

**FARM PLANNING FOR A TYPICAL CROP-LIVESTOCK
INTEGRATED FARM: AN APPLICATION OF A MIXED INTEGER
LINEAR PROGRAMMING MODEL**

ASSIGNMENT

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Amanuel Habte Ghebretsadik

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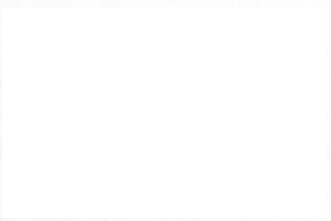
Supervisor: Professor HC. De Kock

Co-Supervisor: Mr S E Visagie

Declaration

I, the undersigned, hereby declare that the work contained in this assignment is my own original work and that I have not previously in its entirety or in part submitted at any university for a degree.

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Abstract

In an integrated crop-livestock production farm, the profitability and sustainability of farm production is dependent on the crop rotation strategy applied. Crop rotations have historically been applied to maintain long-term profitability and sustainability of farming production by exploiting the jointly beneficial interrelationships existing among different crop types and the animal production activity.

Monocrop (specifically wheat) growers in the Swartland area of the Western Cape are struggling to maintain long-term profitability and sustainability of the crop production, challenging them to rethink about the introduction crop rotation in the production planning. By making proper assumptions, this paper develops a mixed integer linear programming model to suggest a decision planning for the farm planning problem faced by an integrated-crop-livestock production farmer. The mathematical model developed includes crop production, dairy production and wool sheep production activities, which permitted the consideration of five crop types within a crop rotation system. By assuming that a farmer uses a cycle of at most three years, the crop rotation model was incorporated in the composite mixed integer linear farm planning model.

In order to demonstrate the application of the mathematical farm planning model formulated, a case study is presented. Relevant data from the Koeberg area of the Swartland region of the Western Cape was applied. For each planning period, the model assumed that the farm has the option of selecting from any of 15 cropping strategies. A land which is not allocated to any of the 15 crop rotation strategies due to risky production situation is left as grass land for roughage purposes of the animal production.

Results of the mathematical model indicated that farm profit is dependent on the cropping strategy selected. Additionally, animal production level was also dependent on the crop strategy applied. Furthermore, study results suggest that the profit generated from the integrated crop-livestock farm production by adopting crop rotation was superior to profit generated from the farm activities which are based on monocrop wheat strategy. Empirical results also indicated that the complex interrelationship involved in a mixed crop-livestock farm operation play a major role in determining optimal farm plans. This complex interrelationships favour the introduction of crop rotation in the crop production activities of the farm under investigation.

Crop production risk is the major risk component of risk the farmer faces in the farm production. In this study, risk is incorporated in the mixed integer programming farm planning model as a deviation from the expected values of an activity of returns. Model solution with risk indicated that crop rotation strategy and animal production level is sensitive to risk levels considered. The Results also showed that the incorporation of risk in the model greatly affects the level of acreage allocation, crop rotation and animal production level of the farm.

Finally, to improve the profitability and sustainability of the farm activity, the study results suggest that the introduction of crop rotation which consist cereals, oil crops and leguminous forages is of paramount importance. Furthermore, the inclusion of forage crops such as medics in the integrated crop livestock production is beneficial for sustained profitability from year to year.

Opsomming

Wisselbou is baie belangrik om volhoubare winsgewindheid te verseker in 'n geïntegreerde lewendehawe / gewasverbouing boerdery in die Swartland gebied van Wes-Kaap. 'n Monokultuur van veral koring produksie het ernstige probleme vir produsente veroorsaak.

In hierdie studie word 'n gemengde heeltallige liniêre programmerings-model gebruik om te help met besluitneming in sulke boerderye. Die wiskundige model beskou die produksie van kontant- en voer-gewasse (5 verskillende soorte) asook suiwel- en wol/vleis-produksie (beeste en skape). Daar word aanvaar dat die boer 'n siklus van hoogstens 3 jaar in die wisselbou rotasie model gebruik..

'n Gevallestudie word gedoen met behulp van toepaslike data van 'n plaas in die Koeberg gebied. Die model aanvaar dat die produsent 'n keuse het uit 16 wisselbou strategieë. Resultate toon dat winsgewindheid afhanklik is van die strategieë gekies en dat wisselbou beter resultate lewer as in die geval van 'n monokultuur.. Dit wys ook dat die wisselwerking tussen diere-produksie en gewasproduksie baie belangrik is in die keuse van 'n optimale strategie.

Die risiko in gewasverbouing is die belangrikste risiko factor vir die produsent. In hierdie studie word risiko ook ingesluit in die gemengde heeltallige model, naamlik as 'n afwyking van die verwagte opbrengs-waardes. Die model toon duidelik dat gewasproduksie en lewendehawe-produksie baie sensitief is ten opsigte van die gekose risiko vlak.

Die studie toon ook dat 'n wisselbou program wat die produksie van graan (veral koring), oliesade asook voere insluit belangrik is vir volhoubare winsgewindheid. Die insluiting van klawers (bv "medics") is veral belangrik hier.

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Table of Contents

Declaration.....	i
Abstract.....	ii
Opsomming.....	iv
Acknowledgments	v
Table of Contents	vi
List of Tables.....	ix
List of Figures.....	x
Chapter I	1
Background Information.....	1
1. Introduction: Integrated crop and livestock production	1
2. Problem Statement and Underlying Assumptions	8
2.1. <i>The Problem</i>	8
2.2. <i>Underlying Assumptions</i>	10
3. Underlying Hypothesis and Objectives.....	14
4. Data	15
5. Sequence of Chapters.....	16
Chapter II	18
Literature Review	18
1. Crop Rotation Modelling Background: A Literature Review	18
2. Review of Modelling Risk and Uncertainty Using Mathematical Programming Techniques: <i>Selected Literature</i>	23
2.1. <i>Game Theoretic Approach</i>	26
2.2. <i>The "Safety First" Approach</i>	34
2.3. <i>The "E-V" Approach (Quadratic Programming)</i>	37

2.4.	<i>MOTAD</i>	43
2.5.	<i>Target-MOTAD</i>	48
Chapter III.....		51
Mathematical Model		51
1.	Introduction	51
2.	Definitions of Decision Variables and Parameters in the model.....	53
3.	Crop Rotation Modelling.....	55
3.1.	<i>Derivation of Crop Rotation Strategies</i>	55
3.2.	<i>Limiting Number and size of Strategies Implemented in the Farm</i>	60
3.3.	<i>Land Constraint</i>	63
4.	Income Variability: <i>as a Source of Farm Risk</i>	64
5.	Animal feed activities.....	68
5.1.	<i>Blended feed (Feed mix)</i>	68
5.2.	<i>Roughage Requirements</i>	70
6.	Availability activities.....	72
7.	Renting Activities	73
8.	Animal Feed Storage Constraint	75
9.	Livestock buying and selling activities	76
10.	The objective function	77
Chapter IV.....		81
Mathematical Model Solution and Sensitivity Analysis: <i>A Case Study</i>		81
1.	Introduction	81
2.	Description of the Agricultural Activities of the Farm: <i>Case Model</i> <i>Empirical Specification</i>	83
2.1.	<i>Designing Feasible Crop Rotation Strategies and Input Data Development for the Mathematical Model</i>	83
2.2.	<i>Dairy and Wool Sheep Production Activities</i>	87
2.3.	<i>Additional Resource Renting Activities</i>	89
3.	Model Solution without Considering Risk: Comparison of Monocropping and Crop Rotation Farming Strategies	90
3.1.	<i>Farming Plan under Normal Year Model Assumption</i>	90
3.2.	<i>Farming Plan under Wet Year Model Assumption</i>	93
3.3.	<i>Farming Plan Assuming an Average of the Three States</i>	95
4.	Model Solution with Risk Considerations.....	98
5.	Sensitivity of Model Solutions to Risk, Strategy and Price Changes	104

5.1. <i>Sensitivity Analysis on Values of Risk Parameter</i>	105
5.2. <i>Sensitivity Analysis on the Number of Strategies</i>	107
5.3. <i>Sensitivity Analysis on Price of Crops</i>	108
Chapter V	110
Conclusion and Future Studies	110
1. Conclusion	110
2. Future Study	115
Reference	117
Appendix A: Mixed Integer Linear Programming Model	125
Appendix B: Cost of growing (Rand/hectare) Crops per strategies	127
Appendix C: Market price of crops (Rand/ton)	128
Appendix D: Yield data (Tons/hectare)	129
Appendix E: deviation values	131
Appendix F: Model Solution for Different values of Risk Levels	132
Appendix G: Optimal Acreage Allocation versus Risk	133
Appendix H: Model solution for Different number of plots	134

List of Tables

Table 1. Aggregated representation of the mixed integer linear programming model....	78
Table 2. Gross income of the different strategies (Rand/hectare).....	86
Table 3. Raw materials (ingredient) and their nutrient content in the feed mix preparation	87
Table 4. Optimal model solutions for wheat monocropping and crop rotation farm planning situation	91
Table 5. Optimal Feed mix results for animal feeding plan for normal year farming plan (ton).....	91
Table 6. Optimal model solutions for animal production plan.....	92
Table 7. Crop production and sell plan under wheat monocropping farming plan for wet year scenario.....	93
Table 8. Optimal Feed mix results for animal feeding plan under wet year assumption for the monocropping strategy (ton)	93
Table 9. Model solution of wheat monocropping strategy for animal production plan under wet year assumption.	93
Table 10. Optimum crop production marketing plan for the wet year crop rotation farm production	94
Table 11. Optimal animal feed mix production plans for the Crop rotation scenario farm plan (ton)	95
Table 12. Animal production plan of crop rotation strategy under wet year assumption.	95
Table 13. Crop Production and sell plan for average data	95
Table 14. Animal Production Plan.	96
Table 15. Optimal animal feed mix production plans (tons)	96
Table 16. Amount of crops (tons) produced and sold to the market applying the minimum risk	99
Table 17. Optimal feed mix under minimum risk (tons).....	99
Table 18. Optimal animal production plans.....	99
Table 19. Model solution -quantity of crops (tons) produced and sold to the market for the constraint level $\lambda=R1,000,000$	100
Table 20. Optimal animal production plans at risk level $\lambda=R1,000,000$	100
Table 21. Optimal feed mix (tons) at risk level $\lambda=R1,000,000$	101

Table 22. Model solution to different risk levels101

List of Figures

Figure 1. A dynamic cropping system.....2
Figure 2. Monocrop (one year) Strategy58
Figure 3. Two crop per year strategy.....58
Figure 4. Three crop per year strategy58
Figure 5. Two strategies per year farm cropping plan.....62
Figure 6. Land allocation under normal year crop rotation scenario92
Figure 7. Cropping plan under wet year crop rotation situation.....94
Figure 8. Farmland allocation under average for crop rotation scenario96
Figure 9. Farmland allocation for the minimum risk situation.....99
Figure 10. Land allocation under $\lambda=R1,000,000$ 100
Figure 11. Profit-Risk Frontiers106
Figure 12. Percentage of acreage allocation for different risk values.....107

Chapter I

Background Information

1. Introduction: Integrated crop and livestock production

Agricultural activity occurs in an environment that is always changing. In every growing season, producers must pay attention to numerous factors that influence their management decisions. Some factors are within the control of the farmers; however, many are not. The weather, market conditions (including input and output prices), new technology, government policy and information represent some of the factors that have an impact on production decisions. As sustainability and profitability of a farm firm is dependent on the management of these broad categories of externalities, accordingly, the farmer must deal with such factors on a continual basis (Tanaka et al., 2002).

The broad externalities create a daunting task to the farmer who is constrained by many challenging factors. To meet these challenging factors farmers must manage externalities by introducing different management options that optimise the outcome. This is a challenging task. In this regard, producers need to possess the ability to integrate the vast amount of information on externalities that are constantly changing. The information that one needs to understand well enough is how to take advantage of situations in which externalities interact. Furthermore, the information must be translated within the context of the resources available to the producer.

To significantly benefit from the agricultural activities, the modern agricultural production has been and is applying sustainable agricultural management. There are several alternative definitions of sustainable agriculture. However, all seem to agree that the definition includes

reductions in the reliance on non-renewable inputs such as fertilizers and pesticide products; reductions in environmental degradation and an increase in management input (Novak, Mitchell & Crews, 1990; Smit, 1997).

As part of a sustainable agriculture, a practice of dynamic cropping system is of paramount importance for long-term sustainable and profitable farming activity (Tanaka et al., 2002) (See Figure 1).

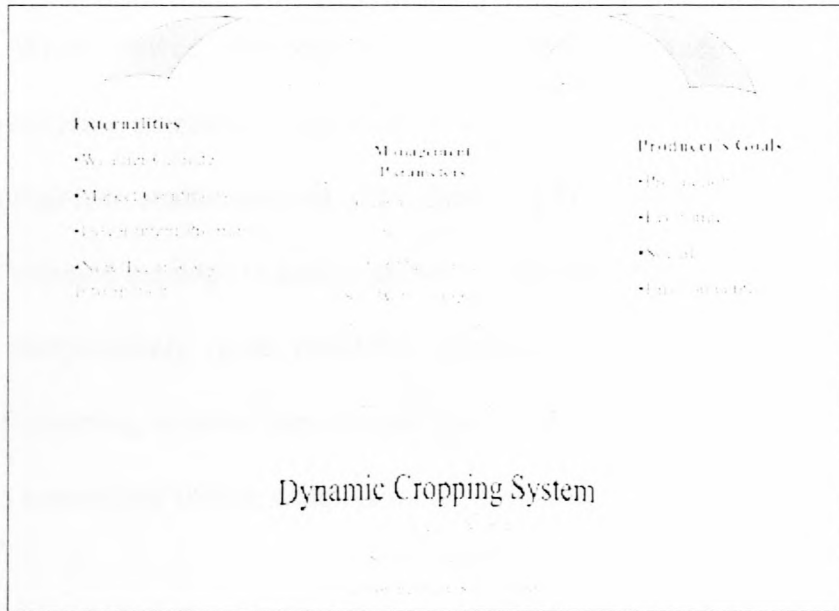


Figure 1. A dynamic cropping system

(Source: Tanaka, et al., 2002)

Cropping systems can be defined as the combination of crops grown and management applied of which monocropping, intercropping and crop rotation systems are a subset (Harper, 1983; Lockhart & Wiseman, 1988). Crop rotation systems are characterized by a defined sequence of crops grown on a given arable land and the associated management practices.

Numerous cropping systems can be technically feasible on a given farmland. However, decision criteria are required to choose among the technically feasible ones. Decision criteria for a cropping system choice can include impact on soil fertility, environmental quality, interdependence with animal activities, and of course farm profitability.

Some cropping systems endow the farmer a better profit through their impact on soil quality and fertility. When interest is directed to the long-term sustainability of the agricultural productivity of the farm those social benefits should be valued and incorporated into a decision criterion which is used to compare the various cropping system alternatives. At the farm level, optimising farmers choose the best cropping system from among feasible alternatives. When viewed from the individual farmer's standpoint, farm profitability becomes the overriding criterion. In addition to being technically feasible, a cropping system needs to be profitable to enable survival of the farm as a firm. Annual profits are accumulated over time into retained earnings to enable growth of the farm business. Among the cropping systems, the comparatively more profitable alternatives would still be preferred. The profitability of cropping systems can change over time due to several aspects and farmers need to adopt a sustainable system which is more profitable.

After the 1940's, especially in the developed world, due to the increase in mechanization and the increase in application of chemical protection, farming activities had been tremendously profitable and have been simplified. This simplification has led the farmers to concentrate their production only on one crop type (monoculture practice). According to Harper (1983), the adoption of monoculture practices was prompted due to the following advantages of applying the strategy.

- ❑ Simplicity of management
- ❑ Reduction in the range of machinery required
- ❑ Low labour requirement
- ❑ Yield levels can be maintained with available fertilizers and crop protection products

However, due to various existing problems in the present agricultural production system, the above mentioned advantages of monocropping are no more relevant to the present agricultural production situation. From time to time the return from monoculture agricultural production is shrinking. The following are some of the factors exacerbating the poor return from monoculture agricultural practices (Harper, 1983).

1. Continuous monoculture cereal cropping does encourage weeds. The control of weeds in the cereal production farming is becoming a problem. Even if chemicals can be used, control is expensive and difficult. In recent times, the pollution of environment because of such practices is becoming a serious issue.
2. The greater incidence of soil born fungal disease and pest is very difficult to control.
3. Continuous monoculture cereal growers are more vulnerable to changes in the market trends and prices: The current serious problem of farmers with regard to this is the surplus production of cereals. This results in falling prices of especially wheat and barley.
4. Decline of soil fertility and organic matter: This problem may require additional input of expensive fertilizers. Consequently, the production cost increases making farming activity unprofitable from time to time.

In order to combat the various challenging problems described above, the farming sector has been and is leaning toward applying dynamic cropping system (Tanaka et al., 2002). According to Tanaka et al. (2002), a dynamic cropping system is a long-term strategy of annual crop sequencing that optimises cropping options and the outcome of production, economic and resource conservation goals by using sound ecological management principles. They further described the following key factors of dynamic cropping system.

- Diversity

- Adaptability
- Reduced input cost
- Multiple enterprise
- Environmental awareness
- Information awareness

One of the important aspects of cropping systems can be the employment of effective crop rotation. Taking into consideration the various biological and climatic conditions, careful selection of crop rotation systems offers the possibility of reducing the trade-off between maintaining long-term profitability and reducing environmental impact. Crop rotations are considered as major cropping system alternative to reduce agriculture's dependence on external inputs. This is indirectly achieved by internal nutrient recycling, maintenance of long-term productivity of the land and breaking the weed and disease cycles (Hardy, 1998). Crop rotation is an additional factor that can help rejuvenate the soil as it has several advantages including enhancement of soil fertility and efficient utilisation of plant nutrients. Additionally, crop rotation helps to build soil fertility by not allowing one single type of crop to remove the same nutrients from the soil over a few seasons, as different crops feed on different nutrients at different rates. Growing one crop on the same field over time will result in total loss of many vital nutrients.

The importance of crop rotation has long been recognised prior to the development of modern farming that relied extensively on external inputs. According to the explanation of Struik & Bonciarelli (1997), the basis of sustainable agriculture is a good rotation, adequate soil, and water management, and proper husbandry of the different crops in the rotation. He further stated that agronomically, farmers should aim at the minimum input of each production resource required to allow maximum utilisation of other resources. Crop rotation serves

multitude purposes including control of pests, weeds, and diseases; reducing soil erosion; maintaining soil fertility and enhancing productivity (Guertal, Bauske & Edwards, 1997; Ikerd, 1991; Hardy, 1998). As dependence on external inputs increased, some believed that the importance of crop rotation would be reduced. However, recent concerns about sustainability of farming profit, environmental impact due to chemical inputs, high rate of use of purchased mineral fertilizers such as nitrogen, acceleration of soil erosion, uncertainty about the long term supply and effectiveness of external inputs and declining yields have brought again crop rotation into the agricultural sector (Ikerd, 1991).

As mentioned above, several advantages of crop rotations have been widely recognised. (Guertal, Bauske & Edwards, 1997). Crop rotations break weed and disease cycles, effectively reduce soil erosion thereby avoiding the long-term decline in the productivity of land and reduce the pollution that could occur otherwise. Crop rotations improve soil quality and improve soil structure thereby enhancing permeability and increase biological activity, increase water storage capacity and the amount of organic matter.

Crop rotations with legumes and oil crops like medics, lupines, canola, and many other types are beneficial to the farm production activity. As most legume types help fix Nitrogen into the soil, introducing legumes in the crop rotation cycle can help to reduce the cost for fertilizer expenses. Moreover, some legumes and oil crops are deep rooted and are excellent for breaking weed and pest cycles (Hardy, 1998).

An indirect but important benefit of crop rotation is that it involves diversification. The risk benefit of crop diversification is generally well understood. In an integrated crop-livestock farm environment, diversification reduces risks by spreading among a number of crops and

animals. That is, diversification provides an economic buffer against fluctuations in income resulting from various factors (Alternative agriculture, 1989).

The use of crop rotations has generally been thought to reduce risk compared with monoculture cropping (Helmert, Langemeier & Atwood, 1986 cited by Helmers, Yamoah, & Varvel, 2001). According to Helmers, Langemeier & Atwood (1986), the benefit of crop rotation in reducing risk involves three distinct effects. These are:

[1]. Conventionally practiced rotations involve diversification, which is an offsetting phenomenon where low returns in one year for one crop are combined with relatively high returns from a different crop. [2.] Crop rotation is generally thought to reduce yield variability compared with monoculture practices. [3]. Crop rotations as opposed to monoculture cropping may result in overall higher crop yield as well as reduced production cost. In addition, assuming that risk is defined as the failure to reach target returns, these influences may reduce risk by reducing the severity of return failures.

To conclude this section, in an integrated crop-livestock farm, applying crop rotation is beneficial to the farmer. Crop rotation is one of the pivotal drivers of sustainable farming. For this reason, introducing crop rotation in agricultural production activity is an indispensable means.

2. Problem Statement and Underlying Assumptions

2.1. *The Problem*

In many studies dependence on monocropping practices in agricultural production activities and acreage allocations by farmers have been identified as one of the causes of decline in farm profitability. Growing only one type of crop in successive years in a given fixed land has been shown to adversely affect soil structure, cause depletion of organic matter and increase the incidence of diseases, weeds and pest problems. (Hardy, 1998) Furthermore, due to the unpredictability of weather changes, a large portion of instability presents in a yield of a single crop grown.

In consequence, the problem as shown in this paper is one of resource (acreage) allocation decisions by farmers in an integrated crop-livestock crop production farm. There is a need for defining a crop production strategy, which takes into consideration the overall integrated agricultural production system. Given the importance of crop rotation, this paper focuses on risk and diversification issues associated with the selection of crop rotation strategies by taking into consideration dairy and wool sheep production activities of the farm. In addition to the selection of strategies, the decision maker's problem is to integrate the complex relationship existing in the crop and livestock production activities in an optimal manner. Moreover, given the importance of dairy and wool sheep production and the reliance of these production activities in the farmland for some ingredients and pasture requirements, the crop rotation planning is affected by this complex interdependence. In light of this interdependence, the introduction of forage and oil crops in the production planning is of paramount importance to the farmer from economical, biological and ecological perspective (Hardy, 1998).

The second component of the integrated crop livestock production decision problem is the activities of animal production. As part of the farm business, the number of livestock the farm owns is dependent on the availability of the space the farmer has for livestock production and availability of the feed supply. Therefore, as part of the integrated farm-planning problem, determining the number of animals, determining the amount of feed necessary for the livestock's maintenance satisfying the necessary nutrients and ingredients requirements for both the dairy cattle and wool sheep in the planning period is indispensable.

In this study, it is hypothesised that the farmer owns an agronomically homogeneous fixed area of land. There are n possible crops that can possibly grow on the land. Actually, it is impossible to grow all types of crops in the fixed land the farmer has. Moreover, it is not profitable and feasible to grow many possible crop types due to management problems, as different varieties of crops need variety of machinery and other tools, tillage practices, etc..., which make the farm more complex and expensive from small and medium scale farming points of view. Therefore, the farmer's specific problem is then to select a profitable combination of crop and livestock production strategies. That is, the farmer's major problem is to implement cropping strategies that maximise his profit and at the same time minimise risks taking into account the various interlinked activities of integrated crop-livestock enterprise.

In view of the selection of cropping strategies, the farmer's main problem is which strategy to implement monoculture or crop rotation. Considering crop rotation strategies, the farmer is assumed to follow well defined cropping sequences, which do not change from time to time. Under this assumption, farm resources are seen as components of a crop production system where the objective is to exploit the mutually beneficial interrelationships among individual

crops. Accordingly, this cropping scheme provides the various benefits described in the previous chapter; namely: lowering the incidence of weeds, insects and plant diseases; improving soil quality, balancing the requirement for resources and stabilising of the level of farm profits overtime (El-Nazer & McCarl, 1986).

2.2. *Underlying Assumptions*

The problem described above is a complex one. It requires a clear recognition of the various factors which have an enormous influence in the decision making process of the farm business. Crop production occurs in a complex, biological, agronomical and market dynamics. Since such a complex system offers a formidable challenge to incorporate it into a decision model, representation of such a comprehensive system with a mathematical model is basically not simple. Hence, it is essential to include the following assumptions in the process of developing a mathematical model.

1. It is assumed that the profit, in real terms, remains constant over the period for which the problem is solved. This implies that the cost coefficients in the mathematical model remain constant.
2. The year-to-year variability of the weather conditions of the farm is assumed to be categorised in three discrete states of nature. The three states considered are normal year, dry year and wet year. In this study, these three states of nature are used as the strategies of nature.
3. With reference to the weather conditions with which the decision maker is operating, it is assumed that the farm operates in three possible states of weather conditions. Moreover, the risk of cropping generated from weather variability is modelled as a deviation from the average of the three states of nature.

4. The profit from a crop is dependent on the crop itself as well as the crop that was planted on the same soil in the previous years. Further, it is also assumed that the crop grown current year is dependent on the crop that was planted on the same soil before two years. However, a crop that grew on the same soil three years ago was assumed to have no effect on the current crop. To highlight this assumption, Wassemann (1982) stated that [as quoted in De Kock & Visagie, 1998] the cultivation of a specific crop on the specific piece of land may influence the crops that are planted on the same land because of direct (indirect) influences on the level of plant nutrients, on erosion, as well as on the presence of weeds, pests and diseases. In this paper, the influence of crops that grew a year ago or two years ago is reflected in the current crop by the cost coefficients.
5. The most important objective of this study is to develop an optimal sequence of crops to be planted in the farm. In developing this sequence, it is assumed that the optimal sequence of crops form a cycle (El-Nazer & McCarl, 1986; De Kock & Visagie, 1998). For the purpose of this study, only cycles of one, two, and three years will be considered. De Kock & Visagie (1998) presented the following assertions to justify the above assumption.
 - The computational effort to solve the mathematical problem rapidly increases as the number of cycles increases. Therefore, it is imperative to limit the number of cycles to a reasonable number that can be handled.
 - From the practical perspective of the farmers and the dynamics of the markets, one can argue that the prices of the relevant crops do not remain constant for a long period of time. With this in mind, it is impractical to consider a long cycle, as it is impossible to predict future prices with certainty. Moreover, the longer the cycle is, the higher the chance that price fluctuations will occur so that the current cycle will not be optimal any more.

6. Area of arable land (A) is assumed to be divided into T unit fields (T plots). It is also assumed that the estimated yield of each crop in each field for the specified state of nature is known.

Regarding the complex interdependence between the crop and animal production activities of the farm, the following assumptions are relevant in the farm planning problem.

7. The farm is assumed to be self sufficient in forage and straw production: that is production of forage and straw of the farm must satisfy the animal's consumption requirement for the given planning period.
8. Availability in this paper is used in the sense that the animals receive the required amount of feed and roughage which satisfies the ingredient and nutrient restrictions set by the decision maker.
9. For the sake of simplicity, it is assumed that animal sale and buy transaction decisions are made at the start of the planning period. For that reason, animals bought are considered in the animal feed intake planning and animals sold are excluded from the animal feed considerations. Moreover, no activity related profit is generated from those animals sold, as they are assumed out of the activity in the planning period. The only return from these animals is of course the return from the sale of these animals.
10. In this study, animal types are categorised into three sets, namely, adult cattle, young cattle and sheep. It is assumed that the number of young cattle is always 80% of the adult cattle. Moreover, the only source of revenue from the animal production is revenue from adult cattle and sheep.
11. The loss from animal death and other natural hazards is assumed to be negligible. Consequently, the cost incurred from such circumstances will not be accounted in the mathematical model.

12. For the purpose of this study, it is assumed that all crops produced at the harvest time are sold or used as animal feed in the feed mix preparation in the period of study. This implies that no cost is incurred other than the production cost.
13. The variability of input prices is assumed to be negligible. Furthermore, the risk resulting from the variability of input prices in the integrated crop-livestock production will not be investigated. Input risk considerations are beyond the scope of this study. Generally, the cost of different input components of the farm activity for cultivating a particular crop or managing an animal is considered as one grand cost component for each particular activity.
14. The risk of planting crops resulting from unpredictability of weather changes is reflected on the variability of yields of crops in the three nature states. The risk due to this yield variability of crops is shown by the differences in the income variation of the same cropping strategy at the three different states from the expected value.

3. Underlying Hypothesis and Objectives

The hypothesis underlying this study is that in an integrated crop-livestock farming environment, cropping strategies which rely on crop rotation practices are superior to cropping strategies which are dependent on the practice of monocropping. Further, it is hypothesized that risk affects the choice of resource combination in the farming activity.

The purpose of this study is to point out how the introduction of crop rotation alternatives influence the decision planning of an integrated crop-livestock farm situation in the absence and presence of risk. To investigate both issues, a mathematical model for farm planning will be developed, incorporating the different activities of the farm under consideration.

The more specific objectives of this study are:

1. To determine the optimum maximum profit farm plan which includes an optimum continuously repeatable cropping sequence mix for 1800 hectare of land growing predominantly wheat, canola, silage, lupines and medics; and an optimum dairy and wool sheep production.
2. To investigate the profitability of cropping strategies that employ wheat monoculture and crop rotations.
3. To explore the effect of risk in decision making of the general farming plan by paying special attention to the different cropping strategy alternatives the farmer has.

4. Data

Three sets of data are used in this study. These three sets of data are:

1. Data for the crop production
2. Data for the animal production activities and
3. Data for resources hiring activities of the farm.

The crop production data includes cost of production, price data of crops and yield data of crops for different strategies (see appendix B, C and D). The data are taken from the study taken by Visagie (2004). The crop yield of the five crops, roughage and straw for each of the cropping strategies are presented in appendix D.

The second set of data dealing with the livestock production activities, refer to the annual animal food consumption requirements, nutrient and ingredient restriction. Furthermore, the restriction on the number of animals the farm can keep, profit earned and cost incurred from each type of animal per annum is required to investigate the farm plan. De Kock (2003) and Perry (1982) are used as source of the data used in the model for animal production data requirements.

The third set of data, which represents the capacity data of the Combine Harvester, and Balling machine the farm owns was taken from De Kock (2003).

5. Sequence of Chapters

The aspects crop rotation and risk and uncertainty programming modelling from literature are outlined chapter 2. The evolution of crop rotation modelling in the past years will be presented in the first section of this chapter.

Since risk and uncertainty play an important role in agricultural decision making, a brief discussion will be presented in section two of this chapter. Furthermore, section two of this chapter will present a preview of different risk and uncertainty mathematical programming models from literature, which are applicable in the farm planning situation.

In chapter 3, a mathematical model for the investigation of the problem stated will be formulated. This chapter consists of 10 sections. Section 1 gives an introduction on the development of a mathematical model. Section 3 will focus on the defining the indices, variables and parameters necessary for the development of the model. In section 3, a mathematical crop rotation model will be developed. Section 4 will introduce risk as a variability of income into the mathematical model as constraint. Sections 5 and 6 focus on the animal feed and availability constraints of the mathematical model. The techniques of mathematical representation of resources renting, storage capacity and animal sale and buy activities are discussed. The final section of this chapter presents the objective function of this study.

Chapter 4 outlines the results of the mathematical model for the problem under investigation. Based on the results of the mathematical model, farming plans for different situations will be analysed. This chapter will present the mathematical model solution solved by What'sBest! ® 7.0 optimization software. In order to investigate the problem for different farm situations

the mathematical model is solved under different assumptions. Section 2 focuses on the development of data used in the mathematical model. Section 3 presents the mathematical model results for the farm plans under assumptions of monocropping and crop rotation for different states of nature conditions without risk. The results of the model when introducing risk in the model is illustrated in section 4. Sensitivity analysis on the mathematical model for risk, number of strategy and crop prices are examined in section 5.

Chapter 5 deals with a short summary of the study and presents some recommendations future for study.

Chapter II

Literature Review

1. Crop Rotation Modelling Background: A Literature Review

According to El-Nazer & McCarl (1986), the economics of rotations have been studied for many years. It was well understood that in order to understand the economic impact of crop rotation in agricultural activities, a mathematical model is required in order to choose the best alternative from the existing feasible alternatives.

The early theoretical discussion of crop rotation selection was done by Heady (1948), as quoted in El-Nazer & McCarl (1986). Following Heady (1948), based on El-Nazer & McCarl (1986) exposition, Hildreth & Reiter (1951) developed crop rotation modelling approaches in one of the first (1949) Linear Programming conferences on Linear Programming applications in USA. In their modelling, they specified alternative linear programming activities as a sequence of crops (rotations). They developed a model to select the optimum combination of crop rotations. Peterson (1955) presented a linear programming model in which crop rotation and a livestock enterprise are selected simultaneously.

One important limitation of the literature on the earliest rotation modelling approaches as El-Nazer & McCarl (1986) described, concerns the flexibility permitted in the choice of rotations. For instance, all the studies carried out following Hildreth & Reiter (1951) crop rotation modelling define activities in terms of explicit crop sequences. In these studies, the following explicit activity definitions were considered rigidly.

- Three years of corn
- Three years of hay

- Two years of corn followed by one year of hay
- One year of corn followed by two years of hay.

In a similar approach, Beneke & Winterboer (1973) [quoted in El-Nazer & McCarl (1986)] presented fixed and rigid sequences of crop rotation activities in their example of crop rotation modelling.

The above examples of crop rotation models are based on an explicit configuration of sequences of crops to grow on a given land. As a result of the explicit sequential method of crop rotation modelling, there is a limit in the choice of crop rotations to the combinations that the modeller wants to develop (El-Nazer & McCarl (1986)). Furthermore, model size and data availability considerations are additional limitations of such models. That is, in such modelling approaches one has no freedom of developing different rotation options.

As explained above, historically crop rotations have been modelled using explicit predetermined rotations. To get rid of the limitations mentioned above, Burt (1963, 1982) suggested an alternative approach [as cited in El-Nazer & McCarl (1986)] defining dynamic programming states provisional on preceding crops.

As pointed out by El-Nazer & McCarl (1986) the choice of crop rotation model can occur in either a dynamic disequilibrium or a timeless equilibrium setting. In this modelling approach, either multiyear linear programming model (Loftsgard & Heady, 1959; Irwin, 1968; Dean & Benedicts, 1964) as cited by El-Nazer & McCarl (1986)) or the dynamic programming model (Burt & Allison (1963); Burt (1956, 1982) cited by El-Nazer & McCarl (1986)) was employed to represent crop rotation in a mathematical model. Both approaches assume the crops chosen in year t to depend on the crops grown in the same land in year $t-1$. In such models, El-Nazer & McCarl (1986) stated that the early period solutions depend upon the

initial conditions. Nevertheless, the models solution tends to stabilize after a few periods. In crop rotations, multi period linear programming models are used to capture the carry over effects of the rotation system on soil fertility, terminal land value overtime, etc... (Baffeo, et al., 1986).

El-Nazer & McCarl (1986) developed a mathematical crop rotation model which allows for rotations to be developed endogenously, with the aims of identifying an optimum long-run crop rotation strategy and its sensitivity to risk attitude. In their modelling approach, they applied an annual, timeless equilibrium model formalized by Throsby (1967) instead of the multiyear linear programming model based on the firm growth model developed by Loftsgard & Heady (1959)[cited by El-Nazer & McCarl (1986)]. The annual equilibrium approach assumes that the present farming environment should not influence rotation choice; that the interrelationship data are not readily available and that the switch to the optimal rotation was short enough to neglect the time path of adjustment. This alternative modelling approach uses an annual, timeless, equilibrium model. In this case, a continuously repeatable crop rotation is chosen. El-Nazer & McCarl (1986) argued that the solution generated by using this approach corresponds to the stabilised solution of the disequilibrium model and noted that the solutions do not depend on the initial conditions, rather giving a long-term plan.

Clark (1989) developed a linear programming model for crop rotation and crop diversification, which is continuously repeatable. Clark's model ignores the agronomic and biological interdependence of crops. That is, the model was built on the assumption that present crop yields are independent of crops grown previously. This is a restrictive assumption, because ignoring the advantages of crop rotation in the model can lead into incorrect choice of crop rotations. However, his model was built based on the fact that crops

take different periods to mature and have different yield properties when planted at different dates during a year.

The model was applied to a subsistence farm data from Bangladesh in conjunction with nutrient constraints. The problem was solved using a linear programming code and an optimal solution was generated with another alternative solution. The linear programming solution generated by the model indicates the sequences of crops that can be grown and the optimal planning horizon.

El-Nazer & McCarl (1986) crop rotation model considers n crops and the major assumption of the model is that the yield of a crop grown in a particular year depends upon the crops grown on the same land in the previous three years. The linear programming rotation model constructed by El-Nazer & McCarl (1986) is represented in the following maximum profit rotation model.

$$\begin{aligned}
 & \text{Max} \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{r=1}^n C_{ijkl} X_{ijkl} \\
 & \text{Subject to} \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{r=1}^n X_{ijkl} \leq A \\
 & \sum_{i=1}^n X_{ijkl} - \sum_{m=1}^n X_{jkrm} \leq 0 \text{ for all } j = 1, 2, \dots, n; k = 1, 2, \dots, n; r = 1, 2, \dots, n \\
 & X_{ijkl} \geq 0
 \end{aligned} \tag{2.1}$$

Where X_{ijkl} is the acreage of crop i , which is planted following crops j , k and r in the preceding years (j in year $t-1$, k in year $t-2$, and r in $t-3$). C_{ijkl} is the return coefficient in the objective function in which the objective sums the returns from planting of all possible four-year crop sequences under the total acreage available (A).

El-Nazer & McCarl (1986) stated that the crop rotation constraints (2nd constraint) are the key elements in the mathematical crop model. Moreover, the timeless equilibrium –continuously repeatable crop plan is an important aspect of the model.

According to the exposition of El-Nazer & McCarl (1986), the above described mathematical model of crop rotation has been used in a number of studies, particularly in USA and Canada. McCarl et al. (1977) [as cited in El-Nazer & McCarl, (1986)] used a variant of the model for double cropping where both the preceding crop, and the timing of the preceding crop influence yield. In another study, which is published in the Purdue University Agricultural Experiment Station Bulletin, McCarl (1982) [as cited by El-Nazer & McCarl (1986)] shows where current return only depends on the immediate preceding crop. The model also includes within year time considerations. The model was also applied by Musser et al. (1981) for vegetation crop rotation modelling, within year crop sequences, double cropping and triple cropping were permitted.

However, due to various factors, the model has a drawback. The major drawback of El-Nazer & McCarl (1986) formulation is that the complexity of the model increases enormously with the increase of crops in the model. Particularly, the number of the rotation linkage constraints increases greatly with the increase of the number of crops in the model. Another drawback of the model is the availability of data, namely that it requires a vast set of data.

Built on the same premise as El-Nazer & McCarl (1986) crop rotation model, De kock & Visagie (1998) developed a linear programming rotation model under the assumption that optimal sequences of crops form a cycle of three years and shorter. This study will follow the crop rotation model formulated by De Kock & Visagie (1998) in developing strategies of crop sequences for the composite crop –livestock mathematical model.

2. Review of Modelling Risk and Uncertainty Using Mathematical Programming Techniques: *Selected Literature*

Risk and uncertainty influence the efficiency of resource use in agriculture and decision making process of farmers in their farming activities. Risk is generally considered as a strong behavioural force affecting decision-making. At present, there is much debate amongst theoreticians and applied researchers on research issues related to risk and uncertainty.

The more specific objective of this subsection is to give a partial review on the literature of risk and uncertainty and their modelling aspects. That is, this section is basically a literature review dealing with the general concepts of risk and uncertainty in the agricultural decision making process and gives weight to the review of mathematical programming models which deal with risk and uncertainty modelling. It is not an exhaustive review, and is not intended to be.

Before discussing literatures on risk programming modelling, it is appropriate to define risk and uncertainty. Risk and uncertainty have been defined in different ways depending on the purpose in the mind of the researcher. According to Knight (1921, reprinted in 1965), the distinction between risk and uncertainty is that risk is a condition where probabilities of outcome are known, whereas uncertainty is a condition in which probabilities associated with the outcome are not known. In delineating the degree of knowledge in a decision situation, he further proposed three major categories of decision-making. These are: perfect knowledge, risk and uncertainty. Roumasset (1974) describes this difference as follows: uncertainty is a state of mind, in which the individual recognises alternatives to a particular action. On the other hand, risk has to do with the degree of uncertainty in a given situation. Barry (1984)

draws on uncertainty to indicate imperfect knowledge on the part of the actor and defines risk as the possibility of incurring a loss of production.

The above viewpoints highlight that there is no straightforward agreed definition of risk and uncertainty. However, current popular usage implies that there is very little distinction between risk and uncertainty (Barry, 1984).

Agricultural production is a risky business. Farmers face a variety of price, yield and resource risks, which make their incomes unstable from year to year. Based on an imperfect information, a farm firm makes a decision under price and output uncertainty. The outcomes of a particular decision are revealed *ex post*, i.e., after the uncertainty is resolved. Since the decision has to be made *ex ante* (i.e., before the uncertainty is resolved) it has to be evaluated based on *ex ante* information (Hardaker, Huirne, & Anderson, 1997).

Based on Anderson, Dillon & Hardaker (1977), static economic analysis is based on simplified assumptions of certainty about the production environment and an objective of profit maximisation. Linear programming models for farm activities are based on this premise. That is, linear programming models are based on expected return rather than sure activity returns. Ideally, the solutions generated by such modelling tools would not satisfy risk-averse farmers. Introduction of risk extends such concepts to include the decision maker's perception of risk and his/her attitude toward risk (Barry, 1984).

The omission of risk in farm level decision models may lead to results that bear little if any similarity to farmers' actual behaviour (Anderson, Dillon & Hardaker, 1977). Agricultural decision models that do not include risk considerations may overestimate outputs of risky activities and fail to recognise the importance of diversification in agricultural productions

systems (Wolgin, 1975). Ignoring risk may also lead to over valuation of some inputs and lead to incorrect prediction of technology choices (Hazell, 1982).

Empirical applications of behavioural models and theoretical considerations indicate the importance of incorporating risk into analysis of agricultural decision making at the farm level. Risk from market, production, environment, and policy factors ...etc will always exist in agricultural decision making (Mapp, et al., 1979). Subsequently, it is appropriate to take into account risk in agricultural decision making (Anderson, Dillon & Hardaker, 1977; Barry, 1984).

Various risk-modelling techniques have been developed in the past 40 years to address risk in agricultural decision-making. A number of risk concepts models and their analytical implementation exist in literature (Anderson, Dillon & Hardaker, 1977). Three approaches to risk and uncertainty programming have been reported. Risk and uncertainty concepts and hence, risk and uncertainty mathematical programming models are classified into three major categories: (1) those requiring no probability information or game theoretic models, (2) safety first approaches and (3) expected utility maximisation (Young, 1984). A brief review of the existing major modelling approaches in the evaluation of risky alternatives in agricultural decision-making, which are based on the above categories, follows below.

2.1. *Game Theoretic Approach*

The conventional game theory formulation is where each player has a number of possible actions, and each set of choices and actions by the players has a consequence, whose 'utility' is typically different for each player. The conventional strategy is that for each player to choose an action for which the worst outcome over all the other player's assignments is best or least bad. This framework can be used to capture uncertainties in agricultural decision-making.

Game theory decision models are one of the main conventional approaches to agricultural decision making under uncertainty (McInnerney, 1967 & 1969). In this approach, the decision maker's problem is described as a two person zero sum games. All the risk and uncertainty components facing the decision maker can be summarised as a composite 'Nature' component (Hazell & Norton, 1986). Such games are called games against Nature. In the game theory modelling framework a clear definition of nature and the decision maker is important. The following can be cited from Hazell (1970) about the definition of nature.

"...All competitive forces and uncertainty facing a farmer can be summarised as a composite "Nature" component. Thus defined, Nature can be considered an opponent in a two person zero sum games, who, perhaps, randomly rather than wilfully, may financially undo a farmer in his selection of a farm plan, each superimposing its own utility assumptions on the model."

Maltitz (1969) also describes nature as complex and all encompassing opponent as follows.

" ...Nature represents the spectrum of uncertainty in the social system and biological complex within which the farmer operates. The farmer has a range

of possible alternative courses of action that he can select a certain combination of enterprises and resource levels. The set of states of nature characterises conditions of weather, resultant prices and other inherent uncertainty phenomena which the farmer can neither control nor predict.”

Following Romero & Rehman (1989), the main features of game theory models (game against Nature) can be summarised as follows.

1. The existence of a decision maker (farmer) who is considered as the only rational player of the game.
2. The decision maker (farmer) has a set of n possible sets of strategies or actions to follow.
3. The existence of a set of h different possible states of nature representing the uncertainties within which the decision maker operates.
4. The game is of the form $n \times h$ matrix whose elements represent the outcome of the game when the decision maker chooses the i^{th} strategy to face the τ^{th} state of nature.

As described above, the aim of a game theoretic model is to find a pure or mixed strategy that optimises the wishes and aspirations of a decision maker under different constraints and limitations of resources. This is based on the idea that game theory assumes all important states can be enumerated but avoids an explicit assumption about the probabilities of future occurrence (Hazell, 1970). This approach was introduced to agricultural decision making by McInnerney (1967, 1969). McInnerney (1967) defined a set of available strategies as those that corresponded with feasible set of an ordinary linear programming problem. He defined the payoff matrix of the games as the observed gross margins over a few past years.

A number of criteria have been used to represent the aspirations of a decision maker in the game theoretic model, chiefly in a game against Nature of agricultural planning (Hazell &

Norton, 1986 and Romero & Rehman, 1989). The most prominent of such criteria are briefly discussed in Hazell & Norton (1986) and Romero & Rehman (1989). Some of the well-known criteria's are presented below.

1. The Maxmin (Wald Criterion)

This criterion assumes that the farmer searches for a strategy, which offers a maximum of the minimum output. That is, the farmer looks for a strategy that maximises the outcome that can be achieved in the worst possible state of nature. In other words, the decision maker examines the worst outcome for each action and then selects the action that maximises the minimum gain from the proposed plan.

In order to represent this criterion in a mathematical model, McInnerney (1969) developed a linear programming model and used this criterion to derive a Maxmin solution for constrained farm planning problem. Following his formulation, the criterion can be represented using the following linear programming model.

$$\begin{array}{ll}
 \text{Max } Z & \\
 \text{Subject to} & \\
 C_i X \geq Z & \\
 AX \{ \leq, \geq, = \} B &
 \end{array} \tag{2.2}$$

Where

Z is the worst possible outcome of farm income,

X is the vector of activity levels X_i ,

A is the matrix of linear programming coefficients,

B is the right hands of the matrix (activity levels),

C_i is the vector of the observed gross margin $C_{i\tau}$ of activity i during state of nature τ ,

$\tau = 1, 2, \dots, h$

According to the investigation of Hazell and Norton (1986), the Maxmin criterion is very conservative, and often leads to farm plans with such low total gross margins on average in relation to overhead costs and decision maker income needs. As a result, it would not be acceptable to the decision maker. However, they stated that the idea of minimising the worst loss is appealing. Moreover, unlike the E-V models, higher incomes in favourable years are not penalised by the Maxmin decision criterion.

Hazell (1970) and Kawaguchi & Maruyama (1972) independently suggested an addition of an expected income constraint to the McInnerney (1969) formulation of the farm planning problem model. This constraint further provides a useful method of analysis to the criterion. Hazell's (1970) parametric formulation of the model is a slight modification of McInnerney's (1969) formulation and is given in the following format.

$$\begin{array}{ll}
 \text{Max } Z & \\
 \text{Subject to} & \\
 C_i X \geq Z & (2.3) \\
 AX \{ \leq, \geq, = \} B & \\
 \bar{C} X = \lambda &
 \end{array}$$

Where \bar{C} is the expected value of C_{it} and λ is the target for the expected total farm income.

This formulation is an analogue of the Parametric Markowitzean (Freundean) model. The model enables the analyst to draw an indifference curve λ versus Z as was done with the E-V model formulation when an indifference curve was drawn between λ and σ^2_Z .

McInnerney's (1969) model and its slight modifications have been applied by many authors throughout the world in agricultural decision analysis. Agrawal & Heady (1968), Tadros & Casler (1969), Hazell (1970) and Kawaguchi & Maruyama (1972) are some of the examples.

2. The Minimax Regret (Savage Criterion)

This criterion is based on the assumption that a decision maker wishes to minimise the regret that he/she experiences after having made a decision (McInnerney, 1969). The first step in this criterion formulation is the construction of the payoff matrix, called the 'regret matrix'. The elements of this matrix represent the difference between the outcome actually achieved and the maximum outcome the decision maker could have achieved had he known the prices and state of nature that would have prevailed. Based on this matrix, the Savage criterion looks for a strategy which involves the maximum possible "regret" that any state of nature can produce. That is, the Savage regret Criterion focuses on the largest of these regrets over all states of nature and calls for the minimum value of the maximum regret.

The original formulation of this game theoretic decision modelling is given by McInnerney (1969); and is given as follows.

$$\begin{array}{l}
 \text{Min } V \\
 \text{Subject to} \\
 R_{\tau}X \leq V \\
 AX \{ \leq, \geq, = \} B \quad [\tau = 1, 2, \dots, h] \\
 aX = b \\
 X, V, b \geq 0
 \end{array} \tag{2.4}$$

Where V is the largest total regret that nature could inflict from any of the h possible states, R_{τ} is the $n \times m$ matrix of regrets in which each elements of the regret matrix are calculated as, $R_{it} = (\max_i C_{it}) - C_{it}$.

In line with McInnerney (1969), this model is designed to identify a feasible farm plan which minimises the largest possible regret that nature could inflict from any of h possible states of nature, through linear programming. However, according to the analysis of Hazell (1970),

McInnerney's (1969) model has definite problems. One of the problems mentioned by Hazell (1970) is that optimisation of the programming problem depends on the acreage level. He further points out that McInnerney (1969) assertion is incorrect. The second problem mentioned by Hazell (1970) in the McInnerney's (1969) formulation is that the regret component formulation is incorporated into the model due only to the farm constraints rather than to uncertainty. Based on Hazell's (1970) analysis V (largest total regret) has no meaning.

To take care of the above mentioned drawback, Hazell (1970) introduced the use of direct measure of regret. The direct measure of risk is based on the ordinary linear programming solved for different states of nature. Let g_τ be the linear programming solution for the problem of the τ^{th} state of nature. The mathematical representation of the model is given below.

$$\begin{aligned}
 & \text{Min } V \\
 & \text{Subject to} \\
 & g_\tau - C_\tau X \leq V \\
 & AX \{ \leq, \geq, = \} B \quad [\tau=1,2,\dots,h] \\
 & a'X = b \\
 & X', V, b \geq 0
 \end{aligned} \tag{2.5}$$

Furthermore, Hazell (1970) proposed the following parametric linear programming adaptation of the Maxmin criterion for farm planning

$$\begin{aligned}
 & \text{Min } V \\
 & \text{Subject to} \\
 & g_\tau - C_\tau X \leq V \\
 & AX \{ \leq, \geq, = \} B \quad [\tau=1,2,\dots,h] \\
 & \bar{C}X = \lambda \\
 & X', V, \lambda \geq 0
 \end{aligned} \tag{2.6}$$

In the above formulation, λ is parameterised to provide an efficient income (E) and regret (V) set of plans.

3. The Benefit Criterion

Agrawal & Heady (1968) have argued that Wald's Maxmin criterion is pessimistic leading to a conservative solution. On the other hand, according to their analysis of Savage's criterion, it is optimistic leading the decision maker to choose a risky solution. As alternative to both approaches, Agrawal & Heady (1968) offered "The Benefit criterion" which combines the properties of both Wald's criterion and Savage's criterion.

In this approach, the benefit matrix is formulated from the payoff matrix. The elements in this matrix represent the differences between the outcome actually received by the decision maker and the minimum he/she could have achieved under the worst state of nature. The next step in this approach is the selection of a strategy that maximises the minimum possible benefit under any state of nature. The Benefit criterion is less optimistic than the regret criterion and less pessimistic than the Wald's criterion, based on the argument of Agrawal & Heady (1968).

The above discussed class of models requiring no probability information are commonly referred to as *game theoretic models*. These types of models assume that decision makers have no objective information or subjective feeling about the probabilities associated with alternate outcomes. On the other hand, they totally ignore whatever information the decision maker may have. The main criticism of such models stems from this point of view. Anderson, Dillon & Hardaker (1977) explain that such models can be criticised on the ground that the decision criteria employed are incompatible with the axioms of rational choice underlying decision analysis.

Game against Nature models assume that nature acts as a conscious opponent of a decision maker, that is it strives to limit the expected payoff to the minimum. Such a view may suit to pessimistic or cautious decision maker, but may not be attractive to an optimistic decision maker. Critics argued that assumption that Nature is malicious is not possibly accurate. According to Kmiettowicz & Pearman (1981) description, Nature neither consciously favours a decision maker nor hinders him. As a result, the application of such models is fairly limited in the farm planning modelling.

2.2. *The “Safety First” Approach*

According to Robinson, et.al (1984), the safety first approach to risk programming is commonly used in risk analysis as a form of lexicographic utility. To be precise, this approach to risk management is applicable if a decision problem aim is first to a preference for safety (such as minimising the probability of bankruptcy) when making decisions in the agricultural activities. This means that only when the safety goal is met at a threshold level the other goals can be addressed. Thus, the highest priority goal serves as a constraint on goals that have successfully lower priorities (Bigman, 1996).

Safety-first mathematical programming methods are particularly applicable where survival of the business is of paramount concern. However, in most business risk management situations, the use of safety-first methods is somehow arbitrary, as no single goal can be clearly dominant from a set of goals the firm has.

As explained by Robinson et al. (1984), the safety first criterion can be specified in various ways of empirical formulation. The first type was introduced by Telser (1955). As described in Robinson et al. (1984), this method assumes that the decision maker maximises expected return ($E(y)$) subject to the constraint that the probability of returns less than or equal to a specified disaster level (Y_{minimum}) does not exceed a given probability. Mathematically Telser’s (1955) approach is expressed as follows.

$$\begin{aligned}
 & \text{Max} E(y) \\
 & \text{such that} \\
 & \text{Pr ob}(Y < Y_{\text{min}}) \leq p
 \end{aligned}
 \tag{2.7}$$

The second safety-first approach was introduced by Kataoka (1963). This approach selects a plan that maximises return at a lower confidence limit (L) subject to the constraint that the probability of return being less than or equal to the lower limit does not exceed a specified value of probability. (Robinson et al. (1984)). Mathematically,

$$\begin{aligned} & \text{Max} L \\ & \text{such that} \\ & \text{Pr ob}(E(Y) < L) \leq p \end{aligned} \tag{2.8}$$

The third type of safety-first approaches was developed by Roy (1952), as described in Robinson, et al (1984) and involves choosing the set of activities with the smallest probability of yielding an expected return below a specified disaster level of return (Y_{\min}). Mathematically this approach can be expressed as in the following format.

$$\min P(E \leq E_{\min}) \tag{2.9}$$

The topics that have been addressed by the above mathematical modelling approaches of the safety-first method vary widely. Optimal Hedging (Telser, 1955), Dynamic cropping decisions (van Kooten, Young & Kran, 1997), farm extension programs (Musser, Ohannesia & Benson, 1981), attitudes toward risk regarding fertilizer applications among peasants in Mexico (Moscardi & Janvry, 1977), a discrete stochastic farm management model with chance constraints to access the risk-income tradeoffs associated with buying, selling and producing at alternative fish growing stages (Hatch, Atwood, & Segar, 1989) are some of the topics investigated by such risk programming model..

The drawback of the first and second safety-first approaches is that they are not generally compatible with the general utility theory (Bigman, 1996). According to the explanation of Bigman (1996), these safety first criteria need not satisfy the continuity and independence

axioms. Even though the third approach (Roy's criterion) can be derived from a utility function, Bigman (1996) pointed out that in general this approach does not strictly rise with a rise in safety threshold.

2.3. *The “E-V” Approach (Quadratic Programming)*

A classic problem in uncertainty and risk analysis involves determining an optimal allocation of resources across a range of risky alternatives. It is well understood that risk-averse investors seek to reduce the effect of uncertainty and risk in the expected returns from a portfolio of assets. This modelling approach is one of the modelling tools based on expected utility maximisation theory (Anderson, Dillon & Hardaker 1977).

The first attempt to take explicit account of risk in mathematical programming formulations of a farming activity-planning problem were by quadratic risk programming. It is a non-linear mathematical programming based on the assumption that utility is maximised in terms of the mean-variance of the probability distribution of total revenue. Markowitz (1952) provides a theoretical foundation for portfolio selection employing first two moments of return distributions. According to Markowitz(1952), given various combinations of mean (E) and variance (V) there exists a set of efficient E-V combinations. His problem was to select an optimal portfolio of stocks under a budget constraint. Markowitz(1952) solved the problem in the context of selecting optimal stock portfolios to find the set of allocation that maximises expected total return. If a decision maker can state which E-V combination from an attainable set he/she prefers, one could then find a portfolio, which gives the desired combination. That is, Markowitz (1952, 1959) provided a means to quantitatively compare potential portfolios and select those with minimum risk given an expected return.

Markowitz (1959) described the portfolio quadratic programming framework and specified the objective to minimise portfolio variance for alternative levels of expected return. To obtain an efficient E-V set, it is required to minimise the variance (V) for each possible level

of expected income (E), while retaining feasibility with respect to available resource constraints and other activities. The relevant programming model to achieve such possible level of expected income is described by the following mathematical model (Anderson, Dillon & Hardaker 1977; Hazell & Norton, 1986).

$$\text{Minimize } V = \sum_{i=1}^n \sum_{j=1}^n X_i \sigma_{ij} X_j$$

Subject to (2.10)

$$\sum_{i=1}^n \bar{C}_i X_i \geq \lambda$$

$$\sum_{i=1}^n a_{ij} X_i \{ \leq = \geq \} b_j, j = 1, 2, \dots, m$$

$$X_i \geq 0 \text{ for all } i$$

Where \bar{C}_i denote the expected gross margin of each risky investment of activity i ,

X_i represents the level of the i^{th} farming activity.

λ is scalar denoting the risk return trade of coefficient.

σ_{ij} is the covariance returns on activity i and j .

a_{ij} is the coefficient in m linear constraints on the activity levels.

b_j levels of linear constraints ($j = 1, 2, \dots, m$).

Alternative formulation of the above mathematical model exists in literature. The formulation of the risk programming problem using quadratic programming differs depending on the choice of the analyst. Another alternative approach (Freund, 1956) is to maximise a quadratic function of activity levels subject to linear constraints (Anderson, Dillon & Hardaker 1977).

That is,

$$\text{Max } \sum_{i=1}^n \bar{C}_i X_i - \lambda \sum_{i=1}^n \sum_{j=1}^n X_i \sigma_{ij} X_j$$

Subject to (2.11)

$$\sum_{i=1}^n \bar{C}_i X_i \geq \lambda$$

$$\sum_{i=1}^n a_{ij} X_i \{ \leq = \geq \} b_j, j = 1, 2, \dots, m$$

$$X_i \geq 0 \text{ for all } i$$

The above mathematical problem is solved iteratively through parametric variations in λ to define the risk efficient (minimum variance) solutions (Hazell & Norton, 1986). This is to say that one has to trace out the E-V frontier and the quadratic programming problem must be solved parametrically as the risk aversion coefficient λ varies from 0 to ∞ . If the decision maker is assumed to be risk neutral ($\lambda=0$), the problem collapses to income maximisation, which can be solved with ordinary linear programming algorithms.

Following Markowitz's (1952) influential article, a large body of literature on portfolio analysis has focused on the securities of markets for which the theory was originally developed. Portfolio theory has also been extended to various decision making problems including agricultural crops (Collins & Barry, 1986; Stovall, 1966). The oldest (Freund, 1956) approach of risk programming in agricultural planning is a straightforward application of the method proposed by Markowitz (1952). In this method, agricultural risk of different agricultural activities is measured by the variability of returns using the variance as the index. According to this method, low risk activities have relatively low variance, which means their returns are concentrated around the mean value. On the other hand, high-risk enterprises have relatively large variance. However, variations in gross margin due to fluctuations in yield, price etc... are considered and are assumed to follow a normal distribution; which is the main drawback of the method.

The E-V efficiency criterion can be used in allocating a farm's resources among risky alternatives. A risk-averse decision maker desires high expected return and low variance of

return. The optimal combination of activities for the farmer occurs at the point on the E-V frontier that provides the preferred combination of expected return and variance of return. On many occasions, this approach has been applied to farming decisions, particularly to decisions about enterprise choice and diversification (Anderson, Dillon & Hardaker, 1977). According to this risk programming method, E-V efficient combinations of crop and livestock enterprises can be identified and the combination that offers the preferred mix of expected return and variability of returns can be chosen.

Freund (1956) was one of the first to apply quadratic programming to a farm firm problem. Freund's model contained four production activities and several resource constraints for a representative farm. His application involved the evaluation of four production activities and several resource constraints on a representative farm in USA. He recognised that the introduction of risk into a programming model reduced both the level and standard deviation of net return. Moreover, diversification was explained as a rational choice of expected utility maximisers.

According to the investigation of Freund (1956), the solution of an ordinary deterministic linear programming was quite different from and much better (in terms of income) than the actual crop and livestock system worked out by the farmers. Moreover, following the conclusion from Freund (1956) the solution of the quadratic programming model was quite similar to the results obtained by the farmers and further concluded that the behaviour of the farmers was rational and optimal and that the differences between the actual plans and those obtained by linear programming were only due to neglect of risk considerations. The risk consideration has a somewhat heavy cost since the mean income of the optimal plan without risk was more than 50% higher than the income of the optimal plan with risk (Freund, 1956).

This was repeatedly confirmed by all empirical studies made about the introduction of risk into a decision model (Anderson, Dillon & Hardaker, 1977).

The use of E-V risk modelling approach has both advantages and disadvantages. Some of the limitations cited by different researchers are briefly mentioned as follows.

First, the necessity of using a quadratic programming algorithm was considered as a severe constraint in studies made in the 1970's and 1980's (Hazell & Norton, 1986). Due to the fast growing of the computational capability of different solvers, however, this is no more considered a bottleneck though the computational complexity is high compared to linear programming solution algorithms.

Second, estimation of the variance-covariance matrix presents numerous methodological pitfalls. Preferably, variance-covariance should be based on the subjective evaluation of the decision maker (Barry, 1974). Most of the studies undertaken have taken objective measure of variability based on historical data. In such modelling activities, key decisions include identifying relevant sources of risk, collecting the appropriate data as in the case of crop yields or prices, selecting the appropriate length of the historical series etc. There is also a need to distinguish between known patterns of variation (trends, cycles, seasonal) and a random variation.

There are different viewpoints from previous studies in the estimating process of the data that are to be used in the quadratic programming model. Some believe that producers base their plan on the long-term mean of historical series of returns and that any deviation from the mean is considered as a random event (Barry, 1984). Measuring variability as variance is consistent with this view. Others have approximated the expected outcome based on linear or polynomial trends. Another approach is to measure the unexpected variation as deviations

from the expected components of a moving average (Brink & McCarl, 1978). Further advanced approaches include measuring random components in terms of first differences of the data series; utilising first through k^{th} order differences and using autoregressive integrated moving average (ARIMA) models, a moving weighted auto regression model or a moving weighted linear time trend model.

Third, specifying the risk aversion coefficient is arbitrary, yet it is very critical in determining a risk efficient farm plan. There is no clear-cut measure of the risk aversion coefficient. One might derive the entire efficient frontier and present the set of farm plans to the decision maker.

Fourth, the assumption that returns are normally distributed about the mean is another drawback of the approach, especially in observations which have skewed distribution. Moreover, the calculation of variance-covariance matrix is a problem (Hazell, 1971).

Fifth, variance being a measure of risk equally penalises both the upside and downside risk. However, from agricultural producers' viewpoint, the downside variation is the important aspect of risk that the farmer needs to minimize (Hazell, 1971).

2.4. MOTAD

A linear programming alternative for the E-V approach that has been widely used in agricultural decision-making practice was developed by Hazell (1971). Hazell (1971) reduced the minimisation of variance to minimisation of mean absolute deviations (MAD). The technique is called MOTAD. The acronym MOTAD stands for minimisation of total absolute deviations. Hazell's (1971) article "*a Linear Alternative to Quadratic and Semivariance Programming for Farm Planning Under Uncertainty*" is the basis for the application of MOTAD.

One concern associated with E-V formulation is that it results in a quadratic objective function. Until the 1980's this had been a concern to some researchers given the complexity of a nonlinear model and the limitation of computational solvers. As a response to this problem, Hazell (1971) developed the MOTAD, which linearly approximates E-V results based on total absolute deviations. An additional concern that has been raised is that the assumption of a quadratic utility function is quite restrictive. If this assumption is imposed, it implies that absolute risk aversion increases with the level of payoff (Hardaker, Huirne, & Anderson, 1997).

This approach closely parallels the quadratic programming (E-V) approach. However, this risk programming model does not need a non-linear algorithm for solution elicitation purposes. It enables one to deal with an ordinary linear programming model rather than solve the quadratic programming model. Hazell (1971) showed that with some manipulation this measure of risk can be incorporated into an enlarged linear programming model of a farm planning problem in such a way that mean absolute deviations can parametrically be

minimised for a given level of expected profit value over the relevant range. Hazell & Norton (1986) further commented that this approach is more relevant when mean absolute deviation of a farm income is estimated using time series (or a cross sectional) of a sample data.

Hazell's (1971) approach in his article was twofold. He first sets out to develop review variance as a good methodology under certain assumptions. Then he raises two major problems within the E-V approach in farm planning. The first problem is the availability of a code (solver) to solve the quadratic programming implied by E-V. The second problem is the estimation problem. Specifically, the data required for the E-V are the mean and variance matrix. However, the variance matrix is an artefact of the assumption of normality.

Following Anderson, Dillon & Hardaker (1977), the MOTAD risk model can be described in the following way.

Let X_i be the level of activity i in the portfolio.

$C_{\tau i}$ ($\tau = 1, 2, \dots, h$; $i = 1, 2, \dots, N$) be the net revenue of observation for the i^{th} activity for the state of nature τ .

Let C_j be the expected return on activity i .

The most important argument that Hazell (1971) used in the estimation problem is the variance in the E-V formulation is estimated by:

$$\sigma^2 = \sum_{i=1}^N \sum_{j=1}^N X_i X_j \left[\frac{1}{h-1} \sum_{\tau=1}^h (C_{\tau i} - C_i)(C_{\tau j} - C_j) \right] \quad (2.12)$$

This equality can be reformulated as:

$$\sigma^2 = \frac{1}{h-1} \sum_{\tau=1}^h \left[\sum_{i=1}^N C_{\tau i} X_i - \sum_{i=1}^N g_i X_i \right]^2 \quad (2.13)$$

Hazell (1971) suggests replacing this objective function of the E-V model with mean absolute deviation as follows.

$$M = \frac{1}{h} \sum_{\tau=1}^h \left| \sum_{i=1}^N (C_{\tau i} - C_i) X_i \right| \quad (2.14)$$

Thus, Hazell (1971) argues that instead of minimising the variance of the farm plan subject to income constraint, one can minimise the absolute deviation subject to an income constraint.

Another formulation for this objective function is to let each observation τ be represented by a single row as follows.

$$L_{\tau} = \sum_{i=1}^N (C_{\tau i} - C_i) X_i \quad \forall \tau = 1, 2, \dots, h \quad (2.15)$$

Where L_{τ} is the deviation from the average.

This deviation can be divided into positive from the average, L_{τ}^{+} and negative deviations from the average, L_{τ}^{-} . Therefore, equation (2.15) can be written as follows.

$$L_{\tau}^{+} - L_{\tau}^{-} = \sum_{i=1}^N (C_{\tau i} - C_i) X_i \quad \forall \tau = 1, 2, \dots, h \quad (2.16)$$

Furthermore, following Hazell (1971), the risk programming problem now can be formulated as the minimisation of the sum of the deviation variables, $\left(\sum_{\tau=1}^h (L_{\tau}^{+} + L_{\tau}^{-}) \right)$, subject to the usual resource constraints and to a parametric constraint on the expected total revenue. That is

$$\text{Min} \sum_{\tau=1}^h (L_{\tau}^{+} + L_{\tau}^{-})$$

Such that

$$\begin{aligned}
& \sum_{i=1}^N (C_{\bar{\pi}} - C_i) X_i - L_{\tau}^+ + L_{\tau}^- = 0 \quad (\tau = 1, 2, \dots, h) \\
& \sum_{i=1}^N C_i X_i = \lambda \\
& \sum_{i=1}^N a_{ij} X_i \{ \leq, =, \geq \} b_j \quad \text{all } j \\
& X_i, L_{\tau}^+, L_{\tau}^- \geq 0
\end{aligned} \tag{2.16}$$

Where L_{τ}^+, L_{τ}^- are positive and negative deviations.

Alternatively, the expected farm profit can be maximised with parametric constraints on the negative and positive deviations. That is,

$$\begin{aligned}
& \text{Max} E(Z) = \sum_{i=1}^N C_i X_i - \lambda \left(\sum_{\tau=1}^s L_{\tau}^+ + L_{\tau}^- \right) \\
& \text{such that} \\
& \sum_{i=1}^N (C_{\bar{\pi}} - C_i) X_i - L_{\tau}^+ + L_{\tau}^- = 0, \quad \tau = 1, 2, \dots, h \quad \lambda \text{ is a parameter.} \\
& \sum_{i=1}^N a_{ij} X_i \{ \leq, =, \geq \} b_j \quad \text{all } j \\
& X_i, L_{\tau}^+, L_{\tau}^- \geq 0
\end{aligned} \tag{2.17}$$

The above model can be solved by a parametric linear programming algorithm to obtain an efficient set of plans satisfying the criteria set by the decision maker.

Based on the assumption that farmers are risk averse, the interest of farmers is the minimisation of downside risk; there is a modification to the above model from Hazell (1971) and Hazell & Norton (1986). It is stated that instead of minimising the total deviation it is sufficient to minimise the sum of the negative deviations.

The advantage of this model is that, unlike the quadratic programming model the MOTAD approach does not require a variance-covariance matrix. However, MOTAD does consider the covariance relationship among activities (Hazell & Norton, 1986). Deviations from the mean of the series for each activity are summed across all activities. Positive deviations in

one activity may cancel out negative deviations in another activity, thus accounting for the correlation between activities. This is one of the advantages of the MOTAD approach in risk programming (Hazell & Norton, 1986).

The MOTAD risk programming approach has been used extensively in different agricultural studies. Brink & McCarl (1978) formulated a MOTAD model for Corn Belt farmers in USA to develop a set of farm plans. They developed this model for individual farm data and applied negative deviations from the expected return as a measure of risk. In this study, a set of farm plans was developed for each farmer by parameterising the scalar λ . Mapp, et al. (1979) developed a MOTAD model for a typical farm situation in South Western Oklahoma, USA, and utilised the risk efficient farm plan in a simulation model to evaluate the effects of alternative economic features. In a separate study, Gebremeskel & Shumway (1979) developed a MOTAD model to investigate the risk reducing forage and cattle management strategies. This model was used to determine forage species, fertilizer rates, herd size and the degree of on farm integration. Moreover, Gebremeskel & Shumway (1979) derived annual calf marketing strategies based on observed data for predicting subsequent calf prices forage yield by integrating statistical decision theory with the programming model.

2.5. Target-MOTAD

Target-MOTAD is another risk programming model related to MOTAD. This risk-programming model was developed by Tauer (1983) as an extension of the MOTAD risk-programming model. Target-MOTAD model offers the additional advantage that the solution sets derived are contained in the set of production plans that are second degree stochastic efficient (SSD) (Tauer, 1983; McCamley & Kliebenstein, 1987).

In this programming model, risk is measured as the expected sum of the negative deviations of a solution results from the target income level. Risk is parametrically varied so that a risk-return frontier is traced out (McCamley & Kliebenstein, 1987).

Following Tauer (1983) the mathematical representation of this modelling approach is given below. Given a target income level T_τ , for the state of nature τ .

$$\begin{aligned}
 & \text{Max } \sum_i^N C_i X_i \\
 & \text{Subject to} \\
 & \sum_{i=1}^N a_{ij} X_i \leq b_j \quad [\text{all } j] \\
 & T_\tau - \sum_{i=1}^N c_{\tau i} X_i - Y_\tau \leq 0 \quad [\tau = 1, 2, \dots, h] \\
 & \sum_{\tau=1}^h p_\tau Y_\tau = \lambda
 \end{aligned} \tag{2.18}$$

Where

T_τ represents the target income level for the state of nature τ ,

Y_τ is the deviation below T_τ for state of nature τ ,

λ is the expected deviation below the target level (maximum average income shortfall permitted).

A closer look at the above mathematical programming model shows that the objective function and the first n constraints are identical to the MOTAD risk-programming model.

The development of the Target-MOTAD requires the definitions of two risk parameters: the target income level (T) and the maximum amount of deviation allowed (λ). In turn, λ can be parameterised to yield different solutions reflecting varying degrees of risk aversion. Low λ values indicate little tolerance for risk bearing combinations of production activities. As λ is allowed to increase, the risk constraint is relaxed and new mixes of production activities associated with larger deviation from T , but with higher potential for profit are selected.

According to the arguments of Tauer (1983), the evolution of Target-MOTAD as a risk-programming model was due to the shortcomings existing in the E-V and MOTAD risk programming models. Tauer (1983) further states that if the returns of the farm activities are normally distributed, the solutions found using the E-V approach are SSD efficient and are consistent with the expected utility theory. However, if the normality assumption is not satisfied, using the E-V model the analyst must determine or assume that the decision maker has a quadratic utility function. In this case, the results derived from the E-V method are not necessarily efficient.

The basic advantage of the MOTAD risk programming model is that solutions can be generated by linear programming algorithms. Despite this important advantage of the MOTAD model, Tauer (1983) argues that the results obtained applying the MOTAD model are not necessarily SSD. Based on the analysis made by Tauer (1983), the Target-MOTAD risk programming model has two important advantages. First, the Target-MOTAD has a linear objective function and linear constraints. Therefore, the model can be solved by any linear programming algorithm. Second, the Target-MOTAD formulation can be useful

because agricultural decision makers often wish to maximise the expected return, but are also concerned about net returns falling below a critical target.

1. Introduction

This study develops a novel crop rotation strategy that maximises the expected return while ensuring that the net returns do not fall below a critical target. The model is based on the assumption that the farmer is risk averse and that the net returns are normally distributed. The model is solved using a stochastic programming approach. In both criteria, the model will be applied to determine the optimal crop rotation strategy under the given assumptions.

The maximisation of the income objective is achieved in a way that balances the trade-off between the gross income derived from the different production activities (maize, wheat, soybean, sunflower) and the risk of net returns falling below a critical target. The specific set of interacting parameters that affect the net returns are: land availability and requirement of animals, availability of water, fertilizer, and other inputs, and the risk of net returns falling below a critical target (diversifying crops), risk constraint, etc.

The concepts of risk employed in this paper focus on the risk of net returns falling below a critical target. The concept of risk is defined as the probability of net returns falling below a critical target. The concept of risk is defined as the probability of net returns falling below a critical target. In this study, the risk of net returns falling below a critical target is defined in terms of the levels of net returns associated with the different states of nature described in the assumption's section of this paper.

Chapter III

Mathematical Model

1. Introduction

This study develops a mixed integer linear programming model, which determines the optimal crop sequences and livestock numbers in a given farm. The competing criteria of profit maximisation and risk minimisation (risk resulting from weather unpredictability) for defining cropping sequences are explored. Cropping risk will be considered as a constraint. In both criteria, the model will be expected to determine optimal crop-livestock production strategy under the given assumptions.

The maximisation of the income objective is defined in real terms, in terms of the difference between the gross income derived from each enterprise (crop production and animal production activities) minus the costs associated with the three activities mentioned under different constraints. The specific set of interacting production constraints includes land size, food availability and requirement of animals, availability of some machinery in the farm, strategy restriction (diversifying crops), risk constraints etc...

The concepts of risk employed in this paper focuses on the randomness or variability of outcomes. This concept of risk finds a theoretical justification in the expected utility maximisation decision model (Robinson & Barry, 1987). In this study, the risk of the crop production is defined in terms of the levels of income variability associated with the different states of nature described in the assumption section of this paper.

In this chapter, a mathematical model describing the various interrelationships of the integrated crop-livestock production will be presented. Mathematical representation of the different activities and constraints will be developed in the following sections of this chapter.

In section two definitions of variables and parameters that are employed in the model will be given. In Section 3, crop rotation modelling will be discussed in detail. More emphasis will be given to the mathematical model representation of crop rotation strategies. Moreover, the aspect of limiting the number of strategies will also be discussed briefly. Risk as a variability of income in the selection of crop rotations due to the randomness of weather states is presented in section 4. Section 5 and 6 present the animal feed formulation and availability activities. Both sections are crucial in developing the relationship existing between the crop and animal production of the farm. Incorporation of renting, storage and buying activities of resources in the mathematical model will also be presented in sections 7, 8 and 9 of this chapter. In the final section of, the objective function of the composite mixed integer-linear programming model for the integrated crop-livestock production will be presented.

2. Definitions of Decision Variables and Parameters in the model

In order to construct a mathematical model for the problem discussed, first let us define some of the variables, coefficients and indices that will be used in this study.

i. Indices

i ($i = 1, 2, 3, \dots, N$) refers to the number of strategy

s ($s = 1, 2, \dots, n$) refers to the crop type.

m ($m = 1, 2, 3, \dots, y$) refers to the raw material type used in the feed mix.

a ($a = 1, 2, \dots, l$) refers to the animal type, where $a=1$ represents adult cattle and $a=2$ represent sheep.

τ ($\tau = 1, 2, \dots, h$) refers to the states of nature type.

w ($w = 1, 2, \dots, R$) refers resource type.

r ($r = 1, 2, \dots, x$) refers to nutrient type.

ii. Decision Variables

X_i = Hectares of farmland planted applying strategy type i .

U_s = amount of crop types sold to the market (tons/year).

Y_m = amount of raw material (crop type) m used in the feed mix (tons/year).

W_a = the number of animal type a initially, $a = 1, 2, \dots, l$.

Z_a = number of animal type a sold to the market, $a = 1, 2, \dots, l$.

N_a = number of animal type a bought from the market, $a = 1, 2, \dots, l$.

R_w = amount of resource w rented, $w = 1, 2, \dots, R$.

iii. Parameters

C_i = Production cost (Rand/hectare) of cropping strategy i .

V_s = Selling price of crop type s (Rand/ton)

d_m = Cost of food staff type m (Rand/ton).

π_a = Blended feed requirement of animal type a (tons/head/year).

μ_a = Roughage requirement of animal type a (tons/head/year)

β_{is} = Yield of crop type s from strategy i (tons/hectare)

K_i = Yield of roughage (tons/hectare) from strategy i.

α_{is} = Proportion of X_i cultivated by crop type s.

$C_{i\tau}$ = Income from strategy i when state of nature τ is prevailed (Rand/hectare/year).

f_a = Income from animal type a (Rand/head/year).

q_a = Income from selling of animal type a (Rand/head-income from interest).

b_a = Cost of buying of animal type a (Rand/head, interest cost).

θ_a = Minimum number of animal type a in the farm.

Θ_a = Maximum number of animal type a in the farm.

n_{rk} = Percentage of nutrient r contained in raw material type k.

P_r = Maximum amount (%) of nutrient r required in the feed mix.

p_r = Minimum amount (%) of nutrient r required in the feed mix.

E_m = Maximum amount of raw material k (%) desired in the feed mix.

e_m = Minimum amount of raw material k (%) desired in the feed mix.

h_w = Cost of rent of resource w.

A = Maximum available area of land in hectares the farm owns.

B = Baling machine capacity (Tons/year).

H = Combine harvester capacity (Hectares/year).

3. Crop Rotation Modelling

3.1. Derivation of Crop Rotation Strategies

As described in the problem formulation section of this paper, the one and major component of the integrated crop–livestock production is the problem of finding feasible optimal crop sequences that are not altered year from year. Let A be the total surface area the farmer has on which crops can be grown. Let there be n possible crops that can be grown in the given area of land. Following El-Nazer & McCarl (1986) and De Kock & Visagie (1998), it is assumed that optimal sequences of crops that can be grown in a given area in successive years form a cycle of three years and shorter.

Let i, j and k be the indices that indicate possible crops in a crop rotation system. Thus i, j and k can have the following possible values.

$$i = 1, 2, 3, \dots, n$$

$$j = 1, 2, 3, \dots, n$$

$$k = 1, 2, 3, \dots, n$$

Based on El-Nazer & McCarl (1986) and De Kock & Visagie (1998), let x_{ijk}^t be the area of the farm in which crop i is grown in year t following crops j and k (crop j grown in year $t-1$ and crop k grown in year $t-2$). Let C_{ijk} be the cost coefficient per unit surface area if crop k (2 years ago) was followed by crop j (one year ago), which was then followed by crop i in the current year. The cost coefficients are assumed independent of time.

In line with the approach set by El-Nazer & McCarl (1986) and De Kock & Visagie (1998), the crop rotation problem can be formulated as a linear programming problem, where the

objective is the maximization of profit for the cycle of year T . De Kock & Visagie (1998) formulated the problem mathematically as follows.

$$\text{Max } Z = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{t=1}^T C_{ijk} x_{ijk}^t \quad (3.1)$$

subject to

$$\sum_{i=1}^n x_{ijk}^{t+1} \leq \sum_{i=1}^n x_{jki}^t \quad \forall j, k, t \quad (3.2)$$

$$\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n x_{ijk}^t \leq A \quad t=1,2,3,\dots,T. \quad (3.3)$$

$$x_{ijk}^t \geq 0 \quad \forall i, j, k, t \quad (3.4)$$

As explained in the literature review section of crop rotation modelling, the first constraint ensures the correct sequences of crops as well as the correct surface areas. The second constraint places a limit on the total surface area on which crops can be grown. Furthermore, the third constraint ensures that the surface area is positive.

As a further refinement of the above linear programming model, in order to exclude the trivial solution $x_{ijk}^t = 0 \quad \forall i, j, k, t$ from the solution set it is crucial to change the inequality constraint into equality constraint (equation 3.3). This constraint will force the linear programming model to assign the whole part of the surface area (the land the farmer has) with a crop. The mathematical model will be written as follows.

$$\text{Max } Z = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{t=1}^T C_{ijk} x_{ijk}^t \quad (3.5)$$

subject to

$$\sum_{i=1}^n x_{ijk}^{t+1} = \sum_{i=1}^n x_{jki}^t \quad \forall j, k, t \quad (3.6)$$

$$\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n x'_{ijk} = A \quad t=1,2,3,\dots,T. \quad (3.7)$$

$$x'_{ijk} \geq 0 \quad \forall i, j, k, t \quad (3.8)$$

According to the study of De Kock & Visagie (1998), with an increase in the number of crops the problem can be quickly too large to handle. For instance, the first constraint (constraint 3.2) alone consists n^2T constraints and n^3 variables.

To remedy the problem mentioned above and to retrieve further information regarding the crop rotation modelling problem, the dual of the above primal problem is considered for further investigation in order to make the problem handy for linear programming solutions methods (De Kock & Visagie (1998)).

Following De Kock & Visagie (1998) let u'_{ijk} and v^t be the dual variables corresponding to constraints 3.2 and 3.3 respectively. The dual problem is given below.

$$\text{Min } W = A \sum_{t=1}^T v^t \quad (3.9)$$

subject to

$$u'^{t-1}_{jk} - u^t_{ij} + v^t \geq C_{ijk} \quad \forall i, j, k, t = 1, 2, 3, \dots, T \quad (3.10)$$

$$u^t_{ij}, u'^{t-1}_{jk} \text{ urs} \quad \forall i, j, k, t = 1, 2, 3, \dots, T \quad (3.11)$$

$$v^t \text{ urs} \quad \forall t \text{ [urs stands for unrestricted in sign]}$$

According to De Kock & Visagie (1998), because of the assumptions made the solution of the above dual problem leads to the formulation of strategies. These strategies are independent of

time, and therefore the subscript t can be omitted. Let S'_{ijk} be a strategy that corresponds to the total surface area to which the strategy is applied. The strategies are defined as follows.

$$S'_i = x_{iii} \quad \forall i \text{ (Continuous monocropping or one-year strategy)} \quad (3.12)$$

$$S'_{ij} = \frac{1}{2}(x_{iji} + x_{jij}) \quad \forall i, j \text{ (Two-year strategy).} \quad (3.13)$$

$$S'_{ijk} = \frac{1}{3}(x_{ijk} + x_{jki} + x_{kij}) \quad \forall i, j, k \text{ (Three-year strategy)} \quad (3.14)$$

The corresponding cost coefficients for each strategy are defined as follows.

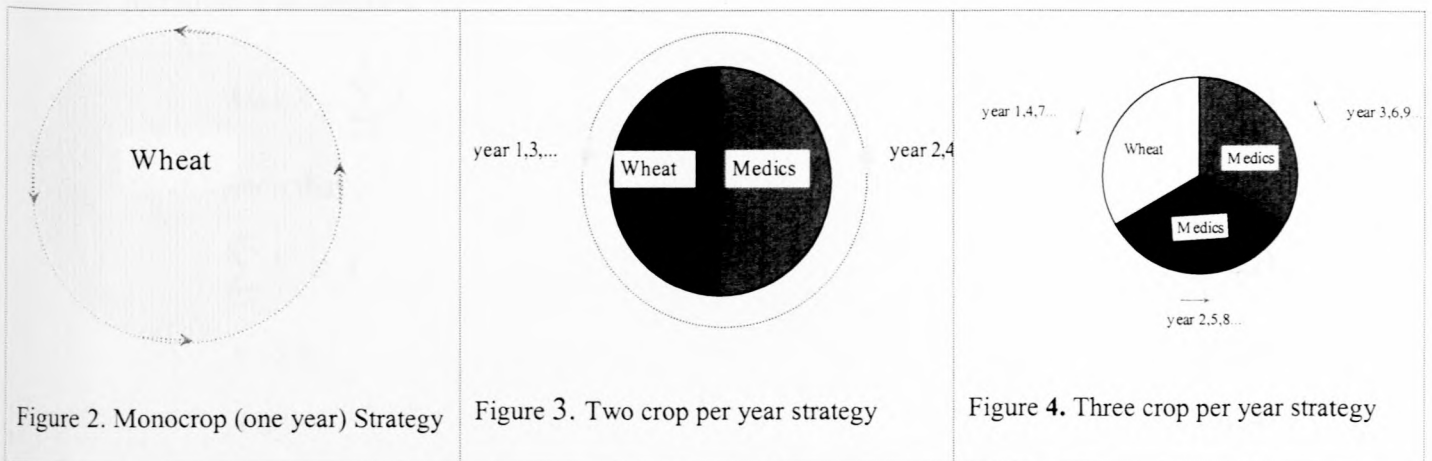
$$\gamma'_i = C_{iii} \quad \forall i \quad (3.15)$$

$$\gamma'_{ij} = \frac{1}{2}(C_{jii} + C_{jij}) \quad \forall i, j \quad (3.16)$$

$$\gamma'_{ijk} = \frac{1}{3}(C_{jik} + C_{jki} + C_{kij}) \quad \forall i, j, k \quad (3.17)$$

The cost coefficients correspond to the net income per year the farmer earns from the relevant strategy per unit of measurement.

Diagrammatically, the three possible crop rotation cycles are portrayed below.



Following De Kock & Visagie (1998), all non-equivalent strategies are grouped together in a partition. Using the principle of reduction partition as described in De Kock & Visagie

(1998), let $S = \{S_i, S_{ij}, S_{ijk}\}$ be the collection of non equivalent strategies with elements S_i, S_{ij}, S_{ijk} and with corresponding cost coefficients γ_i, γ_{ij} , and γ_{ijk} .

The problem can now be formulated as follows.

$$\text{Max } Z = \sum_i \gamma_i S_i + \sum_{ij} \gamma_{ij} S_{ij} + \sum_{ijk} \gamma_{ijk} S_{ijk} \quad (3.18)$$

Such that

$$\sum_i S_i + \sum_{ij} S_{ij} + \sum_{ijk} S_{ijk} = A \quad (3.19)$$

$$S_i, S_{ij}, S_{ijk} \geq 0 \quad (3.20)$$

In theory, all the strategies in the set S are feasible. However, for some agronomic reasons some of the strategies can be agronomically infeasible depending on various agronomic factors. Let $X_i \in \{S_i, S_{ij}, S_{ijk}\}$ such that $X = \{X_1, X_2, X_3, \dots, X_N\}$ be the only agronomically feasible strategies, where the set X includes all the one year, two year and three year agronomically feasible strategies. Assume that the set X satisfies all the properties of the set S . Let C_i be the cost coefficient associated with each X_i . The crop rotation problem is now formulated as follows.

$$\text{Max } Z = \sum_{i=1}^N X_i \quad (3.21)$$

Such that

$$\sum_{i=1}^N X_i \leq A \quad (3.22)$$

$$X_i \geq 0 \quad (3.23)$$

$$A \geq 0 \quad (3.24)$$

3.2. *Limiting Number and size of Strategies Implemented in the Farm*

It is assumed that the integrated crop-livestock farm has a choice of n crops that it can grow in the given fixed land. Furthermore, the farm has N possible crop rotation strategies of decision alternatives from which a combination of strategies is assumed to be implemented. However, due to several management issues it is not possible to implement all the relevant feasible strategies in the fixed farmland.

Suppose the decision maker (farmer) is restricted to a maximum of T strategies (It is assumed that the arable land A is divided into T unit fields (plots)). That is, the restriction on the acreage decision problem is, at most T of the N strategies $\{X_1, X_2, X_3, \dots, X_N\}$ must be satisfied. In order to introduce this restriction to the linear programming model, let us introduce an indicator variable δ_i to link the strategy option with each of the continuous (X_i 's) in the model.

Based on Williams (1999) and Winston (1994), in order to incorporate the above decision restriction into a linear programming farm planning model, an integer programming formulation is required. This formulation is one of the applications disjunctive constraint formulations (William, 1999). This option incorporates an integer into the decision model. Let us introduce the following constraint.

$$X_i - A\delta_i \leq 0 \text{ for } i = 1, 2, \dots, N \quad (3.25)$$

This constraint forces δ_i to take the value 1 when $X_i > 0$. This condition can be written as

$$X_i > 0 \rightarrow \delta_i = 1 \quad (3.26)$$

The above constraint provides a sufficient link between X_i and δ_i . Equation (3.26) also imposes the following condition.

$$X_i = 0 \Rightarrow \delta_i = 0 \quad (3.27)$$

Equation (3.27) can be written as

$$\delta_i = 1 \Rightarrow X_i \geq 0 \quad (3.28)$$

Equations (3.25), (3.26), (3.27) and (3.28) together impose the restriction.

$$\delta_i = 1 \Leftrightarrow X_i \geq 0 \quad (3.29)$$

Therefore, the problem of restricting number of strategies can be handled by the following set of constraints.

$$\begin{aligned} X_i - A\delta_i &\leq 0 \text{ for } i = 1, 2, \dots, N \\ \sum_{i=1}^N \delta_i &\leq T \\ T &\geq 1, \quad X_i \geq 0, \\ \delta_i &= \begin{cases} 0 & \text{if } X_i = 0 \\ 1 & \text{if } X_i > 0 \end{cases} \end{aligned} \quad (3.30)$$

Constraint (3.30) offers two major benefits to the farm decision model. Principally, it restricts the number of strategies that should be used in time of optimisation. In reality, it is impossible to implement all the feasible crop rotation strategies described in this study, as the programming model can allocate strategies which are too small and many stripes of land for implementation. Another benefit, which is an indirect, but an important one is that it involves crop diversification. That is, if more than one strategy is implemented, the notion of crop diversification will be introduced in the decision model. Consequently, in a single year more than one crop will be grown in the land (See Figure 5 below).

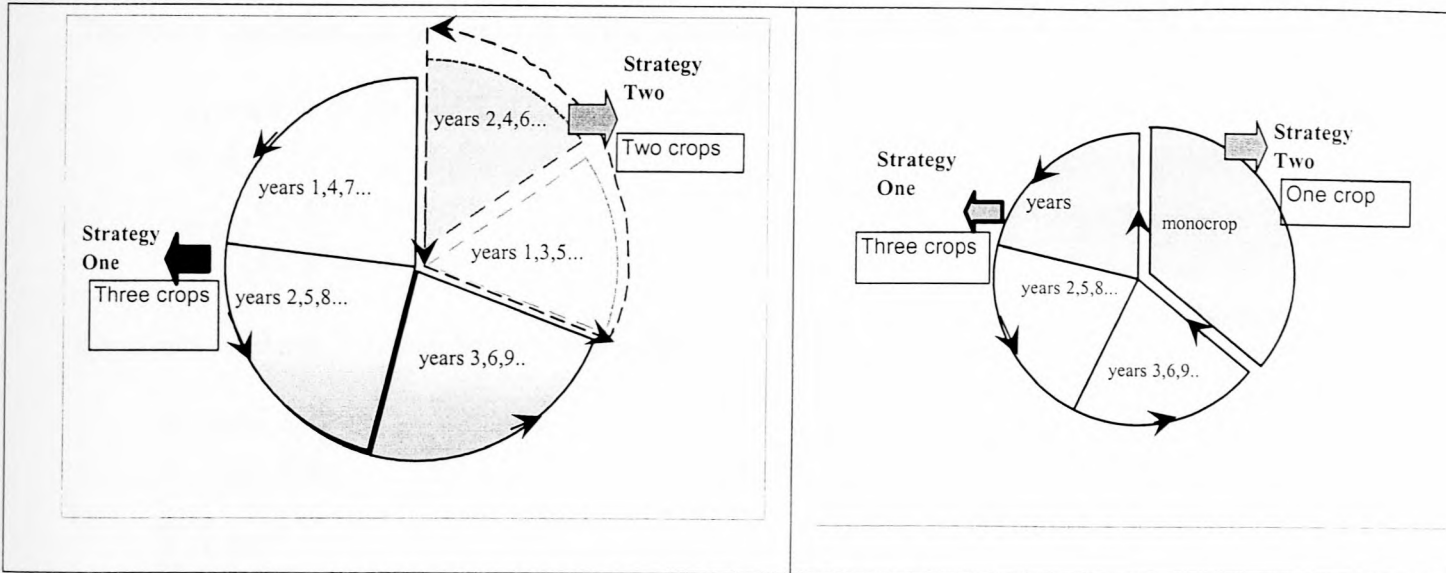


Figure 5. Two strategies per year farm cropping plan

Crop diversification is one of the methods normally applied to manage risks. In general, diversification provides an economic buffer against yield and price fluctuations for crops and production inputs as well as the vagaries of pest infestations, weather etc... (Alternative agriculture, 1989).

From equation 3.30, if $\delta_i = 1$ strategy i is selected for implementation in the farm planning problem. That is, X_i amount of hectares of land must be allocated for cropping strategy i . However, if the amount of land selected by the strategy is very small it is unrealistic to implement from management point of view. Therefore, the model should incorporate a minimum value constraint to eliminate the above mentioned problem from the mathematical model.

If a strategy is selected by the mathematical model for implementation, for each strategy selected it is necessary to define a threshold level (lower bound), below which it will be regarded as 0 level. Let g hectares be the lower limit imposed in the decision model. The following condition must be implemented to impose a lower bound for the strategy selected.

$$\delta_i = 1 \Rightarrow X_i \geq g \tag{3.31}$$

The above condition can be imposed by the following constraint (Williams, 1999).

$$\begin{aligned}
 X_i - g\delta_i &\geq 0 \text{ for } i = 1, 2, \dots, N \\
 X_i &\geq 0, \\
 \delta_i &= \begin{cases} 0 & \text{if } X_i = 0 \\ 1 & \text{if } X_i > 0 \end{cases}
 \end{aligned} \tag{3.32}$$

Equations 3.30 and 3.32 can be written together as follows.

$$\begin{aligned}
 X_i - A\delta_i &\leq 0 \text{ for } i = 1, 2, \dots, N \\
 X_i - g\delta_i &\geq 0 \\
 \sum_{i=1}^N \delta_i &\leq T \\
 T &\geq 1, \quad X_i \geq 0, \\
 \delta_i &= \begin{cases} 0 & \text{if } X_i = 0 \\ 1 & \text{if } X_i > 0 \end{cases}
 \end{aligned} \tag{3.33}$$

3.3. Land Constraint

In an integrated crop-livestock production enterprise land is the vital factor for the different activities. The amount of crop available for the market and animal consumption and the amount of forage production is directly related to the amount of available land the farm has. The land constraint limits the total available area of land allocated to the different cropping strategies. Let A be the maximum available area of land in hectares. The land constraint is given as follows.

$$\sum_{i=1}^N X_i \leq A \tag{3.34}$$

4. **Income Variability: as a Source of Farm Risk**

Farmers never know future events with absolute certainty. Therefore, in order to assess the impact of a decision strategy on the farm planning, a need for an objective measure for a risk associated with the decision strategy is important. Specifically, to evaluate the impact of the unpredictability of the states of nature over a crop rotation strategy selected, consideration of risk is a key factor in selecting an optimal farm plan, as the introduction of risk in a production process affects the pattern of resource allocation and the level of production (Gabriel & Baker, 1980). Risk attitudes may be reflected in the farm plan analysis in different ways.

Assuming that the future income variability which results from variations in weather in the adoption of the different cropping strategies in this study is closely related to past variability, crop income risk can be estimated by income variability over some past time period (Hazell, 1971; Hazell & Norton, 1986). For the purpose of this study, as given in the assumption section, income variation across three states of nature is considered.

There are various categories of sources of risk in agricultural production (Anderson, Dillon & Hardaker, 1977). One of such sources is the biophysical environment which produces yield or production variability, which is termed as the production risk (Gabriel & Baker, 1980). Production risk emanates from the unpredictable nature of weather and uncertainties in the performance of crops and livestock. As indicated in the assumption section, production risk will be incorporated as variability of income due to the variability of the yield of crops across the states of nature and crop rotation strategies which are assumed relevant. Hence, it is necessary to take into consideration income variability levels associated with alternative crop rotation strategies in the mathematical model.

As enumerated in the partial literature review section, there are different mathematical programming techniques available that introduce risk into an optimisation procedure. The approach followed here for introducing risk in the farm-planning problem is a slight modification of Hazell's (1971) risk linear programming model. In adopting the constraints of this model, it is recognised that agricultural production activities take place in a risky environment. Moreover, it is true that a farm does not have a known income level due to the uncertainties in the weather states of each year.

Hazell's (1971) variance estimator is based on the sample mean absolute deviation instead of the more widely used sum of squares error (variance). This is a key point in Hazell's (1971) formulation that allows the incorporation of risk into a linear programming model. As discussed in the literature review section, the objective function of the risk linear programming model formulated by Hazell (1971) is the minimisation of the total absolute deviations. That is the objective was minimising the risk level of an optimal farm plan. However, minimising risk is insufficient by itself and would result in plans with low-income levels.

In this paper, risk is introduced as a constraint (target level of risk) so that the model selects a combination of strategies that achieve a specified target level of risk with highest income.

The mathematical formulation of incorporation of risk as constraint in the mathematical model of maximisation income objective follows below.

Let $C_{i\tau}$ = income from strategy i when state of nature τ is prevailed (Rand/hectare/year)

The average income across the states of nature for strategy i is calculated by the following formula.

$$C_i = \frac{\sum_{\tau=1}^h C_{\tau}}{h} \quad (3.35)$$

Equation (3.35) is the sample mean of net revenue per unit of the i^{th} activity.

Given an appropriate sample of activity net revenues for the τ states of nature, an unbiased estimate of the mean absolute deviations (MAD) of an income from the strategies is given by

$$M = \frac{\sum_{\tau=1}^h \left| \sum_{i=1}^N (C_{\tau} - C_i) X_i \right|}{h} \quad (3.36)$$

In statistics, the expected value of $\left| (C_{i\tau} - C_i) \right|$ is termed as the loss function (Mood, Graybill and Boes, 1974).

Let the sum of deviations of income of strategies from mean in state of nature τ be denoted by

L_{τ}^{+} if it is positive and L_{τ}^{-} if it is negative. Hazell & Norton (1986) showed how the

variance of farm income (i.e. the risk) could be estimated using the sample mean absolute

deviation (MAD) drawn from time-series or cross-sectional data. The attraction of the MAD

estimator is that it can be included in a standard linear programming model. Furthermore, it is

a linear approximation to quadratic programming (Hazell, 1971).

Then the following is true.

$$L_{\tau}^{+} - L_{\tau}^{-} = \sum_{i=1}^N (C_{i\tau} - C_i) X_i \quad \forall \tau \quad (3.37)$$

The interesting property of the above deviations is that both are non-negative and only one of them can be greater than zero in each states of nature, that is the deviation cannot be both positive and negative at the same time. Each measures the size of absolute value of the deviation of income of each strategy in given state of nature from its mean.

As illustrated above, the emphasis in the risk is the downside risk. Therefore, in order to incorporate the downside risk in the model the following constraints are necessary to consider the mathematical model.

$$\sum_{i=1}^N (C_{i\tau} - C_i) X_i + L_{\tau}^{-} \geq 0 \quad \tau = 1, 2, \dots, h. \quad (3.38)$$

$$\sum_{\tau=1}^h L_{\tau}^{-} = \lambda \quad (3.39)$$

Where λ is a parameter and $\lambda = 0 \rightarrow \lambda_{\max}$

5. Animal feed activities

In an integrated crop livestock production farm that is considered, the crops produced by the farm can be used for animal feeding purposes, the remaining being sold to the market at the market price. Other foodstuffs, which are necessary for the livestock production, including the supplies required because of insufficient farm production can be purchased from the market.

As described in the assumption section, the animals in the farm are categorised in three sets. With regard to feed, each type of animal requires a certain amount of blended feed and pastures, which satisfy certain requirements, set by the farm. The quantitative modelling of the two type of food consumed by the animals, feed mix and roughage is discussed below in separate sections.

5.1. *Blended feed (Feed mix)*

Feed is arguably the most important input, next to the actual animals, for a livestock operation in terms of impact on total expenses. Given the importance of feed to livestock operations, the selection of minimum cost feed rations using linear programming has, historically been given considerable attention in agricultural activities.

The animal feeding policy and the crop production strategy influence each other as the land used to produce foodstuffs for animal consumption could be utilised for crop production for sale purposes. In formulating the feed mix, the farmer is assumed to make use of the type of feedstuffs produced in the farm. The remaining is assumed to be supplemented by purchasing from the market. The problem that needs addressing is the design of a version of minimum cost feed mix formulations that satisfy certain requirements set by the farmer. Each of the

possible ingredients had a different price, and each contained different proportions of various nutrients that the cattle need annually. Therefore, the problem that we need to investigate within the feed mix context is which ingredients, in which quantity should be combined to meet the nutritional needs of the adult cattle as inexpensively as possible taken into consideration the interdependence between the crop production and livestock production of the farm.

In order to formulate the mathematical description of the feed mix problem, the definition of some variables and coefficients that are not defined in the introduction section of this chapter is necessary. Let

$F = \{Y_1, Y_2, Y_3, \dots, Y_y\}$ be a set of food stuffs (Raw materials).

$N = \{N_1, N_2, N_3, \dots, N_r\}$ be a set of nutrients.

The feed mix is prepared from the set of raw materials and comprises a set of nutrients satisfying different restriction.

The mathematical model for the least cost feed mix satisfying the nutritional and raw material requirement is given by the following linear programming model (Klein, et al., 1986; De Kock & Sinclair, 1987; Munford, 1989).

$$\text{Max} - \sum_{m=1}^y d_m Y_m \quad (3.40)$$

Subject to

$$p_r \leq \frac{\sum_{m=1}^y a_{rm} Y_m}{\sum_{m=1}^y Y_m} \leq P_r \quad [\text{Nutrient } r \text{ requirement constraint } r = 1, 2, \dots, x] \quad (3.41)$$

$$e_m \leq \frac{Y_m}{\sum_{m=1}^y Y_m} \leq E_m \quad [\text{Raw material } m \text{ requirement constraint, } m = 1, 2, \dots, y] \quad (3.42)$$

$$\sum_{m=1}^M Y_m - \sum_{a=1}^A \pi_a (W_a + N_a - Z_a) = 0 \quad \text{Total feed requirement} \quad (3.43)$$

$$Y_m, P_r, P_r, e_m, E_m, W_a, N_a, Z_a, \pi_a \geq 0 \quad \text{and } W_a, N_a, Z_a \text{ are integers}$$

The livestock feed mix problem for each state of nature is modelled by equations (3.40) through (3.43). Equation (3.41) presents the objective function of minimising cost. Equations (3.41) and (3.42) ensure the nutrient and raw material requirement restrictions respectively. Furthermore, equation (3.43) states that the total consumption of feed mix by the livestock equals the production level.

The above minimum cost feed selection linear programming model will be incorporated with the composite mixed integer linear programming model in order to find a minimum cost feed staff satisfying the different nutrient, ingredient and availability restrictions. In the formulation of the above model, the parameters are assumed to be known with certainty, that is the constraints and objective function coefficients are known with certainty.

5.2. *Roughage Requirements*

Roughage is part of the animal feed. The young cattle and sheep require roughage feed from the farm. Since the roughage requirement of animals is totally supplied from the farm production, it is required that the amount of roughage produced in the farm should satisfy the pasture demand of young cattle and sheep.

In order to incorporate this activity into the general mathematical decision model the formulation of roughage availability constraint is necessary. The pasture availability constraint is given as follows.

$$\sum_{i=1}^N K_i X_i - \sum_a \mu_a [(W_a + N_a - Z_a)] = 0 \quad (3.44)$$

$$X_i - A\delta_i \leq 0 \text{ for } i = 1, 2, \dots, N$$

$$X_i - g\delta_i \geq 0$$

$$\sum_{i=1}^N \delta_i \leq T$$

$$T \geq 1, \quad X_i \geq 0,$$

$$\delta_i = \begin{cases} 0 & \text{if } X_i = 0 \\ 1 & \text{if } X_i > 0 \end{cases}$$

$$K_i, X_i, W_a, N_a, Z_a, \mu_a \geq 0 \text{ and } W_a, N_a, Z_a \text{ are integers}$$

6. Availability activities

As already explained in the previous sections, the farm considered in this study is an integrated crop-livestock production farm. Hence, the farm can produce all or any combination of the crops from the crops considered. The harvested crops can be used either in the feed mix or can be sold to the market if there is an excess amount. This activity plays a pivotal role in linking both the crop and animal production enterprises. Both crop sale and preparation of the feed mix are dependent on the availability of crop yield in the farm's crop production activity. Therefore, this activity is termed as *availability* (Williams, 1999). This activity will be represented by the following availability constraint.

$$\sum_{i=1}^N \beta_{is} X_i - U_s - Y_m = 0 \quad s = 1, 2, \dots, n, \quad m = 1, 2, \dots, y \quad (3.45)$$

$$\beta_{is}, X_i, U_s, Y_m \geq 0$$

These constraints ensure that the total quantity demanded (both crop sale and animal feed) does not exceed the supply from the farm production.

7. Renting Activities

Some of the resources available to the farm are not fixed. During the planning period, the capacity of the available resources of the farm cannot match the demands of some activities. If the resources available are not enough, it is assumed that the shortfalls can be supplemented through hiring or renting activities of additional units of the required resource.

As explained in Hazell & Norton (1986), a resources renting decision can be incorporated into a linear programming farm planning model through renting activities. These activities typically have a negative entry in the objective function of the model representing the total cost incurred and a -1 entry in the relevant resource constraint. The -1 entry indicates the supply of an additional of the resource assumed which is in deficit. This is equivalent to the addition of one unit to the right hand side of a resource activity under consideration.

In this study, the combine harvester and baling machine capacity of the farm is assumed fixed. If the demand for the capacity of such machines is greater than the capacity, additional units are required in order for the farm to cope with the demand. Let us discuss the additional demand constraint for both machines separately.

A. Assume that farm's combine harvester capacity is only H hectares per year. In a given year, if the amount of land cultivated with wheat, canola and lupines is greater than the capacity of the machine, an additional amount of combine harvester capacity is necessary.

Let α_{is} be the proportion of land cultivated by crop type s under strategy i . R_1 be the extra amount of hectares that need extra combine harvester capacity

$$\sum_{i=1}^N \sum_{s=1}^n \alpha_{is} X_i - R_1 \leq H \quad (3.46)$$

$$X_i - A\delta_i \leq 0 \text{ for } i = 1, 2, \dots, N$$

$$X_i - g\delta_i \geq 0$$

$$\sum_{i=1}^N \delta_i \leq T$$

$$T \geq 1, \quad X_i \geq 0,$$

$$\delta_i = \begin{cases} 0 & \text{if } X_i = 0 \\ 1 & \text{if } X_i > 0 \end{cases}$$

$$\alpha_{is}, R_1, H \geq 0$$

B. Another restricting resource in the farm is the baling machine capacity. Silage and medics are either used in the preparation of feed mix or can be sold to the market. Both require baling in the farm. If the amount of silage and medics that need baling is more than the capacity of the baling machine of the farm, extra amount of machine capacity can be hired for extra cost. Assume the capacity of the baling machine is B tons/year and let R_2 tons/year be capacity required. The constraint representing the extra baling machine capacity is given below.

$$\sum_{\text{silage, medics}} U_s + \sum_{\text{silage, medics}} Y_m - R_2 \leq B \quad (3.47)$$

$$U_s, Y_m, R_2, B \geq 0$$

Where U_s is the amount of crop type s sold to the market (Tons) and Y_m is the amount of crop type m (tons) used in the feed mix.

8. Animal Feed Storage Constraint

The storage capacity of the storage area that the farm has for some feed material types is limited. That is, beyond some quantity level, this constraint restricts the amount of the feed type that can be stored in the farm available storage area.

Let Y_m (Tons) be the quantity of feed raw material type m required in the planning period of the feed mix. Let K_m (tons) be the maximum possible storage capacity available for raw material type m . The raw material storage constraint for ingredient type m is illustrated by the following equation.

$$Y_m \leq K_m \quad m = 1, 2, \dots, y \quad (3.48)$$

9. Livestock buying and selling activities

The number of livestock the farm keeps depends on various factors of the farm. Some of the factors include availability of space, profitability, availability of feed and pasture etc.... Because of such restricting factors, the number of animals the farm keeps is constrained between a maximum and a minimum number.

Let

θ_a = minimum number of animal type a the farm keeps in the planning period.

Θ_a = Maximum number of animal type a the farm keeps in the planning period.

For animal type a , the upper and lower bound is given by the following constraints.

$$\theta_a \leq W_a + N_a - Z_a \leq \Theta_a, \quad a = 1, 2, \dots, l \quad (3.49)$$

$$\theta_a, W_a, N_a, Z_a, \Theta_a \geq 0 \text{ and } \theta_a, W_a, N_a, Z_a, \Theta_a \text{ are integers.}$$

10. The objective function

A large body of literatures on applications of mathematical programming to agricultural decision making argues that an agricultural firm operates to maximise its profit from its production activities while meeting different constraints, such as availability, risk, land, etc.... Accordingly, this paper considers the farm management to have an objective of maximising profit, which satisfies the different restricting constraints the farm has.

As described in the problem formulation section of this paper, the main objective of the farm is to maximise profit from both activities of the farm, namely crop production activity and animal production activity. To be precise, the decision maker's problem is to select the optimum maximum profits combination of crop production strategies and number of animals that satisfy the different resource availability, resource restrictions and risk constraints.

Profit is defined as the difference between total income and total expenses. That is,

$$\text{Profit} = \text{Total income} - \text{Total expenses}$$

Total income of the farm from the activities of crop and animal production is calculated as follows.

$$\text{Total income} = \text{Income from crop sale} + \text{Income from animal activity}$$

Income from animals includes the income from dairy production of the adult cattle, wool sheep production and animal sale activities.

The total expenses of the farm are also evaluated arithmetically in the following way.

$$\begin{aligned}
 \text{Total expenses} &= \text{Cost of crop production} + \text{Cost of Animal Feed} + \text{Cost of extra} \\
 &\quad \text{Resources rent} + \text{Cost of Animals bought}
 \end{aligned}$$

Therefore, the objective function of the decision model subject to the different constraints is given by the following.

$$\text{Max } \mathbf{G} = \sum_{s=1}^n v_s u_s - \sum_{i=1}^N C_i X_i + \sum_{a=1}^l f_a [W_a + N_a - Z_a] + \sum_{a=1}^l q_a Z_a - \sum_{a=1}^l b_a N_a - \sum_{m=1}^y d_m Y_m - \sum_{w=1}^R h_w R_w \tag{3.50}$$

Subject to the different constraints discussed in the previous sections of this chapter.

The general composite model of the decision problem is given in the following matrix format in Table 1 below (See the full mathematical model in Appendix A).

	Crop rotation Strategies	Crop sale	Animal Feed	Rent Activities	Animal Activities	Strategy restriction	Crop net returns Deviation		RHS
	X	U	Y	R	Z	T	L		
Objective function	-C	V	-D	-K	F	0	0		
Land	I ₁	0	0	0	0	0	0	≤	A
Crop Availability	B	-I ₂	-I ₃	0	0	0	0	=	0
Pasture Availability	P ₁	0	0	0	-P ₂	0	0	=	0
Straw Constraint	J ₁	0	0	0	-J ₂	0	0	≥	0
Raw Material Constraints	0	0	R	0	0	0	0	≥(≤)	0
Animal Nutrition	0	0	N	0	0	0	0	≥(≤)	0
Total feed	0	0	I ₄	0	-I ₅	0	0	=	0
Combine Harvester rent	Q	0	0	-I ₆	0	0	0	≤	H
Baling machine rent	0	I ₇	I ₈	-I ₉	0	0	0	≤	B
Feed Storage Constraint	0	0	0	I ₁₀	0	0	0	≥	K
Number of Animals (upper bound)	0	0	0	0	Z	0	0	≤	Θ
Number of Animals (Lower bound)	0	0	0	0	Z	0	0	≥	θ
Strategies-Upper bound	I ₁₁	0	0	0	0	-AI ₁₂	0	≤	0
Strategies-Lower bound	I ₁₃	0	0	0	0	-gI ₁₄	0	≥	0
Strategies	0	0	0	0	0	I ₁₅	0	≤	T
Deviation	I ₁₆	0	0	0	0	0	I ₁₇	=	0
Deviation	0	0	0	0	0	0	I ₁₈	=	λ

Table 1. Aggregated representation of the mixed integer linear programming model

Where

C is $1 \times N$ matrix of values production cost (Rand/hectare) of strategies.

V is $1 \times n$ matrix of values crop prices (Rand/ton).

D is $1 \times y$ matrix of values of raw material costs (Rand/ton)

K is $1 \times W$ matrix of resource rent costs (Rand/unit).

F is $1 \times a$ matrix of returns from animal production (Rand/head)

I_1 represents $1 \times N$ matrix of 1's.

B represents $n \times N$ matrix of crop yields (tons/hectare) of strategies.

I_2 represents $n \times n$ matrix of crop produced and sold to the market (Tons)

I_3 is matrix of $n \times n$ of crop produced (tons) and used in the feed mix.

P_1 is $1 \times N$ matrix of pasture yields (Tons/hectare).

P_2 is $1 \times a$ matrix of animal pasture requirements (Tons/year/head).

J_1 is $1 \times N$ matrix of straw yields (Tons/hectare).

J_2 is $1 \times a$ matrix of animal straw requirements (Tons/year/head).

R is $m \times y$ matrix values of raw material restrictions ($m \leq y$)

N is $r \times y$ values of nutrient restrictions ($r \leq x$).

I_4 represents a matrix of $1 \times y$ 1's.

I_5 represents a matrix of $1 \times a$ 1's.

Q represents a matrix of values of $1 \times N$ values of proportion of land requiring combine harvester.

I_6 is a matrix of 1×1 representing coefficient of combine harvester capacity rent.

I_7 and I_8 represents a matrix of 1×2 representing crops sold and used as feed respectively requiring baling

I_9 is a matrix of 1×1 (1) representing coefficient of baling machine capacity rent

I_{10} represents a diagonal matrix of feed storage constraints.

Z and z represents a matrix of an upper and lower bound on the number of animals.

I_{11} , I_{12} , I_{13} and I_{14} are a diagonal matrix of $N \times N$ 1's representing strategies.

A and g represent the upper and the lower bound for the strategies.

I_{15} represents a matrix of $1 \times N$ (a value of 1's).

I_{16} represents a matrix of $h \times N$ (coefficient of deviations for the strategies).

I_{17} represents a matrix of $h \times h$ 1's.

I_{18} represents a matrix of $1 \times h$ 1's.

Chapter IV

Mathematical Model Solution and Sensitivity Analysis: *A Case Study*

1. Introduction

A typical farm situation which is located in the Koeberg area of Western Cape was selected for the case study of the mathematical programming model developed in chapter three. The farm has 1800 hectares of arable land, which is suitable for crop production. The farm's activities include crop production, dairy production and wool sheep production. As illustrated in chapter three, the objective of the mixed integer linear programming model formulated was to elaborate an annual plan of activities conducive to the maximization of the farm's profit.

By and large, this chapter identifies farm-planning alternatives (crop and livestock production options) for the 1800 hectares of land, which can grow cash crops, namely wheat, canola, lupines, silage (oats) and medics. Generally, this chapter is devoted to the application and discussion of the results of the mathematical programming model formulated in chapter three. This paper uses two standard mathematical programming models, one without risk and the other with risk to further explore the issues around the farm planning problem of the firm under consideration.

Section two presents a discussion mainly on the data development issues for the mixed integer linear programming model developed in chapter three. This section gives a discussion on formulation of the different coefficients for the mathematical model.

This chapter presents results of the mathematical model developed in chapter three primarily by determining profit maximizing schemes without considering risk in this section. Considering each state of nature as a separate scenario in the decision making process, the results of the mathematical model for the farming plan under wheat monocropping will be compared with the model results that employ crop rotation strategies, assuming a normal year, a wet year and an average state of nature prevails. The results of this comparison of profit maximization farm plan without considering risk in the whole farm-planning situations is presented in section three of this chapter.

The results of the mixed integer linear programming model for the farm planning decision in the presence of crop yield risk is introduced in section four. For the cropping sequences considered, risk is measured as a sum of negative deviation as explained in chapter three. In the mathematical programming model so far formulated, risk was introduced as a parametric constraint. Using this formulation, the effect of risk in the farming plan will be explored in this section.

The last section of this chapter will give a sensitivity analysis of results for different risk values. The tradeoffs between risk and profit of the whole farm planning activity are investigated.

2. Description of the Agricultural Activities of the Farm: *Case Model Empirical Specification*

2.1. *Designing Feasible Crop Rotation Strategies and Input Data Development for the Mathematical Model*

As described in the problem statement section of this paper, the farm's management main problem is the formulation of an optimal farm plan which takes into account the interrelationships that exist between the crop and animal production in the overall farming situation. In dealing with this problem, one of the major questions that we need to address by making use of the mathematical model is the question of which cropping strategy to follow from the set of feasible alternatives the farm has. To be specific, the question is that should the farm employ monocropping or crop rotation in the farming operation.

The farm can grow wheat, canola, lupines, silage (oats) and medics. Moreover, it is assumed that the farmer has 15 choices of feasible cropping alternatives from which the decision maker can employ based on the decision criteria utilized. The feasible cropping alternatives include monocropping, two-year crop rotation and three-year crop rotation. The feasible cropping alternatives are selected based on the idea that these crops are grown currently in the region where the farm is located (De Kock, 2003; Hardy, 1998). The 15 different feasible cropping alternatives (strategies) are given below. The 16th strategy is included to incorporate the roughage growing possibility if the farmer finds out that growing crop is not a feasible choice.

Strategies

- | | | |
|-----------|---|--------------------|
| 1. Wheat | } | 1-Year
strategy |
| 2. Medics | | |

- | | | |
|----------------------------------------------------------------------------------------|---|--------------------|
| 3. Wheat/Medics | } | 2-Year
Strategy |
| 4. Wheat/Canola | | |
| 5. Wheat/Silage | | |
| 6. Wheat/Lupines | | |
| 7. Medics/Canola | | |
| 8. Wheat/Wheat/Medics | } | 3-Year
Strategy |
| 9. Wheat/Canola/Medics | | |
| 10. Wheat /Canola/Silage | | |
| 11. Wheat/Medics/Medics | | |
| 12. Wheat/Wheat/Lupines | | |
| 13. Wheat/Silage/Medics | | |
| 14. Wheat/Canola/fallow | | |
| 15. Wheat/Medics/Lupines | | |
| 16. Grassland (the land is left uncultivated and will be used for roughage production) | | |

The length and number of crops grown in the given farmland determine the complexity of crop rotation. As described in chapter two and three, lengthy crop rotations make the formulation of the mathematical model more complex. Therefore, due to reasons of further modelling complexities and data availability problems, it is assumed that the maximum possible number of different crops per rotation is three, as indicated in the 15 selected cropping strategies above.

The cost coefficients of the mathematical programming model developed for each of the above 15 strategies was calculated by applying equation 3.15 to 3.17. The cost data (cost coefficients) applicable in the mathematical model are indicated in Appendix B (taken from

Visagie, 2004). The market price of crops, which the farmer receives as revenues from crop sale to the market is also illustrated in Appendix C. All the data for crop production activities which are used in the model are taken from the study carried out by Visagie (2004). If the land is not planted with crops, on average it is assumed that the land provides a roughage yield of 1 ton/hectare/year.

Growing season variation of the farm is approximated by three states of nature. The criteria for classifying states of nature assume that the rainfall levels for the different periods have influence on the on crop yield, forage yield and straw yield. The yield data for each of the three states of nature is given in Appendix D.

In each planning period, the revenue the farmer receives from crop production of each cropping strategy depends on the prevailing state of nature for that particular period. It is assumed that a net return per hectare for each cropping sequence in each state of nature is calculated under a specified market scenario. For each state of nature considered the net return of a given cropping sequence is calculated by multiplying the annual crop yield of each crop, roughage and straw yield per hectare in the sequence times the price of each product and then taking the summation of the return of from the crop, roughage and straw yields of that particular strategy. The values of net return per hectare for each of the 15 strategies are illustrated in Table 2 below.

calculated. The figures shown in appendix E are the coefficients that enter into the rows of risk constraints of the mathematical programming model.

2.2. Dairy and Wool Sheep Production Activities

At present, the farm has 450 adult cattle and 1500 sheep. The number of young cattle is assumed to be 80 % of the number of adult cattle. The number of animals the farm keeps is subject to different criteria. The restricting criteria include availability of food, availability of space, and profitability of the operation.

On average, an adult cattle needs 10 tons of blended feed per year. Approximately the blended feed should contain on average, at least 16.5% protein, 68% energy and 15% fibre. The feed mix is prepared on the farm and may consist of the following raw materials, as shown in Table 3 below (De Kock, 2003; Perry, 1982).

Ingredient	Protein %	Energy %	Fibre %	Cost R/ton
Wheat	12	80	3	80
Canola	20	80	15	80
Lupines	36	82	15	80
Silage (Oats)	5	55	29	10
Medics	15	52	25	10
Am. Straw	8	45	37	190
Lusen	15	52	25	850
Cotton seed oil cake	38	75	13	1100
Fish meal	60	71	0.1	3500
Maize	8.5	82	3	800
Canola oil cake	32	70	12	1000
Bran	14	62	10	480
Molasses	4	60	0.3	680
Cotton seed	20	84	24	750

These crops are produced in the farm (the costs indicated are above the production costs including storage, insurance etc...)

Table 3. Raw materials (ingredient) and their nutrient content in the feed mix preparation

The feed mix prepared should also guarantee the following minimum and maximum restrictions on ingredients that have to be included in the blended feed. That is, the following constraints are applicable in animal feed (De Kock, 2003; Perry, 1982).

Straw	min 7%
Silage	min 10%
Medic	min 19%
Molasses	min 6%
Cotton Seed	max 10%
Canola Oil cake	max 12%
Bran	max 15%

In the farm, the young cattle are fed with both blended feed and roughage from the farm. Each young cattle requires 2 tons of blended feed and 2 tons of roughage per year. The different restrictions on nutrients and ingredients of the blended feed for the young cattle are the same as for the adult ones. Roughage is the only food which the sheep consumes in the farm. A single sheep also needs 0.5 ton of roughage per year (De Kock, 2003; Perry, 1982).

Due to farm space and operational business restrictions, the number of animals the farm keeps is constrained. As the farms space for animal accommodation is fixed, the maximum possible adult cattle the farm can keep is 600 and the lowest possible number of adult cattle the farm keeps due to business operational restrictions is 300. The number of sheep is also governed by the same situation. The maximum and minimum sheep numbers the farm keeps are respectively 500 and 2000.

In the farm business, animals are another source of income to the farmer. It is assumed that on average the farm generates an income of R10450 per year from single adult cattle in the dairy production. Income per single sheep in the wool sheep production activity is assumed to be R250 per year. Another activity in the animal production is the selling and buying activities of animals. From a single cattle and sheep sold, the farm can generate an income of R600 and R50 (the income is the interest on capital) per year respectively. Furthermore, if the condition on the farm business is favourable for buying, the farm has an option of buying

adult cattle and young sheep at the cost of R900 and R75 (the cost is the interest on capital) per year respectively (De Kock, 2003).

2.3. Additional Resource Renting Activities

In the overall farming production activities, the farm's resources are limited. The farm has to use hired resources based on the operational requirement of the farming activities, as the capacities of certain equipments the farm owns are limited. The Combine harvester and Baling machine capacities can be restricting factors in the overall operation of the firm. However, in time of extra capacity need of both machines, extra capacity required can be hired from an external provider.

A combine harvester is required for harvesting wheat, canola and lupines. The capacity of the existing combine harvester is 1200 hectares per year. The extra capacity cost of this machine is R1000/ hectare. As mentioned above, another machine with fixed capacity is the baling machine. Silage and medics are either sold to the market at the existing market price or can be used in the preparation of blended feed for animal consumption. In each case, both crops require baling. Baling is carried out on the farm with the existing machine capacity. The baling capacity of the machine is 2000 tons/year. If the demand for baling is more than the existing machine capacity, the farm should employ a hired additional capacity. The cost of an additional capacity of baling machine is 150 R/ton.

3. Model Solution without Considering Risk: Comparison of Monocropping and Crop Rotation Farming Strategies

The first application of the mathematical programming model formulated in chapter three was to investigate the profitability of crop production and animal production enterprises assuming risk free activities on the part of crop production. The mathematical programming was solved without the risk constraints formulated in chapter three by equations 3.38 and 3.39. The mixed integer linear programming farm planning model developed in chapter three was solved without risk consideration in order to compare the profitability of wheat monocropping farm planning strategy with a farm planning which rely on the different crop rotation strategies regarded as feasible. The evaluations hinge on the basis of the total profit generated from the overall farming activity of the farm. The comparison was made based on the following important aspects.

- Comparison of model results of monocropping and crop rotation farming strategies assuming normal year and wet year states of nature.
- Comparison of model results of monocropping and crop rotation farming strategies considering the average of the three states of nature

The model was solved using the optimisation software Whats'Best! ® 7.0, Copyright©2003, Lindo systems, Inc.

3.1. Farming Plan under Normal Year Model Assumption

Under monocropping strategy, both wheat and medic crop are included in the mathematical programming model as the only crops grown in the available farmland as separate and distinct strategies. However, the analysis of this section will give attention only to the comparison of

wheat monocropping strategy with other strategies that follow crop rotation strategies. Moreover, in evaluating the performance of the strategies, the main criterion of evaluation was the profit generated from the whole farming plan in each of the scenarios. Both scenarios include all the activities mentioned in developing the model.

The model results for a normal year assumption from the risk free profit maximizing mixed integer linear programming model suggest that in the planning period (year), the farm earns the total profit of R3, 035,640 from the total activities of the farm if wheat monocrop is assumed to be the only viable cropping strategy. The optimal model solution for both wheat monocropping and crop rotation strategies for a normal year assumption is given in the following tables (Tables 4-6).

	Wheat monocropping	Crop rotation
Tons of wheat Produced	4500	2684.62
Tons of medics Produced	-	4395.41
Tons of silage (oats) Produced	-	346.00
Tons of wheat sold to the market	4500	2684.62
Tons of medics sold to the market	-	3738.01
Extra combine harvester capacity rented (hectares)	600	-
Extra capacity of baling machine hired (Tons)	-	2741.41

Table 4. Optimal model solutions for wheat monocropping and crop rotation farm planning situation

	Lupines	Silage (Oats)	Medics	Straw	Cotton seed Oil cake	Maize	Braw	Molasses	Cotton seed
Wheat monocropping	820.51	392.4	745.56	274.68	-	692.19	370.82	235.44	392.40
Crop rotation	-	346.00	657.40	242.20	721.52	864.84	74.44	207.60	346.00

Table 5. Optimal Feed mix results for animal feeding plan for normal year farming plan (ton)

	Wheat monocropping strategy		Crop rotation strategy	
The number of adult cattle	340	Sell 110	300	Sell 150
Number of sheep the farm keeps	1000	Sell 500.	1796	Buy 296

Table 6. Optimal model solutions for animal production plan

The mixed integer linear programming model was also solved under the assumption of normal year considering the 15 cropping strategies. The model was restricted only to select a maximum of two strategies. The model solution under this scenario of planning shows, especially, that the value of objective function (profit) is greater than the wheat monocropping farming plan scenario. The solution of the objective function (profit) of the model shows a value of R5, 673,436 which is 46.49% higher than the profit achieved under the monocropping farming plan. The pattern of farmland allocation under normal assumption is given in figure 6 below.

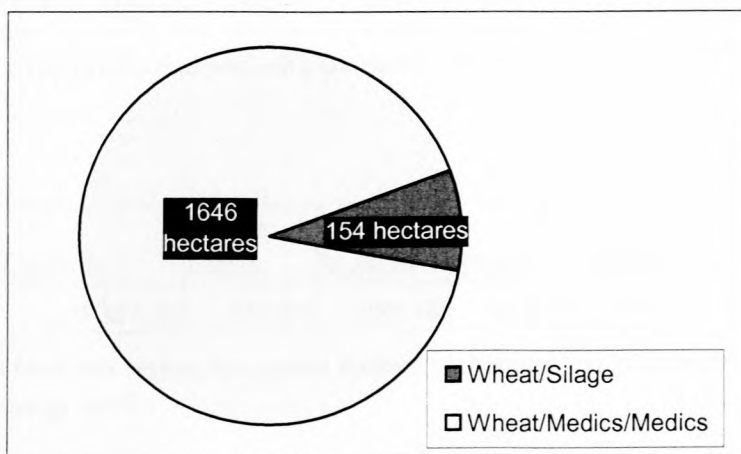


Figure 6. Land allocation under normal year crop rotation scenario

As shown in the above figure 6, the model solution indicates that two sets of crop rotation strategies are selected for implementation. The optimal solution suggests that the decision maker should grow wheat-silage in 154 hectares of land in one of the two plots of land and wheat-medics-medics in 1646 hectares in the second plot of land.

Further results of the model solution for both scenarios are given in Tables 4, 5 and 6 above.

3.2. Farming Plan under Wet Year Model Assumption

The mathematical model was solved under the assumption of a wet year state of nature for monocropping and crop rotation scenarios in order to compare the profitability of the farming plans of each plan. The mixed integer linear programming model for the wheat monocropping strategy of the farm plan offers a profit of R3, 771,663 per year, which is 20% higher than the profit earned under normal year wheat monocropping farming plan scenarios. The model solution for the wheat monocropping-farming plan under wet year situation is given in Tables 7, 8 and 9 below.

Hectares of wheat grown	1800
Tons of Wheat produced	4950
Tons of Wheat sold to the market	4950
Extra combine Harvester capacity rented (hectares)	600

Table 7. Crop production and sell plan under wheat monocropping farming plan for wet year scenario

Raw material type	Lupines	Silage (oats)	Medics	Straw	Maize	Braw	Molasses	Cotton seed
Tons	1087.32	520.00	988.00	364.00	917.28	491.40	312.00	520.00

Table 8. Optimal Feed mix results for animal feeding plan under wet year assumption for the monocropping strategy (ton)

The number of adult cattle	450	Sell 0
Number of sheep the farm keeps	1152	Sell 348.

Table 9. Model solution of wheat monocropping strategy for animal production plan under wet year assumption.

To compare the profitability of wheat the monocropping farm plan with the performance of a farming plan which is based on crop rotation alternatives, the mathematical model was also

solved for a farming plan which consists of 15 choices of cropping strategies for the wet year situation. Figure 7 below shows the optimal allocation of land under wet year crop rotation scenario. The land is divided into two lots. The model solution suggests that the crop sequence wheat-medics-medics is grown in 80% of the available cropping plan. Furthermore, the cropping sequence wheat/silage medics share the remaining 20% of the area. Under this scenario, the profit improves by 45.72% from the farming plan that adopts wheat monocropping as the only option of cropping plan for the farm.

Tables 10, 11 and 12 below present the optimum solution for the crop rotation under wet year assumption.

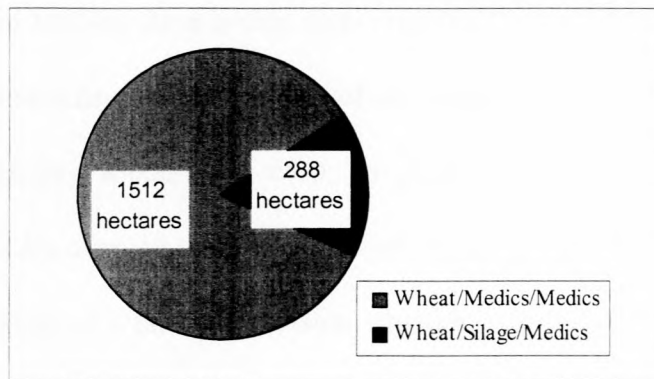


Figure 7. Cropping plan under wet year crop rotation situation

Tons of Wheat produced	2699.03
Tons of Medics Produced	6187.29
Tons of Silage (Oats) Produced	475.92
Tons of Wheat sold to the market	2699.03
Tons of Medics sold to the market	5283.05
Tons of Silage (Oats) Produced	475.92
Extra capacity of baling machine hired (Tons)	4663.21

Table 10. Optimum crop production marketing plan for the wet year crop rotation farm production

Silage (Oats)	Medics	Straw	Cotton seed Oil cake	Maize	Braw	Molasses	Cotton seed
475.92	904.25	333.14	992.44	1189.58	102.39	285.55	475.92

Table 11. Optimal animal feed mix production plans for the Crop rotation scenario farm plan (ton)

The number of adult cattle	412	Sell 38
Number of sheep the farm keeps	1599	Buy 499

Table 12. Animal production plan of crop rotation strategy under wet year assumption.

3.3. Farming Plan Assuming an Average of the Three States

The data considered for this situation is that an average of the three states of nature was used as input rather than considering the data inputs of the three states of nature separately. In the mathematical programming model, the crop, roughage and straw yields applied in the availability constraints are average values. This was implemented in the mathematical model to investigate the outcome of a farming decision situation without paying particular attention to a specific state of nature assuming risk is not present in the farming operation.

The optimal cropping and animal production results of the model farm for wheat monocropping and crop rotation strategies are shown below in Tables 13, 14 and 15 below

	Wheat monocrop	Crop rotation
Tons of Wheat Produced	4350	2490.51
Tons of Medics Produced	-	4220.55
Tons of Silage (Oats) Produced	-	657.40
Tons of Wheat sold to the market	4350	2490.51
Tons of Medics sold to the market	-	3563.15
Extra combine Harvester capacity rented (hectares)	600	-
Extra capacity of baling machine hired (Tons)	-	2566.55

Table 13. Crop Production and sell plan for average data

	Wheat monocropping strategy		Crop rotation strategy	
The number of adult cattle	340	Sell 110	300	Sell 150
Number of sheep the farm keeps	1000	Sell 500.	1696	Buy 196

Table 14. Animal Production Plan.

Figure 8 below shows the land use specified by the mathematical model solution for the crop rotation scenario. The optimal solution indicates that approximately 91 % of the farmland is allocated to the three-year crop rotation strategy (wheat-medics-medics) and the remaining 9% of the land is allocated to the two-year crop rotation strategy (wheat-silage).

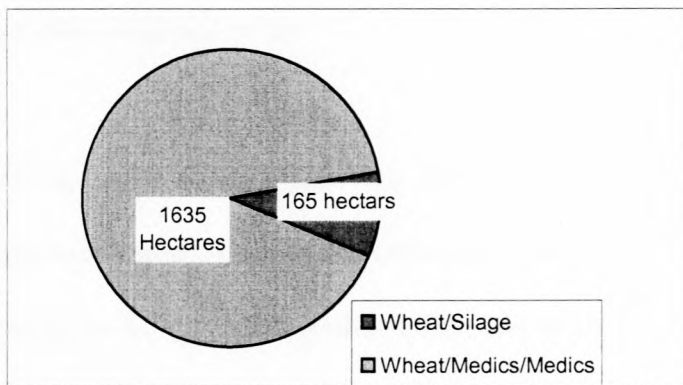


Figure 8. Farmland allocation under average for crop rotation scenario

Silage (Oats)	Medics	Straw	Cotton seed Oil cake	Maize	Braw	Molasses	Cotton seed
346.00	657.40	242.20	721.52	864.84	74.44	207.60	346.00

Table 15. Optimal animal feed mix production plans (tons)

In all the above three cases discussed, a farming plan which adopts a crop rotation is dominant to the monocropping farm planning counterpart if profit maximisation is considered as a performance measure. Based on the profit generated from the activities of the farm, the model results illustrate that the performance of the crop rotation strategy is superior. The

higher profit generated by the crop rotation strategy in all the states of nature considered highlights the benefit of crop rotation in the overall integrated farm planning decision.

Based on the model results, the number of sheep the farm keeps under crop rotation strategy is greater than the wheat monocropping counterpart (see table 6, 9, 12 and 14). This can be accounted to the better availability of roughage in the crop rotation farming scenario. In other words, the favourable condition for forage production created by the rotation strategy allows the increasing stocking of sheep. Consequently, the profit generated from activities that apply crop production is higher. Moreover, the model solution shows that the interdependence between the crop production and the animal production of the farm favours crop rotation strategies that include roughage crops.

Another interesting result when comparing the results of wheat monocropping and crop rotation scenarios in all the above discussed situations, the optimal solution suggests that there is a slight difference in the type of ingredients that constitute the feed mix composition of the farm production. However, the quantity utilized from the farm production in each case differs markedly. The optimal feed mix solution for wheat monocropping strategy indicates that from the total of the feed mix solution the animals consume raw materials produced in the farm represent approximately 7 % of the feed mix. However, in the crop rotation strategy, 36% of raw materials in the feed mix come from the crop produced in the farm whereas the remaining is purchased from the market.

4. Model Solution with Risk Considerations

This section identifies farm plans (especially crop production plans), which maximize profit for different levels of risk. As discussed previously the negative deviation from the expected value of the net return of a cropping sequence is considered as a measure of risk. This measure of risk is parameterised over feasible ranges, which correspond to an arbitrary lower bound of R494,257.05 (λ_{\min}) to an upper bound of R2,237,706.11(λ_{\max}) which is the maximum negative deviation allowed (more investigation of the analysis of risk will be given in section 5 of this chapter). The upper bound corresponds to the maximization problem and the lower bound on risk corresponds to the minimum risk that can be achieved. This minimum risk value can be achieved by considering minimization of risk as the objective function of the model. In order to investigate the effect of risk in the farm planning problem, the mathematical model was solved for the risk levels R494257.05, R600000, R1000000, R1400000, R1600000, R1800000, R2000000, R2200000, and R2237706.11. The summary of the results for such risk levels is shown in Table 22.

The farmland use pattern solution of the mathematical model for the minimum risk situation of a cropping plan is given in Figure 9 below. Under this minimum risk scenario, the model solution suggests to the decision maker to allocate 28 % of the land for the crop production. Due to the conservativeness of this decision scenario, 72% of the land is not allocated to any of the 15 cropping alternatives the decision maker has. Based on the model solution, the remaining land is used to only roughage (grass) production. This is mainly attributed to the high risk aversion nature of the decision maker, as the decision maker prefers production alternatives which are less risky. The maximum profit that can be achieved in such a case is R2, 022,516. Moreover, the optimal model for this scenario specifies that the cattle production activity is carried out in the lowest minimum capacity possible. However, as can

be evidenced from the model solution, due to the availability of land for roughage production, the sheep production is carried at the highest possible level.

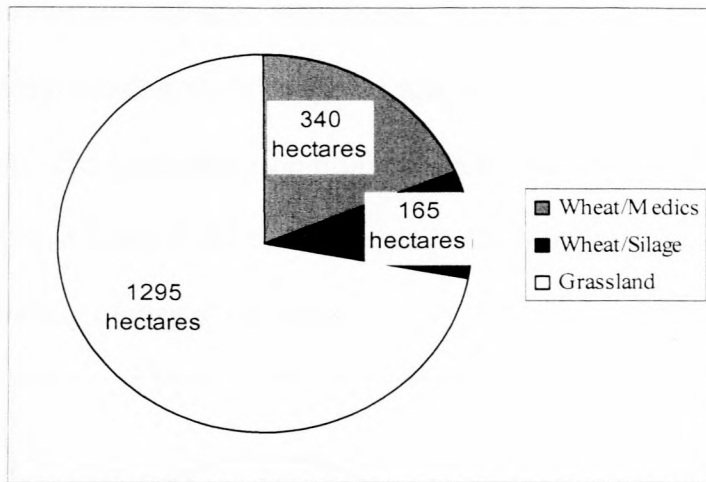


Figure 9. Farmland allocation for the minimum risk situation.

Tables 16, 17 and 18 identifies risk efficient farm plan for the minimum risk situation.

Wheat produced	716
Wheat sold to the market	716
Silage (Oats) produced	346
Medics produced	657
Straw produced	242

Table 16. Amount of crops (tons) produced and sold to the market applying the minimum risk

Silage (Oats)	Medics	Straw	Cotton seed Oil cake	Maize	Braw	Molasses	Cotton seed
346.00	657.40	242.20	721.52	864.84	74.44	207.60	346.00

Table 17. Optimal feed mix under minimum risk (tons)

Type of Animal	Number of Animals the farm keeps	Number sold	Number bought
Adult cattle	300	150	-
Sheep	1955	-	455

Table 18. Optimal animal production plans

If the risk level that the decision maker incurs is allowed to increase from the minimum level to R1000000, the profit level of the farm will increase by approximately 39.47%. Moreover, the basis of the optimal solution will change. Under this condition, the solution from the mathematical programming model suggests that 72% of the land should be allocated for the crop production. The land allocation proposal under this risk scenario is shown in figure 10 below. The optimal solution for the risk level $\lambda=R1,000,000$ are shown in Tables 19, 20 and 21 for the different activities of the farm.

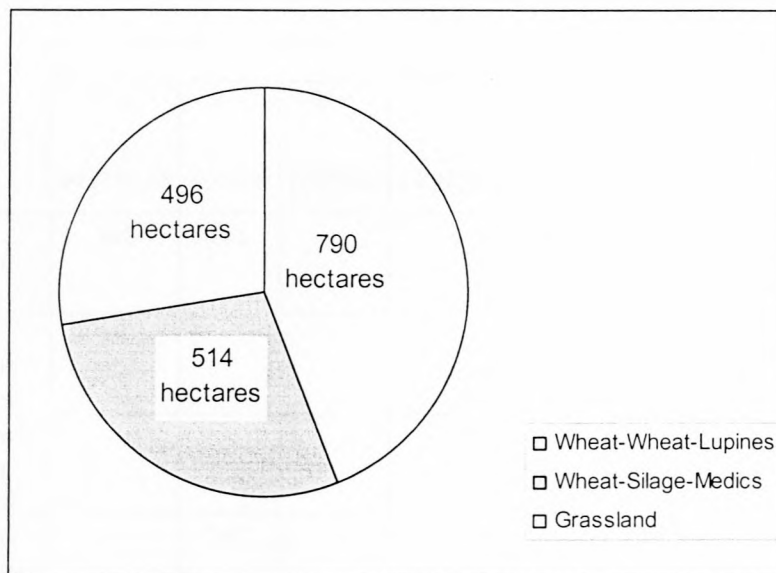


Figure 10. Land allocation under $\lambda=R1,000,000$

Crop type	Produced	Sold to the market
Wheat	2109.07	2109.07
Lupines	234.73	234.73
Silage (Oats)	719.28	373.28
Medics	660.54	3.14

Table 19. Model solution -quantity of crops (tons) produced and sold to the market for the constraint level $\lambda=R1,000,000$

Type of Animal	Number of Animals the farm keeps	Number sold	Number bought
Adult cattle	300	150	
Sheep	1671	-	171

Table 20. Optimal animal production plans at risk level $\lambda=R1,000,000$

Silage (oats)	Medics	Straw	Cotton seed Oil cake	Maize	Braw	Molasses	Cotton seed
346	657.4	242.2	721.52	864.84	74.44	207.6	346

Table 21. Optimal feed mix (tons) at risk level $\lambda=R1,000,000$

The same analysis was also completed for different risk levels. The model solution results are illustrated in Table 22 below.

		Risk constraining levels of λ (Rand)								
		$\lambda =$	$\lambda =$	$\lambda =$	$\lambda =$	$\lambda =$	$\lambda =$	$\lambda =$	$\lambda =$	$\lambda =$
		494257.05	600000	1000000	1400000	1600000	1800000	2000000	2200000	2237706
Cropping Sequence selected	Wheat-Silage	165	285			767	987	612	237	165
	Wheat-Medics	340								
	Wheat-Wheat-Medics					1033				
	Wheat-Medics-Medics		297				813	1188	1563	1635
	Wheat-Wheat-Lupines			790	925					
	Wheat-Silage-Medics			514	824					
	Grassland	1295	1218	496	50					
	Sheep	1955	1999	1671	848	502	581	871	1225	1693
Number of Animals	Adult cattle	300	349	300	447	403	450	450	430	301
	Total Land utilized by crops (hectares)	505	582	1304	1750	1800	1800	1800	1800	1800
Objective function (Rand)		2022516	2382316	3341461	4175168	4428339	4700690	4974153	5246224	5285961

Table 22. Model solution to different risk levels

At the highest risk scenario, the model solution indicates that the choice of a combination of strategies that include the cropping sequences wheat-medic-medic, wheat-silage and wheat-medic are preferred. For the medium risk levels the cropping sequences, wheat-silage-medics and wheat-wheat-lupines are in the optimal solution. This roughly shows that increasing diversified cropping sequences occur for lower risk solutions. The cropping sequences wheat-wheat-medics, wheat-medics-medics and wheat-silage acreage use increased as risk constraints were relaxed in the model. As expected, the value of the objective function increased as risk become less constraining (see Table 22 and Appendix F).

Moreover, one can observe that the choice of risk specification in the mathematical model results (see table 21 and appendix F) in significantly different crop mixes. Regardless of the decision maker's degree of risk aversion, the optimal farm plan included a diversified cropping system. In all the optimisation done for different risk levels, the crop rotation strategies are the only solution in the basis. The optimal solution suggests that forage crops become evident as the yield variability are the important factors both in the forage and crop production plan of the farm. This can be attributed to the interdependence of the crop and livestock activities of the farm. Moreover, one can infer from the model solutions that cropping sequences that are monocrops (wheat and medics) failed to enter to the optimal solution in all the risk levels specified.

The model solutions point out that choice of risk level significantly affects recommendations of crop mix and animal enterprise selection given current market conditions and farm organization. Essentially, in the integrated crop-livestock environment as evidenced from the importance of sustainable agricultural activity, diversification is the best option for profit maximization.

The question, however, remains which strategy would be optimal to follow for the planning period in question, as the differences in the alternatives offered by these different risk specifications are not minor. As there is no direct theory that guarantees an explicit choice from the different solution alternatives, the decision what strategy to adopt will depend on the behaviour of the decision maker to risk. A risk averse decision maker will select strategies which give him some shield from some an adverse situation. On the contrary, a decision maker who is indifferent to risk levels will prefer strategies that give him the maximum profit possible.

5. Sensitivity of Model Solutions to Risk, Strategy and Price Changes

According to Williams (1999), the duality relationship that is true in linear programming problem fails in mixed integer programming circumstances. Though Williams (1999) states that for mixed integer programming problems solved by the branch and bound method, a sensitivity analysis can be performed on the linear programming sub problem at the node giving optimal integer solution, the fact is that, however, for complex problems this approach is not practical. As an alternative, Williams (1999) suggest that by fixing the integer variables at their optimal values sensitivity analysis can be made on the continuous variables. Nevertheless, both methods have drawbacks in deriving meaning full economic information (Williams, 1999).

The only satisfactory method of sensitivity analysis in integer programming (the same as in mixed integer programming) involves solving the model again with changed coefficients (and right hand side) and comparing optimal solutions (Williams, 1999). This study will follow such an approach in performing sensitivity analysis of the risk parameter.

Parametric programming is also one of the most important methods of further analysis after an initial optimum solution is obtained. This option allows one to consider the impact of a sequence of incremental changes in any of the model coefficients. According to Hazell & Norton (1986), there are two approaches of doing sensitivity analysis to problems using a parametric option. These are:

1. Establish a fixed interval of change for the coefficient (right hand side) parameterised and then determine the optimal solution for successive values of that coefficient.
2. Find optimal solutions only at basis changes when a coefficient (right hand side) is parameterised.

In this section, in assessing the effect of risk, the number of strategies selected and the changes in the prices of crops in the decision planning of the farm, a combination of both methods will be employed.

5.1. Sensitivity Analysis on Values of Risk Parameter

To analyse the model results for different values of risk aversion, the expected total sum of negative deviations, λ was parameterised between the minimum value $\lambda_{\min}=\text{R}494,257.05$ and $\lambda_{\max}=\text{R}2,237,706.11$. The model was solved for different risk values between these intervals. Appendix F represents the trade-off between risk and profit of the model result. Figure 11 also shows the model's result reflecting the effect of increasing levels of risk aversion on the maximising profit objective function value.

The parameter λ , which controls the sum of negative deviation, was initially set at a large value. In this case, the mathematical model was equivalent to the mixed integer linear programming model without risk. As λ become smaller (i.e. as decision maker becomes more risk averse), basis change occurred. At each change of basis, the values of λ and the corresponding optimal solution was reported. The optimal cropping plan, number of cattle and sheep the farm keeps, expected profit and the corresponding values of λ are shown in Appendix F.

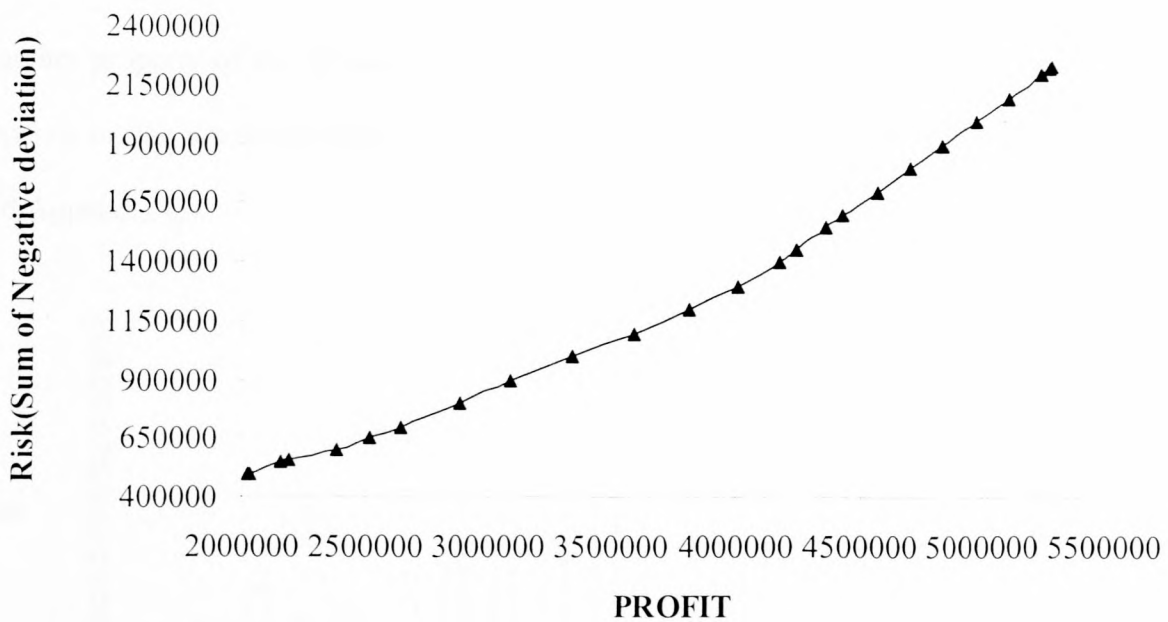


Figure 11. Profit-Risk Frontiers

As the risk level change occurs, a basis change is detected for some values. Specifically, a basis change occurs for the values $\lambda=R494,257.05$, $\lambda=R559,485.98$, $\lambda=R582,966$, $\lambda=R901,016$, $\lambda=R1,499,989.5$ and $\lambda=R1,730,638$ (Appendix F).

The analysis of the optimal farm plans for different levels of risk values showed that the profit level from the farm activities decreases when risk level increases (see figure 11 above). Results of the mathematical model suggest that profit can increase considerably with the decrease of risk aversion level.

One evident property of the optimal solution profit-risk efficient set of the farm plan is that no one plan is superior to another with respect to both the performance measures, namely profit and risk. That is, farm plans with higher profit levels also have high measures of risk. It follows that production plans generating low profit also are associated with low risk levels. As highlighted in the previous section, the selection of a farm plan depends on the decision maker's objective preference.

Another property of the solution set suggests that the acreage utilization for crop production depends on the amount of risk the decision maker incurs (see Figure 12 below, Appendix F and Appendix G).

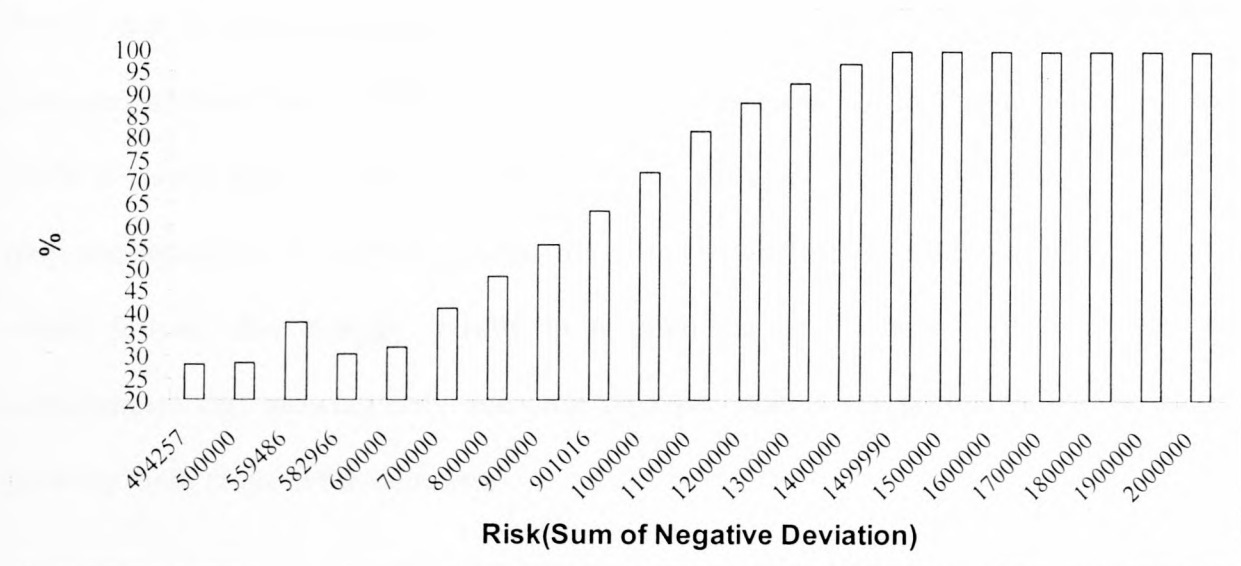


Figure 12. Percentage of acreage allocation for different risk values

In general, based on the sensitivity analysis done for different values of λ , it can be said that the number of adult cattle the farm keeps increases with the increase of λ . This can be attributed to the favourable condition created for the animal production due to the forage crops availability which can replace the feed materials that can be bought from the market. As indicated in section 5, for lower value of λ (if the farmer risk averse, i.e. if the activity is risky) the model solution favours the increase of sheep production. If the farmer is risk averse, part of the land is not allocated for any of the feasible crop alternatives. This allows the increase of forage production, which makes favourable for sheep production.

5.2. Sensitivity Analysis on the Number of Strategies

In all the above model solutions discussed the number of strategies enforced by the strategy constraint was two ($T=2$). In this subsection, to evaluate the effect of number strategies on

the model solution (number of plots), the mixed integer programming for the farming planning model was solved for different numbers of strategies without the risk constraints.

To investigate the effect of number strategies on the farm planning, the model was solved for $T=1, 2, \text{ and } 3, 4$ fixed strategy values (number of plots). Optimal solutions for each of the strategies assumed are illustrated in Appendix H. As presented in appendix H, the highest profit is earned from the farming activity if $T=2$ strategies are implemented. Moreover, if only one strategy ($T=1$) is implemented, the optimal model solution shows that the farm plan, which follows this strategy, results in a minimum profit value. This analysis also demonstrates that growing only one crop type per year is not profitable as compared to growing multi crops in the same year.

5.3. Sensitivity Analysis on Price of Crops.

A number of sensitivity runs were conducted on the prices of wheat, canola, silage and medics assuming a change of price only on one crop occur at a time employing the mixed integer linear programming without taking into account the risk constraints of the model. Such a sensitivity analysis was done to explore the effect of price change of a particular crop on the crop sequence selected.

The sensitivity analysis on the price change (coefficient of objective function, which is R1350/ton) of a crop show that if the price of wheat remains between R1163.3-R2110/ton, the cropping sequences wheat-silage and wheat-medics-medics are selected by the model. However, if the price of wheat is assumed to stay between R2110/ton-R4784.6/ton, the cropping sequence wheat-silage and wheat-wheat -medics is selected. Furthermore, if the price level of wheat is greater than R4784.6/ton monocropping wheat and the cropping

sequence wheat-silage-medic enters into basis. A sensitivity analysis on low wheat price (less than R1163.3/ton) shows that the model solution shifts to growing cropping sequence that are predominantly forage crops (wheat-medics-medics and wheat-silage-medics).

If the price of canola is assumed between R1500-R2427.4/ton, no basis change occurs from the optimal solution. Moreover, the same result is obtained for values less than R1500/ton. In contrast, if the price of canola is greater than R2427.5 three-year sequences, which include canola, enter into the basis (cropping strategies wheat-canola-medics and wheat-canola-silage).

The same analysis was made to investigate the price changes of medic crop on the cropping strategies selected by the mathematical model. As indicated in the optimal solution for an average situation, the cropping sequences of wheat-silage and wheat-medics-medics are in the basis. The basis stays the same for the medic price in the range between R511-R851.8/ton. The cropping sequences wheat-canola-silage and wheat-medic-medic is selected for the medic price less than R511/ton, favouring the sequences wheat-canola-silage as prices lower. On the other hand, if the price of medic is assumed to be more than R851.8/ton, based on the results of the model for the sensitivity analysis, the basis shifts from the cropping sequences wheat-silage and wheat-medic-medic to the cropping sequence wheat-medic-medic and wheat-silage-medic favouring cropping sequence, which consists chiefly of medic crop.

The above sensitivity analysis results underline the fact that for lower market prices of a particular crop the decision maker minimises the acreage allocated for that crop. Similarly, if a favourable condition is created for a particular crop in terms of market price values, the model results indicate that cropping sequences which consist largely of the crop under consideration are good to follow for implementation.

Chapter V

Conclusion and Future Studies

1. Conclusion

In this paper, a mathematical model on a farm planning problem was developed. The paper develops a framework for farmland allocation decision planning in an integrated crop-livestock farm situation by developing a mixed integer linear programming farm planning model. In an integrated crop-livestock farm planning decision problem situation, this paper assumed that crop production decisions are made by taking into consideration the interdependence existing with livestock production. Therefore, the analysis focuses on comparisons of wheat monocropping and multicropping which is based on different crop rotation, risk, profit and other farm planning issues associated with 5 different crop types and animal production activities (dairy and sheep production).

This paper assessed the profitability of the farm production planning by applying wheat monocropping and crop rotation strategies employing the mathematical model formulated. The assessment also included the effect of states of nature, particularly normal year, wet year and average of the three states assumed in the two cropping categories compared (wheat monocropping and multi crop (crop rotation)).

The general perception held by some crop growers is that monocropping wheat is more profitable than introducing crop rotation that allows the incorporation of some oil and leguminous forage crops. However, the results of the comparative profitability performance of continuous wheat versus crop rotation strategies after solving the mathematical model for different states of nature assumptions showed that crop rotations outperform the wheat monocropping in the integrated crop-livestock production context. Furthermore, crop rotation

does seem to respond to the interdependencies that exist in the integrated-crop production situation. That is, the results of the model support the hypothesis made in the first chapter of this paper that in an integrated crop-livestock farming environment, cropping strategies that rely on crop rotation practices are superior to cropping strategies which are dependent on the practice of monocropping. Based on the results of the model, it can be said that the dynamic cropping strategies that are based on crop rotations (two year and three year) are better than their wheat monocropping alternative. One of the reasons for this result is that the interdependence between the crop production and livestock production favours growing a combination of grain and forage crops. The analysis also points out that, assuming the model is adequate; in the integrated crop-livestock farm situation the crop rotation practice is a means of increasing profit.

The concept of crop yield risk as a measure of income variability was incorporated within the context of a farming model to allow the uncertainty (risk) constraints to interact with the specific set of crop production strategies. The paper then developed mixed integer linear programming model which address the following issues.

- What cropping strategy to implement and how much land should be allocated to the crop strategy selected
- Amount of feed mix required
- Amount of crops to sell to the market
- Amount of extra hired resource required
- Risk and
- The number of animals to keep and their feed requirement through out the planning year.

The model was solved assuming the absence of risk and the presence of risk consideration in the farm planning decision-making.

Applying the model in the absence of risk to a representative farm for different states of nature assumptions indicate that the profit level is constrained by the maximum land availability and the cropping strategy chosen. The mathematical programming model identifies wheat-medics-medics and wheat-silage crop rotation strategies with best opportunities for maximum profit cropping under normal year and average year assumption of states of nature. For the wet year condition, the three-year cropping sequences, wheat-medics-medics, and wheat-silage-medics serve as a maximum profit cropping strategy.

The need to consider risk in a farm planning problem allows the decision maker to consider the trade-off between risk and profitability of the strategies selected for implementation. In the mathematical model developed this was performed in building a model that captures the interdependencies between crop and animal production systems, the characterisation of typical years, that is, states of nature must be considered, as the prevailing state of nature directly affects the operation of both crop and livestock activities of the farm. The states of nature allow taking into account favourable and unfavourable conditions for the groups of crops considered. The negative deviation framework considered at the constraint level was used to transfer the risks to the objective function that permitted the trade-off among risk and the profit margin analysed. For different risk profiles, the farming decision plan was investigated.

From a risk averse farmer point of view, conservative cropping strategies, wheat-silage-medic and wheat-canola-fallow are selected. Furthermore, if the state of nature is not favourable for farming activity, the model solutions suggest that the cattle production activity is carried at its lowest limit possible. However, due to the availability of land for grass production (forage) the model indicates the intensification in the wool sheep production.

As discussed previously, diversification is commonly used in risk management strategy that involves participating in more than one activity. The motivation for diversifying is based on the idea that returns from various enterprises do not move up and down jointly, so that when one activity has low returns, other activities likely would have higher returns. A crop farm, for example, may have several productive enterprises (several different crops or both crops and livestock), or may operate disjoint parcels so that localized weather disasters are less likely to reduce yields for all crops simultaneously. The model solution discussed above confirms this idea.

Many factors may contribute to a farmer's decision to diversify. The underlying theory suggests that farmers are more likely to diversify if they confront greater risks in farming. As they are relatively risk averters they face small reductions in expected returns in response to diversification. Other factors may also be important. One of the main cropping sequences selected as optimal strategy in all the above cases is the wheat-medic-medic cropping strategy. Planting medic after wheat may reduce the need for fertilizer because of the nitrogen fixing properties of medic. Moreover, as livestock is part of the enterprise mix of the farm under investigation, crop rotations, which are beneficiary to the feed plan of the animals are favoured by the model. As a result, such rotation strategies are suitable for such a mix of enterprises.

Sensitivity analysis was also performed on the risk parameter, number of plots and price of crops. Overall, the sensitivity analysis for the different coefficients of the mathematical model showed that crop rotation strategies are superior to wheat monocropping cropping strategies in the integrated crop-livestock production.

Therefore, this paper concludes that by having a diversified crop rotation strategy, unpredictable fluctuations of profit from the integrated crop production may be attenuated. To illustrate, for a farm which specialises in an integrated crop-livestock production activities, the implementation of diversified crop rotation strategies based on the combination of grains, oil crops and forage is profitable.

2. Future Study

This research paper identifies the following areas as priorities for further research:

1. Whenever crop and animal production are present together in a farm production, they compete among themselves for the use of production factors, such as, labour, machinery funding. Moreover, as discussed in different sections of this paper, in extensive and semi extensive animal production systems crop production and animal production systems are interdependent. Therefore, in developing an optimum farm plan the decision making process has to consider such factors.
2. The farmer always operates in a competitive market situation. The assumption of a fixed price market scenario overly simplifies the model. Therefore, for future studies, it is imperative that the mathematical model to consider the market change dynamics existing in the input and output prices.
3. The number of states of nature considered in this model was only three. This was due to the problem in the acquisition of data. Therefore, for better results it is imperative to identify many possible states of nature which can possibly occur in the environment in which the farm is operating.
4. The model developed can be easily expanded to consider a broader set of crops, cropping sequences, land preparation and utilization, technology acquisitions and animal production activities.

5. Alternative risk management strategies might be incorporated into the model by defining new variables or by redefining variables. For instance, the weight of financial investments to crop and livestock production activities, insurance, and marketing alternatives are all options, which the model might consider.

6. The farm planning model developed in this paper could be incorporated within the framework of a decision support system. In this way, it would be possible to recompute and update systematically all the coefficients required by the model.

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Appendix A: Mixed Integer Linear Programming Model

$$\text{Max } \mathbf{G} = \sum_{s=1}^n v_s u_s - \sum_{i=1}^N C_i X_i + \sum_{a=1}^I f_a [W_a + N_a - Z_a] + \sum_{a=1}^I q_a Z_a - \sum_{a=1}^I b_a N_a - \sum_{m=1}^Y d_m Y_m - \sum_{u=1}^R h_u R_u$$

Subject to

$$\sum_{i=1}^N X_i \leq A$$

$$\sum_{i=1}^N (C_{i\tau} - C_i) X_i + L_{\tau}^- \geq 0 \quad ; \tau = 1, 2, \dots, h.$$

$$\sum_{\tau=1}^h L_{\tau}^- = \lambda$$

$$p_r \leq \frac{\sum_{m=1}^Y a_{rm} Y_m}{\sum_{m=1}^Y Y_m} \leq P_r \quad [\text{Nutrient } r \text{ requirement constraint } r=1, 2, \dots, x]$$

$$e_m \leq \frac{Y_m}{\sum_{m=1}^Y Y_m} \leq E_m \quad [\text{Raw material } m \text{ requirement constraint, } m=1, 2, \dots, y]$$

$$\sum_{m=1}^Y Y_m - \sum_{a=1}^I \pi_a (W_a + N_a - Z_a) = 0 \quad \text{Total feed requirement}$$

$$\sum_{i=1}^N K_i X_i - \sum_{a=1}^I \mu_a [(W_a + N_a - Z_a)] = 0$$

$$\sum_{i=1}^N \beta_{is} X_i - U_s - Y_m = 0 \quad s = 1, 2, \dots, n \quad , \quad m = 1, 2, \dots, y$$

$$\sum_{i=1}^N \sum_{s=1}^n \alpha_{is} X_i - R_1 \leq H$$

$$\sum_{\text{silage, medics}} U_s + \sum_{\text{silage, medics}} Y_m - R_2 \leq B$$

$$U_s, Y_m, R_2, B \geq 0$$

$$\theta_a \leq W_a + N_a - Z_a \leq \Theta_a, \quad a = 1, 2, \dots, I$$

$$X_i - A\delta_i \leq 0 \text{ for } i = 1, 2, \dots, N$$

$$X_i - g\delta_i \geq 0$$

$$\sum_{i=1}^N \delta_i \leq T$$

$$T \geq 1, \quad X_i \geq 0.$$

$$\delta_i = \begin{cases} 0 & \text{if } X_i = 0 \\ 1 & \text{if } X_i > 0 \end{cases}$$

$$A, Y_m, p_r, P_r, e_m, E_m, K_i, \beta_{is}, X_i, U_s, Y_m, \alpha_{is}, R_1, H, R_2, B, W_a, N_a, Z_a, \theta_a, \Theta_a, \pi_a, \delta_i, \lambda \geq 0$$

and $\delta_i, W_a, N_a, Z_a, \theta_a, \Theta_a$ are integers

Where λ is a parameter and $\lambda = 0 \rightarrow \lambda_{\max}$

Appendix B: Cost of growing (Rand/hectare) Crops per strategies

Wheat	1835
Medics	1100
Wheat/Medics	1250
Wheat/Canola	1660
Wheat/Silage	1340
Wheat/Lupines	1250
Medics/Canola	1300
Wheat/Wheat/Medics	1400
Wheat/Canola/Medics	1433
Wheat /Canola/Silage	1500
Wheat/Medics/Medics	1100
Wheat/Wheat/Lupines	1400
Wheat/Silage/Medics	1266
Wheat/Canola/fallow	1300
Wheat/Medics/Lupines	1200

Appendix C: Market price of crops (Rand/ton)

Wheat	1350
Canola	1500
Lupines	1050
Silage	750
Medics	750

Appendix D: Yield data (Tons/hectare)

1. Wet year yield data

Strategy	Wheat	Canola	Lupines	Silage (oats)	Medics	Roughage	Straw
Wheat	2.75	-	-	-	-	0.72	1.80
Medics	-	-	-	-	5.00	0.72	-
Wheat/Medics	1.65	-	-	-	2.80	0.36	0.90
Wheat/Canola	1.65	0.83	-	-	-	0.96	0.90
Wheat/Silage	1.54	-	-	2.48	0.00	0.48	0.90
Wheat/Lupines	1.65	-	0.50	-	-	0.72	0.90
Medics/Canola	-	0.83	-	-	2.80	0.72	-
Wheat/Wheat/Medics	2.20	-	-	-	1.86	0.72	1.20
Wheat/Canola/Medics	1.43	0.66	-	-	1.86	0.72	0.48
Wheat /Canola/Silage	1.29	0.66	-	1.65	-	0.72	0.48
Wheat/Medics/Medics	1.54	-	-	-	3.74	0.96	0.48
Wheat/Wheat/Lupines	2.20	-	0.33	-	-	0.96	1.20
Wheat/Silage/Medics	1.29	-	-	1.65	1.86	0.72	0.48
Wheat/Canola/fallow	1.29	0.66	-	-	-	0.96	0.48
Wheat/Medics/Lupines	1.32	-	0.33	0.00	1.86	0.72	0.48
Grassland(unused land)	-	-	-	-	-	1.50	-

2. Normal year yield data

	Wheat	Canola	Lupines	Silage (oats)	Medics	Roughage	Straw
Wheat	2.50	-	-	-	-	0.60	1.50
Medics	-	-	-	-	4.00	0.60	-
Wheat/Medics	1.50	-	-	-	2.00	0.30	0.75
Wheat/Canola	1.50	0.75	-	-	-	0.80	0.75
Wheat/Silage	1.40	-	-	2.25	-	0.40	0.75
Wheat/Lupines	1.50	0.00	0.50	-	-	0.60	0.75
Medics/Canola	-	0.75	-	-	2.00	0.60	-
Wheat/Wheat/Medics	2.00	-	-	-	1.33	0.60	1.00
Wheat/Canola/Medics	1.30	0.60	-	-	1.33	0.60	0.40
Wheat /Canola/Silage	1.17	0.60	-	1.50	-	0.60	0.75
Wheat/Medics/Medics	1.40	-	-	-	2.67	0.80	0.40
Wheat/Wheat/Lupines	2.00	-	0.33	-	-	0.60	1.00
Wheat/Silage/Medics	1.17	-	-	1.50	1.33	0.60	0.40
Wheat/Canola/fallow	1.17	0.60	-	-	-	0.80	0.40
Wheat/Medics/Lupines	1.20	-	0.33	-	1.33	0.60	0.40
Grassland(unused land)	-	-	-	-	-	1.00	-

3. Dry year yield Data

	Wheat	Canola	Lupines	Silage (oats)	Medics	Roughage	Straw
Wheat	2.00	-	-	-	-	0.42	1.05
Medics	-	-	-	-	2.00	0.42	-
Wheat/Medics	1.20	-	-	-	1.00	0.21	0.53
Wheat/Canola	1.20	0.53	-	-	-	0.56	0.53
Wheat/Silage	1.12	-	-	1.58	-	0.28	0.53
Wheat/Lupines	1.20	-	0.35	-	-	0.42	0.53
Medics/Canola	-	0.53	-	-	1.00	0.42	-
Wheat/Wheat/Medics	1.60	-	-	-	0.67	0.42	0.70
Wheat/Canola/Medics	1.04	0.42	-	-	0.67	0.42	0.28
Wheat /Canola/Silage	0.94	0.42	-	1.05	-	0.42	0.28
Wheat/Medics/Medics	1.12	-	-	-	1.34	0.56	0.28
Wheat/Wheat/Lupines	1.60	-	0.23	-	-	0.42	0.70
Wheat/Silage/Medics	0.94	-	-	1.05	0.67	0.42	0.28
Wheat/Canola/fallow	0.94	0.42	-	-	-	0.56	0.28
Wheat/Medics/Lupines	0.96	-	0.23	-	0.67	0.42	0.28
Grassland(unused land)	-	-	-	-	-	0.50	-

Appendix E: deviation values

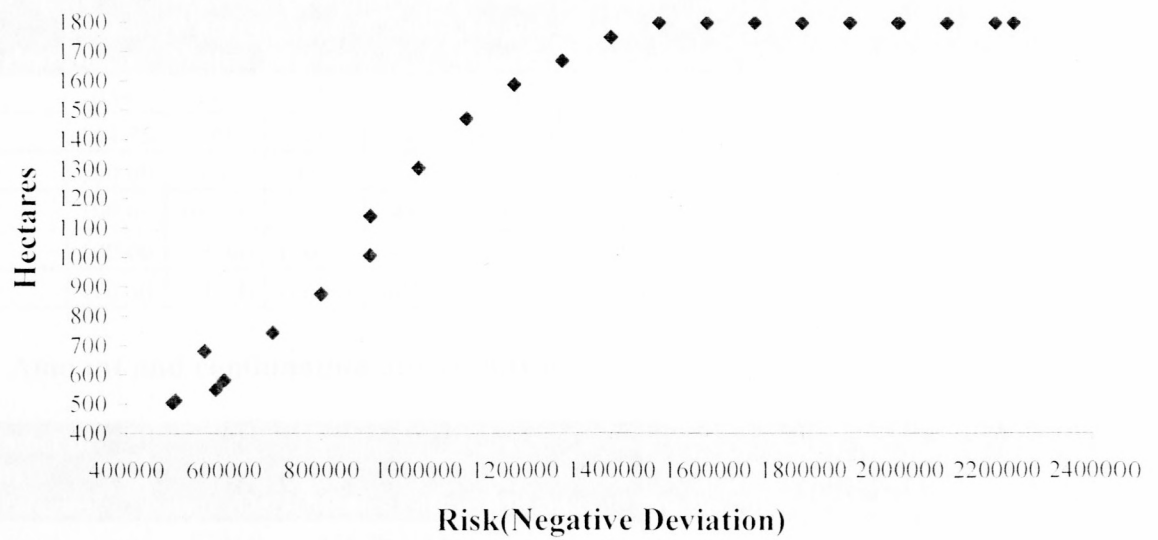
Strategy	State of nature		
	Wet year	Normal year	Dry year
Wheat	534.00	124.50	-658.50
Medics	1014.00	252.00	-1266.00
Wheat/Medics	962.00	123.50	-1085.50
Wheat/Canola	511.17	150.17	-661.33
Wheat/Silage	577.58	181.83	-759.42
Wheat/Lupines	371.50	127.00	-498.50
Medics/Canola	851.50	127.00	-978.50
Wheat/Wheat/Medics	852.92	131.92	-984.83
Wheat/Canola/Medics	848.92	156.42	-1005.33
Wheat /Canola/Silage	557.43	238.98	-796.42
Wheat/Medics/Medics	1157.08	135.08	-1292.17
Wheat/Wheat/Lupines	471.32	125.32	-596.63
Wheat/Silage/Medics	863.02	165.57	-1028.58
Wheat/Canola/fallow	397.93	117.98	-515.92
Wheat/Medics/Lupines	715.57	126.57	-842.13

Appendix F: Model Solution for Different values of Risk Levels

Profit	Risk	Cropping Sequence Selected							
		Wheat-Silage	Wheat-Medics	Wheat-Wheat-Medics	Wheat-Medics-Medics	Wheat-Wheat-Lupines	Wheat-Silage-Medics	Wheat-wheat-medics	Wheat-Medics-Lupines
2022516	494257.05	165	340						
2036070	500000	172	340						
2191848	559485.98	168							513
2336455	582966	241			309				
2382316	600000	285			297				
2640445	700000	488			255				
2883850	800000	620			255				
3091590	900000	751			255				
3095022	901016					629	511		
3341461	1000000					790	514		
3591559	1100000					958	514		
3812093	1200000					1005	583		
4010580	1300000					965	704		
4175168	1400000					925	824		
4286816	1499989.5	1208						592	
4286835	1500000	1208						592	
4428339	1600000	767						1033	
4569247	1700000	321						1478	
4605443	1730638	1116			683				
4700690	1800000	987			813				
4837422	1900000	799			1001				
4974153	2000000	612			1188				
5110785	2100000	424			1376				
5246224	2200000	237			1563				
5285961	2237706.0	165			1635				

N.B: The highlighted values indicate the values where the basis change occurs.

Appendix G: Optimal Acreage Allocation versus Risk



Appendix H: Model solution for Different number of plots

1. Strategies selected and amount of crops produced

Number of plots (Strategies)	Strategy						Amount produced (tons)				
	Wheat-Silage	Wheat-Medics	Wheat-Canola-Medics	Wheat-Canola-Silage	Wheat-Medics-Medics	Wheat-Silage-Medics	Wheat	Canola	Silage (oats)	Medics	Straw
1	0.00	0.00	0.00	0.00	0.00	1800.00	2035.80	0.00	2520.00	2314.20	364
2	164.76	0.00	0.00	0.00	1635.24	0.00	2490.51	0.00	346.00	4220.55	242.2
3	100.00	0.00	0.00	0.00	1600.00	100.00	2467.10	0.00	350.00	4258.17	244.636
4	100.00	100.00	0.00	0.00	1500.00	100.00	2521.77	0.00	350.00	4128.63	244.636
5	100.00	100.00	100.00	0.00	1400.00	100.00	2508.77	56.00	350.00	3999.10	244.636
6	100.00	100.00	100.00	100.00	1300.00	100.00	2483.20	112.00	490.00	3741.00	342.888

2. Amount and combination of feed mix used

Number of plots (Strategies)	Silage (Oats)	Medics	Straw	Cotton seed Oil cake	Maize	Braw	Molasses	Cotton seed
1	520.00	988.00	364.00	1084.36	1299.76	111.87	312.00	520.00
2	346.00	657.40	242.20	721.52	864.84	74.44	207.60	346.00
3	349.48	664.01	244.64	728.78	873.54	75.19	209.69	349.48
4	349.48	664.01	244.64	728.78	873.54	75.19	209.69	349.48
5	349.48	664.01	244.64	728.78	873.54	75.19	209.69	349.48
6	489.84	930.70	342.89	1021.47	1224.38	105.38	293.90	489.84

3. Table showing amount of crops sold to the market, number of animals the farm keep and the profit generated for the different number of plots assumed.

Number of plots (Strategies)	Quantity of crops sold to the market (tons)					Number of animals the farm keep		Profit
	Wheat	Canola	Silage (Oats)	Medics		Adult cattle	Sheep	
1	2035.8	0	2000	1326.2	2834.2	450	648	4,473,440.59
2	2490.508	0	0	3563.15	2566.55	300	1696	5,286,327.46
3	2467.1	0	0.52	3594.155	2608.167	303	1698	5,281,076.61
4	2521.767	0	0.52	3464.621	2478.633	303	1659	5,240,331.61
5	2508.767	56	0.52	3335.088	2349.1	303	1621	5,189,111.61
6	2483.2	112	0.16	2810.304	2231	424	1195	5,129,725.74