Investigation into Cracking in Reinforced Concrete Water-retaining Structures

by
Christina Helen McLeod

Thesis presented in fulfilment of the requirements for the degree of Master of Science in Engineering in the Faculty of Engineering at Stellenbosch University

Supervisors: Prof JA Wium & Prof JV Retief

March 2013

ABSTRACT

Durability and impermeability in a water-retaining structure are of prime importance if the structure is to fulfill its function over its design life. In addition, serviceability cracking tends to govern the design of water retaining structures. This research concentrates on load-induced cracking specifically that due to pure bending and to direct tension in South African reinforced concrete water retaining structures (WRS).

As a South African design code for WRS does not exist at present, South African designers tend to use the British codes in the design of reinforced concrete water-retaining structures. However, with the release of the Eurocodes, the British codes have been withdrawn, creating the need for a South African code of practice for water-retaining structures. In updating the South African structural design codes, there is a move towards adopting the Eurocodes so that the South African design codes are compatible with their Eurocode counterparts. The Eurocode crack model to EN1992 (2004) was examined and compared to the corresponding British standard, BS8007 (1989). A reliability study was undertaken as the performance of the EN1992 crack model applied to South African conditions is not known. The issues of the influence of the crack width limit and model uncertainty were identified as being of importance in the reliability crack model.

Declaration

By submitting this thesis/dissertation electronically, I declare that the entirety of the work contained therein is my own, original work, that I am the sole author thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

Date: March 2013

ACKNOWLEDGEMENTS

First, my sincere thanks my supervisors at Stellenbosch University, Professor Jan Wium and Professor Johan Retief, for their time, guidance and encouragement they always very willingly gave.

Thank you to all those civil engineers in industry for their input into this research – the list is a long one.

Thank you to Professor Geoff Pegram, Professor Derek Stretch and to Rod Rankine for their assistance.

Last but not least, a big thank you to my family and friends for their love, encouragement and support.

LIST OF TABLES

LIST OF FIGURES

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Motivation

The aim of this thesis is to investigate serviceability cracking due to loading for small crack widths, as applied to reinforced concrete water retaining structures (WRS) in a South African context. Historically, South African design codes for structural concrete have been based on the British standards. As a South African design code for WRS does not exist at present, South African designers tend to use the British codes in the design of reinforced concrete water-retaining structures. However, with the release of the Eurocodes, the British codes have been withdrawn, creating the need for a South African code of practice for water-retaining structures. In updating the South African structural design codes, there is a move towards adopting the Eurocodes so that the South African design codes are compatible with their Eurocode counterparts. The code of practice for loading of structures, SANS 10160: *Basis of structural design and actions for buildings and industrial structures* (2011), has already been released. The structural design codes for concrete structures, SANS10100-1 and -2, are currently under revision and will also be compatible with the equivalent Eurocodes, with the likelihood that Eurocode will be adopted. Motivation was thus created to carry out research to ensure that the Eurocodes are compatible with South African parameters and practices in the design and construction of reinforced concrete WRS. The new code for water retaining structures is proposed as SANS10100-3: *Design of concrete water-retaining structures*.

Durability and impermeability in a water-retaining structure are of prime importance if the structure is to fulfill its function over its design life. The control of cracking is therefore important in the design and construction of this type of structure. This means that the serviceability requirements for cracking, measured by a limiting maximum crack width, are more arduous for WRS than in buildings. This study concentrates on load-induced cracking, specifically the load cases of pure bending and pure tension. As the proposed new code of practice for WRS is to be compatible with Eurocode, the Eurocode crack model to EN1992 (2004) was examined and compared to the corresponding British standard, BS8007 (1989). A reliability study was undertaken as the performance of the EN1992 crack model applied to South African conditions is not known.

1.2 Objectives

The main objective of this thesis was to investigate cracking with respect to South African water retaining structures. In the process of the research, three key issues were identified and are summarised as follows:

(i) Serviceability limit state cracking tends to be dominant over ultimate limit state loading in the case of WRS. Research using reliability analysis and limit state design has previously tended to focus on the usually more dominant ultimate limit state of collapse, with limited research on serviceability. The establishment of an appropriate target reliability was therefore investigated.

(ii) Eurocode was found to have a more stringent limiting crack width of 0,05mm, as opposed to the general limit of 0,2mm to BS8007 (BS8007 does allow for a further reduction in the crack width limit to 0,1 mm for aesthetic reasons). The implications of this on the design and construction of WRS obviously require further study.

(iii) Crack models have been developed largely empirically with little data available on the uncertainty in the models and tend to be conservative. An investigation of model uncertainty with respect to load-induced cracking in WRS would then be undertaken, leading to possible future improvements in the crack models.

This research investigated these issues as outlined in the next section.

1.3 Outline of thesis

The structure and summary of the thesis is as follows:

- Chapter 2: Literature and South African (SA) industry review with respect to cracking.
- Chapter 3: Deterministic analysis of the BS8007 and EN1992 design procedures for cracking.
- Chapter 4: Overview of reliability and development of reliability model of the EN1992 crack model.
- Chapter 5: Reliability analysis of the EN1992 crack model using the First Order Reliability Method (FORM).
- Chapter 6: Sensitivity analysis of the EN1992 crack model using reverse-FORM.
- Chapter 7: Reliability calibration of EN1992 crack model.
- Chapter 8: Final conclusions and summary.

- Appendices: Data sheets and graphs not presented in the main text of the thesis.

A summary and the structure of the research is shown in the flow chart presented as Figure 1.1.

The first step of this research was to undertake a literature study and review of the South African industry with regard to the design and construction of water retaining structures, presented in Chapter 2. This resulted in the identification of the key issues in the control of cracking in WRS. Design code formulations and representative structural configurations with respect to load-induced cracking were also investigated.

A deterministic analysis and comparison of the BS8007 and EN1992 design code crack formulations, presented in Chapter 3, was undertaken as a first step in exploring the issues of the degree of importance of serviceability cracking and the implications of using a smaller limiting crack width. The deterministic analysis also aided in ascertaining representative cases for use in the reliability crack models.

Reliability analysis using the First Order Reliability Method (FORM), presented over Chapters 4, 5, 6 and 7, was applied to the EN1992 crack models in order to explore all three key issues. A literature review of reliability analysis was first undertaken, summarised in Chapter 4, along with the development of the reliability crack models. Chapter 5 presents the investigation of the three key issues by means of a forward-FORM analysis and is concerned mainly with the implications of a smaller limiting crack width on the design of a WRS and model uncertainty. Three reliability crack models were required for load-induced cracking, dependent on the load case and the formulation of the effective depth of the tension zone in concrete subject to cracking.

A sensitivity analysis was then performed using a reverse-FORM analysis to investigate the most influential variables of the flexural and tension crack models with respect to the three key issues identified, presented in Chapter 6. The theoretical partial safety factors of the model variables for representative cases were also determined. In order to develop a usable design crack formulation, a calibration of the theoretical partial safety factors is required. This was outside the scope of this thesis. However, a preliminary exercise in calibrating the load-induced crack models was carried out and reported on in Chapter 7.

Chapter 8 presents the final summary of results and conclusions to be made from this research.

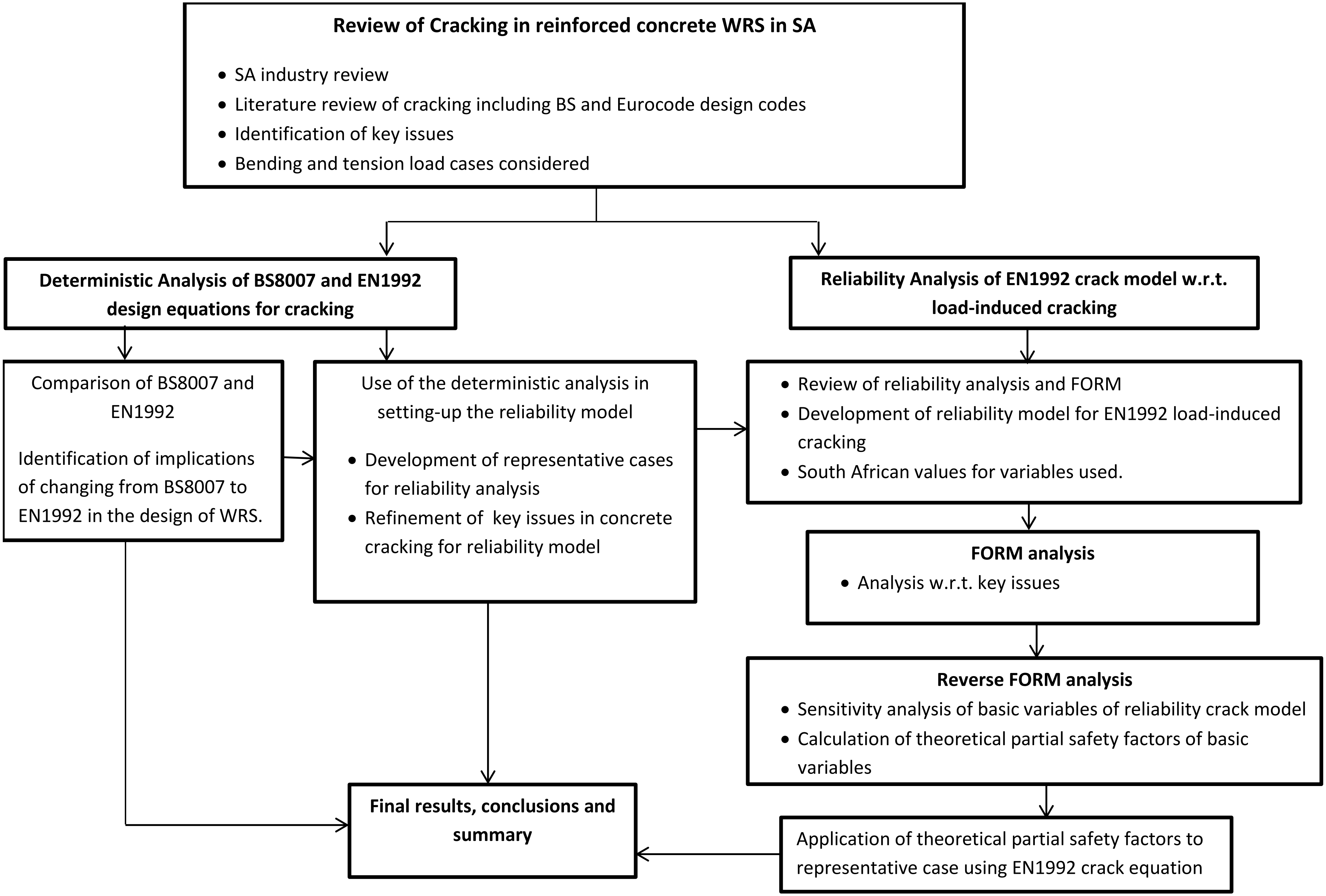


Figure 1.1: *Flow chart of the investigation into cracking in South African WRS*

CHAPTER 2

OVERVIEW OF CRACKING IN CONCRETE WATER RETAINING STRUCTURES

2.1 GENERAL

The serviceability limit state (SLS) of cracking is important in WRS as failure to meet this limit state would result in a loss of function of the structure. Cracking needs to be controlled to maintain water-tightness, for durability and corrosion protection of reinforcement, and for aesthetics. Cracking in concrete has a variety of causes, which in summary, are:

- Cracking due to deformations of a structure under applied forces, namely, flexural tensile and direct tensile cracking in mature concrete.
- Cracking in immature concrete due to drying shrinkage and restrained concrete.
- Thermal and shrinkage effects in restrained concrete.
- Corrosion of reinforcement within concrete, resulting in eventual spalling of the concrete.
- Expansive chemical reactions within the concrete.

As stated in the previous chapter, this research focuses on load-induced cracking specifically that due to pure flexure and direct tension load effects.

In order to identify key issues in the design of reinforced concrete water-retaining structures (WRS) with respect to load-induced cracking, the relevant British and Eurocode standards were reviewed. Crack models used by each code for cracking due to load effects were investigated. The codes concentrated on in investigating SLS cracking are:

- BS EN 1992-3: *Eurocode 2: Design of Concrete Structures – Part 3: Liquid Retaining and Containment Structures* BSI 2006
- BS EN 1992-1-1: *Eurocode 2: Design of concrete structures – Part 1: General requirements* BSI 2004
- BS8007: *British Standards Code of Practice for the Design of Concrete Structures for Retaining Aqueous Liquid,* BSI 1987.
- SANS10160: *Basis for structural design and actions for buildings and industrial structures* and EN1991: *Eurocode 1: Actions on structures* referred to for guidance on load factors.

A literature review on research carried out on parameters used in the crack models was undertaken and presented in this chapter. Any references to water-retaining structures in existing South African codes were included in this review. In addition, a summary of the design and construction practices of WRS in South Africa is given here, with emphasis on configurations and loading cases resulting in tensile and flexural cracking. Information gathered was used in the design calculations comparing EN1992 and BS8007 for load-induced cracking in Chapter 3. These calculations were then used in comparing and developing the reliability models to assess the reliability of the Eurocode crack model.

2.2 SCOPE OF DESIGN CODES RELEVANT TO SOUTH AFRICA

The scope of the following design codes dealing with the design and construction of WRS is summarised here:

(i) The British standard BS 8007: *Design of concrete structures for retaining aqueous liquids* (1987) pertains to aqueous liquid-retaining structures only, that is, tanks, reservoirs and other vessels constructed in pre-stressed or reinforced concrete. The scope does not include other liquids, aggressive waters or granular solids, with the water contained assumed to be at ambient temperatures. BS8007 is read in conjunction with BS 8110: *Structural Use of Concrete* (1997).

(ii) The Eurocode standard EN1992-3 (2006): *Liquid retaining and containment structures*, as the title describes, pertains to the containment of all liquids and granular solids, so has a wider scope than BS8007. The scope of the code also includes retaining and containment structures. Plain, lightly reinforced concrete, reinforced concrete and pre-stressed concrete are considered. Exclusions are materials at very high temperatures, the storage of hazardous materials that could pose a major health risk if leakage occurs, design of liners and coatings, pressurized and floating structures and gas tightness. This code is to be read in conjunction with EN1992-1*: Design of concrete structures – Part 1-1: General rules and rules for buildings* (2004). EN1992-3 allows for stored materials having a temperature range of -40˚C to +200˚C, a greater range than that of BS8007. EN1992-3 states that for durability and leakage, mainly aqueous liquids are considered. Specialist literature is to be consulted for other liquids store in direct contact with structural concrete.

2.3 CRACK MODELS DUE TO LOADING DEFORMATIONS

2.3.1 General

The crack models used by BS8007 and EN1992 respectively assume that the concrete section considered is cracked and linear elastic theory applies as cracking is a serviceability limit state. Cracking due to loading is primarily controlled by the provision of reinforcement to obtain less than a specified maximum expected crack width, assuming proper construction practices and good quality concrete.

The cracking model for a section under loading to BS 8007 was developed empirically, while the EN1992 model is based on a limited bond-slip model. The mechanism of the crack formation is similar in each case. Once the tensile stress induced by the load exceeds the tensile strength of the concrete, a primary crack develops. The reinforcing at the crack carries the tensile force, as the stress in the concrete at the crack is zero. Away from the immediate vicinity of the crack, bond strength transfers and redistributes the tensile stress from the reinforcement into the concrete over a distance, S_o. If the tensile capacity of the concrete is again exceeded as load increases, further cracking results at a distance no less than S_o from the first crack, thus defining the minimum crack spacing. The bond strength and thus the rate of transfer of tensile stress between the reinforcement and the concrete influence the crack spacing. If loading is increased, the inelastic phase is reached when either the steel yields or the concrete is no longer elastic. However the loads required for the structural element to reach this phase usually exceed those of normal service, hence the inelastic phase is not considered for the serviceability state.

The average stresses and strains are calculated using linear elastic theory, modified for tension stiffening. Tension-stiffening is the capacity of the uncracked concrete, between two adjacent cracks, to carry the tensile force which is transferred from the reinforcement to the concrete by bond stress between the reinforcement and the concrete. Eurocode and the British standards differ in their determination of the tension stiffening effect on strain and crack spacing and therefore the calculation of the maximum crack width. This is a point of difference between many crack models used by other countries. These differences are discussed in the following paragraphs.

Under increased loading a condition is reached where no additional cracks form. The crack spacing then remains constant while cracks widen as the load increases further. Eurocode divides the formation of the crack pattern into two phases: the first, the crack formation phase, and the second, the stabilised cracking phase. In the crack formation phase, the first crack occurs when the concrete tensile strength is exceeded at a point. The tensile stresses are transferred from the concrete to the reinforcement at the crack. This transfer of stresses occurs up to a distance S_o (also called the transfer length) away from the crack. A second crack occurs when the load is increased at the next weakest point, at a distance greater than S_o. As load increases, further cracks form until all cracks are a minimum of S_o apart. In the stabilised crack phase, no new cracks are formed and the average crack spacing, S_m, remains constant as load increases. The average crack spacing (S_{rm}) in the stabilized cracking phase is found to be between the initial transfer length (S_0) after the formation of the first crack and twice that initial length, i.e., $S_0 \leq S_{rm} \leq 2S_0$ (*Beeby* (2005)). Stresses in the concrete are relieved by internal crack formation and limited bond slip near crack faces.

The cracking mechanism under load described in the previous paragraph is illustrated in Figure 2.1 (*Narayanan & Beeby*, 2005).

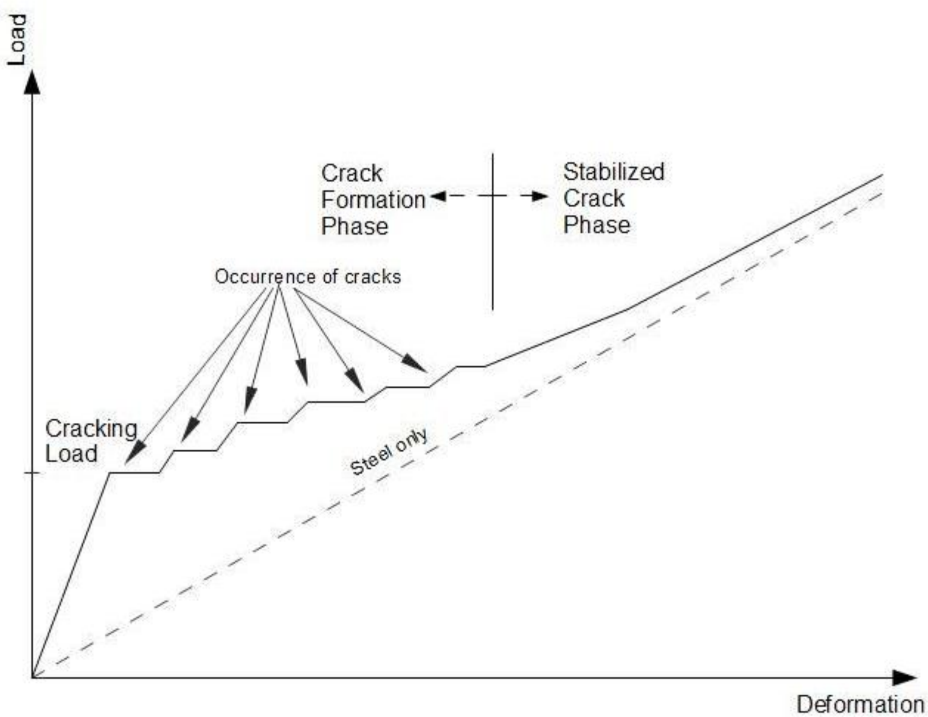


Figure 2.1: *Load-deformation response to steadily increasing load (Source: Narayanan & Beeby, 2005)*

In a controlled load test (as illustrated in Figure 2.2 overleaf, *Narayanan & Beeby, 2005*), the crack widths were found to remain the same in the crack formation phase, whilst the number of cracks increased. Cracks widened at an approximately linear rate once the stabilised crack phase was reached with no new cracks developing.

2.3.2 BS 8007 Equations

The crack model for load effects to BS8007 is an empirical model and takes into account the effect of cover, reinforcement diameter and spacing, and stress in the reinforcement. BS 8007 sets maximum limits for the stresses in the reinforcement and concrete to ensure the structure remains in the elastic phase under loading. Models developed to calculate crack widths have been simplified by using parameters specific to the United Kingdom, such as

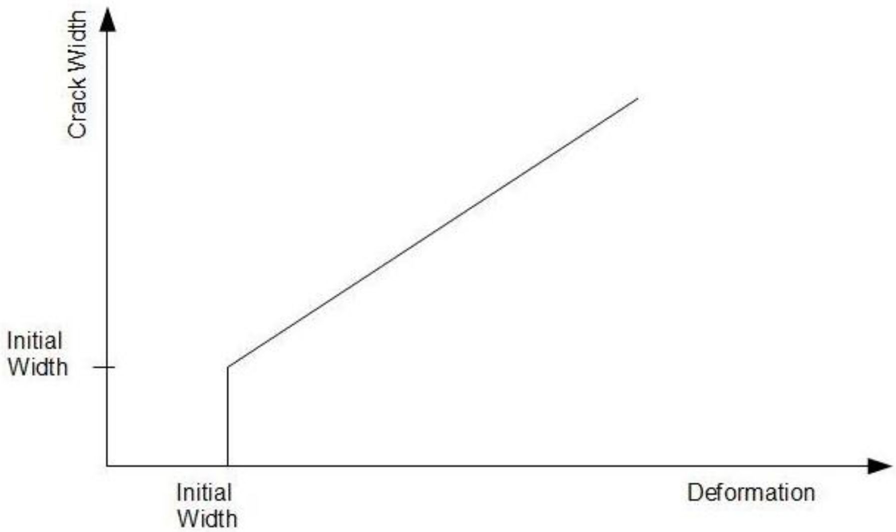


Figure 2.2*: Crack-deformation response in load-controlled test (Source: Narayanan & Beeby, 2005)*

climatic parameters. Awareness of the assumptions made in simplifying crack calculations is necessary to have reliable results for South African conditions. BS8007 differentiates between cracking resulting from temperature and moisture effects (Appendix A) and that in mature concrete due to loads (Appendix B).

The maximum surface crack width, w, for flexure is calculated in BS8007 using:

$$w = \frac{3a_{cr}\varepsilon_m}{1+2\left(\frac{a_{cr}-c_{min}}{h-x}\right)} \qquad (2.1)$$

The maximum crack width for tension is calculated from the expression:

$$w = 3\, a_{cr}\, \varepsilon_m \qquad (2.2)$$

where the parameter a_{cr} is defined as the distance from the point considered to the nearest longitudinal bar (position of the crack), ε_m is mean strain, c_{min} is the concrete cover, h is the section depth and x is the depth from the compression face of the section to the neutral axis.

Expressions (2.1) and (2.2) assume that the crack spacing is a function of the distance from a crack to the nearest reinforcing bar (a_{cr}), as illustrated in Figure 2.3 for a slab or wall section under bending. This distance will be a maximum to a point mid-way between reinforcing bars in the case of a slab or wall. The crack width is taken to be the smallest at the reinforcing bar, widening to the surface. For a typical reservoir wall section a_{cr} is determined from:

$$a = \sqrt{\left(\frac{s}{2}\right)^2 + \left(c + \frac{\varphi}{2}\right)^2}$$

$$a_{cr} = a - \frac{\varphi}{2} \qquad (2.3)$$

where a is the distance from the surface crack to the centre of the reinforcing bar, c is cover to reinforcement, s is spacing of reinforcement and φ is reinforcement diameter

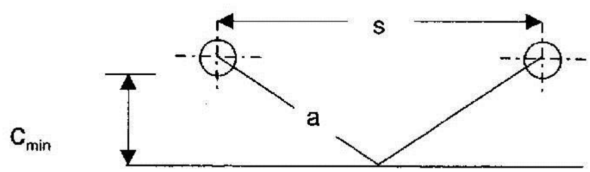


Figure 2.3: *Distance from surface crack to centre of bar for slab or wall section*

The average strain, ε_m, is calculated from:

$$\varepsilon_m = \varepsilon_1 - \varepsilon_2 \qquad (2.4)$$

where ε_1 is the apparent strain at the surface and ε_2 is the tension-stiffening effect of the concrete in tension.

The apparent strain at the level of the tension reinforcement (ε_s) is calculated using elastic theory. The apparent strain at the surface (ε_1) for flexure is then determined from:

$$\varepsilon_1 = \varepsilon_s (h - x)/ (d - x) \qquad (2.5)$$

where d is the effective depth. The average strain is then calculated by deducting the effect of the concrete in tension, i.e., tension stiffening from the apparent strain, ε_1. In the tension case, the apparent strain (ε_1) is equal to the steel strain (ε_s).

The equations to calculate tension stiffening strain for flexure were derived empirically and are dependent on the value chosen for the maximum crack width limit (w_{lim}) as follows:

$$\varepsilon_2 = \frac{b_t(h-x)(a'-x)}{3E_s A_s (d-x)} \qquad \text{for } w_{lim} = 0{,}2\,\text{mm} \qquad (2.6(a))$$

or,

$$\varepsilon_2 = \frac{1{,}5 b_t(h-x)(a'-x)}{3E_s A_s (d-x)} \qquad \text{for } w_{lim} = 0.1\,\text{mm} \qquad (2.6(b))$$

where a' is defined as the distance from the compression face to the level at which the crack width is being considered, and is equal to the depth of section, h, in the case of a reservoir wall under bending, E_s is the steel modulus of elasticity, b_t is the width of the section in tension and A_s is the area of the tension reinforcement.

Tension stiffening strain for the tension case is

$$\varepsilon_2 = \frac{2b_t h}{3E_s A_s} \qquad \text{for } w_{lim} = 0{,}2\text{ mm} \qquad (2.7(a))$$

$$\varepsilon_2 = \frac{b_t h}{E_s A_s} \qquad \text{for } w_{lim} = 0{,}1\text{ mm} \qquad (2.7(b))$$

For a limiting crack width of 0,3 mm, equations 2.6(a) and 2.7(a) are used to calculate ε_2 for flexure and tension respectively (*Bhatt et al* (2006) and *Reynolds& Steedman* (2008)), while no guidance is given for limiting crack widths less than 0,1 mm. On giving the equations for tension stiffening strain relative to the limiting crack width BS8007 states that interpolation or extrapolation may not be done for other limiting crack widths. According to these equations, ε_2 for a w_{lim} of 0,1 mm is 1,5 times that for a w_{lim} of 0,2 mm.

The above expressions for calculating crack width apply if the strain in the tension reinforcement is limited to $0.8f_y/E_s$ where f_y is the characteristic yield strength of the reinforcement. The stress in the concrete is limited to $0.45f_{cu}$ where f_{cu} is the characteristic compressive strength of concrete at 28 days (cube). BS8007 (read in conjunction with BS8110) specifies a minimum tensile reinforcement area, $A_{s,min}$, of 0,35 % for high tensile reinforcement and 0,64 % for mild steel, with a maximum bar spacing of 300mm. The total reinforcement is to be arranged as follows:

(i) Walls and suspended slabs with section thickness h ≤ 500 mm: minimum area of reinforcement required is calculated using the full section depth. Half this area is provided to each face.

(ii) Walls and suspended slabs with section thickness h > 500 mm: minimum area of reinforcement required is calculated using the outer 250mm depth of concrete of each face. Half this area is to be provided to each face.

(iii) Ground slabs with section thickness h < 300 mm: minimum area of reinforcement required is calculated using the top half of the slab depth only. No reinforcement is to be provided in the bottom half.

(iv) Ground slabs with section thickness 300 < h ≤ 500 mm: reinforcement is provided in the top half of the slab as calculated as (iii). Additional reinforcement is provided in the bottom of the slab, using a depth of 100 mm.

(v) Ground slabs with section thickness h > 500 mm: reinforcement is calculated as in (iv) but with a maximum depth considered of 250 mm for the top of the slab.

The code does not specifically give guidance on combined flexure and tension crack calculations. However, elastic stress-strain theory for bending moment and axial tension over the cross section can be applied to determine the stresses and strains induced. An iterative process is required to solve the series of equations for combined tension and bending.

It must be noted that the BS8007 equations for load cracking differ from those for cracking due to thermal and temperature effects in restrained concrete. The latter are similar to those of Eurocode and are based on bond-slip theory. Eurocode uses the same general formula to calculate crack widths for loading and restrained shrinkage. Strain is determined using the appropriate equations.

2.3.3 EN 1992 Equations

The EN1992 crack design equation for cracking was developed from the compatibility relationship for cracking in the stabilised crack phase,

$$w_m = S_{rm}.\ \varepsilon_m$$

where w_m is the mean crack width, S_{rm} is average crack spacing (as determined from experimental data) and ε_m the mean strain. The mean strain ε_m is ε_{sm} - ε_{cm} where ε_{sm} is the mean strain in the reinforcement under loading calculated using linear elastic theory. The mean concrete strain is calculated from:

$$\varepsilon_{cm} = k_t \frac{f_{ct,eff}}{\rho_{p,eff}} \left(1 - \alpha_e \rho_{p,eff}\right) / E_s \qquad (2.9)$$

where α_e is the ratio E_s / E_{cm}, $\rho_{p,eff}$ is the effective reinforcement ratio, $f_{ct,eff}$ is the tensile strength of the concrete at the time of cracking and k_t is a factor dependent on the duration of load. EN1992-1-1 recommends values for k_t of 1,0 for short-term loading and 0,4 for long-term loading. The code also suggests that $f_{ct,eff}$ is taken as the tensile strength of concrete at 28 days, f_{ctm}. This is a mean value, not a characteristic value, and is determined from $f_{ctm} = 0,3.f_{ck}^{(2/3)}$ where f_{ck} is the compressive cylinder strength of concrete which is approximately 0.8 f_{cu} where f_{cu} is the compressive cube strength. A minimum limit of $0,6\sigma_s/E_s$ is placed on the mean strain to ensure that the stabilised cracking stage is reached.

According to Narayanan & Beeby (2005) on the development of the Eurocode crack model, the initial transfer length, S_o, (which is also the minimum crack spacing) and therefore S_{rm}, depends on the rate of transfer of stress from the reinforcement to the concrete, which in turn is affected by bond stresses on the bar surface. The mean crack spacing is assumed to be at $1,5S_o$. Assuming that bond stress, τ, is constant along the length S_o and will be at the tensile strength of the concrete, f_{ct}, at S_o from a crack within an area of concrete of A_c, then

$$\tau \pi \varphi S_o = A_c f_{ct}$$

If the reinforcement ratio is taken $\rho = \pi \varphi^2 /4 A_c$, then

$$S_o = 0,25 f_{ct} \varphi/\rho \tau$$

This equation for S_o was developed to become the equation for mean crack spacing which is:

$$S_{rm} = 0,25 k_1 \varphi/\rho$$

where k_1 is a factor depending on the reinforcement bond characteristics.

More recent research included the effect of cover, c. The equation for mean crack spacing was also modified so that it can be applied to flexural cracking as well as pure tension by the introduction of a strain distribution factor k_2 which takes into account the difference in the distribution of strain between the tension and flexure cases. The EN mean crack spacing formulation then has the form:

$$S_{rm} = k.c + 0,25 k_1 k_2 \varphi/\rho$$

Thus, crack spacing is a function of cover, bond strength, bar diameter and effective reinforcement ratio.

For design, the maximum crack width likely to be exceeded is used rather than the mean width. EN1992 takes this maximum or characteristic crack width to be that having a probability of exceedence of 5%.This maximum crack width is related to the mean crack spacing by the equation:

$$w_k = (\beta_w\, S_{rm}).\ \varepsilon_m$$

where ($\beta_w\, S_{rm}$) is the maximum crack spacing, $S_{r,\,max}$.

Borosnyoi and Balázs (2005) checked and compared various models for flexural cracking, including that used by EN1992-1-1. They found that for crack models of the form

$$w_k = \beta_w\, w_m = (\beta_w\, S_{rm}).\ \varepsilon_m$$

where $\beta_w\, S_{rm} = S_{r,max}$, the ratio $S_{r,max}/\ S_{rm}$ was from 1,33 to 1,54, whilst experimental data indicated ratios of 1,3 to 2,8. They also stated that experiments on reinforced concrete in flexure showed that concrete cover, spacing of reinforcement and size effects were influential factors in determining the average crack spacing. The EN1992-1-1-1 formulation uses a value of 1,7 for the factor β_w. (*Narayanan & Beeby (2005)* and *Holicky& Retief (2010)*).

The EN1992 equation for the maximum expected crack width then becomes:

$$w_k = S_{r,max}.\ \varepsilon_m \qquad (2.10)$$

The EN1992 maximum crack spacing equation is written as:

$$S_{r,max} = k_3.c + k_1 k_2 k_4 \varphi\ /\rho_{p,eff} \qquad (2.11)$$

where φ is the bar diameter (mm), c is the cover to the longitudinal reinforcement and k_1 is a coefficient taking into account of the bond properties of the bonded reinforcement. The coefficient k_1 has a value of 0,8 for high bond bars. The distribution of strain coefficient k_2 has a value of 0,5 for bending and 1,0 for pure tension. For combined tension and bending, intermediate values of k_2 may be calculated from the relation $k_2 = (\varepsilon_1 + \varepsilon_2)/2\varepsilon_1$, where ε_1 and ε_2 are the greater and lesser tensile strains respectively, at the boundaries of the section considered, assessed on the basis of a cracked section. The values of k_3 and k_4 are determined by individual member countries' National Annexes. EN1992-1-1 gives recommended values of 3,4 and 0,425 for k_3 and k_4, respectively.

The effective reinforcement ratio is calculated as the ratio between the reinforcement area, A_s, and the effective area of concrete in tension, $A_{ct,eff}$. The latter is determined following Figure 7.1 of EN1992-1-1, given here as Figure 2.4. In calculating $A_{ct,eff}$, the effective depth of the tension area $h_{c,eff}$ is taken as the lesser of h/2, 2,5(h - d) and (h – x)/3. The limiting equation depends on the type of tensile stress as well as the geometry on the section considered. The last term would apply to members under bending. For members in tension, the first 2 terms apply. The term 2,5(h – d) can be written in the form 2,5(φ/2 + c). In other words, the effective depth in tension in this case is dependent on the diameter of the reinforcement and the cover, and independent of section thickness.

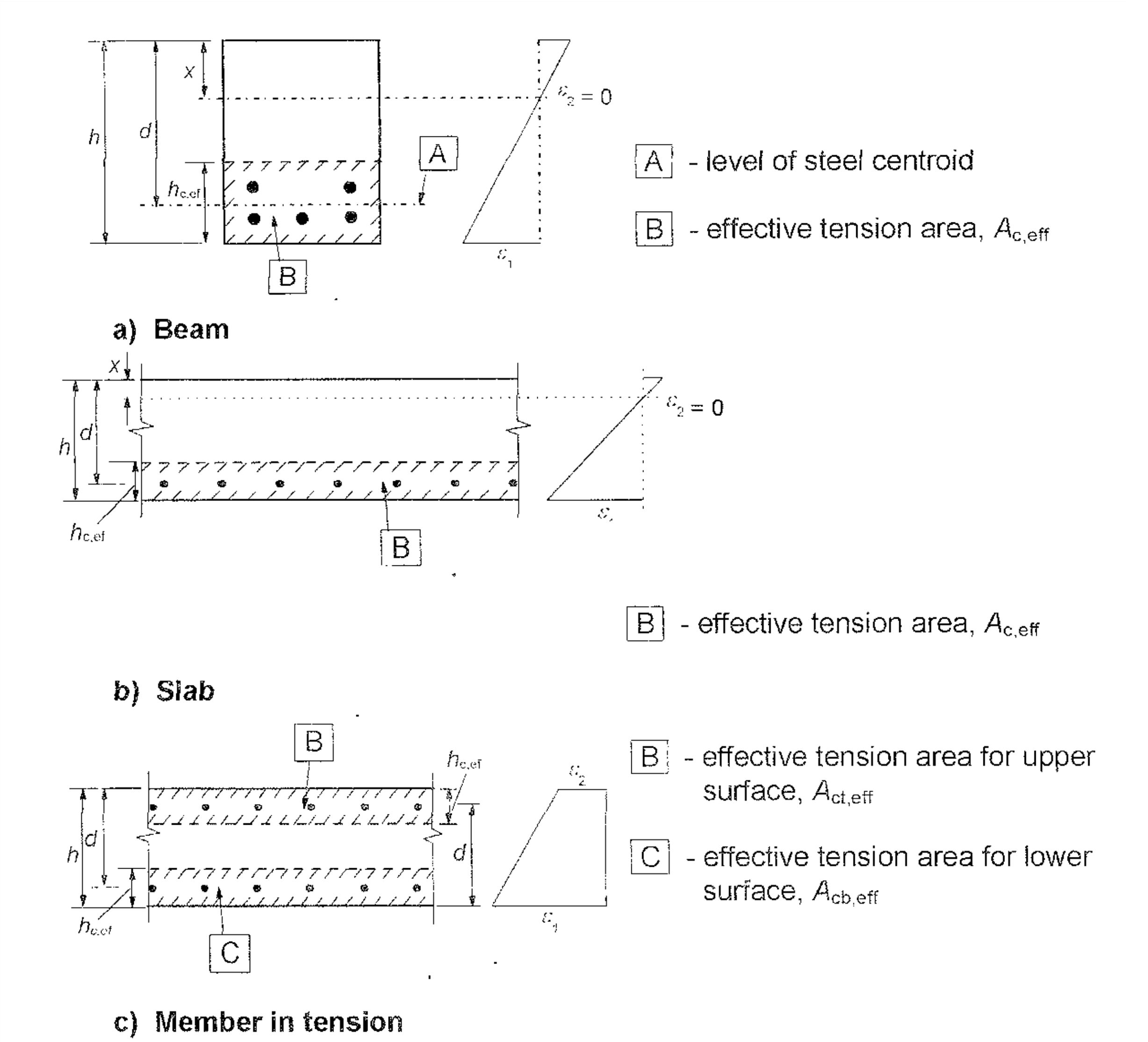


Figure 2.4: *Determination of $A_{ct,eff}$ (Source: Figure 7.1 of EN1992-1-1)*

The influence of the bar spacing on crack behaviour in the Eurocode crack equations is implied in the calculation of crack spacing through the specification of an area of reinforcement for a given bar diameter, in contrast to BS8007 which uses bar diameter and spacing directly. Crack spacing affects the crack widths as it has been found in experimental research that the wider the spacing, the larger the crack widths will be. It is therefore desirable to limit the crack spacing to ensure smaller crack widths. Reinforcement of smaller bar diameters and at closer centres is recommended as this has been found to result in a pattern of finer cracks at a closer spacing.

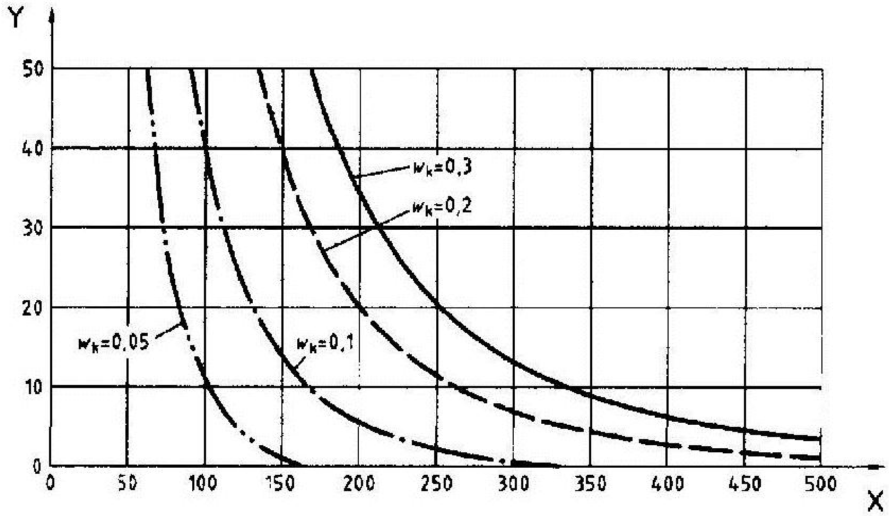
A minimum area of tension reinforcement is required to ensure that the reinforcement remains elastic, particularly after the formation of the first crack. Equation 7.1 of EN1992-1-1 defines this area as:

$$A_{s,\,min} = k_c\, k\, f_{ct,\,eff}\, A_{ct} / \sigma_s$$

where the area of concrete in tension, $A_{ct,}$ is that area just before the formation of the first crack, k_c is a coefficient taking into account the stress distribution across the section just before cracking for different load conditions (recommended as 1,0 for tension and 0,4 for flexure by Narayanan & Beeby (2005)). The maximum stress in the reinforcement (σ_s) is taken as the yield strength (f_y). Non-linear stress distributions resulting in higher tensile stresses at the surface of the section, and thus a lower cracking load, can occur due to internal non-uniform self-equilibrating stresses. These stresses occur if shrinkage or temperature change deformations are restrained. The coefficient k was introduced to account for this effect, with recommended values varying from1,0 for a section depth less than 300 mm deep, to 0,65 for section depths greater than 800 mm (*Narayanan & Beeby* (2005)).

EN1992-3 gives an alternative method to control cracking without directly calculating crack widths. The appropriate maximum bar diameter and spacing may be obtained from Figures 7.103N and 7.104N of EN1992-3 respectively, for a given maximum crack width and the calculated steel stress for a cracked section, as shown here in Figure 2.5. The tables apply to sections under direct tension only. For sections in pure flexure, the maximum bar diameter obtained from Figure 7.103N must be modified using Equation 7.122 of EN1992-3.

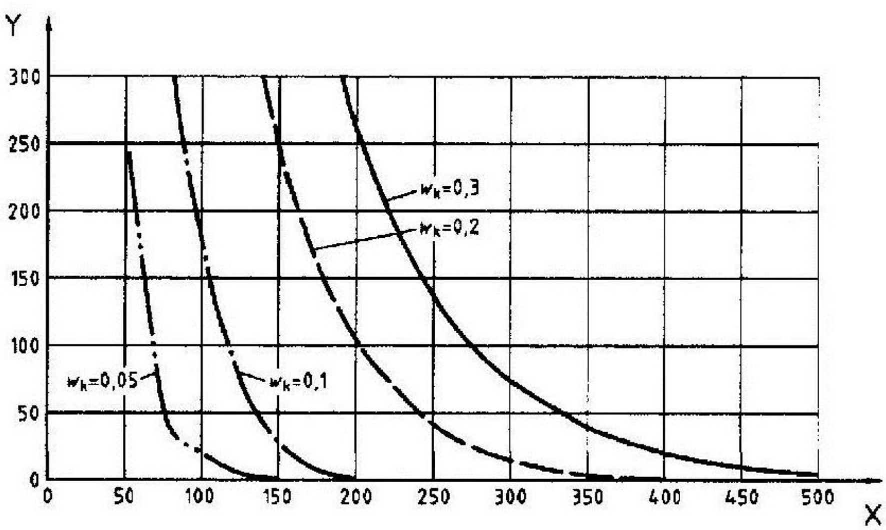
EN1992-1-1 Cl 7.3.4 mentions making provision for the effect of imposed deformations on mean strain under the relevant combination of loads. The research reported on in this thesis looks at loading effects for long term loads and mature concrete only, and where the effect of long term creep is taken into account by the use of the long-term modulus of elasticity of concrete determined using SANS10100-1 (2004).



Key

X reinforcement stress, σ_s (N/mm²)
Y maximum bar diameter (mm)

Figure 7.103N — Maximum bar diameters for crack control in members subjected to axial tension



Key

X reinforcement stress, σ_s (N/mm²)
Y maximum bar spacing (mm)

Figure 7.104N — maximum bar spacings for crack control in members subjected to axial tension

Figure 2.5: *Determination of cracking reinforcement without calculation to EN1992-3 (2004)*

Neither the British nor the Eurocode crack models take the effect of transverse reinforcement into account, such as the case for walls and slabs in WRS. Dawood & Marouk (2010) proposed a model predicting crack spacing of orthogonally reinforced concrete plate elements in axial tension and reinforced concrete slabs in flexure. In their research, experimental results were compared to various crack spacing equations, including EN1992-1-1. It was found that EN1992-1-1 underestimated the crack spacing in the slabs tested, but overestimated the crack spacing in the axially-loaded panels. In general, as the concrete cover and bar spacing were increased, the crack spacing increased for both design code equations and experiments.

In terms of research into possible future improvements of the EN1992-1-1 crack formulation, work is being carried out by, amongst others, the members of the CEB Task Group fib TG 4.1: Serviceability Models (Task Convener Prof. J. Vitek) on the New Model Code (NMC) 2010. The Model Code (MC) provides the basis of the formulation of EN 1992 -1. From the task groups' 1st draft of MC 2010 (2011) Chapter 7.6 Verification of serviceability (SLS) of RC and PC structures, the following equation (7.6-3) is proposed to calculate the maximum crack width in the stabilised cracking phase:

$$w_d = 2l_{s,max}\,(\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs})$$

where ε_{cs} is the strain in the concrete due to shrinkage and $l_{s,max}$ is the maximum length over which slip between concrete and steel occurs . The latter is used as opposed to the maximum crack spacing and is calculated using the equation 7.6-4 which is:

$$l_{s,max} = k.c + 0{,}25 \frac{f_{ctm}}{\tau_{bms}} \frac{\varphi}{\rho_{s,ef}}$$

A value of 1 is proposed for k which is the same as used in the current EN1992 formulation for the maximum crack spacing. τ_{bms} is the mean bond strength between the reinforcement and concrete and is taken as $1{,}8.f_{ctm}$ for the stabilised cracking phase. Bond strength is allowed for by the factor k_1 in the EN1992-1-1 equation for maximum crack spacing. The mean strain is calculated as in EN1992.

The following summarises comments made by Eckfeldt (2011), a member of fib TG4.1, on the proposed MC model:

- MC 90 underestimates crack widths but EN 1992 is over- conservative in some cases and less efficient.
- Regarding 2*1.0*c in calculating $2l_{s,max}$, it was agreed that c has an important influence on $2l_{s,max}$ but that 3.4*c (= 1,7.2c) used by EN 1992-1-1 is too conservative. The value of 1 for k then provides a value in the middle.
- $2^*l_{s,max}$ is proposed instead of $1.7^*S_{r,\,mean}$ because of better theoretical backup.

The final draft of MC2010 (*Eckfeldt* (2012)) gives the design crack width as an upper fractile, w_d, as:

$$w_d = 2(c + ¼*1/1.8*\phi/\rho).\ \Delta\varepsilon,$$

where the max transfer length, $l_{s,\ max}$, is

$$l_{s,max} = 2c + 2*1/4*1/1.8*\ \phi/\rho$$

The concrete tensile strength is not expressed explicitly unlike EN1992, but is implicit in the 1/1,8 ratio.

Eckfeldt (2009) in his research on small crack widths in bond-slip experiments, made the following conclusions:

- Concrete cover influences the effective area in tension.
- In determining the effective area of concrete in tension ($A_{c,eff}$) under a direct tensile force, $h_{c,eff}$ was found to be between 2,7 and 9 times the bar diameter, with an average value of 5.
- The length $l_{s,\ max}$ correlates better with the crack width than the maximum crack spacing as currently calculated in EN1992-1-1.

These conclusions are reflected in the proposed equation 7.6-3 of MC 2010.

2.3.4 Durability and material specifications

Exposure of the reinforcement to moisture results in corrosion, compromising the durability of the structure. The cover to reinforcement and good quality well-compacted concrete, as well as limiting crack widths, are key in ensuring the durability of the concrete. The general crack width limit considered to be sufficient for durability in buildings by EN1992-1-1-1 is 0,3 mm.

BS8007 makes recommendations for concrete grade, cement content, and cement-water ratio. These variables all influence the formation of cracks. A minimum concrete grade (as cube strength, f_{cu}) of 35 MPa at 28 days is specified. This value is the characteristic cube strength (probability of exceedence 5%). Eurocodes express concrete strength in terms of cylinder strength, $f_{ck,}$ where f_{ck} is approximately 0.8 f_{cu}, and is a characteristic strength (probability of exceedence of 5%). The corresponding cube strength values may be obtained from EN1992 -1-1 Table 3.1. BS8007 recommends a maximum cement content (OPC) of 400 kg/m^3 to control the heat of hydration, hence control of the development of cracks due to drying shrinkage.

Guidelines on concrete cover to reinforcement are given in BS8007 and Eurocode for WRS.

Eurocode determination of cover involves a combination of factors to Section 4 of EN1992-1-1. The two main factors in determining cover are the exposure class and the structural class of the structure. Exposure classes are defined in EN1992-1-1 (2004), while structure classes are to be defined by individual member countries. For a given exposure class, a minimum cover is selected for bond (diameter of reinforcing bar) and the appropriate environmental conditions, and class of structure. Suggested concrete grades are given in Annex E of EN1992-1-1 for a given exposure and structure class. The procedure to determine an appropriate cover is more extensive in EN1992-1-1 than that in the South African and the British standards.

Using EN1992-1-1 for an appropriate exposure class and a S3 structure, a minimum concrete grade of C30/37 (f_{ck}/ f_{cu}) and a minimum cover of 45 mm would be selected for a WRS. These values are comparable to those specified by the British code. BS 8007 specifies a nominal minimum cover of 40 mm for durability of the structure. A caution is given on increasing cover, particularly in sections less than 300 mm thick, although a maximum limit is not specified in BS 8007 Cl. 2.7.6. Crack models for flexure and tension imply that the deeper the cover to reinforcement, the wider the crack will be at the surface. This is because crack width is taken as directly proportion to crack spacing which is highly influenced by the cover. Illston & Stevens (1972) stated that crack spacing was approximately twice the cover. The current Eurocode formulation has mean crack spacing equal to twice the cover, with the additional effect of the reinforcement included.

In research aimed at the durability of concrete in beams, Tammo et al (2009) concluded that for durability, a larger cover is appropriate for severe environmental conditions as the time taken for chlorides to reach the reinforcement and carbonation to take place is proportional to the square of the concrete cover. It was also concluded that a better predictor of corrosion was the crack width at the reinforcement, which was found to be approximately half the surface crack width. However, the surface crack width was influenced more by cover than the crack width at the reinforcement. Beeby (2004) proposed that the parameter φ/ρ_{eff} (where φ is bar diameter and ρ_{eff} is reinforcement ratio) did not have much influence on crack widths but that cover did, because of its influence on the crack spacing. This proposition was supported by *Eurocode 2 Commentary* (2008). Beeby introduced the parameter into the crack spacing equation as the term ‘2c’ based on his research conclusions. Figure 2.6 summarises this research, showing the effect of cover on the transfer length, l_t, (also defined as S_o).

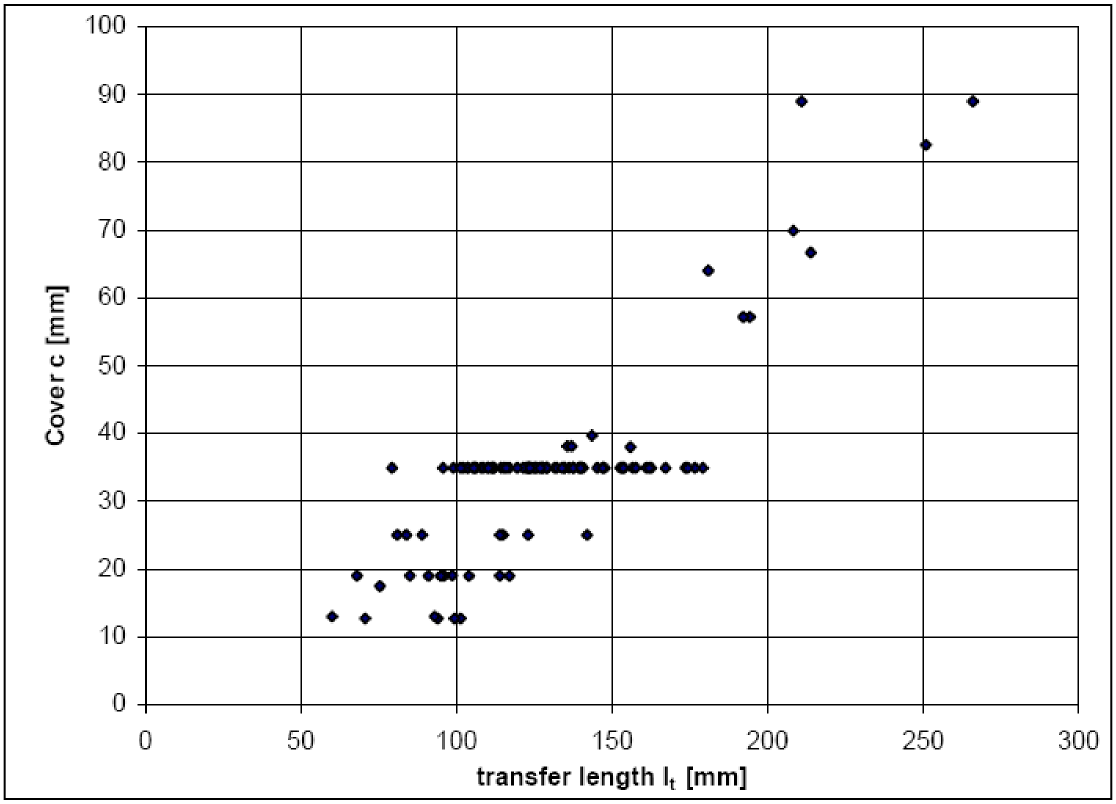


Figure 2.6 *Influence of cover on transfer length to Beeby. (Source: Eurocode 2 Commentary (2008))*

The transfer length can be written as crack width/strain and was found to be proportional to cover. Referring to Figure 2.6, most of the data was obtained for covers of 35 mm and less. At a cover of 35 mm, there is a notable spread of values for the transfer length from about 90 mm to 180 mm. There is also a gap in the data for cover between 35 and about 57 mm.

The influence of cover in determining crack widths needs further investigation, with balance between durability requirements and maximum allowable surface crack widths to ensure the permeability needed is achieved.

2.3.5 South African structural codes of practice

A review of SANS standards for reinforced concrete was done to assess any clauses that may be relevant to WRS. The scope of SANS 10100 (2000): *Structural Use of Concrete: Part 1 (Design)* covers the general rules of design for reinforced and pre-stressed concrete buildings. Crack calculations for members under direct tension or flexure appear in Annex A.3 of SANS 10100-1 and are the same as those found in BS8110 and BS8007. Rules for the minimum area of reinforcement and maximum spacing of reinforcement are laid out in Clauses 4.11.4 and 4.11.8 respectively, for the control of cracking in buildings. Annex B of SANS 10100-1 provides general information on movement joints in reinforced concrete buildings. There are no specifications or clauses that deal explicitly with water-retaining structures.

The South African design codes give some guidance on material specifications for water-retaining structures which affect cracking. Part 2 of SANS 10100 (1994): Materials and Execution of Work specifies a cement-water ratio of more than 2,0 (Cl. 10.6.2) for durability and impermeability. Exposure conditions, as laid out in Cl 3.3, for a water-retaining structure would be severe or very severe depending on the aggressiveness of the water in contact with concrete. Minimum concrete cover to reinforcement for normal density concrete for an exposure condition of severe would be 40mm. Annex A sets out a procedure for determining the aggressiveness of water in contact with concrete.SANS10160 gives guidance on partial load factors incidentally for loads applicable to WRS. These are discussed in Section 2.5 on the current practices for WRS in South Africa.

2.4 LIMITS ON CRACK WIDTHS

In order to limit cracking in a structure, the maximum expected surface crack width and spacing for a given configuration, material properties and reinforcement quantity are determined and compared to a maximum crack width limit. The structure is considered to meet the serviceability limit state (SLS) requirement for cracking if the expected maximum crack width calculated using the relevant design equation is less than the specified maximum crack width. The specified crack width limit is taken as the value that has a probability of exceedence of 5%, as determined from experimental research. Factors influencing the crack width are cover, the position, spacing and diameter of reinforcement, the type of action resulting in cracking, thus the determination of stresses in the section, and material properties. The surface crack width gives an indication of the penetration of any cracks, and therefore the durability and permeability of a structure. Leakage of a WRS would obviously compromise the function of the structure, therefore crack width limits are set to control leakage as well as protect the durability of the structure.

There is debate as to what the crack width limit should be, partly because the cracking mechanism is random, and modelling, testing and measurement of crack widths can be problematic. Beeby (2004) gave a summary of some problems encountered in comparing laboratory research, such as recording either the maximum or the average crack widths, but not both. Historically from research and industry experience (*WRC workshop, 2007)*, it has been found that that cracks less than 0,2 mm in width will self-heal if the cracked concrete element is in contact with water that is not flowing or exerting a pressure. The crack limits are therefore set lower in BS8007 than for buildings (to BS8110), that is from an average value of 0,3 mm to 0,2 mm. BS 8007 specifies a maximum allowable crack width of 0,2mm for severe/very severe exposure conditions, following the assumption that cracks will generally seal themselves for crack widths less than 0,2 mm, in structures not subjected to high pressures. Alternatively, a crack width of 0,1mm for aesthetic considerations may be chosen. Regarding a crack going through the full section in a wall or a slab under liquid pressure, Reynolds & Steedman (2008) states some initial seepage would be expected but "it is assumed that such cracks will heal autogenously within 21 days for a 0,2 mm design crack width, and 7 days for a 0,1 mm design crack width".

Eurocode allows for the maximum allowable crack width, w_{k1}, to be defined in individual member countries' National Annexes. Recommendations for maximum crack width limits for water retaining structures are given in EN1992-3. WRS's are first classified according to a tightness class defined by the requirements for protection against leakage. Table 7.105 of EN 1992-3 defines the tightness classes as follows:

Class 0	Some degree of leakage acceptable, or leakage of liquids irrelevant.
Class 1	Leakage to be limited to a small amount. Some surface staining or damp patches acceptable.
Class 2	Leakage to be minimal. Appearance not to be impaired by staining.
Class 3	No leakage permitted

Crack width limits are then recommended depending on the tightness class required. For Tightness Class 0 structures, EN1992-1-1 Cl 7.3.1 may be followed using recommendations for buildings. Class 0 structures would be those storing dry materials such as silos, thus this class would not apply to a water retaining structure.

A Tightness Class 1 structure may have some leakage, although crack healing is expected to occur where the range for service load strain is less than 150 x 10^{-6}. There may be some cracks through the full section. Cracks are to be assumed to pass through the full section if alternate actions are applied to the section. The recommended maximum crack width for this class depends on the ratio of the hydrostatic pressure (h_D, expressed as head of water) to wall thickness (h). The crack width limits as determined by the hydraulic ratio for sections cracked through the full depth of section are:

$h_D/ h \leq 5$ w_{k1} is 0,2 mm

$h_D/ h \geq 35$ w_{k1} is 0,05 mm.

Autogenous healing of any cracks is expected to take place if these crack width limits are adhered to. For intermediate values of h_D/ h, crack widths may be interpolated. To relate the hydraulic ratio to actual wall heights, for a wall height of 5m (water head of 5m and corresponding water pressure of 50kN/m^2), a wall thickness of 1 m would be required to ensure that a 0,2 mm wide crack would not normally leak. The design wall thickness for a 5m wall would usually be in the range of 400 to 500 mm for a rectangular reservoir, implying a maximum crack width limit of 0,16 to 0,175 mm. Wall thicknesses for circular reservoirs are usually less than this, meaning the maximum crack width limit would be more onerous.

Class 2 and 3 structures are expected to have cracks that do not pass through the full section. To achieve this, the depth of the compression zone is limited to a recommended value, x_{min}, the lesser of 50 mm or 0,2h (h being the section thickness). If class 2 sections do have cracks passing through the section, then it is expected that appropriate measures are taken, such as prestressing and using liners. Class 3 structures require that special measures are taken (such as liners), but no specific guidance is given on the specification of those liners.

A review of available literature was done to attempt to ascertain at what crack width autogenous healing occurs given that there does not appear to be agreement on this. Jones (2008) referred to a graph of the variation of crack width with hydraulic ratio illustrating the point at which autogenous healing was seen to have taken place in research carried out by various researchers for Tightness Class 1 structures, i.e. through cracking. It was also stated that "cracks may be expected to heal when a range of strain under service conditions is less than 150 x 10^{-6}". The graph, shown as Figure 2.7, shows where the crack width limits set by Eurocode are placed compared to other research.

The graph confirms that autogenous healing does occur for cracks less than 0,2 mm when subject to low water pressures. The graph shows that Eurocode has h_D/ h ratios higher than those by Lohmeyer and Meichner for crack widths less than about 0,17 mm. This means that for a given water height, h_D, and section thickness, h, EN1992-3 predicts that self-healing of cracks will occur at a larger crack width limit for crack widths less than 0,17 mm for a given hydraulic ratio.

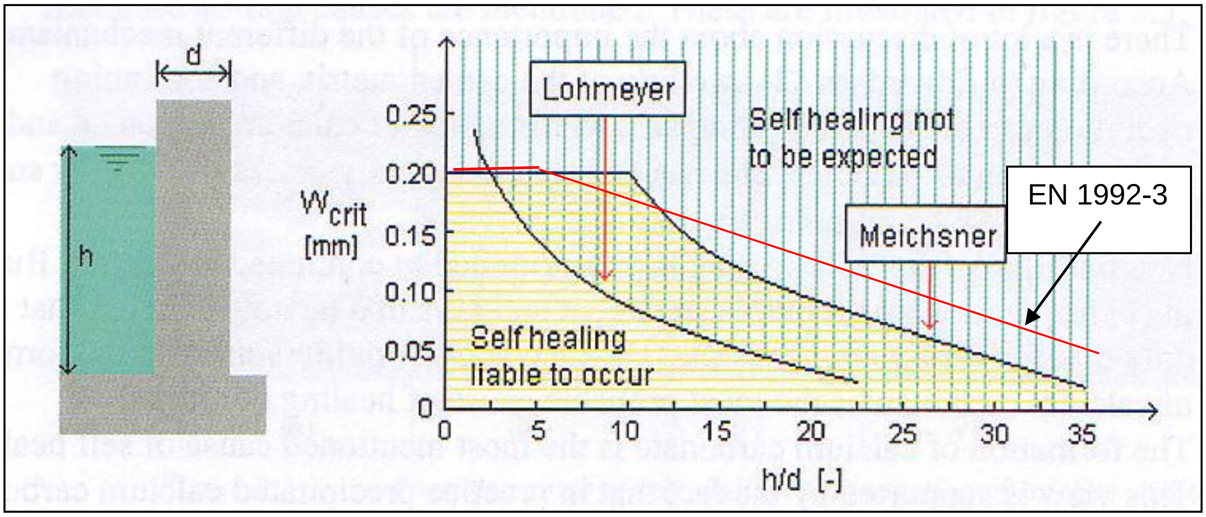


Figure 2.7: *Self-healing of cracks to Jones (2008)*

There are several mechanisms contributing to autogenous healing, such as the precipitation of calcium carbonate, continued hydration of hardened concrete resulting in the growth of hydration products and the deposition of debris, amongst others. These mechanisms and their interactions are not yet fully understood, with research ongoing. The rate decrease of flow through a crack has been found to depend on the initial flow rate and the crack width. Research has showed a rapid decrease in the initial flow through a crack within the first stages of testing (*Allen (1983)* and *Ziari et al (2009(a)& (b)),* with mixed results as to when flow ceases completely and the crack widths at which this occurs.

Ziari & Kianoush (2009a) performed self-healing tests on reinforced concrete panels under direct tension. One of their requirements was that there was to be a continuous flow of water through the crack at the start of the test. This occurred for cracks 0,25 mm and larger. It was found that for 0,25 mm cracks under a constant head of 5m, the flow rate decreased significantly within the first 7 hours and that self-healing did occur. They did however, recommend in their conclusions that direct tensile cracks should be avoided in WRS. Their research on this topic is ongoing.

In testing on panels in flexure where cracks did not go through the depth of section, Ziari &

Kianoush (2009b) found that leakage did not occur even for crack widths of 0,6 mm and depths of 200 mm. They also found that the depth of the compression zone played a key role in leakage, as well as the crack width. The effective width of the crack internally also influences the amount of leakage, and is less than the surface crack width. It should be noted that design codes do not mention the type of cracking relative to a crack limit, when the type of cracking may influence the crack limit chosen. Flexural cracks do not always go through the full section due to the compression zone, whilst cracks due to direct tension do go through the section and are therefore more likely to leak, possibly requiring a more stringent crack limit.

Seong-Tae Yi *et al* (2011) performed laboratory permeability tests on 50 mm thick sections cracked through the section, for crack widths of 0,03 mm to 0,1 mm under varying water pressures. From their results, an allowable crack width of 0,1 mm for a hydraulic pressure of 10 kN/m^2, with a reduction in crack width to 0,05 mm for a hydraulic pressure of 25 kN/m^2, was recommended. It was noted that further experimental testing was needed. In all samples tested, autogenous healing did take place, although not all cracks sealed fully. The effect of the depth of section was not considered in this study, thus the results could not be expressed in terms of a hydraulic ratio as defined by EN 1992-3. However, considering the 50 mm thickness of the samples, this research would suggest that EN1992-3 is on the conservative side for crack widths in the region of 0,1 mm.

Figure 2.8 shows a 5M*l* circular reservoir in northern KwaZulu-Natal undergoing a leakage test, performed by Eyethu Engineers (2010) before backfilling.

Figure 2.8: *Water tightness test for leakage (Source: Eyethu Consulting Engineers)*

Slight leakage was apparent at the horizontal construction joint. Staining was evident but self-healing occurred within 72 hours.

Preliminary testing performed at the University of KwaZulu-Natal over September and October 2012 (*Mans* (2012)) on cracked reinforced concrete samples to a water pressure on one cracked face showed significant self-healing of through-cracks of approximately 0,2 mm width within 72 hours. A hydraulic gradient of 12 was used and the test duration was 250 hours. Figure 2.9 is a summary of results from these tests.

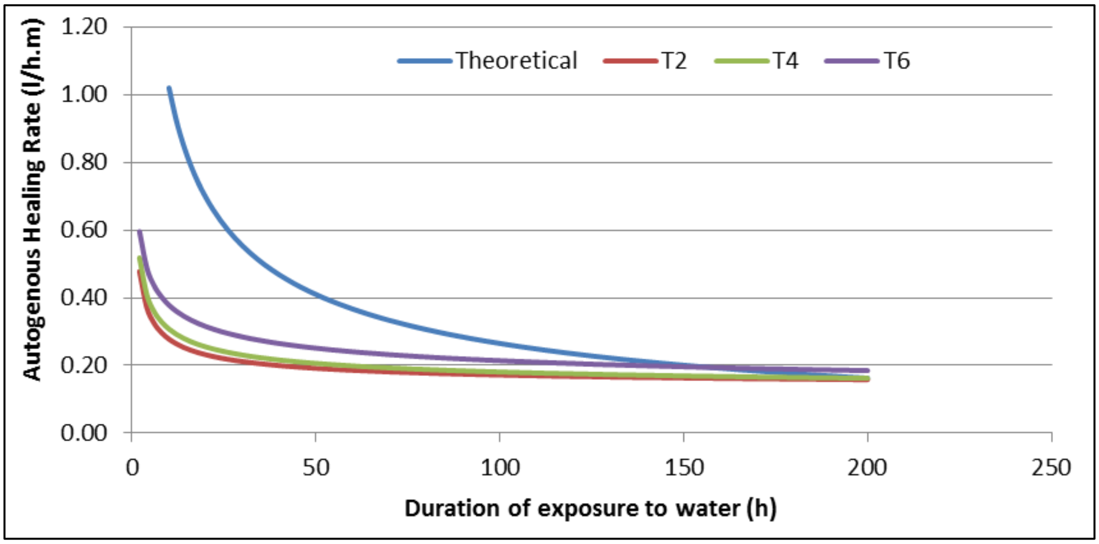


Figure 2.9: *Autogenous healing tests (University of KwaZulu-Natal, 2012)*

Deposition of calcium carbonate was observed over the duration of the tests, as shown in Figure 2.10, which aided the self-healing process.

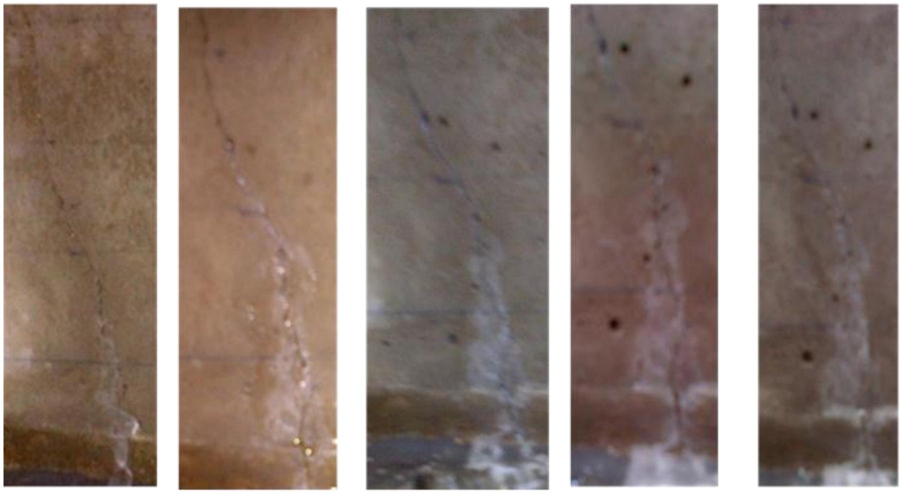


Figure 2.10: *Deposition of calcium carbonate over time during testing (Mans (2012))*

A case study of the circular 5M*l* reservoir, Ofudu, near Marianhill in KwaZulu-Natal was also done as part of the research. According to Bowles (2012), the reservoir developed cracks soon after construction and continued to be active for a period. Surface cracks were found to be from hairline to up to widths of 0,6mm. Autogenous healing did occur in some of the cracks, evident by deposits of white calcium carbonate precipitation. Remediation works were done to seal some of the larger cracks. Figure 2.11 shows the leakage of the reservoir in September 2011. (*Umgeni Water (2012)).*

Figure 2.11: *Ofudu water reservoir leakage observed in September 2011 (Source: Umgeni Water)*

A site visit was made in October 2012 as part of the case study. Self-healing of cracks that had previously experienced leakage was observed. Figure 2.12 shows the calcium carbonate precipitation that had occurred.

Figure 2.12: *Calcium carbonate deposition, Ofudu water reservoir, October 2012.*

The cracks shown were measured using a crack microscope and were found to be approximately 0,2mm in width. Self-healing was also found in cracks as large as 0,4 mm.

Self-healing is further demonstrated in Figure 2.13. An interesting observation made on site was that cracks occurred at a regular spacing of approximately 900 mm in the bottom third of the wall.

Figure 2.13: *Site visit to Ofudu water reservoir, October 2012*

It should be noted that when applying any crack model, the effective width of a crack through a section is not necessarily the crack width on the surface, nor is the crack profile uniform through the section. The effective crack width may be less than the surface crack width (taken as the limiting crack width) which then may make it appear that self-healing has taken place at a larger crack width than is actually the case. A non-uniform profile would not necessarily give the same results as a more uniform one. Researchers therefore need to take note of this in predicting crack widths to ensure water-tightness of a structure.

2.5 CURRENT PRACTICES FOR WATER RETAINING STRUCTURES IN SOUTH AFRICA

2.5.1 General

In order to select a structural element in modelling cracking due to loading in water retaining structures (WRS) appropriate to South African conditions, an investigation into the current industry practices for the design and construction of WRS in South Africa was carried out. Reservoirs only were considered, as structures such as tanks and water towers have combined forces induced in them by loading due to their configurations and geometry. Elements were considered in terms of the type of action effects inducing flexural and direct tension modes of cracking. Cracking due to restrained deformations such as temperature and shrinkage effects do not form part of this research, although later modifications to the model could be made to include these effects. Information was gathered from sources such as industry surveys. These included surveys done as part of a 4th year undergraduate research project at the University of Stellenbosch (*Roux, 2007*) and a workshop held in Stellenbosch, October, 2007 (headed by Prof J. Wium, *WRC Workshop 2007*). Summaries of these industry surveys are found in a progress report to the Water Research Commission (*Wium, 2007b*). Information was also collected from a 4th year undergraduate research project done at the University of KwaZulu-Natal (*Mucambe, 2007*) and various discussions with Mr S. Scruton (City of Durban) and Mr S. Naidu (Naidu Consulting, Durban). Information gathered was also used in the compilation of a draft South African code of practice for WRS (2010) which will become the future SANS10100-3*: Design of water-retaining structures*. Information on South African design practice that is relevant to this research is summarised in this section.

2.5.2 General configurations of WRS in South Africa

Reinforced concrete water retaining structures are commonly either rectangular or circular in plan. Depending on the geology and geographical location in South Africa, and the preference of the client, reservoirs are often buried partially or fully. Under these circumstances, the reservoir is constructed on excavated ground, with a fill embankment constructed after the completion of the reservoir structure. Sizes vary according to the required storage capacity. This study considers capacities greater than approximately 500k*l* and less than 10M*l.* Figure 2.14 overleaf shows some typical reservoir configurations.

2.5.3 Design practice

The current design practice is to follow the guidelines and procedures set out in BS8007 in conjunction with BS8110, as a South African code of practice for WRS does not exist at present. BS8007 was a logical choice, as the South African code of practice for reinforced concrete was historically based on the British codes of practice. South African engineers use their experience

(a) Buried rectangular reservoir, Durban, KwaZulu-Natal (Source: Google Earth (2011)

(b) *Partially buried circular reservoir, Waterfall, KwaZulu-Natal*

Figure 2.14: *Typical configurations for water reservoirs, KwaZulu-Natal*

to adapt BS8007 to local conditions where necessary. For example, in the check for restrained cracking, designers use local temperatures, rather than the British values to Table A2 of BS 8007 (*WRC Workshop, 2007*). A few designers make use of EN1992 at present, but these are in the minority.

The design life for WRS is taken as 50 years, with the corresponding assumptions of good design, durable materials and proper construction practices. Reservoirs are designed to resist the worst ultimate load case or cases by determining the amount of reinforcement required for a particular configuration. Serviceability checks, such as cracking, are then performed. The serviceability limit state for cracking tends to dominate the design, with an increase necessary in the quantity of ultimate limit state reinforcement required.

Actions on WRS in design include self-weight (permanent), imposed loads, geotechnical actions, seismic actions, liquid loads and temperature and restrained shrinkage effects. Liquid loads are considered as quasi-permanent by EN1990 and as permanent by BS8007 and SANS10160, assuming relatively constant water levels and no rapid emptying or filling of the reservoir so as not to have unwanted secondary load effects. Seismic actions are only allowed for in seismically-sensitive areas of South Africa and were not considered in this study. The effect of load actions depends on the configuration and structural element considered. Load factors used are to SANS 10160 or BS 8110. Ultimate load factors for permanent load (DL) are 1,2, 1,35 and 1,4 for SANS, Eurocode and BS respectively, for the ULS load combination of 1,2 DL + 1,6 LL, where LL is the live load. The alternative ULS load combination of 1,35 DL + 1,0 LL can also be considered. Note that EN1990 specifies an ultimate load factor of 1,35 for a quasi-permanent load, such as liquid load. Serviceability load factors for all three codes are commonly taken as 1,0 for liquid loads. A crack limit of 0,2 mm is specified for serviceability, with a limit of 0,1 mm used if aesthetics are a consideration, as per BS 8007.

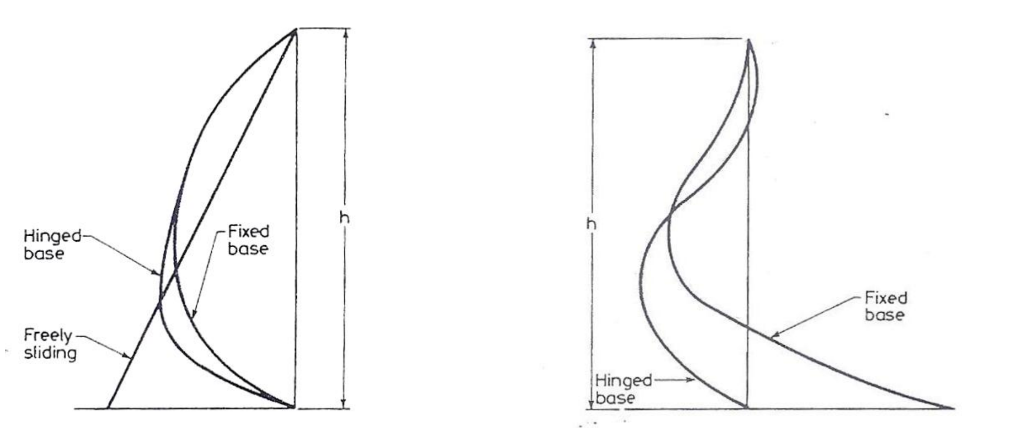
Three main structural elements are considered in the design of a WRS:

(i) Walls

The load cases and effects are dependent on the shape and proportions of the structure, the most common being circular and rectangular, and the type of construction. Walls may be either continuous or jointed vertically. Horizontal joints are generally construction joints. Partially or fully buried reservoirs will obviously have soil action on the exterior face of walls.

Rectangular reservoir walls tend to be designed as cantilever walls with the top of the wall either free or propped. The base of the wall is generally fixed. Ignoring the self-weight of the structure, a cantilever wall (top free) has a bending moment induced about a horizontal axis (max at base of wall) by the liquid pressure. This is obviously at a maximum when the reservoir is full and without soil backfill in the case of buried reservoirs. An alternative load case is the reservoir empty with soil pressure on the exterior face of the wall. Any flexural cracking induced in both load cases would be perpendicular to the vertical reinforcement provided to resist the ultimate bending moment. Bending moments in the horizontal plane are induced at the corners along fixed vertical edges along with a direct tensile force. Any flexural cracking induced would obviously then be in a vertical direction near the corners. Smaller rectangular reservoir walls act as vertical slabs restrained on 3 sides, therefore having horizontal and vertical moments induced. Stability against overturning is also checked.

The dominant force induced in circular reinforced concrete reservoir walls is axial ring tension in the horizontal plane due to hoop stress from the liquid load. The magnitude of the ring tension is dependent on the fixity between the wall stem and base, with the top of the wall free. Walls with sliding joints at the base have the maximum tensile force $T = \gamma_w r z$ induced at the base of the wall, where γ_w is the unit weight of water, r is the internal radius of the reservoir and z is depth of water. Any bending moments induced in the vertical direction are neglible (about a vertical axis). Walls that are pinned or fixed at the base have a distribution of ring tension over the height of the wall, as shown in Figure 2.15 (a). The ring tension is zero at the base of the wall for both fixity conditions. The liquid load on the wall will also induce a vertical bending moment in the wall (about a horizontal axis) due to restraint of the wall by the base, the profile of this vertical bending moment depending on the type of joint between wall and base (fixed or pinned), as shown in Figure 2.15 (b).



(a) Ring Tension (b) Vertical Bending Moment

Figure 2.15: *Distribution of ring tension and vertical bending moment over height of wall (Source: Anchor et al (1983))*

The equations for the ring tension and vertical bending moment are derived from the fundamental equation for the radial displacement at a depth y from the top of the wall, namely,

$$w=\frac{\gamma r^2}{Et}y$$

where E is Young's modulus and t is section thickness. Differentiating with regard to y, the equations for ring tension and vertical bending moment become

$$T=\frac{Et}{r}w \quad \text{and} \quad M = -EI\frac{d^2w}{dy^2}$$

respectively (Bhatt *et al* (2006)).

The ring tension and the bending moment are thus dependent on the ratio $y^2/2rh$ where h is the depth of water. For easier use, charts have been developed from the equations, such as those in Reynolds and Steedman (2008). The vertical bending moment for a circular reservoir is smaller than that in a rectangular reservoir. From the industry survey, it was found that a representative ratio between the diameter of a circular reservoir and the height was 4 for reservoirs having a capacity of less than 10M*l*. Niehaus, at a WRC industry workshop, stated that a sliding joint at the base of the wall stem is usually used in pre-stressed concrete circular reservoirs in South Africa, rather than in reinforced concrete WRS. (*Wium (2007(a)))*.

Shear forces induced at the base of the wall are checked to ensure an adequate depth of section is used, although shear is usually not a dominant action effect. Shear reinforcement is not provided to the wall.

(ii) Floor slabs and Foundations

Floor slabs may be cast such that they are discontinuous with the walls (larger reservoirs) or continuous i.e. no joint at the intersection between floor slab and wall (tanks). For rectangular reservoirs, reinforced concrete slabs are cast in square panels, whilst a radial casting pattern may be utilised for circular reservoirs. Panel casting sequence, joints and reinforcement are used to control the cracking due to the dominant action of serviceability limit state (SLS) restrained deformations due to temperature and moisture effects. Where required, the ultimate limit state (ULS) case of uplift due to groundwater is checked.

(iii) Roof slab

Currently, the most common type of construction for the roof slab of a reservoir is a flat slab supported on columns, with heads if necessary to resist punching shear. Figure 2.16 shows the interior of a reservoir using this type of construction.

Figure 2.16: *Interior of reservoir – flat slab and column construction (Mucambe (2007))*

Loads include self-weight, the permanent load of protection materials (stone, soil etc) and imposed loads due to construction and later maintenance, inducing bending moments in two directions. Serviceability checks for deflection and cracking are done. Serviceability limit state restrained deformations due to temperature and moisture effects may occur, although the slab is usually relatively unrestrained at edges, allowing some movement due to these effects.

2.5.4 Material properties specified

Durability, not strength, is the criterion for the materials chosen for WRS, as durable concrete will have adequate strength resistance and the impermeability necessary for proper functioning of the structure over its design life. A characteristic concrete strength (as cube strength at 28 days) of 35MPa is commonly chosen. To limit shrinkage, slagment in cement is generally avoided, as is rapid-hardening cement. A minimum cement content of 375 kg/m^3 of cementious material is the accepted value for good-quality concrete. The maximum cement content is however limited to 500kg/m^3 to control thermal shrinkage. The water-cement ratio is also key to ensuring concrete that has the specified strength and impermeability, with a maximum ratio of 0,5 usually specified. To ensure adequate protection of reinforcement, the concrete cover generally chosen is 40mm, with a 50 mm cover against soil. High-yield reinforcement with a characteristic strength of 450 MPa is standard in South Africa.

2.6 SUMMARY

The review of cracking models and design codes, and current industry practices in South Africa for serviceability cracking with respect to reinforced concrete WRS revealed the following:

(i) Cracking is defined a serviceability limit state. Cracking is controlled in design by means of a limiting crack width which may not be exceeded. The section dimensions and reinforcement for the structure element considered are then chosen according to that limit. Current design practice in South Africa is the use of BS8007. The design equations and general parameters used in industry to calculate SLS load-induced crack widths to EN1992 and BS8007 have been established.

(ii) The serviceability limit state of cracking is found to be the dominant limit state in design, rather than the ultimate limit state.

(iii) More onerous crack limits are imposed by Eurocode than those of the British design codes, specifically, a lower limit of 0,05 mm for EN1992-3 and 0,1 mm for BS8007. This could have negative economic consequences which need to be investigated, as an increase in reinforcement would be required as the crack width limit decreases. In addition, there is no consensus as to the appropriate crack width for sections under water pressure.

(iv) Research done on cracking in concrete tends to be for a specific structural and loading configuration with crack models largely developed empirically. In addition, as cracking is a random phenomenon which results in a wide variation in results, it is difficult to generalise a crack model for all cracking cases. The uncertainty in the crack models is thus not really known.

(v) The structural configurations resulting in the worst loading case for either tension or bending were investigated. The wall of a reservoir is subject to water pressure and possible leakage, therefore is a critical design element. The structural configuration that will result in the greatest flexural load is a wall section of a large rectangular reservoir under loading due to water pressure, inducing a bending moment about a horizontal axis. A section of wall in a circular reservoir under hoop stress due to water pressure will be a critical configuration for the direct tension load case. Important parameters to consider in the crack model are cover, section thickness and load due to water pressure.

The issue of serviceability cracking in water retaining structures due to loads under South African conditions will be investigated in the remainder of this thesis with respect to the following objectives which have been identified in the literature review:

(i) A comparison between BS8007 and EN1992 design equations for cracking, quantifying the similarities and differences between the two codes.

(ii) The extent to which serviceability cracking dominates the design of WRS, as opposed to the ultimate limit state, by performing calculations to BS8007 and EN1992, and by means of a reliability analysis.

(iii) The effect of the lower crack width limit of 0,05 mm of EN1992 compared to the general limit of 0,2 mm to BS8007.

(iv) The reliability of the EN1992 crack model. This will be investigated by means of a reliability analysis.

CHAPTER 3

DESIGN ANALYSIS FOR CRACKING TO EN1992-1-1 &3 AND BS 8007

3.1 GENERAL

The investigation into South African current practices and Eurocode in the design of WRS, to Chapter 2, identified the following four critical issues with respect to load-induced cracking:

(i) How EN1992 compares to BS8007 when applied to South African conditions.

(ii) The implications of the more onerous crack width limit of EN1992-1-1 compared to that specified by BS8007. This needed to be assessed to establish whether or not this is a critical issue.

(iii) The extent to which serviceability cracking governs the design of water retaining structures (WRS), rather than the ultimate limit state. If the serviceability limit state (SLS) cracking criteria are not met in a WRS, leakage is likely and therefore a greater loss of function of a WRS as opposed to a building structure. This makes serviceability the more important design limit state rather than the ultimate limit state. The motivation for performing a reliability analysis of the EN1992 crack model is thus strengthened if serviceability cracking is the dominant state.

(iv) The reliability of the EN1992 crack model with respect to South African conditions.

Design calculations for SLS load-induced cracking were done to BS8007 and EN1992 to compare the two codes. These are presented in this chapter along with the results and conclusions. The calculations were also used to assess which limit state is dominant, which crack modes are critical and to establish a representative physical model for use in the reliability analysis of the EN1992-1-1 crack model, detailed in Chapters 4 to 7. Certain parameters needed for the reliability evaluation were investigated, the results of which are presented in this chapter.

Performing calculations to satisfy a fixed-value SLS load-induced cracking criterion using a design code in the design of a WRS, in this case BS8007 and EN1992, represents a deterministic analysis, as opposed to a probabilistic analysis. Provision is made in the design code to meet the required level of SLS performance by means of a fixed-value limiting crack width and by using design values for material properties and loads. The deterministic analysis process was begun by establishing a typical structural configuration and material parameters for each load cracking case (tension and flexure) using the information gathered in the review of South African design and construction practices of WRS. The design calculations to calculate the maximum crack widths to EN1992-1-1 and BS8007 for both the tension and flexural cracking cases were then set up using Excel spreadsheets. The crack calculation procedures are summarised in the following section.

3.2 DETERMINISTIC CRACK ANALYSIS MODELS TO BS8007 AND EN1992-1-1

3.2.1 Structural configuration and loading

Typical loading conditions on the main structural elements in circular and rectangular reservoirs leading to cracking in direct tension or flexure were investigated. The structural configurations of a representative reservoir resulting in the worst tensile and flexural cracking conditions could then be chosen for the deterministic analysis models. The wall of a reservoir would be more critical than other elements in the structure with regard to cracking and leakage. Based on this, a 1 m length of wall subject to water pressure only over the full height of the wall (H) for both flexural and tensile load cracking cases was considered. The liquid load was considered as a quasi-permanent load.

(i) Flexure load case

The worst flexure case would be found in the wall of a rectangular reservoir with bending (M) at about the vertical axis due to liquid load (L_k), as shown in Figure 3.1.

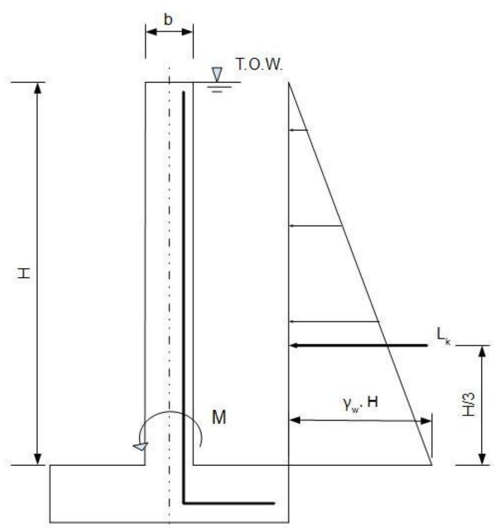


Figure 3.1: *Rectangular reservoir wall configuration*

The proportions of the rectangular reservoir were chosen such that the wall acts as a cantilever wall with a fixed base. The top of the wall is not propped. The liquid load has a triangular distribution with the maximum bending moment occurring at the base of the wall.

(ii) Tension load case

The worst tension case would be in the wall in a circular reservoir, in the horizontal plane. The configuration chosen for the tension model was a circular reservoir with a diameter (D) to wall height (H) ratio in the region of 4, with hoop tension in the horizontal plane due the liquid load, as shown in Figure 3.2. The wall has a sliding base and is unpropped at the top, such that the reservoir behaves as a thin walled cylinder under a constant internal pressure due to the liquid load. This configuration results in the maximum direct tensile force in the wall with no bending. A ratio of 4 between wall height and diameter was identified from the industry survey as being representative of circular reservoirs of up to a 10 M*l* capacity.

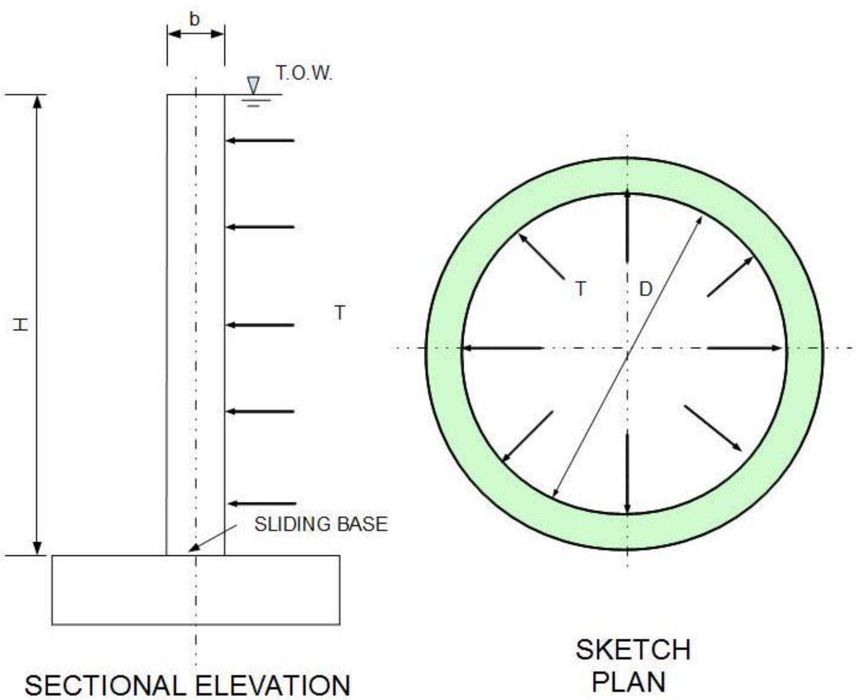


Figure 3.2: *Circular reservoir configuration*

As the fixity of the base (conditions of pinned, fixed or sliding) influences the magnitude and position of the hoop tension, the critical condition resulting in the worst tensile force and bending moment was investigated. The pinned and fixed conditions were compared to the sliding base condition using Table 2.75 of Reynolds and Steedman (2008) to calculate the tensile forces and bending moments induced in the base of the wall stem. The equations calculating the service tensile force and vertical bending moment have the form of $T = \alpha_n \gamma l_z r$ and $M = \alpha_m \gamma l_z^3$, respectively, where α is the coefficient from Reynolds' *et al* (2008) Table 7.5, γ is the unit weight of water of 10 kN/m^3 , l_z is the height of the wall and r is the radius of the reservoir. The coefficients depend on the ratio $l_z^2/2rh$.

The tensile force for the sliding condition is calculated from T = L.D/2 where L is the liquid load equal to γl_z and D is the diameter of the reservoir. This equation can be rewritten as $T_{sliding}$ = 1,0. $\gamma l_z r$. From Reynolds' *et al* (2008) Table 2.75, all α_n are less than 1,0 for both the pinned and fixed conditions, therefore $T_{sliding}$ will be the maximum hoop tension. Results from calculations for a 28m diameter reservoir and wall heights of 5m and 7m are summarised in Table 3.1.

Table 3.1: *Summary of calculations of SLS ring tension and vertical bending moment using Reynolds' et al (2008) Table 2.75 for circular reservoir configuration.*

			α_n		α_m		T (kN)			M (kNm)		
H (m)	h (mm)	$\frac{l_z^2}{2rh}$	Fixed Base	Pinned Base	Fixed Base	Pinned Base	Fixed Base	Pinned Base	Sliding Base	Fixed Base	Pinned Base	Sliding Base
5	250	3.57	0.400	0.553	0.0295	0.0133	280.0	387.1	700.0	36.88	16.63	0
5	450	1.98	0.274	0.434	-0.0436	0.0219	191.8	303.8	700.0	-54.50	27.38	0
7	250	7.00	0.545	0.670	-0.0167	0.0068	534.1	656.6	980.0	-57.28	23.32	0
7	450	3.89	0.421	0.572	-0.0275	0.0120	412.6	560.6	980.0	-94.33	41.16	0

Tensile forces for the pinned and fixed conditions are between about 0,55 and 0,67 $T_{sliding}$, depending on the section thickness. The calculations also confirm that a rectangular reservoir wall has a maximum bending moment greater than that for a circular reservoir by at least a factor of 3,8 for the reservoir dimensions chosen, therefore is the critical configuration for bending. The value of 250 mm for h would be considered as a minimum wall thickness from a practical consideration.

3.2.2 Design parameters

Typical South African values were used for the material properties, as follows:

Concrete compressive strength (characteristic value)	f_{ck}/ f_{cu}	30/37 Mpa
Concrete tensile strength (mean value)	$f_{ct,eff}$	2,90 Mpa
Reinforcement yield strength (characteristic value)	f_y	450 MPa
Modular ratio	α_e	15
Modulus of elasticity, steel	E_s	200 GPa
Density of water	γ_w	9,81 kN/m^3

Values of 30/37 for f_{ck}/ f_{cu} were chosen so as to be consistent with EN 1992-1-1 tabulated values. The characteristic strength of a material corresponds to the mean strength, f_m, less 1,64 times the standard deviation, s, with the mean and standard deviation for a material determined from standard materials testing. However, the tensile strength of concrete is an exception as the mean value is used as opposed to the characteristic value. The concrete tensile strength $f_{ct,eff}$ is taken as f_{ctm} (mean value of tensile strength) and was calculated using the EN1992-1-1 formula:

$$f_{ctm} = 0.3\, f_{ck}^{2/3}$$

The modular ratio is the ratio E_{steel} /$E_{concrete,\ long\ term}$.

The physical parameters of height of wall (H) thus liquid load (L_k), section thickness (h), cover (c), the diameter (φ) and area of the tension reinforcement were chosen as follows:

(i) A wall height of 5m, being reasonable for a reinforced concrete water retaining reservoir, was chosen as the reference height for the analyses using BS8007 and EN1992. In the EN1992 deterministic analysis for later comparison with the reliability analysis, wall heights corresponding to the depth of water of 5m, 6m and 7m were chosen. The wall heights of 5, 6 and 7m result in maximum service liquid loads of 50 kN/m^2, 60 kN/m^2 and 70 kN/m^2 respectively and maximum ultimate liquid loads (γ = 1,2) of 60 kN/m^2, 72 kN/m^2 and 84 kN/m^2 for both the rectangular and the circular reservoir configurations.

(ii) For the comparison between BS8007 and EN1992-3, section thicknesses from 250 mm increasing in 50 mm increments were considered for a wall height of 5m.

(iii) The reinforcement cover was taken as 40 mm in all analyses investigating the effect of wall thickness and bar diameter, as this is the value most commonly used in practice for WRS in South Africa. The effect of concrete cover as a parameter was investigated by carrying out the analyses using a cover of 50 mm and then comparing the crack widths obtained to those calculated for a cover of 40 mm, using a bar diameter of 20 mm and section thickness of 450 mm.

(iv) Bar diameters of 16, 20 and 25 mm were chosen as they are those typically used in reinforced concrete WRS. Diameters larger than 25 mm result in wider cracks forming, while diameters smaller than 16 mm result in excessively small reinforcement spacings.

(v) In defining a representative range of reinforcement area (A_s), a maximum feasible limit was determined by considering a minimum practical spacing of bars, namely, a 75 mm spacing, given single reinforcing bars. The SANS10100-1 design code rules for minimum and maximum areas of reinforcement were also observed. SANS10100 Cl. 4.11.5.3 specifies that the total maximum reinforcement area (as ρ% = % A_s = 100A_s/A_c) for a beam is 4% of the gross cross sectional area, A_c for either the tension or the compression reinforcement. Guidance is not given for the maximum reinforcement to be provided in the horizontal plane of the wall of a structure such as a circular reservoir under tension. Using a minimum feasible bar spacing of 75 mm resulted in areas less than 4%, therefore tended to be the limiting criterion, rather than the maximum reinforcement area. Table 3.2 summarises the total maximum reinforcement that can be specified for a given wall thickness using bar spacing as the limiting criterion, for one face of the wall.

Table 3.2: *Maximum reinforcement area per face (*%A_s*) for minimum 75 mm bar spacing.*

	%A_s		
h mm	Y16 – 75 (2681 mm^2)	Y20 – 75 (4189 mm^2)	Y25 – 75 (6545 mm^2)
250	1.07	1.68	2.62
300	0.89	1.40	2.18
350	0.77	1.20	1.87
400	0.67	1.05	1.64
450	0.60	0.93	1.45
500	0.54	0.84	1.31
550	0.49	0.76	1.19
600	0.45	0.70	1.09
650	0.41	0.64	1.01
700	0.38	0.60	0.94
750	0.36	0.56	0.87

Note: A_s is per single layer of reinforcement

Crack width limits over the range of 0,2 to 0,05 mm were considered, as specified by the BS8007 and EN1992-3 design codes respectively. In the case of EN1992-3, as summarised in Section 2.4.1 of Chapter 2, the crack width limit for cracks through the section is determined by the hydraulic ratio of wall height to wall thickness (h_D/ h) which results in smaller limiting crack widths than 0,2 mm. Limiting crack widths are thus as follows:

h_D/ h ≤ 5 w_{k1} is 0,2 mm

h_D/ h ≥ 35 w_{k1} is 0,05 mm.

3.2.3 Ultimate limit state of loading calculations

The ultimate limit state for both the tension and the flexure load cases was calculated. The section geometry was chosen for given bar diameters and wall heights, given the spacing of reinforcement that can be practically fixed. ULS reinforcement quantities (A_{ULS}) were also calculated in order to confirm that serviceability is the dominant limit state. The ratio of serviceability to ultimate reinforcement required, A_{SLS}/A_{ULS}, was used to indicate the degree of dominance. If serviceability cracking is the dominant limit state in the design of WRS, the A_{SLS}/A_{ULS} ratio will be greater than one. The ULS reinforcement for flexural and tension loading is the same for both BS8007 and Eurocode.

Water pressure on the rectangular reservoir wall in the flexural load case will result in a triangular load distribution from 0 at the top of the wall to a maximum at the base of γ_w H. As already stated, the partial load factor of 1,2 was used for the quasi-permanent liquid load. Ultimate maximum bending moments (M_u) for the flexure condition were then calculated using

$$M_u = 1{,}2\left(\frac{1}{2}L_k H\right).\frac{H}{3} = 1{,}2.\frac{1}{6}\gamma_w H^3$$

The maximum direct tensile force (T_u) for the direct tension case was calculated using

$$T_u = 1{,}2\ L_k\ \frac{D}{2} = 1{,}2\gamma_w H\ \frac{D}{2}$$

where D is the diameter of the reservoir.

The reinforcement required for the ultimate limit state bending moment at the base of the wall for the flexure model was calculated using the equations of SANS 10100-1 Cl 4.3.3, which use the simplified rectangular-parabolic stress block theory for tension reinforcement only. The procedure is as follows:

1. Calculate the lever arm $z = d\left\{0{,}5 + \sqrt{\left(0{,}25 - \frac{K}{0{,}9}\right)}\right\} \leq 0{,}95d$

where d is the effective depth of section and

$$K = \frac{M_u}{bd^2 f_{cu}}.$$

For tension reinforcement only, K ≤ K’ = 0,156 and the redistribution of moments is less than 10%. The width of the section (b) is 1 m.

2. Calculate the tension reinforcement required in the tension face of the wall (i.e. one face) from:

$$A_s = \frac{M_u}{0{,}87\ f_y z}$$

The ULS reinforcement required for the tension model was calculated using:

$$A_s = \frac{T_u}{0{,}87 f_y}$$

Note that A_s for the tension model is the reinforcement required for both faces as T_u is the total ultimate tensile force in the wall.

3.2.4 Serviceability limit state of cracking due to loads

The models were set up in order to calculate the crack widths (w) over a range in reinforcement area for a given bar diameter. Graphs for comparison could then be plotted for the variation of crack width with reinforcement (as $\%A_s$, area in mm^2 or as ratio A_{SLS}/A_{ULS}). The serviceability bending moments (M) were calculated using:

$$M = \left(\frac{1}{2} L_k H\right) . \frac{H}{3} = \frac{1}{6} \gamma_w H^3$$

with the serviceability tension (T) calculated using:

$$T = L_k \frac{D}{2} = \gamma_w H \frac{D}{2}$$

The SLS crack widths were then calculated using the relevant equations in each code for a given reinforcement diameter and spacing, hence area of reinforcement, for both the tension and flexure crack conditions. These equations were presented in 2.3.2 and 2.3.3 of Chapter 2 for BS8007 and EN1992, respectively. Stresses and strains were calculated using linear elastic theory. In the SLS flexural cracking model, the section was designed to have tension reinforcement only. In the SLS tension cracking model, a constant tensile stress distribution was assumed across the full thickness of the wall. The procedures predicting the expected maximum crack width to each code for a given wall geometry and material properties are as follows:

BS8007 crack calculations procedure

(i) Flexure

1. Use the SLS bending moment.
2. Assume a bar diameter (φ) and spacing (s).

3. Calculate the depth of the neutral axis (x) based on the principle of equivalent areas and the lever arm $z = d – x/3$.
4. Calculate the distance from the crack to the nearest reinforcing bar (a_{cr}). For a typical reservoir wall section a_{cr} is determined using equation 2.1.
5. Determine the service steel stress $f_s = M/ A_s z$ and thus the strain at the level of the steel, $\varepsilon_s = f_s/ E_s$. For flexure, the depth of the neutral axis, x, is calculated based on the principle of equivalent areas and thus, the lever arm, $z = d – x/3$.
6. Calculate the apparent strain at the surface (ε_1) using Equation 2.5.
7. Calculate tension stiffening in the concrete in tension using Equation 2.6(a) or 2.6(b), depending on the limiting crack width chosen.
8. Determine the mean strain, $\varepsilon_m = \varepsilon_1 – \varepsilon_2$
9. Calculate the expected maximum crack width for flexure using Equation 2.1.

(ii) Tension

1. Use the SLS tensile force.
2. Assume a bar diameter and spacing.
3. Calculate the distance from the crack to the nearest reinforcing bar (a_{cr}) as for flexure.
4. Determine the service steel stress $f_s = T/ A_s$ and thus the steel strain ε_s (= ε_1) = f_s/ E_s.
5. Calculate tension stiffening in the concrete in tension, using Equations 2.7(a) or 2.7(b), depending on the limiting crack width chosen.
6. Calculate the mean strain, $\varepsilon_m = \varepsilon_1 – \varepsilon_2$.
7. Determine the expected maximum crack width for tension using Equation 2.2.

EN1992-1-1 crack calculation procedure

EN1992 uses the same equations for the crack width and spacing for both tension and flexural cracking. The procedure used for the analysis is as follows:

1. Use the SLS bending moment or tensile force, as relevant.
2. Use the same bar diameter and spacing as the BS8007 analysis to obtain the area of tension reinforcement (A_s) to enter into the crack width equation.
3. Determine the effective depth of the tension area $h_{c,eff}$, taken as the lesser of h/2, 2,5(h - d) and (h – x)/3, and so the effective area in tension $A_{ct,eff}$ with a section width b of 1m. The effective depth for the tension case will either be h/2 or 2,5(h-d).
4. Calculate the maximum predicted crack spacing using equation 2.11.

5. Calculate the steel stress which will be $\sigma_s = M/ A_s z$ for flexure and $\sigma_s = T/ A_s$ for tension, as set out in the procedure above to BS8007. Thus the mean steel strain $\varepsilon_{sm} = \sigma_s / E_s$ is calculated.
6. Determine mean concrete strain using equation 2.9, where $k_t = 0,4$ for long term loading.
7. Determine the maximum crack width using equation 2.10.

Excel spreadsheets were set up to calculate and record the crack widths for varying parameters and reinforcement quantities for both crack models. Graphs showing the variation of crack width with reinforcement area were plotted. The reinforcement area was expressed in terms of ρ as $\%A_s = A_s/A_c$ where A_c is the gross cross sectional area of the wall section, and in terms of the ratio serviceability reinforcement area / ultimate reinforcement area (A_{SLS}/ A_{ULS}). Some representative graphs are presented here. Data sheets and additional graphs are included in Appendix 1.

3.3 RESULTS AND DISCUSSION

Results of the deterministic analysis of BS8007 and EN1992-3 serviceability limit state crack equations for flexural and tensile load-induced cracking in WRS are presented in this section with respect to the following objectives of the deterministic analysis:

(i) A comparison between BS8007 and EN1992 for SLS flexural and tension cracking.
(ii) The implications of the more onerous crack width limits of EN1992-1-1 than BS8007.
(iii) The extent to which serviceability cracking governs the design of WRS, rather than the ultimate limit state.
(iv) Choice of representative parameters for use in the reliability analyses of the EN1992-1-1 crack model, presented in Chapter 5.

Graphs for comparison were plotted as the variation of crack width with $\%A_s$ or A_{SLS}/A_{ULS}. As stated earlier in this chapter, the $\%A_s$ values given are the tension reinforcement only for flexural cracking (i.e. one face of wall only) and in the case of tension cracking, for the reinforcement required in both faces to resist the total tensile force in the wall.

3.3.1 Comparison of EN1992 and BS8007 – direct tensile and flexural load cracking

The reinforcement required to satisfy crack limits of 0,2 mm and 0,1mm for both flexural and tension cracking is used as the basis of comparison between BS8007 and EN1992.

(i) Flexural cracking

Comparisons of the calculated crack widths for flexural cracking in the rectangular reservoir configuration considered could be made directly between BS8007 and EN 1992, for a range of wall thicknesses from 250 to 450 mm, a 20 mm bar diameter and a cover of 40 mm for a wall height of 5 m. Figure 3.3 shows the variation of crack width with %A_s for flexural cracking.

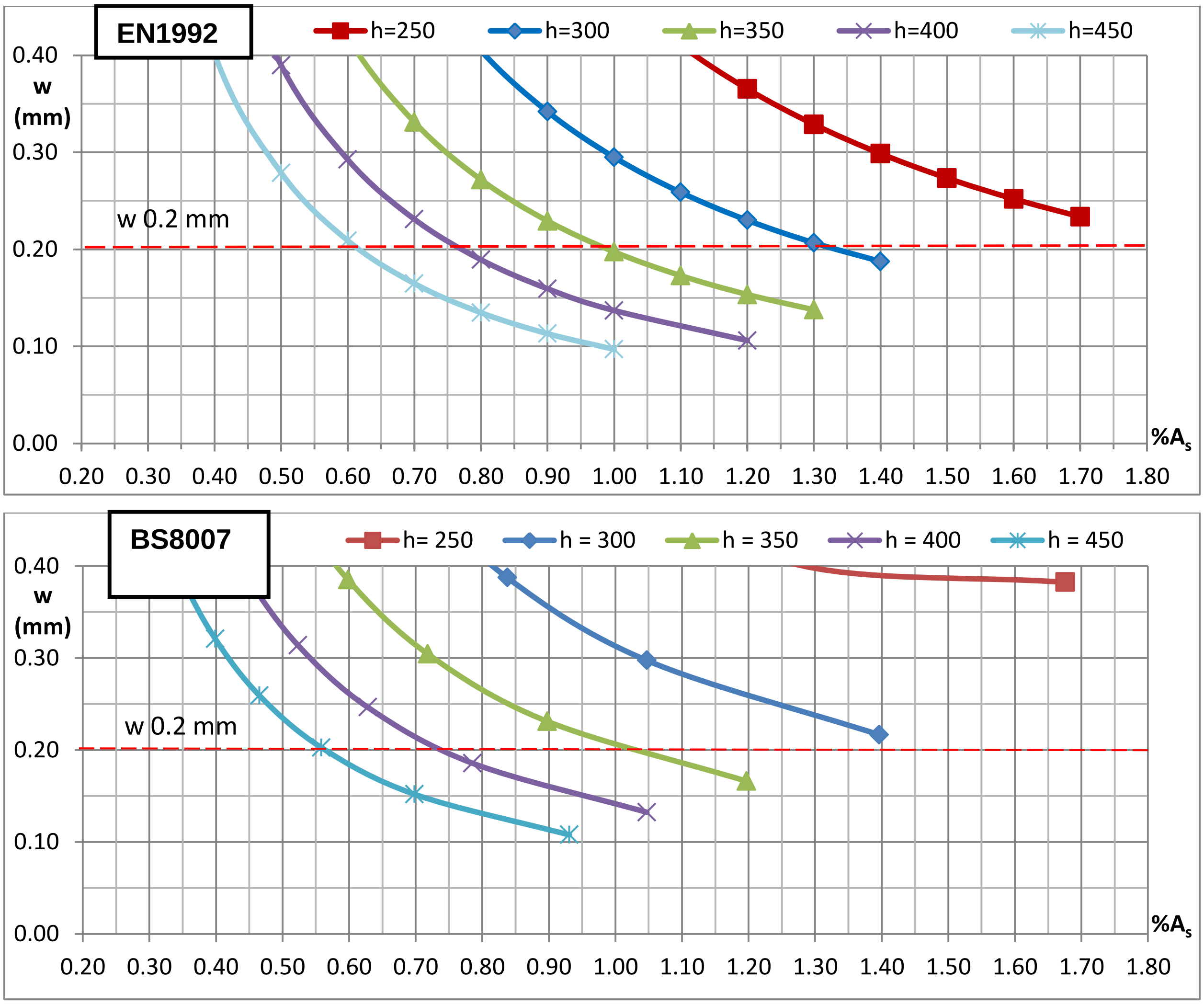


Figure 3.3: *Flexure case - Comparison between EN1992 & BS8007 (using ε_2 for w_{limit} = 0,2 mm)*

From Figure 3.3, the %A_s to be provided to satisfy a 0,2 mm crack width varies from about 1,0% to 0,62% for EN1992 and from about 1,03% to 0,56% for BS8007, for section thicknesses from

350 to 450 mm. It can therefore be concluded that EN1992 is slightly more conservative than BS8007 for flexural cracking at a crack width of 0,2 mm.

Referring to Figure 3.3, for BS8007, a crack width of 0,2 mm will not be achieved using section thicknesses less than 300 mm with a single layer of reinforcement.

(ii) Tension cracking

The calculated EN1992 crack width was found to be independent of section thickness when considering a bar diameter of 20mm and 40 mm cover. This is due to the limiting equation for the effective depth of the tension zone ($h_{c,eff}$) in the EN1992 tension cracking calculations, which was found to be 2,5 (h – d) and can be written as 2,5 (c + φ/2). This means that the effective area in tension, and thus the effective area of concrete, is constant for this bar diameter and cover and is independent of h. A summary of the calculations for $h_{c,eff}$ to EN1992 for bar diameters of 16, 20 and 25 mm is given in Table 3.3. The limiting equation of 2,5(h-d) tends to dominate for wall thicknesses greater than 300 mm, regardless of the combination of cover, bar diameter and wall thickness. The variation of crack width with reinforcement area will therefore be the same for all wall thicknesses in the calculation of EN1992 crack widths.

Table 3.3: *Calculation of effective depth of the tension area (mm) for tension cracking.*

Cover (mm)	h (mm)	Bar dia 16 mm			Bar dia 20 mm			Bar dia 25 mm		
		d	h/2	2,5(h-d)	d	h/2	2,5(h-d)	d	h/2	2,5(h-d)
40	250	202	125	120	200	125	125	197.5	125	131.25
	300	252	150	120	250	150	125	247.5	150	131.25
	350	302	175	120	300	175	125	297.5	175	131.25
	400	352	200	120	350	200	125	347.5	200	131.25
	450	402	225	120	400	225	125	397.5	225	131.25
	500	452	250	120	450	250	125	447.5	250	131.25
50	250	192	125	145	190	125	150	187.5	125	156.25
	300	242	150	145	240	150	150	237.5	150	156.25
	350	292	175	145	290	175	150	287.5	175	156.25
	400	342	200	145	340	200	150	337.5	200	156.25
	450	392	225	145	390	225	150	387.5	225	156.25
	500	442	250	145	440	250	150	437.5	250	156.25

Note that highlighted values given in Table 3.3 are the minimum effective depth of the tension zone in concrete.

The difference between BS8007 and EN1992 tension cracking in the circular reservoir configuration considered is illustrated by Figure 3.4 showing the variation of crack width with $\%A_s$ (both faces) for a 20 mm bar diameter, a cover of 40 mm and a wall height of 5 m over a range of wall thicknesses from 250 to 450 mm.

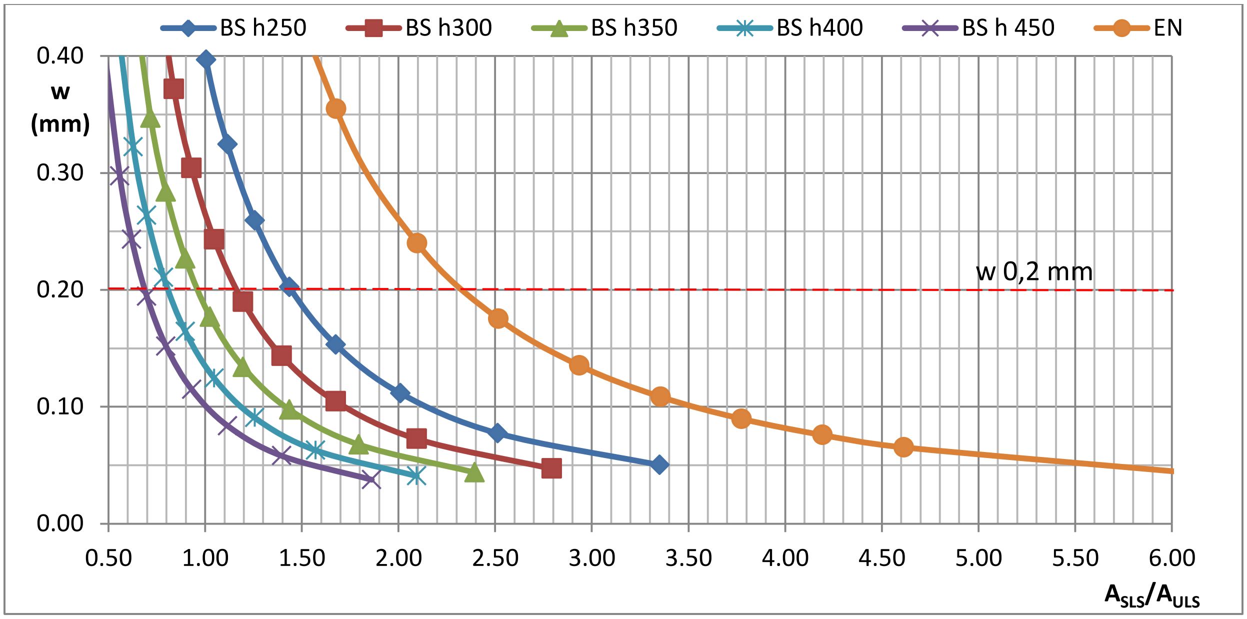


Figure 3.4: *EN1992-1-1 & BS8007 (ε_2 for w_{limit} = 0,2 mm) tension case crack width*

As discussed, the EN1992 calculated crack width is independent of h and is thus represented in Figure 3.4 by a single curve. Using Figure 3.4, for a crack width of 0,2mm, the reinforcement area to be provided according to EN1992 is about 5000 mm^2 regardless of section thickness (which is about 2 $\%A_s$ for an h of 250 mm and 1,11 $\%A_s$ for an h of 450 mm)

The corresponding values of $\%A_s$ to BS8007 for tension cracking to meet a crack width of 0,2mm are less than those of EN1992, that is, in the range of 1,45% to 0,69% for h varying from 250 to 450 mm, respectively, corresponding to reinforcement areas of 3625 mm^2 and 3100 mm^2, respectively. EN1992 is found to be far more severe than BS8007 for tension cracking, predicting larger reinforcement areas by about 38 to 61% as section thicknesses increases over a range of 250 to 450 mm at a crack width limit of 0,2 mm.

Table 3.4 is a summary of the reinforcement calculated to both codes in satisfying crack width limits of 0,2 and 0,1. Values are also given for a 0,05 mm crack width to EN1992. A 40 mm cover and 20 mm bar diameter for both tension and flexural cracking were used.

Table 3.4: *Comparison between BS8007 and EN1992 for varying crack widths*

Load case	h (mm)	Crack width (mm)	A_{ULS} (mm^2)	BS8007		EN1992		EN/BS
				A_{SLS} (mm^2)	A_{SLS}/A_{ULS}	A_{SLS} (mm^2)	A_{SLS}/A_{ULS}	
Flexure	450	0.2	1680	2520	1.50	2777	1.65	1.1
		0.1		4140	2.46	4405	2.62	1.1
		0.05		–	–	7151	4.26	-
	250	0.2	4257	-	-	5000	1.17	-
		0.1		-	-	7500	1.76	-
		0.05		-	-	11500	2.70	-
Tension	450	0.2	2146	3100	1.48	4999	2.33	1.6
		0.1		4120	1.62	7571	3.60	2.2
		0.05		–	–	11776	5.49	-
	250	0.2	2146	3600	1.68	4999	2.33	1.4
		0.1		4815	2.24	7571	3.60	1.6
		0.05		-	-	11776	5.49	-

Note: A_s exceeding Y20@75 (4189 mm^2) highlighted in yellow
A_s exceeding maximum of 4% highlighted in green

Table 3.4 illustrates that EN1992 is slightly more conservative than BS8007 for flexural cracking for both 0,2 mm and 0,1 mm crack widths (h 450 mm), requiring about 1,1 times the reinforcement calculated to BS8007. In the tension case, EN1992 requires about 1,6 times the reinforcement calculated to BS8007 for a crack width of 0,2 mm (h 450 mm). This ratio increases as the crack width limit decreases. A crack width of 0,1 mm could not be met using a section thickness of 250 mm and BS8007.

Referring to Table 3.4 and the A_s values highlighted in yellow, it must be noted that a maximum feasible reinforcement area of Y20@75 (4189 mm^2) is exceeded in all cases considered, except when using a section thickness of 450 mm and a crack width of 0,2 mm are used. The diameter of the reinforcement could in some cases be increased, a double layer of reinforcement may be used or the section thickness can be increased. The latter two alternatives have obvious negative economic consequences on the construction of the WRS. The EN1992 A_s values highlighted in green on Table 3.4 are those exceeding the maximum allowable area of 4% to SANS10100-1 (2004), that is 10 000 mm^2 for h 250 mm and 18 000 mm^2 for h 450 mm.

In summary, the results show that EN1992 is comparable to BS8007 for flexural cracking but is more demanding in the case of tension cracking.

3.3.2 Influence of specified maximum crack width limit to EN1992

As discussed, BS8007 specifies a crack width limit of 0,2 mm which may be reduced to 0,1 mm for aesthetic reasons. In contrast, EN1992-3 states that the crack width limit ranges from 0,2 to 0,05 mm depending on the hydraulic ratio, h_D/h, for a water Tightness Class 1 structure. This means that the crack width limit will depend on the section thickness (h) for a given depth of water (h_D). The depth of water is taken here as the height of the wall (H). Therefore the implication of a lower crack width limit specified by EN1992-3 on the design of WRS was investigated for flexural and tension cracking. The minimum wall thickness for varying crack width limits and wall heights of 5 m and 7 m are given in Table 3.5.

Using the hydraulic ratio to determine the limiting crack width, a section thickness of 1 m is indicated if a crack width limit of 0,2 mm is required for a wall height of 5 m. This is a substantial increase in concrete and not feasible. To keep the section thickness to a reasonable dimension, the crack width limit must then less than 0,2 mm. The effect of this reduced crack width limit from 0,2 mm is now discussed for both flexural and tension cracking.

Table 3.5: *Limiting crack width to EN1992-3 using hydraulic ratio h_D/h.*

H (m)	h_D/h	w_k (mm)	Minimum h (mm)
5	35	0.050	143
	30	0.075	167
	25	0.100	200
	20	0.125	250
	15	0.150	333
	10	0.175	500
	5	0.200	1000
7	35	0.050	200
	5	0.200	1400

(i) Flexural cracking

The results from the flexural cracking calculations to EN1992 for a range of section thicknesses with their associated limiting crack widths were summarised and presented in Table 3.6. The reinforcement required was found to be dependent on both the limiting crack width determined using the hydraulic ratio (w_k) and the section thickness used.

Table 3.6: *EN1992 Flexural cracking – Effect of reduction in crack width limit*

				w_k using h_D/h				w 0.2 mm			
H (m)	h (mm)	h_D/h Ratio	ULS A_s (mm^2)	w_k (mm)	SLS $A_{s,k}$ (mm^2)	A_{SLS}/A_{ULS}	SLS $\%A_s$	SLS $A_{s,0.2}$ (mm^2)	A_{SLS}/A_{ULS}	SLS $\%A_s$	$A_{s,k}/A_{s,0.2}$
5	250	20.0	4257	0.125	7140	1.68	2.86	4819	1.13	1.93	1.48
	300	16.7	2973	0.147	5073	1.71	1.69	3999	1.35	1.33	1.27
	350	14.3	2346	0.154	4189	1.79	1.20	3468	1.48	0.99	1.21
	400	12.5	1970	0.163	3545	1.80	0.89	3084	1.57	0.77	1.15
	450	11.1	1680	0.169	3098	1.84	0.69	2777	1.65	0.62	1.12
	500	10.0	1494	0.175	2731	1.83	0.55	2513	1.68	0.50	1.09
	550	9.1	1344	0.180	2419	1.80	0.44	2265	1.68	0.41	1.07
	600	8.3	1222	0.183	2241	1.83	0.37	2116	1.73	0.35	1.06
7	500	14.0	3964	0.155	7405	1.87	1.48	6104	1.54	1.22	1.21
	550	12.7	3857	0.161	6598	1.71	1.20	5635	1.46	1.02	1.17
	600	11.7	3441	0.167	5955	1.73	0.99	5249	1.53	0.87	1.13
	750	9.3	1659	0.178	4533	2.73	0.60	4189	2.52	0.56	1.08

Note: A_s exceeding maximum practical reinforcement limit of Y20 @75 (4189) highlighted in yellow

Referring to Table 3.6, reinforcement areas obtained are greater than that of Y20 @ 75 (4189) have been highlighted yellow. Using 20mm diameter bars, the section thickness required to achieve the specified crack width limit whilst providing a practical reinforcement quantity must be at least 350 mm. Increasing the bar diameter to 25 mm would result in an increase in bar spacing and a reduction in the section thickness to 300 mm. Alternatively, for thicker sections, 2 layers of reinforcement per face could be used, such that minimum spacing of reinforcement and maximum reinforcement areas are observed. As a comparison, the minimum wall thickness required by ULS load calculations such that compression reinforcement is not needed is about 250 mm considering a 5 m wall height.

An increasing section thickness for a given wall height (increasing h_D/h) results in a corresponding decrease in the amount of reinforcement required, illustrated by Table 3.6. However, the decrease in the limiting maximum crack width from 0,2 mm results in an overall increase in the reinforcement required to meet the specified limit for a given section thickness. This is also shown in Figure 3.5 which is the variation of the reinforcement area required to satisfy a given crack width with the hydraulic ratio. As the graphs diverge with increasing hydraulic ratio, the difference between $A_{s,k}$ and $A_{s,0.2}$ increases.

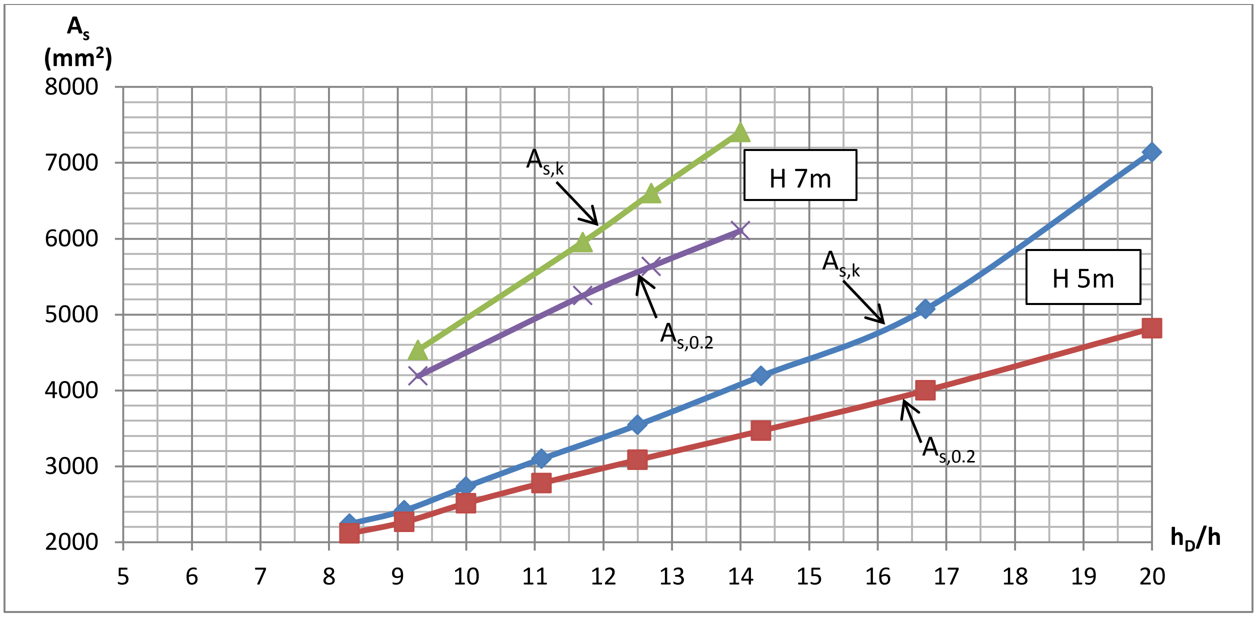


Figure 3.5: *EN1992 Flexural cracking – variation of reinforcement area with hydraulic ratio.*

This increase was measured by calculating the reinforcement ratio $A_{s,k}$ / $A_{s,0.2}$ which is given in the last column in Table 3.6. $A_{s,k}$ / $A_{s,0.2}$ is the ratio of the reinforcement required for w_k ($A_{s,k}$) using the hydraulic ratio (h_D/h) to the reinforcement required at a 0,2 mm crack width ($A_{s,0.2}$) as the hydraulic ratio increases. Figure 3.5, showing the variation of the reinforcement ratio $A_{s,k}$ / $A_{s,0.2}$ with the hydraulic ratio, was then plotted. The figure shows that $A_{s,k}$ / $A_{s,0.2}$ increases with increasing hydraulic ratio, irrespective of the height of the wall.

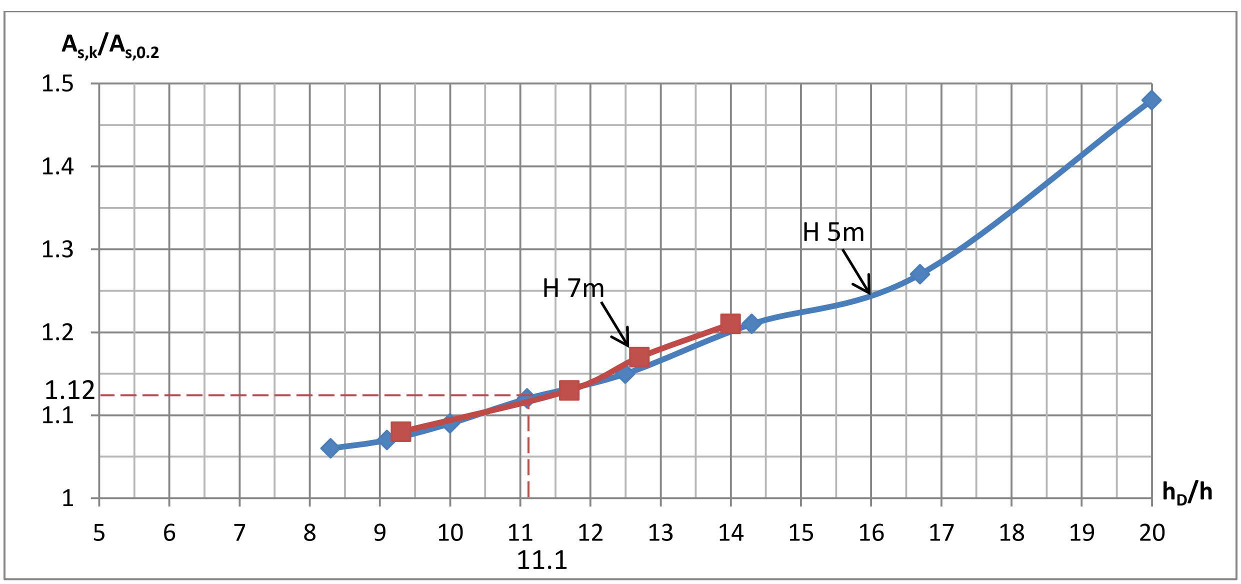


Figure 3.6: *EN1992 Flexural cracking - Variation of ratio of $A_{s,k}$ to $A_{s,0.2}$ with hydraulic ratio, h_D/h*

Referring to Table 3.6 and Figure 3.6, the reinforcement area required increases by a factor of 1,12 for a hydraulic ratio of 11,1 (450 mm section and 5 m wall height). The crack width limit for this ratio is 0,17 mm. Comparing this to the area of reinforcement required of 2520 mm^2 to achieve a 0,2 mm crack width using BS8007 (refer to Table 3.4), the application of EN1992-3 means an increase in reinforcement by a factor of 1,23 to satisfy the reduced crack width limit from 0,2 mm to 0,169 mm.

It must be noted that in the flexural cracking case, cracks do not always go through the section if loading is on one face of the wall only. In this case, EN1992-3 Cl 7.3.1 states that the crack width limit may be taken as 0,2 mm, providing the depth of the compression zone is at least x_{min} considering the quasi-permanent combination of loadings. EN1992-3 recommends values of the smaller of 50 mm and 0,2h for x_{min}. However, this clause also states that where a section is subject to alternate actions, then cracks must be considered as passing through the full section unless it can be proved that some part of the section thickness of at least x_{min} will remain in compression. Thus, for the flexural cracking case, the possibility of through-cracks, whilst not as likely as for direct tension cracking, must still be considered.

(ii) Tension cracking

The reinforcement required to meet the required limiting crack widths to EN1992-3 for tension cracking is summarised in Table 3.7. Using a 20 mm bar diameter and 40 mm cover, the EN1992 crack width equation applied to the tension case is independent of section thickness, as the dominating equation for $h_{c,eff}$ in determining the effective area of concrete in tension is governed by 2,5(c + φ/2). Referring to Table 3.7, this is shown by the constant area of reinforcement obtained (SLS $A_{s,0.2}$) to satisfy a 0,2 mm crack width irrespective of the section thickness. The reinforcement required is thus dependent only on the crack width limit determined by the hydraulic ratio.

It was noted that a wall thickness of 250 mm for a 7 m wall height results in the maximum allowable reinforcement of 4% (10 000 mm^2) being exceeded as 4,15% (10373 mm^2) is obtained to meet the specified crack width of 0,085 mm. The wall thickness must in this case then be increased, decreasing the hydraulic ratio and therefore decreasing the specified crack width limit and the amount of reinforcement needed. Referring to Table 3.7 and considering a 5 m wall height, a wall thickness of 450 mm (h_D/h of 11.1) leads to a crack width limit of 0,17 mm.

Table 3.7: *EN1992 Tension cracking – Effect of reduction in crack width limit (A_s both faces)*

				Reinforcement for w_k to h_D/h				w 0.2 mm			
H (m)	h (mm)	h_D/h Ratio	ULS A_s (mm^2)	w_k (mm)	SLS $A_{s,k}$ (mm^2)	A_{SLS}/A_{ULS}	SLS $\%A_s$	SLS $A_{s,0.2}$ (mm^2)	A_{SLS}/A_{ULS}	SLS $\%A_s$	$A_{s,k}/A_{s,0.2}$
5	250	20.0	2146	0.125	6607	3.08	2.64	4999	2.59	2.23	1.32
	300	16.7	2146	0.147	5994	2.79	2.00	4999	2.59	1.85	1.20
	350	14.3	2146	0.154	5830	2.72	1.67	4999	2.59	1.59	1.17
	400	12.5	2146	0.163	5637	2.63	1.41	4999	2.59	1.39	1.13
	450	11.1	2146	0.169	5518	2.57	1.23	4999	2.59	1.24	1.10
	500	10.0	2146	0.175	5406	2.52	1.08	4999	2.59	1.11	1.08
	550	9.1	2146	0.180	5317	2.48	0.97	4999	2.59	1.01	1.06
	600	8.3	2146	0.183	5266	2.45	0.88	4999	2.59	0.93	1.05
7	250	28.0	3004	0.085	*10373	3.45	4.15	6098	2.03	2.44	1.70
	300	23.3	3004	0.108	8901	2.96	2.97	6098	2.03	2.03	1.46
	350	20.0	3004	0.125	8122	2.70	2.32	6098	2.03	1.74	1.33
	400	17.5	3004	0.138	7369	2.45	1.84	6098	2.03	1.52	1.21
	450	15.6	3004	0.147	7347	2.45	1.63	6098	2.03	1.36	1.20
	500	14.0	3004	0.155	7113	2.37	1.42	6098	2.03	1.22	1.17
	550	12.7	3004	0.161	6950	2.31	1.26	6098	2.03	1.11	1.14
	600	11.7	3004	0.167	6797	2.26	1.13	6098	2.03	1.02	1.11

Note: A_s exceeding maximum practical reinforcement limit of Y20 @75 both faces (8378) highlighted in yellow.

A reinforcement area of 5518 mm^2 is obtained which is an increase by a factor of about 1,1 over that for a 0,2 mm crack width. Comparing to BS8007, reinforcement areas are substantially larger for EN1992. The area determined by BS8007 requires about 2900 mm^2 and 2160 mm^2 for 250 and 450 mm wall thicknesses, respectively, at a 0,2 mm crack width.

As with flexural cracking, Figure 3.7 shows the ratio $A_{s,k}$ / $A_{s,0.2}$ increases with increasing hydraulic ratio, irrespective of the height of the wall, although the increase is slightly smaller at higher h_D/h values in the case of tension cracking.

In general for both flexural and tension cracking, there is an increase in the materials required therefore an increase in costs if a smaller crack width limit is specified. A cost optimisation analysis would determine the best combination of geometry of section (so determining the concrete quantity), reinforcement and limiting crack width. The increase in costs is directly proportional to the increase in materials.

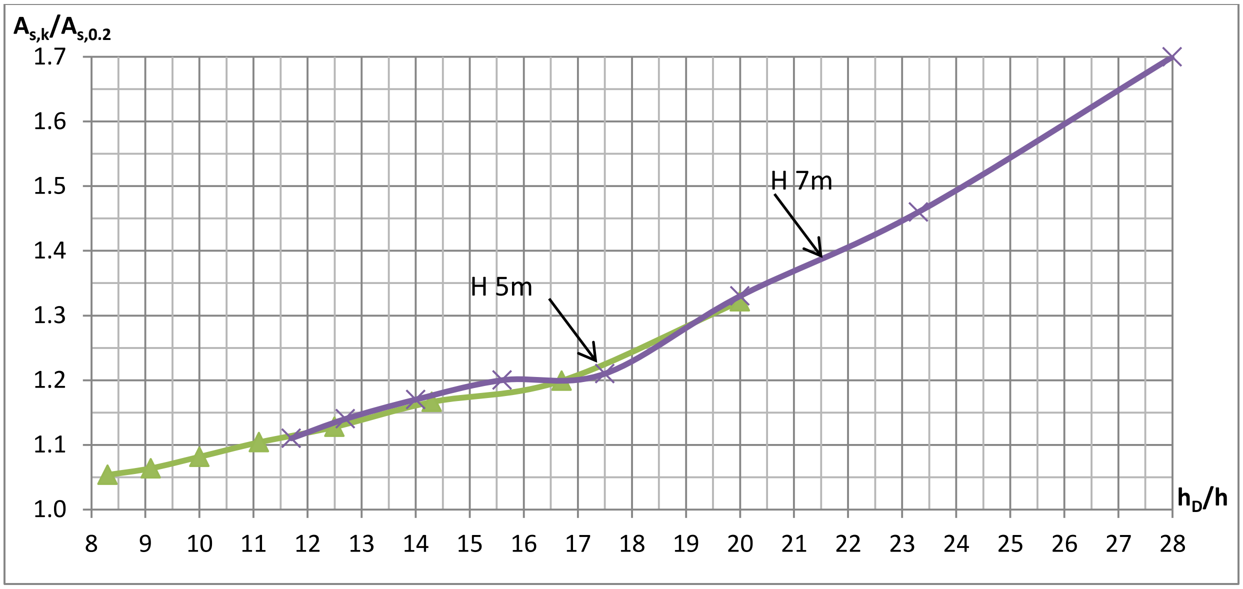


Figure 3.7 *EN1992 Tension cracking - Variation of ratio of* $A_{s,k}$ *to* $A_{s,0.2}$ *with hydraulic ratio,* h_D/h

3.3.4 Influence of SLS cracking (using EN1992)

From the overview of the design of WRS summarised in Chapter 2, SLS cracking is considered in industry to be the dominant limit state, rather than the ULS of loading. To investigate the extent of this dominance, A_{SLS}/A_{ULS} ratios were determined for flexural and tension cracking to EN1992, and graphs were plotted for comparison showing the variation of crack width with A_{SLS}/A_{ULS} It must be noted that the characteristic tensile strength of reinforcement used in the calculation of the ultimate reinforcement to SANS10100-1 is 450 MPa (E_s = 200 GPa) which is about 89% of the typical value of 500 MPa (E_s = 200 GPa) used by Eurocode. This means calculations for ULS loading to SANS10100-1 result in a higher ULS reinforcement area by about 11% compared to Eurocode. As the SLS crack width equations are independent of the steel strength, the enhancement of ultimate reinforcement requirements to meet the serviceability requirement using the South African value for the steel strength is 11% less than that strength used by Eurocode. The A_{SLS}/A_{ULS} ratio will also be influenced proportionally by the ULS partial safety load factor considered for the liquid load. A larger ULS load factor will result in a smaller A_{SLS}/A_{ULS} ratio.

(i) Flexure

Considering crack width limits of 0,2 mm and less for flexural cracking, Figure 3.8 (using

EN1992) shows that the serviceability limit state is the dominant state rather than the ultimate limit state for all wall thicknesses in the deterministic crack width calculations as A_{SLS}/A_{ULS} ratios exceed 1,0. Figure 3.8 also illustrates that the A_{SLS}/A_{ULS} ratio determined using EN1992 increases with increasing wall thickness. It was noted that the ULS flexure reinforcement required decreases as section thickness increases, but at a smaller rate than the increase in SLS reinforcement required as section thickness increases.

The dominance of SLS increases as the limiting crack width for flexural cracking decreases. Referring to Table 3.6 and Figure 3.8, for flexural cracking calculations to EN1992 and a section thickness of 450 mm, a A_{SLS}/A_{ULS} ratio of about 1,65 is obtained to satisfy a crack width limit of 0,2 mm. The ratio increases to about 2,6 for a crack width limit of 0,10 mm (determined by the hydraulic ratio) which is an increase in reinforcement by a factor of about 1,6. A further reduction in the crack width limit to 0,05 mm results in a ratio of about 3,8, an increase by a factor of about 2,3. Note that as discussed in the previous Section 3.3.2, for flexural cracking, these crack width limits will apply to cracking through the full section.

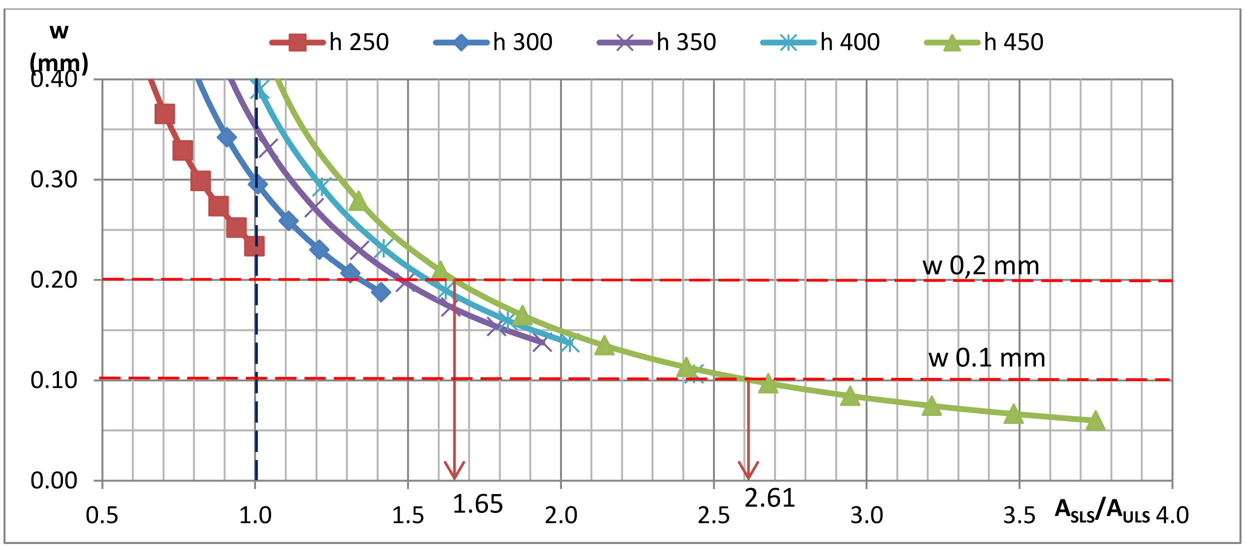


Figure 3.8: *EN1992 flexure load case – influence of SLS cracking*

(ii) Tension

The serviceability limit state is also found to be the dominant limit state for the tension cracking condition as illustrated by Figure 3.9 which shows the variation of crack width with A_{SLS}/A_{ULS} ratios using EN1992. In addition, the ratios are higher for tension cracking than for flexural cracking. This is in part due to the ULS reinforcement calculation for the direct tension case in a wall with a sliding base being dependent on the applied tensile force and steel strength but independent of wall thickness, whereas the ULS reinforcement calculation for an applied flexure is not independent of wall thickness. The ULS reinforcement required for flexural loading therefore decreases as wall thickness increases.

Referring to Table 3.7 and Figure 3.9, EN1992 requires a ratio of about 2,32 for all wall thicknesses to achieve a 0,2 mm crack width. This is higher than the ratio of 1,65 for flexural cracking to EN1992 by a factor of about 1,44. As the crack width limit decreases, the A_{sls}/A_{uls} ratios are found to increase substantially for tension cracking, that is, the dominance of serviceability increases. Tension cracking is thus more critical than flexural cracking in terms of the extent to which SLS dominates the design of a WRS. As the crack width limit decreases to 0,1 mm, there is an increase in the A_{SLS}/A_{ULS} ratios to 3,5 for EN1992. A further increase in reinforcement is required at a crack width limit of 0,05 mm, giving a ratio of about 5.7.

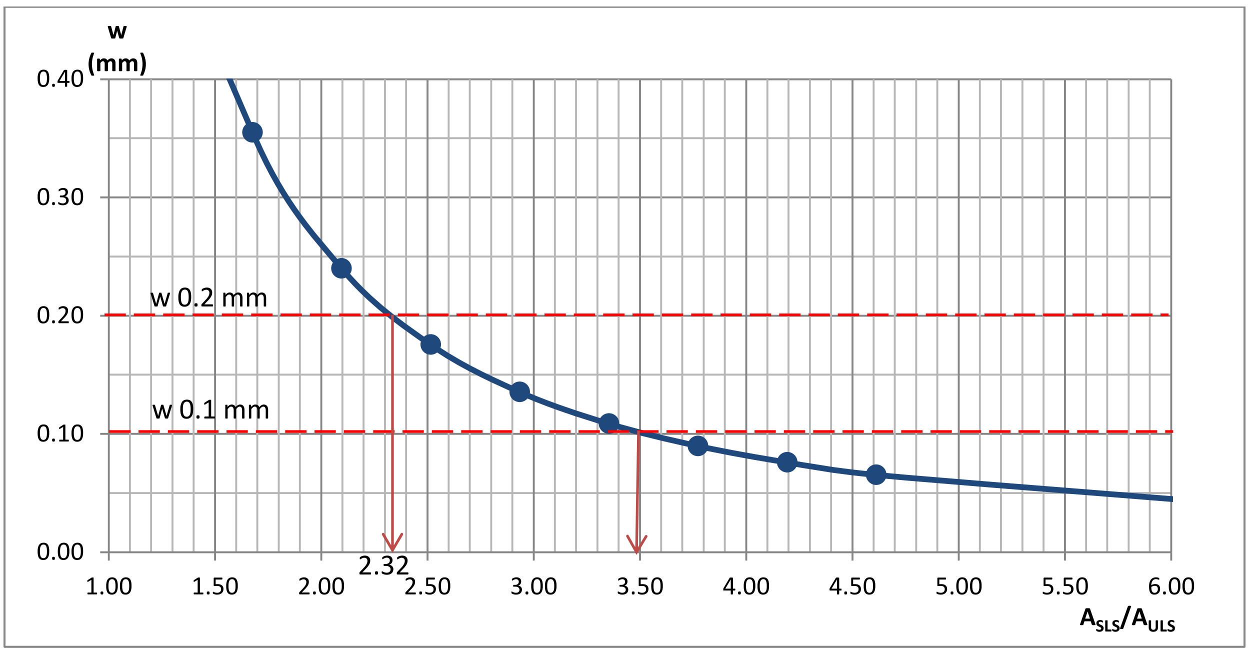


Figure 3.9: *EN1992 tension cracking – influence of SLS cracking*

The ratios obtained from the deterministic analysis of load-induced cracking show that serviceability is the dominant limit state, therefore highlighting the importance of optimum design for serviceability cracking in WRS, particularly at smaller crack widths.

3.3.5 Range of parameters for reliability analysis of EN1992 crack width formulation

One of the aims of the deterministic analysis was to investigate the relative influence of the following parameters on the limiting crack width, such that, together with the investigation of industry practices, a realistic range of values of these parameters for use in the reliability analysis of the EN1992 crack equation could then be selected. These parameters are:

- wall height
- section thickness
- concrete cover
- bar diameter

The reinforcement required to achieve a crack width of 0,2 mm for both ULS and SLS is summarised in Tables 3.8 and 3.9 for flexural and tension cracking, respectively, for various representative values for these parameters. The relative importance of wall height, section thickness, concrete cover and bar diameter is now discussed for both the flexural and tension cracking conditions.

3.3.5.1 Wall height H

The liquid load onto a reservoir wall is related to the height of the wall. For a wall in tension, the load is directly proportional to H with the resulting tensile force proportional to H, whereas for a wall in flexure, the load is proportional to H^2 with the resulting bending moment proportional to H^3. A flexural load therefore has a greater influence on the wall than a tension load. On considering typical configurations of constructed reservoirs, a reference value for wall height of 5m has been used. In increasing the height to 6m and 7m, the load onto the wall is increased proportionally as is seen in Tables 3.8 and 3.9. There was an increase in both the amount of reinforcement and the wall thickness required in order to meet the specified crack width. The wall height and therefore load, is found to be an influential parameter in the calculation of crack widths.

3.3.5.2 Section thickness h

The resistance of the wall to an applied load is in part dependent on section thickness which is affected by the magnitude of that load and therefore the wall height. As previously stated, the ULS reinforcement for flexural loading was calculated such that the section thickness provided ensures that tension reinforcement only is required (one face of the wall). For a wall height of 5m, a 250 mm section thickness was found to be adequate.

Table 3.8: *EN1992 flexural load case: Summary of reinforcement requirements.*

H (m)	Bar dia (mm)	Cover (mm)	h (mm)	ULS A_s (mm)	SLS A_s (mm^2)	A_{SLS}/A_{ULS}	SLS $\%A_s$
5m	16	40	250	4191	4614	1.10	1.85
			300	2940	3797	1.29	1.27
	Max A_s		350	2327	3269	1.40	0.93
	2681		400	1941	2887	1.49	0.72
	mm^2		450	1672	2584	1.55	0.57
			500	1487	2325	1.56	0.47
			550	1339	2086	1.56	0.38
			600	1218	1912	1.57	0.32
5m	20	40	250	4257	4819	1.13	1.93
			300	2973	3999	1.35	1.33
	Max A_s		350	2346	3468	1.48	0.99
	4189		400	1970	3084	1.57	0.77
	mm^2		450	1680	2777	1.65	0.62
			500	1494	2513	1.68	0.50
			550	1344	2265	1.68	0.41
			600	1222	2116	1.73	0.35
5m	25	40	250	4311	5067	1.18	2.03
			300	3015	4235	1.40	1.41
	Max A_s		350	2370	3699	1.56	1.06
	6545		400	1970	3309	1.68	0.83
	mm^2		450	1692	2997	1.77	0.67
			500	1502	2725	1.81	0.55
			550	1351	2508	1.86	0.46
			600	1228	2361	1.92	0.39
5m	20	50	250	4481	5962	1.33	2.38
			450	1728	3097	1.79	0.69
6m	20	40	450	3031	4471	1.48	0.99
			500	2635	4093	1.55	0.82
			550	2336	3778	1.62	0.69
			600	2112	3466	1.64	0.58
7m	20	40	450	5173	6697	1.29	1.49
			500	3964	6104	1.54	1.22
			550	3857	5635	1.46	1.02
			600	3441	5249	1.53	0.87

Note: Values highlighted in yellow exceed maximum feasible area (for single layer of reinforcement).

Referring to both Figure 3.3 and Table 3.8, for SLS EN1992 flexural cracking and considering a practical reinforcement quantity of Y20 at 75 mm spacing, the wall thickness needs to be greater than 250 mm in order to meet a crack width limit of 0.2 mm. An increase in section thickness results in a nearly directly proportional decrease in the quantity of reinforcement required to satisfy SLS flexural cracking crack width criteria. It can then be stated that the section thickness does have an influence on the crack width for flexural cracking as would be expected.

For tension cracking, the crack equation is independent of section thickness if a bar diameter of 20 mm is used. Section thickness was not found to be an influential parameter in crack width calculations for tension loading using 16 and 25 mm bar diameters.

Table 3.9: *EN1992 tension load case: Summary of reinforcement requirements (A_s both faces)*

H (m)	Bar dia (mm)	Max A_{sls} (mm^2)	Cover (mm)	ULS A_s (mm^2)	SLS A_s (mm^2)	A_{SLS}/A_{ULS}	SLS $\%A_s$
5	16	5362	40	2146	4485	2.09	1.00
5	20	8378	40	2146	4999	2.33	1.11
5	20	8378	50	2146	5606	2.61	1.25
5	25	13090	40	2146	5607	2.61	1.25
6	20	8378	40	2575	5564	2.16	1.24
7	20	8378	40	3004	6098	2.03	1.36

3.3.5.3 Concrete cover c

Concrete cover is considered to be an influential parameter in the determination of crack widths, as discussed in Chapter 2, due to its effect on the effective depth of section and distance to the crack location, thus on the crack spacing and corresponding crack width. Values of 40 and 50 mm were chosen to assess and compare the influence of cover. The ULS and the SLS reinforcement quantities required to satisfy a 0,2 mm crack width for a section thickness of 450 mm and 20 mm bar diameter and varying cover are given in Table 3.8 for flexural cracking and Table 3.9 for tension cracking.

The influence of cover is found to be similar for both flexural cracking and tension cracking. There is an increase in reinforcement required as cover increases from 40 to 50 mm therefore decreasing effective depth, as would be expected. This increase was found to be in the order of a factor of 1,12 for both flexural and tension cracking. Concrete cover does have an influence on the calculated crack width in the deterministic analysis, although is not dominant.

3.3.5.4 Bar diameter φ

To assess the effect of reinforcement diameter, crack width calculations were done using bar diameters of 16, 20 and 25 mm. The bar diameter is found to have a lesser influence on the crack width although its influence is connected to the bar spacing. The bar diameter was chosen such that the practical limit of providing sufficient reinforcement at a reasonable spacing was observed.

3.4 SUMMARY

A deterministic analysis of the EN1992 and BS8007 crack formulations was performed with respect to flexural and tension cracking due to loading. A representative configuration of a typical WRS was chosen for both load cracking conditions, namely, a rectangular reservoir wall in the case of flexural cracking and a circular reservoir for the tension cracking condition. Using the industry review, presented in Chapter 2, values were chosen for the parameters of the crack width formulations to BS8007 and EN1992. Conclusions from the deterministic analysis of BS8007 and EN1992-3 are summarised as follows:

- It can be concluded that EN1992 is more conservative than BS8007 for flexural cracking, although reinforcement areas obtained for 0,2 mm crack widths are comparable. However, for tension cracking, EN1992 is found to be substantially more demanding than BS8007. This would result in increased costs of construction, proportional to the increase in the reinforcement required, on moving from the use of BS8007 to EN1992 in the design of WRS.
- EN1992-3 specifies a range of crack width limits from 0,2 mm to 0,05 mm for through-cracks, depending on the hydraulic ratio which in turn is dependent on the wall geometry. Decreasing the limiting crack width results in a substantial increase in reinforcement for both flexural and tension cracking. This is exacerbated by the conservatism of Eurocode compared to the British standards. In addition, an increase in wall thickness and/or increase in bar diameter is required in order to obtain a practical reinforcement spacing. There is a negative effect on the economics of the structure proportional to the decrease in the crack width limit.
- The deterministic analysis shows that the serviceability limit state of cracking dominates the design of a WRS rather than the ultimate limit state of loading, particularly for tension cracking. A reliability analysis is therefore justified to investigate the reliability of the crack models with respect to load-induced cracking, with possible improvement to the efficiency of the EN1992-1-1 crack design model. Both tension and flexural cracking will be considered in the reliability analyses, although the tension case is expected to be more critical.

The effect of the parameters of wall height, wall thickness, concrete cover and bar diameter was investigated and a selection of representative values was made for these parameters in the reliability crack width model. The conclusions from the deterministic parametric analysis are summarised as follows:

- The height of the wall (corresponding to the depth of water) and corresponding liquid load have an influence on the deterministic crack models, especially for flexural cracking. A reference height of 5m will be used in the reliability analysis. Heights of 6 and 7 m will only be considered to investigate the role of load in the reliability analysis.
- The wall thickness h has an influence on the deterministic crack models, particularly in the flexural cracking model, thus will also be considered in the reliability analysis. This parameter's influence on the tension cracking model is dependent on the limiting equation for the effective depth of the tension zone in the concrete. Decreasing reinforcement areas were noted as section thickness increased. Wall thicknesses of 250 and 450 mm will be used as representative values in the reliability analysis.
- Concrete cover needs to be considered in the reliability analysis, although it has a lesser effect than section thickness. An increase in cover has the expected effect of increasing the reinforcement required. Thus a cover of 40 mm will be used as the representative value in the reliability analysis, using a 50 mm cover only selectively to explore its influence.
- The diameter of the tension reinforcement does not have a large influence on the variation in crack width to either design code, except in the way in which it influences reinforcement spacing. A reference value of 20 mm will be used in the reliability analysis.

The reliability analysis would allow the effect of the design variables to be explored in a systematic way.

CHAPTER 4

OVERVIEW OF RELIABILITY ANALYSIS w.r.t. MODELLING OF CRACKING

4.1 GENERAL

Following on from the review of the design and construction of water retaining structures (WRS) presented in Chapter 2 and the deterministic analysis of the BS8007 and EN1992 crack models presented in Chapter 3, a reliability analysis of the EN1992 crack equation was performed with respect to the following key issues that have been identified:

(v) The implications of the more onerous crack width limits of EN1992-1 compared to BS8007 on the design of a WRS.

(vi) The reliability of the EN1992-1 crack model.

(vii) The extent to which serviceability cracking governs the design of WRS, rather than the ultimate limit state. From the deterministic analysis, SLS was found to dominate in both flexural and tension cracking.

In order to set up the reliability model, presented in Chapter 5, of the EN1992 crack equation for tension and flexural loading applied to WRS, a review of reliability-based design was carried out. This chapter is a summary of that investigation. Reliability-based analysis is the basis for limit state design which, in turn, is the basis of modern structural design codes. Reliability methods provide a way of determining the performance of a structure while considering safety and economy of design. The methods aim to determine the reliability or safety of a structure and achieve reasonable probabilities that the structure designed will not become unfit for the function of the structure. They also provide a tool to assess and improve existing structural models which in this case is cracking in reinforced concrete water retaining structures.

To date, most research in reliability-based structural design has been carried out for the ultimate limit state (ULS) with little done on the serviceability limit state, as the ULS usually governs the design of the structure. As the ULS is the collapse limit state, higher levels of reliability are specified than for SLS to ensure an adequate structural safety. However, in the case of WRS, SLS generally governs the design (more specifically SLS cracking), raising the question of what would be an appropriate level of reliability applicable to this case. The physical crack models have been developed mostly through experimental research and tend to be conservative. Empirical factors are selected and applied to the model with an unspecified degree of conservatism, based on the judgment of the researcher and possibly moderated by a code committee.

Therefore the performance of the crack model needs to be assessed in reliability terms and a reasonable reliability determined. In order to formulate a reliability model for the EN1992-1 crack equation applied to a reinforced concrete water-retaining structure, a review of the First Order Reliability Method (FORM) was undertaken and presented here. The development of the EN1992-1 crack width equation in probabilistic terms is discussed and presented. The formulation of the FORM crack model used in the reliability analysis is presented in Chapter 5. The statistical parameters required for the FORM analysis, model uncertainties and target reliabilities were researched and presented in this chapter.

4.2 THE FIRST ORDER RELIABILITY METHOD OF ANALYSIS

This section summarises the investigation carried out on the reliability method of analysis the First Order Reliability Method (FORM). A brief background to the method is given and the FORM algorithm is set out.

4.2.1 Limit state function

In terms of the basis for reliability analysis and limit state design, the structural performance of a structure or its elements is described in terms of a set of limit states which are conditions that describe the fitness of the structure in performing its function. The ultimate limit state is associated with collapse of a structure or any of its elements that affect the safety of human life whilst the serviceability limit state is associated with the conditions of normal use. The limit state may be described by means of a limit state function and is generally expressed as:

$$g(X_i) = R – E$$

where X_i are the probabilistic basic variables of the limit state function. R is the resistance of the structural model and E the action effect.

For a serviceability limit state, the limit state (LS) function expressed in SANS 10160-1 (2011) has the form:

$$g(X_i) = C - E$$

where C is the limiting value of the serviceability criterion. The term E is the action effects of the SLS condition. The LS function defines the boundary between safe and unsafe conditions of the limit state considered. The function $g(X_i)$ will be greater than zero if the structure is safe, equal zero at the limit state and less than zero if the structure is unsafe. The development of the limit state function for the EN1992 crack model is discussed in Section 4.4.

4.2.2 Definition of reliability index, β

To determine the structural reliability as a measure of performance, the probability that the limit state will be met or exceeded (probability of safety, p_s) is assessed using the statistical data for the parameters (X_i) in a structural model. Conversely, the probability of failure ($g < 0$) is $p_f = 1 – p_s$. The measure of the level to which the limit state is met is the safety or reliability of the structure, which can be expressed in terms of the reliability index (β) as defined in SANS 2394:2004 (ISO 2394: 1998)

$$\beta = - \Phi^{-1}(p_f)$$

where Φ^{-1} is the inverse standardised normal distribution. The relationship between the reliability index and the probability of failure is quantified in Table 4.1.

Table 4.1: *Relationship between β and p_f*

p_f	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	1.3	2.3	3.1	3.7	4.2	4.7	5.2

4.2.3 First Order Reliability analysis (FORM)

The First Order Reliability Method provides a recognized suitable method of assessing the probability that the limit state concerned is met and the degree to which this occurs (i.e. the reliability thereof) by means of the reliability index, β. The development of the FORM is well-documented and described in full in literature therefore just a brief summary of the method following the Ang and Tang (1990) description is given here. Figure 4.1 serves as a graphical interpretation of FORM. Diagram (a) represents the limit state function at the failure surface and the joint probability distribution for the original basic variables (not all normal pdf's). Diagram (b) shows the normalised form of the problem to obtain the solution.

The limit state function or performance function is the failure plane on which $g(X_i) = 0$. The parameters of the limit state function are expressed as random or basic variables (RV's) described in terms of their probability distribution function (pdf), mean (μ) and variance (σ).

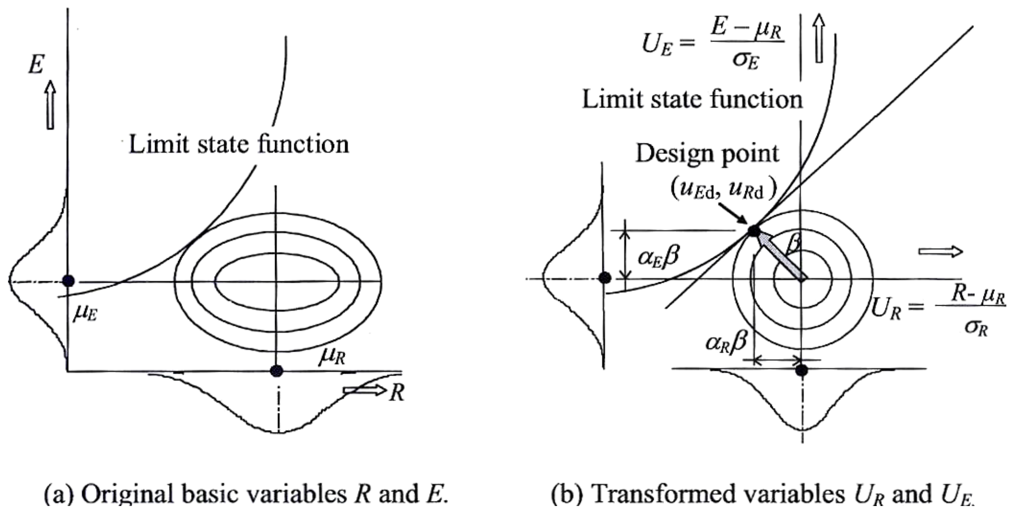


Figure 4.1: *Graphical representation of FORM (Source: Holický (2009))*

The equation to find the probability of failure may be written as:

$$p_f = \int f_{X_i}(X_i)\, dx_i \qquad (4.1)$$

where X_i are the random variables, $f_{xi}(X_i)$ is the joint pdf of the RV's and the integral is over the unsafe region $g(X_i) < 0$. In the case of a non-linear limit state or performance function, the failure plane is non-linear and is approximated by the tangent plane which intersects the actual failure plane at the most probable failure point or the so-called design point. All non-normal pdf's are converted into equivalent normal distributions in the region of the design point.

Given the difficulties of mathematically solving the numerical integration of the above equation, the design point and its associated reliability index are found by means of a first-order approximation of a Taylor series. The mathematical solution is then found through an iterative process. The reliability index is the shortest distance from the design failure point on the failure surface to the origin expressed in units of the standard deviation of $g(X_i)$ or σ_g and has the general equation:

$$\beta = \frac{\sum x'^{*}_i \left(\frac{\delta g}{\delta X'_i}\right)_*}{\sqrt{\sum \left(\frac{\delta g}{\delta X'_i}\right)^2_*}} \qquad (4.2)$$

The FORM algorithm (after *Ang & Tang (1990)*) in reliability analysis to determine the reliability index for a given set of variables is summarised as follows:

1. Define limit state function, $g(X_i)$.

2. Collect statistical data for all variables in the limit state function (pdf, μ, σ). The variables are considered as the random variables (RV), X_i.

3. Any non-Normal pdf's for the RV's are normalised with normalised mean μ^N and coefficient of variation σ^N obtained for each RV at the estimated design point.

4. Initial values for the design point value for each variable (x_i) are assumed. The normalised mean is usually the initial value taken.

5. From the limit state equation, the partial derivatives $\delta g/\delta X'_i$ with respect to the reduced RV's are found.

6. Hence the directional cosines (α*) are calculated from:

   $\alpha_i^* = (\delta g/\delta X_i')_* \;/\; \sqrt{(\Sigma(\delta g/\delta X_i')_*^2)}$

8. Substitute the equation to calculate the new design points $x_i^* = \mu_i^N - \alpha_i^* \beta\, \sigma_i^N$ into the limit state equation $g(X_i) = 0$ and solve for the reliability index, β.

9. Steps 4 to 8 are repeated until the β- value converges.

10. The x_i^* values obtained for the final β will be the design failure point values of the variables for the given limit state equation.

The directional cosines (α_i) are also the sensitivity factors for the variables in the limit state function with values between -1 and 1 for a specific reliability index. These sensitivity factors indicate the negative or positive effect of the RV's on the stochastic model. Thus the most significant RV's can be identified. A value around zero indicates that the variable has little effect on the statistical model, with variable influence increasing as factors approach ± 1. From the expression for α_i, it should be noted that $\sum \alpha^2 = 1$.

The algorithm as set out above may be described as a forward FORM analysis and can be used to explore the effect on reliability of given values of the variables of the reliability model.

4.2.4 Sensitivity analysis and calibration of model for design purposes

The influence of the basic variables of a reliability analysis can be assessed using the directional cosines, as described in the FORM algorithm given in Section 4.2.3. The sensitivity analysis is done using a reverse First Order Reliability method. The procedure for the reverse FORM analysis is philosophically the same as for the forward FORM analysis algorithm, except that in solving the algorithm the reliability level is selected first. The iterations on the limit state equation will then converge at the design point for the chosen reliability level. Thus the values of the basic variables at that design point for the given target reliability are also calculated. The effect of reliability on the model and its basic variables can also be investigated by varying the reliability level. The sensitivity factors of the basic variables obtained for the given target reliability may also be used in the process of calibrating the parameters of the model for design purposes.

The theoretical partial safety factor, γ_x, is the ratio between the theoretical design value obtained from the reverse FORM analysis (x^*) and the mean value of the basic variable considered (μ_x), such that:

$$\gamma_x = \frac{x^*}{\mu_x} \tag{4.3}$$

The theoretical design value of a basic variable is calculated using:

$$x^* = \mu_x - \alpha_x \beta \sigma_x \tag{4.4}$$

where σ_x is the standard deviation of the basic variable considered, and α_x is the sensitivity factor of the basic variable associated with the target reliability β. Equation 4.4 can be written using the CoV (w_x) of the basic variable instead of the standard deviation, i.e.:

$$x^* = \mu_x (1 - \alpha_x \beta w_x) \tag{4.5}$$

The theoretical partial safety factor for the basic variable considered is then expressed by substituting and rearranging equations 4.3 and 4.5, such that:

$$\gamma_x = 1 - \alpha_x \beta w_x \tag{4.6}$$

The basic variable considered is normalised if it as a non-normal pdf.

The design equation for serviceability to SANS 10160-1 (2011) is

$$C_d \geq E_d$$

where C_d is the limiting serviceability criterion and E_d is the design value of the effect of actions. In the case of concrete cracking, C_d is the limiting crack width and E_d the design maximum crack width calculated using the EN1992 crack formulation. The design E_d is achieved for a given target reliability by applying design partial safety factors to each of the variables, taken as characteristic values, in the crack equation. The design partial safety factors and characteristic values are obtained by means of an optimisation process, starting with the theoretical partial safety factors and theoretical design values from FORM.

On reviewing previous research, it was stated by Markova *et al* (2001) that the significant basic variables influencing the design crack width include loads, concrete cover, the tensile strength of concrete and the coefficients such as k_1 and k_2. It was also found that some basic variables such as cover have varying significance depending on the theoretical model determining the crack width.

A sensitivity analysis was performed, and is presented in Chapter 6, to assess the influence of the random variables of the reliability crack model. The theoretical partial safety factors were also calculated for given target reliabilities as a first step in determining the design partial safety factors and final design values of the basic variables.

4.3 TARGET RELIABILITY

The failure probabilities obtained from a probabilistic analysis are compared to a desired failure probability (p_f) which is a maximum value. Alternatively, this desired failure probability may be expressed in terms of the target reliability index (β_t), as a minimum value, which is specified for a given reference period, such that

$$p_f = \Phi\ (-\ \beta_t)$$

where Φ is the cumulative normal distribution function. In choosing the required level of reliability, it is assumed that good quality management practices are followed such that gross errors are avoided.

SANS 2394:2004 states that “Specified maximum acceptable failure probabilities should depend on the consequence and the nature of failure, the economic losses, the social inconvenience, and the amount of expense and effort required to reduce the probability of failure. They should be calibrated against well-established cases that are known from past experience to have adequate reliability. Hence, the specified failure probability should depend on the reliability class.” Target reliability is dependent on the limit state considered as this determines the severity of the consequence of failure. The target reliability for an ultimate limit state is chosen using a cost of safety measure which considers human safety, more specifically the cost of human safety. The target reliability is related to a relative cost of safety and/or the consequences of failure. The JSCC (2008) defines consequence classes by considering the risk to life cost as a ratio of the total costs of failure to construction costs. Serviceability limit states are generally related to performance and loss of use, rather than loss of human life or injury.

The reference period that a failure probability and related reliability index is associated with is defined by SANS 2394 (2004) as being "a certain *a priori* specified period of time". It may be a one-year period (often given as the standard reference period), or the life of the structure, for example. The reliability index for a one-year reference period is related to a reference period of n years by

$$\Phi\ (\beta_{t,n}) = [\Phi(\beta_{t,1})]^n$$

where Φ is the distribution function of a standardized normal distribution. This equation allows a reliability index corresponding to a reference period of the same duration as the design working life to be calculated using the reliability index for a 1 year reference period. In this way, the design working life of a structure can be related to the target reliability.

The design working life of a structure is defined by Holicky (1990) as "the period for which a structure or part thereof is to be used for its intended purpose with anticipated maintenance but without major repair being necessary". The design working life of a structure is categorized in SANS 10160-1 (2011), reproduced here as Table 4.2 overleaf.

The design working life categories are defined according to the type of structure. EN1990-1 and ISO 2394 (2004) define similar categories for the design life of a structure. The design working life of a water retaining structure would generally be taken as 50 years (category 3).

In general, more attention has been given to determining appropriate target reliability levels for ultimate limit states than to serviceability as the former is the more critical state in structures such as buildings. Retief & Dunaiski (2009) stated that "no proper reliability assessment is generally applied to the SLS". A review of recommended values for the target reliability for the serviceability limit state given in various standards was therefore undertaken.

SANS 10160-1 (2011) defines reliability classes and corresponding reliability indexes for the ultimate limit state but not for serviceability. Performance levels for serviceability are differentiated according to the degree of recovery from the consequence of actions exceeding the given serviceability criterion. The levels are irreversible, reversible and long-term and appearance, in order of decreasing severity. A reliability index of 1,5 is recommended by SANS 2394 (2004) for irreversible serviceability states such as cracking in buildings. However, for water-retaining structures, serviceability cracking has a greater importance due to the consequences if cracking results in water leakage in the structure, therefore may require a higher level of target reliability. Retief & Dunaiski (2009) stated that target reliabilities for serviceability, as given in Table 4.4, are an indication of appropriate levels but that 'further refinement of the scheme of target reliabilities may be feasible'.

Table 4.2: *Notional design working life to SANS 10160-1 (2011)*

1	2	3
Design working life category	**Indicative design working life years**	**Description of structures**
1	10	Temporary structures.[a b]
2	25	Replaceable structural parts, for example bearings, agricultural structures and similar structures with low consequences of failure.
3	50	Building structures and other common structures.[c]
4	100	Building structures designated as essential facilities such as having post-disaster functions (hospitals and communication centres, fire and rescue centres), having high consequences of failure[d] or having another reason for an extended design working life.

[a] Structures or parts of structures that can be dismantled with a view to being re-used should not be considered as temporary.

[b] Refer to SANS 10160-8 for the assessment of temporary structures during execution.

[c] The design working life category applies to the reference reliability class referred to in 4.5.2.3.

[d] Consequences of structural failure could be determined in accordance with annex A.

The Joint Committee on Structural Safety (JCSS) (2008) made recommendations for target reliability index values (β_t) for irreversible serviceability limit states such as concrete cracking, given here as Table 4.3, for a one year reference period along with the corresponding failure probabilities. Three levels of relative cost of safety are defined as low, normal and high, with normal being the reference level. As serviceability states are not associated with loss of human life, the relative cost is associated more with performance and use of the structure. The JCSS (2008) states that values chosen for target values may vary by about 0,3 from the β_t values of Table 4.3. The JCSS does not give any guidance on target values for reversible serviceability states, but that irreversible and reversible serviceability states must be distinguished from each other when setting target reliabilities.

Table 4.3: *Target Reliability Indices for Irreversible SLS to JCSS (2008).*

Relative Cost of Safety Measure	SLS Target Index β	Probability of Failure p_f
High	1,3	10^{-1}
Normal	1,7	5.10^{-2}
Low	2,3	10^{-2}

Target reliability levels to ISO 2394: 1998 (SANS 2394:2004) and EN1990: *Eurocode 1: Basis of structural design* were summarized by Retief and Dunaiski (2009) as shown in Table 4.4 (overleaf), for both the ultimate and the serviceability limit states. ISO 2394 recommends β_t levels using a relative safety cost of safety and a consequence of failure. According to SANS 2394:2004, the β-values were derived assuming a normal distribution for permanent load, a Gumbel distribution for imposed load and log-normal or Weibul distribution for resistance. EN1990 (2002) defines reliability classes (or consequence classes) RC1 to RC3. Each consequence class looks at the effects of loss of human life and economic, social and environmental costs where RC1 has a low consequence, RC2 has a medium consequence and RC3 has a high consequence. RC2 is taken as the reference class. Target reliabilities of 2,9 and 1,5 are recommended for reference periods of 1 and 50 years, respectively. In this research, a value of 1,5 was used as the reference target reliability.

A review of any research previously carried out on reliability with respect to serviceability cracking was carried out. It was found that there has been little research done on reliability of serviceability cracking, particularly for small crack widths. A summary of relevant research is now given.

Table 4.4: *Target reliability levels (β) according to ISO 2394 and EN 1990 (Source: Retief and Dunaiski, 2009)*

Relative cost of safety measures	**ISO 2394 Minimum values for β**			
	Consequences of failure			
	Small	Some	Moderate	Great
High	0	1,5 (A)	2,3	3,1 (B)
Moderate	1,3	2,3	3,1(C)	3,8 (D)
Low	2,3	3,1	3,8(D)	4,3(E)

A for serviceability limit states β = 0 for reversible and β = 1,5 for irreversible states
B for fatigue limit states β = 2,3 to 3,1 depending on the possibility of inspection
For ultimate limit states the safety classes: C β = 3,1 D β = 3,8 E β = 4,3

Reliability Class	**EN 1990 Minimum values for β**					
	Ultimate LS		Fatigue		Serviceability LS	
Reference period	1 year	50 years	1 year	50 years	1 year	50 years
RC1	4,2	3,3				
RC2	4,7	3,8(F)		1,5 to 3,8	2,9	1,5
RC3	5,2	4,3(G)				

F With ISO 2394 clause 4.2(b) *moderate safety costs & RC2 consequences* , but EN 1990 is more conservative; EN1990 value agrees with ISO 2394 for either *low safety cost or great consequences*

G The EN1990 value for RC3 agrees with ISO 2394 for *low safety cost and great consequences*

ISO: 2,3 – 3,1 EN: 1,5 – 3,8 Fatigue: ISO 2394 – restricted range; EN1990 – range from *serviceability LS equivalent* to *ultimate LS*

Holicky *et al* (2009) presented a reliability analysis of the EN1992 crack model with cracking due to a direct tensile load as a pilot investigation of cracking in reinforced concrete water-retaining structures. The validity of using the probability level of 5% of exceeding the specified crack width limit as the accepted reliability level, as stated in EN1992-1 for verification of serviceability limit states, was considered rather than the target reliability index, β_t. The conclusion was that this was not an optimum choice of reliability level.

Markova *et al* (2001) performed a reliability analysis of cracking in a reinforced concrete slab for various design code crack formulations and found the ENV 1992-1 crack model (precursor of EN1992-1) was sufficient for a limiting crack width of 0,3 mm (probability of exceedence of 5%). The reliability index was determined to be above 1,5. However, crack widths less than 0,3 mm were not considered.

Quan and Gengwei (2001) investigated the calibration of the reliability index for cracking of reinforced concrete beams by means of an inverse FORM analysis. The crack model used was that to the Chinese code for reinforced concrete and was found to be of similar formulation to the Eurocode concrete cracking model. Therefore, some comparisons could be made in establishing values of the reliability index. For crack widths equal to and greater than 0,2 mm, the reliability index values determined and evaluated were in the range of 0 to 1,8.

4.4 MODEL UNCERTAINTY

In reliability models, uncertainty will always exist and needs to be quantified as far as possible. The first step in doing so is to identify sources of uncertainty. Two main types of uncertainty can be defined, that is, inherent random variability and that due to incomplete knowledge including statistical uncertainty as defined in literature such as that by König *et al* (1985), Ang & Tang (1990) and the JCSS – PMC (2001).

Inherent random variability may or may not be affected by human activities. The uncertainties in strength values of materials is an example of inherent random variability affected by human activity, while uncertainty of load due to precipitation is an example of those not affected by human activity. Uncertainties are influenced by the level of production (e.g. of steel) and quality control during design and construction. Summarising from the literature, uncertainty due to random variability is generally dealt with by means of the coefficients of variation of the random variables in the probabilistic model.

There is also uncertainty due to mathematical simplifications of physical and probabilistic models. In this case, the former would be in the formulation of the deterministic crack model from experimental research, and the latter in the stochastic model on normalizing non-normal distributions and in the first order approximation using FORM for the mathematical solution of the failure probability integral. The statistical parameters of the material, load and geometric properties applying to the crack model are known and available in literature, and are discussed further in Section 4.6 and summarised as Table 4.6.

It is accepted to treat model uncertainty as a random variable having commonly either a normal or log-normal PDF (*JCSS- PMC* (2001 and 2008)). The CoV would be chosen to reflect the degree of uncertainty expected. Table 4.5 gives typical values of model uncertainty as a random variable. The load effect and resistance factors for the general structural model have been included as a comparison to a value that would be appropriate for the crack model.

A review of research into model uncertainty with respect to concrete cracking was done. Quan and Gengwei (2002) investigated the reliability of cracking in reinforced concrete beams in buildings with respect to the Chinese structural concrete code and considered limiting crack widths equal and greater than 0,2 mm.

Table 4.5: *Summary of model uncertainty values from literature applicable to EN1992*

Variable	Symbol	Units	PDF	Mean μx	Std Dev. σx
Model uncertainty, general	θ	-	LN	1	0,10 - 0,3
Load effect factor (Holický (2009))	θ_E	-	N	1	0.05 - 0.1
Resistance factor (Holický (2009))	θ_R	-	N	1-1.25	0.05 - 0.2
Model uncertainty-cracking (Holicky *et al* (2009))	θ_w	-	LN	1	0.1

The limit state equation used had the same form as Equation 4.11 of Section 4.4 to be used in this investigation, with model uncertainty allowed for as a random variable. The Chinese crack model used is similar enough to the Eurocode to be able to draw some comparison for the model uncertainty. On analyzing experimental data on cracking in beams, they concluded that the model uncertainty had a log-normal distribution with a mean of 1,05 and a cov of about 0,3, based on experimental data.

Holicky *et al* (2009) presented a reliability study of the EN1992 crack model with cracking due to a direct tensile load. The model uncertainty was applied as a random variable having a mean of 1,0 and a CoV of 0,1 as an initial value. This is the general value recommended for use in structural models.

It was concluded that model uncertainty would be treated as a random variable, and is further discussed in Section 4.6.

4.5 THE LIMIT STATE FUNCTION FOR THE EN1992-1 CRACK MODEL

The FORM procedure begins with the formulation of the limit state function for the particular model to be analysed as discussed in Section 4.2.3, in this case the SLS of load-induced cracking to EN1992. As cracking is a serviceability limit state, the limit state function for the EN1992 crack width equation has the form:

$$g(X_i) = C - E$$

as expressed in Section 4.2. C will be the limiting crack width desired, w_{lim}. As the action effects in this case are due to load-induced cracking, the effect of actions, E, will be the calculated crack width, w_{calc}. Considering the formulation of the EN1992 crack width calculation, it would be difficult to separate the resistance (R) from the actions (E) and therefore a limit state function in the form of $g(X_i) = R – E$ would not be feasible.

Crack width, in general, may be expressed in terms of the compatibility relationship, as given in Section 2.3.3 of Chapter 2, where

$$w = S_r . \varepsilon_m$$

where S_r is crack spacing and ε_m is nett strain, i.e. steel tensile strain less concrete tensile strain.

However, the EN1992 design crack width equation, as given in Section 2.3.3 of Chapter 2, gives the maximum or characteristic crack width, w_k,

$$w_k = S_{r,max} . \varepsilon_m$$

where $S_{r,max}$ is the maximum crack spacing. In the case of concrete cracking due to load effects, ε_m is the nett strain due to loading. The characteristic crack width is taken as the width having a probability of exceedence of 5%. The derivation of the EN1992 crack width equation is therefore examined to obtain the equation for crack spacing (S_r) and thus the equation for crack width (w) to be applied in the reliability model.

The equation for the net strain ε_m which will be used in the calculation of the crack width in the limit state function, as given in Chapter 2, is:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t f_{ct,eff}\left(1+\alpha_e \rho_{eff}\right)/\rho_{eff}}{E_s} \qquad (4.8)$$

where ε_{sm} is the strain in the reinforcement, ε_{cm} is the concrete strain, σ_s is the stress in the tension reinforcement at the cracked section, k_t is a factor to account for the duration of the load, $f_{ct,eff}$ is the concrete tensile strength and α_e is the modular ratio. In applying this equation to the reliability model, the stresses and strains will be expressed in terms of their basic variables with in turn, their probability distribution functions.

The EN1992 characteristic crack width, w_k, which must not exceed the specified limiting crack width has been found experimentally to correspond to a maximum crack width, $S_{r,max}$, such that the ratio $\beta_w = S_{r,max}/ S_r$ is 1,7. Thus the maximum crack spacing may be written as:

$$S_{r,max} = 1{,}7S_r$$

The characteristic crack width then becomes:

$$w_k = (1{,}7S_{rm}) . \varepsilon_m = 1{,}7\ w$$

Note that this ratio β_w must be distinguished from the FORM reliability index β. The use of the same symbol is coincidental as this ratio is not related to the reliability index.

To obtain the equation for the crack spacing distribution, the EN1992 equation (given as equation 2.11 in Chapter 2) for the maximum crack spacing ($S_{r,max}$) is considered:

$$S_{r,max} = k_3 c + k_1 k_2 k_4 \varphi / \rho_{p,eff}$$

where φ is the bar diameter (mm), c is the cover to the longitudinal reinforcement and k_1 is a coefficient taking into account of the bond properties of the bonded reinforcement. The factors k_3 and k_4 are given the values of 3,4 and 0,425 respectively by EN1992-1. Using the ratio β_w, the maximum crack width can then be expressed as:

$$S_{r,max} = 1{,}7\ (2c + 0{,}25 k_1 k_2 \varphi / \rho_{p,eff})$$

Thus the crack spacing distribution may be written as:

$$S_r = 2c + 0{,}25\ k_1 k_2\ \varphi / \rho_{p,eff} \tag{4.9}$$

The calculated crack width equation is then:

$$w_{calc} = S_r \cdot \varepsilon_m \tag{4.10}$$

such that S_r is calculated using equation (4.9) and ε_m is calculated using equation (4.8).

The SLS limit state function for the reliability crack model may then be expressed as:

$$g(X) = w_{lim} - \theta .\ w_{calc} \tag{4.11}$$

where θ is the model uncertainty applied as a random basic variable.

Further to the discussion on model uncertainty in Section 4.4, uncertainty in the crack model, as allowed for by means of the model uncertainty basic variable, exists in the following:

- The determination of strain as the model for strain is based partly on experimental research and observation.
- The factors k_1 and k_2 appearing in Equation (4.9) taken as deterministic values. They have been derived from experimental research in developing the EN1992 crack equation and therefore may have some degree of uncertainty associated with them which is not known.
- Uncertainty in the crack model itself which is not actually known at present. As cracking is a random phenomenon, the crack model is an approximation of cracking behaviour and is investigated parametrically in the probabilistic analyses.

In order to perform the reliability analysis, the parameters of the crack width equation 4.10 are expressed as random basic variables. In this way, the deterministic design function is converted into the reliability performance function. An investigation into the available statistical data for the basic variables of equation 4.10 for crack width, w_{calc}, was carried out and presented in the following Section 4.6.

4.6 GENERAL DATA FOR PROBABILISTIC PARAMETERS IN CRACKING MODEL

An investigation into the probabilistic parameters of the variables used in the calculation of EN1992-1 crack widths was carried out with the resulting values summarized in Table 4.6 overleaf. These general values were sourced mainly from Holicky (2009), the Joint Committee on Structural Safety JCSS – PMC (2001), Fulton's (2009) and Holicky, Retief & Wium (2009). Data is given in terms of the probability distribution type (pdf), the characteristic value, mean (μ_x) and coefficient of variation (CoV). South African values were found to correspond with the values given in Table 4.6.

Variables which have very small coefficients of variation are taken as deterministic values, for example, geometric properties which generally have small variations compared to actions and material properties. Geometric properties such as section thickness tend to have a normal probability distribution function (pdf).

The statistical data for material properties are determined from standard testing. Material properties tend to have normal or log-normal distributions. The characteristic value of the material property concerned is generally used in design and corresponds to the 5% lower fractile (*Cl. 5.6 of SANS 10160-1(2011)*), taken as 1,64 x standard deviation for a normal pdf.

The target design strength of concrete (not to exceed the characteristic strength by 1,64 the standard deviation) is also related to a degree of quality control (*EN1990-1*). Fulton's (2009) defines 3 degrees of control, namely, poor, average and good with their associated standard deviations. This research assumes a good degree of quality control as would be expected for special classes of structures such as water-retaining structures.

Permanent actions or loads tend to have normal distribution which is the case for the liquid load on the wall of a WRS. Note that the coefficients k_1 and k_2 appearing in Equation (4.2) are have deterministic values and therefore have not been included in Table 4.6. A search was made to find data on the statistical parameters for measured crack widths compared to values calculated using EN1992-1, particularly for small crack widths. Such data would aid in establishing the reliability in probabilistic terms of the EN1992-1 crack model in terms of the model uncertainty.

Table 4.6: *Summary of basic variables for time-invariant reliability analysis, derived from Holicky(2009), JCSS-PMC (2001), Fulton's (2009) & Holicky et al (2009).*

Variable	Symbol	Units	PDF	Mean μ_x	CoV
Permanent Load	G_k	kN/m^2	N	G_k	0.03 – 0.10
Liquid Load	L_k	kN/m^2	N	L_k	0.03 – 0.10
Steel yield point	f_y	MPa	LN	$f_{yk}+2\sigma$	0.07 – 0.10
Concrete compressive strength	f_c	MPa	LN	$f_{ck}+ 2\sigma$	0.10 – 0.18
Concrete tensile strength	$f_{c,t}$	MPa	LN	$f_{ctk}+ 2\sigma$	0.10 – 0.18
Steel modulus	E_s	GPa	Det		
Concrete modulus	E_c	GPa	Det		
Reinforcement diameter	Φ	mm	Det		
Reinforcement area	A_s, A_s'	mm2	Det		
Concrete c/s geometry	b, h	m	N	b_k, h_k	0.005 – 0.01
Cover	c	m	BETA / Γ	c_k	0.005 – 0.015
Distance to centre of bar	a	m	BETA / Γ	$c + \varphi/2$	0.005 – 0.015
Limiting Crack width (average)	w_{lim}	mm	Det	0.05 – 0.2	
Model factor for crack width	θ_w	mm	N/ LN	1.0	0.1 – 0.3

Note: LN = log-normal, N = normal, Det = deterministic, Γ = gamma

It was concluded in *Eurocode 2 commentary (2008)* that from a first analysis, the maximum crack width has a normal PDF and relates to the applied stress level. A comparison between test data for mean crack widths ($w_{m,exp}$) and those calculated ($w_{m,calc}$) was made. The crack models from the CEB model code MC90, PrEN (ENV1992, previous version to EN1992) and EC2 (EN1992) were compared. Results were plotted, as shown in Figure 4.2. Any analyses were related to crack widths of 0,3 mm and larger. Figure 4.2 shows a wide variation in test and model crack widths was obtained PrEN was indicated as correlating well with test crack widths, while EC2 predicted values slightly less than test values. MC90 underestimates the crack width when compared to test values.

The error in the mean crack width ($w_{m,calc} – w_{m,exp}$) was then plotted against the measured crack width, shown in Figure 4.3. The mean and standard deviation for the error was determined and expressed in mm. However, the error was not related in the text to any particular crack width. Figure 4.3 (overleaf) does indicate that there is a wide scatter in the test results compared to the calculated crack widths. On plotting the distribution and density functions of the error, it was concluded that the error had an approximately normal distribution for EN1992.

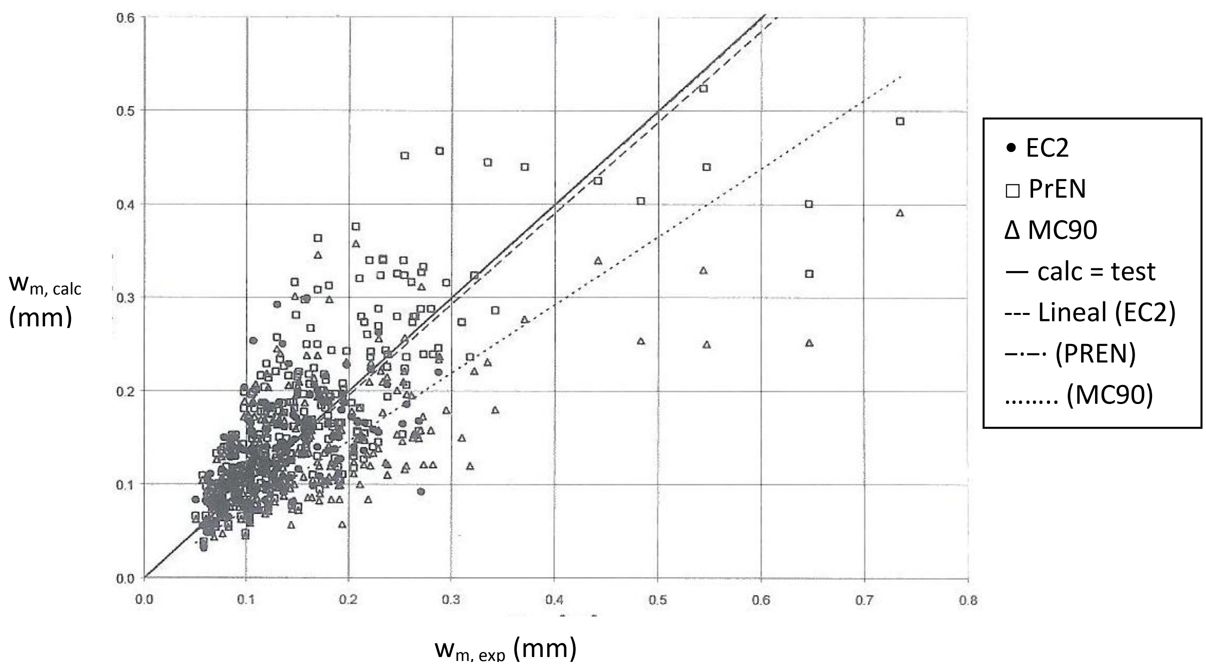


Figure 4.2: *Comparison between test and calculated mean crack widths to EC2, MC90 and PrEN. (Source: Eurocode commentary (2008))*

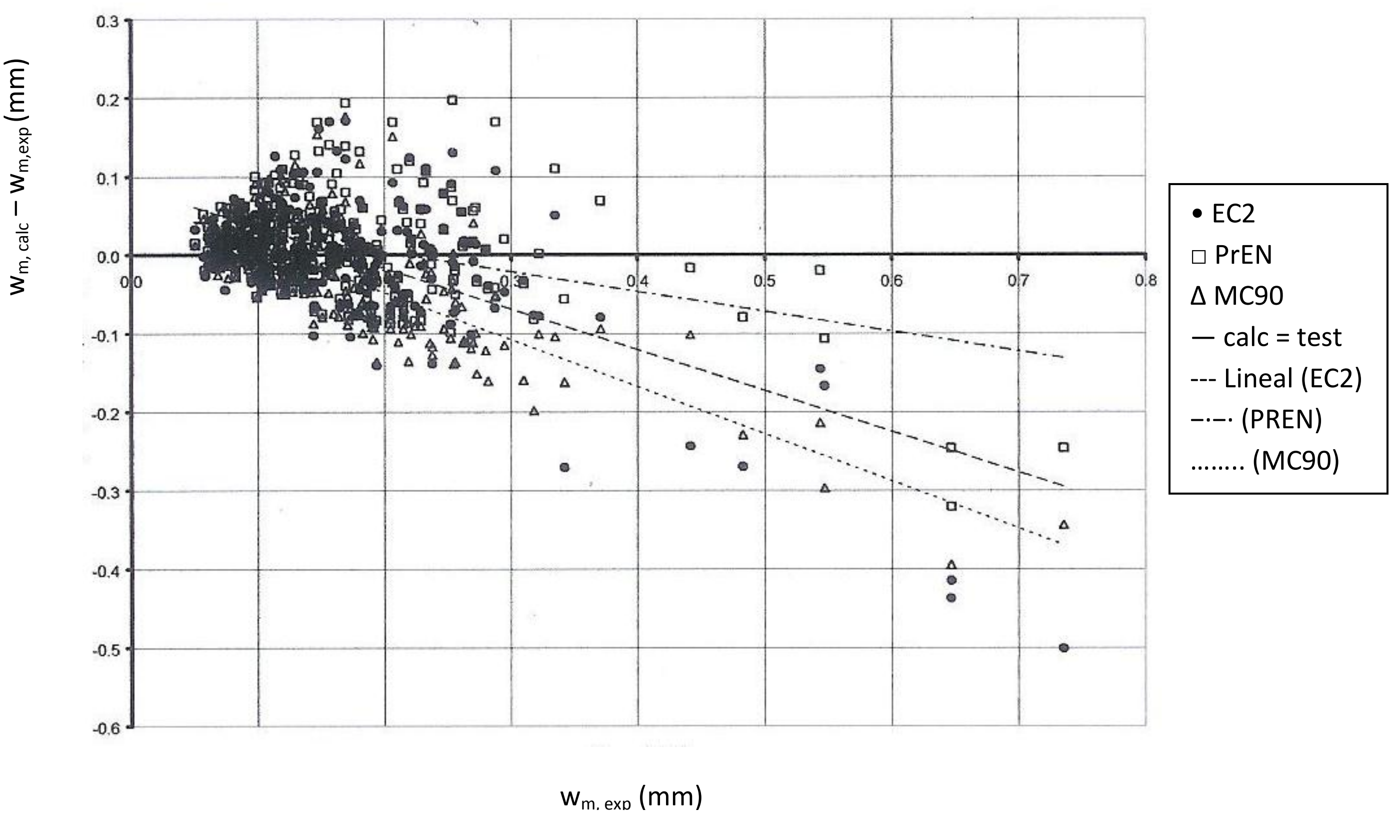


Figure 4.3: *Error crack width (Source: Eurocode commentary (2008))*

Research carried out by Borosnyoi *et al* (2005) on comparing models for flexural cracking found that the distribution of experimental crack widths was log-normal. This conclusion agreed with that reached by Quan *et al* (2001) in carrying out FORM analyses for cracking in reinforced concrete beams. The limit state function used had the same form as that of Equation 4.11 with a deterministic value for w_{lim} used. As there appears to be little more statistical data on measured crack widths, it can be concluded that further research is required on predicted versus actual crack widths. In this research, the limiting crack width in the limit state function for the EN1992 crack reliability model is accepted as a deterministic value. It is recognized that the limiting crack width will result in some uncertainty in the reliability model which will be included in the model uncertainty. However, insufficient data is available at present for the limiting crack width to be modelled as a random variable. This is a topic for further research. Model uncertainty is treated as a random variable in the determination of the crack width, w_{calc}, as discussed in Section 4.4.

The concrete cover to reinforcement is seen as an influential parameter in the calculation of crack widths, as discussed in Chapter 2. Research shows that the variability of concrete cover depends on the quality control during construction (*JCSS (2008)*). Variability obviously decreases with an increasing level of quality control. Suggested coefficients of variation (CoV) for typical British construction for the standards of control were defined as near - laboratory precision, good, moderate and poor, associated with CoV values of 10%, 15%, 20% and 30% respectively (*Ronné* (2006)). Ronné (2006) reviewed local and international cover data and found that variability also decreased with increasing cover. He also stated that South African construction has a higher absolute variability for concrete cover. However, structures such as bridges have a significantly higher quality control therefore a lower variability of cover. It can be concluded that a CoV of 0,15% would be a reasonable representation for the variable of concrete cover in the probabilistic crack model, assuming a good level of quality control. Holicky *et al* (2009) concluded that for cover, limited beta or gamma pdf's, a log-normal distribution would be a reasonable approximation. In this research, the parameters of load, section thickness, concrete cover and concrete tensile strength of the EN1992 crack formulation are modelled as random variables, each with their own pdf. All other material and physical parameters are considered as deterministic values.

4.7 SUMMARY

In order to properly formulate the reliability model for the EN1992 crack equation, the First Order

Reliability Method of analysis (FORM) was investigated and the requirements for this method of analysis were determined with respect to the EN1992 crack model. The FORM algorithm is summarised and was used in the reliability analysis presented in the next chapter. The requirements to set up the FORM analysis model are as follows:

- The limit function for the crack model is defined in equation (4.11) as $g(X) = w_{lim} - \theta.w_{calc}$, where w_{lim} is the specified crack width, w_{calc} is the calculated crack width and θ is the model uncertainty.
- The crack width is determined using the following equations which are expressed in terms of their basic variables in the FORM analysis:

  Crack width $w_{calc} = S_r.\varepsilon_m$

  Crack spacing $S_r = 2c + 0{,}25\ k_1k_2\ \varphi/\ \rho_{p,eff}$

  Strain $\varepsilon_s - \varepsilon_c = \frac{\sigma_s - k_t f_{ct,eff}(1+\propto_e \rho_{eff})/\rho_{eff}}{E_s}$

- The statistical values of the material and geometric parameters of the crack model were established for the crack model. A set of appropriate basic variables was chosen for application in the reliability analysis with general values summarised in Table 4.5. The parameters of load, section thickness, concrete cover and concrete tensile strength of the EN1992 crack formulation will be modelled as random variables, each with its own PDF, with all other parameters taken as deterministic values. The actual values used for the variables of the reliability model are presented in Chapter 5 with the reliability analysis.
- Data on uncertainty of the crack model is limited, therefore needs further investigation. In this study, it is allowed for by means of a random variable θ having a log-normal distribution with a mean of 1, applied to the calculated crack width. The CoV will be varied from 0,1 to 0,3 to investigate the effect of model uncertainty on the reliability of the crack model.
- The limiting crack width is taken as a deterministic value corresponding to the limits set by EN1992, applied to a representative WRS, namely the range from 0,05 mm to 0,2 mm.
- A SLS reliability β of 1,5 for a reference period of 50 years is used as the reference level in the reliability study in keeping with SANS10160-1.
- The reliability of the EN1992-1 crack equation has not been assessed with respect to water-retaining structures in South Africa, providing motivation for the probabilistic analysis in this research. In addition, as SLS cracking is generally the dominant limit state, an appropriate level of reliability for the crack width calculation needs to be determined. Therefore, the effect of varying the reliability index on the crack model will be investigated.

CHAPTER 5
FORM ANALYSIS OF EN1992 CRACK MODEL

5.1 GENERAL

This chapter presents the reliability analysis performed on the EN1992 crack models for flexural and tension cracking. The reliability analysis was used to investigate the EN1992 crack model applied to the design of reinforced concrete water retaining structures (WRS) in South Africa, leading to possible improvements in performance of the crack model. With this objective in mind, the reliability study focused on the following issues:

(viii) The reliability of the EN1992-1-1 crack model and therefore the associated level of reliability for serviceability cracking.

(ix) The extent to which serviceability cracking governs the design of WRS, rather than the ultimate limit state.

(x) The implications of the more severe limiting crack widths less than 0,2 mm specified by EN1992-3.

(xi) The model uncertainty of the EN1992 crack model.

The EN1992 crack model was developed semi-empirically with limited research having previously been done in reliability terms. This study attempts to extend the understanding of modelling the EN1992 crack model in reliability terms. As in the deterministic analysis, cracking is assumed to be through the section.

The serviceability limit state (SLS) is generally concerned with a loss of function or comfort rather than a failure of the structure leading to possible harm to life. Therefore it has a lower general level of reliability than ultimate limit states (ULS). From the deterministic analysis, SLS was found to dominate in both flexural and tension cracking, making serviceability the critical limiting state in the design of WRS. The reliability study will then consider the requirements needed to meet the serviceability cracking criteria for the general reliability level of 1,5 applied to irreversible serviceability states such as cracking, as laid out by SANS10160-1. In addition, the level of reliability for serviceability has been developed for buildings, not the special class of structure of WRS where leakage of the structure is more critical. Therefore, both the reliability of the EN1992 crack equation and the effect of changing the level of reliability for load-induced cracking in a WRS are investigated in this chapter.

The deterministic analysis showed that the use of limiting crack widths less than 0,2 mm had a negative effect on the economics of the structure for both tension and flexural cracking. The effect of a more severe crack width limit on the reliability of the crack model is therefore investigated.

From the literature review on probabilistic analysis presented in Chapter 4, the model uncertainty for cracking is not really known. Therefore it was investigated to determine its influence on the EN1992 crack equation, the results of which are presented in this chapter. Model uncertainty is treated as a random variable in the reliability analysis, as discussed in Chapter 4.

Reliability analyses using the First Order Reliability method (FORM) were then performed to explore these objectives with respect to both the tension and flexural cracking models.

5.2 FORMULATION OF THE RELIABILITY LOAD-INDUCED CRACKING MODELS

The structural configurations used in the reliability analysis of the flexural and tension cracking models were the same as those of the deterministic analysis presented in Chapter 3 investigating the flexure and tension cracking cases to EN1992 (illustrated by Figures 3.1 and 3.2). The representative values used for the parameters of the crack model were taken from the deterministic analysis. The formulation of the limit state function and general values of variables were obtained from the literature review presented in Chapter 4.

5.2.1 Structural configuration for the reliability crack model

A summary of the structural configuration and the physical parameters is given for both tension and flexural cracking cases as follows:

(i) In the flexure case, a 1 m section of wall of a rectangular reinforced concrete water-retaining structure was considered. The wall is subject to flexure due to water pressure about the horizontal axis of the cross section.

(ii) For the tension case, a 1 m section of wall in a cylindrical reservoir was considered. The wall is subject to hoop tension due to water pressure in the horizontal plane. A representative reservoir diameter was taken as 28 m.

5.2.2 Formulation of FORM crack model equations

Summarising from Chapter 4, the limit state equation for the FORM analysis is:

$$g = w_{lim} - \theta .\ w_{calc} \tag{5.1}$$

where θ is the model uncertainty, modelled as a random variable.

The crack width is determined from the basic compatibility equation,

$$w_{calc} = S_r\ .\ \varepsilon_m. \tag{5.2}$$

The crack spacing S_{rm} considered in the probabilistic model is determined from:

$$S_r = 2c + 0{,}25k_1k_2\varphi\ /\rho_{p,eff} \tag{5.3}$$

EN1992-1-1 recommends a value of 0,8 for the coefficient k_1 for high tensile reinforcement bond. The coefficient k_2 is given a value of 0,5 for bending stress distribution and 1,0 for tension. Both coefficients are deterministic.

The mean strain equation as given by EN1992 is:

$$\varepsilon_m = \varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t f_{ct,\ eff}\left(1+\alpha_e\rho_{p,eff}\right)/\rho_{p,eff}}{E_s} \tag{5.4}$$

The coefficient k_t has a value of 0,4 for long-term loading. The modular ratio, α_e, is the ratio E_s/E_c. The parameters in the reliability limit state function are either modelled as random variables or taken as deterministic values, as summarised in Table 4.5 of Chapter 4.

For the reliability models for the flexural and tension load cases, the equations (5.1) to (5.4) are expressed in terms of the basic random variables, which are considered here to be section thickness (h), concrete cover (c), liquid load (L_k) and concrete tensile strength ($f_{ct,eff}$), summarised as follows:

(i) Flexural load case

In the determination of the mean strain, the steel stress σ_s, and therefore the effective depth of section, lever arm z, bending moment M, and the effective reinforcement ratio $\rho_{p,eff}$ (using first principle equations of elastic linear theory) are expressed in terms of the basic random variables h, c and L_k as follows:

- Effective depth of section, $d = h - c - \varphi/2$
- The depth to the neutral axis, x :

$$x = \frac{\sqrt{(\alpha_e A_s)^2 - 2b\alpha_e A_s d} - \alpha_e A_s}{b}$$

where α_e is the modular ratio E_s/ E_c and b is the length of wall section considered.

- Lever arm, $z = d - x/3$
- Liquid load $L_k = H.\ \gamma_w$ where γ_w is 9,81 kN/m^3 per metre length of wall
- Tensile stress due to bending in the reinforcement, $\sigma_s = M/ A_s z = L_k.H^2/(6\ A_s z)$
- Effective depth of the tension zone, $h_{c,eff} = (h - x)/3$ (limiting $h_{c,eff}$ for all h considered)
- Effective area in tension zone of section, $A_{ct,eff} = b.h_{c,eff}$
- Effective reinforcement ratio, $\rho_{p,eff} = A_s/ A_{ct,eff} = A_s/(\ b.h_{c,eff})$

For flexural cracking, the reinforcement provided to the tension face of the wall only is considered in the calculations.

(ii) Tension load case

Steel stress σ_s, tension T and the effective reinforcement ratio $\rho_{p,eff}$ are expressed in terms of the basic random variables h, c, L_k as follows:

- Steel stress, tension $\sigma_s = T/ A_s$ with $T = L_k.D/2$ where D = diameter of reservoir (considered as a deterministic value of 28 m).
- Effective depth of the tension zone in concrete $h_{c,eff} = h/2$ or $2,5(h - d)$ (the latter rewritten as $2,5\ (c + \varphi)$) as per EN1992.
- Effective area in tension $A_{ct,eff} = b.h_{c,eff}$
- Effective reinforcement ratio $\rho_{p,eff} = A_s/ A_{ct,eff}$, as flexural cracking.

Note that the reinforcement area considered for tension cracking is the total amount required for the section due to the applied tensile force, with reinforcement being provided equally to both faces of the wall.

5.2.3 Values used for parameters of the reliability crack models

Table 5.1 summarises the statistical data for all parameters used in the reliability models for tension and flexural cracking. Units of the basic variables are as given in Table 5.1. Water pressure, concrete tensile strength, concrete cross sectional depth (wall thickness), concrete cover and model uncertainty were found to be the basic random variables of the model as all other parameters may be considered as deterministic values.

Table 5.1*: Basic variables used in reliability crack model*

Variable	Symbol	Units	pdf	Characteristic Value	Mean μ_x	Std Dev. σ_x
Height of wall	H	m	Det	5	5	0
Liquid load, L_k	L_k	kN/m^2	N	50	49.05	2.45
Concrete tensile strength	$f_{c,t}$	MPa	LN	2.00	2.89	0.55
Steel modulus	E_s	GPa	Det	200	200	0
Concrete modulus (long term)	$E_{c,eff}$	GPa	Det	13.7	13.7	0
Reinforcement diameter	φ	mm	Det	20	0.02	0
Reinforcement area	A_s	mm^2	Det	3500	3500	0
Concrete c/s depth	h	mm	N	450	450	4.5
Concrete c/s width	b	mm	Det	1000	1.0	0
Concrete cover	c	mm	LN	40	40	6
Concrete modulus	E_c	GPa	Det	27.4	27.4	0
Concrete creep factor	ϕ	-	Det	1	1	0
Limiting crack width	w_{lim}	mm	Det	0.2	0.2	0
Model Uncertainty	θ_w	-	LN	1	1	0.2

Note: LN = log-normal pdf, N = normal pdf, Det = deterministic value.

The depth of water is taken as the height of the wall, H, and is modelled as a deterministic parameter as its variation is considered to be small, as other section dimensions (depth and width). However, as a permanent load, liquid load is modelled as having a normal pdf.

A bar diameter of 20 mm was used and considered as a deterministic value in the reliability analysis as it has been found to have a very small variation. Therefore any influence it has will be proportional directly to the change in the value chosen, rather than due to any statistical variation. The influence of bar diameter on the crack calculations was determined by the deterministic analysis to be of lesser influence on the crack model, further validating treating this variable as deterministic.

The area of reinforcement was taken as a deterministic value and used as the basis of comparison in the analysis. As in the deterministic analysis, the minimum practical spacing of reinforcement was taken as 75 mm. For a given bar diameter, this gave the maximum practical area of reinforcement which will be 4198 mm^2 for a 20 mm bar diameter. Mean values used for concrete cover and wall height were 40 mm and 5 m, respectively. All other values of the parameters such as material properties are summarised in Table 5.1.

The model uncertainty was investigated by considering this parameter as a random variable with a log-normal distribution, mean of 1 and coefficient of variation (CoV) of 0,2 as a reference level, in keeping with the findings of the literature review. A general value used for model uncertainty CoV is 0,1 in structural reliability models, as used by Holicky *et al* (2009), and was taken to be the lower limit for the CoV in the case of concrete cracking. A value of 0,3 has been suggested in research as being appropriate to cracking in reinforced concrete beams (*Quan et al* (2002)) and was therefore taken to be the maximum limit in this study. It was judged that a value of 0,1 would be too low for cracking as the uncertainty for cracking is greater than for general structural models, whilst a value of 0,3 is possibly too conservative for the models considered here. A value of 0,2 for the model uncertainty CoV was therefore considered to be reasonable for use as the reference level. In addition, to consider the effect of model uncertainty variation on the cracking models, reliability analyses were performed for a range of values from 0,1 to 0,3 for the model uncertainty variation (θ_{cov}).

The effect of crack width on the reliability model was investigated for crack width limits (w_{lim}) of 0,3, 0,2, 0,1 and 0,05 mm, in keeping with EN1992 for buildings and water retaining structures.

5.2.4 Formulation of reliability models using Microsoft EXCEL

Analyses following the FORM algorithm (as summarised in the previous chapter) were performed using Microsoft EXCEL to determine the level of reliability for a given set of variables and area of reinforcement for both flexural and tension cracking.

For tension cracking, the formulation of the crack equation in terms of the basic variables depends on the limiting equation for the effective depth of the tension zone, $h_{c,eff}$. Therefore two reliability models were required in the case of tension cracking. Thus the reliability models evaluated using EXCEL were:

(i) Model 1 - Flexural cracking

(ii) Model 2(a) - Tension cracking, $h_{c,eff} = 2,5(c + \varphi/2)$

(iii) Model 2(b) - Tension cracking, $h_{c,eff} = h/2$

The formulation of the reliability models using EXCEL for the tension and flexural cracking cases is illustrated by the flow chart given in Figure 5.1 overleaf and is described as follows:

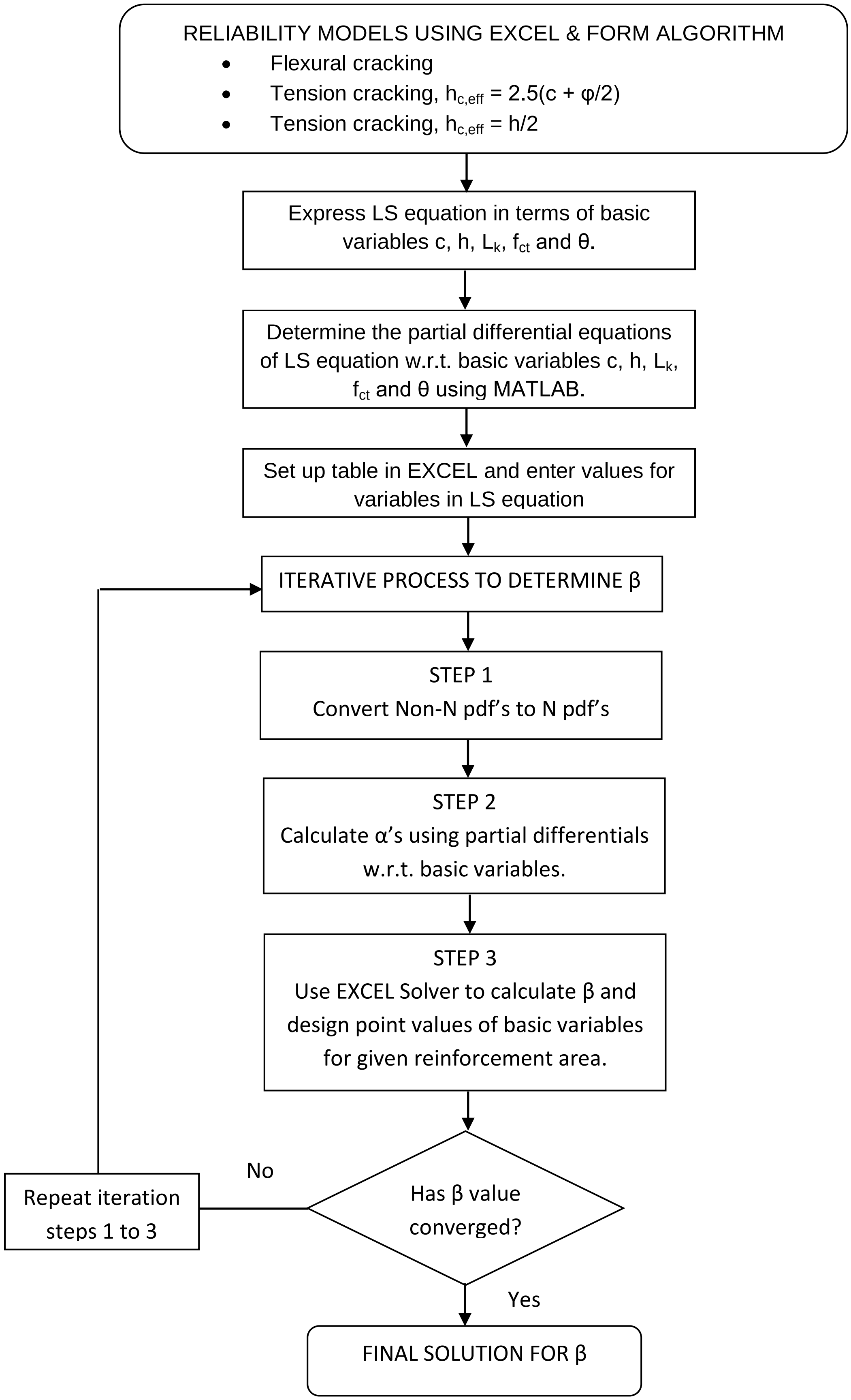


Figure 5.1: *Flowchart of EXCEL process to solve for β in reliability crack models*

- The limit state equation was written in terms of the basic variables. The partial differential equations of the limit state equation with respect to each of the basic variables were calculated using MATLAB.
- A table was set up in EXCEL for each crack model for the variables appearing in the limit state equation, as shown in Figure 5.2. Units of the variables of the model were all expressed in kN and m. The basic variables that were not deterministic were identified as section thickness, concrete cover, liquid load, concrete tensile strength and model uncertainty. The reinforcement area, A_s, was parametrically chosen for a representative range selected from the results of the deterministic analysis.
- The FORM algorithm, as summarised in Chapter 4, was then set up for each crack model in EXCEL. As the FORM algorithm is an iterative process, the EXCEL spreadsheet was set up with sufficient iterations to allow the reliability index value to converge, as shown in Figures 5.3(a) and 5.3(b). For <u>Step 1</u> of the first iteration, and referring to Figure 5.3(a), the non-normal mean and variation were converted to the equivalent normal parameters. In <u>Step 2</u>, the normalised mean and variation were then entered for σ_i and μ_i. The initial mean value of each basic variable was taken as the assumed design point value (assumed x_i*) for that variable. The partial differentials and thus the directional cosines were then calculated. In the final step of the first iteration (<u>Step 3</u>), the new failure point (new design point) and β were calculated. Referring to Figure 5.2(b), Iteration 2 used the failure point calculated in Iteration 1 as the new assumed design point. Steps 1 to 3 were repeated to find the new failure point for Iteration 2. Generally, 5 iterations were performed to ensure convergence of β.
- The software add-in EXCEL Solver was used to calculate β at each iteration step by satisfying the limit state function g = 0. Solver uses a Generalized Reduced Gradient (GRG) nonlinear method for problems that are smooth non-linear to find an optimal value (in this case zero) for a formula in one cell (the objective cell) subject to constraints on the values of other formula cells on the worksheet. In the reliability model this is the limit state function, g. The decision variable cell for each iteration step, set as the reliability index, β, participates in computing the formulas in the objective cell. Solver then adjusts the values in the decision variable cells to satisfy any limits set on the constraint cells and produces the result required for the objective cell. In this case, there were no constraints set. The reliability index was determined using the FORM algorithm for a given quantity of reinforcement.
- The final data generated by each analysis was copied to a data worksheet in EXCEL. Graphs of the variation of the reliability index β with reinforcement required could then be plotted.

Variable	**Symbol**	Dim	Distribution	Characteristic Value	Mean μ_x	Std Dev. σ_x	cov V
Permanent Load	$\mathbf{G}$	kN/m^2	N	0	0		
Water pressure, Lk	$\mathbf{L_k}$	kN/m^2	N	50	49.05	2.453	0.05
Concrete tensile strength	$\mathbf{f_{c,t}}$	MPa	LN	2.00	2890	549.1	0.19
Steel modulus	$\mathbf{E_s}$	GPa	Det	200	200000000	0	0
Concrete modulus, long term	$\mathbf{E_{c,eff}}$	GPa	Det	13.7	13700000	0	0
Reinf dia (HT reinf)	$\boldsymbol{\varphi}$	mm	Det	20	0.020	0	0
Reinf area in tension (vary) at SLS	$\mathbf{A_s}$	mm^2	Det	2250	0.0023	0	0
Concrete c/s depth, h	$\mathbf{h}$	mm	N	450	0.45	0.0045	0.01
Concrete c/s width, b	$\mathbf{b}$	mm	Det	1000	1.0	0	0
cover	$\mathbf{c}$	mm	gamma	40	0.04	0.006	0.15
distance to centre of bar	$\mathbf{a}$	mm	gamma	50	0.05	0.01	0.15
Coefficient k1 (HT reinforcement)	$\mathbf{k_1}$	-	Det	0.8	0.8	0	0
Coefficient k2 (bending only)	$\mathbf{k_2}$ **(kc)**	-	Det	0.5	0.5	0	0
Coefficient kt (duration of load - long terr	$\mathbf{k_t}$	-	Det	0.4	0.4	0	0
Height of wall	$\mathbf{H}$	m	Det	5	5		
Concrete modulus	$\mathbf{E_c}$	GPa	Det	27.4	27400000	0	0
Concrete creep factor	$\boldsymbol{\phi}$	-	Det	1	1	0	0
Limiting crack width	$\mathbf{w_{lim}}$	mm	Det	0.2	0.0002	0	0
Crack width (experimental results)	$\mathbf{w_{m,exp}}, \mathbf{w_{k,exp}}$	mm	N				
Model Uncertainty	$\boldsymbol{\theta_w}$	-	LN		1	0.1	0.1

Note: Mean and standard deviation units – kN and m

Figure 5.2: *Initial input values for variables of reliability crack model*

Variable	**Symbol**	Dim	Distribution	Characteristic Value	Mean μ_x	Std Dev. σ_x	cov V
Crack width	θ_w	-	LN		1	0.1	0.1

mod ratio, α_e = 14.6

Crack limit $w_{m,lim}$ = 0.0002 m

Step 1

FORM Reliability Analysis

Non-linear Limit State Eq

$g(X) = w_{lim} - w_{m,calc}$

Iteration 1

Step 1. Convert nonnormal distributions to N.

	PDF	ξ	λ	y*	μ^N	σ^N	
Model Uncertainty, θ	LN	0.10	-0.01	1.00	1.00	0.10	Check pdf
Concrete tensile strength, f_{ct}	LN	0.19	7.95	2890	2837.84	549.1	
Cover, c - approx. LN pdf	LN	0.15	-3.23	0.04	0.0396	0.006	

LN → N-pdf

$\xi = \Omega$ $\lambda = \ln\mu - 0.5\Omega^2$

$\sigma^N = y^*\xi$ $\mu^N = y^*(1-\ln y^*+\lambda)$

Step 2

Step 2: Numerical algorithm: Iteration 1

Uncorrelated variables c, h, L_k, f_{ct}, θ	X_i	**Assumed** x^*_i	μ_{Xi}	σ_{Xi}	$(\delta g/\delta X'_i)_*$	**Directional Cosines** α^*_{xi}	Failure Point, x^*_i
cover	c	0.04	0.040	0.0060	-7.691E-06	-0.0425	0.040
section thickness	h	0.45	0.450	0.0045	-2.844E-06	-0.0157	0.450
water pressure	L_k	49.05	49.050	2.4525	-1.795E-04	-0.9929	50.728
concrete tensile strength	fct	2890.00	2837.836	549.1000	1.178E-05	0.0651	2813.180
model uncertainity	θ	1.00	0.99500	0.1000	-1.602E-05	-0.0886	1.00
				$\sqrt{\Sigma((\delta g/\delta X'i)^2 *)}$ =	1.808E-04	β =	0.68925

D.F's	
eq 1	0.17
eq 2	0.04
eq 3	0.40
eq 4	0.07

Step 3

Step 3. Optimisation of β Using Excel Solver for g(X) = 0
SUMMARISED FOR ALL ITERATIONS

Equations for New Failure Points (assuming β value)

$x^* = \mu Nx - \alpha^* x\beta\sigma Nx$

Solver Iteration	**β**	**c***	**h***	**L***	**ft***	**θ***	**d**	**x**	**hc**	**Sr (m)**	**ε (-)**	**wm calc (m)**	**g = 0**	input **%As**
1	0.6893	0.0397	0.4500	50.728	2813.18	1.00	0.4003	0.13261	0.1058	0.1735	9.7224E-04	1.687E-04	3.1124E-05	0.500
2	0.6834	0.0397	0.4501	50.721	2814.99	1.00	0.4003	0.13261	0.1058	0.1735	9.7184E-04	1.686E-04	3.1118E-05	As (mm2)
3	0.6835	0.0397	0.4501	50.721	2814.97	1.00	0.4003	0.13261	0.1058	0.1735	9.7185E-04	1.686E-04	3.1118E-05	2250.000
4	0.6835	0.0397	0.4501	50.721	2814.97	1.00	0.4003	0.13261	0.1058	0.1735	9.7185E-04	1.686E-04	3.1118E-05	
5	0.6835	0.0397	0.4501	50.721	2814.97	1.00	0.4003	0.13261	0.1058	0.1735	9.7185E-04	1.686E-04	3.1118E-05	

Figure 5.3 (a): *Iteration 1 of FORM algorithm to calculate β*

Variable	Symbol	Dim	Distribution	Characteristic Value	Mean μ_x	Std Dev. σ_x	cov V
Crack width	θ_w	-	LN		1	0.1	0.1

mod ratio, α_e =	14.6	
Crack limit $w_{m,lim}$ =	**0.0002**	m

FORM Reliability Analysis

Non-linear Limit State Eq

$g(X) = w_{lim} - w_{m,calc}$

Iteration 1

Step 1. Convert nonnormal distributions to N.

Step 1

	PDF	ξ	λ	y*	μ^N	σ^N	
Model Uncertainty, θ	LN	0.10	-0.01	1.00	1.00	0.10	Check pdf
Concrete tensile strength, f_{ct}	LN	0.19	7.95	2890	2837.84	549.1	
Cover, c - approx. LN pdf	LN	0.15	-3.23	0.04	0.0396	0.006	

LN → N-pdf

$\xi = \Omega$ $\quad \lambda = \ln\mu - 0.5\Omega^2$

$\sigma^N = y^*\xi$ $\quad \mu^N = y^*(1-\ln y^*+\lambda)$

Step 2: Numerical algorithm: Iteration 1

Uncorrelated variables c, h, L_k, f_{ct}, θ	X_i	Assumed x^*_i	μ_{Xi}	σ_{Xi}	$(\delta g/\delta X'_i)_*$	Directional Cosines α^*_{xi}	Failure Point, x^*_i
cover	c	0.04	0.040	0.0060	-7.691E-06	-0.0425	0.040
section thickness	h	0.45	0.450	0.0045	-2.844E-06	-0.0157	0.450
water pressure	L_k	49.05	49.050	2.4525	-1.795E-04	-0.9929	50.728
concrete tensile strength	fct	2890.00	2837.836	549.1000	1.178E-05	0.0651	2813.180
model uncertainity	θ	1.00	0.99500	0.1000	-1.602E-05	-0.0886	1.00
				$\sqrt{\Sigma((\delta g/\delta X'i)^2 *)}$ =	1.808E-04	β =	0.68925

D.F's	
eq 1	0.17
eq 2	0.04
eq 3	0.40
eq 4	0.07

Iteration 2

	PDF	ξ	λ	y*	μN	σN
Model Uncertainty, θ	LN	0.100	-0.01	1.001	0.995	0.1001
Concrete tensile strength, fct	LN	0.19	7.95	2813.18	2838.192	534.504
Cover, c - approx. LN pdf	LN	0.15	-3.23	0.04	0.040	0.006

Step 2

Uncorrelated variables c, h, Lk, fct, θ	Xi	Assumed x*i	μXi	σXi	(δg/δX'i) *	Directional Cosines α*xi	Failure Point, x*i
cover	c	0.040	0.040	0.0060	-8.190E-06	-0.0455	0.040
section thickness	h	0.450	0.450	0.0045	-2.985E-06	-0.0166	0.450
water pressure	Lk	50.728	49.050	2.4525	-1.796E-04	-0.9968	50.721
concrete tensile strength	fct	2813.180	2838.192	534.5042	1.144E-05	0.0635	2814.986
model uncertainity	θ	1.00	0.995	0.1001	-1.68862E-05	-0.0937	1.001
				√Σ((δg/δX'i)2 *) =	1.801E-04	β =	0.68345

D.F's	from Matlab eq's
eq 1	1.655E-01
eq 2	4.420E-02
eq 3	4.003E-01
eq 4	6.617E-02

Step 3. Optimisation of β Using Excel Solver for g(X) = 0
SUMMARISED FOR ALL ITERATIONS

Step 3

Equations for New Failure Points (assuming β value)

$x^* = \mu Nx - \alpha^* x\beta\sigma Nx$

Solver Iteration	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	input %As
1	0.6893	0.0397	0.4500	50.728	2813.18	1.00	0.4003	0.13261	0.1058	0.1735	9.7224E-04	1.687E-04	3.1124E-05	0.500
2	0.6834	0.0397	0.4501	50.721	2814.99	1.00	0.4003	0.13261	0.1058	0.1735	9.7184E-04	1.686E-04	3.1118E-05	As (mm2)
3	0.6835	0.0397	0.4501	50.721	2814.97	1.00	0.4003	0.13261	0.1058	0.1735	9.7185E-04	1.686E-04	3.1118E-05	2250.000
4	0.6835	0.0397	0.4501	50.721	2814.97	1.00	0.4003	0.13261	0.1058	0.1735	9.7185E-04	1.686E-04	3.1118E-05	
5	0.6835	0.0397	0.4501	50.721	2814.97	1.00	0.4003	0.13261	0.1058	0.1735	9.7185E-04	1.686E-04	3.1118E-05	

Figure 5.3 (b): *Iteration 2 of FORM algorithm*.

5.3 VERIFICATION OF THE PROBABILISTIC MODEL

The FORM model of this research was verified by setting up the model for cracking due to direct tension and comparing to that of Holicky *et al* (2009). The model used by Holicky et al is detailed in their paper “Probabilistic Design of Concrete Structures”. In that model, a 1m length of wall, 7m high, of a 28 m diameter circular reservoir under direct tension due to water pressure only was considered. Wall thickness was 250 mm. A concrete cover of 40 mm was used. A reliability analysis was carried out to investigate the variation of serviceability probability of failure, p_f, with the reinforcement ratio A_{SLS}/A_{ULS} for w_{lim} of 0,2 mm and 0,05mm using reinforcement diameters of 16 and 20 mm. The area of reinforcement required for the ultimate limit state was calculated using the EN1992-1-1 typical value for the characteristic yield strength of the reinforcement of 500 MPa (E_s = 200GPa). The Eurocode formulation was used to calculate the concrete modulus, giving a value of 33 GPa (as opposed to 27,4 GPa calculated using the SANS10100-1 formula).

Graphs of the variation of failure probability with the ratio A_{SLS}/A_{ULS} were plotted for both 0,2 and 0,05 mm crack width limits. Figure 5.4(a) shows the graphs for the tension cracking model for a crack width limit of 0,2mm.

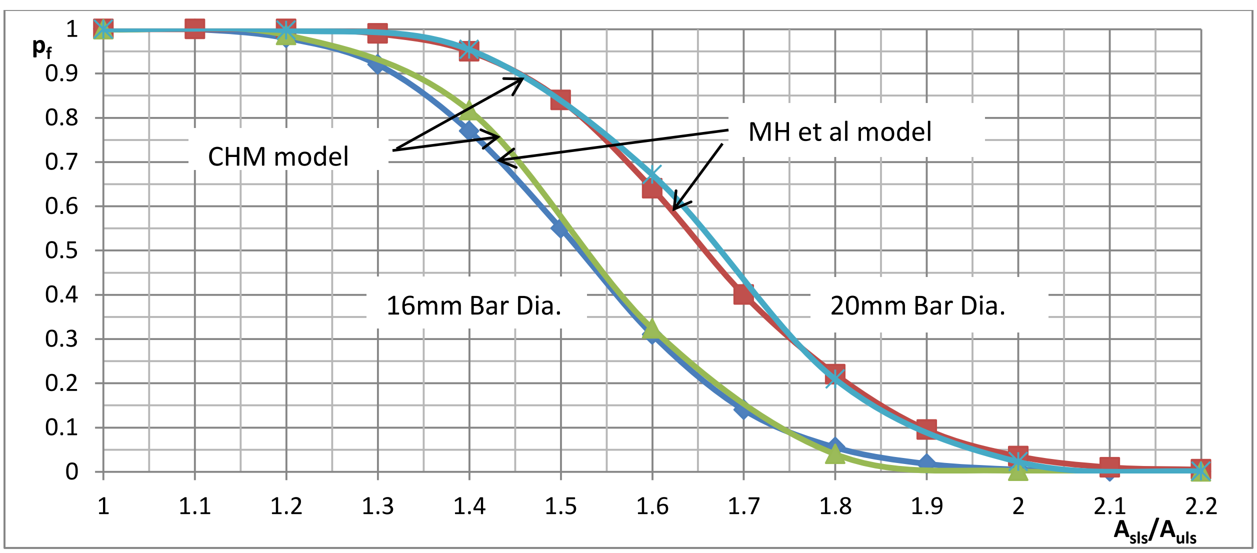


Figure 5.4: *Comparison of reliability models of CHM and MH et al (2009)*

The reliability analysis was concerned with failure probabilities less than 0,5, such that the corresponding reliability levels, as β, will be over a range from 0 to 3,0. Figure 5.5 was then plotted showing this limited range of reliability to assess the correlation between the reliability models.

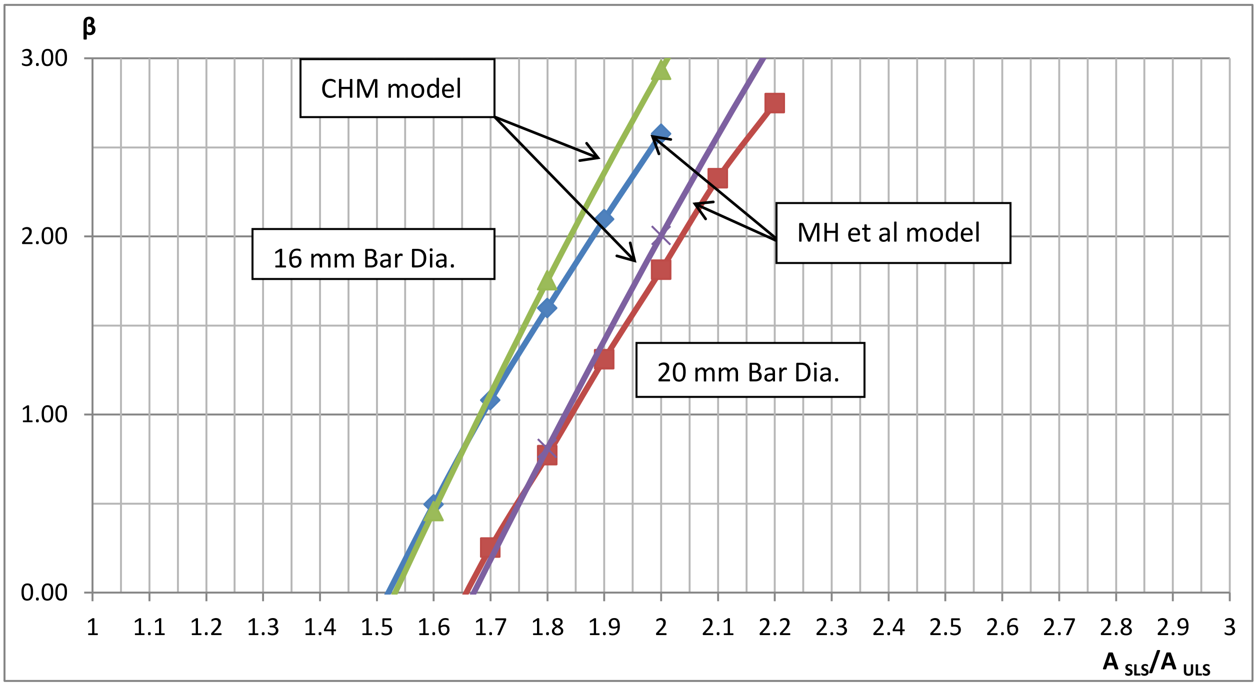


Figure 5.5: *Comparison of reliability models with respect to β*

As the analysis in this research was done using EXCEL which is not the same software used by Holicky *et al* and values for the Holicky *et al* model were interpolated from graphs presented in their paper, a small difference in values was expected, particularly at very small probabilities of failure (that is, as β increases). It was found that the graphs for the two models at a 0,2 mm and 0,05 mm crack widths corresponded sufficiently well enough to conclude that the tension cracking model, and by inference, the flexural cracking model, would be accurate. In addition, hand calculations were carried out to verify both models. Graphs for a crack width of 0,05 mm can be found in Appendix B.

5.4 RESULTS AND DISCUSSION

The results of the FORM analysis for the flexural and tension cracking models are presented and discussed under this heading with respect to the issues set out at the beginning of the chapter, summarised as follows:

(i) The extent to which SLS cracking governs the design of a WRS.
(ii) The effect of the specified crack width limit on the level of reliability.

(iii) The influence of model uncertainty on the reliability of the crack model.

In the case of tension cracking, $h_{c,eff}$ = 2,5(c + φ/2) is limiting for most combinations of concrete cover, diameter and section thickness considered. Therefore Model 2(a) using this equation for $h_{c,eff}$ was taken as the general model for tension cracking. However, for smaller section thicknesses, either $h_{c,eff}$ = h/2 is limiting (Model 2(b)) or both equations for $h_{c,eff}$ apply. Thus, reliability analyses were performed using both equations for $h_{c,eff}$ and comparisons made; the results of which are presented in Section 5.4.4.

As satisfaction of the serviceability cracking condition is dependent on an adequate amount of reinforcement being supplied such that the limiting crack width specified is not exceeded. The quantity of reinforcement required as % reinforcement (or ρ as $\%A_s = A_s/A_c$ where A_c is the gross cross sectional area of the wall section) was therefore used as the basis of comparison in the analysis of the results from the probability analyses. It was anticipated, based on the results of the deterministic analysis, that the reinforcement requirements to satisfy the serviceability cracking requirements would be more severe than for the ultimate load case, with possible economic consequences. The degree to which this was the case was therefore investigated, by expressing results in terms of the A_{SLS}/A_{ULS} ratios. Results obtained were also compared to the corresponding deterministic values. As in the deterministic analysis, a maximum limit was considered for the reinforcement area, determined by a minimum practical spacing of bars, as given in Table 3.2 of Chapter 3.

Selected graphs only are presented here. The data sheets generated by the analyses may be found in Appendix B.

5.4.1 Significance of serviceability limit state (SLS) load-induced cracking

To assess the influence of the SLS of cracking compared to the ULS, graphs were plotted for the variation of reliability with the A_{SLS}/A_{ULS} ratio. As discussed in the deterministic analysis, ratios larger than 1,0 indicate that the SLS dominates the design. The data from both the flexural and tension cracking reliability analyses was analysed for a reference value of 0,2 for θ_{cov} and is discussed, as follows:

(i) Flexural cracking (Model 1)

The influence of SLS for flexural cracking is illustrated by Figure 5.6 showing the variation of reliability with the ratio A_{SLS}/A_{ULS} as the limiting crack width decreases from 0,3 to 0,05 mm for a 450 mm wall thickness. An A_{SLS}/A_{ULS} ratio of approximately 2,5 was calculated to be the upper limit corresponding to the maximum feasible area of reinforcement considered in terms of a practical bar spacing (A_s = 4189 mm^2 for a 20 mm bar diameter). Referring to Figure 5.6, SLS is not dominant for a crack width of 0,3 mm, with A_{SLS}/A_{ULS} ratios just below 1 for a reliability level of 1,5. However, as the crack limit decreases, SLS cracking is increasing dominant, with A_{SLS}/A_{ULS} ratios above 1.

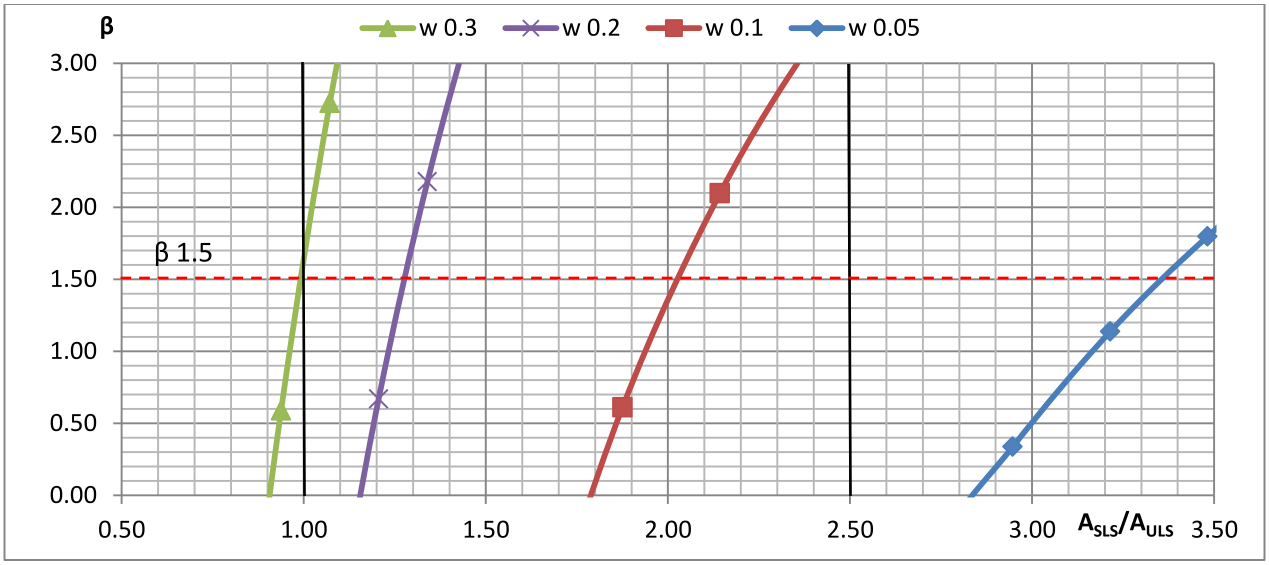


Figure 5.6: *Flexure - Effect of SLS on variation of β with ratio A_{SLS}/A_{ULS}*

Table 5.2 is a summary of the reinforcement (as % A_s and as the ratio A_{SLS}/A_{ULS}) satisfying crack width criteria for flexural cracking at a reliability index (β) of 1,5.

Table 5.2: *Flexural cracking: Reinforcement required for reliability and deterministic analyses.*

			Reliability Analysis β = 1,5		Deterministic Analysis		Ratio of Deterministic/ Reliability
Load Case	w (mm)	θ_{CoV}	%A_s	A_{SLS}/A_{ULS}	%A_s	A_{SLS}/A_{ULS}	
Flexure (Model 1)	0.30	0.10	0.36	0.96	0.48	1.28	1.33
		0.20	0.37	0.99			1.29
	0.20	0.10	0.469	1.26	0.62	1.65	1.31
A_{ULS}		0.15	0.472	1.26			1.31
1680 mm^2		0.20	0.476	1.28			1.29
		0.25	0.482	1.29			1.28
		0.30	0.489	1.31			1.26
	0.10	0.10	0.74	1.98	0.98	2.62	1.32
		0.15	0.75	2.00			1.31
		0.20	0.76	2.04			1.28
	0.05	0.10	1.21	3.24	1.59	4.26	1.32
		0.15	1.23	3.29			1.29
		0.20	1.25	3.35			1.27

Referring to Table 5.2 and Figure 5.6, considering a model uncertainty variation of 0,2, values of about 0,99, 1,28, 2,04 and 3,35 for A_{SLS}/A_{ULS} are required to meet crack width limits of 0,3, 0,2, 0,1 and 0,05 mm, respectively. A target reliability of 1,5 is not reached for a crack width of 0,05 mm and a section thickness of 450 mm as the maximum practical A_{SLS}/A_{ULS} ratio of 2,5 is exceeded for all levels of reliability. The deterministic analysis showed that the SLS is the dominant limit state for all wall thicknesses used, with A_{SLS}/A_{ULS} ratios for a 450 mm thick wall of 1,65 at a 0,2 mm crack width, 2,62 at a crack width limit of 0,1 mm and 4,26 at a 0,05 mm crack width, as summarised in Table 5.2. These ratios are about 25 to 30% higher than those predicted by the probabilistic analysis for a reliability level of 1,5, meaning that the deterministic analysis is more conservative than the probabilistic analysis. For a wall thickness of 450 mm and model uncertainty variation of 0,2, the deterministic analysis is more conservative by a factor of approximately 1,29 at a crack width limit of 0,2 mm and 1,27 at a crack width limit of 0,05 mm.

(ii) Tension cracking (Model 2(a))

In the case of tension cracking, the dominance of the serviceability limit state is illustrated in Figure 5.7 showing the variation of reliability with reinforcement ratio A_{SLS}/A_{ULS} as the crack limit decreases from 0,3 mm to 0,05 mm. The SLS, rather than the ULS, is dominant in tension cracking for all levels of reliability. The significance of SLS increases substantially with decreasing crack width for the same level of reliability.

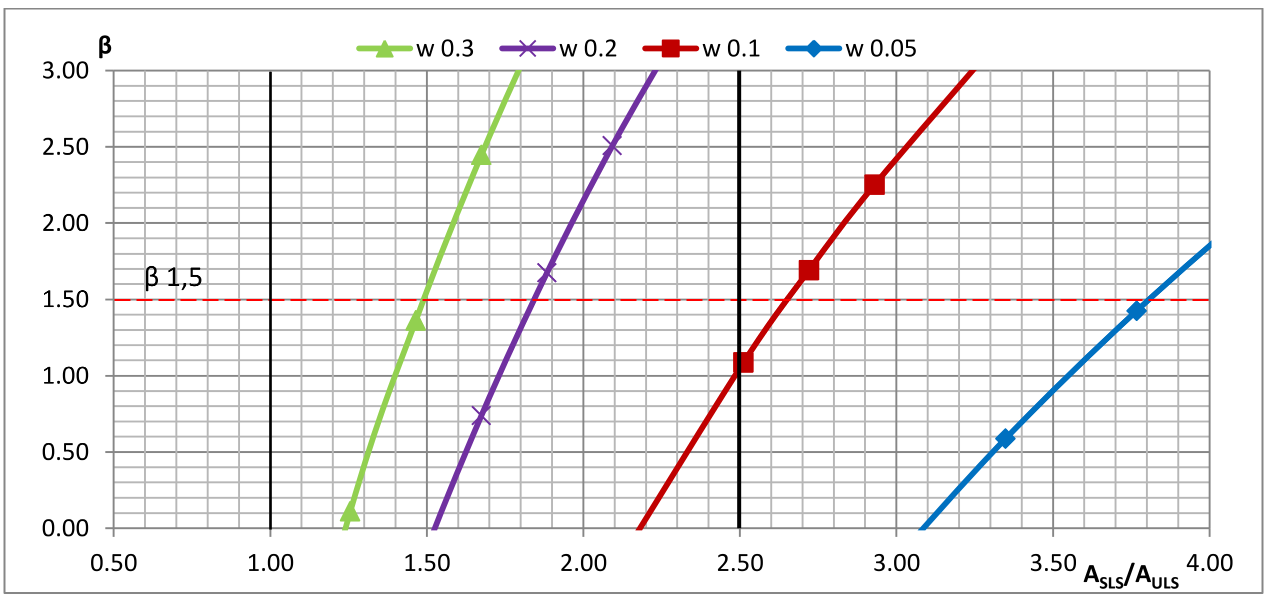


Figure 5.7: *Tension – Significance of SLS on variation of reliability*

Table 5.3 is a summary of the reinforcement (% A_s and A_{SLS}/A_{ULS}) satisfying crack width criteria for tension cracking at a reliability index (β) of 1,5 for crack widths from 0,05 to 0,3 mm.

Table 5.3: *Tension cracking - Reinforcement required for reliability and deterministic analyses*

			Reliability Analysis β = 1,5		Deterministic Analysis		Ratio of Deterministic/ Reliability
Load Case	w (mm)	θ_{CoV}	$\%A_s$	A_{SLS}/A_{ULS}	$\%A_s$	A_{SLS}/A_{ULS}	
Tension (Model 2(a)) A_{ULS} 2150 mm^2	0.30	0.10	0.68	1.42	0.88	1.85	1.30
		0.20	0.71	1.49			1.24
	0.20	0.10	0.84	1.76	1.11	2.33	1.32
		0.15	0.86	1.80			1.29
		0.20	0.88	1.84			1.27
		0.25	0.90	1.88			1.24
		0.30	0.93	1.95			1.19
	0.10	0.10	1.22	2.55	1.68	3.60	1.41
		0.15	1.24	2.60			1.38
		0.20	1.27	2.68			1.34
	0.05	0.10	1.76	3.68	2.62	5.49	1.49
		0.15	1.78	3.73			1.47
		0.20	1.82	3.81			1.44

Referring to Figure 5.7 and Table 5.3, considering a model uncertainty variation of 0,2, ratios of about 1,49, 1,84, 2,68 and 3,81 are required for crack widths of 0,3, 0,2, 0,1 and 0,05 mm at a reliability level of 1,5.

On comparing the probabilistic analysis at a β of 1,5 using a model uncertainty variation of 0,2, with the deterministic analysis for tension cracking, the design analysis is more demanding than the probabilistic analysis with increasing conservatism as the crack width limit decreases. Ratios for deterministic to reliability analysis of about 1,27 and 1,44 for crack widths of 0,2 mm and 0,05 mm respectively. The results from the reliability analyses for both tension and flexural cracking therefore suggest that there are possible improvements that can be made in the design crack model.

5.4.2 Effect of the specified crack width limit, w_{lim}, on reliability

Reliability analyses were performed for crack width limits of 0,3 mm, 0,2 mm, 0,1 mm and 0,05 mm, for varying reinforcement quantities. For both the flexural and tensile cracking models, the level of reliability was found to decrease considerably with a decreasing crack width limit for a given reinforcement quantity and model uncertainty variation, as discussed as follows:

(i) Flexural cracking (Model 1)

The influence of crack width limit for the flexural cracking case is illustrated in Figure 5.8 which shows the variation of reliability as the crack width limit is varied for a model uncertainty variation of 0,2 and section thickness of 450 mm. Referring to Figure 5.8 and Table 5.3, increasing $\%A_s$ values (obtained at β 1,5 and θ_{cov} 0,2) of about 0,37, 0,48, 0,76 and 1,25 for decreasing flexural crack widths of 0,3, 0,2, 0,1 and 0,05 mm, respectively, illustrates this point. Considering the maximum practical limit on the reinforcement required of 0,93% (4189 mm^2) for a wall thickness of 450 mm and 20 mm bar diameter, the target reliability of 1.5 will be met for crack widths down to and including 0,1 mm, for flexural cracking. However, as also predicted by the deterministic analysis, a limiting crack width of 0,05 mm will not be met for the wall configuration considered as the maximum feasible reinforcement area (0,93% A_s) is exceeded for all β. The section thickness and/or bar diameter will need to be increased. Alternatively, for thicker sections, a double layer of bars may be used.

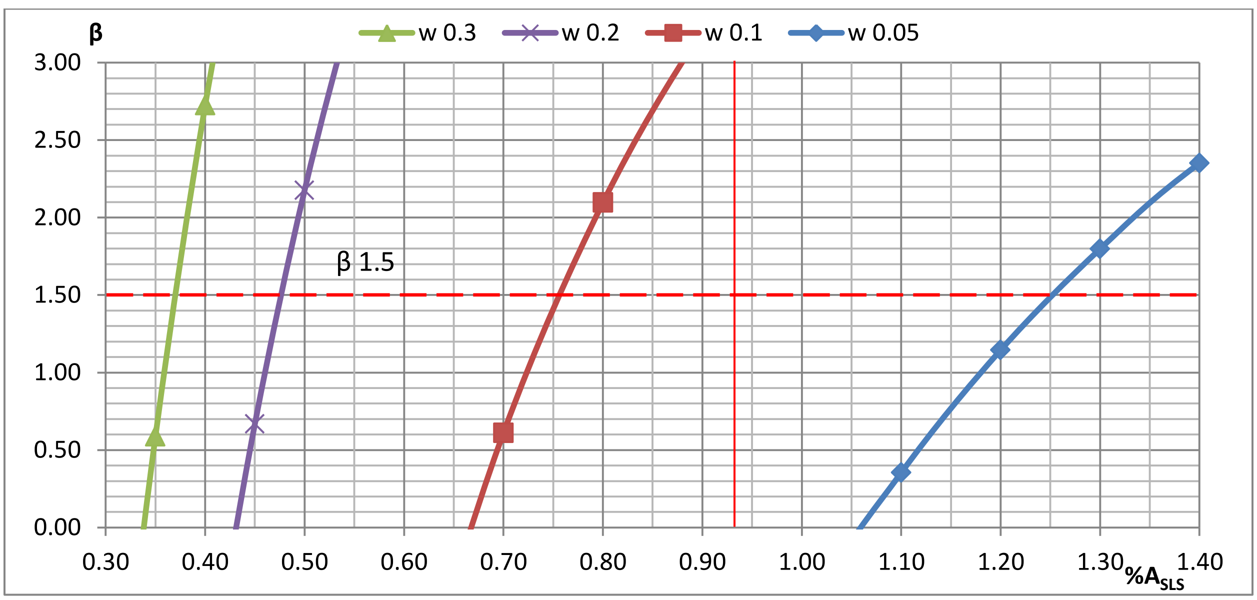


Figure 5.8: *Flexural cracking - Effect of limiting crack width on variation of β with $\%A_s$*

The graphs for each crack width limit are approximately linear at the larger crack widths, with gradients decreasing as the crack width limit decreases. This means that a small change in $\%A_s$ will result in a larger change in reliability for a crack width limit of 0.3 and 0.2 mm than for a crack width limit of 0.05 mm. Therefore, a smaller limiting crack width will require a greater increase in reinforcement to achieve the same increase in reliability as for a larger crack width limit, that is, at a greater cost. To increase reliability from 1,5 to 2,0, for example, at a 0,2 mm crack width limit, requires an increase in reinforcement from about 0,48% to 0,49%, which is very small. To achieve the same increase in reliability at a 0,05 mm crack width limit, a greater increase in reinforcement from about 1,25% to 1,37% is required.

(ii) Tension cracking (Model 2(a))

The influence of the crack width on reliability for tension cracking is illustrated by Figure 5.9 showing the variation of reliability with $\%A_s$ for a 450 mm wall thickness and a decreasing crack width limit from 0,3 mm to 0,05 mm.

Referring to Figure 5.9 and Table 5.3, a target reliability of 1,5 requires increasing $\%A_s$ of about 0,71%, 0,88%, 1,24% and 1,82% to satisfy decreasing crack width limits of 0.3, 0.2, 0.1 and 0.05 mm, respectively, for tension cracking. Considering the maximum practical limit on the total reinforcement ($\%A_s$ for both faces of wall) required of 1,86% (8370 mm^2) for a wall thickness of 450 mm, reliabilities greater than the SLS target reliability of 1,5 for all limiting crack widths considered can be achieved in the case of an applied direct tension.

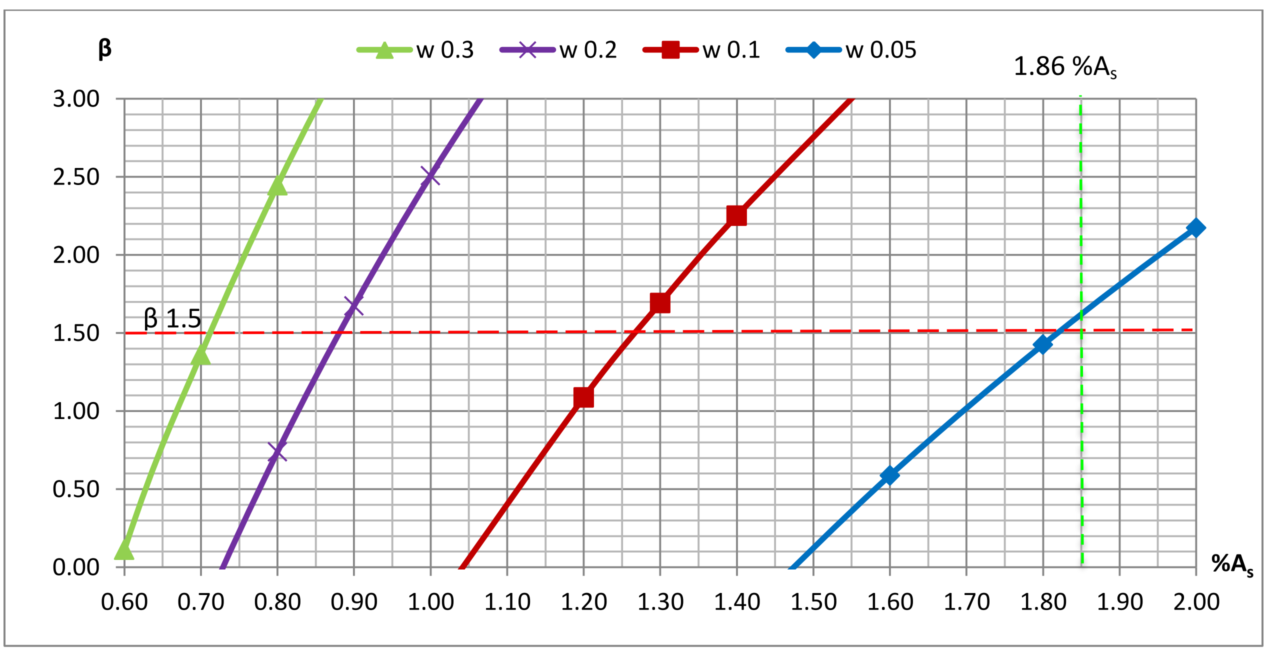


Figure 5.9: *Tension cracking – Effect of crack width limit on variation of β with $\%A_s$*

The graphs for each crack width are approximately linear, with the gradient decreasing as the crack width decreases, as in the case of flexural cracking, meaning that the reinforcement required increases to achieve the same level of reliability as the limiting crack width decreases. For example, referring to Figure 5.9, to increase reliability from 1,5 to 2,0 at a 0,02 mm crack width, an increase in reinforcement from about 0,88% to 0,94% is required (corresponding to a gradient of 8,33), whereas an increase from 1,82% to 1,95% is required at a 0,05 mm crack width (corresponding to a gradient of 3,85). Hence, the cost of increasing reliability at smaller crack widths is greater.

It can be concluded that the crack width limit has a substantial influence on the level of reliability for both flexural and tension cracking. Reducing the crack width limit from 0,2 mm to the more demanding value of 0,05 mm set by EN1992-3 (a decrease by a factor of 4) will result in an increase in the reinforcement required by a factor of about 2,0 for flexural cracking and about 2,6 for tension cracking, at a reliability level of 1,5 and model uncertainty variation of 0,2.

5.4.3 Effect of Model uncertainty, θ, on reliability

The statistical values considered for the model uncertainty were estimated as there is little available data in literature, thus providing motivation to examine the effect of this variable. From the conclusions of the literature review of reliability, presented in Chapter 4, model uncertainty in the crack model was treated as a variable with a mean of 1 and log-normal probability distribution. The θ_{cov} was varied from 0,1 to 0,3 for both flexural and tension cracking, with the resulting reinforcement required and corresponding level of reliability noted. The influence of model uncertainty on reliability for flexural and tensile cracking is discussed as follows:

(i) Flexural cracking (Model 1)

To assess the effect of the model uncertainty, Figure 5.10 was plotted showing the variation of β with reinforcement ratio for flexural cracking. A section thickness of 450 mm and a 0,2 mm crack width were used.

Figure 5.10 shows a decreasing gradient for each graph with increasing θ_{cov}, which means that a greater increase in reinforcement will be required to achieve the same increase in the reliability, as the model uncertainty variation increases. The graphs are roughly linear and intersect at a β of about 0,65 and a A_{SLS}/A_{ULS} of about 1,21 (0,45 $\%A_s$) for a section thickness of

450 mm. It follows that reliability decreases as the model uncertainty variance increases, as was expected, for a given reinforcement above a β of about 0,65.

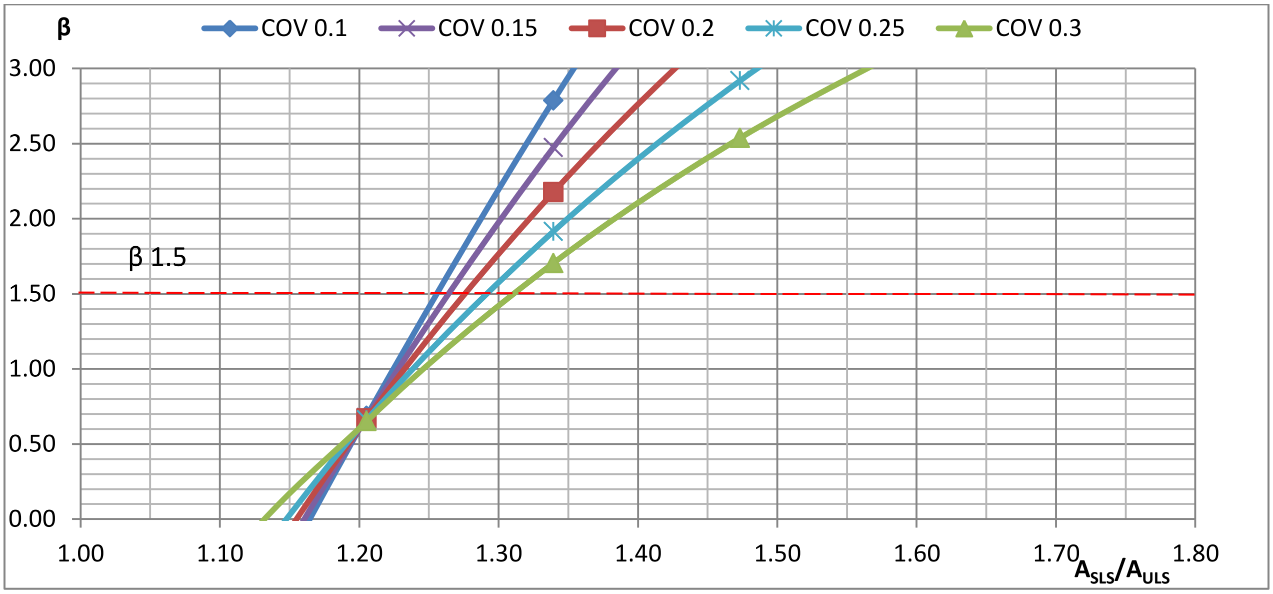


Figure 5.10: *Flexure - Effect of model uncertainty on variation of β with ratio* A_{SLS}/A_{ULS}

The values for reinforcement required at a reliability of 1,5, as summarised in Table 5.2, suggest that model uncertainty has a small influence for a wall thickness of 450 mm, as the A_{SLS}/A_{ULS} ratio increases only slightly with increasing CoV, for flexural cracking and a limiting crack width of 0,2 mm. Using a model uncertainty CoV of 0,1 results in a A_{SLS}/A_{ULS} of about 1,26 (0,47 %A_s), while a value of 0,2 results in a A_{SLS}/A_{ULS} of about 1,28 (0,48 %A_s) which is a small increase. Referring to Table 5.2, a greater increase in reinforcement is observed for crack width limits less than 0,2 mm, although this increase is still relatively small. For a crack width limit of 0,05 mm, a A_{SLS}/A_{ULS} of about 3,24 (1,21 %A_s) is obtained using a CoV of 0,1 and about 3,35 (1,25 %A_s) for a CoV of 0,2. It can be concluded that model uncertainty appears to have some influence on flexural cracking.

(ii) Tension cracking (Model 2(a))

For the tension cracking model, the graph gradients are flatter than for flexure and thus the relative decrease in β is greater for a given %A_s with increasing model uncertainty variation, as is shown in Figure 5.11. The graphs intersect at a β of about 0,30 and a A_{SLS}/A_{ULS} of about 1,57 (0,75%A_s) for a section thickness of 450 mm. The influence of model uncertainty increases as the level of reliability increases above a value of about 0,30.

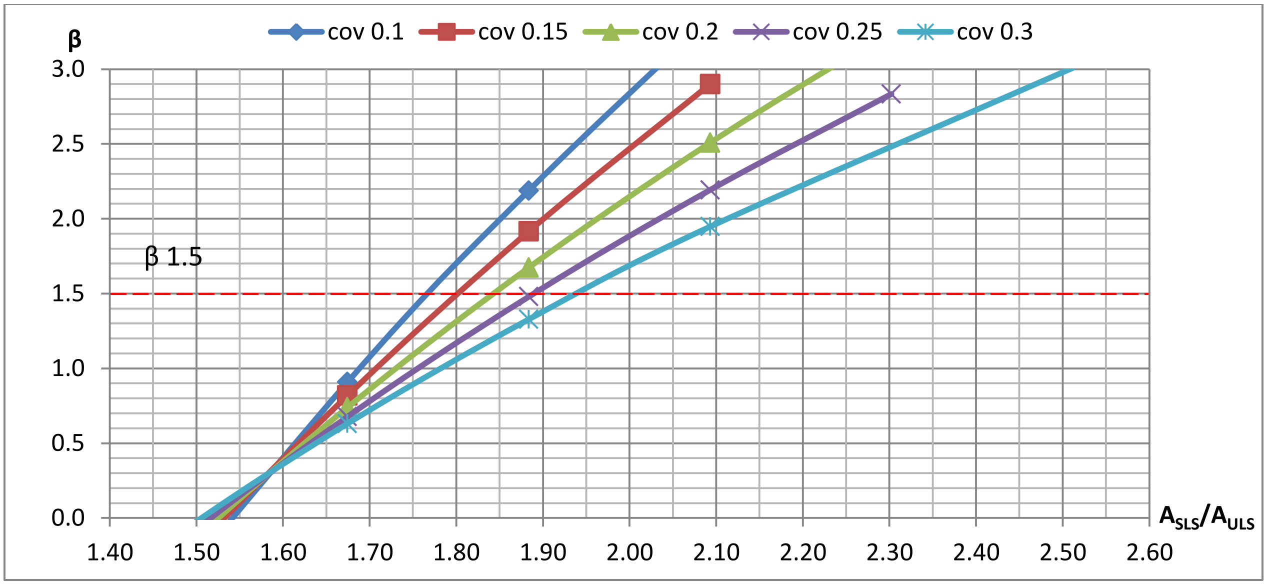


Figure 5.11: *Tension Cracking – Effect of model uncertainty on variation of* β *with* A_{SLS}/A_{ULS}

Referring to Figure 5.11 and Table 5.3, at a reliability of 1,5, a model uncertainty CoV of 0,1 results in a A_{SLS}/A_{ULS} of about 1,76 (0,84 %A_s), while a value of 0,2 results in a A_{SLS}/A_{ULS} of about 1,84 (0,88 %A_s). This is a small increase in reinforcement by a factor of 1,05. It was observed that there is a similar small increase in the reinforcement required for crack width limits less than 0,2 mm. For a crack width limit of 0,05 mm, a A_{SLS}/A_{ULS} of about 3,68 (1,76 %A_s) is needed for a CoV of 0,1 and a A_{SLS}/A_{ULS} of about 3,81 (1,82%A_s) is required for a CoV of 0,2.

Model uncertainty is further investigated by means of a sensitivity analysis, presented in Chapter 6.

5.4.4 Tension cracking: Influence of the effective depth of tension area on reliability.

To assess the influence of the limiting effective depth of the tension zone in concrete, two reliability models were set up. As discussed in the deterministic analysis, the effective depth of the tension zone, $h_{c,eff}$, for cracking due to tension loading is defined by EN1992-1-1 as the minimum of h/2 and 2,5(h - d). The equation $h_{c,eff}$ = 2,5(h – d), where d = h – c - φ/2, reduces to 2,5(c + φ/2). The equation which gives the minimum $h_{c,eff}$, is found to be dependent on the combination of section thickness, concrete cover and bar diameter. Values calculated for $h_{c,eff}$ for the equations h/2 and 2,5(c + φ/2) were given in Table 3.3 and repeated here as Table 5.4 overleaf for different combinations of section thickness, concrete cover and bar diameter. The highlighted values in Table 5.4 are the limiting effective depths to be used in calculating the effective area of the tension zone around the reinforcement.

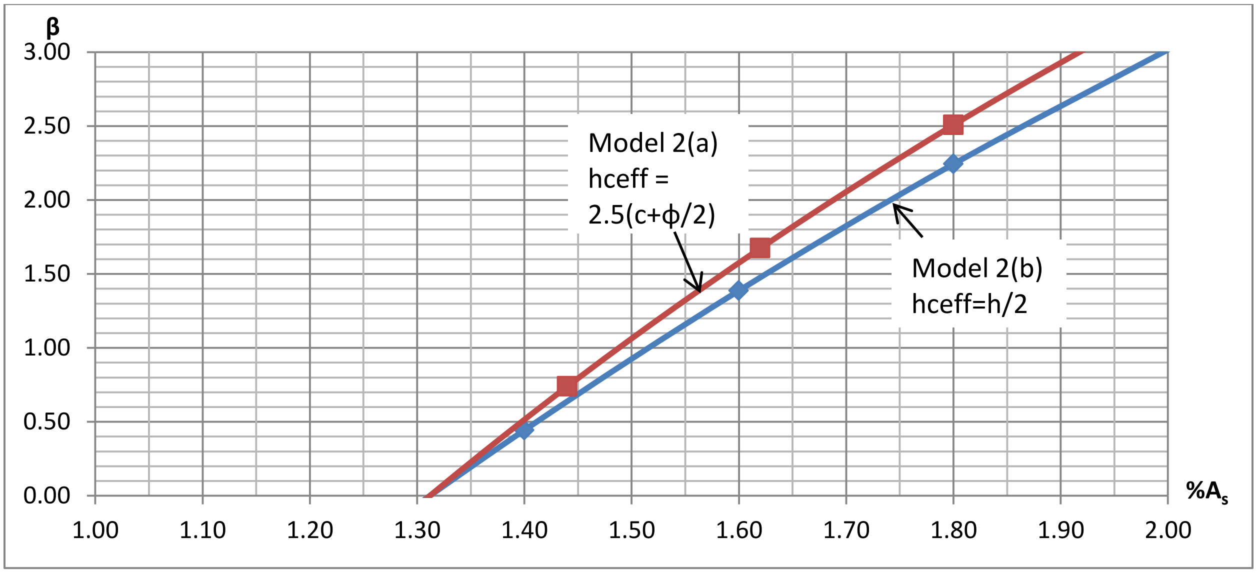
In terms of the reliability analysis of the EN1992 crack equation for tension loading, section thickness, h, and concrete cover, c, are basic random variables whilst bar diameter is taken as a deterministic value. The effective depth of the tension zone determined by h/2 is independent of concrete cover and bar diameter. Bar diameter and concrete cover only appear directly in the equation for the maximum crack width. However, if the effective depth is determined using 2,5(c + φ/2), the effective depth of the tension zone is expressed in terms of concrete cover as a random variable and bar diameter (deterministic value) and is independent of h. The crack width is also then independent of h. The reliability model is then influenced by which equation is used for the effective depth of the tension zone.

Table 5.4: *Calculation of effective depth of the tension area (mm) surrounding the reinforcement for tension cracking, where $h_{c,eff}$ is lesser of h/2 or 2,5(h-d).*

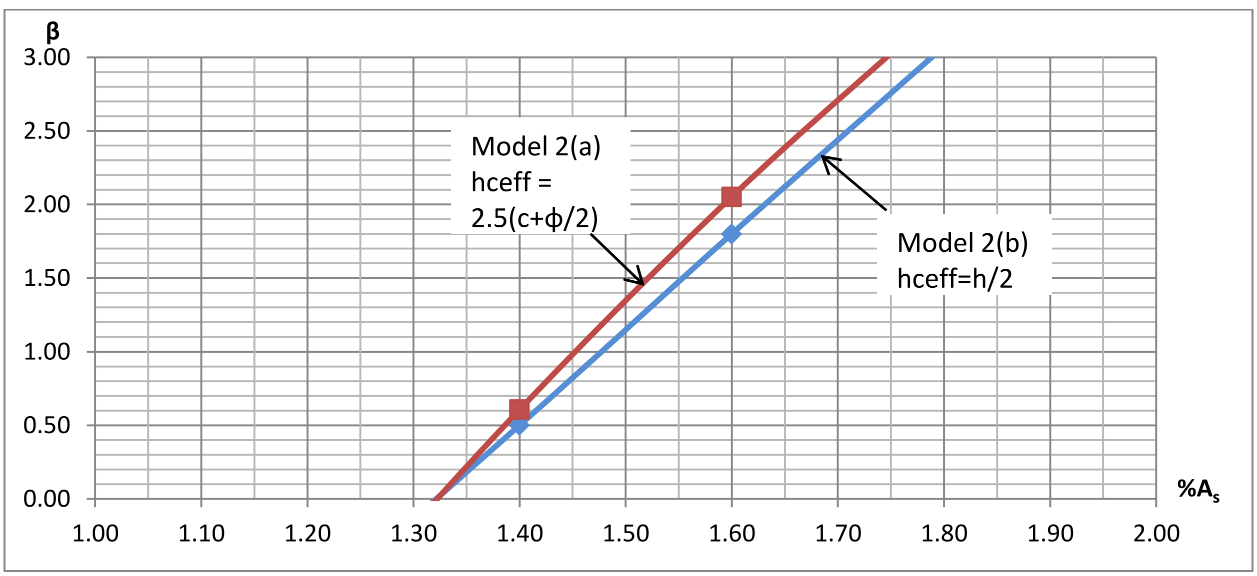
		Bar dia 16 mm			Bar dia 20 mm			Bar dia 25 mm		
Concrete cover (mm)	h	d	h/2	2,5(h-d)	d	h/2	2,5(h-d)	d	h/2	2,5(h-d)
40	250	202	125	120	200	125	125	197.5	125	131.25
	300	252	150	120	250	150	125	247.5	150	131.25
	350	302	175	120	300	175	125	297.5	175	131.25
	400	352	200	120	350	200	125	347.5	200	131.25
	450	402	225	120	400	225	125	397.5	225	131.25
	500	452	250	120	450	250	125	447.5	250	131.25
50	250	192	125	145	190	125	150	187.5	125	156.25
	300	242	150	145	240	150	150	237.5	150	156.25
	350	292	175	145	290	175	150	287.5	175	156.25
	400	342	200	145	340	200	150	337.5	200	156.25
	450	392	225	145	390	225	150	387.5	225	156.25
	500	442	250	145	440	250	150	437.5	250	156.25

Note: highlighted values are the limiting effective depth of the tension zone in concrete

For most combinations of section thickness, concrete cover and bar diameter, only one equation results the minimum $h_{c,eff}$. The reliability analysis is then performed using the appropriate equation for $h_{c,eff}$. However, there are some combinations of section thickness, concrete cover and bar diameter which result in the same value for $h_{c,eff}$ calculated by both equations. Referring to Table 5.4, this occurs for the combination of a 250 mm section thickness, a 20 mm bar diameter and 40 mm concrete cover, giving an $h_{c,eff}$ of 125 mm for both equations. FORM analyses were performed using these values to assess the effect of the way in which $h_{c,eff}$ is determined on the reliability of the crack width model. A crack width limit of 0,2 mm was used. The results of these analyses are illustrated by Figure 5.12 showing the variation of the reliability index with $\%A_s$ for $h_{c,eff}$ determined using h/2 and 2,5(c + φ/2) using model uncertainty variations of 0,2 and 0,1, respectively.



(a) θ_{cov} *of 0,2*



(b) θ_{cov} *of 0,1*

Figure 5.12: *Influence of $h_{c,eff}$ on the reliability of the tension cracking model.*

Figure 5.12(a) shows that $h_{c,eff}$ determined using 2,5(c + φ/2) results in a higher reliability for a given reinforcement area than $h_{c,eff}$ as h/2, the difference in reliability increasing as the supplied reinforcement area increases. For example, for 1,5%A_s of, reliability levels of about 1,05 and 0,91 are obtained for $h_{c,eff}$ determined using 2,5(c + φ/2) and h/2, respectively. Conversely, for a given reliability, using $h_{c,eff}$ = 2,5(c + φ/2) requires slightly less reinforcement than if $h_{c,eff}$ = h/2 is used. At a β of 1,5, values of about 1,57% for $h_{c,eff}$ = 2,5(c + φ/2) and 1,63% for $h_{c,eff}$ = h/2 are obtained. Thus, using 2,5(c + φ/2) results in the crack width model being marginally less conservative by a factor of 1,04 than using h/2 at this reliability level. If a lower model uncertainty was considered, it was noted that for a given %As, using 2,5(c + φ/2) results in a lower reliability than when using h/2, for a given amount of reinforcement.

Referring to Figure 5.12(b), considering a model uncertainty variation of 0,1 and 1,5%A_s, reliability levels of 1,35 and 1,15 are obtained using 2,5(c + φ/2) and h/2, respectively. As model uncertainty variation decreases from 0,2 to 0,1, the %A_s required to meet a particular reliability level decreases for both formulations of $h_{c,eff}$. It can be concluded that reliability in tension cracking is dependent on the relationship between the formulation of $h_{c,eff}$, more specifically the basic variables of concrete cover and model uncertainty. This is investigated further in the sensitivity analysis presented in Chapter 6.

5.5 SUMMARY

The influence of the specified crack width limit, the reliability of the EN1992-1-1 model and the influence of SLS cracking were explored by means of the reliability FORM analysis. The effects of model uncertainty were also studied. Summarising from the results and discussion:

- The crack width limit has a significant effect on the probabilistic model, namely, an increase in reinforcement required and a decrease in reliability, that is, a decrease in the performance of the structure, as the crack width limit reduces. A cost optimisation analysis, as future research, is recommended to obtain the best reliability to cost ratio.
- The SLS dominates the design of a WRS at a reliability level of 1,5 for crack widths less than and equal to 0,2 mm for flexural cracking and all crack widths for tension cracking. This emphasizes the importance of optimizing the design of a WRS for serviceability.
- Model uncertainty has a limited effect on the reliability of the model, particularly for the tension cracking case. However, the actual model uncertainty at this point is not known.

A comparison of the probabilistic analysis to experimental data on flexural and tension cracking needs to be done to improve the statistical data for crack models and their related model uncertainty. A sensitivity analysis was done and reported on in Chapter 6 to better assess the influence of model uncertainty on the reliability crack models.

- The deterministic analysis is more conservative than the probabilistic analysis for both flexural and tension crack models at a reliability level of 1,5 which suggests that there are potential savings to be made for load-induced cracking conditions if this level of reliability is used. This supports the conclusion that Holicky *et al* (2009) made from their investigation into tension cracking to EN1992.
- The reliability model for tension load cracking is influenced by the geometry of the member considered, in particular, the combination of section thickness, bar diameter and concrete cover chosen. This is turn affects which equation is used to calculate the effective depth of the tension zone, $h_{c,eff}$, either h/2 or 2,5(c + φ/2). When both equations apply for a particular combination of concrete cover, bar diameter and section thickness, the reliability obtained using h/2 was less than that using 2,5(c + φ/2).

CHAPTER 6

SENSITIVITY ANALYSIS

6.1 GENERAL

This chapter presents the sensitivity analysis performed at given levels of reliability on the EN1992 flexural and tension crack models, as detailed in Chapter 5, by means of the reverse-FORM procedure, namely:

(i) Model 1 - Flexural cracking
(ii) Model 2(a) - Tension cracking, $h_{c,eff} = 2{,}5(c + \varphi/2)$
(iii) Model 2(b) - Tension cracking, $h_{c,eff} = h/2$

The purpose of the sensitivity analysis was as follows:

(i) In order to determine the influence of the basic variables of cover, section thickness, load, concrete tensile strength and model uncertainty on the statistical model, the sensitivity factors for these variables were determined for a given level of reliability.
(ii) Given the importance of serviceability cracking in water retaining structures (WRS), the reliability level was varied to assess the influence of reliability on the crack models.

The level of reliability (measured by the reliability index, β) was chosen, as well as the mean values of the basic variables. The reverse-FORM analysis was then performed with the result being the quantity of reinforcement required for a given reliability level and chosen values of the basic variables. Sensitivity factors for each variable were determined for a given reliability level in the process of the analysis.

Theoretical partial safety factors for the basic variables of the crack models were also calculated for a given reliability level as a first step in calibrating the crack model for design purposes. It should be noted that the optimisation process of determining the final design model factors is beyond the scope of this thesis.

6.2 FORMULATION OF REVERSE-FORM ANALYSIS CRACK MODELS

6.2.1 Structural configuration of the crack model

The structural configuration used was as that used for the forward- FORM analysis presented in Chapter 5, summarised here as follows:

(iii) In the flexure case, a 1 m section of wall of a rectangular reinforced concrete water-retaining structure was considered. The wall is subject to flexure due to water pressure about the horizontal axis of the cross section.

(iv) For the tension case, a 1 m section of wall in a circular reservoir was considered. The wall is subject to hoop tension due to water pressure in the horizontal plane. The reservoir diameter was taken as 28 m.

6.2.2 Formulation of reliability crack model for reverse-FORM analysis

The limit state equation used in the reverse-FORM analysis was the same as that of the forward-FORM analysis presented in Chapters 4 and 5. The model equations are summarised as follows:

Limit state equation $g = w_{lim} - \theta .\ w_{calc}$

Crack width $w_{calc} = S_r\ .\ \varepsilon_m$

Crack spacing $S_r = 2c + 0{,}25 k_1 k_2 \varphi / \rho_{p,eff}$

Mean strain $\varepsilon_m = \varepsilon_{sm} - \varepsilon_{cm} = \dfrac{\sigma_s - k_t f_{ct,\,eff}\left(1+\alpha_e \rho_{p,eff}\right)/\rho_{p,eff}}{E_s}$

The crack width equation is expressed in terms of the basic variables of concrete cover, section thickness, concrete tensile strength, liquid load and model uncertainty. The parameters in the limit state function are either modelled as random variables or taken as deterministic values.

6.2.3. Choice of values for model parameters

The target level of reliability chosen was 1,5 for a reference period of 50 years, corresponding to that defined by SANS10160 for serviceability cracking in buildings (irreversible state). Given the importance of serviceability in WRS, reverse FORM analysis were performed over a range of values for the reliability index, β, that is, 0.5, 1.5 and 2.0 as a means of investigating the effect of reliability on the crack model and the basic variables of concrete cover, section thickness, load, concrete tensile strength and model uncertainty. A β of 0,5 is the value specified for reversible serviceability limit states whilst a β of 2,0 was taken as a reasonable upper limit for serviceability cracking.

The influence of the basic random variables of cover, section thickness, load, concrete tensile strength and model uncertainty on the crack model was investigated by means of the sensitivity factors. The following mean values were chosen for the initial values of the basic random variables:

Section thickness, h	250 and 450 mm
Cover, c	40 mm
Load, L_k	49,05 kN/m^2
Concrete tensile strength, f_{ct}	2,89 MPa
Model uncertainty	1,0

The CoV of model uncertainty (θ_{cov}) was varied over the range of 0,1 to 0,3 to assess the influence of this parameter on reliability.

Analyses were performed for crack width limits (w_{lim}) of 0,05, 0,1 and 0,2 mm to assess the effect of the crack width limit on the sensitivity factors. All other parameters were the same as the forward FORM analysis as summarised in Table 5.1 of Chapter 5.

6.2.4 Formulation of the reverse-FORM model in Microsoft EXCEL

Step 1 of the reverse-FORM algorithm was set up in Microsoft EXCEL in much the same way as for the forward FORM analysis presented in Chapter 5. The main difference is that in the reverse-FORM process, the reliability index (β) is defined first. Then the design point values of the basic variables (RV's) with their sensitivity factors (directional cosines, α_i) are then calculated using the FORM algorithm as summarised in Chapter 4. The process to determine the design point values (x^*_i) of the RV's and their sensitivity factors for a given β is as follows:

- Select β
- Select initial values for basic variables
- Perform FORM algorithm iterations with known β, calculating the reinforcement area until convergence is reached.
- Use Microsoft EXCEL Solver to determine the reinforcement area set as the decision variable cell, with the limit state function set as the objective cell.
- The final iteration gives the reinforcement area required and the design point values of the basic variables for the chosen β. Sensitivity factors (being the directional cosines), α_i, are calculated in executing the FORM algorithm.

- Copy results to a spreadsheet.
- Generate the theoretical partial safety factors (γ_i) for each RV at the design point using the equation:

$$\gamma_x = 1 - \alpha_x \beta w_x$$

- Generate graphs for γ_i and α_i.

Analyses were performed for the flexural cracking and both tension cracking models.

6.3 RESULTS AND DISCUSSION

The results of the reverse-FORM analyses for each of the EN1992 crack models are presented in terms of the following:

(i) Sensitivity factors (α_i) indicating the influence of the RV's of section thickness, concrete cover, load, concrete tensile strength and model uncertainty on the crack models.
(ii) Theoretical partial safety factors (γ_i) for each of the RV's.
(iii) Influence of reliability on the crack models.

For clarity, sensitivity factors and theoretical partial safety factors for each of the crack model basic variables are defined as follows:

	Sensitivity factor	Partial Safety Factor
Liquid Load, L	α_L	γ_L
Concrete cover, c	α_c	γ_c
Concrete tensile strength, $f_{ct,eff}$	α_{fct}	γ_{fct}
Section thickness, h	α_h	γ_h
Model uncertainty, θ	α_θ	γ_θ

6.3.1 Sensitivity of parameters

The influence of the basic variables of cover, section thickness, load, concrete tensile strength and model uncertainty was assessed by means of the sensitivity factors (α_i) generated by the FORM analysis for given reliability index levels for both flexural and direct tension cracking. The following was observed:

(i) Flexural Cracking

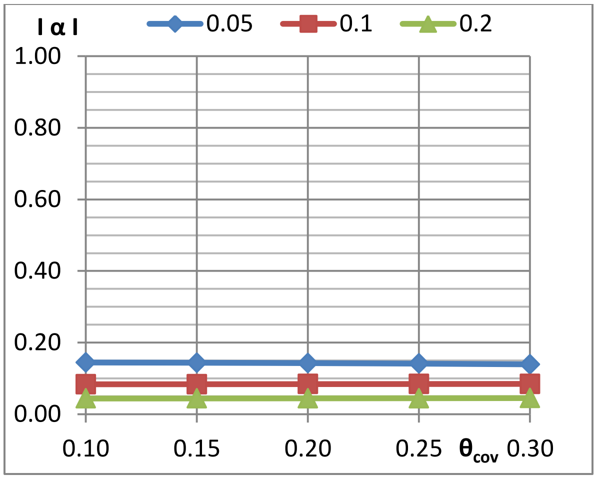
In the case of flexural cracking, the sensitivity factors indicate that load dominates over all other variables, irrespective of the coefficient of variation of the model uncertainty, the reliability level chosen and the crack width limit, with α_L values about -0,84 to -0.99 over all β values, crack width limits and model uncertainty CoV values chosen. Considering the physical model for cracking, the liquid load (in units of kN) is proportional to H^2 where H is the water depth, taken as the height of the wall. Therefore it would be expected that this parameter would be of significance. Load has a negative influence on the model, as expected. Figure 6.1 (overleaf) showing the variation of the sensitivity factors with crack width limit and model uncertainty CoV for a β of 1,5, illustrates the dominance of loading on the flexural cracking model. Table 6.1 below is a summary of the sensitivity factors of the basic variables obtained at a β of 1,5 for crack width limits of 0,2 and 0,05mm and model uncertainty CoV of 0,1 and 0,3, for flexural cracking and further illustrates the influence of loading.

Table 6.1: *Sensitivity factors for flexural cracking at a β of 1,5.*

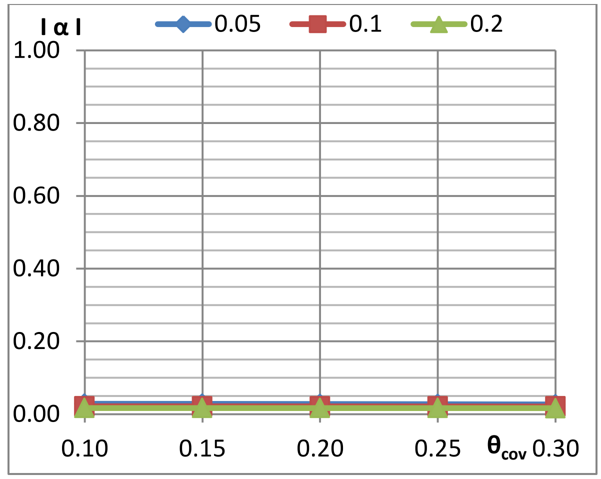
w_{lim} (mm)	θ_{CoV}	α_c	α_h	α_L	α_{ft}	α_θ
0.2	0.1	-0.044	-0.017	-0.993	0.060	-0.095
	0.2	-0.044	-0.017	-0.979	0.060	-0.188
	0.3	-0.045	-0.016	-0.957	0.060	-0.280
0.05	0.1	-0.144	-0.029	-0.966	0.128	-0.170
	0.2	-0.143	-0.028	-0.924	0.127	-0.331
	0.3	-0.139	-0.027	-0.860	0.125	-0.474

Referring to Figure 6.1 and Table 6.1, model uncertainty, although less dominant than load, has a negative influence on the flexural cracking model. This negative influence increases as crack width limit decreases and as the model uncertainty CoV increases. The crack width, in particular, is sensitive to model uncertainty with model uncertainty sensitivity factors increasing by a factor of about 1,75 as the crack width decreases from 0,2 to 0,05mm.

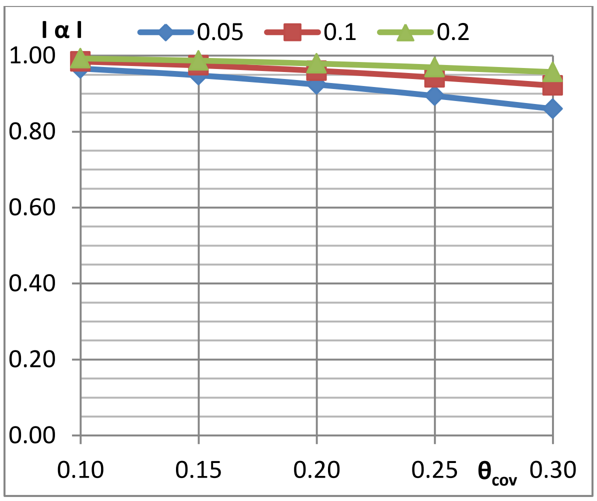
There is little variation in each of the sensitivity factors for section thickness, concrete tensile strength, load and cover. Concrete tensile strength has little to small influence as the crack width limit decreases from 0,2mm to 0,05mm, with a maximum value of 0,133 at the lower crack width limit and θ_{CoV} of 0,3. The effect of concrete cover is also small for the upper crack width limit of 0,2mm. This parameter’s influence increases marginally as the crack width decreases to 0,05mm. The reliability crack model is not influenced by the section thickness (taken as 450 mm) as all sensitivity factors are less than 0,1. .



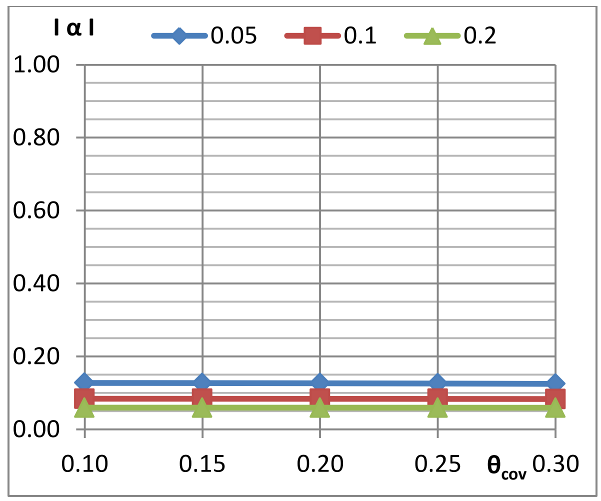
(a) Concrete cover, c



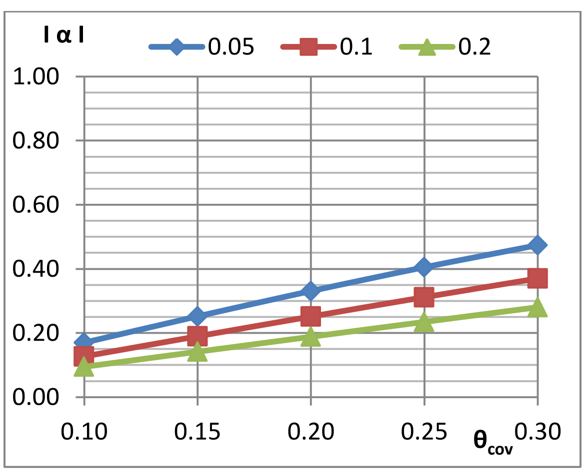
(d) Section thickness, h



(b) Liquid load, L_k



(e) Effective Concrete Tensile Strength, $f_{ct,eff}$



(c) Model Uncertainty, θ_{CoV}

Figure 6.1: *Sensitivity factors for Flexural Cracking (β of 1,5)*

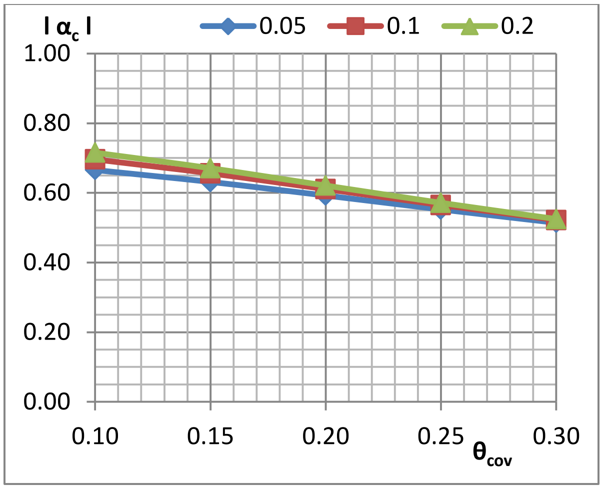
The sensitivity factors for section thickness and load are consistent irrespective of the limiting crack width. Those for concrete cover and tensile strength, as well as model uncertainty, increase as the limiting crack width decreases.

(ii) Tension cracking

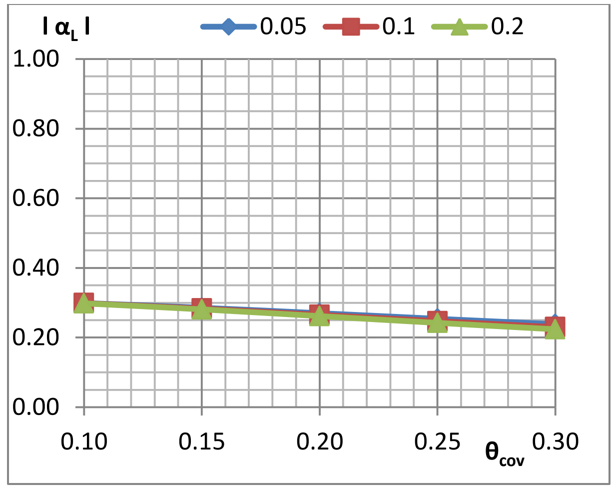
In the case of tension cracking, the reliability model is found to be influenced by the geometry of the physical model, specifically, the combination of section thickness, bar diameter and cover. This decides the equation used to determine depth of the effective tension zone, namely, h/2 or 2,5(c + φ/2). The former tends to be limiting for thinner wall thicknesses. For a 20mm bar and a 40mm cover, 2,5(c + φ/2) is limiting for all section thicknesses greater than 250mm. As discussed in Chapter 5, the effective depth of the tension zone, and thus the crack width, is independent of section thickness using this equation for $h_{c,eff}$. However, if the section thickness is 250mm, combined with a 20mm bar diameter and 40mm cover, both 2,5(c + φ/2) and h/2 apply. Reverse reliability analyses were thus performed using h/2 and 2,5(c + φ/2), for a section thickness of 250mm, 40mm cover and 20mm bar diameter.

Figures 6.2 (overleaf) and 6.3 (page 118) illustrate the variation of the sensitivity factors with model uncertainty CoV for crack widths of 0,05, 0,1 and 0,2 mm at a reliability level of 1,5, using $h_{c,eff}$ of 2,5(c + φ/2) and h/2,respectively. Table 6.2 is a summary of the sensitivity factors of the basic variables obtained at a β of 1,5 for crack width limits of 0,2 and 0,05mm and model uncertainty CoV of 0,1 and 0,3, for tension cracking using both equations for the effective depth of the tension zone.

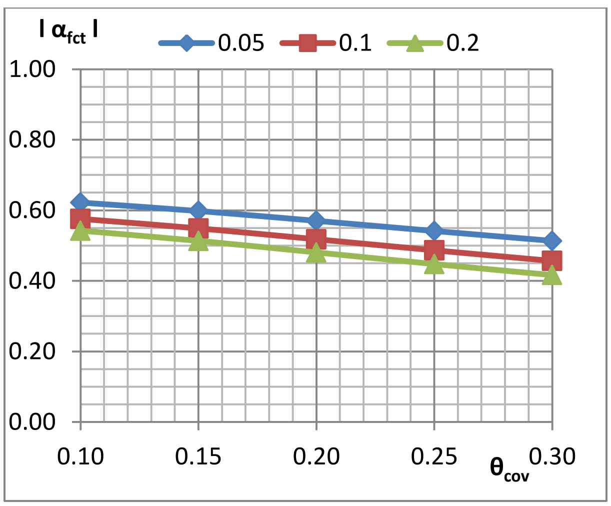
Referring to Figure 6.2 and Table 6.2 for $h_{c,eff}$ = 2,5(c + φ/2), the dominant variable for tension cracking in this case is concrete cover, followed by concrete tensile strength which has a positive influence (as expected). Model uncertainty is the next most influential variable, having a negative effect which increases as θ_{CoV} increases.



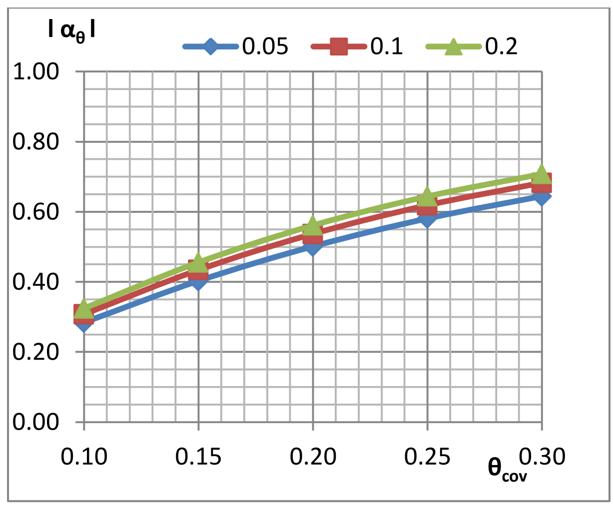
(a) Concrete cover, c



(b) Liquid load, L_k



(c) Effective concrete tensile strength, $f_{ct,eff}$



(d) Model uncertainty, θ_{CoV}

Figure 6.2: *Sensitivity factors for Tension Cracking with $h_{c,eff}$ = 2,5(c + φ/2) (β 1,5).*

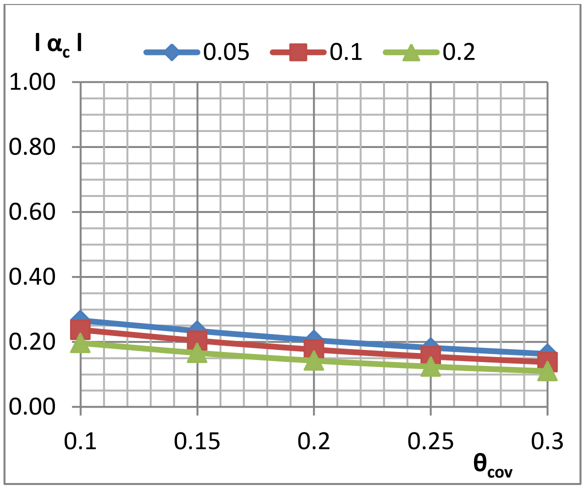
Table 6.2: *Sensitivity factors for tension cracking at a β of 1,5.*

$h_{c,eff}$ (mm)	w_{lim} (mm)	θ_{CoV}	α_c	α_h	α_L	α_{ft}	α_θ
Model 2(a) 2,5(c + φ/2)	0.2	0.1	-0.715	-	-0.298	0.542	-0.324
		0.2	-0.662	-	-0.262	0.481	-0.561
		0.3	-0.525	-	-0.224	0.416	-0.708
	0.05	0.1	-0.666	-	-0.299	0.622	-0.284
		0.2	-0.592	-	-0.270	0.570	-0.501
		0.3	-0.514	-	-0.238	0.513	-0.645
Model 2(b) h/2	0.2	0.1	-0.197	-0.011	-0.427	0.692	-0.539
		0.2	-0.142	-0.006	-0.325	0.542	-0.762
		0.3	-0.110	-0.003	-0.253	0.439	-0.855
	0.05	0.1	-0.266	0.001	-0.406	0.757	-0.437
		0.2	-0.206	0.003	-0.327	0.639	0.666
		0.3	-0.163	0.004	-0.268	0.546	-0.777

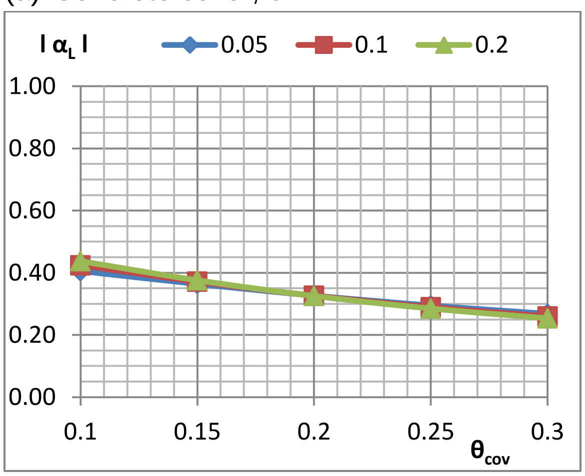
The basic variable of load has some influence, with values between 0,22 and 0,30. Section thickness obviously does not have any effect as it does not appear in the crack width equation for this formulation of $h_{c,eff}$. The crack model is sensitive to model uncertainty with the sensitivity factor for model uncertainty increasing by a factor of about 2,2 as the model uncertainty variation increases from 0,1 to 0,3.

It was also noted that the limiting crack width chosen has little effect on the sensitivity factors of concrete cover and load. There is an increase in the sensitivity factors of concrete tensile strength as crack width decreases, with a corresponding decrease in the sensitivity factors for model uncertainty.

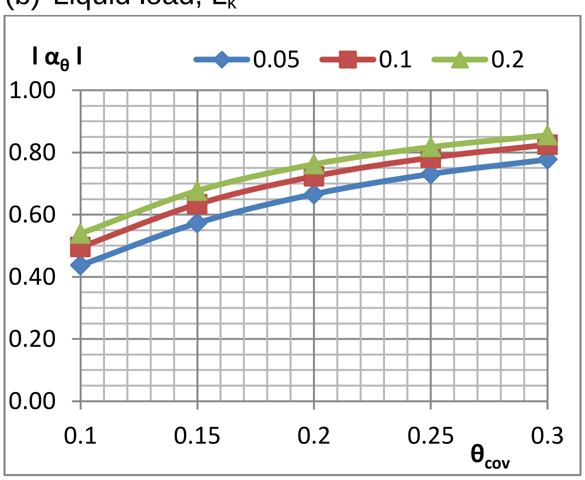
Referring to Figure 6.3 and Table 6.2 with $h_{c,eff}$ as h/2, the dominant variable for tension cracking in this case depends on the value considered for the model uncertainty CoV. At the lower limit considered for θ_{CoV} of 0,1, concrete tensile strength is the dominant variable, with sensitivity factors greater than about 0,7. As θ_{CoV} increases to the maximum value considered of 0,3, model uncertainty becomes more influential than the concrete tensile strength. The sensitivity factors for model uncertainty increase as the model uncertainty variation increases, while the factors for concrete tensile strength decrease proportionately.



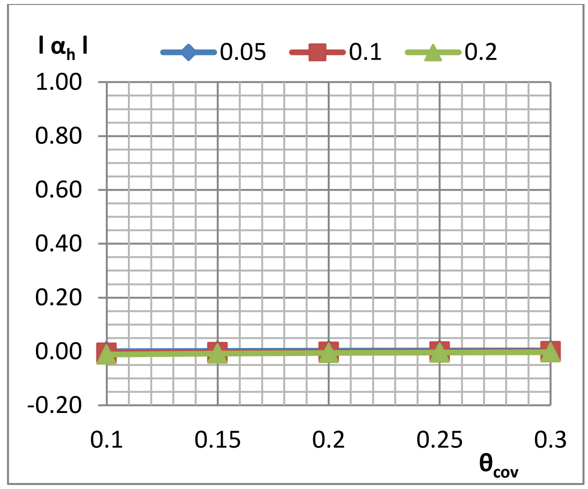
(a) Concrete cover, c



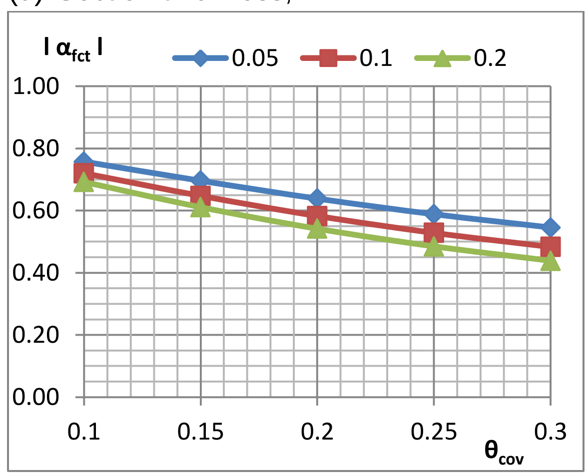
(b) Liquid load, L_k



(c) Model Uncertainty, θ



(d) Section thickness, h



(e) Effective concrete tensile strength, $f_{ct,eff}$

Figure 6.3: *Sensitivity factors for Tension Cracking with $h_{c,eff}$ = h/2 (β of 1,5)*

Referring to Figure 6.3, Load has a moderate effect on the model, depending on the value considered for the model uncertainty variation. Concrete cover has a small influence as sensitivity factors are between 0,11 and 0,27. Section thickness sensitivity factors are around zero therefore this variable has a negligible influence on the reliability crack model. It was also noted that the limiting crack width did not have any effect on the sensitivity factors of section thickness and load. The sensitivity factors for concrete cover and tensile strength increase as the limiting crack width decreases. Those for model uncertainty decrease as the limiting crack width decreases.

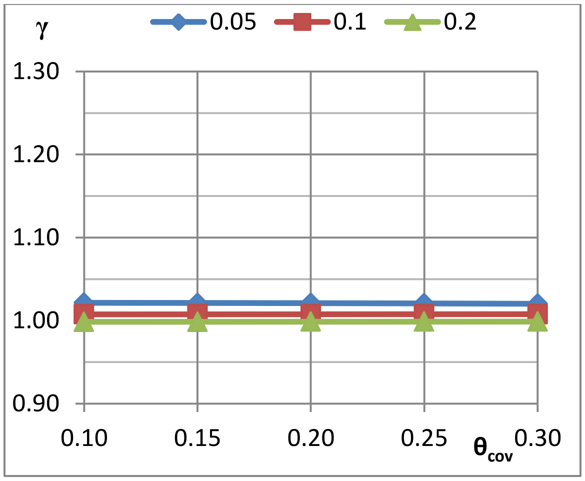
Comparing the two formulations for $h_{c,eff}$ of the tension cracking model, concrete cover is substantially more influential on the crack model using 2,5(c + φ/2) than h/2 by a factor of about 2,5 at a crack width limit of 0,05mm, and by about 3,6 at a crack width limit of 0,2mm (θ_{CoV} of 0,1). This relative influence increases as the model uncertainty variation increases. The sensitivity factors obtained for section thickness confirm the conclusion made in Chapter 5 that section thickness has little influence on the reliability model for tension cracking, irrespective of the formulation of $h_{c,eff}$. The conclusion that the influence of cover depends on the formulation of the effective tension zone in concrete is also confirmed. Model uncertainty is more influential on the tension cracking model when $h_{c,eff}$ is determined using h/2. However, the increase in the influence of model uncertainty as model uncertainty variation increases is greater using $h_{c,eff} = 2,5(c + \varphi/2)$.

6.3.2 Theoretical partial safety factors

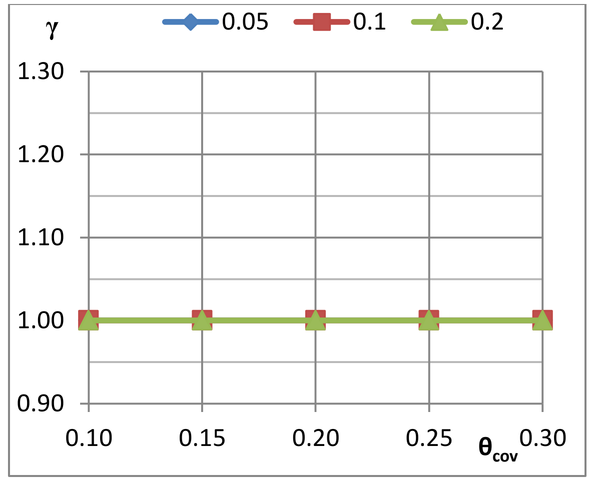
The theoretical partial safety factors for the basic variables of concrete cover, section thickness, load, concrete tensile strength and model uncertainty were calculated for flexural and tension load cracking using equation 4.3 of Chapter 4. The following was observed:

(i) Flexural cracking

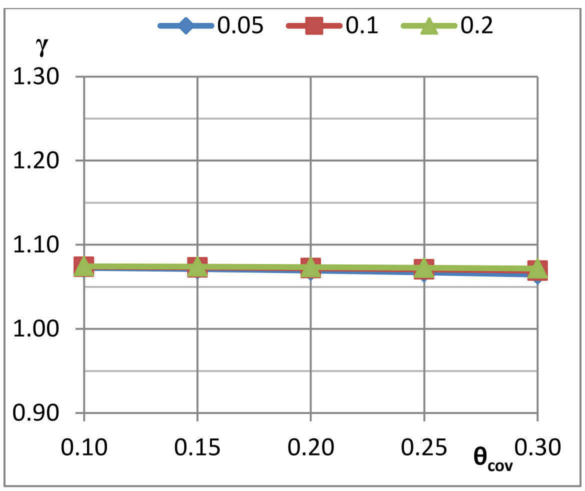
Figure 6.4 shows the partial safety factors obtained for flexural cracking at β 1,5. In the case of flexural cracking, although the sensitivity factors indicated that load was the most influential basic variable, the partial safety factors for load were found to have a maximum value of 1,1 at a β of 2,0. As shown by Figure 6.4, the values obtained for load remain relatively constant as model uncertainty increases. The partial safety factors for cover and section thickness were found to be about 1,0, irrespective of the β value chosen, indicating that these variables do not require any adjustment to achieve the required reliability level.



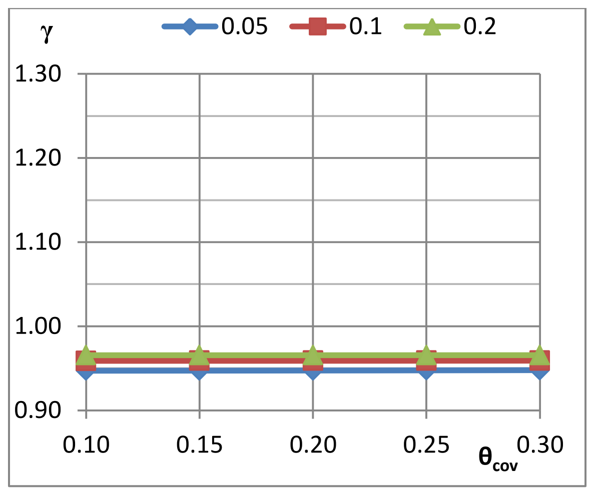
(a) Concrete cover, c



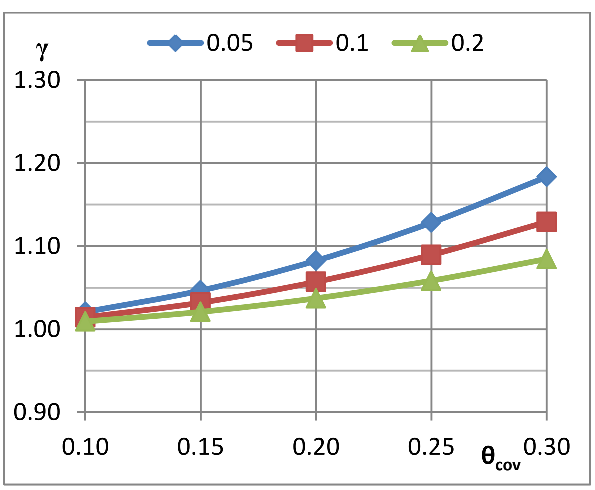
(d) Section thickness, h



(b) Liquid load, L_k



(e) Effective concrete tensile strength, $f_{ct,eff}$



(c) Model uncertainty, θ

Figure 6.4: *Theoretical partial safety factors for flexural cracking (β of 1,5)*

Within the range of crack width limits, model uncertainty CoV and β levels chosen, concrete tensile strength, $f_{ct,eff}$, has a minimum partial safety factor, γ_{fct} of 0,934, where the factored value is $f_{ct,eff}.\ \gamma_{fct}$. As concrete tensile strength constitutes a resistance effect on the crack model, the factored or design value used is conventionally expressed as $f_{ct,eff}.\ (1/\gamma_{fct})$, that is, $f_{ct,eff}/\ 1,07$.

The partial safety factors determined for model uncertainty increase as the variation of this variable increases, following the trend of increasing sensitivity factors as model uncertainty increases. At a CoV of 0,1, the partial safety factor is about 1,0, irrespective of the reliability level and crack width limit chosen. The factor increases to 1,29 at a CoV of 0,3 (β of 2,0 and wlim of 0,05mm). Figure 6.2(c) shows the partial safety factor increasing as model uncertainty increases. It was also observed that the partial safety factors for model uncertainty are sensitive to the crack width limit chosen. The smaller crack width of 0,05mm requires a higher partial safety factor than a crack width limit of 0,2mm, namely 1,18 and 1,08, respectively for β 1,5 and θ_{CoV} of 0,2. Specifying a more stringent crack width limit of 0,05mm therefore results in a more conservative design than at a 0,2mm crack width limit. It appears that the crack with limit does not have a significant effect on the other basic variables. The partial safety factors indicate that the most influential variable on the flexural cracking model is model uncertainty.

Table 6.3 provides a summary of partial safety factors (psf) obtained for flexural cracking for varying crack width and uncertainty variation, and further illustrates the dominant variables of model uncertainty and liquid load.

Table 6.3: *Partial safety factors for flexural cracking case at β 1,5*

			Theoretical partial safety factors					
Load case	Crack width limit (mm)	θ_{CoV}	γ_c	γ_h	γ_L	γ_{ft}	$1/\gamma_{ft}$	γ_θ
Flexure	0.05	0.1	1.02	1.00	1.07	0.95	1.05	1.02
	0.05	0.2	1.02	1.00	1.07	0.95	1.05	1.08
	0.05	0.3	1.02	1.00	1.07	0.95	1.05	1.18
	0.20	0.1	1.00	1.00	1.07	0.97	1.03	1.01
	0.20	0.2	1.00	1.00	1.07	0.97	1.03	1.04
	0.20	0.3	1.00	1.00	1.07	0.97	1.03	1.08

With the exception of model uncertainty, it was noted that there was little or no variation in the psf's of all the variables as the model uncertainty variation increased from 0,1 to 0,3. This trend is illustrated in Table 6.3 and Figure 6.4.

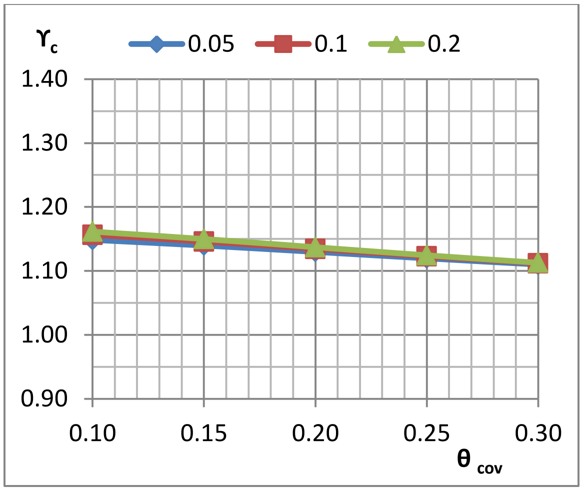
(ii) Tension cracking

For the tension cracking model, Figures 6.5 and 6.6 show the partial safety factors determined using $h_{c,eff}$ of 2,5(c + φ/2) and h/2, respectively, with increasing model uncertainty variation and for crack widths of 0,2, 0,1 and 0,05 mm. Following the trend indicated by the sensitivity factors, the magnitude of the partial safety factors for cover are influenced by the equation used for the effective depth of the concrete tension zone. When using 2,5(c + φ/2), a maximum value of 1,23 (at β 2,0 and model uncertainty CoV of 0,1) is obtained for the partial safety factor for cover which is larger than the maximum value of 1,05 obtained using h/2. This means that to achieve the same reliability, the EN1992 crack model with h/2 limiting will result in a more conservative design. Figures 6.5(a) and 6.6 (a) show the partial safety factors at a β of 1,5 using 2,5(c + φ/2) and h/2 for $h_{c,eff}$, respectively. From Figure 6.5(a) ($h_{c,eff}$ = 2,5(c + φ/2)), the maximum value for γ_c is 1,16 whereas Figure 6.6(a) ($h_{c,eff}$ = h/2) shows that the maximum value for γ_c is 1,03. It is also noted that γ_c decreases with increasing model uncertainty and is not sensitive to the crack width limit.

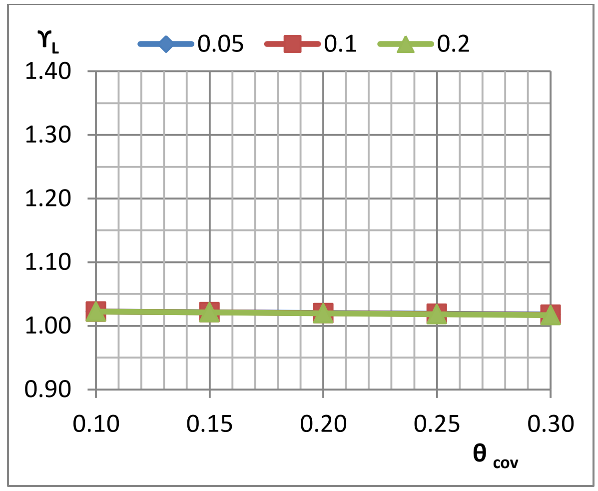
The partial safety factors for load are about 1,03 for both formulations of $h_{c,eff}$ and is not sensitive to increasing model uncertainty or the crack width limit. Using h/2, the safety factors obtained for section thickness are about 1,0, following the conclusion from the sensitivity analysis that this variable does not influence the reliability model.

The partial safety factors obtained for the concrete tensile strength, which has a positive effect on the model, are similar for both formulations of $h_{c,eff}$, such that $1/\gamma_{ft}$ has values in the range of 1,0 to about 1,3. The partial safety factors for concrete tensile strength increase as model uncertainty increases, that is, $1/\gamma_{ft}$ decreases as model uncertainty increases.

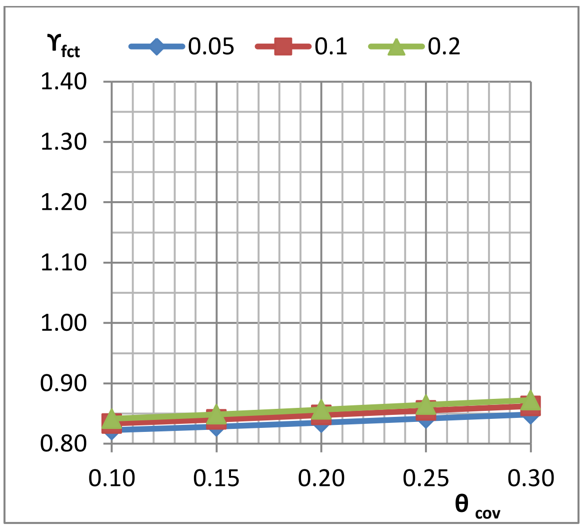
Model uncertainty partial factors for tension cracking using $h_{c,eff}$ of 2,5(c + φ/2) are only just greater than 1,0 for θ_{cov} of 0,1 for all crack widths, being about 1,04. The factors increase as θ_{cov} and reliability increase, with a 0,2mm crack width requiring a slightly higher value factor (maximum value of 1,46 at a θ_{cov} of 0,3 and β of 2,0) than the smaller crack widths of 0,1 and



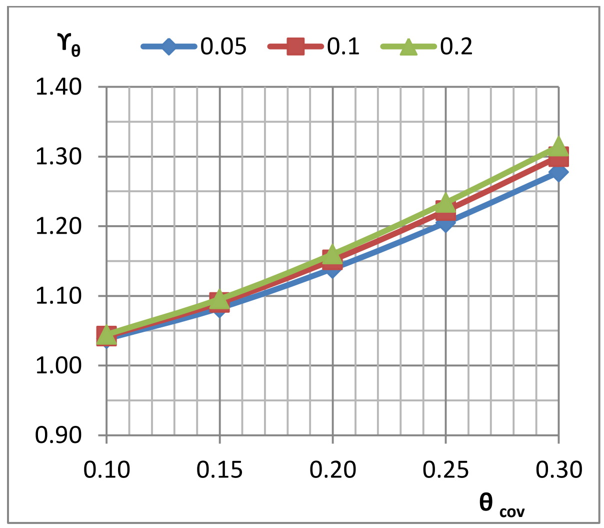
(a) Concrete cover, c



(b) Liquid load, L_k



(c) Effective concrete tensile strength, $f_{ct,eff}$

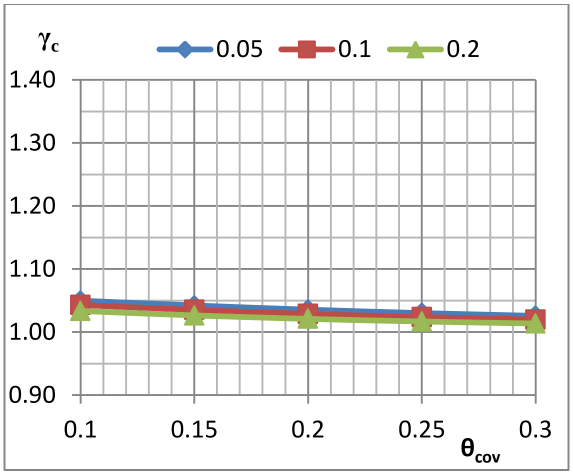


(d) Model uncertainty, θ

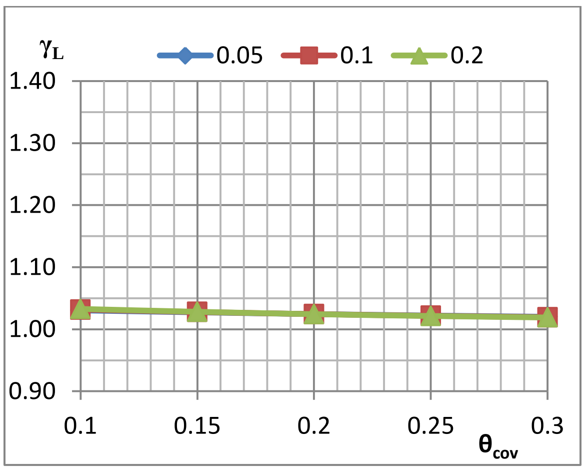
Figure 6.5 : *Theoretical partial safety factors for tension cracking ($h_{c,eff}$ = 2,5(c + φ/2)) (β 1,5)*

0,05mm (maximum value of 1,44 at a θ_{cov} of 0,3 and β of 2,0). The partial safety factors obtained using $h_{c,eff}$ of h/2 are marginally higher than those obtained using 2,5(c + φ) by about a maximum of 10%. These factors are higher than those obtained for flexure as model uncertainty is more influential for tension cracking than for flexural cracking.

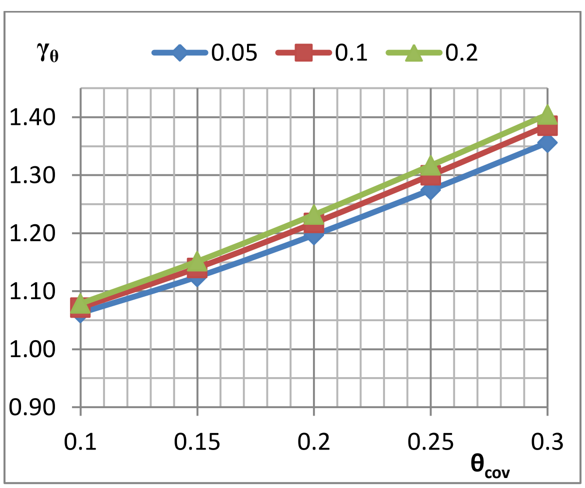
The crack width limit does not have a significant effect on the partial safety factors for tension cracking.



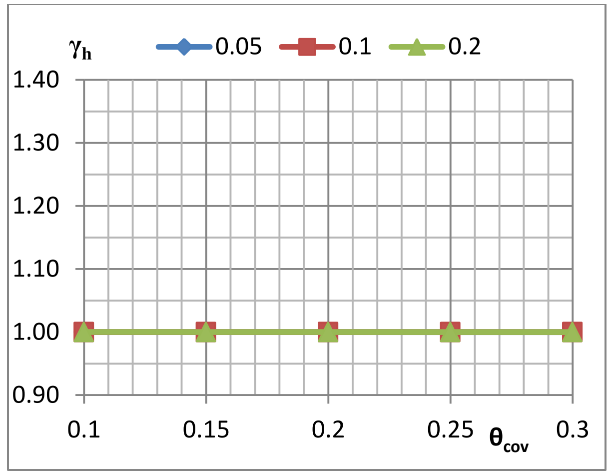
(a) Concrete cover, c



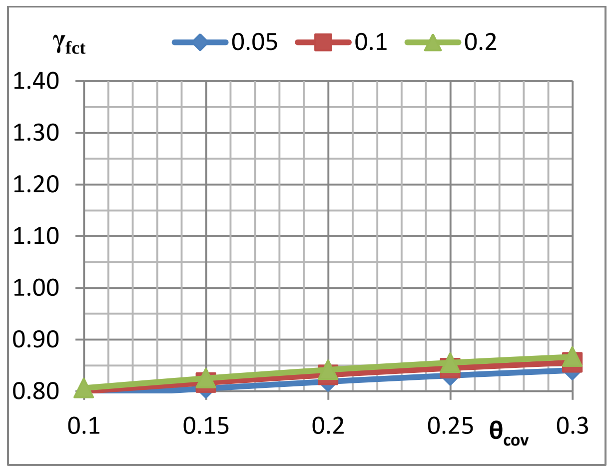
(b) Liquid load, L_k



(c) Model uncertainty, θ



(d) Section thickness, h



(e) Effective concrete tensile strength, $f_{ct,eff}$

Figure 6.6: *Theoretical partial safety factors for tension cracking ($h_{c,eff}$ =h/2) (β 1,5)*

Table 6.4 provides a summary of the partial safety factors obtained at a reliability level of 1,5 for tension cracking. The table illustrates the significant variables for tension cracking with respect to the reliability model, namely, concrete cover, concrete tensile strength and model uncertainty. Section thickness and liquid load are not influential in this load case, regardless of the limiting equation for $h_{c,eff}$.

Table 6.4: *Summary of partial safety factors for tension cracking (β 1,5).*

			Theoretical partial safety factors					
Load case	Crack width limit (mm)	θ_{CoV}	γ_c	γ_h	γ_L	γ_{ft}	$1/\gamma_{ft}$	γ_θ
Tension $h_{c,eff} = 2.5(c + \varphi)$	0.05	0.1	1.15	-	1.02	0.82	1.22	1.04
	0.05	0.2	1.14	-	1.02	0.84	1.19	1.14
	0.05	0.3	1.11	-	1.02	0.85	1.18	1.28
	0.20	0.1	1.16	-	1.02	0.84	1.19	1.05
	0.20	0.2	1.14	-	1.02	0.86	1.16	1.16
	0.20	0.3	1.11	-	1.02	0.87	1.15	1.32
Tension $h_{c,eff} = h/2$	0.05	0.1	1.05	1.00	1.03	0.79	1.27	1.06
	0.05	0.2	1.04	1.00	1.02	0.82	1.22	1.20
	0.05	0.3	1.03	1.00	1.02	0.84	1.19	1.34
	0.20	0.1	1.03	1.00	1.03	0.81	1.23	1.08
	0.20	0.2	1.02	1.00	1.02	0.84	1.19	1.23
	0.20	0.3	1.01	1.00	1.02	0.87	1.15	1.41

6.3.3 Influence of reliability level, β

The influence of the basic variables remains relatively stable over the reliability levels of 0,5, 1,5 and 2 for both flexure and tension load cases. Table 6.5, showing the sensitivity factors for given reliability index values for a 0,2 mm crack width, illustrates this trend.

The partial safety factors obtained for flexural cracking display the same lack of variation as the sensitivity factors as the reliability index increased from 0,5 to 2,0, as shown by the values summarised in Table 6.6. This means that the reliability can be increased at a low cost to the structure. The greater influence was found to be the model uncertainty, more specifically the variation thereof.

Table 6.5: *Influence of basic variables over varying reliability levels (w_{lim} = 0,2mm, θ_{cov} = 0,2)*

Load Case	β	α_c	α_h	α_L	α_{ft}	α_θ
Flexure	0.5	-0.037	-0.015	-0.983	0.058	-0.169
	1.5	-0.044	-0.017	-0.979	0.060	-0.188
	2.0	-0.048	-0.017	-0.977	0.061	-0.199
Tension	0.5	-0.617	-	-0.267	0.493	-0.555
$h_{c,eff}$ = 2,5(c + φ/2)	1.5	-0.662	-	-0.262	0.481	-0.561
	2.0	-0.624	-	-0.260	0.476	-0.563
Tension	0.5	-0.118	-0.002	-0.334	0.598	-0.719
$h_{c,eff}$ = h/2	1.5	-0.142	-0.006	-0.325	0.542	-0.762
	2.0	-0.155	-0.007	-0.321	0.519	-0.777

Table 6.6: *Partial safety factors for varying reliability levels (w_{lim} = 0,2 mm, θ_{cov} = 0,2)*

Load Case	β	γ_c	γ_h	γ_L	γ_{ft}	$1/\gamma_{ft}$	γ_θ
Flexure	0.5	0.992	1.000	1.025	0.977	1.024	0.997
	1.5	0.999	1.000	1.073	0.966	1.035	1.037
	2.0	1.003	1.000	1.098	0.960	1.041	1.061
Tension	0.5	1.035	-	1.007	0.937	1.067	1.036
$h_{c,eff}$ = 2,5(c + φ/2)	1.5	1.137	-	1.020	0.856	1.168	1.160
	2.0	1.192	-	1.026	0.820	1.220	1.228
Tension	0.5	0.998	1.000	1.008	0.928	1.078	1.053
$h_{c,eff}$ = h/2	1.5	1.021	1.000	1.024	0.842	1.188	1.232
	2.0	1.036	1.000	1.032	0.806	1.241	1.338

In the case of tension cracking, the factor for model uncertainty increases as the reliability increases whilst that for concrete tensile strength decreases. The remainder of the basic variables display little or no variation in the values obtained for the theoretical partial safety factors. Model uncertainty for the tension cracking model requires further investigation to improve the accuracy of the model.

6.4 SUMMARY

The sensitivity factors and partial safety factors were determined for each of the basic variables. The following conclusions could be made from the reverse- FORM reliability analysis:

- The sensitivity factors obtained for flexural cracking indicated that load is the dominant variable, dominating the model with values close to 1, followed by model uncertainty. However, when considering the partial safety factors, the most influential variable was found to be model uncertainty with factors increasing in value as the model uncertainty variation increased. The factors for load were about 1,03 to 1,1 for β 0,5 to 2,0.
- Model uncertainty was found to be the most influential variable for tension cracking, indicated by both the sensitivity factors and the partial safety factors. Both sensitivity factors and partial safety factors increased with increasing model uncertainty variation. Model uncertainty is also dependent on the limiting crack width, particularly for flexural cracking, and the model uncertainty variation.
- Section thickness has no or little effect on the crack model for both flexural and tension cracking. This variable could therefore be considered as deterministic. Load was found to have a small influence on the tension crack models. Concrete tensile strength was found to be a positive influence on the crack models.
- Concrete cover was found to have little influence for flexural cracking with partial safety factors around 1,0 to 1,04. For tension cracking, the influence of cover was in turn influenced by the limiting equation for the effective depth of the tension zone as this determined the variables appearing in the limit state function. Using $h_{c,eff}$ of h/2, values of about 1,05 to 1,0, as model uncertainty increases, were obtained for the partial safety factors for cover. Cover was found to be more influential for tension cracking using 2,5(c + φ/2), with higher values obtained for the partial safety factors of about 1,15 to 1,1 as model uncertainty increases.
- Higher partial safety factors for model uncertainty were required for smaller crack widths for the flexural cracking case. However, the crack width limit did not appear to have much influence on the partial safety factors for model uncertainty in the case of tension cracking.
- For flexural cracking, the theoretical partial safety factors were not sensitive to the level of reliability, meaning that the reliability of the crack models could be increased at a low cost to the structure. In the case of tension cracking, the model uncertainty factor increased with increasing level of reliability, whilst the partial safety factors for concrete tensile strength decreased. It was established that model uncertainty requires further investigation.

Select theoretical partial safety factors calculated from the sensitivity analysis will be used in the reliability calibration, presented in the following Chapter 7.

CHAPTER 7

RELIABILITY CALIBRATION OF PARTIAL SAFETY FACTORS

7.1. GENERAL

In formulating a design code equation from a reliability assessment basis, any design partial safety factors to be applied and the design values of the model parameters need to be determined and calibrated for use in that design equation such that the required structural reliability is met. This is an optimisation process. As a first step in obtaining an optimal design format for the EN1992 crack models, a reliability calibration analysis using theoretical partial safety factors, determined in the reverse-FORM analysis as presented in Chapter 6, was performed on the EN1992 crack model for the tension and flexural load cases. In this way, an indication of possible safety factors for each of the variables of the model could be obtained. The full optimisation process to obtain the final design format is beyond the scope of this thesis.

7.2. FORMULATION OF RELIABILITY CALIBRATION MODEL

The calibration analysis consists of a reverse- FORM analysis and was performed on the EN1992 crack equation for the tension and flexural load cases by using the theoretical partial safety factors (psf) determined from the reverse-FORM analyses presented in Chapter 6. The calibration was carried out for a given level of reliability (measured as reliability index, β) and set of values for the crack model parameters and their psf's. The limit state function (or performance function) was first defined. The level of reliability and the set of values of the model variables were then chosen. The reverse-FORM analysis was performed using Microsoft EXCEL, in a similar manner to the reliability analyses presented in Chapter 6, for the following models:

(iv) Model 1 - Flexural cracking
(v) Model 2(a) - Tension cracking with $h_{c,eff} = 2,5(c + \varphi/2)$
(vi) Model 2(b) - Tension cracking with $h_{c,eff} = h/2$

The analyses for each model were performed by determining the partial safety factor for model uncertainty for given values for the remaining partial safety factors and reinforcement area. A target level of reliability of 1,5 was used, which is the reference serviceability level considered in this research.

The representative structural configurations used in the calibration analysis were the same as those used in the forward and reverse-FORM analyses presented in Chapters 5 and 6.

7.2.1 Limit state function

As discussed in Chapter 4, in a design code format, partial safety factors are applied to nominal or characteristic values of the parameters to obtain design values which are then applied in the relevant design equation. In the case of serviceability cracking, this would be the design crack width. The theoretical partial safety factors (γ_x) of the crack model variables were evaluated by determining the probability that the design crack width, $w_d(X_k, \gamma_x)$, would be exceeded by the random crack width, w(X), for a given reliability. The limit state function may then be written as:

$$g(X) = \theta . w(X) - w_d(X_k, \gamma_x) \tag{7.1}$$

where θ is the model uncertainty modelled as a random variable.

Both the design crack width and the random crack width were determined using the basic compatibility equation for cracking, as presented in Chapter 4, namely:

$$w = S_r \,.\, \varepsilon_m.$$

The crack spacing, S_r, is calculated using:

$$S_r = 2c + 0{,}25 k_1 k_2 \varphi / \rho_{p,eff,}$$

The mean strain equation as given by EN1992 is:

$$\varepsilon_m = \varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t f_{ct,\,eff}\left(1 + \alpha_e \rho_{p,eff}\right)/\rho_{p,eff}}{E_s}$$

The values for the deterministic coefficients k_1, k_2 and k_t were as used in the previous reliability analyses, presented in Chapters 5 and 6, that is:

- k_1 = 0,8 for high tensile reinforcement bond
- k_2 = 0,5 for bending stress distribution and 1,0 for tension
- k_t = 0,4 for long-term loading.

The design crack width was determined using nominal values for the parameters of the crack width equation with safety partial factors applied. The random crack width was expressed in terms of the basic random variables, namely, section thickness (h), concrete cover (c), liquid load (L_k) and concrete tensile strength ($f_{ct,eff}$).

7.2.2 Model parameters

In determining the random crack width, the variables of section thickness (h), concrete cover (c), liquid load (L_k) and concrete tensile strength ($f_{ct,eff}$) were modelled using their respective probability distribution functions and mean values, as summarised in Table 7.1. All other variables were considered as deterministic values.

With the design crack width, the design value of the effective concrete tensile strength, $f_{ct,eff}$, is taken as a mean value, not characteristic, as specified by EN1992-1-1. All other parameters were assigned nominal values, as summarised in in Table 7.1.

Values of 450 mm and 250 mm were used for section thickness for flexural and tension cracking, respectively. Concrete cover was taken as 40mm. The wall height considered was 5m, corresponding to the water depth, resulting in a nominal liquid load of 49,05 kN/m^2.

Table 7.1: *Parameters for reliability calibration of EN1992 crack model*

Variable	Symbol	Dim	PDF	Characteristic Value	Mean μ_x	Std Dev. σ_x
Height of wall	H	m	Det	5	5	0
Water pressure, L_k	L_k	kN/m^2	N	49.05	49.05	2.453
Concrete tensile strength	$f_{c,t}$	MPa	LN	2.00	2.89	0.549
Steel modulus	E_s	GPa	Det	200	200	0
Concrete modulus, short term	E_c	GPa	Det	27.4	27.4	0
Concrete creep factor	φ	-	Det	1	1	0
Concrete modulus, long term	$E_{c,eff}$	GPa	Det	13.7	13.7	0
Concrete c/s depth	h	mm	N	450/250	450/250	4.5/2.5
Concrete c/s width	b	mm	Det	1000	1000	0
Concrete cover	c	mm	LN	40	40	6
Reinforcement diameter	φ	mm	Det	20	0.02	0
Reinforcement area	A_s	mm^2	Det	Varied	Varied	0
Model uncertainty	θ	-	LN		1	0.1 – 0.3

Model uncertainty was modelled as a random variable with a lognormal distribution, mean of 1,0 and variation of 0,2. To investigate the influence of model uncertainty, its partial safety factor was determined using a mean of 1,0 and coefficient of variation values of 0,1 and 0,3, in additional to the reference level of 0,2.

7.2.3 Theoretical partial safety factors applied to design crack width, w_d

The theoretical partial safety factors (psf) of the basic variables determined from the reverse-FORM analysis, presented in Chapter 6, were summarised for each load case and crack widths of 0,2 and 0,05 mm, at a reliability level of 1,5. Variables that had partial safety factors close to 1,0 were modelled as deterministic values. Model uncertainty psf's were found to be dependent on the model uncertainty variation in all three crack models. For each model, values were then chosen for the remaining variables, as follows:

(i) Model 1 - Flexural cracking

Summarised values of the psf's chosen for the calibration analyses for flexural cracking are given in Table 7.2. On examining the partial safety factors (psf) for flexural cracking, summarised and given as Table 6.3 of Chapter 6, it was noted that the psf's for the variables of cover (γ_c) and section thickness (γ_h) were either 1,0 or close to 1,0, therefore could be approximately treated as deterministic. The psf's for liquid load (γ_L) were between 1,03 and 1,1. As the variable of concrete tensile strength contributes to the resistance of the structure, the partial safety factor for this variable is applied as $f_{ct,eff}/\ \gamma_{ft}$, as expressed in Table 7.2.

Table 7.2: *Flexural cracking – Summary of theoretical partial safety factors (β 1,5).*

		Theoretical partial safety factors				
Analysis	θ_{CoV}	γ_c	γ_h	γ_L	$1/\gamma_{ft}$	γ_θ
1	0.2	1.00	1.00	1.00	1.00	Calc
2	0.2	1.00	1.00	1.10	1.00	Calc
3	0.2	1.00	1.00	1.10	1.05	Calc
4	0.1	1.00	1.00	1.00	1.00	Calc
5	0.3	1.00	1.00	1.00	1.00	Calc

Note: Calc = calculated in reliability calibration analysis

In performing the calibration analysis and determining the model uncertainty, three combinations of values for the psf's of load and concrete tensile strength were considered, as given in Table 7.2, titled Analyses 1, 2 and 3, using a model uncertainty variation of 0,2. To assess the effect of model uncertainty on the reliability model, two additional analyses (Analyses 4 and 5) were performed using a value of 1,0 for both load and concrete tensile strength psf's and model uncertainty variations of 0,1 and 0,3.

(ii) Model 2(a) – Tension cracking with $h_{c,eff}$ = 2,5(c + φ/2)

The summarised values for tension cracking using $h_{c,eff}$ = 2,5(c + φ/2) are given in Table 7.3. As the psf's obtained for liquid load were close to 1,0, this variable was considered as deterministic. Section thickness does not appear in the crack width equation therefore does not have any influence on the model. Three combinations of values for the psf's of concrete cover and concrete tensile strength were considered, namely, Analyses 1, 2 and 3 as summarised in Table 7.4 for a model uncertainty variation of 0,2.

Two additional analyses (Analyses 4 and 5) were performed using a value of 1,0 for both concrete cover and concrete tensile strength and model uncertainty variations of 0,1 and 0,3 to assess the effect of model uncertainty, as with flexural cracking.

Table 7.3: *Tension cracking, $h_{c,eff}$ = 2,5(c + φ/2) - Summary of theoretical partial safety factors (β 1,5).*

		Theoretical partial safety factors				
Analysis	θ_{CoV}	γ_c	γ_h	γ_L	$1/\gamma_{ft}$	γ_θ
1	0.2	1.00	-	1.00	1.00	Calc
2	0.2	1.10	-	1.00	1.15	Calc
3	0.2	1.10	-	1.00	1.25	Calc
4	0.1	1.00	-	1.00	1.00	Calc
5	0.3	1.00	-	1.00	1.00	Calc

Note: Calc = calculated in reliability calibration analysis

(iii) Model 2(b) - Tension cracking with $h_{c,eff}$ = h/2

The summarised values for tension cracking using $h_{c,eff}$ = h/2 are given in Table 7.4. As the psf's obtained for liquid load and section thickness were close to 1,0, these variables were considered as deterministic. In performing the calibration analysis and determining the model uncertainty, three combinations of values for the psf's of concrete cover and concrete tensile strength were considered (Analyses 1, 2 and 3 given in Table 7.4) for a model uncertainty variation of 0,2.

To assess the effect of model uncertainty, two additional analyses (Analyses 4 and 5) were performed using a value of 1,0 for both concrete cover and concrete tensile strength partial safety factors and values of 0,1 and 0,3 for the model uncertainty variation.

Table 7.4: *Tension cracking, $h_{c,eff}$ = h/2 - Summary of theoretical partial safety factors (β 1,5).*

		Theoretical partial safety factors				
Analysis	θ_{cov}	γ_c	γ_h	γ_L	$1/\gamma_{ft}$	γ_θ
1	0.2	1.00	1.00	1.00	1.00	Calc
2	0.2	1.00	1.00	1.00	1.30	Calc
3	0.2	1.05	1.00	1.00	1.20	Calc
4	0.1	1.00	1.00	1.00	1.00	Calc
5	0.3	1.00	1.00	1.00	1.00	Calc

Note: Calc = calculated in reliability calibration analysis

7.3. RESULTS AND DISCUSSION

Results of the calibration analyses for each of the EN1992 flexural and tension cracking cases were analysed and are presented here. Suitable values for partial safety factors were obtained using the reverse reliability analysis presented in Chapter 6. Results are discussed in terms of:

(i) Interaction between γ_θ and remaining partial safety factors.

(ii) Influence of model uncertainty.

All analyses were performed using a level of reliability of 1,5.

7.3.1. Model 1 - Flexural cracking

In the case of flexural cracking and referring to Table 6.3 of Chapter 6, the partial safety factors for the basic variables of section thickness and concrete cover were found to be about 1,0. Therefore these two variables were modelled as deterministic parameters. Partial safety factors were applied to liquid load (γ_L), concrete tensile strength (γ_{ft}) and model uncertainty (γ_θ). It was found that γ_θ is influenced by the reinforcement ratio chosen and by γ_L. The psf for concrete tensile strength has a lesser influence. Figure 7.1 shows the variation of the model uncertainty partial safety factor with reinforcement for various values of the partial safety factors for liquid load and concrete tensile strength (Analyses 1, 2 and 3).

Referring to Figure 7.1, graphs become approximately linear after about 1,5 %A_s . Thereafter, the increase in γ_θ is approximately proportional to increasing %A_s. As γ_L increases from 1,0 to 1,1, there is a decrease in gradient with a significant decrease in γ_θ for a given reinforcement ratio. Aside from the influence of γ_L and γ_{ft} , γ_θ is dependent on the reinforcement ratio.

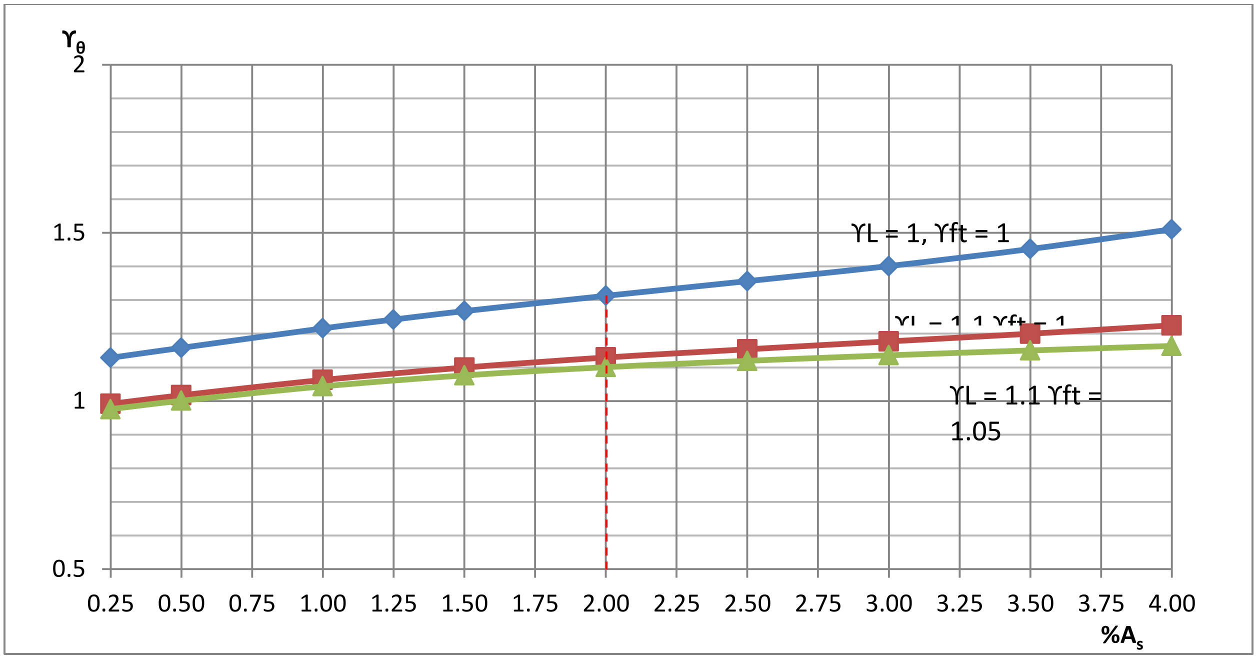


Figure 7.1: *Flexure load case – variation of γ_θ with reinforcement for different model factors (θ_{CoV} 0,2, β 1,5)*

For clarity, Table 7.5 is given, showing the influence of γ_L and γ_{ft} on γ_θ as the reinforcement quantity increases.

Table 7.5: *Flexural cracking – Influence of γ_L and $1/\gamma_{ft}$ on γ_θ*

Analysis	θ_{CoV}	γ_c	γ_h	γ_L	$1/\gamma_{ft}$	%A_s	γ_θ
1	0.2	1.00	1.00	1.00	1.00	2	1.31
						4	1.51
2	0.2	1.00	1.00	1.10	1.00	2	1.13
						4	1.22
3	0.2	1.00	1.00	1.10	1.05	2	1.10
						4	1.16

Referring to Table 7.5 and Figure 7.1, for a %A_s of 2% (ρ of 0,02) and with γ_{ft} of 1,0, γ_θ decreases from 1,31 to 1,13 as γ_L increases from 1,0 to 1,1. Taking a 4% A_s (ρ of 0,04) with γ_{ft} of 1,0, γ_θ decreases from 1,51 to 1,22. As γ_{ft} increases from 1,0 to 1,05 with γ_L of 1,1, the gradient decreases such that γ_θ has a maximum value of 1,16 for a %A_s of 4%. Choosing γ_L of 1,1, γ_{ft} of 1,05 and γ_θ of 1,2 for flexural cracking would mean that a β of 1,5 is achieved over a feasible range of reinforcement areas. However, as γ_θ is dependent on the reinforcement area, this combination would overestimate the reinforcement area, particularly when a small reinforcement area is actually required.

Figure 7.2 shows the variation of γ_θ with $\%A_s$ as model uncertainty variation increases from 0,1 to 0,3 (Analyses 3, 4 and 5). Values of 1,0 for all partial safety factors except model uncertainty were used. Model uncertainty variations of 0,1, 0,2 and 0,3 were chosen. The values thus obtained for γ_θ indicate the degree of uncertainty in the reliability crack model over and above that dealt with by means of the variation of each of the basic variables. Figure 7.2 shows that as model uncertainty variation increases, γ_θ increases substantially for a given $\%A_s$, as would be expected, and as $\%A_s$ increases. Conversely, it can be stated that the reinforcement area is significantly influenced by uncertainty in the crack model. Referring to Figure 7.2, increasing values of 1,43, 1,51 and 1,64 were obtained for model uncertainty variations of 0,1, 0,2 and 0,3, respectively, at a reinforcement area of 4%.

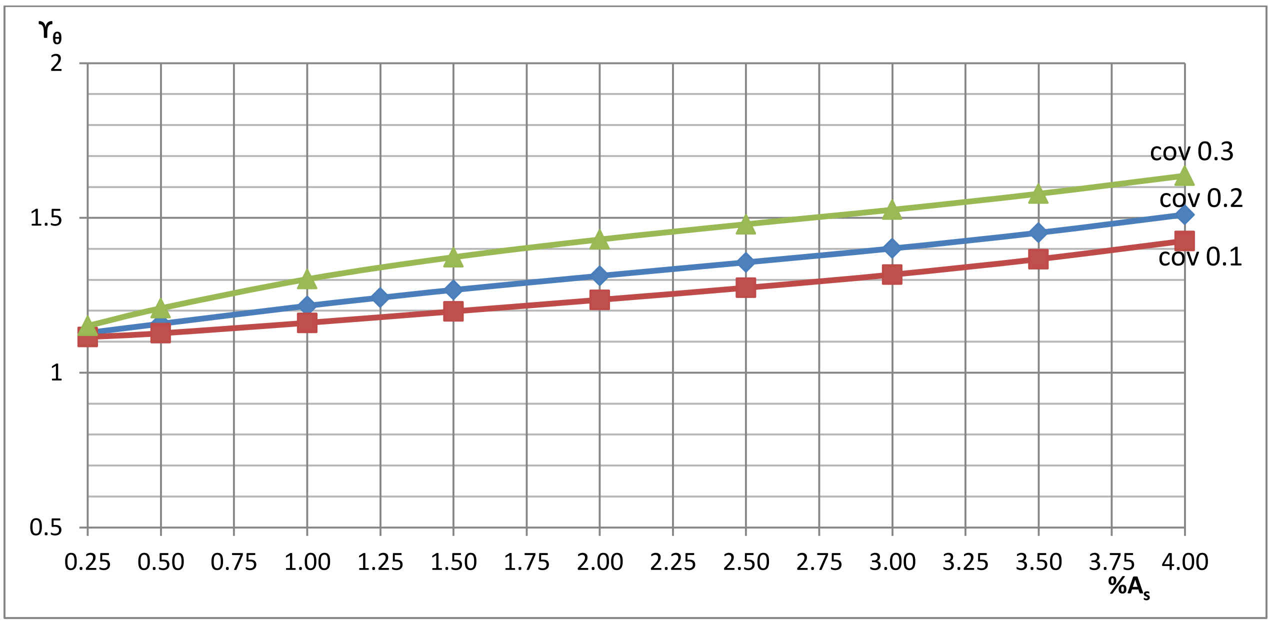


Figure 7.2: *Flexure load case – variation of γ_θ with reinforcement as θ_{CoV} varies and model factors of 1,0 (β 1,5)*

7.3.2. Model 2a - Tension cracking with $h_{c,eff}$ = 2,5(c + φ/2)

For tension cracking such that $h_{c,eff} = 2,5(c + \varphi/2)$, and referring to Table 6.4 of Chapter 6, the partial safety factors for liquid load were found to be about 1,0. Liquid load was therefore modelled as a deterministic parameter. Partial safety factors were applied to concrete cover, concrete tensile strength and model uncertainty. Figure 7.3 shows the variation of γ_θ with $\%A_s$ for various combinations of γ_c and γ_{ft} for tension cracking using $h_{c,eff} = 2,5(c + \varphi/2)$. The partial safety factors for both concrete cover (γ_c) and concrete tensile strength (γ_{ft}) were found to influence the values obtained for γ_θ. For $\gamma_c = 1.1$ with $\gamma_{ft} = 1.15$, γ_θ varies between 1,22 and 1,29 for reinforcement areas of 0,25% to 4%. When $\gamma_c = 1.1$ and $\gamma_{ft} = 1.25$, γ_θ is fairly constant at a value of about 1,15 over the same $\%A_s$ range, and is thus is nearly independent of the reinforcement ratio. When γ_c and γ_{ft} are 1,0, γ_θ is has a greater value, namely,1,39 at $\%A_s$ of 0.25%, increasing with increasing $\%A_s$.

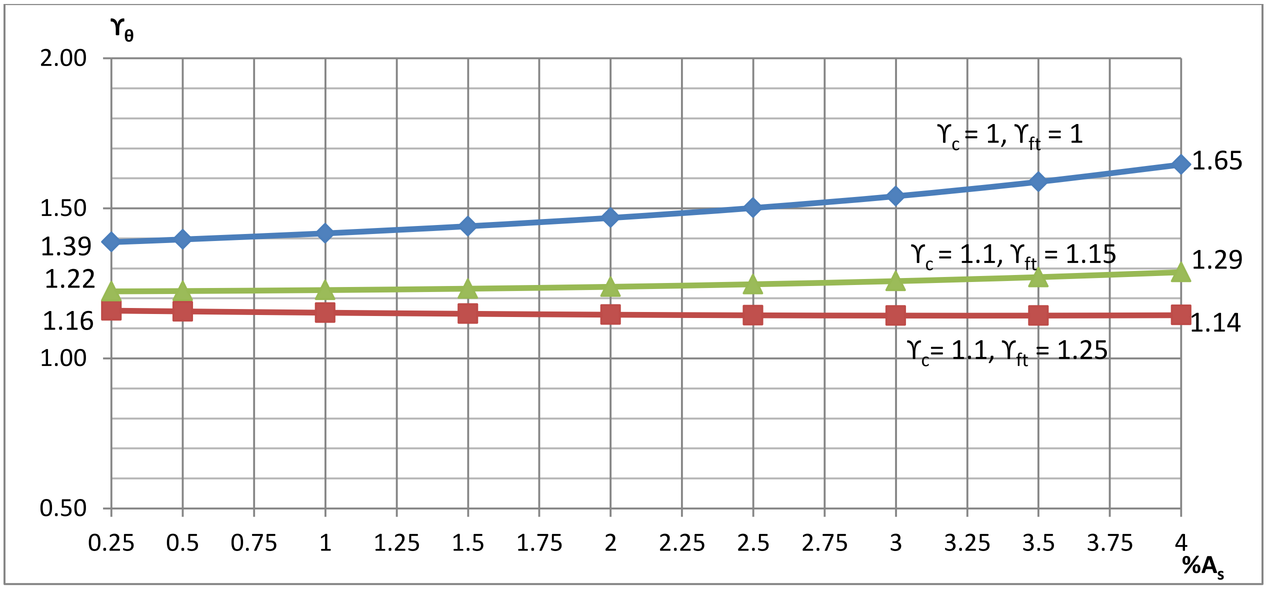


Figure 7.3: *Tension Cracking ($h_{c,eff}$ =2,5(c + φ/2)) – variation of γ_θ with reinforcement for different model factors (θ_{CoV} 0,2, β 1,5)*

To assess the influence of model uncertainty, γ_c and γ_{ft} were taken as 1,0 for model uncertainty variations (θ_{CoV}) of 0,1, 0,2 and 0,3, as in the case of flexural cracking. Figure 7.4 shows the variation of γ_θ with reinforcement as θ_{CoV} varies. For all values of model uncertainty variation, γ_θ increases at a similar rate with increasing $\%A_s$. As model uncertainty variation increases, a significant increase in γ_θ is required to achieve the reliability level of 1,5. Referring to Figure 7.4, considering a $\%A_s$ of 0,25%, γ_θ has values of 1,27, 1,37 and 1,54 for θ_{CoV} of 0,1, 0,2 and 0,3, respectively. These values increase to 1,56, 1,65 and 1,78 for θ_{CoV} of 0,1, 0,2 and 0,3, respectively, as $\%A_s$ increases to 4%. The values required for γ_θ are all greater than 1,27 and thus indicate that the influence of model uncertainty is significant for tension cracking with $h_{c,eff}$ = 2,5(c + φ/2).

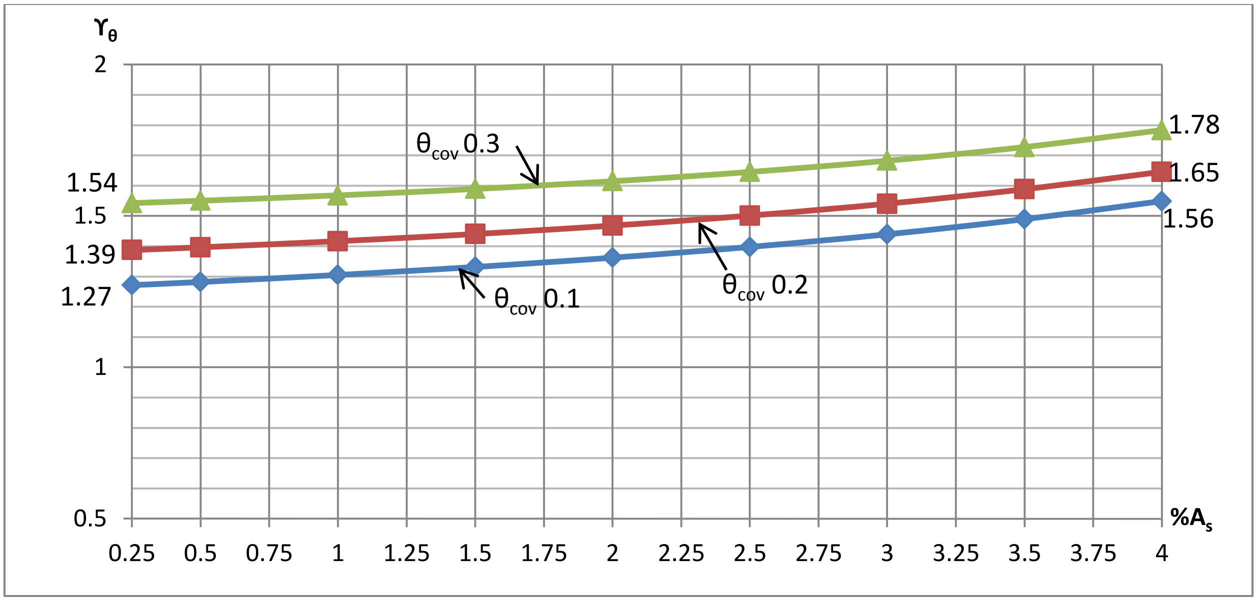


Figure 7.7: *Tension cracking ($h_{c,eff}$ =2,5(c + φ/2)) - variation of γ_θ with reinforcement and θ_{CoV}, model factors of 1,0.*

7.3.3. Model 2b - Tension cracking with $h_{c,eff}$ = h/2

As in the case of tension cracking with $h_{c,eff}$ = 2,5(c + φ/2), liquid load was taken as a deterministic parameter as the partial safety factors obtained for liquid load were found to be about 1,0 (refer to Table 6.4 of Chapter 6). The partial safety factors obtained for concrete cover were less when using $h_{c,eff}$ as h/2 than 2,5(c + φ/2), whilst those for concrete tensile strength were found to be higher. Figure 7.5 shows the variation of γ_θ with $\%A_s$ for various combinations of γ_c and γ_{ft} for tension cracking using h/2 for $h_{c,eff}$.

Considering γ_c of 1 and γ_{ft} of 1,3, γ_θ is nearly constant at about 1,28 as $\%A_s$ increases. This combination of psf's could be therefore applied to tension cracking ($h_{c,eff}$ of h/2) for β 1,5 as this is independent of the reinforcement area. When γ_c is increased to 1,05 and γ_{ft} is 1,2, γ_θ decreases to about 1,23 for a $\%A_s$ of 0,25% with a further small decrease to about 1,19 as $\%A_s$ increases to 4%. Considering γ_c and γ_{ft} of 1, γ_θ increases from 1,44 to 1,75 as the reinforcement area increases from 0,25% to 4%.

When the calibration analysis was performed for model uncertainty variations of 0,1, 0,2 and 0,3, as in the two previous crack models, and γ_c and γ_{ft} equal to 1, γ_θ increases with increasing model uncertainty variation and $\%A_s$. This trend is illustrated by Figure 7.6, showing the variation of γ_θ with $\%A_s$ as model uncertainty variation increases.

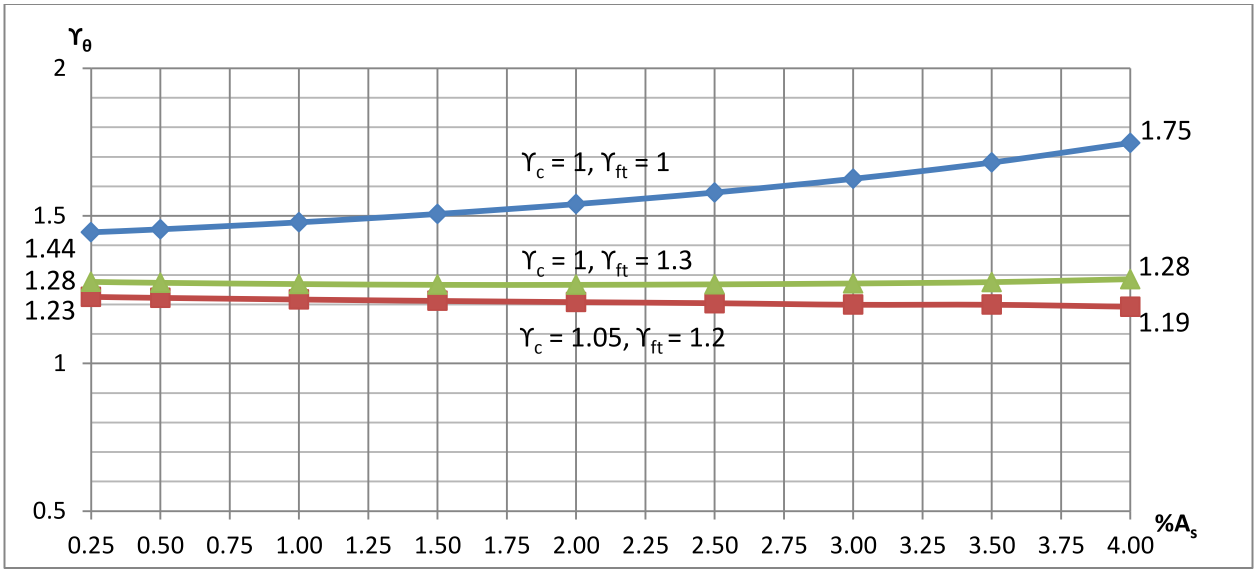


Figure 7.5: *Tension cracking ($h_{c,eff}$ = h/2) - variation of γ_θ with reinforcement for different model factors (θ_{CoV} 0,2, β 1,5)*

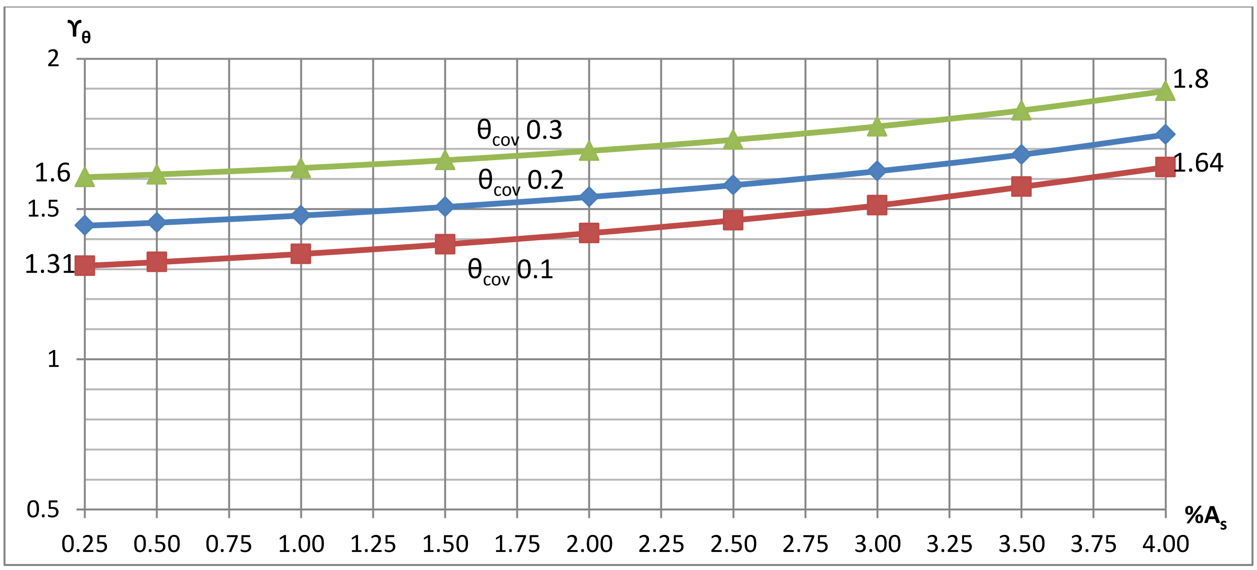


Figure 7.6: *Tension cracking ($h_{c,eff}$ = h/2) - variation of γ_θ with reinforcement as θ_{CoV} varies, model factors of 1,0.*

Referring to Figure 7.6, at a $\%A_s$ of 0,25%, γ_θ increases from 1,31 to1,6 as model uncertainty variation increases from 0,1 to 0,3. For a $\%A_s$ of 4%, γ_θ increases from 1,64 to1,89 as model uncertainty variation increases from 0,1 to 0,3. The values obtained for γ_θ are higher when $h_{c,eff}$ is h/2 than when 2,5(c + φ/2) is limiting. All values obtained for γ_θ, irrespective of model uncertainty variation, are greater than about 1,3, indicating that model uncertainty has a significant influence on the crack model.

Comparing the two models for tension cracking, referring to Figures 7.3 and 7.5, the following combinations of partial safety factors are independent of reinforcement area (θ_{CoV} of 0,2):

(i) $h_{c,eff} = 2{,}5(c + \varphi/2)$: $\gamma_{ft} = 1{,}30$ $\gamma_c = 1{,}0$ $\gamma_\theta = 1{,}28$

(ii) $h_{c,eff} = h/2$: $\gamma_{ft} = 1{,}25$ $\gamma_c = 1{,}0$ $\gamma_\theta = 1{,}15$

Values for these partial safety factors are similar enough to be refined to obtain one scheme for tension cracking without introducing undue conservatism into the design equation.

To compare flexural and tension cracking models, consider a value of 1,0 for all partial safety factors except model uncertainty, as the influencing variables in each load case are not the same. Referring to Table 7.5 and Figures 7.3 and 7.5, the following values for model uncertainty partial safety factors were thus determined for a reinforcement area of 4% (θ_{CoV} of 0,2):

(i) Flexural cracking $\gamma_\theta = 1{,}51$

(ii) Tension cracking, $h_{c,eff} = 2{,}5(c + \varphi/2)$ $\gamma_\theta = 1{,}65$

(iii) Tension cracking, $h_{c,eff} = h/2$ $\gamma_\theta = 1{,}75$

Given the differences in γ_θ between tension and flexural cracking, further research is required if one partial safety factor scheme is to be applied to all load cases, such that undue conservatism is not introduced into the design model.

Effectively applying a psf to model uncertainty only (such that all other psf's are equal to 1) equates to a single model factor being applied to the crack equation. The γ_θ obtained above can be compared to the empirical factor of 1,7 applied to the crack width equation to obtain the EN1992 design characteristic crack width (as discussed in Section 4.5). For a model uncertainty variation of 0,2 and 4% A_s, the EN1992 design equation leads to overdesign when considering flexural cracking. For tension cracking, there is a better correlation, depending on the formulation of the crack model.

7.4. SUMMARY

The aim of the calibration analysis presented in this chapter was not to produce the final partial safety factors to be applied to the design crack width, but to investigate different feasible combinations of partial safety factors (psf) applied to the basic variables to access their effect on the crack models. Feasible partial safety factors for model uncertainty, indicating the influence of this variable, were of particular interest. As in the previous reliability analyses presented in Chapters 5 and 6, model uncertainty was modelled as a basic variable, with a lognormal probability distribution function, mean of 1,0 and variation of 0,1 to 0,3. A β of 1,5 was chosen as the level of reliability.

The following conclusions were made:

- Model uncertainty is more critical for tension cracking than flexural cracking, having a significant influence on the tension crack model, with psf's obtained greater than about 1,1 for a model uncertainty variation of 0,2. For flexural cracking, model uncertainty partial safety factors were found to be around 1,0 for smaller reinforcement areas. However, as noted in Chapter 6, the values obtained for the partial safety factors depend on the model uncertainty variation chosen, with values increasing as uncertainty increased. Further research is required to improve the data on model uncertainty of the crack model applied to South African conditions.
- If all other partial safety factors are assigned values of 1,0, the model uncertainty partial safety factor and the reinforcement required are interdependent, irrespective of the load case. The model uncertainty partial safety factor for flexural cracking was influenced significantly by both model uncertainty variation and reinforcement area.
- The partial safety factors obtained for the basic variables of concrete cover, concrete tensile strength and liquid load are different for each load case using the same general EN1992 crack equation, as the influence of each variable on the crack model is dependent on the particular load case. One partial safety factor scheme applied to all crack models would then by necessity be conservative if the desired reliability level is to be achieved. Further research, applied using South African design values, is required to refine the crack models and devise a reasonable combination of partial safety factors that to be applied to the design equation for crack width, irrespective of the load case.
- The flexural cracking model is dominated by liquid load, depending on the model uncertainty variation. A partial safety factor of 1,1 for liquid load appears to be reasonable to achieve the required reliability, resulting in a maximum partial safety factor of 1,23 (for γ_{ft} of 1,0) although further research is required to refine this value.
- Tension cracking with $h_{c,eff}$ as h/2 is more conservative case than when 2,5(c + φ/2) is limiting in that a higher γ_{θ} is required in the former to achieve the same level of reliability. However, values obtained for the partial safety factors are similar enough that a common partial safety factor scheme could be developed for tension cracking.

CHAPTER 8

FINAL SUMMARY AND CONCLUSIONS

8.1 General

A summary of the findings over the course of this research is presented in this chapter. The research was initially conducted to investigate concrete cracking in South African reinforced concrete water retaining structures (WRS), in particular, load-induced cracking (flexure and direct tension). As the updating of South African structural design codes of practice is currently being done such that they are compatible with Eurocode standards, the focus of the research was on current practices in South Africa regarding WRS and on the relevant Eurocode standards that may be used.

In the process of the research, three main issues were identified, namely:

(i) The importance and dominance of the serviceability limit state of cracking over ultimate limit state loading in the case of WRS. This is of significance in establishing an appropriate level of reliability for serviceability cracking.

(ii) Eurocode was found to have a more stringent limiting crack width of 0,05mm, as opposed to 0,1mm to BS8007. The implications of this on the design and construction of WRS in South Africa was then researched.

(iii) Model uncertainty with respect to load-induced cracking in WRS was not well established.

Research into these three key issues identified took the form of:

(i) A literature and industry review on cracking and related topics in reinforced concrete.

(ii) A deterministic analysis of the load-induced cracking models to BS8007 (1987) and EN1992-3 (2006).

(iii) A reliability study of the load-induced cracking models to EN1992-3, consisting of a FORM analysis, a sensitivity analysis and a reliability calibration.

Chapter 2 presented the literature review and investigation into current industry practices in South Africa for serviceability cracking with respect to reinforced concrete WRS. This confirmed that current design practice in South Africa is the use of BS8007. The design equations and general parameters used in industry to calculate SLS load-induced crack widths to EN1992 and BS8007 were established. The structural configurations resulting in the worst loading case for either direct tension or flexural cracking were investigated. The wall of a reservoir was identified as a critical design element. The structural configuration that results in the greatest flexural load was found to be a wall section of a large rectangular reservoir under loading due to water pressure, inducing a bending moment about a horizontal axis. In the direct tension load case, a section of wall in a circular reservoir under hoop stress due to water pressure was ascertained to be a critical configuration. Important parameters considered in the crack model were found to be cover, section thickness and load due to water pressure. The limiting crack width was found to be 0,05 mm for Eurocode which is more stringent than BS8007 (having a general 0,2 mm limit which may be reduced to 0,1 mm).

A deterministic analysis, presented in Chapter 3, of the EN1992 and BS8007 crack formulations was performed with respect to flexural and tension cracking due to loading. A representative configuration of a typical WRS was chosen for both load cracking conditions, namely, a rectangular reservoir wall in the case of flexural cracking and a circular reservoir for the tension cracking condition. In addition to comparing BS8007 and EN1992, the deterministic analysis was used in aiding the selection of a representative set of values for the reliability analyses. Parameters considered were section thickness, wall height, concrete cover and reinforcement bar diameter.

The reliability analysis of the EN1992 crack model made use of the First Order Reliability Method (FORM) and consisted of three sets of analysis. This took the form of:

(i) A literature review of reliability analysis with emphasis on load-induced reinforced concrete cracking.

(ii) A FORM analysis

(iii) A sensitivity analysis using reverse-FORM

(iv) Calibration of the reliability crack model

The literature review, as summarised by Chapter 4, was first done to properly formulate the reliability model of the EN1992 crack equation using available knowledge. Using the information gathered from the literature review, model uncertainty in the model was treated as a random variable. The FORM analysis, presented in Chapter 5, was then performed. The influence of the specified crack width limit, the reliability of the EN1992-1-1 model and the influence of SLS cracking were explored. Model uncertainty in the crack model was also investigated.

Using reverse-FORM, a sensitivity analysis was done (Chapter 6) to assess the influence of the basic variables of liquid load, concrete cover, concrete tensile strength, section thickness and model uncertainty on the crack model. The theoretical partial safety factors for each of the variables were calculated for a given set of values for the parameters of the crack model. In addition, the influence of the reliability level was investigated. The calibration analysis presented in Chapter 7 looked at different feasible combinations of partial safety factors applied to a given set of values for the basic variables. A β of 1,5 was chosen as the level of reliability.

8.2 Conclusions

Final conclusions are now discussed with respect to the crack width limit, model uncertainty of the EN1992 crack model and the reliability of the EN1992 crack model.

8.2.1 Crack width limit

EN1992-3 specifies a range of crack width limits from 0,2 mm to 0,05 mm for through-cracks, depending on the hydraulic ratio which in turn is dependent on the wall geometry. A decreasing limiting crack width results in a substantial increase in reinforcement and increase in section geometry for both flexural and tension cracking, with a negative effect on the cost to structure on moving from the use of BS8007 to EN1992 in the design of WRS. One of the findings of the deterministic analysis (Chapter 3) was that EN1992 was more conservative than BS8007 by a factor of about 1,1 for flexural cracking and about 1,6 for tension cracking, for a crack width limit of 0,2 mm. This conservatism is exacerbated by the negative effects of a more stringent crack width limit in the case of tension cracking. EN1992 was found to be more demanding than BS8007 at a crack width limit of 0,1 mm by a factor of about 2,2. Comparisons could not be made for a crack width limit of 0,05 mm as BS8007 does not make provision for this crack width limit.

On performing the FORM analysis, as discussed in Chapter 5, it was found that the crack width limit has a significant effect on the probabilistic models for both flexural and tension cracking, namely, an increase in reinforcement required and a decrease in reliability, that is, a decrease in the performance of the structure, as the crack width limit reduces.

Higher partial safety factors for model uncertainty were required for smaller crack widths for the flexural cracking case, as reported in Chapter 6. However, the crack width limit did not appear to have much influence on the partial safety factors for model uncertainty in the case of tension cracking.

8.2.2 Reliability of EN1992 crack model

The deterministic and the reliability analyses showed that the serviceability limit state of cracking dominates the design of a WRS rather than the ultimate limit state of loading, particularly for tension cracking and smaller crack widths. This emphasizes the importance of optimizing the design of a WRS for serviceability cracking. The EN1992-1 crack model had not previously been examined in reliability terms with respect to WRS in South Africa, providing motivation for the probabilistic analysis component of this research. A SLS reliability β of 1,5 for a reference period of 50 years was used as the reference level in the reliability study in keeping with SANS10160-1.

The deterministic analysis was found to be more conservative than the probabilistic analysis for both flexural and tension crack models at a reliability level of 1,5 (Chapter 5) by factors of 1,29 to 1,27 for flexural cracking and 1,27 to 1,44 for tension cracking, using a model uncertainty variation of 0,2 and limiting crack widths from 0,2 mm to 0,05 mm, respectively. This suggests that there are potential savings to be made for load-induced cracking conditions using reliability design at this level of reliability. This supports the conclusion that Holicky et al (2009) made from their investigation into the tension cracking case.

On performing the sensitivity analysis (Chapter 6), it was found that in the case of flexural cracking, except for model uncertainty, the partial safety factors (psf) of the basic variables were relatively insensitive to a changing reliability level. This means that the level of reliability can be increased at a low cost to structure, depending on the model uncertainty variation. For tension cracking, the level of reliability influenced the psf’s for model uncertainty and concrete tensile strength

It was discovered (Chapter 5) that the reliability model for tension load cracking is influenced by the geometry of the member considered, in particular, the combination of section thickness (h), bar diameter (φ) and concrete cover (c) chosen. This in turn affects which equation is limiting in calculating the effective depth of the tension zone, $h_{c,eff}$, being either h/2 or 2,5(c + φ/2). When both equations apply for a particular combination of cover, bar diameter and section thickness, the reliability obtained using h/2 was less than when using 2,5(c + φ/2). On undertaking the sensitivity analysis (Chapter 6), a higher model uncertainty psf (γ_{θ}) was required when $h_{c,eff}$ of h/2 is limiting to achieve the same level of reliability. However, values obtained for the psf's were similar enough that a common partial safety factor scheme could be developed for tension cracking.

The partial safety factors obtained from the sensitivity analysis (Chapter 6) for the basic variables of concrete cover, concrete tensile strength and liquid load were different for each load case using the same general EN1992 crack equation, as the influence of each variable on the crack model is dependent on the particular load case. One partial safety factor scheme applied to all crack models would then, by necessity, be conservative. The reliability analysis showed that the flexural cracking model is dominated by liquid load. A partial safety factor of 1,1 for liquid load appeared to be reasonable to achieve the required reliability level of β 1,5, resulting in a maximum partial safety factor of about 1,22 (for γ_{ft} of 1,0) although further research is required to refine this value.

For tension cracking, the influence of concrete cover was dependent on the limiting equation for the effective depth of the tension zone as this determined the variables appearing in the limit state function. Using $h_{c,eff}$ of h/2, values of about 1,05 to 1,0 were obtained for the partial safety factors for cover as model uncertainty variation increased. Concrete cover was found to be more influential in the model for tension cracking using 2,5(c + φ/2), with higher values obtained for the partial safety factors of about 1,15 to 1,1 as model uncertainty increases.

8.2.3. Model uncertainty in EN1992 crack model

Previous research done on cracking in concrete tended to be for a specific structural and loading configuration with crack models having been largely developed empirically. In addition, as cracking is a random phenomenon which results in a wide variation in results, it is difficult to generalise a crack model for all cracking cases. The uncertainty in the crack models is thus not really known with limited data available. From the FORM analysis (Chapter 5), model uncertainty was found to have some effect on the reliability of the model, particularly for the tension cracking case. Further investigation by means of the sensitivity analysis (Chapter 6) indicated that in the case of flexural cracking, load is the most influential variable, dominating the model with sensitivity factors close to 1, followed by model uncertainty. When considering the partial safety factors, the dominant variable was found to be load at a lower model uncertainty variation. As model uncertainty variation increased, model uncertainty became dominant with factors increasing in value as the model uncertainty variation increased. The factors for load had values of about 1,02 to a maximum value of 1,1.

Model uncertainty was found to be the most influential variable for tension cracking, shown in both the FORM analysis and the sensitivity analysis. Both the sensitivity factors and partial safety factors increased with increasing model uncertainty variation. Partial safety factors obtained for model uncertainty had values greater than 1,15. Model uncertainty was found to be dependent on the limiting crack width, particularly for flexural cracking, and the model uncertainty variation chosen.

On performing the reliability calibration, (Chapter 7), it was noted that if all other partial safety factors are assigned values of 1,0, the model uncertainty partial safety factor (γ_θ) and the reinforcement area obtained are interdependent, irrespective of the load case. The model uncertainty variation was found to affect γ_θ, with values increasing as uncertainty increased. The model uncertainty partial safety factor for flexural cracking was influenced significantly by both model uncertainty variation and reinforcement area.

In summary, model uncertainty as a basic variable in the reliability model is interdependent with the reliability level, the coefficient of variation used for model uncertainty, and to a lesser extent the crack width chosen.

8.3 Recommendations for further research

From this research, several recommendations for further research can be made, as discussed as follows:

- The crack width limit has a significant influence on the reliability and design of a WRS. Given the difference in the lower limit used by EN1992 as determined by the hydraulic ratio, further research is suggested to determine an appropriate limiting crack width, in particular on the topic of autogenous healing.
- The model uncertainty in the EN1992 currently has been dealt with empirically rather than in reliability terms, and has thus not really been quantified. A comparison of the probabilistic

analysis to experimental data on flexural and tension cracking would be useful in improving statistical data for crack models and their related model uncertainty.

- A cost optimisation analysis is recommended to obtain the best reliability to cost ratio for serviceability cracking.
- Further research using South African design values, would refine the crack models and aid in devising a reasonable combination of partial safety factors that could be applied to the design equation for crack width, irrespective of the load case.

REFERENCES

1. Allen AH (1983). *Autogenous healing.* Seminar on design of concrete water retaining structures to BS 5337 held by the Division of Hydraulic and Water Engineering of the South African Institution of Civil Engineers in association with the Concrete Society of South Africa.
2. Anchor RD, Hill AW, Hughes BP. (1983). *Handbook on BS 5337:1976 (The structural use of concrete for retaining aqueous liquids. 2nd Edition.* Viewpoint Publications.
3. Ang AH-S, Tang WT (1990). *Probability concepts in engineering planning and design. Volume II – Decision, risk and reliability.* John Wiley & Sons. ISBN 0-471-03201-8
4. Batty I, Westbrook W (1991). *The design of water-retaining structures.* Longman.
5. Beeby, AW (2004). *The influence of the parameter φ/ρ_{eff} on crack widths.* Structural Concrete (2004) 5 No.2, pgs 71 – 83.
6. Beeby AW, Narayanan RS (1993). *Designer's handbook to Eurocode 2: Part1.1 Design of concrete structures.* Thomas Telford.
7. Bhatt P, MacGinley TJ, Choo BS (2006). *Reinforced concrete design theory and examples.* (3rd Edition). Taylor and Francis.
8. Borosnyoi A, Balazs GL (2005). *Models for flexural cracking in concrete: the state of the art.* Structural Concrete 2005 Vol 6 No.2 pgs 53 – 62.
9. Bowles L (2012). Personal communication. Umgeni Water, Durban, KwaZulu-Natal, South Africa.
10. BS EN 1990: Eurocode 0: Basis of structural design. BSI 2002.
11. BS EN 1992-1: *Eurocode 2: Design of concrete structures – Part 1: General requirements.* BSI 2004.
12. BS EN 1992-3: *Eurocode 2: Design of Concrete Structures – Part 3: Liquid Retaining and Containment Structures,* BSI 2006.
13. BS8007: *British Standards Code of Practice for the Design of Concrete Structures for Retaining Aqueous Liquids,* BSI 1987.
14. Eckfeldt, L (2009). *Answers on open questions from Innesbruck and small crack width research – what could be of importance for the NMC?* Presentation, fib TG4.1 Serviceability models, Kassel April 2009. Civil Engineering Institute of Concrete Structures.

15. Eckfeldt, L (2011). E-mail personal communication, March 2011, subject *fib Journal outline and MC 2010 information.* Academic staff, Faculty 2 Architecture, Civil Engineering and Urban Planning, BTU Cottbus, Germany, 24 March 2010.
16. Eckfeldt, L (2012). E-mail personal communication, November 2011, subject *fib Journal outline and MC 2010 information.*
17. *Eurocode 2 Commentary* (2008). European Concrete Platform ASBL 2008. Editor J-P Jacobs.
18. Faber M, Vrouwenvelder ACWM (2000). *Probabilistic model code*. *12th Draft*. Joint Committee of Structural Safety.
19. Faber M, Vrouwenvelder ACWM (2008). *Interpretation of uncertainties and probabilities in civil engineering decision analysis.* Background documents on risk assessment in engineering. Document No.2. Joint Committee of Structural Safety.
20. Gulvanessian H, Holický M (2005). *Eurocodes: using reliability analysis to combine action effects.* Proceedings of the Institute of Civil Engineers. Structures and Buildings 158. August 2005, Issue SB4, pgs 243 – 252.
21. *Handbook 1: Basis of structural design.* Guide to interpretive documents for essential requirements, to EN1990 and to application and use of Eurocodes. Leonardo da Vinci pilot project CZ/02/B/F/PP-134007.
22. Holický M (2009). *Reliability analysis for structural design.* Sun Press. Stellenbosch, South Africa. ISBN 978-1-920338-11-4.
23. Holický M, Retief JV (2005). *Reliability assessment of alternative Eurocode and South African load combination schemes for structural design.* Journal of the South African Institute of Civil Engineering. Vol 47 No. 1, 2005, pgs 15 – 20.
24. Holický M, Retief JV & Wium JA (2009). *Probabilistic design for cracking of concrete structures.* Proceedings of the 7th International Probabilistic Workshop, Delft 2009. Van Gelder, Proske & Vrijling.
25. Holický M, Retief JV & Wium JA (2010). *Partial factors for selected reinforced concrete members: Background to a revision of SANS 10100-1.* Journal of the South African Institute of Civil Engineering. Vol 52 No.1, 2010, pgs 36 – 44.
26. Hughes BP (1980), *Limit state theory for reinforced concrete design* (3rd Edition), Pitman.
27. Illston JM, Stevens RF (1972). *Long-term cracking in reinforced concrete beams*. Proceedings of the Institute of Civil Engineers, Vol 53, Dec 1972, pp 445 – 459.

28. Joint Committee on Structural Safety (2001). *Probabilistic model code.* JCSS, Zurich 2001. http://www.jcss.ethz.ch/.

29. Joint Committee on Structural Safety (2008). *Interpretation of Uncertainties and Probabilities in Civil Engineering Decision Analysis.* Background documents on risk assessment in engineering. Document No. 2. JCSS, Zurich 2008. http://www.jcss.ethz.ch/.

30. Jones A. (2008) *Eurocode 2 – Design of concrete structures: Part 3. Liquid-retaining and Containment Structures.* Dissemination of information workshop, Brussels February 2008.

31. König G, Hosser D & Wittke B (1985). *Basic note on model uncertainties.* Bulletin d' Information No. 170. Comite Euro International du Beton.

32. Lane JW (1983) *Concrete specification for local conditions.* Seminar on design of concrete water retaining structures to BS 5337, held by the Division of Hydraulic and Water Engineering of the South African Institution of Civil Engineers in association with the Concrete Society of South Africa.

33. Mans R (2012). *Will autogenous healing Seal a Concrete Crack with a Surface Width of 0.2mm?* 4th yr undergraduate dissertation in partial fulfillment for the degree of Bachelor of Science in Engineering, University of KwaZulu-Natal. Supervisor: C.H. M^{c}Leod

34. Markova J, Holický M (2001). *Credibility of design procedures (for limit states design of cracking in reinforced concrete).* Acta Polytechnica Vol 41, No. 4-5/ 2001.

35. Mosley B, Bungley J, Hulse R (2007). *Reinforced concrete design to Eurocode 2. 6th Edition.* Palgrave Macmillan.

36. Mucambe, ESD (2007). *Permeability and durability of concrete water retaining structures in South Africa.* 4th yr undergraduate dissertation in partial fulfillment for the degree of Bachelor of Science in Engineering, University of KwaZulu-Natal. Supervisor: C.H. M^{c}Leod

37. Naidu, S (2008, 2009). Personal communication. Naidu & Assoc Consulting Engineers, Durban, KwaZulu-Natal, South Africa.

38. Narayanan RS, Beeby AW (2005). *Designer's guide to EN1992-1-1 and EN1992-1-2: Design of concrete structures. General rules and rules for buildings and structural fire design.* Thomas Telford.

39. Nowak AS, Collins KR (2000). *Reliability of structures.* McGraw-Hill.

40. Owens G (Ed) (2009). *Fulton's concrete technology (9th Ed)*. Cement and Concrete Institute, SA.

41. Quan Q, Gengwei Z (2002). *Calibration of reliability index of RC beams for serviceability limit state of maximum crack width.* Journal of Reliability Engineering and System Safety. No. 75 (2002), pages 359 – 366.
42. Retief JV, Dunaiski PE (2009). *Background to SANS 10160. Basis of structural design and actions for buildings and industrial structures.* Sun Press. Stellenbosch, South Africa.
43. Reynolds CE, Steedman JC, Threlfall AJ. (2008) *Reynolds's reinforced concrete designer's handbook, 11th Edition.* Taylor and Francis
44. Ronné PD (2006). *Variation in cover to reinforcement: local and international trends.* CSSA.
45. Roux WR (2007). Thesis No. 3-36. *South African practice for the design of water retaining concrete structures.* 4th yr undergraduate dissertation in partial fulfillment for the degree of Bachelor of Science in Engineering, Stellenbosch University. Supervisor: Prof. JA Wium.
46. SANS 10100 -1: *The structural use of concrete: Part 1: Design.* South African Bureau of Standards. Pretoria 2000.
47. SANS 10100 -2: *The structural use of concrete: Part 2: Materials and execution,* South African Bureau of Standards. Pretoria 1984.
48. SANS 10100 Part 3 (Draft) – *Design of concrete water retaining structures.* WRC Project group with the Institute of Structural Engineering. 2010
49. SANS 10160 *General procedures for loading in buildings and industrial structures*. DRAFT. South African Institute for Civil Engineers 2006.
50. SANS 2394: 2004 Edition 1 (ISO 2394: 1998 Edition 2): *General principles on reliability for structures.* South African Bureau of Standards. Pretoria. 2004.
51. Scruton, S (2007, 2008, 2009, 2010). Personal communications. Umgeni Water, Durban, KwaZulu-Natal, South Africa.
52. Seong-Tae Yi, Tae-Yang Hyan & Jin-Keun Kim (2011). *The effects of hydraulic pressure and crack width on water permeability of penetration crack-induced concrete.* Journal of Construction and Building Structures 25 (2011) Pages 2576 – 2583.
53. Sundararajan C (Ed) (1995) *Probabilistic structural mechanics handbook – Theory and industrial applications.* Chapman and Hall.
54. Tammo K, Lundgren K, Thelandersson S (2009). *Non-linear analysis of crack widths in reinforced concrete.* Magazine of Concrete Research 2009, Vol 61, No. 1 Feb, pp 23 - 34.

55. Vitek, J (Task convener) (2011). MC 2010 Improved 1st draft, Ch 7.6 *Verification of serviceability (SLS) of RC and PC Structures.* Fib TG 4.1: Serviceability models. February 2011.
56. Wium, JA (2007a), project leader. *A code for the design of water retaining structures in South Africa.* Workshop held at University of Stellenbosch, October 2007.
57. Wium, JA (2007b). *Interim report December 2007: The development and calibration of South Africa's National Standards for water retaining structures.* WRC Project No. K5/1764, Water Research Commission of South Africa, Pretoria, SA.
58. Wium, JA (2008). *Report April 2008: Basis of design and materials. The development and calibration of South Africa's National Standards for water retaining structures.* WRC Project No. K5/1764, Water Research Commission of South Africa, Pretoria, SA
59. Ziari A, Reza Kianoush M (2009a). *Investigation of direct tension cracking and leakage in RC elements.* Journal of Engineering Structures 31 (2009) pp 406 – 474.
60. Ziari A, Reza Kianoush M (2009b). *Investigation of flexural cracking and leakage in RC liquid containing structures.* Journal of Engineering Structures 31 (2009) pp 1056 – 1067.

APPENDIX A

DETERMINISTIC ANALYSIS

for

FLEXURAL AND TENSION CRACKING

Appendix A.1: Ultimate Limit State loading calculations for reinforcement.

Cantilever Wall. Water pressure only considered			
Concrete, f_{ck} =	30	MPa	
Concrete f_{cu} =	37	MPa	
Reinf. f_y =	450	MPa	
mod ratio α_e =	15		
overall depth of section h =	750	mm	
effective depth d =	700	mm	
Length of section b =	1000	mm	
For Wall, H wall =	7	m	
Reinf. E_s =	200	kN/mm^2	
cover to reinf., c =	40	mm	
Dia reinf =	20	mm	
		SANS	
	Mu =	432.0	kNm
	Muc=	2828.3	kNm
	Mu/Muc =	0.153	
	z /d=	0.97	
	As, req =	1659.3	mm^2

Figure A.1.1: *Ultimate limit state reinforcement calculation to SANS10100*

Table A.1.1: *ULS reinforcement to SANS010*

						Asuls, req to SANS, fy = 450MPa			Max As = 4%
H (m)	MULS (kNm)	cover (mm)	h (mm)	φ = 16 mm	% As	φ = 20 mm	% As	φ = 25 mm	% As
5	250	40	250	4191.0	1.68	4257.1	1.70	4311.02	1.72
			300	2940.3	0.98	2973.0	0.99	3015.05	1.01
			350	2326.8	0.66	2345.8	0.67	2370.06	0.68
			400	1940.9	0.49	1969.8	0.49	1969.85	0.49
			450	1672.1	0.37	1680.4	0.37	1691.73	0.38
			500	1487.1	0.30	1493.7	0.30	1502.07	0.30
			550	1339.0	0.24	1344.4	0.24	1351.11	0.25
			600	1217.7	0.20	1222.1	0.20	1227.72	0.20
		50	250			4481.2	1.79		
			300			3150.5	1.05		
			350			2446.4	0.70		
			400			2020.3	0.51		
			450			1728.0	0.38		
6	432	40	450			3030.5	0.67		
			400			3588.4	0.90		
			350			4465.2	1.28		
			300			5885.1	1.96		
			250			req compression reinf.			
7	686	40	750			1659.3	0.22		
			700			2843.0	0.41		
			650			3111.5	0.48		
			600			3440.8	0.57		
			550			3856.8	0.70		
			500			3963.9	0.79		
			450			5172.8	1.15		
			400			6386.5	1.60		
			350			7787.7	2.23		
			300			require compression reinf.			
			250			require compression reinf.			

Red – highlighted cells: reinforcement spacing exceeds minimum feasible.

Appendix A.2 SLS cracking calculations and data for Flexural cracking to BS8007

	INPUT			
Concrete f_{cu} =	37	MPa		
Reinf. fy =	450	MPa		
mod ratio α_e =	15			
overall depth of section h =	750	mm		
effective depth d =	700	mm		
Length of section b =	1000	mm		
Area of Tension reinf for bending A_s =	2513	mm^2/m wall		
Reinf dia, Φ =	20	mm		
Reinf. Spacing s =	125.0	mm		
For Wall, H wall =	7	m		
Reinf. E_s =	200	kN/mm^2		
cover to reinf., c =	40	mm		
Bending Moment M =	571.7	kNm/m		
1. Flexural Cracking Only (SLS, Elastic Theory, Steel yields, T = C)				
(i) NA Depth x =	195.1	mm		
Lever arm z =	635.0	mm		
(ii) Stresses in Steel & Concrete				
f_s =	358.26	MPa	Max f_s =0.8*fy	360.0
f_c =	9.23	MPa	Max fc = 0.45*fcu	16.7
(iii) Strain at steel level				
ε_s =	1.79E-03		Max $\varepsilon_s = 0.8fy/E_s$	1.8E-03
(iv) Apparent strain at surface				
ε_1 =	1.97E-03			
(v)Tension stiffening effect	0.2mm	0.1mm		
ε_2 =	4.04E-04	6.07E-04	Note: a' = h	
(vi) Average strain				
$\varepsilon_m = \varepsilon_1 - \varepsilon_2$	1.56E-03	1.36E-03		
(vii) Calc a_{cr}				
a =	80.0	mm		
a_{cr} =	70.0	mm		
(viii) Width of Crack (mm)				
w =	0.297	0.258		
Stress/ Strain Theory: BS 8007/ 8110: Cracked Section: Bending: Singly Reinforced Section				

Figure A.2.1: *SLS Flexural cracking calculation to BS 8007 using EXCEL spreadsheet*

Table A.2.1: *SLS flexural cracking results to BS8007: H = 5m*

dia (mm)	20		
s	125.0	spacing of reinf	
Area reinf	2513	0.2mm	0.1mm
h (mm)	750	0.297	0.258

H = 5m	Cover = 40mm				h 250 too small!!!		H = 5m	Cover = 40mm					
Dia = 20 mm					Crack width (mm)		Dia = 20 mm					too small	too small
h	%As	As (mm2)	Asls/Auls	s (mm)	w = 0.2 mm	w = 0.1 mm	h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm
250	1.676	4189	0.984	75.0	0.383	0.374	300	1.396	4189	1.409	75.0	0.217	0.210
Auls	1.257	3142	0.738	100.0	0.405	0.396	Auls	1.047	3142	1.057	100.0	0.297	0.287
4257	1.005	2513	0.590	125.0	0.519	0.507	2973	0.838	2513	0.845	125.0	0.388	0.374
	0.838	2094	0.492	150.0	0.641	0.626		0.698	2094	0.704	150.0	0.485	0.467
	0.718	1795	0.422	175.0	0.770	0.751		0.598	1795	0.604	175.0	0.590	0.567
	0.628	1571	0.369	200.0	0.904	0.881		0.524	1571	0.528	200.0	0.700	0.672
	0.558	1395	0.328	225.0	1.044	1.017		0.465	1395	0.469	225.0	0.816	0.782
	0.503	1257	0.295	250.0	1.186	1.154		0.419	1257	0.423	250.0	0.933	0.894

H = 5m	Cover = 40mm						H = 5m	Cover = 40mm					
Dia = 20 mm						too small	Dia = 20 mm						too small
h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm	h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm
350	1.197	4189	1.786	75.0	0.166	0.159	400	1.047	4189	2.126	75.0	0.132	0.124
Auls	0.898	3142	1.340	100.0	0.231	0.220	Auls	0.786	3142	1.595	100.0	0.186	0.172
2345	0.718	2513	1.072	125.0	0.305	0.288	1970	0.628	2513	1.276	125.0	0.246	0.227
	0.598	2094	0.893	150.0	0.385	0.364		0.524	2094	1.063	150.0	0.314	0.288
	0.513	1795	0.765	175.0	0.472	0.444		0.449	1795	0.911	175.0	0.387	0.354
	0.449	1571	0.670	200.0	0.564	0.530		0.393	1571	0.797	200.0	0.465	0.425
	0.399	1395	0.595	225.0	0.662	0.621		0.349	1395	0.708	225.0	0.548	0.499
	0.359	1257	0.536	250.0	0.762	0.713		0.314	1257	0.638	250.0	0.633	0.576

H = 5m	Cover = 40mm						H = 5m	Cover = 40mm					
Dia = 20 mm							Dia = 20 mm						
h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm	h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm
450	0.931	4189	2.493	75.0	0.108	0.098	500	0.838	4189	2.804	75.0	0.089	0.078
Auls	0.698	3142	1.870	100.0	0.152	0.137	Auls	0.628	3142	2.103	100.0	0.126	0.109
1680	0.558	2513	1.496	125.0	0.203	0.181	1494.0	0.503	2513	1.682	125.0	0.168	0.144
	0.465	2094	1.246	150.0	0.259	0.230		0.419	2094	1.402	150.0	0.216	0.183
	0.399	1795	1.068	175.0	0.321	0.284		0.359	1795	1.201	175.0	0.268	0.225
	0.349	1571	0.935	200.0	0.387	0.341		0.314	1571	1.052	200.0	0.324	0.271
	0.310	1395	0.830	225.0	0.458	0.401		0.279	1395	0.934	225.0	0.344	0.288
	0.279	1257	0.748	250.0	0.531	0.464							

H = 5m	Cover = 40mm						H = 5m	Cover = 40mm					
Dia = 16(mm)					h 250 too small		Dia = 16(mm)					too small	too small
h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm	h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm
250	1.072	2681	0.640	75.0	0.427	0.417	300	0.894	2681	0.912	75.0	0.312	0.301
	0.804	2011	0.480	100.0	0.584	0.570		0.670	2011	0.684	100.0	0.435	0.418
Auls	0.643	1608	0.384	125.0	0.757	0.738	Auls	0.536	1608	0.547	125.0	0.572	0.549
4191	0.536	1340	0.320	150.0	0.944	0.918	2940	0.447	1340	0.456	150.0	0.722	0.691
	0.460	1149	0.274	175.0	1.140	1.109		0.383	1149	0.391	175.0	0.882	0.843
	0.402	1005	0.240	200.0	1.346	1.308		0.335	1005	0.342	200.0	1.051	1.004
								0.298	894	0.304	225.0	1.226	1.170
								0.268	804	0.273	250.0	1.410	1.343

H = 5m	Cover = 40mm						H = 5m	Cover = 40mm					
Dia = 16(mm)					too small	too small	Dia = 16(mm)						too small
h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm	h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm
350	0.766	2681	1.153	75.0	0.242	0.229	400	0.670	2681	1.381	75.0	0.193	0.178
	0.575	2011	0.865	100.0	0.340	0.321		0.503	2011	1.036	100.0	0.274	0.251
Auls	0.459	1608	0.691	125.0	0.452	0.425	Auls	0.402	1608	0.828	125.0	0.366	0.335
2326	0.383	1340	0.576	150.0	0.575	0.539	1941	0.335	1340	0.690	150.0	0.469	0.427
	0.328	1149	0.494	175.0	0.708	0.662		0.287	1149	0.592	175.0	0.581	0.527
	0.287	1005	0.432	200.0	0.850	0.793		0.251	1005	0.518	200.0	0.700	0.633
	0.255	894	0.384	225.0	0.998	0.929		0.224	894	0.461	225.0	0.826	0.745
	0.230	804	0.346	250.0	1.153	1.073		0.201	804	0.414	250.0	0.959	0.863

H = 5m	Cover = 40mm						H = 5m	Cover 50					
Dia = 16(mm)					too small		Dia = 20 mm						
h	%As	As (mm2)	Asls/Auls	s	w = 0.2 mm	w = 0.1 mm	h	%As	As (mm2)	Asls/Auls	s (mm)	w = 0.2 mm	w = 0.1 mm
450	0.596	2681	1.603	75.0	0.157	0.141	450	0.931	4189	2.493	75.0	0.136	0.124
	0.447	2011	1.203	100.0	0.224	0.199	Auls	0.698	3142	1.870	100.0	0.186	0.168
Auls	0.357	1608	0.962	125.0	0.301	0.265	1680	0.558	2513	1.496	125.0	0.243	0.217
1672	0.298	1340	0.801	150.0	0.387	0.339		0.465	2094	1.246	150.0	0.305	0.272
	0.255	1149	0.687	175.0	0.481	0.419		0.399	1795	1.068	175.0	0.373	0.330
	0.223	1005	0.601	200.0	0.582	0.505		0.349	1571	0.935	200.0	0.445	0.393
	0.199	894	0.535	225.0	0.689	0.596		0.310	1395	0.830	225.0	0.522	0.459
	0.179	804	0.481	250.0	0.802	0.691		0.279	1257	0.748	250.0	0.601	0.528
								0.233	1047	0.623	300.0	0.771	0.673

Table A.2.2: *SLS flexural cracking results to BS8007: H = 7m*

H= 7m	Cover = 40mm					
Dia = 20 mm						
h	%As	As (mm2)	Asls/Auls	s (mm)	w = 0.2 mm	w = 0.1 mm
450	0.8378	4189	0.810	75.0	1.177	1.129
Auls	0.6284	3142	0.607	100.0	1.304	1.251
5173	0.5026	2513	0.486	125.0	1.437	1.378
	0.4188					

H= 7m	Cover = 40mm					
Dia = 20 mm						
h	%As	As (mm2)	Asls/Auls	s (mm)	w = 0.2 mm	w = 0.1 mm
500	0.8378	4189	1.2	75.0	0.530	0.505
Auls	0.6284	3142	0.9	100.0	0.587	0.560
3441	0.5026	2513	0.7	125.0	0.647	0.618
	0.4188	2094	0.6	150.0	0.707	0.674

H= 7m	Cover = 40mm					
Dia = 20 mm						
h	%As	As (mm2)	Asls/Auls	s (mm)	w = 0.2 mm	w = 0.1 mm
750.0	0.5585333	4189.0	2.5	75.0	0.14948384	0.1326
Auls	0.4189333	3142.0	1.9	100.0	0.21681189	0.1903
1659.0	0.3350667	2513.0	1.5	125.0	0.29656388	0.2582

Appendix A.3: SLS flexural cracking to EN1992: Crack width calculation

For given As, calc wlim						
		INPUT				
Concrete, 28 days	f_{ck} =	30	MPa			
Concrete, 3 days	$f_{ck,3}$ =		MPa			
Reinf.	fyk =	500	MPa			
mod ratio	α_e =	15.0				
overall depth of section	h =	450	mm			
effective depth	d =	400	mm			
Length of section	b =	1000	mm			
Area of Tension reinf for bending	A_s =	2700	mm^2/m wall/face			
Reinf dia,	φ =	20	mm			
Reinf. Spacing	s =		mm			
H wall	=	6	m			
Reinf.	E_s =	200	GPa			
cover to reinf.,	c =	40	mm			
Service Bending Moment	M =	360.00	kNm			
Service Applied tensile force	N =	0	kN			
Mean axial tensile strength conc.						
$f_{ct,eff}$ (Table 3.1, = f_{ctm})		2.896	MPa			
	Ecm =	32.84	MPa			
Coefficients						
Bond prop's reinf. Coeff	k1 =	0.8	High bond			
Distribution strain coeff , bending	k2 =	0.5	Bending = 0.5			
Coeff (recommended)	k3 =	3.4				
Coeff (recommended)	k4 =	0.425				
Duration of load factor	kt =	0.4	Long term			
1. Imposed Load Cracking Only (EN2-1 Cl 7.3)						
(i) Check As,min			%As			
	k =	1				
	σc =	0	MPa			
	kc, k1 =	1,5				
	h* =	450				
	kc, bending =	0.4	2.5(h-d)	h/2	(h-x)/3	
	bt =	125.0	125	225		
	depth tension area =	125.0		225		
	Act =	102000.0	mm2/m	225		
	Min As,min =	236.4	mm2/m			
(ii) sr,max for reinf spacing <= 5(c+dia./2)			Check: s (reinf) =	250	mm	ok
	ρp,eff =	0.02647				
Max crack spacing	sr,max =	264.4	mm	≤ 5(c+φ/2)		
Mean crack spacing β = 1,7	srm =	155.6	mm			
(iii) Strain (εsm - εcm)						
	depth to neutral axis x =	144.00	mm			
	z=	352.00	mm			
	σs =	378.8	MPa	**2.5(h-d)**	**(h-x)/3**	**h/2**
	depth of tension area h,eff =	102.0	depth T- zone	125	102	225
	Act,eff =	102000.0	mm2			
Tension stiffening due to Concrete,	εcm =	0.000306				
	Steel strain =	0.001894				
k width	wk =	0.420	mm			
Mean crack width,	wm =	0.247	mm			

Figure A 3.1: *SLS flexural cracking to EN1992 – Calculation of crack width*

Table A.3.1: *SLS Flexural cracking to EN1992 – Data for crack width for given reinforcement*

dia (mm)	20													
%As	0.6	wk (mm)	es	ecm	Sr (mm)	Note: reinforcement chosen such that spacing in range of 75 to 300/400 mmm								
h (mm)	450	0.4200	0.00189	0.00031	264.4									
H = 5m	c = 40 mm							H = 5m	c = 40 mm					
Dia = 25 mm								Dia = 25 mm						
%As	h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)	h		Asls/Auls	wk (mm)	es	ecm	Sr (mm)
0.2	250	500	0.115982	6.2111	0.01132	0.00087	594.4	450	900	0.531915	1.3372	0.00313	0.00086	587.7
0.3		750	0.173974	3.0327	0.00767	0.00058	428.0		1350	0.797872	0.6532	0.00212	0.00058	422.6
0.4	Auls	1000	0.231965	1.8674	0.00583	0.00045	346.9	Auls	1800	1.06383	0.4014	0.00161	0.00044	342.2
0.5	4311	1250	0.289956	1.3022	0.00472	0.00037	299.3	1692	2250	1.329787	0.2789	0.00130	0.00036	295.2
0.6		1500	0.347947	0.9806	0.00397	0.00031	268.2		2700	1.595745	0.2090	0.00110	0.00031	264.4
0.8		2000	0.463929	0.6398	0.00303	0.00025	230.3		3600	2.12766	0.1347	0.00084	0.00024	227.1
1.0		2500	0.579912	0.4673	0.00245	0.00021	208.3		4500	2.659574	0.0970	0.00068	0.00021	205.4
1.2		3000	0.695894	0.3653	0.00207	0.00019	194.0		5400	3.191489	0.0745	0.00057	0.00018	191.4
1.4		3500	0.811877	0.2987	0.00179	0.00017	184.0		6300	3.723404	0.0599	0.00049	0.00016	181.6
1.6		4000	0.927859	0.2520	0.00158	0.00016	176.8		7200	4.255319	0.0495	0.00044	0.00015	174.5
1.8		4500	1.043841	0.2176	0.00142	0.00015	171.2		8100	4.787234	0.0419	0.00039	0.00014	169.1
2.0		5000	1.159824	0.1912	0.00129	0.00014	166.9		9000	5.319149	0.0360	0.00035	0.00014	164.9
2.4		6000	1.391788	0.1535	0.00109	0.00013	160.6							
2.8		7000	1.623753	0.1279	0.00094	0.00012	156.2							

H = 5m	c = 40 mm							H = 5m	c = 40 mm					
Dia = 20 (mm)								Dia = 20 (mm)						
%As	h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)	h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)
0.2	250	500	0.117454	6.2111	0.01132	0.00087	594.4	300	600	0.201816	3.9477	0.00753	0.00086	591.8
0.3		750	0.17618	3.0327	0.00767	0.00058	428.0		900	0.302725	1.9267	0.00510	0.00058	425.9
0.4	Auls	1000	0.234907	1.8674	0.00583	0.00045	346.9	Auls	1200	0.403633	1.1856	0.00388	0.00044	345.1
0.5	4257	1250	0.293634	1.3022	0.00472	0.00037	299.3	2973	1500	0.504541	0.8262	0.00314	0.00036	297.7
0.6		1500	0.352361	0.9806	0.00397	0.00031	268.2		1800	0.605449	0.6216	0.00264	0.00031	266.8
0.7		1750	0.411088	0.7776	0.00343	0.00027	246.4		2100	0.706357	0.4924	0.00228	0.00027	245.1
0.8		2000	0.469814	0.6398	0.00303	0.00025	230.3		2400	0.807265	0.4047	0.00201	0.00025	229.1
0.9		2250	0.528541	0.5411	0.00271	0.00023	218.0		2700	0.908174	0.3419	0.00180	0.00022	216.8
1.0		2500	0.587268	0.4673	0.00245	0.00021	208.3		3000	1.009082	0.2949	0.00163	0.00021	207.2
1.1		2750	0.645995	0.4104	0.00224	0.00020	200.4		3300	1.10999	0.2587	0.00149	0.00019	199.4
1.2		3000	0.704722	0.3653	0.00207	0.00019	194.0		3600	1.210898	0.2300	0.00138	0.00018	193.0
1.3		3250	0.763448	0.3288	0.00192	0.00018	188.6		3900	1.311806	0.2067	0.00128	0.00017	187.7
1.4		3500	0.822175	0.2987	0.00179	0.00017	184.0		4200	1.412714	0.1875	0.00119	0.00017	183.1
1.5		3750	0.880902	0.2734	0.00168	0.00016	180.1							
1.6		4000	0.939629	0.2520	0.00158	0.00016	176.8							
1.7		4250	0.998356	0.2336	0.00150	0.00015	173.8							
2.0		5000	1.174536	0.1912	0.00129	0.00014	166.9							
1.9		4800	1.127555	0.2010	0.00134	0.00014	168.5							

H = 5m c = 40 mm							H = 5m c = 40 mm						
Dia = 20 (mm)							Dia = 20 (mm)						
h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)	h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)
350	700	0.29838	2.6642	0.00538	0.00086	590.0	400	800	0.406091	1.8666	0.00403	0.00086	588.7
	1050	0.44757	1.3003	0.00364	0.00058	424.5		1200	0.609137	0.9113	0.00273	0.00058	423.4
Auls	1400	0.59676	0.7997	0.00277	0.00044	343.9	Auls	1600	0.812183	0.5602	0.00207	0.00044	343.0
2346	1750	0.745951	0.5568	0.00224	0.00036	296.6	1970	2000	1.015228	0.3897	0.00168	0.00036	295.8
	2100	0.895141	0.4185	0.00188	0.00031	265.8		2400	1.218274	0.2925	0.00141	0.00031	265.0
	2450	1.044331	0.3311	0.00163	0.00027	244.1		2800	1.42132	0.2311	0.00122	0.00027	243.5
	2800	1.193521	0.2718	0.00143	0.00024	228.2		3200	1.624365	0.1893	0.00107	0.00024	227.6
	3150	1.342711	0.2292	0.00128	0.00022	216.0		3600	1.827411	0.1594	0.00096	0.00022	215.4
	3500	1.491901	0.1974	0.00116	0.00021	206.4		4000	2.030457	0.1370	0.00087	0.00021	205.8
	3850	1.641091	0.1729	0.00106	0.00019	198.7		4400	2.233503				
	4200	1.790281	0.1534	0.00098	0.00018	192.3		4800	2.436548	0.1060	0.00073	0.00018	191.8
	4550	1.939471	0.1377	0.00091	0.00017	187.0		5200	2.639594				

Table A.3.2: *SLS Flexural cracking to EN1992 – Data for crack width continued*

H = 5m c = 40 mm
Dia = 20 (mm)

h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)
450	900	0.535714	1.3372	0.00313	0.00086	587.7
	1350	0.803571	0.6532	0.00212	0.00058	422.6
Auls	1800	1.071429	0.4014	0.00161	0.00044	342.2
1680	2250	1.339286	0.2789	0.00130	0.00036	295.2
	2700	1.607143	0.2090	0.00110	0.00031	264.4
	3150	1.875	0.1648	0.00095	0.00027	242.9
	3600	2.142857	0.1347	0.00084	0.00024	227.1
	4050	2.410714	0.1131	0.00075	0.00022	214.9
	4500	2.678571	0.0970	0.00068	0.00021	205.4
	4950	2.946429	0.0845	0.00062	0.00019	197.7
	5400	3.214286	0.0745	0.00057	0.00018	191.4
	5850	3.482143	0.0665	0.00053	0.00017	186.1
	6300	3.75	0.0599	0.00049	0.00016	181.6
	6750	4.017857				
	7200	4.285714	0.0495	0.00044	0.00015	174.5

H = 5m c = 40 mm
Dia = 16 (mm)

%As	h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)
0.2	250	500	0.119303	6.2111	0.01132	0.00087	594.4
0.3		750	0.178955	3.0327	0.00767	0.00058	428.0
0.4	Auls	1000	0.238607	1.8674	0.00583	0.00045	346.9
0.5	4191	1250	0.298258	1.3022	0.00472	0.00037	299.3
0.6		1500	0.35791	0.9806	0.00397	0.00031	268.2
0.7		1750	0.417561	0.7776	0.00343	0.00027	246.4
0.8		2000	0.477213	0.6398	0.00303	0.00025	230.3
0.9		2250	0.536865	0.5411	0.00271	0.00023	218.0
1.0		2500	0.596516	0.4673	0.00245	0.00021	208.3
1.1		2750	0.656168	0.4104	0.00224	0.00020	200.4
1.2		3000	0.71582	0.3653	0.00207	0.00019	194.0
				Section too small			

H = 5m c = 40 mm
Dia = 16 (mm)

h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)
450	900	0.538278	1.3372	0.00313	0.00086	587.7
	1350	0.807416	0.6532	0.00212	0.00058	422.6
Auls	1800	1.076555	0.4014	0.00161	0.00044	342.2
1672	2250	1.345694	0.2789	0.00130	0.00036	295.2
	2700	1.614833	0.2090	0.00110	0.00031	264.4
	3150	1.883971	0.1648	0.00095	0.00027	242.9
	3600	2.15311	0.1347	0.00084	0.00024	227.1

H = 5m Cover = 50
Dia = 20mm

%As	h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)
0.2	450	900	0.520833	1.4684	0.00322	0.00086	623.3
0.3		1350	0.78125	0.7328	0.00218	0.00058	457.9
0.4	Auls	1800	1.041667	0.4582	0.00165	0.00044	377.4
0.5	1728	2250	1.302083	0.3230	0.00134	0.00036	330.2
0.6		2700	1.5625	0.2450	0.00113	0.00031	299.4
0.7		3150	1.822917	0.1951	0.00097	0.00027	277.8
0.8		3600	2.083333	0.1609	0.00086	0.00024	261.9
0.9		4050	2.34375	0.1362	0.00077	0.00022	249.7
1.0		4500	2.604167	0.1175	0.00070	0.00021	240.1

H=7m Cover =40
Dia 20mm

%As	h	Asls	Asls/Auls	wk (mm)	es	ecm	Sr (mm)
0.2	450	900	0.17398	4.5471	0.00859	0.00086	587.7
0.4		1800	0.347961	1.3631	0.00442	0.00044	342.2
0.6	Auls	2700	0.521941	0.7145	0.00301	0.00031	264.4
0.8	5173	3600	0.695921	0.4655	0.00229	0.00024	227.1
1		4500	0.869901	0.3395	0.00186	0.00021	205.4
1.2		5400	1.043882	0.2651	0.00157	0.00018	191.4
1.4		6300	1.217862	0.2164	0.00136	0.00016	181.6
1.6		7200	1.391842	0.1823	0.00120	0.00015	174.5
1.8		8100	1.565823	0.1572	0.00107	0.00014	169.1
2		9000	1.739803	0.1379	0.00097	0.00014	164.9

Appendix A.4: SLS flexural cracking to EN1992: Reinforcement calculation

Bending Only

To compare to P&R analysis: calculate As for given wk

SLS Reinforcement check using prEN2-1

Input

		Units
Concrete, f_{ck} =	30	MPa
Concrete f_{cu} =	37	MPa
Reinf. fy =	450	MPa
mod ratio α_e =	15	
overall depth of section h =	750	mm
Length of section b =	1000	mm
For Wall, H wall =	7	m
Reinf. E_s =	200	kN/mm^2
cover to reinf., c =	40	mm
Dia reinf =	20	mm
k_1	0.8	
k_2	0.5	Bending
k_t	0.4	
k_3	3.4	
k_4	0.425	
Water p =	70	kN/m2
SLS M =	571.7	kNm
wk max =	0.3	mm

prEN 2-1 Equations for Cracking

$$w_{m,calc} = s_{rm}(\varepsilon_{sm} - \varepsilon_{cm})$$

$$s_{rm} = k_3 c + k_4 k_1 k_2 \frac{A_{c,eff}}{A_s}\phi$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}}(1+\alpha_e \rho_{p,eff})}{E_s} \geq 0{,}6\frac{\sigma_s}{E_s} \qquad (7.9)$$

where:

σ_s is the stress in the tension reinforcement assuming a cracked section. For pretensioned members, σ_s may be replaced by $\Delta\sigma_p$ the stress variation in prestressing tendons from the state of zero strain of the concrete at the same level.

α_e is the ratio E_s/E_{cm}

$\rho_{p,eff}$ $(A_s + \xi_1^2 A_p')/A_{c,eff}$ (7.10)

A_p' and $A_{c,eff}$ are as defined in 7.3.2 (3)

ξ_1 according to Expression (7.5)

k_t is a factor dependent on the duration of the load

Calcs

			h/2	(h-x)/3	2.5(h-d)
effective depth d =	700	mm			
x =	247.94	mm	h/2	(h-x)/3	2.5(h-d)
heff	125.0	mm	375	167.352	125
Ac,eff =	125000.00	mm2			
z =	617.35	mm			
fctm =	2.896	MPa			
ρ eff =	0.0363				

Use Excel Solver to optimise As for given wk

As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es
4533	229.75	0.001021	0.000247	0.00077	0.0008	0.17800	0.0006128

Figure A.4.1: *SLS Flexural cracking to EN1992 – Calculation of reinforcement*

Table A.4.2: *SLS Flexural cracking to EN1992 – Data - reinforcement for given crack width*

			reinforcement spacing too close, increase section size										
			reinf. spacing ok										
wk = 0.2 mm			Asls/Auls> 1: Asls dominates										
dia = 16mm			reinf area less than min. reinf = 0,13%									Sr, max = 1.7Sr, ave	
H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	250	4191.0	1.101	1.845	4614	163.17	0.00137	0.00014	0.00123	0.00123	0.200	0.00082	95.98
	300	2940.3	1.291	1.266	3797	178.55	0.0013	0.00018	0.00112	0.00112	0.200	0.00078	105.03
	350	2326.8	1.405	0.934	3269	197.05	0.00123	0.00022	0.00101	0.00101	0.200	0.00074	115.91
	400	1940.9	1.487	0.722	2887	218.74	0.00118	0.00026	0.00091	0.00091	0.200	0.00071	128.67
	450	1672.1	1.546	0.574	2584	244.07	0.00114	0.00032	0.00082	0.00082	0.200	0.00068	143.57
	500	1487.1	1.564	0.465	2325	273.99	0.00111	0.00038	0.00073	0.00073	0.200	0.00067	161.17
	550	1339.0	1.558	0.379	2086	292.51	0.0011	0.00042	0.00068	0.00068	0.200	0.00066	172.06
	600	1217.7	1.570	0.319	1912	306.75	0.00109	0.00045	0.00064	0.00065	0.200	0.00065	180.44

dia = 20mm													
H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	250	4257.1	1.132	1.928	4819	168.34	0.00133	0.00014	0.00119	0.00119	0.200	0.0008	99.02
	300	2973.0	1.345	1.333	3999	186.08	0.00125	0.00017	0.00107	0.00107	0.200	0.00075	109.46
	350	2345.8	1.478	0.991	3468	207.19	0.00117	0.00021	0.00097	0.00097	0.200	0.0007	121.88
	400	1969.8	1.565	0.771	3084	231.73	0.00111	0.00025	0.00086	0.00086	0.200	0.00067	136.31
	450	1680.4	1.653	0.617	2777	260.22	0.00107	0.0003	0.00077	0.00077	0.200	0.00064	153.07
	500	1493.7	1.682	0.503	2513	293.71	0.00104	0.00036	0.00068	0.00068	0.200	0.00062	172.77
	550	1344.4	1.685	0.412	2265	323.63	0.00102	0.00041	0.00062	0.00062	0.200	0.00061	190.37
	600	1222.1	1.732	0.353	2116	336.83	0.00099	0.00043	0.00056	0.00059	0.200	0.00059	198.14

dia = 25mm													
H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	250	4311.02	1.175	2.027	5067	174.25	0.00129	0.00014	0.00115	0.00115	0.200	0.00077	102.50
	300	3015.05	1.405	1.412	4235	194.59	0.00119	0.00017	0.00103	0.00103	0.200	0.00072	114.46
	350	2370.06	1.561	1.057	3699	218.54	0.00111	0.0002	0.00092	0.00092	0.200	0.00067	128.55
	400	1969.85	1.680	0.827	3309	246.19	0.00105	0.00024	0.00081	0.00081	0.200	0.00063	144.82
	450	1691.73	1.771	0.666	2997	278.13	0.001	0.00028	0.00072	0.00072	0.200	0.0006	163.61
	500	1502.07	1.814	0.545	2725	315.51	0.00097	0.00033	0.00063	0.00063	0.200	0.00058	185.59
	550	1351.11	1.856	0.456	2508	356.60	0.00093	0.00039	0.00055	0.00056	0.200	0.00056	209.76
	600	1227.72	1.923	0.394	2361	372.23	0.0009	0.00041	0.00049	0.00054	0.200	0.00054	218.96

wk=0,2mm	cover = 40, dia = 20mm												
H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
6m	450	3030.5	1.475	0.99	4471	205.94	0.00118	0.00021	0.00097	0.00097	0.200	0.00071	121.14
	250	-	-	3.23	8069	152.84	0.00142	0.00012	0.00131	0.00131	0.200	0.00085	89.91
	500	2635.0	1.553	0.82	4093	224.19	0.00113	0.00024	0.00089	0.00089	0.200	0.00068	131.88
	550	2336.0	1.617	0.69	3778	244.60	0.00109	0.00027	0.00082	0.00082	0.200	0.00065	143.88
	600	2112.0	1.641	0.58	3466	258.62	0.00107	0.0003	0.00077	0.00077	0.200	0.00064	152.13
7m	450	5172.8	1.295	1.49	6697	178.26	0.00128	0.00016	0.00112	0.00112	0.200	0.00077	104.86
	250	-	-	5.10	12743	145.34	0.00148	0.0001	0.00138	0.00138	0.200	0.00089	85.49
	500	3964.0	1.54	1.22	6104	189.89	0.00123	0.00018	0.00105	0.00105	0.200	0.00074	111.70
	550	3857.0	1.46	1.02	5635	202.72	0.00119	0.0002	0.00099	0.00099	0.200	0.00071	119.25
	600	3441.0	1.53	0.87	5249	216.77	0.00115	0.00022	0.00092	0.00092	0.200	0.00069	127.51

wk = 0.2 mm													
cover = 50, dia = 20mm													
H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	250	4481	1.330578	2.38493	5962	195.57	0.00115	0.00013	0.00102	0.00102	0.200	0.00069	115.04
	450	1728	1.792482	0.68831	3097	279.95	0.00099	0.00027	0.00071	0.00071	0.200	0.00059	164.68

w=0.3 mm													
cover = 40, dia = 20mm													
H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	250	4257.1	0.819	1.395	3488	184.25	0.0018	0.00017	0.00163	0.00163	0.30000	0.00108	108.38
	300	2973.0	0.997	0.988	2963	208.24	0.00165	0.00021	0.00144	0.00144	0.30000	0.00099	122.49
	350	2345.8	1.116	0.748	2618	235.93	0.00153	0.00026	0.00127	0.00127	0.30000	0.00092	138.78
	400	1969.8	1.199	0.590	2361	267.53	0.00143	0.00031	0.00112	0.00112	0.30000	0.00086	157.37
	450	1680.4	1.280	0.478	2150	303.83	0.00136	0.00037	0.00099	0.00099	0.30000	0.00082	178.72
	500	1493.7	1.314	0.393	1963	346.21	0.00131	0.00045	0.00087	0.00087	0.30000	0.00079	203.66

wk = 0.1 mm

dia = 16mm

Sr, max = 1.7Sr, ave

H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	250	4191.0	2.002	3.356	8390	148.71	0.00079	0.00011	0.00067	0.00067	0.100	0.00047	87.48
	300	2940.3	2.266	2.221	6663	156.99	0.00077	0.00013	0.00064	0.00064	0.100	0.00046	92.35
	350	2326.8	2.385	1.586	5550	167.70	0.00075	0.00015	0.0006	0.0006	0.100	0.00045	98.65
	400	1940.9	2.455	1.191	4764	180.92	0.00074	0.00018	0.00055	0.00055	0.100	0.00044	106.42
	450	1672.1	2.491	0.925	4164	196.92	0.00072	0.00022	0.00051	0.00051	0.100	0.00043	115.84
	500	1487.1	2.470	0.735	3673	216.27	0.00072	0.00026	0.00046	0.00046	0.100	0.00043	127.22
	550	1339.0	2.475	0.602	3314	234.50	0.00071	0.0003	0.00041	0.00043	0.100	0.00043	137.94
	600	1217.7	2.531	0.514	3082	241.90	0.00069	0.00031	0.00038	0.00041	0.100	0.00041	142.30

dia = 20mm

H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	250	4257.1	2.029	3.456	8639	151.43	0.00077	0.00011	0.00066	0.00066	0.100	0.00046	89.08
	300	2973.0	2.322	2.301	6902	161.22	0.00075	0.00013	0.00062	0.00062	0.100	0.00045	94.83
	350	2345.8	2.468	1.654	5790	173.70	0.00073	0.00015	0.00058	0.00058	0.100	0.00044	102.18
	400	1969.8	2.541	1.251	5006	188.95	0.00071	0.00018	0.00053	0.00053	0.100	0.00042	111.14
	450	1680.4	2.622	0.979	4405	207.22	0.00069	0.00021	0.00048	0.00048	0.100	0.00041	121.90
	500	1493.7	2.618	0.782	3911	229.16	0.00068	0.00025	0.00044	0.00044	0.100	0.00041	134.80
	550	1344.4	2.667	0.652	3586	251.60	0.00066	0.00028	0.00038	0.0004	0.100	0.0004	148.00
	600	1222.1	2.756	0.561	3368	262.18	0.00064	0.0003	0.00033	0.00038	0.100	0.00038	154.22

dia = 25mm

H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	250	4311.02	2.075	3.579	8947	154.64	0.00076	0.00011	0.00065	0.00065	0.100	0.00046	90.96
	300	3015.05	2.385	2.397	7191	166.11	0.00073	0.00013	0.0006	0.0006	0.100	0.00044	94.83
	350	2370.06	2.562	1.735	6073	180.58	0.0007	0.00015	0.00055	0.00055	0.100	0.00042	102.18
	400	1969.85	2.683	1.322	5286	198.05	0.00068	0.00017	0.0005	0.0005	0.100	0.00041	111.14
	450	1691.73	2.768	1.040	4682	218.82	0.00066	0.0002	0.00046	0.00046	0.100	0.00039	121.90
	500	1502.07	2.784	0.836	4182	243.59	0.00064	0.00023	0.00041	0.00041	0.100	0.00039	134.80
	550	1351.11	2.858	0.702	3861	268.43	0.00062	0.00027	0.00035	0.00037	0.100	0.00037	148.00
	600	1227.72	3.025	0.619	3714	286.21	0.00058	0.00029	0.00029	0.00035	0.100	0.00035	154.22

wk=0,1mm cover = 40, dia = 20mm Vary H

H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
6m	450	3030.5	2.465	1.66	7471	172.77	0.00073	0.00015	0.00058	0.00058	0.100	0.00044	101.63
	250	Mu>Muc		6.03	15071	143.51	0.0008	1E-04	0.0007	0.0007	0.100	0.00048	84.42
7m	450	5172.8	2.266	2.60	11720	156.46	0.00076	0.00012	0.00064	0.00064	0.100	0.00046	92.04
	250	Mu>Muc		9.72	24311	140.04	0.00081	9.4E-05	0.00071	0.00071	0.100	0.00048	82.37

wk = 0.05mm cover = 40, dia = 20mm

H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	250	4257.1	3.65	6.210	15526	143.22	0.00045	9.9E-05	0.00035	0.00035	0.050	0.00027	84.25
	450	1680.4	4.26	1.589	7151	174.88	0.00044	0.00015	0.00029	0.00029	0.050	0.00026	102.87
	500	1493.7	4.16	1.243	6213	188.71	0.00044	0.00018	0.00026	0.00026	0.050	0.00026	111.00
	550	1344.4	4.35	1.064	5855	199.64	0.00042	0.00020	0.00022	0.00025	0.050	0.00025	117.43
	600	1222.1	4.56	0.929	5576	211.00	0.00039	0.00021	0.00018	0.00024	0.050	0.00024	124.12
	650	1120.3	4.66	0.803	5220	217.42	0.00038	0.00023	0.00016	0.00023	0.050	0.00023	127.89
	700	1034.1	4.74	0.700	4897	222.78	0.00037	0.00023	0.00014	0.00022	0.050	0.00022	131.05
	750	960.3	4.81	0.616	4621	227.98	0.00037	0.00024	0.00012	0.00022	0.050	0.00022	134.10

wk = 0.05mm cover = 40, dia = 25mm

H (m)	h (mm)	ULS As (SABS)	Calculated Asls/Auls	SLS %As	SLS As (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es	Sr, ave (mm)
5m	650	1012.5	5.61	0.873	5677	234.26	0.00036	0.00022	0.00013	0.00021	0.050	0.00021	137.80
	700	934.3	5.71	0.762	5335	240.55	0.00035	0.00023	0.00012	0.00021	0.050	0.00021	141.50
	750	867.3	5.81	0.672	5041	246.65	0.00034	0.00024	0.0001	0.0002	0.050	0.0002	145.09

Appendix A.5: SLS Tension cracking calculations and data to BS8007

	INPUT			
Concrete f_{cu} =	35	MPa		
Reinf. fy =	450	MPa		
mod ratio α_e =	15			
overall depth of section h =	350.0	mm		
effective depth d =	300	mm		
Length of section b =	1000	mm		
Area of Tension reinf for bending , one face A_s =	1047	mm^2/m wall	Each face	
Reinf dia, φ =	20	mm		
Reinf. Spacing s =	300.0	mm		
For Wall, H wall =	5	m		
Reinf. E_s =	200	kN/mm^2		
cover to reinf., c =	40	mm		
Reservoir diameter	28	m		
L =	50	kN/m2		
Applied tensile force N =	700	kN/m of wall		
As in equations is total As				
2. Direct Tension Cracking Only	(BS 8007)			
NB Stress in steel limited to $0.8f_y$				
(i) Stress and strain in steel				
f_s =	334.3	Max f_s = 0.87.fy	360	MPa
$\varepsilon_s = \varepsilon_1$ =	1.67E-03	1.67E-03		
(ii) Tension stiffening	0.2mm	0.1mm		
ε_2 =	5.6E-04	8.4E-04		
(iii) Average Strain				
ε_m =	1.11E-03	8.36E-04		
(iv) Calc a_{cr}				
a =	158.1			
a_{cr} =	148.1	148.1		
(viii) Width of Crack (mm)				
w =	4.95E-01	3.71E-01	mm	

Figure A.5.1: *SLS Tension cracking to BS8007 - calculations*

Table A.5.1: *SLS Tension cracking to BS8007 – data*

H = 5m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s (mm)	As (mm2)	As Both	Asls/Auls	%As Both	w = 0.2 mm	w = 0.1 mm
250	75.0	4189	8378	3.904	3.351	0.0501	0.0423
Auls	100.0	3142	6284	2.928	2.514	0.0773	0.0652
2146	125.0	2513	5026	2.342	2.010	0.1115	0.0941
	150.0	2094	4188	1.952	1.675	0.1531	0.1292
	175.0	1795	3590	1.673	1.436	0.2023	0.1707
	200.0	1571	3142	1.464	1.257	0.2592	0.2187
	225.0	1395	2790	1.300	1.116	0.3243	0.2737
	250.0	1257	2514	1.171	1.006	0.3966	0.3346
	300.0	1047	2094	0.976	0.838	0.5659	0.4774

H = 5m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	Asls/Auls	%As	w = 0.2 mm	w = 0.1 mm
300	75.0	4189	8378	3.904	2.793	0.0470	0.0376
Auls	100.0	3142	6284	2.928	2.095	0.0725	0.0580
2146	125.0	2513	5026	2.342	1.675	0.1045	0.0836
	150.0	2094	4188	1.952	1.396	0.1435	0.1148
	175.0	1795	3590	1.673	1.197	0.1896	0.1517
	200.0	1571	3142	1.464	1.047	0.2430	0.1944
	225.0	1395	2790	1.300	0.930	0.3041	0.2432
	250.0	1257	2514	1.171	0.838	0.3718	0.2974
	300.0	1047	2094	0.976	0.698	0.5305	0.4244

H = 5m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	Asls/Auls	%As both	w = 0.2 mm	w = 0.1 mm
350	75.0	4189	8378	3.904	2.394	0.0439	0.0329
Auls	100.0	3142	6284	2.928	1.795	0.0676	0.0507
2146	125.0	2513	5026	2.342	1.436	0.0975	0.0732
	150.0	2094	4188	1.952	1.197	0.1339	0.1005
	175.0	1795	3590	1.673	1.026	0.1770	0.1328
	200.0	1571	3142	1.464	0.898	0.2268	0.1701
	225.0	1395	2790	1.300	0.797	0.2838	0.2128
	250.0	1257	2514	1.171	0.718	0.3470	0.2603
	300.0	1047	2094	0.976	0.598	0.4951	0.3713

H = 5m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	Asls/Auls	%As	w = 0.2 mm	w = 0.1 mm
400	75.0	4189	8378	3.904	2.095	0.0407	0.0282
Auls	100.0	3142	6284	2.928	1.571	0.0628	0.0435
2146	125.0	2513	5026	2.342	1.257	0.0906	0.0627
	150.0	2094	4188	1.952	1.047	0.1244	0.0861
	175.0	1795	3590	1.673	0.898	0.1644	0.1138
	200.0	1571	3142	1.464	0.786	0.2106	0.1458
	225.0	1395	2790	1.300	0.698	0.2635	0.1824
	250.0	1257	2514	1.171	0.629	0.3222	0.2231
	300.0	1047	2094	0.976	0.524	0.4598	0.3183

H = 5m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	Asls/Auls	%As	w = 0.2 mm	w = 0.1 mm
450	75.0	4189	8378	3.904	1.862	0.0376	0.0235
Auls	100.0	3142	6284	2.928	1.396	0.0580	0.0362
2146	125.0	2513	5026	2.342	1.117	0.0836	0.0523
	150.0	2094	4188	1.952	0.931	0.1148	0.0718
	175.0	1795	3590	1.673	0.798	0.1517	0.0948
	200.0	1571	3142	1.464	0.698	0.1944	0.1215
	225.0	1395	2790	1.300	0.620	0.2432	0.1520
	250.0	1257	2514	1.171	0.559	0.2974	0.1859
	300.0	1047	2094	0.976	0.465	0.4244	0.2652

H = 5m	c = 40 mm						
Dia = 16(mm)	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	Asls/Auls	%As	w = 0.2 mm	w = 0.1 mm
250	75.0	2681	5362	2.499	2.145	0.0789	0.0666
	100.0	2011	4022	1.874	1.609	0.1220	0.1029
Auls	125.0	1608	3216	1.499	1.286	0.1761	0.1486
2146	150.0	1340	2680	1.249	1.072	0.2419	0.2041
	175.0	1149	2298	1.071	0.919	0.3196	0.2697
	200.0	1005	2010	0.937	0.804	0.4096	0.3456
	225.0	894	1788	0.833	0.715	0.5115	0.4315
	250.0	804	1608	0.749	0.643	0.6264	0.5285
	300.0	670	1340	0.624	0.536	0.7437	0.6275

H = 5m	c = 40 mm						
Dia = 16(mm)	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	Asls/Auls	%As	w = 0.2 mm	w = 0.1 mm
300	75.0	2681	5362	2.499	1.787	0.0740	0.0592
	100.0	2011	4022	1.874	1.341	0.1143	0.0915
Auls	125.0	1608	3216	1.499	1.072	0.1651	0.1321
2146	150.0	1340	2680	1.249	0.893	0.2268	0.1814
	175.0	1149	2298	1.071	0.766	0.2996	0.2397
	200.0	1005	2010	0.937	0.670	0.3840	0.3072
	225.0	894	1788	0.833	0.596	0.2397	0.1918
	250.0	804	1608	0.749	0.536	0.2936	0.2349
	300.0	670	1340	0.624	0.447	0.4184	0.3347

e	Total reinf (both Faces)			Crack width (mm)	
As (mm2)	As both	Asls/Auls	%As (Total)	w = 0.2 mm	w = 0.1 mm
2681	5362	2.499	1.532	0.0691	0.0518
2011	4022	1.874	1.149	0.1067	0.0800
1608	3216	1.499	0.919	0.1541	0.1156
1340	2680	1.249	0.766	0.2117	0.1588
1149	2298	1.071	0.657	0.2796	0.2097
1005	2010	0.937	0.574	0.3584	0.2688
894	1788	0.833	0.511	0.4475	0.3356
804	1608	0.749	0.459	0.5481	0.4111
670	1340	0.624	0.383	0.7809	0.5857

H = 5m	c = 40 mm						
Dia = 16(mm)	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As total	Asls/Auls	%As	w = 0.2 mm	w = 0.1 mm
400	75.0	2681	5362	2.499	1.341	0.0641	0.0444
	100.0	2011	4022	1.874	1.006	0.0991	0.0686
Auls	125.0	1608	3216	1.499	0.804	0.1431	0.0991
2146	150.0	1340	2680	1.249	0.670	0.1966	0.1361
	175.0	1149	2298	1.071	0.575	0.2597	0.1798
	200.0	1005	2010	0.937	0.503	0.3328	0.2304
	225.0	894	1788	0.833	0.447	0.4156	0.2877
	250.0	804	1608	0.749	0.402	0.5089	0.3523
	300.0	670	1340	0.624	0.335	0.7252	0.5020

H = 5m	c = 40 mm						
Dia = 16(mm	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As total	Asls/Auls	%As	w = 0.2 mm	w = 0.1 mm
450	75.0	2681	5362	2.499	1.192	0.0592	0.0370
	100.0	2011	4022	1.874	0.894	0.0915	0.0572
Auls	125.0	1608	3216	1.499	0.715	0.1321	0.0826
2146	150.0	1340	2680	1.249	0.596	0.1814	0.1134
	175.0	1149	2298	1.071	0.511	0.2397	0.1498
	200.0	1005	2010	0.937	0.447	0.3072	0.1920
	225.0	894	1788	0.833	0.397	0.3836	0.2397
	250.0	804	1608	0.749	0.357	0.4698	0.2936
	300.0	670	1340	0.624	0.298	0.6694	0.4184

H = 6m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
250	75.0	4189	8378	3.254	3.351	0.0633	0.0555
	100.0	3142	6284	2.440	2.514	0.0976	0.0855
Auls	125.0	2513	5026	1.952	2.010	0.1407	0.1233
2575.000	150.0	2094	4188	1.626	1.675	0.1933	0.1693
	175.0	1795	3590	1.394	1.436	0.2554	0.2238
	200.0	1571	3142	1.220	1.257	0.3272	0.2867
	225.0	1395	2790	1.083	1.116	0.4095	0.3588
	250.0	1257	2514	0.976	1.006	0.5007	0.4387
	300.0	1047	2094	0.813	0.838	0.7144	0.6260

H = 6m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
300	75.0	4189	8378	3.254	2.793	0.0602	0.0508
Auls	100.0	3142	6284	2.440	2.095	0.0927	0.0783
2575.000	125.0	2513	5026	1.952	1.675	0.1338	0.1129
	150.0	2094	4188	1.626	1.396	0.1837	0.1550
	175.0	1795	3590	1.394	1.197	0.2427	0.2048
	200.0	1571	3142	1.220	1.047	0.3110	0.2624
	225.0	1395	2790	1.083	0.930	0.3892	0.3284
	250.0	1257	2514	0.976	0.838	0.4759	0.4015
	300.0	1047	2094	0.813	0.698	0.6790	0.5729

H = 6m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	AsIs/Auls	%As (Total)	w = 0.2 mm	w = 0.1 mm
350	75.0	4189	8378	3.254	2.394	0.0570	0.0461
Auls	100.0	3142	6284	2.440	1.795	0.0879	0.0710
2575.000	125.0	2513	5026	1.952	1.436	0.1268	0.1024
	150.0	2094	4188	1.626	1.197	0.1741	0.1406
	175.0	1795	3590	1.394	1.026	0.1972	0.1593
	200.0	1571	3142	1.220	0.898	0.2948	0.2381
	225.0	1395	2790	1.083	0.797	0.3689	0.2980
	250.0	1257	2514	0.976	0.718	0.4511	0.3644
	300.0	1047	2094	0.813	0.598	0.6437	0.5199

H = 6m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As total	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
400	75.0	4189	8378	3.254	2.095	0.0539	0.0414
Auls	100.0	3142	6284	2.440	1.571	0.0831	0.0638
2575	125.0	2513	5026	1.952	1.257	0.1198	0.0920
	150.0	2094	4188	1.626	1.047	0.1646	0.1263
	175.0	1795	3590	1.394	0.898	0.2175	0.1669
	200.0	1571	3142	1.220	0.786	0.2786	0.2138
	225.0	1395	2790	1.083	0.698	0.3487	0.2676
	250.0	1257	2514	0.976	0.629	0.4263	0.3272
	300.0	1047	2094	0.813	0.524	0.6083	0.4668

H = 6m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As total	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
450	75.0	4189	8378	3.254	1.862	0.0508	0.0367
Auls	100.0	3142	6284	2.440	1.396	0.0783	0.0565
2575	125.0	2513	5026	1.952	1.117	0.1129	0.0815
	150.0	2094	4188	1.626	0.931	0.1550	0.1119
	175.0	1795	3590	1.394	0.798	0.2048	0.1479
	200.0	1571	3142	1.220	0.698	0.2624	0.1895
	225.0	1395	2790	1.083	0.620	0.3284	0.2372
	250.0	1257	2514	0.976	0.559	0.4015	0.2900
	300.0	1047	2094	0.813	0.465	0.5729	0.4138

H = 7m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
250	75.0	4189	8378	2.789	3.351	0.0765	0.0686
	100.0	3142	6284	2.092	2.514	0.1179	0.1058
Auls	125.0	2513	5026	1.673	2.010	0.1700	0.1526
3004	150.0	2094	4188	1.394	1.675	0.2335	0.2095
	175.0	1795	3590	1.195	1.436	0.3085	0.2769
	200.0	1571	3142	1.046	1.257	0.3953	0.3548
	225.0	1395	2790	0.929	1.116		
	250.0	1257	2514	0.837	1.006		
	300.0	1047	2094	0.697	0.838		

H = 7m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
300	75.0	4189	8378	2.789	2.793	0.0733	0.0639
Auls	100.0	3142	6284	2.092	2.095	0.1130	0.0985
3004	125.0	2513	5026	1.673	1.675	0.1630	0.1421
	150.0	2094	4188	1.394	1.396	0.2239	0.1952
	175.0	1795	3590	1.195	1.197	0.2959	0.2579
	200.0	1571	3142	1.046	1.047	0.3791	0.3305
	225.0	1395	2790	0.929	0.930	0.4743	0.4135
	250.0	1257	2514	0.837	0.838	0.5800	0.5057
	300.0	1047	2094	0.697	0.698	0.8276	0.7215

H = 7m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	AsIs/Auls	%As (Total)	w = 0.2 mm	w = 0.1 mm
350	75.0	4189	8378	2.789	2.394	0.0702	0.0592
Auls	100.0	3142	6284	2.092	1.795	0.1082	0.0913
3004	125.0	2513	5026	1.673	1.436	0.1561	0.1317
	150.0	2094	4188	1.394	1.197	0.2143	0.1808
	175.0	1795	3590	1.195	1.026	0.2832	0.2390
	200.0	1571	3142	1.046	0.898	0.3629	0.3062
	225.0	1395	2790	0.929	0.797		

H = 7m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As total	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
400	75.0	4189	8378	2.789	2.095	0.0671	0.0545
Auls	100.0	3142	6284	2.092	1.571	0.1034	0.0841
3004	125.0	2513	5026	1.673	1.257	0.1491	0.1212
	150.0	2094	4188	1.394	1.047	0.2047	0.1665
	175.0	1795	3590	1.195	0.898	0.2706	0.2200
	200.0	1571	3142	1.046	0.786	0.3467	0.2819
	225.0	1395	2790	0.929	0.698	0.4338	0.3527

H = 5m	Cover = 50mm						
Dia 20mm	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As both	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
250	75.0	4189	8378	3.904	3.351	0.0580	0.0489
Auls	100.0	3142	6284	2.928	2.514	0.0867	0.0732
2146	125.0	2513	5026	2.342	2.010	0.1220	0.1029
	150.0	2094	4188	1.952	1.675	0.1644	0.1387
	175.0	1795	3590	1.673	1.436	0.2141	0.1807
	200.0	1571	3142	1.464	1.257	0.2715	0.2291
	225.0	1395	2790	1.300	1.116	0.3369	0.2843
	250.0	1257	2514	1.171	1.006	0.4094	0.3454
	300.0	1047	2094	0.976	0.838	0.5790	0.4885

H = 5m	Cover = 50mm						
Dia 20mm	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As total	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
450	75.0	4189	8378	3.904	1.862	0.0435	0.0272
Auls	100.0	3142	6284	2.928	1.396	0.0650	0.0406
2146	125.0	2513	5026	2.342	1.117	0.0915	0.0572
	150.0	2094	4188	1.952	0.931	0.1233	0.0770
	175.0	1795	3590	1.673	0.798	0.1606	0.1004
	200.0	1571	3142	1.464	0.698	0.2036	0.1273
	225.0	1395	2790	1.300	0.620	0.2527	0.1579
	250.0	1257	2514	1.171	0.559	0.3071	0.1919
	300.0	1047	2094	0.976	0.465	0.4343	0.2714

H = 7m	c = 40 mm						
Dia 20	Reinf one face		Total reinf (both Faces)			Crack width (mm)	
h	s	As (mm2)	As total	AsIs/Auls	%As	w = 0.2 mm	w = 0.1 mm
450	75.0	4189	8378	2.789	1.862	0.0639	0.0498
Auls	100.0	3142	6284	2.092	1.396	0.0985	0.0768
3004	125.0	2513	5026	1.673	1.117	0.1421	0.1108
	150.0	2094	4188	1.394	0.931	0.1952	0.1521
	175.0	1795	3590	1.195	0.798	0.2579	0.2010
	200.0	1571	3142	1.046	0.698	0.3305	0.2576
	225.0	1395	2790	0.929	0.620	0.4135	0.3223
	250.0	1257	2514	0.837	0.559	0.5057	0.3941

Appendix A.6: SLS tension cracking to EN1992: Crack width calculation

Tension Only

SLS Reinforcement check using prEN2-1

Input			
		Units	
fck	30	MPa	
Concrete f_{cu} =	37	MPa	
Reinf. fy =	450	MPa	
mod ratio α_e =	15		
overall depth of section h =	450	mm	
Length of section b =	1000	mm	
For Wall, H wall =	5	m	
Reinf. E_s =	200	kN/mm^2	
cover to reinf., c =	40	mm	
Dia reinf =	20	mm	
k_1	0.8		
k_2	1	Tension	
k_t	0.4		
k_3	3.4		
k_4	0.425		
Water p =	50	kN/m2	
Tension axial force	700.0	kN	total
wk max =	0.2	mm	

prEN 2-1 Equations for Cracking

$$w_{m,calc} = s_{rm}\left(\varepsilon_{sm} - \varepsilon_{cm}\right)$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}\left(1 + \alpha_e \rho_{p,eff}\right)}{E_s} \geq 0{,}6\frac{\sigma_s}{E_s} \qquad (7.9)$$

where:

σ_s is the stress in the tension reinforcement assuming a cracked section. For pretensioned members, σ_s may be replaced by $\Delta\sigma_p$ the stress variation in prestressing tendons from the state of zero strain of the concrete at the same level.

α_e is the ratio E_s/E_{cm}

$\rho_{p,eff}$ $(A_s + \xi_1^2 A_p')/A_{c,eff}$ (7.10)

A_p' and $A_{c,eff}$ are as defined in 7.3.2 (3)

ξ_1 according to Expression (7.5)

k_t is a factor dependent on the duration of the load

$$s_{rm} = k_3 c + k_4 k_1 k_2 \frac{A_{c,eff}}{A_s}\phi$$

Calcs							
effective depth d =	400	mm					
			h/2	2.5(h-d)			
heff	125.0	mm	225	125	**per face**	125	
Ac,eff =	250000.00	mm2	**total**				
As =	5013.00		**total**				
fctm =	2.896	MPa					
ρ eff =	0.0201		**total reinf**				

Use Excel Solver to calculate As for given wk

As (Total) (mm2)	Sr, max (mm)	εs	εc	ε	ε final	wk = Sr.ε (mm)	Min strain 0.6*σs/Es
5013	475.12	0.00070	0.000376	0.00032	0.00042	0.19903	0.0004189

Figure A.6.1: *SLS Tension cracking to EN1992 – Calculation of crack width*

Table A.6.1: *SLS Tension cracking to EN1992 – Data - crack width for given reinforcement*

H 5m
cover 40
Dia = 16 (mm) NOTE for cover = 40mm, hc,eff is independent of h for dia =16 mm. %As varies with h, but As (mm2) for given w the same.

%As per face	h	Asls	Asls total	% As total	Asls/Auls	wk (mm)	es	ecm	Sr (mm)	h	Asls	%As	Asls/Auls	wk (mm)
0.2	450	900	1800	0.4	0.8387698	1.0049	0.00194	0.00086	861.3	250	1800	0.72	0.83877	1.0049
0.3		1350	2700	0.6	1.25815471	0.4819	0.00130	0.00060	619.6		2700	1.08	1.258155	0.4819
0.4	Auls	1800	3600	0.8	1.67753961	0.2909	0.00097	0.00047	498.7		3600	1.44	1.67754	0.2909
0.5	2146	2250	4500	1	2.09692451	0.1989	0.00078	0.00040	426.1		4500	1.8	2.096925	0.1989
0.6		2700	5400	1.2	2.51630941	0.1469	0.00065	0.00034	377.8		5400	2.16	2.516309	0.1469
0.7		3150	6300	1.4	2.93569432	0.1144	0.00056	0.00031	343.2		6300	2.52	2.935694	0.1144
0.8		3600	7200	1.6	3.35507922	0.0926	0.00049	0.00028	317.3		7200	2.88	3.355079	0.0926
0.9		4050	8100	1.8	3.77446412	0.0770	0.00043	0.00026	297.2		8100	3.24	3.774464	0.0770
1.0		4500	9000	2	4.19384902	0.0656	0.00039	0.00024	281.1		9000	3.6	4.193849	0.0656
1.1		4950	9900	2.2	4.61323392	0.0568	0.00035	0.00023	267.9		9900	3.96	4.613234	0.0568

H 5m
cover 40
Dia = 20 (mm) NOTE for cover = 40mm, hc,eff is independent of h for dia = 20mm. %As varies with h, but As (mm2) for given w the same. Total reinf

%As per face	h	Asls	Asls total	% As total	Asls/Auls	wk (mm)	es	ecm	Sr (mm)	h	Asls	%As	Asls/Auls	wk (mm)
0.2	450	900	1800	0.40	0.839	1.2605	0.00194	0.00089	1080.4	250	1800	0.72	0.839	1.2605
0.3		1350	2700	0.60	1.258	0.5955	0.00130	0.00062	765.6		2700	1.08	1.258	0.5955
0.4	Auls	1800	3600	0.80	1.678	0.3548	0.00097	0.00049	608.2		3600	1.44	1.678	0.3548
0.5	2146	2250	4500	1.00	2.097	0.2398	0.00078	0.00041	513.8		4500	1.8	2.097	0.2398
0.6		2700	5400	1.20	2.516	0.1753	0.00065	0.00036	450.8		5400	2.16	2.516	0.1753
0.7		3150	6300	1.40	2.936	0.1353	0.00056	0.00032	405.8		6300	2.52	2.936	0.1353
0.8		3600	7200	1.60	3.355	0.1085	0.00049	0.00029	372.1		7200	2.88	3.355	0.1085
0.9		4050	8100	1.80	3.774	0.0897	0.00043	0.00027	345.9		8100	3.24	3.774	0.0897
1.0		4500	9000	2.00	4.194	0.0758	0.00039	0.00025	324.9		9000	3.6	4.194	0.0758
1.1		4950	9900	2.20	4.613	0.0653	0.00035	0.00023	307.7		9900	3.96	4.613	0.0653
1.5		6750	13500	3.00	6.291	0.0407	0.00026	0.00019	261.9					

H 5m
cover 40
Dia = 25 mm NOTE for cover = 40mm, hc,eff is independent of h for dia =25 mm and h > 250 mm. %As varies with h, but As (mm2) for given w the same.

%As per face	h	Asls	Asls total	%As total	Asls/Auls	wk (mm)	es	ecm	Sr (mm)	h	Asls	Asls total	%As total	Asls/Auls	wk (mm)	es	ecm	Sr (mm)
0.2	250	500	1000	0.40	0.46598322					450	900	1800	0.4	0.83877				
0.3		750	1500	0.60	0.69897484	2.1737	0.00233	0.00105	1552.7		1350	2700	0.6	1.258155	1.6048	0.00194	0.00093	1375.6
0.4	Auls	1000	2000	0.80	0.93196645	1.2584	0.00175	0.00081	1198.5	Auls	1800	3600	0.8	1.67754	0.7485	0.00130	0.00065	962.4
0.5	2146	1250	2500	1.00	1.16495806	0.8282	0.00140	0.00067	986.0	2146	2250	4500	1	2.096925	0.4409	0.00097	0.00051	755.8
0.6		1500	3000	1.20	1.39794967	0.5910	0.00117	0.00057	844.3		2700	5400	1.2	2.516309	0.2949	0.00078	0.00042	631.8
0.7		1750	3500	1.40	1.63094129	0.4459	0.00100	0.00050	743.1		3150	6300	1.4	2.935694	0.2136	0.00065	0.00037	549.2
0.8		2000	4000	1.60	1.8639329	0.3503	0.00088	0.00045	667.3		3600	7200	1.6	3.355079	0.1634	0.00056	0.00033	490.2
0.9		2250	4500	1.80	2.09692451	0.2838	0.00078	0.00041	608.2		4050	8100	1.8	3.774464	0.1301	0.00049	0.00030	445.9
1.0		2500	5000	2.00	2.32991612	0.2356	0.00070	0.00038	561.0		4500	9000	2	4.193849	0.1067	0.00043	0.00027	411.5
1.1		2750	5500	2.20	2.56290774	0.1994	0.00064	0.00035	522.4		4950	9900	2.2	4.613234	0.0896	0.00039	0.00026	383.9
1.2		3000	6000	2.40	2.79589935	0.1716	0.00058	0.00033	490.2		5400	10800	2.4	5.032619	0.0767	0.00035	0.00024	361.4
1.3		3250	6500	2.60	3.02889096	0.1496	0.00054	0.00031	462.9		5850	11700	2.6	5.452004	0.0666	0.00032	0.00023	342.6
1.4		3500	7000	2.80	3.26188257	0.1319	0.00050	0.00029	439.6		6525	13050	2.9	6.081081	0.0586	0.00030	0.00022	326.7
1.6		4000	8000	3.20	3.7278658	0.1054	0.00044	0.00027	401.6						0.0494	0.00027	0.00020	307.0
1.8		4500	9000	3.60	4.19384902	0.0868	0.00039	0.00025	372.1									
2		5000	10000	4	4.65983225	0.0732	0.00035	0.00023	348.5									

H 5m
cover50
Dia = 20 (mm) NOTE for cover = 50mm, hc,eff is independent of h for dia = 20mm and h >250 mm. %As varies with h, but As (mm2) for given w the same.

%As per face	h	Asls	Asls total	% As total	Asls/Auls	wk (mm)	es	ecm	Sr (mm)	h	Asls	As total	% As total	Asls/Auls				
0.2	250	500	1000	0.40	0.4660	3.9270	0.00350	0.00154	1870.0	450	900	1800	0.40	0.839	wk (mm)	es	ecm	Sr (mm)
0.3		750	1500	0.60	0.6990	1.8247	0.00233	0.00105	1303.3		1350	2700	0.60	1.258				
0.4	Auls	1000	2000	0.80	0.9320					Auls	1800	3600	0.80	1.678	0.7199	0.00130	0.00073	925.6
0.5	2146	1250	2500	1.00	1.1650	0.7140	0.00140	0.00067	850.0	2146	2250	4500	1.00	2.097	0.4297	0.00097	0.00057	736.7
0.6		1500	3000	1.20	1.3979	0.5157	0.00117	0.00057	736.7		2700	5400	1.20	2.516	0.2909	0.00078	0.00047	623.3
0.7		1750	3500	1.40	1.6309	0.3934	0.00100	0.00050	655.7		3150	6300	1.40	2.936	0.2130	0.00065	0.00041	547.8
0.8		2000	4000	1.60	1.8639	0.3124	0.00088	0.00045	595.0		3600	7200	1.60	3.355	0.1646	0.00056	0.00036	493.8
0.9		2250	4500	1.80	2.0969	0.2556	0.00078	0.00041	547.8		4050	8100	1.80	3.774	0.1322	0.00049	0.00033	453.3
1.0		2500	5000	2.00	2.3299	0.2142	0.00070	0.00038	510.0		4500	9000	2.00	4.194	0.1094	0.00043	0.00030	421.9
1.1		2750	5500	2.20	2.5629	0.1829	0.00064	0.00035	479.1		4950	9900	2.20	4.613	0.0926	0.00039	0.00028	396.7
1.2		3000	6000	2.40	2.7959	0.1587	0.00058	0.00033	453.3		5400	10800	2.40	5.033	0.0798	0.00035	0.00026	376.1
1.3		3250	6500	2.60	3.0289	0.1394	0.00054	0.00031	431.5		5850	11700	2.60	5.452				
1.4		3500	7000	2.80	3.2619	0.1239	0.00050	0.00029	412.9		6300	12600	2.80	5.871				
1.5		3750	7500	3.00	3.4949	0.1111	0.00047	0.00028	396.7									
1.6		4000	8000	3.20	3.7279	0.1004	0.00044	0.00027	382.5									
1.8		4500	9000	3.60	4.1938	0.0837	0.00039	0.00025	358.9									

H7m									
cover 40									
Dia = 20 (mm)									
%As per face	h	Asls	Asls total	% As total	Asls/Auls	wk (mm)	es	ecm	Sr (mm)
0.2	450	900	1800	0.40	0.599				
0.3		1350	2700	0.60	0.899	2.8107	0.00327	0.00105	1269.3
0.4	Auls	1800	3600	0.80	1.198				
0.5	3004	2250	4500	1.00	1.498	1.0558	0.00196	0.00067	816.0
0.55		2475	4950	1.10	1.648				
0.6		2700	5400	1.20	1.798	0.7474	0.00163	0.00057	702.7
0.7		3150	6300	1.40	2.097	0.5591	0.00140	0.00050	621.7
0.8		3600	7200	1.60	2.397	0.4354	0.00123	0.00045	561.0
0.9		4050	8100	1.80	2.696	0.3495	0.00109	0.00041	513.8
1.0		4500	9000	2.00	2.996	0.2872	0.00098	0.00038	476.0
1.1		4950	9900	2.20	3.296	0.2407	0.00089	0.00035	445.1

Key

w > 0.2

0,1<w<0.2

0.05<w<0.1

w<0.05

APPENDIX B

SELECT DATA FOR

FORM ANALYSIS

FLEXURAL CRACKING

Appendix B.1: Flexural cracking to EN1992 - Model Uncertainty Variation of 0,2

Table B.1.1: *SLS Flexural cracking to EN1992 – FORM analysis data with θ_{CoV} 0,2, w_{lim} 0,2 mm*

Analysis includes model uncertainty (cov = 0.2)					**wk = 0.2mm**												
As (ULS) calculated using SABS 0100				**Ec, eff to SABS 0100**													
All units kN and m																	
Reinf. dia = 20 mm																	
cover = 40 mm																	
H = 5m		SABS															
h	%As	Asls/Auls	Vary A_s	β	c*	h*	L*	ft*	**θ***	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
450	0.40	1.07	1800.00	-1.1732	0.0394	0.4499	46.206	2873.03	0.95	0.4006	0.12117	0.1096	0.2005	1.0511E-03	2.107E-04	6.0829E-08	0.87964
	0.45	1.21	2025.00	0.6682	0.0397	0.4500	50.660	2817.48	1.00	0.4003	0.12710	0.1076	0.1857	1.0741E-03	1.995E-04	-8.2851E-08	0.25201
Auls =	0.50	1.34	2250.00	2.1760	0.0402	0.4502	54.258	2767.22	1.07	0.4000	0.13254	0.1059	0.1745	1.0708E-03	1.869E-04	-6.1614E-08	0.01478
1680	0.55	1.47	2475.00	3.3967	0.0408	0.4503	57.118	2722.35	1.15	0.3995	0.13757	0.1042	0.1659	1.0508E-03	1.743E-04	1.8574E-08	0.00034
	0.60	1.61	2700.00	4.3902	0.0416	0.4504	59.391	2682.09	1.23	0.3989	0.14223	0.1027	0.1592	1.0215E-03	1.627E-04	-1.7985E-08	0.00001

Table B.1.2: *SLS Flexural cracking to EN1992 – FORM analysis data with θ_{CoV} 0,2, w_{lim} 0,05 mm*

Analysis includes model uncertainty (cov = 0.2)					**wlim = 0.05mm**												
As (ULS) calculated using SABS 0100				**Ec, eff to SABS 0100**													
All units kN and m																	
Reinf. dia = 20 mm																	
cover = 40 mm			check	1.0708	0.0399	0.4501	51.664	2796.68	1.02	0.4001	0.14251	0.1025	0.1179	8.3639E-04	9.858E-05		
H = 5m		SABS															
h	%As	Asls/Auls	Vary A_s	β	c*	h*	L*	ft*	**θ***	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
450	0.80	2.14	3600.00	-3.2205	0.0385	0.4498	41.329	3010.73	0.87	0.4013	0.15944	0.0968	0.1307	4.3721E-04	5.715E-05	4.2237E-08	0.99936
	1.00	2.68	4500.00	-0.6189	0.0392	0.4499	47.599	2875.29	0.95	0.4007	0.17299	0.0923	0.1194	4.4039E-04	5.260E-05	3.3069E-08	0.73200
	1.10	2.95	4950.00	0.3533	0.0398	0.4500	49.868	2816.16	1.00	0.4002	0.17887	0.0904	0.1161	4.3110E-04	5.006E-05	-6.6252E-08	0.36193
Auls =	1.20	3.21	5400.00	1.1451	0.0405	0.4501	51.664	2763.27	1.05	0.3997	0.18428	0.0886	0.1138	4.1791E-04	4.754E-05	-9.8905E-08	0.12609
1680	1.30	3.48	5850.00	1.7979	0.0412	0.4502	53.096	2715.46	1.11	0.3990	0.18928	0.0870	0.1121	4.0282E-04	4.516E-05	-8.0880E-08	0.03609
	1.40	3.75	6300.00	2.351	0.0419	0.450	54.26	2671.27	1.17	0.398	0.194	0.085	0.1110	3.87E-04	0.00004	-8.705E-08	0.00936
	1.60	4.29	7200.00	3.234	0.0435	0.450	56.01	2591.67	1.28	0.397	0.202	0.083	0.1099	3.57E-04	0.00004	-8.912E-08	0.00061

Appendix B.2: Flexural cracking to EN1992: Model Uncertainty Variation of 0,1

Table B.2.1: *SLS Flexural cracking to EN1992 – FORM analysis data with θ_{CoV} 0,1, w_{lim} 0,2 mm*

Analysis includes model uncertainty (cov = 0.10)					**wlim = 0.2mm**												
As (ULS) calculated using SABS 0100				**Ec, eff to SABS 0100**													
All units kN and m																	
Reinf. dia = 20 mm																	
cover = 40 mm																	
H = 5m		SABS															
h	%As	Asls/Auls	Vary A_s	β	c*	h*	L*	ft*	**θ***	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
450	0.40	1.07	1800.00	-1.6095	0.0393	0.4499	45.119	2886.62	0.98	0.4006	0.12118	0.1096	0.2004	1.0140E-03	2.032E-04	1.9693E-08	0.94625
	0.45	1.21	2025.00	0.6861	0.0397	0.4500	50.722	2816.69	1.00	0.4003	0.12710	0.1076	0.1857	1.0760E-03	1.999E-04	-5.2771E-08	0.24633
Auls =	0.50	1.34	2250.00	2.7863	0.0404	0.4502	55.820	2746.79	1.03	0.3998	0.13251	0.1059	0.1750	1.1146E-03	1.951E-04	7.2105E-09	0.00267
1680	0.55	1.47	2475.00	4.6332	0.0415	0.4505	60.265	2680.19	1.06	0.3989	0.13746	0.1043	0.1674	1.1324E-03	1.895E-04	-3.7088E-08	0.00000

Table B.2.2: *SLS Flexural cracking to EN1992 – FORM analysis data with θ_{CoV} 0,1, w_{lim} 0,05 mm*

Analysis includes model uncertainty (cov = 0.10)																	
As (ULS) calculated using SABS 0100				**Ec, eff to SABS 0100**			**wlim = 0.05mm**										
All units kN and m				**cov = 0.1**													
Reinf. dia = 20 mm																	
cover = 40 mm							corrected feb2013										
H = 5m		SABS															
h	%As	Asls/Auls	Vary A_s	β	c*	h*	L*	ft*	**θ***	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
450	0.80	2.14	3600.00	-4.0460	0.0383	0.4497	39.224	3063.37	0.96	0.4014	0.15947	0.0968	0.1304	3.9771E-04	5.187E-05	7.3358E-08	0.99997
too small!	1.00	2.68	4500.00	-0.9408	0.0390	0.4499	46.787	2896.56	0.98	0.4009	0.17303	0.0923	0.1191	4.2774E-04	5.093E-05	-7.8889E-08	0.82660
Auls =	1.20	3.21	5400.00	1.4383	0.0408	0.4502	52.459	2741.42	1.02	0.3994	0.18420	0.0887	0.1144	4.2873E-04	4.906E-05	-8.9207E-09	0.07517
1680	1.40	3.75	6300.00	3.1757	0.0433	0.4505	56.440	2608.21	1.06	0.3972	0.19355	0.0856	0.1137	4.1394E-04	4.708E-05	-4.0623E-08	0.00075

APPENDIX B.3: Flexural cracking to EN1992 – MATLAB equations for partial differentials

Symbols used in MATLAB routine		**MATLAB**
Cover	c	c
Section thickness	h	h
Liquid load	L_k	L
Concrete tensile strength	f_{ct}	ft
Crack width limit	w_{lim}	wl
Width of section	b	b
Area of reinf. (tension)	A_s	A
Bar diameter	φ	phi
Modular ratio	α_e	alfa
Height of wall/water	H	H
Elastic modulus steel	E_s	E
Strain	ε	e
Model Uncertainty	θ	th
Effective depth	d	d
Depth to neutral axis	x	x
Effective depth of tension zone	$h_{c,eff}$	hc
Crack spacing	S_r	s
Strain	ε	e
Limit state function	g	g

NB: ALL UNITS TO BE IN kN & m

Mathlab Routine to find Partial Derivatives

```
syms c h L ft wl b A phi alfa H E k1 k2 kt th
d=h-phi/2-c
x=(-alfa*A+(alfa^2*A^2+2*b*alfa*A*d)^0.5)/b
hc=(h-x)/3
s=2*c+0.25*k1*k2*b*phi*hc/A
e=((H^2*L/A/(d-x/3)/6)*10^6-kt*(1+alfa*A/b/hc)*b*hc*ft/A)/E
g=wl-th*s*e
dc=diff(g,c)
dh=diff(g,h)
dL=diff(g,L)
dth=diff(g,th)
```

Equations

```
d = h-1/2*phi-c
x = (-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b
hc =1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b
s = 2*c+1/4*k1*k2*b*phi*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/A
e = (1/6*H^2*L/A/(h-1/2*phi-c-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)-kt*(1+alfa*A/b/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b))*b*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)*ft/A)/E
g = wl-th*(2*c+1/4*k1*k2*b*phi*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/A)*(1/6*H^3*L/A/(h-1/2*phi-c-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)-kt*(1+alfa*A/b/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b))*b*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)*ft/A)/E
```

Partial Derivatives using MATLAB

```
dg/dc = -th*(2+1/12*k1*k2*b*phi/(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2)*alfa)*(1/6*H^2*L/A/(h-1/2*phi-c-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)-kt*(1+alfa*A/b/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b))*b*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)*ft/A)/E-(2*c+1/4*k1*k2*b*phi*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/A)*(-1/6*H^2*L/A/(h-1/2*phi-c-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)^2*(-1+1/3/(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2)*alfa*A)+1/3*kt*alfa^2*A/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2)*ft-1/3*kt*(1+alfa*A/b/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b))*b/(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2)*alfa*ft)/E)

dg/dh =th*(-1/4*k1*k2*b*phi*(1/3-1/3/(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2)*alfa*A)/A*(1/6*H^2*L/A/(h-1/2*phi-c-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)-kt*(1+alfa*A/b/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b))*b*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)*ft/A)/E-(2*c+1/4*k1*k2*b*phi*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/A)*(-1/6*H^2*L/A/(h-1/2*phi-c-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)^2*(1-1/3/(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2)*alfa*A)+kt*alfa/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)*(1/3-1/3/(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2)*alfa*A)*ft-kt*(1+alfa*A/b/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b))*b*(1/3-1/3/(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2)*alfa*A)*ft/A)/E)

dg/dL =th*(-1/6*(2*c+1/4*k1*k2*b*phi*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/A)*H^2/A/(h-1/2*phi-c-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/E)

dg/dft = th*(2*c+1/4*k1*k2*b*phi*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/A)*kt*(1+alfa*A/b/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b))*b*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/A/E)

dg/dth = -(2*c+1/4*k1*k2*b*phi*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)/A)*(1/6*H^2*L/A/(h-1/2*phi-c-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)-kt*(1+alfa*A/b/(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b))*b*(1/3*h-1/3*(-alfa*A+(alfa^2*A^2+2*b*alfa*A*(h-1/2*phi-c))^(1/2))/b)*ft/A)/E
```

Figure B.3.1: *EN1992 flexural cracking - Partial differential equations*

APPENDIX C

SELECT DATA FOR

FORM ANALYSIS

TENSION CRACKING

Appendix C1: Verification of CHM reliability model – tension cracking model

Table C.1.1: *CHM reliability model using MH/JVR model*

As (ULS) calculated using EN & SABS 0100					Ec = 33	(phi = 2)	α = 18.3									
All units kN and m					All values as MH/JR model											
wk = 0.2mm					fy =500 MPa		in calc of Tu									
cover = 40 mm																
H = 7m	**0.9**				**NBB**	**h=250mm**	**hceff = h/2**									
	0.2															
Reinf. dia = 16 mm	**SANS**	**EN**		check	-3.7668	0.0385	0.2502	65.250	5448.73	0.93	0.2502	0.4476	4.8261E-04	2.160E-04	2.9335E-08	
h	Asls/Auls	Asls/Auls	%As	Vary A_s	β	c*	h*	L*	ft*	**θ***	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
250	0.90	1.00	1.08	2700.00	-3.3731	0.0381	0.2502	65.496	5012.56	0.92	0.2502	0.3726	5.8601E-04	2.184E-04	7.0282E-08	0.99963
A uls = (EN)	1.08	1.20	1.30	3240.00	-2.2241	0.0378	0.2500	66.7048	3999.80	0.9145	0.2500	0.3226	0.0007	0.0002	0.0000	0.98693
2700	1.26	1.40	1.51	3780.00	-0.9018	0.0385	0.2500	68.586	3205.78	0.95	0.2500	0.2886	7.2895E-04	2.104E-04	-3.0532E-08	0.81641
Auls (SANS)	1.44	1.60	1.73	4320.00	0.4580	0.0403	0.2500	70.723	2689.54	1.02	0.2500	0.2658	7.3637E-04	1.957E-04	-1.5398E-08	0.32349
3000	1.62	1.80	1.94	4860.00	1.7529	0.0431	0.2501	72.714	2363.48	1.11	0.2501	0.2508	7.1770E-04	1.800E-04	-1.6407E-08	0.03981
	1.80	2.00	2.16	5400.00	2.9350	0.0467	0.2502	74.413	2147.25	1.20	0.2502	0.2416	6.8720E-04	1.660E-04	5.2953E-08	0.00167
	1.98	2.20	2.38	5940.00	4.0015	0.0510	0.2502	75.824	1995.15	1.29	0.2502	0.2369	6.5255E-04	1.546E-04	-2.4955E-08	0.00003
	2.16	2.40	2.59	6480.00	4.9481	0.0560	0.2502	76.966	1892.74	1.38	0.2502	0.2356	6.1607E-04	1.451E-04	4.3542E-08	0.00000
Reinf. dia = 20 mm	**SABS**	**EN**														
h	Asls/Auls	Asls/Auls	%As	Vary A_s	β	c*	h*	L*	ft*	**θ***	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
250	0.90	1.00	1.08	2700.00	-3.7670	0.0385	0.2502	65.250	5448.98	0.93	0.2502	0.4476	4.8255E-04	2.160E-04	5.1826E-08	0.99992
A uls = (EN)	1.08	1.20	1.30	3240.00	-2.8151	0.0381	0.2501	66.065	4499.66	0.91	0.2501	0.3849	5.6826E-04	2.187E-04	-6.8595E-09	0.99756
2700	1.26	1.40	1.51	3780.00	-1.6834	0.0382	0.2500	67.445	3645.19	0.93	0.2500	0.3409	6.3360E-04	2.160E-04	-8.8520E-08	0.95386
Auls (SANS)	1.44	1.60	1.73	4320.00	-0.4436	0.0390	0.2500	69.301	3008.63	0.97	0.2500	0.3095	6.6477E-04	2.058E-04	-9.2071E-10	0.67133
3000	1.62	1.80	1.94	4860.00	0.8100	0.0408	0.2500	71.279	2582.93	1.04	0.2500	0.2874	6.6649E-04	1.915E-04	-1.6940E-08	0.20898
	1.80	2.00	2.16	5400.00	2.0043	0.0433	0.2501	73.113	2299.84	1.13	0.2501	0.2720	6.5067E-04	1.769E-04	-1.5132E-08	0.02252
	1.98	2.20	2.38	5940.00	3.1087	0.0466	0.2502	74.709	2102.59	1.22	0.2502	0.2616	6.2646E-04	1.639E-04	-1.2097E-08	0.00094
	2.16	2.40	2.59	6480.00	4.1136	0.0505	0.2502	76.059	1960.22	1.31	0.2502	0.2554	5.9859E-04	1.529E-04	2.7679E-08	0.00002

Analysis includes model uncertainty (cov = 0.10)					**Tension Case**											
As (ULS) calculated using EN & SABS 0100					Ec = 33											
All units kN and m					All values as MH/JR model											
wk = 0.05 mm					fy = 500MPa											
cover = 40 mm																
H = 7m			SABS													
Reinf. dia = 16 mm	**SANS**	EN														
h	Asls/Auls	Asls/Auls	%As	Vary A_s	β	c*	h*	L*	ft*	**θ***	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
250	1.98	2.20	2.38	5940.00	-2.5943	0.0377	0.2501	66.687	4413.32	0.94	0.2501	0.2102	2.5292E-04	5.317E-05	7.7850E-08	0.99526
A uls =	2.34	2.60	2.81	7020.00	-1.6175	0.0378	0.2500	67.785	3679.69	0.95	0.2500	0.1896	2.7931E-04	5.295E-05	-8.3162E-08	0.94712
2700	2.7	3.00	3.24	8100.00	-0.6014	0.0386	0.2500	69.141	3103.41	0.97	0.2500	0.1760	2.9253E-04	5.150E-05	-3.9468E-08	0.72623
3000	3.06	3.40	3.67	9180.00	0.4096	0.0404	0.2500	70.592	2684.09	1.01	0.2500	0.1679	2.9400E-04	4.935E-05	-2.8104E-08	0.34105
	3.42	3.80	4.10	10260.00	1.3698	0.0428	0.2500	71.963	2388.16	1.06	0.2500	0.1636	2.8731E-04	4.701E-05	-4.1458E-08	0.08537
	3.78	4.20	4.54	11340.00	2.2551	0.0458	0.2500	73.172	2175.41	1.12	0.2500	0.1622	2.7625E-04	4.481E-05	-5.9926E-08	0.01206
	4.14	4.60	4.97	12420.000	3.0606	0.0492	0.2500	74.209	2015.62	1.17	0.2500	0.1628	2.6343E-04	4.288E-05	-8.0589E-08	0.00110
	4.5	5.00	5.40	13500.00												0.50000
	SANS															
	Asls/Auls	Asls/Auls	%As	Vary A_s	β	c*	h*	L*	ft*	**θ***	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
	1.98	2.20	2.38	5940.00	-2.8934	0.0381	0.2502	66.452	4691.49	0.94	0.2502	0.2447	2.1652E-04	5.297E-05	2.3212E-08	0.99809
Reinf. dia = 20 mm	2.34	2.60	2.81	7020.00	-2.0210	0.0379	0.2501	67.338	3976.37	0.94	0.2501	0.2184	2.4285E-04	5.304E-05	-5.0826E-08	0.97836
h	2.70	3.00	3.24	8100.00	-1.0863	0.0383	0.2500	68.487	3366.84	0.96	0.2500	0.2001	2.6098E-04	5.221E-05	-4.9807E-08	0.86132
250	3.06	3.40	3.67	9180.00	-0.1224	0.0394	0.2500	69.824	2889.51	0.99	0.2500	0.1877	2.6945E-04	5.057E-05	-5.8183E-08	0.54870
A uls =	3.42	3.80	4.10	10260.00	0.8239	0.0412	0.2500	71.193	2538.58	1.03	0.2500	0.1798	2.6924E-04	4.840E-05	-4.6841E-08	0.20500
2700	3.78	4.20	4.54	11340.00	1.7236	0.0436	0.2500	72.477	2282.91	1.08	0.2500	0.1753	2.6329E-04	4.616E-05	-6.1538E-08	0.04239
3000	4.14	4.60	4.97	12420.00	2.5564	0.0464	0.2500	73.613	2093.36	1.14	0.2500	0.1734	2.5410E-04	4.405E-05	-5.7027E-08	0.00529
	4.50	5.00	5.40	13500.00	3.3214	0.0496	0.2501	74.600	1947.20	1.19	0.2501	0.1733	2.4352E-04	4.221E-05	-6.4052E-08	0.00045

Appendix C.2: Tension cracking to EN1992

Table C.2.1: *Tension cracking to EN1992 – θ_{CoV} 0,2*

Analysis includes model uncertainty (cov = 0.20)				**Tension Case**		to compare to bending									
As (ULS) calculated using SABS 0100				Ec =	27.4										
All units kN and m				All material values as MH/JR model except Ec											
wk = 0.2mm				fy = 450 Mpa		affects Auls but not Asls									
cover = 40 mm															
H 5m															
	SANS	SANS		-0.5475	0.0392	0.2500	49.389	3112.60	0.89	0.2500	0.4117	5.4274E-04	2.234E-04	3.7287E-08	
Reinf. dia = 20 mm	Asls/Auls	%As	Vary As	β	c*	h*	L*	ft*	**θ***	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
h	1.16	1.00	2500.00	-1.6352	0.0390	0.2501	47.672	3628.00	0.83	0.2501	0.4780	5.0312E-04	2.405E-04	6.49E-08	0.94900
250	1.40	1.20	3000.00	-0.5799	0.0392	0.2500	48.563	3059.82	0.91	0.2500	0.4118	5.3380E-04	2.198E-04	-7.60E-08	0.71901
A uls =	1.63	1.40	3500.00	0.4442	0.0399	0.2500	49.415	2697.72	1.04	0.2500	0.3654	5.2414E-04	1.915E-04	-6.90E-08	0.32845
2150	1.86	1.60	4000.00	1.3873	0.0407	0.2500	50.160	2457.08	1.21	0.2500	0.3315	4.9890E-04	1.654E-04	-3.16E-08	0.08267
h/2	2.09	1.80	4500.00	2.2430	0.0417	0.2500	50.802	2284.35	1.39	0.2500	0.3057	4.6970E-04	1.436E-04	-1.75E-08	0.01245
	2.33	2.00	5000.00	3.0168	0.0429	0.2501	51.358	2152.35	1.59	0.2501	0.2859	4.4088E-04	1.260E-04	2.17E-08	0.00128

H = 5m	Asls/Auls	%As	Vary As	β	c*	L*	ft*	**θ***	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f	h 250 %As
Reinf. dia = 20 mm	1.26	0.60	2700.00	-1.5296	0.0345	48.006	3315.27	0.834	0.2224	0.3983	6.0177E-04	2.397E-04	6.3598E-09	0.93694	
h	1.47	0.70	3150.00	-0.3195	0.0384	48.838	2926.86	0.947	0.2421	0.3842	5.4997E-04	2.113E-04	-9.2531E-09	0.62534	1.08
450	1.67	0.80	3600.00	0.7391	0.0423	49.531	2649.82	1.064	0.2617	0.3755	5.0042E-04	1.879E-04	-8.7950E-09	0.22993	1.26
A uls =	1.88	0.90	4050.00	1.6738	0.0462	50.121	2437.31	1.183	0.2812	0.3702	4.5668E-04	1.691E-04	-3.3466E-09	0.04708	1.44
2150	2.09	1.00	4500.00	2.5076	0.0501	50.631	2266.05	1.300	0.3003	0.3671	4.1894E-04	1.538E-04	-2.0342E-09	0.00608	1.62
2.5(c+phi/2)	2.30	1.10	4950.00	3.2579	0.0538	51.079	2122.66	1.415	0.3192	0.3656	3.8663E-04	1.413E-04	6.8897E-09	0.00056	1.80

h	Asls/Auls	%As	Vary As	β	c*	h*	L*	ft*	**θ***	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	p_f
250	1.16	1.00	2500.00	-1.6352	0.0390	0.2501	47.672	3628.000	0.8313	0.2501	4.7802E-01	5.031E-04	2.4050E-04	6.49E-08	0.94900
A uls =	1.16	1.00	2500.00	-1.6354	0.0390	0.2501	47.672	3628.130	0.8313	0.2501	4.7802E-01	5.031E-04	2.4049E-04	8.00E-08	0.94902
2150	1.40	1.20	3000.00	-0.5799	0.0392	0.2500	48.563	3059.818	0.9101	0.2500	4.1182E-01	5.338E-04	2.1983E-04	-7.60E-08	0.71901
hceff = h/2	1.63	1.40	3500.00	0.4442	0.0399	0.2500	49.415	2697.721	1.0445	0.2500	3.6544E-01	5.241E-04	1.9154E-04	-6.90E-08	0.32845
cov 0,2	1.86	1.60	4000.00	1.3873	0.0407	0.2500	50.160	2457.080	1.2097	0.2500	3.3146E-01	4.989E-04	1.6536E-04	-3.16E-08	0.08267
	2.09	1.80	4500.00	2.2430	0.0417	0.2500	50.802	2284.345	1.3928	0.2500	3.0575E-01	4.697E-04	1.4361E-04	-1.75E-08	0.01245
	2.33	2.00	5000.00	3.0168	0.0429	0.2501	51.358	2152.351	1.5867	0.2501	2.8586E-01	4.409E-04	1.2603E-04	2.17E-08	0.00128

Analysis includes model uncertainty (cov = 0.20)				**Tension Case**										
As (ULS) calculated using SABS 0100				Ec =										
All units kN and m				All material values as MH/JR model except Ec										
wk = 0.05mm				fy = 450 Mpa		affects Auls but not Asls								
cover = 40 mm				h250	hc = h/2	same as 2.5(h-d) = 2.5(c+phi)								
H = 5m				h450	hc=2.5(h-d)									
Reinf. dia = 20 mm	SANS	SANS												
h	Asls/Auls	%As	Vary As	β	c*	L*	ft*	θ*	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	pf
450	2.93	1.40	6300.00	-0.3542	0.0384	48.808	2956.83	0.948	0.2418	0.2302	2.2903E-04	5.273E-05	6.7142E-10	0.63842
A uls =	3.35	1.60	7200.00	0.5881	0.0417	49.445	2658.48	1.038	0.2583	0.2268	2.1238E-04	4.816E-05	6.5494E-10	0.27824
2150	3.77	1.80	8100.00	1.4257	0.0449	49.995	2431.43	1.131	0.2744	0.2253	1.9630E-04	4.423E-05	-4.5567E-10	0.07697
	4.19	2.00	9000.00	2.1744	0.0480	50.474	2250.49	1.222	0.2902	0.2251	1.8174E-04	4.090E-05	-7.3654E-10	0.01484
hc=2.5(h-d)	4.60	2.20	9900.00	2.8483	0.0511	50.897	2100.45	1.312	0.3054	0.2256	1.6895E-04	3.811E-05	-3.3836E-10	0.00220

Appendix C.3: Tension cracking to EN1992 – MATLAB equations for partial differentials

Symbols used		**Mathlab**
cover	c	c
thickness section	h	h
liquid load	Lk	L
concrete tensile strength	fct	ft
crack width limit	wlim	wl
width of section	b	b
area of reinf. (tension)	As	A
dia. of reinf.	φ	phi
modular ratio	αe	alfa
height of wall	H	H
Elastic modulus steel	Es	E
Strain	ε	e
Model Uncertainty	θ	th
Tensile load	Ns	N
Diameter of reservoir	D	D
NB: ALL UNITS IN kN and m		
Mathlab Routine to find Partial Derivatives		
syms c h L ft wl b A phi alfa E k1 k2 kt th D		
s=k3*c+k4*k1*k2*b*h*phi/A		
e=((D*L/A/2)-kt*(1+alfa*A/b/h)*b*h*ft/A)/E		
g=wl-th*s*e		
dc=diff(g,c)		
dh=diff(g,h)		
dL=diff(g,L)		
dth=diff(g,th)		
Equations		
g = wl - th*(k3*c+k4*k1*k2*b*h*phi/A)*(D*L/2/A- kt*b*h*ft*A-kt*ft*alfa)		
Partial Derivatives (all to be x relevant std dev)		
dg/dc = -th/E*k3*(D*L/2/A-kt*b*h*ft*A-kt*ft*alfa)		
dg/dh = th/E(k3*c*kt*b*ft*A-k4/2*k1*k2*b*phi*h*D*L/A^2-2*k4*k1*k2*b^2*h*phi*ft*kt/A^2-k4*k1*k2*b*phi*kt*ft*alfa/A)		
= th/E*(k3*c*kt*b*ft*A-k4*k1*k2*b*phi/A*(D*L/A+2*bb*h*ft/A+kt*ft*alfa))		
dg/dL = -th/E*(k3*c*D/2/A+k4/2*k1*k2*b*phi*h*D/A^2)		
dg/dth = -1/E*(k3*c+k4+k1*k2*b*h*phi/A)*(D*L/2/A-kt*ft*b*h/A+kt*ft*alfa)		
dg/dft = th/E*(k3*c*kt*(b*h/A-alfa)+k4*k1*k2*b*h*phi*kt/A*(b*h/A-alfa))		
= th/E*(k3*c*kt+k4*k1*k2*b*h*phi*kt/A)*(b*h/A-alfa)		

Figure C.3.1: *Tension cracking to EN1992 – MATLAB equations for partial differentials*

APPENDIX D

SENSITIVITY ANALYSIS

using

REVERSE-FORM ANALYSIS

Appendix D.1: Sensitivity analysis for flexural cracking

Table D.1.1: *Flexural cracking: Sensitivity analysis using reverse-FORM*

Reverse FORM

Note:
1 variable changed and g solved for target beta to find As
solution stable gen after 3 iterations

Fixed variables
Fct = 2.89
h = 450
dia = 20
H5m

This run: wlim = 0.2 mm

VARY θ cov

Target beta 0.5

																Sensitivity Factors									Partial safety factors					
θ cov	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	α*xi c		α*xi h		α*xi L		α*xi ft	α*xi θ		γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.10	0.5	0.039663	0.450034	50.26868	2822.609	0.999	0.4004	0.126635	0.1078	0.186777	0.001071579	0.000200147	-3.5302E-11	2006.481581	0.445885	-0.037159936	0.037159936	-0.0152295	0.01522953	-0.99382974	0.99382974	0.0583668	-0.08534	0.08534	1.001	0.994	1.000	1.025	0.975	1.001
0.15	0.5	0.0397	0.4500	50.263	2822.69	0.998	0.4004	0.12660	0.1078	0.1869	1.0721E-03	2.003E-04	9.5031E-11	2005.1	0.446	-0.03694	0.036943406	-0.01515	0.01515228	-0.98935	0.989350265	0.05808	-0.12735	0.12735	1.002	0.992	1.000	1.025	0.977	0.998
0.20	0.5	0.0397	0.4500	50.256	2822.79	0.997	0.4004	0.12655	0.1078	0.1870	1.0730E-03	2.006E-04	2.9982E-10	2003.0	0.445	-0.03664	0.036642611	-0.01505	0.0150457	-0.98320	0.983199135	0.05767	-0.16859	0.16859	1.003	0.992	1.000	1.025	0.977	0.997
0.25	0.5	0.0397	0.4500	50.246	2822.93	0.995	0.4004	0.12648	0.1079	0.1872	1.0741E-03	2.010E-04	5.8764E-10	2000.2	0.444	-0.03626	0.036259891	-0.01491	0.01491129	-0.97549	0.975491111	0.05717	-0.20884	0.20884	1.005	0.992	1.000	1.024	0.977	0.995
0.30	0.5	0.0397	0.4500	50.235	2823.09	0.992	0.4004	0.12638	0.1079	0.1874	1.0757E-03	2.016E-04	9.6990E-10	1996.5	0.444	-0.03580	0.035798233	-0.01475	0.01475089	-0.96637	0.966366392	0.05656	-0.24787	0.24787	1.008	0.991	1.000	1.024	0.977	0.992

Target Beta 1.5

θ cov	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	α*xi c		α*xi h		α*xi L		α*xi ft	α*xi θ		γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.10	1.5	0.0399	0.4501	52.702	2790.28	1.009	0.4002	0.12919	0.1070	0.1813	1.0930E-03	1.982E-04	3.4693E-09	2109.5	0.469	-0.04370	0.043701063	-0.01673	0.01672594	-0.99262	0.992619995	0.05988	-0.09450	0.09450	0.991	0.999	1.000	1.074	0.965	1.009
0.15	1.5	0.0399	0.4501	52.681	2790.31	1.021	0.4002	0.12954	0.1069	0.1805	1.0853E-03	1.959E-04	-5.8372E-10	2123.8	0.472	-0.04389	0.043894977	-0.01669	0.01669012	-0.98701	0.987008914	0.05984	-0.14152	0.14152	0.980	0.999	1.000	1.074	0.966	1.021
0.20	1.5	0.0399	0.4501	52.652	2790.35	1.037	0.4002	0.13003	0.1067	0.1794	1.0747E-03	1.928E-04	4.0737E-10	2143.9	0.476	-0.04416	0.044161491	-0.01664	0.01663906	-0.97916	0.979164631	0.05978	-0.18824	0.18824	0.964	0.999	1.000	1.073	0.966	1.037
0.25	1.5	0.0400	0.4501	52.615	2790.42	1.058	0.4002	0.13066	0.1065	0.1781	1.0613E-03	1.890E-04	1.3622E-09	2169.7	0.482	-0.04449	0.04449281	-0.01657	0.01657152	-0.96910	0.969104986	0.05970	-0.23456	0.23456	0.945	0.999	1.000	1.073	0.966	1.058
0.30	1.5	0.0400	0.4501	52.570	2790.51	1.085	0.4002	0.13143	0.1062	0.1764	1.0453E-03	1.844E-04	1.3289E-09	2201.4	0.489	-0.04488	0.044879782	-0.01649	0.01648622	-0.95686	0.956861487	0.05959	-0.28032	0.28032	0.922	0.999	1.000	1.072	0.966	1.085

Target beta 2.0

θ cov	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	α*xi c		α*xi h		α*xi L		α*xi ft	α*xi θ		γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.10	2.0	0.0401	0.4502	53.915	2773.63	1.015	0.4000	0.13047	0.1066	0.1788	1.1023E-03	1.971E-04	-3.6350E-09	2162.8	0.481	-0.04726	0.047255037	-0.01750	0.01750166	-0.99193	0.991929004	0.06065	-0.09929	0.09929	0.985	1.003	1.000	1.099	0.960	1.015
0.15	2.0	0.0401	0.4502	53.885	2773.51	1.034	0.4000	0.13106	0.1064	0.1775	1.0896E-03	1.934E-04	-3.3500E-09	2187.1	0.486	-0.04775	0.047745351	-0.01749	0.01749485	-0.98566	0.985656203	0.06077	-0.14901	0.14901	0.967	1.003	1.000	1.099	0.960	1.034
0.20	2.0	0.0401	0.4502	53.841	2773.34	1.061	0.4000	0.13188	0.1061	0.1758	1.0721E-03	1.884E-04	-4.0402E-09	2221.6	0.494	-0.04843	0.048428327	-0.01748	0.0174828	-0.97679	0.976792384	0.06093	-0.19878	0.19878	0.942	1.003	1.000	1.098	0.960	1.061
0.25	2.0	0.0401	0.4502	53.785	2773.13	1.098	0.4000	0.13295	0.1057	0.1736	1.0500E-03	1.822E-04	-8.5885E-09	2266.9	0.504	-0.04930	0.049298438	-0.01746	0.01746286	-0.96527	0.965265862	0.06113	-0.24858	0.24858	0.911	1.004	1.000	1.097	0.960	1.098
0.30	2.0	0.0402	0.4502	53.715	2772.89	1.143	0.4000	0.13427	0.1053	0.1709	1.0233E-03	1.749E-04	-8.7572E-10	2323.8	0.516	-0.05035	0.05035114	-0.01743	0.01743221	-0.95100	0.950996551	0.06136	-0.29833	0.29833	0.875	1.004	1.000	1.095	0.959	1.143

This run: wlim = 0.1

VARY θ cov

Target beta 0.5

θ cov	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	α*xi c		α*xi h		α*xi L		α*xi ft	α*xi θ		γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.10	0.5	0.0398	0.4500	50.261	2816.44	1.001	0.4003	0.15076	0.0998	0.1435	6.9654E-04	9.994E-05	-9.8352E-09	3119.7	0.693	-0.07092	0.070924113	-0.01968	0.01967721	-0.98743	0.987433	0.08139	-0.11373	0.11373	0.999	0.994	1.000	1.025	0.974547	1.00068653
0.15	0.5	0.0398	0.4500	50.251	2816.61	1.001	0.4003	0.15078	0.0998	0.1435	6.9609E-04	9.986E-05	1.0358E-10	3120.8	0.694	-0.07037	0.070366121	-0.01952	0.01951883	-0.97955	0.979545849	0.08077	-0.16921	0.16921	0.999	0.994	1.000	1.024	0.974605	1.00144211
0.20	0.5	0.0398	0.4500	50.238	2816.84	1.002	0.4003	0.15080	0.0997	0.1434	6.9561E-04	9.977E-05	3.0178E-10	3121.7	0.694	-0.06959	0.069589797	-0.01930	0.01930142	-0.96883	0.968826494	0.07991	-0.22310	0.22310	0.998	0.994	1.000	1.024	0.974684	1.00231275
0.25	0.5	0.0398	0.4500	50.222	2817.13	1.003	0.4003	0.15081	0.0997	0.1434	6.9515E-04	9.969E-05	5.7961E-10	3122.3	0.694	-0.06861	0.068613946	-0.01903	0.01903071	-0.95557	0.955570516	0.07884	-0.27495	0.27495	0.997	0.994	1.000	1.024	0.974784	1.0031241
0.30	0.5	0.0398	0.4500	50.203	2817.47	1.004	0.4003	0.15081	0.0997	0.1434	6.9478E-04	9.963E-05	9.4453E-10	3122.1	0.694	-0.06745	0.067454261	-0.01871	0.01871266	-0.94013	0.940128098	0.07757	-0.32441	0.32441	0.996	0.994	1.000	1.024	0.974901	1.00366828

Target Beta 1.5

θ cov	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	α*xi c		α*xi h		α*xi L		α*xi ft	α*xi θ		γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.10	1.5	0.0403	0.4501	52.672	2770.96	1.014	0.3998	0.15422	0.0986	0.1401	7.0387E-04	9.861E-05	-1.7655E-09	3316.4	0.737	-0.08343	0.083431959	-0.02173	0.0217287	-0.98455	0.984553377	0.08425	-0.12698	0.12698	0.986	1.008	1.000	1.074	0.959	1.014
0.15	1.5	0.0403	0.4501	52.635	2771.04	1.032	0.3998	0.15486	0.0984	0.1393	6.9558E-04	9.691E-05	-9.3615E-10	3353.0	0.745	-0.08365	0.083653567	-0.02162	0.02162437	-0.97441	0.974412722	0.08415	-0.18966	0.18966	0.969	1.008	1.000	1.073	0.959	1.032
0.20	1.5	0.0403	0.4501	52.583	2771.16	1.06	0.3998	0.15576	0.0981	0.1383	6.8426E-04	9.461E-05	3.2790E-10	3404.2	0.756	-0.08393	0.083929416	-0.02148	0.02147577	-0.96035	0.96034555	0.08400	-0.25135	0.25135	0.946	1.008	1.000	1.072	0.959	1.057
0.25	1.5	0.0403	0.4501	52.517	2771.33	1.089	0.3998	0.15689	0.0978	0.1370	6.7022E-04	9.179E-05	1.4303E-09	3469.9	0.771	-0.08423	0.084225705	-0.02128	0.02128085	-0.94251	0.942508017	0.08379	-0.31163	0.31163	0.918	1.008	1.000	1.071	0.959	1.089
0.30	1.5	0.0403	0.4501	52.439	2771.56	1.129	0.3998	0.15824	0.0973	0.1354	6.5382E-04	8.856E-05	7.3151E-10	3549.9	0.789	-0.08450	0.08450061	-0.02104	0.02103773	-0.92113	0.921134017	0.08349	-0.37008	0.37008	0.886	1.008	1.000	1.069	0.959	1.129

Target beta 2.0

θ cov	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	α*xi c		α*xi h		α*xi L		α*xi ft	α*xi θ		γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.10	2.0	0.0406	0.4502	53.871	2747.29	1.022	0.3996	0.15599	0.0981	0.1386	7.0598E-04	9.785E-05	-5.2027E-09	3421.8	0.760	-0.09033	0.090331147	-0.02281	0.02280909	-0.98285	0.982851971	0.08577	-0.13404	0.13404	0.978	1.016	1.000	1.098	0.950619	1.02204694
0.15	2.0	0.0406	0.4502	53.814	2747.03	1.050	0.3996	0.15703	0.0977	0.1374	6.9291E-04	9.523E-05	-4.5184E-09	3482.0	0.774	-0.09113	0.091129152	-0.02275	0.02274518	-0.97134	0.971341624	0.08602	-0.20069	0.20069	0.952	1.016	1.000	1.097	0.950529	1.0501738
0.20	2.0	0.0407	0.4502	53.735	2746.69	1.091	0.3995	0.15847	0.0972	0.1358	6.7506E-04	9.170E-05	-4.5858E-09	3567.8	0.793	-0.09219	0.09218713	-0.02265	0.0226476	-0.95518	0.955177427	0.08634	-0.26678	0.26678	0.917	1.017	1.000	1.096	0.950411	1.09058069
0.25	2.0	0.0407	0.4502	53.633	2746.29	1.144	0.3995	0.16032	0.0966	0.1339	6.5291E-04	8.740E-05	-9.3999E-09	3680.0	0.818	-0.09344	0.093438283	-0.02251	0.02250865	-0.93438	0.934377306	0.08673	-0.33193	0.33193	0.874	1.017	1.000	1.093	0.950272	1.14421268
0.30	2.0	0.0407	0.4502	53.509	2745.85	1.212	0.3995	0.16257	0.0959	0.1316	6.2699E-04	8.250E-05	-1.4579E-09	3820.5	0.849	-0.09481	0.094809784	-0.02232	0.02232145	-0.90904	0.909041509	0.08715	-0.39568	0.39568	0.825	1.017	1.000	1.091	0.950121	1.21216404

This run: wlim = 0.05

VARY θ cov

Target beta 0.5

θ cov	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	α*xi c		α*xi h		α*xi L		α*xi ft	α*xi θ		γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.10	0.5	0.0399	0.4501	50.243	2805.76	1.003	0.4001	0.17966	0.0901	0.1158	4.3078E-04	4.988E-05	-5.8253E-09	5013.7	1.114	-0.12237	0.122369575	-0.02566	0.02565517	-0.97312	0.973116712	0.12141	-0.15057	0.15057	0.998	0.998	1.000	1.024	0.97085	1.00253173
0.15	0.5	0.0399	0.4501	50.227	2806.15	1.005	0.4001	0.17976	0.0901	0.1157	4.2982E-04	4.973E-05	-5.8423E-09	5021.8	1.116	-0.12079	0.120785549	-0.02531	0.02530963	-0.95956	0.959559934	0.11991	-0.22279	0.22279	0.995	0.998	1.000	1.024	0.970988	1.00547429
0.20	0.5	0.0399	0.4501	50.204	2806.69	1.009	0.4001	0.17990	0.0901	0.1156	4.2854E-04	4.954E-05	4.5553E-09	5032.6	1.118	-0.11867	0.118666224	-0.02485	0.02484804	-0.94147	0.94147179	0.11791	-0.29160	0.29160	0.991	0.998	1.000	1.024	0.971172	1.00920185
0.25	0.5	0.0399	0.4501	50.178	2807.35	1.013	0.4002	0.18003	0.0900	0.1155	4.2725E-04	4.934E-05	-3.3560E-09	5042.9	1.121	-0.11604	0.116038313	-0.02428	0.02428296	-0.91966	0.919663231	0.11542	-0.35615	0.35615	0.987	0.997	1.000	1.023	0.971402	1.0133571
0.30	0.5	0.0399	0.4501	50.147	2808.11	1.018	0.4002	0.18017	0.0900	0.1154	4.2586E-04	4.914E-05	1.6873E-09	5053.8	1.123	-0.11303	0.113033706	-0.02364	0.02363977	-0.89496	0.8949585	0.11258	-0.41598	0.41598	0.983	0.997	1.000	1.022	0.971664	1.01754942
																														0

Target Beta 1.5

θ cov	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	α*xi c		α*xi h		α*xi L		α*xi ft	α*xi θ		γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.10	1.5	0.0409	0.4502	52.604	2737.01	1.021	0.3993	0.18451	0.0886	0.1143	4.2841E-04	4.899E-05	-1.0228E-09	5428.0	1.206	-0.14433	0.144332548	-0.02857	0.02857381	-0.96604	0.966044924	0.12750	-0.16986	0.16986	0.980	1.021	1.000	1.072	0.947064	1.0206897
0.15	1.5	0.0409	0.4502	52.538	2737.29	1.046	0.3993	0.18558	0.0882	0.1137	4.2032E-04	4.778E-05	-4.3124E-10	5518.1	1.226	-0.14365	0.143646308	-0.02827	0.02827031	-0.94811	0.948109081	0.12716	-0.25197	0.25197	0.956	1.021	1.000	1.071	0.947158	1.04649065
0.20	1.5	0.0408	0.4502	52.449	2737.69	1.082	0.3993	0.18703	0.0877	0.1128	4.0954E-04	4.619E-05	7.0657E-09	5642.9	1.254	-0.14261	0.142610668	-0.02785	0.02785018	-0.92383	0.923834974	0.12664	-0.33072	0.33072	0.924	1.021	1.000	1.069	0.947296	1.08243865
0.25	1.5	0.0408	0.4502	52.339	2738.26	1.128	0.3994	0.18881	0.0871	0.1117	3.9671E-04	4.432E-05	2.9534E-09	5798.8	1.289	-0.14113	0.14112998	-0.02732	0.02731567	-0.89411	0.894109394	0.12591	-0.40503	0.40503	0.886	1.021	1.000	1.067	0.947495	1.12821566
0.30	1.5	0.0408	0.4502	52.214	2739.01	1.183	0.3994	0.19086	0.0864	0.1105	3.8236E-04	4.226E-05	-1.9923E-09	5983.5	1.330	-0.13918	0.139180619	-0.02668	0.02667988	-0.86001	0.860007884	0.12495	-0.47402	0.47402	0.845	1.020	1.000	1.065	0.947754	1.18330022

Target beta 2.0

θ cov	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	α*xi c		α*xi h		α*xi L		α*xi ft	α*xi θ		γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.10	2.0	0.0415	0.4503	53.767	2700.59	1.032	0.3988	0.18707	0.0877	0.1139	4.2560E-04	4.848E-05	-9.1084E-09	5660.3	1.258	-0.15653	0.156529406	-0.03014	0.0301434	-0.96174	0.961743523	0.13089	-0.18030	0.18030	0.970	1.036	1.001	1.096	0.934459	1.03154723
0.15	2.0	0.0415	0.4503	53.666	2700.13	1.072	0.3988	0.18874	0.0872	0.1129	4.1318E-04	4.666E-05	1.9083E-09	5808.1	1.291	-0.15651	0.15650995	-0.02988	0.02987755	-0.94107	0.941068217	0.13133	-0.26787	0.26787	0.933	1.036	1.001	1.094	0.9343	1.07155482
0.20	2.0	0.0415	0.4503	53.528	2699.60	1.128	0.3988	0.19102	0.0864	0.1116	3.9689E-04	4.431E-05	1.0114E-09	6013.8	1.336	-0.15620	0.156198484	-0.02949	0.0294901	-0.91290	0.912901004	0.13185	-0.35208	0.35208	0.886	1.036	1.001	1.091	0.934119	1.1284357
0.25	2.0	0.0414	0.4503	53.358	2699.08	1.203	0.3988	0.19382	0.0855	0.1101	3.7759E-04	4.158E-05	-5.9225E-09	6276.7	1.395	-0.15541	0.155412228	-0.02898	0.02897876	-0.87819	0.878187751	0.13236	-0.43160	0.43160	0.832	1.036	1.001	1.088	0.933938	1.2026751
0.30	2.0	0.0414	0.4503	53.162	2698.61	1.294	0.3988	0.19709	0.0844	0.1084	3.5625E-04	3.863E-05	-5.4332E-09	6595.1	1.466	-0.15400	0.153998689	-0.02835	0.02834989	-0.83825	0.838248399	0.13281	-0.50515	0.50515	0.773	1.036	1.001	1.084	0.933777	1.29445919

Appendix D.2: Sensitivity analysis for tension cracking

Table D.2.1: *Tension cracking, $h_{c,eff}$ = 2,5(c + φ/2): Sensitivity analysis using reverse-FORM*

Reverse FORM

Fixed variables

Fct = 2.89 Mpa

h = 450 mm

dia = 20 mm

H = 5m

Tension case

hceff = 2,5(c + phi)

Note:

1 variable changed and g solved for target beta to find As

													h=450	h=250	Sensitivity Factors				Theoretical Partial safety factors				
θ cov	**wlim**	**β**	c*	L*	ft*	θ*	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	%As	α*xi c	α*xi L	α*xi ft	α*xi θ	γ wlim	γ c	γ L	γ ft	γ θ
0.10	0.05	0.5	0.0415	49.421	2669.64	1.01	0.2576	2.288E-01	2.1676E-04	4.959E-05	-9.2741E-09	7071.520182	1.5714	2.8286	-0.64788	-0.30243	0.64488	-0.27003	0.992	1.038	1.008	0.924	1.009
0.10	0.1	0.5	0.0416	49.422	2682.16	1.01	0.2581	2.913E-01	3.3985E-04	9.901E-05	7.5466E-10	4961.851747	1.1026	1.9847	-0.68136	-0.30349	0.59562	-0.29811	0.990	1.041	1.008	0.928	1.010
0.10	0.2	0.5	0.0417	49.423	2691.40	1.01	0.2585	3.817E-01	5.1827E-04	1.978E-04	1.0015E-09	3465.421626	0.7701	1.3862	-0.70262	-0.30385	0.55942	-0.31787	0.989	1.042	1.008	0.931	1.011
0.10	0.05	1.5	0.0459	50.150	2377.24	1.04	0.2797	2.334E-01	2.0637E-04	4.816E-05	-8.6677E-09	7907.255269	1.7572	3.1629	-0.66554	-0.29906	0.62198	-0.28418	0.963	1.149	1.022	0.823	1.038
0.10	0.1	1.5	0.0463	50.149	2408.79	1.04	0.2813	2.976E-01	3.2248E-04	9.597E-05	3.3547E-12	5486.795255	1.2193	2.1947	-0.69609	-0.29873	0.57573	-0.30783	0.960	1.156	1.022	0.833	1.042
0.10	0.2	1.5	0.0465	50.147	2431.80	1.04	0.2823	3.899E-01	4.9107E-04	1.915E-04	1.7614E-12	3802.54302	0.8450	1.5210	-0.71533	-0.29823	0.54237	-0.32434	0.957	1.161	1.022	0.841	1.045
0.10	0.05	2.0	0.0484	50.507	2248.79	1.05	0.2920	2.366E-01	2.0042E-04	4.743E-05	-2.5293E-09	8351.792788	1.8560	3.3407	-0.67285	-0.29703	0.61267	-0.28928	0.949	1.210	1.030	0.778	1.054
0.10	0.1	2.0	0.0488	50.503	2287.49	1.06	0.2941	3.017E-01	3.1299E-04	9.444E-05	6.4868E-12	5765.24548	1.2812	2.3061	-0.70226	-0.29617	0.56776	-0.31107	0.944	1.221	1.030	0.792	1.059
0.10	0.2	2.0	0.0491	50.499	2315.60	1.06	0.2955	3.951E-01	4.7659E-04	1.883E-04	3.7591E-12	3980.911269	0.8846	1.5924	-0.72072	-0.29534	0.53563	-0.32623	0.942	1.227	1.030	0.801	1.062
0.15	0.05	0.5	0.0414	49.405	2676.26	1.02	0.2572	2.280E-01	2.1551E-04	4.913E-05	-7.0444E-09	7089.700435	1.5755	2.8359	-0.61942	-0.28963	0.61881	-0.38665	0.983	1.036	1.007	0.926	1.018
0.15	0.1	0.5	0.0415	49.403	2689.61	1.02	0.2576	2.900E-01	3.3785E-04	9.798E-05	-7.2061E-09	4977.878413	1.1062	1.9912	-0.64574	-0.28809	0.56641	-0.42332	0.980	1.038	1.007	0.931	1.021
0.15	0.2	0.5	0.0416	49.401	2699.37	1.02	0.2578	3.796E-01	5.1520E-04	1.956E-04	-7.6709E-09	3478.338582	0.7730	1.3913	-0.66151	-0.28650	0.52828	-0.44860	0.978	1.039	1.007	0.934	1.023
0.15	0.05	1.5	0.0456	50.100	2393.58	1.08	0.2780	2.296E-01	2.0115E-04	4.619E-05	-7.0676E-09	8030.070568	1.7845	3.2120	-0.63142	-0.28551	0.59796	-0.40279	0.924	1.140	1.021	0.828	1.083
0.15	0.1	1.5	0.0458	50.091	2427.38	1.09	0.2792	2.917E-01	3.1439E-04	9.172E-05	3.0711E-12	5582.7874	1.2406	2.2331	-0.65602	-0.28307	0.54875	-0.43403	0.917	1.146	1.021	0.840	1.090
0.15	0.2	1.5	0.0460	50.084	2451.99	1.10	0.2800	3.811E-01	4.7902E-04	1.826E-04	1.5256E-12	3873.607553	0.8608	1.5494	-0.67080	-0.28102	0.51336	-0.45555	0.913	1.150	1.021	0.848	1.096
0.15	0.05	2.0	0.0479	50.440	2268.41	1.12	0.2894	2.313E-01	1.9346E-04	4.474E-05	-8.1588E-09	8540.866146	1.8980	3.4163	-0.63637	-0.28335	0.58981	-0.40851	0.895	1.197	1.028	0.785	1.118
0.15	0.1	2.0	0.0482	50.426	2310.10	1.13	0.2911	2.934E-01	3.0223E-04	8.869E-05	4.9688E-12	5910.256364	1.3134	2.3641	-0.66044	-0.28058	0.54189	-0.43756	0.887	1.205	1.028	0.799	1.128
0.15	0.2	2.0	0.0484	50.415	2340.39	1.13	0.2921	3.828E-01	4.6060E-04	1.763E-04	2.4478E-12	4086.725962	0.9082	1.6347	-0.67490	-0.27835	0.50760	-0.45757	0.882	1.211	1.028	0.810	1.134
0.20	0.05	0.5	0.0413	49.386	2684.21	1.03	0.2566	2.270E-01	2.1402E-04	4.858E-05	3.8200E-09	7111.391388	1.5803	2.8446	-0.58556	-0.27435	0.58756	-0.48644	0.972	1.033	1.007	0.929	1.029
0.20	0.1	0.5	0.0414	49.381	2698.35	1.03	0.2569	2.885E-01	3.3551E-04	9.678E-05	-1.5146E-09	4996.732181	1.1104	1.9987	-0.60441	-0.27015	0.53228	-0.52762	0.968	1.035	1.007	0.934	1.033
0.20	0.2	0.5	0.0414	49.377	2708.54	1.04	0.2571	3.772E-01	5.1168E-04	1.930E-04	1.2532E-09	3493.287061	0.7763	1.3973	-0.61469	-0.26668	0.49259	-0.55533	0.965	1.035	1.007	0.937	1.036
0.20	0.05	1.5	0.0452	50.043	2412.63	1.14	0.2760	2.253E-01	1.9487E-04	4.389E-05	-6.3143E-09	8184.526838	1.8188	3.2738	-0.59235	-0.26988	0.57013	-0.50124	0.878	1.130	1.020	0.835	1.139
0.20	0.1	1.5	0.0454	50.026	2448.78	1.15	0.2769	2.850E-01	3.0473E-04	8.683E-05	7.8002E-09	5703.256194	1.2674	2.2813	-0.61077	-0.26527	0.51795	-0.53696	0.868	1.134	1.020	0.847	1.152
0.20	0.2	1.5	0.0455	50.012	2474.98	1.16	0.2774	3.710E-01	4.6473E-04	1.724E-04	-2.9240E-09	3962.216468	0.8805	1.5849	-0.62100	-0.26164	0.48061	-0.56118	0.862	1.137	1.020	0.856	1.160
0.20	0.05	2.0	0.0473	50.363	2290.98	1.20	0.2864	2.250E-01	1.8515E-04	4.167E-05	-8.4982E-09	8780.813869	1.9513	3.5123	-0.59510	-0.26776	0.56375	-0.50630	0.833	1.182	1.027	0.793	1.200
0.20	0.1	2.0	0.0475	50.340	2335.92	1.22	0.2877	2.840E-01	2.8949E-04	8.221E-05	-5.0820E-09	6093.428233	1.3541	2.4374	-0.61359	-0.26298	0.51263	-0.53996	0.822	1.189	1.026	0.808	1.217
0.20	0.2	2.0	0.0477	50.322	2368.54	1.23	0.2885	3.688E-01	4.4171E-04	1.629E-04	4.6140E-09	4220.295869	0.9378	1.6881	-0.62393	-0.25928	0.47614	-0.56282	0.815	1.192	1.026	0.820	1.228
0.25	0.05	0.5	0.0412	49.366	2692.78	1.04	0.2561	2.261E-01	2.1252E-04	4.804E-05	2.4039E-09	7132.348477	1.5850	2.8529	-0.54959	-0.25805	0.55402	-0.56958	0.961	1.030	1.006	0.932	1.041
0.25	0.1	0.5	0.0413	49.358	2707.49	1.05	0.2563	2.869E-01	3.3313E-04	9.557E-05	5.1390E-09	5015.692735	1.1146	2.0063	-0.56165	-0.25153	0.49668	-0.61203	0.956	1.031	1.006	0.937	1.046
0.25	0.2	0.5	0.0413	49.352	2717.94	1.05	0.2564	3.748E-01	5.0816E-04	1.905E-04	7.8970E-09	3508.203218	0.7796	1.4033	-0.56716	-0.24650	0.45612	-0.63994	0.952	1.032	1.006	0.940	1.050
0.25	0.05	1.5	0.0448	49.984	2432.51	1.21	0.2739	2.206E-01	1.8806E-04	4.149E-05	-3.6871E-09	8360.720988	1.8579	3.3443	-0.55241	-0.25380	0.54134	-0.58085	0.830	1.120	1.019	0.842	1.205
0.25	0.1	1.5	0.0449	49.959	2470.80	1.22	0.2746	2.779E-01	2.9438E-04	8.181E-05	7.3355E-10	5839.558306	1.2977	2.3358	-0.56519	-0.24719	0.48655	-0.61865	0.818	1.123	1.019	0.855	1.222
0.25	0.2	1.5	0.0450	49.941	2498.37	1.23	0.2749	3.606E-01	4.4946E-04	1.621E-04	2.2552E-09	4062.53941	0.9028	1.6250	-0.57137	-0.24216	0.44761	-0.64385	0.810	1.124	1.018	0.864	1.234
0.25	0.05	2.0	0.0467	50.286	2314.16	1.30	0.2835	2.186E-01	1.7622E-04	3.852E-05	-4.6126E-09	9057.087398	2.0127	3.6228	-0.55347	-0.25193	0.53726	-0.58443	0.770	1.167	1.025	0.801	1.298
0.25	0.1	2.0	0.0469	50.253	2362.24	1.32	0.2844	2.742E-01	2.7587E-04	7.565E-05	-1.8738E-09	6304.289597	1.4010	2.5217	-0.56680	-0.24526	0.48315	-0.62060	0.757	1.172	1.025	0.817	1.322
0.25	0.2	2.0	0.0470	50.228	2397.03	1.34	0.2849	3.545E-01	4.2165E-04	1.495E-04	8.8740E-09	4373.625865	0.9719	1.7495	-0.57342	-0.24021	0.44468	-0.64479	0.747	1.174	1.024	0.829	1.338
0.30	0.05	0.5	0.0411	49.346	2701.41	1.05	0.2555	2.251E-01	2.1110E-04	4.753E-05	1.6070E-10	7151.069573	1.5891	2.8604	-0.51385	-0.24176	0.52033	-0.63778	0.951	1.028	1.006	0.935	1.052
0.30	0.1	0.5	0.0411	49.336	2716.45	1.06	0.2556	2.854E-01	3.3096E-04	9.447E-05	-4.1160E-10	5032.511566	1.1183	2.0130	-0.52023	-0.23341	0.46188	-0.67937	0.945	1.028	1.006	0.940	1.059
0.30	0.2	0.5	0.0411	49.329	2726.98	1.06	0.2557	3.726E-01	5.0496E-04	1.882E-04	9.4884E-09	3521.620156	0.7826	1.4086	-0.52193	-0.22722	0.42116	-0.70610	0.941	1.028	1.006	0.944	1.063
0.30	0.05	1.5	0.0444	49.926	2452.00	1.28	0.2720	2.161E-01	1.8115E-04	3.914E-05	-8.8164E-09	8549.79537	1.9000	3.4199	-0.51411	-0.23826	0.51335	-0.64452	0.783	1.110	1.018	0.848	1.278
0.30	0.1	1.5	0.0445	49.896	2492.05	1.30	0.2724	2.710E-01	2.8389E-04	7.693E-05	2.3709E-09	5986.273463	1.3303	2.3945	-0.52207	-0.22996	0.45649	-0.68277	0.769	1.112	1.017	0.862	1.300
0.30	0.2	1.5	0.0445	49.873	2520.70	1.31	0.2726	3.505E-01	4.3410E-04	1.521E-04	3.7212E-09	4169.983499	0.9267	1.6680	-0.52489	-0.22379	0.41638	-0.70784	0.761	1.113	1.017	0.872	1.315
0.30	0.05	2.0	0.0461	50.212	2336.48	1.41	0.2807	2.123E-01	1.6720E-04	3.549E-05	-3.5898E-09	9357.753074	2.0795	3.7431	-0.51403	-0.23684	0.51199	-0.64618	0.710	1.154	1.024	0.808	1.409
0.30	0.1	2.0	0.0463	50.171	2387.41	1.44	0.2814	2.648E-01	2.6218E-04	6.942E-05	-8.5679E-09	6533.953539	1.4520	2.6136	-0.52292	-0.22850	0.45526	-0.68344	0.694	1.157	1.023	0.826	1.441
0.30	0.2	2.0	0.0463	50.140	2424.11	1.46	0.2816	3.407E-01	4.0155E-04	1.368E-04	-8.2132E-09	4540.544126	1.0090	1.8162	-0.52638	-0.22229	0.41511	-0.70795	0.684	1.158	1.022	0.839	1.462

Table D.2.2 *Tension cracking, $h_{c,eff} = h/2$: Sensitivity analysis using reverse-FORM*

Reverse FORM

Note:
1 variable changed and g solved for target beta to find As

Fixed variables
Fct = 2.89 Mpa
h = 250 mm
dia = 20 mm
H = 5m

hceff = h/2 cover = 40mm, dia=20mm, h=250mm

θ cov	wlim	β	c*	h*	L*	ft*	θ*	hc	Sr (m)	ε (-)	wm calc (m)	g = 0	As	%As	Sensitivity Factors α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ	Theoretical Partial safety factors γ wlim	γ c	γ h	γ L	γ ft	γ θ
0.1	0.05	0.5	0.0402	0.2500	49.535	2626.69	1.01	0.2500	0.2201	2.2445E-04	4.940E-05	-7.5760E-08	7153	2.86	-0.2026	0.0068	-0.3958	0.8156	-0.3702	0.988	1.004	1.000	1.010	0.909	1.014
0.1	0.05	1.5	0.0420	0.2500	50.542	2287.25	1.06	0.2500	0.2068	2.2726E-04	4.700E-05	6.9939E-08	8143	3.26	-0.2663	0.0011	-0.4055	0.7574	-0.4371	0.940	1.050	1.000	1.030	0.791	1.062
0.1	0.05	2	0.0432	0.2500	51.046	2151.52	1.09	0.2500	0.2024	2.2666E-04	4.587E-05	-7.6687E-08	8627	3.45	-0.2970	-0.0010	-0.4069	0.7290	-0.4633	0.917	1.081	1.000	1.041	0.744	1.092
0.15	0.05	0.5	0.0401	0.2500	49.498	2641.20	1.03	0.2500	0.2197	2.2191E-04	4.875E-05	-7.1746E-08	7171	2.87	-0.1855	0.0066	-0.3654	0.7576	-0.5080	0.975	1.003	1.000	1.009	0.914	1.027
0.15	0.05	1.5	0.0417	0.2500	50.390	2327.62	1.12	0.2500	0.2042	2.1759E-04	4.443E-05	2.2380E-08	8277	3.31	-0.2340	0.0021	-0.3642	0.6960	-0.5729	0.889	1.042	1.000	1.027	0.805	1.125
0.15	0.05	2	0.0427	0.2500	50.824	2201.07	1.18	0.2500	0.1985	2.1343E-04	4.236E-05	-8.6264E-08	8848	3.54	-0.2567	0.0006	-0.3617	0.6691	-0.5962	0.847	1.068	1.000	1.036	0.762	1.182
0.2	0.05	0.5	0.0401	0.2500	49.459	2656.59	1.04	0.2500	0.2192	2.1922E-04	4.805E-05	-7.3598E-08	7190	2.88	-0.1679	0.0064	-0.3337	0.6964	-0.6127	0.961	1.001	1.000	1.008	0.919	1.042
0.2	0.05	1.5	0.0414	0.2500	50.251	2366.10	1.20	0.2500	0.2014	2.0763E-04	4.181E-05	-4.7278E-08	8437	3.37	-0.2055	0.0029	-0.3265	0.6385	-0.6660	0.836	1.036	1.000	1.024	0.819	1.197
0.2	0.05	2	0.0423	0.2500	50.632	2246.32	1.29	0.2500	0.1942	1.9987E-04	3.882E-05	-1.4406E-08	9121	3.65	-0.2233	0.0018	-0.3225	0.6156	-0.6835	0.776	1.057	1.000	1.032	0.777	1.288
0.25	0.05	0.5	0.0400	0.2500	49.423	2671.28	1.06	0.2500	0.2185	2.1626E-04	4.726E-05	5.9443E-08	7218	2.89	-0.1517	0.0061	-0.3038	0.6384	-0.6907	0.945	1.000	1.000	1.008	0.924	1.057
0.25	0.05	1.5	0.0412	0.2500	50.133	2400.15	1.27	0.2500	0.1984	1.9789E-04	3.927E-05	-5.9487E-08	8618	3.45	-0.1821	0.0035	-0.2945	0.5883	-0.7307	0.785	1.030	1.000	1.022	0.831	1.275
0.25	0.05	2	0.0420	0.2500	50.474	2285.41	1.41	0.2500	0.1902	1.8726E-04	3.561E-05	-3.9576E-08	9412	3.76	-0.1968	0.0028	-0.2902	0.5702	-0.7429	0.712	1.049	1.000	1.029	0.791	1.405
0.3	0.05	0.5	0.0400	0.2500	49.390	2684.78	1.07	0.2500	0.2182	2.1416E-04	4.674E-05	8.4937E-10	7229	2.89	-0.1372	0.0058	-0.2770	0.5853	-0.7495	0.935	0.999	1.000	1.007	0.929	1.070
0.3	0.05	1.5	0.0410	0.2500	50.035	2429.55	1.36	0.2500	0.1955	1.8867E-04	3.689E-05	-3.3222E-08	8812	3.52	-0.1631	0.0039	-0.2677	0.5456	-0.7772	0.738	1.026	1.000	1.020	0.841	1.356
0.3	0.05	2	0.0417	0.2500	50.345	2318.36	1.53	0.2500	0.1862	1.7544E-04	3.266E-05	-5.6211E-09	9727	3.89	-0.1758	0.0034	-0.2640	0.5325	-0.7848	0.653	1.042	1.000	1.026	0.802	1.531
0.1	0.1	0.5	0.0401	0.2500	49.559	2635.02	1.02	0.2500	0.2796	3.5207E-04	9.844E-05	-5.6897E-08	5015	2.01	-0.1836	0.0014	-0.4150	0.7823	-0.4268	0.984	1.003	1.000	1.010	0.912	1.016
0.1	0.1	1.5	0.0417	0.2500	50.609	2311.66	1.07	0.2500	0.2614	3.5722E-04	9.338E-05	-8.0956E-08	5620	2.25	-0.2372	-0.0052	-0.4238	0.7201	-0.4955	0.934	1.043	1.000	1.032	0.800	1.072
0.1	0.1	2	0.0428	0.2500	51.134	2182.74	1.10	0.2500	0.2544	3.5611E-04	9.059E-05	-6.0797E-08	5926	2.37	-0.2635	-0.0076	-0.4248	0.6911	-0.5219	0.906	1.070	1.000	1.042	0.755	1.104
0.15	0.1	0.5	0.0400	0.2500	49.509	2652.93	1.03	0.2500	0.2788	3.4770E-04	9.695E-05	-6.7054E-08	5032	2.01	-0.1643	0.0017	-0.3743	0.7110	-0.5722	0.969	1.001	1.000	1.009	0.918	1.032
0.15	0.1	1.5	0.0414	0.2500	50.416	2360.08	1.14	0.2500	0.2572	3.4069E-04	8.763E-05	7.7304E-08	5735	2.29	-0.2040	-0.0031	-0.3712	0.6474	-0.6336	0.876	1.035	1.000	1.028	0.817	1.140
0.15	0.1	2	0.0423	0.2500	50.856	2242.24	1.20	0.2500	0.2486	3.3445E-04	8.314E-05	-7.8544E-08	6098	2.44	-0.2231	-0.0048	-0.3682	0.6203	-0.6556	0.831	1.057	1.000	1.037	0.776	1.204
0.2	0.1	0.5	0.0400	0.2500	49.460	2670.76	1.05	0.2500	0.2780	3.4324E-04	9.543E-05	-7.5224E-08	5049	2.02	-0.1458	0.0020	-0.3347	0.6404	-0.6757	0.954	1.000	1.000	1.008	0.924	1.049
0.2	0.1	1.5	0.0412	0.2500	50.250	2404.05	1.22	0.2500	0.2529	3.2489E-04	8.218E-05	-6.7671E-08	5861	2.34	-0.1765	-0.0015	-0.3261	0.5826	-0.7232	0.822	1.029	1.000	1.024	0.832	1.218
0.2	0.1	2	0.0419	0.2500	50.628	2294.42	1.32	0.2500	0.2424	3.1318E-04	7.592E-05	-1.7956E-08	6304	2.52	-0.1916	-0.0026	-0.3218	0.5598	-0.7392	0.759	1.047	1.000	1.032	0.794	1.317
0.25	0.1	0.5	0.0399	0.2500	49.417	2686.97	1.06	0.2500	0.2773	3.3915E-04	9.403E-05	-8.4208E-08	5066	2.03	-0.1295	0.0021	-0.2993	0.5768	-0.7490	0.940	0.998	1.000	1.007	0.930	1.064
0.25	0.1	1.5	0.0410	0.2500	50.114	2441.62	1.30	0.2500	0.2485	3.0979E-04	7.699E-05	-9.4543E-08	6002	2.40	-0.1549	-0.0004	-0.2894	0.5282	-0.7831	0.770	1.024	1.000	1.022	0.845	1.300
0.25	0.1	2	0.0416	0.2500	50.448	2338.18	1.44	0.2500	0.2363	2.9353E-04	6.938E-05	-1.2570E-08	6529	2.61	-0.1677	-0.0011	-0.2849	0.5101	-0.7940	0.694	1.040	1.000	1.028	0.809	1.442
0.3	0.1	0.5	0.0399	0.2500	49.380	2701.14	1.08	0.2500	0.2765	3.3542E-04	9.275E-05	6.8611E-09	5083	2.03	-0.1156	0.0022	-0.2690	0.5214	-0.8015	0.927	0.997	1.000	1.007	0.935	1.078
0.3	0.1	1.5	0.0408	0.2500	50.005	2473.27	1.39	0.2500	0.2442	2.9577E-04	7.221E-05	-6.2304E-08	6152	2.46	-0.1378	0.0004	-0.2595	0.4830	-0.8248	0.722	1.020	1.000	1.019	0.856	1.386
0.3	0.1	2	0.0414	0.2500	50.304	2374.54	1.57	0.2500	0.2305	2.7553E-04	6.350E-05	-3.2214E-09	6768	2.71	-0.1493	-0.0001	-0.2557	0.4695	-0.8318	0.635	1.034	1.000	1.026	0.822	1.575
0.1	0.2	0.5	0.0400	0.2500	49.576	2641.67	1.02	0.2500	0.3657	5.3696E-04	1.964E-04	-4.3964E-08	3500	1.40	-0.1542	-0.0037	-0.4292	0.7557	-0.4699	0.982	1.000	1.000	1.011	0.914	1.019
0.1	0.2	1.5	0.0413	0.2500	50.659	2330.18	1.08	0.2500	0.3402	5.4506E-04	1.854E-04	-6.7785E-08	3883	1.55	-0.1966	-0.0111	-0.4374	0.6921	-0.5393	0.927	1.034	1.000	1.033	0.806	1.079
0.1	0.2	2	0.0422	0.2501	51.200	2206.10	1.11	0.2501	0.3299	5.4423E-04	1.795E-04	-4.2045E-08	4076	1.63	-0.2174	-0.0139	-0.4384	0.6631	-0.5662	0.898	1.055	1.000	1.044	0.763	1.114
0.15	0.2	0.5	0.0400	0.2500	49.516	2662.11	1.04	0.2500	0.3645	5.2989E-04	1.932E-04	-5.2527E-08	3514	1.41	-0.1355	-0.0027	-0.3800	0.6746	-0.6182	0.966	0.999	1.000	1.009	0.921	1.036
0.15	0.2	1.5	0.0411	0.2500	50.432	2384.75	1.15	0.2500	0.3343	5.1981E-04	1.738E-04	-8.2638E-08	3966	1.59	-0.1662	-0.0079	-0.3757	0.6109	-0.6767	0.869	1.026	1.000	1.028	0.825	1.151
0.15	0.2	2	0.0418	0.2500	50.877	2273.21	1.22	0.2500	0.3215	5.1048E-04	1.641E-04	-6.8764E-08	4203	1.68	-0.1813	-0.0097	-0.3725	0.5842	-0.6978	0.821	1.044	1.000	1.037	0.787	1.219
0.2	0.2	0.5	0.0399	0.2500	49.460	2681.50	1.05	0.2500	0.3632	5.2297E-04	1.900E-04	-6.9902E-08	3528	1.41	-0.1184	-0.0019	-0.3343	0.5982	-0.7186	0.950	0.998	1.000	1.008	0.928	1.053
0.2	0.2	1.5	0.0408	0.2500	50.246	2432.31	1.23	0.2500	0.3278	4.9527E-04	1.624E-04	-1.3611E-08	4063	1.63	-0.1424	-0.0056	-0.3251	0.5416	-0.7620	0.812	1.021	1.000	1.024	0.842	1.232
0.2	0.2	2	0.0414	0.2500	50.623	2330.15	1.34	0.2500	0.3126	4.7841E-04	1.496E-04	-5.6892E-08	4352	1.74	-0.1545	-0.0068	-0.3206	0.5191	-0.7770	0.748	1.036	1.000	1.032	0.806	1.338
0.25	0.2	0.5	0.0399	0.2500	49.412	2698.43	1.07	0.2500	0.3619	5.1657E-04	1.870E-04	5.6918E-08	3543	1.42	-0.1040	-0.0013	-0.2952	0.5320	-0.7868	0.935	0.997	1.000	1.007	0.934	1.069
0.25	0.2	1.5	0.0407	0.2500	50.098	2471.93	1.32	0.2500	0.3213	4.7268E-04	1.519E-04	7.0752E-11	4168	1.67	-0.1242	-0.0039	-0.2850	0.4849	-0.8174	0.759	1.017	1.000	1.021	0.855	1.317
0.25	0.2	2	0.0412	0.2500	50.426	2376.87	1.47	0.2500	0.3038	4.4901E-04	1.364E-04	1.0940E-09	4516	1.81	-0.1347	-0.0047	-0.2806	0.4669	-0.8277	0.682	1.030	1.000	1.028	0.822	1.466
0.3	0.2	0.5	0.0398	0.2500	49.372	2712.81	1.08	0.2500	0.3610	5.1142E-04	1.846E-04	-1.7551E-09	3555	1.42	-0.0920	-0.0009	-0.2626	0.4760	-0.8342	0.923	0.996	1.000	1.007	0.939	1.083
0.3	0.2	1.5	0.0405	0.2500	49.981	2504.73	1.40	0.2500	0.3149	4.5210E-04	1.424E-04	1.3072E-09	4277	1.71	-0.1101	-0.0027	-0.2532	0.4387	-0.8552	0.712	1.014	1.000	1.019	0.867	1.405
0.3	0.2	2	0.0410	0.2500	50.273	2415.19	1.60	0.2500	0.2953	4.2240E-04	1.247E-04	7.4678E-09	4689	1.88	-0.1197	-0.0032	-0.2493	0.4248	-0.8620	0.624	1.025	1.000	1.025	0.836	1.604

APPENDIX E

SELECT DATA FOR

RELIABILITY CALIBRATION

using

REVERSE-FORM ANALYSIS

Appendix E.1: Reliability calibration of flexural cracking model

Table E.1.1: *Flexural cracking – Reliability calibration – reverse-FORM determination of crack width*

COV 0.2																							
γL	γft	%As	γθ	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	wd	g = 0	As	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ
1	1	0.25	1.128536	1.5	0.039672263	0.4500768	52.6975619	2806.981	1.0169555	0.400404494	0.0994287	0.116882686	0.287135968	0.00199196	0.000571963	0.000515412	-1.85615E-16	1125	-0.01343	-0.01137	-0.99152	0.038937	-0.12271
		0.50	1.158091	1.5	0.039978118	0.4501154	52.6467133	2788.706	1.0390078	0.400137318	0.13257789	0.105845849	0.174041436	0.00102571	0.000178516	0.000160169	-1E-08	2250	-0.04757	-0.0171	-0.9777	0.061856	-0.19422
		1.00	1.215875	1.5	0.040589324	0.4501677	52.5173825	2754.763	1.0707672	0.399578418	0.1726655	0.092500747	0.122290092	0.00051719	6.32475E-05	5.57075E-05	-1E-08	4500	-0.115	-0.02485	-0.94254	0.104824	-0.29458
		1.25	1.242158	1.5	0.040838613	0.4501877	52.4496256	2737.954	1.0822743	0.39934908	0.18682502	0.087787559	0.11289058	0.00041091	4.63873E-05	4.04166E-05	4.14572E-16	5625	-0.14222	-0.02781	-0.92413	0.1263	-0.33022
		1.50	1.267261	1.5	0.041045974	0.4502052	52.3840638	2721.088	1.0915584	0.399159198	0.19874183	0.083821115	0.106927835	0.00033862	3.62082E-05	3.11959E-05	-1E-08	6750	-0.16473	-0.0304	-0.9063	0.147982	-0.35869
		2.00	1.312764	1.5	0.041350121	0.450235	52.2645648	2687.113	1.1048872	0.398884856	0.21801114	0.077407944	0.099902007	0.00024619	2.45949E-05	2.07003E-05	9.35429E-17	9000	-0.19754	-0.03481	-0.87382	0.192067	-0.39914
		2.50	1.356376	1.5	0.041536414	0.45026	52.1623559	2652.882	1.1129425	0.398723582	0.23318342	0.07235886	0.095936626	0.00018935	1.81653E-05	1.49051E-05	-3.90618E-17	11250	-0.21752	-0.03852	-0.84604	0.237052	-0.42336
		3.00	1.40121	1.5	0.041634345	0.4502816	52.0755143	2618.586	1.1171223	0.398647257	0.24560659	0.068225003	0.093376098	0.00015077	1.40781E-05	1.12238E-05	3.92217E-16	13500	-0.22798	-0.04172	-0.82243	0.282709	-0.43585
		3.50	1.451943	1.5	0.041665078	0.4503005	52.0006846	2584.473	1.1183348	0.398635428	0.25605708	0.06474781	0.091552101	0.00012285	1.12472E-05	8.66984E-06	-1E-08	15750	-0.23126	-0.04452	-0.80209	0.328719	-0.43947
		4.00	1.510606	1.5	0.041643464	0.4503171	51.9345165	2550.817	1.1171911	0.398673626	0.26502542	0.06176389	0.090149582	0.00010171	9.16931E-06	6.78792E-06	-1E-08	18000	-0.22896	-0.04698	-0.7841	0.374712	-0.43606
1.1	1	0.25	0.991475	1.5	0.039672263	0.4500768	52.6975619	2806.981	1.0169555	0.400404494	0.0994287	0.116882686	0.287135968	0.00199196	0.000571963	0.000586662	3.58437E-16	1125	-0.01343	-0.01137	-0.99152	0.038937	-0.12271
		0.50	1.016965	1.5	0.039978118	0.4501154	52.6467133	2788.706	1.0390078	0.400137318	0.13257789	0.105845849	0.174041436	0.00102571	0.000178516	0.000182386	3.70065E-16	2250	-0.04757	-0.0171	-0.9777	0.061856	-0.19422
		1.00	1.062547	1.5	0.040589324	0.4501677	52.5173825	2754.763	1.0707672	0.399578418	0.1726655	0.092500747	0.122290092	0.00051719	6.32475E-05	6.37368E-05	-3.63004E-16	4500	-0.115	-0.02485	-0.94254	0.104824	-0.29458
		1.50	1.099387	1.5	0.041045974	0.4502052	52.3840638	2721.088	1.0915584	0.399159198	0.19874183	0.083821115	0.106927835	0.00033862	3.62082E-05	3.59504E-05	3.8105E-16	6750	-0.16473	-0.0304	-0.9063	0.147982	-0.35869
		2.00	1.129317	1.5	0.041350121	0.450235	52.2645648	2687.113	1.1048872	0.398884856	0.21801114	0.077407944	0.099902007	0.00024619	2.45949E-05	2.40717E-05	-1E-08	9000	-0.19754	-0.03481	-0.87382	0.192067	-0.39914
		2.50	1.153762	1.5	0.041536414	0.45026	52.1623559	2652.882	1.1129425	0.398723582	0.23318342	0.07235886	0.095936626	0.00018935	1.81653E-05	1.75226E-05	2.8047E-17	11250	-0.21752	-0.03852	-0.84604	0.237052	-0.42336
		3.00	1.1772	1.5	0.041634345	0.4502816	52.0755143	2618.586	1.1171223	0.398647257	0.24560659	0.068225003	0.093376098	0.00015077	1.40781E-05	1.33681E-05	-1E-08	13500	-0.22798	-0.04172	-0.82243	0.282709	-0.43585
		3.50	1.200078	1.5	0.041665078	0.4503005	52.0006846	2584.473	1.1183348	0.398635428	0.25605708	0.06474781	0.091552101	0.00012285	1.12472E-05	1.04894E-05	-1E-08	15750	-0.23126	-0.04452	-0.80209	0.328719	-0.43947
		4.00	1.22498	1.5	0.041643464	0.4503171	51.9345165	2550.817	1.1171911	0.398673626	0.26502542	0.06176389	0.090149582	0.00010171	9.16931E-06	8.37064E-06	-1E-08	18000	-0.22896	-0.04698	-0.7841	0.374712	-0.43606
1.1	1.05	0.25	0.975847	1.5	0.039672263	0.4500768	52.6975619	2806.981	1.0169555	0.400404494	0.0994287	0.116882686	0.287135968	0.00199196	0.000571963	0.000596048	1E-08	1125	-0.01343	-0.01137	-0.99152	0.038937	-0.12271
		0.50	1.000765	1.5	0.039978118	0.4501154	52.6467133	2788.706	1.0390078	0.400137318	0.13257789	0.105845849	0.174041436	0.00102571	0.000178516	0.000185338	-9.15338E-17	2250	-0.04757	-0.0171	-0.9777	0.061856	-0.19422
		1.00	1.043536	1.5	0.040589324	0.4501677	52.5173825	2754.763	1.0707672	0.399578418	0.1726655	0.092500747	0.122290092	0.00051719	6.32475E-05	6.49076E-05	-1E-08	4500	-0.115	-0.02485	-0.94254	0.104824	-0.29458
		1.50	1.076084	1.5	0.041045974	0.4502052	52.3840638	2721.088	1.0915584	0.399159198	0.19874183	0.083821115	0.106927835	0.00033862	3.62082E-05	3.67289E-05	5.95091E-17	6750	-0.16473	-0.0304	-0.9063	0.147982	-0.35869
		2.00	1.100568	1.5	0.041350121	0.450235	52.2645648	2687.113	1.1048872	0.398884856	0.21801114	0.077407944	0.099902007	0.00024619	2.45949E-05	2.46914E-05	3.00148E-16	9000	-0.19754	-0.03481	-0.87382	0.192067	-0.39914
		2.50	1.119476	1.5	0.041536414	0.45026	52.1623559	2652.882	1.1129425	0.398723582	0.23318342	0.07235886	0.095936626	0.00018935	1.81653E-05	1.80593E-05	-3.02669E-16	11250	-0.21752	-0.03852	-0.84604	0.237052	-0.42336
		3.00	1.135854	1.5	0.041634345	0.4502816	52.0755143	2618.586	1.1171223	0.398647257	0.24560659	0.068225003	0.093376098	0.00015077	1.40781E-05	1.38547E-05	-1E-08	13500	-0.22798	-0.04172	-0.82243	0.282709	-0.43585
		3.50	1.150332	1.5	0.041665078	0.4503005	52.0006846	2584.473	1.1183348	0.398635428	0.25605708	0.06474781	0.091552101	0.00012285	1.12472E-05	1.0943E-05	-1E-08	15750	-0.23126	-0.04452	-0.80209	0.328719	-0.43947
		4.00	1.163731	1.5	0.041643464	0.4503171	51.9345165	2550.817	1.1171911	0.398673626	0.26502542	0.06176389	0.090149582	0.00010171	9.16931E-06	8.80109E-06	1.77185E-09	18000	-0.22896	-0.04698	-0.7841	0.374712	-0.43606
cov 0.1																							
γL	γft	%As	γθ	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	wd	g = 0	As	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ
		0.25	1.115056	1.5	0.039673033	0.4500772	52.7183269	2806.805	1.0042699	0.400404203	0.09942866	0.116882859	0.287137815	0.00199305	0.00057228	0.000515412	1E-08	1125	-0.01352	-0.01144	-0.99717	0.039156	-0.06174
1	1	0.50	1.127216	1.5	0.039985036	0.4501173	52.6986324	2788.006	1.0098464	0.400132224	0.13257688	0.105846794	0.174056112	0.00102716	0.000178784	0.000160169	1.5084E-16	2250	-0.04834	-0.01737	-0.99181	0.062736	-0.09865
		1.00	1.160998	1.5	0.040629795	0.450174	52.635537	2752.025	1.0180946	0.399544228	0.17265608	0.092505981	0.12237336	0.00051904	6.3517E-05	5.57075E-05	-1E-08	4500	-0.11943	-0.02578	-0.97466	0.108314	-0.15289
		1.50	1.197886	1.5	0.041142765	0.4502178	52.5712125	2714.85	1.0238711	0.399075014	0.19871392	0.083834619	0.107125416	0.00034079	3.65077E-05	3.11959E-05	1E-08	6750	-0.17445	-0.03213	-0.95309	0.155368	-0.18979
		2.00	1.235392	1.5	0.041497084	0.4502529	52.4879428	2677.333	1.0276728	0.398755831	0.21796262	0.077430098	0.100200856	0.00024834	2.48843E-05	2.07003E-05	9.35497E-17	9000	-0.21239	-0.03731	-0.93055	0.203986	-0.21439
		2.50	1.274193	1.5	0.04172291	0.4502824	52.4062678	2639.613	1.0300627	0.398559516	0.23311561	0.072388938	0.096314964	0.00019143	1.84377E-05	1.49051E-05	-3.90618E-17	11250	-0.23641	-0.04166	-0.90844	0.253559	-0.22981
		3.00	1.316697	1.5	0.041846641	0.4503074	52.327324	2602.212	1.0313593	0.398460779	0.24552357	0.068261285	0.093806064	0.00015275	1.4329E-05	1.12238E-05	3.92217E-16	13500	-0.24952	-0.04535	-0.88707	0.303417	-0.23816
		3.50	1.366733	1.5	0.041889642	0.4503285	52.2506233	2565.595	1.0317886	0.398438896	0.25596409	0.064788151	0.092006352	0.00012472	1.14746E-05	8.66984E-06	-1E-08	15750	-0.25406	-0.04846	-0.86631	0.352929	-0.24093
		4.00	1.42577	1.5	0.04186848	0.4503462	52.1752016	2530.156	1.0315173	0.398477716	0.26492787	0.061806107	0.090604305	0.00010345	9.37261E-06	6.78792E-06	-1E-08	18000	-0.25183	-0.05107	-0.8459	0.401525	-0.23918
cov 0.3																							
γL	γft	%As	γθ	β	c*	h*	L*	ft*	θ*	d	x	hc	Sr (m)	ε (-)	wm calc (m)	wd	g = 0	As	α*xi c	α*xi h	α*xi L	α*xi ft	α*xi θ
		0.25	1.15136288	1.5	0.039671524	0.4500763	52.6793197	2807.134	1.0380483	0.400404784	0.09942874	0.116882523	0.2871342	0.001991	0.000571685	0.000515412	1E-08	1125	-0.01329	-0.01126	-0.98235	0.038581	-0.1822
1	1	0.50	1.208043	1.5	0.039967338	0.4501126	52.5654652	2789.8	1.0863759	0.400145253	0.13257947	0.105844375	0.174018564	0.00102344	0.000178097	0.000160169	-1E-08	2250	-0.04637	-0.01668	-0.95561	0.060479	-0.28411
		1.00	1.302071	1.5	0.040531492	0.4501587	52.3467214	2758.73	1.1537645	0.399627246	0.17267896	0.092493259	0.122171099	0.00051452	6.28596E-05	5.57075E-05	-1E-08	4500	-0.10867	-0.02352	-0.89615	0.099776	-0.41784
		1.50	1.37307	1.5	0.040926693	0.4501896	52.1516218	2728.863	1.1952108	0.399262882	0.19877619	0.083804461	0.106684338	0.00033593	3.58382E-05	3.11959E-05	-1.6839E-16	6750	-0.15179	-0.02809	-0.84312	0.13797	-0.49627
		2.00	1.430147	1.5	0.04117621	0.4502136	51.9962798	2698.94	1.220378	0.39903739	0.2180685	0.077381701	0.099548353	0.0002436	2.42502E-05	2.07003E-05	-1E-08	9000	-0.17881	-0.03164	-0.80089	0.176657	-0.54258
		2.50	1.47956	1.5	0.041322509	0.450234	51.8769312	2668.57	1.2349023	0.398911508	0.23326107	0.072324315	0.095502673	0.00018691	1.785E-05	1.49051E-05	-1E-08	11250	-0.19457	-0.03467	-0.76845	0.216364	-0.56887
		3.00	1.526337	1.5	0.041395922	0.4502522	51.7858798	2637.671	1.2420455	0.39885632	0.24569966	0.068184196	0.092893206	0.00014848	1.37929E-05	1.12238E-05	3.92217E-16	13500	-0.20246	-0.03737	-0.7437	0.257228	-0.58169
		3.50	1.577879	1.5	0.041415916	0.4502689	51.7159232	2606.322	1.2437314	0.398853024	0.25616001	0.064702975	0.091048082	0.00012072	1.09911E-05	8.66984E-06	-1E-08	15750	-0.2046	-0.03984	-0.72468	0.299181	-0.5847
		4.00	1.636266	1.5	0.041394951	0.4502844	51.6611867	2574.689	1.2411388	0.398889454	0.26513286	0.061717181	0.089647366	9.9734E-05	8.94085E-06	6.78792E-06	-1E-08	18000	-0.20235	-0.04213	-0.7098	0.342027	-0.5800652

Appendix E.2: Reliability calibration of tension cracking model

Table E.2.1: *Tension cracking, $h_{c,eff} = 2,5(c + \varphi/2)$ – Reliability calibration – reverse-FORM determination of crack width*

cov 0.2																	
γ_c	γ_{ft}	%As	γ_θ	β	c*	L*	ft*	θ*	hc	Sr (m)	ε (-)	w calc (m)	wd	g = 0	α*xi c	α*xi L	α*xi ft	α*xi θ
1	1	0.25	1.38752	1.5	0.045607315	49.985266	2526.515694	1.17581545	0.278036573	1.870648699	0.0032767	0.006129552	0.00519432	-1E-08	-0.6330607	-0.254235	0.4082995	-0.60654
		0.50	1.39656	1.5	0.045593092	49.99039682	2516.774342	1.172829333	0.277965459	0.980675652	0.00160666	0.00157561	0.001323207	-1.000E-08	-0.6316745	-0.255629	0.4218542	-0.598064
		1.00	1.41678	1.5	0.045551637	50.00068516	2497.39697	1.166887896	0.277758185	0.535516371	0.00077216	0.000413507	0.000340578	-1E-08	-0.6276316	-0.258426	0.4489738	-0.581134
		1.50	1.44044	1.5	0.045494786	50.01080729	2478.215574	1.160937059	0.277473931	0.386961766	0.00049444	0.000191328	0.00015421	-1E-08	-0.6220812	-0.261178	0.4760271	-0.564092
		2.00	1.46830	1.5	0.045424294	50.02056915	2459.292396	1.154934764	0.277121469	0.312545763	0.00035587	0.000111227	8.74957E-05	-1E-08	-0.6151894	-0.263831	0.5029222	-0.546813
		2.50	1.50134	1.5	0.045341481	50.02981654	2440.685124	1.148853902	0.276707403	0.267775699	0.00027296	7.30919E-05	5.59378E-05	-1E-08	-0.6070793	-0.266345	0.5295708	-0.529216
		3.00	1.54057	1.5	0.045247396	50.03842426	2422.447232	1.142677875	0.276236981	0.237821182	0.00021785	5.18097E-05	3.84285E-05	4.29724E-16	-0.5978474	-0.268685	0.5558883	-0.511249
		3.50	1.58843	1.5	0.045142919	50.04628931	2404.628175	1.136397759	0.275714597	0.216326798	0.00017862	3.86406E-05	2.76506E-05	-1E-08	-0.5875733	-0.270823	0.5817935	-0.492878
		4.00	1.64594	1.5	0.045028822	50.0533265	2387.273457	1.130010399	0.27514411	0.20011529	0.0001493	2.98778E-05	2.05125E-05	-1.1882E-16	-0.5763259	-0.272736	0.6072088	-0.47409
1.1	1.25	0.25	1.159123173	1.5	0.045607315	49.98526604	2526.515694	1.17581545	0.27803657	1.870648699	0.0032767	0.006129552	0.006217814	1E-08	-0.6330607	-0.254235	0.4082995	-0.60654
		0.50	1.156703551	1.5	0.045593092	49.99039682	2516.774342	1.172829333	0.27796546	0.980675652	0.00160666	0.00157561	0.001597576	6.20164E-17	-0.6316745	-0.255629	0.4218542	-0.598064
		1.00	1.152352732	1.5	0.045551637	50.00068516	2497.39697	1.166887896	0.277758185	0.535516371	0.00077216	0.000413507	0.000418722	-3.2564E-16	-0.6276316	-0.258426	0.4489738	-0.581134
		1.50	1.148700521	1.5	0.045494786	50.01080729	2478.215574	1.160937059	0.277473931	0.386961766	0.00049444	0.000191328	0.000193366	3.02492E-16	-0.6220812	-0.261178	0.4760271	-0.564092
		2.00	1.1457271	1.5	0.045424294	50.02056915	2459.292396	1.154934764	0.277121469	0.312545763	0.00035587	0.000111227	0.000112112	1E-08	-0.6151894	-0.263831	0.5029222	-0.546813
		2.50	1.143652133	1.5	0.045341481	50.02981654	2440.685124	1.148853902	0.276707403	0.267775699	0.00027296	7.30919E-05	7.34156E-05	1E-08	-0.6070793	-0.266345	0.5295708	-0.529216
		3.00	1.142724867	1.5	0.045247396	50.03842426	2422.447232	1.142677875	0.276236981	0.237821182	0.00021785	5.18097E-05	5.18076E-05	3.64699E-17	-0.5978474	-0.268685	0.5558883	-0.511249
		3.50	1.142724867	1.5	0.045142919	50.04628931	2404.628175	1.136397759	0.275714597	0.216326798	0.00017862	3.86406E-05	3.84256E-05	1.2399E-09	-0.5875733	-0.270823	0.5817935	-0.492878
		4.00	1.144044739	1.5	0.045028822	50.0533265	2387.273457	1.130010399	0.275144112	0.20011529	0.0001493	2.98778E-05	2.95113E-05	2.82712E-16	-0.5763259	-0.272736	0.6072088	-0.47409
1.1	1.15	0.25	1.2234347	1.5	0.045607315	49.98526604	2526.515694	1.17581545	0.278036573	1.870648699	0.0032767	0.006129552	0.005890983	-1E-08	-0.6330607	-0.254235	0.4082995	-0.60654
		0.50	1.224512356	1.5	0.045593092	49.99039682	2516.774342	1.172829333	0.277965459	0.980675652	0.00160666	0.00157561	0.001509108	-1.9776E-16	-0.6316745	-0.255629	0.4218542	-0.598064
		1.00	1.227702895	1.5	0.045551637	50.00068516	2497.39697	1.166887896	0.277758185	0.535516371	0.00077216	0.000413507	0.000393032	-1E-08	-0.6276316	-0.258426	0.4489738	-0.581134
		1.50	1.232380544	1.5	0.045494786	50.01080729	2478.215574	1.160937059	0.277473931	0.386961766	0.00049444	0.000191328	0.000180245	-1E-08	-0.6220812	-0.261178	0.4760271	-0.564092
		2.00	1.238665962	1.5	0.045424294	50.02056915	2459.292396	1.154934764	0.277121469	0.312545763	0.00035587	0.000111227	0.000103708	2.4278E-16	-0.6151894	-0.263831	0.5029222	-0.546813
		2.50	1.247113508	1.5	0.045341481	50.02981654	2440.685124	1.148853902	0.276707403	0.267775699	0.00027296	7.30919E-05	6.7341E-05	-1E-08	-0.6070793	-0.266345	0.5295708	-0.529216
		3.00	1.25755336	1.5	0.045247396	5.004E+01	2.422E+03	1.14267788	0.276236981	0.237821182	0.00021785	5.18097E-05	4.7077E-05	-3.1548E-16	-0.5978474	-0.268685	0.5558883	-0.511249
		3.50	1.27084736	1.5	0.045142919	5.005E+01	2.405E+03	1.13639776	0.275714597	0.216326798	0.00017862	3.86406E-05	3.45526E-05	1.29616E-16	-0.5875733	-0.270823	0.5817935	-0.492878
		4.00	1.28737646	1.5	0.045028822	5.005E+01	2.387E+03	1.1300104	0.275144112	0.20011529	0.0001493	2.98778E-05	2.62256E-05	-2.6365E-16	-0.5763259	-0.272736	0.6072088	-0.47409
1.15	1.2	0.25	1.173306656	1.5	0.045607315	49.98526604	2526.515694	1.17581545	0.27803657	1.870648699	0.0032767	0.006129552	0.00614265	1E-08	-0.6330607	-0.254235	0.4082995	-0.60654
		0.50	1.172652954	1.5	0.045593092	49.99039682	2516.774342	1.172829333	0.27796546	0.980675652	0.00160666	0.00157561	0.001575838	1E-08	-0.6316745	-0.255629	0.4218542	-0.598064
		1.00	1.172223022	1.5	0.045551637	50.00068516	2497.39697	1.166887896	0.277758185	0.535516371	0.00077216	0.000413507	0.000411616	1E-08	-0.6276316	-0.258426	0.4489738	-0.581134
		1.50	1.173136334	1.5	0.045494786	50.01080729	2478.215574	1.160937059	0.277473931	0.386961766	0.00049444	0.000191328	0.000189347	-1E-08	-0.6220812	-0.261178	0.4760271	-0.564092
		2.00	1.175378692	1.5	0.045424294	50.02056915	2459.292396	1.154934764	0.277121469	0.312545763	0.00035587	0.000111227	0.000109301	-1E-08	-0.6151894	-0.263831	0.5029222	-0.546813
		2.50	1.17902163	1.5	0.045341481	50.02981654	2440.685124	1.148853902	0.276707403	0.267775699	0.00027296	7.30919E-05	7.12216E-05	-8.8037E-17	-0.6070793	-0.266345	0.5295708	-0.529216
		3.00	1.184498378	1.5	0.045247396	50.03842426	2422.447232	1.142677875	0.276236981	0.237821182	0.00021785	5.18097E-05	4.99805E-05	3.2316E-17	-0.5978474	-0.268685	0.5558883	-0.511249
		3.50	1.191986708	1.5	0.045142919	50.04628931	2404.628175	1.136397759	0.275714597	0.216326798	0.00017862	3.86406E-05	3.68386E-05	-4.7133E-16	-0.5875733	-0.270823	0.5817935	-0.492878
		4.00	1.201828727	1.5	0.045028822	50.0533265	2387.273457	1.130010399	0.275144112	0.20011529	0.0001493	2.98778E-05	2.80924E-05	2.17254E-16	-0.5763259	-0.272736	0.6072088	-0.47409
1.15	1.15	0.25	1.209319728	1.5	0.045607315	49.98526604	2526.515694	1.17581545	0.278036573	1.870648699	0.0032767	0.006129552	0.005959725	1E-08	-0.6330607	-0.254235	0.4082995	-0.60654
		0.50	1.210672043	1.5	0.045593092	49.99039682	2516.774342	1.172829333	0.277965459	0.980675652	0.00160666	0.00157561	0.00152636	3.15286E-16	-0.6316745	-0.255629	0.4218542	-0.598064
		1.00	1.214587067	1.5	0.045551637	50.00068516	2497.39697	1.166887896	0.277758185	0.535516371	0.00077216	0.000413507	0.000397268	-2.3766E-16	-0.6276316	-0.258426	0.4489738	-0.581134
		1.50	1.220308877	1.5	0.045494786	50.01080729	2478.215574	1.160937059	0.277473931	0.386961766	0.00049444	0.000191328	0.000182028	-1E-08	-0.6220812	-0.261178	0.4760271	-0.564092
		2.00	1.227985445	1.5	0.045424294	50.02056915	2459.292396	1.154934764	0.277121469	0.312545763	0.00035587	0.000111227	0.000104619	-1E-08	-0.6151894	-0.263831	0.5029222	-0.546813
		2.50	1.237929794	1.5	0.045341481	50.02981654	2440.685124	1.148853902	0.276707403	0.267775699	0.00027296	7.30919E-05	6.78406E-05	-1E-08	-0.6070793	-0.266345	0.5295708	-0.529216
		3.00	1.25051403	1.5	0.045247396	5.004E+01	2.422E+03	1.14267788	0.276236981	0.237821182	0.00021785	5.18097E-05	4.735E-05	-1E-08	-0.5978474	-0.268685	0.5558883	-0.511249
		3.50	1.26592871	1.5	0.045142919	5.005E+01	2.405E+03	1.13639776	0.275714597	0.216326798	0.00017862	3.86406E-05	3.46868E-05	-2.9207E-16	-0.5875733	-0.270823	0.5817935	-0.492878
		4.00	1.28527807	1.5	0.045028822	5.005E+01	2.387E+03	1.1300104	0.275144112	0.20011529	0.0001493	2.98778E-05	2.62684E-05	3.21486E-16	-0.5763259	-0.272736	0.6072088	-0.47409

Table E.2.2 *Tension cracking, $h_{c,eff}$ = h/2 – Reliability calibration – reverse-FORM determination of crack width*

COV 0.2

γ_c	γ_{ft}	%As	Asls	γ_θ	β	c*	h*	L*	ft*	θ*	hc	Sr (m)	ε (-)	wm calc (m	wd (m)	g = 0	$\alpha^*_{xi\,c}$	$\alpha^*_{xi\,h}$	$\alpha^*_{xi\,L}$	$\alpha^*_{xi\,ft}$	$\alpha^*_{xi\,\theta}$
1	**1**	0.25	625	1.4444	1.5	3.982E-02	0.250061	50.227012	2491.410844	1.25697615	0.25006096	1.680021502	0.003559069	0.005979	0.00520333	1E-07	-0.02947	-0.0162568	-0.319949	0.4573941	-0.82903
		0.50	1250	1.4545	1.5	4.005E-02	0.250052	50.232224	2480.055678	1.25286018	0.25005234	0.880272597	0.001748364	0.001539	0.00132565	1E-07	-0.05584	-0.0139573	-0.321366	0.4734227	-0.818097
		1.00	2500	1.4782	1.5	4.045E-02	0.250037	50.239915	2458.231511	1.2439171	0.25003698	0.480967968	0.000843225	0.000406	0.00034129	-3.5237E-15	-0.1002	-0.0098615	-0.323456	0.504436	-0.794218
		1.50	3750	1.5065	1.5	4.077E-02	0.250024	50.245016	2437.394457	1.2343909	0.25002385	0.348236562	0.000541725	0.000189	0.000154574	-1.0872E-15	-0.13499	-0.00636	-0.324843	0.5343046	-0.768592
		2.00	5000	1.5399	1.5	4.102E-02	0.250013	50.24833	2417.388976	1.22455724	0.25001261	0.28204908	0.000391145	0.00011	8.77297E-05	2.20895E-15	-0.16186	-0.0033613	-0.325744	0.5632225	-0.741931
		2.50	6250	1.5792	1.5	4.121E-02	0.250003	50.250306	2398.126458	1.21457106	0.25000294	0.242417715	0.000300933	7.3E-05	5.61073E-05	-2.6405E-15	-0.18223	-0.0007837	-0.326281	0.5912934	-0.714637
		3.00	7500	1.6256	1.5	4.135E-02	0.249995	50.251175	2379.56353	1.20451667	0.24999459	0.216024935	0.000240901	5.2E-05	3.85608E-05	2.28802E-15	-0.19723	0.0014423	-0.326517	0.6185589	-0.686928
		3.50	8750	1.6807	1.5	4.145E-02	0.249987	50.25104	2361.685612	1.19443898	0.24998735	0.197170841	0.000198107	3.91E-05	2.77591E-05	-2.0708E-15	-0.20777	0.0033734	-0.32648	0.6450201	-0.658922
		4.00	10000	1.7471	1.5	4.151E-02	0.249981	50.249933	2344.495366	1.18436194	0.24998104	0.183010488	0.000166081	3.04E-05	2.06047E-05	-1.6551E-16	-0.21459	0.005056	-0.32618	0.6706531	-0.630681
1	**1.3**	0.25	625	1.2256	1.5	3.982E-02	0.250061	50.227012	2491.410844	1.25697615	0.25006096	1.680021502	0.003559069	0.005979	0.006132388	3.73746E-15	-0.02947	-0.0162568	-0.319949	0.4573941	-0.82903
		0.50	1250	1.2222	1.5	4.005E-02	0.250052	50.232224	2480.055678	1.25286018	0.25005234	0.880272597	0.001748364	0.001539	0.001577542	1E-07	-0.05584	-0.0139573	-0.321366	0.4734227	-0.818097
		1.00	2500	1.2166	1.5	4.045E-02	0.250037	50.239915	2458.231511	1.2439171	0.25003698	0.480967968	0.000843225	0.000406	0.000414661	-2.6991E-15	-0.1002	-0.0098615	-0.323456	0.504436	-0.794218
		1.50	3750	1.2119	1.5	4.077E-02	0.250024	50.245016	2437.394457	1.2343909	0.25002385	0.348236562	0.000541725	0.000189	0.000192151	1.24632E-15	-0.13499	-0.00636	-0.324843	0.5343046	-0.768592
		2.00	5000	1.2078	1.5	4.102E-02	0.250013	50.24833	2417.388976	1.22455724	0.25001261	0.28204908	0.000391145	0.00011	0.000111856	-1.6596E-15	-0.16186	-0.0033613	-0.325744	0.5632225	-0.741931
		2.50	6250	1.2041	1.5	4.121E-02	0.250003	50.250306	2398.126458	1.21457106	0.25000294	0.242417715	0.000300933	7.3E-05	7.35856E-05	-5.0882E-16	-0.18223	-0.0007837	-0.326281	0.5912934	-0.714637
		3.00	7500	1.1989	1.5	4.135E-02	0.249995	50.251175	2379.56353	1.20451667	0.24999459	0.216024935	0.000240901	5.2E-05	5.22E-05	1E-07	-0.19723	0.0014423	-0.326517	0.6185589	-0.686928
		3.50	8750	1.1989	1.5	4.145E-02	0.249987	50.25104	2361.685612	1.19443898	0.24998735	0.197170841	0.000198107	3.91E-05	3.89465E-05	-3.7739E-08	-0.20777	0.0033734	-0.32648	0.6450201	-0.658922
		4.00	10000	1.1922	1.5	4.151E-02	0.249981	50.249933	2344.495366	1.18436194	0.24998104	0.183010488	0.000166081	3.04E-05	3.0112E-05	1E-07	-0.21459	0.005056	-0.32618	0.6706531	-0.630681
1.05	**1.2**	0.25	625	1.2764	1.5	3.982E-02	0.250061	50.227012	2491.410844	1.25697615	0.25006096	1.680021502	0.003559069	0.005979	0.005888303	-1E-07	-0.02947	-0.0162568	-0.319949	0.4573941	-0.82903
		0.50	1250	1.2732	1.5	4.005E-02	0.250052	50.232224	2480.055678	1.25286018	0.25005234	0.880272597	0.001748364	0.001539	0.001514425	-2.2963E-16	-0.05584	-0.0139573	-0.321366	0.4734227	-0.818097
		1.00	2500	1.2689	1.5	4.045E-02	0.250037	50.239915	2458.231511	1.2439171	0.25003698	0.480967968	0.000843225	0.000406	0.000397566	-2.6366E-15	-0.1002	-0.0098615	-0.323456	0.504436	-0.794218
		1.50	3750	1.2663	1.5	4.077E-02	0.250024	50.245016	2437.394457	1.2343909	0.25002385	0.348236562	0.000541725	0.000189	0.00018381	1E-07	-0.13499	-0.00636	-0.324843	0.5343046	-0.768592
		2.00	5000	1.2663	1.5	4.102E-02	0.250013	50.24833	2417.388976	1.22455724	0.25001261	0.28204908	0.000391145	0.00011	0.000106656	3.23585E-08	-0.16186	-0.0033613	-0.325744	0.5632225	-0.741931
		2.50	6250	1.2680	1.5	4.121E-02	0.250003	50.250306	2398.126458	1.21457106	0.25000294	0.242417715	0.000300933	7.3E-05	6.9876E-05	-2.8712E-15	-0.18223	-0.0007837	-0.326281	0.5912934	-0.714637
		3.00	7500	1.2710	1.5	4.135E-02	0.249995	50.251175	2379.56353	1.20451667	0.24999459	0.216024935	0.000240901	5.2E-05	4.9319E-05	1.1139E-15	-0.19723	0.0014423	-0.326517	0.6185589	-0.686928
		3.50	8750	1.2756	1.5	4.145E-02	0.249987	50.25104	2361.685612	1.19443898	0.24998735	0.197170841	0.000198107	3.91E-05	3.65767E-05	1.18196E-15	-0.20777	0.0033734	-0.32648	0.6450201	-0.658922
		4.00	10000	1.2855	1.5	4.151E-02	0.249981	50.249933	2344.495366	1.18436194	0.24998104	0.183010488	0.000166081	3.04E-05	2.80815E-05	-1E-07	-0.21459	0.005056	-0.32618	0.6706531	-0.630681

COV 0.1

γ_c	γ_{ft}	%As	Asls	γ_θ	β	c*	h*	L*	ft*	θ*	hc	Sr (m)	ε (-)	wm calc (m	wd (m)	g = 0	$\alpha^*_{xi\,c}$	$\alpha^*_{xi\,h}$	$\alpha^*_{xi\,L}$	$\alpha^*_{xi\,ft}$	$\alpha^*_{xi\,\theta}$
1	**1**	0.25	625	1.3112	1.5	3.995E-02	0.250103	50.747499	2372.918545	1.09265847	0.25010321	1.680561747	0.003715319	0.006244	0.00520333	-1.6896E-15	-0.04451	-0.0275224	-0.461434	0.6283714	-0.624091
		0.50	1250	1.3230	1.5	4.030E-02	0.250089	50.733443	2363.395852	1.09010562	0.25008909	0.880890763	0.001826373	0.001609	0.00132565	-1E-07	-0.08352	-0.0237579	-0.457613	0.6424806	-0.608497
		1.00	2500	1.3503	1.5	4.088E-02	0.250064	50.699747	2346.545276	1.0847592	0.25006427	0.481859291	0.000881651	0.000425	0.00034129	-3.5237E-15	-0.14653	-0.017138	-0.448453	0.6675872	-0.57572
		1.50	3750	1.3823	1.5	4.130E-02	0.250044	50.663119	2331.678786	1.0794071	0.25004368	0.349320336	0.000566689	0.000198	0.000154574	-1.0872E-15	-0.19253	-0.0116493	-0.438497	0.6898876	-0.542746
		2.00	5000	1.4194	1.5	4.161E-02	0.250027	50.626589	2318.071527	1.07424484	0.25002673	0.283236293	0.000409259	0.000116	8.77297E-05	2.20898E-15	-0.22511	-0.0071289	-0.428567	0.7104242	-0.510786
		2.50	6250	1.4621	1.5	4.182E-02	0.250013	50.591469	2305.325977	1.06934318	0.25001276	0.243639719	0.00031488	7.67E-05	5.61073E-05	-2.6405E-15	-0.24731	-0.0034028	-0.41902	0.7297698	-0.480297
		3.00	7500	1.5117	1.5	4.195E-02	0.250001	50.558168	2293.240889	1.06471175	0.25000119	0.217232876	0.000252037	5.48E-05	3.85608E-05	2.28801E-15	-0.26149	-0.0003173	-0.409968	0.7482121	-0.451361
		3.50	8750	1.5734	1.5	4.202E-02	0.249992	50.526667	2281.72363	1.0603342	0.24999155	0.198331442	0.000207214	4.11E-05	2.77591E-05	-1E-07	-0.26947	0.0022531	-0.401405	0.7658784	-0.423894
		4.00	10000	1.6387	1.5	0.0420543	0.249983	50.496772	2270.738192	1.05618565	0.24998347	0.184102019	0.000173649	3.2E-05	2.06047E-05	-1.6552E-16	-0.27259	0.0044078	-0.393278	0.7828124	-0.39776

COV 0.3

	γ_{ft}	%As	Asls	γ_θ	β	c*	h*	L*	ft*	θ*	hc	Sr (m)	ε (-)	wm calc (m	wd (m)	g = 0	$\alpha^*_{xi\,c}$	$\alpha^*_{xi\,h}$	$\alpha^*_{xi\,L}$	$\alpha^*_{xi\,ft}$	$\alpha^*_{xi\,\theta}$
1	**1**	0.25	625	1.6056	1.5	0.0397434	0.250041	49.92981	2567.203458	1.4362461	0.25004063	1.67974693	0.003463087	0.005817	0.00520333	1E-07	-0.0214012	-0.01083563	-0.23915997	0.3522433	-0.904517
		0.50	1250	1.6148	1.5	0.0399164	0.250034	49.939354	2556.394409	1.43156823	0.25003446	0.879943096	0.001699266	0.001495	0.00132565	-1E-07	-0.0407	-0.0091899	-0.241754	0.3670479	-0.897268
		1.00	2500	1.6361	1.5	0.0402143	0.250023	49.957429	2534.931698	1.42119937	0.25002339	0.480466035	0.000817762	0.000393	0.00034129	-3.5237E-15	-0.07375	-0.0062371	-0.246668	0.3966309	-0.881114
		1.50	3750	1.6621	1.5	0.0404568	0.250014	49.974442	2513.649635	1.40977899	0.25001377	0.347594946	0.000524293	0.000182	0.000154574	-1.0872E-15	-0.10047	-0.0036721	-0.251292	0.4262131	-0.863184
		2.00	5000	1.6932	1.5	0.0406527	0.250005	49.99059	2492.544044	1.39753656	0.25000536	0.281309775	0.000377834	0.000106	8.77297E-05	2.20895E-15	-0.12194	-0.0014291	-0.255682	0.4557984	-0.843803
		2.50	6250	1.7302	1.5	0.0408094	0.249998	50.005952	2471.634709	1.38460706	0.24999795	0.241617454	0.000290173	7.01E-05	5.61073E-05	-2.6405E-15	-0.13903	0.0005455	-0.259858	0.4853566	-0.823148
		3.00	7500	1.7744	1.5	0.0409325	0.249991	50.020534	2450.958555	1.37107075	0.24999139	0.215193811	0.000231906	4.99E-05	3.85608E-05	2.28802E-15	-0.15243	0.0022948	-0.263822	0.514832	-0.801316
		3.50	8750	1.8276	1.5	0.0410269	0.249986	50.034296	2430.56452	1.35697788	0.24998555	0.196332869	0.000190428	3.74E-05	2.77591E-05	-2.0708E-15	-0.16266	0.0038531	-0.267563	0.5441497	-0.778357
		4.00	10000	1.8923	1.5	0.0410962	0.24998	50.047171	2410.509719	1.34236391	0.24998032	0.182184525	0.000159434	2.9E-05	2.06047E-05	-1.6552E-16	-0.17016	0.0052478	-0.271062	0.5732205	-0.754295