enjoyed the experience. This could possibly be because it was something new to them or that gave them opportunity to work on the computer, which they found fascinating (as found in qualitative data). There were mixed feelings, however, with regard to usefulness of spreadsheets in promoting understanding of algebra; some learners believed that spreadsheets helped them to understand algebra better, while others disagreed to that. It appeared that learning algebra through spreadsheets provided learners opportunity to acquire technical (mechanical and conceptual) knowledge and skills. Learners indicated that they had learned to use the fill down command correctly. They were able to develop formulas for use in spreadsheets. Some had learned how to choose suitable scales for their graphs, although it was still a problem to others.

Learners developed positive attitudes towards algebra learning due to their engagement with it in spreadsheets. They appreciated the usefulness of spreadsheets in creating graphs. Spreadsheets provided a quicker and more accurate way of producing graphs. Learners also appreciated the role of spreadsheets in performing computations. They indicated that they believed they learned algebra better through spreadsheets.

Learning algebra through spreadsheets engaged learners into manipulating the artifact towards achievement of lesson objectives. The practical nature of the environment promoted interaction between the learners. This promoted a conducive learning environment, where learners were free to share their difficulties as well as achievements. All learners were happy to learn in such an environment.

Cooperative learning and collaboration played a very significant role in the classrooms. All learners indicated that they were happy being allowed to work, talk and discuss their ideas with their peers. Peer discussions enhanced learning as learners talked among themselves in a language that was most understood by them. In sharing ideas they were able to refine their constructions of knowledge.

Even though they appreciated learning with spreadsheets, learners also valued doing some work with pencil-and-paper. Both techniques were used in the study. It is important to note that technology alone, in mathematics education, should not be viewed as having the potential to help in equipping learners with the kind of algebra that we expect our learners to know, rather, they should be seen as tools that may help learners to do the tasks that may also be accomplished without using the technologies, but can more easily, quickly and accurately be completed with these tools (Harvey, Waits, & Demana, 1995, p. 103). Technological tools should be used to enhance learning.

5.4 Conclusion

This chapter presented findings from data collected using quantitative methods. Findings indicate that learners encountered challenges, some of which they were able to overcome with time, and benefitted from learning algebra through spreadsheets. The challenges included those associated with physical manipulation of the hardware of the computer, knowledge about what spreadsheets can offer and conceptual aspects related to the spreadsheets notation and the formal algebraic language.

Apart from the acquiring the targeted algebraic knowledge and skills, learners were happy that their engagement with the computer and spreadsheets in particular, helped them to improve their skills in using the technology. They pointed out that their confidence in learning mathematics was also increased. Learners attested to the important role played by individual work, group work and discussions in their algebra learning within the spreadsheets environment. These findings are consistent with those obtained from the qualitative data.

The next chapter provides the summary of findings, conclusion and recommendations arising from the study.

CHAPTER 6

SUMMARY OF FINDINGS, CONCLUSION AND RECOMMENDATIONS

6.1 Introduction

The main research question for this study was: "*How do Grade 9 learners in Lesotho experience teaching and learning of algebra through Excel spreadsheets?.*" The empirical study followed a mixed-methods approach, where both qualitative and quantitative approaches to data collection, presentation and analysis were used. Chapter 4 presented findings from the qualitative part while Chapter 5 was on the quantitative part of the study (Creswell &Plano Clark, 2011, p. 234). This chapter provides a summary of the findings from both parts of the study. Triangulation of findings is done and collaboration, corroboration and correspondence between the results is sought in this chapter.

Triangulation here involved looking across both qualitative and quantitative results , assessing how these answered the main question of the study (Creswell & Plano Clark, 2011, p. 212). The mixed-methods approach had advantages over single methods (qualitative or quantitative only designs) in that, it added insights and enhanced better understanding, which may have, otherwise, been missed when a single method was used. Mixed methods were used to bring about *complementarity*, which "seeks elaboration, enhancement, illustration and clarification of the results from one method with the results from the other"(Creswell& Plano Clark, 2011, p. 62). Combining qualitative and quantitative methods has an advantage in that the strengths of one method were used to overcome the weaknesses of the other. In this way the methods complemented one another. (Johnson & Christensen, 2008, p. 444).

Conclusions about the study are drawn and recommendations are also made based on the discussion of the findings.

6.2 Triangulation of Findings

The triangulation of findings from the qualitative and quantitative methods of data collection is presented according to how the sub-questions for the empirical study are addressed.

6.2.1 Challenges encountered in the teaching and learning of Grade 9 algebra through Excel spreadsheets in Lesotho

From the study it was found that there were two classes of challenges that were encountered, the school-based and instruction-based challenges.

6.2.1.1 School-based challenges

These were related to the physical conditions in the learning environment, which resulted from insufficient support by the school authorities. These findings came from the qualitative part of the study. *(See Section 4.2.1.1)*. Two categories of such challenges were identified:

i) Lack of adequate facilities. It was also found that the computer laboratories were very small and did not allow free movement of the teacher in monitoring progress of learners as they worked through the tasks. There was lack of additional, yet useful, facilities such as projectors which could have made the teaching and learning more effective, particularly with regard to aiding interventions in the form of illustrations and demonstrations by the teacher.

ii) Lack of maintenance of facilities. It was found that most of the computers used in the school laboratories were old and thus took a long time to open and function. This gave the teacher an extra responsibility of acting as a laboratory technician as well. There were electrical power cuts experienced during the teaching sessions, due to lack of maintenance of electrical facilities in the schools.

6.2.1.2 Instruction-based challenges

These were challenges related to the events that took place during the teaching-learning process. The instruction-based challenges were categorized as technical (related to the artifact and the mathematical content), personal(related to behavior and attitudes of the learners), and those related to the LoTL.

i) Technical challenges: This category of challenges include those that were mechanical, that is , related to physical manipulation of the computer and use of spreadsheets, and those that were conceptual, relating to the understanding of the algebraic concepts involved. Some of these challenges were brought about by constraints imposed by the use of the artifact. These findings came from both qualitative and quantitative data. *(See Sections 4.2.1.2 (i) and 5.3.1(i))*. Areas that presented challenge to learners included:

- mechanical manipulation of the hardware of the computer such as handling the mouse in highlighting and dragging;
- accessing available commands;
- lack of necessary mathematical (arithmetic) knowledge and skills;
- strict syntax in spreadsheets for formulating spreadsheets formulas for relationships;
- implicit processing of data by spreadsheets;
- confusing multiple representations provided by spreadsheets.

Three types of constraints imposed by spreadsheets were identified. These were organizational constraints, which relate to knowledge about access and organization of available commands on the artifact. Some of the learners had difficulty in operating the computer because they were not yet familiar with the location of different keys on the keyboard. Learners also experienced the command constraints, which related to the knowledge about the commands available on the spreadsheets and the strict syntax associated with them. They were seen to be zapping between windows, trying to access commands, exploring what commands were available, and test-trying the order of operations they should follow to get the spreadsheets perform the intended tasks. This was mostly evident when learners had to create graphs. The internal constraints, were those that related to the knowledge the learner had about what the spreadsheets could do.

Some of the affordances provided by spreadsheets became sources of difficulties for learners. While the spreadsheets notation is very much similar to the formal algebraic notation and thus may have promoted better formulation of expressions or equations for the relationships presented in spreadsheets, learners expressed that they found translating from spreadsheets notation to formal algebraic language difficult. Although the ambiguity with which a cell argument, say A1, functions in spreadsheets could enhance learners' development of the concept of variable, this created confusion to the learners and became an obstacle in their attempts to formulate generalizations. The implicit data processing done by spreadsheets made it difficult for learners to follow how the products came about. The dynamic nature of spreadsheets also became a source of learners difficulties, as all operations happened so quickly that learners were not able to connect the effect of changing parameters on the different forms of representations. The learners' levels of growth in the process of instrumental genesis greatly influenced the efficiency with which they used the artifact to achieve algebra learning objectives.

Learners were found to be lacking in the basic algebraic thinking skills and conceptual knowledge about number patterns and number behaviors developed at primary school level. Lack of these skills were a source of learners difficulties in areas such generalizing patterns of numbers and in choosing suitable scales for their graphs. Profound knowledge and skills in arithmetic provided a basis upon which formal algebra learning met in the secondary school level could be built. Learners with a lower mathematical background were found to struggle more with using spreadsheets than those whose mathematical background was higher.

ii) Personal challenges: Learners also experienced challenges related to their usual classroom practices, behavior and attitudes. The following were personal challenges encountered by learners:

a) Clash between learning styles and teaching approach. Some learners felt unhappy with the way lessons were conducted, it was different from what they were used to. The lessons were geared to helping learners discover knowledge as opposed to situations where they were provided with procedures for solving algebraic problems and then drilled on implementing such procedures. Learners were used to the teacher showing them how to solve problems and they, as learners, only had to practice the skill; not to grapple with the problems to find the solutions, working on their own. Some of the learners did not like group work, they preferred working on their own. *(See Sections 4.2.1.1i(i) and 5.3.1(ii).*

b) Distractive effect of the computer. For other learners, their source of distraction was the exposure to the computer itself, they had very little experience with the machine and the only interaction they had was during computer lessons, this was not enough, so they were overcome by anxiety. *See section 4.2.1.1(ii*).

c) Lack of motivation and focus. Some of the learners seemed to lack commitment in the work they were supposed to be doing. They confessed that one of their challenge was lack of concentration. This was evident, more particularly, with learners from Palo High School, where the morale for learning was low; probably the long-standing history of poor examination results in the school gave birth to a culture where there is no urge to work hard. Learners who are challenged during instruction, also have a tendency to procrastinate (Wirth &Perkins, 2008).

d) Lack of cooperation and collaboration within groups. Some of the learners were not cooperative in their groups. They would engage in non-task related discourses, leaving one member of the group to carry on the work alone; only when the teacher approached that they would pretend to be engaged with learning. This is why some learners felt that they did not like being in groups. In other cases, some learners were not happy with the long debates that took place within their groups, and felt that group work was holding them back. *(See section 4.2 .1.1 (ii) and 5.3.1(ii)).*

e) Lack of freedom in the classroom. Some learners confessed that at the beginning of the project, they found it difficult to ask or answer questions in class, as they were not familiar with the teacher-researcher. They however indicated that the situation improved as time went on. *(See section 4.2 .1.1 (ii).* It is also worth noting that, in some cases, learners who find difficulty in coping with lesson demands may not feel free to disclose their struggles to their peers or even their teacher.

iii) Language related challenges: These were related to teaching in a second language.

a) Learners found it difficult understanding instructions provided in the worksheets and in some cases, they were not able to follow what the questions required. They found it difficult to express themselves in those tasks that required them explanations. (*See Section 4.2.1.1. (ii)).*

b) Learners also encountered difficulties when they had to translate mathematical statements from ordinary language to the symbolic spreadsheets or formal algebraic language as may be required by the tasks.(*See Section 5.3.1 (iii*). Learners who were able to verbalize the relationships between variables were able to translate within spreadsheets notation and the formal algebraic notation.

6.2.2 Pedagogical strategies that enabled effective teaching and learning of Grade 9 algebra through Excel spreadsheets?

Although teaching and learning of algebra through spreadsheets presented some challenges to both the teacher and the learners, by the end of the study, learners had been able to acquire some knowledge of the targeted algebraic concepts. Strategies that enabled acquisition of knowledge were those that were initiated by the learners and teacher.

6.2.2.1 Learner- initiated strategies

It was found important that learners should have intention to learn and engage in active participation towards achieving learning objectives. From the study it was gathered that learners were able to learn through the following practices:

i) Participating fully in classroom activities. This involved trying, testing and exploring their ideas, practicing learned skills, asking questions when they experienced some challenges, and engaging in peer and whole–class discussions, sharing what they knew with others. (*See Section 4.2.2.2 and 5.3.2)*

ii) Developing positive attitudes and behavior. Learners indicated that attending lessons regularly and cooperation with other members of the group helped them to learn successfully. (*See Section 4.2.2.2).*

6.2.2.2 Teacher-initiated strategies

Instrumental orchestration is the process in which the teacher steered learners' progress through instrumental genesis, in order to enable them to acquire the targeted algebraic concepts. This consists of all the processes that the teachers engaged in, from planning for the teaching to implementation of the plan, including all ad hoc actions that s/he carried out in the classroom while attending to learners' needs. Effective teaching and learning was dependent among other factors, on didactical configurations, exploration modes and didactical performances.

i) Didactical configurations: These refer to the arrangement or organization of the learning environment that the teacher made in preparation for effective teaching and learning. This included organization of the computers and /or learners into small groups while at the same time arranging that each individual should have access to the technology. This was helpful to the learners in that they had opportunity to share their challenges and achievements, while at the same time were able to have hands-on experience with the artifact. *(See section 4.2.2.1).*

ii) Exploitation modes: This refers to the plan that the teacher had in order to develop conceptual understandings in the learners. This included the following:

a) Organization of the content. the nature and sequencing of tasks through worksheets,
b) Fostering understanding. Use of examples and teaching-learning aids helped to lead learners into discovering new knowledge moving from known to unknown. Use of realistic problems helped to capture learners' interest in learning of algebra. Through interviews and responses to the questionnaire, learners confirmed that use of contextualized problems made algebra learning interesting to them.
c) The amount of time that was allocated for the activities was important to consider in planning, as learners needed opportunity to engage with the content and the spreadsheets. *(see sections 4.2.2.1 and 5.2.2.1).*

Didactical performances include all unplanned actions that the teacher undertook in the classroom in attending to learners' needs related to acquisition of targeted concepts. The main form of intervention that the teacher provided was through use of different types of discourses. Examples comprise use of discourse for both cognitive and enrollment functions. Cognitive discourses included procedural, conceptual and contextual discourses such as illustrations with examples, technical-demos, explain-the-screen, discuss-the-screen (questioning, probing), link-screen-board, enhanced effective teaching and learning within the spreadsheets environment. Contextual discourses are those that involved explaining tasks to learners without actually providing hints to the solution. Code switching was also found to be very helpful in promoting discourse in the classrooms. Enrolment functions involved directing learners' attention and focus to classroom activities. *(See Section 4.2.2.1)*

The quality of TPCK the teacher possessed greatly determined the quality of preparation made and nature of interventions provided during the lessons conducted in the spreadsheets-assisted teaching/learning environment. Knowledge of the affordances and potentialities of spreadsheets, as well as the constraints that the use of the artifact may impose, helped the teacher to make appropriate decisions and act accordingly to assist learners in attaining the classroom objectives. Cooperation between the teacher and the learners was very necessary for enhancing effective teaching and learning. The teacher needed to have interests of learners at heart and use all resources and strategies at her disposal to help them learn effectively.

6.2.3 Perceptions on the use of spreadsheets for teaching and learning of Grade 9 Algebra

This section discusses the overall impressions that learners had with regard to the teaching and learning of algebra through spreadsheets. Learners' impressions can be classified into benefits that learners derived from learning algebra through spreadsheets and their general opinions regarding their usual algebra learning experience through pencil-and-paper compared to the spreadsheets–enhanced instruction.

6.2.3.1 Benefits derived from use of spreadsheets for the teaching and learning of algebra

The benefits that learners derived are categorized into technical and personal.

i) Technical benefits: Learners indicated that they had acquired technical knowledge and skills from algebra instruction based in spreadsheets environment. They were able to learn how to formulate algebraic expressions and equations for mathematical relationships situated in real life contexts. The dragging power of spreadsheets helped learners to develop the concept of variable and thus they were able to make generalizations through use of affordances provided by spreadsheets, learners were able to appreciate the concept of equivalence of expressions and solve linear equations. They were able to create graphs, explore and make generalizations regarding properties such as gradient, parallelism and perpendicularity for straight lines. *(See sections 4.2.3.1 and 5.2.2.3).*

Beside the algebraic content that they learned, learners were also happy about the computer skills they had acquired during the project. They indicated that they had gained more knowledge regarding use of computers and of spreadsheets in particular. For example, they mentioned that they were able to use the dragging command properly, they had learned that they could use the backspace key to delete, that they could delete many items, all at once, through highlighting. They said that they did not have many of their computer skills before participation in the project. They were happy to be more knowledgeable than their colleagues, who were not in the project, about using a computer. (*See Section 4.2.3.1, 5.2.2.1 and5.3.2*).

ii) Personal benefits: Learners indicated that through learning algebra through spreadsheets, they had developed certain attributes and characteristics that contributed to their success in learning. They had learned that perseverance was necessary in problem-solving; that they should test and try out their ideas, improving their hypotheses as they carried on the solution processes until they reached more convincing solutions. They also learned group dynamics, that is, cooperation, collaboration, tolerance, patience, and listening to each other's point of view to achieve a common goal. (S*ee section 4.2.2, 2 4.2.3.1,and 5.3.2).*

Through learning algebra in a spreadsheets-enhanced environment, learners indicated that they had "developed passion" for algebra. They had developed positive attitude and confidence towards algebra learning and spreadsheets. They mentioned that they liked spreadsheets for performing long calculations and when creating graphs, that these were done very accurately and quickly. According to the learners, the experience was full of surprises, for example they did not know that they could learn mathematics through spreadsheets and that they could also create a calendar through spreadsheets. Learners indicate that they enjoyed learning algebra through spreadsheets. (*See Section 4.2.3.1 and 5.3.2).*

6.2.3.2 Spreadsheets-enhanced algebra instruction compared with pencil-and-paper instruction

In this study, use of spreadsheets was made to complement the usual pencil-and-paper Grade 9 algebra instruction. Data from both quantitative and qualitative methods indicated that all learners were positive about use of spreadsheets for the teaching of algebra, but that it should be done together with pencil-and paper instruction. Some learners pointed out that they still needed the algebraic manipulation skills. They also pointed out that as some of them may not be confident with the use of computers, particularly spreadsheets, they would need to be given opportunity to learn in the mode in which they were comfortable *(See Sections 4.2.3.2,5.2.2.1 and 5.3.2).*

6.3 Conclusion

The main purpose of this study was to investigate the experiences that learners had when they were taught algebra through *Excel* spreadsheets. The research was conducted following the mixed-methods approach in a multiple-case study. The mixed methods complemented each other, findings from data collected through quantitative methods strengthened those from the qualitative data. The multiple cases used in the study provided opportunity to identify similarities in the views expressed by learners with regard to their experiences in learning algebra through spreadsheets. This, therefore, provided possibility for generalizability of results obtained from this study, even though the main purpose of conducting the case study was not to generalize but gain a deeper understanding of the phenomenon under study. The limited number of participants in the quantitative phase however, made it difficult for making any generalizations from the study.

Learners' experiences were investigated in terms of challenges, benefits and the general impressions that learners had developed regarding use of spreadsheets for teaching and learning of algebra. The challenges that were encountered in the teaching and learning of algebra through spreadsheets, and ways in which those challenges were managed, were also identified.

The study also identified the affordances and enablement provided, and the constraints associated with the use of spreadsheets for the teaching of grade 9 algebra. It was confirmed that from the way the *Excel* spreadsheets are designed, they have the potential for enhancing effective teaching and learning of algebra. While use of spreadsheets for algebra holds great potential for bringing about success, the challenges associated with the practice need careful consideration, if the approach were to be adopted. Despite the challenges that are already associated with algebra learning, use of the spreadsheets also carries with it a compilation of challenges which further complicate learning of algebra for those learners who have poor mathematical background. Effective use of spreadsheets relies on ability of users to formulate the involved mathematical relationships using the symbolic spreadsheets notation, and from the study, this is the area where learners were challenged.

Working on the spreadsheets itself also provided further challenge to some learners; the dynamic nature of spreadsheets created difficulties to learners, when they had to observe the impact of varying parameters in mathematical relationships. This property of spreadsheets permitted the variations to occur simultaneously and so quickly that learners were not able to follow what was happening.

Creating graphs with spreadsheets involves a series of steps which according to learners were too many to remember, learners were challenged with locating the relevant commands and in some cases the order of function for those commands created difficulties. With the current situation, of limited access to computers by learners, teaching algebra through spreadsheets would put heavy demands on the mathematics teachers; their focus would not only be on facilitating acquisition of algebraic concepts, but of the associated computer and spreadsheets skills as well.

Some of the challenges that were encountered were beyond classroom level; electricity failures due to use of faulty appliances within school premises led to the power cuts that were experienced. This situation created stress not only to the teacher, but the learners as well, as some of them lost their work and had to go over the tasks, after the electricity problem was attended to.

Despite the challenges that were encountered, learners also benefitted from the experience. They gained both conceptual and mechanical knowledge and skills associated with learning algebra within spreadsheets. Learners developed interpersonal skills through the interactions that they had in the classroom. They appreciated the role of cooperation and perseverance in learning. They realized the importance of attending classes regularly and participating fully in lesson activities.

The study established that the role played by the teacher, the learner and the nature of the context within which teaching takes place, were factors that shaped learners' experiences when they were taught algebra through *Excel* spreadsheets in Lesotho. It is only when the mentioned factors are in favorable conditions that effective teaching and algebra through spreadsheets may be realized.

The teacher's role. Through the study, it was found that the role of the teacher in the classroom where algebra is learned through spreadsheets is of vital importance, in shaping the experiences learners may have. Instrumental orchestration is the process through which the teacher steers learners' progression as they grapple with the algebra in the spreadsheets environment. The type of orchestrations that the teacher would deploy in his/her teaching is determined by the quality of his/her TPCK. When the teacher is aware of the opportunities, affordances and enablement provided by spreadsheets, as well as possibilities in terms of constraints that may be imposed by the artifact, s/he would be in a better position to plan and conduct the lessons in a manner that the constraints are managed to enhance effective teaching and learning. The teacher's mathematical content knowledge within the subject domain, together with the associated challenges in its learning, are very important in deciding on how the artifact should be used in order to enhance learning of the targeted concepts. The teacher's competence in pedagogical knowledge related to mathematics and algebra, in particular, is of vital importance in planning and executing his orchestration strategies within the spreadsheets environment.

Didactical configurations, exploitation mode and didactical performances are the key areas that contribute to the success of the instrumental orchestration process. These involve organization of the physical space, the computers and related equipment; learners (individual or group work) and subject matter (how concepts are to be developed, the nature of tasks). The teachers' ability and flexibility in responding to learners' needs, during the teaching-learning process, is very important. Several strategies that the teacher may deploy, to enhance effective teaching and learning of Grade 9 algebra within spreadsheets environment, have been identified. Discourse, in its different forms (conceptual, contextual, procedural and regulatory), is the main strategy through which the teacher may orchestrate learning of algebra through spreadsheets under the current laboratory conditions in Lesotho. The teacher also needs to be aware of the dynamics of providing instruction to second-language learners; code switching plays a vital role in such cases. In a spreadsheets-enhanced algebra classroom, the teacher's roles were increased to include that of a computer technician. It was also established that pencil-and-paper techniques still had a significant role and thus should be used together with spreadsheets as complementary techniques in the teaching and learning of algebra, as was the case in the study.

The learner's role. The study also revealed the necessary traits that learners should possess in order to learn algebra effectively in a spreadsheets environment. *Instrumental genesis* is the process through which learners develop facility with the artifact, spreadsheets in this case, as they interact with it to achieve algebra learning objectives. Learners' attitudes and behaviors towards learning through the technology greatly influence their progression in instrumental genesis. Learners who have a positive attitude towards the artifact and mathematics would show perseverance and would strive towards overcoming their difficulties; while those with negative attitudes would persevere less and resort to withdrawal and disengagement behaviors. While it was the responsibility of the teacher to encourage and motivate learners into learning, learners ought to have intention to learn and take active participation in the classroom activities directed towards their learning.

The context. It was also found that even though computer laboratories were there in the schools, the available infrastructure did not allow for smooth running of the lessons. Computer laboratories need to be equipped with adequate well-functioning computers and have adequate space to allow free movement to enable the teacher to monitor learning and attend to individual learners' needs. There has to be proper maintenance of the equipment, including electrical equipment outside the computer laboratory, to and constant supply of electricity to facilitate smooth running of the lessons. Learners should have more access to the computers, this would provide them opportunity to interact with the technology and hence accelerate their instrumental genesis.

Use of technology in the teaching and learning of mathematics is increasingly becoming universally recommended, but since use of dynamic software is a new thing in Lesotho, it was important that their use be investigated. Identifying the challenges and benefits that Grade 9 learners in Lesotho experienced when they were taught algebra through spreadsheets is very important to teachers as the knowledge would help them make informed decision concerning use of the technology. Under the prevailing conditions, particularly regarding scarcity of facilities, and perhaps manpower, in the form of laboratory technicians, to support use of computers for mathematics teaching and learning, it is also important that teachers are aware of the demands that the practice would put on them. Both teachers and learners can benefit from the use of spreadsheets for the teaching and learning of Grade 9 algebra. These can only be realized if the challenges associated with the use of the tool are managed.

6.4 Recommendations

In view of the issues discussed in this study, there are recommendations I would like to make. Use of spreadsheets for the teaching and learning of algebra carries with it both challenges and benefits. Some of the challenges can be overcome depending on the teachers knowledge and facility with the artifact, and being able to identify those topics for which the artifact can be used. This means that spreadsheets may be beneficial to both the teacher and the learner if appropriately used. In order for this to happen it is recommended that

1. Education authorities should support teachers by

   a) providing opportunities for training in order to equip them with necessary knowledge and skills (TPCK) related to use of spreadsheets for the teaching of mathematics and of algebra in particular. Lack of knowledge usually leads to rejection because teachers would not want to risk failure in front of their learners. It is also important to note that this idea of technology integration, in the form of computers, in the schools particularly in Lesotho, is still at its infant stage, so some teachers, may not be willing to move away from their comfort zones and engage in some new practices that require them to undergo training. Motivation of why use spreadsheets when their usual strategies still "work" has to be considered.

   b) providing suitable infrastructure in the form of laboratories, with enough space and well-functioning computers and accompanying technologies such as projectors for the smooth running of the lessons;

   c) Reviewing the present curriculum assessment procedures so as they would provide opportunity for use of the technology.

   d) Reviewing the language policy in education. According to the Lesotho Education Policy, all children are taught all subjects, except English, in their mother tongue, Sesotho, from their first day up to end of their Grade 3 in formal school. This late start in learning English contributes to their lack of fluency in the language of teaching and learning (English), even at Grade 9, hence hindering their learning of algebra and mathematics as a whole.

2. As learners are individuals, even though they learn as a community, use of technology should be done along with the usual pencil-and paper instruction, so that they would be able to use techniques of their choice in solving problems. It is also important to note that, not all topics are suited for the use of spreadsheets in their teaching and learning, so there is still room for pencil-and-paper instruction. Even in those topics where spreadsheets may be used, the two techniques should be used to complement one another.

3. Cooperation and collaboration is needed between teachers of Mathematics and those of Computer Studies, so that Mathematics teachers, in planning for teaching, may just tap on the skills that learners may have acquired from Computer Studies' lessons This may necessitate re-organization of the curricula.

6.5 Directions For Further Research

From the issues that came out of the study I will now present a number of areas that may be considered for future research in this section.

Effective teaching and learning of beginning algebra builds on algebraic thinking skills that learners should have acquired in their arithmetic lessons in the primary school. It is on the basis of this that I propose that a study be carried out to investigate the primary school mathematics teachers' perception of algebraic thinking, and what they do in their mathematics lessons to develop learners' algebraic thinking skills.

Even though learners indicated they had acquired technical knowledge and skills while they were taught through spreadsheets, and some even said that spreadsheets had helped them to understand those topics they had already done in their usual mathematics lessons, I suggest that a comparative experimental study be carried out in Lesotho and compare performance of learners in a common test in algebra after one group is taught through spreadsheets and another received the usual paper-and-pencil instruction.

I also propose that further research be carried out to investigate into teacher education programs and find out how much preparation there is, for student teachers, for integration of technology (computers) in their mathematics teaching. Use of computers should not only be restricted to the teacher demonstrating certain concepts, but should include manipulation of the technology by learners to attain mathematical learning goals.

Studies may also be conducted to investigate the perceptions of stake holders in education (education authorities, school administrators, parents and learners) with regard to introduction of computers in the classrooms. This issue has implications on the availability of support and use of computers for teaching and learning of mathematics in the schools.

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APPENDICES

APPENDIX A: Sample Worksheet

ALGEBRA **WORKSHEET 6**

NAME______________________________ **SCHOOL____________________________**

Equivalence: Solving Equations

Objectives

By the end of the lesson, you should be able to:

- Formulate expressions for situations arising in real life situations;
- Use Excel spreadsheets to construct a table of values for formulated expressions;
- Draw graphs of the linear functions;
- Solve problems using tabular and/or graphical representations.

Instructions

a) Answer all questions.

b) Write your responses in the spaces provided.

c) Submit your worksheet and spreadsheets books at the end of the session.

---

1. Two brothers, Lele and Thato are to contest a 100m race. The younger brother, Thato is allowed a 20m head start and runs at 5m/min. Lele runs at 15m/min.

   Let x = time in minutes

   y = distance covered in metres

a) Construct a table of values to represent the distances covered by the two brothers between 0 – 7 minutes.

b) Draw a graph to represent this information and determine :

   i) the distance and time when Lele will overtake Thato;

   ____

   ____

   ii) The time Lele will take to finish the race.

   ____

   ____

2. A video shop offers two plans for renting videos. In Plan A, you pay a subscription fee of M66.00 per year and pay M5.00 for every video hired. In plan B, there is no subscription fee, but only pay M8.00 for every video hired.

   Let x = the number of video rentals.

   e) Make a table of values for the two plans

   f) Formulate an expression for the amount paid for video rentals according to each plan.

Plan A: ____

Plan B____

   g) Draw graphs showing the cost for the rentals according to each plan.

   h) For how many video rentals will: i) payment in both plans equal? _______________

   ii) Plan A be cheaper?_______________________

3. A water and sewage company has a standing charge of M100.00 and charges M3.00 per unit of water. The charges are on monthly basis.

   a) How much would be paid for using 165 units of water?_________________________

   b) Determine the number of units used if the bill is M250.00.________________________

   c) If Mr. Tumor's family went away from their home on holiday for one month, how much would they have to pay for that month? Explain.

   ____

   ____

What have you learned and understood in this session?

____

____

____

4. What do you think helped you to learn successfully?
5. What challenges did you encounter in working through this worksheet?

____

____

____

____

6. Why do you think were the causes of your problems?

____

What do you think could have been done to help you?

THANK YOU!!!!

APPENDIX B: Observation Schedule

CLASSROOM OBSERVATION SCHEDULE

Date:________________________________ **Group:________________**

Topic: ________________________________

INSTRUCTIONS

1. The schedule consists of a list of open–ended questions that relate to learners' interaction with the subject matter, the teacher and the technology, spreadsheets.
2. Please read the questions before the lessons as this would guide your observations.
3. Please answer the questions as honestly as possible regarding your observations.
4. You may write down your responses for each of the lessons observed and only hand in the form upon completion of each worksheet.

LESSON OBSERVATIONS

1. Which parts of this lesson did you like? Why?

____

____

2. Do you think spreadsheets helped learners to understand the algebra better? Why?

____

____

____

3. a) Which parts of the lesson seemed difficult for the learners? Explain.

____

____

____

b) What initiatives were taken by learners to address their own difficulties? Explain

____

____

____

____

4. What forms of intervention were provided by the teacher ?

____

____

5. What do you suggest could have been done to improve this lesson?

____

____

____

6. Would you encourage teachers to use spreadsheets in teaching this particular topic ? Why?

a) ____

____

____

____

b) Full names of observer:_____________________

c)

d) Signature of observer: _______________________

THANK YOU VERY MUCH!!!

APPENDIX C: Learners Semi-Structured Interview Schedule

Thank you very much for having agreed to be part of this research, which is aimed at investigating how teaching and learning of Grade 9 algebra through use of spreadsheets can be done successfully in Lesotho.

1. You have been involved in this project , what is your opinion about the learning experience you have had?
2. Which part or parts of your learning experience did you like? Why?
3. Which part or parts of your learning experience did you not like? Explain?
4. What do you think could have been done differently?
5. Some of the topics you were taught were not new to you. Has your participation in this project benefitted you in any way?
6. To what extent have you understood the topics that were covered in this project?
7. Which topic or topics did you enjoy most? Why?
8. Which topics still remain a challenge to you? Explain?
9. If a similar project is organized would you encourage other Grade 9 learners to participate in it? Explain.
10. Thank you very much for your time and your cooperation.

APPENDIX D: Assistant Observer's Semi-Structured Interview Schedule

Q1. What is your opinion about the use of spreadsheets for the teaching and learning of mathematics in Lesotho?

Q2. Do you think learners were coping with the skills involved?

Q3. What challenges did learners experience during this learning period? (In all the worksheets)

Q4 How did the learners overcome their challenges?

Q5. Was the teacher aware of the learners' difficulties? Explain.

Q6. Did the teacher address the problems of the learners?

Q7. What strategies did the teacher use to address learners' problems?

Q8. Were the teacher's intervention strategies helpful? Explain.

Q9. In your opinion, what else could have been done to address the problems?

Q10. Considering the nature of tasks that were designed for the learners, would you say they were appropriate or not? In what ways?

Q11. What alternative suggestions do you have concerning the design of the tasks?

Q12. From your observation, how was the learners' attitude towards the whole exercise?

Q13. Which behavioral patterns displayed by learners reflected their attitude?

Q14. What do you think could be done to promote the positive attitude towards learning of algebra in a spreadsheets environment?

Q16. What do you think could be done to combat the undesirable behavior?

Q17. If you were to tell other mathematics educators and teachers about teaching of algebra through spreadsheets, which positive aspects would you mention?And which negative aspects would you mention?

Q18. If teachers were to use spreadsheets to teach algebra, what would you suggest should be done in order to help them to do so effectively?

Thank you very much for your cooperation.

APPENDIX E: Learners' Questionnaire

INSTRUCTIONS.

1. Answer all questions.
2. All answers will be treated with strictest confidence.
3. When presented with a choice, circle the number next to the alternative chosen.

SECTION A

PERSONAL AND GENERAL INFORMATION

1. Sex

Male	1
Female	2

2. Age in years

14	1
15	2
16	3
Above 16	4

3 Grade obtained in Mathematics in the PSLE

1	**1**
2	**2**
3	**3**
F	**4**

SECTION B

INSTRUCTIONS

1. Please answer all questions as honestly and carefully as you can.
2. Circle 1,2,3,4,or 5 to indicate how strongly you disagree or agree with each statement.

Ratings are;

1 – strongly disagree; 2 – disagree;3 – not sure; 4 – agree; 5 – strongly agree.

STATEMENTS	RATINGS				
1. Spreadsheets helped me to recognize patterns in sequences easily	1	2	3	4	5
2. I enjoyed learning algebra using spreadsheets	1	2	3	4	5
3. Spreadsheets helped me to understand algebra	1	2	3	4	5
4. I can develop formulas for use in the spreadsheets successfully	1	2	3	4	5
5. I actively participate in the algebra lessons	1	2	3	4	5
6. I can solve algebra problems more quickly using spreadsheets	1	2	3	4	5
7. I like working alone than in a group	1	2	3	4	5
8. I enjoy creating graphs in spreadsheets	1	2	3	4	5
9. The activities in spreadsheets helped me to “make sense” of algebra	1	2	3	4	5
10. I like drawing graphs with pencil-and—paper than with spreadsheets	1	2	3	4	5
11. Doing computations using spreadsheets helped me to solve mathematical problems successfully	1	2	3	4	5
12. I was happy to be allowed to work, talk and discuss ideas with my peers in the algebra classroom	1	2	3	4	5
13. I feel more confident in doing mathematics after using spreadsheets	1	2	3	4	5
14. I can always choose a suitable scale for the graph I draw	1	2	3	4	5
15. I prefer to perform calculations using pencil-and-paper than in excel spreadsheets	1	2	3	4	5
16. I can translate expressions in ordinary algebra language to spreadsheets notation	1	2	3	4	5
17. I learn algebra better through using pencil-and-paper than spreadsheets	1	2	3	4	5
18. When different graphs are drawn in spreadsheets, they can be easily compared	1	2	3	4	5
19. Knowledge of algebra to solve problems in other subjects	1	2	3	4	5

THANK YOU !

APPENDIX F: Cronbach's Alpha Test Results

Item	Items included					
	Round 1	Round 2	Round 3	Round 4	Round 5	Round 6
	(24 items)	(23 items)	(22 items)	(21 items)	(20 items)	(19 items)
Spreadsheets helped me to recognize patterns in sequences easily	√	√	√	√	√	√
I enjoyed learning algebra using spreadsheets	√	√	√	√	√	√
Spreadsheets helped me to understand algebra	√	√	√	√	√	√
I can develop formulas for use in the spreadsheets successfully	√	√	√	√	√	√
I can use the "fill down" command correctly	√	√	√			
I actively participate in the algebra lessons	√	√	√	√	√	√
I can solve algebra problems more quickly using spreadsheets	√	√	√	√	√	√
I like working alone than in a group (ITEM REVERSE-SCORED)	√	√	√	√	√	√
I enjoy creating graphs in spreadsheets	√	√	√	√	√	√
The activities in spreadsheets helped me to "make sense" of algebra	√	√	√	√	√	√
I like drawing graphs with pencil-and-paper than with spreadsheets (ITEM REVERSE-SCORED)	√	√	√	√	√	√
I learn better when I work and discuss problems with my peers	√	√				
Doing computations using spreadsheets helped me to solve mathematical problems successfully	√	√	√	√	√	√
I was happy to be allowed to work, talk and discuss ideas with my peers in the algebra classroom	√	√	√	√	√	√
I feel more confident in doing mathematics after using spreadsheets	√	√	√	√	√	√
I can always choose a suitable scale for the graph I draw	√	√	√	√	√	√
I was able to interpret the results I got on the Excel spreadsheets	√	√	√	√		
I prefer to perform calculations using pencil-and-paper than in excel spreadsheets (ITEM REVERSE-SCORED)	√	√	√	√	√	√
Solving real life problems in spreadsheets makes algebra learning interesting	√					
I can translate expressions in ordinary algebra language to spreadsheets notation	√	√	√	√	√	
I learn algebra better through using pencil-and-paper than through spreadsheets (ITEM REVERSE-SCORED)	√	√	√	√	√	√
I can translate expressions in spreadsheets notation to ordinary algebra language	√	√	√	√	√	√
When different graphs are drawn in spreadsheets, they can be easily compared	√	√	√	√	√	√
Knowledge of algebra can help me to solve problems in other subjects	√	√	√	√	√	√
Value of Cronbach's Alpha Coefficient	**0.74**	**0.76**	**0.77**	**0.78**	**0.79**	**0.80**
An Alpha coefficient of 0.70 is normally considered an acceptable cut-off for reliability. All items were included in the final version of the questionnaire						

APPENDIX G :Assistant Observer's Interview

I: Thank you very much 'M'e for having agreed to be part of this study and also having agreed to participate in this interview.

R: You are welcome 'M'e.

I: You saw the learners as they progressed through the worksheets, working through the different tasks contained in those worksheets. Do you think the learners were coping with the skills involved?

R: Ok. At the beginning they lacked the necessary computer skills but as time went on , as they worked through the tasks they gained a lot.

I: Can you give examples of those skills that learners did not have?

R: Some of them included not being able to generalize patterns, to identify the pattern in some cases, generating a formula from that. It was difficult for them to use the spreadsheets notation, if ever there was a formula to write about a pattern, they were not able to do that.

But it did not take a long time to recuperate, in no time they were able to do such things. There was also a problem with a "dragging command" in filling the rows and columns.

I: May I follow up on that, you said there was a problem with generalizing and using the spreadsheets notation. What do you think could have been a source of that lack of knowledge?

R: May be that was due to their lack of necessary computer skills and poor arithmetic skills. I think if they do not have knowledge about number patterns and number behaviors then it will be difficult for them to identify the relationships in the sequences.

I: And you said with time they were able to …

R: Yes once they acquired the necessary skills they were able to identify patterns and write their generalization in spreadsheets notation.

I: Now do you think there is some contribution that spreadsheets made or a role that spreadsheets played towards helping them in this regard?

R: Very much so , very much so. They gained a lot of skills through this project which they can even use in the future. It was very helpful to them. And meaningful too.

I: Ok. In as far as this generalization is concerned and now using spreadsheets notation and then coming up with the usual algebraic formula for the expressions, when they were taught in the normal pencil-and-paper how would you compare the manner in which they understood those generalizations and formulations? Do you think spreadsheets was somehow helpful in a way?

R: Very much so. Not only were they gaining computer skills they were also understanding algebra as well. Their emphasis on understanding and making sense of the whole thing. You see when these kids were using the spreadsheets notation, the cell references were already in place for the variable. So they were able to move from the spreadsheets notation to writing the formula using x and y, interpreting what the formula is saying, so there was that interaction and it really helped.

I: Ok 'Me'. We were still on the skills that learners acquired.

R: skills acquired. Should I go through all the worksheets?

I: No no. I had just asked if the learners were coping with the skills involved in the whole project. And some of your answers, in fact your answer was yes, that at first they had some problems but were able to cope with time.

R: Yes they were able to cope with time.

I: What challenges did the learners experience in the whole learning period while they were working through the worksheets. What kind of challenges did you actually see?

R: I have already mentioned some.

I: ok.

R: one of them is generalizing, put whatever they have in spreadsheets form. That was the first one. The second formulating a formula out of a pattern and so shifting from formulating a formula can then be able to….three, I mentioned again using spreadsheets notation to write the formula itself. Four, the use of the dragging command and also five, understanding the meaning of the entries in the cells, for example if there was a 3 in a cell, then they did not understand the meaning of the entries, this project helped them to understand these things.

OK, in some worksheets there was this exercise where they had to find relationship between two variables x and y, expressing the relationship between the numbers using letters, using letters for numbers. That was a challenge also but like I said spreadsheets helped With the generation of more values they were finally able to ..

I: ok. So these challenges as you mentioned…

R: Sorry. I did not mention the drawing of graphs.

I: They also had that as a challenge?

R: They also had a problem with drawing graphs at first, but with time they managed. They actually excelled.

I: Was the teacher aware of the learners' problems?

R: Yes the teacher was aware of that.

I: Can you explain a bit on that?

R: That is why in most cases she intervened, tried to discuss. There was this whole class discussions, between the teacher and the whole group. The teacher also encouraged peer discussions among members of a group. There was also use of the chalk board to elaborate and clarify some concepts. So that is how the teacher intervened.

I: so what strategies did the teacher use to address the learners' problems? You have already mentioned some.

R: Yes I mentioned use of the chalk board, peer group discussions and the whole class discussions.

I: ok were these strategies helpful?

R: very much so. They helped some of them to , I mean we all learn from others. If one understands something better and you allow him or her to share the idea with others , then these others can grab the idea and perform better.

I: ok. About this strategies, in your opinion what else do you think the teacher could have done to address the problems the learners had? Is there anything else that you could have suggested to the teacher may be?

R: Yes if time allowed. The teacher knew some of the important spreadsheets skills that were needed in this programme. So if time allowed the teacher could have checked on the learners background knowledge and then taught those computer and algebraic skills that learners lacked.

I: OK. Now how did the teacher address this issue?

R: She somehow taught the skills along, they learned them as they required them and it was fine. It is not like the learners did not know all the required skills, some of the skills were there but since they were not used regularly, so she just tapped on the skills and *(clapped*) they surfaced.

I: Hmm. So this means learners were not immediately able to apply the computer skills they learned from their computer lessons into algebra?

R: yes, yes.

I: Now considering the nature of tasks that were designed for the learners, would you say they were appropriate or not and in what ways? In other words would you say they were helpful in helping learners acquire the targeted learning outcomes?

R: Yes the tasks were valuable. They were at their level , because here we were talking about generating sequences, they were helpful because they covered the syllabus, the content of grade 9 level such as drawing graphs, understanding algebra, solving equations.

I: so it was appropriate?

R: They were also about things learners know and meet, there was enough background upon which the activities were based and as such they were meaningful.

I: From your observation how was the learners' attitude towards the whole exercise?

R: They were very very excited about the whole thing. It was hands-on and that made them to be happy ; and promoted positive attitude; it was challenging and normally it is exciting for learners to get challenged. It was exposing them to the unknown making them realize what they were knew but were not aware of, so such things help in exciting them. that was their attitude.

I: what do you think could be done to promote a positive attitude in the learning of algebra in a spreadsheets environment?

R: Sorry what's the question again?

I: you said the whole exercise was exciting to learners, but our observation is that the number of learners in the groups dropped as time went on. There were those that ran away from the programme. I want to believe some of them had some problems. so my question is what could be done to promote a positive attitude towards learning algebra in a spreadsheets environment? so that we would end up having none of the learners running away from programmes such as this?

R: Ok what I will attempt to respond to is what could have been the source of learners running away from the project.

I: Ok.

R: There are many reasons one may think of. One could be that they did not consider it as one of their normal classes. So if it was accommodated in the school time table, they may not have had any problems. as it were it was like depriving them of their leisure time.

I: so you are thinking that the very fact that this was kind of too liberal, there was too much liberty for the students that they could stay or leave whenever they felt like was somehow a source of a problem?

R: yes, and it's not that they had a negative attitude. They were interested but it was taking their leisure time. If it was done during their normal class-time they may have not run away or left.

I: What do you think could be done to combat the undesirable behavior that was going to be my next question, but you have already come up with what could have solved the problem.

R: The other thing could be people are different, some do not like to show when they do not know, so some of them had not acquired some of the spreadsheets skills and so they were not ready to show that they did not know so they decided to run away.

I: may be as you said it this required learners to lay hands-on and such …

R: Such learners would be exposed and they were not ready for that so they ran away.

I : So if you were to tell other teachers and mathematics educators about the use of spreadsheets to teach algebra, which positive aspects would you mention and which negative ones would you also mention?

R: can you repeat the question please?

I: If you were to tell other Mathematics teachers and mathematics educators about the use of spreadsheets in the teaching of algebra, which positive aspects would you mention and which negative ones would you also mention?

R: use of spreadsheets. Wow! Really what we are going to talk about is what I have already mentioned. -- It helps them to make sense of algebra,

- to make sense of letters for numbers;
- It helps them to represent information in different ways, tabular, equation and graphically and helps them to make sense of those representations;
- And in the end you are going to translate the information and use it into your everyday life, actually we are solving problems every day and also
- It helps them to compare functions
- And where there are variables , say x and y, and those are related, you can represent that nicely on the graph and interpret what it actually says.

I: Now are there any negative aspects you can mention about this spreadsheets usage?

R: Aaachee! I can even add more of the positive. Spreadsheets help learners to understand both algebra and the computer.

Negative aspect? *Pause.* I do not have anything to say. Ok. I think I can think of one. At one stage children may go through a worksheet for fun! I don't know if this is really negative. They go through the worksheet without really relating what they are doing to learning. Or what do you think?

I: I don't think I understand whether you are coming to that situation where one learner in the middle of the programme asked "madam , are we doing maths or computer, are teaching us mathematics or computer?

R: But then in that case I would say that brings another positive aspect of teaching algebra with spreadsheets.

I: Why?

R: It is nice to learn in that environment. To have children learn without them being aware and this would even change their negative attitude towards mathematics. It would draw learners to looking forward to the next lesson. I really find it difficult to get any negative aspects.

I: what keeping learners attention?

R: like in any other class, learners need to be monitored as they work through tasks. So I would not say spreadsheets would be bringing anything new.

I: Ok. Now like you said it is difficult to find negative aspects, there is this group of people who we can say they fear technology, they are scare even to touch the computer. Would you say algebra should just be taught with spreadsheets alone or spreadsheets should be used together with pencil and paper?

R: I would say they should be done together hand-in-hand. At some point learners would still need those manipulation skills, those pencil-and-paper skills. It would be much better to have the two together.

I: If teachers were to teach algebra through spreadsheets what would you suggest should be done in order for them to teach effectively?

R: Ok one challenging thing for them to be successful in their classes is organization of their classroom, they should know how to organize like pupils and their computers. Each learner should have own workstation such that each has a feel of the tasks being done, they should have interaction with the computer. That interaction is important. So they should be able to organize their classes that way.

I: so mean each learner should have….

R: Yes, yes. If possible the ratio of computer to learner must be 1:1.

I: Ok what about group discussions?

R: Oh no! those should always be there. Each student will always have a neighbor and should be encouraged that should they encounter any difficulties first talk to your neighbor. Because that is very helpful.

Again there should always be collaboration between the maths and computer teachers. So what the other is doing this side must be communicated to the other. That collaboration can help a lot, a lot of content can be covered in a short period of time . also we may not confuse the kids as we would be knowing how much computer skills they have been taught and what can we tap from which may be used in maths.

So we would also encourage the teachers to attend relevant workshops where they will be equipped with enough skills to help the learners go through the process. Which is being done nowadays as they are encouraging teachers to use ICT for teaching their relevant subjects.

I: So more workshops should be organized for teachers?

R: Yes more workshops should be organized to equip teachers with the necessary ICT skills.

I: ok. Is there anything else you may want to add?

R: Pause. There is nothing much to say really, but from me, for those learners out there who managed to go through this exercise, it was very meaningful and helpful, but I would also recommend that this be extended not only to algebra but to all mathematics topics. I have also gained a lot as a mathematics teacher so I recommend that this kind of exercise to all maths topics where spreadsheets could be applied, like that statistics and angles in the other worksheets.

I: ok. Thank you very much ‘m’e for your time and cooperation and of course your patience throughout the entire project. Your contribution to the success of this study is invaluable.

R: welcome. Oh what a lengthy interview!

THE END

APPENDIX H: R1 Interview Transcriptions

I: Thank you much R1 for having agreed to take part in this research which is aimed at investigating how teaching and learning of grade can be done successfully in Lesotho. Now, you have been involved in this project, what your opinion about the learning experience you have had?

R: I would like to advice all teachers to use spreadsheets when teaching algebra because it makes it easier for students to understand better than when we were using just pen-and-pencil.

I: Does it mean you had been taught some algebra first**?**

R: Yes madam.

I: So working in spreadsheets helped you to understand those things better? Is that what you are saying?

R: Yes madam.

I: Ok. I guess that it was not all so easy, all so enjoyable, when you were working through this project. There were some parts that you liked and there are those that you did not like. So which part or parts of your learning experience did you like?

R: When we were dealing graphs madam.

I: You liked graphs?

R: Yes madam. When we were doing equations of the graphs, with the y-intercepts.

I: drawing graphs or when you were learning about the behavior of graphs, changing the gradients and the y-intercepts? What in particular?

R: when we were drawing graphs madam.

I: Ok. What about those parts you did you not like?

R: Pause.

I: can you try to remember as we were working through the project which ones were challenging to you or you did not like?

R: there were those that were challenging but I cannot say I did not like them because they are good when they are challenging so that you can determine whether you understand or not.

I : ok. So which ones were challenging to you?

R: when we were doing with the last worksheet, when we were required to change from ordinary language to mathematical language madam

I: You found that challenging?

R: Yes madam

I: Ok. Those questions as you mentioned in that worksheet, worksheet 6, required that you to be able to identify the relationships expressed in those problem statements in that ordinary language and write them in mathematical symbolism. I want to get this very clearly, were you able to understand the relationships as they were there in the ordinary language?

R: Some but not all of them madam

I: Ok. so the problem was writing those relationships now into mathematical symbolism

R: Yes madam.

I: if the table of values was already generated would you be able to draw the graph from that and answer questions based on the graph?

R: yes madam

I: Ok, now I get you. Now this exercise required that you have some certain basic computer skills, right from worksheet 1 up to worksheet 6. Did you have all the necessary skills?

R: Not all madam. Some I learned them as we were working through the worksheets.

I: Can you give me examples of such skills?

R: Madam, when we were to draw a graph, at first I was not able to draw it using spreadsheets.

I: What about writing spreadsheets formulas

R: I knew that madam before we started the project.

I: ok. I see. Were you able to do all the tasks in the worksheets without any difficulties?

R: Not in all cases. In some we had difficulties.

I: what kind of difficulties can you remember?

R: in some I did not understand what the question was saying.

I: You had difficulty with the language

R: yes madam

I: So what did you do under those circumstances?

R: When I did not understand the question I would ask my group members and when I also did not understand them I would ask the teacher and she would explain.

I: the teacher would give you answers?

R: Not answers but the explanations madam. she would explain more so that I understand better.

I: and then you would tackle the problem yourself?

R: Yes madam.

I: Is that what you were expecting from the teacher? Explanations or you expected the teacher should give you answers?

R: Not answers madam but explanations.

I: Ok. Now you often used the fill down or drag down command as we would sometimes say, when you were generating sequences or the table of values. Can you explain to me what you think the command was telling the computer to do?

R: It tells the computer to apply the formula you have entered in the first cell to all cells that you have dragged down to.

I: ok. So you understand the mathematics behind that dragging command.

R: Yes madam

I: Now, in other cases after identifying the relationship between the variables in your problem situation, like in that Mpho's restaurant problem, on the number of tables and numbers of chairs, you came up with a spreadsheets formula that you would use and the changed that to the normal algebraic formula in the form y =2x +1. Isn't it?

R: Yes madam.

I: You had such cases , even when you were to express the rate of growth of those plants in Lebo's garden, yes how did you find that shifting from spreadsheets notation to normal algebraic form? Was that problematic?

R:No madam**,** it wasn't a problem.

I: ok. So it wasn't a problem?

R: Yes madam.

I: if you had to draw the graph of y =2x +3 using spreadsheets, can you remember the spreadsheets formula you would enter, suppose this is cell…

R: A1

I: yes suppose it is that A1, what would be the formula you would enter in that cell which you are going to drag to get all other values in that column?.

R: It would be A1×2 +3 .

I: just like that?

R: yes madam?

I: Don't you think you have left something that is very important?

R: Oh. The equal sign. It would be = A1×2 +3.

I: OK. So you still remember. And what is this?

R: it's a star.

I: a star and what does it mean?

R: times.

I: Ok so you know that it stands for multiplication? I am happy to see that you still remember this things.

R: yes madam.

I: Ok. Now when you drew graphs using spreadsheets, was it easy or difficult for you?

R: It was easy because it helps you to draw the graph faster than when using pencil-and paper.

I: So you like it because it is faster? But was it easy?

R: Yes it was easy madam.

I: From the beginning?

R: Yes madam. But not from the beginning because at first I wasn't able to draw a graph using spreadsheets but after learning how to draw it was easy madam.

I: ok. Did you find anything fascinating about that? interesting about it? Helpful?

R: Yes madam.

I: What is it that you found very satisfying when drawing the graph with spreadsheets rather than when you draw with pen and paper?

R: It's because once you draw the graph with spreadsheets, when you have already got the table, you don't have to think of where to plot the points, when you have highlighted the table when you draw the graph it's going to draw the line for you.

I: ok so it becomes easy?

R: yes madam.

I: And faster?

R: Yes madam.

I: Were you able to format the axes?

R: Yes madam.

I:Even the gridlines were you able to insert them?

R: yes madam

I: Now you also used spreadsheets to learn about the behavior of graphs when the gradients or y-intercepts are changed. Do you remember?

R: Yes, madam.

I: Where we used the scroll bars to change the values from 0, 1,2 3,4 etc. to see how the position of the graph was changing with different y-intercepts?

R: Yes madam

I: What can you say about that exercise?

R: When the number of y-intercept is changed the line will cut the y-axis at that point which I have selected. And the gradient determines the slope of the line.

I: So did you like the exercise or did you find the exercise helpful?

R: I liked it madam.

I: Have you drawn graphs now in your mathematics class.

R: Yes madam

I: Now how do they compare, where was the spreadsheets more helpful? Did you like it when you had to draw every single graph with pencil and paper or using spreadsheets to draw the graphs?

R: I preferred it on spreadsheets madam

I: Lazy? *Laughs.*

R: *laughs*. No madam. It's not that I am lazy?

I: *laughs***.** But….?

R: It makes the work easier.

I: Ok. Did you have any difficulties during the lessons, during the entire project?

R: yes there were some problems?

I: How did you deal with your problems?

R: If I had a problem, I would ask my colleagues to help me and if they were not able to help me I would ask for help from the teacher.

I: And she would give you answers?

R: Not answers, but she would explain madam.

I: Oh. Now in your opinion, was the way the teacher was responding to your problems satisfactory to you?

R: Yes madam.

I: Did you like it?

R: Yes madam.

I: and were the hints she gave helpful?

R: yes

I: What in your opinion do you think your teacher should do to help you learn even better?

R: I think after doing a certain topic, the teacher should always ask if students have problems and then explain. That would help

I: Ok. So your teacher should have done that?

R: Yes madam.

I: Ok what can you say about the kind of activities you carried out in your worksheets?

R: pause

I: You used worksheets isn't it?

R: yes madam

I: that's not what you are used to in your normal class.

R: yes madam.

I: So what can you say about the manner in which the lessons were conducted, that is using worksheets? It was new to you. What is your opinion about this use of worksheets?

R: I think it is easier when you are using worksheets because it helps students to learn faster because with worksheets you are able to do one thing at a time.

I: But even in class when you don't use worksheets you still do one thing at a time?

R: Yes But here madam, when you are using worksheets you are more focused madam and you learn the topic more faster.

I: Now you were also taught or you were working in groups and each one of you was using their own computer. Did you like group work?

R: Yes because sometimes it happened that one of the members did not understand but when we discussed then the task it was easy for the members to understand.

I: Ok while still on groups, suppose you were given one computer for the whole group, which would you prefer one computer for the whole group or each one having their own computer.

R: Each one having a computer because even after explaining if you are not doing the task yourself it would not be easy to tackle the task even when you are alone. but if you are using your own computer when the group members are not around you will be able to tackle the task alone.

I: If they are there?

R: When they are there they would still be able to help you madam.

I: ok so it gives you opportunity to assess individual understanding?

R: yes madam.

I: Now what about the activities themselves. did you feel like you were learning maths or the computer?

R: mathematics madam.

I: so think it was very straight that you were learning mathematics?

R: Yes madam

I: the tasks themselves, you dealt with the calendar which is something that you know, you also dealt with the time stable, you also learned about growing seedlings, you also learned about brothers playing racing. You learned about the cell-phone bills, the water bills, those are the kind of problems that you dealt with. So there was maths based on real life problems. well there are other situations where you are given mathematics, dry and without any attachment to things you have in your environment. so what is your opinion with regard to the problems that you had in the activities? the real life problems. did you like them or did you find them to be complicating the mathematics or did you find them to be making maths to be answering real life problems?

R: I prefer the problems that are related to real life because they help us understand what we are taught madam.

I: understand what you are being taught because they are based on real life experiences?

R: Yes madam.

I: oh. What have you learned throughout this project. I am sure you learned something.

R: yes madam

I: So what is it that you find very valuable that you learned in this project?

R: I was able to determine the equation used when drawing the graphs and also to draw the graph myself, creating the equation.

I: ok. Now you have also said you would encourage that algebra be taught through spreadsheets.

R: yes madam.

I : And through pencil-and-paper, did you say?

R: yes it can also be taught through pen-and paper but it is easier when we are using spreadsheets madam.

I: so do you think that it should just be algebra on spreadsheets or with pencil-and-paper or with both?

R: I think both madam, because when you are using a computer and sometimes when you have not entered the correct values it will guide you with what to do but with pen-and–paper you will be able to do the values yourself.

I: you can still improve your mathematics ?

R: yes madam

I: and not just getting answers or solutions from the computer?

R: yes madam.

I: Ok. Thank you very much R1 for your cooperation and support. This has been very informative. Thank you very much.

R: You are welcome madam.

-------------THE END-------------

APPENDIXI: P2 Interview Transcriptions

I: Thank you very much **P2** for having agreed to take part in this study which is aimed at investigating the kind of difficulties and the successes that Grade 9 learners in Lesotho that may experience when they are taught algebra through spreadsheets.

Now what experiences did you like when you were working through this project?

P2: Madam I liked working in groups and working alone. Working alone helps you to see how much you understand. It makes you aware of yourself , because you are working on your own computer.

I:And what about working in groups? How was working in groups helpful to you?

P2: It was helpful because when you do not understand others, your colleagues will help you understand the questions.

I: Now apart from group work and working alone what other things did you like?

P2: [Pause]

I: You were most of the time confronted with problems to answer, and gaining knowledge as you worked through them. Now can you remember one worksheet or a problem that you worked through where you felt like aha! after doing it?

P2: Madam, it was worksheet 5, no worksheet 4.

I: Worksheet 4? What were you doing in that worksheet?

P2: We asked to find the kilometers and the hours the plane was moving.

I:Oh the question on the plane moving between two towns?

P2: Yes madam. I was not aware of those things the Km and the hours , I can now see them on cars and now I understand what it is all about.

I: OK. what other things or let me ask you, would you say you have benefited from being in this project?

P2: Yes madam. I can give the formular. Given the line on the graph I can give the formular for that line.

I: You mean the equation of the line?

P2: Yes madam.. And if I am given some numbers and other on the other side(indicating table of values), I can give the formular for that and also I have seen that computers are not like us, they show that the answers are not the same if you don't use brackets.

I: So you have learned the importance of using brackets from the computer?

P2: Yes madam.

I : You saw difference in your answers when you used brackets and when you do not have brackets from the computer?

P2 Yes madam.

I: OK. What else have you learned?

P2: I have learned how to make a graph. I was taught with a ruler how to make those columns and now I can see the x-axis, the y-axis and now I can see where that graph comes from. That time when we were taught with paper and pencil I did not understand now I understand.

I:OK. Now what challenges did you encounter while working through the whole project.

P2: Madam, the gradient when finding the gradient.

I: Oh! You dealt with change in y, which is the vertical change, and change in x, the horizontal change, what exactly was difficult with the gradient?

P2: Madam it was difficult at the beginning when I calculated that but now I understand it.

I: Oh. So it was difficult at the beginning when you were still learning.

P2: Yes. Madam.

I: OK. So that is why you said you can now give the equation of a line because you can now calculate the gradient?

P2: Yes, Madam.

I: What other things did you find challenging. The background knowledge that you had, do you think it was enough for you to be able to cope in the project?

P2: No madam.

I: what do you think you should have been taught before you could come to this spreadsheets programme?

P2: Madam I did not know how to drag, so our teacher should have made us aware of the dragging and also of entering the formulas.

I: But you learned that now?

P2: Yes madam

I: are you happy with that?

P2 yes madam.

I: OK. Do you think having participated in this project will enable you to perform better in your computer tests?

P2: Yes madam. I am going to perform better because I am going to save a lot of time. With that dragging I am going to save a lot of time also giving a formular.

I: OK. If a similar project is organized, would you encourage other learners to take part in it?

P2: Yes, Madam.

I: Why?

P2: Madam, with this one I just volunteered thinking it's about computers am going to be typing and typing and ask questions, but this was a very serious one that taught us about everything.

I: So you learned more than you expected.

P2: Yes, madam.

I: Are you happy with that?

P2: Very happy madam.

I: OK . what do you think teachers should do to attract more learners into liking mathematics?

P2: By organizing trips. I think teachers should organize trips and say were are going to do this and that. They will like maths.

I: What do you think could be done to attract more learners into projects such as this?

P2: Madam, they were attracted into this particular project. They were many madam, just the word computer madam, we came we were attracted.

I: OK. Thank you very much for your contribution, you have given me some information that is quite useful.

APPENDIX J: Triangulation of Findings from Qualitative Data

Colour Key: blue =data excerpts from Observation schedule

Black= data excerpts from learners' worksheets

Red= data excerpts from Learners' interviews

Green = data excerpts from Assistant observers interviews

Challenges Encountered In Teaching and & Learning Algebra Through Spreadsheets

SCHOOL-BASED

R11: All computers should be working well. …..I hated moving up and down changing computers.

Use of overhead projector could be more helpful to the teacher in demonstrating certain aspects on spreadsheets (Retha H.S)

Use of an overhead would enable the teacher to do demonstrations much easier than on the board with pen or doing same thing for each and every group.(Palo HS.)

INSTRUCTION-BASED

TECHNICAL

Organizational constraints

R13: I think the sources are caused by my typing speed and my slow work with a lot of time when typing.

R1: I think the sources is I don't know how to insert graph using spreadsheets

P9: The source of my problem is how to make a graph on the computer

R6: expanding the graph

R5:entering the formulas, inserting the vertical lines and the horizontal lines to the graphs; inserting the minor units, major units, the maximum and minimum units

R6:The challenge in working through this worksheet is when finding distance and time when Lele overtake Thato

Learners found it difficult to follow steps in getting the graph, particularly where adjusting scales were necessary.(Retha H.S)

Creating the graph was problematic. Too many steps, adjusting scales was more serious. (Palo H.S)

R3: The problem was creating the graph, the steps after highlighting the table, madam. The steps to take in order to get the correct graph

Command constraints

At the beginning they lacked the necessary computer skills"

There was also a problem with a "dragging command" in filling the rows and columns

R9: I couldn't get how possible it is(if it is) to not enter a formula in each and every cell

P4:…. Dragging down was a problem to me.

P2: Madam I did not know how to drag, so our teacher should have made us aware of the dragging and

They lacked basic computer skills and spreadsheets in particular (Palo H.S.)

Handling the mouse to drag. (Retha H.S)..

They are too slow in operating the mouse and remain behind. (Retha H.S).

Learners had a problem handling the mouse. They spend a lot of time trying to select and drag (Palo H.S)

They were not aware they could open new books in the same window. When starting a new task they reopened Excel spreadsheets anew (Palo H.S.)

Internal constraints

Also they did not understand the meaning of the "drag command (Retha H.S)

Learners found it rather challenging to observe what was happening in the table of values and the graphs at the same time when the values in the scroll bars were changed. This involved more than one graph and more than one column in the table (Retha H.S)

Being able to realize the effect of changing the parameters on the values in the table and the corresponding graphs was a bit challenging. With one click all changes happened at the same time (Palo H.S)

R3: Sometimes when you highlight after entering the formula and you draw the graph, may be the formula you entered was wrong then you get the wrong graph

R3: …this fill down command tell the computer to give more answers, it tells us that the computer should .. it means that we should highlight down so that we can get more answers

R7: When I supposed to enter the formula for multiplication it gives me the correct answer but when I drag to the place I want it write the answer that I don't want to.

R9: I did not calculate /enter the correct values on the table of values, so my graph was very different from the one that the teacher expected

P9: I encountered the challenge of comparing the properties of graphs

P10: I was not able to see what was going on

P9: The graphs confused me.

R12:I met the challenges of finding the differences of the graphs and my group

Syntax/Notation

It was difficult for them to use the spreadsheets notation, if ever there was a formula to write about a pattern, they were not able to do that.

understanding the meaning of the entries in the cells, for example if there was a 3 in a cell, then they did not understand the meaning of the entries, this project helped them to understand these things.

R7: I didn't know how to use the $A2*B$1 but madam explain it to us and we used it for writing multiplication table

P9: my problem is that it is extremely hard to find the correct formula in the spreadsheets

R3: I met the challenge of forgetting how is the formula used

R15: it was difficult to find the formula for Pascal's triangle

P8: multiplication, Pascal triangle and square, creating a calendar and entering formulas.

P1: Sometimes not easy to get the formulas to give to the computer.

Creating formulas for the patterns was very difficult as a result. (Palo H.S)

They syntax followed was not mastered. They usually forgot the equal sign at the beginning.(Palo H.S)

They were not aware of the absolute referencing

P1:I knew the relationship well but I did not know how to guide the computer to give other figure

I: What was problematic there?

P1: Finding the formulas

I: you said what was really difficult?

R2: When using the formulas.

I: The formulas were a bit problematic?

R2: Yes, madam.

I: if you had to draw the graph of y =2x +3 using spreadsheets, can you remember the spreadsheets formula you would enter, suppose this is cell…

R1: A1

I: yes suppose it is that A1, what would be the formula you would enter in that cell which you are going to drag to get all other values in that column?.

R1: It would be A1×2 +3 .

I: just like that?

R1: yes madam?

I: Don't you think you have left something that is very important?

R1: Oh. The equal sign. It would be = A1×2 +3.

Conceptual: Lack of number sense

P10: identifying patterns and sequences. The formulas used.

P7: the challenge is how to find relationship between numbers

R9:trying to find out how the sequences are build up

R10:…when I had to write the relationship between people accommodated and the tables…

R14:when we had to write relationship between people accommodated and the tables.

R10:…To find the formula for the height of the seedling

P2: in this session I did not understand how to find relationship between the dependent and independent values and sometimes I become confused of multiplying values by letters

R10:... When we had to find the formula to use in sequence 2 and 4

I think the main problem here was identifying relationships between consecutive numbers in a sequences. (Retha H.S).

Pascal's triangle was the most difficult to recognize (Palo H.S

I:..Can just give me examples of difficulties you encounter?

P3 The relationships between numbers in Pascal's triangle

poor arithmetic skills. I think if they do not have knowledge about number patterns and number behaviors then it will be difficult for them to identify the relationships in the sequences.

Learners difficulties lie on not knowing number patterns and not being able to identify relationships between sets of numbers. They lack number sense. Getting relationship between numbers in a sequence is much easier to them, so they rather use this to fill up columns in the tables.(Retha H.S)

The main challenge in this worksheet is making connections between sets of values (Palo H.S)

They found writing relationships between pairs of numbers difficult (Palo HS)

Learners could not see 2A + 2B and 2B + 2A as two ways of writing the same thing hence were frustrated by not getting the second formula or expression. (Retha H.S)

The main problem is that of lack of necessary background knowledge(number relationships) (Palo H.S)

This was due to learners lack of numbers sense. they are not aware of number behaviors.(Palo H.S.)

II PERSONAL

a)Anxiety on computer use

P12:….I don't listen to what the teacher is teaching and at that time the computers are on, I don't listen I am doing something on the computer.

P8: The source of my problem is that every time I lose concentration because I will always look at the computer without listening what you are saying and it is extremely hard for me to ignore it because I like it.

P10: Not asking questions where I did not understand and less discussions with the group members

R10:paying little attention to what I am doing. Not asking when having a problem. Being fearful when I have to ask a question.

P8:I was afraid because I met with people especially teachers who I was not familiar with but as time went I became familiar and comfortable with them.

R7:First lack of concentration, second was speed

Some displayed lack of commitment towards learning.(Palo HS)

Learners were also not focused on their work. .(Palo HS)

Learners were expecting the teacher to give them the correct formula.(Palo HS).

R2:It was sometimes boring because I am used to talking to the teacher , not solving problems on my own.

P1: I was afraid that if I enter a wrong formula and then I would have to start all over again

P1: I liked the easy questions

R2:but some of them I did not like where I encountered a problem and I got stuck and did not know what to do.

R3:Yes some of them, some people get sad when they ask a question and the teacher may be returns another question to you, instead of answering him or her.

III LANGUAGE

R6:I did not understand the meaning of independent value and dependent values

R4: The challenges I encountered are of understanding the meening of the words flatter and steeper

R2: I was not able to answer some questions because of language used here. It is difficult English and I couldn't understand some of the questions.

R3:Sometimes I do not understand what we are asked to do and I do things without asking;

R5:in some I did not understand what the question was saying

P6: I did not understand question. The English was difficult.

R7: understanding the English in the question

P2: May be not understanding the question.

R8:…. When not understanding the questions and what it requires.

Learners seem to have difficulty with understanding the statements expressing the problem. But once the statement is explained they are able to solve the problem (Retha H.S).

They found it difficult to ask questions, they could not express themselves clearly and were not free to speak aloud. (Palo H.S).

R1:In some I did not understand what the question was saying.

P3: Sometimes madam, the English that was written in the worksheets was difficult. We could not understand that question, if she used simpler English, we could understand easily.

I: ……what basically was the problem with that worksheet?

R2: Drawing the graph for the given questions the information given.

I: So at which state did you find it difficult to do, was is it on translating the information into the symbols or moving from the graph to the table or what exactly?

R2: Translating the information to the table to the graph.

I: If the table was there, would you be able to draw the graph from the table?

R2: Yes, madam.

ENABLING PEDAGOGICAL STRATEGIES

I TEACHER –INITIATED STRATEGIES

a) Didactical configurations and exploitation modes

R9:The teacher's supervision, the discussions with my group members and the time provided.

Use of teaching aids in the form of calendar (Retha H.S)

TASKS

Use of contexts familiar to learners… Contexts say to learners- mathematics is real and applicable(. Retha H.S)

R1:I prefer the problems that are related to real life because they help us understand what we are taught madam.

P3: Yes madam. And even something that the person cannot be aware of sometimes she can be aware of it when it is now appearing in the activities or in the test because he or she could know that this thing had happened and I know this thing.

They were also about things learners know and meet, there was enough background upon which the activities were based and as such they were meaningful.

Yes the tasks were valuable. They were at their level...

The teacher foresaw that and took advantage of what they were familiar with, like the multiplication table to develop expressing relationships between sets of numbers. (Palo HS)

I liked the introduction on the Calendar. It was quite realistic. Learners could relate with it.(Palo HS)

I liked the tasks. They were simple and learners could easily understand them(Palo HS)

Learners are given autonomy over their work (Retha HS)

Learners did more independent work and only consulted each other when they got stuck (Palo H.S)

This idea of worksheet is helpful in engaging learners, focusing their attention to the business of the lesson.(Palo HS)

USE OF WORKSHEETS

R2: I think it is easier when you are using worksheets because it helps students to learn faster because with worksheets you are able to do one thing at a time

R1:Yes But here madam, when you are using worksheets you are more focused madam and you learn the topic more faster.

P3: I like it when you gave us worksheets madam, you did not tell us everything that was there madam, some of them we were thinking for ourselves.

INDIVIDUAL COMPUTERS

I: Ok while still on groups, suppose you were given one computer for the whole group, which would you prefer one computer for the whole group or each one having their own computer?

R1:Each one having a computer because even after explaining if you are not doing the task yourself it would not be easy to tackle the task even when you are alone. but if you are using your own computer when the group members are not around you will be able to tackle the task alone.

P1: I think it was better that each one had their own computer…. A chance to do our own thing

P2:…. Working alone helps you to see how much you understand. It makes you aware of yourself , because you are working on your own computer.

Each learner should have own workstation such that each has a feel of the tasks being done

MONITORING LEARNING

I: Was the teacher aware of the learners' problems?

R: Yes the teacher was aware of that.

Attending to individual groups,

b) Didactical performances

R12:You should teach us slowly and you must ask us questions so that we could hear whether we are wrong or right

R10:I think we should be given a quiz at the end so that we can be able to see if we understand

EXAMPLES/HINTS

P5: Making more examples.

P8: The examples that my teacher gave to me helped me to learn successfully

P13: and making some simple examples which we can meet in daily life

Use of several examples help learners to identify which are dependent and which are independent variables. (Retha H.S)

R3:she gave us the first step so that we can think for ourselves

NURTURE

R3: That she helped us, that she come to us, and speak politely to us not I such way that a person can say the teacher is not happy or stressed, not in such a manner that a person cannot like.

QUESTIONING

P3: she asked more questions and sometimes she would make an example like that that one in the question

P1: Questions should be explained before we do anything.

using questioning and probing to lead them to the solutions (Retha HS)

I: Now was your teacher aware of the problems you encountered?

P1: Yes, she was but we had to figure it out ourselves. But if all of us did not know the answer we would ask the teacher

I: And what did she do. Did she give you solutions?

P1: No she would not just give us right away. She would ask us what if you did this and this or that, then we would get the answer.

I: So she would ask you questions ….

P1 : That would lead us to the answers.

I: Did you like that or you would have loved if the teacher gave you the answer straight away?

P1: I liked it because it teaches me much about that we should not always ask and be told I should try out first for myself

CODE-SWITCHING

P11: I think I can be helped by talking English and explain them in Sesotho. So that I can ask if there is anything I don't understand

DISCUSSIONS

Interactions-group discussions, and whole -class discussions.(Retha. H.S)

She would probe them trying to help them connect the situation with what they had learned (Palo H.S)

She would call the attention of the whole class to share (Palo HS)

That is why in most cases she intervened, tried to discuss. There was this whole class discussions, between the teacher and the whole group. The teacher also encouraged peer discussions among members of a group. There was also use of the chalk board to elaborate and clarify some concepts.

EXPLAINING

P13: I have learned successfully in the help of and the teacher herself by explaining the questions in words that I understand very well

P3: she just told us what the question needs and then we shall find answers.

The teacher had to provide explanations for the tasks (Palo HS)

She also provided explanations to words learners found difficult(Palo HS)

Explaining how to read the gradient and y-intercept from the graph and how to obtain the gradient from coordinates by calculations.(Palo HS)

The explanations about the meaning conveyed by point of intersection of the graphs (Palo HS) Explanations (Palo HS)

Explanations/illustrations on the board (Retha

R1: When I did not understand the question I would ask my group members and when I also did not understand them I would ask the teacher and she would explain.

I: the teacher would give you answers?

R1: Not answers but the explanations madam. she would explain more so that I understand better.

I: and then you would tackle the problem yourself?

R1: Yes madam.

I: Is that what you were expecting from the teacher? Explanations or you expected the teacher should give you answers?

R1: Not answers madam but explanations.

DEMONSTRATION

Sometimes she would show them what they should do on their computers (Palo HS)

Demonstrating how to check solution by algebraic manipulations.(Palo HS)

INSTRUCTIONS

On how to create the graph, the teacher had to give instructions slowly and repeatedly (Palo HS)

She somehow taught the skills along, they learned them as they required them.

REVOICING/ REPHRASING/VERBALISING

she asked learners to instruct one learner to do it.(Palo HS)

Asking learners to express relationships in their own words helps them reflect on their thinking(Retha HS)

Asking learners to reiterate the steps one after the other until the graph was produced. Probing to justify the next steps(Retha HS)

Encouraging learners to read the questions aloud (Retha HS)

II LEARNERS –INITIATED STRATEGIES

Intention

P8: I have to see by myself that I must do my work quickly and save it and to ask question.

R9: enjoying what I was doing with full dedication and love, discussing with my group members and using the teachers word helped me to learn successfully.

P13: A lot has helped me to learn successfully today, like concentrating in class enjoying the day.

Active Participation

Asking questions and discussing

R9:Asking question where I did not understand and following instructions and working together with my group or partners

P10: Discussion with the group members and asking questions

P4: to ask for help from my colleagues, and to ask help from my teacher and concentrating to what the teacher is saying.

P4: What I think can be done is to ask for help and don't be lazy to think, and to work in groups so that we can talk about one another's problem

P13: I have learned successfully in the help of discussion with my classmates and the teacher herself by explaining the questions in words that I understand very well and making some simple examples which we can meet in daily life

Interactions-group discussions, and whole -class discussions (Retha HS)

They would try, talk with their neighbors, then ask the teacher. (Palo HS)

Discussions and asking questions.(Palo HS)

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They only ask the teacher after attempts were made(Palo H.S)

R1: When I did not understand the question I would ask my group members and when I also did not understand them I would ask the teacher and she would explain.

R1: Yes because sometimes it happened that one of the members did not understand but when we discussed then the task it was easy for the members to understand.

R1:I brainstormed with my group members. Sometimes got help from them but when we got stuck we would consult other groups. from there we would call the teacher

P2: Madam I liked working in groups and working alone. Working alone helps you to see how much you understand. It makes you aware of yourself , because you are working on your own computer.

P3: well madam, my opinion is that when we were doing this we had different minds madam. And some could understand so we would have information.

P2: It was helpful because when you do not understand others, your colleagues will help you understand the questions.

P1: I like working in groups particularly working in groups of three, because unlike when in groups of two the other one will have their opinion and the other one will have their own opinion and we will never end but if in threes then the majority will rule.

R1: Yes because sometimes it happened that one of the members did not understand but when we discussed then the task it was easy for the members to understand.

Focusing/concentrating

R1: Concentrating and participating in my group

P12: I have to stop doing what I am not expected to do in the lesson so that I can concentrate to what the teacher is teaching me.

Re-voicing

R9:The best was also to read questions carefully and swallow my pride and where I did not understand ask questions

R5:I could have read the question over and over till I can say it in my own language

iii)Practicing/ testing &trying

R9:I could have practiced more, by making different examples of what I have been taught

P3: I should have practiced the speed and I should have concentrated.

P11: I think trying has helped me to learn successfully because it is better than doing nothing at the moment

P7: I think if we could be allowed during our spare time to use the computers alone without teachers next to us, we could be able to work out numbers without the help of the teachers

Trying out and testing their thinking (Retha HS)

They also tested what they came up with (Palo HS)

R1: In such cases we just look at our first step that we did when we do the graph and then we can see where is our problems and then we check our formulas whether they are correct or wrong and when we see that they are wrong, then we start another formula and then create another graph with another formula.

P3: Before I could ask I had to think about the question and the answer. If I can't understand I could ask the group members and then the teacher

Attending lessons

R13:By attending classes correctly

P10: Attending class and listening in class

P11: The sources of my problem is that the first day my teacher was teaching us about this topic I was upset now it was difficult to learn some as my classmates have more knowledge than I have

THE END