Applying the phi ratio in designing a musical scale

Konrad van Zyl Smit

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Supervisors:
Mr Theo Herbst
Prof Johan Vermeulen

DECLARATION

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

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Signature Date

ABSTRACT

In this thesis, an attempt is made to create an aesthetically pleasing musical scale based on the ratio of phi. Precedents for the application of phi in aesthetic fields exist; noteworthy is Le Corbusier's architectural works, the measurements of which are based on phi.

A brief discussion of the unique mathematical properties of phi is given, followed by a discussion of the manifestations of phi in the physical ratios as they appear in animal and plant life.

Specific scales which have found an application in art music are discussed, and the properties to which their success is attributable are identified. Consequently, during the design of the phi scale, these characteristics are incorporated. The design of the phi scale is facilitated by the use of the most sophisticated modern computer software in the field of psychacoustics.

During the scale's design process, particular emphasis is placed on the requirement of obtaining maximal sensory consonance. For this reason, an in-depth discussion of the theories regarding consonance perception is undertaken.

During this discussion, the reader's attention is drawn to the difference between musical and perceptual consonance, and a discussion of the developmental history of musical consonance is given.

Lastly, the scale is tested to see whether it complies with the requirements for successful scales.

OPSOMMING

In die tesis word ‘n poging aangewend om ‘n toonleer te skep wat gebaseer is op die phi verhouding. Die phi verhouding het al voorheen toepassings in die estetiese velde gevind; noemenswaardig is Le Corbusier se argitektoniese werke, waarvan die afmetinge gebaseer is op dié verhouding.

‘n Kort bespreking volg oor die unieke wiskundige eienskappe waaroor die phi verhouding beskik, waarna ‘n vlugtige bespreking oor die manifestasies van dié verhouding in die natuur plaasvind.

Spesifieke toonlere wat suksesvolle aanwendings gevind het in kunsmusiek word bespreek, en die eienskappe waaroor die toonlere beskik wat tot hul gewildheid gelei het word geïdentifiseer. Daar word dan gepoog, m.b.v. die gesofistikeerdste hedendaagse rekenaarprogrammatuur, om ‘n toonleer te skep wat ook oor dié eienskappe beskik.

In die skep van die nuwe toonleer word daar veral klem gelê op die vereiste van die optimalisering van die toonleer se perseptuele konsonans. Om die rede word verskeie teorieë aangaande die persepsie van konsonans in diepte bespreek.

In die bespreking word die leser se aandag gevestig op die verskil tussen perseptuele en musikale konsonans, en ‘n bondige bespreking oor die geskiedenis van musikale konsonans word gevoer.

Laastens word die nuutgeskepte toonleer getoets teen die vereistes wat vantevore daaraan gestel is, en daar word vasgestel dat die toonleer wel voldoen aan die meeste van die vereistes.

ACKNOWLEDGMENTS

I would like to thank my promoters.

I would also like to thank my father, my mother, Mario, Ben, Don and John-Paul. After all "What are friends are for!"

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INTRODUCTION

1.1 AIM

Good or bad, correct or incorrect, ratios are pervasive in many fields of human enquiry. In geometry, the sum of degrees of all squares is the same, as are the sum of all triangles and any other two-dimensional linear figures. Similar ratios are found in the squares of sides of certain triangles.

In physics, an interchangeable relationship exists between energy, mass and the speed of light. In nature, ratios generally exist of physical measures of different bodily attributes of different species although a measure of variability does exist due to environmental and other influences (Livio, 2002: 113).

Since time immemorial, man has puzzled over the meaning of these ratios, and tried to assess whether or not they are the product of some sort of reasoning, or whether they exist per chance. In their fascination of oft occurring ratios, the Greeks have imitated nature with their architecture. Certain ratios were held to be good or bad, or at least more or less pleasing to the eye.

In music too, ratios are applied to create what is generally perceived to be beautiful. Usually when notes with frequencies with small integer number ratios are sounded together, they create the most pleasing effect.

Over the ages, ratios have assisted artists in the creation of aesthetically pleasing works in the most diverse of artistic fields, including Greek architecture and their Dorian ratio, Charles-Édouard Jeanneret Le Corbusier's (1887-1965) architecture based on the golden ratio and Maurits Cornelis Escher's (1898-1972) drawings based amongst other things on the underlying ratios prevalent in the golden triangle. It was the Italian scholastic philosopher St. Thomas Aquinas (1225-1274) who said "The senses delight in things duly proportioned" (Livio, 2002: 37). His belief in a strong correlation between aesthetics and mathematics is certainly worthy of consideration. It can therefore be meaningful to investigate whether phi can be applied to create a musically pleasing scale.

1.2 RESEARCH PROBLEM AND APPROACHES TO ITS SOLUTION

This thesis will investigate whether the phi ratio can be used to create a musically appealing scale.

First, we shall investigate the mathematical properties of phi (φ), all its presentations in nature and the possible reasons for its manifestations. Thereafter we will investigate applications of the golden and other ratios in human fields; mostly in applications of an aesthetic nature.

If the task is set of using φ to create something of beauty in a musical context, it has to be asked "What is beauty in the musical context?" Can one define or quantify it? Is there a scientific basis for it or should one proceed on a hit-or-miss basis, a basis which one can hardly ascribe the adjective 'scientific' to.

Fortunately, scientific research has been done which can be used as a basis for creating a beautiful scale. Extant research indicates intervals are chosen on the basis of their sensory consonance. Papers by Plomp & Levelt (1965) Kameoka & Kurigayawa (1969), Sethares (1997) and Leman (2000), amongst others have shown how one can successfully create intervals for new scales.

After investigating the scientific basis of scale intervals, alternative scales to the widely-used western equal-tempered scale will be discussed, to illustrate that a precedent does exist for the creation of alternative tunings. In fact, the western equal-tempered scale was also once considered an alternative scale!

In the next chapter, some further considerations in creating a scale will be discussed. At the conclusion of these actions, a musically appealing scale using phi will be suggested. We shall use Sethares' (1997) method, which is topical, in constructing our scale.

Chapter 2

THE MATHEMATICS OF PHI

2.1 INTRODUCTION

The golden ratio is an irrational number defined to be (1+√5)/2. This number has been at the centre of much discussion over the centuries. Often referred to by other names, such as the golden mean, the golden section, the golden cut, the divine proportion, the Fibonacci number and the mean of Phidias, it surfaces in a multitude of fields.

An irrational number is one which cannot be expressed as a ratio of finite integers. What, however, warrants a discussion on φ (φ, the Greek letter for "p", denotes this ratio, presumably because the mathematician Phidias studied its properties), as opposed to other irrationals? The justification is the number of important properties which this number possesses. It is a number which appears frequently in the study of nature, which would suggest some unique properties, and also finds numerous applications in human endeavour, mainly in aesthetic considerations.

First, let us start with an exposition of some of φ's most interesting mathematical properties, and then its appearance in nature and then finally some applications which man has found for it.

2.2 MATHEMATICAL EXPOSITION

The following material is derived from Livio (2002), Vajda (1989), Dunlap (1998) and Le Corbusiers' (1958; 1973) books.
Firstly, we shall prove phi is indeed an irrational number. For the source of the proof given below see the *Fibonacci Quarterly*, volume 13, 1975, p.32 in "A simple Proof that Phi is Irrational" by J Shallit and corrected by D Ross. Researcher's comments are inserted in brackets.

2.2.1 Property 1: Phi is irrational

Presuming phi can be written as A/B where A and B are two integers, we could choose the simplest form for phi and write phi = p/q. There will be no factors in common for p and q (except for 1). The proof will show that this is a contradiction. (Consequently phi cannot be rational, as rational numbers can be written as p/q, where both p and q are integers.)

The definition of phi and – phi is that it satisfies the equation:

phi ^{2} - phi = 1 (henceforth "equation 1")

Assuming that phi is rational, i.e. that it can be written as p/q, we can substitute the following:

$$(p/q)^2 - p/q = 1$$

Since q is not zero, we can multiply both sides by q^2 to get:

$$p^2 - pq = q^2 \quad \text{(henceforth “equation 2”)}$$

Now we can factorize the left hand side, giving:

$$p\,(p - q) = q^2$$

Since the left hand side has a factor of p then so must the right hand side. In other words p is a factor of q^2.

Recall, however, we said p and q had no factor in common except 1. Consequently, p must be 1.

After rearranging equation 2, we arrive at the following:

$$\begin{aligned} p^2 &= q^2 + pq \\ &= q\,(q + p) \end{aligned}$$

Since the left hand side has a factor of q then so must the right hand side. In other words q is a factor of p^2. Recall, however, we said p and q had no factor in common except 1. Consequently, q must be 1. If both p and q are 1, p/q is 1, and this does not satisfy equation 1. We have a logical impossibility if we assume phi can be written as a proper fraction. Therefore phi cannot be written as a proper fraction, and must be irrational.

2.2.2 Property 2: The Golden rectangle

Given a rectangle with sides in the ratio 1: φ, the golden ratio can be defined by partitioning the original rectangle into a square and a rectangle. The rectangle that forms will have sides in ratio 1: φ. This rectangle is called a “golden rectangle”.

Euclid (phi was first defined in his ‘*Elements’* around 300 B.C.) constructed golden rectangles with the following method: Draw square ABDC, call E the midpoint of AC, so that AE = EC ≡ x. Now draw segment BE. BE will have length of x √5. This is found with Pythagoras’ theorem of the square of the hypotenuse of a right-angled triangle (ABE) being equal to the sum of the squares of the other two sides. Now construct EF with length = BE. Complete the rectangle CFGD. So far we have the following:

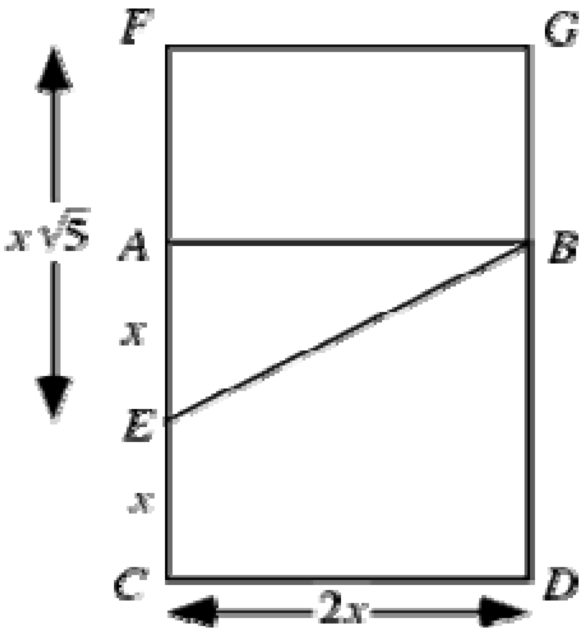

Figure 1. Construction of the golden rectangle

This rectangle is golden as can be seen from the following:

$$\phi = \frac{FC}{CD} = \frac{EF + CE}{CD} = \frac{x(\sqrt{5}+1)}{2x} = \tfrac{1}{2}(\sqrt{5}+1).$$

The longer segment of the golden rectangle is approximately 1.618054 times the length of the shorter segment, while the shorter segment is 0.618054 times the length of the longer. These numbers are remarkable in that they are reciprocals of each other and the figures after the decimal point are identical in both.

Successive points dividing a golden rectangle into squares lie on a logarithmic spiral (cf.Wells, 1986: 39). The spiral is not actually tangent at these points, however, but passes through them and intersects the adjacent side, as illustrated in Figure 2.

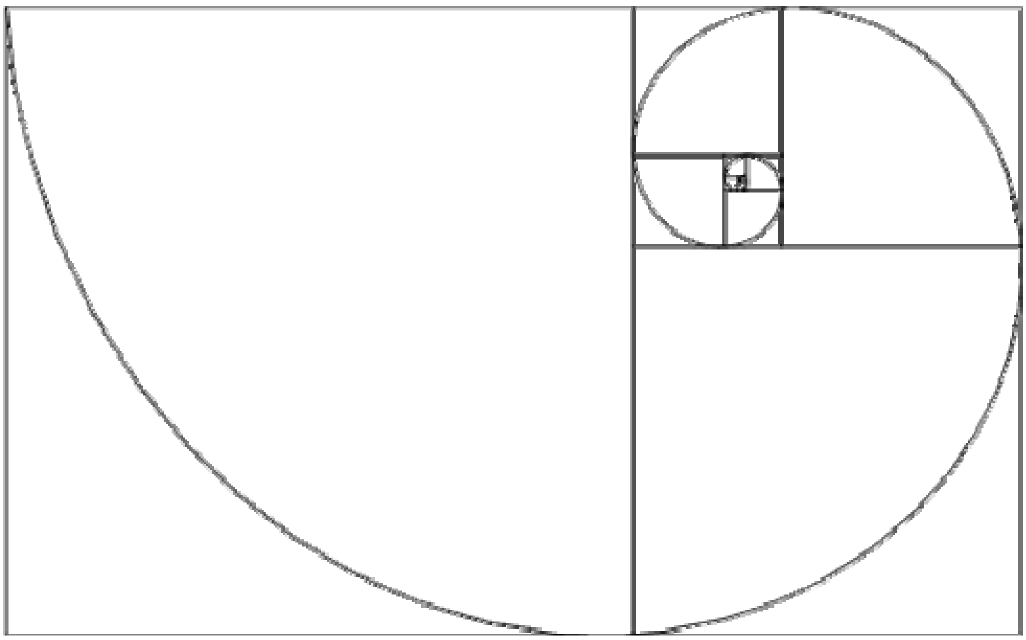

Figure 2. The golden spiral

2.2.3 Property 3: The convergence of Fibonacci ratios towards phi

Leonardo Fibonacci was born in Pisa, Italy, in approximately 1170. His father, Guilielmo Bonacci, held the position of secretary of the Republic of Pisa. Guilielmo's responsibilities included directing the Pisan trading colony in Bugia, Algeria.

Guilielmo intended having Leonardo become a merchant, and took his son with him to Bugia in 1192 to receive instruction in calculation techniques. In Bugia Leonardo became acquainted with the Hindu-Arabic numerals which had not yet made its way to Europe.

After Leonardo's instruction, his father enlisted his services on behalf of the Republic of Pisan, and consequently Leonardo was sent on trips to Egypt, Syria, Sicily, Provence and Greece. During his travels, Leonardo took the opportunity of acquainting himself with the mathematical techniques employed in each of these regions.

After his extensive travels, Leonardo returned to Pisa around 1200, where, for at least the next twenty-five years, he worked on his mathematical compositions. Only five of these have come down to us, namely the '*Liber Abaci* '(1202), the '*Practica geometriae*' (1220/1221), a letter to emperor Frederick II's imperial philosopher Theodorus, the collection of solutions to problems posed in the presence of Frederick II, entitled '*Flos*' and the '*Liber quadratum*' (1225) which dealt with the problem of the simultaneous solution of equations which are quadratic in two or more.
As a result of these works, Leonardo gained a reputation as a great mathematician, a reputation which led to such a noted person as Frederick summoning him for an audience when Frederick was in Pisa around 1225.

Very little is known of Leonardo's life after 1228, except that he was awarded a yearly salary by the Republic of Pisa in return for his pro bono advice to the Republic on accounting and mathematical matters. He presumably died some time after 1240 in Pisa (http://www.lib.virginia.edu/science/parshall/fibonacc.html and Tatlow, 2004).

In '*Liber Abaci*' (1202) Leonardo investigated how fast rabbits could breed in ideal circumstances. Suppose a newly-born male and female rabbit are placed in a field. Rabbits can mate after a month, and consequently the female can produce another pair of rabbits. Supposing that these rabbits are immortal, and the female produces one new pair consisting of a male and a female every month, the sequence which represents the number of pairs after each year is known as Fibonnacci's sequence. The following figure illustrates this sequence graphically:

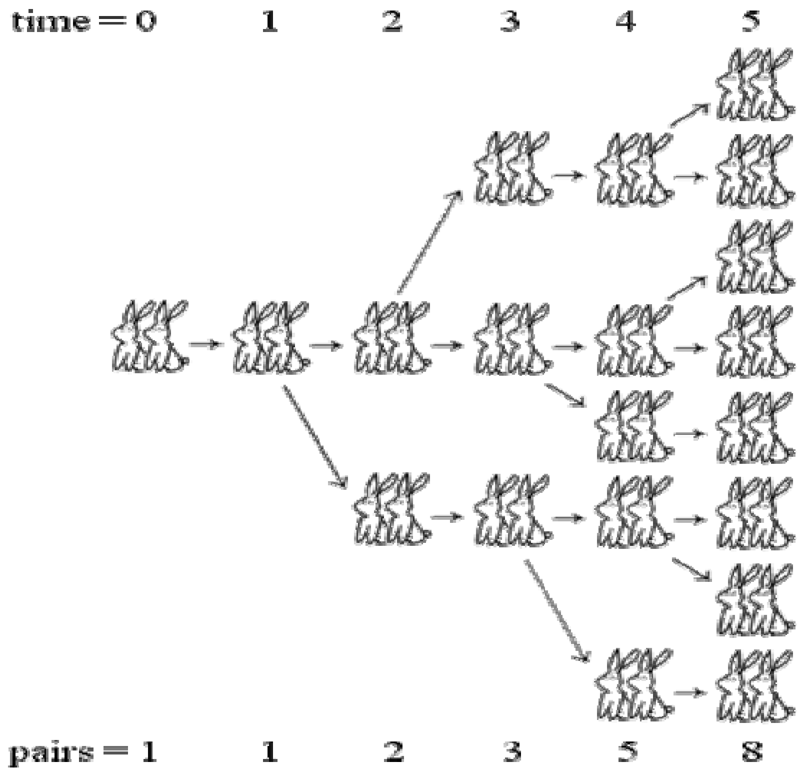

Figure 3. Fibonacci's rabbits
(http://www.jimloy.com/algebra/fibo.htm)

At the start of each month, the sequence representing the number of pairs of rabbits in the field is 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 and so forth (http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html#bees).

If one takes two successive Fibonacci numbers and divide the larger by the smaller number, we seem to get an approximation of φ. Let us, for instance, take the first few numbers of the series:

$1/1 = 1$, $2/1 = 2$, $3/2 = 1.5$, $5/3 = 1.6°$, $8/5 = 1.6$, $13/8 = 1.625$, $21/13 \approx 1.61538$

The ratio of these numbers is converging towards phi. This characteristic is well illustrated by the following graph:

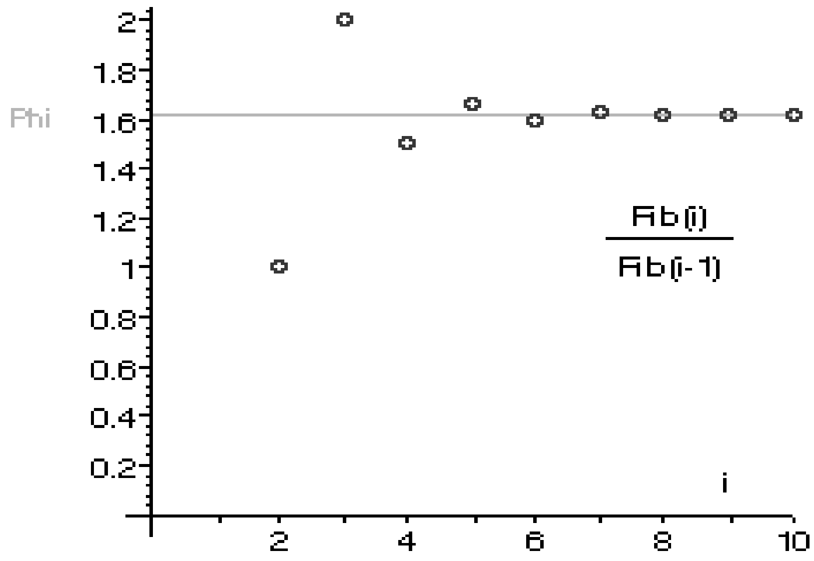

Figure 4. The convergence towards phi
(http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/The Golden Section.html)

Though this visual representation is convincing, it is also possible to represent this with an equation. The following proof relies on the assumption that the ratio of numbers are converging towards a certain number, let us say Y. Keep in mind the basic Fibonacci relationship which is F(x+2) = F (x+1) + F(x).

If we take three neighbouring Fibonacci numbers, F(x), F(x+1) and F(x+2), for very large values of x, the ratio of F(x+1) /F(x) will be almost equal to the ratio F(x+2)/F(x+1).

Therefore F(x+1) /F(x) = F(x+2)/F(x+1) = Y.

Substituting F(x+2) with F (x+1) + F(x) as given by the basic Fibonacci relationship, we get:

F(x+1) /F(x) = F (x+1) + F(x)/F(x+1) = Y

Then, from F (x+1) + F(x)/F(x+1) = Y, we derive 1 + F(x)/F(x+1). The last fraction is 1/x, and so we have an equation purely in terms of x, namely:

x = F(x+1) /F(x) = 1 + F(x)/F(x+1) = 1 + 1/x

Multiplying both sides by x gives $x^2 = x + 1$, which is phi (See equation 1 above)
(http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/The Golden Section.html).

Chapter 3

BIOLOGICAL OCCURRENCES OF PHI

3.1 PHYLLOTAXIS

The following exposition is taken mainly from Livio (2002) and Dunlaps' (1998) books. Leaves along a twig of a plant, or the stems along a branch tend to grow in positions that would optimize exposure to sun, rain and air. If one assumes the stem is vertical, and the leaves are viewed from above, it would be beneficial to avoid the situation where leaves are positioned directly above each other.

As a vertical stem grows, it produces leaves at quite regular spacing. This phenomenon is called phyllotaxis, and means 'leaf arrangement' in Greek. The naming phyllotaxis was coined in 1754 by the Swiss naturalist Charles Bonnet, who lived from 1720-1793. In basswoods, for example, leaves generally grow opposite each other, which corresponds to half a turn around the stem. This then is known as a ½ phyllotactic ratio. In other plants, such as the beech, blackberry and hazel, a 1/3 phyllotactic ratio, which translates to a one-third turn from one leaf to the next, can be observed. The apple, coast live oak and apricot trees have leaves every 2/5 of a turn, and the pear and the weeping willow every 3/8 of a turn, which corresponds to a 2/5 and 3/8 phyllotactic ratio respectively. It is noticeable that all these phyllotactic ratios comprise of consecutive Fibonacci numbers. These are all rough approximates of the golden section! Some other trees with their Fibonacci leaf arrangement numbers are:

1/2 elm, linden, lime, grasses
1/3 grasses
2/5 cherry, holly, plum, common groundsel
3/8 poplar, rose,
5/13 pussy willow, almond

Though not all trees display phyllotactic ratios comprising of Fibonacci numbers, there does exist a definite tendency for these ratios to do so. One estimate is that 90% of all plants exhibit this pattern of growth.

Theophrastus (ca. 370 B.C. - ca 285 B.C.) was first in noticing that leaves of plants follow certain patterns, and first made note of his discovery in his treatise entitled '*Enquiry into Plants'*. Here he remarked "those that have flat leaves have them in a regular series." In his monumental work, '*Natural History*', Pliny the Elder (A.D. 23-79) made a similar observation and referred to regular intervals between leaves arranged circularly around the branches (Adler, Barabe & Jean, 1997: 232).

Only in the fifteenth century did the study of phyllotaxis venture beyond these early, qualitative observations, when Leonardo da Vinci (1452-1519) added a quantitave description to phyllotaxis by noting that the leaves were arranged in spiral patterns, with cycles of five, corresponding to an angle of 2/5 of a turn.

Number of leaf	1	2	3	4	5	6
Total turn around stem	2/5	4/5	6/5	8/5	10/5	12/5
Fractoral part	.4	.8	.2	.6	.0	.4

Table 1. Phi in phyllotaxis

The first person to discover (though only intuitively) the relationship between phyllotaxis and the Fibbonacci numbers was the astronomer Johannes Kepler (1571-1630), who wrote “It is in the likeness of this self-developing series (referring to the recursive property of the Fibbonacci sequence) that the faculty of propagation is, in my opinion formed, and so in a flower the authentic flag of this faculty is shown, the pentagon” (The pentagon is one of the platonic solids, a shape which has a close relationship with phi.)

The first to initiate serious studies in observational phyllotaxis, was Charles Bonnet, who in his 1754 work ‘*Recherches sur l’Usage des Feuilles dans les Plantes*’ (Research on the use of leaves in plants), gives a clear description of 2/5 phyllotaxis. It is also presumed that Bonnet discovered the sets of spiral rows, now known as “parastichies”, appearing in some plants, whilst working in collaboration with mathematician G.L.Calandrini.

The history of truly mathematical research into phyllotaxis, as opposed to the purely descriptive approaches up to then, only commenced in the nineteenth century with the works of botanist Karl Friedric Schimper (published in 1830), his friend Alexander Braun (published in 1835), and the crystallographer Auguste Bravais (1811-1863) and his botanist brother Louis (published in 1837). These researchers were the first to discover the general rule that phyllotactic ratios could be expressed by ratios of terms of the Fibbonacci series, like 2/5; or 3/8; and also that parastichies of pinecones and pineapples feature the appearance of consecutive Fibbonacci numbers. Pineapples specifically provide an easily observable manifestation of a Fibbonacci based phyllotaxis.

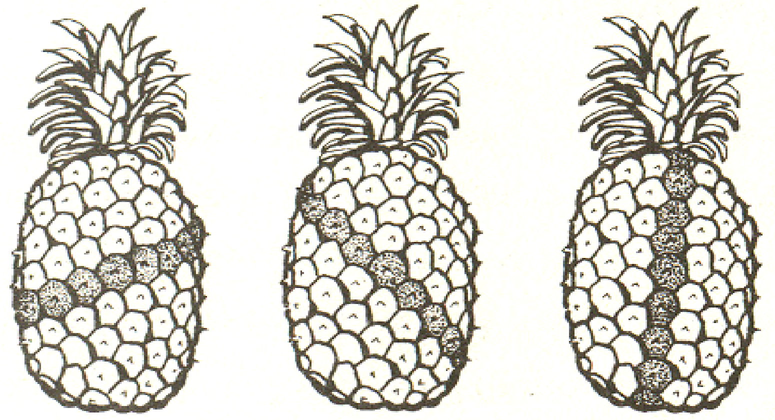

Figure 5. Phi in parastichies
(Livio, 2002: 111)

Each hexagonal scale on the surface of the pineapple forms part of three different spirals. In fig.5, these spirals are indicated. One of eight parallel rows sloping gently from lower left to upper right, one of thirteen parallel rows that slope more steeply from lower right to upper left, and one of twenty-one parallel rows sloping that are very steep, running from lower left to upper right are observable. Most pineapples have 5, 8, 13 or 21 spirals of increasing steepness on the surface. These are all Fibonacci numbers.

Phyllotaxis often occurs in Fibonacci patterns. Though it does not occur invariably, it does occur regularly enough to suggest a tendency towards it.

The growth of a plant takes place at the tip of the stem. This is called the meristem, and has a conical shape, being thinnest at the tip. Leaves further down the tip, which grew earlier, tend to be radially farther out from the stem's centre when viewed from the top, because the stem is thicker there. It would be interesting to see whether the prevalence of irrational number phyllotaxis increases inversely proportionally as the difference between the thicknesses from top to bottom of the stem increases. Figure 33 from page 111 of Livio's book reproduced below, shows such a view of the stem from the top, the leaves being numbered according to their order of appearance.

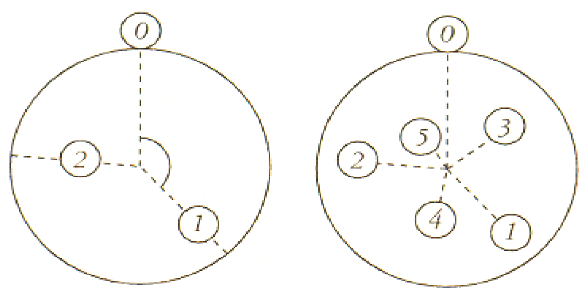

Figure 6. Phi optimizing spacings

The leaf numbered '0', which appeared first, is by now the furthest down from the meristem, and the furthest from the centre. A.H.Church (Livio, 2002: 119) first emphasized the importance of this type of representation for the understanding of phyllotaxis in his 1904 book '*On the Relation of Phyllotaxis with Mechanical Laws'*. By imagining a curve that connects leaves 0 to 5 in figure 6, we find the leaves sit along a tightly wound spiral, known as the generative spiral. The most important quantity that characterizes the location of the leaves is the angle between the lines connecting the stem's centre with successive leaves.

In 1837, the Bravais brothers discovered, amongst other things, that new leaves advance roughly by the same angle around the circle and that the angle, known as the divergence angle, is usually close to 137.5 degrees. The angle that divides a complete turn in a Golden Ratio is 360°/φ= 228.5°. This is more than ½ a circle, so we rather measure it going in the opposite direction around the circle, giving us the observed angle of 137.5°, which has been christened the Golden Angle. To achieve optimal spacing, (with maximum exposure to the elements) the following leaf arrangement, based on φ, can be considered ideal (Adler, Barabe & Jean, 1997: 234).

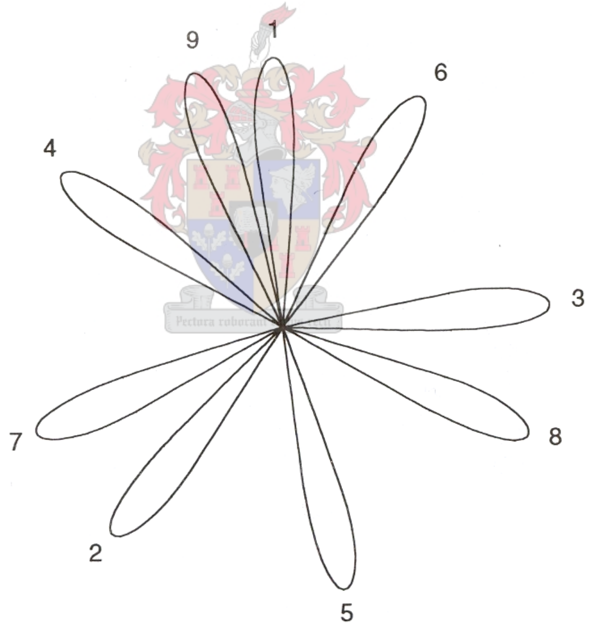

Figure 7. Phi in leaf arrangement
(Dunlap, 1998: 128)

It is quite amazing that a single fixed angle can produce the optimal design no matter how big a plant grows. Once the angle is fixed for a leaf, that leaf will least obscure the leaves below and be least obscured by any future leaves above it.

3.2 COMPOUND FLOWERS

Similarly, once a seed is positioned on a seed head, the seed continues out in a straight line pushed out by new seeds, retaining the original angle on the seed head (http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html#bees). This arrangement seems to form an optimized packing of the seeds so that, no matter how large the seed head, they are uniformly packed at any stage, with no crowding in the centre and not too sparse at the edges.

In 1907, German mathematician G. van Iterson showed that if you closely packed successive points separated by 137.5° on tightly wound spirals, the eye would pick out two families of spiral patterns, one winding clockwise, and the other anti-clockwise. The number of spirals in the two families tends to be consecutive Fibonacci numbers, since the ratio of these numbers approach the golden ratio. (Nature cannot have fractions of seed heads, or an irrational number of seed heads.) This arrangement of counter winding spirals is most spectacularly exhibited by the florets in sunflowers, as illustrated below.

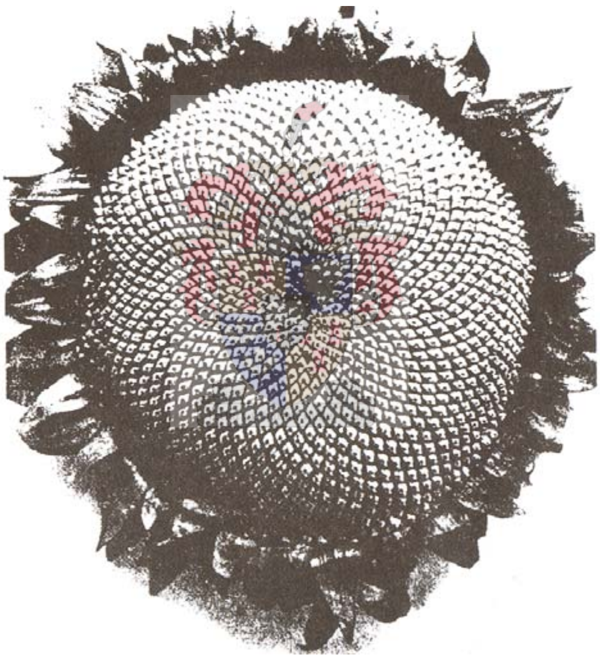

Figure 8. Phi in seed head arrangements
(Dunlap, 1998: 132)

The spirals are not merely patterns which the eye discerns, but are manifest of an underlying mathematical algorithm, an algorithm based on the golden angle. Curvier spirals appear near the centre, whilst flatter spirals, and more of them, appear further from the centre. The number of discernable spirals depends on the flower heads' size; however, the number of spirals in each direction is almost always neighbouring Fibonacci numbers.

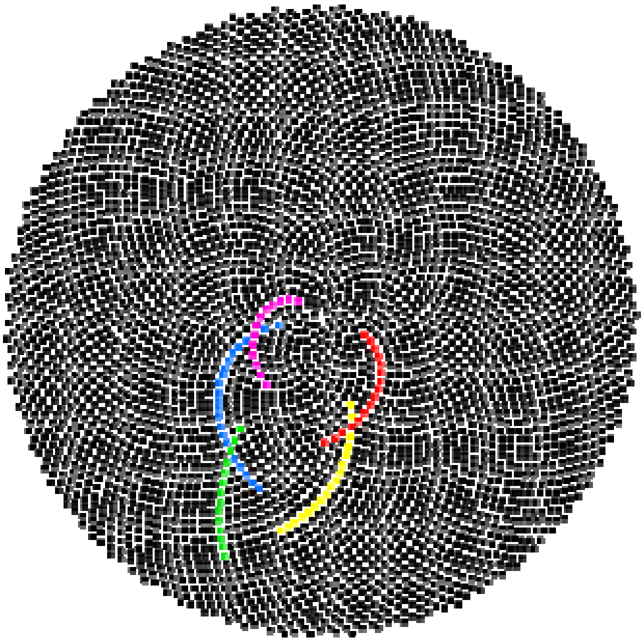

Figure 9. Seed head spirals
(http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/phi.html#fibratio)

In sunflowers, we most commonly observe 34 spirals spiralling one way and 55 the other, but larger sunflowers exhibit ratios of spirals of 89/55, 144/89 and even 233/144.

3.3 THEORIES REGARDING PHI SPACINGS

Experts have come up with two types of explanations for the reason why phyllotaxis and seed placements follow the Golden Ratio. Firstly, theories that concentrate on the geometry of the configuration, and on simple mathematical rules that can generate this geometry (presumably evolutionary), and secondly models which suggest a dynamical cause for the observed behaviour (physics).
Mathematicians Harold S.M. Coxeter and I.Adler showed that buds which are placed along the generative spiral separated by the Golden Angle are close packed most efficiently (Livio, 2002: 113).

This is readily understandable if you consider the effect of using any rational multiple of 360° as a growth angle. Let us for instance consider 30°. The leaves would align radially along 20 (360°/n i.e. 360°/30° = 20) lines, leaving large spaces in between them. On the other hand, a divergence angle like the golden angle, which is an irrational multiple of 360°, ensures the buds do not line up along any specific radial direction, and thus spaces are filled up more efficiently.

The golden angle proves to be the best irrational multiple of 360°, because, as can be seen from its rational approximation, it converges more slowly than other continued fractions. This presumably ensures better spacing, though no irrational angle would lead to the formation of radially aligned leaves.

The best proposal for a dynamical cause of phyllotaxis came from experiments in physics by L.S.Levitov (cf.1991), and Stephane Douady & Yves Couder (cf.1992; 1996). Douady & Couder held a dish full of silicone oil in a magnetic field that was stronger near the dish's edge than at the centre. Drops of magnetic fluid, which act like bar magnets, were dropped periodically at the centre of the dish. The tiny 'magnets' repelled each other, and were pushed radially by the magnetic field gradient. Douady & Couder found patterns that generally converged to a spiral on which the golden angle separated successive drops. Physical systems usually settle into states where energy is minimized. The conclusion drawn from the experiment was therefore that phyllotaxis simply represents a state of minimal energy for mutually repelling buds.

3.4 ARRANGEMENT OF PETALS

Not only has the arrangement of leaves around the stem showed a special relationship with phi. The arrangement of petals of roses is also based on the golden ratio. By dissecting the petals of a rose, one discovers the angle defining the positions, in fractions of a full turn, of the petals are the fractional part of simple multiples of φ. Petal 1 is at 0.618 (fractal of 1*φ) from petal 0, petal 2 is 0.236 (fractal 2*φ) of a turn from petal 1 et cetera.

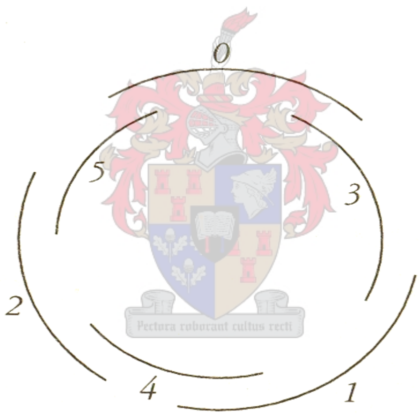

Figure 10. Phi in petal arrangements
(Livio, 2002: 113)

3.5 ANIMAL LIFE

Biological occurrences of the golden ratio are not restricted to plant life, and a most striking example of spiral growth which is related to the golden ratio can be observed in the chambered nautilus, as pictured below (Nautilus pompilius).

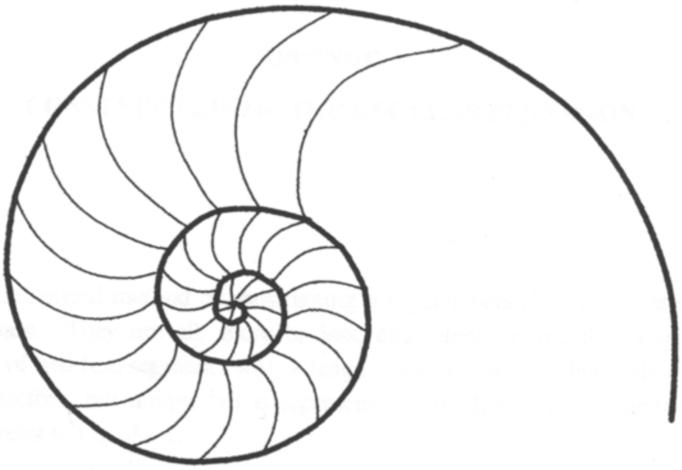

Figure 11. Phi spiral in animal life
(Dunlap, 1998: 135)

The nautilus is in the class Cephalopoda, in which squid and octopuses also fall.

Chapter 4

APPLICATIONS OF Φ AND OTHER RATIOS IN ART AND ARCHITECTURE

In this chapter, the researcher will show how phi found its application in another field which is concerned with aesthetics, namely architecture. The similarities between the application of phi in architecture and in music are numerous, as both music and architecture had previously been governed by other ratios; in architecture we had amongst others the Dorian and Ionic ratios, which found its application in Greek architecture, and in music we have had the Pythagorean and equal-tempered scale currently in use in the western music. Phi found an aesthetically pleasing application in architecture, which is further justification for the assumption that phi could find an aesthetically pleasing application in music.
The earliest evidence of human appreciation for the pleasing qualities of these proportions is found in the pyramids at Giza, which appear to have been built with a 5 to 8 ratio between height and base. This is a close approximation (0.625) to the "perfect" ratio, although scholars disagree over whether the Egyptians were actually aware of it.

4.1 THE ARCHITECTURE OF LE CORBUSIER

The Swiss-French architect and painter Le Corbusier was one of the strongest advocates for the application of the Golden Ratio to art and architecture. Born in La Chaux-de-Fonds, Switzerland, he quickly distinguished himself as an artist and engraver. His mother was a music teacher and encouraged his studies in this field.
He began his studies in architecture in 1905, and eventually became one of the most influential figures in modern architecture. During the winter of 1916, Jeanneret moved to Paris, where he became acquainted with the Cubists, and consequently absorbed an interest in proportional systems and their role in aesthetics from Juan Gris.

Jeanneret only took the name Le Corbusier at the age of 33, a name which was derived from his mother's surname Lecorbesier. At first, Le Corbusier expressed scepticism towards the use of the Golden Section, and prior to 1927, he never used the ratio. Following the publication of Matila Ghyka's (1881-1965) influential book '*Aesthetics of proportion in Nature and in the Arts*' (1933), this changed. Le Corbusier's interest was sparked, and his consequent fascination with the Golden Section can be ascribed to his interest in basic forms and structures underlying natural phenomena, and also to his sympathy towards the Pythagorean craving for a harmony achieved by number ratios.
Le Corbusier was an industrious drawer, sketcher, writer, and most notably a builder. Le Corbusier's search for a standardized proportion culminated in the introduction of a new proportional system he called the "Modulor", which was based on the human body and mathematics.

He envisioned that the Modulor would provide “a harmonic measure to the human scale, universally applicable to architecture and mechanics”(Le Corbusier, 1973: 20) This quote echoes Protagoras’ famous quote from the 5th century B.C., “Man is the measure of all things.”
His Modulor was based on his boyhood experiences when drawing shells, rocks, trees, plants, pine cones and other natural phenomena. He advised others to draw inspiration from nature, thus “… How can we increase our creative power? Not by subscribing to architectural journals, but by adventuring into the inexhaustible realm of natural riches. This is where we can really learn architecture and, to begin with, grace! Yes, flexibility, precision, the unquestionable reality of all those harmonious creations, apparent everywhere in nature. From inside to outside serene perfection prevails; in plants, animals, sites, seas, plains or mountains; even in the perfect harmony of natural catastrophes, geological cataclysms, et cetera. If you wholeheartedly commit yourself to this study of the reason of things, you will inevitably arrive at architecture” (cf. Guiton, 1981).

His system of related proportions were based upon the height of a man with an upraised arm, divided into segments at the positions determining his position in space, namely his feet, solar plexus, his head and his fingertips. These three intervals produce a series of the Golden Section.

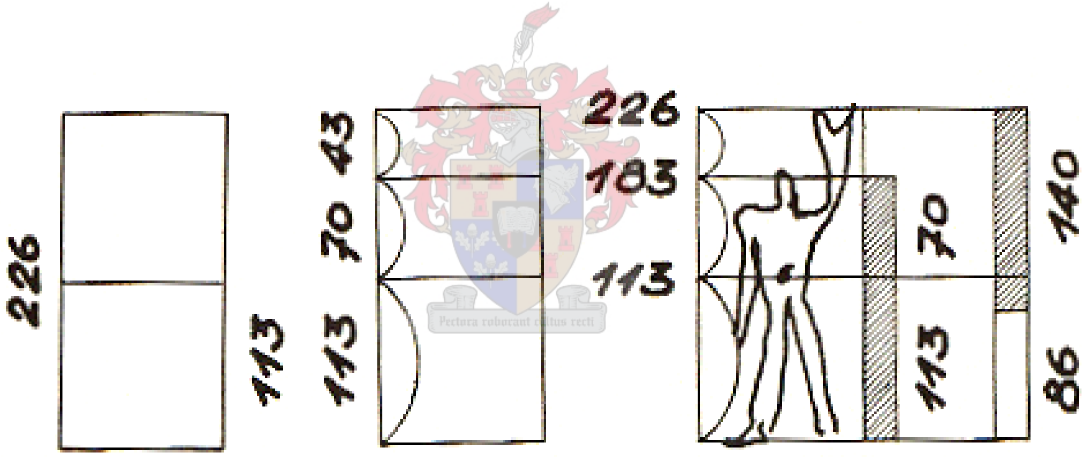

Figure 12. The Modulor
(Le Corbusier, 1958: 66)

Thus two series of dimensions are derived from the human figure. The first, which he named the “Blue Series”, is based on the height of a standing man with upraised arm at 2.26 meters, which translates to 7 ft. 5 inches. The other series, which he named the “Red Series”, is based on the height of the same man measured from his feet to the top of his head, which is 1, 83 meters, or 6 ft.

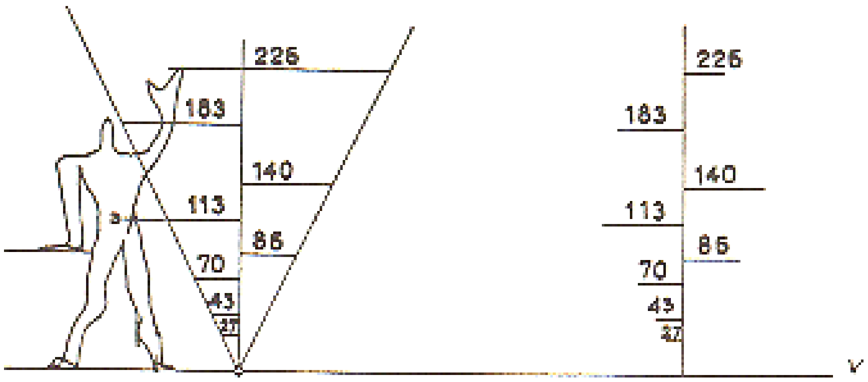

FIG. 25

They may be drawn as follows:

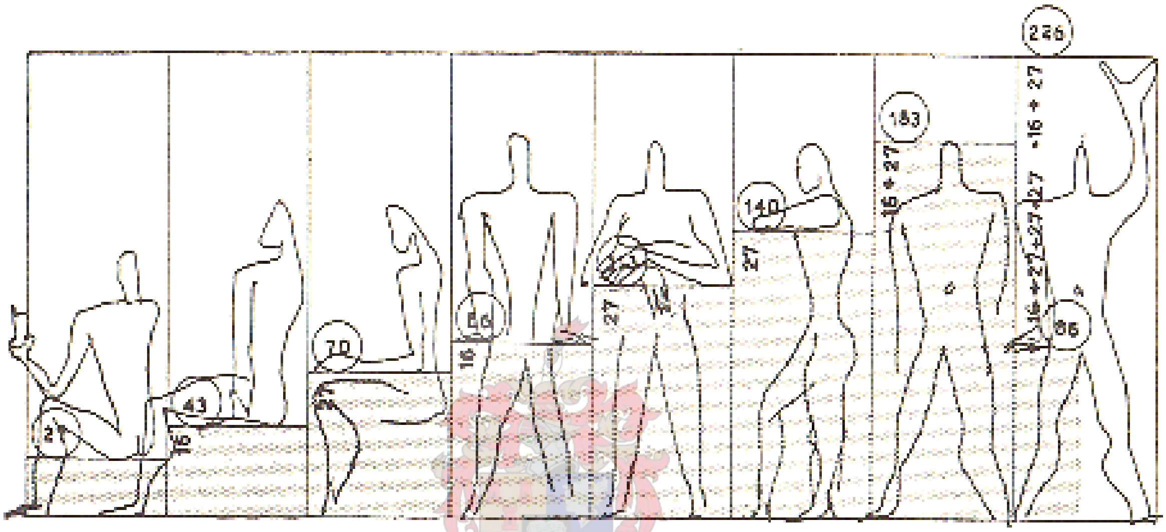

Figure 13. The Modulor scales
(Le Corbusier, 1958: 67)

Le Corbusier reasoned that, since architecture has as its main purpose the creation of structures which serve as extensions and/or containers of men, the Modulor, with its measuring units relating to the dimensions of man, would serve as an invaluable tool to architects in designing their structures. Though these "divisions" seem rather contrived, they do, however, provide a better tool for conceptualization than does the meter, within the context of architectural design.

"The numbers of the Modulor, which are chosen from an infinite number of possible values, are measures, which is to say real, bodily facts. To be sure, they belong to and have the advantages of the number system. But the construction whose dimensions will be determined by these measures are containers or extensions of man. We are more likely to choose the best measurements if we can see them, appraise them with outstretched hands, not merely imagine them"(Le Corbusier, 1973: 52).

Le Corbusier did not intend for the Modulor to replace other systems of measurement, but merely to facilitate the architect in choosing sensible dimensions in their structures.

“The Modulor is a working tool for those who create, such as planners or designers, and is not meant for those who build, such as masons, carpenters and mechanics”(Le Corbusier, 1958: 55).
With these words, one can see Le Corbusier realized the advantage of other systems, such as the metre system, in building, as these systems lend themselves more readily to arithmetic manipulations such as multiplications and divisions. However, just as a composer composes his works in scales, as opposed to crudely referring to Hertz, so should a creator of living spaces “compose” with the Modulor, instead of working with the metre, which is difficult to relate to the human form. The music scale is also divided into proportions which have been found to be pleasing to the human ear.

Le Corbusier often employed the Golden Series in his structures, by having lengths of A, B and then (A+B) as bases next to each other. Note how this is also the start of the Fibonacci series, where each number is the sum of the two preceding numbers. Recall that the ratio between two consecutive numbers in the golden series tends towards φ. Thus, by employing this series, Le Corbusier ensures a relationship between the measures, and thus creates a sense of continuity between the divisions of the structure, which is a necessary element in a structure which is said to be harmonious.

Figure 14. The “Villa at Garches”

Two of Le Corbusier’s most noteworthy creations with the Golden Section are to be found in his designs for the “Villa at Garches”, and his “Cité d’Affairs” in Algiers. Le Corbusier suggests the use of the Golden Section in these structures give them a combination of unity and variety that exists in natural organisms. This claim is justifiable if we recall the biological occurrences of the Golden Section which we had above. Surely man, a product of nature (or at least part of nature) cannot deny the beauty inherent in its (nature’s) design. Denying that such beauty exists, would serve as a denunciation of all beauty, as all beauty presumably comes from nature.

According to Le Corbusier “Natural organisms teach us a valuable lesson: unified forms, pure silhouettes. The secondary elements are distributed on a graduated scale that ensures variety as well as unity. The system, which branches out to its furthest extremities, is a whole” (Le Corbusier, 1931: 36).

Speaking of his “Cité d’Affairs” “Here the Golden Section prevails, it has supplied the harmonious envelope and sparkling prism, it has regulated the cadence on a human scale, permitted variations, authorized fantasies and governed the general character from top to bottom. This 150-meter-high (492ft.) building is insured against all risks: it is harmonious in every part. And it is bound to harmonize with our sensibility.” (Le Corbusier, 1931: 37).

Le Corbusier had the opportunity to present the Modulor to Albert Einstein, in a meeting at Princeton in 1946. Consequently, he later received a letter from Einstein, in which was said “It is a scale of proportions which makes the bad difficult and the good easy” (cf. Dunlap, 1998: 175).

4.2 CONCLUSION

It is clear that phi can be applied in human endeavours of an aesthetic nature, as Le Corbusier did. His work serves as a, albeit somewhat tenuous, precedent for our attempt at applying phi in the creation of a musical scale.

Chapter 5

CONSONANCE AND DISSONANCE

5.1 DEFINITIONS

In dealing with this topic, the researcher will primarily investigate sensory consonance of simultaneous intervals. In other words the perception of pleasantness in musical intervals will be investigated. Specifically, the perceived pleasantness of intervals between two notes will be investigated, with reference to the major theories regarding why these intervals are considered pleasant.

Two tones chosen at random most often sound dissonant, in other words unpleasant. Therefore most subjects would judge most simultaneous tones as unpleasant.

Only a few intervals are judged pleasant by most people. These intervals are said to be relatively consonant. We say “relatively consonant”, since consonance can be measured in degrees. This is relevant in the treatment of consonance in the scientific manner, which is primarily concerned with pleasantness judgments of simultaneously played tones. We refer to this treatment of consonance as sensory consonance.

Consonance as it is used in musical terms was always considered in absolutes. Composers of serious music accepted a few intervals as consonant, and effectively deem all other intervals dissonant. Which intervals were deemed consonant was decided by the current musical style and its needs. The history of this phenomenon, which we refer to as musical consonance, will be investigated.

The importance of identifying the pleasant intervals lies in its usefulness in aiding in the decision of which intervals to use in given scales. The belief is that an optimization of the number of pleasing intervals would help the composer in the composition of pleasant music. Optimizing does not necessarily imply maximizing, as there exists degrees of consonance, and thus a compromise has to be struck between number and quality, where quality refers to the degree of consonance.

There are numerous definitions of consonance. The term ‘consonance’ comes from the Latin ‘consonare’, meaning ‘sounding together’, but in early western music theory the term became synonymous with a harmonic interval (Cazden, 1980: 126).

Palisca & Moore (2001: 324-328) define consonance as follows "Acoustically, the sympathetic vibration of sound waves of different frequencies related as the ratios of small whole numbers; psychologically, a harmonious sounding together of two or more notes, that is with an 'absence of roughness', 'relief of tonal tension' or the like. Dissonance is then the antonym to consonance with corresponding criteria of 'roughness' or 'tonal tension', The 'roughness' criterion…implies a psychoacoustic judgment, whereas the notion of 'relief of tonal tension' depends on a familiarity with the 'language ' of Western tonal harmony. There is a further psychological use of the term to denote aesthetic preferences, the criterion generally used being 'pleasantness' or 'unpleasantness' ".

Terhardt (1974: 1061) is also of the opinion that consonance implies the aspect of pleasantness. He mentions, however, that this pleasantness is not restricted to musical sounds, but that it is a general psychological attribute, and that the criterion for consonance applies to both musical and non-musical sounds. According to Terhardt, Helmholtz (Hermann Ludwig Ferdinand von Helmholtz, 1821-1894) considered consonance as representing the aspect of 'sensory pleasantness'.

Researchers have used many terms in their writings and research regarding consonance, considering most of these words synonymous or opposites, or at least that the difference in meaning was negligible. These terms include: pleasant, unpleasant, euphonious, beautiful, ugly, rough, smooth, fused, pure, diffuse, tense and relaxed (http://dactyl.som.ohio-state.edu/Music829B/notes.html).

Harry Partch (1901-1974), in "*Genesis of a music*", has the following to say with regard to the various terms, "…they are justified in objecting to the common acoustical terms 'pleasant' for consonance and 'unpleasant' for dissonance, terms which are indefinite if not actually misleading. Nor are the terms of the psychologists very clarifying. The criteria, and associated terms, for consonance in their writings include: mechanism of synergy, micro-rhythmic sensation, conscious fusion, fusion, smoothness, purity, blending, and fractionation. So many terms confuse the issue…" (1949: 153).

In 1962 Van de Geer, Levelt & Plomp set out to clarify the situation, and managed to group the appropriate synonyms together, and to identify those terms which thus far had incorrectly been used synonymously (http://dactyl.som.ohio-state.edu/Music829B/notes.html).

According to "Cognitive Foundations of Musical Pitch"(Krumhansl, 1990: 96), tonal consonance refers to the "…attribute of particular pairs of tones that, when sounded simultaneously in isolation, produce a harmonious or pleasing effect. Although the precise definition of this property varies in its many treatments in the literature, there is general consensus about the ordering of the intervals along a continuum of tonal consonance."

For the purposes of this project, we will assume consonance refers to the ‘subjective agreeability of a sound or of simultaneous sounds’(http://dactyl.som.ohio-state.edu/Music829B/notes.html) and that dissonance is the opposite. ”Pleasantness” is thus the operative word. Distinction will also be made between sensory and musical consonance.

According to Palisca & Moore (2001), sensory consonance, refers to the “immediate perceptual impression of a sound as being pleasant or unpleasant”, and it may be judged out of musical context in isolation, and by the musically untrained.

According to them, musical consonance is “related to judgments of the pleasantness or unpleasantness of sounds presented in a musical context”, and depends on musical training and experience. The musical training depends on the accepted and used consonances of the day, and thus a history of musical consonance is given below.

The task which scientists have set themselves concerning consonance, tends to focus solely on causes. The evolutionary purpose of consonance is a question which is addressed less often. For the purposes of this endeavour, the possible causes of consonance shall be concentrated on.

5.2 History of Musical Consonance

5.2.1 Introduction

Consonance in a musical theoretical context is different to consonance in the scientific context, or sensory consonance, which we have been dealing with up to now. Charles Rosen had the following to say: “Which sounds are to be consonances is determined at a given historical moment by the prevailing musical style, and consonances have varied radically according to the musical system developed in each culture. Thirds and sixths have been consonances since the fourteenth century; before that they were considered unequivocally dissonant. Fourths, on the other hand, used to be as consonant as fifths; in music from the Renaissance until the twentieth century, they are dissonances. By the fifteenth century, fourths had become the object of theoretical distress; the harmonic system, defined above all by the relation of consonance to dissonance, was changing, and the ancient, traditional classification of fourths as consonances could no longer be maintained. It is not, therefore the human ear or nervous system that decides what is a dissonance, unless of course we are to assume a physiological change between the thirteenth and fifteenth century” (Storr, 1993: 62).

It is evident that those intervals which had been accepted as consonances have been subject to change over time. This is not to say that sensory consonance has necessarily changed in that time, but only that the acceptance and subsequent usage in composition has changed. As Harry Partch writes: "…the story of man's acceptance of simultaneous sounds as consonances" (Partch, 1949: 90), referring to the intervals which have found acceptance as consonances. It is the researcher's belief that consonance, i.e. the pleasantness of tones, is dependant on various factors, some of which are certainly cultural in nature.

In view of this, it is true that consonance has changed over the centuries, as culture has changed, but not to the extent to which Rosen argues. The most important factors determining consonance are physiological in nature, in other words we believe Helmholtz's beating, the placement of the tones on the basilar membrane (place theory) and perhaps Huron's numerosity conjecture to be of the greatest significance. These physiological factors have not changed over the last few centuries, as Rosen implies, and thus consonance judgments most probably would have been similar to contemporary judgments.

The association of consonance with simple ratios has its origin with the Pythagoreans in the 5th century BC. They regarded consonances, or "symphonies" as they called them, to comprise of the ratios formed from numbers between 1 and 4. These consisted of the octave (2:1), the 5th (3:2), the octave plus fifth (3:1), the 4th (4:3) and the double octave (4:1). The Pythagoreans' acceptance of these four numbers as their consonances was in part due to their fascination with the "tetraktys of the decad", which can be represented as follows (Ferreira, 2002: 4):

```
      *
    *   *
  *   *   *
*   *   *   *
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From this triangle, which is said to have been discovered by Pythagoras, the string lengths 2/1, 3/2 and 4/3 can be derived. The tetraktys stood for the four elements: fire, water, air and earth, the number of seasons and the number of vertices needed to construct a tetrahedron, or pyramid, the simplest regular polyhedron. It was also the symbol upon which the Pythagoreans swore their oath (Nolan: 2002: 273).

Plato believed the harmonic ratios to be engraved into the soul at its creation, and that representations of the harmonic ratios consequently excite it (Cohen, 1984: 108).

It should be noted that the Greeks played notes sequentially; experiments in polyphony only started in the 9th century AD.

Euclid also declared the intervals 2/1, 3/2 and 4/3 consonant in the fourth century B.C. (Partch, 1949: 91).

5.2.2 From B.C. to A.D.

The following text is mainly based on Palisca & Moore (2004) and Tenney (1988). Aristoxenus of Tarent was, according to current knowledge, the first to dissociate consonance from numbers, instead considering consonance as a sensory effect. He considered intervals derived from adding one or more octaves to the Pythagorean consonances as consonances in their own right (Ferreira, 2002: 10). Euclid, in his “Division of the canon”, rejected Aristoxenus’ sensory approach. In the first century AD, Cleonides, in his treatise “Introduction to Harmonics” (Eisagōgē harmonikē), supported Aristoxenus’ opinion.

According to Partch (1949: 91), the four ratios of 5, namely 5/4, 5/8, 6/5 and 5/3 were already recognised as scale degrees by Archytas in the fourth century B.C., but they were not yet accepted as consonances.

Ptolemy, though critical of Aristoxenus’ sensory approach, retained these in his new classification (cf. his “Harmonics”), which embraced homophonic intervals, namely the octave and its intervals, symphonic intervals, which are the fourths and fifths and their combinations with the homophonic or octaves, emmelic, which are intervals smaller than the fourth used in melody, and ekmelic which are the intervals not used in melody. Presumably ekmelic intervals are those not recognised as scale degrees (Palisca & Moore, 2001: 332).

Boethius returned to the set of consonances produced by the numbers 1 to 4, but reported Ptolemy’s opinion that the octave-plus-fourth was a consonance. He classified consonances into three categories, namely “equisonae”, “consonae”, and “unisonae”. Equisonae referred to notes an octave apart, unisonae to notes of exactly the same frequency, and consonae to the “diapente” and “diatessaron”, the fifth and fourth respectively.

In the tenth century the Flemish monk, composer and writer Hucbald proposed his own system. He categorised notes that sound the same or are octaves apart “equisonae”, and “consonae” was the category for all consonances; diatessaron, diapente, diapason (octave), diatessaron-diapente and diapente-diapason.

Hucbald made the distinction between simultaneously played notes, and separately played notes. Consonance referred to simultaneous notes, and “intervallum” or “spatium” to a melodic interval.

Johannes de Garlandia, in the 13th century, categorized consonances and dissonances into three categories each. The three types of consonant chords were perfect (unison and octave), in-between (4th and 5th) and imperfect (major and minor thirds). Dissonance categories consisted of the imperfect major 6th and minor 7th, the in-between major 2nd and minor 6th and the perfect minor 2nd, tritone and major 7th (Palisca & Moore, 2001: 335).

Around 1300, composers started to use pure thirds and sixths as consonances. Previously, the accented chords in polyphonic music had sounded harmoniously in the Pythagorean consonances (unison, octave, fourth and fifth), but now composers used 3rd's and 6th's on accented places. The Pythagorean intervals of the major 3rd (81:64) and minor 3rd (32:27) were perceived to be too large and too small respectively, and were starting to fall into disfavour (Nolan, 2002: 276). Previously, 3rd's and 6th's had been used, but they were always required to resolve in another, "consonant", interval. Henceforth they were not required to be resolved.

Anonymus 4 noted that 3rds were considered the best consonances in the western parts of England. In the anonymous "*Ars contrapunctus secundum Phillipum de Vitriaco*" of the 14th century, minor 6th's were admitted as consonances, and in yet another anonymous treatise entitled "*Ars discantus secundum Johannem de Muris*" the minor and major 6th's were admitted as consonances, but the 4th's were rejected (Palisca & Moore, 2001: 335).

Walter Odington, in his "*Summa de speculatione musicae*", first associated minor and major 3rd's with their superparticular ratios 5/4 and 6/5, rather than those of the Pythagorean tuning of 81/64 and 32/27. In 1518 the verdict was still out on whether major and minor 3rd's were consonant, as Franchinus Gaffurius (1451-1522) only recognised their Pythagorean tuning, and consequently considered these intervals irrational.

Gioseffe Zarlino (1517-1590) subsequently accorded the major and minor 3rd's full recognition as consonances in his 1558 treatise "*Le istitutioni harmoniche*", as well as the major 6th (5:3), but had to justify the minor 6th as a composite of a perfect 4th and a minor 3rd (4/3*6/5 = 8/5). Zarlino believed the reason why the numbers in 6 are consonant, and only they, is because 6 is the first perfect number, being a number which is the sum of all factors into which they can be resolved. Thus his theory of consonance was based on numerology (Cohen, 1984: 5; 78).

Giovanni Battista Benedetti proposed that concordance of intervals depended on the periodicity of coincidence of their vibrations. Thus, by multiplying the terms of the ratio, one can come across the relative consonance of an interval. This seems very significant to me, as it marks the point were consonances were regarded in degrees rather than in absolutes. This theory is known as the coincidence theory, and was the first theory of consonance explained in physical terms (Cohen, 1984: 178).

Vincenzo Galilei (1520-1591), in his 1589 treatise entitled "*Discorso intorno all'opere di Messer Gioseffo Zarlino*", opposed numerical limits imposed on consonance, and insisted that all intervals were equally consonant, and that there were consequently an infinity of consonant intervals (Palisca & Moore, 2001: 336).

Marin Mersenne, (1588-1648), in the 17th century, consequently introduced ratios with 7 as consonances (7/4, 8/7, 7/6, 12/7, 7/5, 10/7)."Since prolonged exercise tends to make sweet and easy what at first seemed rude and annoying, I do not doubt at all that […] the ratios of 7:6 and 8:7, that divide the Fourth, may become agreeable if one gets accustomed to hearing and enduring them…"(Cohen,1984: 108-9) Though Ptolemy had already given these intervals in the reorganisation of the Greek modes in the 2nd century, Mersenne was the first to describe them, though tentatively, as consonances.

Christiaan Huygens (1629-1695), in the 17th century, also supported the acceptance of ratios with 7. He discovered that these intervals exist in almost pure form in mean tone temperament, and observed "…it emits a sound that is agreeable to the ear, and it does not beat…" (Cohen, 1984: 226).

Tartini reinforced Mersenne and Huygens' claims in the 18th century with his own theories and also in his violin playing.

Since the inclusion of 7's as consonances, many theorists have opposed, and many theorists have accepted this claim. The number 7 still does not feature in our music theory or practice (Partch, 1949: 93).

The ratios of 9 were theorized in ancient China in the 27th century BC (Partch, 1949: 93), but were not considered consonant. The ratios of 9 and 11 are considered consonances in monophony. According to Partch, Ptolemy used ratios of number 13 and greater in his scales.

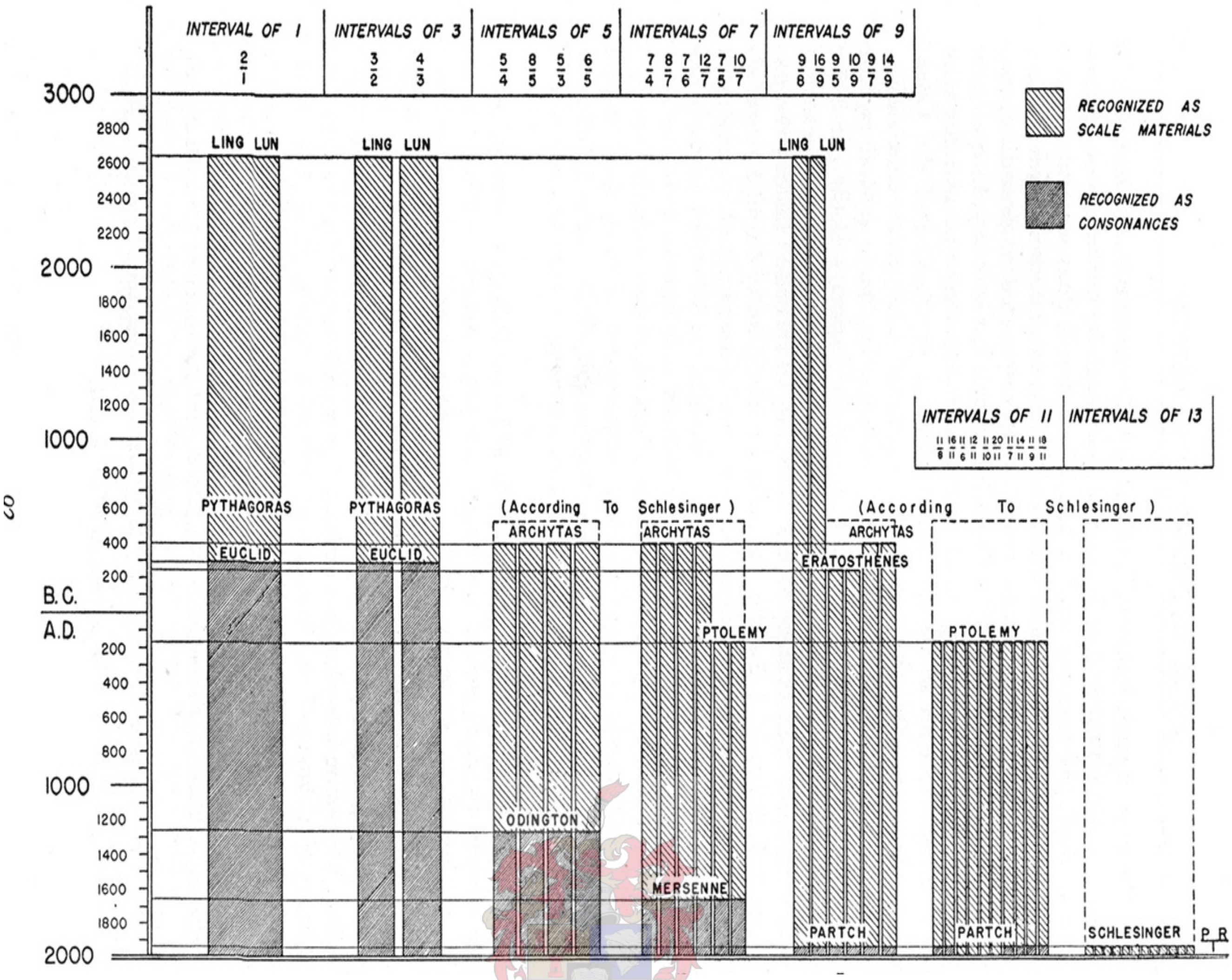


Figure 15. The recognition of intervals
(Partch, 1949)

Harry Partch (1949: 59) summarizes as follows "…from the earliest times it (musical consonance) has progressed from the unison in the direction of the great infinitude of dissonance."

It seems that musical consonance definitions impose an undesirable constraint upon composers.

5.3 THE METHODOLOGY

Generally speaking, scientific fields concerned with the relationship between the objective, physical properties of sensory stimuli in our environment and the subjective, psychological responses evoked by them are termed psychophysics. When the aforementioned stimuli are of an acoustic nature, we are dealing specifically with psychoacoustics. Since we shall examine psychoacoustics applied in auditory stimuli, we shall specifically be dealing with musical psychoacoustics (Rasch & Plomp, 1999: 89).

As an example of how our perception of the objective, physical properties of sensory stimuli differ from the psychological responses evoked by them, one need only consider the experiment done by Houtgast (cf.1962), by which he evoked a low pitch perception by presenting his subjects with only one partial (http://dactyl.som.ohio-state.edu/Music829B/notes.html).

This he achieved by adhering to the following conditions of filling the low-frequency region with noise, ensuring that the single partial has a low signal-to-noise ratio, and that the subject's attention is drawn to the non-existent fundamental frequency's region by prior stimulation. The effect of this situation is to create a perceptual uncertainty with regard to the presence or non-presence of the fundamental frequency, and thus our minds infer the presence of such fundamental (Rasch & Plomp, 1999: 98).

Psychoacoustics is an interdisciplinary field of research, including neurophysiology, acoustics, ethnomusicology and biology, amongst others.

Psychoacoustic experiments are usually done by a stimulus-response procedure, whereby the subject is presented with a certain stimulus, to which the subject is required to illicit some kind of response. The scientist draws conclusions based upon relationships noticed between stimulus and response. Psychoacoustics is therefore an experimental science (Krumhansl, 47: 1990).

The use of computers constitutes an integral part of psychoacoustic research. Computers lend themselves to the description of scientific results which are difficult to describe verbally. The computer also affords the scientist the use of new technologies, such as the recording of neuronal firing patterns in the auditory system (see its application in Chapter 8 below), and the imaging of the working human brain (Gjerdingen, 2002: 977). Furthermore, the precision of the ear in distinguishing fine deviations in stimuli such as sound pressure levels, time differences and frequency differences, is much greater than our ability to produce these differences ourselves, so we rather rely on the computer's precision and sensitivity (Rasch & Plomp, 1999: 90). Computers also afford scientists the opportunity to test their conjectures, due to its great and accurate calculating capabilities.

Choice and adjustment methods are the methods most commonly employed by the psychoacoustician. Of course one could certainly ask the subject to verbally explain his perception of the stimuli he has been subjected to, but unfortunately this is rather insufficient when our sensations allow much finer distinctions than our vocabulary does (Rasch & Plomp, 1999: 90).

The simplest choice method used is the two-alternative-forced-choice (2AFC), where the subject is forced to choose between two well-defined alternatives.

The other most popular method is the adjustment method, where the subject controls the stimulus variable himself, and has to choose an optimal value (Rasch & Plomp, 1999: 93).

5.4 HEARING AND PSYCHOACOUSTICS

5.4.1 The source of sound

All wave motions originate as a result of vibrations, and wave motions are a means for energy to be propagated through a medium. A vibration is a regular, repetitive back and forth or up and down motion. Sound is a longitudinal wave, which means the direction of particle-vibration is the same as the direction of movement of the wave. This vibration has an effect on its surroundings by way of a pattern of pressure changes, consisting of a series of consecutive compressions and rarefactions which move in the direction of propagation. These pressure changes are what we perceive as sound.

The frequency of the wave motion is defined as the number of full waves which move past a fixed point in a unit of time (usually a second), and the amplitude of the wave motion as the maximum displacement undergone by any part of the surface from its equilibrium position, the equilibrium position being the position of rest, or no vibration. The period of a vibration is the time taken in seconds for one complete cycle of vibrations, and its relationship to frequency is simply illustrated with the equation frequency = 1/period.

A vibrating object needs to be in contact with a solid, gas, or a liquid in order for sound waves to be transmitted. It cannot propagate in a vacuum.
In the musical context, the medium is usually air, an "invisible gaseous substance surrounding the earth, a mixture mainly of oxygen and nitrogen" (*The Concise Oxford Dictionary*) although one should keep in mind that sound waves set the fluids of one's inner ear in motion.

5.4.2 How we hear

Our ears are perpetually bombarded by sound waves, as all objects are perpetually vibrating, with many transferring enough energy for our ears to pick up the sound. The ear, in broad terms, converts changes in air pressures which it experiences, into nerve impulses which are sent to the brain.

The earth's atmosphere exerts a constant pressure on our ears, at sea level this is 100 kilopascal, which corresponds to a one kilogram weight's pressure on a surface of one squared centimetre.

The pressure in the middle ear should be more or less the same as the atmospheric pressure outside the ear drum, and for this purpose the inner ear is connected with the throat by the Eustachian tube. This structure allows air pressure to stabilize between the middle ear and the atmosphere. This, then, is the reason why yawning helps when travelling to a place of different atmospheric pressure.

Sound is “caught” by the external ear, or pinna, and channelled down the auditory canal, or maetus. The pinna, which is rather small and pressed back against the side of the head, is not the most effective structure for capturing sound waves, and can be improved by putting a hand behind it. The much larger pinna of an Alsatian, or a fox, together with their ability to seemingly move their ears, gives these animals an advantage. It would be interesting to see if these animals can withstand higher sound pressure levels. From their reaction to firecrackers one would assume not significantly so.

Across the inner end of the auditory channel is stretched a tough membrane, the ear drum, or tympanic membrane as it is called on account of its shape. Sound waves cause the tympanic membrane to vibrate. On the other side of the ear drum is a chamber called the middle ear, where these vibrations are transmitted by three small bones, collectively called the ossicles, consisting of the hammer (malleus), anvil (incus) and stirrup (stapes).

These bones run from the inner side of the ear drum to a membrane-covered opening, the oval window, which leads to the inner ear, the cochlea. The inner ear consists of a series of fluid-filled chambers and canals. The vibration of the ossicles causes the oval window to vibrate, which in turn brings the fluid in the cochlea to motion (Roberts, 1986: 38).

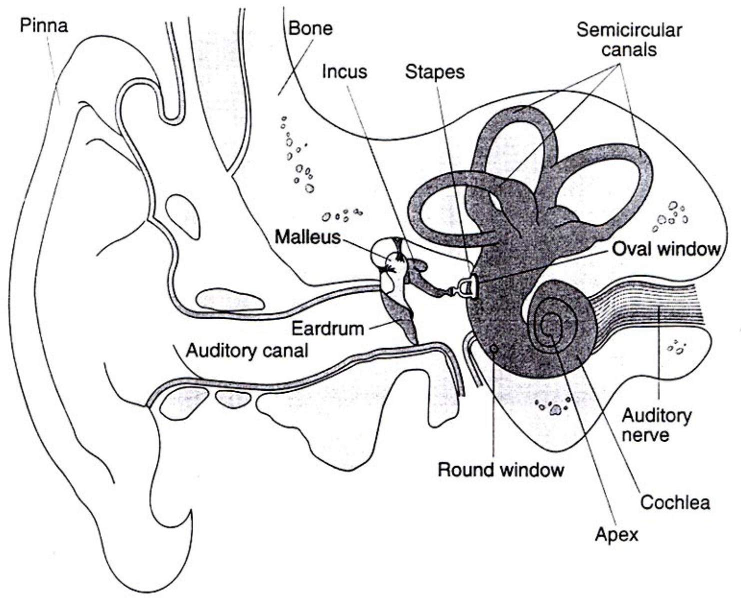


Figure 16. The ear
(Moore, 2001: 294)

The cochlea is divided along its length into two parts by the basilar membrane (Moore, 2004).

Ohm, in 1844, was first to illustrate the ears' ability to perform a sort-of Fourier analysis by separating sounds into its sinusoidal components. Georg von Békésy, (1899-1972) in 1942, was the first to observe that at every point along the length of the basilar membrane, the membrane vibrates with maximum amplitude for a specific frequency. In effect, "the frequency scale of the sound is converted into a spatial scale along the (length of the) basilar membrane" (Rasch & Plomp, 1999: 92).

High-frequency sounds, in the order of 15kHz., cause the basilar membrane to vibrate with maximum amplitude near the oval window, which is situated close to the base, whereas lower frequency-sounds, in the order of 50Hz, would cause a maximum amplitude of vibration towards the other end of the membrane, near the apex of the cochlea. Intermediate frequencies would cause maximum amplitude of vibration at intermediate places. Frequencies below 50 Hz do not affect the distribution of the excitation on the basilar membrane, consequently 50 Hz is regarded as the lower limit of our ear's frequency analyzing capability (von Békésy, 1963a: 589).

Von Békésy demonstrated his finding in an experiment done in 1963. Since the high frequency tones produce maximum amplitude of vibration near the stapes, and the low frequency tones produce maximum amplitude of vibration near the helicotrema, it could be expected that there would be a time delay between the high frequency tones and low frequency tones reaching their respective points of maximum amplitude vibration on the basilar membrane.

A modulated tone of 800 Hz was fed to the left ear, whilst similarly modulated tones of 750, 780, 800, 835 and 880 Hz respectively were, simultaneously with the first tone, fed in turn to the right ear. The time difference between the notes reaching their points of maximum stimulation could be observed by directional hearing, as illustrated below.

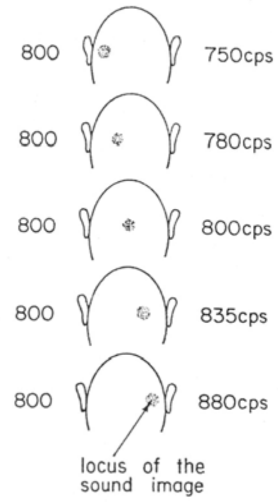


Figure 17. von Békésy experiment
(von Békésy, 1963b: 606)

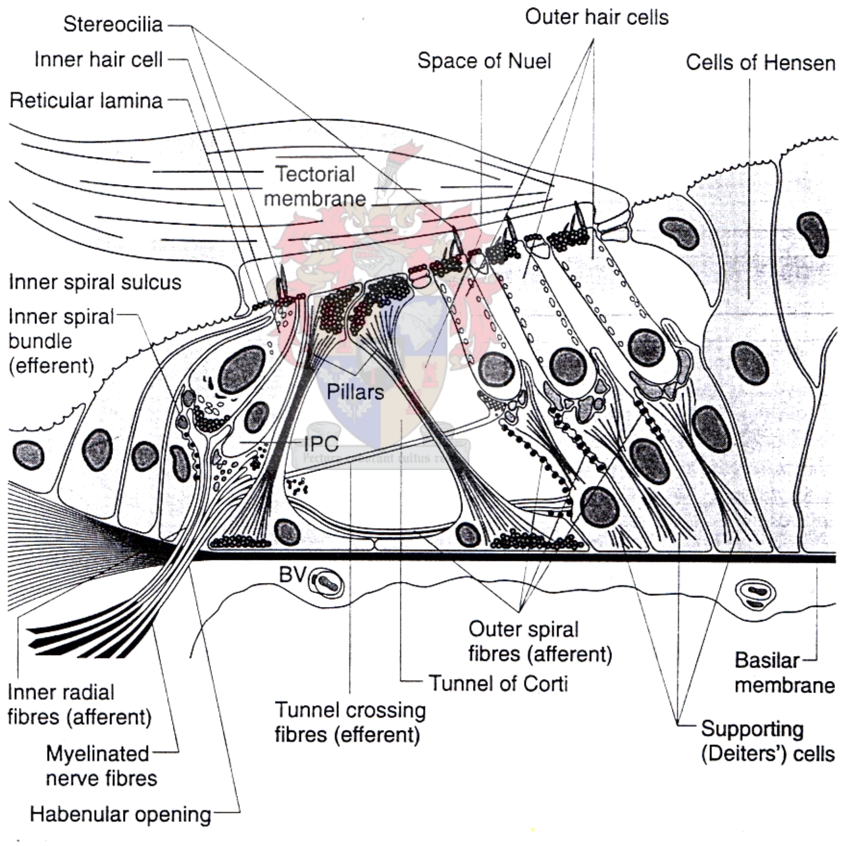


Figure 18. The basilar membrane
(Moore, 2001: 295)

The above figure illustrates the anatomy of the basilar membrane. According to recent measurements of basilar membrane vibration, it has been found that the membrane is even more selectively tuned than von Békésy originally hypothesized in 1960. Khanna & Leonard found, in 1982, that the basilar membrane's selectivity is stronger when the membrane is in a better physiological condition (cf. Moore, 2001: 296). Each point on the basilar membrane is actually only sensitive to a limited range of frequencies, and greater sound intensities are required for higher and lower frequencies. This can be illustrated visually, as in the figure below.

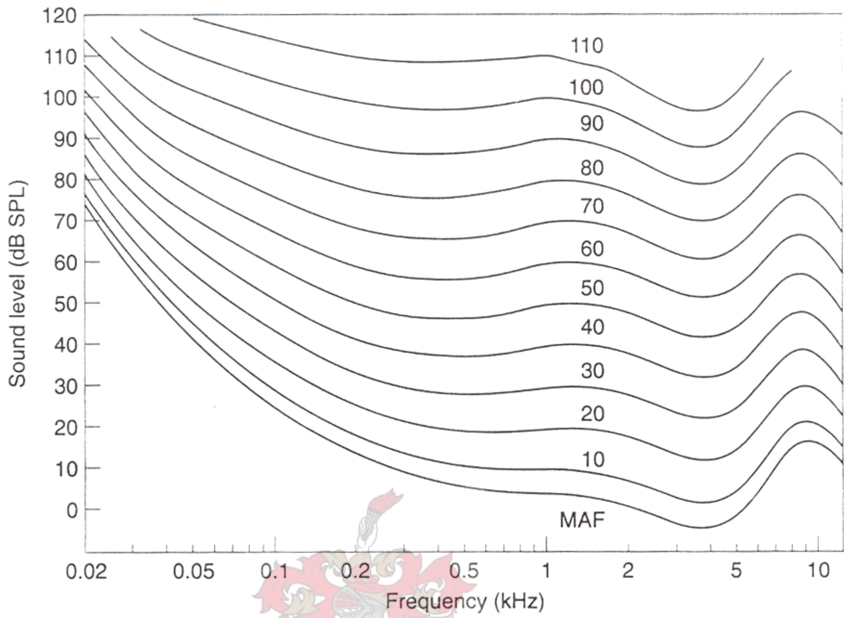


Figure 19. Variability of loudness
(Moore, 2001: 296)

This extreme selectivity and higher differing sound-pressure level requirements lead Yates (cf.1995) to believe that there is an active process in place, since these phenomena cannot be brought about only by means of the mechanical properties of the membrane and the surrounding fluids (Moore, 2001: 294-299).

Above the basilar membrane lies the tectorial membrane, and in between these two structures we find hair cells, which are part of a structure called the organ of Corti (presumably named after the person who discovered it). The hair cells are divided into the inner and outer hair cells, of which there are 3500 and 25 000 respectively. The up and down movement of the basilar membrane causes a shearing motion between the two membranes, and results in the movement of the outer hair cells, which in turn excites the inner hair cells. This excitation causes brief electrical spikes or impulses, called action potentials, in the nerve cells of the auditory nerve, which travel along the nerve towards the brain.

In studies done by Khanna, Ulfendahl and Flock (cf.1989a; 1989b), the outer hair cells have also been shown to act as harmonic enhancers. These cells responded to simple sine component stimuli by generating harmonics not present in the original input signal (Schneider, 1939: 133).

According to Yates (cf.1995), the purpose of the outer hair cells could be to influence the mechanics of the cochlea so as to produce different sensitivities and selective tuning. The ear's tuning phenomenon is maintained by having each nerve cell derive its activity from one or more specific hair cells which occupy a specific place along the basilar membrane, resulting in tuned nerve-cells. Another peculiar characteristic of the ear is that the nerve-firings are phase-locked to the time pattern of the stimulating waveform. This results in a given neuron not firing on every cycle of the stimulus, but ensures that the nerve fires at the approximately the same point or phase of the wave (Palisca & Moore, 2001: 296).

5.4.3 What we hear

What we hear does not correlate exactly with the sound entering the ear. Non-linear distortions in the ear canal and the addition of harmonics in the organ of Corti already insure this. Our mind is perpetually bombarded with perceptual information from our five senses. It would be impossible to process all this information, consequently the brain reduces the amount of processing necessary by searching for recurring patterns in the waveforms. This further accentuates the discrepancy between the sound entering the ear, and what we hear (Fiske, 1992: 138-139). The ear's frequency-analysing capabilities deteriorate due to the ageing process. The first frequency-analysing capabilities to worsen are those in the range of 4 kHz, and subsequent deterioration takes place on lower frequency perception.

The ear's ability to analyse frequencies is what allows us to perceive simultaneous tones. The extent to which the ear can separate simultaneous tones can be studied in various ways. One method is to ascertain how many harmonics the ear can identify in a complex tone. Subjects are presented with a complex tone, and are required to decide which complex tones are present. Plomp, amongst others, has found that listeners are able to distinguish between the first five to eight harmonics.

Another method, by which we can test the ears' ability to separate complex tones, is by measuring the minimum sound pressure level necessary for a probe tone to be audible when presented simultaneously with a complex tone. By varying the probe tone's frequency, we obtain the so-called "masked-threshold".

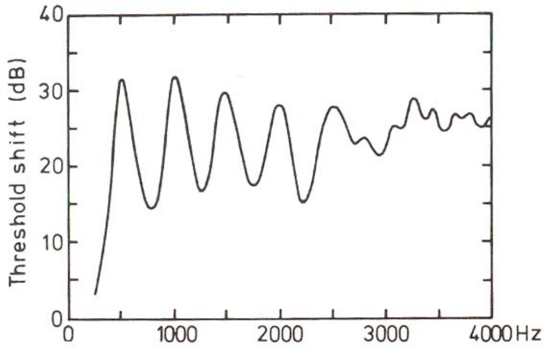


Figure 20. This figure illustrates the masking pattern of a tone of 500Hz with 12 harmonics. (Rasch & Plomp, 1999: 93)

The measure of the ability of a system to analyse complex signals is called its bandwidth. The fact that the fifth harmonic can be distinguished from the fourth and sixth would lead us to believe that the bandwidth of the hearing mechanism is roughly a minor third. In psychophysical literature, this is known as the critical bandwidth. Huron defines the critical bandwidth as the "frequency region within which tones interact" (http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html).

A critical bandwidth of roughly a minor third implies a bandwidth which rises as we raise the frequency, (for a minor third results in a larger hertz difference in the higher registers than in the lower registers) and this makes sense since we perceive a change in pitch according to a logarithmic scale of change in Hertz, and so one would anticipate the critical bandwidth also to have a logarithmic function.

The effect of phase on what we hear

In his masterpiece "*Genesis of a music*", Harry Partch writes: "More frequently than not the like and opposite phases of two tones do not exactly coincide, but are slightly 'askew,' with one delayed in time relation to the other. Fortunately for music, however, this fact has no bearing on comparative consonance (and here an acknowledgement 'Meyer, *Mechanics of the Inner Ear*, 47'). Two tones which are consonant when they have like phases that exactly coincide are just as consonant in a 'different 'phase, since their vibrational ratio is unchanged, and the wave period which the simultaneous sounding of the two tones creates in the air is of exactly the same frequency regardless of phase difference" (1949: 142).

This belief of Partch's could perhaps be ascribed to an earlier theory called the "telephone theory", which held that displacements on the basilar membrane are all in phase (von Békésy, 1963a: 589).

Scientific research has shown, however, that phase does have an effect on the perception of complex tones. Von Helmholtz (1854) was the first to research this problem, but experienced difficulties due to technological restrictions. He did however conclude that phase does not have a negligible effect on the perception of simultaneous tones.

Unlike von Helmholtz, Plomp & Steeneken, in 1969, had a digital computer at their disposal for their experiments on the effect of phase on the timbre of complex tones. Perhaps, to avoid possible confusion of terms, we shall give definitions of the terms "complex tone" and "timbre". David Huron defines 'complex tones' as tones consisting of more than one pure frequency component (http://csml.som.ohio-state.edu/Resources/Handbook/index.html). 'Timbre', according to the American Standards Association (1960), refers to "that attribute of auditory sensation in terms of which the listener can judge that two sounds similarly presented and having the same loudness and pitch are dissimilar." (cf. Plomp & Steeneken, 1969) This definition seems rather circular, but Huron's is no more insightful, so we shall stand by it.

Plomp & Steeneken found that the maximum effect that phase can have on timbre is when the one tone consists only of sine or cosine partials, and the other tone of alternative sine and cosine partials. This maximal effect appears quantitatively comparable with the timbre difference between the vowels [ø], [e] and [œ]. They also concluded that the effect could be considered as arising from the correlation between the vibration patterns at various places on the basilar membrane and the time patterns of the action potentials in the nerve cells of the auditory nerve (Plomp & Steeneken, 1969: 420-421).

5.4.4 Our perception of sound

Certain perceptual characteristics are associated with the perception of certain sounds, such as consonance and dissonance. The perceptual characteristics of consonance and dissonance will be discussed in detail.

Dissonance

Huron (http://www.music-cog.ohio-state.edu/Huron/Talks/Sydney.2002/Dissonance/dissonance.abstract.html) defines dissonance as a "negatively valenced limbic response akin to annoyance, evoked by recognizable stimulus-engendered degradation of the auditory system".

He further gives an evolutionary account for dissonance, and draws parallels with vision. One experiences annoyance when one's vision is obscured by a physical obstruction, or from glare, or from poor focus, which is analogous to one's fear of the dark, where vision is completely "obscured".

According to Huron, the parallel to this phenomenon in audition is masking, of which he distinguishes three degrees, c) no masking, a) complete masking and b) partial masking.

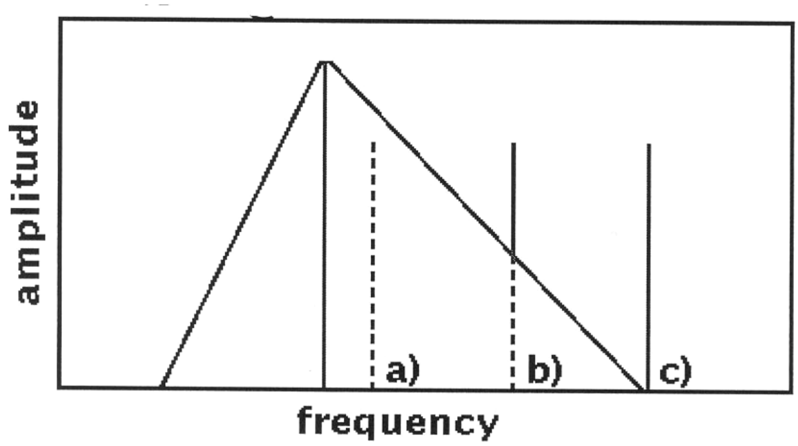


Figure 21. Huron's masking principles
(http://www.music-cog.ohio-state.edu/Huron/Talks/Sydney.2002/Dissonance/dissonance.abstract.html)

"Sounds that are recognized as reducing our capacity to hear other sounds tend to evoke an unpleasant phenomenal experience which in turn leads to stimulus-aversive behaviours."
Survival depended on one's ability to sense danger, and obscured hearing affects that ability. Consequently we experience annoyance at degradation of our audition system.

5.5 Theories regarding consonance and dissonance perception

5.5.1 Frequency ratio theory

The Greek civilisation was apparently first in considering small-number ratios between intervals preferable when seeking consonance. This theory is generally known as the "simple frequency ratio" theory, and is still the most commonly held view by most musicians and music theorists (cf. Bibby, 2003: 13). This belief is, however, not without good reason, as the consonance of a musical interval can roughly be gauged by the simplicity of its frequency ratio. Intervals with small frequency ratios are relatively more consonant as their lower, most important components (on account of them being louder) are either widely spaced or coincide. Less simple frequency ratios give rise to partials which only differ a little, and this leads to dissonance, caused by rapid amplitude fluctuations, otherwise known as beating.

Unfortunately the small frequency ratio theory cannot account for all dissonance, as can be illustrated by comparing the relative dissonance of the just-tuned major third (Ab2/B2), with frequency ratio 4:5, and the just-tuned minor second (A#2/B2), with frequency ratio 11:12. The listener will be struck by the relative dissonance of the major third. Consequently, small frequency ratios should only be regarded as a rough indicator of consonance.

Galileo Galilei (1564-1642) rejected Pythagorean consonance as it is based on numerology. Galileo argued that a regular pattern of vibrations hitting the ear-drum would create a pleasant listening experience, whereas an irregular pattern would make it difficult for the ear-drum to follow the motion, and would ultimately cause an unpleasant listening experience. Galileo implored investigators to focus on the physical properties of sound (Pacey, 1993: 71).

Galileo's theory is known as the co-incidence theory, which graded consonance on the time taken by the tones to "hit" the eardrum at the same time. The co-incidence theory of Galileo is flawed as it does not explain why tempered intervals are still perceived as consonant. They seldom "hit" the eardrum at the same time.

5.5.2 Roughness theory

Euler's theory of consonance, which was based on the idea of a conscious feeling for orderly as opposed to disorderly relations of tone, was also motivated by the commensurability of numbers, as the aforementioned frequency ratio theory was. Euler, a celebrated mathematician, published his findings in 1793 in his book entitled "*An Attempt at a New Theory of Music*".

Helmholtz, in criticizing Euler's theory in his classic volume "*On the Sensations of Tone*" (1863), said "A man that has never made physical experiments has never in the whole course of his life had the slightest opportunity of knowing anything about pitch numbers or their ratios. And almost everyone who delights in music remains in this state of ignorance from birth to death."(1954: 143) Thus Helmholtz argues that one cannot delight in the small number ratios if one is ignorant of them.

Helmholtz believed that music theory could only be understood if it could be shown that its elements had their origin in the perceptual characteristics of the ear. This is clear from the subtitle of his book: "*As a Physiological Basis for the theory of music* ".

Helmholtz believed consonance arises due to the absence of beating of complex tones, and also from the alignment of the upper partials (Butler & Green, 2002: 260). He held that, if all pitch intervals between two tones were tested, those intervals exhibiting the least beating would be preferred by listeners.

Helmholtz' complex tones consist of fundamental, partials and their combinational tones. Combinational tones include difference tones and summation tones. Difference tones were discovered by the German organist Sorge. This phenomenon arises when two tones are sounded simultaneously, loudly and continuously. Difference tones are named thus since their frequency is equal to the difference between the two initial frequencies. Difference tones also exist amongst the sum and difference tones of the sum and difference tones, as first discovered by Hallstroem. Since the fundamentals of the tones are generally louder than their respective partials, the difference tones between the fundamentals are the loudest, and consequently the easiest to discern by ear.

Figure 22. Difference tones
(Helmholtz, 1954: 154)

Summation tones were discovered by Helmholtz (Butler & Green, 2002: 255), and are discerned with much greater difficulty than difference tones on account of their lower loudness. Summation tones are named thus since their frequency is equal to the sum of the frequencies of the two simultaneous tones.

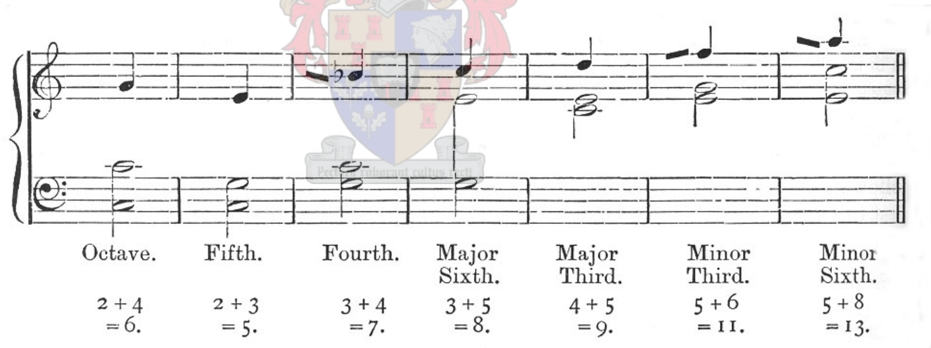


Figure 23. Summation tones
(Helmholtz, 1954: 156)

Combinational tones are believed to arise from distortions created in the ear. In Preyer's experiments using tuning forks combination tones were shown to cause no resonance on sympathetically tuned tuning forks. It is not held, however, that combination tones are imagined, but simply that they do not exist externally to the ear (cf. Helmholtz, 1954: 531-532).

Cohen's illustration shows the result of both combinational tones, as they arise from an octave and a major third.

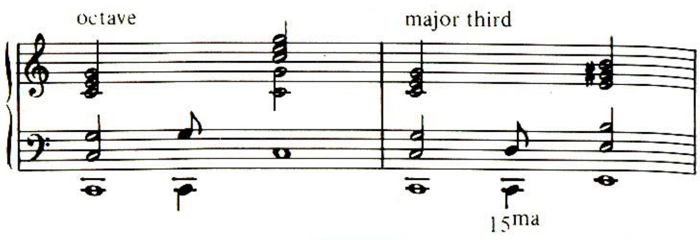


Figure 24. Combinational tones
(Cohen, 1984: 240)

The first and third chords in every beat consist of the fundamentals and their harmonics, and the second chord of the difference and summation tones. It can be seen that the octave has no frequencies which are slightly out, and thus no dissonance is perceived. The third, however, does give rise to some dissonance. This theory also explains why mistuning the octave gives rise to more dissonance than mistuning the third does, as so many of the octaves partials overlap, a slight mistuning would give rise to small frequency differences between the partials of the two notes in many places in the frequency spectrums.

If two simultaneously sounding tones, henceforth primary tones, have equal frequencies, they fuse into one tone, in which the intensity is dependent on the phase relationship of the tones. Slightly differently tuned tones would give rise to a signal with periodic frequency and amplitude variations, with a frequency equal to the frequency between the primary tones. The resultant amplitude variations can be considerable, resulting in a fluctuating intensity and consequently also a fluctuating perceived loudness, which is known as beats. These beats are only discernible by the ear if their frequency is less than 20Hz (Rasch & Plomp, 1999: 103).

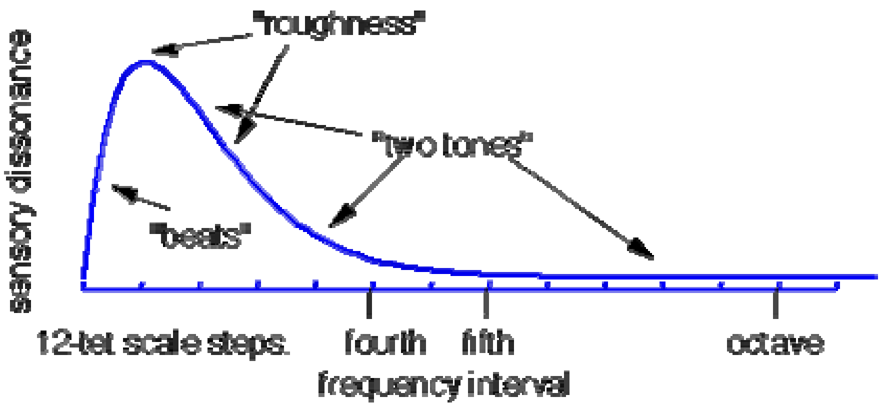


Figure 25. Sensory dissonance
(http://eceserv0.ece.wisc.edu/~sethares/consemi.html#anchor149613)

As mentioned above, Helmholtz attributed dissonance to this beating phenomenon exhibited between adjacent harmonics of complex tones. Helmholtz proposed that maximum dissonance occurs between two tones when the beat rate is roughly 33 cycles per second. (Later research has shown maximum dissonance to vary somewhat in frequency terms, as it is a function of critical bandwidth, but more on that later.) He believed beating at less than 6 Hz to be tolerable and beating at more than 132 Hz as imperceptible.

Helmholtz was aware of the fact that beats alone could not account for dissonance," On the other hand we have seen that distinctness of beating and the roughness of the combined sounds do not depend solely on the number of beats. For if we could disregard their magnitudes all the following intervals, which by calculation should have 33 beats, would be equally rough:

The Semitone	528 – 495Hz
The whole Tones	297 - 264Hz and 330 – 297Hz
The minor Third	198 – 165Hz
The major Third	165 – 132Hz
The Fourth	132 -99Hz
The Fifth	99 -66Hz

and yet we find these intervals are more and more free from roughness" (cf. Helmholtz 1954:171-172; http://dactyl.som.ohio-state.edu/Music829B/notes.html) The phenomenon von Helmholtz observed is due to critical bandwidth decreasing faster than the frequencies of the intervals as frequency decreases.

He also noted that beating for a fixed interval size is highly dependent on the register, "Observation shows us, then, on the one hand, that equally large intervals by no means give equally distinct beats in all parts of the scale. The increasing number of beats in a Second renders the best in the upper part of the scale less distinct. The beats of a Semitone remain distinct to the upper limits of the four-times accented octave [say 4000 vib.], and this is also about the limit for musical tones fit for the combinations of harmony. The beats of a whole tone, which in deep positions are very distinct and powerful, are scarcely audible at the upper limit of the thrice-accented octave [say 2000 vib.]. The major and minor Third, on the other hand, which in the middle of the scale [264 to 528 vib.] may be regarded as consonances, and when justly intoned scarcely show any roughness, are decidedly rough in the lower octaves and produce distinct beats" (Helmholtz, 1954: 171) (http://dactyl.som.ohio-state.edu/Music829B/notes.html).

Harry Partch also made mention of this in his “*Genesis of a music*”: “So far as can be ascertained, the enhancement of consonance that results when an interval is extended from the close to the extended form (for example, from 16/15 to 32/15, by simply raising the upper tone a 2/1) results from a doubling of frequency of the wave period. Whatever the reason, it is generally true, throughout the musical register, that consonance can be enhanced in this manner if consonance is the desideratum of the moment “(Partch, 1949: 153).

The following graph by Plomp & Levelt illustrates the frequency dependency of the dissonance of intervals.

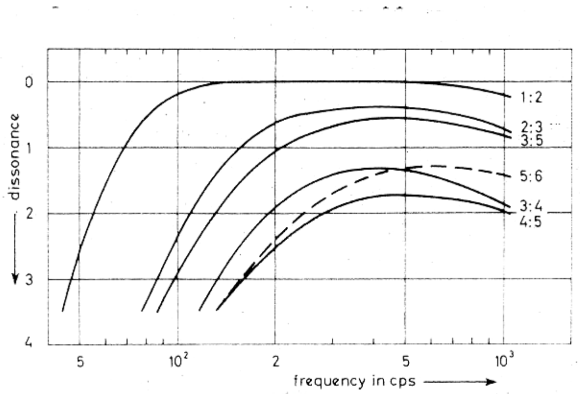


Figure 26. Frequency dependence of dissonance of intervals
(Plomp & Levelt, 1965: 557)

It seems odd to expect the size of the interval to be the deciding parameter in the determination of roughness, as the concept of “intervals” is already a psychoacoustic effect.

Partch’s quotation also lies in a paragraph named “The Variability of Consonance’ (“across the frequency range” is implied), but a more accurate heading might have been “The Variability of fixed Interval Consonance” (“across the frequency range“).

Advances in technology and increased knowledge acquired over the last century have made it possible to ascertain to a greater extent the causes of roughness. Jian-Yu did a study of roughness in 1995 at Michigan State University, and found that roughness is caused by the fluctuations in excitation of the auditory filter. This relies on the spectral components exciting the same auditory filter, which indicates a connection between roughness and critical bandwidth. Another factor is the temporal modulation transfer function, which basically refers to the ear’s inability to detect rapid fluctuation rates.

These two factors would lead one to expect maximum roughness at the lowest possible modulation frequencies or beat rates. There exists, however, a third factor whereby increased fluctuation rates cause an increased sense of roughness. This factor is known as the "speeding factor". Jian-Yu consequently found that 70Hz is the point of maximum roughness, and not Helmholtz' estimation of 35Hz, and that any fluctuation rate faster than 70Hz causes the listener to experience the fluctuation rate as a low pitch. The presence of the low tone leads to a dramatic decrease in perceived roughness. This 70Hz limit was also found in a study done by Zwicker & Fastl (1990) (http://dactyl.som.ohio-state.edu/Music829B/notes.html).

Slaymaker (1970), and Pierce (cf.1966), found that consonance and dissonance is indeed dependent on the coincidence of partials. If the number of partials, or the strength of the higher frequency partials, which are more narrowly spaced, is increased, higher dissonance values are experienced.

5.5.3 Tonotopic theory

As already mentioned above, von Békésy showed different frequencies cause maximum displacement at different points on the basilar membrane of the cochlea. Von Békésy, and later Skarstein as well, measured the distance from the stapes of the point where maximum displacement for specific frequencies occur. This mapping of the correspondence between input frequency and place of maximum displacement of the basilar membrane is known as tonotopic mapping, or as a cochlear map. (http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html, p.11).

Greenwood (cf.1961a) gave the following function indicating the point of maximum excitation on the basilar membrane: $F=A\ (10^{ax}\ -k)$, were F is the frequency in hertz, x is the position of maximum displacement in mm from the apex, A is the constant 165, a is the constant 0.06 and k the constant 1.0 (http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html, 56).

Fletcher, in his 1940 paper, subsequent to von Békésy's research, showed that a close correspondence between regions of masking and distances along the basilar membrane exists, and coined the phrase "critical band", which referred to frequency-domain regions which exhibit equivalent or proportional behaviour (http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html, 12).

A subsequent paper by Zwicker, Flottorp & Stevens proved the importance of critical bandwidths empirically. Their paper showed that loudness perception remains constant when the spacing between groups of pure tones is increased, up to a certain point, after which the loudness increases. They also outlined four types of experiments which provide direct measures of critical bandwidth, namely thresholds; masking, phase and their loudness summation (cf. Zwicker, Flottorp & Stevens, 1957: 548-557).

The following table gives some more recent of measurements of critical bandwidth in frequency terms:

Critical band rate. Bark	Frequency Hz	Critical bandwidth Hz	Center frequency Hz
0	0		
		100	50
1	100		
		100	150
2	200		
		100	250
3	300		
		100	350
4	400		
		110	450
5	510		
		120	570
6	630		
		140	700
7	770		
		150	840
8	920		
		160	1000
9	1080		
		190	1170
10	1270		
		210	1370
11	1480		
		240	1600
12	1720		
		280	1850
13	2000		
		320	2150
14	2320		
		380	2500
15	2700		
		450	2900
16	3150		
		550	3400
17	3700		
		700	4000
18	4400		
		900	4800
19	5300		
		1100	5800
20	6400		
		1300	7000
21	7700		
		1800	8500
22	9500		
		2500	10 500
23	12 000		
		3500	13 500
24	15 500		

Table 2. Critical bandwidths
(Zwicker & Terhardt, 1980: 1523-1525)

Huron gives an estimation of critical bandwidth in terms of music notation.

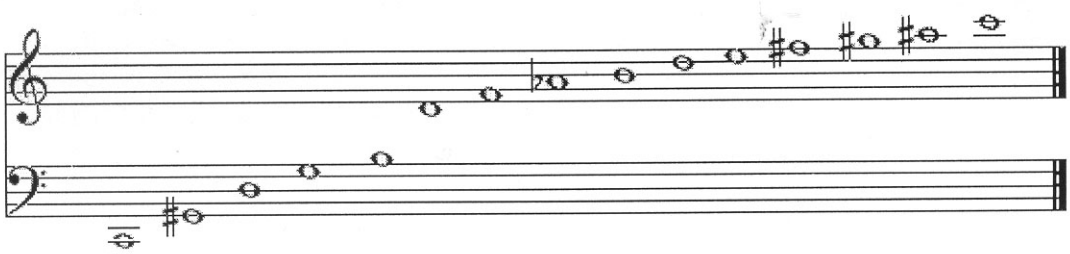


Figure 27. Critical bandwidth in music notation
(http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html, 12)

Huron points out that critical bandwidth increases in frequency terms as frequency increases, but in log terms (or semitones), it decreases as frequency is increased (http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html, 12).
This explains the phenomenon noted by Helmholtz and Partch above, of consonance of fixed musical intervals, such as semitones or whole tones, worsening in lower registers.

Greenwood (cf.1961b) showed that a linear relationship exists between the psychoacoustic measures of critical bandwidth and the frequency-place co-ordinates of Békésy's tonotopic map, with one critical bandwidth approximately equivalent to 1mm. on the basilar membrane (http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html, 12).

Greenwood further suggested a relationship between critical bandwidth and judgments of consonance and dissonance. Greenwood tested his hypothesis by plotting data which Mayer had collected in 1894, against the estimated size of the critical bandwidth.

Mayer had asked listeners to judge the smallest consonant interval between two pure tones. Thus the listeners were required to indicate the minimum frequency where no dissonance was perceived. Greenwood's achievement was his realization that dissonance is judged as absent when the distance between pure tones is roughly the size of a critical bandwidth.

Later on, Mayer's data was questioned, as Mayer had to use tuning forks to generate his stimuli, for he had no electroacoustic means of producing them, and this raised doubts regarding the purity of his 'pure' tones. The range of tunings of his tuning forks was also suspect, his lowest frequency being 256Hz. He asked his friend, König, to do some additional testing in the lower frequencies, and König obliged but only used one subject, thus making the findings very subjective indeed.
In lieu of this Plomp & Steeneken, in 1968, collected data from 20 listeners, and these results are plotted against Greenwood's estimation of the width of critical bands below.

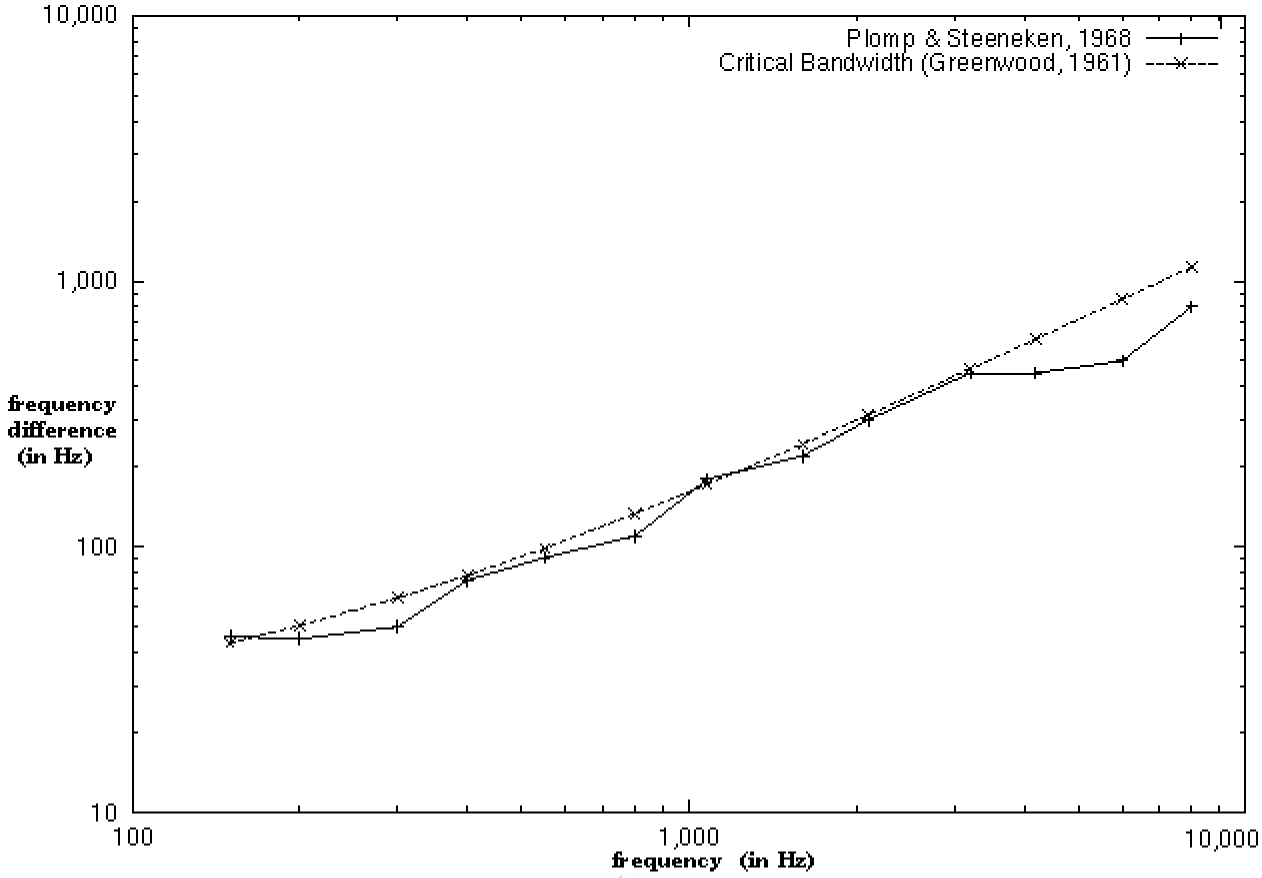


Figure 28. Early critical bandwidth measurements
(http://dactyl.som.ohio-state.edu/Music829B/notes.html)

Plomp & Levelt did extensive work complementing Greenwood's findings. The following illustrations are taken from their much referenced work "Tonal Consonance and Critical Bandwidth, (1965)". They show the consonance value for a sine tone with fundamental frequencies of 125, 250, 500, 1000 and 2000 Hz respectively, as it changes while a second sine tone is played simultaneously with the first and increased in frequency.

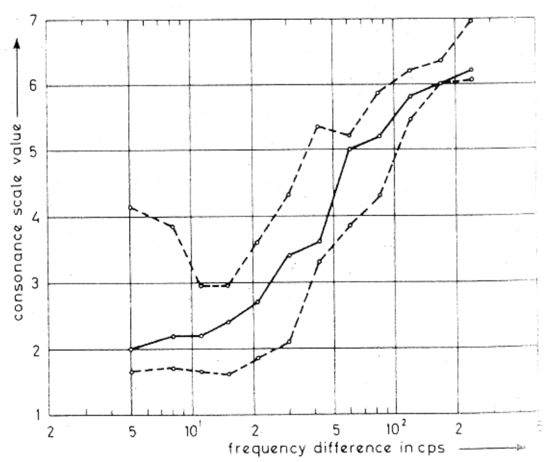


Figure 29. The 125 Hz fundamental sine tone
(Plomp & Levelt, 1965: 552)

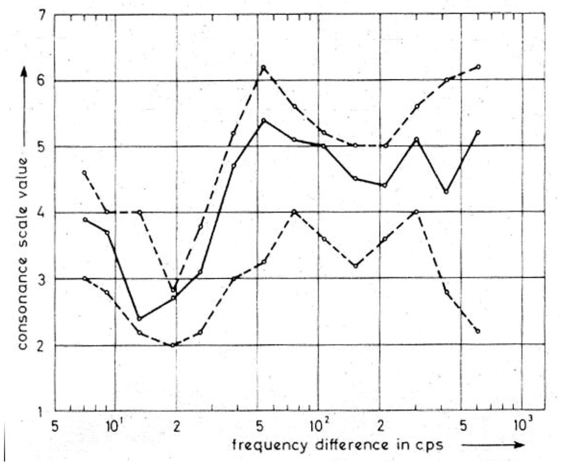


Figure 30. The 250 Hz fundamental sine tone
(Plomp & Levelt, 1965: 553)

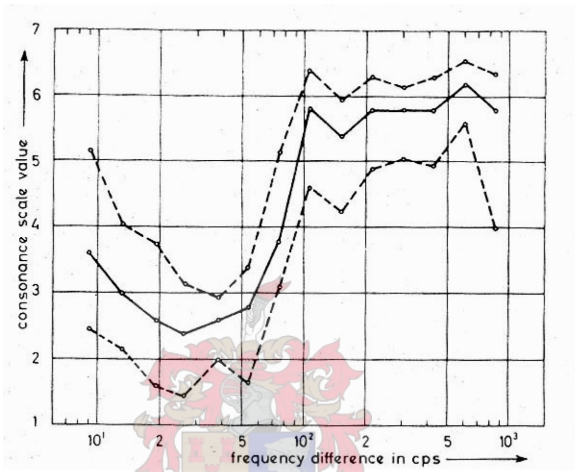


Figure 31. The 500Hz fundamental sine tone
(Plomp & Levelt, 1965: 553)

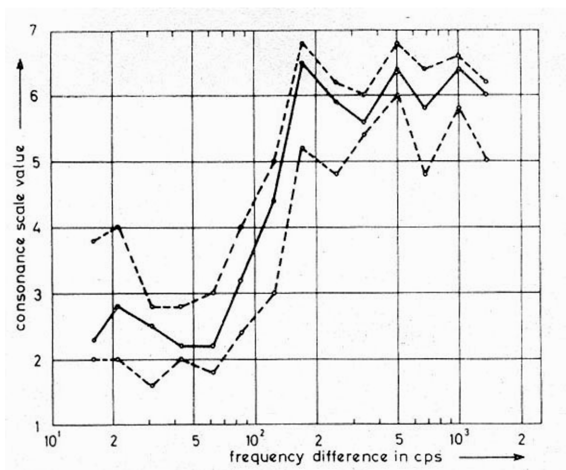


Figure 32. The 1000Hz fundamental sine tone
(Plomp & Levelt, 1965: 554)

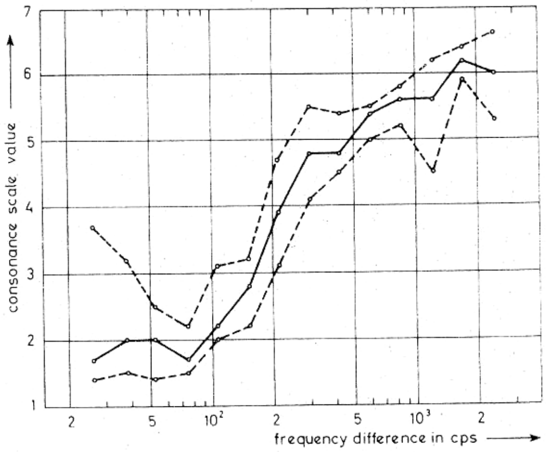


Figure 33. The 2000Hz fundamental sine tone
(Plomp & Levelt, 1965: 554)

Plomp & Levelt show how the consonance of an interval changes according to the change in frequency of a variable tone. The constant tone is of 250Hz, and both constant and variable tones have 6 harmonics.

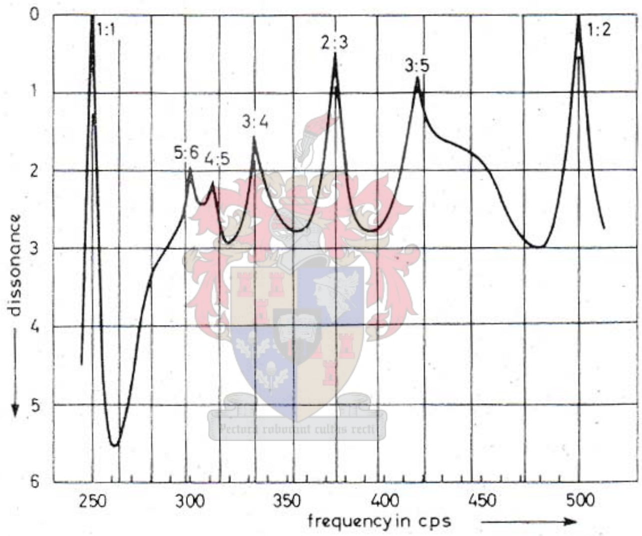


Figure 34. Consonance of intervals
(Plomp & Levelt, 1965: 556)

The curves all have the same shape, and Plomp & Levelt estimated maximum sensory dissonance of pure tones to arise at 25% of the critical bandwidth. The following illustration shows their estimation of sensory dissonance as a function of critical bandwidth:

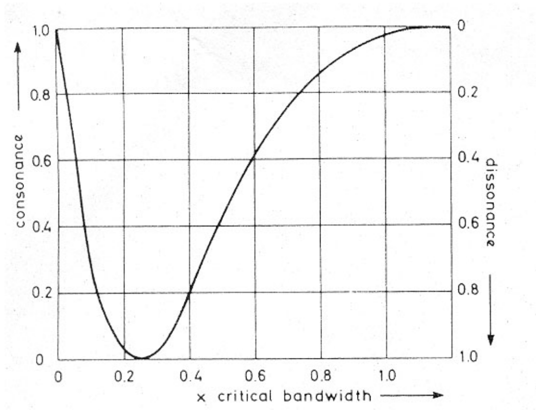


Figure 35. Consonance as a function of critical bandwidth
(Plomp & Levelt, 1965: 556)

Subsequent research by Glasberg & Moore (cf.1990) inadvertently showed Greenwood's 1961 critical bandwidth estimation to be accurate, (http://www.musiccog.ohio-state.edu/Huron/Publications/huron.voice.leading.html) and also that Plomp & Levelts' calculations, which in hindsight were based on an inaccurately estimated critical bandwidth, to be less accurate. Greenwood (1991) consequently estimated maximum sensory dissonance of pure tones to arise at 30-40% of critical bandwidth.

Huron mentions that the maximum dissonance which occurs at 40% of the critical bandwidth is equivalent to a distance of 0.4mm on the basilar membrane. (http://dactyl.som.ohio-state.edu/Music829B/notes.html) This means the point of maximum dissonance cannot be expressed as a constant frequency.

Plomp & Levelt (1965) also found that frequency separations for pure tones larger than the critical band cause no sensory dissonance. Furthermore, they hypothesized that composers would attempt to distribute the total dissonance of a chord homogenously across the span of the chord. Since the tonotopic theory holds that dissonance is linked to critical bandwidth, one would expect the aforementioned distribution to follow critical bandwidth. Huron found Franz Joseph Haydn (1732-1809) and J.S. Bach's (1685-1750) works do in fact adhere to this hypothesis. It appears as if composers attempt to distribute spectral components uniformly across the basilar membrane (http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html, 14).

Figure 36. Spectral distribution correlation with critical bandwidths
(http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html, 14)

Another prediction which follows from the tonotopic theory is that the perception formed by simultaneously presenting two pure tones which fall within a critical bandwidth, each separately to one of the ears, should be free of dissonance. This is on account of each ear analysing the frequencies independently, and the two basilar membranes being stimulated separately. In fact, Sandig had already found in 1939 that playing each tone to a separate ear results in a more "neutral" sounding interval (http://dactyl.som.ohio-state.edu/Music829B/notes.html).

Kameoka & Kurigayawa model

Kameoka & Kurigayawa (1969b: 1460-1469) proposed a mathematical model by which the total dissonance of an interval can be calculated. This method requires summation of the dissonance of all neighbouring partials. The consonance is then inversely related to the summated dissonance. Plomp & Levelt proposed a similar method, but, as Kameoka & Kurigayawa pointed out, their model only took into account dissonances arising from adjacent partials, whereas Kameoka & Kurigayawa's model summates the dissonances between all partials. Kameoka & Kurigayawa's model also takes internal noise and environmental noise into account. In contrast to Plomp & Levelt, Kameoka & Kurigayawa's model took the effect of amplitude on dissonance judgments into consideration. Consequently, greater weight is attached to the lower harmonics, because they are more intense than the higher ones in most musical instruments. Kameoka & Kurigayawa also found that perceived dissonance of two single partial tones was greater when the low frequency tone was of greater amplitude than the higher frequency tone, as opposed to the inverse. (Kameoka, A., & Kurigayawa, M., 1969b: 1460-1469). Their algorithm's results correlated reasonably well with subjective judgments.

After entering values in a Humdrum[1] simulation of Kameoka & Kurigayawa's model, we could see some interesting results.

The input of 400Hz at 50dB, gave a dissonance value of 65. This would appear to be the minimum value attainable, as no critical bands could be breached by a single tone. This value represents the dissonance arising from internal noise.

After adding another tone of 2000Hz at 50dB to the original tone, the dissonance value was still 65. This is due to the two tones being sufficiently far apart, and consequently no critical bandwidths being breached.

Adding, instead of the 2000Hz tone, a 510Hz tone at 50dB, gives rise to a dissonance value of 105. This is despite the fact that the frequency separation between the two tones does not fall within the critical bandwidth value, given by Zwicker & Terhardt above, of 110.

[1] Humdrum is a multi-faceted software program which aims to assist in music research. It can be downloaded at http://dactyl.som.ohio-state.edu/HumdrumDownload/downloading.html .

Inputting an octave interval, with each fundamental having two partials at 50dB, i.e. 440, 880, 1320Hz and 880, 1760 and 1640Hz, all at 50dB, gave the dissonance value of 149. A semitone interval of 440, 880, 1320Hz and 466, 932 and 1398 Hz all at 50 dB gave rise to a dissonance value of 182. This result is in line with sensory judgments of listeners, as the semitone is judged more dissonant than the octave.

Increasing the dB level of the previous input of all the frequencies to 70dB, gives rise to a dissonance value of 274. This is in line with extant research, which holds that increased level increases dissonance value.

The order in which the frequencies are entered into the model is irrelevant, as all frequencies are compared to all other frequencies despite the order. This can readily be observed by rearranging the inputs.

Entering the values 1270Hz at 50dB and 1720Hz at 50dB gave rise to the dissonance value of 92. The maximum dissonance for a 1270Hz tone, according to Greenwood (see above) should arise at a value of between 30-40% of the critical bandwidth. The critical bandwidth for a tone of 1270Hz, according to Zwicker & Terhardt's table above, is 210Hz. Consequently a greater dissonance value can be expected at 1354 Hz (1270 + 210*0.4). The dissonance value given by the model obliges us with a dissonance value of 131. Increasing the second tone's frequency to 1390 Hz lessens the dissonance value, which is what one would expect when moving further away from the point of maximum dissonance.

To test whether a lower tone at higher dB level gives rise to higher dissonance ratings than having a higher tone at higher dB level, an input of 500Hz at 60 dB together with 550Hz at 70dB were entered. This gave a dissonance value of 152. When swapping the dB of the two tones, i.e. rather having the lower tone as the louder one, we are given a dissonance value of 180. This result is in keeping with Kameoka & Kurigayawa's theory outlined above.

Some values inputted do not give the desired results, and this could be due to either the critical bandwidth estimations differing, or the prediction of maximum dissonance value differing. This is understandable given the time lapse, and the consequent amount of new research which has been done in between Kameoka & Kurigayawa (1969a, 1969b), Zwicker & Terhardt (1980) and Greenwood's (cf.1991) research.

Jasba Simpson provided further evidence supporting the tonotopic theory of dissonance with the aid of computer models for the cochlea. Simpson input the stimuli of five perceptual experiments where listeners were required judgments on degrees of consonance and dissonance, into these cochlea models. Simpson then explored the output from these models, in the hope of finding a correlation between the consonance and dissonance judgments and neurophysiologic responses. "Simpson found that the squared distance of the maximums and minimums from the mean of the maximum and minimums accounted for 58% of the variance in the stimuli used in the five experiments. In effect, the squared distance measure used by Simpson amounts to a measure of tonotopic spread" (http://dactyl.som.ohio-state.edu/Music829B/notes.html). His findings suggested dissonance cues at the periphery of the auditory system.

A strong correlation exists between the most consonant intervals indicated by the consonance curve of Plomp & Levelt, (see figure 36 above) and the intervals chosen in the equal-tempered scale (and for mean-tone, Pythagorean, just-intonation scales, et cetera).

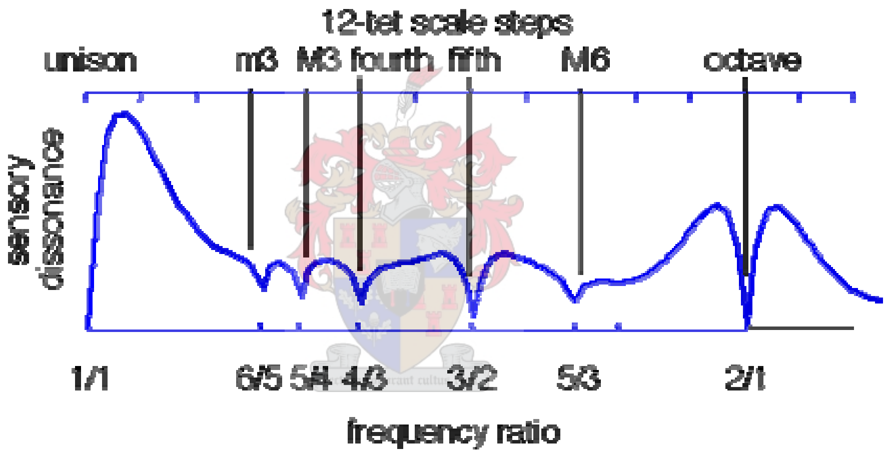


Figure 37. Correlation between local minima and intervals chosen (http://eceserv0.ece.wisc.edu/~sethares/consemi.html#anchor149613)

Sethares drew similar consonance curves for instruments with inharmonic tone complexes, and found that the intervals used in the music of these instruments correspond strongly to the local dissonance minima on their respective consonance maps (cf. Sethares, 1997: 184).[2]. In chapter 8, where we will illustrate how our scale is built, we will make use of Sethares' program for mapping consonance curves for tones with inharmonic spectra (http://eceserv0.ece.wisc.edu/~sethares/consemi.html#anchor149613).

[2] Further discussions on the relationship between roughness and inharmonic tones can be found in Jacobssen & Jerkert. 1999: *Consonance of non-harmonic complex tones: Testing the limits of the theory of beats* and Swallowe 1997: On consonance: Pleasantness and interestingness of four component complex tones *Acustica* 83: 897-902.

The tonotopic theory, which is heavily reliant on critical bandwidth theory, cannot be regarded as an all encompassing theory, as research has shown some results which cannot be explained by this theory alone. The ability of expert listeners to discriminate between pitches differing 2-5%, for instance, is much finer than can be expected from the critical band theory. Furthermore, the ability to identify very high spectral components in complex sounds cannot be explained by way of the critical band theory.

5.5.4 Cultural theory

Some scholars, such as Lundin (cf.1947) and Cazden (1980 cf. below) maintain that consonance and dissonance is a learnt experience. To a certain extent this certainly does seem to be true. The amount of work done on the effect of culture on consonance and dissonance is very limited. This is due to psychoacousticians' and ethnomusicologists' unwillingness, or perhaps their inability for inter-disciplinary research.

Psychoacousticians assume cross-cultural experiments are unnecessary as the hearing organ differs very little across cultures, and ethnomusicologists consider such studies unnecessary as cultural differences to them are so blatantly obvious. In effect, the psychoacousticians mainly consider the physical similarities, whereas the ethnomusicologists consider only the cultural differences.

The effects of culture on consonance and dissonance judgments can clearly be seen from the results of a study done by Joos Vos in 1987 (http://dactyl.som.ohio-state.edu/Music829B/notes.html). He had 18 Western musicians judge the acceptability of tunings of various tempered fifths. He presented his subjects with stimuli of 0.25 seconds (fast presentation) and 0.5 seconds (slow presentation).

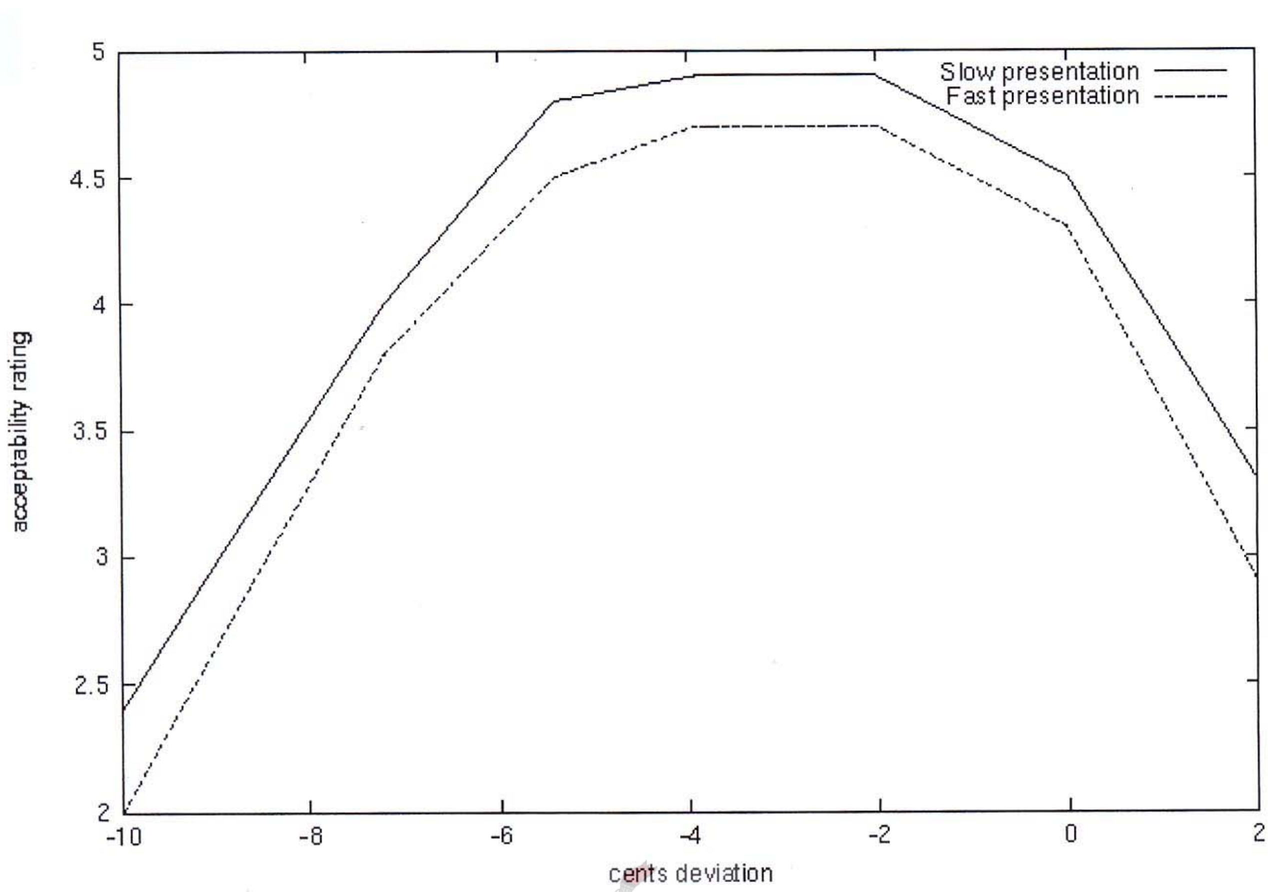


Figure 38. Cultural aspects of perception
(http://dactyl.som.ohio-state.edu/Music829B/notes.html)

The results of this experiment could indicate a learnt preference arising from culture, as the results are skewed towards the left of the perfectly just fifth, which indicates a preference for the tempered fifth, which is two cents too small.

Some questions regarding the correctness of the conclusions drawn from this experiment have been raised, as the subjects were not asked to judge the consonance of the interval, but the acceptability. Also, in some other studies listeners have judged the just intoned intervals as boring, and have shown a preference for slightly out of tune intervals.

Butler and Daston also carried out a study of consonance and dissonance where they compared American and Japanese listeners. They concluded that if there are significant differences between the music listening of Japanese and American listeners, that difference is not to be found in judgments of the consonance of isolated dyads. (http://dactyl.som.ohio-state.edu/Music829B/notes.html)

5.5.5 Cazden's expectation dissonance theory

Norman Cazden (1914-1980), a music theorist, was of the opinion that the motivations for acoustic and physiological causes of consonance and dissonance were lacking. He was, though, unaware of the work of Plomp & Levelt (1965) and Kameoka & Kurigayawa (1969a, 1969b). Cazden acknowledged the existence of a low-level sonorous unpleasantness, but regarded this "euphony", as he called it, insignificant.

He believed that one could not evaluate consonance outside its context, thus "… in principle, euphony refers rather to the overall psychoacoustic quality of a sonority isolated from any musical context "

"With euphony thus distinguished, and defined as a composite of all those psychoacoustic criteria capable of affecting a gradation of isolated sonorities, the terms consonance and dissonance proper may be reserved instead for those particular musical distinctions observed in the practice of Western tonal music"(Cazden, 1980: 155).

Cazden therefore places greater importance on the concept of expectation dissonance, rather than the dissonance perceived of intervals in an isolated and out-of-context light.

"(Dissonance) identifies rather the functional moment of any sonorous event that is expected to resolve, while the moment to which it ultimately resolves is then deemed consonant. Should the framework for the normative expectations of this kind not be present, or should the apparent resolution tendencies and outcomes be thwarted consistently, as may happen in some compositional styles of twentieth century art music, neither consonance nor dissonance can be said to exist"(Cazden, 1980: 157).

In the light of other research, it is believed that Cazden should have to change his references to consonance to 'expectation consonance', since, though it can be believed that his dissonance exists, Cazden should permit the existence of other forms of dissonance as well.

Cazden argued that even a single tone could give rise to dissonance. The assumption is that Cazden's tone exists within some kind of musical context, or perhaps we should presume someone with perfect pitch would experience a somewhat flat or sharp tone as dissonant.

Cazden categorized three expectation-related dissonances:

"Dissonating tone": a non-harmonic or non-chordal tone has a tendency to resolve within the framework of an underlying chord or harmony.

"Dissonant Chord Moment": A chord can be dissonant if it arouses the expectation of resolving to another chord within a harmonic progression.

"Tonal Centre Dissonance": Dissonance may arise from a passage outside its tonal centre. This dissonance may be resolved when for instance the dominant tonal centre eventually moves to the original tonic area.

It is understood that these dissonances are not mutually exclusive, and that at a given moment all three of them could exist.
Cazden implies dissonance to be of a cultural nature. His contempt for the psychoacoustic theories of consonance is clear from the following quote "the raw psychoacoustic or sonorous properties of the musical signal can provide at most certain limiting natural conditions for the art of music, just as there are broad natural limits and conditions for language" (Cazden, 1980: 154).

One has to wonder how Cazden accounts for the existence of the numerous similarities between the music of so many cultures. Nonetheless, Cazden's attributation of consonance to cultural phenomena is justifiable considering the bewilderment people often express at other culture's music. "It may explain why listeners conditioned to Western music sense a somewhat directionless indecision of harmonic moment when they attend to the heterophonic gamelan music of Bali or to the highly mannered Gagaku music of Japan" (Cazden, 1980: 161).

5.5.6 Virtual pitch theory

This theory is the brainchild of Ernst Terhardt. Terhardt's theory (1974) argues that dissonance arises from pitch ambiguity. When partials conform to the harmonic series, a strong pitch sensation will be evoked. Competing pitch sensations are evoked when the spectrum deviates significantly from the harmonic series. According to Terhardt's view, "dissonance is a negative valenced sensory experience that arises when a sound scene evokes highly ambiguous pitch perceptions" (http://dactyl.som.ohio-state.edu/Music829B/notes.html).

Terhardt gives the following illustration showing how the hearing system identifies the spectral components, and then compares it to a spectral template, before the complex sounds' pitch is perceived.

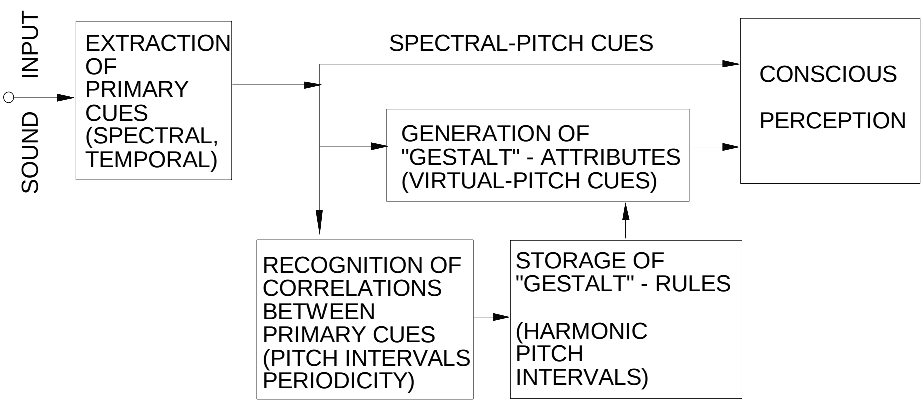


Figure 39. Terhardt's virtual pitch system
(Terhardt, 1974: 1063)

5.5.7 Tonal fusion theory

Carl Friedrich Stumpf's (1848-1936) theory of "Tonverschmelzung", or in English "tonal fusion", is based on the tendency for some sound combinations to fuse perceptually into a single sound image. Listeners judging simultaneously sounding tones were asked to say whether they hear a single tone or two separate tones. Stumpf's experiment indicated the aptly named unison as the most fused interval, followed by the octave and then the perfect fifth (Stumpf, 1890). There is as yet no general consensus concerning the rest of the rung-order, with some scholars considering the perfect fourth (3:4), and others the double octave (1:4) as the next most fused. De Witt & Crowder even suggest that major sevenths tend towards fusion more than perfect fourths (Schneider, 1997: 177).

Extant research has established that fusion is dependent on spectral energy distribution, the harmonicity as opposed to inharmonicity of the spectral components, the listener's familiarity with the spectral envelope of the sounds, and any modulation affecting all, none or some of the spectral components (Schneider, 1997: 177). Stumpf also mentioned some other factors which contribute towards the experience of integral hearing, as opposed to plural hearing, namely that listeners tend to experience a complex sound as more fused when it appears to have one source (timbral similarity) and come from one direction and that the spectral components onset times should be synchronized.

The strong correlation between judgments of consonance and judgments of fusion lead Stumpf to believe that fusion was the cause of consonance ('Tonpsychologie'). The degree to which sounds tended to fuse was thus synonymous with their degree of consonance. Stumpf's theory seems very plausible considering that harmonically related partials (which would have less roughness) would tend to fuse on account of their identical components.

Stumpf rejected Helmholtz' mainly negativistic definition of consonance, as a perception when roughness sensation is absent. Stumpf even argued that there was not necessarily a strong link between perception of dissonance and the experience of roughness. Stumpf gave the following chords in support of this argument.

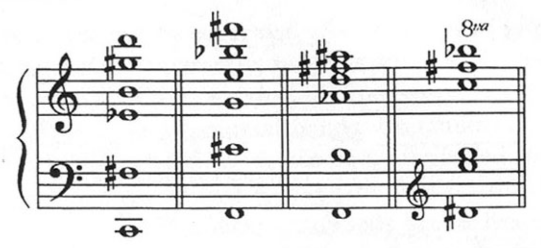


Figure 40. Stumpf's chords
(Schneider, 1997: 117-143)

Schneider did a modern retesting of these chords. Here follows a table of the frequencies proposed and a table of the results of the perceptual judgments.

	Chord 1[Hz]	Chord 2[Hz]	Chord 3[Hz]	Chord 4[Hz]
Component No1	66	88	88	309.38
"" 2	183.3	275	247.5	396
"" 3	316.8	396	422.4	495
"" 4	495	660	594	1056
"" 5	825	950.4	733.34	1466.67
"" 6	1188	1466.67	916.67	1900.8

Table 3. Frequencies of Stumpf's chords
(Schneider, 1997: 117-143)

Despite Stumpf's attempt at avoiding roughness by spacing the notes of the chords wider than critical bandwidth, the perceptual quality of dissonance was still associated with the sensation of roughness. Schneider proposes that roughness perception is not wholly dependent on critical bandwidth interferences. Here follows the table of Schneider's results:

	Degree of consonance (1-9)				Roughness (1-9)			
Chord number	1	2	3	4	1	2	3	4
Mean [arith.]	4.88	3.88	4.44	3.26	4.11	5.31	4.78	6.2
Median	5	3.5	4.5	3	4	5	4.5	7
Standard Dev.	1.3	1.63	1.54	1.75	1.82	1.75	1.82	1.92
Range	1-8	1-7	1-7	1-7	1-8	2-8	2-9	2-9

Table 4. Schneider's results on Stumpf's chords
(Schneider, 1997: 117-143)

It is clear that lower degrees of consonance perceptions correlate with higher degrees of roughness perception.

Various other experiments served to disprove Stumpf's theory, and he eventually abandoned it himself. Take, for instance, any harmonically complex tone and feed it through a phaser or chorus device. Phasing involves duplicating a signal and putting it out of phase in relation to the first copy. Their relative state of phase is also altered over time. The effect of this effect is twofold, firstly, listeners judge the sound more pleasant, euphonious and consonant than before, and the listener also judges the sound as less fused, as the numerosity in the sound field seems to have increased.

Another example which brings the validity of the fusion hypothesis into question is the following: when presenting subjects with the complex tone consisting of the following partials 100, 200, 300,400,500,600,700,800,900 and 1000Hz, they will find this more fused but less consonant than the complex tone consisting of the partials 200 and 300Hz only.

In 1991, Huron did a multiple regression analysis of the interval distributions in the polyphonic writings of J.S.Bach. Huron found that Bach was aiming to achieve maximum consonance in his writings, whilst at the same time avoiding tonal fusion. Unisons, perfect fifths and octaves occur less frequently in Bach's polyphonic works than they would in a random juxtaposition of intervals. Bach also used the less fused intervals more often than he used the more fused intervals, in other words he used the relatively less fused fifth more often than the octave, and the relatively less fused octave more than the unison.

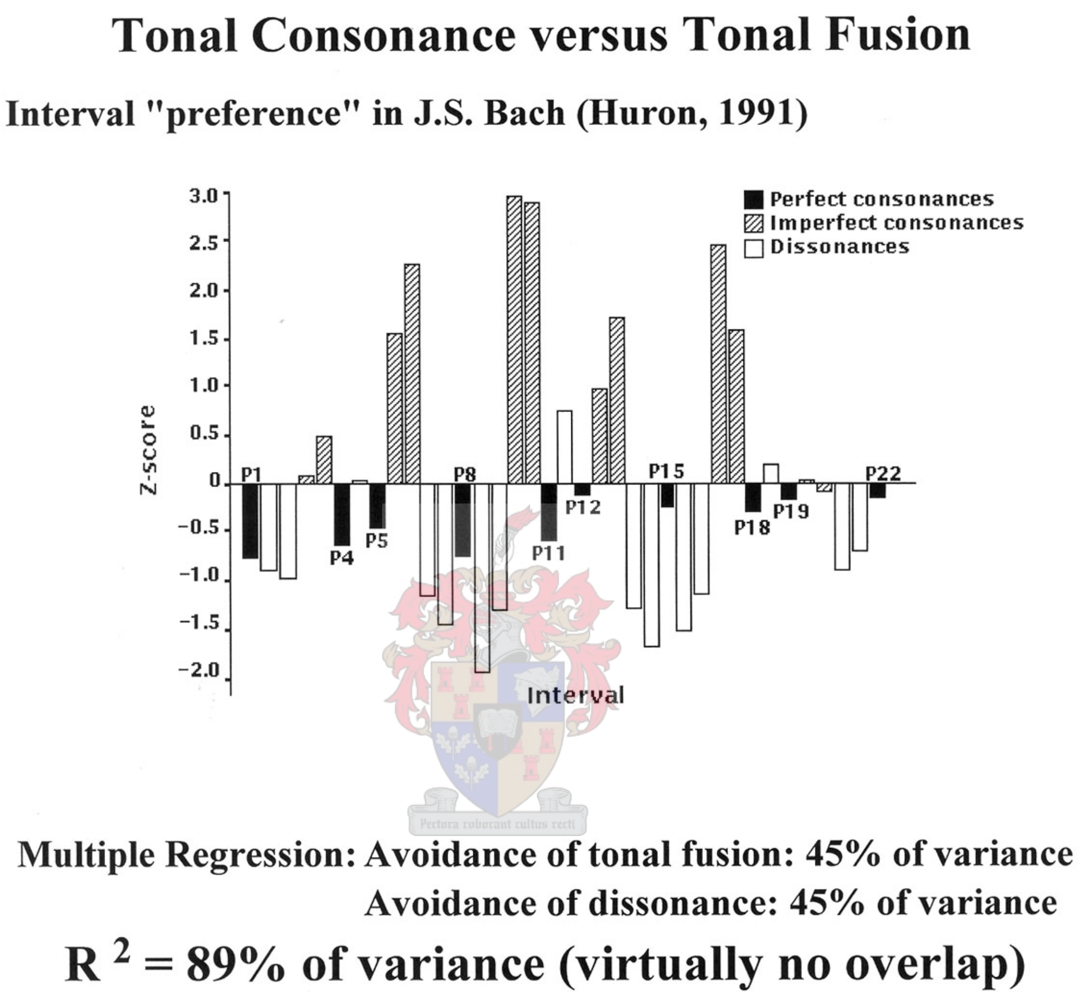


Figure 41. Tonal consonance versus tonal fusion
(http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html,)

5.5.8 Huron's numerosity conjecture

A problem identified with the tonotopic theory was that increasing the number of spectral components in a sonority should lead to an increase in dissonance, on account of the increased number of partials in the critical bandwidth. However, this does not correlate with listener's experience.

Huron proposes that the increase in the perceived number of sound sources would lead to an increase in the consonance, and consequently to a decrease in perceived dissonance. Huron was not the first to notice this phenomenon, as Mersenne (1588-1648), who could distinguish between the fundamental and four upper partial notes, noted ”the sound of any string is the more harmonious and agreeable, the greater the number of different sounds it makes heard at a time” (Cohen, 1984: 103).

Huron’s proposal explains the preference listeners show towards the chorus effect, unison doubling, phasing amplitude modulation and reverberated sounds. Perhaps this might be an evolutionary consequence of greater survival capability when in a big group. It could also be an effect of familiarity, as most complex sounds have partials.

Huron proposes a duplex perception theory of dissonance, consisting of a tonotopic and a perceived numerosity component. Huron points out that numerosity is a non-linear auditory perception. Here is his proposed formula: dissonance = tonotopic dissonance/perceived numerosity, and its inverse, which implies dissonance and consonances as inversely proportionate: consonance = perceived numerosity / tonotopic dissonance.

Huron cites evidence supporting his theory, namely that it effectively predicts the effect of pitch height and timbre on consonance and dissonance judgments, and it also successfully predicts the distribution of pitches in musical chords. His theory also successfully predicts the distinction between “sounding as one” and “sounding smooth”, as indicated in his Bach experiment. The fact that common scales exhibit optimum sensory consonance according to his model is further evidence for the validity of the theory.

5.5.9 Synchrony of neural firings theory

Boomsliter & Creel (cf. 1961) and Meyer (cf. 1898) proposed that simultaneous tones may sound consonant when the two tones share common intervals between nerve impulses. There is no sensation of pitch when a sound is very short. Periodic sounds of less than 10 cycles appear click-like rather than tone-like. The perception of tonality increases as the duration of the note increases. Boomsliter & Creel argued that both pitch and consonance between notes require synchronisation of the neural firing to individual cycles of the sound, and that this synchronization needed to persist for a reasonable time before it could be analyzed.

Ward (cf. 1954) and Attneave & Olson (cf. 1971) found that our ability to make octave matches, and our pitch perception in general, largely disappeared above 5 kHz. This is also the frequency at which neural synchrony no longer operates, according to Palmer & Russel (cf. 1986). These findings support the validity of the neural synchrony theory (Moore, 2001: 294).

5.5.10 Minor theories

Lundin (cf.1947) attributed the effect of consonance to the learning of arbitrary cultural patterns. This seems highly unlikely considering the preference which infants show for consonant versus dissonant intervals. Cheung, Trainor and Tsang (2002: 187) tested 2-and 4-month-old infants' preferences by using a looking-time preference procedure. Infants of both ages preferred listening to consonant rather than dissonant intervals, and lost interest after a sequence of dissonant intervals were played. Presumably, sensitivity to consonance and dissonance is found before any learning regarding scale structure. This would suggest an inherent cause, perhaps a psychoacoustic cause, such as the tonotopic theory, for consonance and dissonance perception. However, the auditory system is already functioning in the 6^{th} prenatal month, and environmental sounds do filter through to the foetus. The child could already be learning about consonance and dissonance at this stage, as most environmental sounds have energy at integer multiples of the fundamental.

On the other hand, a genetic component is present in the structure of the basilar membrane with its critical bands.

The psychoacoustic theories mentioned above do not account for all aspects of the perception of consonance. When notes are presented sequentially to the ear, beats cannot be formed, yet a certain amount of dissonance is experienced. This also runs counter to the tonotopic theory, since successive tones cannot interfere on the basilar membrane. Also, some dissonance may be experienced when two tones are presented, one to each ear. These effects may be the effect of learning. Dissonant intervals will have become familiar to the listener under conditions where he hears both notes simultaneously in both ears. The dissonant intervals are then automatically associated with dissonance, and this perception may persist when tones are presented sequentially, or one to each ear.

5.6 THE EFFECT OF PERSONALITY ON CONSONANCE AND DISSONANCE PERCEPTION

Identical stimuli often illicit different reactions from different subjects. It has been shown that differences in personality can account for this. One can cite the example of differing reactions of subject's heart-rates within four seconds of hearing an unexpected tone. There are usually two possible reactions: the heart-rate can increase, or the heart-rate can slow down, and then increase. The first response is typical of a defence reflex, and the second response is indicative of an orientating response, which signifies interest in the stimuli (cf. Graham, 1979). According to studies done by Orlebeke & Feij (cf. 1979), and Ridgeway & Hare (cf. 1981), the individuals whose heart-rates decelerate and then accelerate, tend to be individuals who score higher on "sensation seeking" personality characteristics. Thus certain types of personality are less likely to experience unpleasantness when confronted by stimuli than other personality types. It is therefore likely that different types of personality would have different reaction to consonance, perhaps affecting the tolerance the most (Burns, 1999: 242).

5.7 THE EFFECT OF AGE ON CONSONANCE AND DISSONANCE PERCEPTION

Perhaps a subject's response would change as he or she ages, as one's tolerance for dissonance could also change. It has been shown, at least amongst schoolchildren, that preferences for musical intervals change with age. Valentine's study showed that children under the age of 9 did not show a preference for tones with simple frequency ratios as opposed to tones with complex frequency ratios, whereas children over 12 did. These changes in perception can either be attributed to the development of sensory and perceptual skills, or to the effect of accustomization to the musical "grammar" in their culture (Moore, 2001: 295).

5.8 SCALING OF INTERVALS ACCORDING TO THEIR CONSONANCE RATINGS

Malmberg, in his extensive study of consonance of musical intervals, provided a useful table of ranking, derived from ten different treatments on this topic from the twelfth to the twentieth century. These treatments used a wide variety of methods, some having a theoretical or mathematical basis, and others being based more upon perceptual judgments. Different timbres were employed, and varied criteria of consonance were employed (Krumhansl, 1990).

Studies of Malmberg's findings lead to the establishment of this table in which the correlation between smallness of the integer ratio and its consonance ratio are shown.

Name	Smallness of integer ratio	Malmberg's consonance ranking
Octave [octave]	(2:1) 1st	1
Perfect fifth [P5]	(3:2) 2nd	2
Perfect fourth [P4]	(4:3) 3rd	3
Major sixth [M6]	(5:3) 4th	5
Major third [M3]	(5:4) 5th	4
Minor third [m3]	(6:5) 6th	7
Minor sixth [m6]	(8:5) 7th	6
Major second [M2]	(9:8) 8th	10
Major seventh [M7]	(15:8) 9th	11
Minor seventh [m7]	(16:9) 10th	9
Minor second [m2]	(16:15) 11th	12
Tritone [TT]	(45:32) 12th	8

Table 5. Correlation between smallness and consonance

It can be seen from this table that the ranking of integer smallness is consistent with the Malmberg ranking for pleasantness within a difference of two positions, with the exception of the tritone. The first three rankings correlate exactly, and the first five positions on both rankings correlate very strongly, with only the 4th and 5th positions being swapped on the rankings.

Chapter 6

TUNINGS

6.1 INTRODUCTION

There are many scales in use today,[3] and an in-depth discussion of all of these would fall outside the scope of this thesis. A long historical evolution has led to the scale westerners use most frequently today, the equal-tempered scale (Cohen 2002: 307). We have found it meaningful to limit our investigations to some of the tunings which have formed as precursors to the equal-tempered scale; of which the lineage can be traced in section 5.2, 'The history of musical consonance'. The following is a brief discussion of the Pythagorean, just-intonation and mean-tone tuning systems. Then follow a few more scales, further illustrating the wealth of current tunings. The main purpose of this chapter is to offer a brief introduction to some of the scales with which comparisons with the newly designed phi scale might be meaningful. Furthermore, it is considered useful to investigate the existence of other tunings, as they show that precedents for the creation of new scales exist. First, however, it is necessary to note the difference between the concepts of a 'tuning system' and a 'modal scale'.

According to Dowling & Harwood (1986: 113-118), "There are certain regularities in the way a culture fills in the octave and in the way it uses pitches in melodies that are best disclosed by considering melodic scales to be constructed out of the underlying tonal material through a process involving several levels of psychological analysis." These levels are illustrated by the figure below.

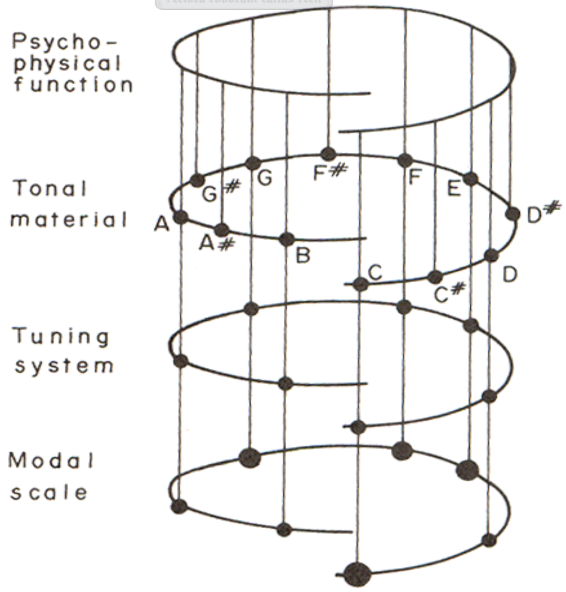


Figure 42. The levels of analysis of musical scales
(Dowling & Harwood, 1986: 114)

3 cf. Rasch, R. 2002: *Tuning and Temperament.*

Musical pitches are understood to be mentally represented distinct sonic points, which occupy higher or lower positions on a vertical two-dimensional continuum (Cohen 2002: 307). The psychophysical pitch function is responsible for assigning pitches to frequencies. The tonal material level consists of all the possible pitches that could be used in melodies. This level is responsible for dividing the undifferentiated continuum of the psychophysical scale into discrete categories.

The tuning system consists of a subset of the tonal material and consists of the pitches which can be used in modal scales. One should not imagine the tuning system as specific pitches, as it only prescribes the interval pattern between successive notes. Imagine for example a circular interval pattern. The modal scale implies a hierarchical organization of the pitches with a tonal centre, and these pitches are the ones used in melodies. Imagine choosing 8 sequential digits starting from any point in the circular interval pattern, and applying this interval pattern to the sequence of notes, after choosing a starting pitch, or 'tonal centre'. The figure below shows (A) the major modal scale in western music, (B) the minor modal scale in western music and (C) an illustration of the generation of the minor mode on C by using the interval sequence of B.

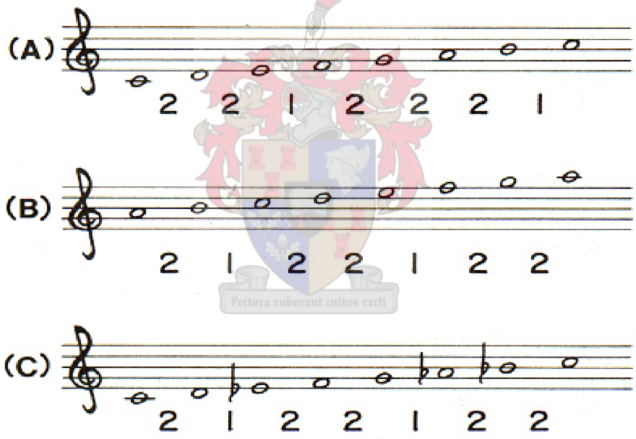


Figure 43. Major and minor modes
(Dowling & Harwood, 1986: 117)

Adhering to a certain interval sequence but starting at different pitches in the scale yields different modes, but the tuning system stays the same. The following figure shows various modes derived from the same tuning system.

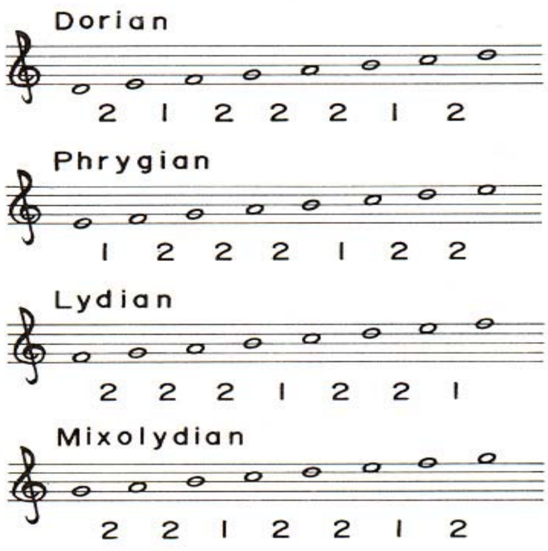


Figure 44. Various modes
(Dowling & Harwood, 1986: 118)

Evidently, various modes can be derived from one tuning system (Dowling & Harwood, 1986: 113-118).

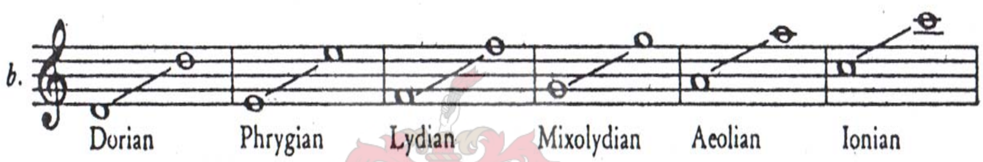


Figure 45. Modes derived from tuning system
(http://www.hugohein.com/links/musical.octave.specs.htm)

For a more detailed discussion of modes, see Cohen (2002).

6.2 PYTHAGOREAN TUNING

The Pythagorean tuning is the the oldest scale construction known, much older than Pythagoras (c.550 BC) himself. Pythagoras' name has, however, become synonymous with this tuning (Bibby, 2003: 13). Pythagoras noticed that if the ratio between note frequencies can be represented by small rational numbers, i.e. x/y where x and y <5, the interval formed by these is consonant. Consequently, Pythagorean intonation is based on a scale where all octaves, fifths and fourths are perfect and untempered (Herlinger, 2002: 176). 'Temperament' refers to the tuning of a scale in which some or all of the concords are made slightly impure in order that few or none will be left distastefully so (Lindley: 2001). The Pythagorean untempered scale is often said to be 'natural' or 'perfect', since its intervals are also present in the harmonic series.

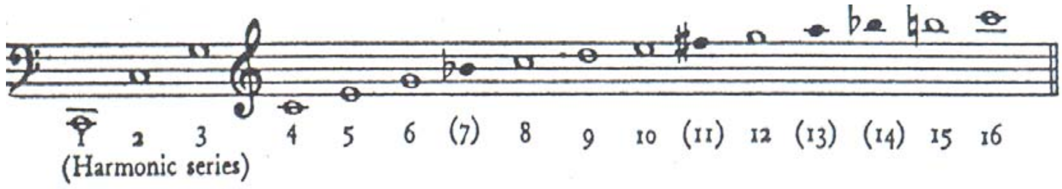


Figure 46. The harmonic series
(Cazden, 1980: 130)

The difference between the fifth and the fourth is expressed by the ratio 9:8 (⅔ * ¾) (Mathiesen, 2002: 116), and the octave is obtained by adding the 4th to the 5th (3/2 * 4/3 = 2/1) (Nolan, 2002: 274). Consequently the major third, consisting of two ‘tones’, is represented by the dissonant ratio 64/81 (Cohen, 1984: 39 and Bibby, 2003: 16). The difference between this ratio (64/81) and the just third (64/80, or 4/5) is 80/81, a value known as the syntonic comma. Its logarithmic size is 21.506 cents (each octave contains 1200 cents, comprising 12 semitones of 100 cents each (Rasch, 2002: 210)), which is about 1/5th of a tempered semitone (Rasch, 2002: 196). Consequently, the thirds (and sixths) of the Pythagorean scale are imperfect, and give rise to beats (Lindley, 2001: 643). Another problem arising from the Pythagorean tuning is that the circle of fifths used to construct the scale cannot be closed. In other words, if 11 5th’s are just, the twelfth 5th is too small by a diatonic comma, of size 524,288: 531,441, or 23.46 cents (Rasch, 2002: 198).

The frequency ratios of the pitches to the tonic in the Pythagorean scale are given in the table below:

Note no.	1	2	3	4	5	6	7	8
Freq. ratio	1:1	9:8	81:64	4:3	3:2	27:16	243:128	1:2

Table 6. Pythagorean frequency ratios

During medieval times, western art music theorists attached great importance to the consonant use of fifths, and 3rd’s were subjugated to a secondary role. The Pythagorean tuning system, with its pure fifths, was predominantly used (Rasch, 2002: 203). Round about 1300 A.D., composers started using 3rd’s and 6th’s as consonant intervals. In other words, these intervals were now being used on accented beats, without the requirement of having to be resolved (Cohen, 1984: 4). The introduction of the consonant 3rd’s and 6th’s created a problem for the division of the octave, as Pythagorean 3rd’s and 6th’s were insufficient (Cohen, 1984: 40 and Nolan, 2002: 276). Furthermore, musical intervals were played melodically, in other words they were not sounded simultaneously, but successively, and this did much to hide the Pythagorean tuning’s faults (Bibby, 2003: 20). Polyphonic music had risen by the early Renaissance, and further highlighted the Pythagorean tuning’s deficiencies.

6.3 JUST INTONATION TUNING

Before the start of the 16th century, another tuning, known as just intonation had already come into use. This division of the octave held that all intervals were pure. This was indeed true, provided one kept to the right keys (Cohen, 1984: 40). Just intonation, however, is not a perfect system, as there invariably exists a 'wolf fifth', referring to the property of a scale's intervals to never divide an octave exactly. One of the fifths is necessarily going to be compromised. The responsibility lies with the tuner in deciding where this 'wolf fifth' will lie. Consequently, in this tuning, some fifths are left distastefully smaller than pure in order that the other fifths and most of the thirds will not beat (Lindley, 2001: 290). Therefore, Galilei was correct in asserting that just intonation was inherently unstable, in that not all intervals were pure (Cohen, 1984: 80).

Just intonation shares the perfect tunings of the octave (2:1), fifth, (3:2) and fourth (4:3) with Pythagorean tuning. Pythagorean tuning does not, however, allow for consonances with frequency ratios of 5 or larger. Just intonation theory admits 5 in order to build pure thirds and sixths (Lindley, 2001: 291). Here follows a table, derived from Gouk (2002: 236) and Bibby (2003: 21), indicating the ratios used in the construction of the just intonation scale:

Note	Frequency ratio relative to the tonic
Tonic	1:1
2nd	9:8
3rd	5:4
4th	4:3
5th	3:2
6th	5:3
7th	15:8
Octave	2:1

Table 7. Just intonation frequency ratios

Since the interval between the fifth and the fourth is 9:8, one would expect the third to have the frequency ratio of 81:64, being $(9/8)^2$. However, the perfect third has the frequency ratio of 5:4, which is the same as 80:64. The discrepancy between these two frequency ratios is 81:80, which, as mentioned above, is known as the syntonic comma. Just intonation consequently has two sizes of whole tone, for instance the second between the tonic and 2nd have a ratio of 9:8, and the second between the 2nd and the 3rd have a ratio of 10:9. The ratio between the 3rd and the 4th gives rise to yet another interval size with frequency ratio 16:15 (Bibby, 2003: 21 and Rasch, 2002: 201). Intonation problems arise if one strays too far from the tonic key. The Oxford Companion to Music (Scholes, 1941: 926) states that just intonation could be accurate only for one key with the ordinary 12-note to the octave keyboard. Therefore, despite the theoretical prestige it enjoyed during the 16th century, this tuning system was considered inappropriate for keyboard tunings (Rasch, 2002: 201).

6.4 MEAN-TONE TUNING

Lindley (2004b) claims the earliest form of mathematically explicit mean-tone temperament was Zarlino's 1558 proposal for 2/7-comma mean-tone, were the major 3rd's are slightly smaller than pure. Various mean-tone tunings were proposed, and no one specific form was particularly favoured to others. In this tuning system, it is attempted to keep the 3rd's as close to their 'purest' form as possible, i.e. frequency ratio 5:4.

In achieving equivalent whole tones, the tuner would often temper the fifths and fourths, making the fifths smaller by a ¼ of the syntonic comma, and the fourth a ¼ of the syntonic comma larger. On account of this tempering, the mean-tone tuning is also known as '¼–comma mean-tone' (Lindley, 2004b). This tuning system was particularly favoured by organists, which could explain why most organ music from the 16th to the 19th century was written in this temperament.

Lindley states that mean-tone tuning, in the broader sense, can also refer to 'any Renaissance or Baroque keyboard tuning in which a major 3rd slightly smaller or, more often, slightly larger than pure is divided into two equal whole tones' (2004b). Dolmetsch online (http://www.dolmetsch.com/theoryintro.htm) claims the earliest thorough description of mean-tone tuning is Salinas' 1/3 mean-tone tuning found in Salinas' *'De Musica libri septem'*, written in 1577, with pure minor 3rd's at the expense of slightly small major 3rd's. The whole tone is calculated with regard to the purity of the 3rd, and the 3rd consists by definition of two equal tones. The tempered tone is the mean proportional of the two tones, thus the squared root of them. It is after this property which the mean-tone system is named (Cohen, 1984: 42).

The number of keys available to the composer was less restricted than in just intonation, but some restrictions were still necessary. The *Oxford Companion to Music* (Scholes, 1941: 926) states that mean-tone temperament was reasonably accurate in about eight keys, counting major and minor, and impossible in others. Renaissance keyboard music did not overstep these limits, and mean-tone temperament consequently became the dominating tuning system during this period until another stylistic revolution in music occurred around 1600 (Cohen, 1984:43).

6.5 EQUAL-TEMPERED TUNING

According to Cohen, temperament is based on the "empirical observation that to a certain extent the human ear is willing to put up with small deviations from absolute purity in the consonances."(1984:41) Temperament means "an adjustment in tuning in order to get rid of gross inaccuracy in the intervals between certain notes - an adjustment by the distribution of the amount of this inaccuracy over the intervals in general (or some of them), so that smaller disturbance to the ear results." (Scholes, 1941: 923) Rasch defines temperament as "the slight alteration of just tunings" (Rasch, 2002: 201) (Also see Lindley's definition of 'temperament' in section 6.2 above.) Basically, temperament is necessary since an octave with all intervals "pure", i.e. in their smallest possible integer relationships to each other, is mathematically incommensurable. Consequently, a compromise has to be struck, and many theoreticians have made attempts at striking the best compromise.

The scale which most westerners are accustomed to is the so-called western equal-tempered scale. As mentioned above, the exact mathematics of equal-temperament was developed in China around 1580 by the scholar Chu Tsai-Yü (Dowling & Harwood, 1986: 94) although the debate around this fact is still alive.[4] Apparently the first writings on equal-temperament only commenced 50 years after Prince Chu Tsai-Yü's introduction of equal-temperament in China (Gamer & Wilson, 2003: 149).

Equal-temperament divides the octave into twelve equal semitones, with the consequence that the 3rd's and 6th's are tempered much more than the 4th's and 5th's. (Lindley, 2001: 275).Equal-temperament allows unrestricted modulation between keys, as all intervals are equal in size.

4 cf. http://www.dolmetsch.com/theoryintro.htm, http://www.cechinatrans.demon.co.uk/ctm-psm.html and Cho, G.J. 2003: *The discovery of musical equal temperament in China and Europe in the sixteenth century* Edwin Mellen Press Ltd.

The rise of equal-temperament during the 18th century as the most prominent tuning for keyboard instruments cannot be separated from the use of all 24 major and minor keys becoming standard in musical composition (Rasch, 2002: 207). Wagner's music, which drifts from key to key, would be incomprehensible lest it were for equal-temperament. Debussy's use of the whole tone scale and Schönberg's (1874 – 1951) use of the dodecuple system are possible because of the rigid mathematical division of the octave into 12 equal portions. Hába's quarter tones are merely an extension of the principles of equal-temperament, and thus he is also indebted to this system (Scholes, 1941: 926). Rasch (2002: 204) makes the interesting observation that the rise of equal-temperament coincided with the appearance of mathematical tools such as root extraction methods, which make this tuning system easily applicable.

J.S. Bach was supposedly a supporter of this temperament, composing two series of Preludes and Fugues he called "*The Well-tempered Clavier*" (BWV 846-869; 870-893), each series including all twelve major and minor keys (Scholes, 1941: 924). However, in light of new research done by Barnes (cf.1979), it is now believed that he wrote these pieces for a temperament similar to one devised by Werckmeister, known as Werckmeister III,[5] where the distribution of fifths were unequal, some being perfect and others being 6 cents smaller.

Use of equal-temperament has grown in popularity, and today is widely regarded as the standard tuning of the western 12-note chromatic scale.

Though very popular, equal-temperament still has and always had its opponents. Helmholtz (1954: 323) stated in 'Sensations of tone': "When I go from my justly-intoned harmonium (a reed organ cf. Fowler, 2003: 78) to a grand pianoforte, every note of the latter sounds false and disturbing" and "The music based on the tempered scale must be considered as an imperfect music… If we suppose it or even find it beautiful, it means that our ear has been systematically spoiled since childhood."

The following table illustrates the aforementioned scale's deviation in cents from the simple frequency ratios of just intonation. This table consequently serves as an indication of deviation from small integer number ratios, and can be construed as a rough indication of relative dissonance between the scales' intervals. This table's results were in great part derived from Herlinger (2002: 177).

5 cf. Barbour (1972).

	C	D	E	F	G	A	B	C'
Just intonation	0	0	0	0	0	0	0	0
Pythagorean	0	0	+22	0	0	+22	+22	0
Mean tone	0	- 11	0	+5.5	- 5.5	+5.5	-5.5	0
Equal-temperament	0	-4	+14	+2	-2	+16	+12	0

Table 8. Cents deviation from simple integer ratios (http://www.hugohein.com/links/musical.octave.specs.htm)

6.6 OTHER TUNINGS

6.6.1 Microtonal tunings

According to Griffiths, Lindley, Zannos, (2001: 625), two basic approaches to microtonality can be distinguished. Microtonal intervals are either introduced as finer divisions within the regular 12-note equal-temperament, or they arise as a necessary condition of different tunings.

The former approach was taken by Carillo, Ives (1874-1954), Hába (1893-1973) and Vïshnegradsky (1893-1973). The 19-tone, 31-tone and 53-tone equal-tempered systems have enjoyed particular interest, as the intervals in these scales often give a closer approximation of the just ratios than does the 12-tone variant The 19-tone and 31-tone equally tempered systems were known in the 16th century, as they were studied by the mathematicians Mersenne and Huygens (Gamer & Wilson, 2003: 149).

The second approach to microtonality was taken by amongst others Harry Partch. Partch is best known for employing a 43-interval octave using frequency ratios involving the primes up to 11 and their multiples, and also for building his own instruments (Griffiths, Lindley & Zannos, 2001: 625).

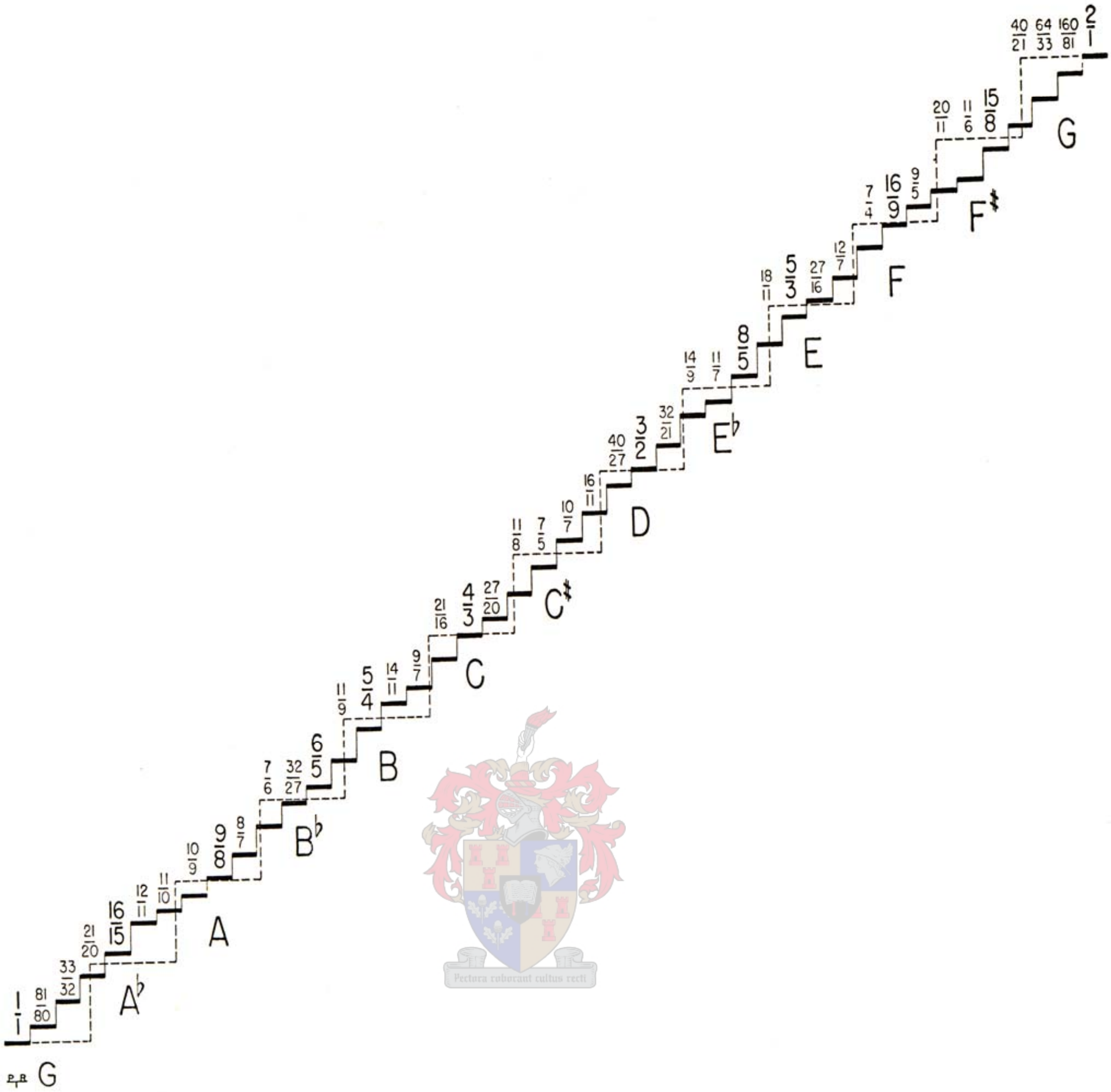


Figure 47. Partch's forty-three tone scale
(Partch, 1949)

In addition, composers have developed alternative equal temperaments. Joseph Yasser (1893-1981), for instance, argued for a 19-interval octave. Since electronic instruments have become available, it has been applied to the performance of microtonal music. In the 1940's, Percy Grainger (1882-1961) used 6th and 8th tone tunings respectively for the tunings of his 'Cross-Grainger Free Music Machine'. In the 50's, Murzin developed a photoelectric composition machine tuned to 72 equal octave divisions, i.e. 12th tones. In the 60's Moog constructed electronic keyboards with 43 and 31 notes to the octave respectively (Davies, 2001: 621).

Particular difficulties have to be overcome when working in microtonal tuning systems. For one, a singer's intonation requires a well trained memory, and furthermore a new notation system has to be devised (Moreno, 1992: 50). Nevertheless, microtonal music enjoys a loyal following.[6]

6.6.2 Dodecaphonic tuning

In the early 20th century, Josef Matthias Hauer (1883-1959) made the first attempt at developing a 12-note tuning system for compositional purposes. Schönberg and Alban Berg (1885-1935) (Covach, 2002: 603), independently developed their dodecaphonic tuning for composition at approximately the same time as Hauer developed his. In dodecaphonic theory, all twelve notes of the chromatic scale hold equal stature. The dodecaphonic scale is the chromatic scale as far as the actual notes are concerned, but under the dodecaphonic system all notes in the octave are given equal importance. Since the tuning system is one with a circular pattern of 1's, it is considered as a separate tuning to the western equal-tempered scale, which has a circular interval pattern of 2-2-1-2-2-2-1.

6.6.3 Chinese tuning

The Chinese scale also has the octave, fifth and an interval resembling a fourth. The following table shows the intervals in the Chinese scale. The Chinese call their fundamental the "Huang Chung". The Chinese rejected equal-temperament on account of the harsh dissonances which result due to the tempering (http://www.hugohein.com/links/musical.octave.specs.htm).

Note no.	1	2	3	4	5	6	7	8
Freq. to "Huang Chung"	1/1	9/8	81/64	729/512 (4/3)	3/2	27/16	243/128	2/1

Table 9. Chinese frequency ratios

The 4th and 7th intervals are not used in transpositions, or as fundamentals. However, the Chinese use the pentatonic scale for most of their modes, so although the approximated fourth appears in their tonal material, it does not feature in the scales they use for making melodies.

The Chinese build their scale by multiplying a base frequency by 3/2, then by ¾ and so on until a series of 12 frequencies is produced. Each of the frequencies thus obtained is then used as the fundamental of a new scale. In total, there are 144 (12^2) frequencies, though many duplications exist. They also make use of the phenomenon of octave equivalence, i.e. at some point in the progression the frequency will be halved. The following table illustrates the frequencies produced by using 260Hz as the first base.

[6] cf. http://www.justintonation.net.

The 12 Lu ("rules") of the Chinese Musical Gamut. Each column is based on a value from column one.

	base 260	base 277.65	base 292.5	base 312.35	base 329.06	base 351.4	base 370.2	base 390	base 416.47	base 438.75	base 468.53	base 493.59
太簇 tai` cu`	260	277.6	292.5	312.4	329	351.4	370.2	390	416.5	438.8	468.5	493.6
夾鐘 jia` zhong¯	277.6	296.5	312.4	333.6	351.4	375.2	395.3	416.5	444.7	468.5	500.3	527.1
姑洗 gu¯ xianˇ	292.5	312.4	329.1	351.5	370.2	395.3	416.5	438.8	468.5	493.6	527.1	555.3
中呂 zhong¯ lüˇ	312.4	333.6	351.4	375.3	395.3	422.1	444.7	468.5	500.3	527.1	562.9	593
蕤賓 rui´ bin¯	329.1	351.4	370.2	395.4	416.5	444.7	468.5	493.6	527.1	555.3	593	624.7
林鐘 lin´ zhong¯	351.4	375.2	395.3	422.2	444.7	474.9	500.3	527.1	562.9	593	633.2	667.1
夷則 yi´ ze´	370.2	395.3	416.5	444.8	468.5	500.3	527.1	555.3	593	624.7	667.1	702.8
南呂 nan´ lüˇ	390	416.5	438.8	468.6	493.6	527.1	555.3	585	624.7	658.1	702.8	740.4
無射 wu´ yi`	416.5	444.7	468.5	500.4	527.1	562.9	593	624.7	667.1	702.8	750.5	790.6
應鐘 ying` zhong¯	438.8	468.5	493.6	527.2	555.3	593	624.7	658.1	702.8	740.4	790.6	832.9
黃鐘 huang´ zhong¯	468.5	500.2	527.1	563	593	633.2	667.1	702.8	750.5	790.6	844.3	889.5
大呂 da` lüˇ	493.6	527	555.3	593.1	624.7	667.1	702.8	740.4	790.6	832.9	889.5	937.1
	527.1	562.8	593	633.3	667.1	712.3	750.5	790.6	844.3	889.5	949.8	1000.7

Table 10. The Chinese scale
(http://www.hugohein.com/links/musical.octave.specs.htm)

The Chinese use five pentatonic scales in their music. These scales, together with their names, frequency ratios to the tonic and frequencies, are given in the table below.

12 Lü Names	Freq Hertz Num	Denom	Calculate from 440.00	Calculate from 469.86	Calculate from 495.00	Calculate from 528.64	Calculate from 556.88	Calculate from 594.39	Calculate from 626.48	Calculate from 660.00	Calculate from 704.79	Calculate from 742.50	Calculate from 792.86	Calculate from 835.31	
Huang Zhong	1	1	440.00	469.86	495.00	528.64	556.88	594.39	626.48	660.00	704.79	742.50	792.86	835.31	黄鐘
Da Lü	2187	2048	469.86	501.75	528.60	564.52	594.67	634.73	669.00	704.79	752.63	792.89	846.67	892.01	大呂
Da Cu	9	8	495.00	528.60	556.88	594.73	626.48	668.68	704.79	742.50	792.89	835.31	891.97	939.73	大蔟
Jia Zhong	1968	1638	528.64	564.52	594.73	635.15	669.07	714.13	752.70	792.97	846.79	892.09	952.59	1003.60	夾鐘
Gu Xian	81	64	556.88	594.67	626.48	669.07	704.79	752.27	792.89	835.31	892.01	939.73	1003.46	1057.19	姑洗
Zhong Lü	1771	1311	594.39	634.73	668.68	714.13	752.27	802.94	846.30	891.58	952.09	1003.03	1071.06	1128.40	仲呂
Rui Bin	729	512	626.48	669.00	704.79	752.70	792.89	846.30	892.01	939.73	1003.51	1057.19	1128.90	1189.34	蕤賓
Lin Zhong	3	2	660.00	704.79	742.50	792.97	835.31	891.58	939.73	990.00	1057.19	1113.75	1189.29	1252.97	林鐘
Yi Ze	6561	4096	704.79	752.63	1.60	846.79	892.01	952.09	1003.51	1057.19	1128.95	1189.34	1270.01	1338.01	夷則
Nan Lü	27	16	742.50	792.89	835.31	892.09	939.73	1003.03	1057.19	1113.75	1189.34	1252.97	1337.95	1409.59	南呂
Wu Yi	5905	3277	792.86	846.67	891.97	952.59	1003.46	1071.06	1128.90	1189.29	1270.01	1337.95	1428.70	1505.19	無射
Ying Zhong	243	128	835.31	892.01	939.73	1003.60	1057.19	1128.40	1189.34	1252.97	83.63	1409.59	1505.19	1585.79	應鐘
			Yu scale on Huang Zhong base freq.		Shang scale on Da Cu base freq.			Gong scale on Zhong Lü base freq.			Jue scale on Yi Ze base freq.		Zhi scale on Wu Yi base freq/		

Table 11. The Chinese scale
(http://www.hugohein.com/links/musical.octave.specs.htm)

The Chinese scale shows the closest resemblance to the Pythagorean scale, as can be seen from the table below, which indicates Hz.

Reordered Chinese	Notation name	Pythagorean	Equal-Temperament
440.00	A	440.00	440.00
469.86	A#	463.54	466.16
495.00	B	495.00	493.88
528.64	C	521.48	523.25
556.88	C#	556.88	554.36
594.39	D	586.67	587.33
626.48	D#	626.48	622.25
660.00	E	660.00	659.25
704.79	F	695.31	698.45
742.50	F#	742.50	739.98
792.86	G	782.22	783.99
835.31	G#	835.31	830.60
880.00	A	880.00	880.00

Table 12. Resemblance between Chinese and other scales
(http://www.dolmetsch.com/theoryintro.htm, http://www.wfu.edu/~moran/Cathay_Cafe/G_tar.html)

An acquaintance of mine recently returned from a two year stay in Beijing. When asked whether she had grown accustomed to their music, we were greeted by a look of confusion. Apparently the music which enjoys the most media exposure is based on the western equal-tempered scale. Storr writes “This may bear witness to the bulldozing effect of Western dominance, which tends to destroy every indigenous aspect of the cultures with which it comes into contact, but it is not a good argument for the superiority of European music. American popular music has swept the world; but few musicians consider it better than the varieties of music which it has displaced”(1993: 59).

This is certainly an unfortunate effect, but there is some irony in the fact that Chu Tsai-Yü was perhaps instrumental in the development of equal-temperament. Interestingly, the dictionary meaning of Chinese words are often determined by their pitch and accent, which complicates the translation of songs written in English into Chinese (Fiske, 1996: 66).

6.6.4 North Indian tuning

North Indian music’s tuning system consists of the following tonal material.

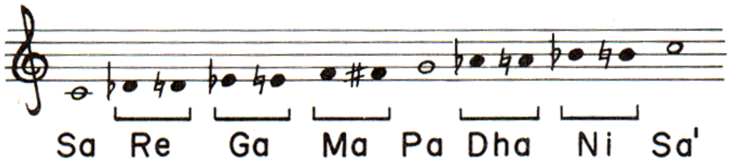


Figure 48. North Indian scale
(Dowling & Harwood, 1986: 115)

The brackets indicate pairs of pitches of which only one pitch usually appears in the scale. Consequently, there are a large number of possible tuning systems available, 2 to the power 5 in fact. Also, there are three instances in which the F appears in both its sharp and natural forms, giving us a sum total of 35 tuning systems. Ten of these tuning systems are in general use, and another ten are used occasionally. The figure below shows three of the thirty-five tuning systems.

Figure 49. North Indian tuning systems
(Dowling & Harwood, 1986: 116)

6.6.5 Hungarian scale

The so-called Hungarian scale is equivalent to the harmonic minor with the fourth raised.

Figure 50. The Hungarian scale
(Boyden, 1978: 37)

Notice that coherence is lost on account of the 3rd interval, from E flat to F sharp (3 semitones) being larger than the 4th and 5th intervals combined (2 semitones).

6.6.6 Pentatonic scale

This scale has already been mentioned in "Building a scale".

Figure 51. The pentatonic tuning
(Boyden, 1978: 38)

The most common forms of the pentatonic scale can be obtained by playing only the black keys of the piano. Starting on C#, we obtain the major form of the pentatonic scale, which the Japanese call "male", and using D# as our fundamental, we obtain the minor pentatonic or female mode. The intervals for the two scales are 2-2-3-2-3 for the major and 3-2-2-3-2 for the minor.

Notice that, though the 3rd interval of the scale has a 3 semitone size, coherence is maintained by virtue of there being no semitone steps in this scale

6.6.7 Whole tone scale

In this scale, the octave interval is divided into equally-sized whole tones. As all intervals between consecutive notes are equally sized, the scale is tonally unstable. Debussy has used this scale as a means of suspending tonality (Andrews, 2004).

Figure 52. The whole tone tuning
(Boyden, 1978: 36)

6.6.8 Harmonic tuning

Since harmonics feature so prominently in music, a scale has been constructed using only a fundamental note and its harmonics. Once again the phenomenon of octave equivalence has been used to construct the scale. The 7th, 11th and 13th harmonics are the most out-of-tune with equal-temperament (Parncutt: 1989: 8).

Notes	C	D	E	F#	G	A-	Bb	B	C'
Freq. ratios	1/1	9/8	5/4	11/8	3/2	13/8	7/4	15/8	2/1

Table 13. Frequency ratios of the harmonic tuning

6.7 CONCLUSION

It can be argued that new scales are often a mere adaptation of existing ones. Consider for instance the various tunings for mean tone temperament, and how it is perhaps not too unlike equal-temperament. The equal-tempered scale, for instance, is actually equal to a mean-tone tuning by 1/12th comma. This implies every fifth is narrowed by 1/12 of the syntonic comma. This phenomenon is understandable in light of Helson's (cf.1964) conceptualization of adaptation level, which predicts the adaptation level along any sensory continuum as the level we are used to (Dowling & Harwood, 1986: 87). In other words, Helson has shown that we prefer that which diverges only slightly from our current adaptation level. Helson's conceptualization can also be a basis for explaining the acceptance of dissonant intervals, which is slowly proceeding towards greater dissonant values.

Chapter 7

FURTHER CONSIDERATIONS WHEN BUILDING A SCALE

7.1 HOW SCALE INTERVALS ARE CHOSEN

This chapter will investigate how the intervals comprising a scale are chosen, and whether these intervals are chosen at random, or whether they are pre-programmed, or learnt.

A scale is a long enough sequence of notes in ascending or descending order of pitch, which is capable of unambiguously defining a linear musical construct such as a mode or tonality (Drabkin, 2004). According to Scholes (1941: 833-4) there are three processes by which scales come about. These processes are based on intuition, scientific reasoning or chance. By way of exclusion, we have decided to develop the scale on the basis of scientific reasoning, as intuition depends on great knowledge, or great natural talent, neither of which we presume to possess, and we would not entrust the thesis to chance.

A number of prominent music theorists such as Leonard Bernstein, Heinrich Schenker, Paul Hindemith and Deryck Cooke claim that the intervals used in the western equal-tempered scale are not man-made but natural, thus arising from intuition. They presume the existence of an innate interval template within every person's mind, or that there are natural frequency ratios inherent in the way the auditory system processes tonal stimuli (Storr, 1993: 52).

These intervals are said to be unique and are the ones used in our scale. Bernstein, for instance, said "…the overriding fact that emerges from all this is that all music – whether folk, pop, symphonic, modal, tonal, atonal, polytonal, microtonal, well-tempered or ill-tempered, music from the distant past or the imminent future – all of it has a common origin in the universal phenomenon of the harmonic series. And that is our case for musical monogenesis, to say nothing of innateness." He thus presumes that the intervals sprout from something inherent (Storr, 1993: 60-61). If this were true, we would be hard pressed justifying a new scale which does not have these intervals.

However, the weight of current evidence suggests that musical interval categories are learned, and are not the direct result of the auditory system's characteristics.

7.1.1 Evidence against innate intervals

Variability of scales

If scale intervals are innate, one would expect scales of different cultures to be the same. The Indian, Chinese and Arab-Persian musical systems indeed have inclusive scales approximately equivalent to the Western 12-interval scales, and thus also have the propensity for perfect consonances (4^{th}, 5th and octave). Some cultures have approximately equal-tempered 5- and 7- interval scales in which the 4^{th} and 5^{th} are far from "perfect", presumably they also use the octave. Malm noted that Southeast Asian cultures use seven-interval scales. Morton measured the tuning of a Thai xylophone that varied approximately 5 cents from an equally tempered 7-interval tuning. It has also been established that the central African peoples of the Ouldeme tribe do not use the octave as a reference interval (Marandola, 2003: 34-41).

Haddon reported, in 1952, a xylophone from the Chopi tribe in Uganda which was tuned in 171-cent steps. The gamelan orchestras in Java and Bali, comprising of tuned gongs and xylophone –type instruments, typically use 240-cent step-size, 5-interval scales. Thus they still use the octave, as 240*5 = 1200cents. Hood (cf.1966) and McPhee (cf.1966) found that there were large deviations in the tunings of gamelan, so much so that McPhee said " Deviations in what is considered the same scale are so large that one might with reason state that there are as many scales as there are gamelans." Wachsmann, in 1950, reported a 5-interval, 240-cent step tuning for a Ugandan harp. Notice once again the octave is present, i.e. 5*240 = 1200 cents (Burns & Ward, 1982).

In pre-instrumental cultures, variability of scales is also found. Boiles (cf.1969) reports a South American scale with intervals of 175 cents. Ellis (cf.1965) found pitch distributions that follow arithmetic scales in the Australian aboriginal pre-instrumental cultures.

According to Burns & Ward "there seems to be a propensity for scales that do not use perfect consonances and that are in many cases highly variable, in cultures that either are pre-instrumental or whose main instruments are of the xylophone type. Instruments of this type produce tones whose partials are largely inharmonic and whose pitches are often ambiguous" (1982: 249).

The only interval size that occurs in almost all cultures' music is that of the octave (Dowling & Harwood, 1986: 93). Therefore octaves might still be innate.

Intonation in performance

An innate sense of intervals would presumably lead musicians to intone their instruments closely to these intervals when playing. Greene (cf.1937), Nickerson (cf.1948), Mason (cf.1960) and Shackford (cf.1961; 1962) have all made measurements of the intonation of musicians playing instruments of which the tuning is variable. Ward summarized these measurements, and found large variability for the tuning of a given interval in a given performance, up to 78 cents. There was also a noticeable tendency to contract the semitone and stretch all other intervals relative to equal-temperament tuning.

This variability is a cross-cultural phenomenon, as measurements of intonation during the performance of Indian classical music (Hindustani) showed variations of up to 38 cents of a given interval in a given piece by one musician. Callow and Shepard (cf.1972) found no correlation between these variations and melodic context. Morton (cf.1974) found variability of up to 50 cents in the intonation of a Thai vocalist (Burns & Ward, 1982: 247).

Just noticeable differences

Houtsma (cf.1968) tested the just-noticeable-difference (JND) of frequency of three subjects. They were required to judge which of two melodic intervals were larger. The JND at the octave was 16 cents, and the JND at intervals in the vicinity of the octave was not significantly different. An innate sense of the octave interval could perhaps be conjectured from a smaller just noticeable difference at the octave. However, the most noticeable tendency is that the JND is smaller at narrower intervals, an observation also noted in an experiment of Raskowski's (cf.1976). Zipf (cf.1949) found that this smaller JND at smaller intervals correspond to the smaller JND in intervals that occur most often in melodies. Thus this phenomenon could be an effect of familiarity. One would expect JND to increase as the interval size increases, as the critical bandwidth increases as frequency increases, implying less sensitivity at higher frequencies. Whether this is a relevant factor in the preceding studies would depend on the frequency domain the experiments were done in.

Adjustment procedures

Many experiments have been done where one frequency is fixed and the other is under the control of the subject. (See "methodology" under "Consonance and Dissonance") The subject is required to adjust the interval under his/her control until the relationship between the two notes corresponds to some specific interval. Ward (cf.1953), Walliser (cf.1969), Terhardt (cf.1969) and others have primarily concentrated on the octave interval in their experiments.

Burns & Ward (1982) observed significant day-to-day variability in intrasubject judgments, significant inter-subject variability and once again the tendency to stretch larger intervals and compress smaller ones relative to equal-temperament.

7.2 CONCLUSION

The results of all these experiments seem to indicate that musical interval categories are learnt, rather than being innately due to a mental template or to characteristics of the auditory system.

According to Burns & Ward, there are three alternative hypotheses in explaining how the musician's ability to reproduce intervals when intoning is learnt. Either the categories are learnt from the scale of a given culture, and the intervals are chosen at random, or the intervals are learned from the scale of a given culture, were the intervals are derived originally from considerations of sensory consonance (Plomp & Levelt, 1965), or lastly, the intervals are based on early unconscious learning of relationships between the partials in complex sounds, mainly those in speech (Terhardt, 1974).

Burns & Ward reject the first hypothesis on the grounds that perfect consonances occur fairly universally in scales, especially the consonance of the octave. They reject Terhardt's theory, as it would entail that even the musically untrained have a sense of the basic musical intervals, which according to Burns & Ward (1982), is not the case. This leaves only one hypothesis, namely that sensory consonance has led to the determination of the intervals in most scales, and that the ability of musicians to intone is a function of their learning of these categories.

It seems unnecessary to reject Terhardt's theory in its entirety based on one inconsistency. Burns & Ward admit that Terhardt's hypothesis of unconscious learning could explain the phenomenon of octave stretching. In general, the pitches of the individual components of a complex tone are slightly altered by the presence of other components. This alteration usually takes the form of a downward pitch shift for lower components and an upward pitch shift for higher components. Dowling & Harwood (1986: 103) argue that the Western musician's closer adherence to the octave stretching phenomenon, rather to that of the piano's, is further evidence in support of Terhardt's theory.

7.3 CONSIDERATIONS

We have seen that we cannot base our scale on the assumption that there exist certain innate intervals. Now we shall see what we should base our scale on.

7.3.1 Discriminability of intervals

According to Helmholtz, discrete pitch levels provide the listener with a standard by which he can measure melodic motion. “The individual parts of a melody reach the ear in succession. We cannot perceive them all at once. We cannot perceive backwards and forwards at pleasure. Hence, for a clear and sure measurement of the change of pitch, no means was left but progression by determinate degrees. This series of degrees is laid down in the musical scale.” (Helmholtz, 1954: 252) The listener’s task of comprehending the amount of change is eased by having discrete steps.

Discrete pitches allow the listener to discriminate between notes played in succession. The semitone of Western music is represented by a frequency difference of about 5.9%. According to Shower & Biddulph (cf.1931), humans are capable of discriminating between frequencies differing by only 1%. Consequently, intervals smaller than the semitone could be acceptable. (Dowling & Harwood, 1986: 92-3) The violinist Maud MacCarthy could sing up and down the Hindu scale of twenty-two notes to the octave, and it is said that Alois Hába, after much practice, could accurately sing five divisions to the octave, thus being able to distinguish between 60 divisions in the octave (Scholes, 1941: 575).

7.3.2 Octave equivalence

Tones with fundamental frequencies standing in the ratio 2:1 are considered equivalences. This makes them ideal for a starting and ending of tones, as they provide an extra point of reference. The perception of octaves as equivalences is a universal effect (Burns & Ward: 1978). Most individuals in a population can tell whether two consecutive tones are in an octave relationship.

According to Demany & Armand, (cf.1984) this ability already exists in infancy. Blackwell & Schlosberg (cf.1943) found that even rats perceive tones an octave apart as similar. Presumably they trained the rats to act a certain way upon hearing a certain tone, and when presenting the rats with the tone’s octave, the rats acted as in the first instance.

According to Burns & Ward, there are two possible reasons for the octave as the “basis for an assumed circularity of relative pitch”. The first is the fact that the adding of a tones’ octave would not increase its perceived dissonance, as the partials of tones in an octave relationship coincide exactly (actually, increase in level does affect perceived dissonance, but perhaps they felt the effect was negligible). The other explanation for octave similarity is based on the perception of complex tones.

Complex tone pitch perception models (cf.Terhardt, 1974; Gerson & Goldstein, 1978) are based on the assumption that there exists a central pitch processor which attempts to match the partials of a complex tone to the best fitting harmonic series, inadvertently causing some octave ambiguity. Gerson & Goldstein and Houtsma (cf.1979) have indeed found such ambiguity in their complex-tone-pitch-perception experiments.

7.3.3 Limited pitch number

There should only be a moderate number of different pitches within the space of the octave, due to the cognitive limitation on the number of different values along a psychological dimension people can handle without confusion. Miller (cf.1956) believes this limitation along a given dimension is 7 ± 2. Over this limit, discrimination would become troublesome. Accordingly, most cultures use five to nine pitches in their scales.

This does not imply a maximum of 9 intervals within the octave, but only a maximum of 9 pitches being chosen within the octave (Dowling & Harwood, 1986: 93).

7.3.4 Uniform modulor pitch interval

In many cultures of the world, it is necessary that the octave's division occurs in a series of equal intervals. Western music divides the scale into 12 semitones. The advantage of such a system is that a given melody can be transposed to start on any interval without distorting the melody. Some scholars hold that the exact mathematics of equal-temperament was developed in China around 1580 by the scholar Chu Tsai-Yü (cf.Needham, 1962). By 1630 this system had made its way to Europe, where only practical approximations of equal-temperament had been developed. Over the next century, equal-temperament came into more common use, and today the modulation possibilities afforded by equally-tempered scales are considered highly desirable (Dowling & Harwood, 1986: 94).

7.3.5 Maximal intervallic variety

It is preferable to have a scale which has a maximal number of pitch interval sizes, in order to afford the composer a larger variety of tools. In the Western equal-tempered-scale we have the semitone and the whole tone, which is preferable to the whole tone scale which obviously only consists of whole tones.

Another requirement for the sake of variety is one which Dowling & Harwood describe as intervallic completeness. In the pentatonic scale, which is commonly used in folk music, only 8 interval sizes smaller than the octave occur.

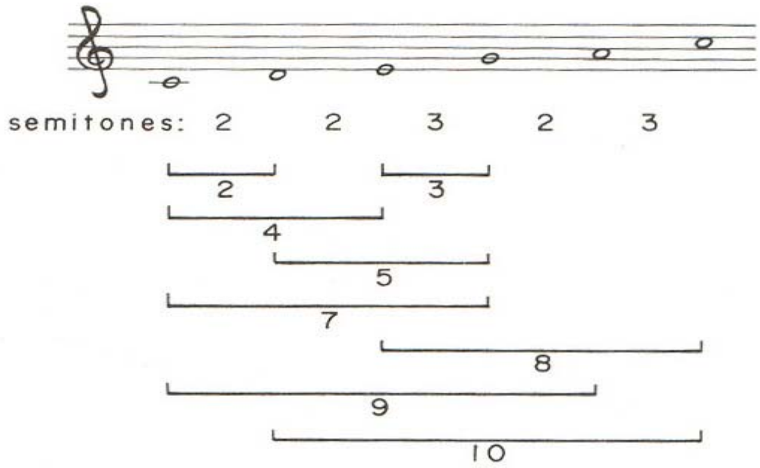


Figure 53. Pentatonic intervallic variety
(Dowling & Harwood, 1986: 100)

In the equal-tempered major scale, there are 11 interval sizes available to the composer.

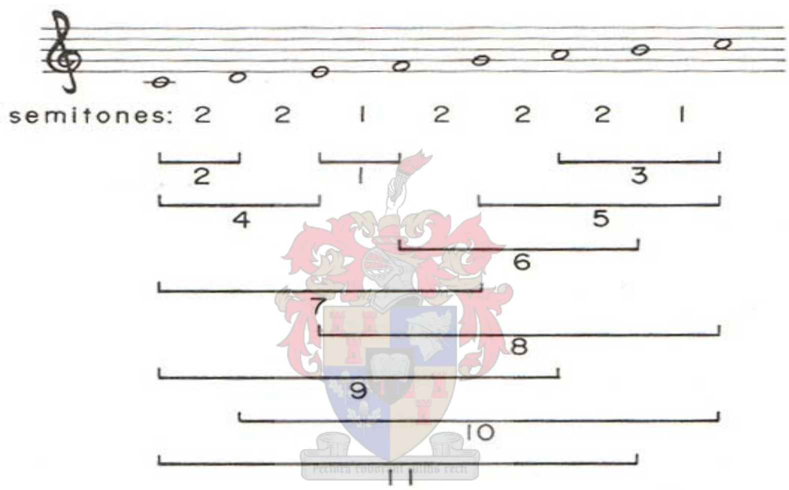


Figure 54. Equal-tempered intervallic variety
(Dowling & Harwood, 1986: 101)

Balzano (cf.1980) has shown that the smallest number of pitches that provide all the possible intervals is seven (Dowling & Harwood, 1986: 99-100).

7.3.6 Preserving coherence

Balzano also describes another desirable property of all scales, which he calls coherence. Meeting the requirements of coherence requires any two scale step to be larger than any one scale step, and any three scale step to be larger than any two scale step and so forth. Scales based on semitones meet the requirements of coherence, and in fact so do most other scales in use over the world. Boring (cf.1929) and Grout (cf.1960) found an exception occurring in the lyre tunings of ancient Greece. The figure below shows a scale described by Aristoxenus.

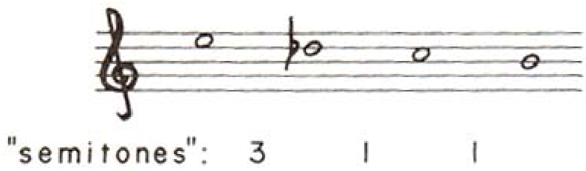


Figure 55. Incoherence
(Boyden, 1978: 37)

Evidently, the requirements of coherence are not met here, as the two-step interval from B to D-flat is smaller than the one-step interval from D-flat to E (Dowling & Harwood, 1986: 101).

7.3.7 Optimizing consonance during interval choice

As mentioned in the chapter consonance and dissonance, it is the belief of the researcher that an optimization of the number of pleasing intervals would facilitate the construction of an aesthetically pleasing musical scale. This optimization will be attained by the minimalisation of roughness as indicated by the computer models in chapter 8. If correct in assuming that sensory consonance is important in music, we should be able to verify our assumption by testing music itself. Extant literature shows that we are indeed correct in our assumption (http://dactyl.som.ohio-state.edu/Music829B/notes.html).

Seen in light of this, Storr's opinion that "all scales are arbitrary inventions governed by the necessity of defining musical relations within the octave", (Storr, 1993: 54) seems harsh. Storr was most probably reacting out of contempt for some of the less plausible theories regarding the cause of the composition of the intervals of scales. As Roger Sessions said, "A great deal of musical theory has been formulated by attempting to codify laws governing musical sound and musical rhythm, and from these to deduce musical principles. Sometimes these principles are even deduced from what we know of the physical nature of sound, and as a result are given what seems to me an essentially specious validity. I say 'essentially specious' because while the physical facts are clear enough, there are always gaps, incomplete or unconvincing transitions, left between the realm of physics and the realm of musical experience, even if we leave 'art' out of account . . . Such speculations have been in many cases the product of brilliant minds . . . Yet it would be quite easy to point out that each author, in a manner quite consistent with his musical stature, found in the overtone series a tool he could adapt to his individual and peculiar purpose" (cf. Storr, 1993: 62-63).

Yet, had Storr made a thorough study of psychoacoustic phenomenon and its effect on consonance judgments, we are sure he would agree with us in assuming that sensory consonance is important in music and hence scale composition. In light of the above quote of Sessions', which we quoted from Storr's book, we find the following quote of Storr's ironic, "The universality of music depends upon basic characteristics of the human mind" (from p.64 of his book entitled 'Music and the Mind').

7.4 Reference tone

Building a scale requires the selection of a reference tone. The second International Standard Pitch Conference held in London in 1938, set this pitch to 440Hz. This is higher than the 435Hz pitch set by the French government in 1859 after consulting, amongst others, Berlioz, Meyerbeer and Rossini. (http://www.hugohein.com/links/musical.octave.specs.htm) For our purposes, the reference pitch of 440Hz will suffice.[7]

It is interesting to note the importance the ancient Chinese attached to the tuning of their fundamental, or 'Kung'. According to Hightower it was a matter of "utmost importance for their civilization and had to be in alignment with the cosmic tone so the celestial influence could be channelled into the society by the music."

[7] For further reading on reference tones, see http://www.dolmetsch.com/theoryintro.htm, and Barbour 1972: *Tuning and Temperament* Da Capo Printers.

Chapter 8

PRACTICAL APPLICATION

8.1 INTRODUCTION

As mentioned above in section 7.3.7, it is desirable, when designing a new scale, to optimize the number of consonant intervals available to the composer. In our efforts to maximise consonance, we would do well to keep the consonance and dissonance theories of 5.5 in mind. The main contributor towards consonance, as extant research would indicate, is the absence of roughness. Many programs have been designed around the principle of minimizing roughness, in order to facilitate the design of a scale with consonant sounding intervals. These programs most often rely on models of the auditory system which includes the modelling of neural processing, in establishing their results. The programs we have chosen, have been chosen on the basis of their incorporation of the latest scientific results, and on the basis of their sanctioning by the leading experts in the field of cognitive musicology such as Huron, Leman and Sethares. The outline of this chapter will be followed in the demonstrations during the aural examination.

8.2 ROUGHNESS MODELLING

8.2.1 Types of roughness modelling

There are two classes of roughness modelling, i.e. curve-mapping and auditory modelling models. Curve-mapping models map all frequency intervals of the complex tone (or timbre or sound spectrum) out on a psychoacoustical curve of sensory dissonance. The calculation of the dissonances is done by summating all the dissonance values of all the partials with regard to each other. The following procedure is typical of the first class of computational models: firstly all the frequencies of the partials are determined; secondly all the dissonance values of all intervals between all frequency components are read off a psychoacoustical chart similar to figure 35 above; lastly all dissonance values are summated, and this value is taken as the dissonance value of the composite sound. Sethares' 1997, Plomp & Levelts' 1965 and Kameoka & Kurigayawas' 1969 models' (see the Humdrum implementation in 5.5.3 above) all fall within the first class of roughness modelling (Leman: 41).

The second type of roughness modelling is done by means of computer auditory modelling. Auditory modelling simulates the non-linearity of the neural encoding of the auditory nerve. Temporal fluctuations, or beats, are effectively introduced as frequencies into the spectrum represented by the neural rate-code patterns. Leman's (2000b) roughness model calculates the extent to which the neurons synchronise with these non-linearities in establishing roughness levels. Though the results of these two different types of modellings are much the same, it would seem the second type allows a better understanding of the reason for our results, as the first type of modelling has an input and an output stage, virtually treating the ear and cerebral functions as a 'black box', whilst the second type attempts to indicate results at the points were they occurred (for instance, during the neural encoding stage).

8.2.2 Practical applications of roughness modelling

Two practical applications of roughness modelling Leman (43) suggests are the building of a tone scale for a given sound, and also for comparing the relative roughness of two sounds. Leman has compared, with his modelling, the roughness of different sets of bells at the carillon in Bruges, and has consequently established which set would illicit the greatest sensory pleasure from the listeners (for the reader with an interest in bells, see Roaf & White, 2003: 113-130). Similarly, the well-known South African audio technician and producer Helmut Meijer could establish which combination of drums in his drum-kit would show the least roughness. Record companies might also want to establish the relative roughness of would-be singers with the aid of this modelling. Furthermore, Sethares' has suggested creating a timbre for a given tone scale, which is basically the inverse application of Leman's first.

8.3 MATLAB[8]

As mentioned in previous chapters, we will use Sethares' (1997) model of consonance curve mapping in establishing the points of local minima of our inharmonic tone. Cartwright, Gonzalez and Piro (2002) have previously attempted to derive a musical scales' fundamental frequencies from the phi ratio, but their methods, especially their use of "mirror images", seem suspect. Attempts at deriving a scale's fundamental frequencies from the timbre of a tone where the timbre is based on the phi ratio seem more logical, yet it has never been attempted before. The "instrument" will consist of a tone with fundamental and inharmonic partials related to each other and to the fundamental by phi. Sethares' program runs on a MATLAB platform, and is capable of analysing a complex tone and then suggesting points of roughness minima. The user specifies the number of partials and the frequency and amplitude of each partial. These values are entered in the computer program, which then maps the level of roughness of every possible interval (with increments of 0.01) for the given timbre starting from the unison (1:1) and ending at the interval with frequency ratio to the fundamental of 2.2:1. The points of roughness minima which can be read off this chart can be considered to be points of consonance maxima, and these points serve as ideal points at which to place successive intervals, as maximizing consonance is desirable when constructing a new scale (cf. 7.3.7 above). Start and end ratios can be set by the user, and the increments can also be adjusted, perhaps if greater accuracy is required.

Sethares' curve mapping program is readily available at http://eceserv0.ece.wisc.edu/~sethares/comprog.html. Initially the timbre we selected had sequential fundamental and partials related to each other by the root of phi, but Sethares' curve mapping program indicated very few points of local roughness minima which could be used as scale points. Through experimentation it was established that decreasing the logarithmic frequency distance between successive partials led to an increase in the number of local roughness minima points, whilst also resulting in a minimal increase in overall roughness over most frequency spectrum bands on account of a greater concentration of spectral energy. The selected timbre has sequential fundamental and partials related to each other by the third root of phi. This, in the researcher's opinion, optimizes the trade-off between increasing the number of local roughness minima points available for scale points (cf. 7.3.7 Miller (1956) believes this limitation along a given dimension is 7 ± 2) and the concomitant increase in spectral energy. Using the third root of phi gives 5 points of local consonance between 440 Hz and 833 Hz. Every third frequency component of the instrument is related to one another by phi.

After reading the points of local roughness minima off the Sethares curve-mapping graph, it was possible to generate the optimal phi scale in Microsoft Excel. This list of fundamental frequencies and partials appear in Appendix 1. The reference tone was set to 440 Hz to facilitate comparisons with other scales.

8 MathWorks' MATLAB® is an interactive programming environment. For further details see http://www.mathworks.com and

8.4 CSOUND

Once the fundamental frequencies together with their harmonics were available, all the notes of the scale had to be generated. For this purpose, the program Csound (available at http://www.csounds.com/) was used, as it is a program sanctioned by David Huron himself. The 'score' and 'instrument' files of the program appear in Appendix 2[9].

Since the upper limit of the human ears' hearing ability is approximately 18 kHz (Taylor, 2003: 51), the sample rate chosen for the sounds generated in Csound was set to 48 kHz, and all frequency components above 24 kHz were excluded during tone generation in accordance with the Nyqvist theory, which requires a sampling rate of at least twice the highest frequency component in order to avoid any aliasing from occurring. The notes were generated in the 'wav' format, as this is a standard format which is used by most programs, including the IPEM toolbox.

8.5 GIGASTUDIO

The scale notes so generated were then mapped across a virtual keyboard synthesizer called Nemesys GigaStudio,[10] a program sanctioned by many composers, notably Hans Zimmer. Consequently, all consecutive intervals of the phi scale were played, and their relative roughness was tested in the IPEM toolbox.

8.6 IPEM TOOLBOX[11]

The IPEM toolbox consists of a number of programs which run in the MATLAB programming environment. These programs are all aimed at feature extraction of sound with the aim of assisting empirical studies in music perception. In our application, the toolbox specifically extracts information and feeds this information through its computer model of the human auditory system. The IPEM toolbox differs from previous programs on feature extraction, as it concentrates on analysing sound, instead of musical scores or other forms of musical notation.

http://www.math.ufl.edu/help/matlab-tutorial.

[9] Csound is a software synthesis program which allows the generation of scales and 'instruments' in a virtual. 'computer' world, and was created by Barry Vercoe at Massachusetts Institute of Technology.

[10] cf. http://www.nemesysmusic.com and http://www.tascam.com/company.

[11] As stated on http://www.ipem.ugent.be/index.html the IPEM Toolbox is "an auditory toolbox for perception based musical analysis" and is available at http://www.ipem.rug.ac.be/Toolbox.

The IPEM toolbox was required to test the roughness of two consecutive notes in the phi scale. The 'IPEMRoughnessFFT' function was used for this purpose. The interval between two consecutive notes in the phi scale is roughly equivalent to a minor third in the equal-tempered scale. Consequently, we also asked the IPEM toolbox to test the roughness of all consecutive minor thirds in equal-temperament, enabling us to establish their comparative roughness.

The IPEM toolbox 'IPEMRoughnessFFT' function generates a three-panelled visual image of its results. Figure 56 illustrates the results of entering the equal-tempered minor third interval with 440 Hz fundamental and 15 harmonics into the Toolbox. The top panel illustrates the beating energy (the energy which might give rise to beats) distribution over the auditory channels, the middle panel shows the same beating energy, indicating the level of neuronal synchronisation to the beating frequencies, and the lowest panel illustrates the roughness of the sound as a function of time. The lower panel can be drawn from either the top or second panel, as they give the same result (Leman: 42).

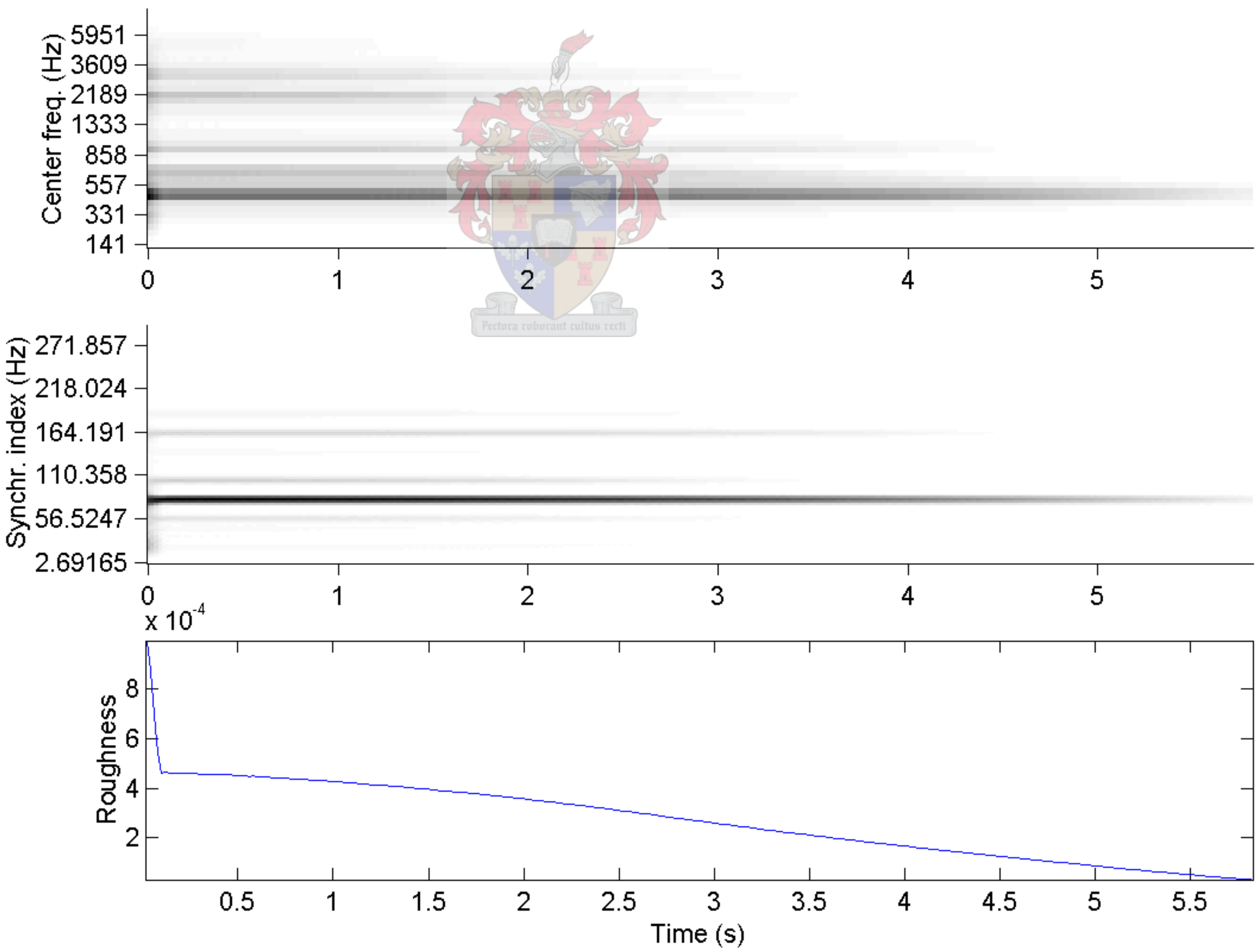


Figure 56 440 Hz equal-tempered minor third interval roughness map

8.7 RESULTS

The phi scale is found to comply with most of the considerations in chapter 8. The consecutive intervals, which correspond to approximate equal-tempered 3^{rd}'s, are well above the minimum human discriminability range of 1 % (cf. 7.3.1). The difference in pitch height is clearly discernable, as illustrated in Example 1 of the compact disc provided. Keep in mind that the phi scale does not enjoy the benefit of a resonating box, or the natural acoustic reverb which any real instrument enjoys. The phi "instrument" is after all still a "virtual computer" instrument. In employing the phi instrument in artistic endeavours, it would obviously be sensible to simulate these aforementioned effects by means of virtual software plug-ins such as Reverb and Delay. Example 1 illustrates the consecutive phi notes with fundamental frequencies of 440 Hz and 516.6 Hz respectively.

The phi scale, however, does not make use of the phenomenon of octave equivalence (cf. 7.3.2). This should not be considered a shortcoming, for precursors for this phenomenon exist, for instance in the scale of the central African people of the Ouldeme tribe (cf. 7.1.1 above). It can be heard from Example 2 that the interval of the 5^{th} in the phi scale is not an exact octave.

The phi scale could be considered to have the desired number of limited pitches within an octave frequency range (cf. 7.3.3), having approximately 4 pitches within each equal-tempered octave-sized interval. Example 3 illustrates 5 consecutive notes in the phi scale. The 1st and 5^{th} notes are slightly less than a traditional octave sized-interval apart. When using harmonic timbres, intervals with size slightly deviating from the octave would give rise to harsh dissonances, but when using the non-harmonic phi timbre, no harsh dissonance is experienced.

The phi scale also complies with the proposed requirement set out by 7.3.4 of having uniform modular pitch interval, each interval being exactly equal logarithmically. Hear how the intervals in Example 4 increase by approximately equally-tempered sized minor third intervals.

The phi scale does, however, lack the intervallic variety of the equal-tempered scale (cf. 7.3.5), having variety equal to that of the whole tone scale (cf. 6.6.7), which also has only one interval size between consecutive notes.

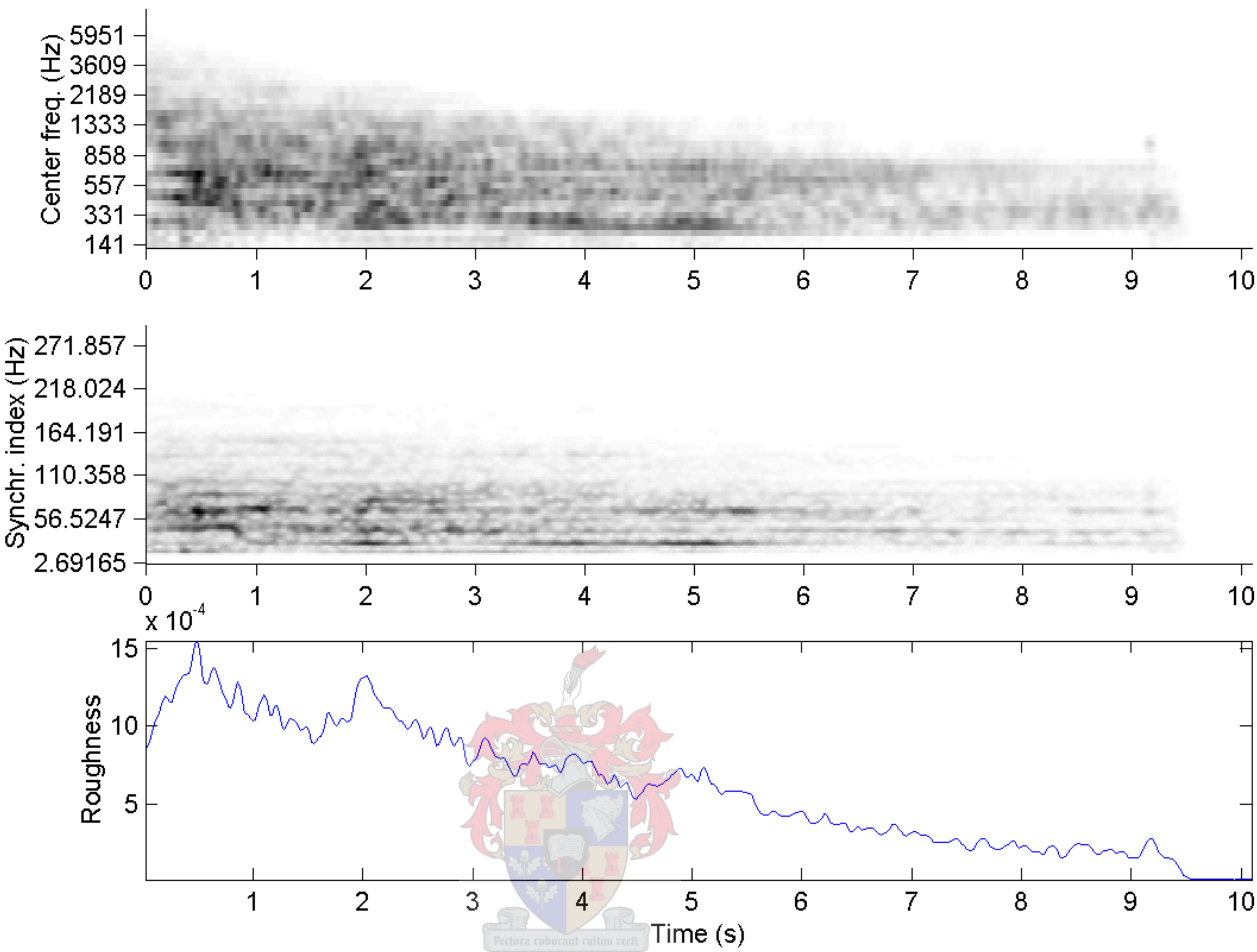
As can be heard from Example 4, the phi scale conforms to the requirement of preserving coherence (cf. 7.3.6), as all intervals are logarithmically equal in size.

Lastly, the requirement of optimizing consonance (cf. 7.3.7) was met by constructing the scale with Sethares' Matlab application. We consider the application of Sethares' Matlab program to be a fool-proof method of ensuring consonance. Included in Appendix 3 is a printout of all the roughness curves of two consecutive notes in the phi scale, and a comparative printout of consecutive minor thirds in the equal-temperament scale. Comparing the roughness curves of randomly chosen chords played on both the piano and on the phi "instrument" shows that the phi "instrument" has lower roughness. An example of a randomly chosen chord which was used in testing is the following:

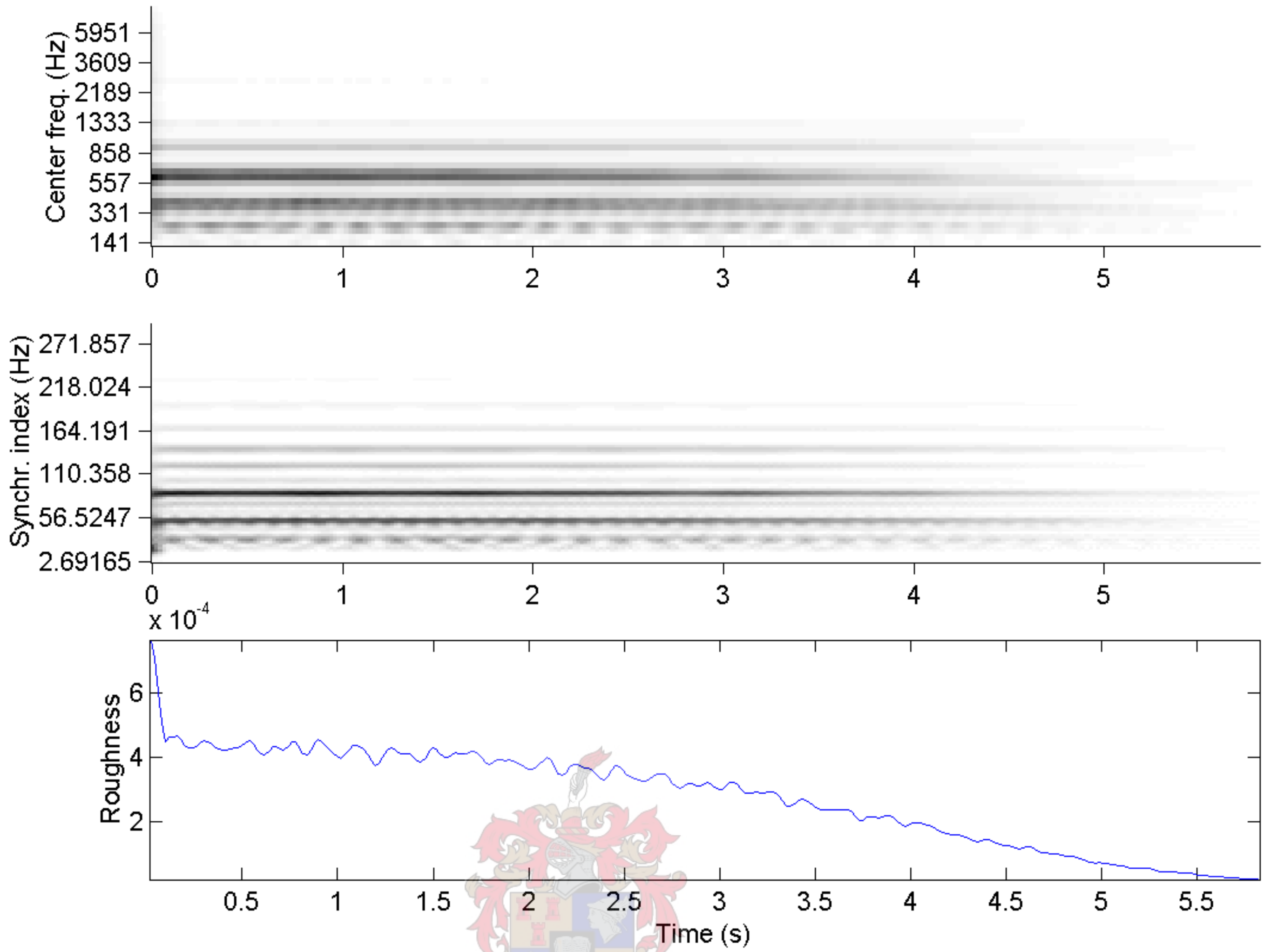
note number	"phi instrument" (fundamental in Hz.)	piano (closest note frequency in Hz.)	note on piano
1	88.5	87.3	F2
2	143.2	138.6	C#3
3	197.3	196.0	G3
4	319.2	311.1	D#4
5	516.6	493.9	B4
6	835.8	830.6	G#5
7	1151.9	1108.7	C#6
8	1587.6	1568.0	G6
9	2188.2	2093.0	C7
10	4156.5	3951.1	B7

Table 14. Random chord example

This chord can be heard as it sounds on a piano in Example 5, and as it sounds on the phi instrument in Example 6. The following is the roughness map for the chord on the piano:



and the following is the roughness curve as taken from the phi instrument:



Results taken from the IPEM toolbox verify that the phi scales' roughness measurements fall within acceptable levels for a scale, having levels comparable to the equal-tempered scales' roughness measurements.

8.8 Future research

8.8.1 Empirical Research

Further research could take the form of testing the phi scale's perceptual qualities with other psychoacoustic models. Zwicker & Fastl (1990), in their "Psychoacoustics, facts and models", present models for testing fluctuation strength, nonlinear distortions, sharpness, loudness and an ambitious all-encompassing model for sensory pleasantness. We have mainly concentrated our efforts on the perceptual aspect of roughness, but other perceptual attributes such as sharpness also have an effect on sensory consonance, albeit not as much as roughness. Testing the sharpness of simultaneous tones of the phi scale in this empirical manner, and comparing results to results taken from other scales, could facilitate in improving the phi scale.

Parncutt, (1989) also proposes an all-encompassing model of sensory pleasantness against which one could test the phi scale. The effect of phase on the perceived sensory pleasantness of simultaneous tones has only recently been incorporated in psychoacoustic models, notably in the IPEM-toolbox used in the above experiments. Further developments and fine-tuning in this regard are expected.

In Leman, Lesaffre & Tanghe's '*Introduction to the IPEM Toolbox for perception based Music Analysis*' (http://www.ipem.rug.ac.be/Toolbox: 2) various other perception based models which warrant attention are mentioned, amongst others Malcolm Slaney's 1993 auditory modelling toolbox (available at http://rv14.ecn.purdue.edu/~malcolm/interval/1998-010/), and John Culling's 'PIPEWAVE' software (available at http://www.cf.ac.uk.psych/CullingJ/pipewave.html) for auditory modelling, which is, however, mostly concerned with speech processing. The 'Centre for the neural basis of hearing' in Essex has also released 'The Development System for Auditory Modelling' (DSAM) (available at http://www.essex.ac.uk/psychology/hearinglab/lutear/) which supports auditory modelling. Roy Patterson's 'Auditory Image Model' (AIM) (available at http://www.mrc-cbu.cam.ac.uk/personal/roy.patterson/aim/) simulates the movement of the basilar membrane in the human cochlea as it responds to sound. These results are then entered as input in the next stage, were the basilar membrane's motion is converted by hair cells into a neural activity pattern in the auditory nerve (Leman, Lesaffre & Tanghe, 2004: 1). Since these programs provide outputs at different stages of the auditory processes, testing our scale against all these models would provide us with more data for evaluating, testing and enhancing our scale.

Attempts can be made to introduce greater intervallic variety and greater pitch number in the phi scale, whilst minimizing the affect it might have on roughness criterion. Since the harmonics of the phi scale overlap, it might be sensible to decrease the dB levels of these harmonics, which would lead to a decrease in perceived roughness, or to skip the second harmonic of each note, modelling our instrument after the clarinet, an instrument which is characterised by the prominence of all of its odd numbered harmonics (http://www.ccnmtl.columbia.edu/services/showcase/harmonic.html).

8.9 ARTISTIC APPLICATIONS

It is the researcher's point of view that the phi scale is best applied in artistic endeavours together with another scale. Preferably, the two scales would occupy two different areas of the frequency spectrum, to avoid the optimized spectral energies of the two scales to interfere with each other. In Example 7, the phi scale is generally applied in the higher frequency range in the presence of melodic content arising from another scale. It is not the intent of the researcher that Example 7 should be regarded as a definitive example of what can be accomplished with the phi scale, for that purpose an accomplished musician's efforts would be required.

It is the researcher's belief that the phi scale can find application in various genres of music, for instance "trance" music and "horror" music. Horror music is often effective when the mid-frequency spectrum is left empty. This creates a great distance between the high frequency and low frequency components, which contribute to an eerie effect. This is an ideal situation for the phi scale, as it allows for the use of dual scale systems, thus increasing the intervallic variety and pitch number available to the composer.

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3. Internet Resources

http://csml.som.ohio-state.edu/Resources/Handbook/index.html#K.

http://csml.som.ohio-state.edu/Resources/Handbook/index.html.

http://dactyl.som.ohio-state.edu/Huron/Publications/huron.voice.leading.html.

http://dactyl.som.ohio-state.edu/Music829B/notes.html.

http://eceserv0.ece.wisc.edu/~sethares/comprog.html.

http://eceserv0.ece.wisc.edu/~sethares/consemi.html#anchor149613.

http://www.ccnmtl.columbia.edu/services/showcase/harmonic.html.

http://www.cechinatrans.demon.co.uk/ctm-psm.html.

http://www.dolmetsch.com/theoryintro.htm.

http://www.grovemusic.com.

http://www.hugohein.com/links/musical.octave.specs.htm.

http://www.ipem.rug.ac.be/Toolbox.

http://www.jimloy.com/algebra/fibo.htm.

http://www.lib.virginia.edu/science/parshall/fibonacc.html.

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html#bees.

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/phi.html#fibratio.

http://www.music-cog.ohio-state.edu/Huron/Talks/Sydney.2002/Dissonance/dissonance.abstract.html.

http://www.wfu.edu/~moran/Cathay_Cafe/G_tar.html.

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/The Golden Section.html.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	54.7	64.2	75.4	88.5	103.9	121.9	143.2	168.1	197.3	231.6	271.9	319.2	374.8	440.0	516.6	606.4
1	64.2	75.4	88.5	103.9	121.9	143.2	168.1	197.3	231.6	271.9	319.2	374.8	440.0	516.6	606.4	711.9
2	75.4	88.5	103.9	121.9	143.2	168.1	197.3	231.6	271.9	319.2	374.8	440.0	516.6	606.4	711.9	835.8
3	88.5	103.9	121.9	143.2	168.1	197.3	231.6	271.9	319.2	374.8	440.0	516.6	606.4	711.9	835.8	981.2
4	103.9	121.9	143.2	168.1	197.3	231.6	271.9	319.2	374.8	440.0	516.6	606.4	711.9	835.8	981.2	1151.9
5	121.9	143.2	168.1	197.3	231.6	271.9	319.2	374.8	440.0	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4
6	143.2	168.1	197.3	231.6	271.9	319.2	374.8	440.0	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6
7	168.1	197.3	231.6	271.9	319.2	374.8	440.0	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9
8	197.3	231.6	271.9	319.2	374.8	440.0	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2
9	231.6	271.9	319.2	374.8	440.0	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9
10	271.9	319.2	374.8	440.0	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8
11	319.2	374.8	440.0	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5
12	374.8	440.0	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5
13	440.0	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7
14	516.6	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7
15	606.4	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4
16	711.9	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5
17	835.8	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2
18	981.2	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9
19	1151.9	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2
20	1352.4	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8
21	1587.6	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2
22	1863.9	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6
23	2188.2	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0
24	2568.9	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1
25	3015.8	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8
26	3540.5	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9
27	4156.5	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3
28	4879.7	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4
29	5728.7	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9
30	6725.4	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5
31	7895.5	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2
32	9269.2	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2	102796.7
33	10881.9	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2	102796.7	120681.8
34	12775.2	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2	102796.7	120681.8	141678.6
35	14997.8	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2	102796.7	120681.8	141678.6	166328.6
36	17607.2	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2	102796.7	120681.8	141678.6	166328.6	195267.3
37	20670.6	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2	102796.7	120681.8	141678.6	166328.6	195267.3	229240.8
38	24267.0	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2	102796.7	120681.8	141678.6	166328.6	195267.3	229240.8	269125.3
39	28489.1	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2	102796.7	120681.8	141678.6	166328.6	195267.3	229240.8	269125.3	315949.1
40	33445.8	39264.9	46096.3	54116.4	63531.9	74585.5	87562.2	102796.7	120681.8	141678.6	166328.6	195267.3	229240.8	269125.3	315949.1	370919.5

Table 15. Phi Scale

Csound Orchestra file

```
     sr = 44100
     kr = 4410
     ksmps = 10
     nchnls = 1

     instr    1

iamp = 20000
isust = 10000

k3   expseg   0.01, 0.1, iamp, 5, 100
k2   linseg   0, 0.01, 1, 5.98, 1, 0.01, 0

k1 = k2 * k3

a1   oscil    k1 * 1, p4, 1
a2   oscil    k1 * 0.25, p5, 1
a3   oscil    k1 * 0.1111, p6, 1
a4   oscil    k1 * 0.0625, p7, 1
a5   oscil    k1 * 0.04, p8, 1
a6   oscil    k1 * 0.02778, p9, 1
a7   oscil    k1 * 0.02041, p10, 1
a8   oscil    k1 * 0.01563, p11, 1
a9   oscil    k1 * 0.01235, p12, 1
a10  oscil    k1 * 0.01, p13, 1
a11  oscil    k1 * 0.00826, p14, 1
a12  oscil    k1 * 0.00694, p15, 1
a13  oscil    k1 * 0.00592, p16, 1
a14  oscil    k1 * 0.0051, p17, 1
a15  oscil    k1 * 0.00444, p18, 1
a16  oscil    k1 * 0.00391, p19, 1


at = a1+a2+a3+a4+a5+a6+a7+a8+a9+a10+a11+a12+a13+a14+a15+a16

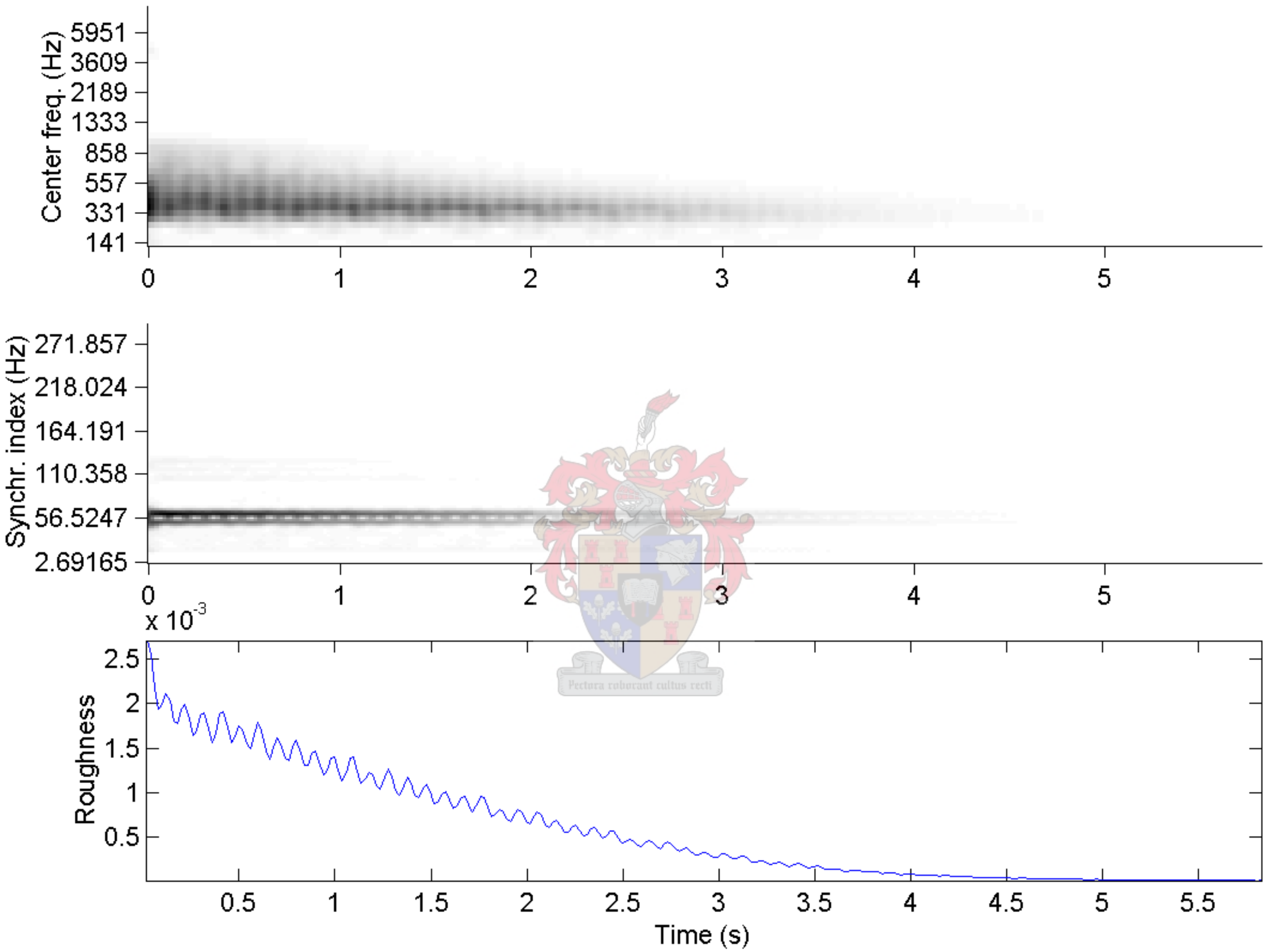
     out  at

endin
```

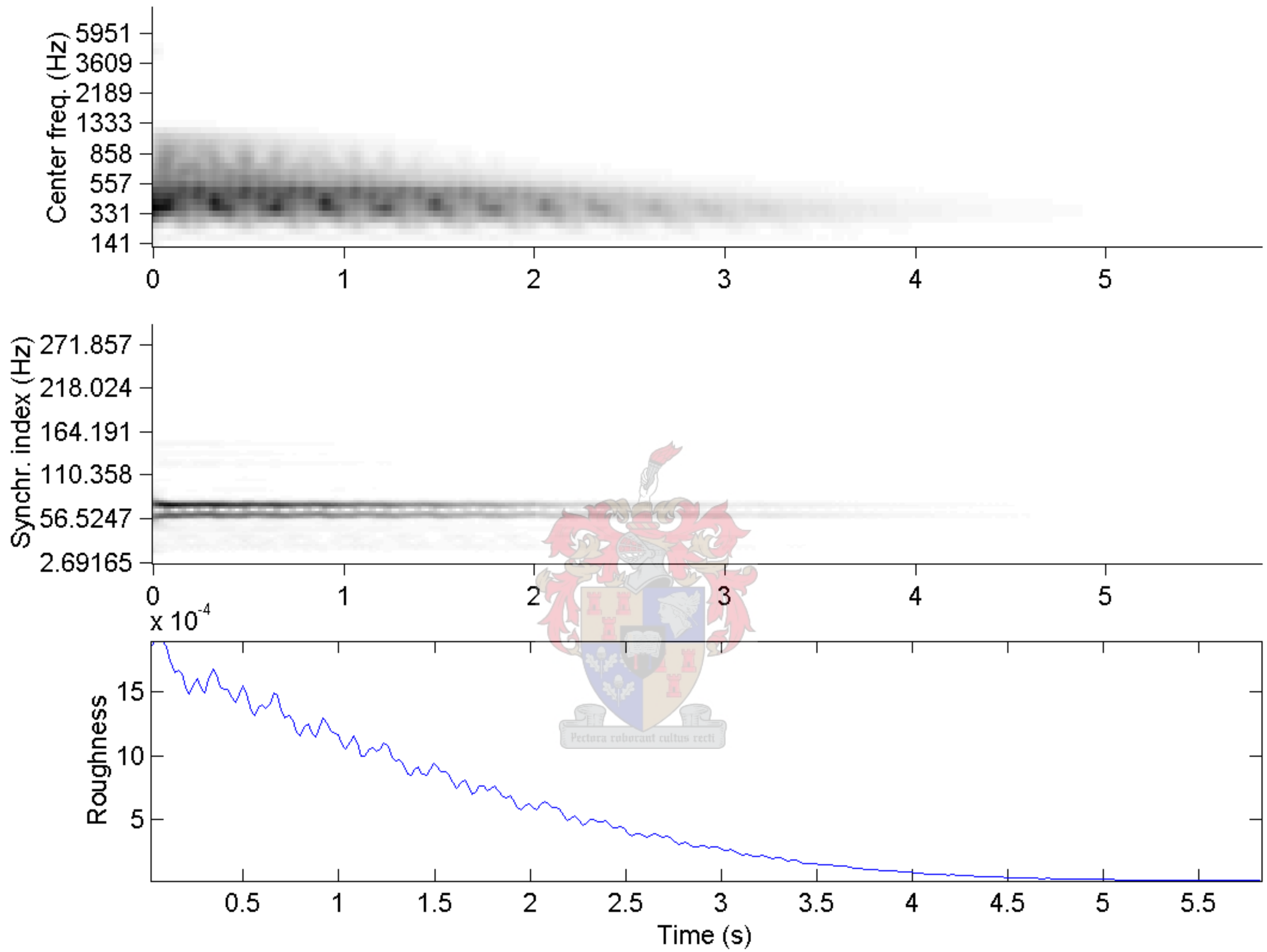
<u>Csound Score file</u>

```
f1 0 4096 10 1

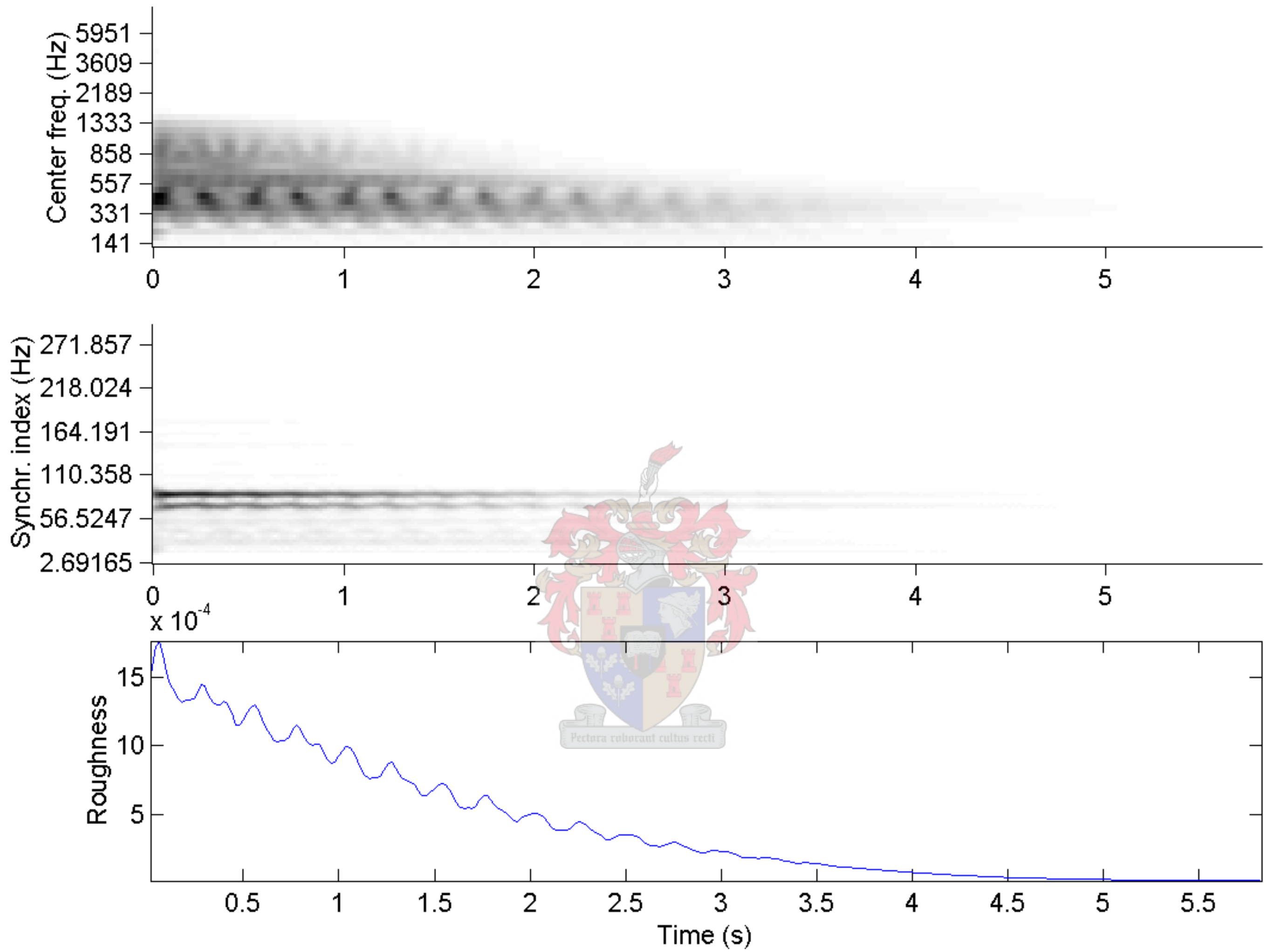
;i1   0  6  54.7     64.2     75.4     88.5     103.9    121.9    143.2    168.1    197.3    231.6    271.9    319.2    374.8    440.0    516.6    606.4
;i1   0  6  64.2     75.4     88.5     103.9    121.9    143.2    168.1    197.3    231.6    271.9    319.2    374.8    440.0    516.6    606.4    711.9
;i1   0  6  75.4     88.5     103.9    121.9    143.2    168.1    197.3    231.6    271.9    319.2    374.8    440.0    516.6    606.4    711.9    835.8
;i1   0  6  88.5     103.9    121.9    143.2    168.1    197.3    231.6    271.9    319.2    374.8    440.0    516.6    606.4    711.9    835.8    981.2
;i1   0  6  103.9    121.9    143.2    168.1    197.3    231.6    271.9    319.2    374.8    440.0    516.6    606.4    711.9    835.8    981.2    1151.9
;i1   0  6  121.9    143.2    168.1    197.3    231.6    271.9    319.2    374.8    440.0    516.6    606.4    711.9    835.8    981.2    1151.9   1352.4
;i1   0  6  143.2    168.1    197.3    231.6    271.9    319.2    374.8    440.0    516.6    606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6
;i1   0  6  168.1    197.3    231.6    271.9    319.2    374.8    440.0    516.6    606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9
;i1   0  6  197.3    231.6    271.9    319.2    374.8    440.0    516.6    606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2
;i1   0  6  231.6    271.9    319.2    374.8    440.0    516.6    606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9
;i1   0  6  271.9    319.2    374.8    440.0    516.6    606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8
;i1   0  6  319.2    374.8    440.0    516.6    606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5
;i1   0  6  374.8    440.0    516.6    606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5
;i1   0  6  440.0    516.6    606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7
;i1   0  6  516.6    606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7
;i1   0  6  606.4    711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4
;i1   0  6  711.9    835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5
;i1   0  6  835.8    981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2
;i1   0  6  981.2    1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9
;i1   0  6  1151.9   1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2
;i1   0  6  1352.4   1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8
;i1   0  6  1587.6   1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2
;i1   0  6  1863.9   2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6
;i1   0  6  2188.2   2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0
;i1   0  6  2568.9   3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1
;i1   0  6  3015.8   3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8
;i1   0  6  3540.5   4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9
;i1   0  6  4156.5   4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3
;i1   0  6  4879.7   5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4
;i1   0  6  5728.7   6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9
;i1   0  6  6725.4   7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5
;i1   0  6  7895.5   9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2
;i1   0  6  9269.2   10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2  102796.7
;i1   0  6  10881.9  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2  102796.7 120681.8
;i1   0  6  12775.2  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2  102796.7 120681.8 141678.6
;i1   0  6  14997.8  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2  102796.7 120681.8 141678.6 166328.6
;i1   0  6  17607.2  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2  102796.7 120681.8 141678.6 166328.6 195267.3
;i1   0  6  20670.6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2  102796.7 120681.8 141678.6 166328.6 195267.3 229240.8
;i1   0  6  24267.0  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2  102796.7 120681.8 141678.6 166328.6 195267.3 229240.8 269125.3
;i1   0  6  28489.1  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2  102796.7 120681.8 141678.6 166328.6 195267.3 229240.8 269125.3 315949.1
;i1   0  6  33445.8  39264.9  46096.3  54116.4  63531.9  74585.5  87562.2  102796.7 120681.8 141678.6 166328.6 195267.3 229240.8 269125.3 315949.1 370919.5
e
```



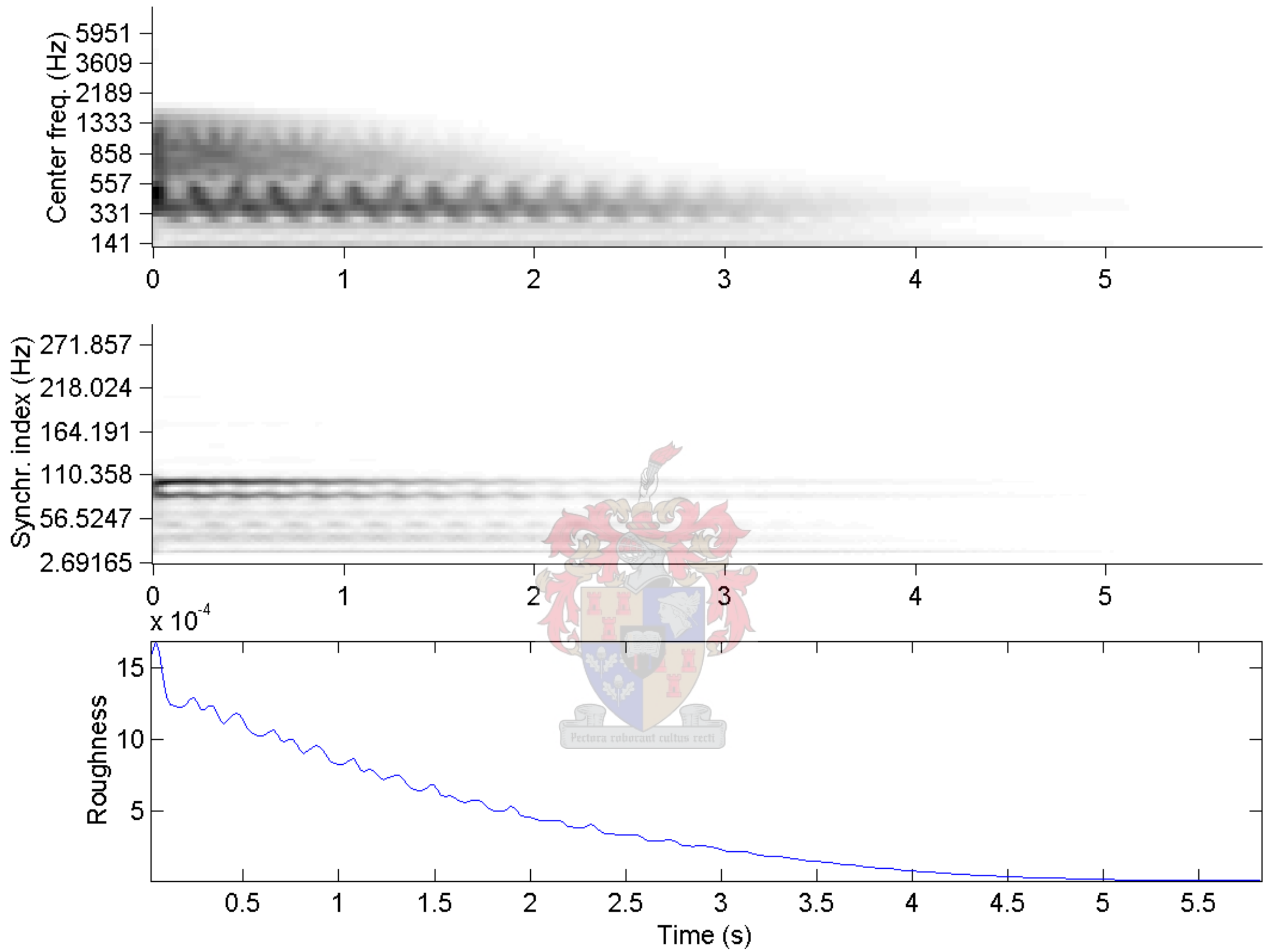
Equal-temperament (et) 54.7 Hz


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
15
10
5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 64.2


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
15
10
5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 75.4


Center freq. (Hz)
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3609
2189
1333
858
557
331
141
0
1
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4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
15
10
5
0.5
1
1.5
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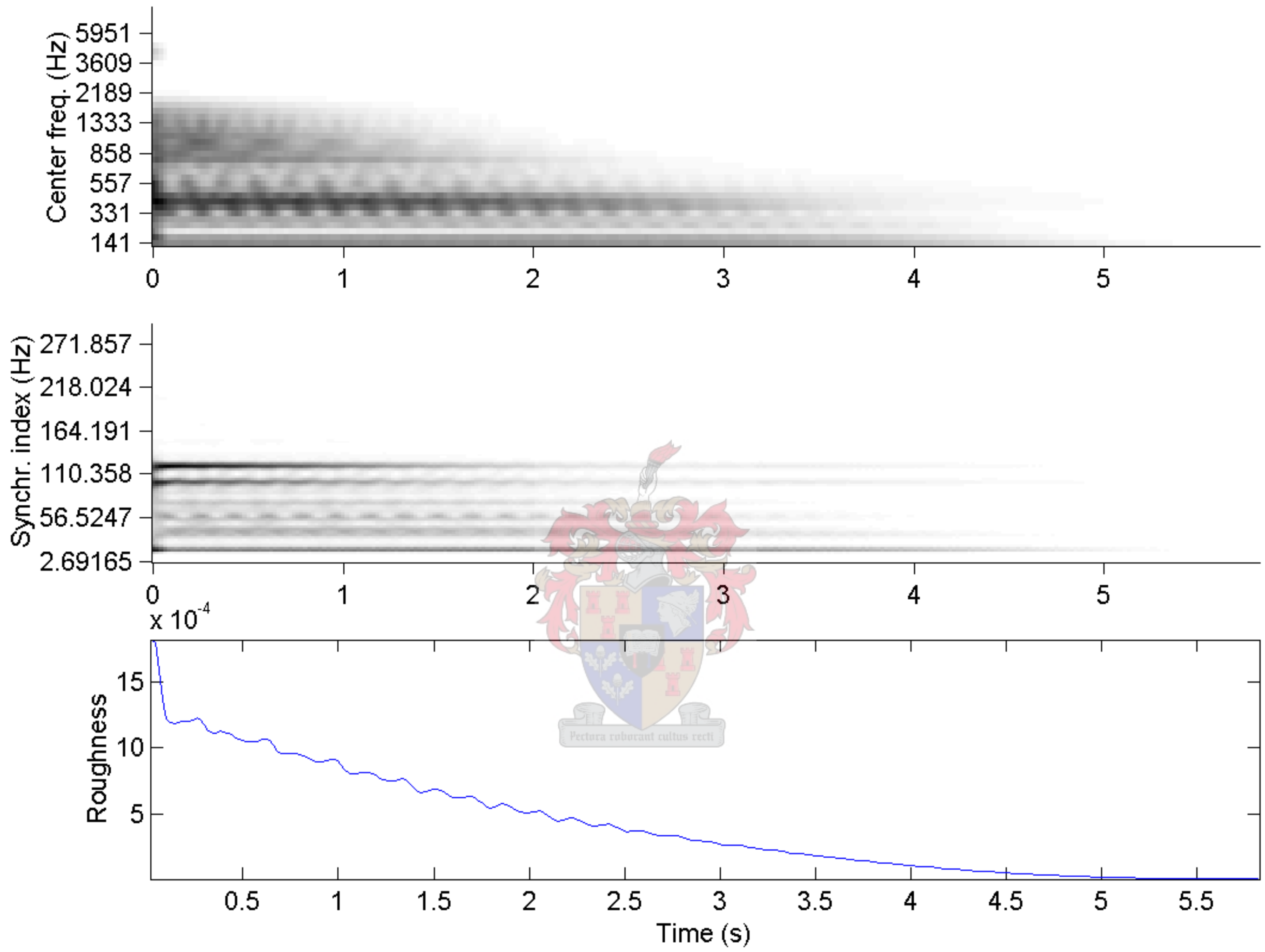
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 88.5



Center freq. (Hz)
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3609
2189
1333
858
557
331
141
0
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2
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5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10-4
Roughness
15
10
5
0.5
1
1.5
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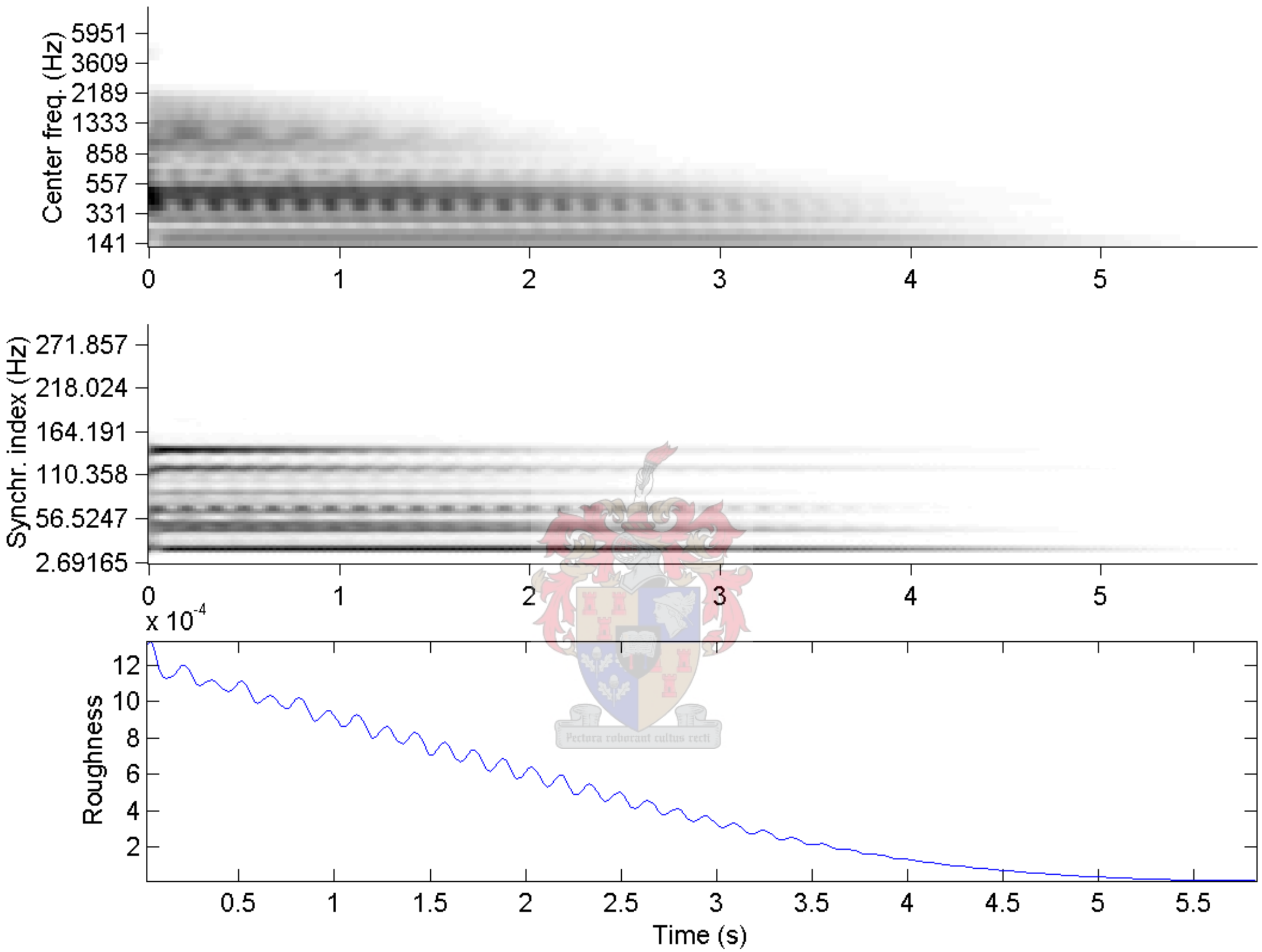
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 103.9



Center freq. (Hz)
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3609
2189
1333
858
557
331
141
0
1
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4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
x 10^-4
Roughness
12
10
8
6
4
2
0.5
1
1.5
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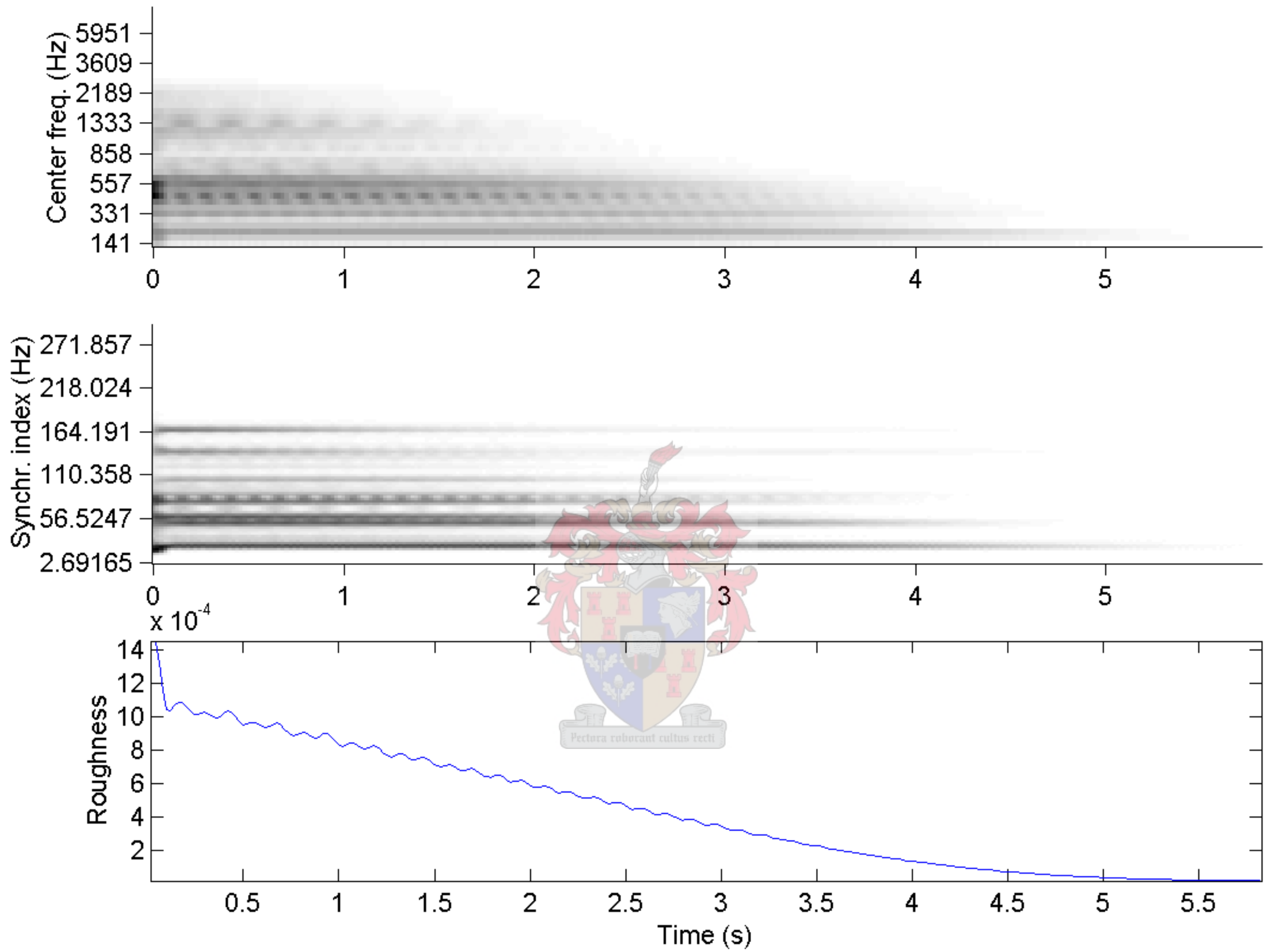
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 121.9


Center freq. (Hz)
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3609
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1333
858
557
331
141
0
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
14
12
10
8
6
4
2
0.5
1
1.5
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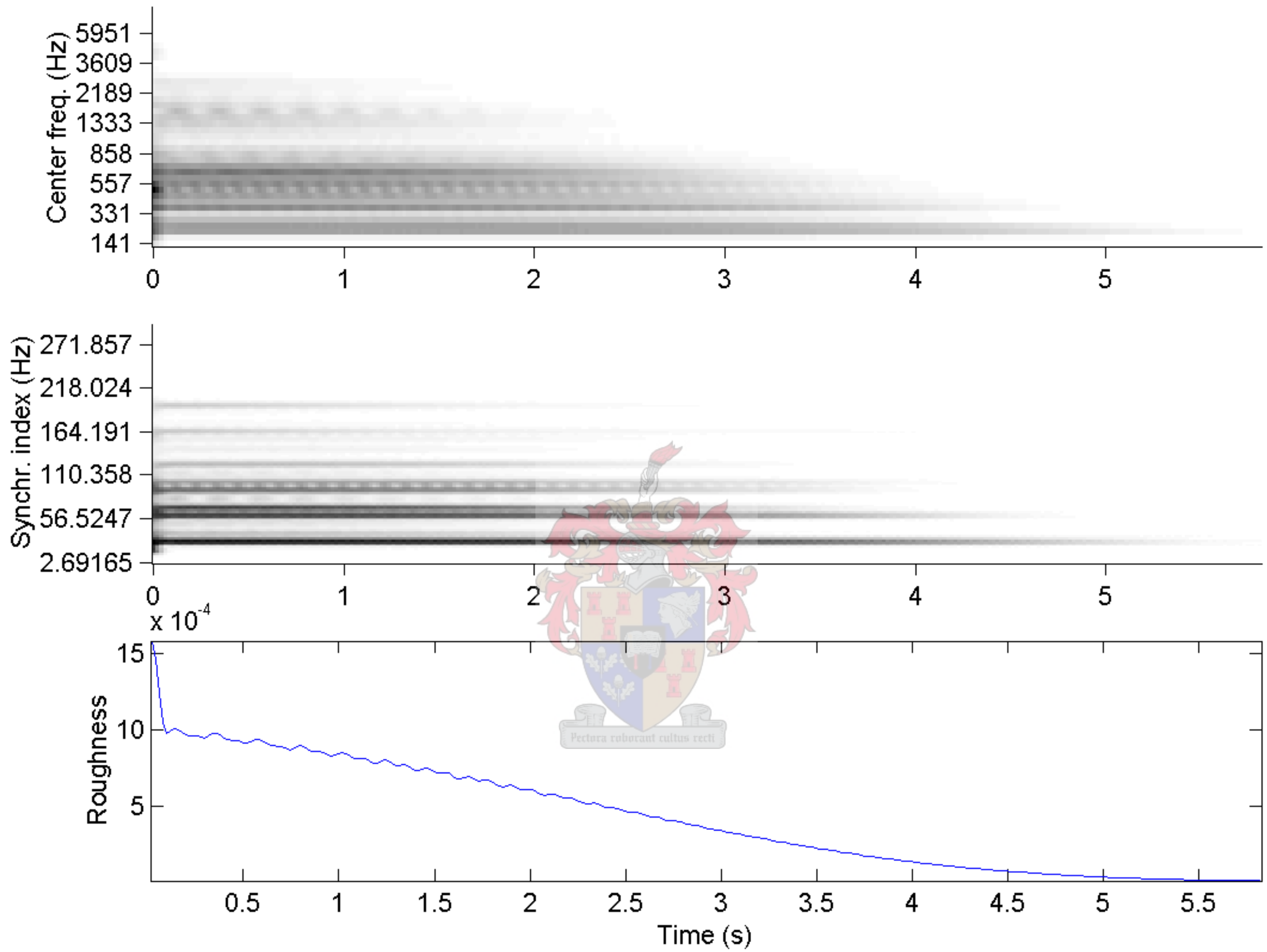
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 143.2


Center freq. (Hz)
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3609
2189
1333
858
557
331
141
0
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4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
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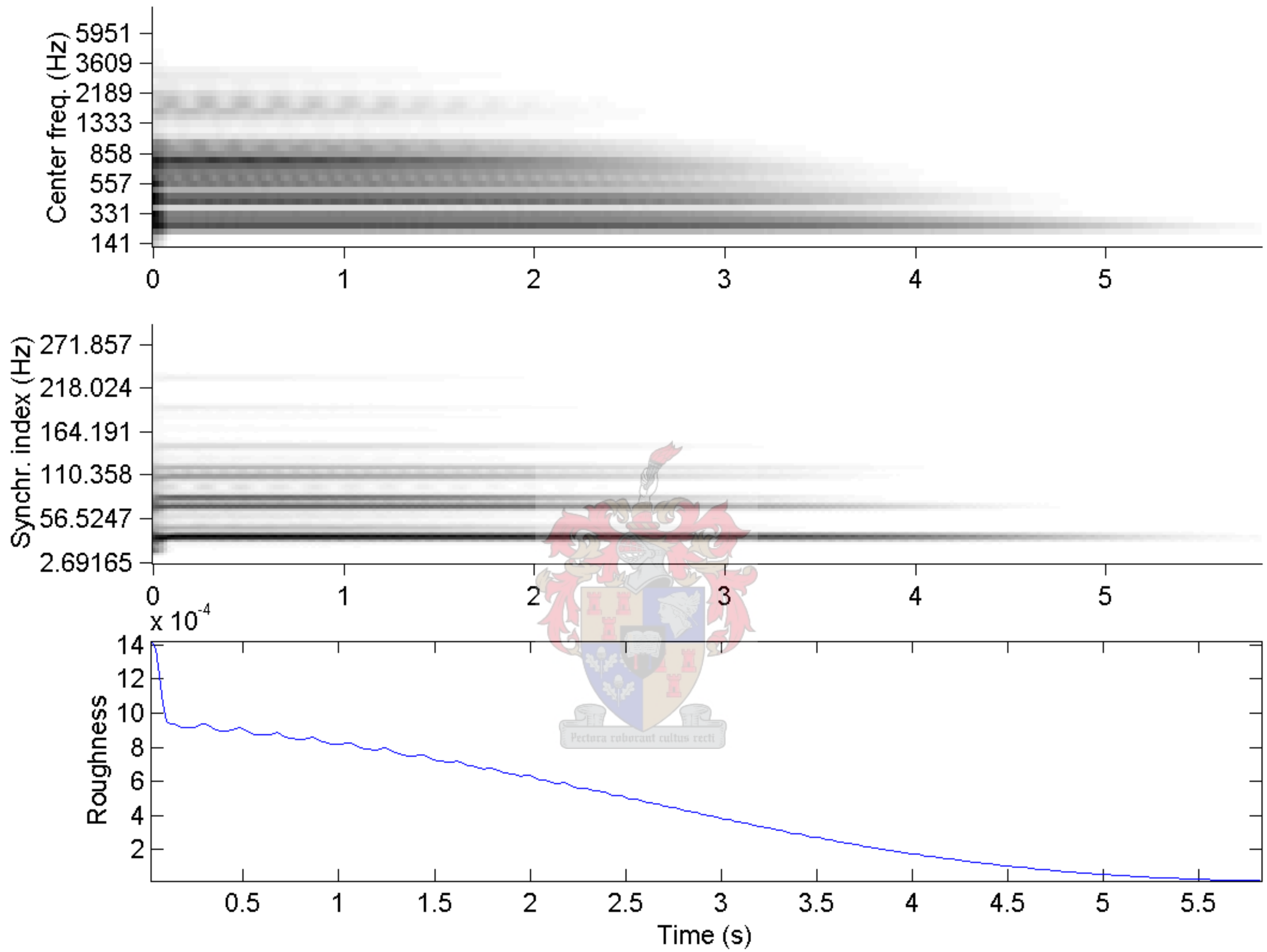
3
4
5
x 10^-4
15
10
5
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 168.1


Center freq. (Hz)
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3609
2189
1333
858
557
331
141
0
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4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
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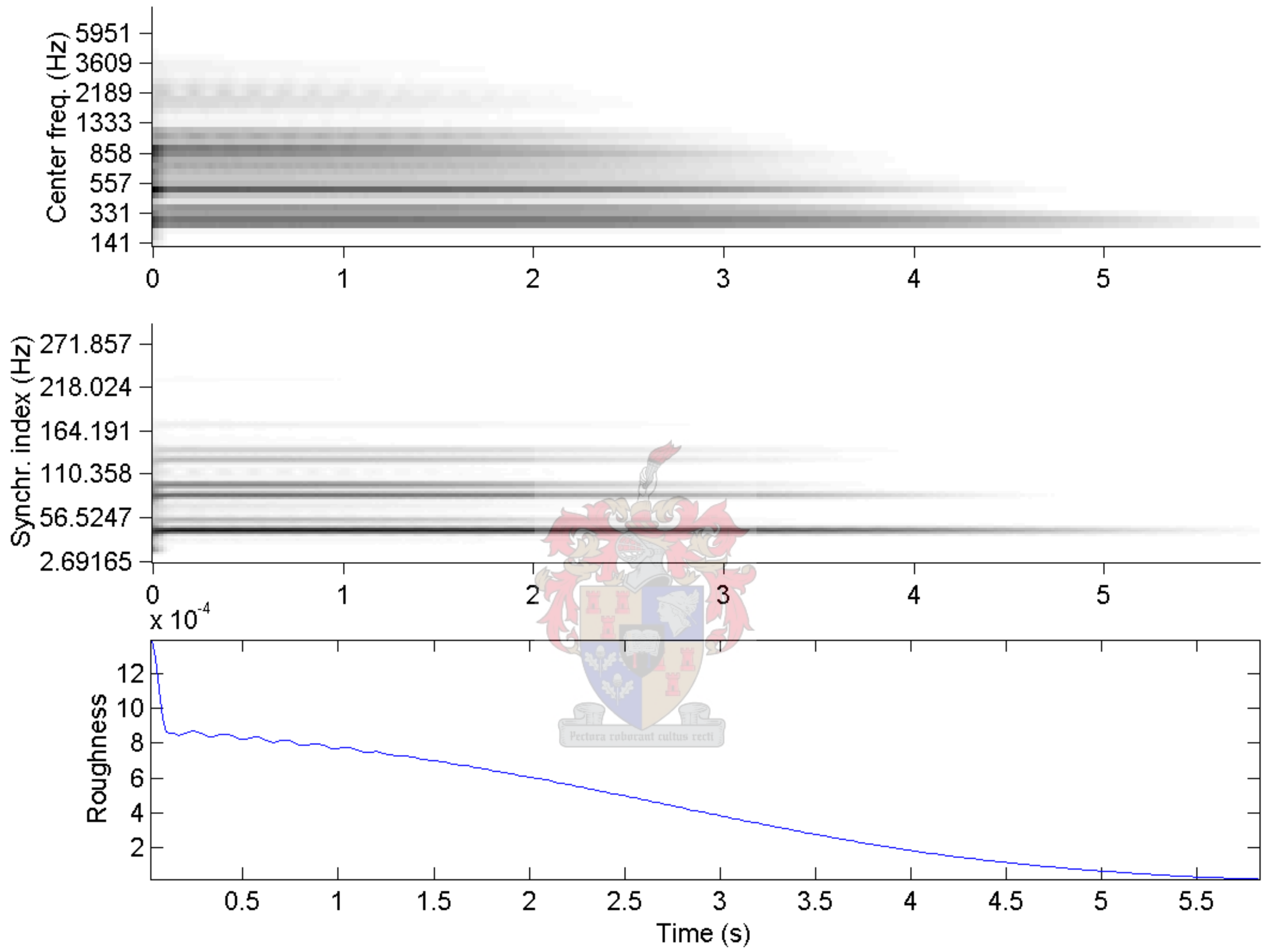
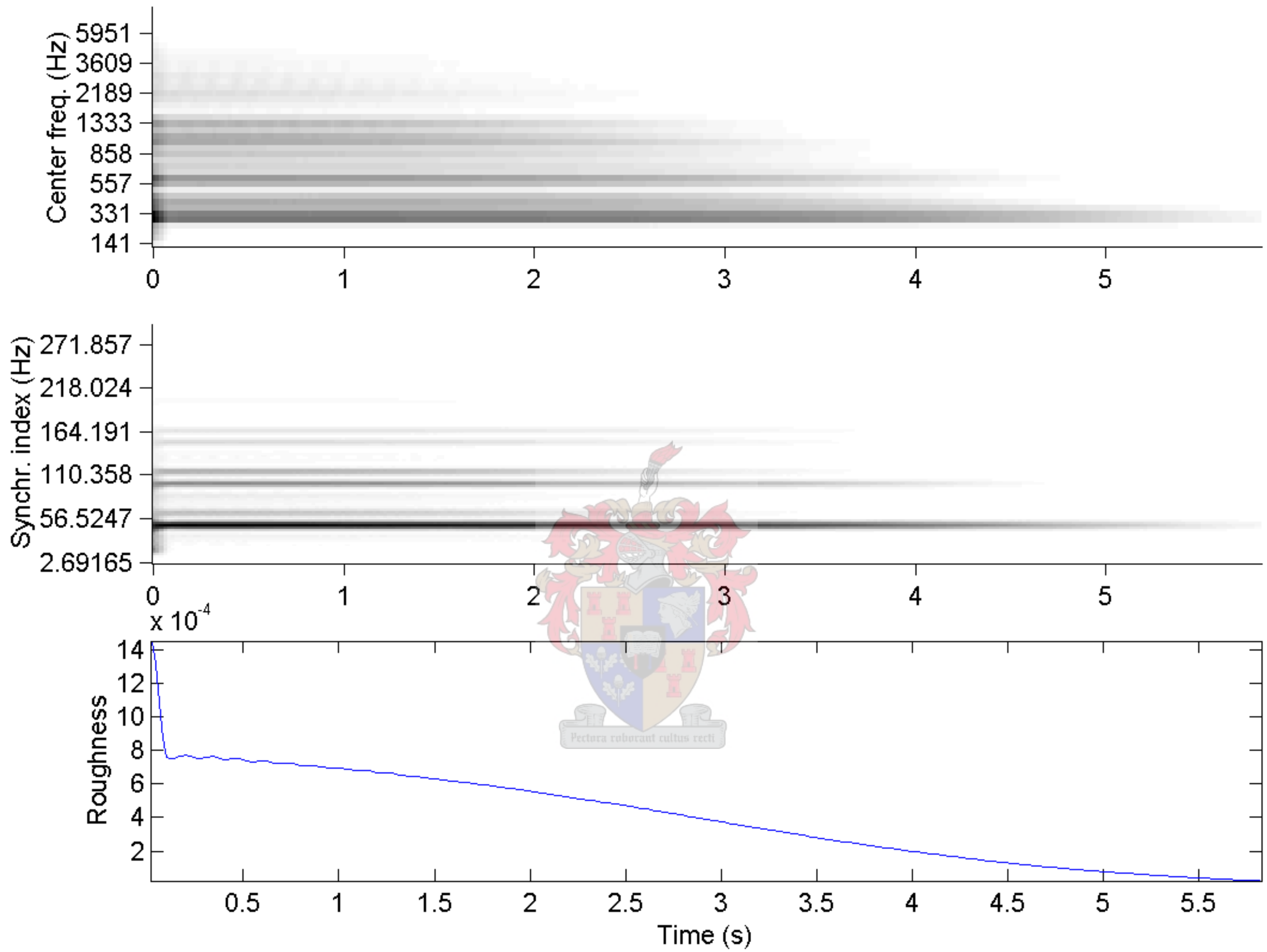
5
x 10-4
14
12
10
8
6
4
2
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 197.3


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0 1 2 3 4 5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0 1 2 3 4 5
x 10^-4
Roughness
12
10
8
6
4
2
0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5
Time (s)

et 231.6



Center freq. (Hz)
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3609
2189
1333
858
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331
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
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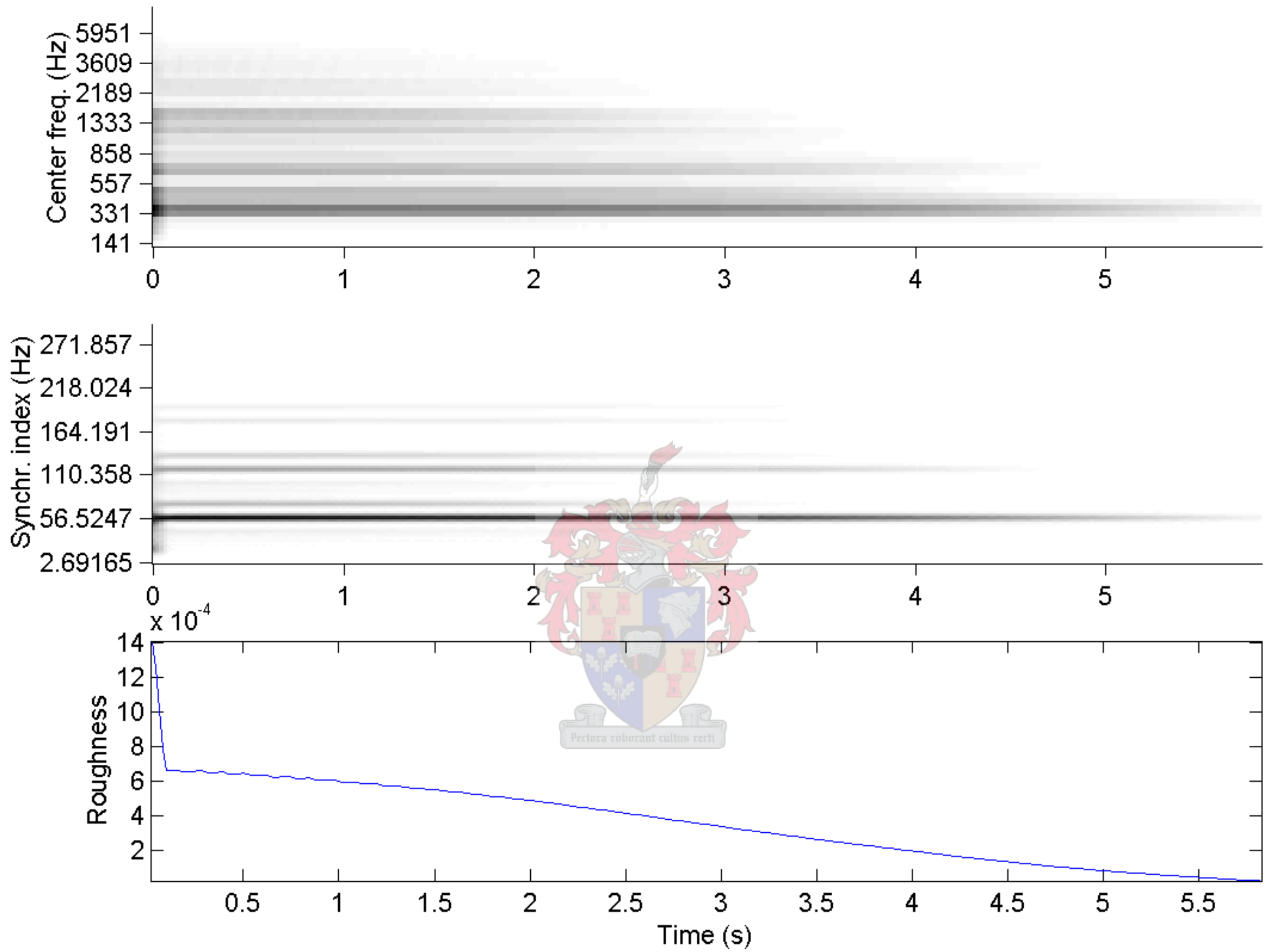
4
5
x 10^-4
Roughness
14
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 271.9


Center freq. (Hz)
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3609
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1333
858
557
331
141
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5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
x 10^-4
Roughness
14
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2
0.5
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1.5
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2.5
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3.5
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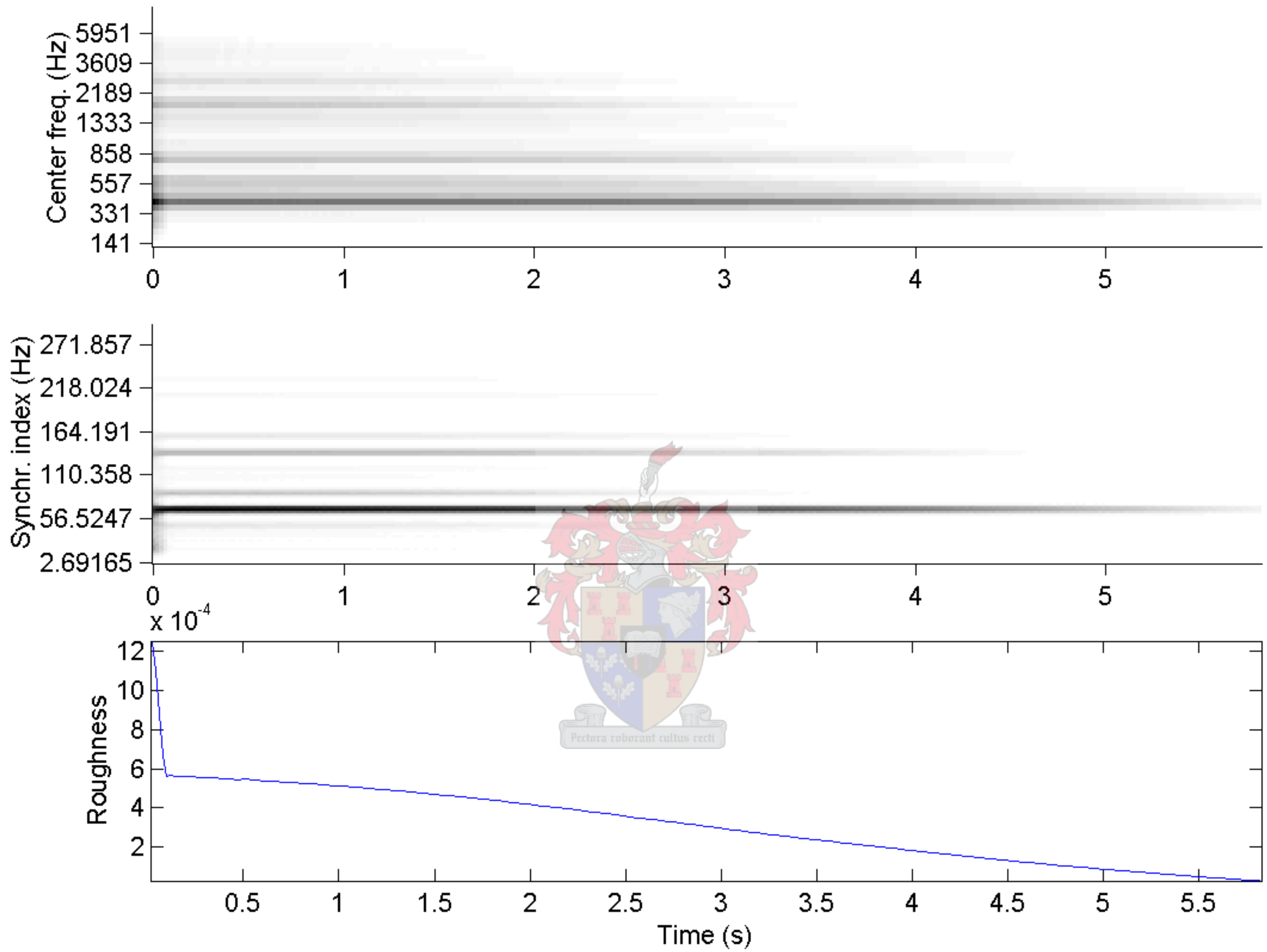
4.5
5
5.5
Time (s)

et 319.2


Center freq. (Hz)
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3609
2189
1333
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331
141
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1
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4
5
Synchr. index (Hz)
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218.024
164.191
110.358
56.5247
2.69165
0
1
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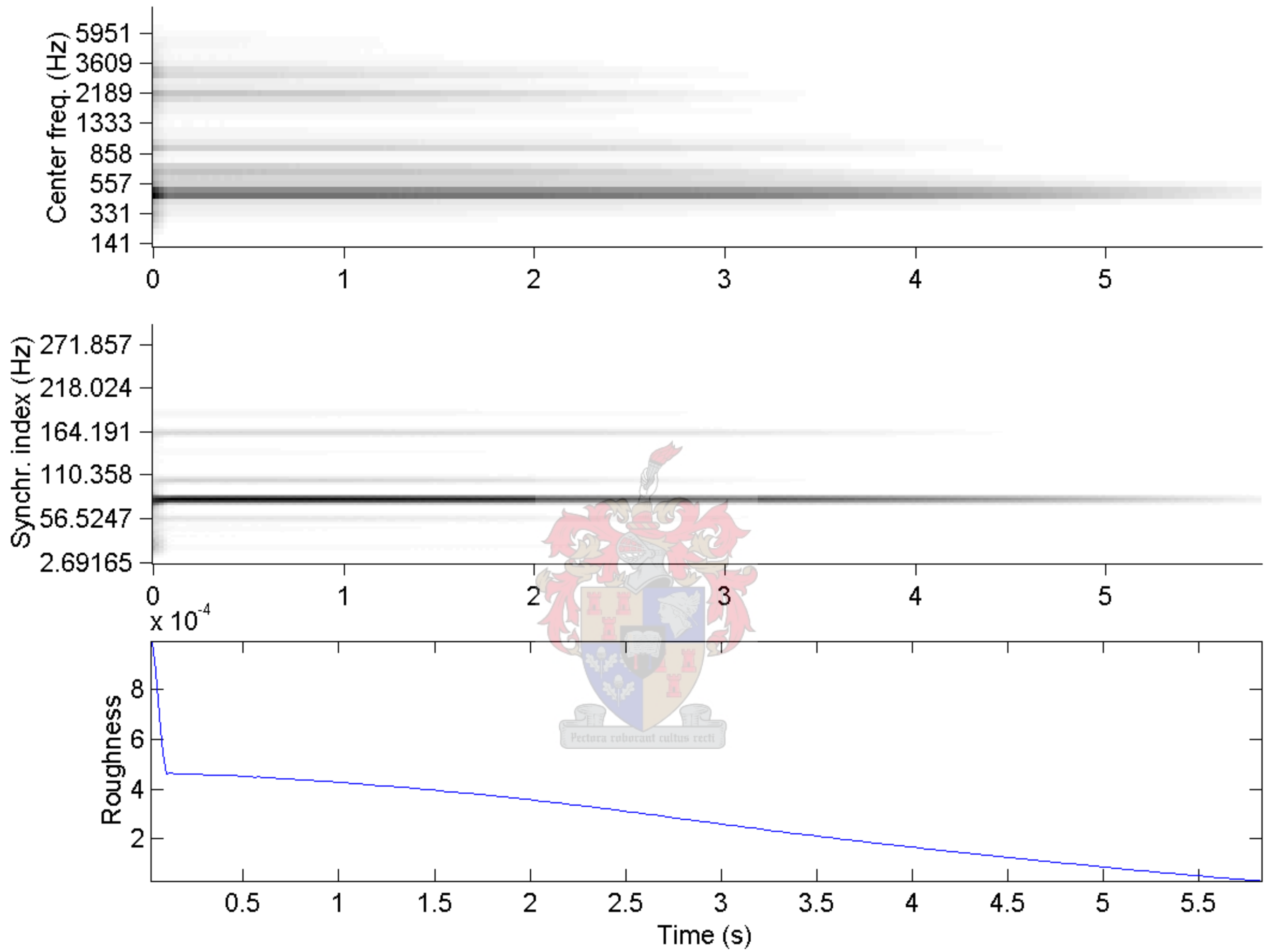
3
4
5
x 10^-4
12
10
8
6
4
2
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 374.8


Center freq. (Hz)
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3609
2189
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331
141
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
x 10^-4
Roughness
8
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0.5
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1.5
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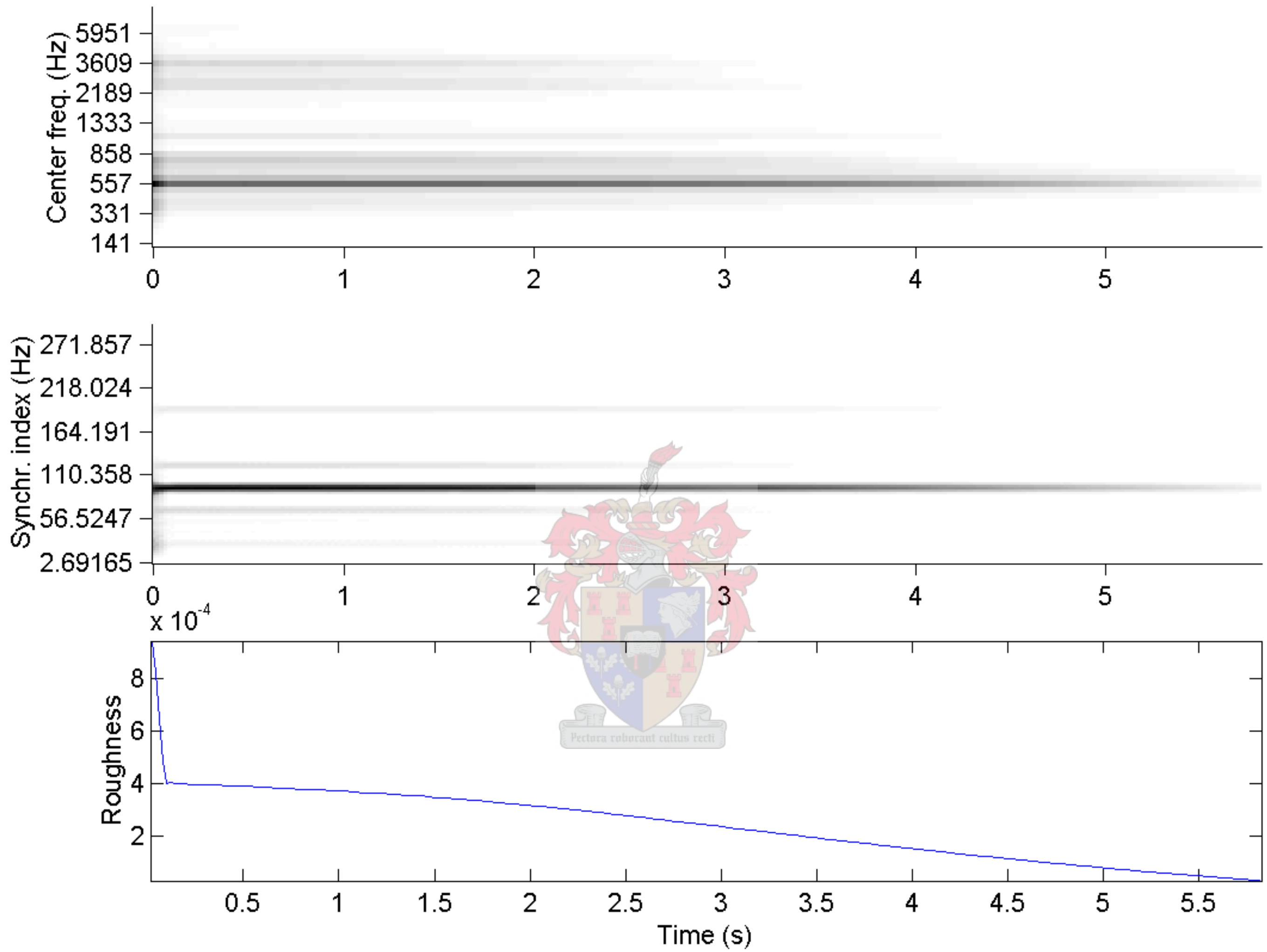
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 440.0


Center freq. (Hz)
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858
557
331
141
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Synchr. index (Hz)
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164.191
110.358
56.5247
2.69165
0
1
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4
5
x 10^-4
Roughness
8
6
4
2
0.5
1
1.5
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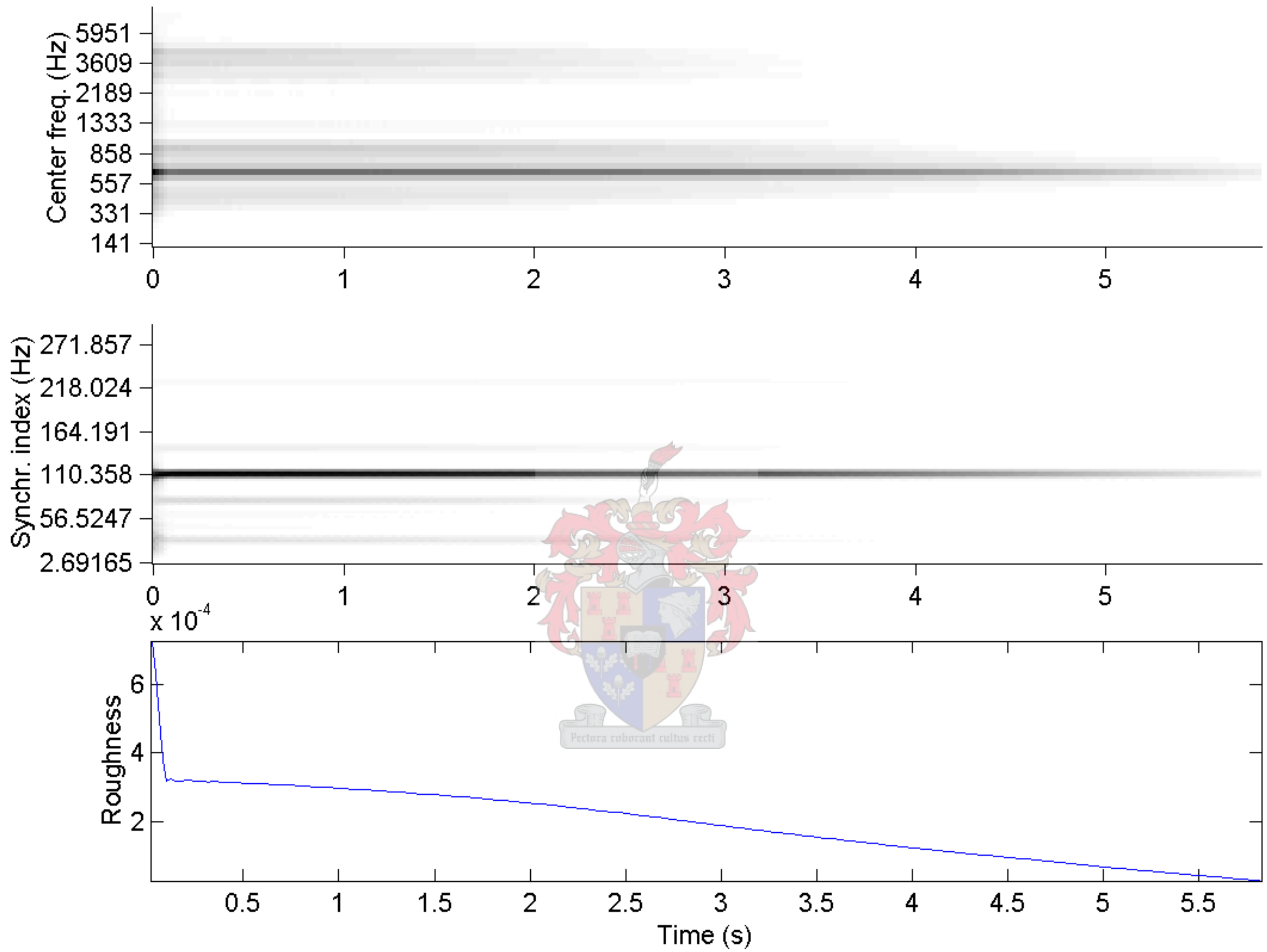
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 516.6



Center freq. (Hz)
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3609
2189
1333
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
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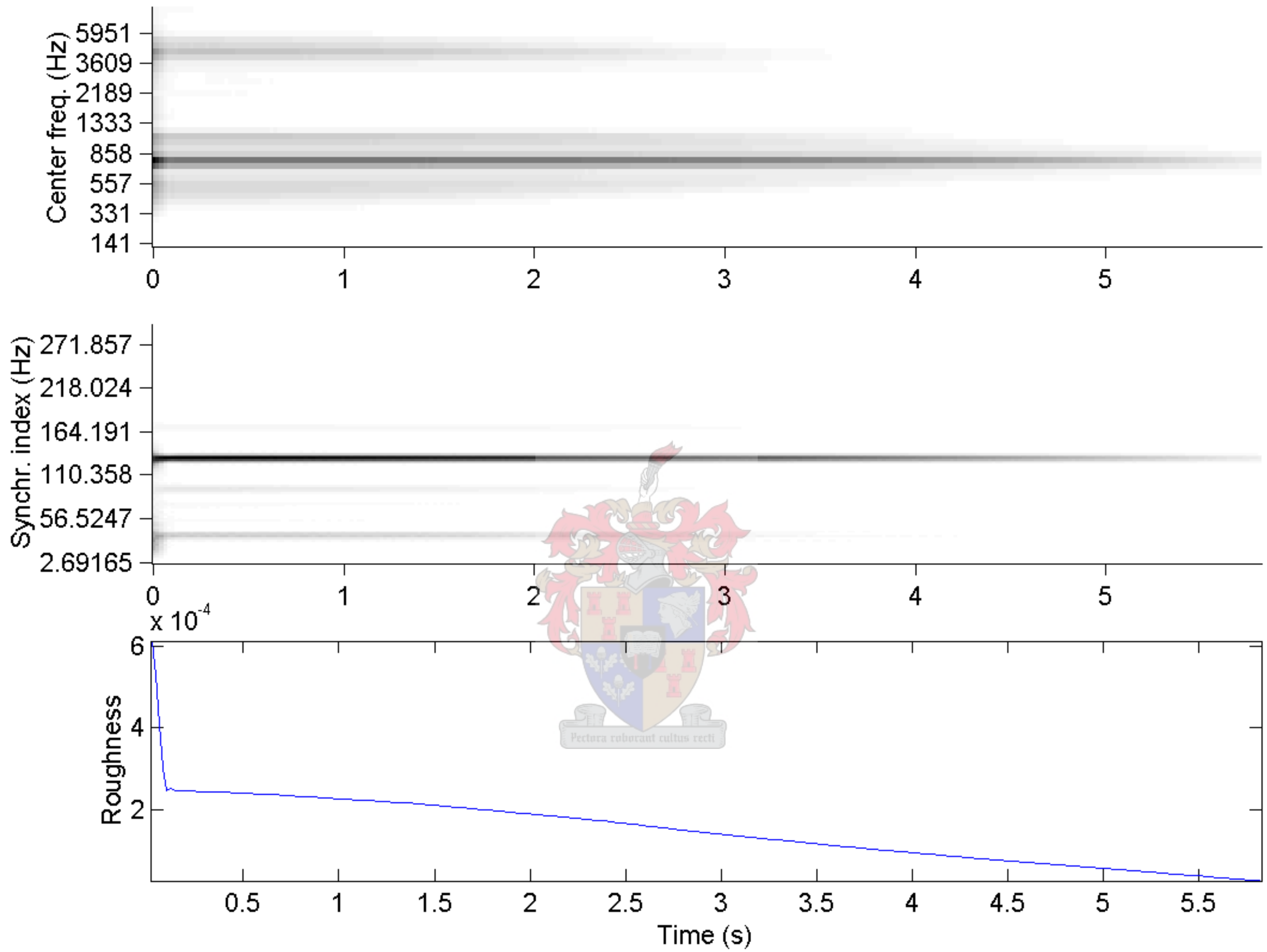
x 10^-4
Roughness
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 606.4


Center freq. (Hz)
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858
557
331
141
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4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
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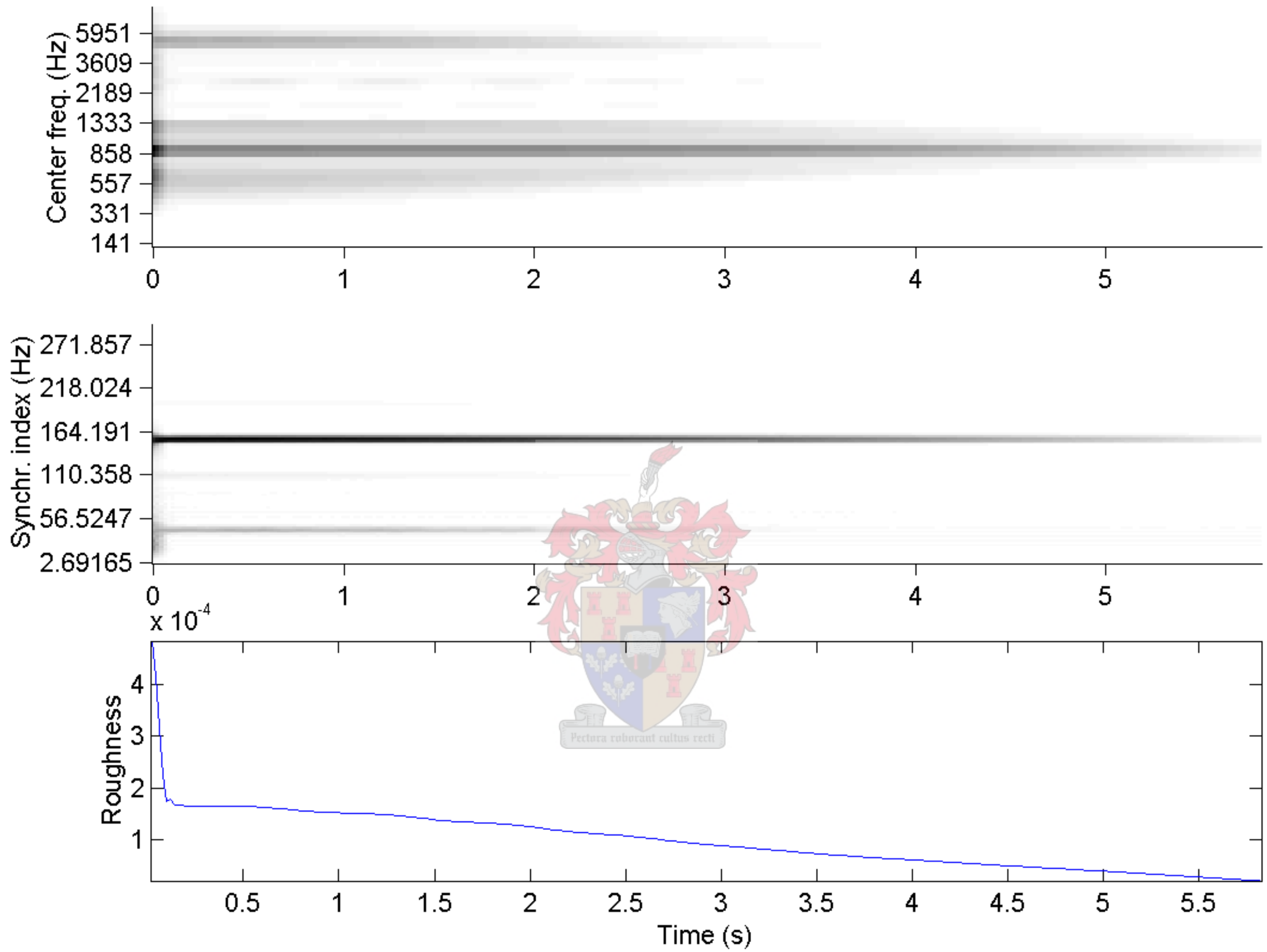
4
5
x 10^-4
6
4
2
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 711.9



Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
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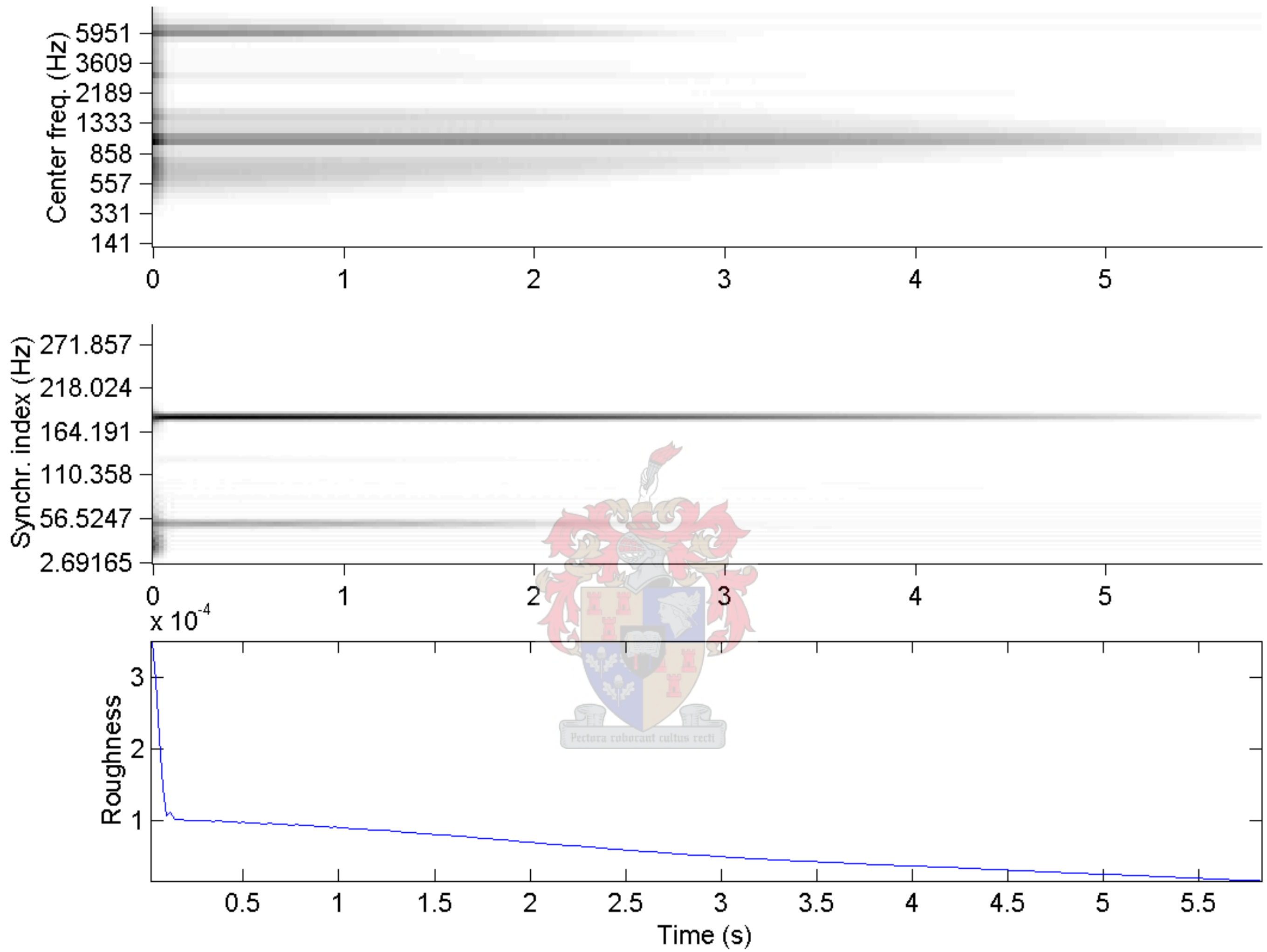
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
x 10^-4
Roughness
4
3
2
1
0.5
1.5
2.5
3.5
4.5
5.5
Time (s)

et 835.8


Center freq. (Hz)
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
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x 10^-4
Roughness
3
2
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0.5
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1.5
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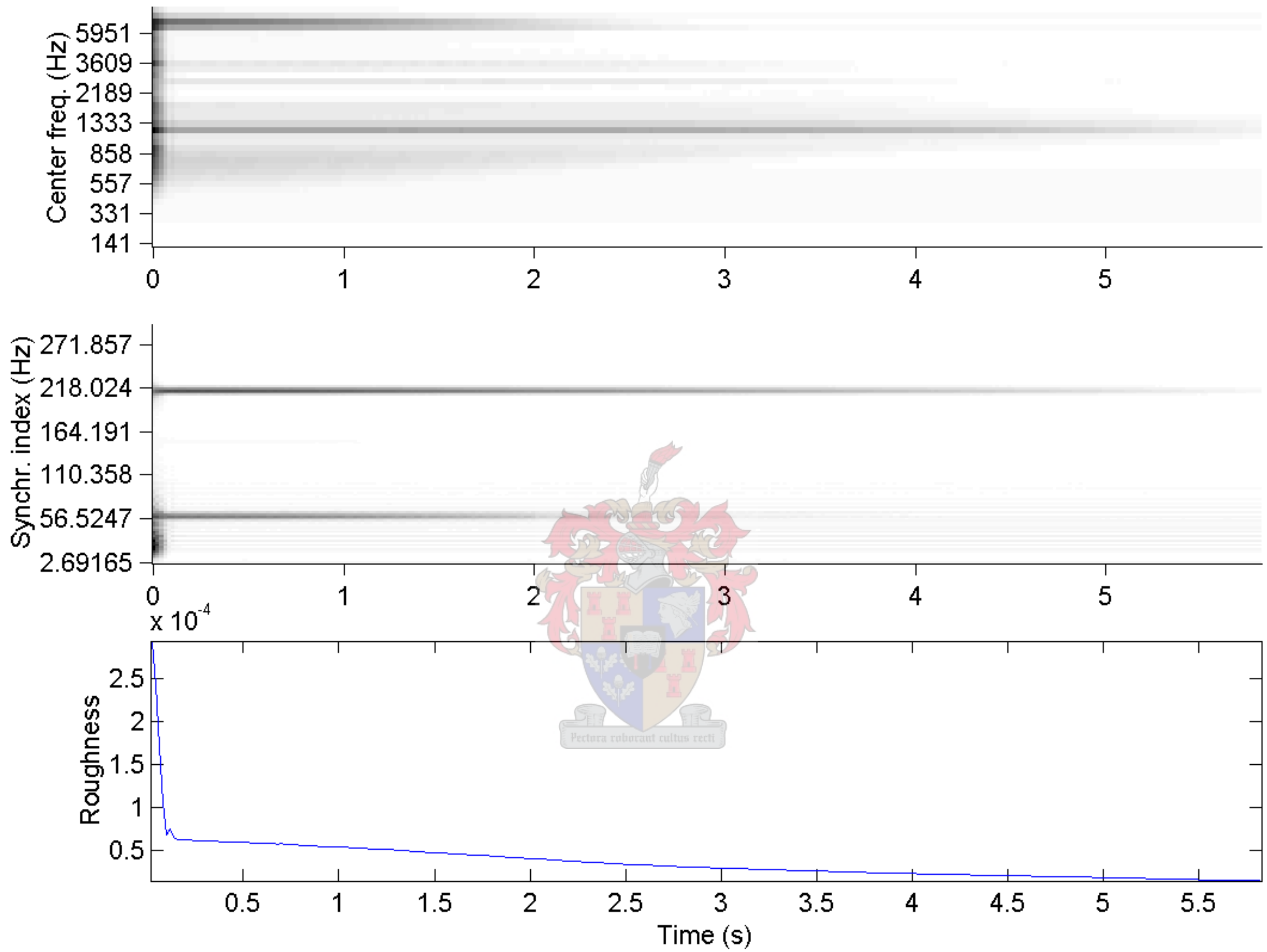
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 981.2


Center freq. (Hz)
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3609
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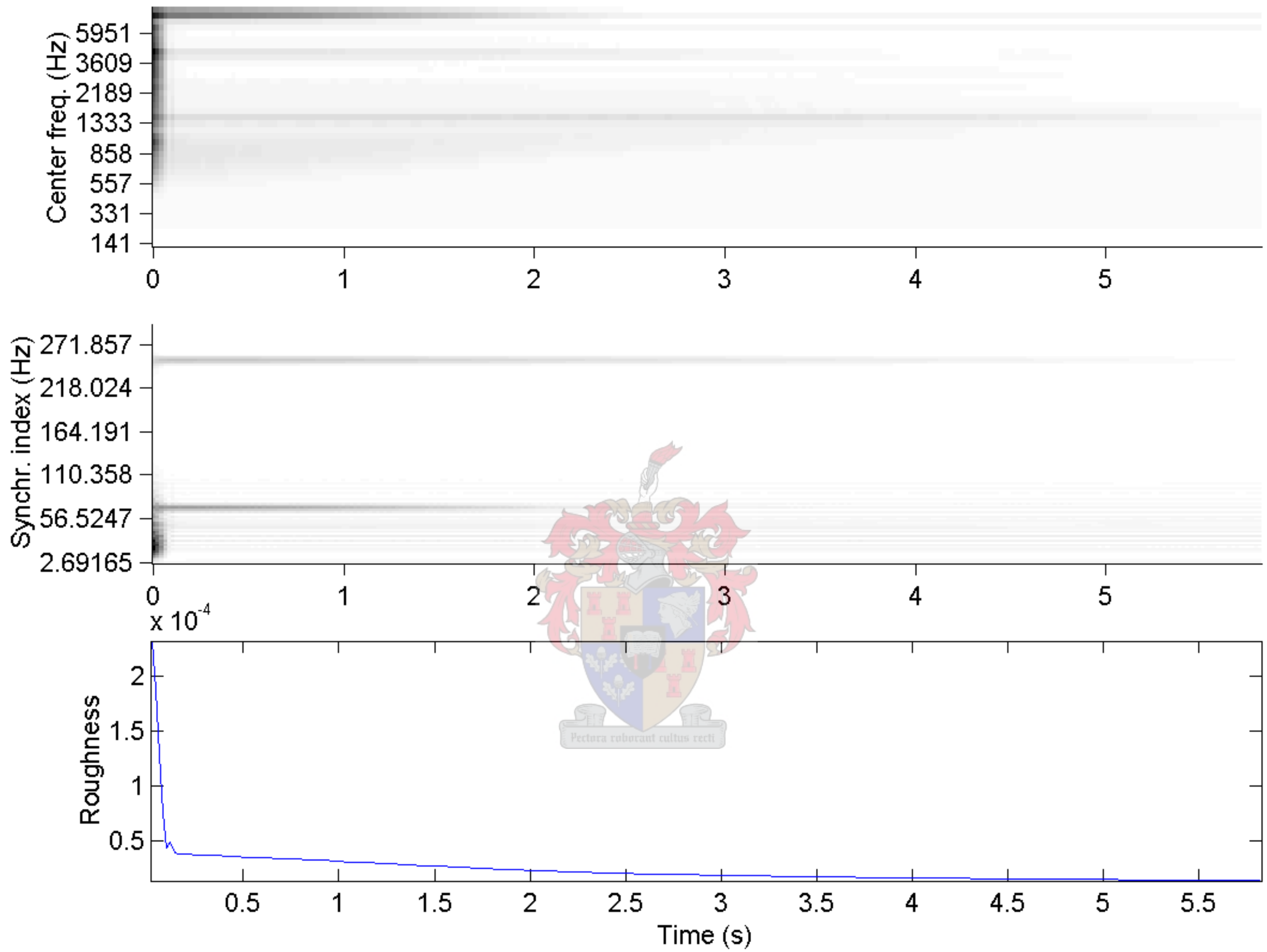
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
x 10^-4
Roughness
2.5
2
1.5
1
0.5
Time (s)

et 1151.9



Center freq. (Hz)
5951
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Synchr. index (Hz)
271.857
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164.191
110.358
56.5247
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x 10^-4
Roughness
2
1.5
1
0.5
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1.5
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2.5
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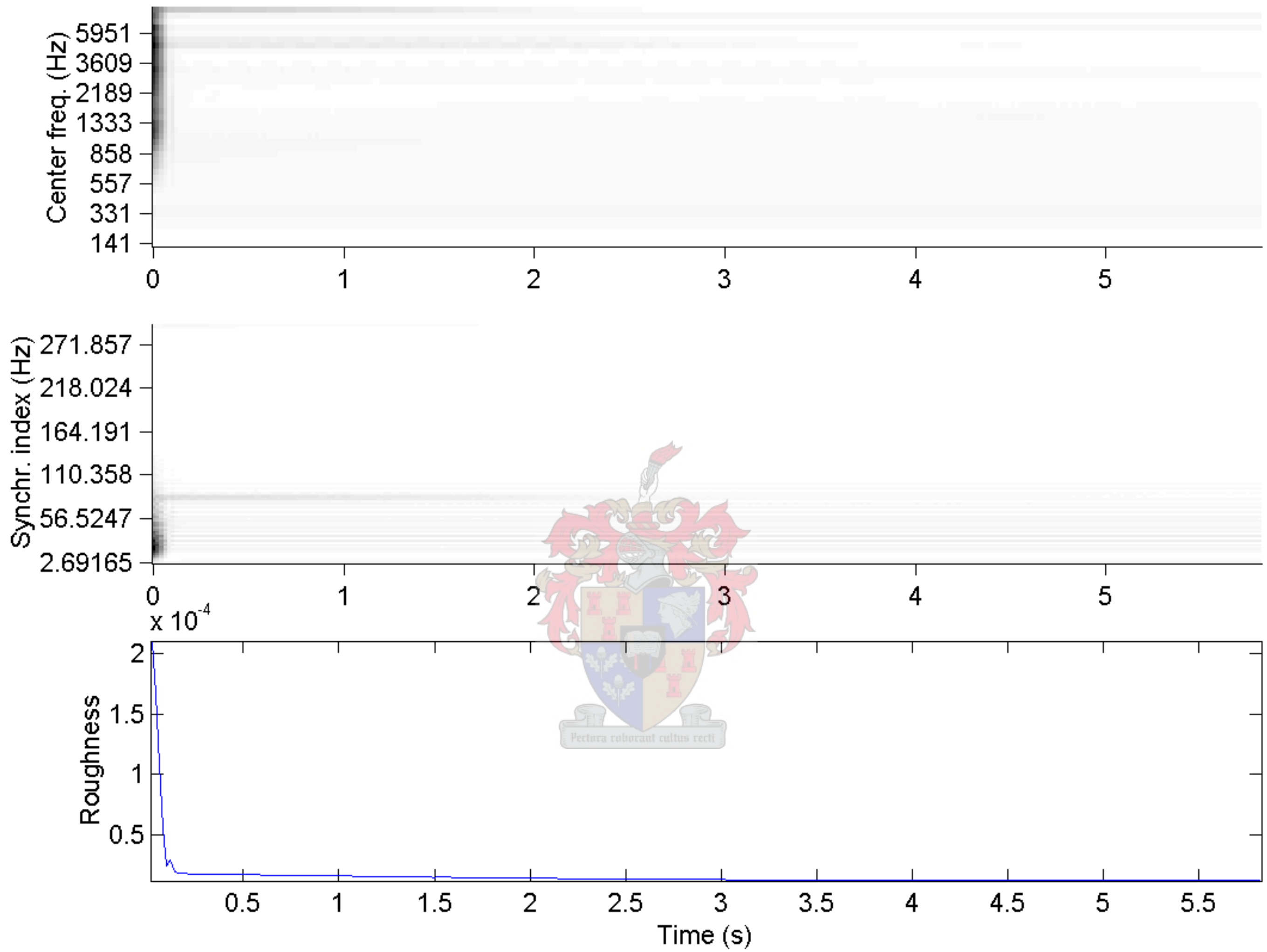
3.5
4
4.5
5
5.5
Time (s)

et 1352.4



Center freq. (Hz)
5951
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Synchr. index (Hz)
271.857
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164.191
110.358
56.5247
2.69165
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x 10^-4
2
1.5
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0.5
Roughness
0.5
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1.5
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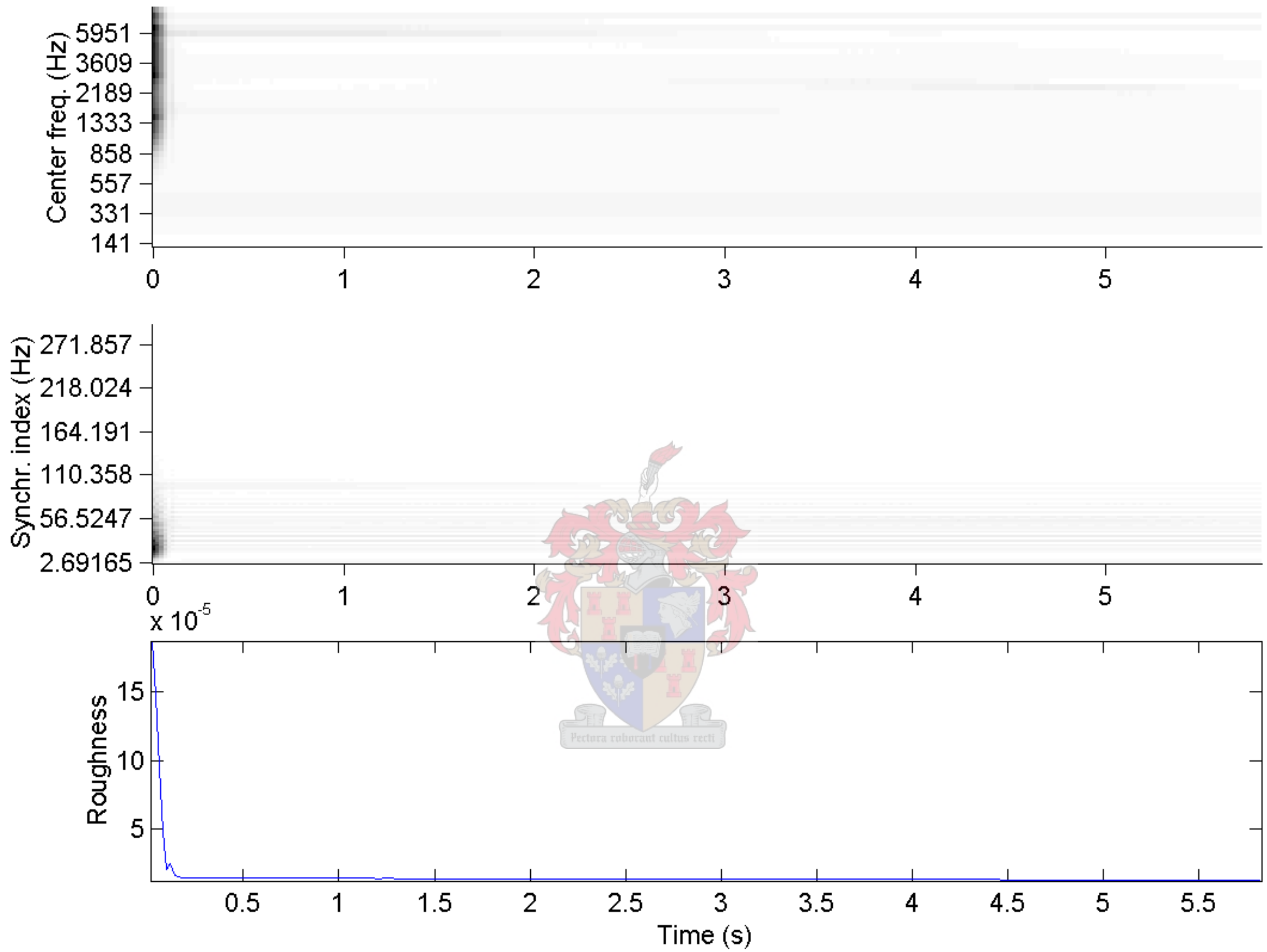
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 1587.6



Center freq. (Hz)
5951
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
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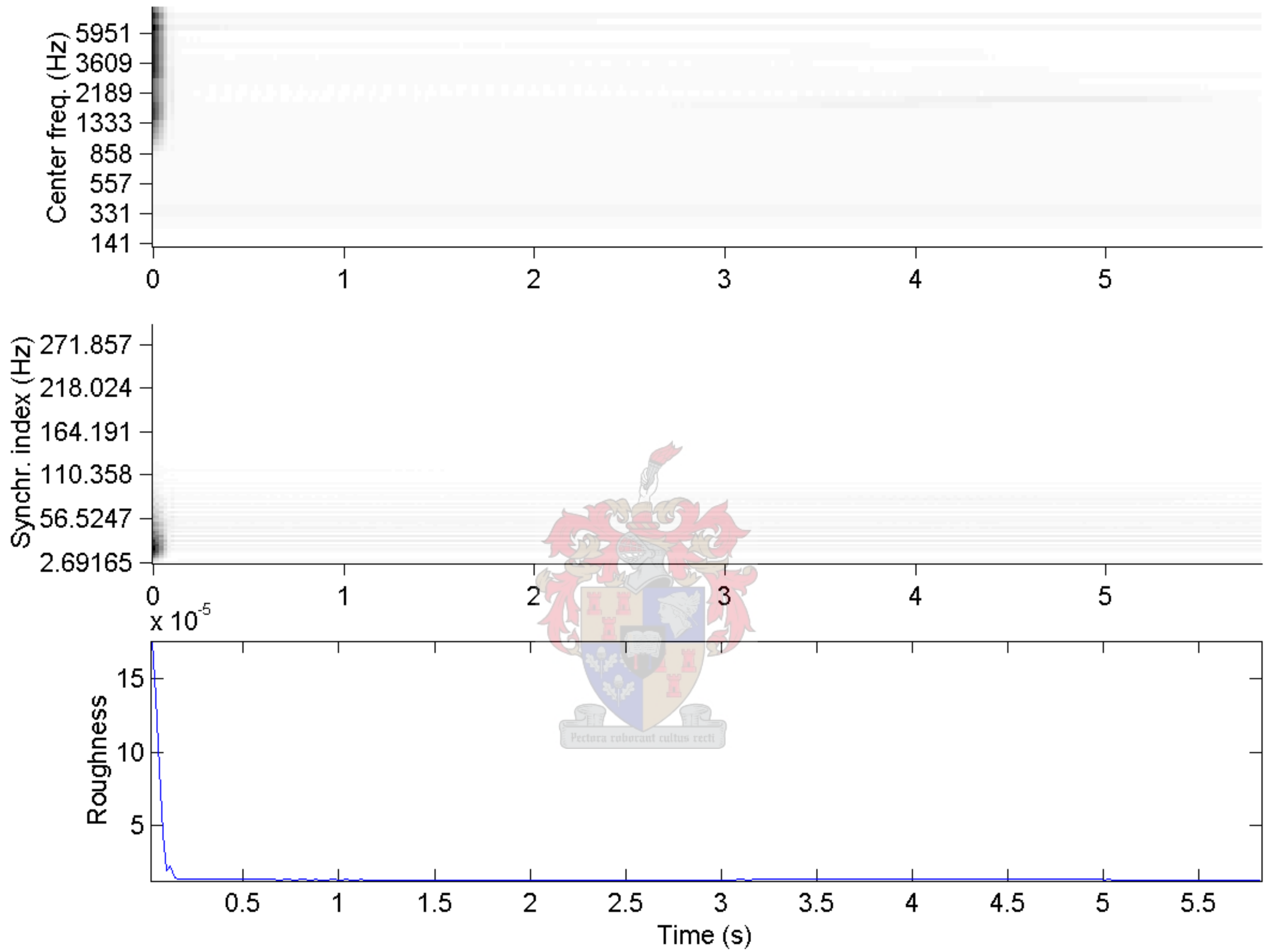
5
x 10-5
Roughness
15
10
5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 1863.9


Center freq. (Hz)
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3609
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Synchr. index (Hz)
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218.024
164.191
110.358
56.5247
2.69165
0
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x 10^-5
Roughness
15
10
5
0.5
1
1.5
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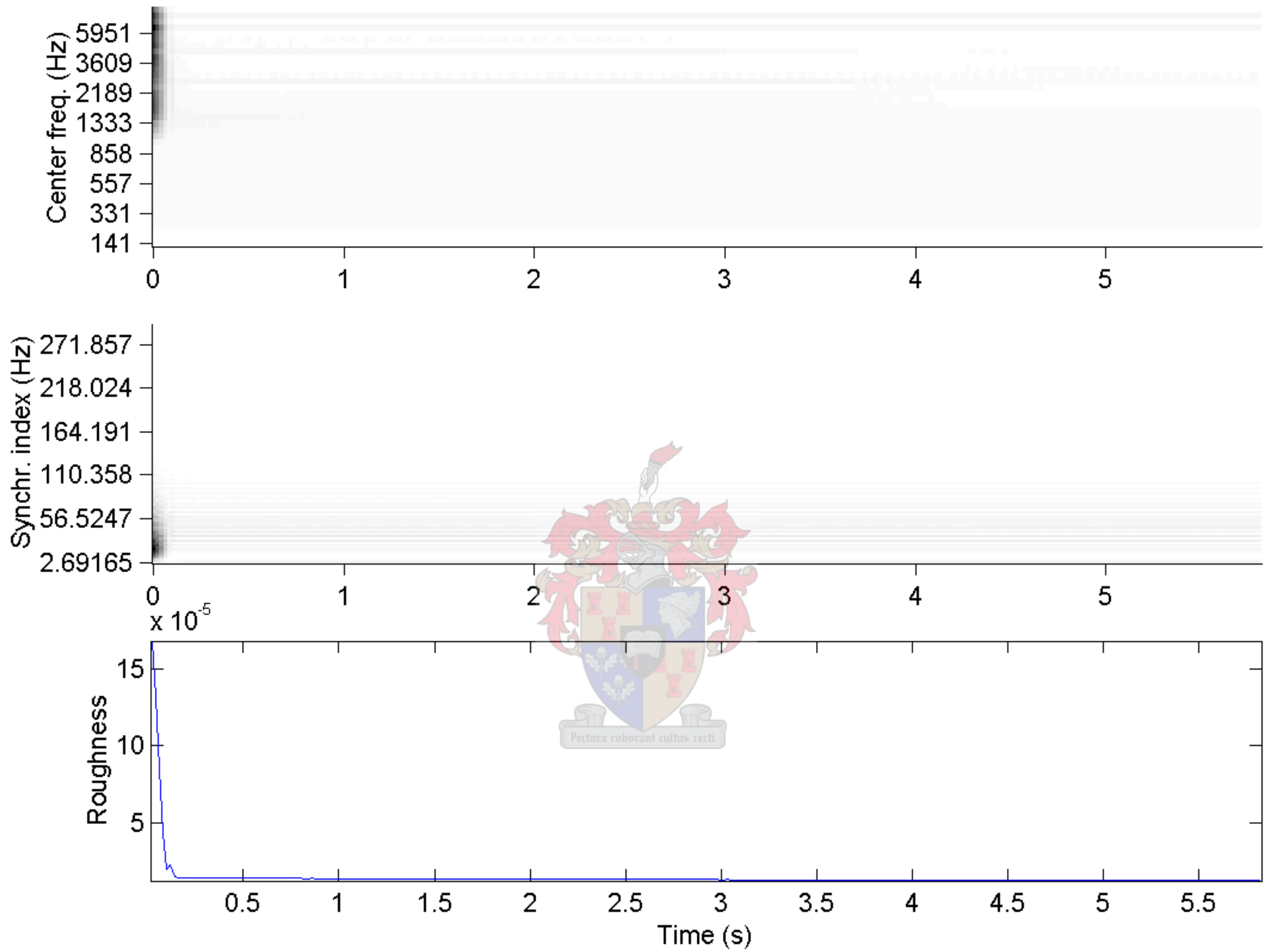
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 2188.2


Center freq. (Hz)
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3609
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4
5
Synchr. index (Hz)
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164.191
110.358
56.5247
2.69165
0
1
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x 10^-5
Roughness
15
10
5
0.5
1
1.5
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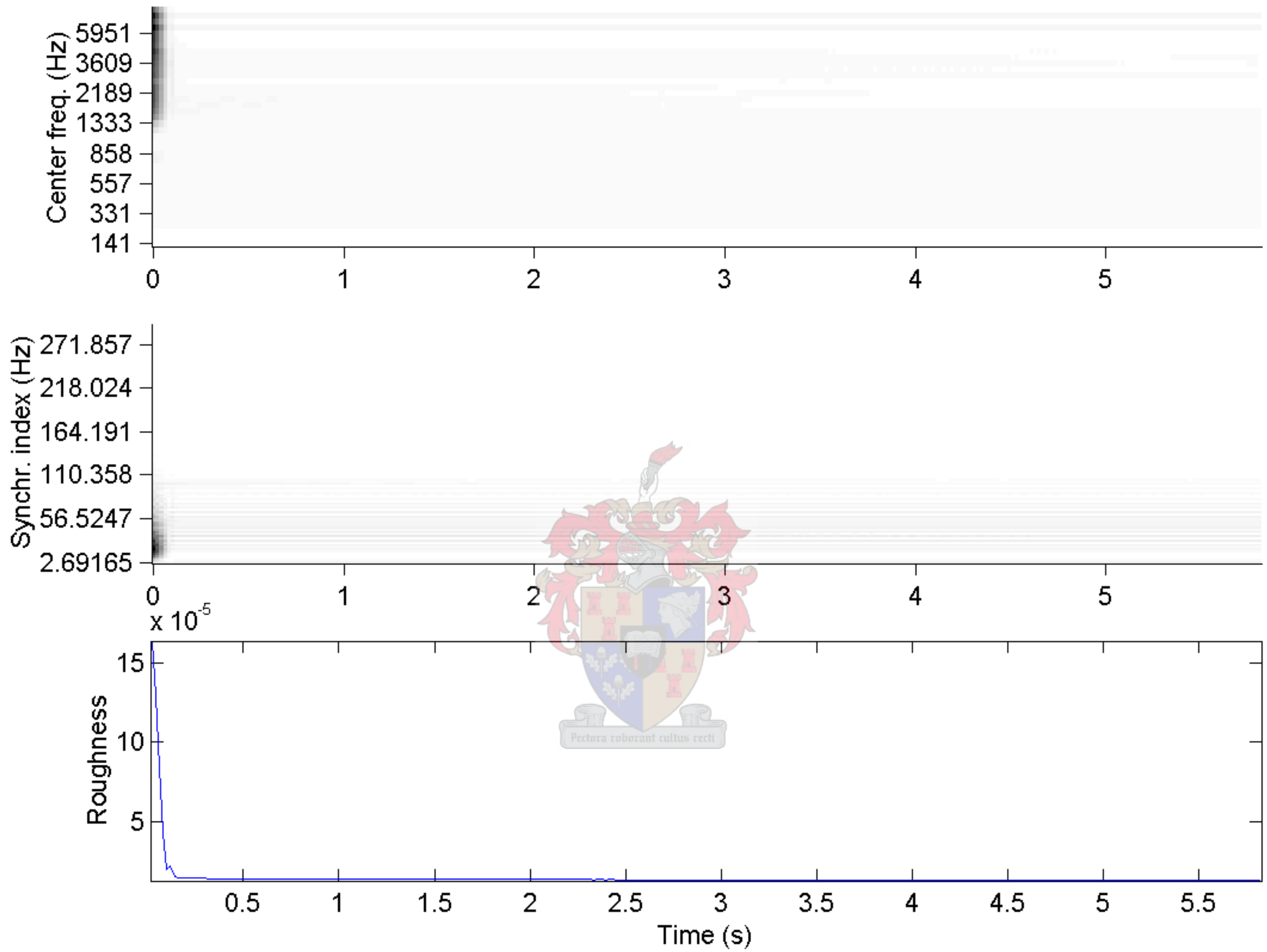
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 2568.9


Center freq. (Hz)
5951
3609
2189
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
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x 10^-5
Roughness
15
10
5
0.5
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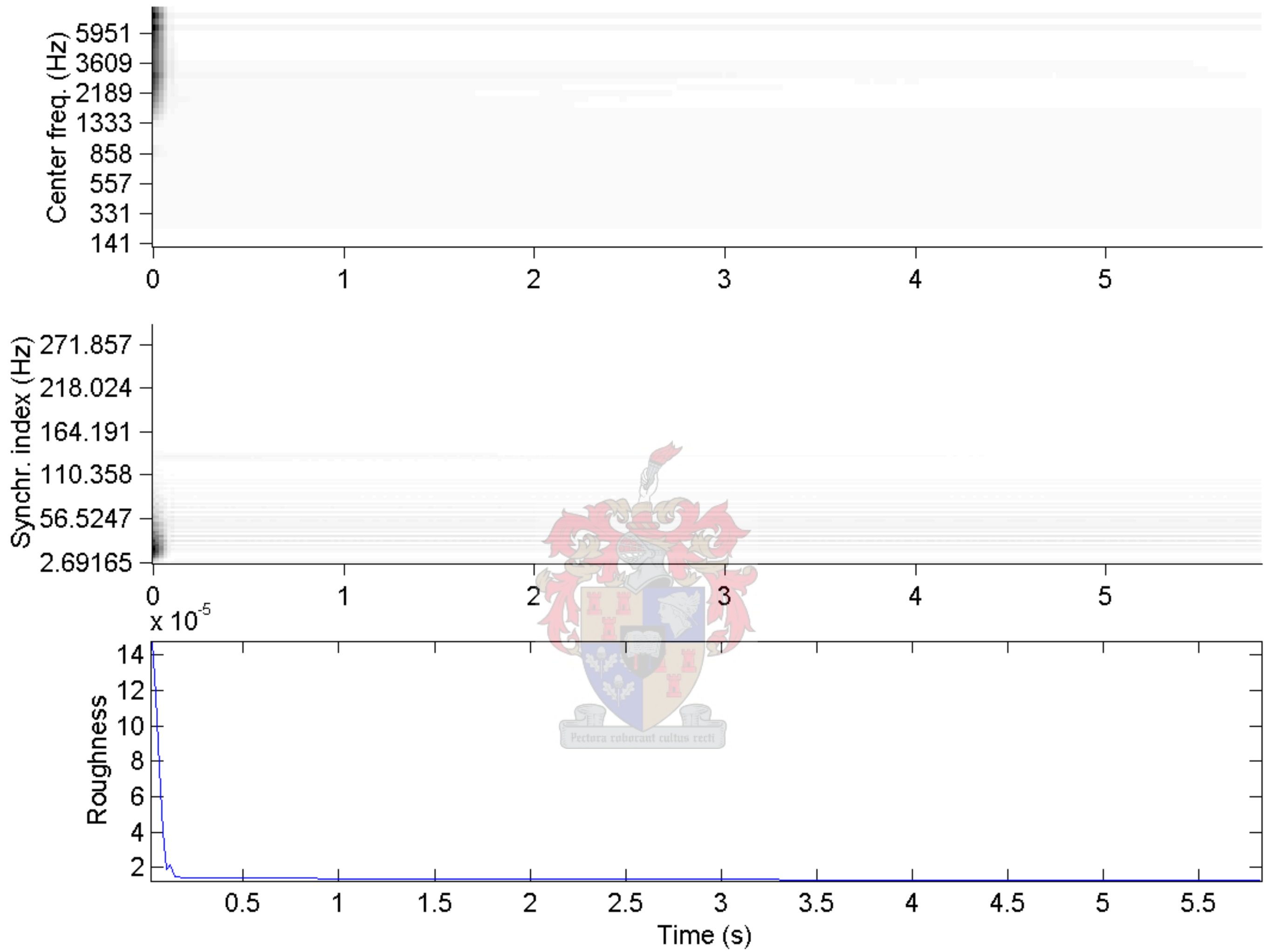
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 3015.8


Center freq. (Hz)
5951
3609
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
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4
5
x 10^-5
Roughness
14
12
10
8
6
4
2
0.5
1
1.5
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2.5
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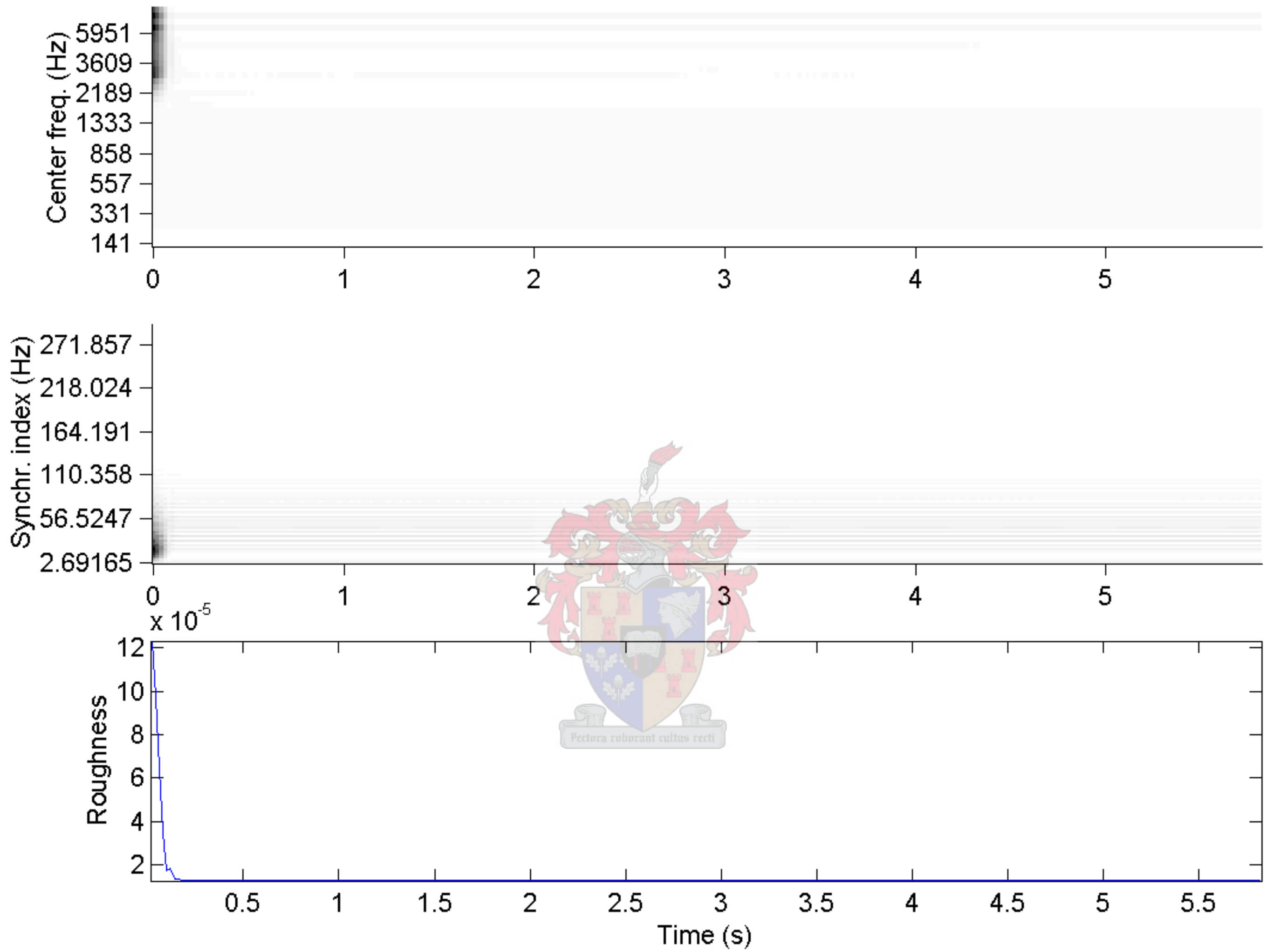
3.5
4
4.5
5
5.5
Time (s)

et 3540.5



Center freq. (Hz)
5951
3609
2189
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331
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
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4
5
x 10^-5
Roughness
12
10
8
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4
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0.5
1
1.5
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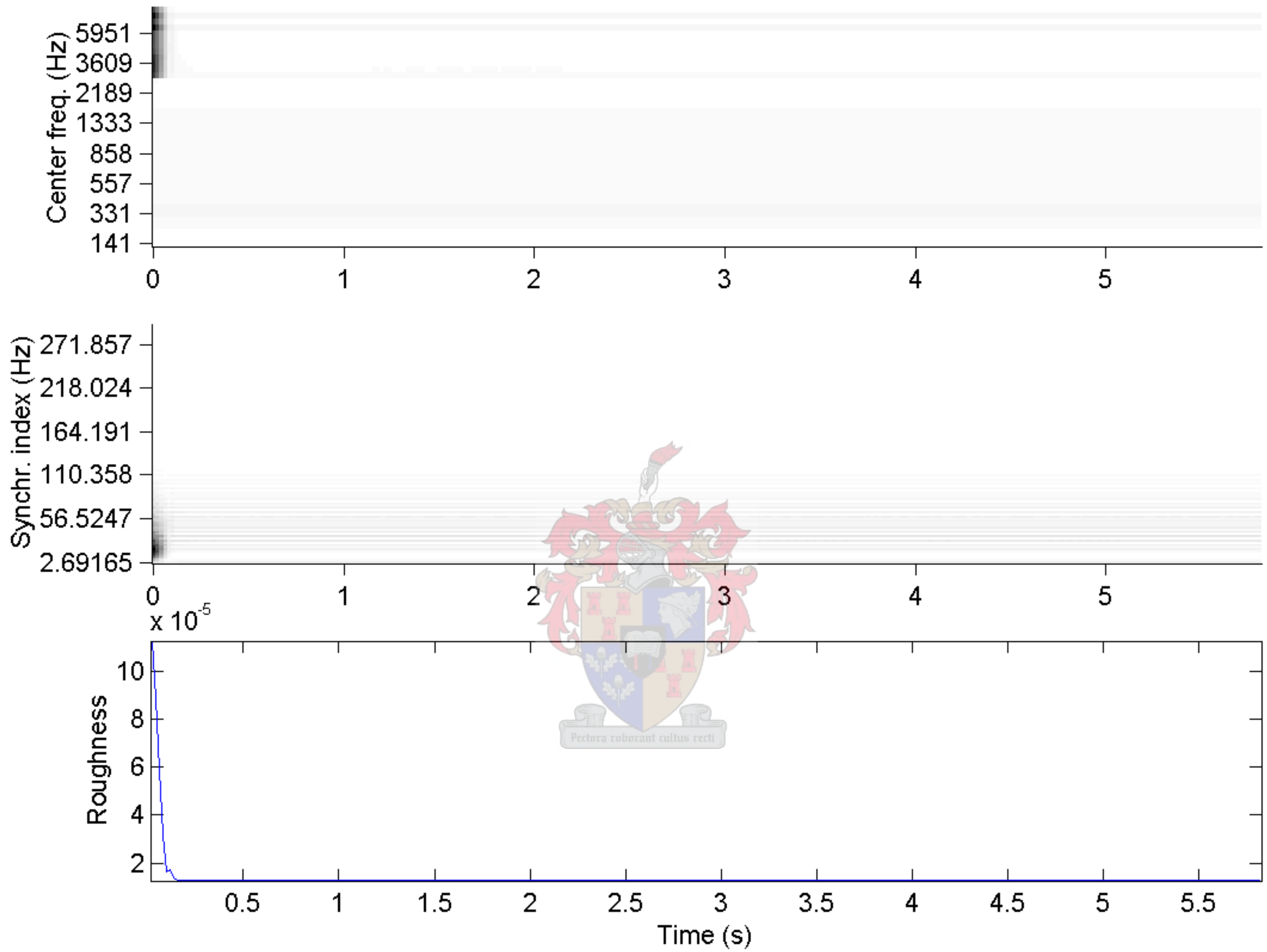
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 4156.5


Center freq. (Hz)
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3609
2189
1333
858
557
331
141
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
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x 10^-5
Roughness
10
8
6
4
2
0.5
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1.5
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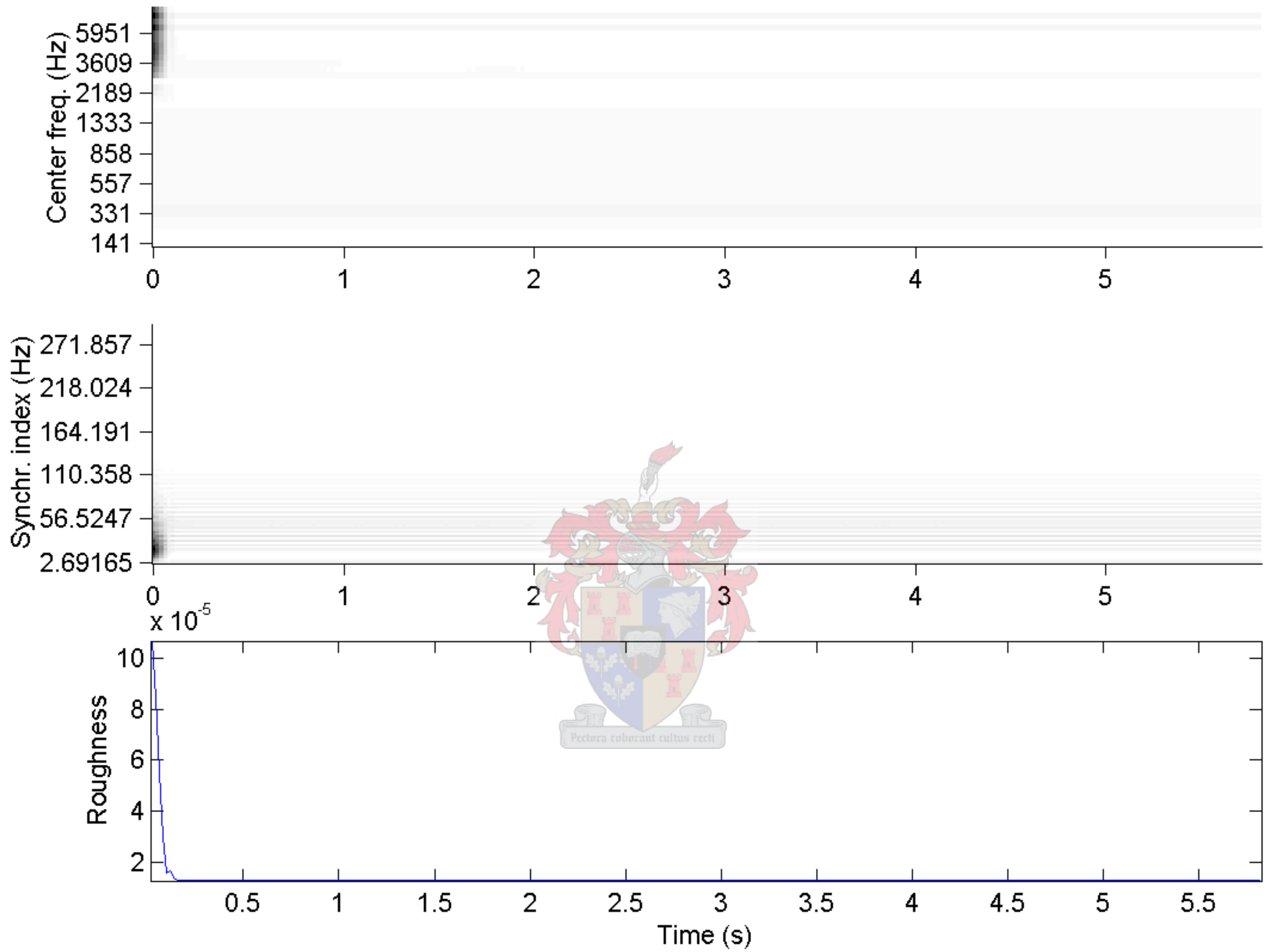
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 4879.7


Center freq. (Hz)
5951
3609
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1333
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331
141
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Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
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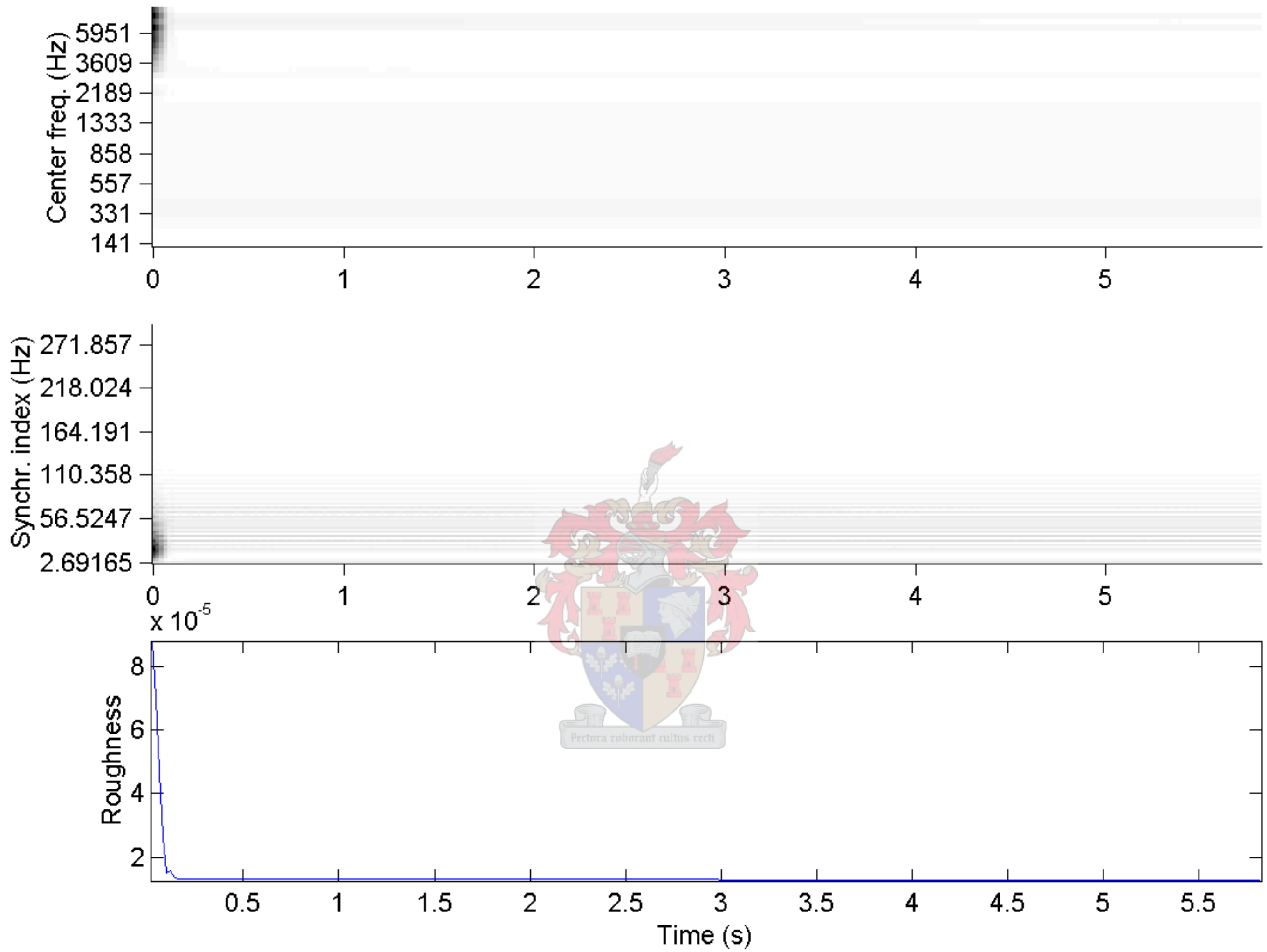
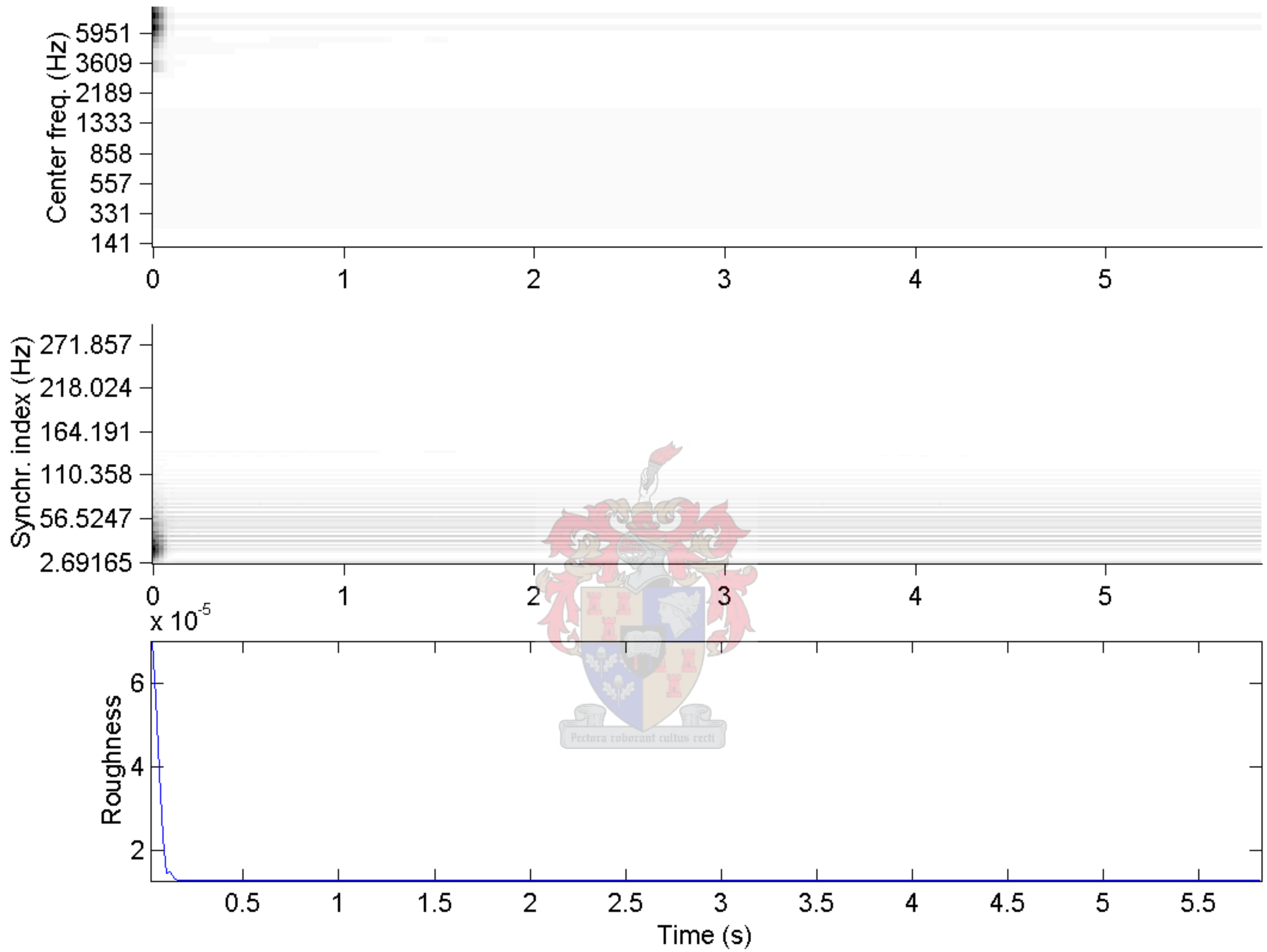
3
4
5
x 10^-5
Roughness
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

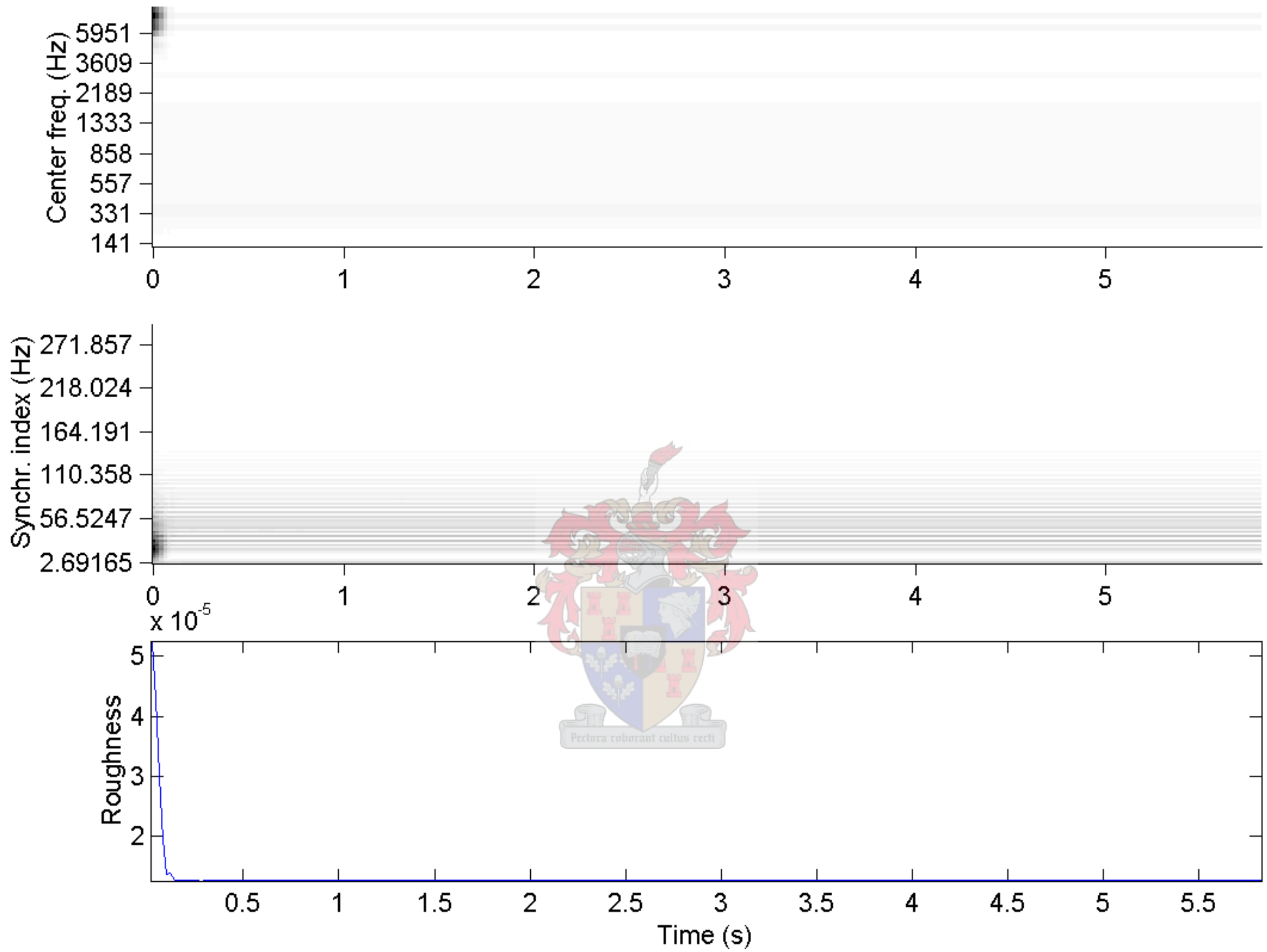
et 5728.7


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 6725.4

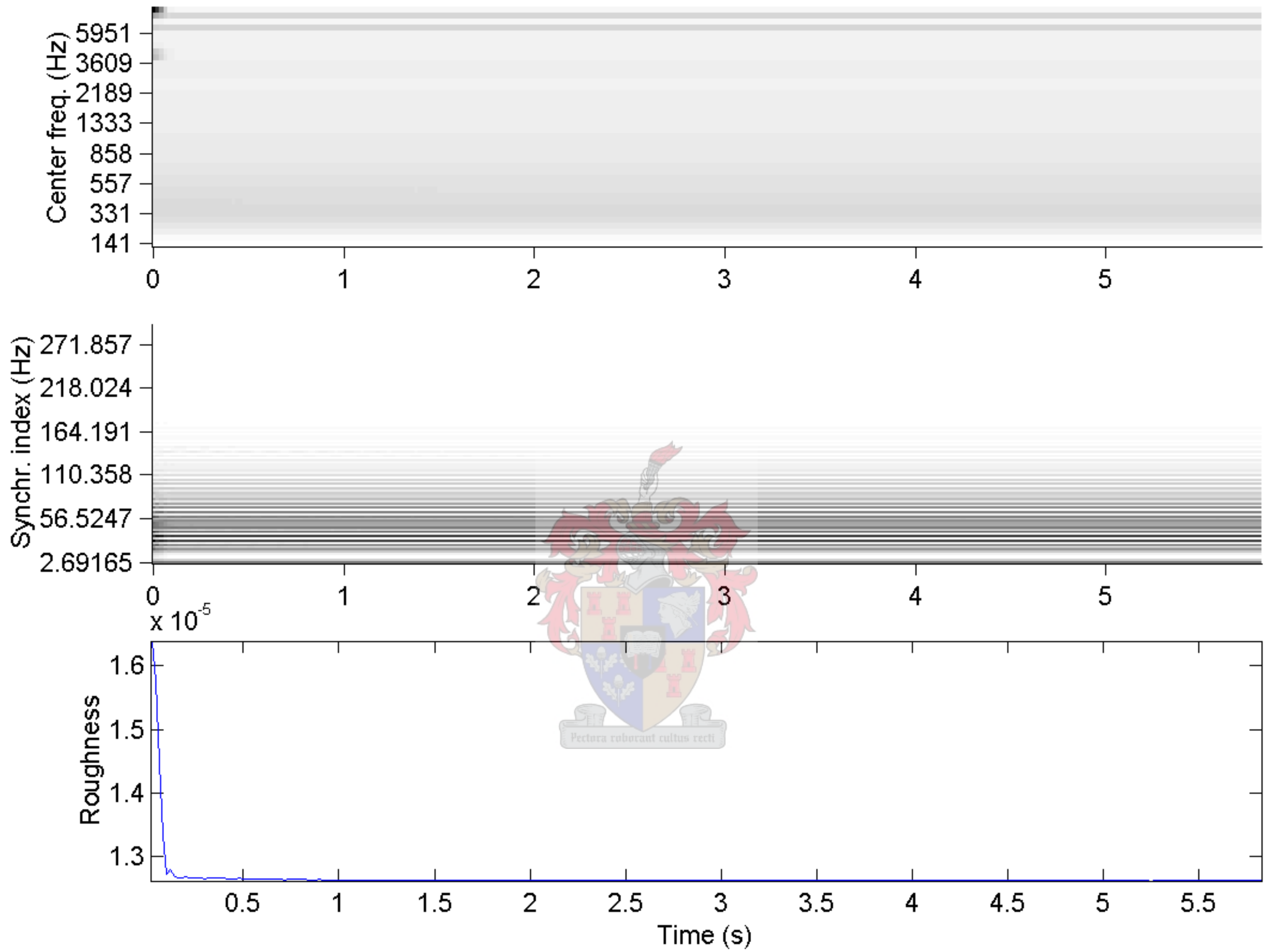

Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 7895.5

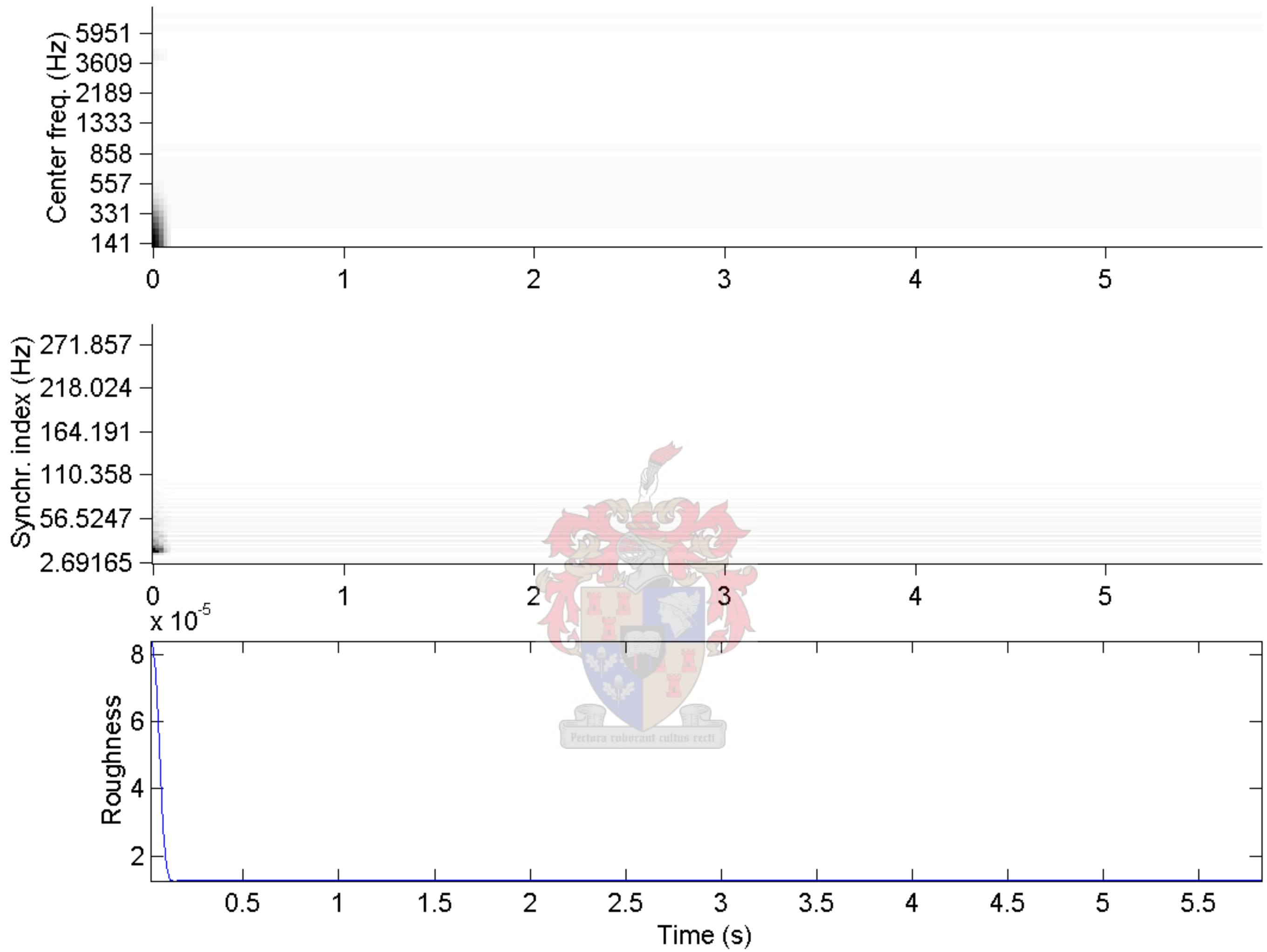


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
5
4
3
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

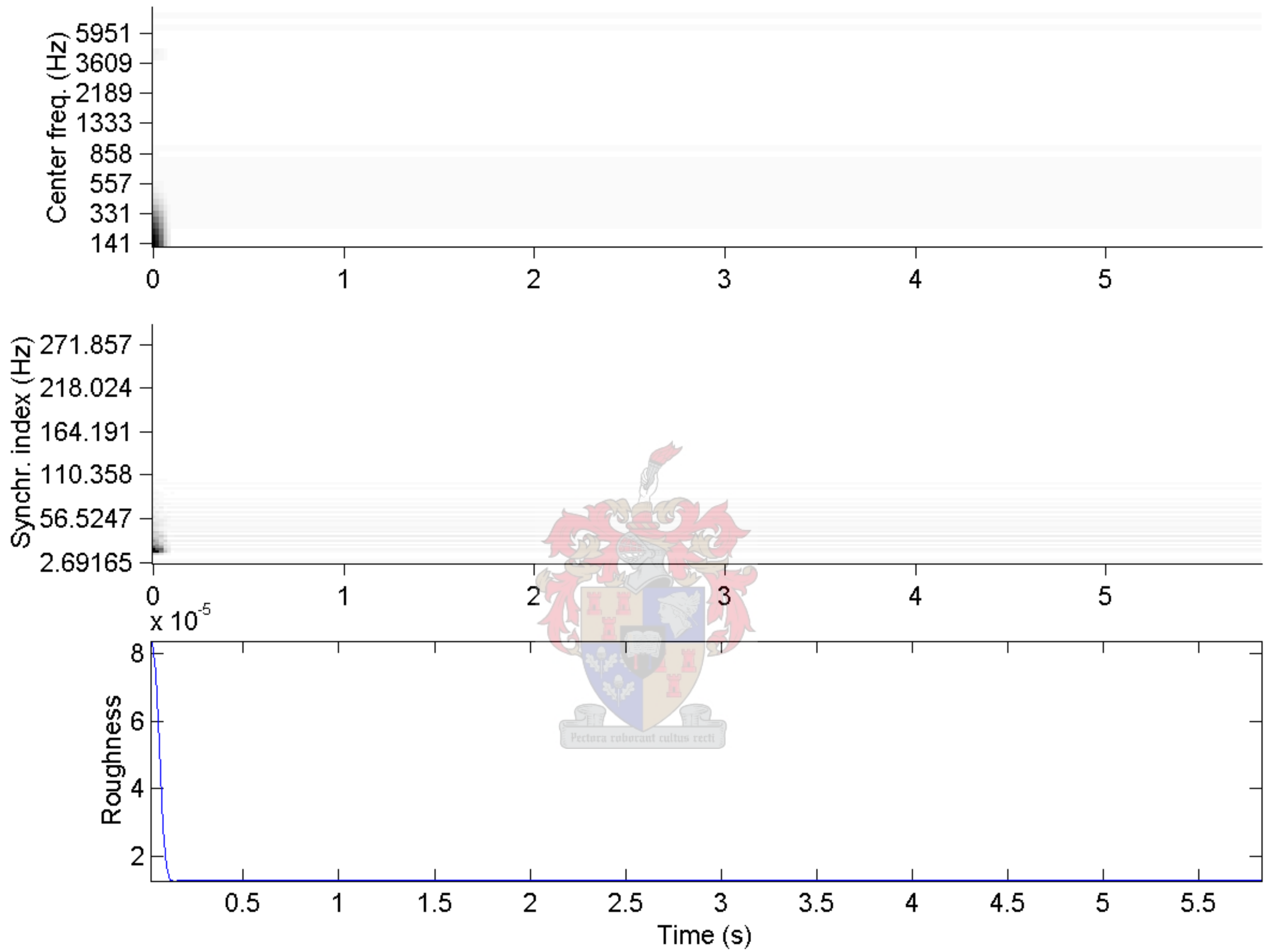
et 9269.2


5951
3609
2189
1333
858
557
331
141
Center freq. (Hz)
0
1
2
3
4
5
271.857
218.024
164.191
110.358
56.5247
2.69165
Synchr. index (Hz)
x 10^-5
1.6
1.5
1.4
1.3
Roughness
0.5
1.5
2.5
3.5
4.5
5.5
Time (s)

et 10881.9


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 12775.2


Center freq. (Hz)
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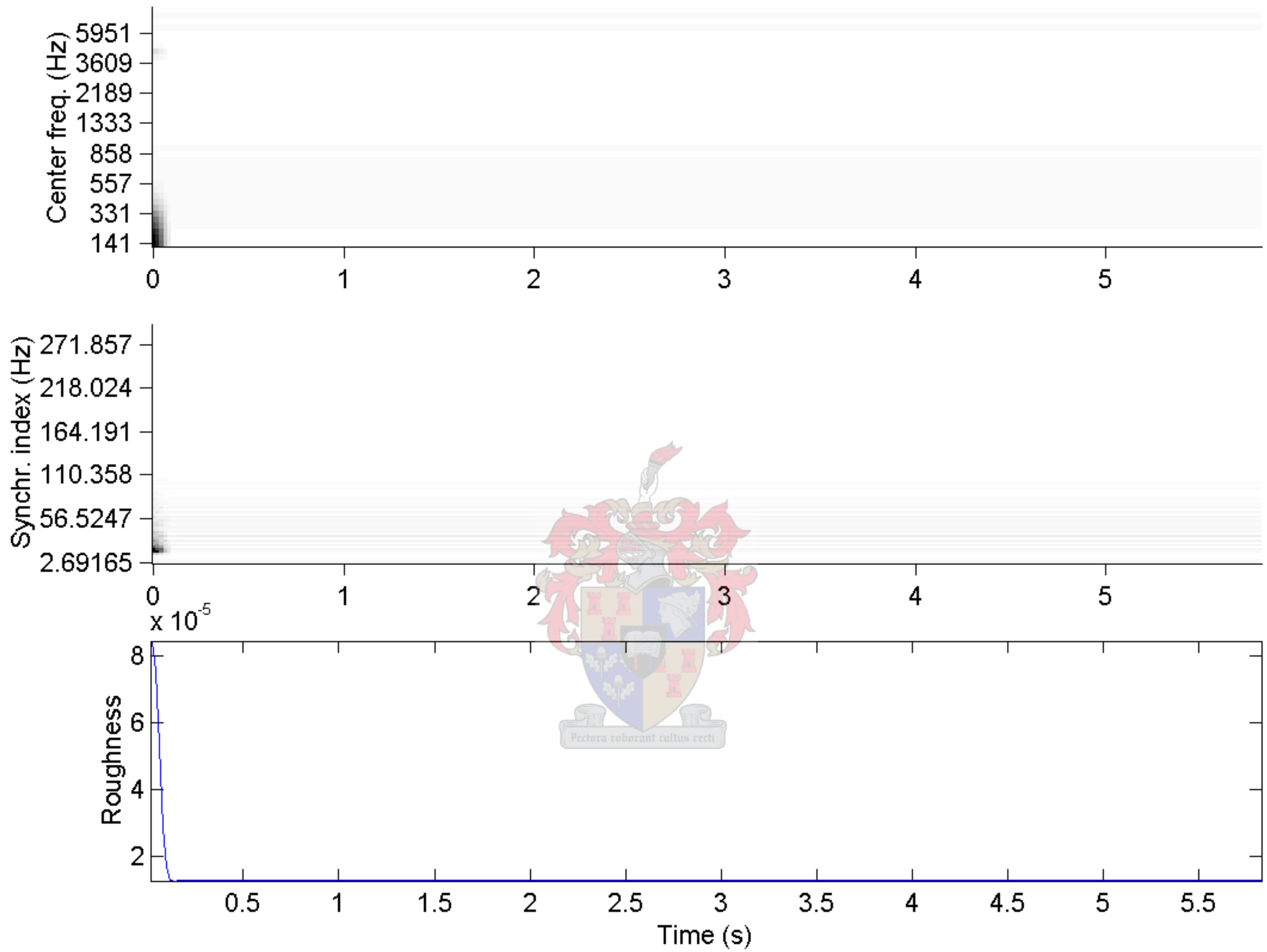
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
8
6
4
2
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 14997.8


Center freq. (Hz)
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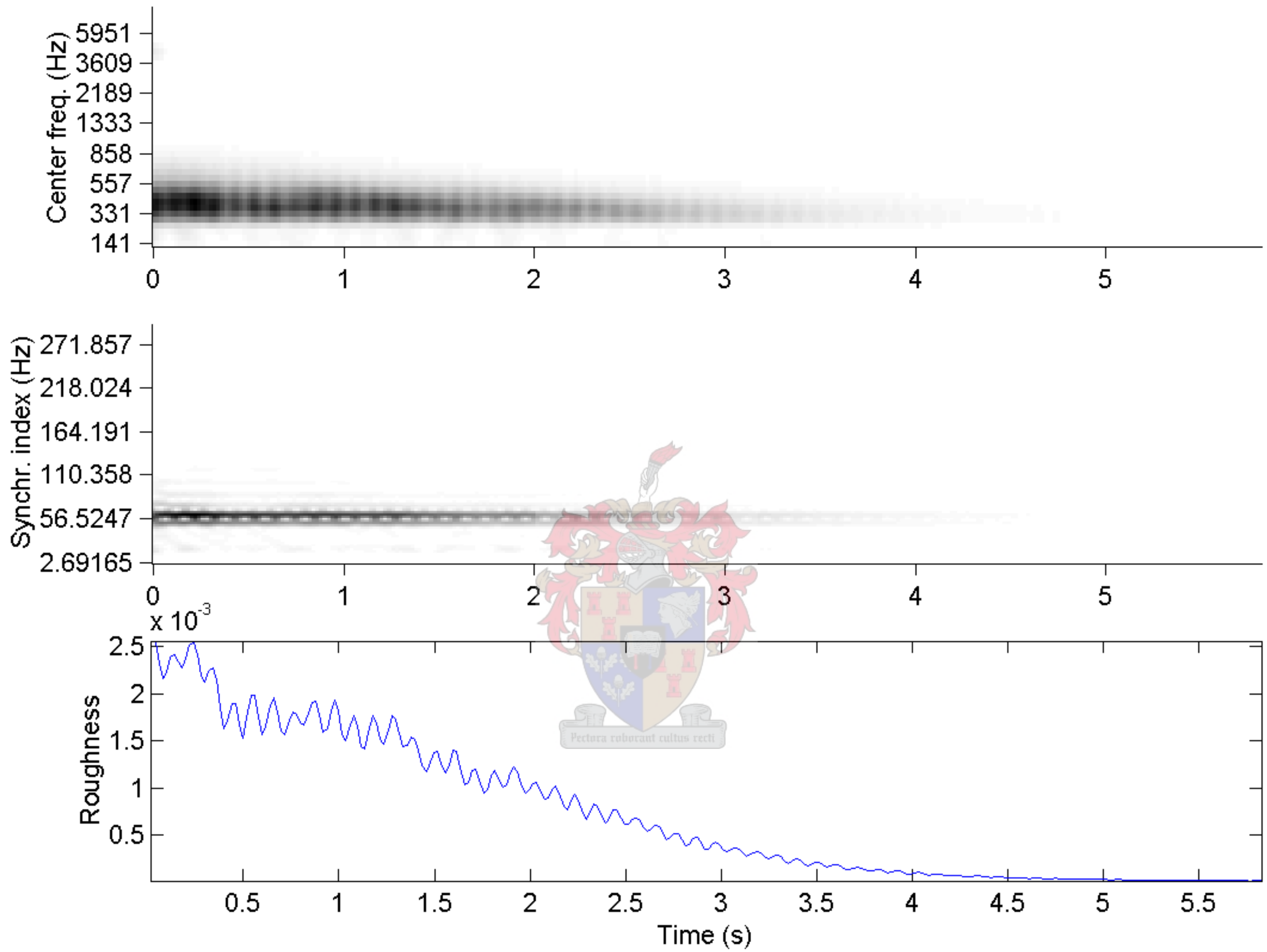
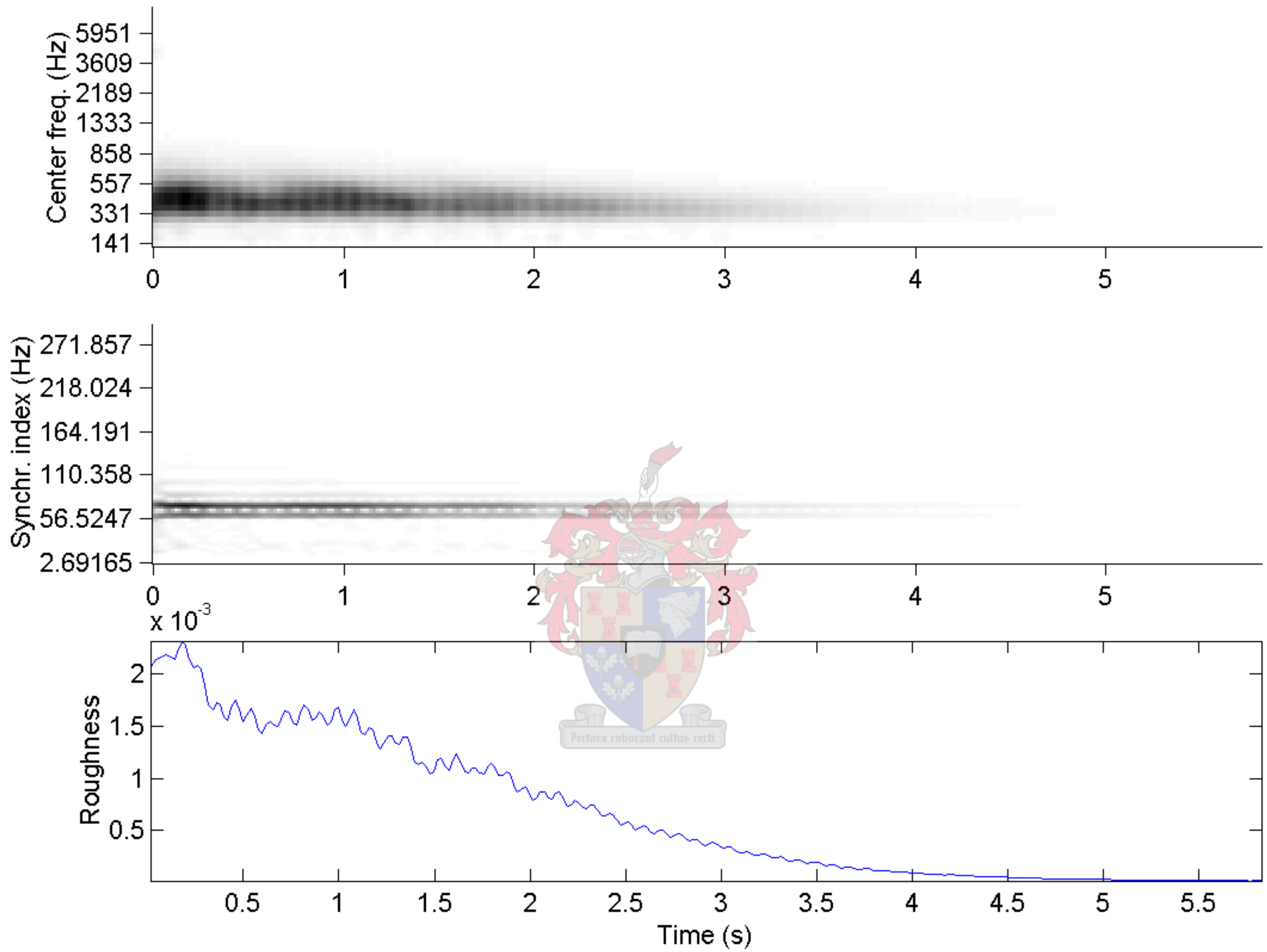
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

et 17607.2

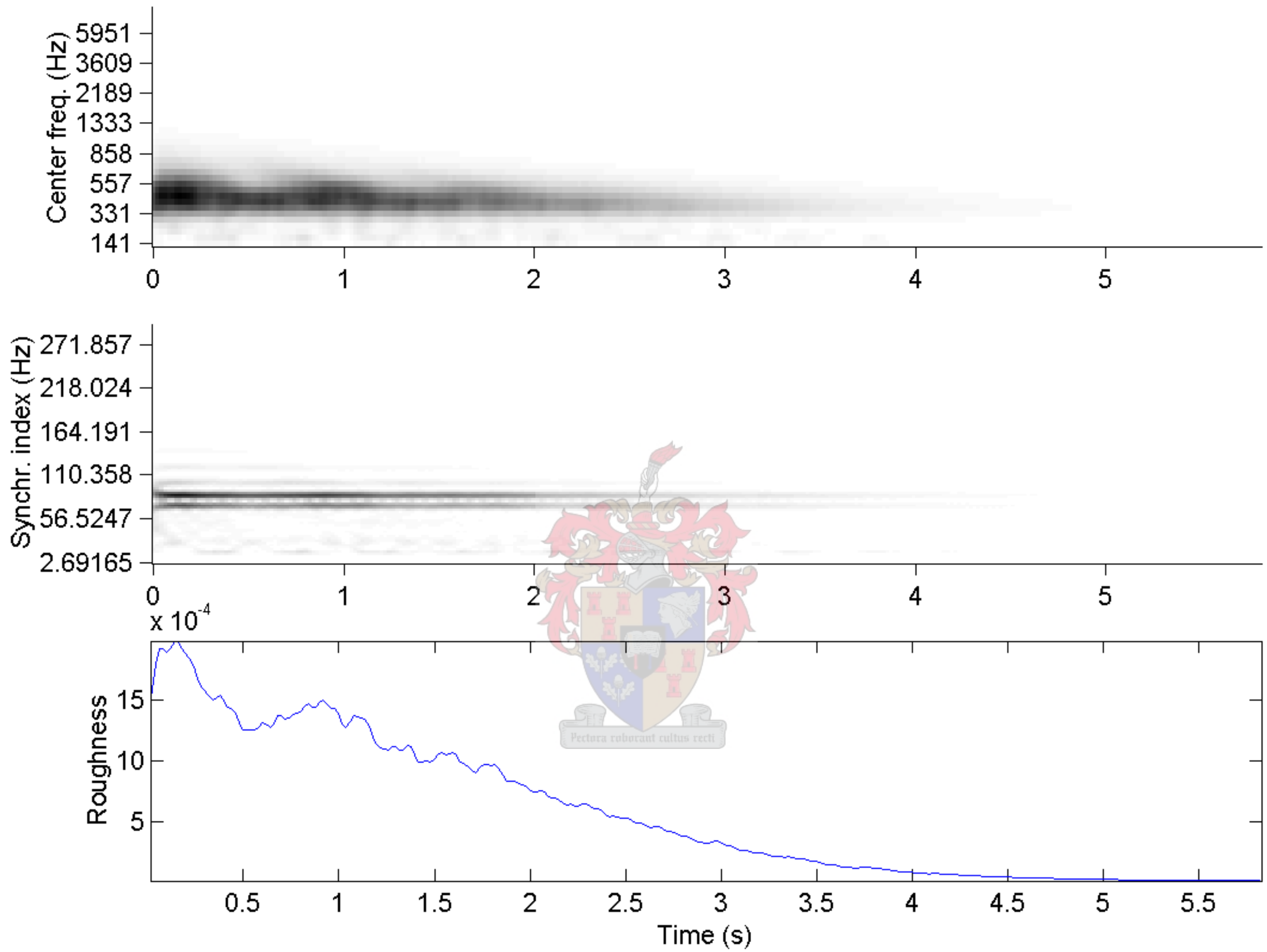


Phi Scale (phi) 54.7 Hz



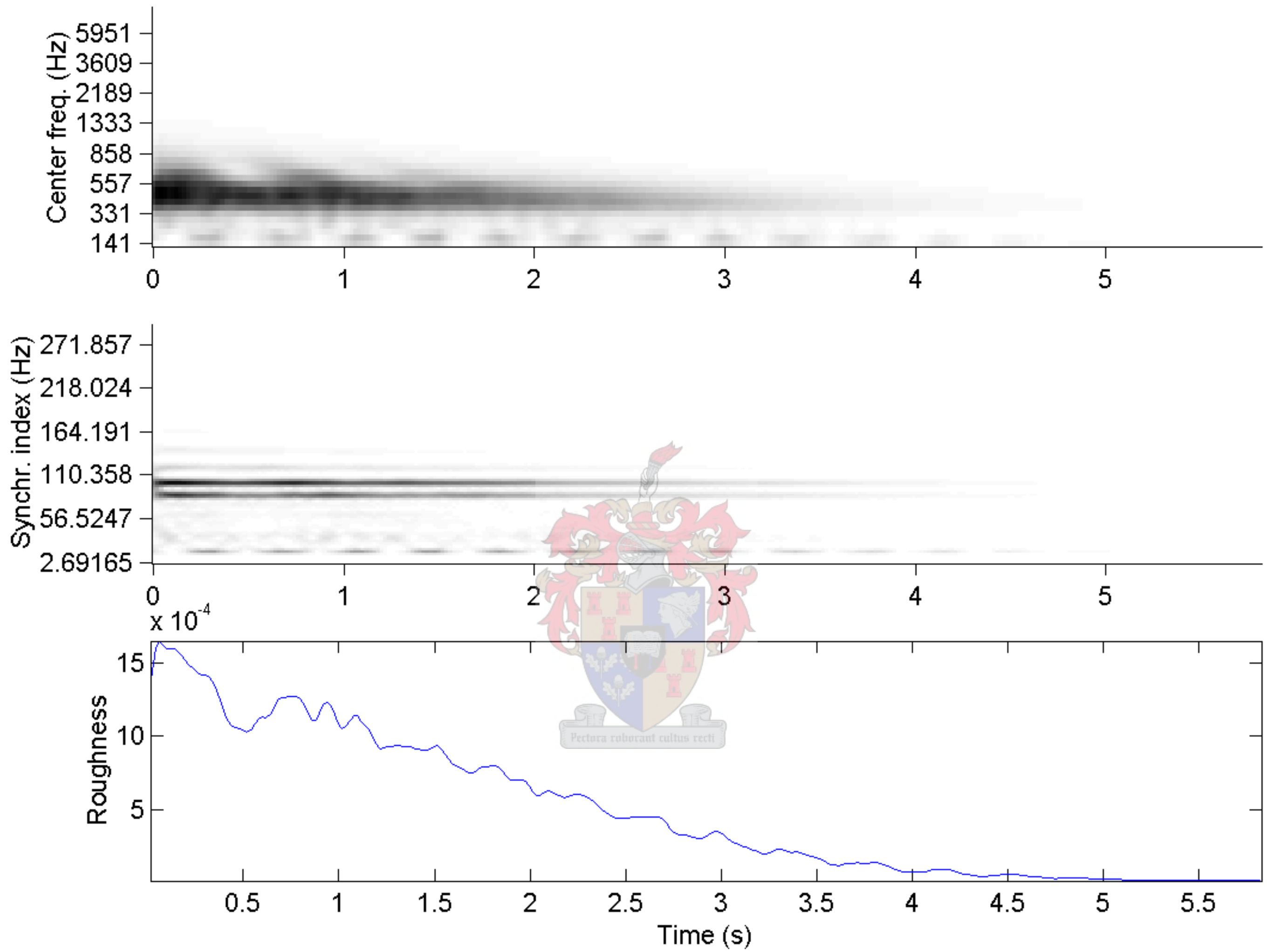
Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10-3
Roughness
2
1.5
1
0.5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 64.2

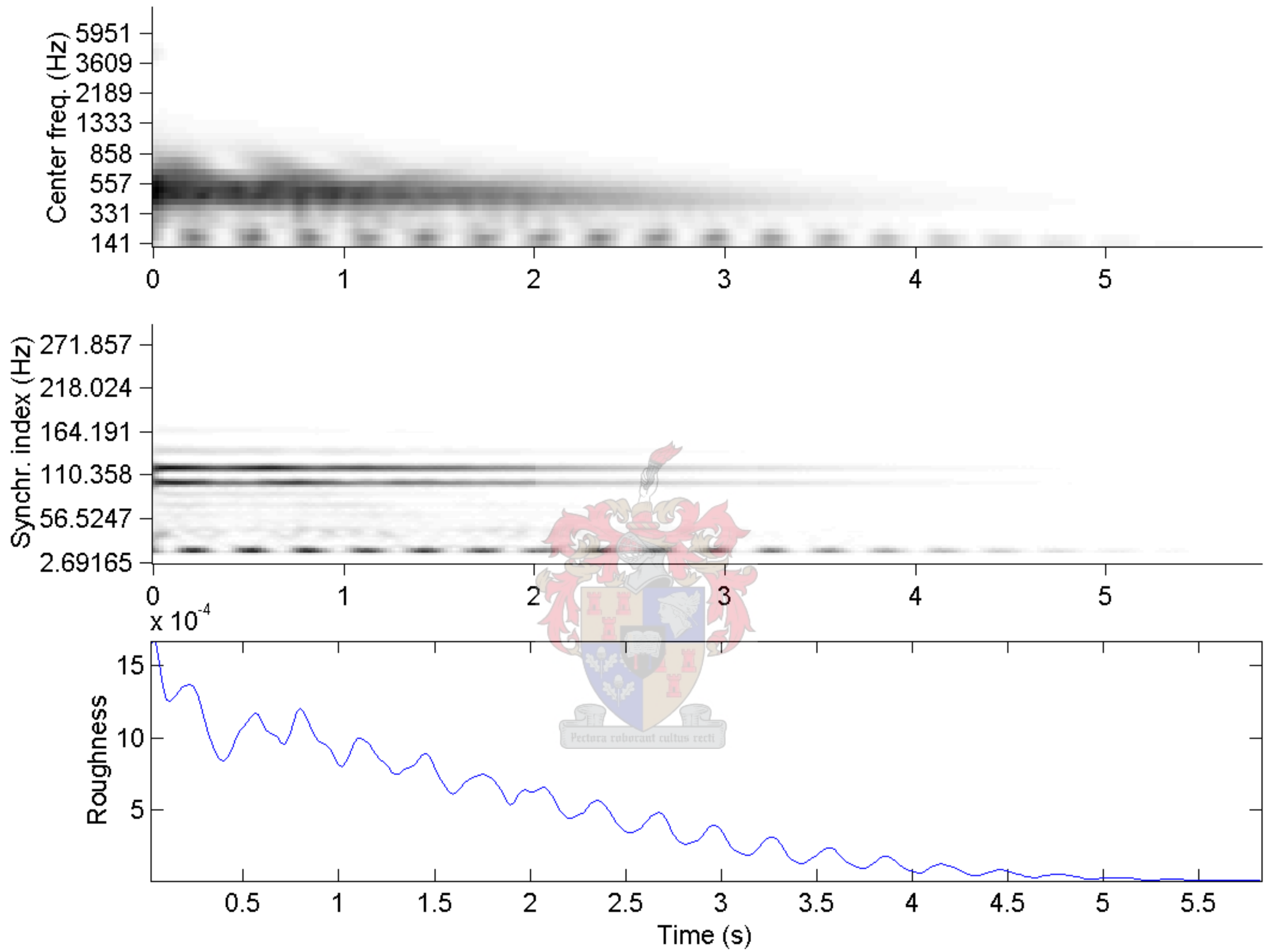


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10-4
Roughness
15
10
5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 75.4

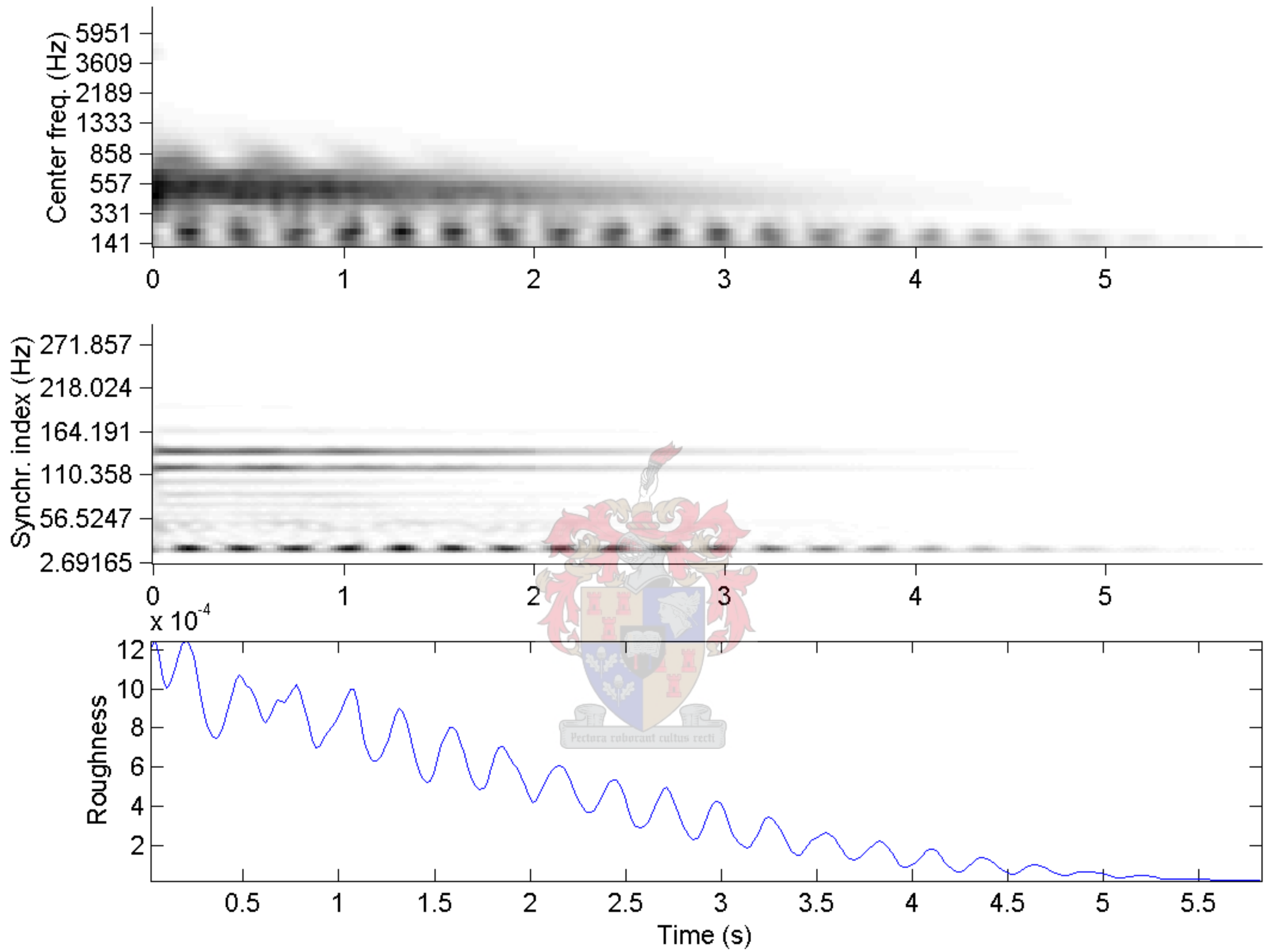

Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10-4
Roughness
15
10
5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 88.5

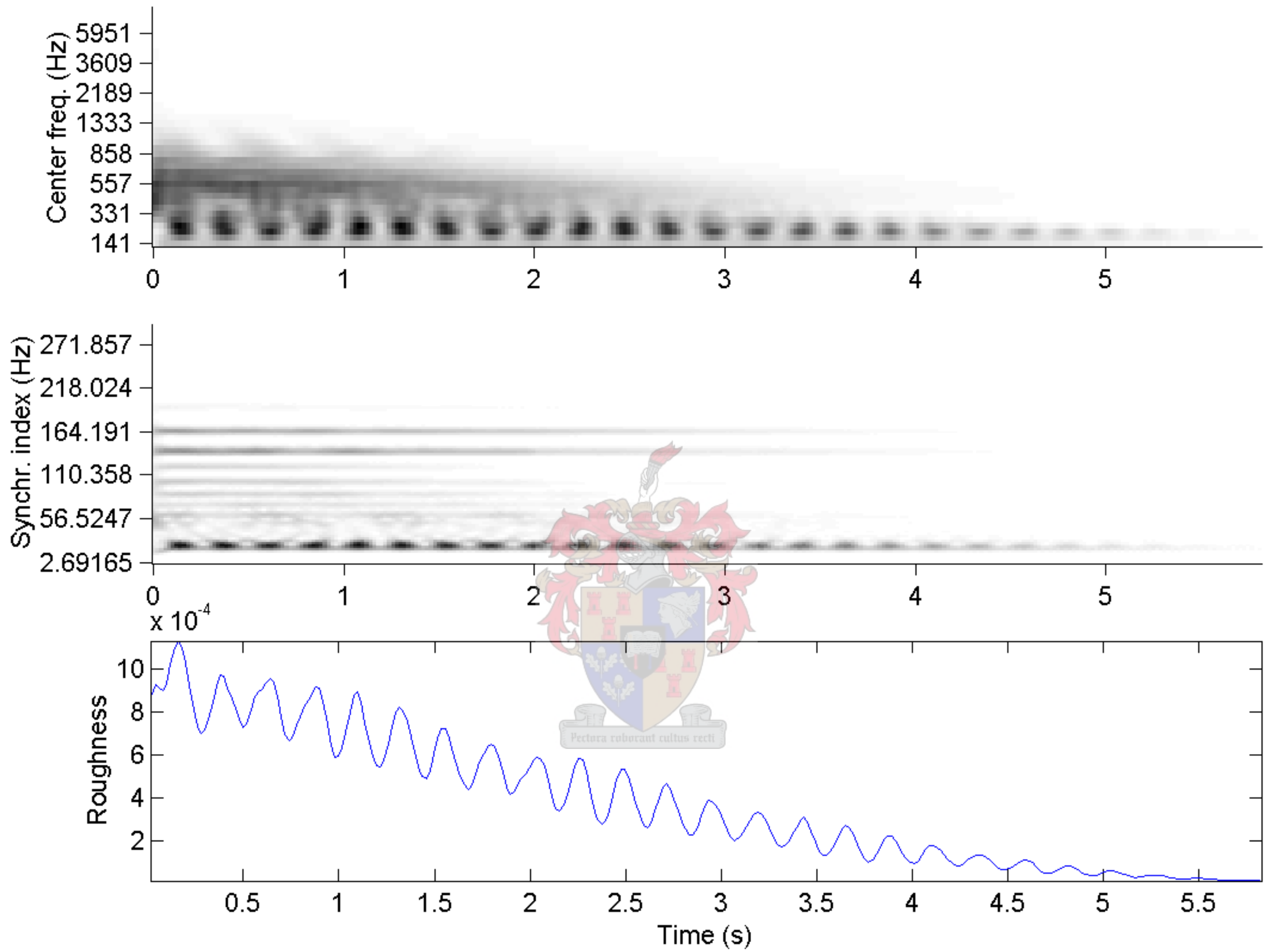


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
15
10
5
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 103.9

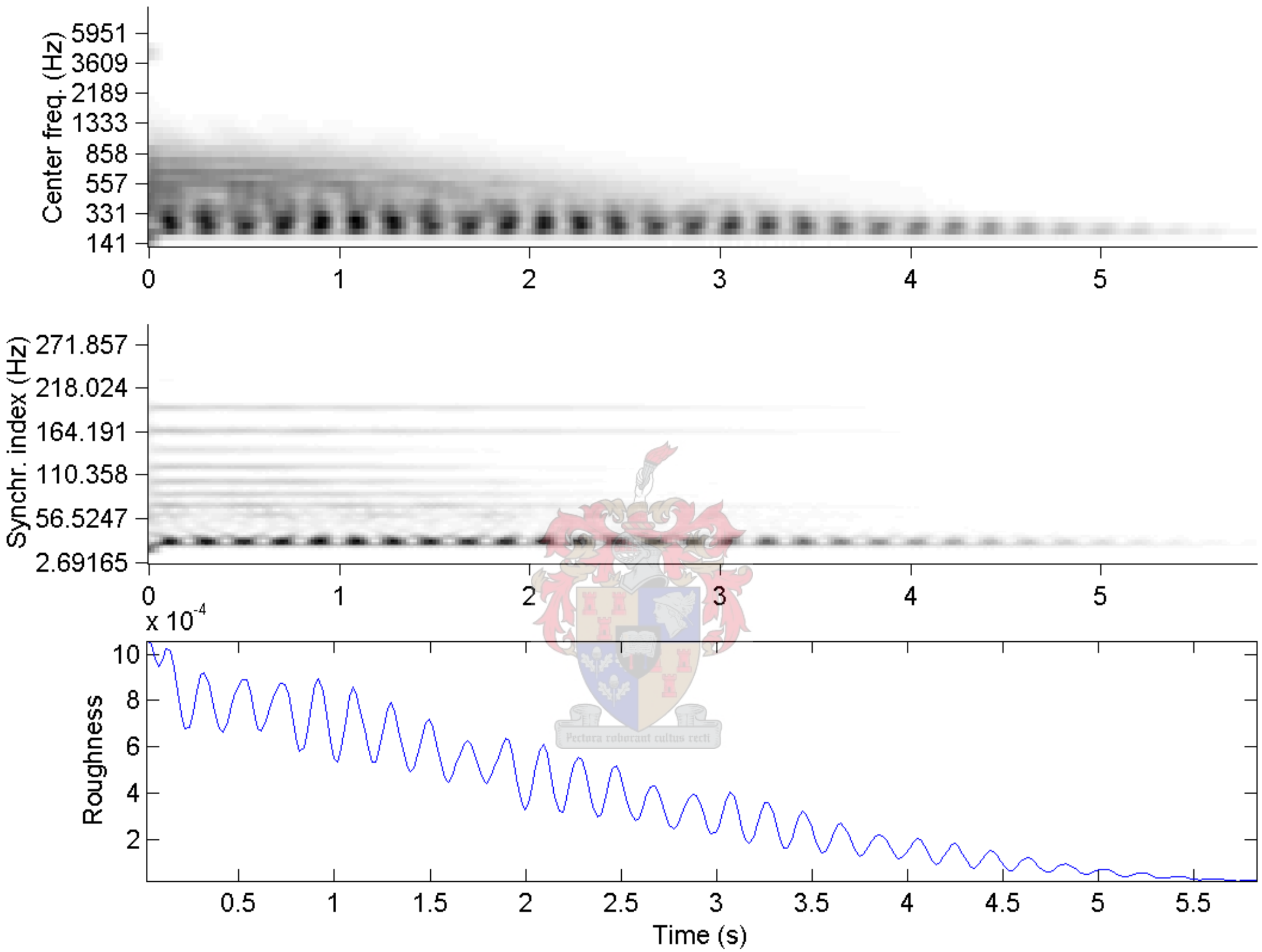

Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 121.9

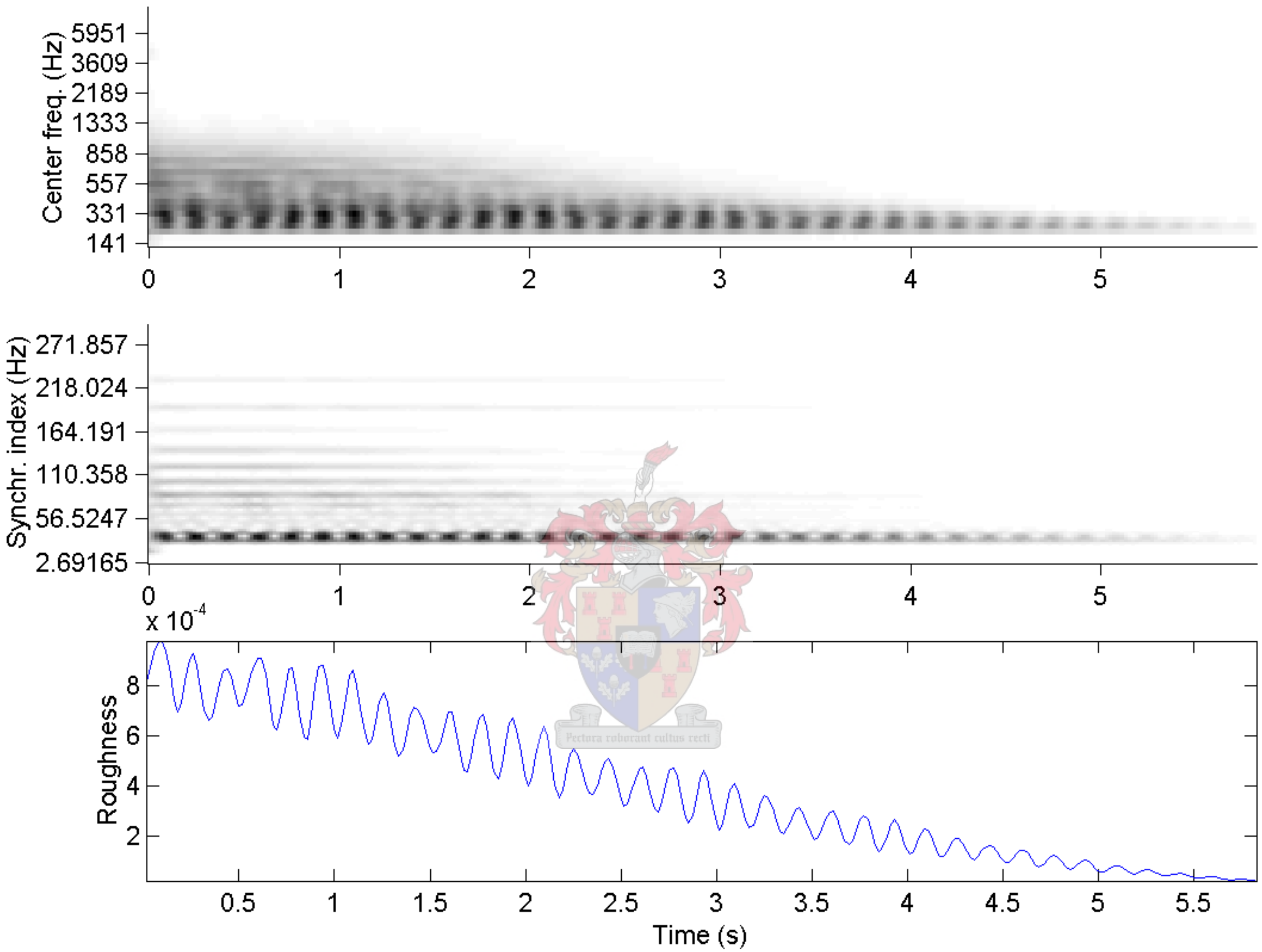


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10⁻⁴
Roughness
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 143.2

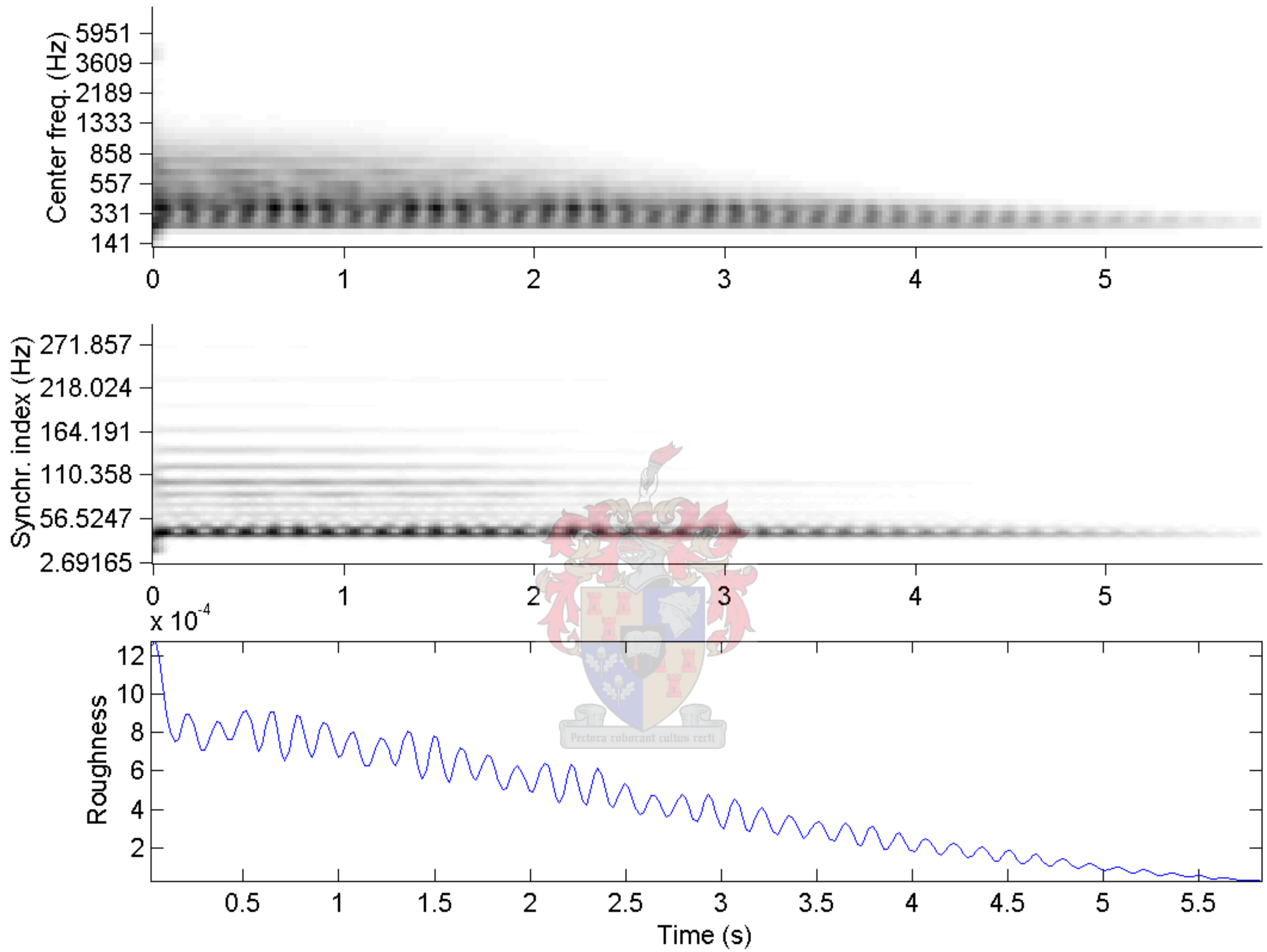

Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 168.1



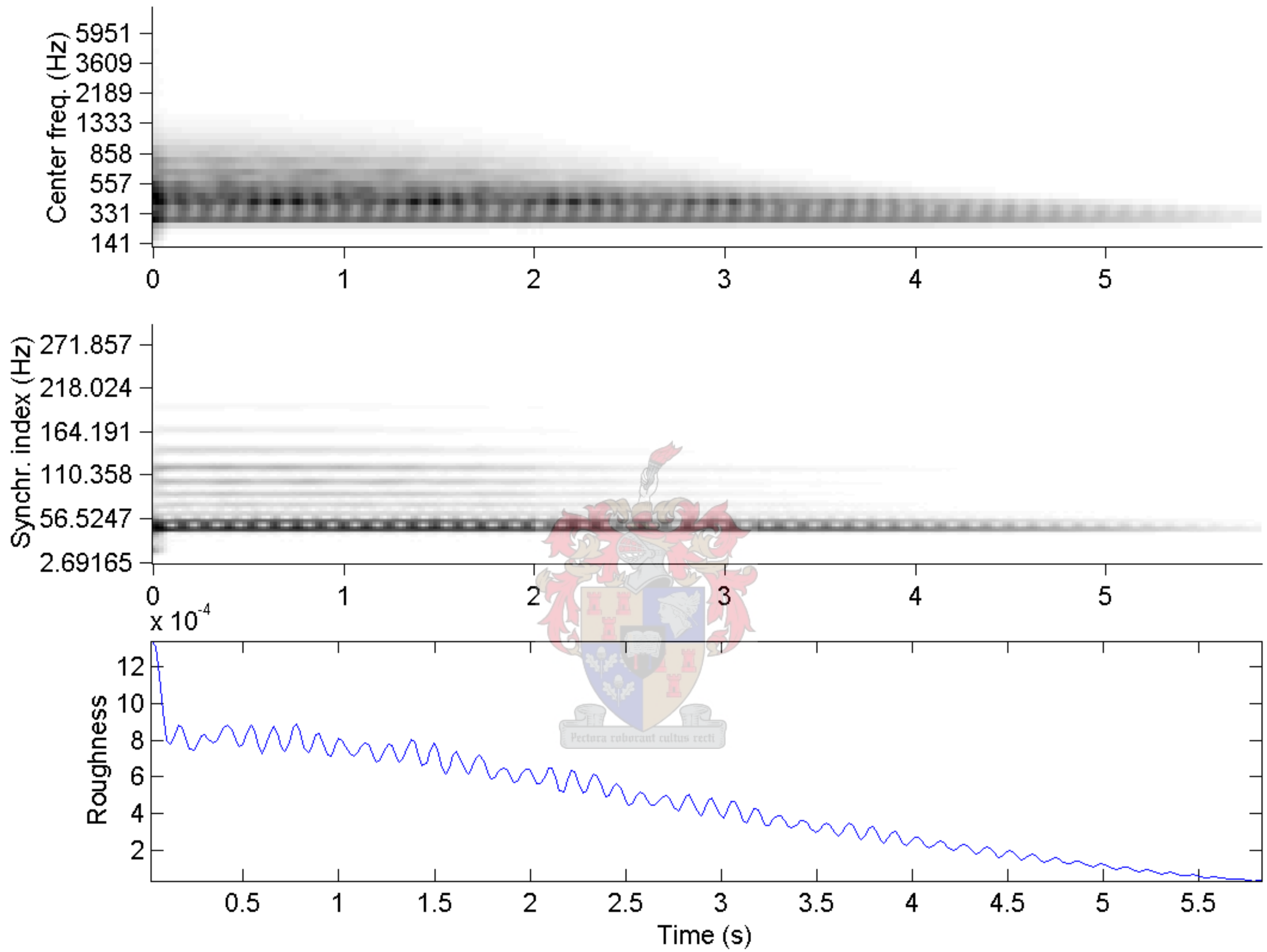
Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 197.3



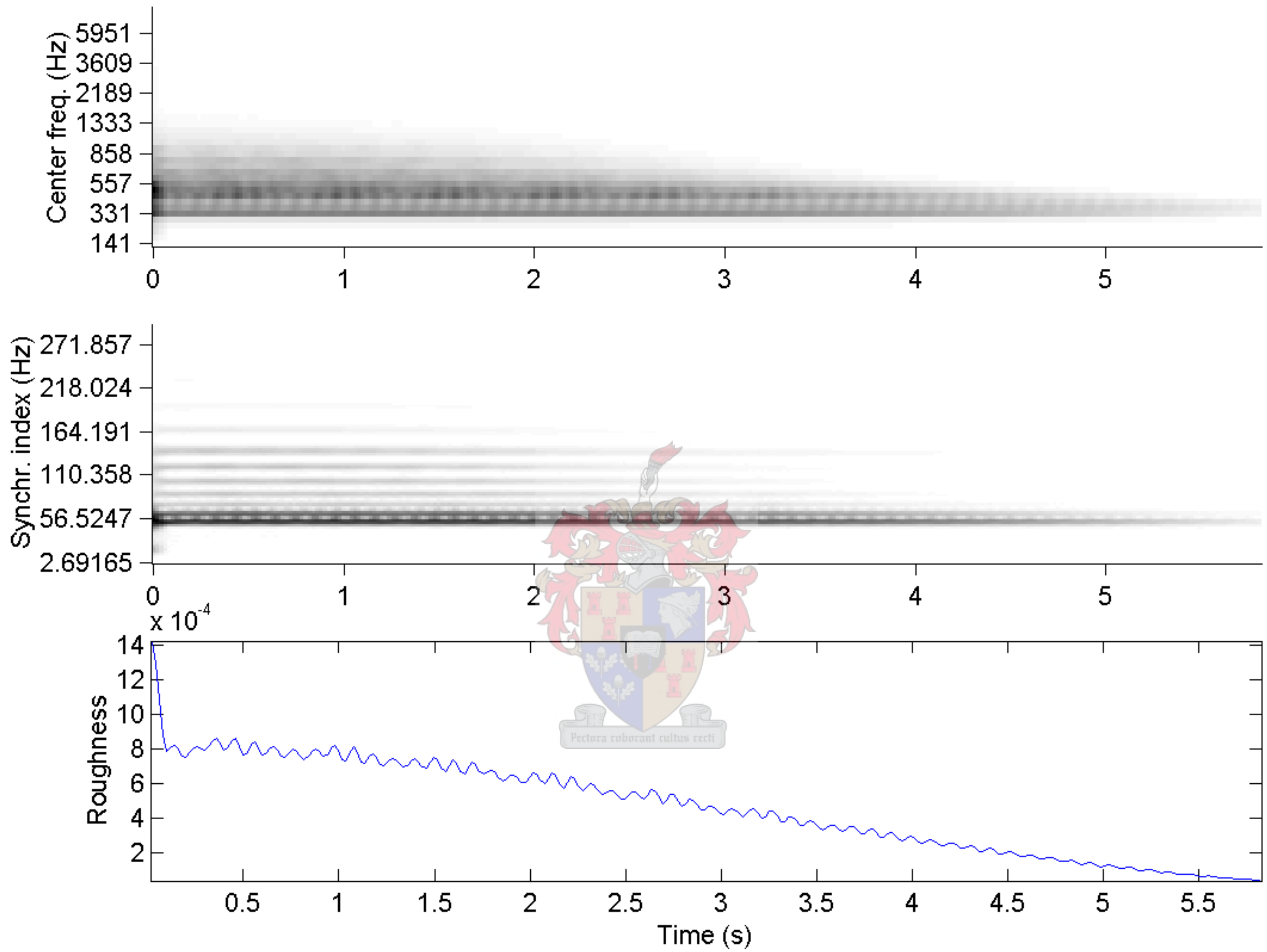
Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 231.6



Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 271.9


Center freq. (Hz)
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3609
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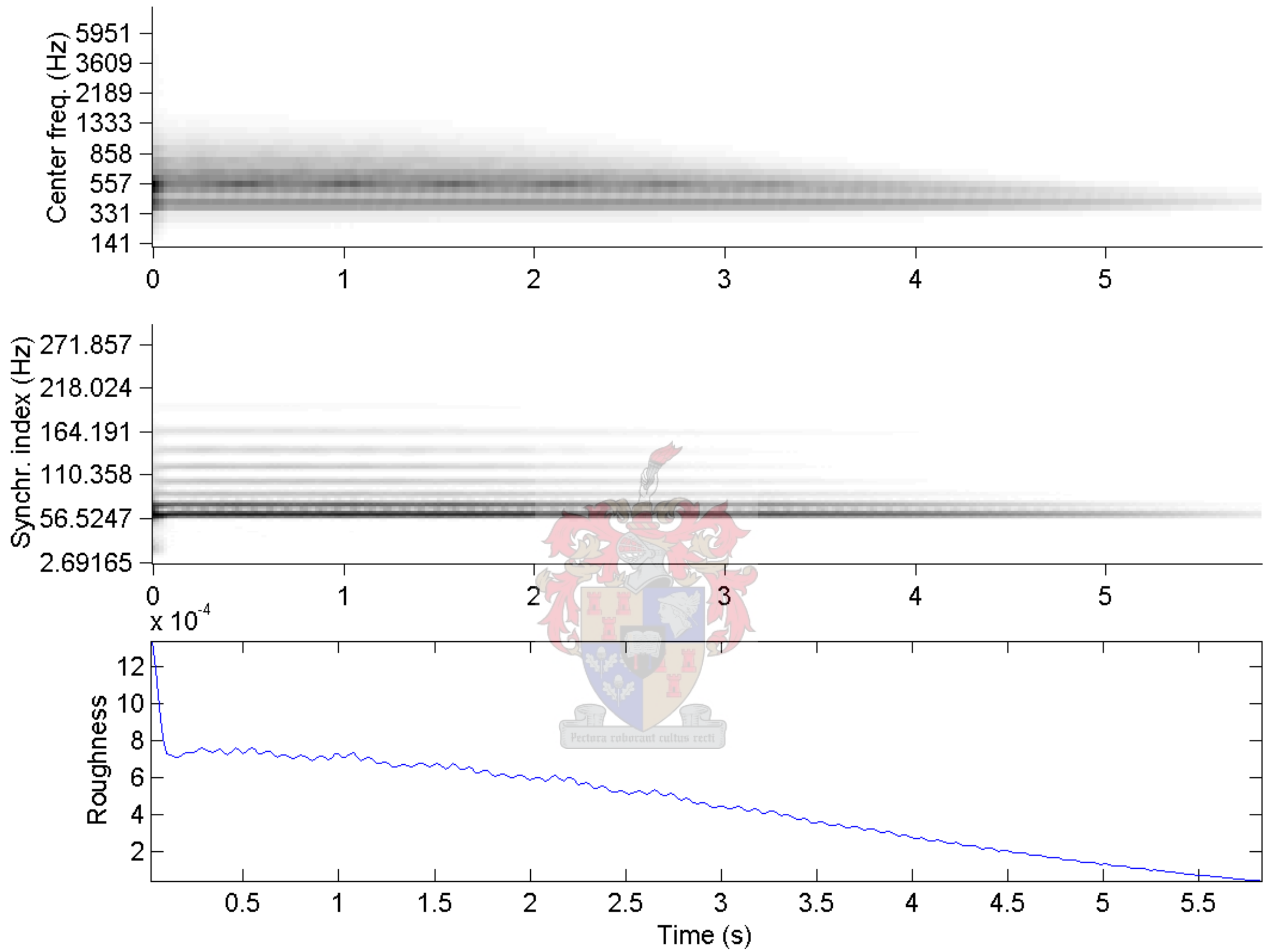
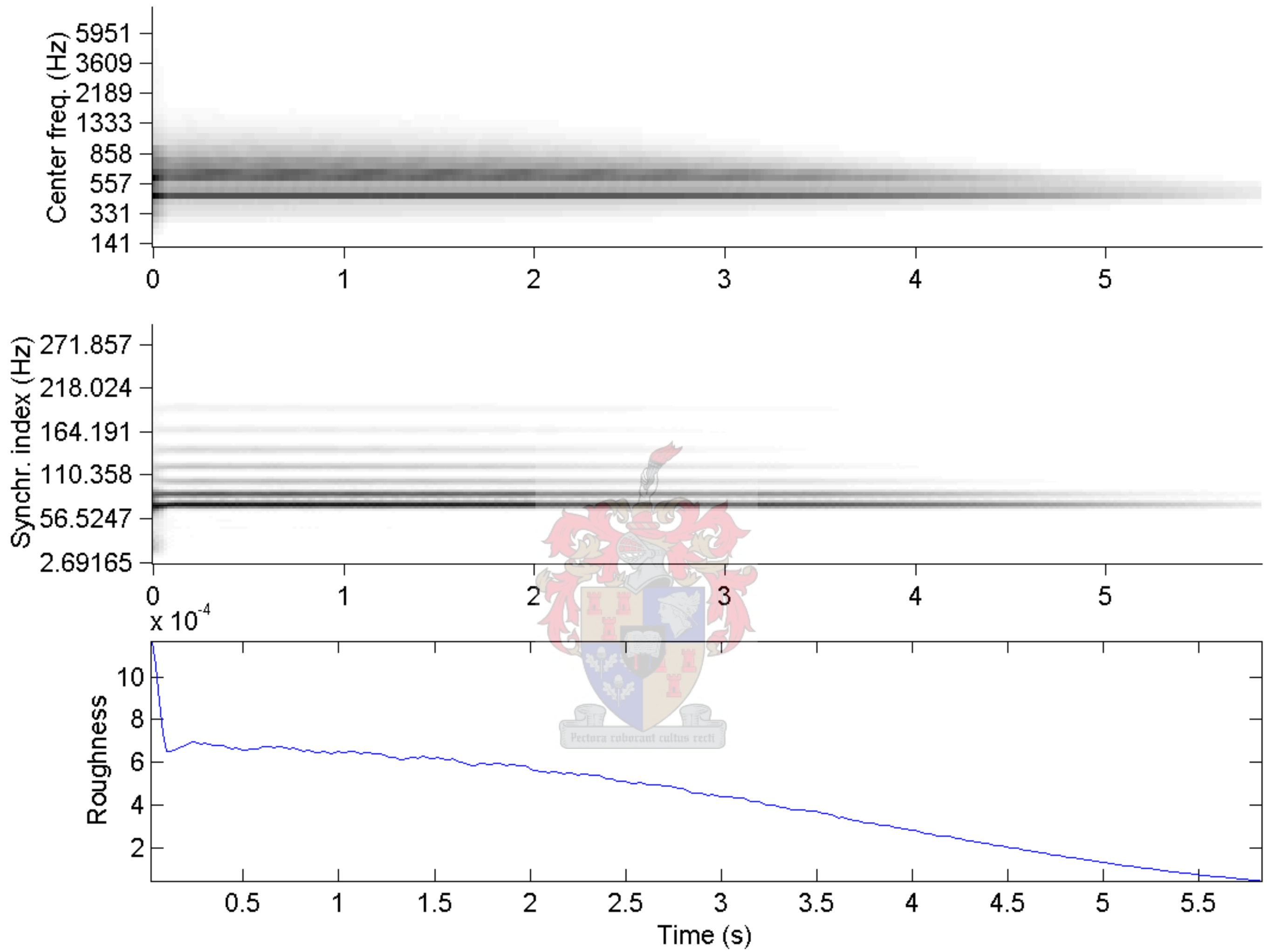
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
14
12
10
8
6
4
2
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 319.2



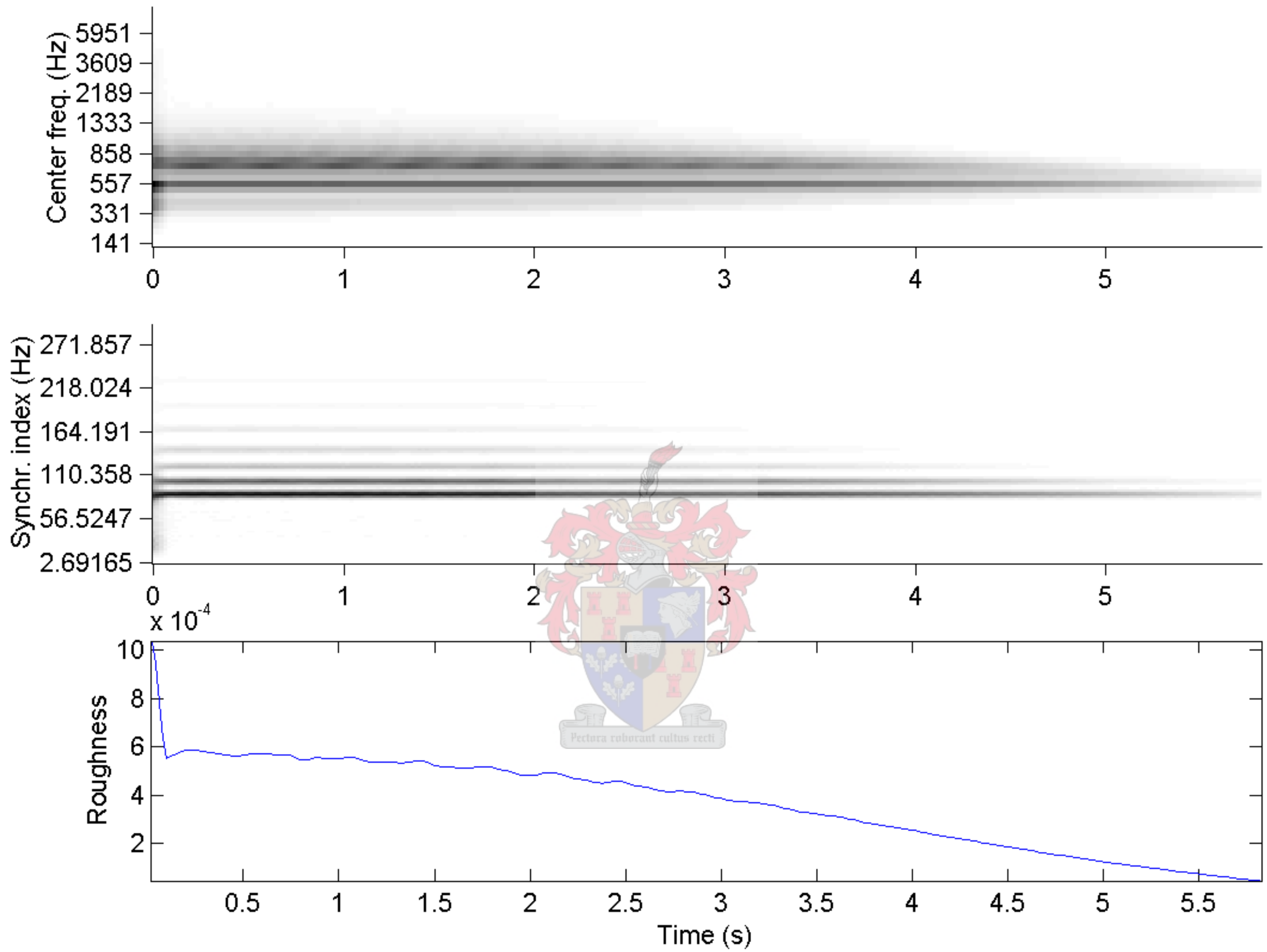
Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 374.8

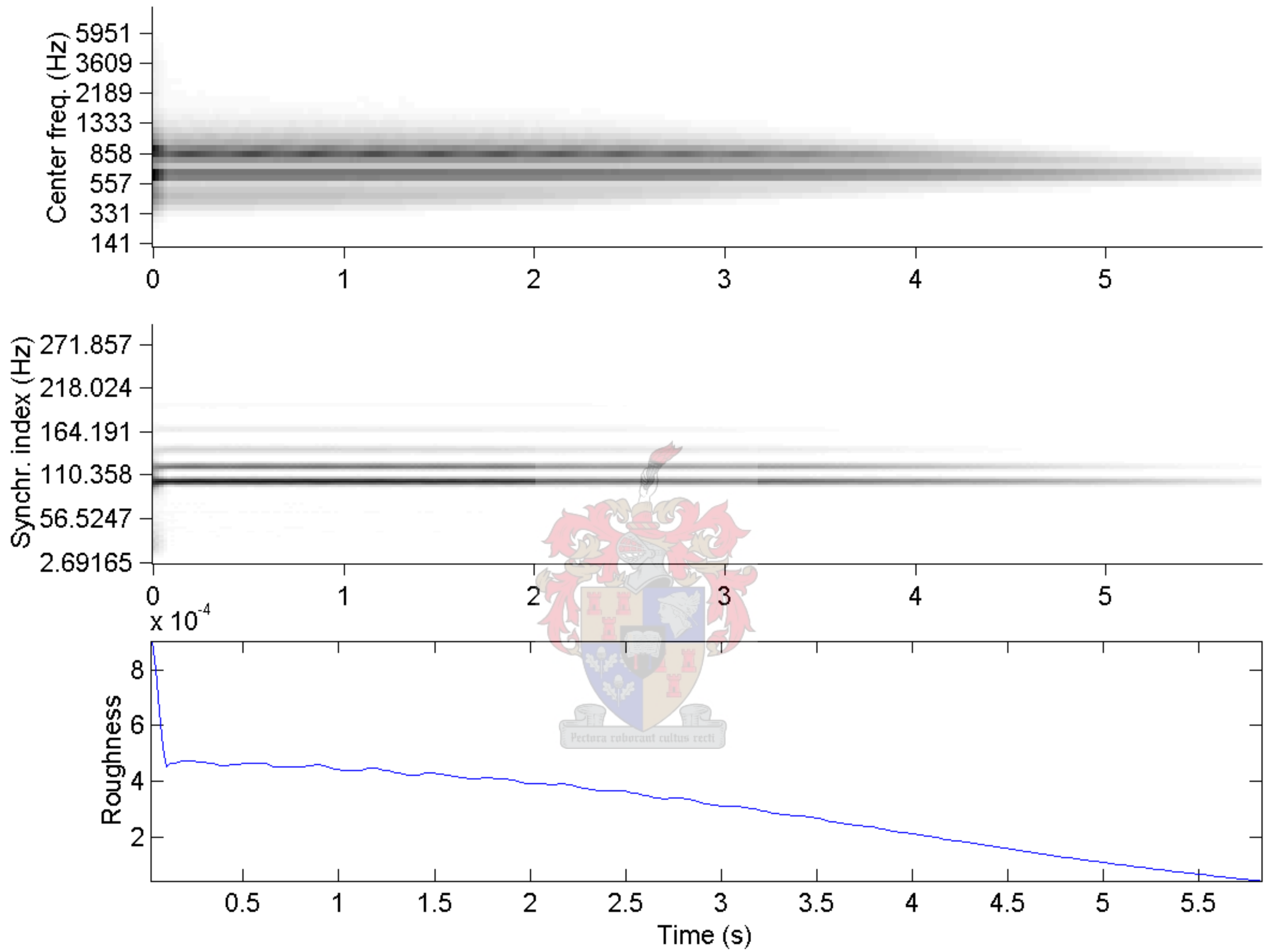


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 440.0


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0 1 2 3 4 5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0 1 2 3 4 5
x 10^-4
Roughness
10 8 6 4 2
0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5
Time (s)

phi 516.6



Center freq. (Hz)
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3609
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1333
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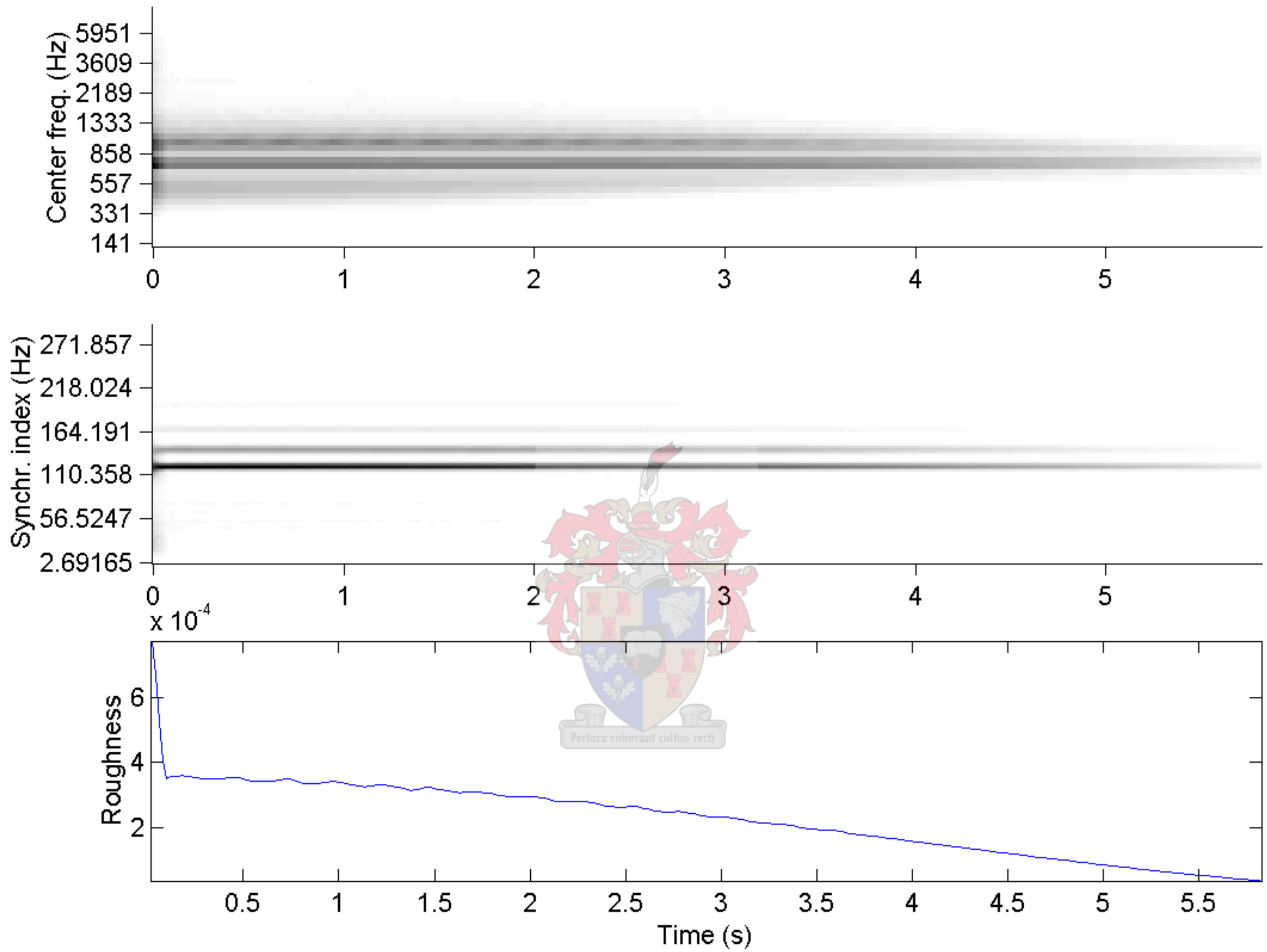
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 606.4


Center freq. (Hz)
5951
3609
2189
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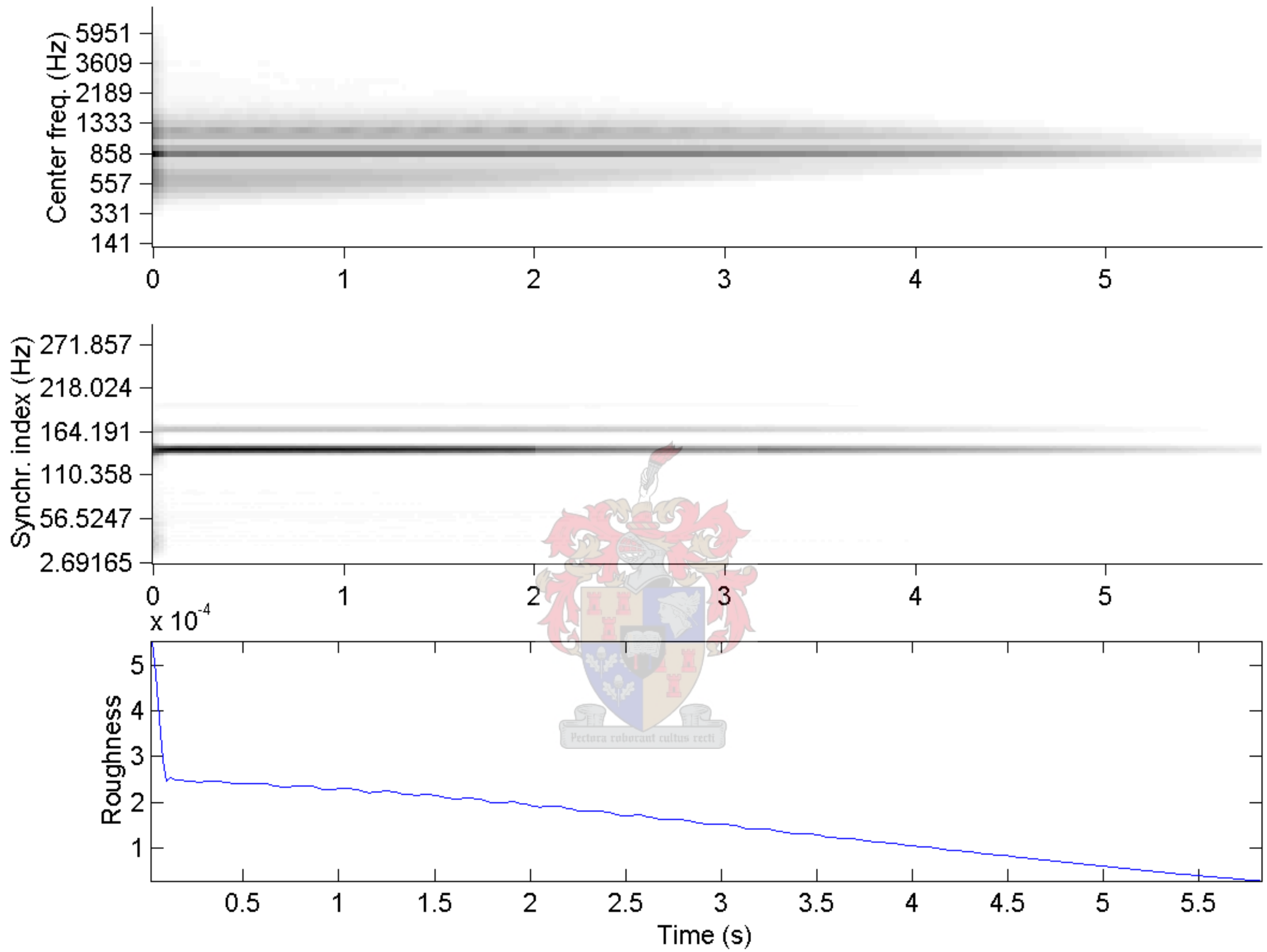
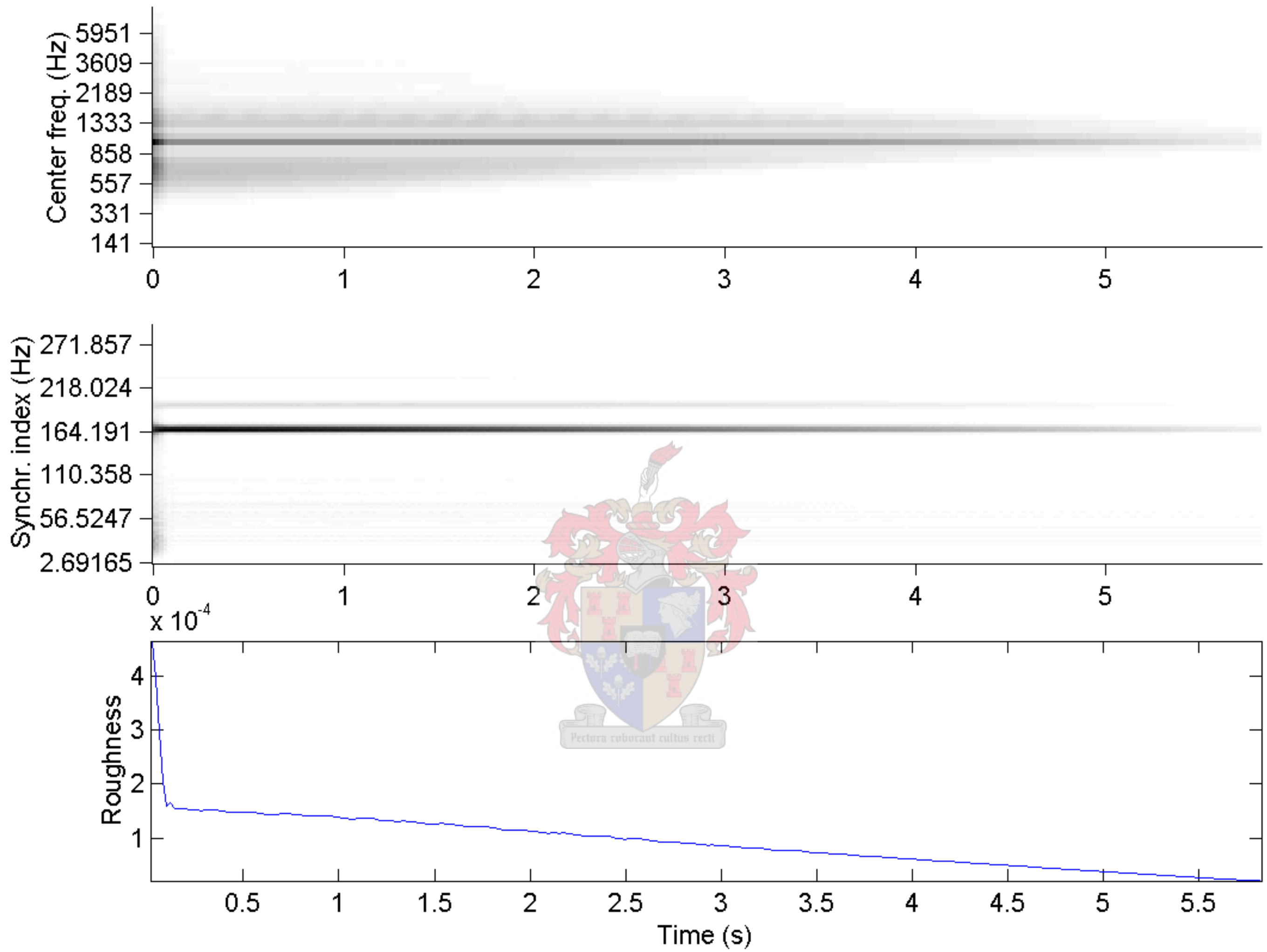
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 711.9



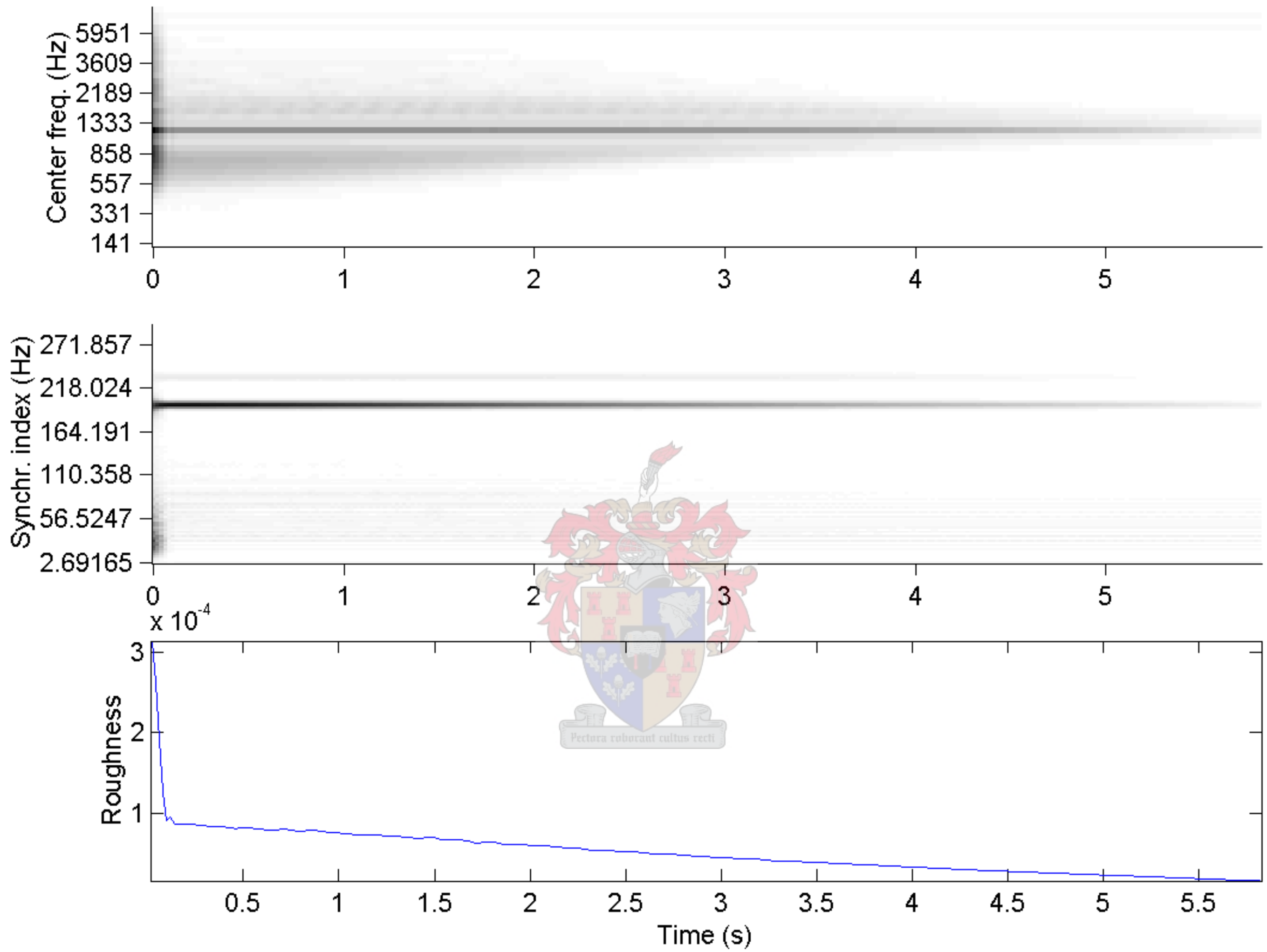
Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
x 10^-4
Roughness
5
4
3
2
1
0.5
1.5
2.5
3.5
4.5
5.5
Time (s)

phi 835.8



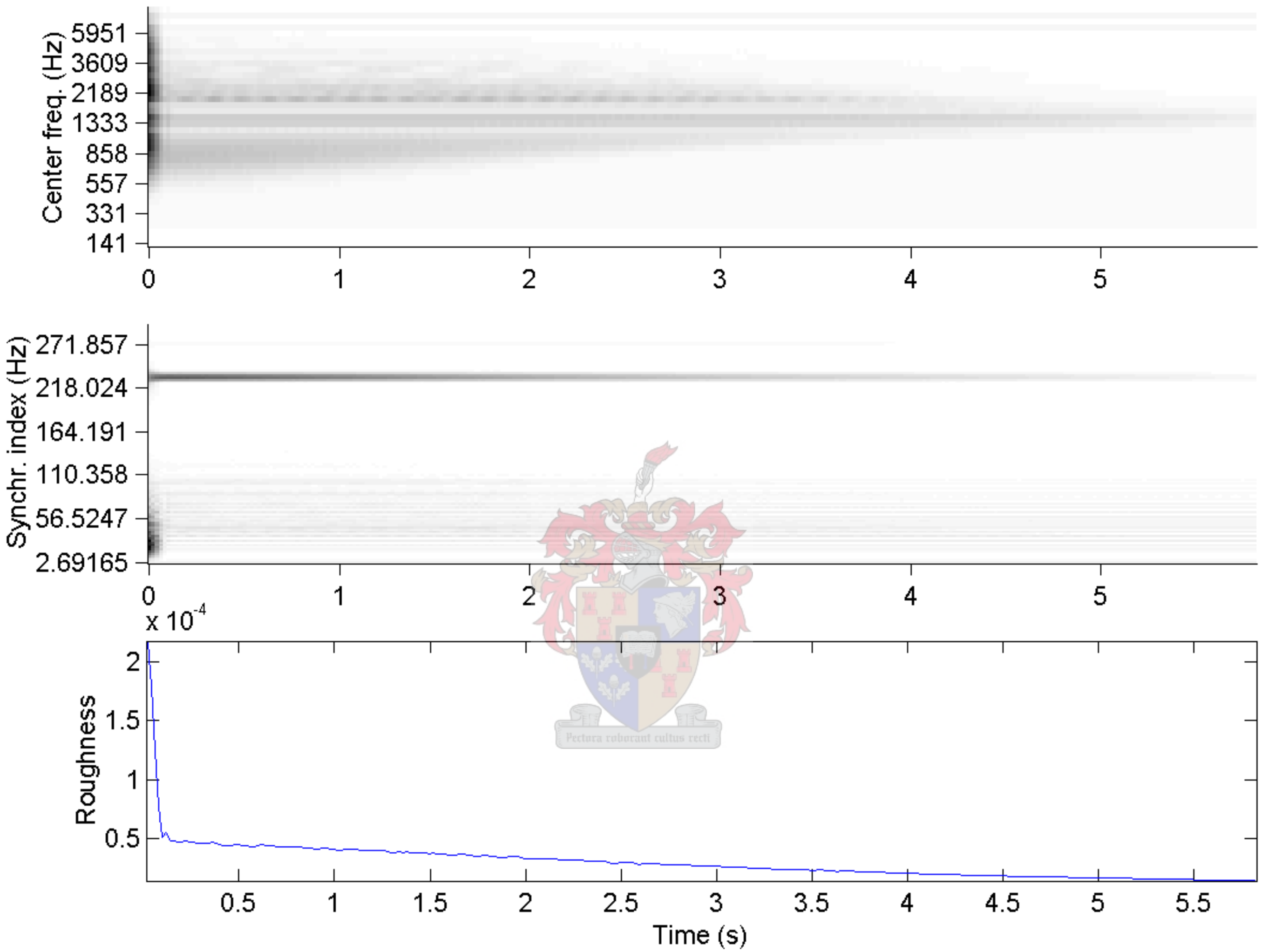
Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
4
3
2
1
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 981.2



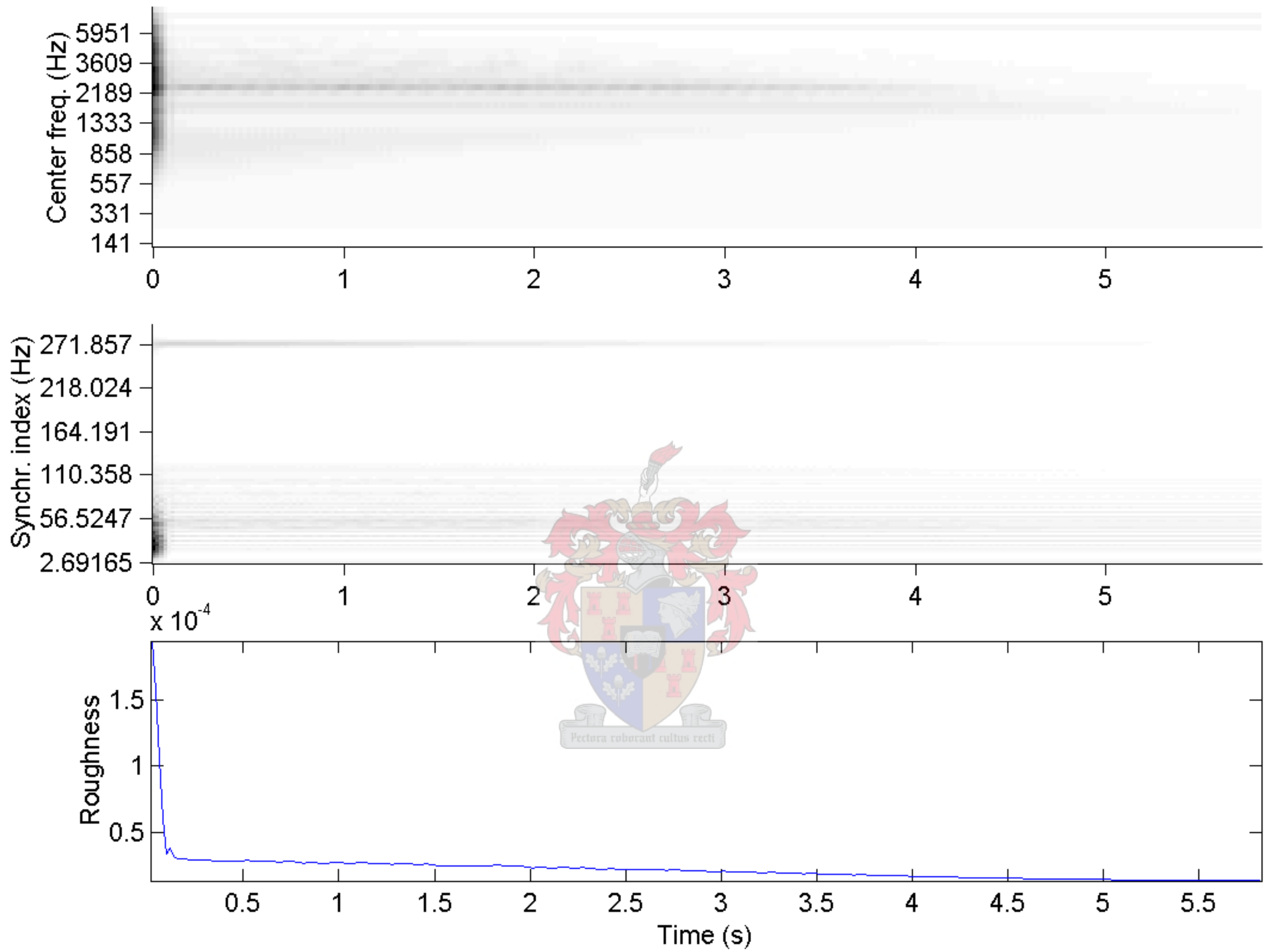
Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
3
2
1
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 1151.9



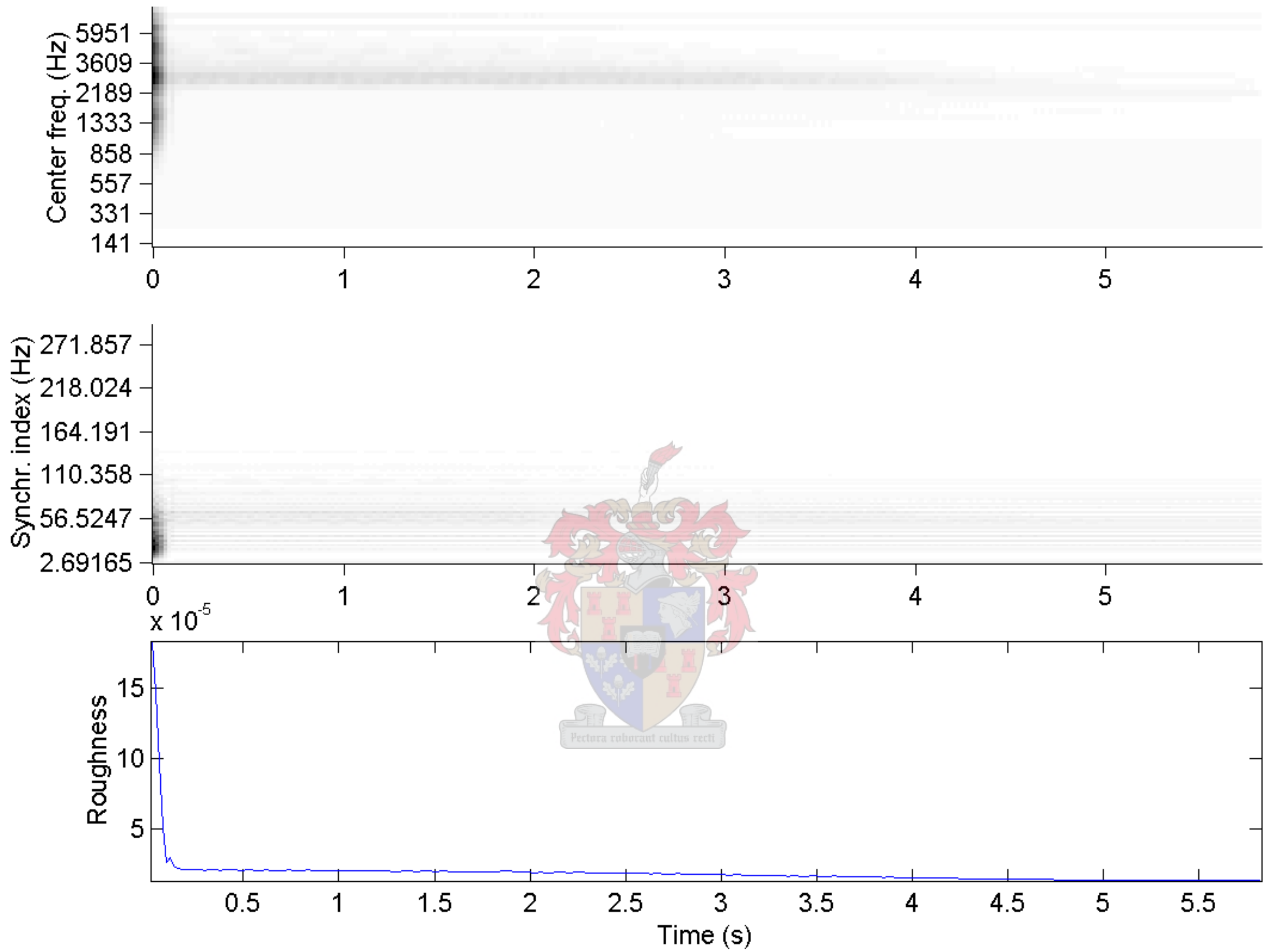
Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
2
1.5
1
0.5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 1352.4



Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-4
Roughness
1.5
1
0.5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 1587.6



Center freq. (Hz)
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3609
2189
1333
858
557
331
141
0
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2
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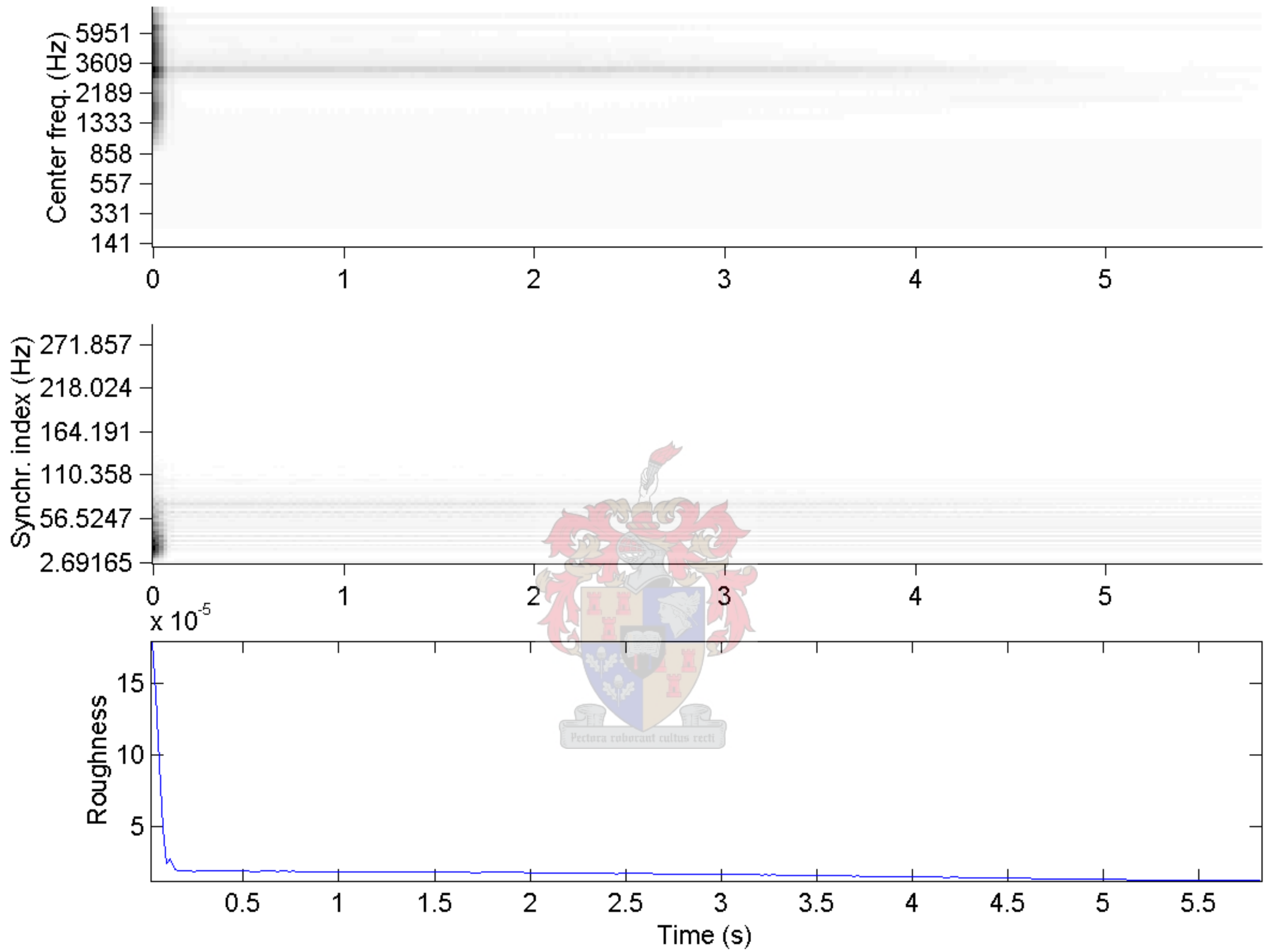
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
15
10
5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 1863.9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557
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141
0
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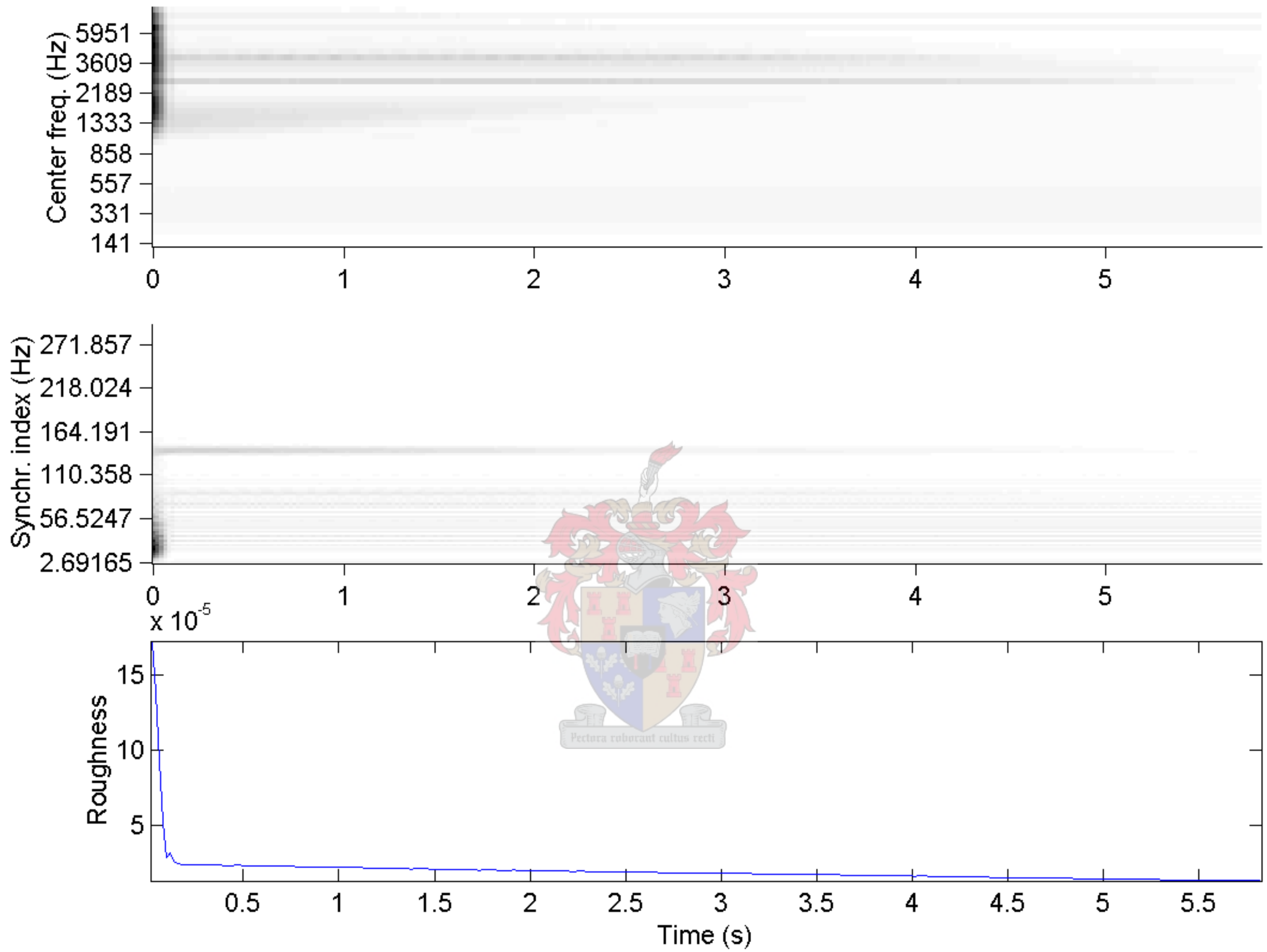
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
15
10
5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 2188.2



Center freq. (Hz)
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3609
2189
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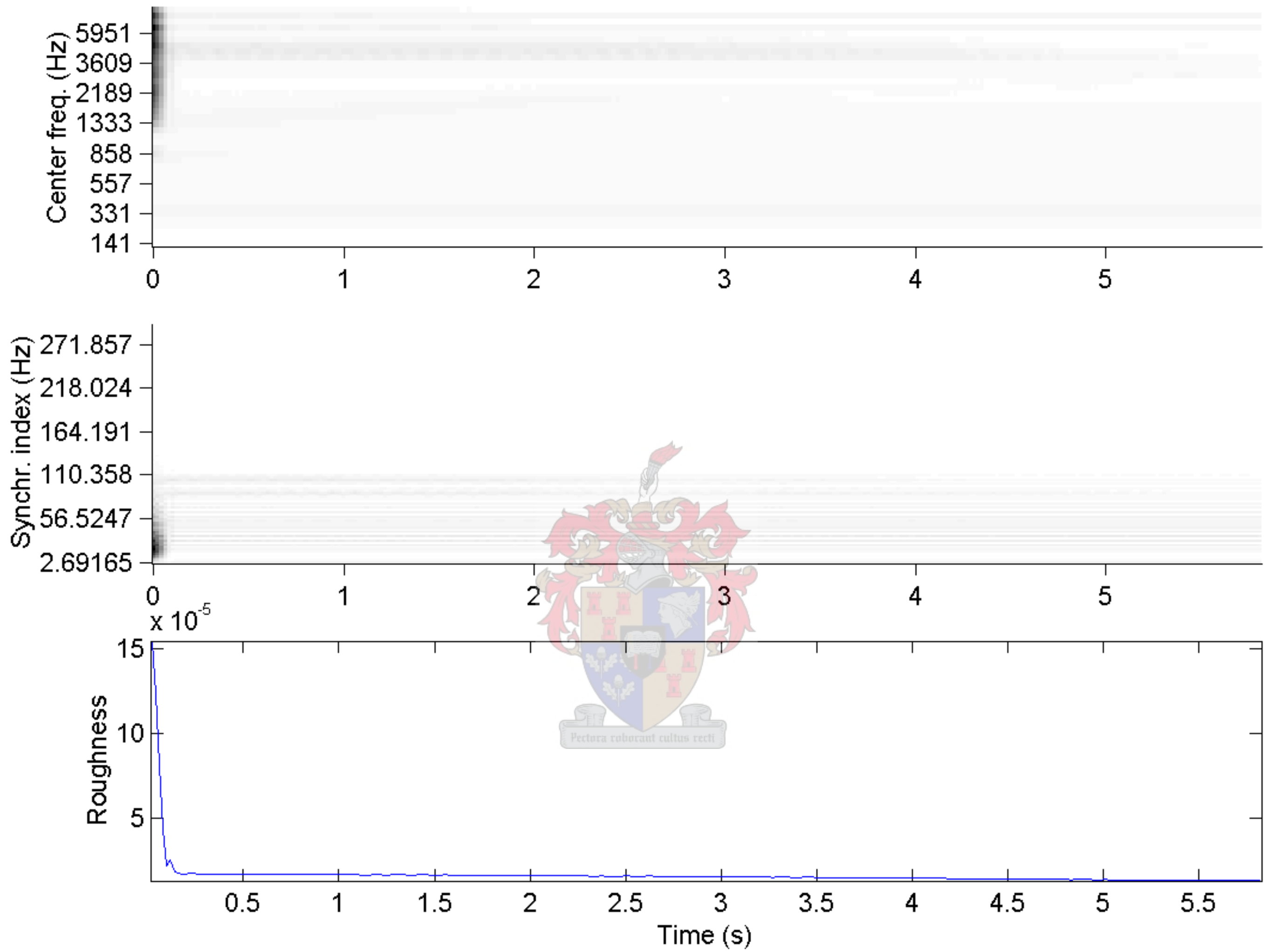
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10-5
Roughness
15
10
5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 2568.9



Center freq. (Hz)
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858
557
331
141
0
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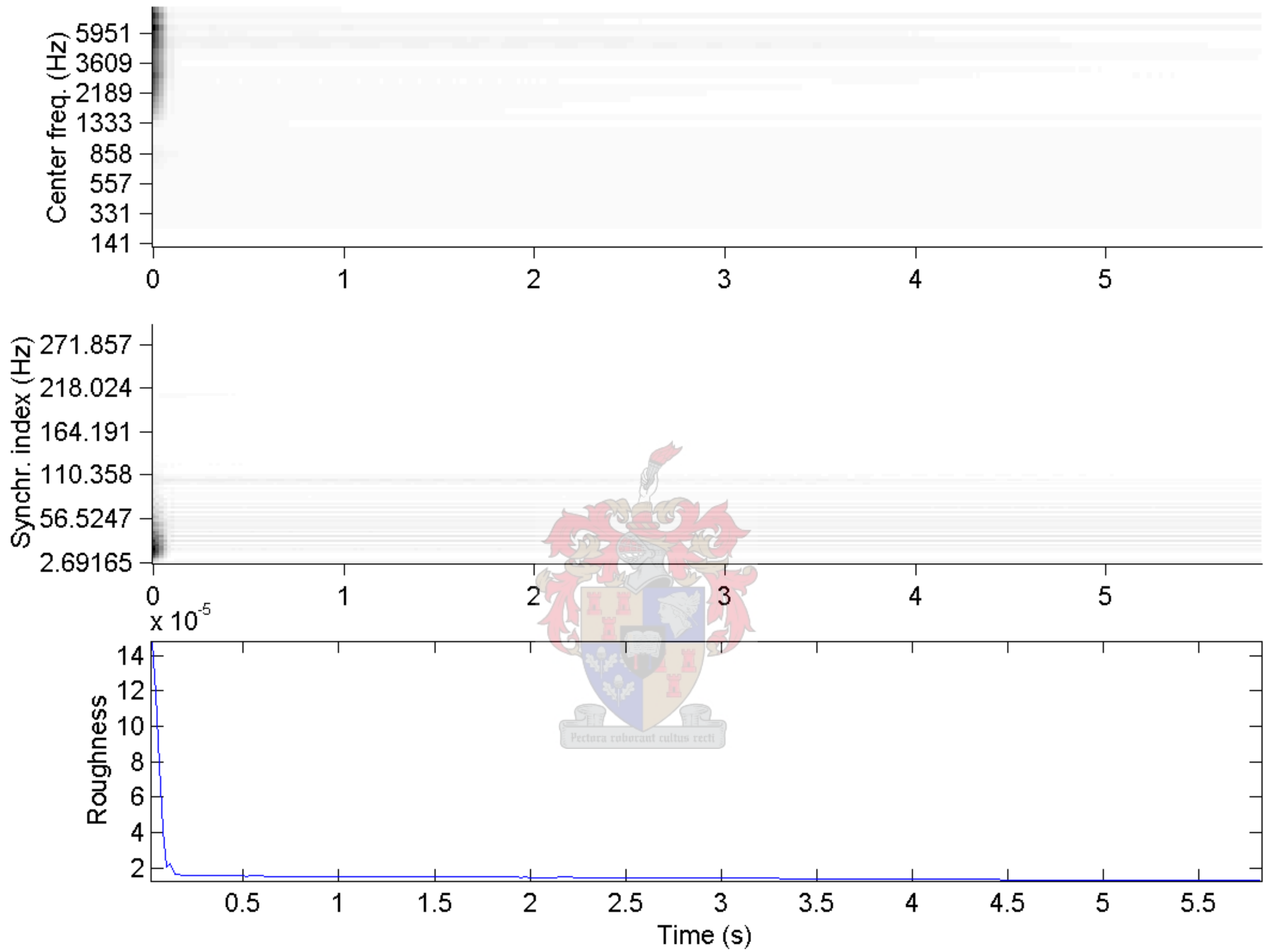
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
15
10
5
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 3015.8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858
557
331
141
0
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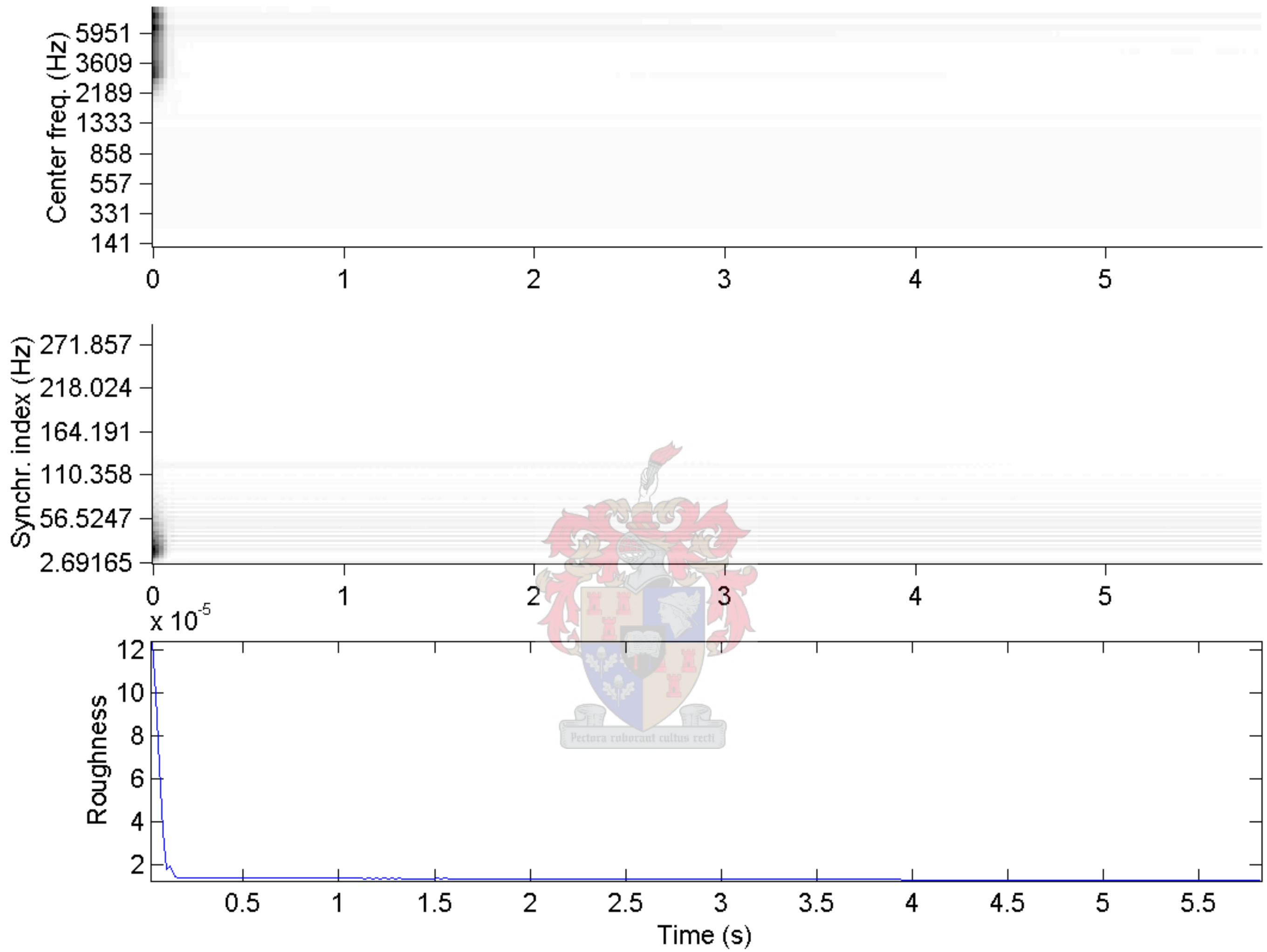
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
14
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 3540.5



Center freq. (Hz)
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858
557
331
141
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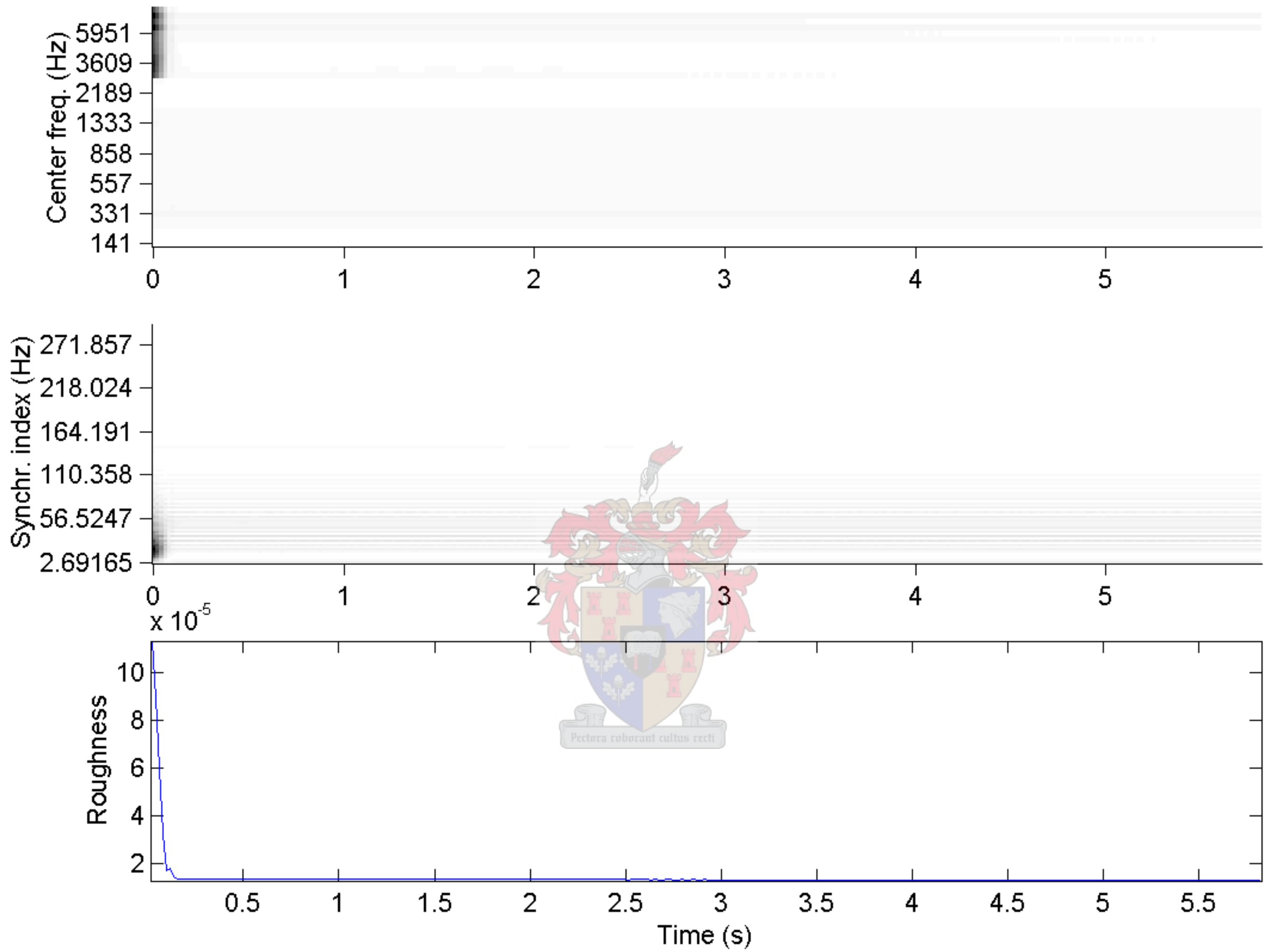
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
12
10
8
6
4
2
Roughness
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 4156.5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858
557
331
141
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4
5
Synchr. index (Hz)
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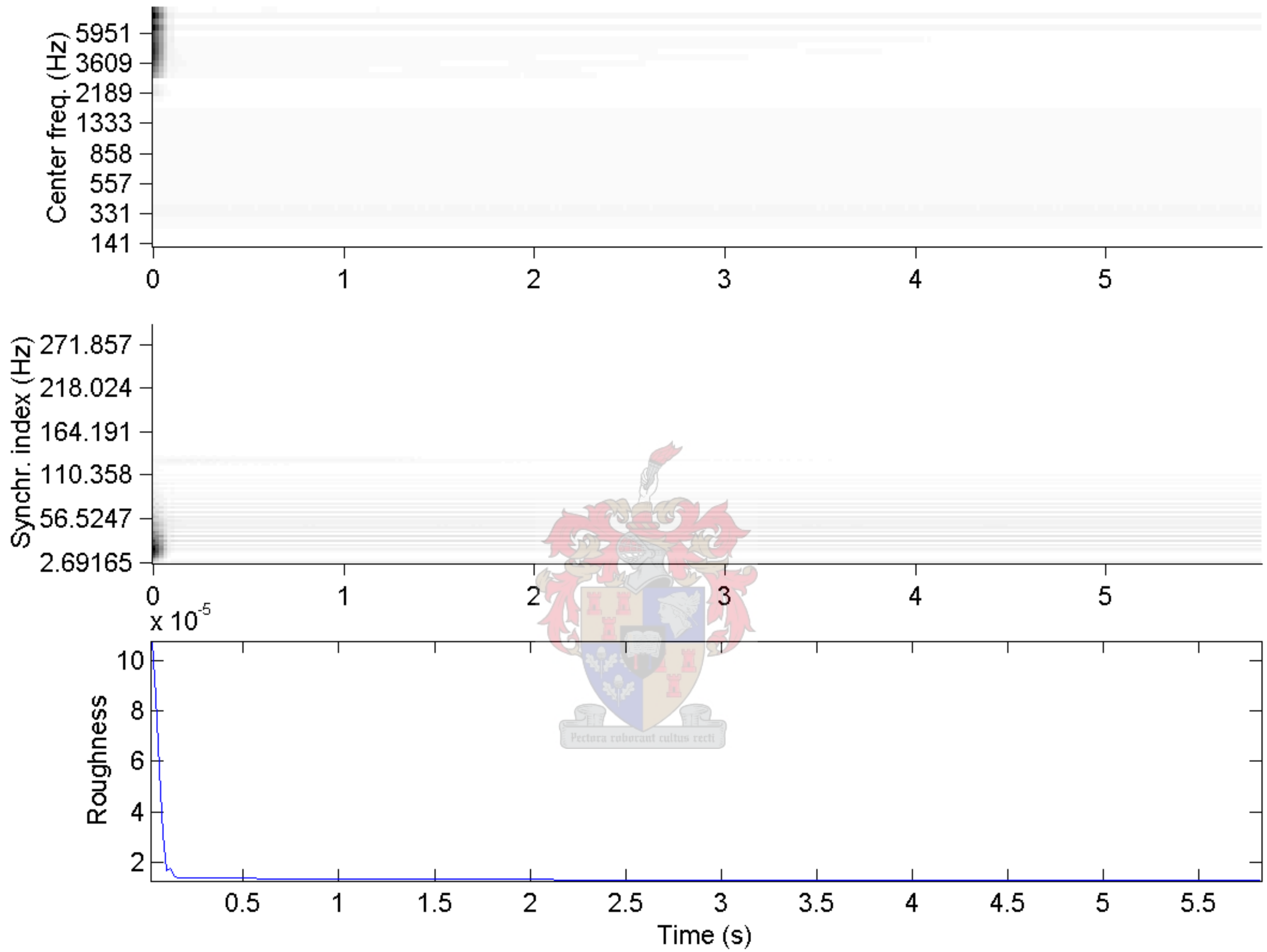
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 4879.7



Center freq. (Hz)
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858
557
331
141
0
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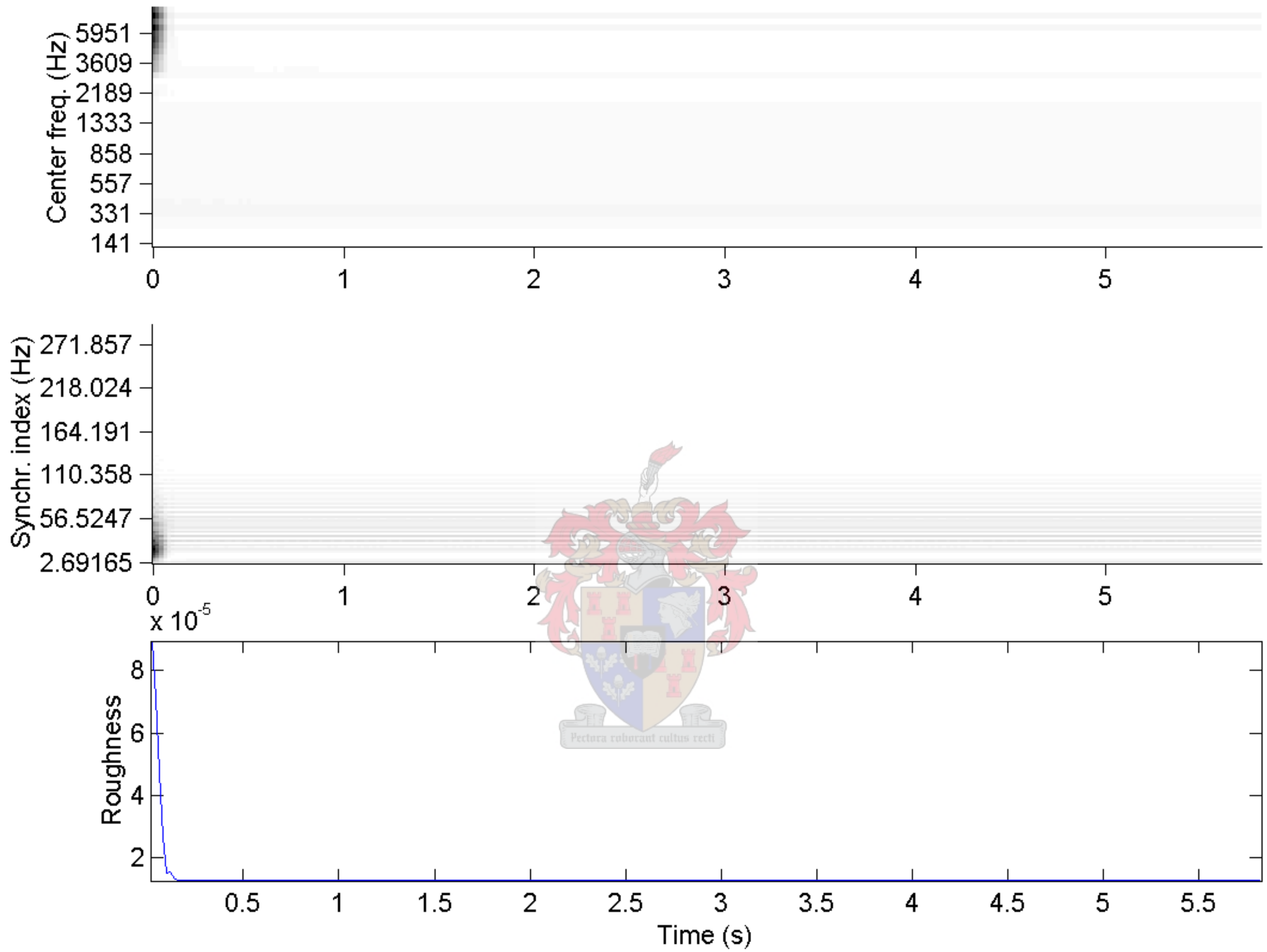
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
x 10^-5
Roughness
10
8
6
4
2
0.5
1.5
2.5
3.5
4.5
5.5
Time (s)

phi 5728.7


Center freq. (Hz)
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557
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4
5
Synchr. index (Hz)
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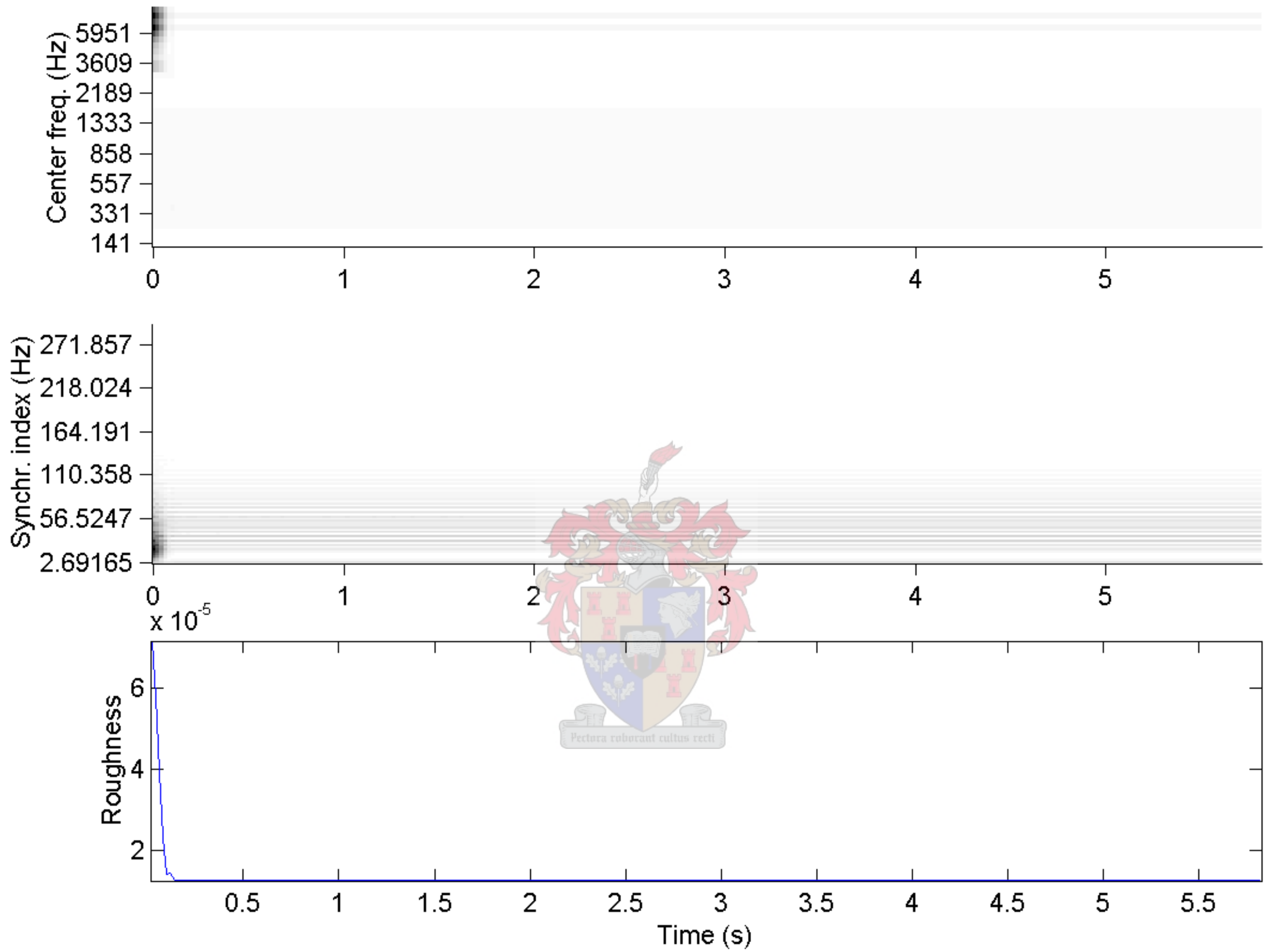
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 6725.4


Center freq. (Hz)
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3609
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858
557
331
141
0
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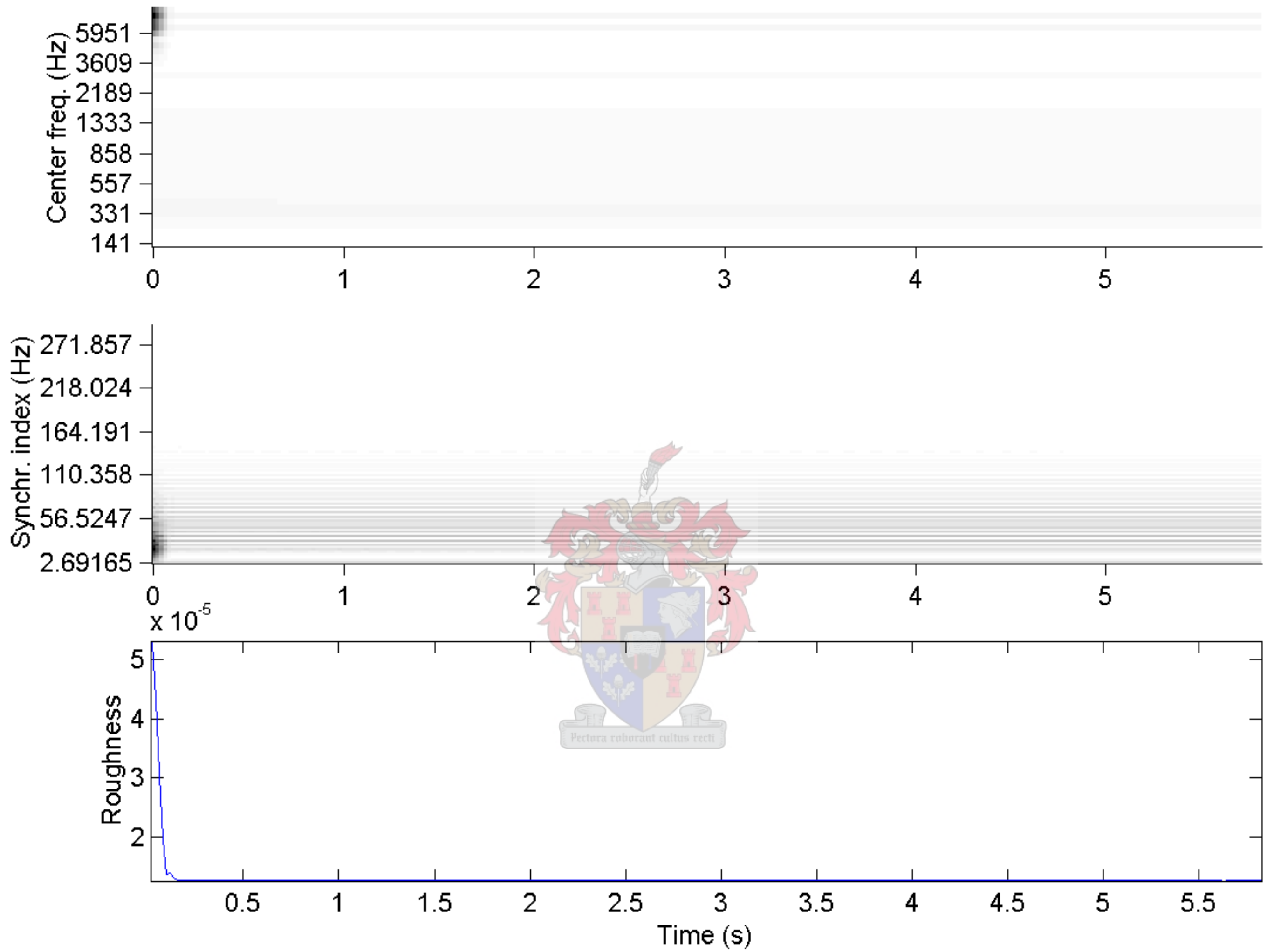
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 7895.5


Center freq. (Hz)
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858
557
331
141
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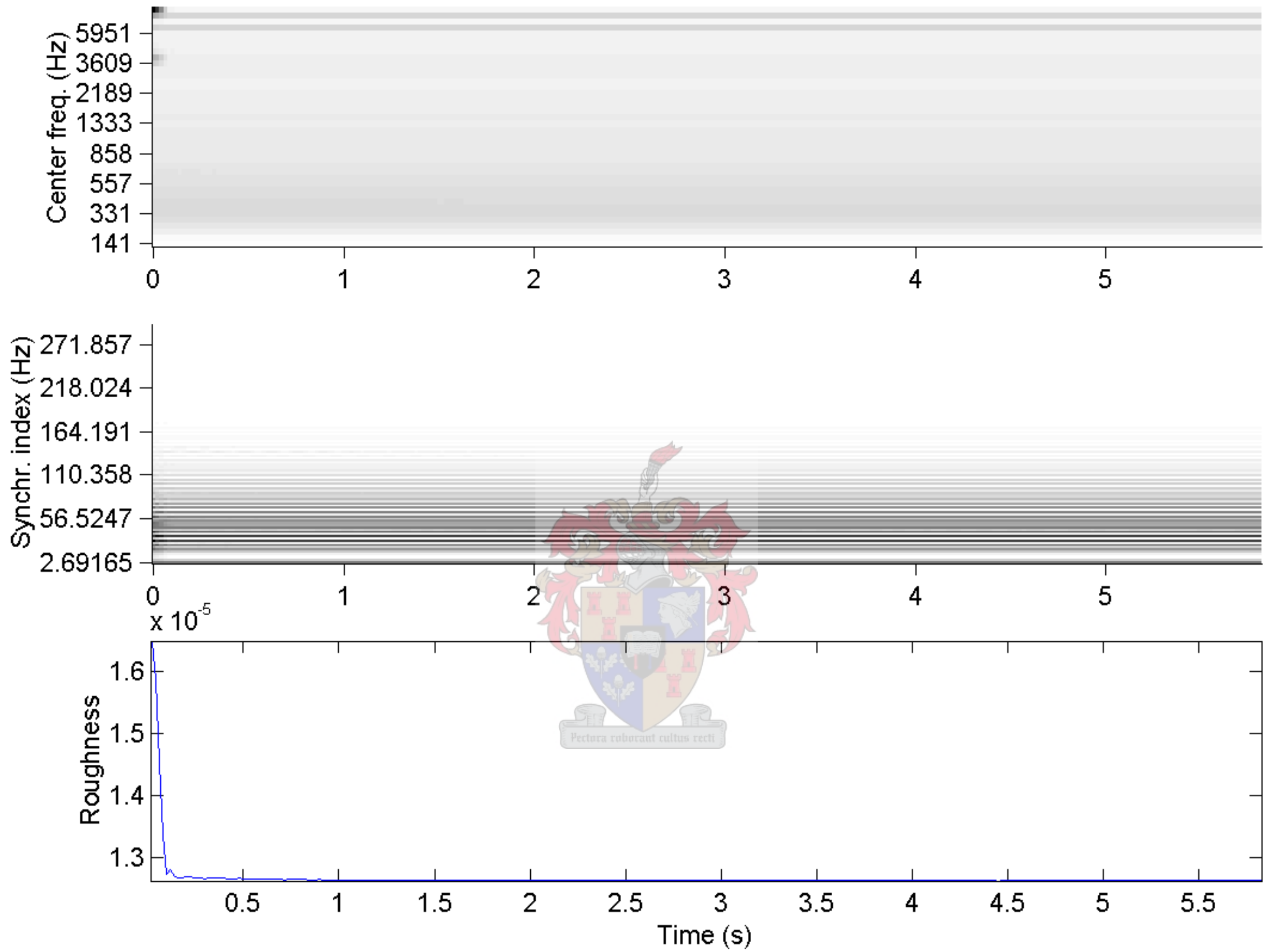
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10-5
Roughness
5
4
3
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 9269.2



Center freq. (Hz)
5951
3609
2189
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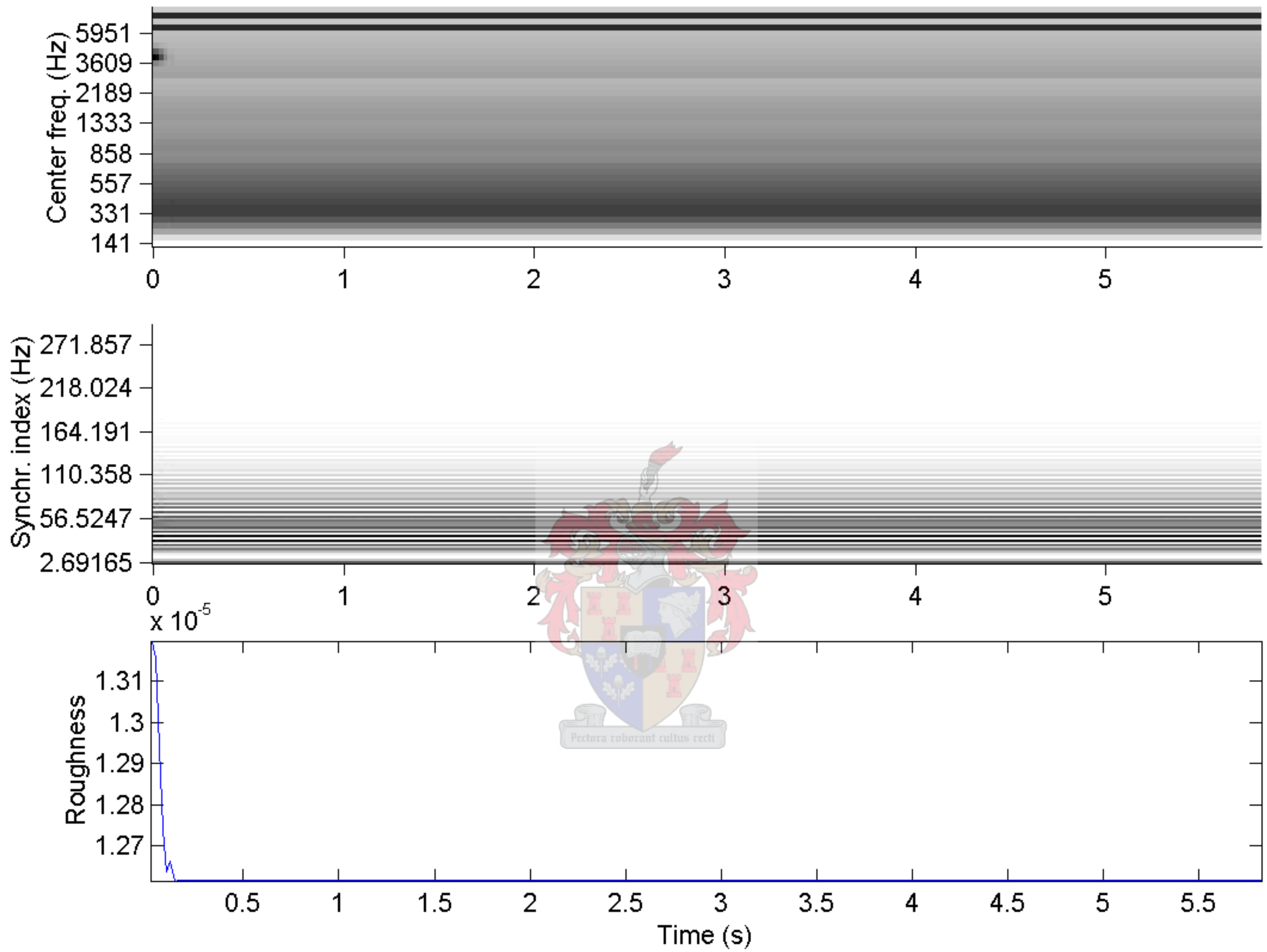
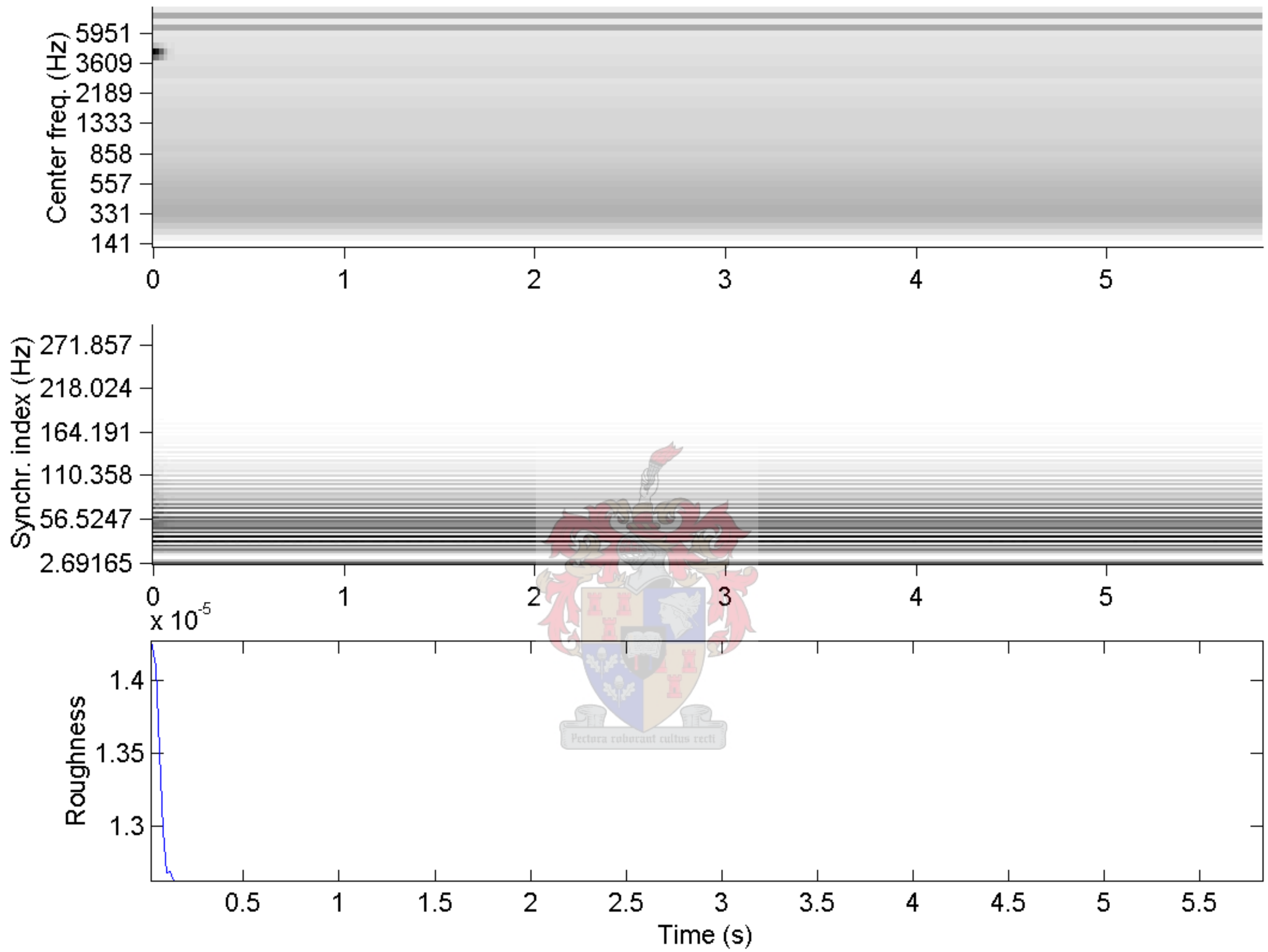
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
1.6
1.5
1.4
1.3
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 10881.9



Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
1.31
1.3
1.29
1.28
1.27
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 12775.2



Center freq. (Hz)
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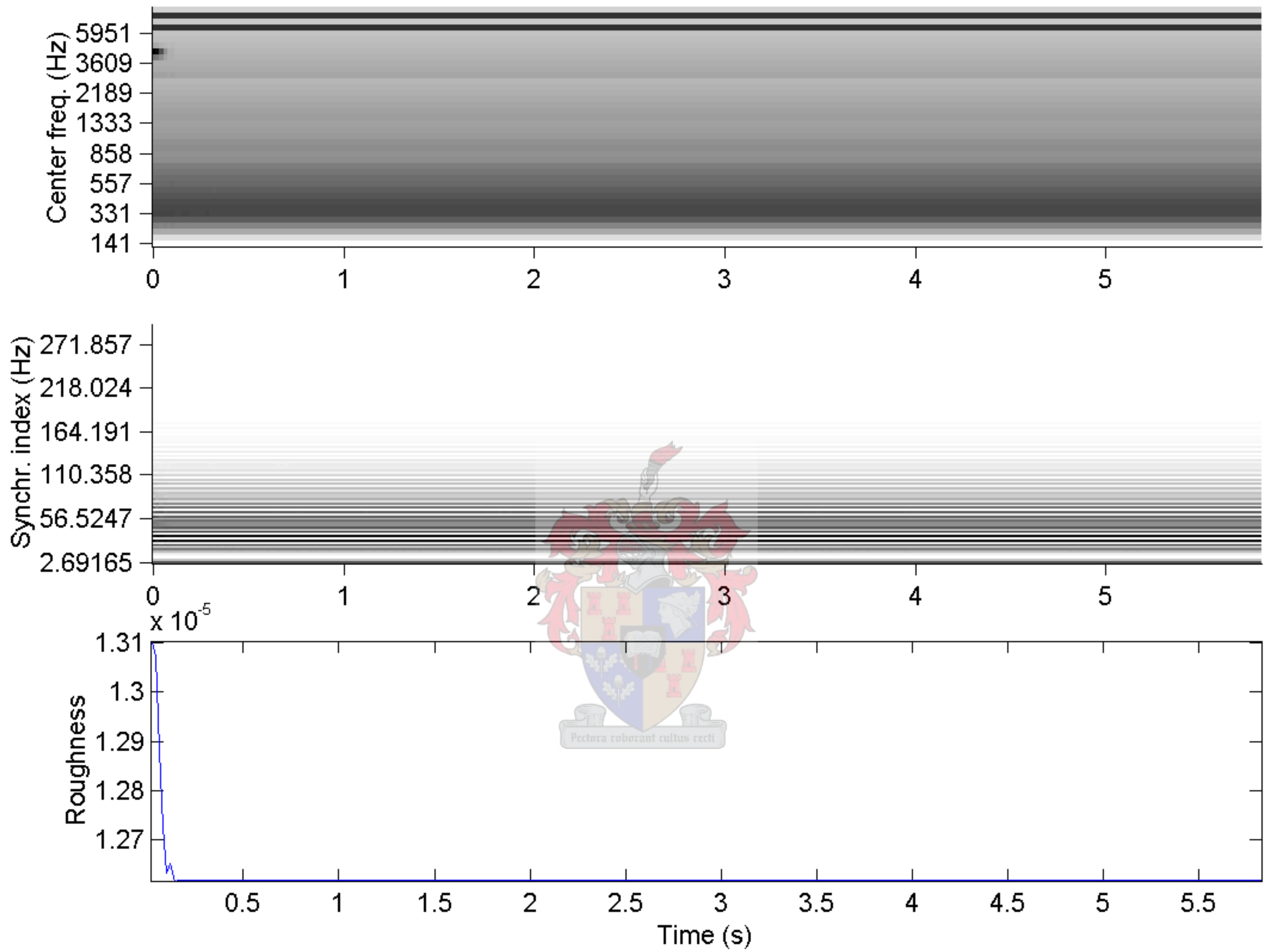
557
331
141
0
1
2
3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
1.4
1.35
1.3
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 14997.8


Center freq. (Hz)
5951
3609
2189
1333
858
557
331
141
0
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3
4
5
Synchr. index (Hz)
271.857
218.024
164.191
110.358
56.5247
2.69165
0
1
2
3
4
5
x 10^-5
Roughness
1.31
1.3
1.29
1.28
1.27
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
Time (s)

phi 17607.2