THE RELATIONSHIPS BETWEEN THE PRICE-EARNINGS RATIO AND SELECTED RISK AND RETURN AND VALUATION MODELS

By

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Assignment presented in partial fulfilment of the requirements for the degree of Master in Accounting.

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Date:

December 2002

Declaration

I, the undersigned, hereby declare that the work contained in this assignment is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

Lastly, as a result of accounting changes and the different accounting rules in force today, the price-earnings ratio also assists in the identification and elimination of the effects of accounting on investment decisions.

It is apparent that the price-earnings ratio possesses the capabilities to assist investors significantly with the analysis of investment opportunities.

Summary

The price-earnings ratio is one of a series of benchmarks developed after the Great Depression, to measure the fair value of shares on a relative basis. It originated from the idea that investors buy the earnings of a company and that the price-earnings ratio provides a consensus indication of the future growth potential of a company. Therefore, the price-earnings ratio is a rating of a company's future profitability.

The price-earnings ratio developed, over the years, firstly, into an indicator of the relative risk associated with a company as the market anomalies associated with the ratio were investigated and clarified, and the theoretical background of the ratio integrated with the portfolio theory. It is now clear that the price-earnings ratio can be a useful indicator of the risk associated with an investment and the uncertainty associated with the duration of the growth phase of a company.

Secondly, the price-earnings ratio is also a growth and valuation model with a theoretical background that can be linked to popular dividend discount models and the growth opportunities approach to investment valuation. With the use of the price-earnings ratio it is easy to visualise the relative profitability and the total investment required to raise a company's rating of future profitability. This simplicity allows one the opportunity to evaluate the reasonableness and likelihood of the investment reaching its projected potential profit targets.

visualiseer. Hierdie eenvoud verskaf die geleentheid om die redelikheid en die waarskynlikheid van 'n belegging om sy voorsiene winsgewendheidsdoelwitte te bereik, te evalueer.

Laastens, as 'n resultaat van die rekeningkundige veranderinge, en die verskillende rekeningundige reëls huidiglik van toepassing in die wêreld, help die prys-verdienste verhouding ook met die identifikasie en die eliminasie van rekeningkundige komplikasies op beleggingsbesluite.

Dit is duidelik dat die prys-verdienste verhouding die vermoë het om die belegger by te staan met die ontleding van beleggingsgeleenthede.

Opsomming

Die prys-verdienste verhouding is een van 'n reeks relatiewe maatstawwe ontwikkel na die Groot Depressie om die redelike waarde van aandele te bepaal. Dit is gebaseer op die idee dat beleggers die winste van 'n maatskappy koop en dat die prys-verdienste verhouding 'n konsensus aanduiding verskaf van die toekomstige groeipotensiaal van 'n maatskappy. As gevolg hiervan is die prys-verdienste verhouding 'n aanduiding van die relatiewe toekomstige winsgewendheid van 'n maatskappy.

Die prys-verdienste verhouding het oor die jare ontwikkel, eerstens as 'n aanwyser van die relatiewe risiko verbonde aan 'n maatskappy soos abnormaliteite wat daaraan verwant is ondersoek en verklaar is, en die teorieë onderliggend aan die verhouding ontwikkel het saam met die portefeulje teorie. Dit is nou duidelik dat die prys-verdienste verhouding 'n bruikbare aanduider is van die risiko wat geassosieer word met 'n belegging en die onsekerheid wat gepaard gaan met die duur van die groeifase van 'n maatskappy.

Tweedens is die prys-verdienste verhouding ook 'n waardasie- en groeimodel met 'n teoretiese agtergrond wat verband hou met die populêre dividend verdiskonteringsmodelle en die groeigeleenthede-benadering tot waardasie. Met die gebruik van die prys-verdienste verhouding is dit maklik om die relatiewe winsgewendheid en die totale belegging wat benodig word om die waarde van die relatiewe winsgewendheid van 'n maatskappy te verhoog, te

I would like to thank everyone that made the completion of this project possible for me, or assisted me with this project, specifically Professor William Brown, Professor Willie de Jager and Mrs Liza Jacobs of the University of Stellenbosch.

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Chapter 1: Introduction

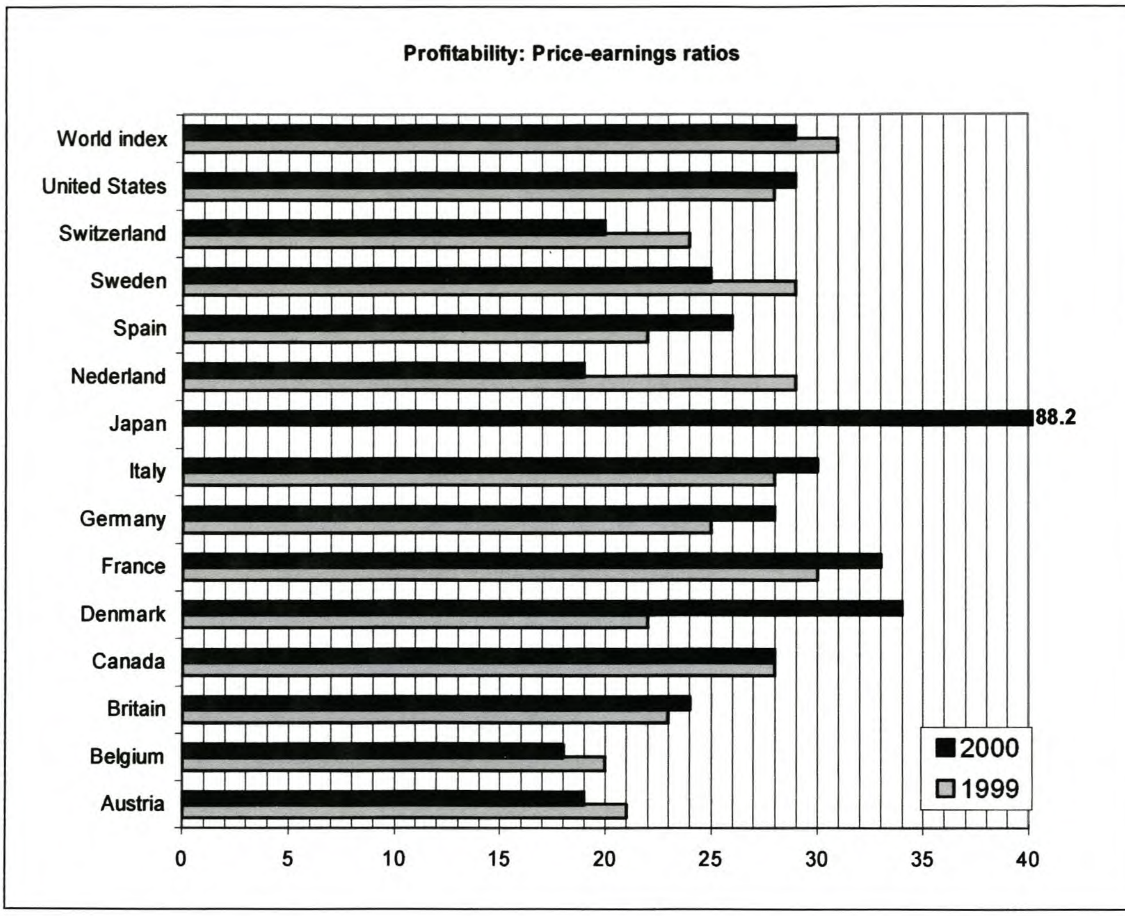
1.1 Development of the research question

The price-earnings ratio is one of a series of benchmarks developed after the Great Depression, to measure the fair value of shares of a company on a relative basis. It was formalised by Benjamin Graham, a lecturer at the Colombia University in his 1934 book termed Security Analysis (Marshall, 11 March 2000). The price-earnings ratio is defined as the price for every unit of annual earnings attributed to a share. Consequently, it originated from the assumption that investors buy a share based on a company's underlying economic health.

Price-earnings ratios differ significantly in different countries, in different sectors of the economy within countries and for different economic entities. They are widely used to value companies and this could result in inappropriate comparisons, and thus conclusions. There are a number of reasons for the widespread use of the price-earnings ratio, according to Aswath Damodaran (1996:291), associate Professor of Finance at New York University's Leonard N Stern School of Business. Firstly, price-earnings ratios are appealing as they relate the price paid to earnings, seen as a proxy for cash flows. Secondly, they are simple to calculate and are widely available, enabling the user to make comparisons between shares effortlessly. Thirdly, price-earnings ratios can be an indicator of other characteristics that include risk, growth and accounting differences (Beaver & Morse, 1978:65).

According to the article "Markets & Data: Closer looks" Economist.com (14 October 2000), price-earnings ratios have, over the past few years, risen to dizzy heights in many countries. The suggestion that share-markets over the world are overvalued is generally accepted these days. Nevertheless, the article identifies the limitation of the calculation of the price-earnings ratio, calculated as the ruling market price divided by the current historical earnings, as temporary small corporate profits in some countries appear to inflate the ratio by reducing the denominator in equation 1.4.1. Variations of the price-earnings ratio address this shortcoming, but are too cumbersome to apply to worldwide ratios. Figure 1.1.1 provides a graphical illustration of these ratios. Note the sharply contrasting differences between different countries and the overall high current value. Japan is an interesting outlier as during 1999 it experienced corporate net losses and therefore no ratio was recorded, however, during 2000 it had the highest ratio as a result of small corporate profits.

Figure 1.1.1 (Price-earnings ratios) (ibid.)



It is clear from the above that the price-earnings ratio eliminates the need to make some assumptions required for discounted cash flow, like growth and discount rates, to value companies as it only requires the current level of earnings and the price-earnings ratio to complete the valuation (Damodaran, 1996:291). Alternatively, this ratio holds the key to a significant amount of fundamental information which, when integrated with the standard valuation models, enables the comparison of this data with the valuation assumptions to determine market perceptions. According to Reilly and Brown (1997:21), the current price-earnings ratio indicates the existing attitude of the market toward a share's prospects for growth.

Observations from market data appear to mislead investors, specifically in an overvalued or undervalued market, and in order to profit from the other misguided market participants, price-earnings ratio data must be linked to fundamentals to assess the reasonability of the market's perception of a company's prospects. According to Malkiel and Cragg (1970:612-616), anticipations affect analysts' valuation of shares, similar to the Keynes beauty contest (where one selects the contestant that is perceived to be the favourite of others and not necessarily the most beautiful). Analysts believe that a rise in the price-earnings ratio of a share will continue as long as it is justified by popular opinion. The analyst simply adjusts the company's growth prospects to justify the price-earnings ratio, without verification of the reasonability of this action.

Contrary to the Keynes beauty contest, it is widely accepted that fluctuations in asset prices are attributable to changes in fundamentals. According to Cutler, Poterba and Summers (1989:4), voluminous evidence demonstrated that share prices react to announcements about corporate control, regulatory policy, and macroeconomic conditions that plausibly affect fundamentals. This provided further concern in how the price-earnings ratio interacts with the above events or announcements, beyond being simply a reasonability check of the variables associated with the valuation of the shares. These events or economic conditions are, in their turn, integrated in the standard valuation equations, as the variables of these equations.

It is also important to analyse the impact of corporate announcements, regulatory amendments and changing macroeconomic conditions such as market timing, or in other words, as the effective anticipation of the above events provide superior returns through effective market timing (Wagner, Shellans & Paul, 1992:89). Fundamental analysis aims to provide the value of assets by the examination of the effects of corporate announcements, regulatory amendments and changing macroeconomic conditions on key drivers, such as earnings, risk and growth (Baruch & Thiagarajan, 1993:190).

Fundamental analysis combined with the efficient market hypothesis, an assumption that security prices fully reflect known information, furthermore contradicts the contention and observation that shares with relatively low price-earnings ratios yield superior returns (Levy & Lerman, 1985:31). Koehler (1994:55) indicated in his study, which used natural networks (a sophisticated statistical technique) that the assumption that modest price-earnings ratios yield superior returns compared to other indicators is correct.

According to Bidoli (8 January 1999), the differences in growth opportunities between companies generally justify differences in price-earnings ratios in South Africa. These differences in growth opportunities indicate how highly a company's shares is regarded by the market or its rating ("Glossary: Price /

Earnings Ratio (P.E. Ratio)", 13 June 2000). Furthermore, it is also accepted that a company should grow at a rate equivalent to the value of its price-earnings ratio and, therefore, the price-earnings ratio is normally used to compare the value of one share with another (Spoormaker, 22 October 2000).

It would appear that a thorough understanding of the price-earnings ratio can be profitable and that the ratio summarises a significant amount of information in a single number that is easy to use for security analysis.

1.2 Rationale for the study

It is evident from the background on the price-earnings ratio that much information is obtainable from the analysis of a company's price-earnings ratio.

An empirical study by Fairfield (1994:28) indicated that price-earnings ratios tend to remain constant as almost seventy percent of companies categorised as high ratio entities continued to remain in that category after one year; five years later half the number of companies classified with high price-earnings ratios still remained in that category. He also noted that the price-earnings ratio reflects earnings growth, compared to earnings potential, reflected by the price-to-book value ratio. Therefore, the price-earnings ratio was slightly more volatile in nature when compared to the price-to-book value ratio, but not very volatile. According to Durand (1957:348), when the growth potential of a company becomes widely recognised its share price is expected to react favourably and it will advance ahead of the shares of companies lacking this growth appeal. It is this recognition of growth that inflates a company's price-earnings ratio.

Beaver and Morse (1978:65-70) observed that variations among portfolios of different price-earnings ratios persist for up to fourteen years. Contrary to other studies they believe that the growth rate does not explain this observation as investors only forecast short-term (less than five years) earnings and earnings growth. According to them, the most likely explanation for their observation is the effect of different accounting methods on earnings.

In order to use the price-earnings ratio as an aid to successful investment decision making one must thoroughly analyse the returns that can be expected from a company's current book of business. A second step in the effective use of the price-earnings ratio would be to identify prospects for growth that can be achieved through the perusal of new investments. Lastly, the investor has to determine the rate of return on the new investments. The price-earnings ratio is valuable in both the identification of companies for further analysis, and the analysis and valuation of these companies for investment purposes in accordance with the guidelines provided above. Leibowitz and Kogelman (1994) also indicated that for a company to merit a high price-earnings ratio it must have significant prospects of earnings growth derived through investment in new projects and that the return on these new investments should provide the company with extraordinary high returns. Accordingly, the analysis of companies for investment should focus on the identification of these factors.

Goldsticker (1994) studied the combination of statistical portfolio selection based on variables such as the price-earnings ratio and the combination of these results with traditional valuation, as indicated by Leibowitz and Kogelman. His results indicated that on average the lowest forward price-earnings ratios (current price divided by one year forward earnings per share) portfolios outperformed all the portfolios included in his study; portfolios were constructed based on different share selection methods. This observation provides support for using the price-earnings ratio as an indicator of profitable investment opportunities.

Nevertheless, in order for forecasts of earnings or the calculation of values to be profitable it must not only be correct, it must also differ from the consensus forecasts, as the consensus is presumably already reflected by share prices (Mitchell, 1995:21). This is where the price-earnings ratio becomes a useful aid, and the primary rationale for this study. The price-earnings ratio is capable of providing the user with the consensus implied risk, value and earnings potential of a share. The aim of this study is to analyse the relationship between the price-earnings ratio and standard risk and valuation models, as well as the relationship between earnings and the price-earnings ratio, in order to determine the consensus market information that the ratio can provide. It further aims to provide information on the effect of a change in earnings on the price-earnings ratio.

1.3 Identification of the research question

According to Leibowitz and Kogelman (1994:1), the decade of the eighties is associated with the start of the flow of products, capital and expertise across corporate and national boundaries at an unprecedented pace. This sudden abundance of resources breached the traditional constraints on growth and development. This allowed some companies to grow at an unprecedented pace and achieve very high price-earnings ratios.

This process of continuously compounding earnings growth appeared to continue for too long, accompanied by phenomenal price-earnings ratios associated with this level of profitable growth. This phenomenon cannot continue indefinitely, as economic variables normally follow a pattern of reversion towards the mean (Beaver & Morse, 1978:68). This observation is explained by two factors. First, a variable's ranking normally contains a transitory component, in other words, price-earnings ratios during a given year will result in part from factors associated with that year and these factors will dissipate in subsequent years. Secondly, the underlying permanent value normally reverts towards the average, or the price-earnings ratio is expected to decrease to its previous value after the extraordinary growth is utilised.

Griffis (1998) stated that as a rule of thumb, a share is fairly valued if the growth rate of earnings per share equals the value of the price-earnings ratio; if earnings grow at a higher rate, the share appears undervalued and vice versa. This rule is crude and requires further investigation. Nevertheless, it provides a guideline.

Huagen and Baker (as cited in Harris & Marston, 1994:19) explained the comparatively poor performance of high price-earnings ratio shares as the result of possible market overreaction. They indicated that prices for companies with profitable growth prospects are bid up to unrealistic levels in anticipation for profits from the expected high growth rates. In order to evaluate the reasonableness of these growth forecasts the price-earnings ratio, as an indicator of growth, must be used in conjunction with traditional valuation techniques.

This study project might assist in understanding the informational content of price earnings ratio and its determinants in order to evaluate the underlying factors, as indicated above. Furthermore, in achieving its objective the project will also indicate common mistakes that may accompany the use of the ratio. The project will also indicate the effects of different accounting conventions and policies on the price-earnings ratio. It is suggested that some form of normalised earnings per share should be used in ratio analysis ("Equity Valuation Ratios." The Cost of Capital Center, 13 June 2000).

Reference will be made to selected literature on the subject in order to gain a better understanding of the subject field and illustrate how the conclusions of the selected research relate. Therefore, an empirical study is not required to achieve the objective of the project. Simple scenarios of hypothetical companies are sufficient to illustrate the theoretical soundness of current theories.

1.4 Basic concepts and definitions

The primary term to define is the price-earnings ratio. The price-earnings ratio tends to take on many different forms for calculation to accommodate the objectives of the computation.

The main form of the price-earnings ratio is the ruling market price divided by the net earnings during the previous twelve months. This price-earnings ratio is represented by Equation 1.4.1. The forward price-earnings ratio is calculated as the ruling market price over the expected net earnings for the next twelve months. The forward price-earnings ratio is calculated in accordance with Equation 1.4.2.

$$P/E_0 = \text{Current market price per share} / \text{Net earnings during the past 12 months per share}$$

Equation 1.4.1

$$P/E_1 = \text{Current market price per share} / \text{Net expected earnings for the next 12 months per share}$$

Equation 1.4.2

The next important distinction between the different types of price-earnings ratios is included in the denominator. Earnings per share can be obtained from the financial statements of companies. Nevertheless, earnings per share calculations are not computed on equal terms between different companies, and in different countries, as accounting practices and conventions vary. Some countries, like South Africa, provide a form of sustainable earnings per share, termed headline earnings per share; however, not all countries require disclosure of this information. Nevertheless, it is important to use earnings that are sustainable and not very volatile in nature, or that possess the possibility to exclude accounting bias in the calculation of the price-earnings ratio. Accordingly, a distinction should be made between normalised (sustainable) and accounting earnings per share. This subject is covered in further detail during the study.

Another important concept to understand for interpretation of this study is that of perfect capital markets. In such a market, no single transaction is large enough to influence the ruling market price. Furthermore, transactions and information costs are equivalent for all market participants. In addition, all the participants in the market possess the same information about the prospects for profit. Lastly, no preference exists for dividends or capital gains, as both are equivalent. Later, some of these restrictions will be relaxed.

A further important assumption is that all investors behave rationally. In other words, the participants in the market prefer more to less wealth and they are indifferent about the form (dividends or capital gains) of the distributions. The last concept is that of perfect certainty. Under perfect certainty, investors have all the information about planned investments and the profits that are associated with these investments. Perfect certainty eliminates the differences between bonds and shares. Perfect certainty does not exist in the market-place; instead an equity premium is required by equity investors to compensate for uncertainty.

1.5 Outline of the study

The study will follow the process normally associated with the valuation of investments. Accordingly, the first objective is to compute and evaluate the required rate of return, then to compare and contrast this with information obtainable from price-earnings ratios. The second step is to select the appropriate valuation model, and to compare and contrast the models with public price-earnings ratio information and evaluate the results for investment purposes. The last step, covered in chapter 4, is to calculate normalised earnings per share, and other earnings forecasts as the results of these calculations are included as input in the valuation models. These processes, covered in chapter 4, require attention before the valuations are completed.

Accordingly, the study will commence with a discussion about the relationship between risk and return models and the price-earnings ratio in chapter 2. In order to introduce the capital asset pricing model and the arbitrage pricing theory, the Markowitz portfolio theory will be covered as an introduction to the above model and theory as the Markowitz portfolio theory forms the basis of both the capital asset pricing model and the arbitrage pricing theory. Both these models and theories will be covered, including their strengths and weaknesses, and their relationship to the price-earnings ratio. The chapter will continue to analyse the importance and association between factor models and the price-earnings ratio. It will conclude with a discussion about stochastic dominance models, as the ultimate test for the effectiveness of investment portfolios constructed from price-earnings ratio classes, and a discussion about the effect of the interest rate term structure on the evaluation of risk and return rates.

Chapter 3 will examine the valuation models and growth. The chapter will commence with a discussion about the standard dividend discount model, as the basis for all valuation models. More complex dividend valuation models, which include the Gordon constant growth dividend discount model, the two-stage dividend discount model, the H-model, and the three-stage dividend discount model, will follow the discussion of the standard dividend discount model. All these models, their advantages and disadvantages, applicability, and association to the price-earnings ratio, will be covered. Lastly, the franchise value valuation model will be discussed together with its association with the price-earnings ratio and the strengths and weakness of this approach. The chapter will conclude with a discussion about the duration of growth, as a very important aspect of the price-earnings ratio.

Chapter 4 will conclude the discussion by examining the denominator of the price-earnings ratio. The chapter will firstly elucidate the effects of the different accounting rules and applications in the world and the divergent calculation methods that apply to the calculation earnings per share. Then, it will emphasise the importance of earnings as a proxy for cash flows and attempt to accentuate the importance of computing a sustainable earnings stream for valuation purposes. The effects of the distribution policy of a company on its market value will also receive some attention in addition to the effects of items included in the financial statements on the market value and the price-earnings ratio of a company.

The last chapter, chapter 5, will conclude the study project and summarise its findings; it also aims to provide some recommendations from the findings of the project.

Chapter 2: Risk and return

According to Damodaran (1996:20), the questions of how risk is measured, and how it is rewarded, are fundamental to every investment decision, from asset allocation to valuation. It is also an area of furious debate between theorists and practitioners about the right model to use. This chapter aims to present a broad outline of a variety of well-known alternative risk and return models and theories, and elucidate how these models and theories explicate some empirical observations. Furthermore, it will show how these models, theories, and tests interrelate with the price-earnings ratio and what useful information from these models is obtainable from the various types of price-earnings ratios.

As a prelude to the examination of risk and return, it is worth defining the ingredients of a good risk and return model or theory, according to Damodaran (1996:20-21).

Such a model should contain a measure of risk that applies to all types of investments. It should specify what types of risk are rewarded and what types are not, as it is an accepted part of modern investment theory that not all risks are rewarded. Furthermore, the model or theory should be capable of translating its risk-measure into an expected or required rate of return, as the objective of risk analysis is to obtain a discount rate for valuation purposes.

Lastly, a universal requirement for all models and theories is that they should be applicable in practice (ibid.).

2.1 Markowitz portfolio theory

Before the development of the modern portfolio theory discussed here, which is before the nineteen fifties, models for risk and return were largely subjective, and varied widely across investors (ibid.). With the development of the modern portfolio theory, these models became more quantitative, and specific in their predictions. They also gained general acceptance from investors.

The portfolio theory was developed by Harry Markowitz during nineteen fifty-nine, for which he won the Nobel Prise in Economics in nineteen ninety (Reilly & Brown, 1997:21). Markowitz assumed that any asset or portfolio could be described by its expected rate of return, and the risk related to that portfolio. The return of a portfolio, according to Markowitz, is calculated as the weighted average of the individual investment's expected returns. Risk, on the other hand, is computed from the variance of the returns earned on these assets or portfolios (Reilly & Brown, 1997:279).

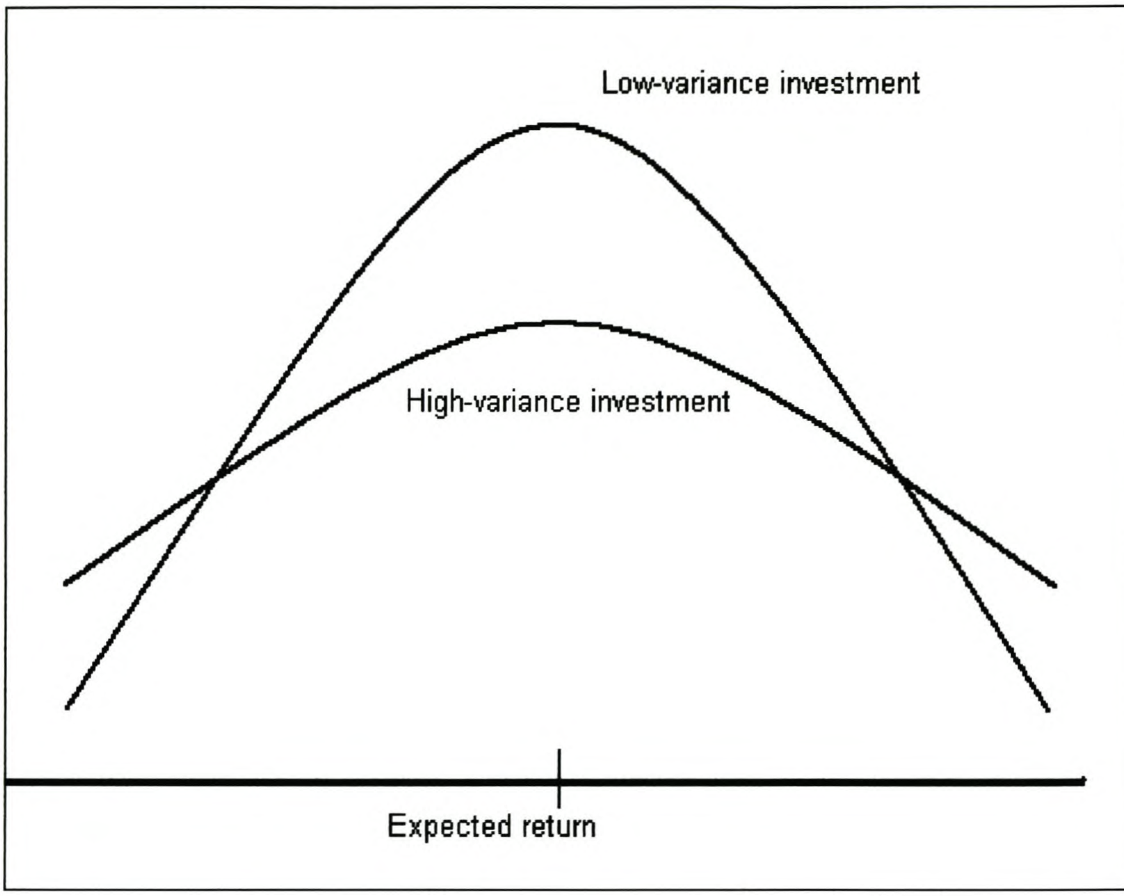
The Markowitz portfolio theory assumes firstly that investors consider every investment's expected rate of return over its holding period. According to

Damodaran (1996:22), investors also consider return skewness. In addition, each investor's utility curve demonstrates diminishing marginal utility for wealth, and investors seek to maximise their expected utility. Investor utility is defined in terms of wealth or consumption (Barlev, Denny & Levy, 1988:70).

Secondly, the Markowitz portfolio theory assumes investors measure portfolio risk based on return variability, calculated as an investment's variance. Investors prefer to reduce their risk as this raises their utility. Lastly, it should be noted that for a given level of risk, investors prefer higher returns to lower ones. Similarly, for a given level of expected return, investors prefer less to more risk, according to Reilly and Brown (1997:279-281).

Figure 2.1.1 is an illustration of the variability of return. It follows that the low-variance investment appears to have much more certainty of reaching the expected rate of return than the high-variance investment. The high-variance investment has a greater possibility of obtaining some other rate of return, even though both investments have the same expected value. The reason is that the low-variance investment possesses thinner tails or less return possibilities and, therefore, a lower frequency of occurrence outside the centre of the graph.

Figure 2.1.1 (Return distributions on two investments)

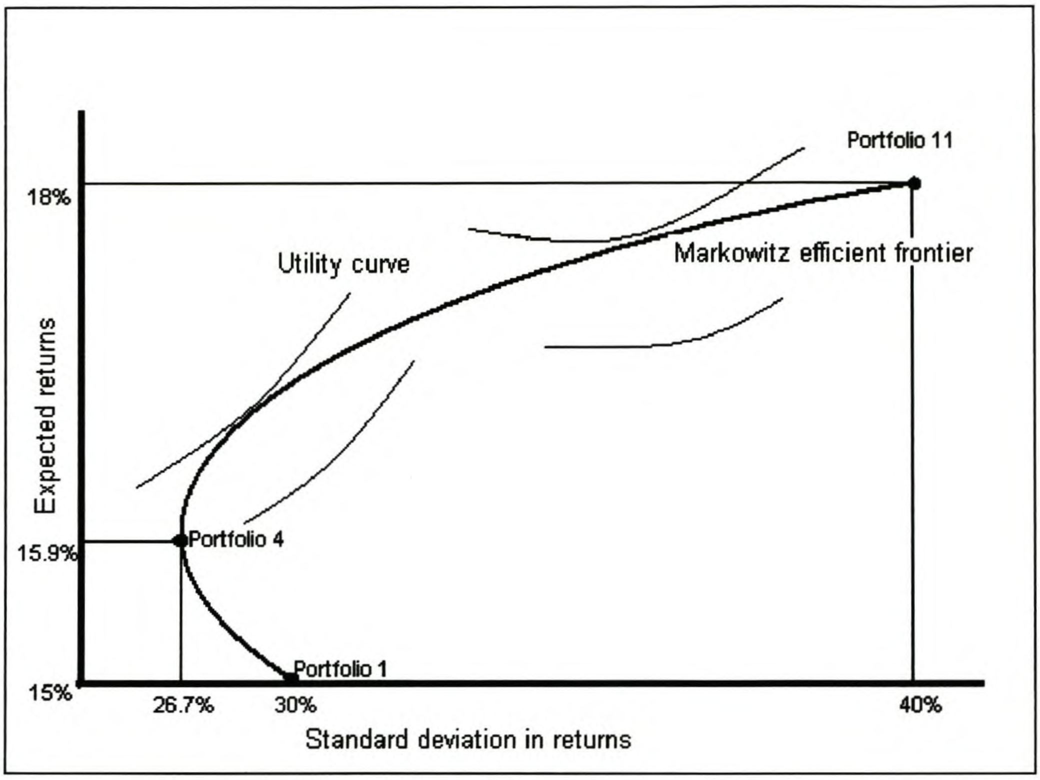


The next step following the explanation of variance, as covered by the Markowitz portfolio theory, is the introduction of more than a single risky asset. This provides the investor with a choice of investment opportunities, and introduces the Markowitz efficient frontier. Assume two assets with the characteristics as per Table 2.1.1. It is notable, that as a result of the correlation of the assets with one another, the relationships of different portfolios of these two assets do not simply follow a straight line from one extreme to another. A portfolio (Portfolio 4) that invests seventy percent of its value in asset A and the rest in asset B would have a lower standard deviation than asset A (Portfolio 1) and a higher rate of return. Portfolio 4 would be preferable to asset A for any Markowitz investor (refer to Figure 2.1.2).

Table 2.1.1 (Expected returns and standard deviation)

Assets	Return	Standard deviation in returns	The covariance between the two assets is assumed to be equal to twenty five percent for the calculation of the standard deviation in returns.	
Asset A	15%	30%		
Asset B	18%	40%		
Portfolios	**Weight of asset A**	**Weight of asset B**	**Expected returns**	**Standard deviation in returns**
Portfolio 1	100.0%	0.0%	15.0%	30.0%
Portfolio 2	90.0%	10.0%	15.3%	28.3%
Portfolio 3	80.0%	20.0%	15.6%	27.1%
Portfolio 4	70.0%	30.0%	15.9%	26.7%
Portfolio 5	60.0%	40.0%	16.2%	26.9%
Portfolio 6	50.0%	50.0%	16.5%	27.8%
Portfolio 7	40.0%	60.0%	16.8%	29.4%
Portfolio 8	30.0%	70.0%	17.1%	31.5%
Portfolio 9	20.0%	80.0%	17.4%	34.0%
Portfolio 10	10.0%	90.0%	17.7%	36.9%
Portfolio 11	0.0%	100.0%	18.0%	40.0%

Figure 2.1.2 (Markowitz efficient frontier)



The combination of all risky assets results in a graph similar to that depicted by Figure 2.1.2. Markowitz termed the line, starting with portfolio 4 and ending with portfolio 11, the efficient frontier. Any portfolio on the efficient frontier would be preferable to other portfolios below the line, and no portfolios exist above the line (Damodaran, 1996:30). Therefore, the specific position the investors prefers on the Markowitz efficient frontier depends on their utility functions, as indicated in Figure 2.1.2.

The benefits of diversification as indicated continue as long as the correlation coefficients between the assets are not equal to one. The first setback associated with the Markowitz portfolio theory is that the number of calculations involved increase exponentially with the number of assets. Secondly, the Markowitz approach ignores one very important alternative available to investors, namely investment in a risk-free asset. The capital asset pricing model addresses these two limitations, and comes up with a simpler solution for the construction of optimal portfolios, with the consideration of the risk-free asset (Damodaran, 1996:26-29).

2.2 Capital asset pricing model

2.2.1 The model

Capital market theory extends the Markowitz portfolio theory to yield a model for the pricing of all risky assets. This is achieved by the application of the capital market theory, or the capital asset pricing model, as it allows the investor to determine the required rate of return for any asset.

The capital market theory is simply an extension of the Markowitz portfolio model, and accordingly, requires many of the same assumptions as outlined earlier. However, relaxation of some of the above assumptions does not change the main implications of the theory (Reilly & Brown, 1997).

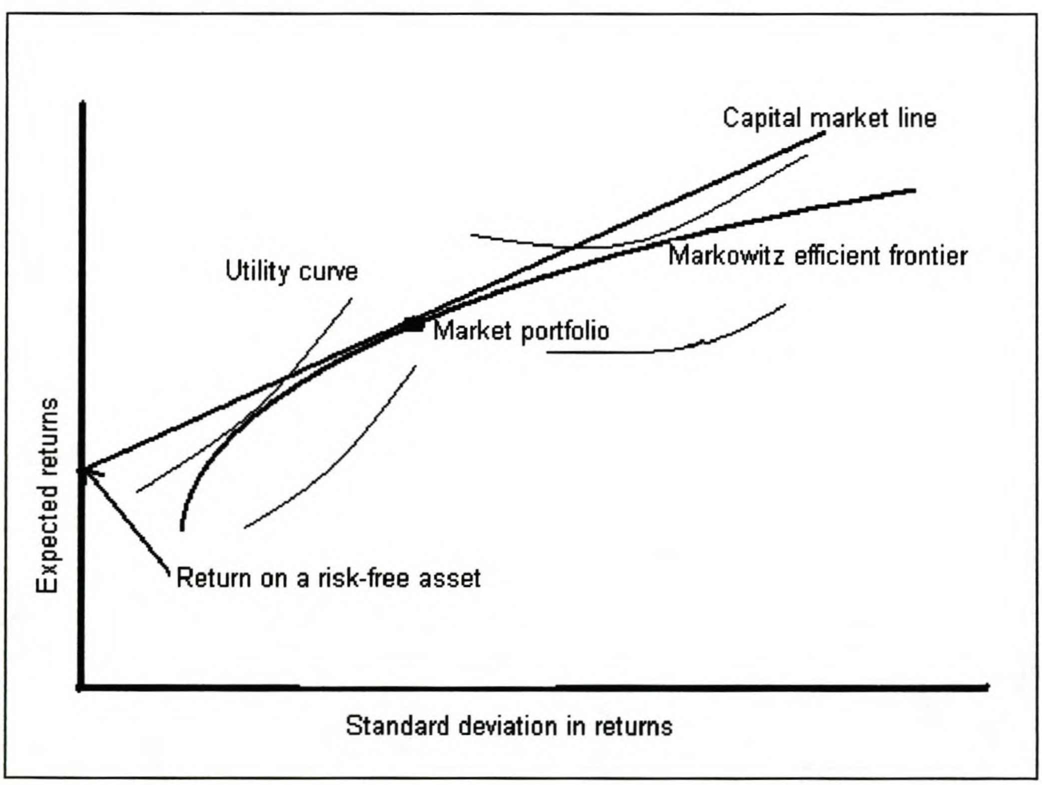
A central assumption to the capital market theory is the existence of a risk-free asset. The risk-free rate of return earned on these risk-free assets is equal to the long-run growth rate of the economy, adjusted for short-run liquidity. William Sharpe introduced the original capital asset pricing model in 1964 based on these assumptions (Sharpe, 1984:22). According to Ross (1976:342), Sharpe, Lintner, Treynor and Mossin independently derived capital asset pricing models based on the assumption of a risk-free asset. These models are collectively known as the Sharpe-Lintner-Mossin capital asset pricing models (Reilly & Brown, 1997:280).

The standard deviation, a risk measure, of the risk-free asset's return is zero as the returns are certain (ibid.). Consequently, it also follows that the covariance, a measure of relative risk, and correlation of the risk-free asset with any other assets or portfolios will always be zero. Alternatively, the risk-free asset can be replaced by a zero-beta return (the return on all portfolios uncorrelated with the market portfolio), as shown by Ross (1976:341-360). Sharpe (1964:425) wrote that the market presents him with two prices, firstly the price of time, as represented by the risk-free asset, and the price of risk that is represented as the beta of the capital asset pricing model.

The expected return of a portfolio that combines a risk-free asset, and a portfolio of risky assets, is computed as the weighted average of the returns of the risk-free asset and the portfolio of risky assets. The standard deviation of such a portfolio (its risk) thus equals the portfolio weight on the risky portfolio, multiplied by the standard deviation of the risky asset portfolio. There is no risk attached to the risk-free asset since its standard deviation of returns is zero (Reilly & Brown, 1997:281).

The combinations of risk and return derived from the addition of the Markowitz efficient frontier with the capital market theory described above, results in the capital market line as the line of tangency between the rate of return on the risk-free asset and the efficient frontier. Refer to Figure 2.2.1.

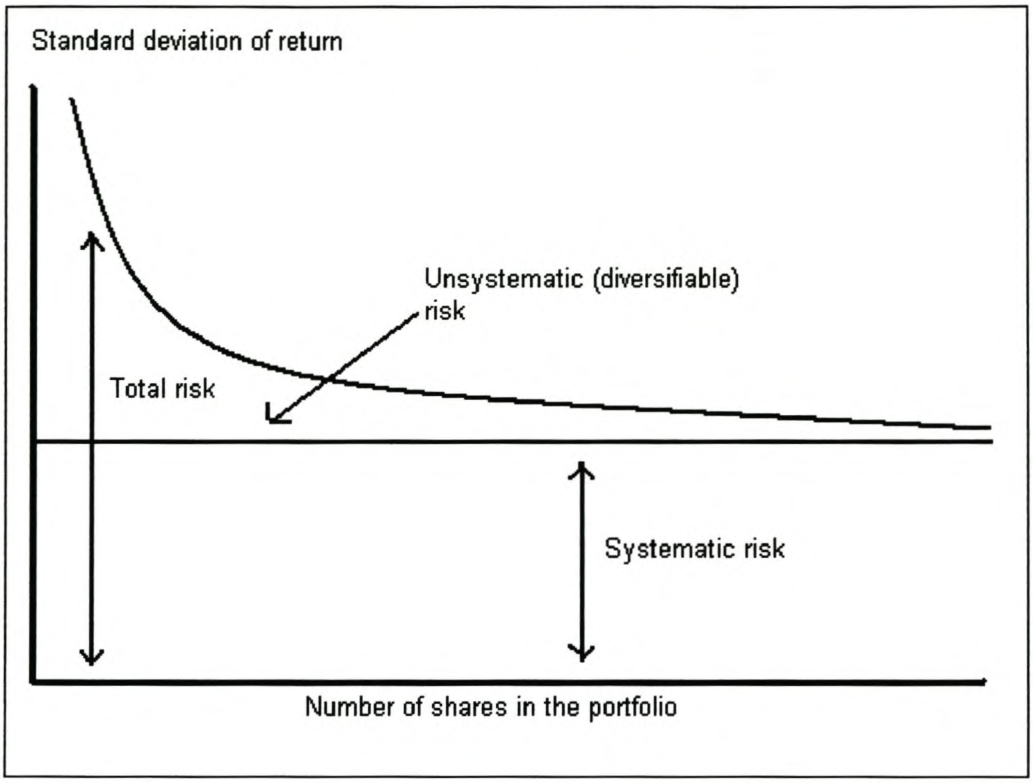
Figure 2.2.1 (Capital market line)



The risk inherent to any individual risky asset, according to the capital asset pricing model, comprises two elements. Namely, a systematic component, that reflects the common movement of a single asset's return with that of all other assets, and an individualistic component, that moves independently of the market's rate of return (Beaver, Kettler & Scholes, 1970:654-682). The events that form the source of these risks result from either the influence of the whole economy (macro-economic changes that affect the rates of return on all securities) or specific factors that influence only the rate of return on specific assets (unsystematic factors).

Risks that affect macroeconomic variables are known as systematic risks, as these risks are evident in the rates of return on all assets. Other events that do not influence the whole economy, but only affect a particular asset are known as unsystematic risks, as these risks are asset-specific and not part of the macro-economic system. The unsystematic risk (diversifiable risk) has no impact on the value of the market portfolio. Refer to Figure 2.2.2 for a graphical depiction of the effects of diversification on risk. The market portfolio is the point of tangency between the efficient frontier and the capital market line in Figure 2.2.1 (Reilly & Brown, 1997:283).

Figure 2.2.2 (Diversification) (Reilly & Brown, 1997:285)



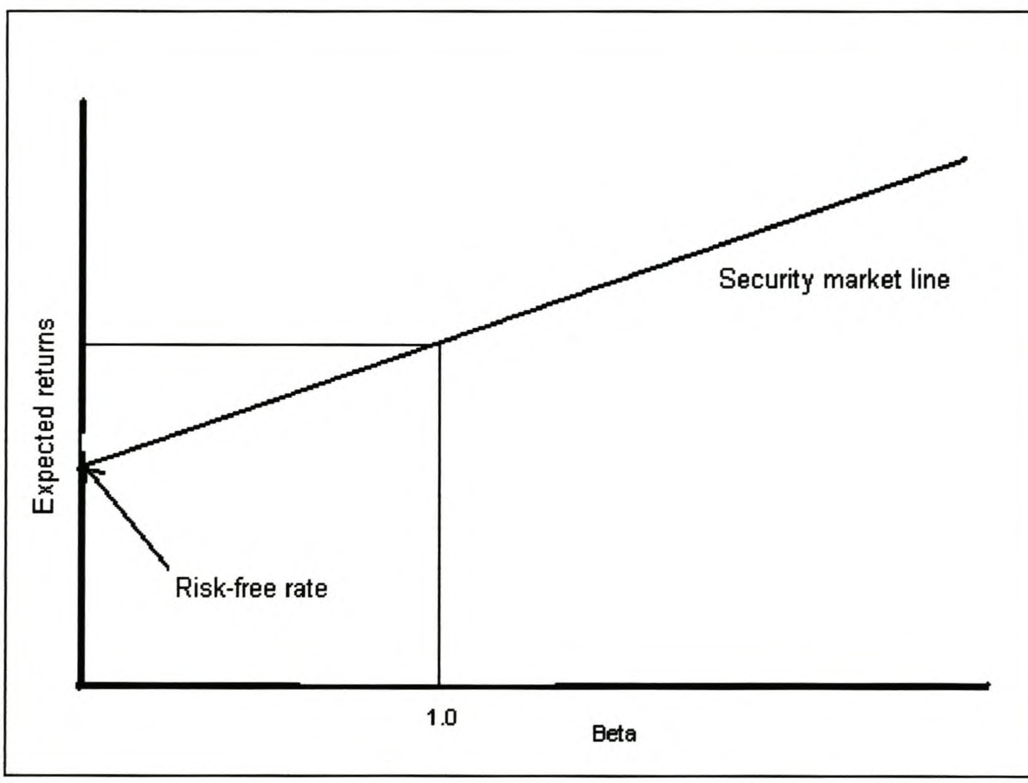
The capital market line is a straight line and that implies that all portfolios on this line are perfectly positively correlated with one another. The reason being that all portfolios on the capital market line combine the risky market portfolio with the risk-free asset. However, since the market portfolio includes all risky assets, it is impossible to invest in this portfolio. Therefore, the only practical alternative is to invest in a large amount of shares, in order to simulate the diversified market portfolio. As a result, the covariance of an asset with the market portfolio is the only relevant measure of risk for the capital asset pricing model (ibid.).

Every investor who wishes to invest in the market portfolio need to choose a point along the capital market line, as described above. This decision is known as the financing decision. According, to Reilly and Brown (1997:286), the investment decision involves determination of the market portfolio on the investment frontier. The parting of the decisions is known as the separation theorem.

The covariance with the market portfolio, as the relevant risk measure, is used to determine the expected return for any risky asset. This objective to determine the expected rate of return is achieved with the use of the capital asset pricing model. The model predicts the expected rate of return on all assets. This expected rate of return can be used as a discount rate for valuation purposes. Alternatively, if the return on an asset has already been calculated with the assistance of valuation models, the estimated return from the valuation models can be compared against the required rate of return, as determined by the capital asset pricing model, in order to identify undervalued and overvalued assets (ibid.).

To complete the development of the capital asset pricing model a new line is introduced, namely the security market line. The security market line visually presents the relationship between systematic risk and the expected or required rate of return on an asset. The difference between the security market line and the capital market line is that the security market line relies on systematic risk as its risk measure, where as the capital market line uses variance to represent risk. This systematic risk is expressed by a factor termed beta. William Fouse pioneered the security market line of Sharpe to compute a measure for the trade-off between risk and return in the market at any given time. Refer to Figure 2.2.3 for a graphical depiction of the security market line as represented in the capital asset pricing model (ibid.).

Figure 2.2.3 (The capital asset pricing model) (Reilly & Brown, 1997:288)



As previously indicated, beta is a measure of systematic risk. Furthermore, beta is also a standardised measure of risk, since it is calculated by dividing the assets' covariance with that of the market portfolio by the variance of the market portfolio (Beaver et al., 1970:658).

$\beta \approx \sigma(R_i, R_M) / \sigma(R_M)$ **Equation 2.2.1** (ibid.)

The numerator represents the covariance of the share's return with that of the market, and the denominator the variation of the rates of return on the market portfolio.

This standardisation in the calculation of beta with the market portfolio resulted in a beta of one for the market portfolio (refer to Figure 2.2.3), according to Reilly and Brown (1997:289). A justifiable conclusion is that an asset with a beta greater than one is more volatile than the market, or alternatively, it possesses added systematic risk when compared to the market, and vice versa. Alternatively, beta can be derived from a regression model known as the asset's characteristic line with the market portfolio. Equation 2.2.2 represents such a regression model.

$R_i = \alpha_i + \beta_i R_M + \varepsilon_i$ **Equation 2.2.2** (Beaver et al., 1970:659)

The capital asset pricing model is represented by the Equation 2.2.3.

$R_i = R_F + \beta_i (R_M - R_F)$ **Equation 2.2.3** (ibid.)

2.2.2 Discussion of tests, advantages and critique

The model is not a new development. Consequently, it has been comprehensively tested and criticised. Nevertheless, it is widely accepted and used in practice.

A study by King (as cited in Beaver et al. ibid.) of monthly security returns found that, on average, variation of a security's return could be explained by the variation of the market wide returns (the coefficient of correlation was 52%). The study, furthermore, revealed that the regression equation conforms to the assumptions of a linear model.

Another study, by Shefrin and Statman (1995:27), indicates that asset prices in the behavioural capital asset pricing model are the outcome of an interaction between two kinds of traders, namely, information and noise traders. According to the authors, information traders know the relationship between the characteristics of companies and the return distributions of the shares of these companies. Noise traders, however, make systematic errors in their assessment of these relationships. A central element to this is a cognitive error that leads noise traders to believe that good shares are shares of good companies. Moreover, Shefrin and Statman showed that the effects of information traders persisted (the market is rational) as a strong relationship exists between share returns and beta.

According to Ross (1976:342) the simplicity of the capital asset pricing model and the evident appeal of a linear relationship between risk and return helped to ensure its popularity. Further evidence of this positive linear relationship assumed by the capital asset pricing model came from Black, Jensen and Scholes (as cited in Reilly & Brown, 1997:312-314) when they examined the risk and return relationships for portfolios of shares and found a positive linear relationship between monthly excess return and portfolio beta. The security market line of Black et al. had a positive slope. Also, as can be expected, it changed over time as a result of macro-economic changes. In addition, the Black et al. study noted that the intercept was not zero, and that it changed over time with short-term liquidity changes in the treasury system.

These results of Black et al. were confirmed by a study of Fama and MacBeth (as cited ibid.), who examined the relationship between the rates of return during a given month and beta. The overall results of Fama and MacBeth also supported the capital asset pricing model, as the intercept in their tests was about equal to the risk-free rate and the systematic risk coefficient was positive and significant.

Kothari, Shanken, and Sloan (as cited in Reilly & Brown, 1997:317), in their study, found that systematic risk is rewarded with substantial returns by the market. They further noted that a definite problem existed with the inclusion of the amount of data in the calculation of the beta coefficient. The impact of the time interval on the calculation of beta, or the number of observations used (amount of data), varies in practice. Weekly rates over the last five years are the most common example, while some services use only monthly observations. The object of beta is to compute the future risk associated with an asset. According to Reilly and Brown (1997:311-317), the shorter weekly intervals caused a larger beta for large companies, and a smaller beta for small companies.

Statman (1981:41-44) indicated that differences in the number of observations resulted in significantly different betas for individual shares. This will result in divergent valuations of the same company. Statman also confirmed that these divergent valuations result from the amount of observations used in the calculation of beta, and suggested that an adjustment be made to the calculation of beta to accommodate these regression tendencies. Blume (as cited in Reilly & Wright, 1988:64-69) then suggested adjustment equations based on Statman's research. Reilly and Wright (ibid.) indicated that non-synchronous trading biases beta estimates downward. This problem is greater for weekly than monthly data, as companies with a lack of liquidity do not experience the same amount of weekly volatility as large companies that trade frequently.

This observation was also made by Dimson (as cited in ibid.) who showed that the interval effect affected shares of companies with a smaller than average market capitalisation in that they experienced a decline in beta when the interval is shortened and vice versa. Therefore, shorter intervals can lead to lower betas for small companies and result in overvaluation.

A further limitation of the model is the enormous amount of historical data required in order to calculate beta (Beaver et al., 1970:657). As stated, the objective is to calculate a future beta. Beta coefficients for individual shares are not stable over time, complicating the use of historical data for the calculation of an expected future beta. Beta is more stable at a portfolio level, provided the investments are held for a sufficient time span, and adequate trading volume exists for the individual shares (Reilly & Brown, 1997:312-322). Historical beta is, therefore, not a true representation of future beta, as it measures the relationship between the share's returns and that of the market over a specific historical interval (Rosenberg, 1985:5-7). Rosenberg (1984) aimed to reduce the problems associated with historical beta with the introduction of a multifactor model that included some company and market fundamentals. As a result, he obtained better forward beta predictions with his model.

Another limitation of the capital asset pricing model is the difficulty associated with the calculation of beta as the model depends upon a market portfolio that comprises all risky assets. Roll (1978:1051-1055) criticised this issue, as no such a portfolio exists. In order to compensate for this shortcoming, a proxy is used to simulate the market portfolio. This proxy complicates the capital asset pricing model further. A relatively complete proxy for all risky assets is the

Morgan Stanley World Equity Index (Reilly & Brown, 1997:1014). A fourth disadvantage associated with the capital asset pricing model is the prediction of liquidity changes in the treasury system as these affect the risk-free rate used and consequently the asset value.

The simplicity and intuitive appeal of a linear relationship between risk and return is difficult to justify on theoretical grounds, as linearity require normally distributed returns. According to Ross (1976:358), the normality of returns was never empirically proven. Sharpe and Cooper (as cited in ibid.) confirmed this observation. They indicated that the relationship between risk and return is not strictly linear, specifically as it relates to high-risk assets. McEnally (as cited in Reilly & Brown, 1997:314) also found that the returns on high-beta shares are associated with a significant amount of positive skewness.

Further confirmation of support for the small price-earnings ratio effect is a study by Shefrin and Statman (1995:34), where they concluded that the relationship between share returns and beta is not significant. Nevertheless, according to Keim (1986:19-25), the absence of a relationship between risk and return would constitute a rejection of the efficient market hypothesis, and not the capital asset pricing model.

According to Shefrin and Statman (1995:26-34), share returns are negatively related to size, and positively to book-to-market ratios. The motivation is that size and book-to-market ratios are indicators of risk and profitability. The authors also noted that investors overreact to recent returns. The observation that investors overreact to recent returns, results in companies that are perceived favourably by the market being in high demand amongst investors and this leads to overvaluation, and vice versa. The study of Shefrin and Statman served as confirmation of the findings of Harris and Marston (1994).

The capital asset pricing model is based on a large number of assumptions. When these assumptions are relaxed, the essence of the model persists, but the practical implications can significantly affect its effectiveness. When the assumption that the investor can borrow at the risk-free rate is relaxed, it results in two tangent lines to the market portfolio. One line originates from the risk-free intercept on the y-axis, and the other from the applicable borrowing rate that applies to the investor. The product is a flatter slope for the security market line to the right of the market portfolio, and therefore the investor that utilises leverage can expect a slightly more subdued rate of return on his or her total portfolio than the investor that invests unleveraged funds (Reilly & Brown, 1997:318). Nevertheless, the zero-beta capital asset pricing model addresses this issue, as it assumes that risk-free borrowing is impossible. Therefore, the zero-beta capital asset pricing model has a slightly higher intercept than the original capital asset pricing model (Sharpe, 1984:23).

When transaction costs are introduced to the capital asset pricing model, securities would be plotted near the security market line but not necessarily on it. The result would be a band around the security market line wherein transactions would not occur, as it would not be profitable. Divergent planning periods would also result in a similar narrow band, as changes in the planning period lead to different risk-free rates, and differing betas apply to the same investment. In addition, the tax burden differs amongst investors, again leading to divergent security market lines, again this result in a narrow band around the original line (Reilly & Brown, 1997:317-322).

2.2.3 Significance and relation to the price-earnings ratio

Research suggests that low price-earnings ratio shares generally outperform high price-earnings ratio shares; refer to Basu (as cited in ibid.); Peavy and Goodman (as cited in ibid.); Jaffe et al. (1989:135-148); and Arbel et al. (as cited in Shefrin & Statman, 1995:26-34).

A possible reason for this suggestion is that the capital asset pricing model does not adequately measure risk. Supporting evidence for this suggestion exists (McEnally, as cited in Reilly & Brown, 1997:315); Fama and French (as cited in ibid.); Reilly and Wright (1988:64-69). McEnally (as cited in Reilly & Brown, 1997:315) found that shares with relatively high betas are normally associated with a high degree of positive skewness. This positive skewness indicates a higher required rate of return, and therefore a lower value or price, leading to a lower price-earnings ratio. The observation conforms to the capital asset pricing model, which indicates that as risk increases so should return. The capital asset pricing model, nevertheless, has difficulty in accounting for the observed positive skewness, as it relies on normally distributed returns around the security market line (linear relationship). Fama and French (as cited ibid.) found that the shares of companies with small sizes and high book-equity to market-equity ratios experienced the highest returns. These observations support the hypothesis that small companies are omitted from analysis and recommendations by analysts and suffer a temporary lack of demand, and, when recognised, achieve significant returns on a risk-adjusted basis as demand increases (Arbel et al., as cited in Shefrin & Statman, 1995:26-34). Reilly and Wright (1988:64-69) suggested adjusted equations for the calculation of beta to compensate for this characteristic of low price-earnings ratio shares.

An alternative explanation for the low price-earnings ratio phenomenon is that returns on low price-earnings ratio shares are higher on a pre-tax basis. A higher tax burden diminishes the returns of these shares, as they normally pay substantial dividends, and these dividends are taxed. This argument is, nevertheless, not clear, as the shares of companies with low price-earnings ratios should not pay high dividends, when these companies represent growing or new companies. The companies just described need cash to fund their growth, be it profitable or not, and cannot pay dividends. If these shares represent mature companies, with few growth opportunities and significant market liquidity, the argument might be valid in jurisdictions that tax dividends, as these companies can afford to declare substantial dividends (Reilly & Brown, 1997:306-322).

Possibly, the last suggestion as to why the returns on low price-earnings ratio companies exceed that of high price-earnings ratio companies is that investors overestimate the value of growth. The investors are, therefore, willing to pay too much to purchase possible growth. This alternative explanation is not consistent with semi-efficient markets, as indicated by Peavy and Goodman (as cited in Reilly & Brown, 1997:314-322). Although, even after adjustment for company size, the Peavy and Goodman study concluded that the risk-adjusted returns for low price-earnings shares were superior. Basu (as cited in ibid.) came to the same conclusion as Peavy and Goodman, and continued to add that price-earnings ratios possess valuable information. Basu indicated that one could use publicly available price-earnings data to outperform the market if the argument above is valid.

A study by Malkiel and Cragg (1970:609) showed that beta had a strong influence on price-earnings ratios following a large decline in share prices. It would appear that investors prefer shares that move independently of the market (zero or negative beta) after a market downturn. The observation indicates that shares with negative betas, in order to oppose market direction or low-betas (low correlation with the market) are popular during such times.

This would affect the normal return patterns associated with the price-earnings ratio, as investors turn to beta for assistance. This results in temporarily skewed demand for shares with low price-earnings ratios, and affects the normal return patterns.

By contrast, Jaffe et al. (1989:138) indicated that companies with high price-earnings ratios are associated with the highest betas. The results of the Jaffe et al. study indicate that the returns on shares with high price-earnings ratios should be in excess of those with lower price-earnings ratios. Empirical studies, however, have shown that companies with low price-earnings ratios tend to provide higher returns. The timing of the study might have affected its results, according to the authors, as it included a period of recession.

A study by Fortune magazine (as cited in Shefrin & Statman, 1995:26-34) required participants to rate companies on certain qualitative aspects, such as quality of management. It is interesting to note that the respondents choose as if they believe that good shares are the shares of good companies, bearing in mind the type of reader associated with this magazine. The respondents further appeared to rate size as a very important factor, but were indifferent to beta. The observations of the Fortune study are inconsistent with the Bayes rule (one derives probabilities associated with future events from historic experience), as the participants ignored evidence that the proportion of shares of sound companies that do well is smaller than the proportion of shares of bad companies that do well. According to Shefrin and Statman, Arbel et al.

explained the observation when they implied that the analysts omit specific companies from their recommendations to focus on large companies with high tradability that sell investment reports, as previously explicated.

Despite all the criticisms of the capital asset pricing model, it is widely used to obtain the relevant discount rate for shares. The alternatives are complex and not without their own complications. The relevant discount rate can also be calculated from the price-earnings ratio for a mature company using Equation 2.2.4. One can restate the valuation equations provided in chapter 3 in terms of their discount factors to accommodate the more complex growth patterns investigated in that chapter.

$$R_i = \left(\text{Payout ratio} \big/ \text{Price - earnings ratio} \right) - g \text{ .}$$ **Equation 2.2.4**

Based on the criteria of Damodaran (1996:20-21) for good risk and return measures, the capital asset pricing model can apply to all types of investments. On the other hand, the price-earnings ratio can also be calculated for all types of investments similar to the capital asset pricing model. Consequently, there is no difference between the price-earnings ratio as a risk and return measure, and the capital asset pricing model based on the requirement of universal application.

The capital asset pricing model makes a distinction between the types of risks facing investors and continues to indicate which risks are rewarded and which not. The justification of the method used to derive this distinction is, nevertheless, debatable. Satisfaction of this requirement by the price-earnings ratio is not as clear-cut as with the capital asset pricing model, as the price-earnings ratio does not provide for this distinction between the different types of risk. Nevertheless, critique against the capital asset pricing model indicated that a true market portfolio is not available, and one can safely assume (as a result of market efficiency) that the price-earnings ratio only includes risks rewarded within the investment spectrum available to investors.

It would also be somewhat difficult to apply the capital asset pricing model to real estate, or art valuations, but the price paid for a specified or expected return still applies as its application is universal. Both models are capable of translating risk into a required or expected rate of return, and both models appear to apply in practice, even though the capital asset pricing model cannot clarify some of the market anomalies identified.

It appears that the price-earnings ratio, a comparative measure, can indeed explain asset returns just as well, if not better, than the capital asset pricing model, a generally accepted risk and return measure. To conclude, the price-earnings ratio provides one with the ability to calculate the required rate of return, based on the market consensus. During the discussion of the discount models in chapter 3, it will be shown that the price-earnings ratio can also be calculated in the absence of a market price with the assistance of some valuation models. The price-earnings ratio is, therefore, the applicable risk-return measure where no market price is available (for example, in usual public offerings and venture capital initiatives).

2.3 Arbitrage pricing theory

2.3.1 The model

The arbitrage pricing theory is an alternative to the capital asset pricing model. The objective of the arbitrage pricing theory, as with the capital asset pricing model, is to determine a suitable required rate of return for a security. The arbitrage pricing theory considers multiple factors to explain the relationship between risk and return of an asset, in contrast to the single-factor capital asset pricing model, which uses just beta as its systematic risk factor. Ross (1976:341) developed the theory as an alternative to the capital asset pricing model and published it in a working paper in 1971.

The arbitrage pricing theory has fewer assumptions than the capital asset pricing model, and its assumptions are not as complex. Firstly, the theory assumes perfect competition exists in the capital markets. The second assumption is that more wealth is preferable to less (Reilly & Brown, 1997:323). The last base assumption is that a multi-factor model represents the random process that generates asset returns (Ross, 1976:342). These assumptions about investor preferences allow the model to exist without any arbitrary factors, as will be observed from the section on factor models (Sharpe, 1984:22). Therefore, the arbitrage pricing theory is not a simple regression model of multiple factors without any significant rationale.

The model also implicitly assumes limited liability. Limited liability normally applies to listed companies. In addition, if a risk-less asset exists, its return is the risk-free rate. This requirement is, however, not a prerequisite to the application of the arbitrage pricing theory. Another implicit assumption, based on investors' wealth preferences and natural risk aversion is that the aggregate expectations of all investors constitute the consensus expectation of an asset's returns. This notion of consensus asset returns is similar to the rate of return calculated from the price-earnings ratio with the use of Equation 2.2.4. The last implicit assumption of the arbitrage pricing theory relates to the extent of disequilibrium experienced in the market. Ross (1976:341-354) assumed that only situations exist where demand exists for assets. Based on these assumptions the excess returns for any risky asset is represented by its return excess above the risk-free rate.

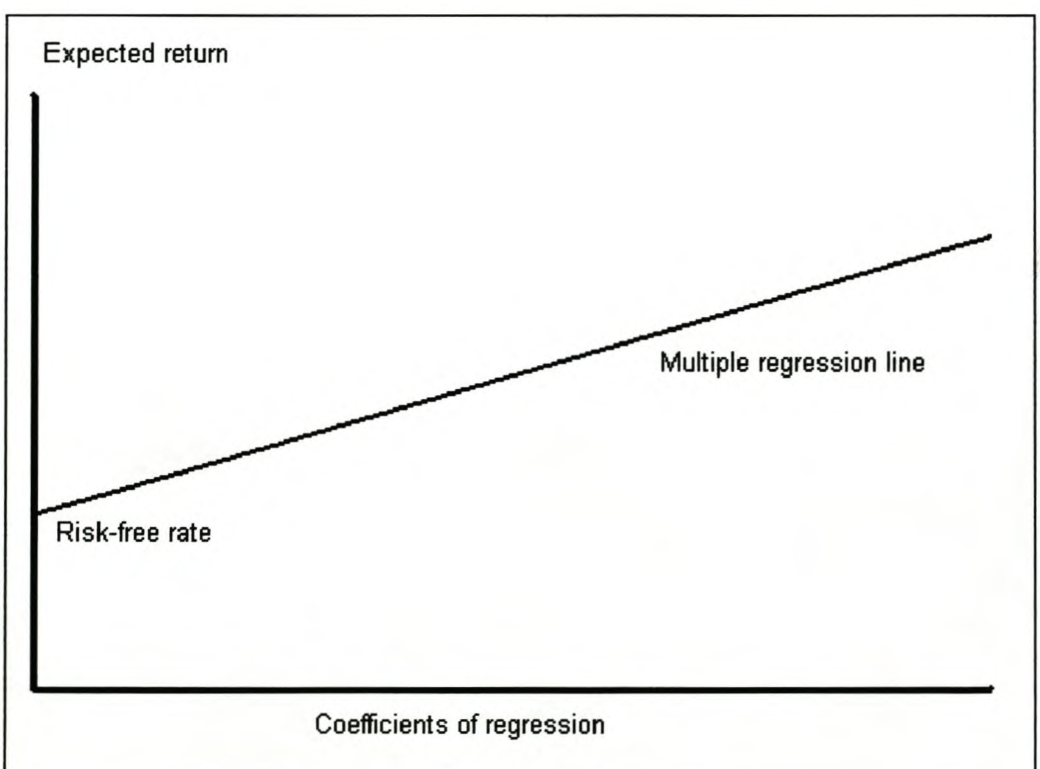
Several factors influence the returns of all assets according to the arbitrage-pricing theorem. This is the fundamental difference of the arbitrage pricing theory when compared to the capital asset pricing model, which assumes only beta as its risk factor. Equation 2.3.1 represents the arbitrage pricing theory (ibid.).

$$R_i = E_i + \varepsilon_i + \sum_{i=1}^{k} b_i \delta_i$$ **Equation 2.3.1**(ibid.)(Adapted)

R_i indicates the required return rate on investment i. E_i is an indication of the intercept of the regression for investment i, or the risk-free rate associated with investment i. Alternatively, the expected returns on a zero-beta portfolio are used, as it is also an indication of the risk-free rate of return. The b_i is an indication of the reaction of investment i to the changes in factor i. An example of such a factor could be the covariance of the asset's change in its rate of return with changes in the inflation rate, divided by the variance of the inflation rate. Therefore, b_i is a standardised measure of the specific risk of the investment in relation to the factor identified and regressed. This approach is similar to the capital asset pricing model, as elucidated by Beaver et al (1970). δ_i indicates the return premium over and above the intercept or risk-free of return associated to the specified risk factor used in the regression equation. With the introduction of inflation as a factor, δ_i would represent the net inflation rate, and the intercept would signify the real risk-free rate. The arbitrage pricing theory can comprise as many factors as required to explain the relationship between risk and return for any asset.

The error term ε_i is sufficiently independent to permit the law of large numbers to hold, and does not require further detailed analysis. As explained during the discussion of the capital asset pricing model, the law of large numbers simply implies that for a well-diversified portfolio the independent noise (unexplained terms) becomes insignificant. The arbitrage pricing theory would result in an asset's returns to plot on a two dimensional space, which is spanned by a vector that originates from an intercept on the y-axis. Refer to Figure 2.3.1 for a graphical depiction of the arbitrage pricing theory. According to Ross (1976:342), if a risk-free asset and a risky asset's returns are independently and normally distributed, the error term would be the only term that signifies the randomly generated returns associated with the model.

Figure 2.3.1 (The arbitrage pricing theory)



The result of the arbitrage pricing theory is an equation where the required rate of return is based on the factors included in the regression, as per Equation 2.3.1. The expected rate return of an asset is computed as the weighted average of the product of the risk premiums and their associated risk factors, added to the risk-free rate of return (Reilly & Brown, 1997:279-300).

Examples of factors include changes in the inflation rate, gross national product growth, interest rates, company size, and political upheavals. The regression terms determine how each asset reacts to the common factors identified (ibid.).

The name of the model is derived from the presumption that investors would short sell overpriced assets and then acquire under-priced assets with the proceeds generated from the short positions created in order to generate a risk-less profit as the market corrects the exploited imbalances. The assumptions of arbitrage relate to the efficient market hypothesis, which assumes competing investors transact promptly on the release of new information. The value of assets swiftly changes to reflect new information. Therefore, the arbitrage pricing theory is an equilibrium theory of expected security returns. The "no arbitrage" assumption is reflected most directly by the intercept or the risk-free rate of return, as it is the equivalent for all assets (Sharpe, 1984:23).

2.3.2 Discussion of tests, advantages and critique

The model is a new development compared to the capital asset pricing model, though it has been around for some thirty years. The arbitrage pricing theory is not as widely used in practise as the capital asset pricing model owing to some limitations and complexities associated with the current theory.

The first advantage of the model is that it requires fewer assumptions than the popular capital asset pricing model. The arbitrage pricing theory better reflects reality than the capital asset pricing model, as it imposes fewer restrictions on its simulated environment than the capital asset pricing model in the form of assumptions. The result is a more practical solution for the calculation of a required rate of return for an asset. This arbitrage pricing theory is considered superior to the capital asset pricing model, owing to the observation that its less restricting assumptions create a more realistic hypothesis (Reilly & Brown, 1997:279-300).

The assumptions of the capital asset pricing model that are not required by the arbitrage pricing theory include the one that investors have quadratic utility functions. Other assumptions of the capital asset pricing model, which is not required by the arbitrage pricing theory, is that asset returns are normally distributed and the existence of a fictitious market portfolio that comprises all assets combined with its associated mean-variance coefficient, beta (ibid.). Consequently, the theorem holds for any risk adverse agent, and not just one with quadratic utility functions. In addition, there is no need to define a representative market portfolio (Ross, 1976:341).

Ross (1976:355) indicated that another strength of the arbitrage pricing theory is that it does not require the same homogeneity of anticipations from market participants as the mean-variance theory contained in the capital asset pricing model. This homogeneity of anticipations is what allows the normal distribution of the error term in the capital asset pricing model.

Studies by Roll and Ross, and Chen (as cited in Reilly & Brown, 1997: 322-330) proved that the arbitrage pricing theory does in fact explain different rates of return, and in some cases, its results are better that the capital asset pricing model. Roll and Ross further showed that, after adjustment for skewness, a share's rate of return does not relate to its own standard deviation of returns, but to external factors, as indicated by the arbitrage pricing theory. If a share's rate of return related to its own standard deviation, it would have provided support for the capital asset pricing model, as the capital asset pricing model computes return as a function of a share's standard deviation of returns (refer to Section 2.2.1). The authors further indicated that the model applied to a cross-section of economically and fundamentally different companies.

Cho, Elton and Gruber (as cited in ibid.) also indicated that a multi-factor model is required to explain returns, similar to the arbitrage pricing theory.

Dhrymes, Friend, and Gultekin (as cited in ibid.) attempted to identify the number of factors required to characterise the risk-return relationship. They observed that a positive relationship exists between the number of assets in a portfolio and the number of factors required to characterise the risk-return relationship. Roll and Ross (as cited in ibid.) indicated that this positive correlation between the amount of factors and the amount of assets, relates to the increase in the probability of obtaining more relevant factors, as the number of relationships that arise in larger samples increases. Such factors include, but are not limited to, industry factors, economic variables and political events.

Burmeister and McElroy (as cited in ibid.) attempted to clarify the January anomaly with the use of the arbitrage pricing theory. The January anomaly simply relates to the observation that returns during the month of January are significantly higher than for any other month. The authors concluded that neither the arbitrage pricing theory, nor the capital asset pricing model, could explain this observation. Nevertheless, Burmeister and McElroy rejected the capital asset pricing model in favour of the arbitrage pricing theory as it provided them with better results.

The most profound disadvantage to the arbitrage pricing theory is that it fails to identify the multiple factors required by the model. This is possibly the cause for its lack of acceptance, combined with the complexities that are normally associated with multi-factor regression. Friend and Gultekin (as cited in ibid.) indicated that one of these complexities is the difficulty associated with the determination of significant factors. The Friend and Gultekin problem can, however, be solved by more complex statistical tests, for example the f-test for factor significance.

Dhrymes, Friend and Gultekin (as cited in ibid.) repeated their previous tests and identified the fact that the number of time-series observations affected the amount of factors required to clarify the risk-return relationship. The number of observations also affected the beta calculations of the capital asset pricing model, as indicated in section 2.2.2. The quantity of data used for regression purposes appears to be a common problem for all factor models, as the discussion on factor models will also indicate. In order to address this shortcoming of the traditional risk and return models, a consensus rate of return for all types of risky assets can be obtained from publicly available data, as per Equation 2.2.4, where the price-earnings ratio is a vital element of the calculation.

Another problem of the arbitrage pricing theory is that it is not time-period specific. In other words, the investors do not all have the same one-period time horizon as the capital asset pricing model. The problem with divergent investment horizons is firstly that all participants envisage different risk-free rates of return that relate to the interest rate profile of their investment horizon. Secondly, they will include different parameters in order to address the investment risks they foresee during the duration of their investment.

Therefore, the arbitrage pricing theory does not consider the fact that investment horizons are not equivalent (Ross, 1976:341-356).

To conclude, the arbitrage pricing theory is still a relatively new development, and according to Ross (1976:343), it is a reasonably robust model, which is unlikely to be proven incorrect. In contrast to other models that rely on an equilibrium condition, it is much more of an arbitrage relation. In addition, the arbitrage pricing theory, unlike the capital asset pricing model, deals with items that might be easily measured, but it neglects to identify these items or factors. According to Sharpe (1984:23), the arbitrage pricing theory is considered to apply to a broad range of possible risk-return relationships.

2.3.3 Significance and relation to the price-earnings ratio

The anomaly, that on a relative basis, low price-earnings ratio shares (normally associated with smaller companies) outperform portfolios of medium to high price-earnings ratio shares lead to various tests of the risk-return relationship with the arbitrage pricing theory with the aim of clarifying this anomaly. The theory is that the arbitrage pricing theory (based on less restrictions) closer reflects reality and should clarify the observation that companies with low price-earnings ratios yield superior results.

Reinganum (as cited in Reilly & Brown, 1997:327), tested the arbitrage pricing theory's ability to explain the small company effect (small companies earn higher risk adjusted returns than the market average). He contended that this anomaly, which could not be explained by the capital asset pricing model, should be explained by the arbitrage pricing theory if it were superior to the capital asset pricing model. Reinganum's tests included the construction of portfolios of securities with similar risk characteristics. Assuming equal risk, the portfolios should have equal excess returns in theory and these excess returns must be close to zero.

According to Reinganum (as cited in ibid.), the results were inconsistent with the arbitrage pricing theory, irrespective of the number of factors used. The small-company portfolios experienced statistically significant positive average excess returns. The mean difference in return between the small-company portfolios and the portfolios of large companies was about 25 percent per year. Reinganum acknowledged the existence of several hypotheses' implicit in the arbitrage pricing theory, and with his tests, concluded that it was impossible to pinpoint the error.

Chen (as cited in Reilly & Brown, 1997:328) subsequently tested the small-company anomaly and his work supported the arbitrage pricing theory, in contrast to Reinganum's findings. Chen concluded that problems, which have to do with limited sample sizes, and the existence of multiple factors, relate to the testing of the theory and not its applicability. He also indicated that the capital asset pricing model did not clarify the small-company anomaly. According to him the missing risk and return information was included in the arbitrage pricing theory. He added that neither the share's own variance nor its company size had sufficient explanatory power of the risk-return relationship. Chen's observations supported the arbitrage pricing theory and assist in the clarification of the small company anomaly.

Arbel et al. (as cited in Shefrin & Statman, 1995:26) might just clarify the inability of both the capital asset pricing model and, in some instances, the arbitrage pricing theory to explain the small-company anomaly. The authors implied that some companies, as previously indicated, are omitted by analysts from research reports and consequently devaluate to achieve significant returns when finally included again. The suggestion is, however, contradicts even the weak form of the efficient market hypothesis. The weak form of the efficient market hypothesis requires prices to reflect all historical information. The only logical assumption is that not all-historical information is included in a share's market price, or at least the price of some shares.

On the other hand, the price-earnings ratio, as with the capital asset pricing model, can be used to calculate market consensus rate of return for an asset. This calculation is achieved with the use of Equation 2.2.4, as discussed in section 2.2.3.

$$R_i = \left(\text{Payout ratio} \big/ \text{Price - earnings ratio} \right) - g \,.$$ **Equation 2.2.4**

The rate of return derived from Equation 2.2.4 can then be used in Equation 2.3.1 to solve for meaningful regression coefficients. Elton, Gruber, and Rentzler (as cited in Reilly & Brown, 1997:328) contended that it was important to develop arbitrage pricing theory regression models with statistically identifiable factors that have economic significance. Economically meaningful factors can then be tested for statistical significance and autocorrelation (normally a problem with economic factors). Examples of meaningful economic factors include changes in the gross domestic product, the inflation rate, and various business confidence indexes.

One can furthermore extend Equation 2.2.4 to accommodate other growth patterns or valuation techniques (unlike the Gordon model used for Equation 2.2.4). These growth patterns normally relate to the different stages of the Gompertz growth curve, as this reflects the profit cycle of new products (Mao, 1966). Equation 2.2.4 can also be adapted to accommodate tests that identify the consensus levels and changes of economic factors incorporated by market prices, in order to evaluate the reasonableness of those expected economic factors associated with the valuation. In order to test expected changes in economic factors more complex valuation models (as per chapter 3) are used, that embody multiple growth phases in a single model. In order to assess the reasonableness of a valuation, as just described, the value must be backwardly induced right through to the risk and return model (arbitrage pricing theory).

If the efficient market hypothesis does not apply to the market even in its weak form (as suggested by Arbel et al. as cited in Shefrin & Statman, 1995:31-33), Equation 2.2.4 provide its user with the opportunity to evaluate consensus market information, identify deficiencies, and make transactions accordingly. Therefore, the price-earnings ratio itself also assists in the clarification of the anomaly that low price-earnings ratio shares normally outperform higher ratio shares, on a relative basis. The answer might be found in the presumption that the market overlooks some companies, as concluded by Arbel et al. (as cited in ibid.). Further evidence that the efficient market hypothesis does not apply to some shares during specific times came from Shefrin and Statman (ibid.). They contended that investors overreact to new information, resulting in companies that are associated with bad news releases becoming undervalued (as supply exceed demand for these shares) and vice versa.

From the discussion above, it is safe to assume that analysts frequently exclude companies associated with bad news releases for some time, decreasing demand for their shares, and dropping their value. Then, as soon as the bad news wears off, and the shares are scrutinised, the observation that these companies represent good value becomes public knowledge. Demand for the shares rapidly increases, raising their price. Both the sell-off and the repurchase actions are associated with overreaction, resulting in significant gains for those that buy cheap earnings (low price-earnings ratio).

Another plausible alternative is that investors overestimate the value of growth and are, therefore, willing to pay too much to procure high growth rates. The section on the franchise value and the growth opportunities approach in section 3.6 will clarify this possibility. It will demonstrate that a significant investment, that offers a higher rate of return than the company's average, is required to raise the price-earnings ratio.

Lastly, an evaluation will now follow of the arbitrage pricing theory as a risk-return model, according to the requirements set out by Damodaran (1996:20-21). The arbitrage pricing theory fulfils the first requirement (universal application) as set out by Damodaran, as it applies to the evaluation of the risk-return relationship of all types of assets. However, it will be rather more complex to apply the arbitrage pricing theory to real estate or art valuations, as a result of a lack of information. Nonetheless, a lack of information is a market deficiency, and not a shortcoming of the arbitrage pricing theory. The price-earnings ratio, as explained, also applies to all types of assets. This universal application of the price-earnings ratio is a result of the presumption that it is a relative measure and, therefore, not limited by a lack of data for regression purposes.

With reference to the second requirement, namely the classification of the types of risk, the arbitrage pricing theory fails to provide an indication. However, the arbitrage pricing theory is based on market regression analysis and, therefore, it should only include risks rewarded by the market. The same logic applies to the price-earnings ratio. As indicated during the discussion of the capital asset pricing model (section 2.2.2), which identifies the types of risk facing an investor, a true market portfolio and its related risks are impossible to generate and, therefore, this requirement is not a reasonable prerequisite for risk-return models.

According to the third requirement, that the identified risks should be related to returns, both the arbitrage pricing theory and the price-earnings ratio are useful, as indicated during this discussion. Furthermore, both models apply in practice, albeit not with the same ease of application, as the arbitrage pricing theory is not as widely used as the price-earnings ratio. The emphasis of this study will now turn to similar factor models, but these models are based more on a statistical approach than plain economic theory, in comparison to the arbitrage pricing theory, which is based more on economic theory.

2.4 Factor models

2.4.1 The model

According to Damodaran (1996:39), the arbitrage pricing theory's failure to identify any factors may be beneficial from a statistical standpoint, but it is a definite weakness from an intuitive perspective. The solution to this problem is simple. Replace the unidentified factors with economic factors. The result is an intuitive model that retains most of the strengths of the arbitrage pricing theory; however, it is dubbed a factor model.

Factor models represent the behaviour of security prices and they attempt to capture the major sources of correlation associated with security risks and returns. The goal, according to Sharpe (1984:21), is to find pervasive factors that affect significant numbers of securities.

Factor models are categorised as either single or multiple factor models. Single factor models use only one independent variable to predict the dependent variable. Multi-factor models include more than a single independent variable, in an attempt to better explain the relationship that exists between the dependent and the independent variables (Mason, Lind & Marchal, 1999:467). Furthermore, these models are not required to represent only linear relationships. Factor models can be exponential, polynomial, or in any other form. Also, multi-factor models can comprise as many factors as required to characterise a desired relationship. For simplicity, and relevance to the objectives of this study, only linear models will be discussed.

Linear multi-factor models are represented by the Equation 2.4.1. A single factor model is also accommodated by Equation 2.4.1, whenever it only has one b_i factor.

$$R_i = E_i + \varepsilon_i + \sum_{i=1}^{k} b_i \delta_i$$ **Equation 2.4.1** (ibid.) (Adapted)

Equation 2.4.1 is essentially equivalent to Equation 2.3.1 that represents the arbitrage pricing theory. The symbol R_i indicates the required rate of return on the investment, similar to the arbitrage pricing theory. The character E_i is an indication of the intercept of the regression for investment i, but not necessarily the risk-free rate, as with the arbitrage pricing theory. The symbol b_i is an indication of the reaction of investment i to changes in the independent variables. It is these independent variables, represented by δ_i, that one substitutes with the expected variables to investigate different circumstances. The remaining term ε_i represents the error term or excess returns.

From a statistical perspective, and for the model to be a true linear relationship between the variables, four statistical assumptions must be satisfied (Mason et al., 1999:475). The first of these assumptions is that for each independent variable, there exists a group of dependent variables. Furthermore, these dependent variables must be normally distributed. Secondly, these normally distributed dependent variables must have means that lie on the straight line produced by the regression equation. The third requirement is that these normal distributions have equivalent standard deviations. The last requirement is that the dependent variables are statistically independent of one another. In other words, the outcome for a particular set of circumstances must not depend on the outcome obtained from another set of independent variables. This requirement is also known as autocorrelation.

Statisticians firstly test a regression equation with a global test, also referred to as the F-test (based on the F-distribution), to determine if the dependent variable, in this case the rate of return, occurred by chance (ibid.). If the relationship occurred by chance, the regression is meaningless. However, if it is not a random variable, they proceed with hypothesis testing of the individual independent variables. Here it is also important to test for multicollinearity, or correlation among the independent variables. Economic factors that depend upon one another might lead to incorrect conclusions about the relevance of the factors included in the analysis. The objective of these tests is simply to ensure that only statistically significant and relevant variables are retained.

The ultimate aim of factor models is to describe the return generation process. Therefore, the identification of the relevant factors of independent variables should proceed from an analysis of the economics of the securities involved, according to Sharpe (1984:21). These factors include aspects of macroeconomics, microeconomics, and other fundamental aspects of the shares, like the relative price-earnings ratio.

According to Sharpe (ibid.), a factor model must be employed either implicitly or explicitly, as it would be impossible to think about the interrelationship of every security with every other security. Sharpe suggested that with the development of factor models, one firstly identifies important factors from the economy and marketplace. Then the user must assess the extent to which securities respond to changes observed in the identified factors. Furthermore, these factors need not remain constant, so there is no reason to assume that a well-defined factor model will remain valuable. The reason for this last observation is that the risks and returns associated with various factors might change over time.

2.4.2 Discussion of tests, advantages and critique

Factor models have not been tested for effectiveness by academics to the same extent as the capital asset pricing model or the arbitrage pricing theory. This is probably a result of the diversity of possible regression models and regression relationships. Furthermore, most models in use today are proprietary, and therefore, not available for testing. Tests that have been done on factor models relate only to the specific models and are not an indication of the effectiveness of other models.

The greatest disadvantage of factor models is that the models are dynamic (the factors are not stationary). Consequently, a specific factor model will not necessarily apply during different economic cycles and other changed circumstances. Generally, factor models are very specific in application. The implication is that a factor model regressed on growth shares will not automatically apply to shares of companies in the retail sector, or any other mature sector for that matter. These sectors require other regression models. The second disadvantage of factor models is that the factors are open to interpretation and no specific guidelines exist for the identification of factors.

An advantage of the factor models is that market efficiency is not a requirement for successful application, according to Sharpe (ibid.). Unlike the capital asset pricing model or the arbitrage pricing theory, that require at least the weak form of market efficiency, factor models do not require any sort of equilibrium to exist in the marketplace. Factor models simply represent the co-movement of some factors (fundamental and economic variables) with the returns earned on equity investments (or any other relationship for that matter). A second advantage associated with the use factor models is that they can be used to regress any relationship. An example is a regression equation with the price-earnings ratio as its dependent variable, and economic fundamentals, like growth in gross domestic product, inflation, and business confidence as its independent variables. The abundance of factor relationships available is part of the strength associated with a model that fails to identify its parameters.

2.4.3 Significance and relation to the price-earnings ratio

Factor models and the price-earnings ratio share several similarities. Both factor models and the price-earnings ratio can be used to characterise risk-return relationships, and as valuation models. Similar to the capital asset pricing model and the arbitrage pricing theory, output (the dependant variable) from factor models can represent the required rate of return for the calculation of price earnings ratios, or other valuation techniques. On the other hand, factor models can also be used to directly calculate the price-earnings ratio, based on regression models that incorporate economic and other fundamental data.

To solve the relationship between the required rate of return and risk with factor models the price-earnings ratio is used, just as with the capital asset pricing model and the arbitrage pricing theory. Equation 2.2.4 characterises the relationship between the price-earnings ratio and the required rate of return. The required rate of return obtained from Equation 2.2.4 is used to solve an appropriate regression equation with the relevant economic factors that underlie the risk-return relationship. More complex price-earnings valuation equations from chapter 3 can be substituted with Equation 2.2.4 to accommodate different growth patterns, as previously indicated.

$$R_i = \left(\text{Payout ratio} \big/ \text{Price - earnings ratio} \right) - g \,. \quad \textbf{Equation 2.2.4}$$

A simpler calculation than the one described above would be to use the ruling market price, instead of the price-earnings ratio. However, this action would deprive one of the opportunity to relate the price paid to historical accounting earnings, expected earnings or some form of normalised earnings. This relationship of the price with earnings assists the user in obtaining a relative perspective of the return generation process. The result of the price-earnings based rate of return is some form of standardised risk-return relationship for the category to which the regression applies. According to Damodaran (1996:20), it is important to have a relative perspective of risk, as risk is a relative measure.

The price-earnings ratio assists in the categorisation of companies according to their accounting practises into two groups. The first of these groups represents companies that misuse accounting conventions and rules in order to achieve inflated results. The second group applies accounting rules prudently and possibly understate their earnings. This classification is achieved through the comparison of the historical price-earnings ratio to the normalised and forward ratios. The logic is that the historical price-earnings ratio will be much lower than the normalised and forward ratios as a result of the inflated earnings figures for companies that overstate earnings and vice versa.

Beaver et al. (1970:659-668) made use of regression analysis to obtain an accounting based measure of risk. These risk measures included (independent variables) the dividend payout ratio, growth rate, percentage of leverage, market liquidity, relative asset size, variability of accounting earnings, and co-variability of earnings with the sector. From these variables, the authors constructed an accounting based measure of risk with the use of a factor model. Beaver et al. concluded that a measure of total variability, such as the price-earnings ratio, performs just as well or better than the accounting based risk measures included in their quantification of the risk-return relationships investigated.

The study by Beaver et al. discussed above, provided support for use of the price-earnings ratio as a relative risk measure. The intuitive appeal of the price-earnings ratio as a risk measure is based on the presumption that it already includes the results of a market-wide valuation process, which is based on a consensus view of risk. Similar studies (such as Myers, Brenner & Smidt, as cited in Gahlon & Gentry, 1982:16) also utilised regression analysis to calculate accounting based risk measures.

The ability of the price-earnings ratio to provide historical, normalised and forward ratios, all based on the same price, provides its user with yet another useful tool. The price can be either the market price, or a price the user deems appropriate under the circumstances. An example that illustrates the application of this advantage is when the forward price-earnings ratio (dependent variable) is regressed against expectations of inflation, changes in the gross domestic product and market penetration predictions (independent variables) to obtain a relative forward-looking price-earnings ratio. The intuitive benefit associated with the forward price-earnings ratio is the advantage of much better regression results when compared to the results obtainable from just the market price, or yield earned during the past year.

A further strength associated with the use of price-earnings ratios is that results of financial regression equations are almost always improved upon by a focus on economic changes rather than levels of activity. In this regard, it is clear that the price-earnings ratio offers additional benefits when compared to price-levels (market prices). The price-earnings ratio possesses this added strength as a result of its origin as a relative measure of risk and return. Section 3.6 will demonstrate that price-earnings ratios change only in relation to the ability of a company to acquire new profitable investment opportunities. It is clear that the price-earnings ratio is not influenced by factors that affect price levels. The result is a more stable factor relationship (dependent or independent variable) and a better indication of activity than a share's market price.

Regression analysis is used to obtain variables that best describe the relationships between independent and dependent factors. Furthermore, the price-earnings ratio provides the possibility of obtaining relative regression relationships in the search for better regression results.

As specific regression models (factor models) are based only on the samples being analysed, it might, depending on the regression coefficients, not apply to the full spectrum of investments available to investors. Regression analysis does not make a distinction between the types of risk an investor faces, as does the capital asset pricing model. However, one can again safely assume that only market (systematic) risk remains relevant for representative regression equations.

Lastly, regression models are widely applied in practice, especially by economists. Regression models also achieve the objective of translating indicators of risk into a required rate of return.

In order to conclude the discussion on factor models, it should be emphasised that factor models do not fulfil all the requirements of a good risk and return model, as set out by Damodaran (1996:20), and discussed at the beginning of this chapter. It should be emphasised that factor models primarily fail to fulfil the requirement of universal application stipulated by Damodaran. However, most other requirements are met. Nevertheless, factor models still provide very useful information, especially when used in conjunction with the price-earnings ratio.

The next model or test is termed stochastic dominance models, as this model, like factor models, do not depend on economic theory. Stochastic dominance is based upon statistics and intuitive logic.

2.5 Other factors affecting return rates

2.5.1 Stochastic dominance models (rules)

Stochastic dominance models are divided into three categories, namely first, second and third degree dominance rules. These performance tests are free from most of the assumptions of the capital asset pricing model, and therefore, do not place the same constraints on the analyst as the capital asset pricing model (Barlev et al., 1988:72-73). Third degree dominance rules involve mathematical techniques that are beyond the scope of this study and do not provide additional information on the relationship between risk and the price-earnings ratio. Therefore, third degree dominance rules will not receive any further attention in this study.

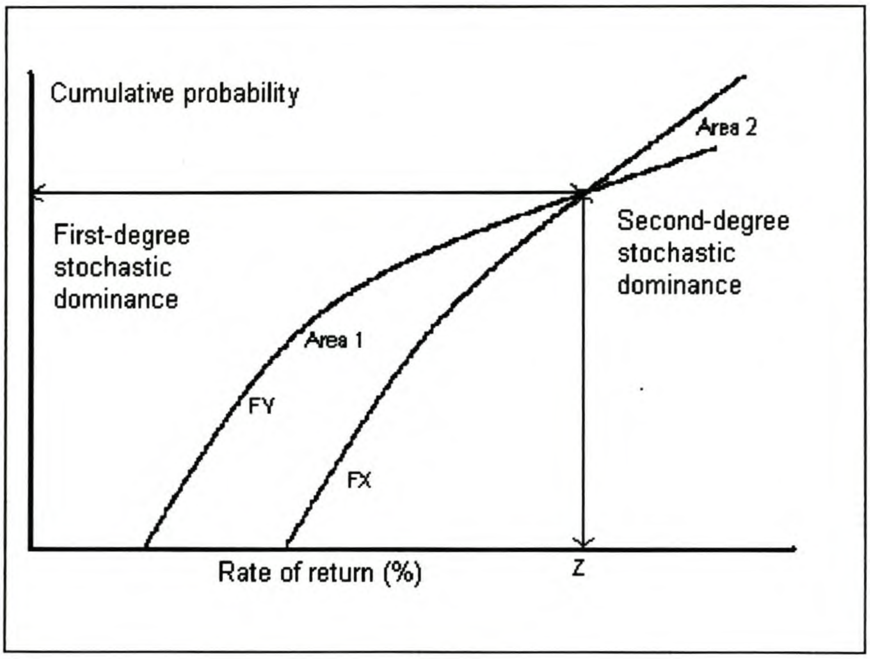
The mean-variance rule (component of the capital asset pricing model) coincides with the second-degree stochastic dominance rule when returns follow a normal distribution (ibid.). The second-degree dominance rule provides an approximation for many types of risk-averse utility functions, even non-normal distributions. The introduction of stochastic dominance models will commence with a discussion on the mean-variance rule, as the basis of first and second-degree dominance.

To elucidate the mean-variance rule, assume two random variables of return, namely X and Y. X dominates Y by the mean-variance rule if, firstly, the expected value of X is equal or greater than that of Y indicated as ($EX \geq EY$). Then secondly, the variance of X must be less than that of Y, as indicated by the inequality $\sigma_X^2 < \sigma_Y^2$ (ibid.). At least one of these inequalities above is strict. If an investor accepts non-decreasing utility functions that are represented by U, investment X will dominate Y, as indicated by the inequality $EU(x) \geq EU(y)$. The investor would always prefer investment X to Y, when the normally distributed cumulative probability function of X is below that of Y, represented as $F_X(z) < F_Y(z)$.

When the last inequality ($F_X(z) \leq F_Y(z)$) is satisfied, X dominates Y by the first-degree stochastic dominance rule (ibid.). The first-degree stochastic dominance rule implies that the cumulative distribution of X is located to the right of Y, indicating that higher utility is obtained from investment X. Refer to

Figure 2.5.1 for a graphical depiction of the first-degree stochastic dominance rule, as represented by the area to the left of the Z on the X-axis. The only trivial requirement of the first-degree stochastic dominance rule is that investors require more rather than less wealth.

Figure 2.5.1 (Stochastic dominance)(Barlev et al., 1988:73)



If the cumulative probability distributions cross each other, as in Figure 2.5.1 to the right of the Z, then there is no dominance by the first rule (ibid.). In order to describe the second-degree stochastic dominance rule the assumption of risk aversion is required. Under this rule X dominates Y, if and only if, the function of X starts to the right of Y, as represented by Figure 2.5.1 and the accumulated enclosed area between the two distributions is not negative (area 2 in Figure 2.5.1) up to any accumulation point. Negative is defined as an area where Y is to the right of X. Even though the graphs cross but the area remain positive (Figure 2.5.1) X dominates Y by the second-degree of stochastic dominance.

Stochastic dominance rules circumvent the problems inherent in the usual capital asset pricing model-based tests, according to Levy and Lerman (1985:31). Their study showed that a low price-earnings ratio portfolio strategy is the preferred route (most profitable), with the assistance of stochastic dominance models. The second-degree stochastic dominance rule applied to the tests of Levy and Lerman, as the low price-earnings ratio portfolios were located to the right of the medium and high ratio portfolios. Also, the positive area associated with the low price-earnings ratio shares exceeded the negative area. The test provides support for the premise that shares with low price-earnings ratios tend to be ignored by market participants. It also disregards the efficient market hypothesis in its weak form.

The weak form of the efficient market hypothesis assumes that security prices reflect all historical market information. However, if investors overreact to bad news and analysts consequently fail to investigate these shares, the associated prices are definitely not a fair reflection of all historical information. Keim (1986:25-30) came to the same conclusion after tests of the capital asset pricing model revealed that for shares with comparatively low-price earnings ratios, the relationship between risk and return is not strict. The suggestion that the analysts might omit some companies from their recommendations came from Harris and Marston (1994:19), and was supported by Shefrin and Statman (1995:26).

Levy and Lerman (1985:31-32), in their tests described above, suggested that stochastic dominance rules are theoretically unimpeachable, because they are universally valid and apply to all investors. The use of these rules allowed Levy and Lerman to consider the total risk associated with portfolios constructed of different categories (high, medium, and low) price-earnings ratios. They insisted that stochastic dominance rules represent the best risk-return criteria under uncertainty, as their tests confirmed the benefits associated with low price-earnings ratio investment selection strategies, which were not evident from the capital asset pricing model. According to Levy and Lerman, stochastic dominance models maximise utility for any given risk preference structure with the assistance of mathematical integration.

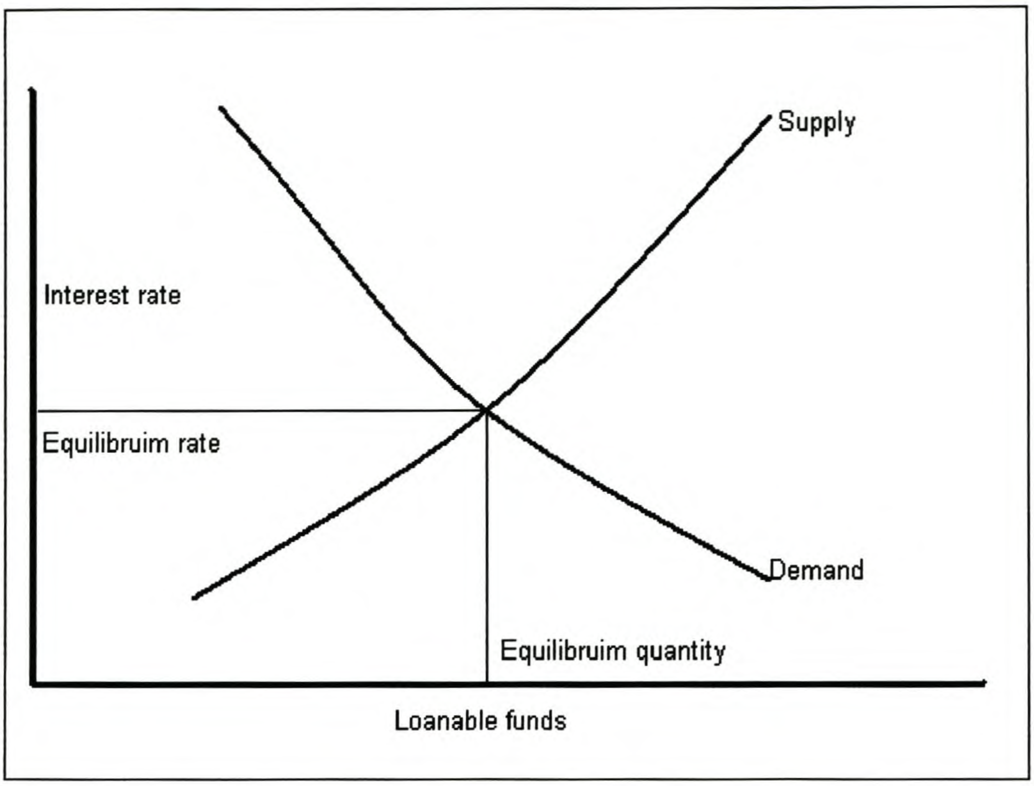
Stochastic dominance performance tests do not provide a required rate of return, such as the previous models. However, stochastic dominance models assist in the ex-post assessment of investment actions. Therefore, stochastic dominance models fail to fulfil the requirements set out by Damodaran (1996:20) for a good risk-return model. However, it proved that shares with relatively low price-earnings ratios yield better investment returns on a risk adjusted basis than random methods of portfolio construction, and conform to research conclusions based on the other models and theories.

2.5.2 Interest rates movements

Treasury securities are of great interest to all market participants as they reflect the minimum rate of return investors are willing to accept. Therefore, the return on risky securities like shares comprises two elements, a risk-free return, and a risk premium (Farrell, 1985). The risk free return again consists of a real return and an inflation premium.

The concept of interest rates allows investors to place a current value on future income. In essence, an interest rate is the price to currently obtain future investment earnings. From the lender's perspective, an interest rate is the reward to delay current consumption (utilisation) into the future. Therefore, interest rates are often referred to as the price of loanable funds or the price for the early procurement of goods and services (Gwartney & Stroup, 1997:660-663).

Figure 2.5.2 (Determination of interest rate) (ibid.).



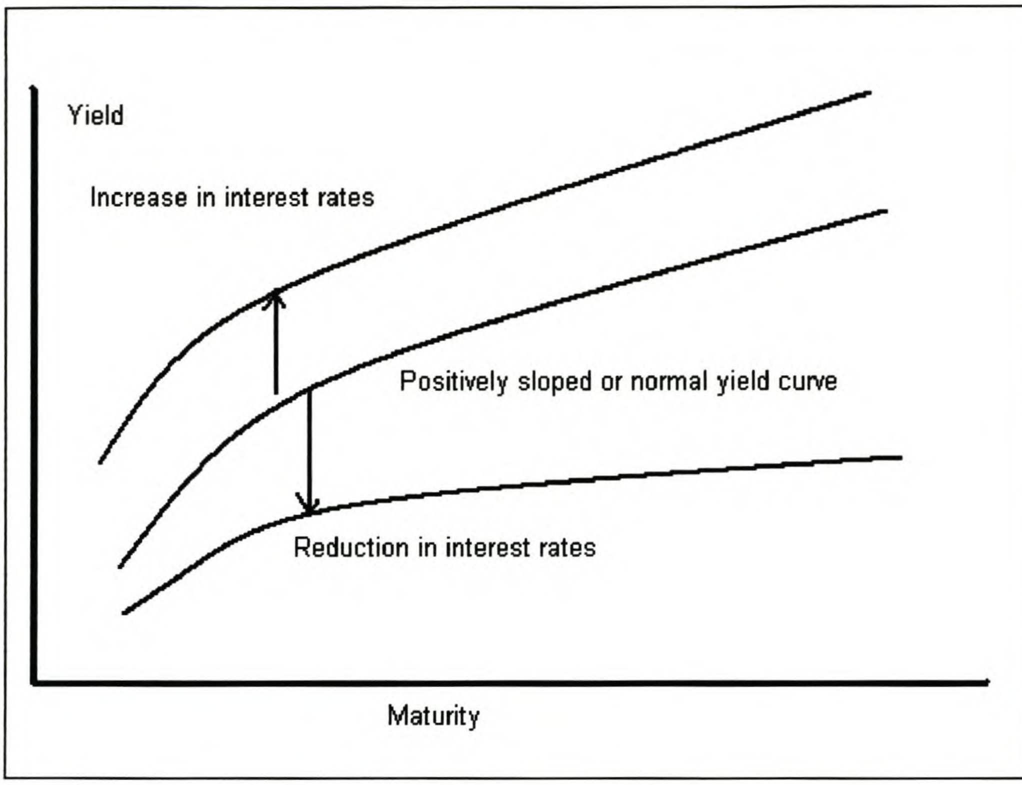
Interest rates are determined by the supply and demand for loanable funds, as depicted by Figure 2.5.2. Investors who demand funds to finance capital investment hope to earn a profit on their investment that covers the value of the loan and the interest, and includes an additional profit for the productive application of capital, before they consider borrowing funds (ibid.). On the other hand, lenders own funds that are in excess of their requirements and they are willing to supply these excess funds to the investors if they can earn something from the transaction. This something the lenders want to earn is the price of funds or the interest rate. Market forces allow participants to determine a consensus rate. The X in Figure 2.5.2 depicts this consensus interest rate (ruling rate). The ruling interest rate is the interest rate at which the quantity of funds borrowers demand for investment or consumption equals the quantity of funds lenders supply.

The risk-free rate of return that applies to the analysis of investment opportunities comprises a real rate of interest and an inflation premium (Gwartney & Stroup, 1997:664-665). The inflation premium reflects the compensation to the lender for the expected decrease, due to changes in the level of prices, in the purchasing power of the principal and the interest during the term of the loan. Therefore, the inflation premium is determined by the expected rate of inflation. The real interest rate is represented by the nominal or market rate less the expected rate of inflation, approximating the generally known Fisher equation. Therefore, the real interest rate indicates the interest premium, in terms of real goods and services, which reimburses one for deferred consumption or investment.

In order to relate the nominal interest rate, observed in financial markets, to the risk-free rate of return it should be noted that nominal interest rates are made up of three components (ibid.). These components are the pure interest premium (real cost of deferred consumption), the inflationary premium (expected changes in price levels) and a risk premium (probability of default). Generally, the risk premium will be substantial when the probability of default is significant. This is why treasury securities represent the risk-free rate of return as it includes only a minor risk premium. Treasury securities are essentially acknowledgements of debt from government. These acknowledgements of debt should carry only a minor risk premium, as government is not expected to default on its liabilities and the market for government instruments is very liquid.

It has now been established that yields on treasury securities reflect the minimum rate of return lenders and investors are willing to accept, as such treasury rates are of great interest to all participants in the marketplace. A focus on the on-the-run issues of treasury securities is particularly informative since the effects of any liquidity premiums (ability to transact) are likely to be smaller than on any other securities as indicated (Fabozzi, as cited in Study Session 15, Reading Highlights, 2001). Liquidity premiums are normally included as part of the risk premium as they are interwoven with the ability to recoup the invested principal.

Figure 2.5.3 (Treasury security yield structure)(ibid.).



When one focuses on the relationship between the yields on treasury securities and their respective maturities, typically longer-maturity issues have higher yields than short-term maturities (ibid.). This situation is referred to as a positively sloped or normal yield curve, and is depicted in Figure 2.5.3. The yield curve forms an important component of the valuation of securities, firstly because an investor's risk-free rate of return is based on his or her investment horizon, as obtained from the treasury yield curve. Secondly, shifts in the yield curve (when interest rates change) also affect the risk-free rate of return during the investment horizon and the value of the investment accordingly.

Therefore, an increase in interest rates (upward shift of the yield curve) would increase the discount rate and decrease the value of securities. It would also decrease the price-earnings ratio, if all other things remain equal. A decrease in interest rates would increase the value of securities and the price-earnings ratio, all other things being equal. Similarly, longer investment horizons would require higher yields under a normal yield curve and result in lower valuations and price-earnings ratios.

The effects of changes in interest rates on security prices and the price-earnings ratio are measured by the duration of an investment. The concept of effective duration (first mathematical derivative with respect to the discount rate), and its association with the price-earnings ratio, will be clarified further in chapter 3. Interest rates changes also have a secondary effect on valuation and the price-earnings ratio. These secondary effects relate to the change in profitability and cash flows a company can expect from changes in interest rates. When interest rates rise, the company will firstly pay more interest on its own debt. Similarly, its customers will also pay more interest, which will in effect reduce their ability to buy new products and services from the company. Thus, a rise in interest rates reduces the value of a company's shares on two grounds. Firstly, an increased discount rate and secondly reduced profitability. The effect on the price-earnings ratio, all things considered, depends on the interest rate exposure of the company and its customers.

Spiro (1990:63-64) showed that interest rates affect security prices with the assistance of a factor model that relates the volatility of share prices to the volatility of interest rates. According to Spiro, many popular investment strategies consist, to some extent, of allocating assets to the instruments that yield returns in excess of their historical norms. As a result of asset reallocation, returns on all types of assets relate closely to one another. The regression model further indicated that the average dividend yield on shares closely reflects the treasury security yield.

Spiro (1990) also suggested the use of real interest rates for valuation purposes, as it reduces the amount of variables in the valuation process, because the expected inflation premium in the risk-free rate is excluded. Another observation Spiro made is that stock exchanges are extremely sensitive to recent fluctuations in interest rates. According to him, this demonstrates a degree of investor shortsightedness and irrationality. Typically, increases in share prices (reduction in interest rates) stimulate investment and increase the demand for credit, resulting in higher interest rates to kerb inflationary pressure. Lastly, Spiro indicated that reserve bank intervention (changing the overnight lending rate to banks) is unlikely to have lasting effects on the stock markets.

2.6 Conclusion

Chapter 2 focused on the relationship between risk and return, and the relevance of the price-earnings ratio to this relationship. Therefore, the chapter firstly defined the characteristics of a good risk and return model.

It then focused on the capital asset pricing model, its development, and association with the price-earnings ratio. The arbitrage pricing theory and its association with the price-earnings ratio followed, after which factor models and their relation to the price-earnings ratio were introduced, followed by stochastic dominance models. The chapter concluded with a focus on the effects of interest rates on share prices and the price-earnings ratio. Interest rate movements form an integral part of the required rate of return calculations discussed during this chapter.

Chapter 3 will build on the foundations laid by chapter 2 as it applies the discount rates obtained from this chapter to various valuation models.

Chapter 3: Valuation and growth

For investment purposes, the valuation of a company is the next logical step after the appropriate discount rate for the company has been determined. According to Damodaran (1996:4), valuation is useful for a wide range of tasks. Nevertheless, the role it plays differs according to the objectives of the valuation.

Valuation is important for active portfolio management, as certain investors focus on value-investing (the search for undervalued companies as possible investments) and others focus on growth-investing (the search for companies with significant growth potential). Secondly, valuation is useful for franchise buyers (a group of active investors who focus their investment decisions on a few undervalued companies). Normally franchise buyers actively participate in the management of the companies they acquire. Lastly, investment bankers and other dealmakers profit from mergers and acquisitions that are a very important component for growth in the economy. Merger and acquisition activity rely very heavily on valuation. From a financial management perspective, valuation also plays a central role in corporate financial management.

The focus of this chapter is on the relationship of the price-earnings ratio to different valuation models, and the effects of the duration of extraordinary growth normally reflected in the price-earnings ratio. Note that Annexure A contains information about the figures and Annexure B is a summary of the equations and symbols used throughout this study.

3.1 Dividend discount model

3.1.1 The model

One can generally expect two types of cash flows from an investment in the shares of a company. The first is a stream of dividends during the holding period and an expected price at the end of that period (Damodaran, 1996:191). This projected price at the end of the holding period is in itself determined by the projected future dividends. Therefore, the value of a company is calculated as the present value of all future dividends, since a company is expected to continue business indefinitely, in accordance with its charter. The dividend discount model is the most cited model for share valuation and, consequently, for the calculation of the relevant price-earnings ratios.

According to Leibowitz and Kogelman (1994:1), the model was first proposed by John Burr Williams in 1938. The dividend discount model is an entirely valid approach to valuation (Williams, 1938:418). However, emphasis on the franchise value method of valuation will clearly illustrate that it is just another valuation approach.

The rationale for the model is the present value rule. The present value rule states that the value of any asset is the present value of the expected future cash flows of that asset discounted at the appropriate rate of return (Damodaran, 1996:191). According to Miller and Modigliani (1961:419), the dividend discount model is formulated as all future dividends during the holding period combined with the terminal cash price received on the sale of the security. The model is expressed by Equation 3.1.1, where DPS_t represents dividends per share, all other variables remaining as previously defined.

$$\text{Value per share} = \sum_{t=1}^{t=\infty} \frac{DPS_t}{(1+R_i)^t} + \frac{\text{terminal price}}{(1+R_i)^t}$$

Equation 3.1.1 (Damodaran, 1996:192)

Equation 3.1.1 is similar in structure to the formula used for the valuation of fixed income securities. It is simply the present value of all future cash flows. Therefore, Equation 3.1.1 applies in a similar manner to the valuation of bonds. The only difference is that the interest and a principal repayment are substituted by the dividends received and the terminal value. Unlike bonds, that have a limited lifetime, the dividend discount model is consistent with the assumption that a company is assumed to enjoy a perpetual life, in accordance with its charter or articles of incorporation as it allows continuous discounting (Farrell, 1985:18).

The appropriate discount rate is determined as indicated in chapter 2. However, the projection of future dividends and the terminal price is a more complex process which merits further attention. To simplify the projection of future dividends the maximum sustainable dividend a company can afford to pay must first be obtained; then management influences should be considered. The maximum dividend a company can afford to distribute is a function of its planned sales growth, margins on sales and investment requirements without violating its target capital structure. This relationship is captured by Equation 3.1.2 as it illustrates the sources of cash and its uses (Rappaport, 1986:53). Refer to section 4.2.1 regarding the influence of dividends on share prices.

$$\begin{aligned}&\text{Net Income} + \text{Depreciation} + \text{Other non-cash charges} + \text{Change in debt capacity} = \\&\text{Capital expenditure} + \text{Working capital investment} + \text{Dividends}\end{aligned}$$

Equation 3.1.2 (ibid.)

The capital expenditure and working capital investment figures affect not only the cost of real growth, but also price changes, changes in the product mix, and regulation and technological improvements. With revision and restatement the maximum dividend payout rate is obtainable from Equation 3.1.3.

$$D = 1 - \frac{g(f + w)}{(1+g)(r)(1-T)(1+L)}$$ **Equation 3.1.3** (Rappaport, 1986:54)

3.1.2 Discussion of tests, advantages and critique

According to Leibowitz and Kogelman (1994:2), the dividend discount model has a lot of appeal as a result of its fundamental simplicity. However, this simplicity belies a complex set of parameter projections. A further attraction of the model is the intuitive logic it is based on (Damodaran, 1996:192). The instinctive logic of the present value rule and the simple equation above made it the most popular model in the literature of valuation, according to Miller & Modigliani (1961:422). The price-earnings ratio, if calculated based on this model, share most of its advantages and some of the criticisms levelled against it.

A major advantage of the model is its ability to provide estimates of the value of individual shares and the aggregate market (Farrell, 1985:17). The model possesses the ability to allow one to evaluate the relative attractiveness of individual shares and the aggregate market on a comparative basis. This ability is a result of the accessibility of average dividends and return rates within the market and its various sub-sectors. The result is an added advantage of the model compared to the capital asset pricing model when the dividend discount model is used to determine which shares are undervalued or overvalued in monetary terms, and by how much. The value of projected gains can be used for effective asset allocation decisions, not just a ranking as with the capital asset pricing model.

According to Damodaran (1996:211-212), a false perception exists that the model cannot be used for the valuation of shares that pay no dividends at the time of valuation. However, companies that pay no dividends at the time of valuation will inevitably declare and pay dividends in order to reduce their cash position as their growth rate declines and excess cash starts to build up. All successful companies will eventually distribute excess resources, and Equation 3.1.1 ultimately will apply to them.

Damodaran (1996:212-214) claims that the model is inherently stable, and, therefore, it will indicate inflated share values. The model will not overvalue in an overvalued market and vice versa. Valuation models based on a relative valuation basis are prone to such systematic errors. The price-earnings ratio calculated from market data ($^{\text{Market price per share}}/_{\text{Earnings per share}}$) is an ideal example of such a relative valuation model. As a result Damodaran (ibid.) indicates that increases in market value as a result of speculative action will be observed with the use of this model as the dividend discount model is based on underlying fundamentals. This hypothesis was supported by Sorensen and Williamson in 1985, as cited by Damodaran (ibid.). Sorensen and Williamson indicated that undervalued shares according to the dividend discount model experienced positive excess returns and vice versa.

The empirical results of Sorensen and Williamson are explained by the fact that the dividend discount model applies more weight to near-term expected dividends than distant dividends, which are less predictable (ibid.). Furthermore, the dividend discount model is biased toward finding low price-earnings ratio shares with high dividend yields undervalued and vice versa (Damodaran, 1996:214-216). The findings are consistent with empirical irregularities observed in the market by Basu (as cited in Reilly & Brown, 1997:314-322); Peavy and Goodman (as cited in ibid.); Jaffe et al. (1989); and Arbel et al. (as cited by Shefrin & Statman, 1995:26-34) and presented in section 2.3.3.

According to Miller and Modigliani (1961), as an investor purchase dividends and the dividend policy affects the amount of these dividends, it is wrongly assumed that the dividend policy affects the current price of a share. The renowned authors indicated that this presumption is a criticism associated with all forms of the dividend discount model. The view of Miller and Modigliani leads to the observation by Nagorniak (1985:13-15) that the model assumes management preferences and also indicates the rationality of management. The model implies that all companies will at some point start to pay dividends and that the dividends will continue indefinitely. As previously noted, the primary disadvantage of the model is found in the substantial errors that can be made with the projection of future dividends. The projection of dividends is on its own based on the payout ratio of the company (Rappaport, 1986:52-57). As indicated in Section 3.1.1, the maximum dividends a company can afford to pay depends on the cash position of the company.

Nagorniak (1985:13-15) further noted that the model implies a constant rate of return per cash flow. The implication is that a single cash flow is reinvested at the discount rate that relates to that cash flow for the period it is discounted. However, the advantage of this implication is that the model allows the application of time varying discount rates per cash flow, necessitated by anticipated interest rate changes. Lastly, Nagorniak (ibid.) noted that the model implies that all shares will always be traded at their fair values. Therefore, the criticism against the dividend discount model is that it does not consider market inefficiencies and allows the existence of arbitrage in an efficient market, as described in section 2.3.1. It is important to consider the shortcomings of the application of this model in order not to place reliance on misleading results.

3.1.3 Significance and relation to the price-earnings ratio

The dividend discount model provides a value for a share. However, it neglects to indicate whether the company is adequately priced compared to its peers. Nor can it provide a ranking of the company for comparative purposes. It has been empirically proved that inexpensive (undervalued) shares, by large, outperform pricey shares on a comparative basis; refer to Basu (as cited in Reilly & Brown, 1997:314-322); Peavy and Goodman (as cited in ibid.); Jaffe et al. (1989); and Arbel et al. (as cited by Shefrin & Statman, 1995:26-34). See Section 2.2.3.

According to Damodaran (1996:191-192), the dividend discount model possesses the advantage that it is not a comparative valuation model and, therefore, the model will always provide a value derived from fundamentals and not market reaction or perception. It is apparent that the price-earnings ratio calculated from market data has a relative disadvantage in this respect. The price-earnings ratio based on market data will follow an overvalued market and continue to increase on average, providing an acceptable investment tool during the build-up to the investment bubble (market phenomenon associated with an abnormally low equity risk premium), but it will not indicate the existence of the bubble and investors that rely on it, therefore, share in the losses when this inconsistency is rectified.

Damodaran (1996:195) also noted that the dividend discount model is biased toward finding low price-earnings ratio shares with high dividend yields undervalued and vice versa. Fama and French (as cited in Shefrin & Statman, 1995:31-33) found that companies with small sizes and high book-equity to market-equity ratios provided the highest returns. High book-equity to market-equity ratios indicates that companies are valued near book value; these companies would consequently have low forward price-earnings ratios.

This supports the hypothesis that small companies are "omitted" by analysts from their recommendations and when recognised as undervalued achieve significant returns on a risk-adjusted basis, as indicated by Arbel et al. (as cited in ibid.) and presented in section 2.2.3.

Reilly and Brown (1997:445-447) stated that shares with low price-earnings ratios only offer higher returns on a pre-tax basis in countries where dividends are taxed. The low price-earnings ratio phenomenon however, also appears to exist in countries where dividends are not taxed, such as South Africa. High-dividend paying shares with accompanying low price-earnings ratios normally represent mature companies that do not need cash to fund future growth. These companies are commonly classified as blue chip stocks and as a result are thoroughly scrutinised by analysts. Therefore, the hypothesis that low price-earnings ratio stocks are "omitted" from analysis by analysts is not universal. It might be that analysts continuously project the performance of these blue chip shares too cautiously and that positive surprises result in abnormal returns. Blue chip companies are relatively few and as a result empirical tests of the performance of low price-earnings ratio shares might include more companies that are "omitted" by analysts than blue chips.

The advantages of the dividend discount model can, nonetheless, be combined with those of the price-earnings ratio to create a price-earnings ratio that would share the advantages of the dividend discount model. According to Fairfield (1994:23-24), the dividend discount model can be restated directly in terms of accounting data. With the use of clean surplus accounting (see section 4.1.1) the dividends in the dividend discount model can be replaced with earnings and book values, to calculate a payout ratio. When the price is restated in terms of earnings and book value the valuation problem focuses on the fundamental process of creating wealth, rather than the distribution thereof. Fairfield (ibid.) claims that earnings and book values are complementary indicators of wealth and, therefore, the price-earnings and price to book value ratios should provide complementary information about expected future earnings. The author argued that the price-earnings ratio is a function of expected changes in future profitability and the price to book value ratio is a function of the expected level of future profitability.

The value of a company is its ability to create wealth, as measured by the aggregate accounting earnings, rather than the distribution of its accumulated wealth, as measured by its dividends. Fairfield (ibid.) replaced dividends in the dividend discount model formula with Equation 3.1.4 and Equation 3.1.5 representing the results of clean surplus accounting and abnormal earnings respectively. The aim of this substitution is to restate the price in terms of the current book value and the future abnormal (extraordinary) earnings. The result is Equation 3.1.6. Equation 3.1.6 represents a discount model that is free from the limitations of current dividend policy. The symbol y_t represents the value of a company at time t, and x_t represents the earnings.

$y_t = y_{t-1} + x_t - d_t$ **Equation 3.1.4** (Fairfield, 1994:24)

$x_t^a = x_t - R_i y_{t-1}$ **Equation 3.1.5** (ibid.) (Adapted)

$\text{Value per share}_t = y_t + \sum_{t=1}^{t=\infty} x_1^a \Big/ (1+R_i)^t$ **Equation 3.1.6** (ibid.) (Adapted)

The price-earnings ratio can be expressed as a function of capitalised current earnings plus the capitalised present value of changes in future abnormal earnings. Substituting Equation 3.1.4 and a function of the present value of normal earnings into Equation 3.1.6 results in the forward price-earnings ratio based upon fundamental accounting data, as represented by Equation 3.1.7. The price-earnings ratio calculated by Equation 3.1.7 is equal to the capitalisation factor plus the capitalised present value of expected growth in abnormal earnings. Companies with temporarily depressed earnings that are expected to increase in the future will have high price-earnings ratios, as will companies with abnormally high current earnings that are expected to increase. This concept is reflected by the franchise value valuation method, included in this section 3.6.

$$P \Big/ E_{t+1} = \left\{ \left. \left(1 + R_i \Big/ R_i \right) \left(x_i \sum_{t=1}^{t=\infty} x_{t+1}^a - x_t^a \Big/ (1+R_i)^t - d_t \right) \right/ x_t \right\}$$

Equation 3.1.7 (Fairfield, 1994:25) (Adapted)

The price-earnings ratio relates directly to the expected change in abnormal earnings. According to Fairfield (1994:25-27) a high price to book value ratio implies above-average return rates on the book value of the company, whereas a high price-earnings ratio implies earnings growth in excess of the normal rate expected from the growth in book value. The combination of the book value and the price-earnings ratio reveals the market's expectation of future profitability relative to current profitability. Equation 3.1.7 represents the sum of an infinite series, but the advantage of Equation 3.1.7 over Equation 3.1.1 is that the series of abnormal earnings represented by Equation 3.1.7 will converge to zero over time. This convergence cannot be expected of a dividend series or a series of cash flows. This reversion towards a mean (termed the mean reversion) value is common in economics and is represented visually by Figure 3.1.3 and by Table 3.1.3. The relationship is usually described by an autoregressive function.

Figure 3.1.3 (Fairfield, 1994:25)(Adapted)

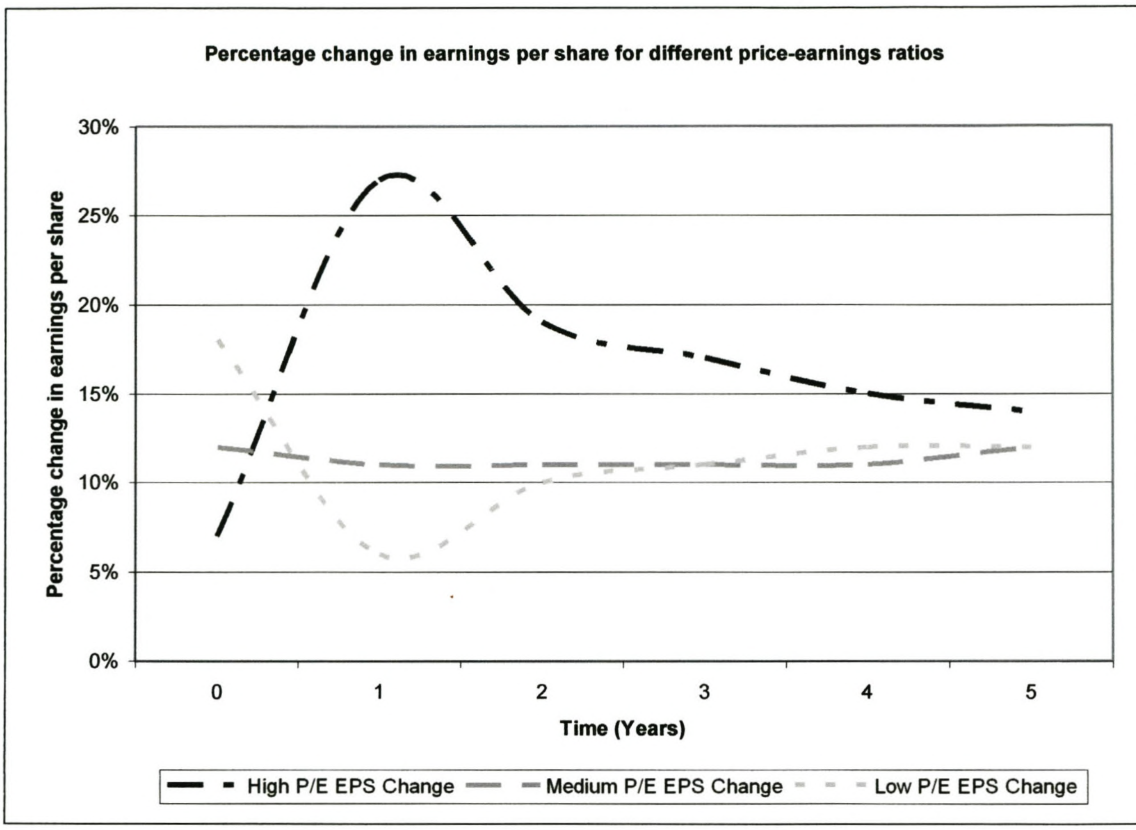


Table 3.1.2 (ibid.)

Median percentage earnings changes and future return on equity (ROE) for companies ranked by price-earnings ratio (P/E) and price-to-book value ratio (P/B), 1970-1984 (n=22,741)									
Panel A: companies ranked by P/B ratio									
	High P/B			Medium P/B			Low P/B		
t	P/B	P/E	ROE	P/B	P/E	ROE	P/B	P/E	ROE
0	2.25	12.68	17%	1.07	7.86	13%	0.65	6.14	9%
1	2.25	12.68	17%	1.07	7.86	13%	0.65	6.14	9%
2	2.25	12.68	16%	1.07	7.86	13%	0.65	6.14	10%
3	2.25	12.68	16%	1.07	7.86	13%	0.65	6.14	11%
4	2.25	12.68	16%	1.07	7.86	13%	0.65	6.14	11%
5	2.25	12.68	16%	1.07	7.86	13%	0.65	6.14	11%
Panel B: companies ranked by P/E ratio									
	High P/E			Medium P/E			Low P/E		
t	P/B	P/E	EPS change	P/B	P/E	EPS change	P/B	P/E	EPS change
0	1.64	14.51	7%	1.18	9.1	12%	0.83	5.76	18%
1	1.64	14.51	27%	1.18	9.1	11%	0.83	5.76	6%
2	1.64	14.51	19%	1.18	9.1	11%	0.83	5.76	10%
3	1.64	14.51	17%	1.18	9.1	11%	0.83	5.76	11%
4	1.64	14.51	15%	1.18	9.1	11%	0.83	5.76	12%
5	1.64	14.51	14%	1.18	9.1	12%	0.83	5.76	12%

From Table 3.1.2 it is clear that companies with high price-earnings ratios and high price to book value ratios are high growth companies according to the empirical tests of Fairfield (ibid.), or that these companies offer robust earnings for at least several consecutive years. A further conclusion is that companies with low price-earnings ratios and high price to book value ratios possess above average future profitability, but these companies tend to grow slowly as they are at the peak of their earnings potential.

Equation 3.1.7 is complex and an uncomplicated solution to the problem exists. Equation 3.1.7 can be restated in terms of earnings per share, the payout ratio, and the growth rate to the following period (Damodaran, 1996:292). The logic is that the dividends in the next period can be restated in terms of the identified components. Equation 3.1.8 represents this relationship. When Equation 3.1.8 is substituted into Equation 3.1.1 and after division by the earnings per share component, the forward price-earnings ratio is derived from simple fundamentals. The resulting equation is presented as Equation 3.1.9. The current price-earnings ratio is obtained by simply discounting Equation 3.1.9 by the growth rate.

$$\text{DPS}_1 = \text{EPS}_0(\text{Payout ratio})(1+g_0)$$ **Equation 3.1.8** (ibid.) (Adapted)

$$P/E_1 = \sum_{t=1}^{t=\infty} \text{Payout ratio}\,(1+g_t)/(1+R_i)^t$$ **Equation 3.1.9** (ibid.) (Adapted)

A primary criticism of the dividend discount model and the fundamental price-earnings ratios derived from the dividend discount model is that it represents the sum of an infinite series. The problem is that projections cannot be made infinitely. The need arose for the development of dividend discount models based upon some assumptions about uniform future growth. The models presented here are, however, handy for short-term detailed projections. Longer-term projections require the use of the models presented in the remainder of this chapter.

3.2 Gordon constant growth dividend discount model

3.2.1 The model

The model is an extension and a simplification to the dividend discount model previously discussed. It shares the advantage of the intuitive appeal of the dividend discount model and extends on the simplicity of the prior model (Leibowitz & Kogelman, 1994:2).

The constant growth model relates the value of a share to its expected future dividends, the required rate of return, and the expected dividend growth rate (Damodaran, 1996:192). The model is a very convenient choice for researchers, as they tend to envisage growth in a simplistic manner proceeding smoothly at a constant rate, self-funded by retained earnings, and generating added earnings with every growth increment.

The appealing concept of constant growth forms the basis of this model and its derivatives. Most of these models, but certainly not all, are derived from the assumption that dividends, earnings, and book values grow at the same rate ad infinitum. Deviations of this model split the growth rate down to two or three different growth rates covering consecutive time spans. An additional assumption is that the growth rate in dividends is solely the consequence of retained earnings, and that outside funding is assumed to grow at the same constant growth rate of the earnings and dividends for the capital structure to remain intact. The model is represented by Equation 3.2.1.

$$\text{Value of share} = \frac{DPS_1}{R_i - g}$$ **Equation 3.2.1** (ibid.)

DPS_1 is the expected dividend for the next period. The R_i in Equation 3.2.1 represents the required rate of return for equity investors over the relevant investment horizon and the g represents the perpetual growth rate.

The notion of a constant growth rate of dividends indicates that net earnings increase at exactly the same rate, and since the capital structure does not vary, the entire enterprise value must also increase at the same pace. Whenever this balance is altered, the capital structure will adjust accordingly, with the changes reflecting in the profits and dividends of the companies. Miller and Modigliani (1961:411-425) showed that if financing is exclusively from internal sources the growth rate of the company equates the growth rate of dividends. Furthermore, they proved that whenever debt was introduced the growth rate of the value of the company and its dividends, assuming a constant dividend payout ratio, would always be less than the rate of return on the value of the company.

3.2.2 Discussion of tests, advantages and critique

The model provides a simple yet powerful approach to the valuation of shares, but its uses are limited (Damodaran, 1996:194). From Equation 3.2.1, it is apparent that the model relates the valuation of a company to only a few variables, and that the calculation process is uncomplicated. However, as with many popular models, the Gordon constant growth dividend discount model is the object of sharp criticisms.

The primary advantage of the model is its simplicity and the few variable estimates required. The projection of the stable growth rate is possibly the most complex element required by the model. According to Damodaran (1996:192-194), two primary insights are required for the projection of a suitable growth rate.

Firstly, the company must be in a state of equilibrium with regard to dividend growth, earnings growth and growth of its cash position, as the effect of an imbalance between these variables will result over time in either reserve shortages or excess under-utilised resources (ibid.). The requirement, according to Miller and Modigliani (1961:411), is that earnings, dividends, and the value of the company must grow at approximately the same steady rate. Consequently, the renowned authors indicated that earnings and dividends must be interchangeable for valuation purposes.

Secondly, the growth rate must be a realistic long-term estimate of the growth of the company (Damodaran, 1996:192-194). The assumption that growth will continue at the chosen rate infinitely establishes rigorous constraints on reasonableness. No company can grow perpetually at a rate that is significantly greater than the growth rate of the economy in which it operates. Under these circumstances, the company would eventually exceed the size of the economy in which it operates. Conversely, no logical or mathematical limits exist on the downside; a diminutive growth rate signifies that the company will become a smaller and smaller part of the economy.

The assumption that the growth rate has to be constant is an increasingly unobtainable requirement to meet, especially given the volatility of earnings. Nonetheless, according to Damodaran (ibid.), whenever a company grows at an average growth rate that is approximately similar to the stable growth rate, the model can be used with little real effect on value, as the mathematical effects would be minimal.

Gordon, Gordon and Gould (1989:50-55) identified four factors that investors can rely on as an indication of the expected long-term growth rate. Firstly, they identified the growth in earnings per share in the past as a simple readily available estimation. Secondly, they suggested historical growth of dividends. The third approximation, namely the consensus growth approximation of analysts, is the best indication according to Gordon et al., but it is not readily available. The final suggestion is an average of the historical product of the retention rate and the return on owner's equity. Conversely, according to Miller and Modigliani (1961:411-425), growth calculations based on retained earnings would in general not provide an acceptable approximation of growth. Miller and Modigliani (ibid.) demonstrated that the historical growth rate does not equal the growth opportunities available to the company, as it excludes the consideration of the application of debt. These growth opportunities are normally quantified by the price-earnings ratio. A further assumption of Gordon regarding the use of debt is that all financing is internal, or alternatively that external financing is utilised in strict proportion to internal financing for the state of equilibrium to persist.

Miller and Modigliani (1961:424) articulate that as a consequence of increased uncertainty the discount rate applied by an investor to any future receipts should augment the further the revenues are projected into the future. Clearly this is logical, as a greater amount of risk pertains to these cash flows. According to Miller and Modigliani, Gordon's view with the construction of the model incorporated this premise and the constant growth dividend discount model might be seen as somewhat of a bird in the hand fallacy. The discount rate applied is effectively an average of the related true rates weighted by the size of the expected returns. The result is that the constant growth dividend discount model weighs nearby cash flows higher than further-off cash flows. It appears that, contrary to Miller and Modigliani (1961), the dividend policy indeed affects the value of a company, although it only applies to this model in reality and not in theory.

A profound limitation of the model is its sensitivity to changes in the growth rate (Leibowitz & Kogelman, 1994:7-8). As the growth rate converges on the discount rate the value of the company approaches infinity. Leibowitz and Kogelman (1994:139-165) proved the sensitivity of the model to changes in growth rate with the use of effective duration. Effective duration is the first derivative of a model in respect of the discount rate, and its aim is to establish sensitivity to changes in rates (Farrell, 1985:22-24). Refer to Equation 3.2.2. Leibowitz and Kogelman (1994:139) empirically proved that the effective duration of share prices, calculated with the use of the constant growth dividend discount model, ranges from twenty to fifty years; the larger number being associated with high-growth companies. Conversely, the effective duration of shares is about seven to fourteen years, and Leibowitz and Kogelman (ibid.) termed this inconsistency the equity duration paradox. They addressed the problem with the use of franchise value or growth-opportunities valuation, refer to Section 3.6.

$\text{Effective duration} = 1/R_i - g$ **Equation 3.2.2** (Farrell, 1985:23) (Adapted)

Farrell (1985:23-24) empirically proved that share prices (associated with flexible cash flows) do not necessarily change in reaction to unanticipated inflation changes. The logic is that dividends can increase to compensate for the increase in the inflation rate, ruling out the requirement to adjust the share price in response to unexpected changes in the inflation rate. According to

Leibowitz and Kogelman (1994:160-164), a company must be able to vary its growth rate in line with inflation. The variation in the growth rate, accompanied by the increased discount rate, allows the net discount rate of the model to remain more or less constant. Alternatively, a partial adjustment should be expected based upon the inflation flow-through rate of a company. Farrell (1985:23-24) concludes that as a result of the ability of companies to pass inflation on to customers, shares are less susceptible than long-term bonds and preference shares to purchasing power risk.

Whenever a company pays no dividends at the time of valuation, Malkiel and Cragg (1970:602) argued that the model would not apply to the company, as the resultant value would be zero. Miller and Modigliani (1961:418-423) however, suggested that earnings and dividends must be interchangeable for valuation purposes in companies that are in a steady, mature state. The logic is that inconsistencies between dividend and value growth of the company would require corrective action to re-establish growth equilibrium, as elucidated above, and in the end, dividends will be paid.

Malkiel and Cragg (1970:602) further stated that the model would result in a negative value under the illogical circumstances where the growth rate is greater than the discount rate over the long-term. Circumstances where the long-term growth rate exceeds the discount rate is clearly unfounded and the observation supports the fundamental integrity of the model. Empirical tests by the authors confirmed that the model appears reasonably relevant to the market for mature, stable industries.

The Gordon constant growth dividend discount model is best suited to companies that grow at a rate that is expected to remain stable over the long term (Damodaran, 1996:195). Companies and industries to which the model applies are those growing at a rate comparable to or lower than the nominal growth rate of the economy, with well-established dividend policies. These companies are normally utilities, banks, life insurers, and other well established companies.

As a product of the interchangeability of dividends and earnings suggested by Miller and Modigliani (1961:418-423), the assumption is not that an increase in the dividend payout ratio should result in an increased price-earnings ratio, *ceteris paribus*. An individual company can, therefore, not increase its price-earnings ratio by raising its dividend payout ratio (Malkiel & Cragg, 1970:604). The effect of an increased dividend payout ratio can result in a more subdued future growth rate per share to such an extent that the price per share will remain constant and decrease future earnings per share, reducing the price-earnings ratio.

3.2.3 Significance and relation to the price-earnings ratio

The price-earnings ratio can be calculated from the Gordon constant growth dividend discount model with some substitution and division (Damodaran, 1996:292). The value or price of the company per share is calculated from Equation 3.2.1. Since the next dividend is required for the calculation of Equation 3.2.1, Equation 3.2.3 can be substituted into Equation 3.2.1 to present Equation 3.2.4. After rearrangement, the result is the price-earnings ratio as per Equation 3.2.5. The forward price-earnings ratio is stated in terms of expected earnings in the next time period and is represented by Equation 3.2.6. Equation 3.2.6 is a simplified version of Equation 3.2.5 as the payout ratio is assumed to remain stable in accordance with the Gordon constant growth dividend discount model.

$DPS_t = EPS_t(\text{Payout ratio})(1+g)$ **Equation 3.2.3** (ibid.)

$P = {EPS_0(\text{Payout ratio})(1+g)} / {R_i - g}$ **Equation 3.2.4** (ibid.)

$P / EPS = P / E = {\text{Payout ratio}(1+g)} / {R_i - g}$ **Equation 3.2.5** (ibid.)

$P / E_1 = {\text{Payout ratio}} / {R_i - g}$ **Equation 3.2.6** (ibid.)

It is evident from the above equations that the price-earnings ratio is an increasing function of the payout ratio and growth rate, and a decreasing function of the relative riskiness of the company. As stated previously, a company cannot increase its price-earnings ratio by merely raising its dividend payout ratio; the action will influence its growth rate, and the net discount rate will consequently increase to decrease the company's value (Miller & Modigliani, 1961:418-423).

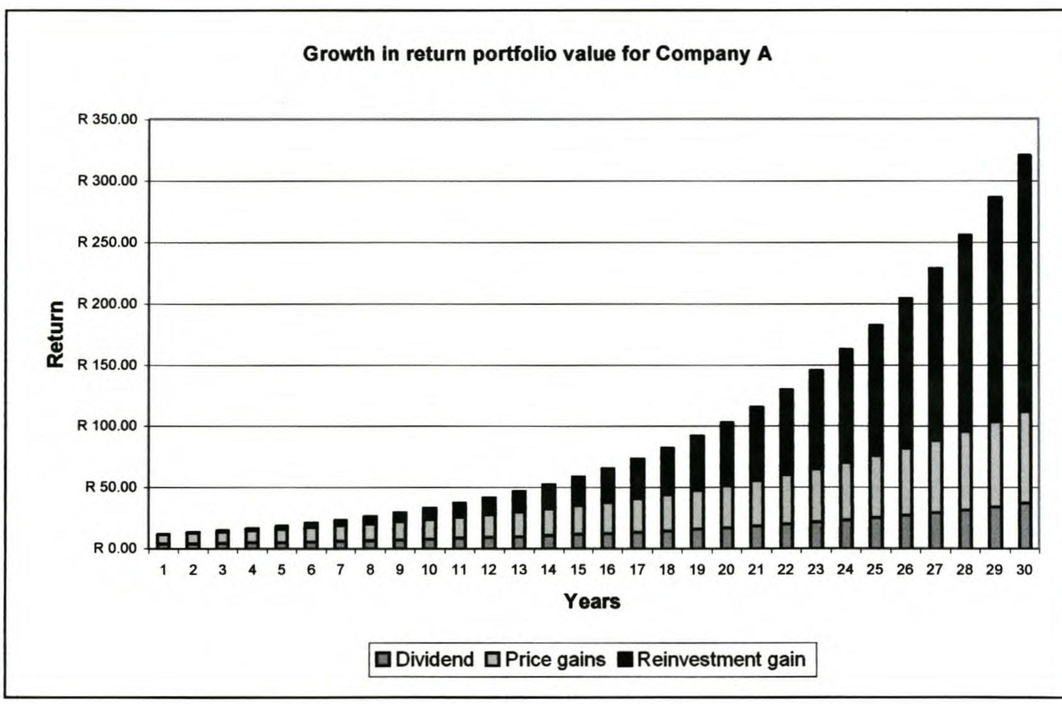
According to Leibowitz and Kogelman (1994:9), return comprises three components, namely dividend return, price return, and reinvestment return, or reinvestment gain whenever an investor reinvests at the market rate of return. In order to test the effect and the relationship between these components, the price-earnings ratio and the Gordon constant growth dividend discount model, four imaginary companies are examined as per Table 3.2.1.

Table 3.2.1 (Leibowitz & Kogelman, 1994)

Financial characteristics of companies				
Characteristic	**Company A** **Stable growth**	**Company B** **No growth**	**Company C** **Market return on equity**	**Company D** **Reinvestment**
Book value of equity	R100	R100	R100	R100
Return on equity	12%	12%	15%	15%
Earnings	R 12.00	R 12.00	R 15.00	R 15.00
Payout ratio	33%	100%	100%	33%
Dividend	R 4.00	R 12.00	R 15.00	R 5.00
Market rate	12%	12%	12%	12%
Gordon constant growth dividend discount model price	R 100.00	R 100.00	R 125.00	R 250.00
Dividend yield	4%	12%	12%	2%
Growth rate	8%	0%	0%	10%
Price-earnings ratio	8.33	8.33	8.33	16.67

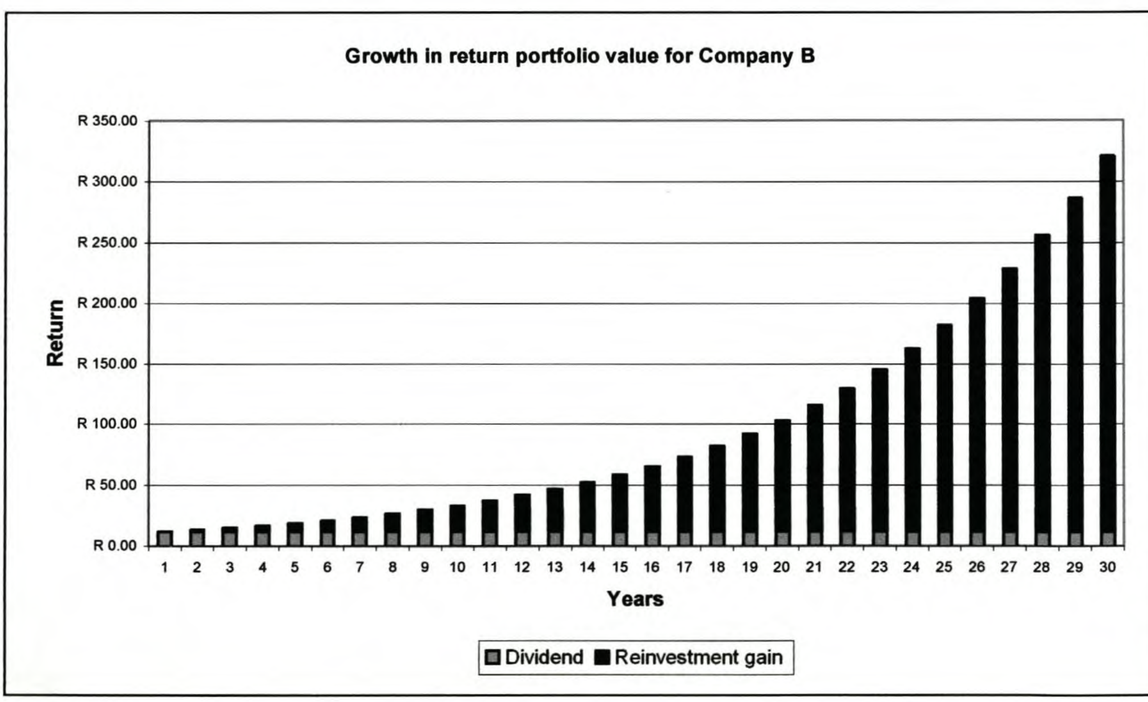
With company A, the dividend and earnings per share will grow at the company's growth rate of eight percent. As a result of the eight percent growth rate the price return will keep pace with the dividend growth at eight percent. Reinvestment return consists of earnings on the dividends received that are reinvested at the market's rate of return and, consequently, grow at the same rate as the market. At the start of the investment, price growth is the dominant component of return, in time though; reinvestment return will begin to dominate. The pattern is consistent with return observations for fixed income-securities (Leibowitz & Kogelman, 1994:10). Refer to Figure 3.2.1. As a result of the constant growth and equilibrium status of the dividend discount model the price-earnings ratio remains unaltered, refer to Annexure A 3.2.1.

Figure 3.2.1



Company B is the opposite of company A, it does not include any price gains as a result of its high dividend payout ratio. Company B is normally referred to as a cash cow, as it distributes all its earnings. During year 15, the dividend of company A surpasses that of company B as a result of its growth. The advantage company B possesses over company A is that unlike the dividends of company B, the price appreciation of company A is not at the discretion of the shareholder. Company A as a result partially controls the investment decisions of the financier. Both companies are fairly priced at the outset of the investment and neither company offers above market returns; accordingly compounding investors (continuous long position) should be indifferent in their selection of investment based on return only. The different compositions of the return permit the price-earnings ratio to remain identical for both investments; refer to Figure 3.2.2 and Annexure A Table 3.2.2.

Figure 3.2.2

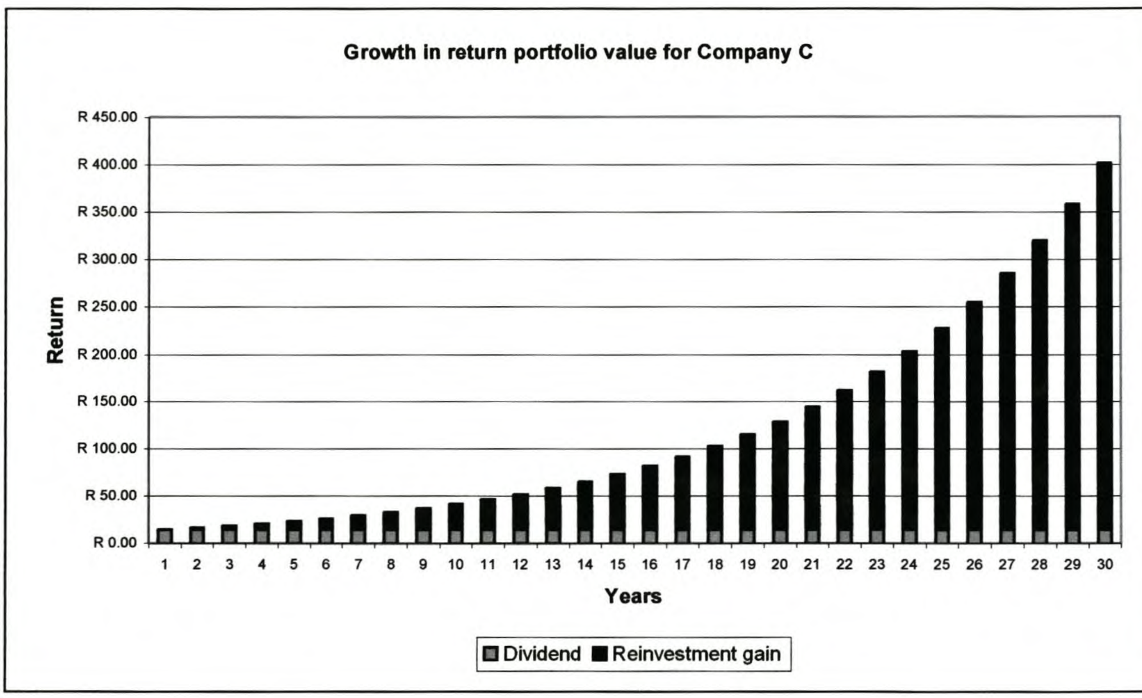


Further insight of the price-earnings ratio is accessible by a comparison of companies A and B with a twelve percent perpetual coupon bond. The price of the bond is calculated by dividing the earnings by the bond yield. The approach is analogous when it is required that the price-earnings ratio represents the reciprocal (reverse) of the yield of the investment. The price-earnings ratio represents the price required for one unit of return. Accordingly, the price-earnings ratio of both companies can be obtained by dividing one by the discount rate of twelve percent in order to find the price-earnings ratio of eight and a third.

Company A retains two thirds of its earnings and adds the amount to its book value. Therefore, the book value of A grows with its growth in earnings in order to retain its twelve percent return on equity. As a result, the price-earnings ratio will remain stable at its base value of eight and a third. A company similar to company A, which reinvests only at the market rate of return, is not providing a special service to investors. Similarly to company B, the investors can reinvest their proceeds at the same market rate of return. Accordingly, reinvestment at the market rate is tantamount to a cash cow scenario, represented by company B; the return remains identical and the price-earnings ratio is the same for both companies.

Company C represents an above market return on equity with no expectation of future growth. The share is fairly priced at the market rate of return and, accordingly, company C trades at a premium to its book value, equivalent to a perpetual bond with an above-market coupon rate. Therefore, company C offers no advantage when compared to A and B. The only difference is its inflated price. Consequently, the price-earnings ratio remains equal for all companies discussed. As a consequence of the inflated price of company C the nominal value of the portfolio of return will be greater than that of A and B; the return rate however, will remain the same. Refer to Figure 3.2.3 and Annexure A Table 3.2.3.

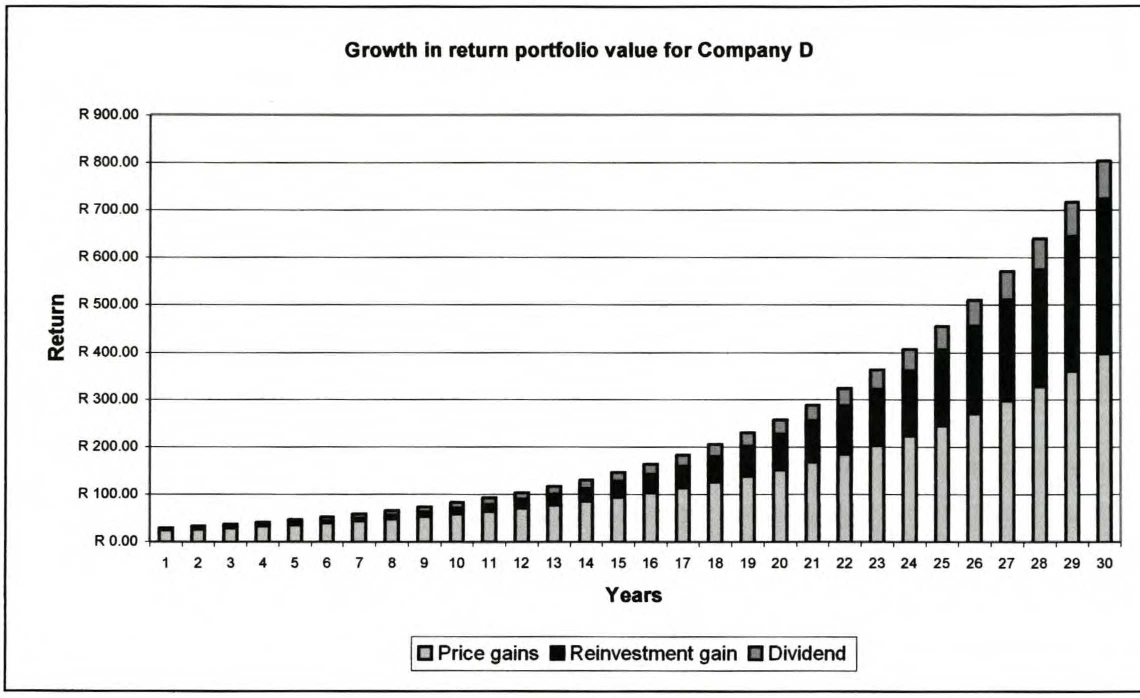
Figure 3.2.3



A new scenario is introduced by the examination of company D. Company D offers an above market return on equity to two thirds of its earnings. Therefore, company D requires a larger share price than any of the previous companies. An investor in company D would require the annual increases in share price to keep pace with the higher dividend yield and lower growth rate of company A, and this is then reflected in the ten percent increase in the book value and earnings per annum of company D. The price gain of ten percent per annum and the dividend yield of two percent provide the equivalent twelve percent market return to an investor in company D. Company D is furthermore equivalent to a perpetual bond with a principal that increases by ten percent per year that carries a two percent coupon rate. As a result of the comparatively small dividend, the reinvestment return comprises only a small percentage of the total return.

The positive impact of the high growth rate combined with an above market return on equity is clearly represented by the price-earnings ratio as it reflects the future opportunities available to the company. The rate of return equals the market rate as all securities are fairly priced. The price-earnings ratio doubled as a result of a base ratio of eight and third calculated as before, and an additional eight and a third pertaining to the future above-market return on reinvested earnings. Company C did not reinvest its earnings and consequently did not share in this added opportunity. Refer to Figure 3.2.4 and Annexure A, Table 3.2.4.

Figure 3.2.4



It is evident from the discussion of the Gordon constant growth dividend discount model that the price-earnings ratio represents an excellent aid to understand share price movements. It is furthermore apparent from the discussion of company D above that obtaining a high price-earnings ratio requires something special. In addition the discussion supports the hypothesis that some shares are omitted from analysis and recommendation by the analysts during certain periods and that, when included again, achieve above average market returns in an attempt to rectify the apparent inequality, as noted by McEnally and Fama and French (as cited in Reilly & Brown, 1997:315) and discussed in section 2.3.3. When these companies are recognised they achieve significant returns on a risk-adjusted basis as indicated by Arbel et al. (as cited in Shefrin & Statman, 1995:31-33).

Furthermore, it proves that the Gordon constant growth dividend discount model can add value for the investor when used and applied to companies that demonstrate stable growth near or at the growth rate of the economy. In order to gain an understanding of companies that do not show the indicated growth pattern alternatives of the constant growth model will be used.

3.3 Two-stage dividend discount model

3.3.1 The model

The two-stage dividend discount model is a more realistic variant of the dividend discount model family intended for companies with special investment opportunities (Miller & Modigliani, 1961:422). The model allows for special investment opportunities to continue for some finite period.

Therefore, the model is also a more complex variant of the Gordon model. It involves additional parameter estimates, which are the terms of the extraordinary growth opportunities, different return rates for the different phases, and the growth rates for the perpetual phase and the extraordinary growth term. The extraordinary growth stage is the initial phase of the model. Subsequently, a steady phase follows, where the growth rate is stable and expected to continue indefinitely, resembling the Gordon model (Damodaran, 1996:294).

The model is the sum of the present value of dividends during the extraordinary phase and the present value of the terminal phase (ibid.). The two-stage dividend discount model is represented by Equation 3.3.1 and the symbol definitions are set out in Annexure A, equation 3.3.1. Whenever the extraordinary growth rate and the payout ratio remain constant for the initial phase the formula simplifies to yield Equation 3.2.2.

$$P = \sum_{t=1}^{t=n} \frac{DPS_t}{(1+R_i)^t} + \frac{DPS_{n+1}/(R_{in} - g_n)}{(1+R_i)^n}$$ **Equation 3.3.1** (ibid.)

$$P = \frac{DPS_0(1+g)\left(1 - \frac{(1+g)^n}{(1+R_i)^n}\right)}{R_i - g} + \frac{DPS_{n+1}}{(R_{in} - g_n)(1+R_i)^n}$$

Equation 3.3.2 (Damodaran, 1996:197)

The first advantage of the model is its ability to accommodate high-growth companies with a specified high-growth term (Miller & Modigliani, 1961:422). The second is that the equation allows the use of different return rates and dividend payout ratios for the subsequent growth phases (Damodaran, 1996:198-199).

The same constraints that applied to the Gordon model apply to the terminal growth rate of this model (ibid.). Accordingly, the terminal growth rate should be comparable to the growth rate of the economy and estimation processes of Gordon apply to this parameter. The model, furthermore, introduces the complexity of approximating a payout ratio consistent with the relevant growth-phases (Rappaport, 1986:52). Whenever it is anticipated that the growth rate would decline significantly following the initial growth phase, the payout ratio should increase for the stable growth phase.

The first dividend for the stable growth phase can be projected with the use of Equation 3.1.3 (Rappaport, 1986:52-54). Companies within a stable growth environment possess the ability to distribute a larger portion of their earnings, compared to a companies that require cash to utilise investment opportunities. Therefore, a change in dividend policy is required whenever a high-growth company enters a stable growth phase. The stable company possesses cash in excess of its requirements and is prepared to distribute this excess cash to the shareholders. These shareholders can find profitable investment opportunities by themselves that are not available to the company with this cash.

The change that occurs with the dividend policy of the company affects other characteristics (Damodaran, 1996:198-199). It is reasonable to assume that the relatively high beta and discount rate that relate to the high-growth company will remain unchanged as the risk profile of the company reduces to that of a stable growth company. Furthermore, return on equity will reduce to levels that are commensurate with that of a stable growth company. This change is another ground for the alteration in the risk profile of the company.

3.3.2 Discussion of tests, advantages and critique

The model possesses the ability to accommodate companies with high-growth opportunities in the near future. According to Damodaran (1996:199), this newfound ability results in three areas of critique.

The first practical problem relates to the duration of the extraordinary growth period (ibid.). Since the extraordinary growth rate is expected to decline, the value of the company will increase as the duration of the extraordinary growth period extends. In theory, the duration of the growth phase can relate to product life cycles or other barriers of entry that prevent competitors from sharing in the unusual profits. Nevertheless, in practice, it is difficult to relate qualitative considerations to specific time-spans.

Secondly, the model assumes that the extraordinary growth rate will change rapidly to a lower stable rate, which will continue indefinitely (ibid.). Instantaneous transformations can occur. Nevertheless, it is more realistic to assume that the change will occur gradually with the passage of time.

The last disadvantage identified by Damodaran (ibid.), relates to the fact that a significant part of the value is obtained from the terminal price, calculated by the Gordon constant growth dividend discount model. The discussion of the Gordon model emphasised its sensitivity to changes in the growth or discount rate assumptions and the difficulties that exist in the projection of these rates.

An advantage of the two-stage model is that it allows for the calculation of the value of extraordinary growth (ibid.). Equation 3.3.2 can be restated as Equation 3.3.3 that specifically provides the opportunity to divide the value of a company between its extraordinary growth phase, its stable growth phase, and the value of the assets of the company, without any growth assumptions. This beneficial aspect allows one the opportunity to determine whether the value placed on growth appears reasonable. As stated previously (Section 2.3.3), refer to Basu (as cited in Reilly & Brown, 1997:223); Peavy and Goodman (as cited in ibid.); Jaffe et al. (1989:135-148); and Arbel et al. (as cited in Shefrin & Statman, 1995:31-33), empirical evidence proved that shares with comparatively low price-earnings ratios command a return premium over those with relatively high price-earnings ratios. As a result the model assists the investor not to fall victim to the hypothesis that shareholders overpay to acquire growth (Damodaran, 1996:315).

$$P=\left[\frac{DPS_0(1+g)\left(1-\frac{(1+g)^n}{(1+R_i)^n}\right)}{R_i-g}+\frac{DPS_{n+1}}{(R_{in}-g_n)(1+R_i)^n}-\frac{DPS_1}{R_{in}-g_n}\right]$$
$$+\left[\frac{DPS_1}{R_{in}-g_n}-\frac{DPS_0}{R_{in}}\right]+\frac{DPS_0}{R_{in}}$$

Equation 3.3.3 (Damodaran, 1996:203)

Equation 3.3.3 is broken down into three terms. The first term represents the value of extraordinary growth, the second the value of stable growth and the last term the value of the assets of the company, or alternatively the value of a company with no growth opportunities. The value of extraordinary growth is calculated by deducting the value of a company with stable growth, calculated with the use of the Gordon constant growth dividend discount model, from the value obtained by the two-stage dividend discount model. Accordingly, the value of stable growth is the remainder of the difference between the cost of the assets in place and the Gordon model value. The cost of assets with no foreseeable growth is a perpetuity.

Damodaran (1996:203-204) also identified four determinants of the value of growth. The first determinant of the value of growth is the growth rate during the extraordinary growth period. A higher extraordinary growth rate typically relates to a large value for growth, *ceteris paribus*. Secondly, an extended extraordinary growth period relates to a larger value of growth, all other determinants being equal. The third determinant is the profitability of projects.

Profitability determines both the growth rate during the initial phase and the terminal value. Increased profitability enhances both extraordinary and terminal growth. Lastly, risks increase the discount rate and, therefore, reduce the value of growth, if no other changes occur.

3.3.3 Significance and relation to the price-earnings ratio

As with the Gordon constant growth dividend discount model, the price-earnings ratio is calculated directly from the two-stage dividend discount model. With the replacement of future dividends by Equation 3.2.3 and some cancellation of common elements and after dividing by the earnings per share, Equation 3.3.2 yields Equation 3.3.4.

$$P/E_1 = \frac{\text{Payout ratio}\left(1 - \frac{(1+g)^n}{(1+R_i)^n}\right)}{R_i - g} + \frac{\text{Payout ratio}_{n+1}}{(R_{in} - g_n)(1+R_i)^n}$$

Equation 3.3.4

Equation 3.3.4 represents the forward price-earnings ratio. A price-earnings ratio representing the current ratio, i.e. current year earnings divided by the ruling market price, is calculated in Equation 3.3.5.

$$P/E_0 = \frac{\text{Payout ratio}(1+g)\left(1 - \frac{(1+g)^n}{(1+R_i)^n}\right)}{R_i - g} + \frac{\text{Payout ratio}_n (1+g)^n (1+g_n)}{(R_{in} - g_n)(1+R_i)^n}$$

Equation 3.3.5 (Damodaran, 1996:294)

3.4 The H-model

3.4.1 The model

Similar to the two-stage dividend discount model, the H-model is a member of the dividend discount model family that relies on dividends for valuation purposes. An additional similarity shared with the two-stage model is that both define the growth pattern as two stages. Unlike the previous model, the growth rate during the initial phase of the H-model declines gradually toward the stable growth rate. The growth rate of the H-model declines over the first phase of the model to achieve a perpetual stable growth rate. Fuller and Hsia first introduced the model in 1984 (Damodaran, 1996:205).

The primary assumption of the model is that the earnings growth rate starts at a high rate, and then continues to decline linearly over the extraordinary growth phase, to reach the infinite stable growth rate (ibid.). Extraordinary growth is assumed to last 2H (time-frame donation) periods, thus the name of the model. Furthermore, it implicitly only allows a single discount rate throughout all growth phases. A further assumption is that the dividend payout ratio remains constant over time, and is by no means affected by changing growth rates. The model is represented by Equation 3.4.1.

$$P = \frac{DPS_0(1+g_n)}{R_i - g_n} + \frac{DPS_0(H)(g_a - g_n)}{R_i - g_n}$$ **Equation 3.4.1** (ibid.)

3.4.2 Discussion of tests, advantages and critique

Primarily, the model avoids the abrupt decline of growth associated with the two-stage dividend discount model when it moves from its first to its second phase (Damodaran, 1996:205-206). As a consequence of this advantage, new limitations are introduced.

Firstly, the decline in growth is not expected to follow the rigid pattern of a declining linear model (ibid.). The model assumes this decline in growth is a function of the initial growth rate, the stable growth rate, and the length of the extraordinary growth period. Clearly, the identified determinants would not be sufficient to elucidate growth patterns experienced by most companies. Small deviations from the assumptions of the model should not affect the valuation process significantly.

Secondly, the model does not account for increased dividend payout ratios as a consequence of diminishing growth rates that result in excess cash (ibid.). As explained with the dividend discount model (Section 3.1.1), dividends should approximate Equation 3.1.3 over the long-term. Small deviations are allowed as a result of management decisions (Rappaport, 1986:56). These deviations should not influence the value of the company whenever they are transient, and subsequently rectified in accordance with Equation 3.1.3.

Furthermore, the current payout ratio should not be accepted as normal and ought to be tested by Equation 3.1.3 in order to determine reasonability. The dividends required by the H-model should accurately reflect the company's ability to declare and pay dividends in accordance with its available opportunities and the efficient utilisation of cash resources.

Finally, as with the Gordon constant growth dividend discount model and all models derived from it, the H-model requires the projection of a reasonable perpetual growth rate. The guidance provided in Section 3.2.2 in this regard should likewise apply.

According to Damodaran (1996:206), the model is relatively limited in its capacity to apply to a wide spectrum of companies. It is most suited to companies that currently enjoy growth rates above the market rate, but which are still capable of declaring and paying dividends. A further prerequisite to the application of the model is that the growth rates of these companies should be expected to decline to market levels in the near future.

3.4.3 Significance and relation to the price-earnings ratio

Similar to the other derivations of the dividend discount model the price-earnings ratio can be calculated as a direct product of the model. With the use of Equation 3.1.8 that defines the relationship between dividends and earnings per share Equation 3.4.1 is altered to present Equation 3.4.2. Equation 3.4.2 characterises the price-earnings ratio in terms of the H-model.

$$P/E_1 = \text{Payout ratio}\,(1+g_n)/R_i - g + \text{Payout ratio}\,(H)(g_a - g_n)/R_i - g$$

Equation 3.4.2

3.5 Three-stage dividend discount model

3.5.1 The model

The three-stage dividend discount model is the most complex variant of the dividend discount models. It is a combination of the two-stage dividend discount model and the H-model. Accordingly, the three-stage model allows for an initial period of extraordinary growth, a transitional period when growth declines to the final perpetual stable growth phase (Damodaran, 1996:207).

Furthermore, the model does not impose any restrictions on the dividend payout ratio. Consequently, this advantage allows the model to apply to general market conditions, and it should result in a more accurate reflection of the growth pattern of the majority of companies.

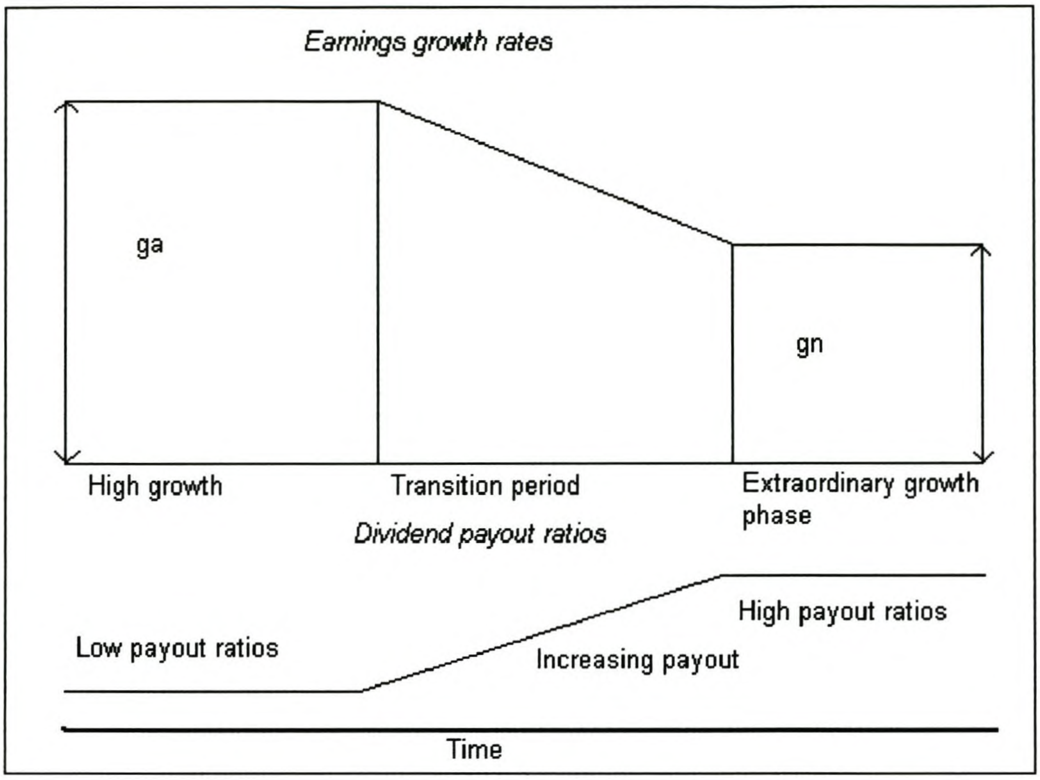
The value per share is calculated as the sum of the present value of expected dividends during the extraordinary growth phase, the transitional period, and the terminal price at the start of the perpetual growth phase. Equation 3.5.1 represents the three-stage dividend discount model.

$$P = \sum_{t=1}^{t=n1} \text{EPS}_0 (1+g_a)^t (\text{High growth phase payout ratio}) \Big/ (1+R_i)^t + \sum_{t=n1+1}^{t=n2} \text{DPS}_t \Big/ (1+R_i)^t + \text{EPS}_{n2}(1+g_n)(\text{Terminal phase payout ratio}) \Big/ (R_i - g_n)(1+R_i)^n$$

Equation 3.5.1 (ibid.) (Adapted)

The dividend payout ratio will generally remain relatively low during the extraordinary growth phase, as the company requires resources for expansion. It will increase in accordance with the decreasing growth rate during the transitional phase, whilst capital requirements slowly diminish. Lastly, the perpetual growth phase does not require the resources of the prior phases and the dividend payout ratio will accordingly increase to its highest level compared to the previous phases. Equation 3.1.3 represents the relationship between growth and the expected dividend payout ratio, as explained in this chapter. The relationship depends upon the sources of cash and the appropriate employment of this capital for investment purposes by the company (Rappaport, 1986:52-54). Figure 3.5.1 below represents a graphical depiction of the relationship.

Figure 3.5.1 (Damodaran, 1996:208) (Adapted)



3.5.2 Discussion of tests, advantages and critique

The primary advantage this model possesses is that it eradicates the majority of constraints imposed by prior derivatives of the dividend discount model (ibid.). Firstly, the model allows a more reasonable depiction of the normal transitional phases that a high-growth company typically follows to achieve its perpetual growth phase.

It does not exhibit the abrupt decline in growth associated with the two-stage dividend discount model. In addition, it allows for some period of extraordinary growth, in contrast to the H-model. Furthermore, it retains the advantage of the Gordon constant growth dividend discount model to facilitate the calculation of perpetual growth. This contrasts the normal dividend discount model that already incorporates the advantage of accommodating various growth patterns, but results in tedious calculation.

Conversely, a trade-off of benefits exists and the model requires numerous additional inputs compared to other derivations of the dividend discount family. The three-stage dividend discount model requires year-specific payout ratios, growth rates, and discount rates with the related additional parameters associated with these factors (ibid.). Whenever noise traders cause substantial market confusion the estimation of the required parameters is prone to error as a result of the abundance of required estimates (Shefrin & Statman, 1995:270). Inaccuracies in the estimation of these parameters pose the ability to overwhelm the benefits that were acquired from the additional flexibility of the model.

Flexibility within the model allows it to apply to any company that, in addition to variations in its growth pattern, is expected to change on other fronts, in particular, payout policies and risk profile (Damodaran, 1996:208). Different discount rates, a consequence of changes in the company's risk profile, applicable to the different phases of the growth, are accommodated within the model. The dividend discount model allows the use of different discount rates, but it requires tedious calculations.

The model is the most appropriate dividend discount model for companies that are experiencing abnormally high earnings and very high growth rates (ibid.). Furthermore, these companies are expected to continue to enjoy this growth rates for some time, as entry barriers exist that prevent entry to new participants. When these barriers of entry are removed, for example, costs or patent rights, growth and returns will slowly start to decline to levels observed for the aggregate economy.

3.5.3 Significance and relation to the price-earnings ratio

Similar to the other derivations of the dividend discount model the price-earnings ratio can be calculated as a direct product of the model, but the calculation would still be a more complex and tedious process, compared to previous models, similar to Equation 3.5.1. The formula is provided by Equation 3.5.2.

$$
\begin{aligned}
P/E_1 &= \sum_{t=n0}^{t=n1} (1+g_a)^t (\text{High growth phase payout ratio}) \Big/ (1+R_i)^t \\
&+ \sum_{t=n1+1}^{t=n2} \text{Individualistic payout ratio}_t \Big/ (1+R_i)^t \\
&+ (1+g_n)(\text{Terminal phase payout ratio}) \Big/ (R_i - g_n)(1+R_i)^n
\end{aligned}
$$

Equation 3.5.2

3.6 Franchise value and growth opportunities

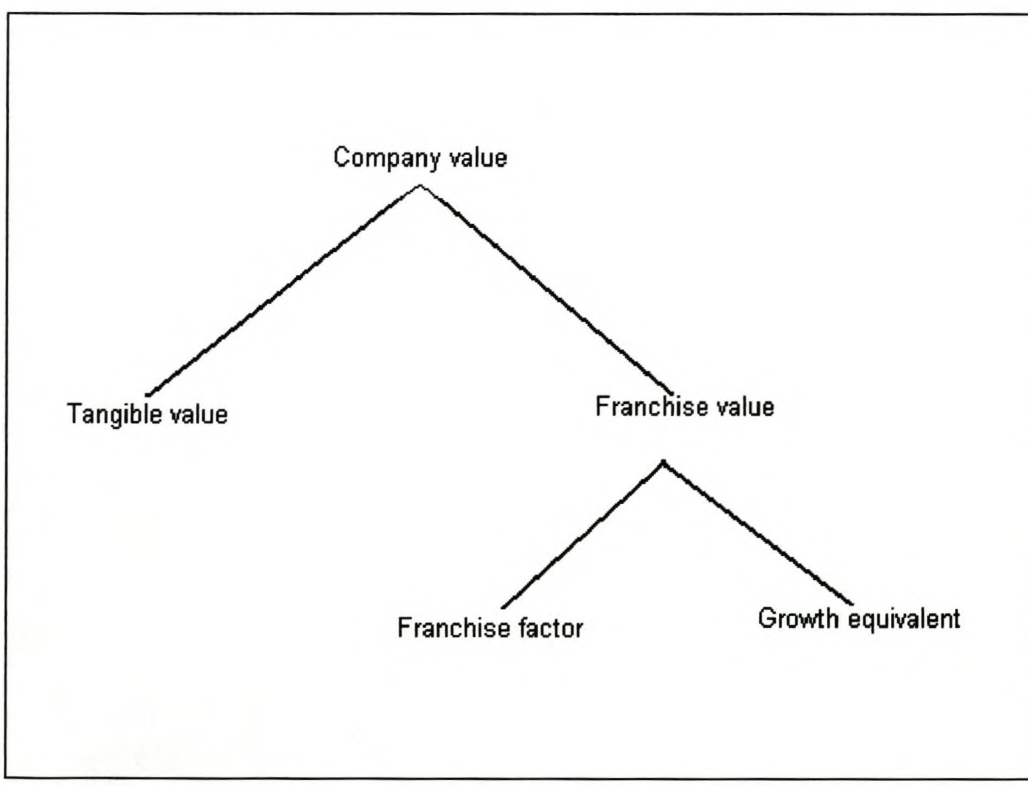
3.6.1 General background

According to Leibowitz and Kogelman (1994:2), this approach to valuation is original, insightful, and retains a practical bent. This appeal is not reliant on the unrealistic assumptions associated with the dividend discount models as noted. Yet, the simplicity and intuitive appeal of the Gordon constant growth dividend discount model is retained.

The franchise value approach is based on three characteristics of value and their interaction with one another (ibid.). The first important component of value is the sustainable returns that are expected to be generated constantly from current business. The second characteristic relates to the prospects that exist for expansion through the pursuit of fresh investment opportunities available to the company. Lastly, the attainable returns on these new investment opportunities are a significant supplementary element to the value of a company.

The franchise value approach breaks company value down into two key components (Leibowitz & Kogelman, 1994:7-27). Initially, a value is placed on a company's existing business or earnings. This value is deemed to persist indefinitely and is termed the tangible value of a company. The second component of value, known as a company's franchise value, is derived from prospective new investments, as explained above, and is further subdivided into two categories. The first of these components captures the return levels associated with these possible investment opportunities, and is termed the franchise factor. Lastly, the second term captures the present value of the opportunities available for productive new investment or the magnitude of accessible investment opportunities. This variable is termed the growth equivalent. A graphical representation is included in Figure 3.6.1 below.

Figure 3.6.1 (Leibowitz & Kogelman, 1994)



In addition, according to Leibowitz and Kogelman (1994:2), the franchise value approach is better attuned to current market realities then the dividend discount models. The rationale behind this statement is firstly, that the return from new investments is separated from the return on equity generated by current business. Secondly, earnings can follow any pattern of erratic movement for any defined time span. Furthermore, growth does not always signify profitable investments; value-depriving investments, indeed, can generate an increased asset base and revenue growth.

The franchise value approach views productive new investments as a scarce resource that is limited by the availability of superior investment opportunities. This contrasts with the financing options available to the company as it is normally the limit placed on investment opportunities by other models. Lastly, the approach does not relate the retained earnings to potential excess profit or the assumption that excess returns will continue and that earnings are not expected to decline. According to the franchise value approach, whenever sound investment projects are not accessible to the company, earnings retention cannot generate such projects. An implicit assumption of the work of Miller and Modigliani (1961:416) is that a company requires retained earnings to generate growth as a cheaper alternative to external financing.

Miller and Modigliani (ibid.) termed this approach the investment opportunities approach, as it accentuates the investment opportunities available to the company and the individual project's respective profitability. According to Miller and Modigliani, the model is representative of an investor who intends to purchase the company with all its opportunities. The authors also emphasised the ability of the franchise value approach to distinguish between current business, future opportunities, and their associated profitability. Lastly, the authors related the franchise factor, as explained above, to the common accounting term "goodwill", as the capacity to acquire value generating investment opportunities.

According to Mao (1966:95), the method best explains the nature of growth. True growth is distinguished from mere expansion, commonly incorrectly termed as growth. True growth requires expansive prospects which are capable of delivering returns beyond the overall market opportunities. Moreover, the author emphasised that companies in general follow a Gompertz growth curve. This curve divides growth into three distinct phases. The first of these phases is an exponential growth phase. The second represents constant growth while the last phase represents declining growth.

3.6.2 The franchise factor

The primary objective of the most companies is to generate value for the benefit of their shareholders. The key concern with all new investments, from a company's perspective, should be to obtain opportunities that present an above-market return, and not the funding of projects, according to Leibowitz and Kogelman (1994:8). Investment opportunities that offer an above-market return should be pursued, regardless of the consequences to corporate finance.

The franchise value approach distinguishes between current business and future investment opportunities (Leibowitz & Kogelman, 1994:9-18). Furthermore, it presents all the components of value in today's monetary terms (present value). The franchise factor provides information about the impact of all embedded (included in the price) investment opportunities currently available to the company. In addition, the base value is an indication of the value of assets in use. Consequently, it is simple to analyse the impact of the different opportunities and current business on the price-earnings ratio.

In order to facilitate comparison between the franchise value approach and the Gordon constant growth dividend discount model, Table 3.2.1 as presented with the Gordon model is used. The equilibrium market within Table 3.2.1 allows investors to earn only the market's rate of return. Therefore, all shares reflect their fair value, based on the market's rate of return. Nevertheless, company D possess the unique capability to earn above-market rate returns, and to reinvest those earnings from its investments in a similar way to the Gordon model (growing reinvestment), in projects that also earn the same above-market returns. This excess return generates a pool of incremental value beyond that of the alternatives presented directly to the investors. These excess returns signify genuine value to the investors, excess return above the market's rate of return is referred to as the franchise value of a company (Leibowitz & Kogelman, 1994:9-10).

Leibowitz and Kogelman (1994:9-22) introduced a variable they labelled the growth equivalent to measure the value of all franchise investments regardless of whether those investments occur at irregular intervals or in the smooth stream implied by the Gordon model. The growth equivalent is merely the present value of all future franchise opportunities at the market's rate of return expressed as a percentage of the original book value. It is similar to an instant investment opportunity that devotes the identified percentage of resources of the book value to an investment that will earn this return on equity in perpetuity. The growth equivalent is valuable in providing insight into the magnitude of investments implicitly assumed in a constant growth assumption.

Equation 3.6.1 is used to calculate the growth equivalent of a constant growth company; the result is expressed as a percentage. This would indicate that company D must invest an amount equal to five times its current book value, and earn a return on equity of fifteen percent, three percent more that the market rate of return on that investment in perpetuity. This level of investment is staggering in relation to growth in the price-earnings ratio as a result thereof. The resultant company is a cash cow with returns similar to company D, and both these companies will share the same present value (ibid.).

who are prepared to compensate the existing shareholders accordingly to acquire control of these resources. The price-earnings ratio reflects the value of these resources. An additional eight and a third units of investment, above and beyond the eight and a third associated with normal returns, are required to obtain one unit of earnings of company D. This additional investment is the premium associated with the franchise opportunities of company D (ibid.).

It is also evident from Table 3.2.1 that, whenever a company's return on equity equates the rate of return on the market portfolio, the price-earnings ratio remains at its base level, regardless of its dividend policy or growth rate. Whenever an above-market return on equity is achieved, with no growth, the price-earnings ratio remains constant; refer to company C in Table 3.2.1. Contrary to company D above, whenever a company invests in projects that offer a return below the market's level of return, the franchise factor will be negative, resulting in a price-earnings ratio below the base ratio. The base ratio is calculated as the inverse of the rate of return (ibid.).

The premium on company D is created through investment at a return in excess of that of the market. Consequently, the value of company D can be divided into two separate components. Firstly, the original value, and in addition, for company D the original value equates the book value. Secondly, a value created through the reinvestment of earnings. According to the Gordon constant growth dividend discount model, both the original value and the reinvestment value each represent half the value of company D. This

$$\text{Growth equivalent} = \frac{g}{(R_i - g)}$$

Equation 3.6.1 (Leibowitz & Kogelman, 1994:21)

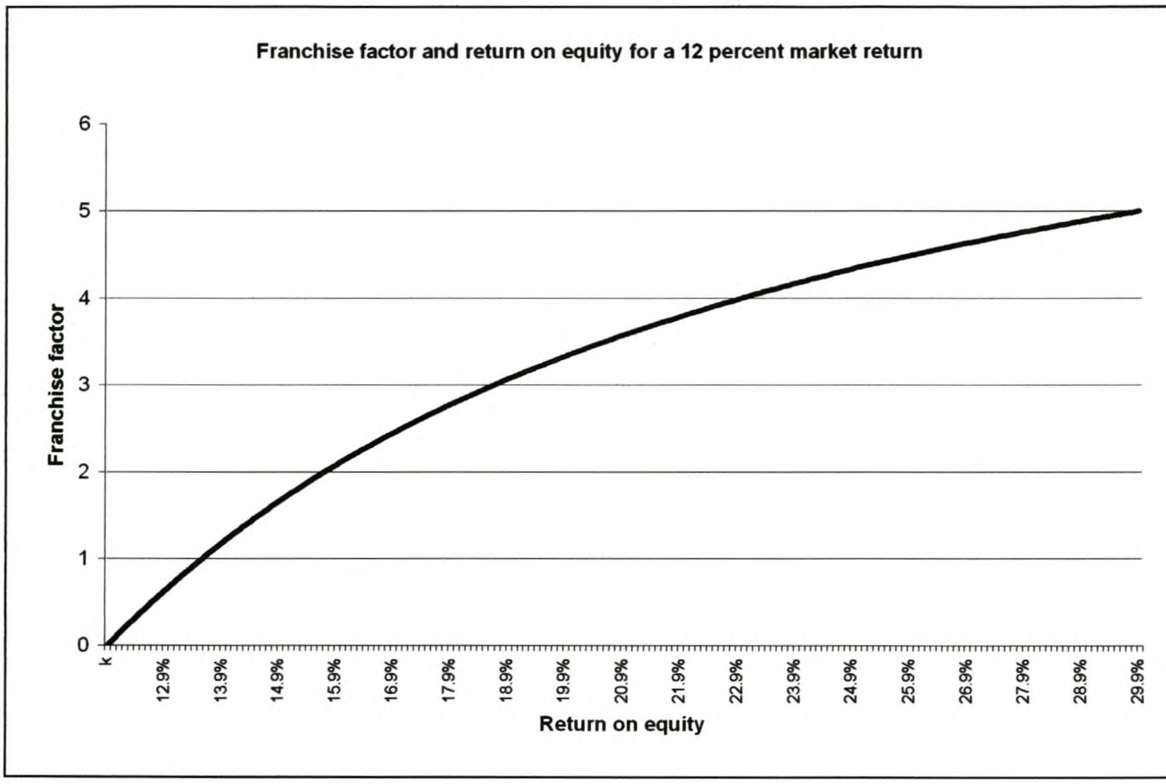
Leibowitz and Kogelman (1994:23-25) defined the franchise factor as a direct measure of the impact of investments that earn an above-market rate of return on the price-earnings ratio. In a stable market, the franchise factor depends only on a company's return on equity on all of its investments. Equation 3.6.2 depicts the computation of the franchise factor. In essence, the franchise factor is the return premium offered by new investments as the numerator, and the product of the company's return on equity for existing business and the market's rate of return. The effect of the return on equity and the market's return rate on the franchise factor is illustrated by Figure 3.6.1 and set out in Annexure A Table 3.6.1.

$$\text{Franchise factor} = \frac{(ROE - R_i)}{(ROE)(R_i)}$$

Equation 3.6.2 (Leibowitz & Kogelman, 1994:23)

The franchise factor of company D is one and two thirds. The indication is that the price-earnings ratio will grow by one and two thirds on each occasion an amount equivalent to the original book value is added to the current investment opportunities.

Figure 3.6.2



The price-earnings ratio is consequently represented by Equation 3.6.3. It is expressed in terms of the market's return, the growth equivalent, and the franchise factor. The second term in Equation 3.6.3 captures the change in the price-earnings ratio from a combination of growth rates and the return on equity. Figure 3.6.1 demonstrates the difficulty companies face to increase their price-earnings ratio. As the return on equity in Figure 3.6.1 approaches infinity, the franchise factor approaches the inverse of the market rate of return.

$$P/E_1 = 1/R_i + (\text{Franchise factor x Growth equivalent})$$ **Equation 3.6.3** (ibid.)

3.6.3 The franchise portfolio

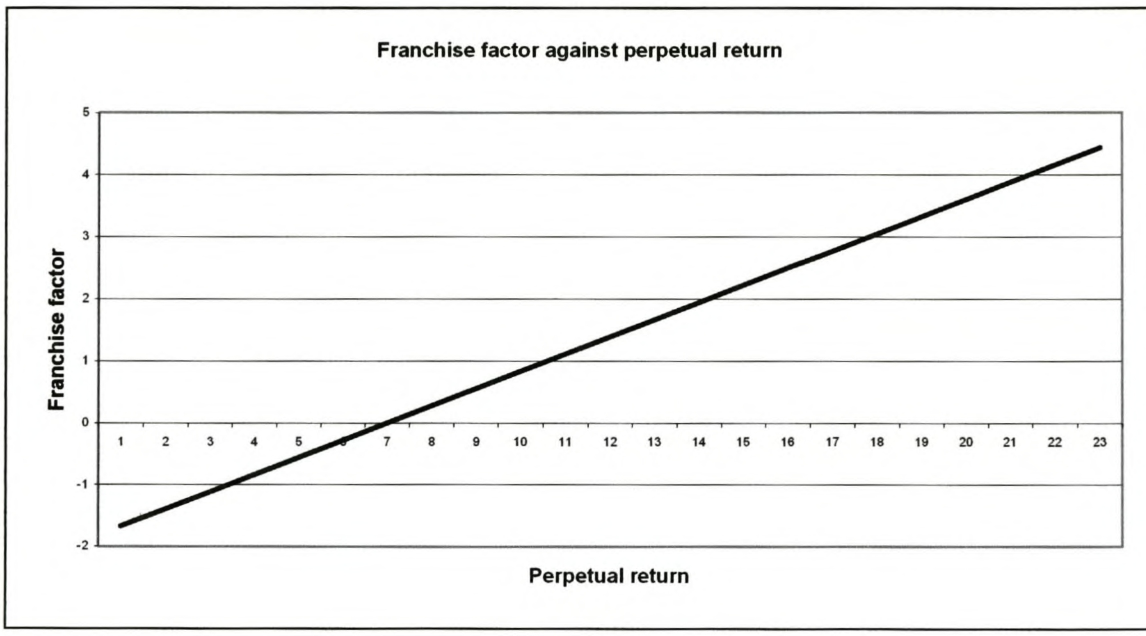
The prior section dealt with simple growth patterns in order to facilitate comparison with the Gordon model. Conversely, Leibowitz and Kogelman (1994:24) elucidated that the franchise value approach applies to all possible growth patterns, as an advantage over the other explained valuation models.

The franchise factor captures the price-earnings producing power of an investment opportunity. Consequently, some alteration is required to adapt Equation 3.6.2 to accommodate these diverse growth patterns. The change, however, relates more to the calculation of the return on invested equity than to Equation 3.6.2, as the return on equity in that equation is simply substituted with the perpetual equivalent return of the investment opportunity (Leibowitz & Kogelman, 1994:36-38). Perpetual equivalent return restates an investment opportunity's growth pattern into an infinite rate of return, in order to correspond to the assumptions of the franchise factor. The franchise factor is consequently calculated by Equation 3.6.4, where the return on equity is replaced with the perpetual equivalent return in the numerator. Figure 3.6.3 represents the linear relationship that exists between the franchise factor and the perpetual return rate.

$$\text{Franchise factor} = \frac{(ROE_p - R_i)}{(ROE)(R_i)}$$

Equation 3.6.4 (Leibowitz & Kogelman, 1994:37)

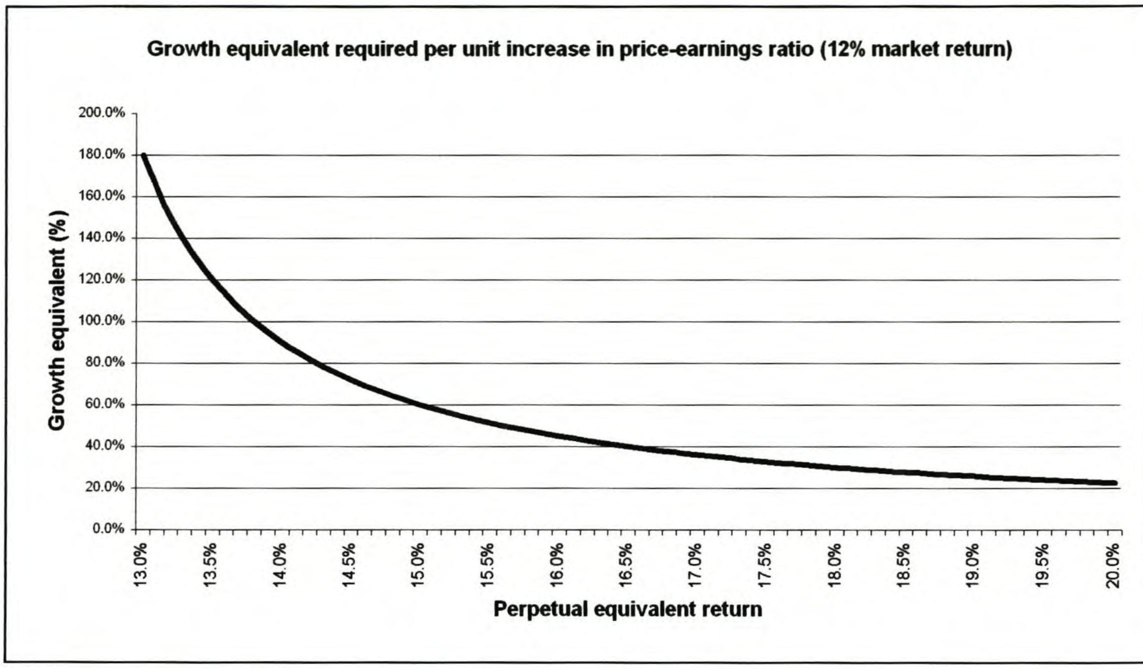
Figure 3.6.3



The advantage of utilising perpetual equivalent return in the calculation is that it applies universally and is consequently suited to evaluate all types of investment opportunities (Leibowitz & Kogelman, 1994:36-38). Furthermore, provided that there is a fixed amount of capital, the project with the highest perpetual equivalent return will make the greatest contribution to the price-earnings ratio. This observation is consistent with the net present value approach to project valuation, and results in optimal project selection.

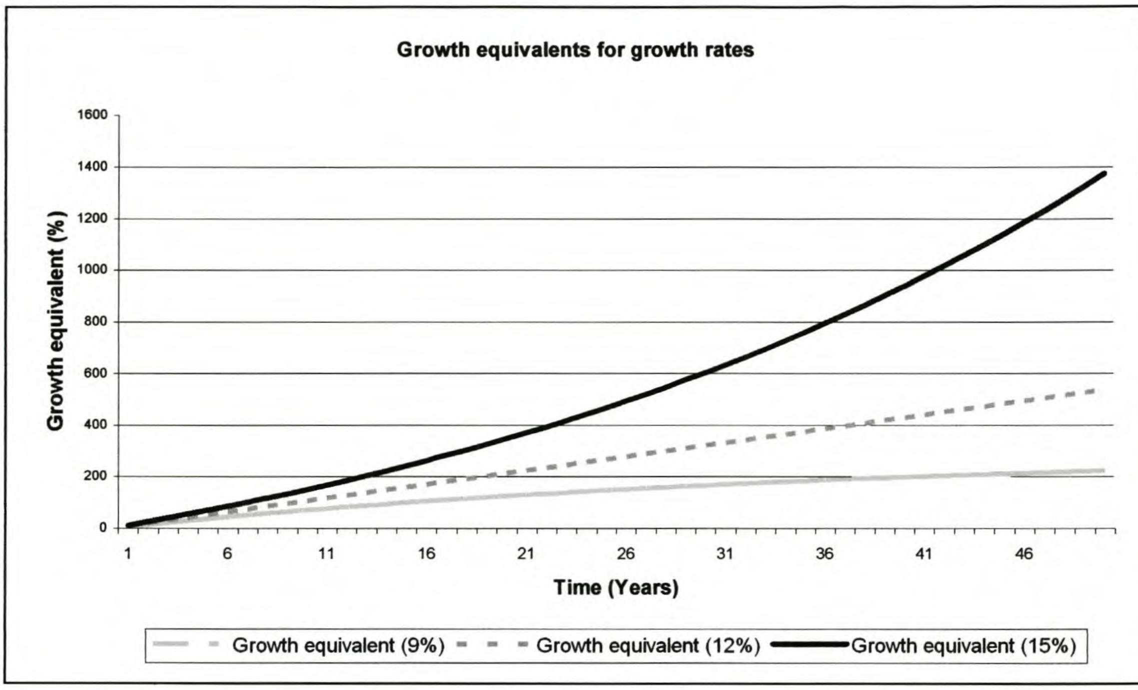
The growth equivalent together with the franchise factor presents the franchise value. This relationship is depicted by Figure 3.6.4 for a company with a twelve percent market return. As a result of the irregular investment patterns allowed, the growth equivalent shares some of the weight that caused an increase in the price-earnings ratio. Accordingly, the growth equivalent of future opportunities is discounted at the market rate of return in order to express these opportunities at their present value, and then expresses that as a percentage of the current book value of equity. Alternatively, this indicates that the company can invest the resources immediately and earn the same rate of return on that investment indefinitely. The discount rate that applies is the return on equity of current business activities, as this is the minimum rate at which the company should reinvest its returns.

Figure 3.6.4



The growth equivalent also shares a relationship with the growth of the opportunities available to the company. As the number of years of high-growth and the growth rate increases, so does the growth equivalent. Whenever the growth rate equates the market rate of return, the growth equivalent will increase in direct relation to the market's rate of return. Under altered circumstances, the growth rate will increase or decrease exponentially. The logic behind this concept is that a company can grow at a rate that differs from its market related rate of return. Companies can only grow with the growth of opportunities available to them to invest profitably. The relationship is depicted by Figure 3.6.5 for a company with a market related return of twelve percent.

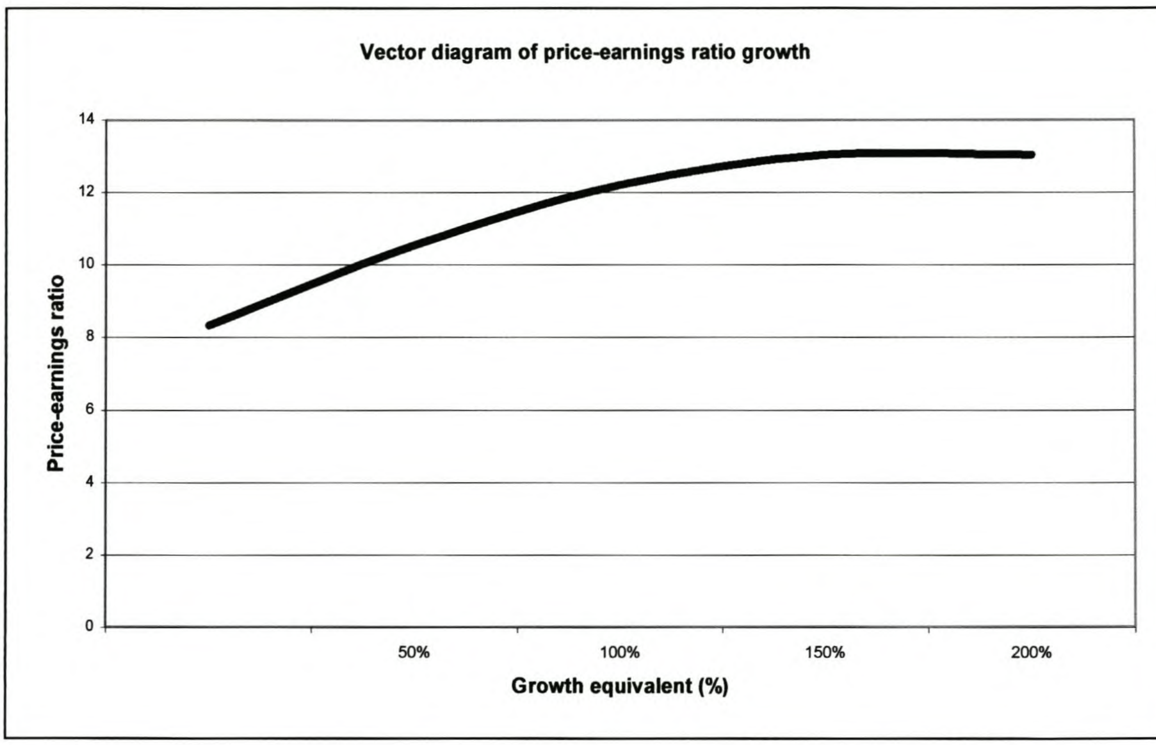
Figure 3.6.5 (Leibowitz & Kogelman, 1994:41) (adapted)



It is clear that the growth equivalent incorporates both the magnitude and the time of occurrence; any pattern of investment and return is accommodated by calculating the sum of the products of the different franchise factors and their corresponding growth equivalents in order to obtain the total price-earnings ratio increment. The relevance of the timing of investments applies only to the extent that it affects the value of the growth equivalent (Leibowitz & Kogelman, 1994:41-42). Once the different growth phases are reduced to their growth equivalents and franchise factors, the sequence is irrelevant.

Whenever the price-earnings ratio is plotted against the growth equivalent the franchise factor represents the slope of the graph and the growth equivalent the duration of the growth opportunity (Leibowitz & Kogelman, 1994:42-44). This is evident from Figure 3.6.6.

Figure 3.6.6 (Leibowitz & Kogelman, 1994:42)(adapted)



The advantage this approach is that it offers a company the opportunity to increase its price-earnings ratio based on the franchise value approach or to guide management with their choice of investments. The company should also manage its investment alternatives to utilise retained earnings effectively; unutilised resources should be returned to the shareholders to invest effectively. Furthermore, this approach can accommodate any possible growth pattern. Some insight was consequently provided as to the inherent difficulty of raising a price-earnings ratio.

According to Leibowitz and Kogelman (1994:45), franchise opportunities tend to erode over time, but some companies seem to compound their franchise positions, as if they had a franchise opportunity in generating new franchise opportunities.

3.6.4 The franchise factor for leveraged companies

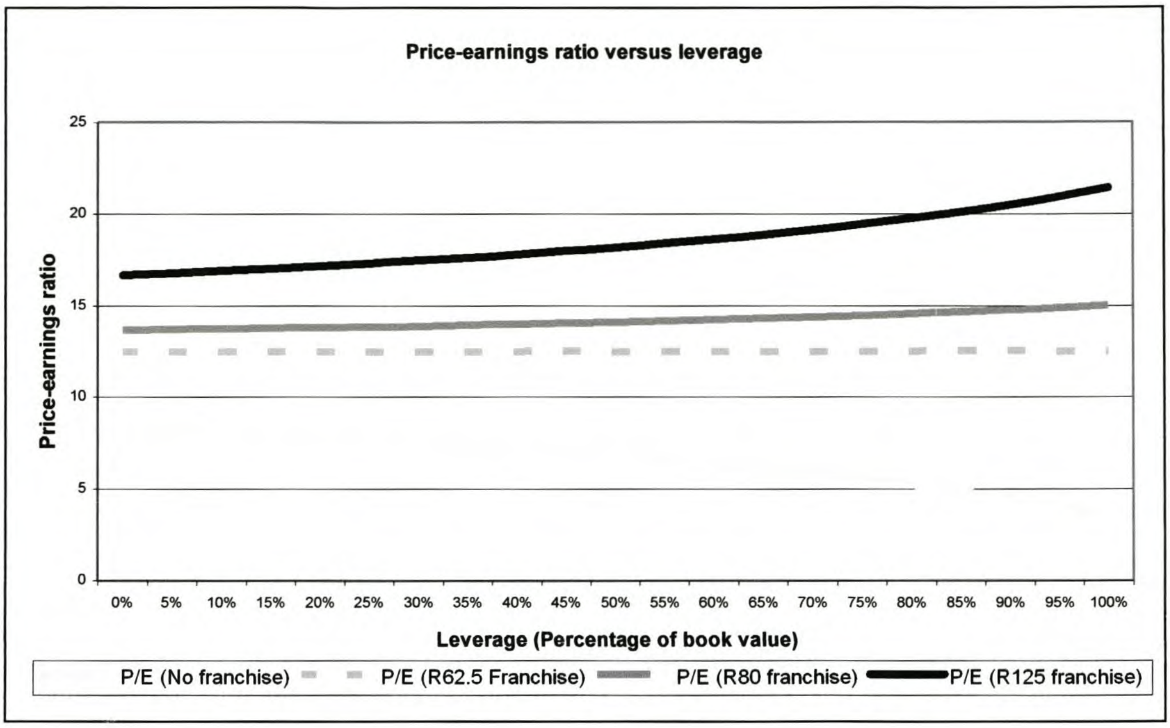
Leverage shrinks both the shareholder's equity of a company and its earnings; intuition regarding the net impact of leverage on the price-earnings ratio is consequently unreliable.

With the introduction of two assumptions, firstly, that all companies are fairly valued, and secondly, that debt will not expand the capital structure, it merely changes the structure and one can examine the effects of leverage (Leibowitz & Kogelman, 1994:67-69). The utilisation of debt results in two immediate effects, initially, the shareholders claim to the company's value is reduced by the value of debt. Also, the net earnings available to the shareholders are reduced by the cost of interest.

The effects of the introduction of debt are elucidated with the use of company C in Table 3.2.1. The cost of debt is eight percent irrespective of the magnitude of leverage. Half of the book value of the company of hundred Rand is assumed to constitute debt. This will result in interest reducing earnings by four Rand to yield a net profit of eleven Rand. This profit of eleven Rand will befall the seventy-five Rand of equity remaining, after it was reduced by fifty Rand from the previous one hundred and twenty five Rand. It is assumed that the unleveraged market value will remain constant. The resultant price-earnings ratio for the company is six point eight two. According to Leibowitz and Kogelman (1994:69-70), leverage on the current book (current business with no expansion) of business produces a declining price-earnings ratio.

Whenever a more representative scenario is sketched, where company C possess opportunities for future growth through investment at rates of return above that of the market, the situation changes. With the introduction of a franchise opportunity valued at eighty Rand presently, the total market value of the company increases to two hundred and five Rand. If one excludes debt, the price-earnings ratio is equal to thirteen point six five units. The relationship between the price-earnings ratio and leverage is depicted by Figure 3.6.7 (Leibowitz & Kogelman, 1994:70-72).

Figure 3.6.7 (Leibowitz & Kogelman, 1994:71) (Adapted)



The directional effect of leverage on the price-earnings ratio depends on the company's structure of value, or opportunities. For companies with franchise opportunities that exceed sixty two Rand and fifty cents, the threshold in the current example, the use of debt financing increase the price-earnings ratio and vice versa.

The basic franchise value model applies equally to leveraged and unleveraged companies. The only required alteration relates to the definitions provided previously. The leveraged base price-earnings ratio is calculated as the adjusted tangible value pertaining to shareholders divided by the annual earnings, net of interest. This decreased shareholders' value is a consequence of the inclusion of debt in the base price-earnings ratio. As a result, the franchise value can be seen to remain constant in the face of leverage.

Accordingly, the relation of debt to the current base value means firstly that the current shareholders are entitled to the full benefit of future franchise opportunities (ibid.). Secondly, it emphasises that the value of franchise opportunities reflects the excess of the return on new investments above the cost of future capital. Lastly, it indicates that the weighted-average cost of future capital theoretically equates the market's capitalisation rate, regardless of the utilisation of debt. This derivation conforms to the theory of Miller and Modigliani (1961:411-433), as the current market capitalisation rate of the company includes all the future information pertaining to the company.

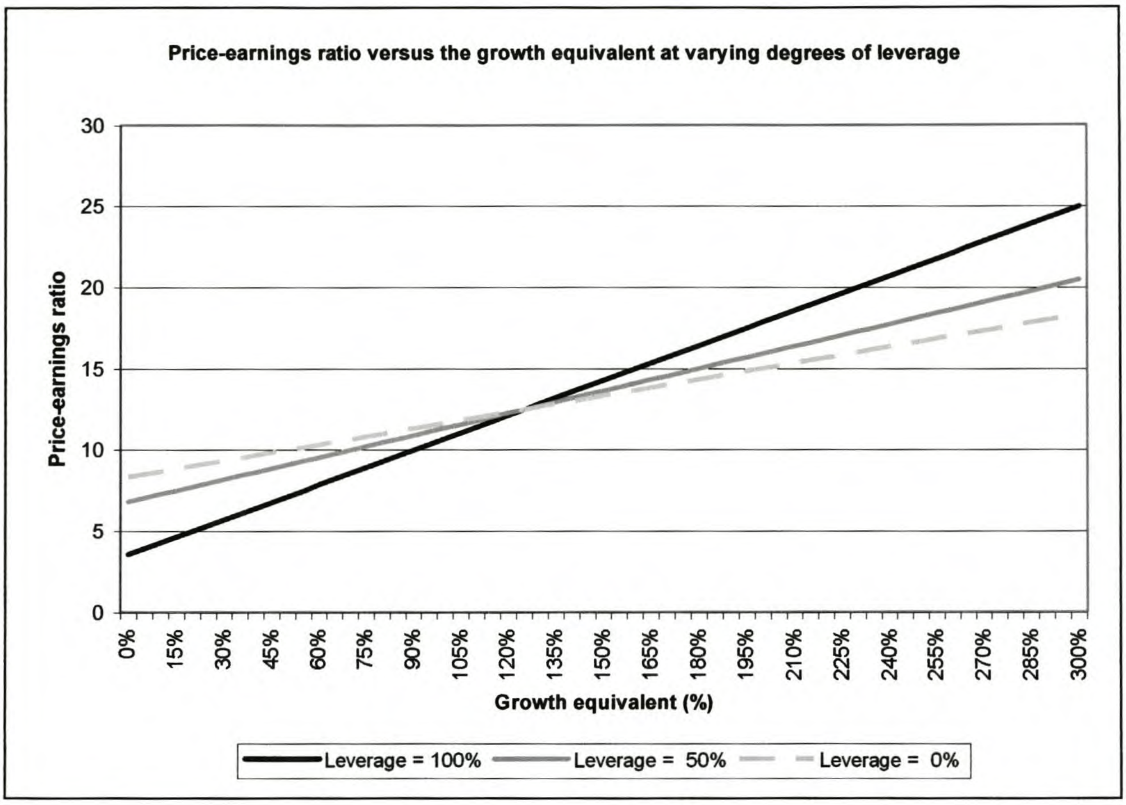
The price-earnings ratio increment from franchise opportunities is obtained by dividing the franchise value by net earnings (Leibowitz & Kogelman, 1994:72-74). Accordingly, as net earnings decrease with concurrent increase in the value of debt, the price-earnings ratio increment of a given franchise opportunity is at all times larger for a company with debt than an unleveraged one. The growth equivalent is not affected by the use of debt, as the entire impact of leverage is effectively loaded in the amplified franchise factor. The leveraged franchise factor is calculated by Equation 3.6.5, where the symbol i represents the interest rate on debt, and the h leverage as a percentage of book value. It is evident from the equation that as leverage increase the franchise factor would increase at an increasing rate. Consequently, the implication is that the incremental price-earnings ratio from a given opportunity grows in proportion to the percentage of debt funding.

$$\text{Leveraged franchise factor} = \frac{ROE_p - R_i}{(ROE - ih)R_i}$$

Equation 3.6.5 (Leibowitz & Kogelman, 1994:73) (Adapted)

Accordingly, the base price-earnings ratio of a company declines with the introduction of leverage. Nevertheless, the incremental price-earnings ratio, which pertains to the franchise value, exhibits an ascending pattern. Therefore, the above emphasises that the effect of debt on the price-earnings ratio cannot be based upon intuition. The relationship of the price-earnings ratio and leverage is represented by Figure 3.6.8 as it relates to the example in this text.

Figure 3.6.8



Two relevant taxation effects affect the employment of debt financing on the price-earnings ratio. The first effect is that earnings are reduced by the after-tax interest payments and secondly, the total value of the company is enhanced with the use of debt. Miller and Modigliani (1961:423-425) demonstrated this issue as they proved that alternate forms of financing could be interchanged for one another in an efficient market, and that a company's cost of capital is lowered by the use of leverage.

The value enhancement is calculated through the above alterations as these incorporate the assumption that the additional value is just the magnitude of the tax wedge, which is the tax rate multiplied by the debt value. The added value of the tax wedge is incorporated into the base price-earnings ratio and the franchise factor by the replacement of the normal leverage with the after-tax leverage. According to Leibowitz and Kogelman (1994:77-78), the introduction of tax serves to diminish the impact of the use of debt as the lines in Figure 3.6.8 will be closer to one another.

3.6.5 Growth patterns and conclusion

Some generalisations apply regarding the behaviour of a company that can be observed with the use of the franchise value approach.

Firstly, franchise consumption will constitute extraordinary earnings growth. This extraordinary growth will end abruptly with the full exploitation of the existing opportunities. Consequently, with the exploitation of the available opportunities, the price-earnings ratio will erode toward its base ratio, although earnings growth can remain high. During the franchise consumption period, price appreciation will lag behind earnings growth; this gap should roughly equate the rate at which the price-earnings ratio decline. Lastly, whenever the franchise is completely consumed, earnings, dividends, and price appreciation will grow at a rate determined by a company's retention policy and its return on equity, as defined by Miller and Modigliani (1961:416-418).

With the passage of time, both the tangible value and franchise value of a company generate interest at the market rate of return (Leibowitz & Kogelman, 1994:83). Without franchise consumption, the franchise value will increase at the market rate of return. Yet, when the franchise investment is in fact funded, the total present value of the residual franchise investment will fall by the amount of the outflow. These realisations are tantamount to payments from the franchise value to the tangible value. Thus, franchise investment consumes franchise value, as future potential becomes current reality.

Again, assume company C in Table 3.2.1, only the market value of the company increases to two hundred and twenty five Rand. This would provide for a franchise value of one hundred Rand and a price-earnings ratio of fifteen. Furthermore, the franchise value is based on the ability to make new investments that provide twenty percent return perpetually. Capitalising this value at the market's rate of return will produce an additional tangible value of one Rand and sixty-seven cents.

Consequently, the net value added is only sixty-seven cents, as one Rand is required for the investment. Therefore, with a total net present value of fifteen Rand, the franchise value will be exactly one hundred Rand. The total of all franchise investments is defined in present value terms, consequently the value of the franchise remain unaltered whether the one hundred and fifty Rand of investments is made immediately or spread out over time. The assumption is nevertheless that the company will consume the franchise in accordance with the prospective schedule of investment opportunities generated by its retained income, resulting in the depletion of the franchise value in the eleventh year in the example.

Initially the present value of the future franchise investments, as a result of the interest, grows faster than the book value of the business. Nevertheless, when the incremental franchise investments begin to exceed the earnings of the time shifting (interest) of the franchise opportunities, the present value of the future franchise investments start to decrease. Refer to Figures 3.6.9 to 3.6.15 to observe the changes associated with the consumption of franchise opportunities as they relate to the company's consumption of franchise opportunities within the limitations of its corporate financing activities.

Figure 3.6.9

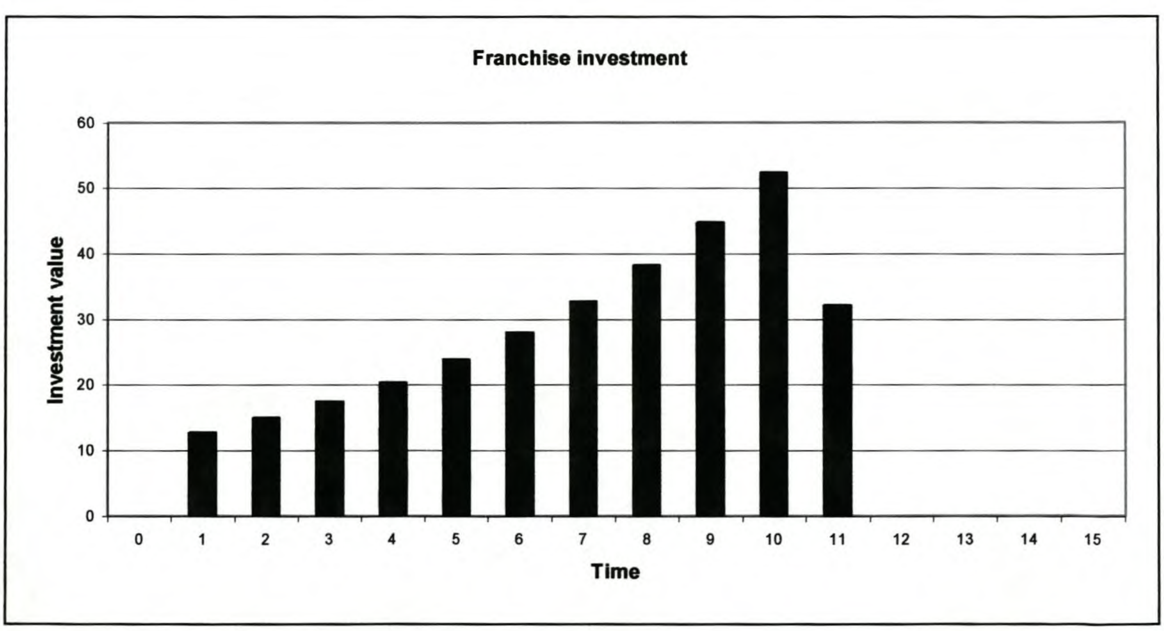


Figure 3.6.10

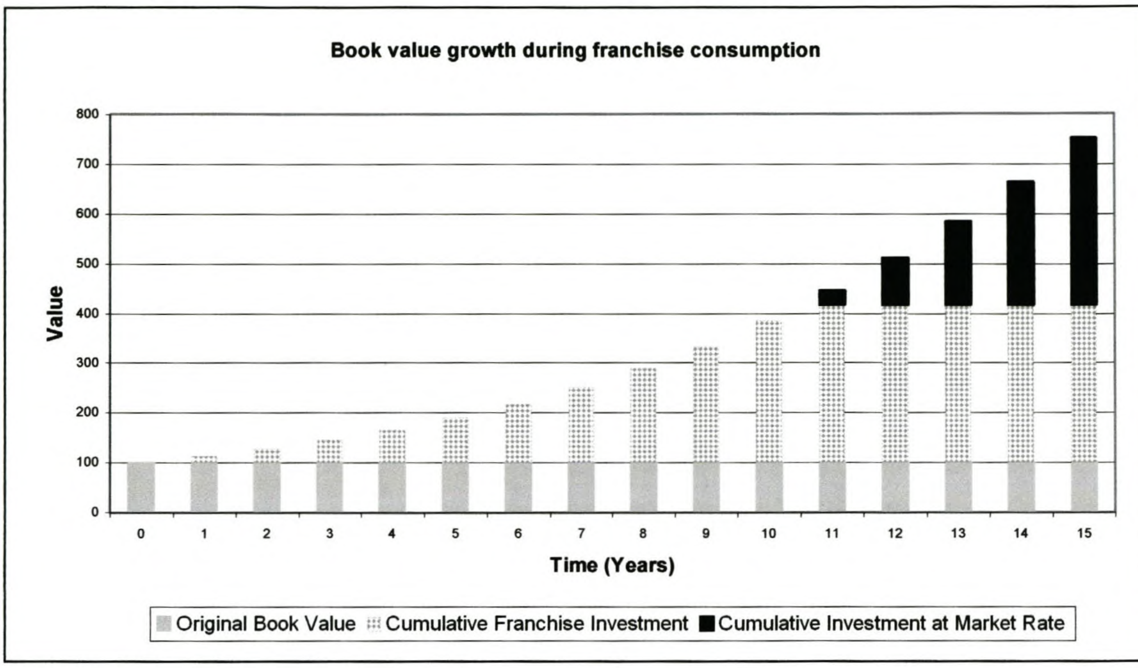


Figure 3.6.11

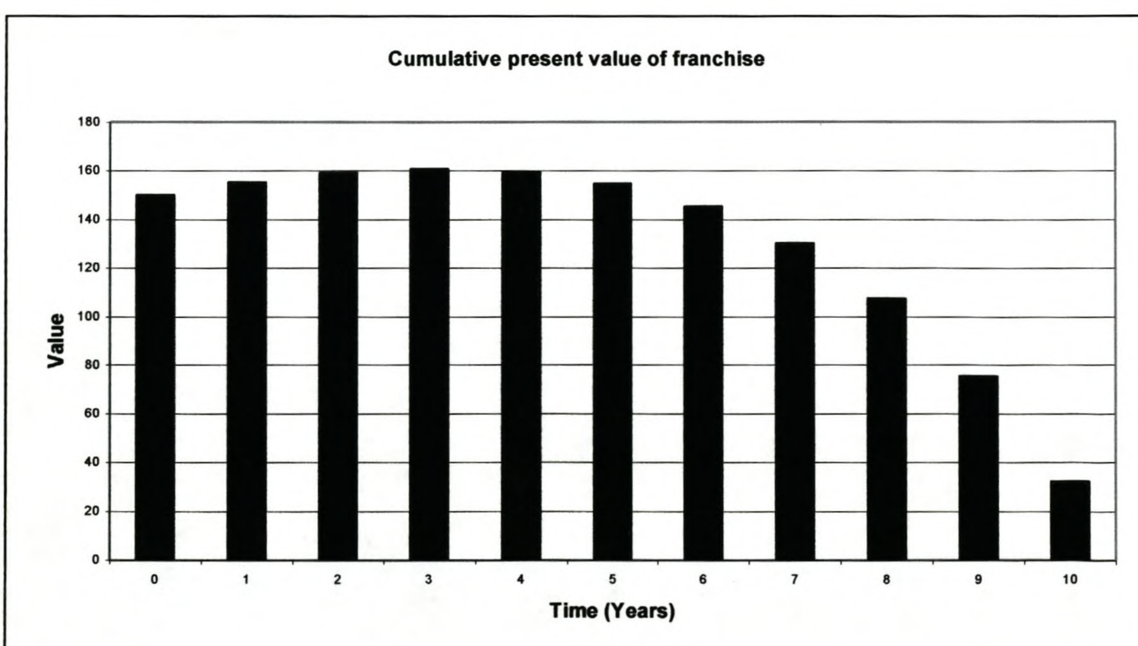


Figure 3.6.12

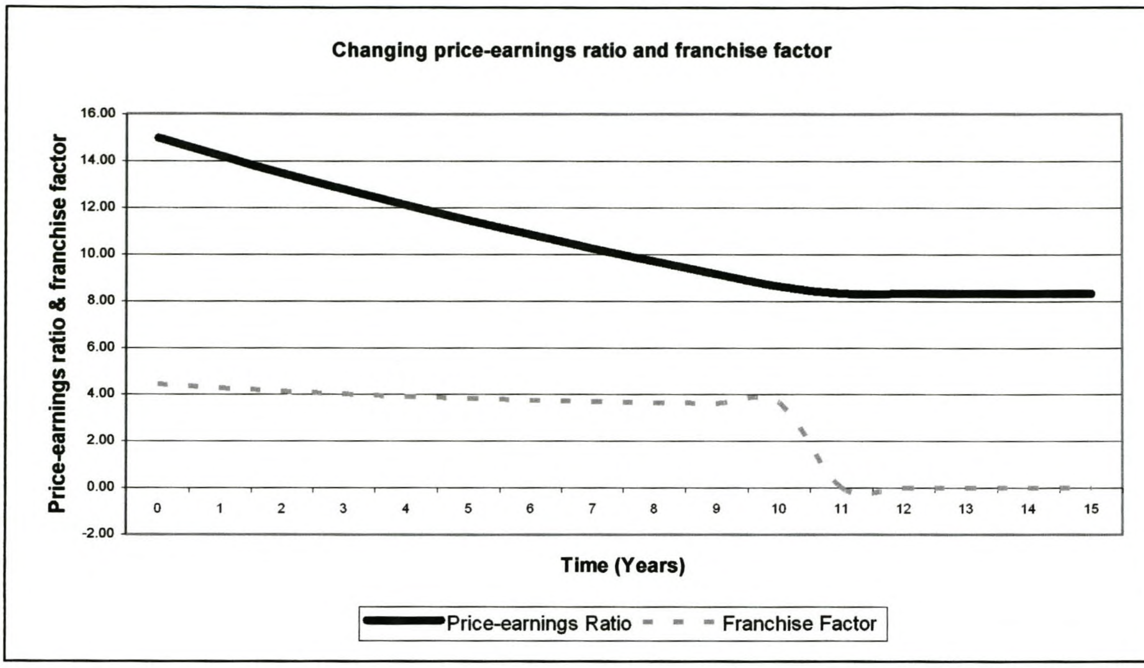


Figure 3.6.13

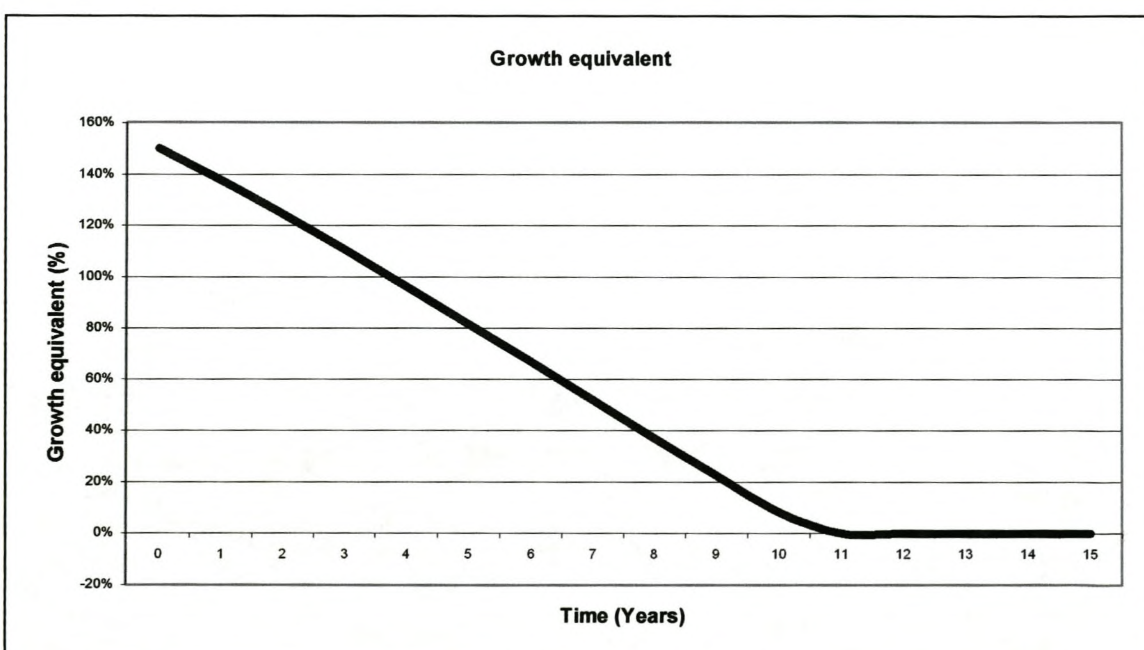


Figure 3.6.14

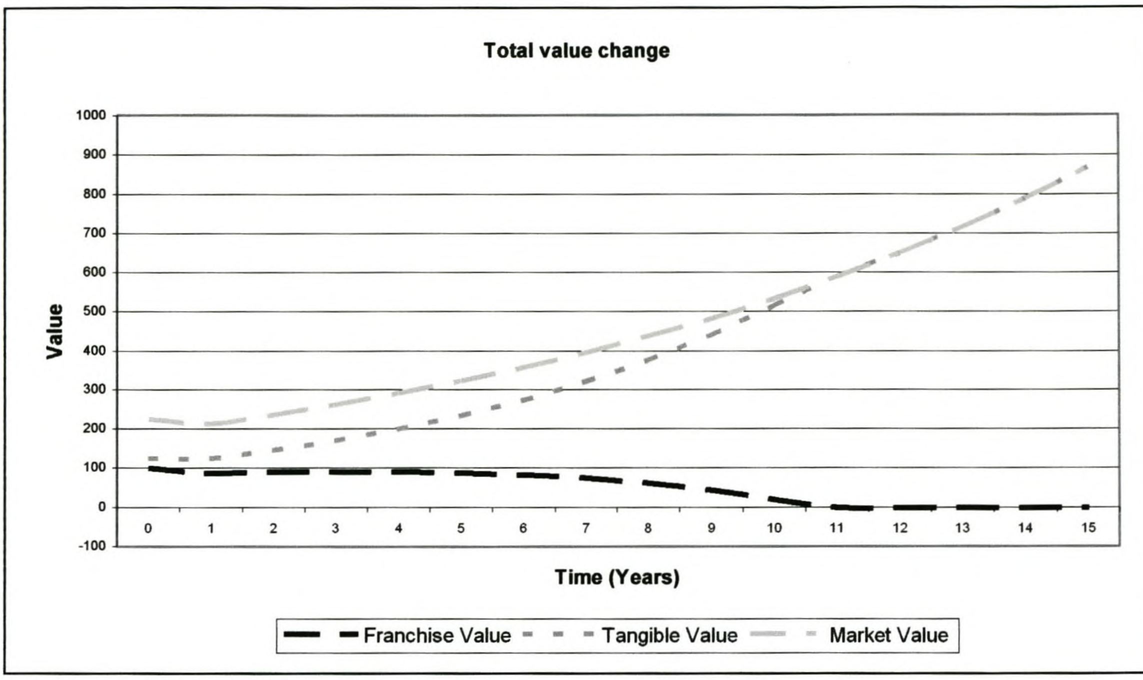
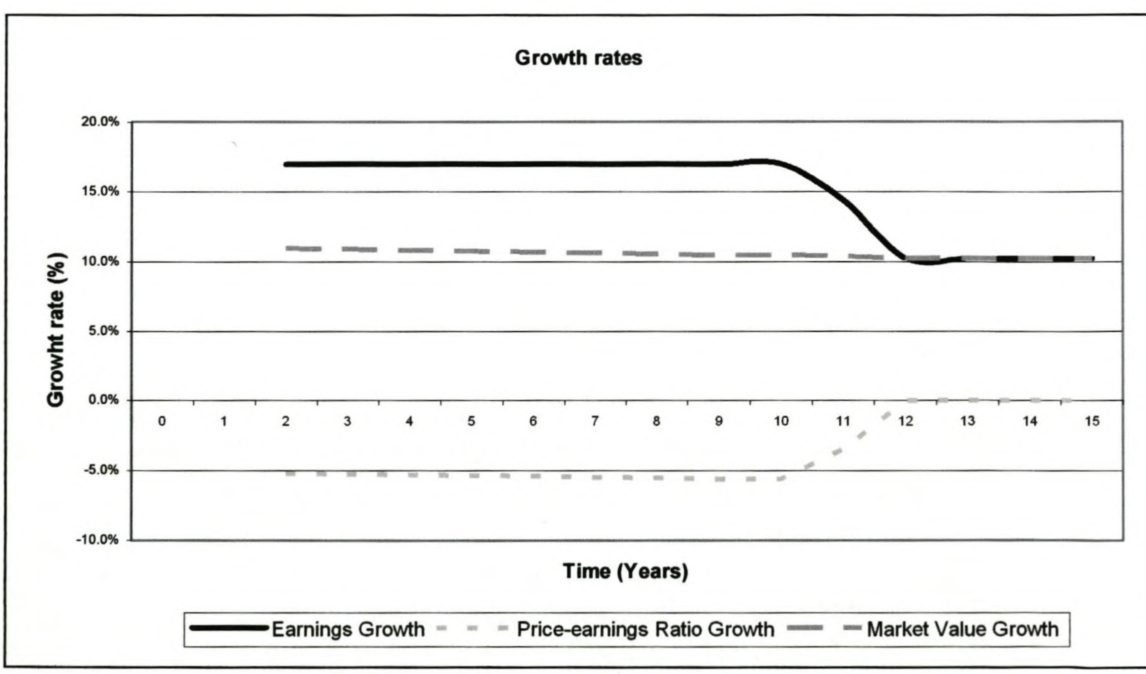


Figure 3.6.15



The decrease in the franchise factor in Figure 3.6.12 is explained by the change in return on book equity as the new investments increase the return on equity slightly. The resultant return on equity is a weighted average of the old and the new returns, consequently a blended rate arises. Furthermore, the price-earnings ratio will, over time, reflect the rapid decline of the growth factor, as evident by the fact that during the eleventh year the price-ratio increment is fully utilised.

Homogeneous growth enjoys intuitive appeal, but it does not indicate how equity value develops over time in the real world (Leibowitz & Kogelman, 1994:91-93). Uniform growth therefore temporarily creates a self-fulfilling prophecy. As depicted in Figure 3.6.15 above, during the franchise consumption phase, earnings growth is fixed at seventeen percent; nevertheless, price appreciation remains fixed at eleven percent. As a result a widening gap in growth rates is experienced during the franchise consumption period. Afterwards, both price and earnings growths follow the intuitive equivalent growth.

Whenever a company follows a homogeneous growth pattern, the price-earnings ratio would remain stable with the passage of time (ibid.). Nonetheless, the only stable price-earnings ratio within the franchise value approach is the base ratio. The incremental ratio that is associated with franchise opportunities continuously fluctuates as opportunities are consumed and new ones arise. Whenever unanticipated events occur, a spike (unanticipated rise) will occur in the price-earnings ratio. The prospects will range from those that are immediate and clearly visible to those that are distant and only possible. Theoretically, this entire range of scenarios is incorporated in the company's franchise value, but both positive and negative surprises are frequent. These discoveries do not affect the tangible value of the company, as the surprises only relate to future earnings in most cases.

As stated previously, this approach to valuation is clearly more practical than the previous discount models and relate directly to observed changes in the price-earnings ratio. The combined information from all these valuation methods provides more information about changes in the price-earnings ratio and how to utilise these changes to profit in the market.

3.7 Duration of growth

As of late, companies with sustained levels of abnormal growth now face fierce competition which threatens to eradicate their superior earnings performance, according to Leibowitz and Kogelman (1994:1). Consequently, it has become increasingly important to analyse the duration of extraordinary growth so as not to overvalue companies. The price-earnings ratio can assist in the determination of the implied growth inherent in a share price.

It was apparent from the discussion of franchise value approach (Section 3.6) that routine investments which deliver the market's rate of return do not add value to a share. These investments may still contribute to nominal earnings growth. When a specified growth rate is evaluated, it may, on the surface, appear admirable, but it in fact reveals nothing exceptional about the company if that company fails to obtain added value from these investments.

According to Miller and Modigliani (1961:417) the essence of growth is not expansion, but the exploitation of opportunities to invest significant funds that earn extraordinary returns. This view was reinforced by Babcock (1970: 108-109) when he emphasised that the basic elements of price appreciation are higher earnings and higher multiples.

Babcock (ibid.) expressed earnings per share as a product of the margin of profit on sales, the turnover ratio of sales to total capital, the percentage of leverage employed, the tax wedge, and book value per share. He added that changes in the sales margins would always occur, but that upward movements are limited by the degree of competition in the industry. Changes in turnover will be restricted by the state of the economy or technology. Likewise, the degree of leverage is constrained by the willingness of creditors or the prudence of management and the government controls taxes paid. Consequently, these changes cannot sustain growth over the long term, and only new profitable ideas can continuously benefit the company with the aid of investment in such ideas, resulting in an ever increasing book value.

Babcock (ibid.) used the DuPont formula developed by the DuPont company in the United States to evaluate the changes in the components identified above to obtain the sustainable earnings level. The DuPont formula is provided in Equation 3.7.1. This formula distinguishes between return, profits in relation to investment, the margin achieved, and profit in relation to sales. It furthermore emphasises return as the primary objective of any investor, and margin on sales as a means to accomplish that objective. Leverage benefits the investors only when the rate of return on the total investment exceeds the interest cost of borrowing. Lastly, the author multiplied the result of the DuPont formula with the book value of the shares to obtain the return in value per share. The function of the DuPont equation is to elucidate the underlying changes in earnings per share.

$$\text{Return on equity} = \left(\text{EBIT}/_{\text{Sales}} \text{ x } \text{Sales}/_{\text{Total assets}} - \text{Interest expense}/_{\text{Total assets}} \right) \left(\text{Total assets}/_{\text{Common equity}} \right) \left(100 - \text{Income taxes}/_{\text{Net income before taxes}} \right)$$

Equation 3.7.1 (Reilly & Brown, 1997:396)

Babcock (1970:110-111) also emphasised that total book value growth from one year to the next is misleading as it incorporates an element of debt financing or repayment. Consequently, the DuPont equation aims to represent sustained growth, although it might be added that the DuPont formula can still incorporate elements of non-recurring income; an example is changes to pension plan contributions.

According to Damodaran (1996:196), the short-term sustainable growth rate of a company can be calculated from the product of the return on equity, as calculated by Equation 3.7.1 above and the retention rate of the company, as per Equation 3.7.2 below. A vital reality frequently omitted is that extraordinary short-term growth must always end, according to Holt (1962:465-475).

Growth = Retention rate x Return on equity

Equation 3.7.2 (Damodaran, 1996:196)

Holt (1962:465) noted that investors seek out growth shares. The result is that the prices of these identified growth shares appear to become inflated, as supported by the high price-earnings ratios associated by these shares. This supports the postulate that some companies are omitted from analysis by analysts and, when finally recognised, achieve significant returns on a risk-adjusted basis as indicated by Arbel et al. (as cited in Shefrin & Statman, 1995:26-34) (See Section 2.3.3). Investors should consequently be aware of the magnitude of growth associated with these shares as they generally become overvalued and resultantly achieve subdued returns, as supported by Peavy and Goodman (as cited in Reilly & Brown, 1997:716). According to Holt (1962:467), the growth implicit in the market price may be valuable in the determination of the suitability of a specified market price. It is an empirical fact that low price-earnings ratio shares outperform high price-earnings ratio shares on a risk-adjusted basis; refer to McEnally, as cited in Reilly and Brown (1997:314).

Holt (1962:467-475) identified the greatest risk for growth investors as the calculation of the duration of the extraordinary growth phase. Companies with lengthy extraordinary growth durations should be more valuable, compared to companies with comparatively shorter growth durations, all other things being equal. The author further indicated that both the duration and the growth rate should be incorporated in the valuation of growth shares, as the Gordon model is applied subsequent to the extraordinary growth rate. Holt also noted the risk associated with securities as another significant oversight that tends to make high-growth companies more attractive. Another problem that can affect returns is taxes, either on dividends or on capital gains.

Solomon, (as cited in Mao, 1966:95) cautions against indefinite growth projections, as according to him a company's opportunity for growth is limited both with respect to the amount of funds that can be invested (although according to Leibowitz and Kogelman (1994:8) limitation of funds is not a criterion for growth) and the period over which these investments are available. Solomon added that realistically growth is likely to follow a Gompertz curve, in which a period of rapid growth is followed by a time of stability after which business declines.

Malkiel and Cragg (1970:605) identified three techniques to approximate the short-term growth rate of a company that are superior to other alternatives, or no worse than these alternatives. First, they identified the geometric growth rate of cash earnings per share, or earnings per share plus non-cash charges. The second was the geometric mean of earnings per share during the previous decade. The last is the ten-year growth rate of cash earnings plus taxes calculated from a regression of the logarithms of earnings over time or, in other words, the exponential average ranked according to empirical effectiveness.

According to Durand (1957:349), the statistical concept of ever increasing growth is normally associated with sales growth, dubbed the Petersburg Paradox. This is explained by Peter, who tosses a coin until it lands heads to the ground. Peter continue to promise Paul something if he gets heads on the very first throw, double the advantage on the second, in such a way that with each additional throw the promise to Paul is doubled. Accordingly, Paul's expectation is the sum of the products of probability by payment or $\frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \ldots$

Therefore, the assumption by Durand (1957:354-360) is that the company will thus pay dividends at an ever increasing rate. It is not foreseeable that this perpetual growth rate will ever decrease or cease to continue. Clearly, this scenario is impracticable, and according to Clendenin and Van Cleave (as cited in ibid.), continued rapid growth will only occur on the assumption of long-run price inflation.

A company that is expected to pay an ever increasing dividend which grows at a greater rate than the discount rate applicable to the company is required to earn a return on capital that exceeds the discount rate dividend by the retention rate, with the assumption that no external financing is allowed (ibid.). The scenario poses an anomaly for equilibrium theorists, who argue that the marginal rate of return on capital must equate interest rates over the long run. Whenever this state of equilibrium ceases to exist, funds will flow from interest-bearing investments to shares, consequently driving share prices up, and those of interest-bearing investments down. Therefore, the opposite of the above action will follow, to reverse the process and reinstate equilibrium. The logic is correct, but the author does not allow compensation for the riskiness of shares in this approximation. Growth shares which characterise business situations in which finite amounts of capital are invested at rates that exceed the equilibrium return pose problems for the analyst who must estimate the duration of this imbalance.

Macaulay (as cited in ibid.) introduced the concept of duration of an investment, mentioned in Section 3.2.2. The Macaulay duration on an investment is the arithmetic mean of the security's cash flows and its maturity, each weighted by their present value. The objective is to determine the sensitivity to changes in the discount rate. As indicated with the Gordon dividend discount model it is very sensitive to the difference between the interest rate and the growth rate (See Section 3.2.2). Consequently, as the growth rate and the discount rate approach one another, increasingly larger durations result or, as stated above, the value will become more sensitive to changes in the discount rate.

The essential characteristic of an elongated duration is that the security owner must expect to wait for an extended time before the security begins to pay a significant return. Clendenin and Van Cleave (as cited in ibid.) suggested that to accommodate uncertainties associated with distant dividends an increased discount rate must apply to those dividends. The suggestion is one alternative to account for the decreasing utility of money and a reason to use more complex valuation methods than the Gordon model for the valuation of growth shares.

Solt and Statman (1989:39-40) argued that an important distinction must be made between growth shares, a share with a high-expected risk-adjusted return, and shares of growth companies (companies with high relative growth). Growth companies possess growth opportunities, as per the franchise value approach, and these opportunities can be measured by Tobin's Q. Tobin's Q is the ratio of the company's market value, with the inclusion of all shares and debt, to the replacement value of its assets. It is an alternative to the price to book value ratio. According to Damodaran

(1996:334-335), Tobin's Q is an excellent determinant to identify inflated or deflated prices where inflation exaggerates asset prices or technology reduces the price of assets. A ratio of one signifies a company with no growth opportunities; as the ratio increases, so does the value of growth opportunities compared to the value of the company.

A positive relationship exists between Tobin's Q, the market value of equity, and the price-earnings ratio. This relationship suggests that the price-earnings ratio serves as a proxy for growth opportunities. Miller and Modigliani (1961:416-418) demonstrated that growth opportunities are one determinant of the price-earnings ratio. Leibowitz and Kogelman (1994:8) identified another determinant as the return on investment. Accordingly, it is clearly proven that the market value serves as a proxy for growth opportunities and return on investment. Solt and Statman (1989:40-42) empirically proved that companies with high multiples for Tobin's Q and the high price-earnings ratio provided lower returns than those of companies lacking high variables. The above provide further proof for the hypothesis that some companies are omitted from analysis by the market, until they provide results that draw the market's attention, and deliver above market returns to their existing investors, as demand for these shares increase.

Kahneman and Tversky (as cited in Solt & Statman, 1989:42-43) showed that people who follow representative intuition, evaluate the probability of an uncertain event by the degree to which it is similar to its parent population, and reflects the salient features by which it was generated. The situation occurs where event A is estimated to be more probable than event B, when A appears more representative than B of the situation. The suggestion is that investors overestimate the probability that the share of a growth company is a growth share because they rely on their representative heuristic intuition. An empirical study by Harris and Marston (1994:18) indicated that a return advantage based solely on growth forecasts for companies does not explain half of the growth in their share prices. This adds empirical support to Kahneman and Tversky.

Another simple method that utilises the price-earnings ratio to indicate whether the growth signified by the price-earnings ratio is expected to follow shortly and indicates the duration of the expected growth is the duration price-earnings growth ratio ("DPEG"). The use of this ratio is nonetheless limited; as it requires very specific information, refer to Equation 3.7.3.

$$\text{DPEG} = \text{Price - earnings ratio} \Big/ \text{5 year estimated growth rate} + \text{expected dividend yield}$$

Equation 3.7.3 (" The DPEG ratio explained", 2000)(Online)

Equation 3.7.3 consequently aims to indicate the value shares from the growth companies; value shares will typically experience high growth over the short term, while growth companies will experience more growth in the long run. Therefore, a value of less than one indicates value shares while a higher value indicates growth companies.

3.8 Conclusion

It is evident from this chapter that it is imperative to value growth properly, and as indicated throughout, simple and complex valuation techniques are required for this purpose. The chapter further emphasised that market prices are determined not only by future cash flows, but also by market sentiment and investor reaction, as seen from chapter 2. The price-earnings ratio assists the user to evaluate the market fundamentals about several aspects associated with market prices in order to determine the reasonability (acceptability) of these market factors, and profit from these changes. Furthermore, the approaches range from simple to complex and quick to thorough analyses of the companies, to allow the user to act on new information as promptly as required. Consequently, each of these methods has some advantage over the other models.

Chapter 4: Other factors and misconceptions

The last important remaining element of the price-earnings ratio, not covered thus far, is the earnings component, or the denominator in the calculation. Malkiel and Cragg (1970:601) identified normal earnings power as one of the most important and unpredictable components of the price-earnings ratio. It is often more difficult to project the normal earnings of a company and the expected growth associated with these earnings than it is to estimate the value, given the earnings.

According to Benesh and Peterson (1986:29-35), earnings forecasts are important for the reason that they play a central role in theoretical share valuation models. Their study suggests that the forecasts of analysts are superior to those estimated earnings produced by time series models. The empirical results indicate that share returns vary directly with both changes in actual earnings and unexpected additional earnings.

It is important to note that the market's concept of earnings is future orientated. Therefore, it is defined in terms of the expectations of the market participants, and not of the accounting rules. As such, the market's concept of earnings is not clearly observable and represents some form of expected permanent earnings per share attributable to the current equity investment (Beaver & Morse, 1978:65-76). Another important aspect is that earnings are not calculated in the same way throughout the world, and even between different companies, which complicates the calculation of comparable price-earnings ratios.

4.1 Accounting issues

4.1.1 General accounting issues

The price-earnings ratio is often cited as an indicator of transitory earnings; high ratios are associated with unusually low current earnings and low ratios are associated with abnormally high current earnings (Fairfield, 1994:24).

Friend, Blume and Friend (as cited in Barlev et al., 1988:70) indicated that investors base investment decisions primarily on accounting data. They further added that the reliance on accounting data is perceptible considering that the quantity of accounting information supplied to investors exceeds the quantity of market data supplied to investors. The accounting data is also in a more comprehensible format and neatly summarised and it, therefore, follows that this data will be the primary source used for decision-making.

The market does not necessarily capitalise the reported accounting earnings of a company during the previous period to obtain a value (Malkiel & Cragg, 1970:604). Non-recurring factors are likely to be excluded as investors seek to calculate an appropriate price-earnings multiple. Consequently, the price-earnings ratios that are used for valuation purposes use some form of normalised earnings rather than the ratio of prices to reported earnings for the preceding accounting period. These normalised earnings are estimated to be the earnings that the company would achieve at a normal operating level, in other words, when the company does not experience abnormal expenses or receive extraordinary profits from certain transactions. According to Nagorniak (1985:13-15), accounting standards do not indicate a company's ability to pay dividends, as identical earnings between two companies will often imply a different ability to pay dividends.

In order to assist users of accounting statements from different countries with the comparison of these statements the Accounting Standards Board introduced Accounting Bulletin 81. The bulletin compares and contrasts the differences between International Accounting Standards (adopted by most countries except the United States of America and Japan) and generally accepted accounting practice, as in force in the United States of America. A summary comparative table of the differences between the two accounting standards is included in Table 4.1.1.

Table 4.1.1 (Accounting Bulletin #81)

Issue covered	International Accounting Standards	U. S. GAAP	Comments
Asset revaluation	Allowance is made for revaluation on most assets.	Revaluation of assets is generally not allowed. U.S. GAAP normally adheres to the historical cost principle.	When assets are revalued under IAS, the historical cost must be disclosed. Consequently, IAS assists comparison between the two methods.
Pooling of interests	This is only allowed if no purchaser can be identified. It therefore applies only to a true merger of equals. Rules cover the identification.	Allowed if 12 criteria are met, the most critical is that in excess of 90% of the target company's common shares must be acquired using common shares of the acquiring company. Therefore, it represents a share purchase.	The IAS standards are clearly more stringent than the U.S. GAAP.
Goodwill	Maximum amortisation period of 20 years is allowed, but can be increased if a company can prove the appropriateness thereof in the statements.	Maximum amortisation period of 40 years is allowed, this is expected to change.	Currently, IAS is slightly more stringent than U.S. GAAP. The U.S. is expected to become more stringent than IAS in this regard, considering future changes. It is however simple to restate the amortisation amount for comparative purposes.

Statement of cash flows	The statements are basically similar, but IAS classify dividends paid as part of either the operating or financing sections, and interest paid and received may be classified as either operating, investing or financing cash flows.	Dividends paid are classified as financing cash flows. Interest and dividends received are classified as operating cash inflows, and interest paid is an operating cash outflow.	The statements are almost similar and it is easily restated to a comparable basis. Other accounting rules may, however, still affect the composition of amounts in the statements.
Development costs	Under the IAS, research costs should be recognised as an expense, but development costs are capitalised if it meets the recoverability requirements set out by the standards.	Under the U.S. GAAP, almost all costs are recognised as an expense as it is incurred. The exceptions to the rule are software development costs, which can be capitalised if the software development costs meet certain criteria.	The IAS standards are clearly more lenient, allowing the creation of assets and periodic expenditure through the income statement. U.S. GAAP easily identifies non-recurring costs in this manner.
Asset impairment	Assets are written down if the carrying value is exceeds the recoverable value. The recoverable value is defined as the higher of either the net selling price or the present value of future expected cash flows. The revaluation of impaired assets is permitted.	Assets are written down if the carrying value is less than the undiscounted sum of future expected cash flows. It is, nonetheless, written down to the present value of future expected cash flows. The upward revaluation of impaired assets is not permitted.	The comparison of the effect of these statements is complicated and an estimate at best. The IAS allows (to some extent) the adoption of a big bath methodology (take all losses at once), as assets can be impaired and subsequently revalued upwards.

Other intangible assets	As with goodwill the maximum amortisation period is 20 years. These assets may be revalued if the price can be obtained with reference to an external active market.	The maximum amortisation period is 40 years and assets may not be revalued.	The comparison of the effects of these statements is complex to say the least, considering the time span involved (the time the effects persist in the books). The issue is further complicated by the revaluation allowed under IAS, although details of such a revaluation must be presented in the statements.

The most important influence of accounting on the price-earnings ratio is the calculation of normalised earnings per share and the estimation of the future normalised earnings per share. According to White, Sondhi and Fried (1998:174-176), the Financial Accounting Services Board in the United States and the International Accounting Standards Board in the United Kingdom embarked on a joint project to harmonise international accounting standards during the nineteen nineties. The aim of the project, as far as it concerns earnings per share, is to simplify the computation of earnings per share by firstly requiring all companies to report basic and diluted earnings per share. Currently the two approaches produce a more or less similar result, and the important differences are mitigated by the accompanying disclosure requirements of each statement.

The most prominent differences that remain between the Accounting Principals Board Opinion (APB) 15 and International Accounting Standard (IAS) 33 lie in the objectives of these accounting rules (ibid.). APB 15 proposes incorporating the potential dilutive effect of all securities that may have a claim on earnings, whereas IAS 33's objective is to warn of the risk or variability of earnings per share. As such, it is clear that both these rules aim to provide future information about earnings per share. This is in line with the orientation of the securities markets, as identified by Beaver and Morse (1978:65). Both accounting rules also provide information about the current earnings per share.

Another important difference stems from the numerator used for the calculation of diluted earnings per share (White et al., 1998:174-176). In line with the future orientated view of the market APB 15 uses only income from continuing operations, whereas IAS 33 proposes the use of net income attributable to ordinary shareholders. Consequently, earnings per share computations based on APB 15 should be more informative, or more comparable over time and across companies, as they will not be affected by management discretion over the amount and timing of voluntary accounting changes, different methods used with the adoption of mandatory accounting changes and discontinuance of operations or extraordinary items.

Furthermore, APB 15 requires the disclosure of earnings per share for all of the components of income. Conversely, IAS 33 only requires the disclosure of earnings per share based on net income attributable to ordinary shareholders.

An additional minor similarity is that both APB 15 and IAS 33 require the computation of basic and diluted earnings per share using the new number of shares outstanding following a capitalisation issue, bonus issue, shares split, or reverse share split that occur after the balance sheet date but before the statements are issued. IAS 33 requires the disclosure of this adjustment in the notes to the financial statements. APB 15 does not share this requirement. Consequently, the adjustment is made without the knowledge of the user of American statements.

Furthermore, APB 15 mandates the use of the effective treasury stock method for rights offers containing a bonus element. IAS 33 does not share this requirement as it adjusts for the fair value of the theoretical ex-rights value. This difference is unlikely to be material unless the rights issue is for a substantial number of shares.

Accordingly, it is clear that the denominator in the calculation of earnings per share only differs on some theoretical grounds. The denominator is mostly representative of all shares and is adjusted for events that have changed the number of ordinary shares outstanding, without a corresponding change in resources. It is, furthermore, calculated as a weighted average to assist comparison. According to White et al. (1998:174-176), comparability of the numerator in the calculation (earnings) still requires detailed analysis, as seen from Table 4.1.1 above and the calculation of earnings per share.

Accounting attempts to calculate normalised earnings cannot be standardised, as these attempts are bound to fail, according to AC 306 (1995). Nonetheless, the South African Accounting Concept attempts to calculate some form of maintainable earnings by separating trading earnings from the profit and losses on capital items. AC 306 is based on a statement of the Institute of Investment Management and Research (in the United Kingdom) and it can serve as a standard to calculate some theoretical standardised form of earnings for the calculation of earnings per share. The accounting rule should be encouraged as it aims to facilitate the calculation of the identified required maintainable earnings used for investment analysis. The statement is, nonetheless, correct in its assumption that the calculation of maintainable future earnings requires some subjective intervention, and if this is left in the hands of the accountants representing these companies, it may not represent the truthful amounts required. Accordingly, this process must remain in the hands of the independent investment analysts.

A further requirement that most accounting standards currently fulfil, is the use of the clean surplus relationship (Nagorniak, 1985:13-15). The clean surplus relationship requires that all changes in book value, other than the transactions with shareholders, flow through the income statement. Nonetheless, exceptions exist for both United States Generally Accepted Accounting Practice and International Accounting Standards. Examples of these exceptions are the recognition of unrealised gains and losses on available-for-sale marketable securities in the United States, exchange rate gains and losses under the all-current method of foreign exchange conversion, and the revaluation of assets under International Accounting Standards.

Beaver et al. (1970:659-663) emphasise the importance of accounting data, and in particular earnings per share. This is consistent with the hypothesis that earnings per share reflect the underlying events that determine the differences in price between the shares of different companies. Further research by Beaver et al. (ibid.) suggests that investors react directly to new accounting data. The authors continued that accounting data could lead to an improvement in the valuation of companies. Moreover, accounting data might be the only available data, examples are companies that "go public" or multi-division companies that want to disinvest significant holdings. The price-earnings ratio can assist with the determination of the value of these companies or divisions.

Malkiel and Cragg (1970:606) identified the most valuable accounting proxies of risk from their empirical work to be the semi-deviation of earnings around the line of best fit of the earnings. Benesh and Peterson (1986:34) proved empirically that a strong direct relation exists between unexpected earnings changes and share price fluctuations. It is clear that accounting earnings play an important role in the valuation of shares. The release of earnings results and the calculations involved are manipulative and as a result companies utilise these weaknesses to temporarily influence their share prices. For the purposes of valuation the user of this information needs to guard against these actions in order to obtain a true and fair value.

Consequently, different companies apply accounting standards according to their own interpretation and in a manner that best presents their objectives. Furthermore, different countries tend to follow dissimilar accounting practices. The resultant effect complicates the comparison of earnings per share between separate companies and across borders. Moreover, noise is added to earnings per share by the inclusion of capital profits, discontinuing operations and other non-recurring transactions and decisions. Whenever the price-earnings ratio is used for the valuation of companies and to base investment decisions, the user must exercise great care to account for the items identified above that influence earnings per share in order to compare the entities on an equal footing.

4.1.2 Accounting earnings and cash flows for valuation

According to Page and Hooper (1979:50), any company symbolises the sum of the cash flows that represents its individual assets. Claims against these cash flows or the company exist in two forms. The first category is debt capital claims or creditors. Debt capital claims are promised specific cash flows at specified dates. The remaining claims against cash flows are equity capital claims. Equity capital claims are the residual claim to future cash flows after the satisfaction of all other claims.

The value of debt is equivalent to the present value of future cash outflows to debt suppliers, discounted at the appropriate discount rate (ibid.). Equity, on the other hand, is the present value of future cash flows to equity suppliers or the net value of the company as a whole after provision for debt.

According to Page and Hooper (ibid.), the difference between published accounting information and cash flow profits for valuation purposes is that accounting information contains some fictive equalisations. Accounting, for example, includes a mixture of real economic and paper transactions, grouping schemes, ambiguous titles, netting of transactions and splitting. In contrast, cash basis accounting would have a misleading and volatile effect on earnings. Accordingly, accrual accounting is required to present a fair picture of a company's activities. This fair picture is not entirely suitable for valuation purposes. As stated above, some adjustment is required to obtain comparative earnings in order to value companies on an equal footing by the use of the price-earnings ratio.

The primary difference between cash flows and accrual accounting is indicated in the fact that accrual accounting records transactions as they occur in substance rather than when they occur in form (ibid.). Accrual accounting further matches revenues and expenses in such a manner that costs are not recognised as an expense until the related revenue is recognised. The rationale is to address earnings in such a way that the advantages of accrual accounting are combined with the reality of cash flows. Normalised forward price-earnings ratios provide the user with this advantage, provided the normalisation of earnings is representative of reality. The advantage of this price-earnings ratio is shown in the fact that the volatility of cash flows is removed, matching occurs but that the reality of cash flow profits persists. Once-off profits and cash flows are also removed.

The primary differences between cash flows and accounting profits are that, in order for accounting to achieve its objective to match revenues with expenses and account for transactions as they occur in substance, accounting firstly allows the company to recognise expenses before suppliers are remitted in cash, when procurement occurs on a credit basis (ibid.). These expenses represent future cash flows and normally fall due within a short time-frame. This is an advantage of accrual accounting above cash flows, and an advantage normalised earnings per share possess above cash flow valuation. Similarly, some minor transactions are recognised before the physical cash flows, for example, interest accruals.

Furthermore, potential future losses are sometimes recognised as contingencies. These items require proper disclosure and can subjectively be excluded if not representative of normal operations. In order to present normalised earnings vague intangible accounting assets like goodwill can be excluded from charges against income.

The primary difficulty to overcome with the analysis of published financial statements is that the effects of accrual accounting are inextricably combined with more concrete transactions not subject to management judgment or allocation (ibid.). A solution to the problem lies in the segregation of the impact of each important transaction area on earnings and the analysis of these transactions to include the effects into or exclude the effects from normalised earnings.

It should be emphasised that the principal intention of accounting is not to project a single value for a company (Page & Hooper, 1979:55). The primary objective is to provide information to all the stakeholders of a company. Accounting ought to provide the investor with the information to value a company appropriately. If future cash flows were available with certainty the financial statement cash flows would be completely descriptive of the company for valuation purposes.

4.2 Other influences

4.2.1 Distribution policy

According to Miller and Modigliani (1961:421-432), three items affect the value of a company when the required rate of return is known. Firstly, distributions of the company or dividends, and alternatively, share re-purchases. Secondly, the market value of a company is affected by the current distribution whenever it reveals information previously unavailable regarding future distribution policy. This scenario applies only when the value of a company is obtained with reference to its future dividend policy. Refer to Chapter 3 regarding this popular approach. Lastly, current dividends affect the value of new shares sold, as a greater amount external capital is required to finance the desired projects.

Miller and Modigliani (ibid.), however, argued that in an ideal world the value of a company is determined solely by the earnings power of assets under its control and the effectiveness of its investment policy, and not the manner by which the earnings are structured for distribution. The problem with this ideal world of Miller and Modigliani (ibid.) lies in the fact that transaction cost raises the cost of external capital above the use of own capital, whenever equity financing is utilised.

As a consequence of transaction costs associated with both the distribution of dividends and the issue of new shares, an increase in current dividends will, provided the investment policy is known and capital is required, reduce the terminal value of existing shares (ibid.). The rationale is that part of the future dividend-stream that would otherwise belong to the existing shareholders must be relinquished to attract outside capital, from which, in effect, the increased current dividends are paid. Nonetheless, as stated, it should be emphasised that dividends are also a function of the availability of investment opportunities.

Whenever a company does not have the opportunity to invest in profitable projects, namely projects with a return that exceeds the company's cost of capital, excess funds should be returned to the shareholders. Consequently, dividends provide the market with powerful signals. According to Brickley (as cited in Born, Moser & Officer, 1988:56), companies that increase dividends by more than twenty percent on the previous annual amount show an increase in earnings at the time of the announcement. Furthermore, a sharp decline in growth follows in the subsequent year. Born et al. (1988:60-62) continued to note that companies that make alternative distributions, or non-cash distributions, may decline in value at the time of the announcement. Nonetheless, these companies continue or increase their growth rates. This provides support to the hypothesis of Miller and Modigliani.

Born et al. (ibid.) continued that the market is generally not capable of distinguishing between superior and inferior earnings performance solely on dividend policy. This provides evidence that a thorough analysis of sustainable earnings is required for valuation purposes. It was further noted that announcements of changes in dividend policy are associated with abnormal returns, suggesting that the change conveys some information about management expectations. Accordingly, it is apparent from the research by Born et al. (ibid.) that the signalling hypothesis does in fact exist and normal earnings estimations need to incorporate changes in dividend policy.

Miller and Modigliani (1961:430-431) also acknowledged the signalling hypothesis by stating that a change in dividend rate is often followed by a change in the market price of the company. They indicated that this is a reflection of what they termed the "informational content" of dividends. To explain the concept, whenever a company adopts a target payout ratio, investors have reason to interpret a change in dividends as a change in the company's future prospects. Management of the company can consequently use the payout ratio to manipulate the normal earnings expectations of investors and as a result the market value of the company.

In order to emphasise the importance of current dividends, Miller and Modigliani (1961:431-432) assumed the existence of some market imperfections. Market imperfections would lead investors to have a systematic preference for a unit of current dividends compared to that same unit of future capital gains, accumulated at the company's return on equity. As a consequence of market imperfections each corporation would attract a clientele of investors that prefer specific dividend policies. As a result, all companies would be valued fairly on their returns and not on their distribution preferences.

It is clear that the investing public incorporate a large range of different signals in their prediction of the normal earnings power of a company. The user of the price-earnings ratio has the advantage of deriving the normalised earnings from the ratio and comparing and contrasting that in order to evaluate investment opportunities. Furthermore, the price-earnings ratio is simple to apply to companies or divisions that do not have a current market value but where earnings per share and normalised earnings per share can be calculated.

4.2.2 Informational content of public earnings forecasts

Beaver et al. (as cited in Waymire, 1984:704) empirically showed that a positive relationship exists between unexpected additional earnings and returns on company shares during the same time. The authors found that earnings announcements are indeed associated with significant return deviations surrounding the date of the announcement.

Waymire (ibid.) also noted that a considerable positive association exists between the magnitude of forecast deviation, the difference between management forecast and expected earnings, and the extent of abnormal returns immediately surrounding the forecast disclosure date. The user of the price-earnings ratio has the advantage of comparing and contrasting earnings realised and forecasted by management before they are announced based on trend analysis of the price-earnings ratio in an attempt to profit from this observation.

Furthermore, it was noted that whenever management forecasts subdued earnings, a tendency exists that some additional counteracting good news is disclosed simultaneously (Waymire, 1984:714-716). This observation applied to approximately two-thirds of companies with bad news earnings announcements. Presumably, this is an attempt by management to reduce the impact of the bad news and create some bias in the prediction of future normal earnings. The normalised price-earnings ratio should assist the user to analyse the impact of the bad news and the goods news on earnings in an attempt to profit from market bias created by the management disclosure. This objective is reached by analysing the different price-earnings ratios from the different components of earnings.

Waymire's (ibid.) observations also apply to analyst's forecasts and revisions of these forecasts, according to research by Imhoff and Lobo (1984:549-553).

This observation persisted despite the potential confounding information released from both the company and other sources. The observation provides evidence that analysts act as substitutes for management regarding the release of information when information is not disclosed from management's side. Again thorough analysis of earnings with the use of the price-earnings ratio should assist the user to identify profit opportunities in advance. The price-earnings ratio possesses this advantage in that it relates prices to earnings, or alternatively, it is capable of comparing what is earned per unit of investment.

Waymire (1984:717) indicated that the results of his empirical research should not be generalised to companies that are not listed or followed by security analysts. As stated previously, the price-earnings ratio provides an alterative method of valuation for companies with divisions and companies that are not publicly traded.

4.2.3 The T-model

The aim of the T-model is to provide a bridge between accounting return and the return that the ownership of a share offers the investor (Born et al., 1988:35). With this model, it is possible to turn financial forecasts directly into expected investment return forecasts. The price-earnings ratio can be used, as described previously, to forecast earnings, and the T-model can be used for periodical return forecasts, since the price-earnings ratio does not possess the ability to forecast short-term returns in a dynamic market where companies tend to frequently reclassify between growth and value investments.

The total return for a period can be calculated with the use of Equation 4.2.1. The factors required by the equation are the return on equity over the period, the growth rate of equity and the price-to-book ratio.

$$T = g + \left(\text{Return on equity - g} \big/ \text{Price - to - book ratio} \right) + \left(\Delta\, \text{Price - to - book ratio} \big/ \text{Price - to - book ratio} \right)(1+g)$$

Equation 4.2.1 (Born et al., 1988:35)

The g symbol represents the growth in shareholders equity over the period. Return on equity is the normalised net income for the period divided by the beginning of the period shareholder's equity. The price-to-book value is calculated by the division of the price per share of the company by the book value per share at the start of the period. The delta price-to-book value is simply the beginning of the period value subtracted from the end value.

Born et al. (1988:35-36) achieved satisfying results with the equation when it was tested under the assumption of perfect hindsight. Accordingly, the results are assured when the user possesses the ability to accurately estimate the required parameters.

4.2.4 Financial statement analysis

The financial statements in general should support the prediction of normalised profits. Baruch and Thiagarajan (1993:190-215) made some general observations regarding trends apparent from the published financial statements that might indicate advantages or difficulties in generating profits.

According to Baruch and Thiagarajan (1993:192-194) inventory increases that exceed the increase in cost of sales are frequently considered a negative signal. The observation appears to indicate that the company is not capable of generating sufficient sales to reduce its stock level, or that production is in excess of sales and accordingly capital is not efficiently utilised within the company. Furthermore, it might indicate the inability of the company to market its products or co-ordinate production and sales efforts. Alternatively, it might, in some rare circumstances, indicate high expected future sales by management. The last alternative should be considered with caution.

Baruch and Thiagarajan (ibid.) maintained that disproportionate inventory increases might suggest the existence of slow-moving or obsolete items that will be revalued in the future. Such an inventory build-up increases current earnings at the expense of future earnings by absorbing overhead costs within a manufacturing environment. The opposite is also true. Inventory decreases appear to indicate higher than expected sales and a decrease in the overhead cost absorption in inventory. Consequently, the situation would lower overhead cost accumulation in inventory and diminish the profit effects of the increased sales. As a conclusion, the market generally views inventory increases as unconstructive.

Analysts normally see a disproportionate increase in accounts receivable as a signal that conveys detrimental information (ibid.). It may suggest difficulty in selling the company's products, triggering credit extensions, or an increase in the likelihood of future reductions in earnings as a result of provision for doubtful receivables.

Furthermore, relative decreases in capital expenditure and in research and development are perceived as negative signals (Baruch & Thiagarajan, 1993:195). These expenses are largely discretionary, so a reduction is suspect. It appears to convey the idea that management is concerned with the ability of the company to generate sufficient profit and its associated cash flows to sustain previous levels of investment and growth. In addition, this might indicate an attempt by management to boost earnings through a reduction of capital investment, and not an increase in sales or decrease in running expenses. The attempt to increase profitability should correspond with the company's apparent generic strategy. On the other hand, an increase in capital expenditure is normally seen to convey good news about future profitability.

A disproportionate decrease in the sales margin normally conveys the information that the company faces elastic demand for its products and can not pass increases in input prices to the customer (ibid.). Therefore, the company will face future difficulty in retaining its profitability. Gross profit is a less volatile indicator of future profitability than net profit. As a result, a price-earnings ratio can be calculated on the gross margin for comparative purposes and trend analysis between similar companies. This price-gross profit ratio will also indicate a company's ability to control expenditure when compared to the price-earnings ratio.

Selling and administrative expenses are mostly fixed; a disproportionate increase in these expenses is considered as a suggestion that management has lost control over these expenses (Baruch & Thiagarajan, 1993:196-197). On the other hand, it can indicate an unusual sales effort and increased future profitability.

An unusual increase in the effective tax rate is generally considered a negative signal when no legal changes were enforced on the company (ibid.). An increase in the effective tax rate indicates a decrease in normalised earnings. It might also indicate the inability of the company to incorporate effective tax structures in its transactions that would hamper profitability over time.

Lastly, according to Baruch and Thiagarajan (1993:197-198), financial analysts view restructuring as a positive short-term sign. It is usually associated with a reduction in the labour force that reduces the salaries and wages expense, and consequently increases profits. The effectiveness of employees is usually defined in terms of sales per employee as it captures the change in the efficiency of labour and accounts for changes in the work force. Therefore, all actions that increase the sales per employee are generally regarded as good news.

Baruch and Thiagarajan (1993:214) validated the hypothesis that investors use fundamentals to assess the extent of earnings persistency and growth, as all these indicators described above turned out to be statistically significant. Accordingly, the effective evaluation and calculation for normalised profit for the calculation of a normalised forward price-earnings ratio holds a key to profitable investing with the use of the price-earnings ratio.

4.3 Conclusion

The earnings and accounting aspects of the price-earnings ratio were discussed in this chapter. Earnings per share, the denominator in the calculation, represent a central component of the price-earnings ratio irrespective of the format of the price-earnings ratio calculated.

In order to address this important aspect, the emphasis was firstly on the calculation of earnings under different accounting systems. The calculation of earnings possesses the ability to influence earnings per share as it represents the numerator in this calculation. Furthermore, it is apparent that companies are in a position to manipulate their earnings to some extent. The issue is complicated by the different sets of accounting rules in force around the world.

Secondly, the attention turned to the calculation of earnings per share and the weighted number of shares used as the denominator for calculating earnings per share. As with the calculation of earnings, different accounting rules stipulate different methods for the calculation of the weighted number of shares. These methods were highlighted in order to compare earnings per share on equal footing all over the world.

The difference between earnings per share and cash flows for normal valuation purposes was then clarified. An attempt was then made to clarify the relationship between distributions from the company and the influence this exerts on the share price. In theory, the investor should be indifferent to capital gains or dividends, and shares should be valued on an equal footing. However, this only applies in the long term, as other factors appear to influence the short term. The influence of forecasts available to the general investing public and its effect on the return on investments and the calculation of predicted earnings were then investigated. An attempt to explain the relationship between returns on shares with the profits of the company followed. To conclude the effect and meaning of some market interpretations of trends observed from financial information closed the discussion on earnings per share and the calculation of projected numbers.

This chapter covered the last aspects of the influence and effect of fundamental factors on the price-earnings ratio since chapter 2 dealt with the effects and associations of risk and return and chapter 3 with valuation models. A general conclusion will follow in chapter 5.

Chapter 5: Conclusion

This chapter intends to succinctly summarise the previous chapters. Secondly, the goal is to interpret the information obtained throughout the study in order to identify the primary observations and conclusions made from the literature study. Lastly, some recommendations and areas for further study will follow.

5.1 Summary

The study followed the process normally associated with the valuation of investments. Accordingly, the first objective was to calculate and evaluate the required rate of return and to compare and contrast this with information obtained from the various calculation methods of the price-earnings ratio. The second step was to select the appropriate valuation model, and to compare and contrast the models with public price-earnings ratio information and evaluate these results for investment purposes. The last step, covered in chapter 4, was to calculate normalised earnings per share and other earnings forecasts, as the results of these calculations are included as input in the valuation models.

According to Damodaran (1996:20), the question about the relationship between risk and return is fundamental to every investment decision and consequently, forms the base of fundamental analysis and valuation. Chapter 2 identified a variety of alternative methods to recognise the investments that would satisfy the investor's requirements. It then continued to indicate the existence of relationships between these methods and the price-earnings ratio.

The objectives for a good risk and return model followed, as identified by Damodaran (1996:20-21). These objectives were used to evaluate the methods, and the indication was that the price-earnings ratio possesses the ability to satisfy the requirements set out by Damodaran, specifically if used in conjunction with the capital asset pricing model. Furthermore, it was clear that the capital asset pricing model also satisfied most of Damodaran's requirements. Nevertheless, in order to evaluate the capital asset pricing model and the arbitrage pricing theory, the Markowitz portfolio theory was included as an introduction to these two models.

A discussion about factor models followed the examination of the capital asset pricing model and the arbitrage pricing theory. The objective of factor models is to analyse the co-movement of different variables. These models are not only used as indicators of risk and return, but also as valuation models. Factor models proved to be immensely popular amongst academics as a tool to evaluate the relationships that exists between the different economic variables that influence share prices. Examples include Brickley, as cited in Born et al. (1988:56-62), in the discussion of dividend policy (section 3.1.3) and Fama and French (as cited in Reilly & Brown, 1997:217) regarding the dividend discount model. Nevertheless, according to Sharpe (1984:21), the objective of factor models is to describe the return generating process. These models generally do not comply with Damodaran's (1996:20-21) recommendations for good risk-return models as they apply only to very specific areas and are not useful enough for general application, such as the capital asset pricing model.

Stochastic dominance rules were discussed as a decisive ex-post test to indicate that the aforementioned unbiased tests exist. Furthermore, it indicated that the price-earnings ratio is in fact a valuable investment tool, and to reject theories which indicate that shares with modest price-earnings ratios outperform relative higher priced investments only on a pre-tax basis, as suggested by Reilly and Brown (1997:306-322).

A discussion on interest rates concluded the chapter about risk and return as the structure of interest rates is an essential element of the analysis of returns, and the calculation of expected returns. Changes in the interest rate structure help the growth investor to identify reasonable growth rates.

The first section covered in chapter 3 involved discount models and covered the dividend discount model in its basic form as the most basic valuation model. Valuation models assist the investor to identify whether assets are undervalued or overvalued, and to allocate funds available for investment accordingly. Nonetheless, the usefulness of the dividend discount model is limited to the projected figures available to the investor. The relationship of the dividend discount model to the price-earnings ratio indicated the close correlation that exists between valuation models in general, and how these relationships assist with the evaluation of companies for investment purposes. Nevertheless, the model forms the basis for the alternatives that followed.

The first of these is the well-known Gordon model. According to Leibowitz and Kogelman (1994:8), the Gordon model is a predominantly convenient choice for researchers, as they possess the tendency to envisage growth in a simplistic manner, proceeding smoothly at a constant rate. As indicated at the end of chapter 2, this model's primary disadvantage is the projection of the long run growth rate. Empirical tests by Malkiel and Cragg (1970:602) confirmed that the model appears to be reasonably relevant to the market for mature and stable industries. The relationship of this model to the price-earnings ratio and the implications of empirical tests of this model on the price-earnings ratio were also examined.

As a result of the limitations placed on growth patterns by the Gordon model, the two-stage model, the H-model, and the three-stage model were also discussed. As with other models, all three models have their own their advantages and disadvantages. The relationships of the price-earnings ratio to these models were also discussed. The three-stage model is the most appropriate of the dividend discount models for the valuation of the shares of growth companies, as it allows time for an extraordinary initial growth period, followed by a declining growth phase and ending with a sustainable growth rate that continues indefinitely.

The dividend discount models were followed by a discussion of the franchise value valuation approach. This approach aims to identify growth opportunities and then proceeds to place a value on these opportunities. In theory, it is closely related to the goals of the price-earnings ratio, as the price-earnings ratio also aims at providing information about future growth opportunities. These approaches were contrasted and the changes associated with different opportunities and different growth rates were also examined. The effects of leverage and different growth patterns on the price-earnings ratio were also investigated. To conclude the discussion about valuation models, the duration of extraordinary growth was investigated, as it was indicated in Chapter 2 that investors tend to value growth too highly. Furthermore, it was shown that the price-earnings ratio possesses the ability to highlight this very important aspect of valuation for reasonableness, specifically if used in conjunction with the franchise value valuation approach.

To conclude, the calculation of earnings and earnings per share was examined, as the earnings calculations based on different accounting rules tend to inhibit comparability. Furthermore, because of the fact that companies are capable of manipulating accounting earnings and that earnings per share calculations tend to be volatile, it was suggested that some form of normalised earnings be used for valuation purposes. Differences between profits and cash flows were highlighted, and the effect of distribution policies on share prices discussed. Finally, the association between returns per share and accounting profits received attention, as well as the market's perception of certain trends in the financial statements.

5.2 Interpretation

According to Damodaran (1996:291), there are numerous reasons for the widespread use and popularity of the price-earnings ratio. Firstly, price-earnings ratios are appealing as they relate the price paid to earnings, seen as a proxy for cash flows that represent returns. Secondly, they are easy to calculate and are widely available, enabling the user to make comparisons across shares effortlessly. Thirdly, they can be an indicator of other characteristics that include risk, growth and different accounting techniques (Beaver & Morse, 1978:65).

The observation by Beaver and Morse (1978:65-73) that the price-earnings ratio is an indicator of risk was highlighted in chapter 2. The ability of the price-earnings ratio to elucidate some observations of empirical studies and to relate these observations to the models and theories discussed, clearly illustrated that a relationship exists between the price-earnings ratio and risk and return models and theories. It was also shown that the price-earnings ratio is in fact a simple valuation model as it relates the price paid for an investment to the earnings that investment generates, as observed by Damodaran (1996:291) and shown in chapter 3. The association of the price-earnings ratio with the franchise value approach indicated the magnitude of investment and profitability required to improve the price-earnings ratio. Investors, such as technology investors in the United States, appear to neglect this important area, and paid a significant price for this mistake as a result. This observation is evident from the abrupt decline in average share prices of technology companies in the United States, referred to as the bursting technology stock bubble.

The price-earnings ratio was also able to explain empirical observations about the efficient market hypothesis and the effect of market liquidity. This was apparent from the observations by Basu (as cited in Reilly & Brown, 1997:314-322); Peavy and Goodman (as cited in ibid.); and Arbel et al. (as cited in Shefrin & Statman, 1995:31-33) and clarified in chapter 2. Arbel et al. (as cited in ibid.) suggested that small companies are omitted from analysis by analysts and, when recognised, achieve significant returns on a risk-adjusted basis. It is apparent that shares of smaller companies tend to be omitted by analysts, and that as a result of thin market liquidity associated with these companies, market reactions significantly influence the returns earned on the shares of these companies when analysts issue buy recommendations on them.

In chapter 4 it was shown how valuable the price-earnings ratio can be with the evaluation of earnings and earnings consistency through the use of forward and normalised price-earnings ratios. Accounting earnings and earrings per share tend to include a fair amount of volatility over time. Consequently, accounting results have a propensity to add some noise to actual sustainable earnings and these items are usually not incorporated in share prices as share prices represent future profitability. Moreover, the market regards information about future earnings as very important, as indicated in chapter 4 by the market's reactions toward news releases.

It is apparent that the price-earnings ratio possesses the capabilities to assist investors significantly with the analysis of investment opportunities, especially if combined with the related models and theories discussed. The objective of this study was to determine the relationship between standard risk and valuation models and the price-earnings ratio, as well as the relationship between earnings and the price-earnings ratio, in order to determine the consensus market information the ratio can provide. The ability of the price-earnings ratio to convey consensus information about companies was illustrated throughout the study.

It was shown that the price-earnings ratio provides its user with a magnitude of important and relevant information and assists with the interpretation of this information. Some common pitfalls that are associated with the use of the price-earnings ratio by uninformed users also received attention. These include the tendency of the ratio, if calculated from market data, not to indicate the existence of overvalued markets, as discussed in Chapter 2. The effect of market perceptions and accounting issues was also addressed.

5.3 Recommendations

This study provided some conclusions, but also identified some issues that remain unanswered and which are beyond the scope of this study. A succinct summary of these items concludes this literature study.

This study provided insight into the workings and fundamental factors that influence price-earnings ratios of a company and indicated the changes that occur as companies pass through different growth phases. The study did not investigate the effectiveness of the formulae provided empirically on a share market in order to determine whether they can provide information that yield superior results. It is recommended that all the equations be tested empirically with different parameter limits, in order to determine the effectiveness of the information provided by these equations and to establish whether the additional effort required is worth the additional returns earned. The fundamentals derived from the analysis of the price-earnings ratio assisted in confirming results of some previous empirical studies.

The research on the price-earnings ratio and the franchise value approach by Leibowitz and Kogelman (1994) did not provide any empirical proof about the effectiveness and applicability of the approach. This study also did not aim to provide information in this regard, as it had a different objective. The recommendation is, therefore, that some empirical research be carried out in this regard. The application of the franchise value approach requires a fair amount of subjective input and the calculation of abstract values. These calculations appear to only apply in an environment of perfect knowledge about the future expectations of companies.

Chapter 4 also raised some unanswered questions about the comparability of earnings and earnings per share. It is the intention of the international accounting harmonisation project to assist with the comparison and convergence of financial statements or bulletins. Nevertheless, the introduction of new accounting guidelines is not without its limitations. New guidelines normally require a substantial grace period before acceptance, and when finally approved, require specific procedures and time-frames to comply with the rules. The application of these rules is further influenced by conflicting legal and regulatory requirements that exist throughout the world. Additional research in the area of comparability and the effects of the application of these guidelines will certainly assist users in a better understanding of their divergent impact. The chapter also indicated that the calculation of a form of sustainable normal earnings is not standardised and comparable between analysts.

Chapter 4 indicated that accounting earnings are, as a result of different interpretations of the accounting guidelines, very volatile. One of the aims of modern accounting is to match income with expenditure. The concept of matching and the deferral of income and expenses appear to add a significant element of noise or bias to earnings and earnings per share. Research in this area would assist in the interpretation of financial statements and the calculation of sustainable earnings.

Lastly, this study does not explain the macro-economic factors that produce the different discount and growth rates experienced in different countries, as this is largely an economic question. Nevertheless, these factors are dynamic and previous research produced regression equations to compare cross-border price-earnings ratios (Damodaran, 1996:299-308). The question about these divergent return, growth and risk factors remain relevant and an answer that links these factors to the economic theories on an all-inclusive basis has yet to be found.

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Annexure A

Figure 1.1.1 (Price-earnings ratios) ("Markets & Data: Closer looks" Economist.com, 14 October 2000)

Profitability Price-earnings ratios		
	1999	**2000**
Austria	21	19
Belgium	20	18
Britain	23	24
Canada	28	28
Denmark	22	34
France	30	33
Germany	25	28
Italy	28	30
Japan	0	88.2
Nederland	29	19
Spain	22	26
Sweden	29	25
Switzerland	24	20
United States	28	29
World index	31	29

Figure 3.1.3

Change in earnings per share: companies ranked by price-earnings ratio			
Time	High	Medium	Low
0	7%	12%	18%
1	27%	11%	6%
2	19%	11%	10%
3	17%	11%	11%
4	15%	11%	12%
5	14%	12%	12%

Figure 3.2.1

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Characteristics										
Book equity (beginning)	100.00	108.00	116.64	125.97	136.05	146.93	158.69	171.38	185.09	199.90
Return on equity	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Earnings	12.00	12.96	14.00	15.12	16.33	17.63	19.04	20.57	22.21	23.99
Payout ratio	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
Dividend	4.00	4.32	4.67	5.04	5.44	5.88	6.35	6.86	7.40	8.00
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Dividend discount model price	100.00	108.00	116.64	125.97	136.05	146.93	158.69	171.38	185.09	199.90
Dividend yield	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%
Growth rate	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
Price-earnings ratio	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Price gain	8.00	8.64	9.33	10.08	10.88	11.75	12.69	13.71	14.81	15.99
Reinvestment gain		0.48	1.06	1.74	2.56	3.52	4.64	5.96	7.50	9.29
Total gain	12.00	13.44	15.05	16.86	18.88	21.15	23.69	26.53	29.71	33.28
No growth										
Dividend (0% growth)	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Reinvestment gain (0% growth)		1.44	3.05	4.86	6.88	9.15	11.69	14.53	17.71	21.28
Percentages										
Dividend	4.00%	4.32%	4.67%	5.04%	5.44%	5.88%	6.35%	6.86%	7.40%	8.00%
Price gain	8.00%	8.64%	9.33%	10.08%	10.88%	11.75%	12.69%	13.71%	14.81%	15.99%
Reinvestment gain	0.00%	0.48%	1.06%	1.74%	2.56%	3.52%	4.64%	5.96%	7.50%	9.29%

Figure 3.2.1 (continued)

	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17	Year 18	Year 19	Year 20
Characteristics										
Book equity (beginning)	215.89	233.16	251.82	271.96	293.72	317.22	342.59	370.00	399.60	431.57
Return on equity	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Earnings	25.91	27.98	30.22	32.64	35.25	38.07	41.11	44.40	47.95	51.79
Payout ratio	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
Dividend	8.64	9.33	10.07	10.88	11.75	12.69	13.70	14.80	15.98	17.26
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Dividend discount model price	215.89	233.16	251.82	271.96	293.72	317.22	342.59	370.00	399.60	431.57
Dividend yield	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%
Growth rate	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
Price-earnings ratio	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Price gain	17.27	18.65	20.15	21.76	23.50	25.38	27.41	29.60	31.97	34.53
Reinvestment gain	11.36	13.76	16.53	19.73	23.40	27.62	32.45	37.99	44.33	51.56
Total gain	37.27	41.74	46.75	52.36	58.65	65.68	73.56	82.39	92.28	103.35
No growth										
Dividend (0% growth)	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Reinvestment gain (0% growth)	25.27	29.74	34.75	40.36	46.65	53.68	61.56	70.39	80.28	91.35
Percentages										
Dividend	8.64%	9.33%	10.07%	10.88%	11.75%	12.69%	13.70%	14.80%	15.98%	17.26%
Price gain	17.27%	18.65%	20.15%	21.76%	23.50%	25.38%	27.41%	29.60%	31.97%	34.53%
Reinvestment gain	11.36%	13.76%	16.53%	19.73%	23.40%	27.62%	32.45%	37.99%	44.33%	51.56%

Figure 3.2.1 (continued)

	Year 21	Year 22	Year 23	Year 24	Year 25	Year 26	Year 27	Year 28	Year 29	Year 30
Characteristics										
Book equity (beginning)	466.10	503.38	543.65	587.15	634.12	684.85	739.64	798.81	862.71	931.73
Return on equity	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Earnings	55.93	60.41	65.24	70.46	76.09	82.18	88.76	95.86	103.53	111.81
Payout ratio	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
Dividend	18.64	20.14	21.75	23.49	25.36	27.39	29.59	31.95	34.51	37.27
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Dividend discount model price	466.10	503.38	543.65	587.15	634.12	684.85	739.64	798.81	862.71	931.73
Dividend yield	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%
Growth rate	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%	8.00%
Price-earnings ratio	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Price gain	37.29	40.27	43.49	46.97	50.73	54.79	59.17	63.90	69.02	74.54
Reinvestment gain	59.82	69.24	79.97	92.17	106.05	121.82	139.72	160.04	183.08	209.19
Total gain	115.76	129.65	145.20	162.63	182.14	204.00	228.48	255.90	286.61	321.00
No growth										
Dividend (0% growth)	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Reinvestment gain (0% growth)	103.76	117.65	133.20	150.63	170.14	192.00	216.48	243.90	274.61	309.00
Percentages										
Dividend	18.64%	20.14%	21.75%	23.49%	25.36%	27.39%	29.59%	31.95%	34.51%	37.27%
Price gain	37.29%	40.27%	43.49%	46.97%	50.73%	54.79%	59.17%	63.90%	69.02%	74.54%
Reinvestment gain	59.82%	69.24%	79.97%	92.17%	106.05%	121.82%	139.72%	160.04%	183.08%	209.19%

Figure 3.2.2

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Characteristics										
Book equity (beginning)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Return on equity	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Earnings	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Payout ratio	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Dividend	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Gordon constant growth dividend discount model price (beginning)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Dividend yield	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Growth rate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Price-earnings ratio	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Price gain	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Reinvestment gain		1.44	3.05	4.86	6.88	9.15	11.69	14.53	17.71	21.28
Total gain	12.00	13.44	15.05	16.86	18.88	21.15	23.69	26.53	29.71	33.28
Percentages										
Dividend	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Price gain	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Reinvestment gains	0.00%	1.44%	3.05%	4.86%	6.88%	9.15%	11.69%	14.53%	17.71%	21.28%

Figure 3.2.2 (continued)

	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17	Year 18	Year 19	Year 20
Characteristics										
Book equity (beginning)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Return on equity	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Earnings	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Payout ratio	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Dividend	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Gordon constant growth dividend discount model price (beginning)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Dividend yield	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Growth rate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Price-earnings ratio	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Price gain	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Reinvestment gain	25.27	29.74	34.75	40.36	46.65	53.68	61.56	70.39	80.28	91.35
Total gain	37.27	41.74	46.75	52.36	58.65	65.68	73.56	82.39	92.28	103.35
Percentages										
Dividend	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Price gain	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Reinvestment gains	25.27%	29.74%	34.75%	40.36%	46.65%	53.68%	61.56%	70.39%	80.28%	91.35%

Figure 3.2.2 (continued)

	Year 21	Year 22	Year 23	Year 24	Year 25	Year 26	Year 27	Year 28	Year 29	Year 30
Characteristics										
Book equity (beginning)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Return on equity	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Earnings	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Payout ratio	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Dividend	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Gordon constant growth dividend discount model price (beginning)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Dividend yield	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Growth rate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Price-earnings ratio	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Price gain	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Reinvestment gain	103.76	117.65	133.20	150.63	170.14	192.00	216.48	243.90	274.61	309.00
Total gain	115.76	129.65	145.20	162.63	182.14	204.00	228.48	255.90	286.61	321.00
Percentages										
Dividend	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Price gain	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Reinvestment gains	103.76%	117.65%	133.20%	150.63%	170.14%	192.00%	216.48%	243.90%	274.61%	309.00%

Figure 3.2.3

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Characteristics										
Book equity (beginning)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Return on equity	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Earnings	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Payout ratio	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Dividend	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Gordon constant growth dividend discount model price (beginning)	125.00	125.00	125.00	125.00	125.00	125.00	125.00	125.00	125.00	125.00
Dividend yield	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Growth rate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Price-earnings ratio	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Price gain	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Reinvestment gain		1.80	3.82	6.07	8.60	11.44	14.61	18.16	22.14	26.60
Total gain	15.00	16.80	18.82	21.07	23.60	26.44	29.61	33.16	37.14	41.60
Percentages										
Dividend	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Price gain	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Reinvestment gains	0.00%	1.44%	3.05%	4.86%	6.88%	9.15%	11.69%	14.53%	17.71%	21.28%

Figure 3.2.3 (continued)

	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17	Year 18	Year 19	Year 20
Characteristics										
Book equity (beginning)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Return on equity	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Earnings	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Payout ratio	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Dividend	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Gordon constant growth dividend discount model price (beginning)	125.00	125.00	125.00	125.00	125.00	125.00	125.00	125.00	125.00	125.00
Dividend yield	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Growth rate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Price-earnings ratio	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Price gain	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Reinvestment gain	31.59	37.18	43.44	50.45	58.31	67.10	76.96	87.99	100.35	114.19
Total gain	46.59	52.18	58.44	65.45	73.31	82.10	91.96	102.99	115.35	129.19
Percentages										
Dividend	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Price gain	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Reinvestment gains	25.27%	29.74%	34.75%	40.36%	46.65%	53.68%	61.56%	70.39%	80.28%	91.35%

Figure 3.2.3 (continued)

	Year 21	Year 22	Year 23	Year 24	Year 25	Year 26	Year 27	Year 28	Year 29	Year 30
Characteristics										
Book equity (beginning)	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Return on equity	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Earnings	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Payout ratio	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Dividend	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Gordon constant growth dividend discount model price (beginning)	125.00	125.00	125.00	125.00	125.00	125.00	125.00	125.00	125.00	125.00
Dividend yield	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Growth rate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Price-earnings ratio	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Price gain	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Reinvestment gain	129.69	147.06	166.50	188.29	212.68	240.00	270.60	304.87	343.26	386.25
Total gain	144.69	162.06	181.50	203.29	227.68	255.00	285.60	319.87	358.26	401.25
Percentages										
Dividend	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Price gain	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Reinvestment gains	103.76%	117.65%	133.20%	150.63%	170.14%	192.00%	216.48%	243.90%	274.61%	309.00%

Figure 3.2.4

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
Characteristics										
Book equity (beginning)	100.00	110.00	121.00	133.10	146.41	161.05	177.16	194.87	214.36	235.79
Return on equity	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Earnings	15.00	16.50	18.15	19.97	21.96	24.16	26.57	29.23	32.15	35.37
Payout ratio	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
Dividend	5.00	5.50	6.05	6.65	7.32	8.05	8.86	9.74	10.72	11.79
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Gordon constant growth dividend discount model price (beginning)	250.00	275.00	302.50	332.75	366.03	402.63	442.89	487.18	535.90	589.49
Dividend yield	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%
Growth rate	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Price-earnings ratio	16.67	16.67	16.67	16.67	16.67	16.67	16.67	16.67	16.67	16.67
Price gain	25.00	27.50	30.25	33.28	36.60	40.26	44.29	48.72	53.59	58.95
Reinvestment gain		0.60	1.33	2.22	3.28	4.55	6.07	7.86	9.97	12.45
Total gain	30.00	33.60	37.63	42.15	47.21	52.87	59.21	66.32	74.28	83.19
Percentages										
Dividend	2.00%	2.20%	2.42%	2.66%	2.93%	3.22%	3.54%	3.90%	4.29%	4.72%
Price gain	10.00%	11.00%	12.10%	13.31%	14.64%	16.11%	17.72%	19.49%	21.44%	23.58%
Reinvestment gains	0.00%	0.24%	0.53%	0.89%	1.31%	1.82%	2.43%	3.14%	3.99%	4.98%

Figure 3.2.4 (continued)

	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17	Year 18	Year 19	Year 20
Characteristics										
Book equity (beginning)	259.37	285.31	313.84	345.23	379.75	417.72	459.50	505.45	555.99	611.59
Return on equity	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Earnings	38.91	42.80	47.08	51.78	56.96	62.66	68.92	75.82	83.40	91.74
Payout ratio	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
Dividend	12.97	14.27	15.69	17.26	18.99	20.89	22.97	25.27	27.80	30.58
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Gordon constant growth dividend discount model price (beginning)	648.44	713.28	784.61	863.07	949.37	1,044.31	1,148.74	1,263.62	1,389.98	1,528.98
Dividend yield	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%
Growth rate	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Price-earnings ratio	16.67	16.67	16.67	16.67	16.67	16.67	16.67	16.67	16.67	16.67
Price gain	64.84	71.33	78.46	86.31	94.94	104.43	114.87	126.36	139.00	152.90
Reinvestment gain	15.36	18.76	22.73	27.34	32.69	38.89	46.06	54.35	63.90	74.91
Total gain	93.18	104.36	116.88	130.90	146.61	164.21	183.91	205.98	230.70	258.38
Percentages										
Dividend	5.19%	5.71%	6.28%	6.90%	7.59%	8.35%	9.19%	10.11%	11.12%	12.23%
Price gain	25.94%	28.53%	31.38%	34.52%	37.97%	41.77%	45.95%	50.54%	55.60%	61.16%
Reinvestment gains	6.15%	7.51%	9.09%	10.93%	13.08%	15.56%	18.43%	21.74%	25.56%	29.96%

Figure 3.2.4 (continued)

	Year 21	Year 22	Year 23	Year 24	Year 25	Year 26	Year 27	Year 28	Year 29	Year 30
Characteristics										
Book equity (beginning)	672.75	740.02	814.03	895.43	984.97	1,083.47	1,191.82	1,311.00	1,442.10	1,586.31
Return on equity	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Earnings	100.91	111.00	122.10	134.31	147.75	162.52	178.77	196.65	216.31	237.95
Payout ratio	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
Dividend	33.64	37.00	40.70	44.77	49.25	54.17	59.59	65.55	72.10	79.32
Market rate	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Gordon constant growth dividend discount model price (beginning)	1,681.87	1,850.06	2,035.07	2,238.58	2,462.43	2,708.68	2,979.54	3,277.50	3,605.25	3,965.77
Dividend yield	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%	2.00%
Growth rate	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
Price-earnings ratio	16.67	16.67	16.67	16.67	16.67	16.67	16.67	16.67	16.67	16.67
Price gain	168.19	185.01	203.51	223.86	246.24	270.87	297.95	327.75	360.52	396.58
Reinvestment gain	87.56	102.11	118.80	137.94	159.87	184.96	213.66	246.45	283.89	326.61
Total gain	289.39	324.12	363.01	406.57	455.36	510.00	571.20	639.75	716.52	802.50
Percentages										
Dividend	13.45%	14.80%	16.28%	17.91%	19.70%	21.67%	23.84%	26.22%	28.84%	31.73%
Price gain	67.27%	74.00%	81.40%	89.54%	98.50%	108.35%	119.18%	131.10%	144.21%	158.63%
Reinvestment gains	35.03%	40.84%	47.52%	55.18%	63.95%	73.98%	85.46%	98.58%	113.55%	130.64%

Figure 3.6.2

Franchise factor for a k of 12 percent							
k	FF	k	FF	k	FF	k	FF
12.00%	0.0000	16.80%	2.3810	21.80%	3.7462	26.80%	4.6020
12.10%	0.0689	16.90%	2.4162	21.90%	3.7671	26.90%	4.6159
12.20%	0.1366	17.00%	2.4510	22.00%	3.7879	27.00%	4.6296
12.30%	0.2033	17.10%	2.4854	22.10%	3.8084	27.10%	4.6433
12.40%	0.2688	17.20%	2.5194	22.20%	3.8288	27.20%	4.6569
12.50%	0.3333	17.30%	2.5530	22.30%	3.8490	27.30%	4.6703
12.60%	0.3968	17.40%	2.5862	22.40%	3.8690	27.40%	4.6837
12.70%	0.4593	17.50%	2.6190	22.50%	3.8889	27.50%	4.6970
12.80%	0.5208	17.60%	2.6515	22.60%	3.9086	27.60%	4.7101
12.90%	0.5814	17.70%	2.6836	22.70%	3.9280	27.70%	4.7232
13.00%	0.6410	17.80%	2.7154	22.80%	3.9474	27.80%	4.7362
13.10%	0.6997	17.90%	2.7467	22.90%	3.9665	27.90%	4.7491
13.20%	0.7576	18.00%	2.7778	23.00%	3.9855	28.00%	4.7619
13.30%	0.8145	18.10%	2.8085	23.10%	4.0043	28.10%	4.7746
13.40%	0.8706	18.20%	2.8388	23.20%	4.0230	28.20%	4.7872
13.50%	0.9259	18.30%	2.8689	23.30%	4.0415	28.30%	4.7998
13.60%	0.9804	18.40%	2.8986	23.40%	4.0598	28.40%	4.8122
13.70%	1.0341	18.50%	2.9279	23.50%	4.0780	28.50%	4.8246
13.80%	1.0870	18.60%	2.9570	23.60%	4.0960	28.60%	4.8368
13.90%	1.1391	18.70%	2.9857	23.70%	4.1139	28.70%	4.8490
14.00%	1.1905	18.80%	3.0142	23.80%	4.1317	28.80%	4.8611
14.10%	1.2411	18.90%	3.0423	23.90%	4.1492	28.90%	4.8731
14.20%	1.2911	19.00%	3.0702	24.00%	4.1667	29.00%	4.8851
14.30%	1.3403	19.10%	3.0977	24.10%	4.1840	29.10%	4.8969
14.40%	1.3889	19.20%	3.1250	24.20%	4.2011	29.20%	4.9087
14.50%	1.4368	19.30%	3.1520	24.30%	4.2181	29.30%	4.9204
14.60%	1.4840	19.40%	3.1787	24.40%	4.2350	29.40%	4.9320
14.70%	1.5306	19.50%	3.2051	24.50%	4.2517	29.50%	4.9435
14.80%	1.5766	19.60%	3.2313	24.60%	4.2683	29.60%	4.9550
14.90%	1.6219	19.70%	3.2572	24.70%	4.2848	29.70%	4.9663
15.00%	1.6667	19.80%	3.2828	24.80%	4.3011	29.80%	4.9776
15.10%	1.7108	19.90%	3.3082	24.90%	4.3173	29.90%	4.9889
15.20%	1.7544	20.00%	3.3333	25.00%	4.3333	30.00%	5.0000
15.30%	1.7974	20.10%	3.3582	25.10%	4.3493		
15.40%	1.8398	20.20%	3.3828	25.20%	4.3651		
15.50%	1.8817	20.30%	3.4072	25.30%	4.3808		
15.60%	1.9231	20.40%	3.4314	25.40%	4.3963		
15.70%	1.9639	20.50%	3.4553	25.50%	4.4118		
15.80%	2.0042	20.60%	3.4790	25.60%	4.4271		
15.90%	2.0440	20.70%	3.5024	25.70%	4.4423		
16.00%	2.0833	20.80%	3.5256	25.80%	4.4574		
16.10%	2.1222	20.90%	3.5486	25.90%	4.4723		
16.20%	2.1605	21.00%	3.5714	26.00%	4.4872		
16.30%	2.1984	21.10%	3.5940	26.10%	4.5019		
16.40%	2.2358	21.20%	3.6164	26.20%	4.5165		
16.50%	2.2727	21.30%	3.6385	26.30%	4.5311		
16.60%	2.3092	21.40%	3.6604	26.40%	4.5455		
16.70%	2.3453	21.50%	3.6822	26.50%	4.5597		
		21.60%	3.7037	26.60%	4.5739		
		21.70%	3.7250	26.70%	4.5880		

Annexure A

Figure 3.6.3

Perpetual return	Franchise factor
9.00%	-1.67
9.50%	-1.39
10.00%	-1.11
10.50%	-0.83
11.00%	-0.56
11.50%	-0.28
12.00%	0.00
12.50%	0.28
13.00%	0.56
13.50%	0.83
14.00%	1.11
14.50%	1.39
15.00%	1.67
15.50%	1.94
16.00%	2.22
16.50%	2.50
17.00%	2.78
17.50%	3.06
18.00%	3.33
18.50%	3.61
19.00%	3.89
19.50%	4.17
20.00%	4.44

Figure 3.6.4

Growth equivalent	Perpetual equivalent return		Growth equivalent	Perpetual equivalent return		Growth equivalent	Perpetual equivalent return
180.00%	13.00%		52.94%	15.40%		31.03%	17.80%
163.64%	13.10%		51.43%	15.50%		30.51%	17.90%
150.00%	13.20%		50.00%	15.60%		30.00%	18.00%
138.46%	13.30%		48.65%	15.70%		29.51%	18.10%
128.57%	13.40%		47.37%	15.80%		29.03%	18.20%
120.00%	13.50%		46.15%	15.90%		28.57%	18.30%
112.50%	13.60%		45.00%	16.00%		28.13%	18.40%
105.88%	13.70%		43.90%	16.10%		27.69%	18.50%
100.00%	13.80%		42.86%	16.20%		27.27%	18.60%
94.74%	13.90%		41.86%	16.30%		26.87%	18.70%
90.00%	14.00%		40.91%	16.40%		26.47%	18.80%
85.71%	14.10%		40.00%	16.50%		26.09%	18.90%
81.82%	14.20%		39.13%	16.60%		25.71%	19.00%
78.26%	14.30%		38.30%	16.70%		25.35%	19.10%
75.00%	14.40%		37.50%	16.80%		25.00%	19.20%
72.00%	14.50%		36.73%	16.90%		24.66%	19.30%
69.23%	14.60%		36.00%	17.00%		24.32%	19.40%
66.67%	14.70%		35.29%	17.10%		24.00%	19.50%
64.29%	14.80%		34.62%	17.20%		23.68%	19.60%
62.07%	14.90%		33.96%	17.30%		23.38%	19.70%
60.00%	15.00%		33.33%	17.40%		23.08%	19.80%
58.06%	15.10%		32.73%	17.50%		22.78%	19.90%
56.25%	15.20%		32.14%	17.60%		22.50%	20.00%
54.55%	15.30%		31.58%	17.70%			

Figure 3.6.5 (Leibowitz & Kogelman, 1994:41) (adapted)

Time	Value (9%)	Growth equivalent (9%)	Value (12%)	Growth equivalent (12%)	Value (15%)	Growth equivalent (15%)
1	9.00	803.57%	12.00	1071.43%	15.00	1339.29%
2	9.81	1585.62%	13.44	2142.86%	17.25	2714.45%
3	10.69	2346.72%	15.05	3214.29%	19.84	4126.44%
4	11.66	3087.43%	16.86	4285.71%	22.81	5576.25%
5	12.70	3808.30%	18.88	5357.14%	26.24	7064.90%
6	13.85	4509.87%	21.15	6428.57%	30.17	8593.43%
7	15.09	5192.64%	23.69	7500.00%	34.70	10162.90%
8	16.45	5857.12%	26.53	8571.43%	39.90	11774.40%
9	17.93	6503.81%	29.71	9642.86%	45.89	13429.07%
10	19.55	7133.17%	33.28	10714.29%	52.77	15128.07%
11	21.31	7745.67%	37.27	11785.71%	60.68	16872.57%
12	23.22	8341.77%	41.74	12857.14%	69.79	18663.80%
13	25.31	8921.90%	46.75	13928.57%	80.25	20503.01%
14	27.59	9486.49%	52.36	15000.00%	92.29	22391.48%
15	30.08	10035.96%	58.65	16071.43%	106.14	24330.54%
16	32.78	10570.71%	65.68	17142.86%	122.06	26321.53%
17	35.73	11091.14%	73.56	18214.29%	140.36	28365.86%
18	38.95	11597.63%	82.39	19285.71%	161.42	30464.95%
19	42.45	12090.55%	92.28	20357.14%	185.63	32620.26%
20	46.27	12570.27%	103.35	21428.57%	213.48	34833.30%
21	50.44	13037.13%	115.76	22500.00%	245.50	37105.62%
22	54.98	13491.50%	129.65	23571.43%	282.32	39438.81%
23	59.93	13933.69%	145.20	24642.86%	324.67	41834.49%
24	65.32	14364.04%	162.63	25714.29%	373.37	44294.34%
25	71.20	14782.86%	182.14	26785.71%	429.38	46820.08%
26	77.61	15190.46%	204.00	27857.14%	493.78	49413.48%
27	84.59	15587.14%	228.48	28928.57%	567.85	52076.34%
28	92.21	15973.20%	255.90	30000.00%	653.03	54810.53%
29	100.50	16348.92%	286.61	31071.43%	750.98	57617.95%
30	109.55	16714.57%	321.00	32142.86%	863.63	60500.57%
31	119.41	17070.43%	359.52	33214.29%	993.18	63460.41%
32	130.16	17416.76%	402.66	34285.71%	1,142.15	66499.53%
33	141.87	17753.81%	450.98	35357.14%	1,313.48	69620.05%
34	154.64	18081.83%	505.10	36428.57%	1,510.50	72824.16%
35	168.56	18401.07%	565.71	37500.00%	1,737.07	76114.09%
36	183.73	18711.76%	633.60	38571.43%	1,997.63	79492.15%
37	200.26	19014.12%	709.63	39642.86%	2,297.28	82960.69%
38	218.28	19308.38%	794.78	40714.29%	2,641.87	86522.14%
39	237.93	19594.77%	890.16	41785.71%	3,038.15	90178.98%
40	259.34	19873.48%	996.97	42857.14%	3,493.87	93933.77%
41	282.68	20144.73%	1,116.61	43928.57%	4,017.95	97789.14%
42	308.13	20408.71%	1,250.61	45000.00%	4,620.65	101747.78%
43	335.86	20665.62%	1,400.68	46071.43%	5,313.74	105812.45%
44	366.08	20915.64%	1,568.76	47142.86%	6,110.80	109986.00%
45	399.03	21158.97%	1,757.01	48214.29%	7,027.43	114271.34%
46	434.95	21395.79%	1,967.85	49285.71%	8,081.54	118671.47%
47	474.09	21626.26%	2,203.99	50357.14%	9,293.77	123189.45%
48	516.76	21850.55%	2,468.47	51428.57%	10,687.84	127828.45%
49	563.27	22068.84%	2,764.69	52500.00%	12,291.01	132591.72%
50	613.96	22281.28%	3,096.45	53571.43%	14,134.66	137482.57%

Figure 3.6.6 (Leibowitz & Kogelman, 1994:42)

Investment	Perpetual return	Franchise factor	Growth equivalent	Price-earnings ratio increment
				8.33
1	20.00%	4.44	50.00%	10.55
2	15.00%	1.67	100.00%	12.22
3	13.00%	0.56	150.00%	13.05
4	12.00%	0.00	200.00%	13.05

Figure 3.6.7 (Leibowitz & Kogelman, 1994:71) (Adapted)

Price-earnings ratio and leverage				
Price-earnings ratio (No franchise)	Price-earnings ratio (R62.5 franchise)	Price-earnings ratio (R80 franchise)	Price-earnings ratio (R125 franchise)	Lever -age
8.33	12.50	13.67	16.67	0%
8.22	12.50	13.70	16.78	5%
8.10	12.50	13.73	16.90	10%
7.97	12.50	13.77	17.03	15%
7.84	12.50	13.81	17.16	20%
7.69	12.50	13.85	17.31	25%
7.54	12.50	13.89	17.46	30%
7.38	12.50	13.93	17.62	35%
7.20	12.50	13.98	17.80	40%
7.02	12.50	14.04	17.98	45%
6.82	12.50	14.09	18.18	50%
6.60	12.50	14.15	18.40	55%
6.37	12.50	14.22	18.63	60%
6.12	12.50	14.29	18.88	65%
5.85	12.50	14.36	19.15	70%
5.56	12.50	14.44	19.44	75%
5.23	12.50	14.53	19.77	80%
4.88	12.50	14.63	20.12	85%
4.49	12.50	14.74	20.51	90%
4.05	12.50	14.86	20.95	95%
3.57	12.50	15.00	21.43	100%

Figure 3.6.8

Growth equivalent (Percentage of book value)	Leverage = 0%	Leverage = 50%	Leverage = 100%
0%	8.33	6.82	3.57
5%	8.50	7.05	3.93
10%	8.67	7.27	4.29
15%	8.83	7.50	4.64
20%	9.00	7.73	5.00
25%	9.17	7.95	5.36
30%	9.33	8.18	5.71
35%	9.50	8.41	6.07
40%	9.67	8.64	6.43
45%	9.83	8.86	6.79
50%	10.00	9.09	7.14
55%	10.17	9.32	7.50
60%	10.33	9.55	7.86
65%	10.50	9.77	8.21
70%	10.67	10.00	8.57
75%	10.83	10.23	8.93
80%	11.00	10.45	9.29
85%	11.17	10.68	9.64
90%	11.33	10.91	10.00
95%	11.50	11.14	10.36
100%	11.67	11.36	10.71
105%	11.83	11.59	11.07
110%	12.00	11.82	11.43
115%	12.17	12.05	11.79
120%	12.33	12.27	12.14
125%	12.50	12.50	12.50
130%	12.67	12.73	12.86
135%	12.83	12.95	13.21
140%	13.00	13.18	13.57
145%	13.17	13.41	13.93
150%	13.33	13.64	14.29
155%	13.50	13.86	14.64
160%	13.67	14.09	15.00
165%	13.83	14.32	15.36
170%	14.00	14.55	15.71
175%	14.17	14.77	16.07
180%	14.33	15.00	16.43
185%	14.50	15.23	16.79
190%	14.67	15.45	17.14
195%	14.83	15.68	17.50
200%	15.00	15.91	17.86

Figure 3.6.8 (continued)

Growth equivalent (Percentage of book value)	Leverage = 0%	Leverage = 50%	Leverage = 100%
205%	15.17	16.14	18.21
210%	15.33	16.36	18.57
215%	15.50	16.59	18.93
220%	15.67	16.82	19.29
225%	15.83	17.05	19.64
230%	16.00	17.27	20.00
235%	16.17	17.50	20.36
240%	16.33	17.73	20.71
245%	16.50	17.95	21.07
250%	16.67	18.18	21.43
255%	16.83	18.41	21.79
260%	17.00	18.64	22.14
265%	17.17	18.86	22.50
270%	17.33	19.09	22.86
275%	17.50	19.32	23.21
280%	17.67	19.55	23.57
285%	17.83	19.77	23.93
290%	18.00	20.00	24.29
295%	18.17	20.23	24.64
300%	18.33	20.45	25.00

Figure 3.6.9 - 3.6.15

The franchise consumption process												
Time	Investment	Franchise investment	Cumulative investment	Cumulative franchise investment	Earnings (Tangible value)	Earnings (Franchise value)	Total earnings	Earnings growth	Book value	Present value of franchise opportunity	Present value of franchise consumed	Cumulative present value of franchise
0	0.00	0.00		0.00	15.00		15.00	0.0%	100.00	150.00		150.00
1	12.75	12.75	12.75	12.75	15.00	0.00	15.00	0.0%	112.75	168.00	12.75	155.25
2	14.92	14.92	27.67	27.67	15.00	2.55	17.55	17.0%	127.67	173.88	14.92	158.96
3	17.45	17.45	45.12	45.12	17.55	2.98	20.53	17.0%	145.12	178.04	17.45	160.58
4	20.42	20.42	65.54	65.54	20.53	3.49	24.02	17.0%	165.54	179.85	20.42	159.43
5	23.89	23.89	89.43	89.43	24.02	4.08	28.11	17.0%	189.43	178.57	23.89	154.67
6	27.95	27.95	117.39	117.39	28.11	4.78	32.89	17.0%	217.39	173.24	27.95	145.28
7	32.71	32.71	150.09	150.09	32.89	5.59	38.48	17.0%	250.09	162.72	32.71	130.01
8	38.27	38.27	188.36	188.36	38.48	6.54	45.02	17.0%	288.36	145.61	38.27	107.34
9	44.77	44.77	233.13	233.13	45.02	7.65	52.67	17.0%	333.13	120.23	44.77	75.45
10	52.38	52.38	285.51	285.51	52.67	8.95	61.63	17.0%	385.51	84.51	52.38	32.13
11	59.91	32.13	345.42	317.64	64.06	6.43	70.48	14.4%	445.42	0.00	0.00	0.00
12	66.02	0.00	411.44	317.64	77.67	0.00	77.67	10.2%	511.44	0.00	0.00	0.00
13	72.75	0.00	484.20	317.64	85.59	0.00	85.59	10.2%	584.20	0.00	0.00	0.00
14	80.18	0.00	564.37	317.64	94.32	0.00	94.32	10.2%	664.37	0.00	0.00	0.00
15	88.35	0.00	652.73	317.64	103.95	0.00	103.95	10.2%	752.73	0.00	0.00	0.00

Figure 3.6.9 - 3.6.15 (continued)

The franchise consumption process											
Time	Growth equivalent	Original book value	Cumulative franchise investment	Cumulative investment at market rate	Price-earnings ratio	Franchise factor	Franchise value	Tangible value	Market value	Price-earnings ratio growth	Market value growth
0	150%	100.00	0.00	0.00	15.00	4.44	99.95	125.00	224.95	0.0%	0.0%
1	138%	100.00	12.75	0.00	14.23	4.28	88.41	125.00	213.41	0.0%	0.0%
2	125%	100.00	27.67	0.00	13.49	4.15	90.52	146.25	236.77	-5.2%	10.9%
3	111%	100.00	45.12	0.00	12.79	4.03	91.43	171.11	262.55	-5.2%	10.9%
4	96%	100.00	65.54	0.00	12.11	3.93	90.77	200.20	290.97	-5.3%	10.8%
5	82%	100.00	89.43	0.00	11.47	3.84	88.04	234.24	322.28	-5.3%	10.8%
6	67%	100.00	117.39	0.00	10.85	3.77	82.67	274.06	356.73	-5.4%	10.7%
7	52%	100.00	150.09	0.00	10.26	3.70	73.95	320.65	394.60	-5.5%	10.6%
8	37%	100.00	188.36	0.00	9.69	3.65	61.01	375.16	436.17	-5.5%	10.5%
9	23%	100.00	233.13	0.00	9.15	3.60	42.82	438.93	481.75	-5.6%	10.5%
10	8%	100.00	285.51	0.00	8.63	3.65	18.52	513.55	532.07	-5.6%	10.4%
11	0%	100.00	317.64	27.78	8.33	0.00	-0.23	587.35	587.12	-3.5%	10.3%
12	0%	100.00	317.64	93.80	8.33	0.00	-0.26	647.26	647.00	0.0%	10.2%
13	0%	100.00	317.64	166.56	8.33	0.00	-0.29	713.28	713.00	0.0%	10.2%
14	0%	100.00	317.64	246.73	8.33	0.00	-0.31	786.04	785.72	0.0%	10.2%
15	0%	100.00	317.64	335.09	8.33	0.00	-0.35	866.21	865.86	0.0%	10.2%

Equation 1.4.1

The price-earnings ratio

$$P/E = \text{Current market price per share} / \text{Net earnings during the past 12 months per share}$$

Equation 1.4.2

The forward price-earnings ratio

$$P/E_1 = \text{Current market price per share} / \text{Net expected earnings for the next 12 months per share}$$

Equation 2.2.1

Beta

$\beta \approx \sigma(R_i, R_M) / \sigma(R_M)$ (Beaver et al., 1970:658)

β	Beta
$\sigma(R_i, R_M)$	Co-variance of the return of asset i with that of the market portfolio
$\sigma(R_M)$	Variance of the return of the market portfolio

Equation 2.2.2

An asset's characteristic line with the market portfolio

$R_i = \alpha_i + \beta_i R_M + \varepsilon_i$ (Beaver et al., 1970:659)

R_i	Return on investment i
α_i	Intercept of the regression equation
β_i	Beta of investment i
R_M	Return on the market portfolio
ε_i	Random error term of the regression

Annexure B

Equation 2.2.3

$$R_i = R_F + \beta_i (R_M - R_F)$$

R_i	Return on investment i
R_F	Risk-free rate of return
β_i	Beta of investment i
R_M	Return on the market portfolio

Equation 2.2.4

$$R_i = \left(\text{Payout ratio} \middle/ \text{Price - earnings ratio} \right) - g$$

R_i	Return on investment i
g	The growth rate in earnings

Equation 2.3.1

$$R_i = E_i + \varepsilon_i + \sum_{i=1}^{k} b_i \delta_i$$ (Ross, 1976:341-354)

R_i	Return on investment i
E_i	The intercept of the regression for investment i
ε_i	Random error term of the regression
b_i	The reaction (covariance) of investment i to the changes in factor i
δ_i	The return premium over and above the intercept or risk-free of return associated to the specified risk factor used in the regression equation

Equation 2.4.1

$$R_i = E_i + \varepsilon_i + \sum_{i=1}^{k} b_i \delta_i$$

R_i	Return on investment i
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ε_i	Random error term of the regression
b_i	The reaction (covariance) of investment i to the changes in factor i
δ_i	The return premium over and above the intercept or risk-free of return associated to the specified risk factor used in the regression equation

Equation 3.1.1

$$\text{Value per share} = \sum_{t=1}^{t=\infty} \frac{DPS_t}{(1+R_i)^t} + \frac{\text{terminal price}}{(1+R_i)^t}$$ (Damodaran, 1996:192)

DPS_t	Dividends per share during time t
R_i	Return on investment i

Equation 3.1.2

$$\text{Net Income} + \text{Depreciation} + \text{Other non-cash charges} + \text{Change in debt capacity} = \text{Capital expenditure} + \text{Working capital investment} + \text{Dividends}$$

(Rappaport, 1986:53)

Equation 3.1.3

$D = 1 - \frac{g(f+w)}{(1+g)(r)(1-T)(1+L)}$ (Rappaport, 1986:54)

D	The dividend payout ratio
g	The growth rate of sales
f	The capital expenditure less depreciation per unit of sales increase
w	Increase of the working capital investment required per unit of sales increase
r	The ratio of earnings before tax to sales
T	The cash income tax rate
L	The target debt to equity ratio

Equation 3.1.4

$y_t = y_{t-1} + x_t - d_t$ (Fairfield, 1994:24)

y_t	The value of a company at time t
y_{t-1}	The value of a company at time t – 1
x_t	The earnings during time t
d_t	The dividend for time t

Equation 3.1.5

$x_t^a = x_t - R_i y_{t-1}$ (Fairfield, 1994:24)

x_t^a	The abnormal earnings during time t
x_t	The earnings during time t
R_i	Return on investment i
y_{t-1}	The value of a company at time t – 1

Equation 3.1.6

$$\text{Value per share}_{t} = y_t + \sum_{t=1}^{t=\infty} x_1^a \Big/ (1+R_i)^t$$ (Fairfield, 1994:24)

y_t	The value of a company at time t
x_1^a	The abnormal earnings during time 1
R_i	Return on investment i

Equation 3.1.7

$$P/E_{t+1} = \left\{ \left(\left(1+R_i\right/R_i \right) \left(x_i \sum_{t=1}^{t=\infty} x_{t+1}^a - x_t^a \Big/ (1+R_i)^t - d_t \right) \Big/ x_t \right\}$$

(Fairfield, 1994:25) (Adapted)

P/E_{t+1}	The forward price-earnings ratio for time t+1
R_i	Return on investment i
x_{t+1}^a	The abnormal earnings during time t+1
x_t^a	The abnormal earnings during time t
d_t	The dividend for time t
x_t	The earnings during time t

Annexure B

Equation 3.1.8

$DPS_1 = EPS_0(\text{Payout ratio})(1+g_0)$ (Damodaran, 1996:292)

DPS_t	The dividend per share for time t
EPS_0	The earnings per share for time 0
g_0	The growth rate in earnings for time 0

Equation 3.1.9

$P/E_1 = \sum_{t=1}^{t=\infty} \text{Payout ratio}(1+g_t) / (1+R_i)^t$ (ibid.)

P/E_1	The forward price-earnings ratio
g_t	The growth rate in earnings for time t
R_i	Return on investment i

Equation 3.2.1

$\text{Value of share} = DPS_1 / R_i - g$ (Damodaran, 1996:192)

DPS_1	The dividend per share for the next period
R_i	Return on investment i
g	The growth rate in earnings

Equation 3.2.2

$$\text{Effective duration} = \frac{1}{R_i - g}$$ (Farrell, 1985:23) (Adapted)

R_i	Return on investment i
g	The growth rate in earnings

Equation 3.2.3

$$DPS_t = EPS_t(\text{Payout ratio})(1+g)$$ (Damodaran, 1996:292)

DPS_t	The dividend per share for time t
EPS_t	The earnings per share for time t
g	The growth rate in earnings

Equation 3.2.4

$$P = \frac{EPS_0(\text{Payout ratio})(1+g)}{R_i - g}$$ (ibid.)

P	The price per share
EPS_0	The earnings per share for time 0
g	The growth rate in earnings
R_i	Return on investment i

Equation 3.2.5

$$\frac{P}{EPS} = \frac{P}{E} = \frac{\text{Payout ratio}(1+g)}{R_i - g}$$ (ibid.)

P	The price per share
EPS	The earnings per share
$\frac{P}{E}$	The price-earnings ratio
g	The growth rate in earnings
R_i	Return on investment i

Equation 3.2.6

$$P/E_1 = \text{Payout ratio}/R_i - g \quad \text{(ibid.)}$$

P/E_1	The forward price-earnings ratio
R_i	Return on investment i
g	The growth rate in earnings

Equation 3.3.1

$$P = \sum_{t=1}^{t=n} DPS_t / (1+R_i)^t + \frac{DPS_{n+1}/(R_{in} - g_n)}{(1+R_i)^n} \quad \text{(Damodaran, 1996:294)}$$

P	The price per share
DPS_t	The dividend per share for time t
R_i	Return on investment i
DPS_{t+1}	The dividend per share for time t + 1
R_{in}	Required rate of return in steady state
g_n	Permanent growth rate after year n

Equation 3.3.2

$$P = \frac{DPS_0(1+g)\left(1-\frac{(1+g)^n}{(1+R_i)^n}\right)}{R_i - g} + \frac{DPS_{n+1}}{(R_{in}-g_n)(1+R_i)^n}$$

(Damodaran, 1996:197)

P	The price per share
DPS_0	The dividend per share for time 0
g	Extraordinary growth rate for the first n years
R_i	Required rate of return (cost of equity) in high-growth period
DPS_{n+1}	Expected dividends per share in year n+1
R_{in}	Required rate of return in steady state
g_n	Permanent growth rate after year n

Annexure B

Equation 3.3.3

$$P = \left[\frac{DPS_0(1+g)\left(1-\frac{(1+g)^n}{(1+R_i)^n}\right)}{R_i - g} + \frac{DPS_{n+1}}{(R_{in} - g_n)(1+R_i)^n} - \frac{DPS_1}{R_{in} - g_n} \right]$$

$$+ \left[\frac{DPS_1}{R_{in} - g_n} - \frac{DPS_0}{R_{in}} \right] + \frac{DPS_0}{R_{in}}$$

(Damodaran, 1996:203)

P	The price per share
DPS_0	The dividend per share for time 0
g	Extraordinary growth rate for the first n years
R_i	Required rate of return (cost of equity) in high-growth period
DPS_{n+1}	Expected dividends per share in year n+1
R_{in}	Required rate of return in steady state
g_n	Permanent growth rate after year n
DPS_1	The dividend per share for time 1

Equation 3.3.4

$$P/E_1 = \frac{\text{Payout ratio}\left(1 - (1+g)^n/(1+R_i)^n\right)}{R_i - g} + \frac{\text{Payout ratio}_{n+1}}{(R_{in} - g_n)(1+R_i)^n}$$

P/E_1	The forward price-earnings ratio
g	Extraordinary growth rate for the first n years
R_i	Required rate of return (cost of equity) in high growth period
R_{in}	Required rate of return in steady state
g_n	Permanent growth rate after year n

Annexure B

Equation 3.3.5

$$P/E_0 = \frac{\text{Payout ratio}(1+g)\left(1 - \frac{(1+g)^n}{(1+R_i)^n}\right)}{R_i - g} + \frac{\text{Payout ratio}_n (1+g)^n (1+g_n)}{(R_{in} - g_n)(1+R_i)^n}$$

(Damodaran, 1996:294)

P/EPS_0	The price-earnings ratio
P/E_0	The price-earnings ratio
g	Extraordinary growth rate for the first n years
R_i	Required rate of return (cost of equity) in high growth period
R_{in}	Required rate of return in steady state
g_n	Permanent growth rate after year n
n	The exponent indicate the time of extraordinary growth

Equation 3.4.1

$$P = \frac{DPS_0(1+g_n)}{R_i - g_n} + \frac{DPS_0(H)(g_a - g_n)}{R_i - g_n}$$

(Damodaran, 1996:205)

P	The price per share
DPS_0	The dividend per share for time 0
g_n	The normal growth rate
R_i	Return on investment i
H	Half the time of the high growth period
g_a	The abnormal growth rate

Equation 3.4.2

$$P/E_1 = \text{Payout ratio}\,(1+g_n)/(R_i - g) + \text{Payout ratio}\,(H)(g_a - g_n)/(R_i - g)$$

P/E_1	The forward price-earnings ratio
g_n	The normal growth rate
R_i	Return on investment i
g	The growth rate of earnings
H	The time of the high growth period / 2
g_a	The abnormal growth rate

Equation 3.5.1

$$P = \sum_{t=1}^{t=n1} EPS_0 (1+g_a)^t (\text{High growth phase payout ratio}) / (1+R_i)^t$$

$$+ \sum_{t=n1+1}^{t=n2} DPS_t / (1+R_i)^t + EPS_{n2}(1+g_n)(\text{Terminal phase payout ratio}) / (R_i - g_n)(1+R_i)^n$$

(Damodaran, 1996:207)

P	The price per share
EPS_0	The earnings per share for time 0
g_a	The abnormal growth rate
R_i	Return on investment i
DPS_t	The dividend per share for time t
EPS_{n2}	The earnings per share for the normal growth
g_n	The normal growth rate

Equation 3.5.2

$$\frac{P}{E_1} = \sum_{t=n0}^{t=n1} \frac{(1+g_a)^t (\text{High growth phase payout ratio})}{(1+R_i)^t} + \sum_{t=n1+1}^{t=n2} \frac{\text{Individualistic payout ratio}_t}{(1+R_i)^t} + \frac{(1+g_n)(\text{Terminal phase payout ratio})}{(R_i - g_n)(1+R_i)^n}$$

P/E_1	The forward price-earnings ratio
g_a	The abnormal growth rate
R_i	Return on investment i
g_n	The normal growth rate

Equation 3.6.1

$$\text{Growth equivalent} = \frac{g}{(R_i - g)}$$ (Leibowitz & Kogelman, 1994:21)

g	The growth rate of earnings
R_i	Return on investment i

Equation 3.6.2

$$\text{Franchise factor} = \frac{(ROE - R_i)}{(ROE)(R_i)}$$ (Leibowitz & Kogelman, 1994:23)

ROE	The return on equity
R_i	Return on investment i

Equation 3.6.3

$$P/E_1 = 1/R_i + (\text{Franchise factor x Growth equivalent})\ \text{(ibid.)}$$

P/E_1	The forward price-earnings ratio
R_i	Return on investment i

Equation 3.6.4

$$\text{Franchise factor} = (ROE_p - R_i)/(ROE)(R_i)$$ (Leibowitz & Kogelman, 1994:37)

ROE_p	The perpetual equivalent return on equity
R_i	Return on investment i
ROE	The return on equity

Equation 3.6.5

$$\text{Leveraged franchise factor} = ROE_p - R_i/(ROE - ih)R_i$$

(Leibowitz & Kogelman, 1994:73) (Adapted)

ROE_p	The perpetual equivalent return on equity
R_i	Return on investment i
ROE	The return on equity
ih	The interest rate multiplied with the leverage percentage

Annexure B

Equation 3.7.1

$$\text{Return on equity} = \left(\text{EBIT}/\text{Sales} \text{ x } \text{Sales}/\text{Total assets} - \text{Interest expense}/\text{Total assets}\right)$$
$$\left(\text{Total assets}/\text{Common equity}\right)\left(100 - \text{Income taxes}/\text{Net income before taxes}\right)$$

(Reilly & Brown, 1997:396)

EBIT The earnings before interest and taxes

Equation 3.7.2

Growth = Retention rate x Return on equity (Damodaran, 1996:196)

Equation 3.7.3

$$\text{DPEG} = \text{Price - earnings ratio}/\text{5 year estimated growth rate} + \text{expected dividend yield}$$

(" The DPEG ratio explained", 2000)(Online)

Equation 4.2.1

$$T = g + \left(\text{Return on equity - g}/\text{Price - to - book ratio}\right) +$$
$$\left(\Delta \text{ Price - to - book ratio}/\text{Price - to - book ratio}\right)(1 + g)$$

(Born et al., 1988:35)

T The total return for the period

g The growth rate of earnings

Δ Change