REDUCING THE TOTAL COST OF OWNERSHIP OF MINING HAUL TRUCKS

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Declaration

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

Signature: Date:

ABSTRACT

The diesel consumption of haul trucks deployed on opencast mines was investigated as a means of reducing the Total Cost of Ownership (TCO) of mining haul trucks. The conceptualisation of TCO and an introduction to the mining operation was presented as an introduction to the field of research. Thereafter, a review of the available literature revealed that linear programming, queueing theory and coast-down testing were applicable means of investigation. The relevant engineering sciences were applied and correlated with experimental and measured data from the Grootegeluk, Sishen and Thabazimbi mines operated by Kumba Resources Ltd (formerly known as ISCOR Mining).

A cost-driver model for diesel consumption was formulated by exploiting the expert judgement of role players in the mining operation. A cost-driver model was developed for the Sishen, Thabazimbi and Grootegeluk mines. The cost-driver models were then modelled as a linear programming problem and solved using the student version of LINDO Optimization Software. The results were discussed and a universal diesel cost driver model was formulated by consolidating the individual diesel cost driver models.

The operational cycle of haul trucks was simulated in order to quantify equipment utilisation and reduce diesel consumption of the mining vehicles. The operational cycle of haul trucks was modelled utilising queueing theory. The simulation of the queue network was implemented in Matlab using the next event advance method and was called Q_Sim. Q_Sim was utilised to investigate optimal fleet size and the economies of scale of haul truck capacity.

The results of coast down tests were analysed in order to determine the effect of treating mining roads, with a bitumen product, on rolling resistance coefficient.

Finally, recommendations for further research are proposed. This includes further refinement of the diesel cost-driver model, expanding the scope of application of Q_Sim in the mining operation and further investigation of dust reduction by bitumen products.

OPSOMMING

Ondersoek is ingestel na die dieselverbruik van myntrokke sodoende die Totale Koste van Eienaarskap (TKE) van myntrokke te verminder. Die konsep rondom TKE is bespreek en 'n inleiding tot die mynybedryf is aangebied sodoende die verskeie aspekte in verband te bring. `n Literatuur studie het gevolg, wat aangetoon het dat lineêre programmeering, toustaan-en loswieltoetse toepaslike navorsingsmetodes is om die nodige resultate te verkry. Daarna is hierdie ingenieurswetenskappe toegepas en `n vergelyking is tussen die eksperimentele en gemete data van die Grootegeluk, Sishen en Thabazimbi myne getref.

`n Diesel koste-drywer model is opgestel met die insette van kundige rolspelers in die mynbou bedryf. Dit het gelei tot Koste-drywer modelle vir die Sishen, Thabazimbi en Grootegeluk myne. Die modelle is met lineêre programmeering as probleemstelling daargestel en is deur middel van die studente weergawe van die LINDO optimaliseringssagteware opgelos. Die resultate was toe bespreek en daarvolgens is `n universele diesel koste-drywer model opgestel deur die reeds-geskepte modelle te konsolideer.

Die operasionele siklus van myntrokke was gesimuleer sodoende die benutting van toerusting te kwantifiseer en die dieselverbruik van myntrokke te verminder. Hierdie operasionele siklus was gemodelleer deur middel van die toustaan-teorie. Die simulasie van `n toustaan-netwerk was in Matlab gevoer deur Q_Sim te gebruik. Hierdie metode was gebruik om die optimale vloot grootte en die invloed van myntrokkapasiteit te ondersoek.

Die ontleding van die loswieltoetse was gedoen om die invloed van 'n bitumen produk op rolweerstand te bepaal.

Ten slotte is aanbevelings vir toekomstige navorsing bespreek. Dit behels die verder aansuiwerings van die diesel koste-drywer model, die uitbreiding van aanwending van die Q_Sim in die mynbou en `n verdere ondersoek om stofvoorkoming in die mynbou te bewerkstellig deur die gebruikmaking van bitumen produkte.

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1 INTRODUCTION

Minerals are extracted from the earth by mining operations for beneficial utilisation. These minerals occur at various depths below and amongst other rock and soil of lesser importance. Opencast, or surface mining is the removal of the overlying rock and soil in order to extract the mineral. Haul trucks are used to transport ore as well as displaced soil and rock (overburden) in opencast mines. The hauling of overburden and ore from the open pit is one of the largest expenses incurred during the mining operation. In some instances it can be the greatest single expense, amounting to as much as 28 percent of the total direct operating cost being spent on haul trucks [Farquier 1989]. The Total Cost of Ownership of these commodities must be managed and minimised in order to sustain economic feasibility of any mining operation.

Total Cost of Ownership can be thought of as the integrated operating cost of the mining operation i.e. the cost per ton of product, be it iron ore or coal. For the purposes of this study overhead and administrative costs such as salaries, insurance and consumable office supplies will be assumed to be a constant and therefore not considered.

The mining industry in South Africa have committed themselves to better management of their fuels and lubricants with the aim of driving down the Total Cost of Ownership (TCO) of these commodities. This has led to considerable cost savings [Milne 2000, Milne 2001]. One of the measures to reduce TCO is to outsource *"the supply, storage and dispensing of fuel and lubricants on the mine"* [Milne 2000, Milne 2001]. In addition the fuel and lubricant contractor (Contractor) is responsible for *"a fuel management system and the supply of various data processing services and management information to the mine"* [Milne 2000, Milne 2001].

The Unit for Vehicle Propulsion (Eenheid vir Voertuig Aandrywing) (EVA) situated at the Department of Mechanical Engineering, University of Stellenbosch in conjunction with the Stellenbosch Automotive Engineering (Pty) Ltd (trading as CAE) is responsible for providing technical support to the Contractor. Technical support encompasses the capture of raw data, the collation as well as the processing and analysis of the raw data with the aim of determining the dominant cost drivers of fuel consumption and the modelling thereof for optimisation and prediction purposes.

1.1 Objective.

The aim of this study is to determine suitable advanced analysis techniques and to perform knowledge based analysis of the fuel consumption and mining production data in order to reduce the TCO of mining haul trucks.

1.2 Scope and Outline.

Chapter 2 begins with an outline of the mining operation will be briefly discussed to develop a sound understanding of the topic. As a result of this outline the following engineering sciences were identified to fulfil the aim of this study:

- Cost driver Modelling
- Wheel and Tyre Theory
- Resource and Production Scheduling

Secondly, a review of the related engineering sciences is presented in the remainder of Chapter 2.

The relevant engineering sciences are then applied and correlated with experimental and measured data from the Grootegeluk, Sishen and Thabazimbi mines operated by Kumba Resources Ltd (formerly known as ISCOR Mining).

A Linear Programming (LP) formulation of the dominant cost drivers of diesel consumption will be presented, in Chapter 3, for the Thabazimbi, Sishen and Grootegeluk mines. The LP problems will be based on Diesel Forums facilitated by CAE at the mines. The aim of the forum was to identify the dominant cost drivers of diesel consumption. The subsequent LP problem is solved using the "Optimizer" tool in Corel Quattro Pro 8 and the Student Version of LINDO Optimization Software. The results for the three mines are compared and interpreted at a qualitative level.

Chapter 4 covers a "scheduling"- type problem for the daily operating cycle of a mining vehicle in the pit. The physical layout of the mines can be extremely dynamic. As the mining operation progresses the working area changes and the optimum deployment of mining vehicles, to achieve operational requirements, has to be continually revised. It was discovered that the problem leant itself to queueing theory. The daily operational cycle of mining vehicles is simulated using queueing theory coded in the Student Edition of Matlab 6.

All the roads in the operational zone of the mine are dirt roads; however frequently used roads on the Thabazimbi mine have been sprayed with a bitumen compound, called Dust-A-Side. Dust-A-Side reduces the amount of dust kicked up by the haul trucks conveying unearthed material. Vehicles travelling on roads treated with Dust-A-Side may sustain a change in rolling losses compared to travelling on unsprayed roads.

The individual resistive forces acting on a vehicle are [Phelps et al. 1977]:

- Aerodynamic
- Grade
- Cornering
- Inertia
- Chassis friction
- Rolling

Of these resistive forces Rolling resistance is the predominant force at speeds below 60 km/h [Thompson et al.1977, Phelps et al. 1977].

Available literature on Tyre Rolling Resistance utilises results from tests with passenger cars and passenger buses, where measurements are taken after tyres are allowed to warm up to reach an equilibrium temperature. A typical warm-up involved driving 30 km at vehicle speeds greater than 50 km/h [Thompson et al. 1977]. This differs from the mining vehicle situation where the vehicles utilised are larger and operating speed and cycle differs. Previous studies utilising coast-down tests recommended research be furthered into different vehicles, especially heavier vehicles [Shear 1987 and Du Plessis 1993].

Coast-down tests were therefore performed using haul trucks to investigate the change in rolling losses on the two surfaces. The results of these coast-down tests will be discussed in Chapter 5.

On Grootegeluk and Sishen mines the trucks carry the ore or coal uphill and make use of a Trolley Assist Technique commonly referred to as a Pantograph to power the haul truck during the ascent. Pantograph enables the vehicle to draw electricity directly from overhead power supply lines and an extendable trolley mounted on the roof of the haul truck. This configuration is similar to that of an electric locomotive. The relevance of pantograph will be investigated in the literature review of Chapter 2.

In short, the aim of this study is twofold. Firstly *determining* suitable techniques for the analysis of fuel consumption and mining production data and, secondly *performing* knowledge based analysis using the techniques identified. The above is done with the intention of reducing the TCO of mining vehicles. It is evident that the outline discussed will meet these aims.

2 REVIEW OF AVAILABLE LITERATURE

In this Chapter we discuss the engineering sciences relevant to this study. In section 2.1 we give a definition of Total Cost of Ownership (TCO) and in section 2.2 an introduction to the mining operation is presented. The remainder of Chapter is devoted to the review of available literature regarding optimisation and linear programming (section 2.3), tyre and wheel theory (section 2.4) and simulating the operational cycle of mining vehicles (section 2.5). The relevance of optimisation to the study is discussed in section 2.3 and the theory in Appendix G. An overview of statistical analysis is presented in Appendix H.

2.1 The Total Cost of Ownership.

The Total Cost of Ownership (TCO) is a concept utilised to account for all costs associated with owning, operating and maintaining a product, system or service [Rockwell 2002-1, Trane 2002]. When referring to the TCO it is common to refer to 'Cradle to Grave' costs [Rockwell 2002 - 1], implying taking into account long-term costs which are often neglected in 'once off' capital purchases.

Initial purchase cost, installation cost, financing cost, commissioning cost, energy costs, repair costs and maintenance costs are the main elements of a TCO [Trane 2002]. Also included are productivity costs, risk cost and disposal costs. TCO is always more than the initial purchase price and can be several times larger than the initial purchase price. The Initial Purchase Cost is often only 20 – 30 % of the TCO [Rockwell 2002-2]. This poses a challenge as initial costs are well documented but it is difficult to measure the long-term costs of products hence the proliferation of commercial software aimed at tracking these 'hidden' costs.

When a decision is made to acquire a new product, system or service, a TCO analysis is an efficient method of determining the acquisition with the lowest total cost. It is however necessary to state that a TCO analysis does not reflect the financial benefits that come from increased incomes, increased business volume, improved customer satisfaction or increased competitiveness, or improved public image. Thus a TCO comparison of options is applicable when all possible options differ only in cost [business case FAQ's: Solutionmatrix.com 1999]. The TCO should thus be compared with the Total Benefit of Ownership to differentiate between options.

TCO analysis of an existing asset is an efficient manner in reducing the TCO of that asset. A TCO analysis will enable the user to identify hidden costs and identify relevant energy costs. This will enable the user to identify opportunities for improvement. These opportunities can then be weighed up by their respective contribution to the TCO. In light of the above, initial purchase cost, installation cost, financing cost, commissioning cost, energy costs, repair costs and maintenance costs as well as disposal costs and productivity costs are the main elements of TCO for mining vehicles. Of these elements, initial purchase cost, installation cost, financing cost, commissioning cost, and disposal costs are fixed (or may be assumed to be fixed) parameters in a TCO analysis. Energy costs, repair costs and maintenance costs as well as productivity costs are the remaining elements that can lead to a reduction in TCO. Productivity is defined as output divided by input [Thomas et al. 1994]. Productivity costs reflect a loss of productivity due to an act/omission of owning and deploying an asset and are not as tangible as the other remaining elements thus this element requires careful quantification.

Diesel Consumption of the Haul Trucks is the predominant Energy Cost of TCO. Factors that affect the cost of diesel consumption are known as *cost drivers* [Roy et al. 1999-1]. To reduce the cost of diesel consumption it is necessary to identify and manage the cost drivers [Roy et al. 1999-1].

Cost drivers may be quantitative or qualitative. The effect of quantitative cost drivers can be modelled by physical values whereas the effect of qualitative cost drivers is determined by approximate methods and expert judgement [Roy et al. 1999–1]. Although the qualitative cost drivers are assigned numerical values by the application of approximations, they are however still only qualitative measures [Roy et al. –2]. Since quantitative cost drivers are tangible, unbiased quantities, deterministic relations between quantitative cost drivers and the affected cost can be derived. Cost reduction efforts therefore tend to focus on quantitative cost drivers.

Qualitative cost drivers represent tacit (hidden) cost implications that are overlooked. It is for this reason, that the qualitative cost drivers of diesel consumption will be investigated. The investigation into the qualitative cost drivers is based on the philosophy of a systems approach. This means that when something is explained it is viewed as part of a larger system and is described in terms of its role in the larger system [Blanchard et al. 1998]. This approach differentiates between events, patterns, and structure.

"We tend to focus on *events* rather than think about their causes or how they fit into a larger pattern …you can only react to each new event rather than anticipate and shape them.

Patterns are … changes in events over time. You focus on exploring how the series of events are related and … what caused them.

'Why is this pattern happening?' It is the *structure* level that holds the key to lasting, high leverage change. You adapt to the patterns that you've observed. Actions taken at this level are creative" [Retief 2001].

A graphical representation of the above concept is given below. This is analogous with an iceberg, where events represent the tip that is visible but it is patterns and structure that is the crux.

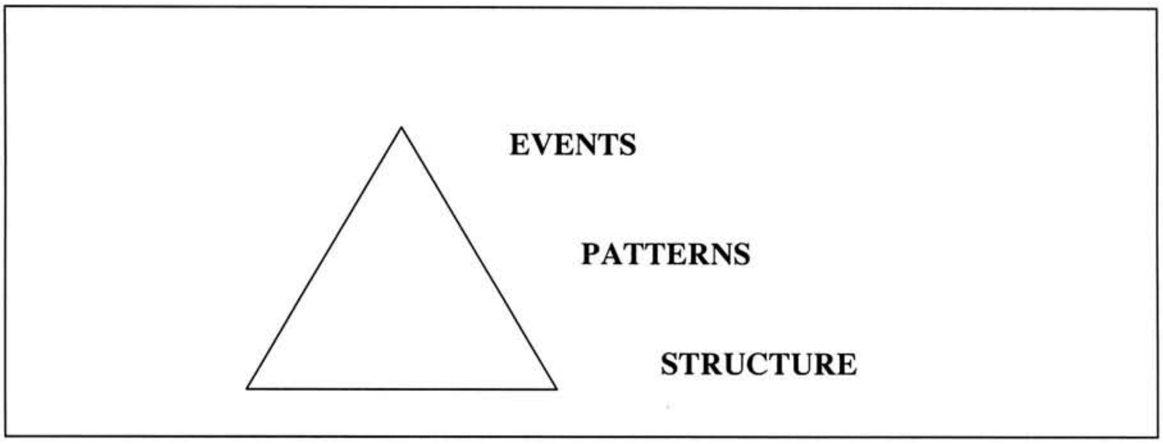

Figure 2.1 Schematic Of The Systems Approach

The purpose of the Systems Approach is to transcend from reacting on event level to a proactive stance by investigating the cost drivers at the structural level.

Qualitative cost drivers and the relative importance thereof, is captured by exploiting role players' expert judgements [Roy et al. 1999-2]. This may be done through survey questionnaires or participative workshops.

Such a Systematic Model for Diesel Consumption on the Mines was developed during participative workshops. This was done in conjunction with mine personnel from different facets of the mining operation; each with different perspectives and technical backgrounds regarding diesel consumption in order to formulate a comprehensive model.

With the input from mine personnel, factors influencing diesel consumption, that is the cost drivers of diesel consumption, are identified and common factors are grouped under a general heading and then referred to as one cost driver. Then the interrelationship of cause and influence between these cost drivers is identified. To utilise the cost-driver model in order to reduce costs associated with diesel consumption, it is necessary to apply optimisation techniques. Optimisation will be discussed in Section 2.3.

2.2 Introduction To The Mining Operation.

Opencast or surface mining is the "removal of soil and rock (overburden) above a seam of ore and the extraction of the exposed mineral. Surface mining is most economical where flat terrain and horizontal seams permit a large area to be stripped. Where deposits occur in rolling or mountainous terrain, a contour method (of mining) is used that creates a shelf with a slope on one side and an almost vertical wall on the other… Source: Britannica " [Jansen van Vuuren 2001].

Figure 2.2 Opencast Coal Mine at Grootegeluk

Opencast mines in South Africa are found at Sishen, Ellisras (called the Grootegeluk mine) and Thabazimbi. The Mining Operation at each mine is unique in that, at Sishen and Thabazimbi iron ore is mined whilst at Grootegeluk Coal is mined. Further, mining at Grootegeluk and Sishen is "into the earth" i.e. carrying ore from the pit up to the plant whilst at Thabazimbi the ore is mined out of the surrounding high ground i.e. carrying ore from the pit down to the plant. The above is simplified graphically hereunder.

Table 2.1 Comparison of the Mines.

Mine	Product	Route to Plant
Grootegeluk	Coal	Uphill to plant
Sishen	Iron ore	Uphill to plant
Thabazimbi	Iron ore	Downhill to plant

Nevertheless the daily operating cycle in the pit of each of these mines is fundamentally the same. Prior to any mine operation surveyors will drill exploration holes to determine the composition of the underlying earth (see Figure 2.3). This provides initial information on the size, shape and quality of ore or coal available albeit approximate. Based on the approximate topology of the ore deposit, the terrain is divided into 3-dimensional 'blocks'. A set of blocks on a common vertical coordinate is referred to as a level (Figure 2.4). Mining then proceeds block by block from upper to subsequent lower levels (Figure 2.5 & 2.6). It is common that the first level(s) do not yield substantial ore deposits but rather waste or overburden (or interburden). This leads to the concept of *strip-ratios* [Bezuidenhout 2001].

Strip ratio is the ratio of ore yield to waste product, and are calculated for the entire mine but also on each level. Referring to Figure 2.4 it is evident that in level 3 there is a very small yield of ore hence the strip ratio is low (e.g. 1 : 20) whereas in level 4 the yield of ore is greater and the strip ratio is for example 1 : 2.

For each level the potential mining area is also divided into blocks on a contour map, called shooting blocks. Each shooting block is prepared properly before explosives are used. The explosion loosens and displaces the underlying earth so that the shovels can commence loading the haul trucks (see Figure 2.7) with ore, coal or waste. Ore is transported to a plant whereas waste is removed to a pre-determined dumpsite. The dumpsite is located close to the loading point to reduce the cost of removing waste [Bezuidenhout 2001].

More detail on underlying ore is acquired once deeper levels are mined and the mining operation could be further refined. For example, new information concerning the quality of ore available, could influence the order in which these blocks are mined [Bezuidenhout 2001].

The physical layout of the mines can also be extremely dynamic. At Thabazimbi, there are currently four different pits, scattered in the surrounding hills which are between 5 km and 12 km from the plant. At each pit different levels are mined simultaneously. As mining is started on a new level, the shovels have to create sufficient workspace by clearing waste or ore. So initially the surface area of the level is restricted implying the need for only one shovel and perhaps two trucks and the remainder are not needed and have to be parked. That means at times some equipment is not productive. But as the one shovel and two trucks work, more

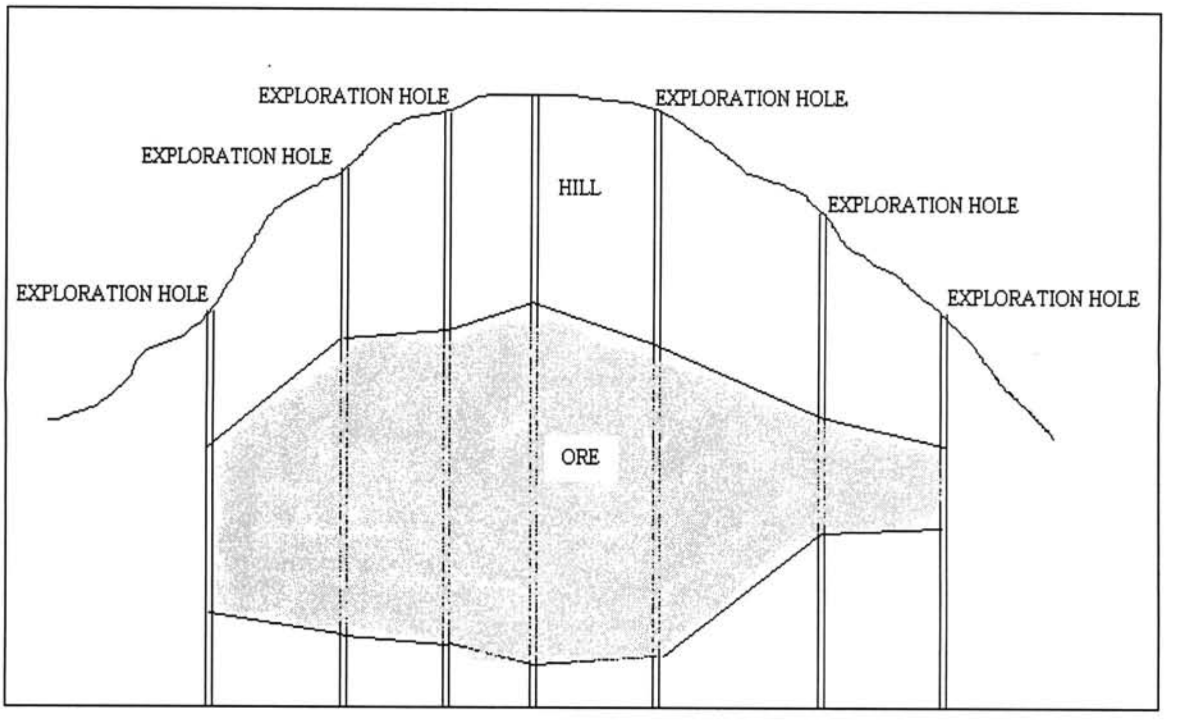

Figure 2.3 Exploration Phase

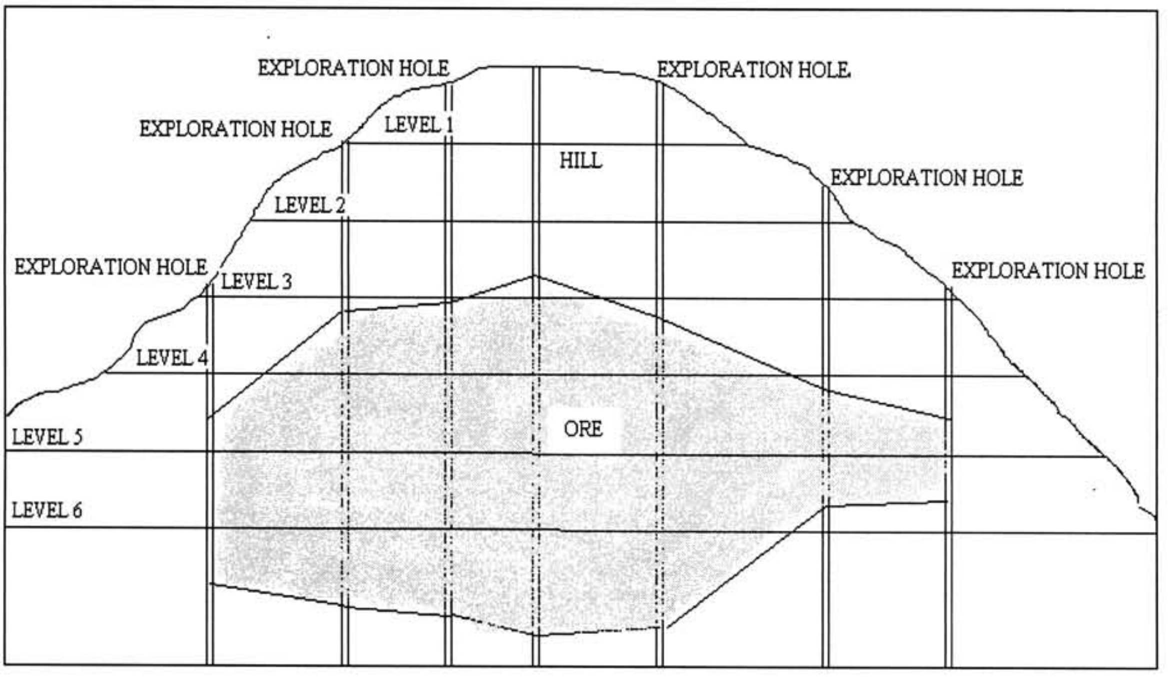

Figure 2.4 Division of Ore Deposit into Levels.

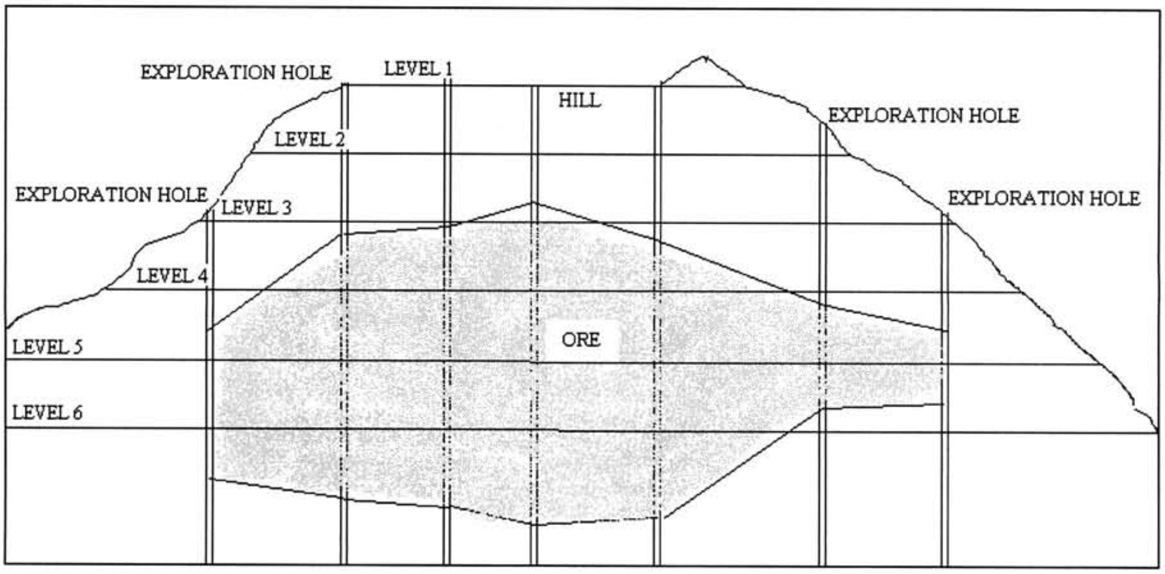

Figure 2.5 Mining on Level 1.

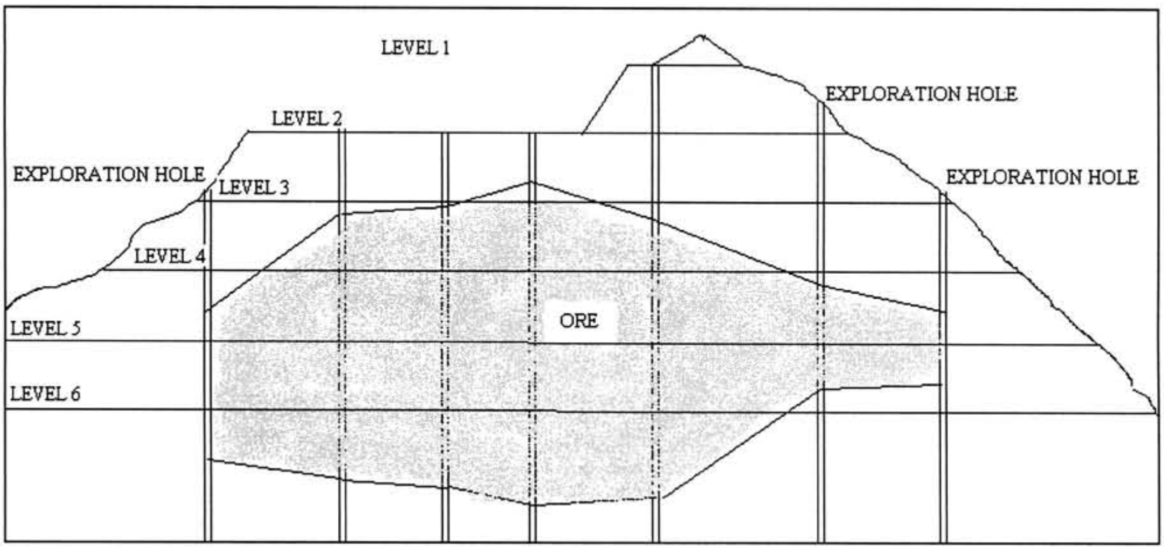

Figure 2.6 Mining on Level 2.

working space is created necessitating more trucks and shovels. This can lead to an under-resourced situation where it would be optimal to deploy more trucks/ shovels than are available [van den Brink 2001].

The mine planning is further complicated when the operating cycle of the haul trucks is considered. Typically more than one haul truck is assigned to a shovel. The haul truck reverses aside the shovel which then begins loading ore or waste onto the haul truck. Meanwhile a second haul truck reverses on the other side of the shovel waiting to be loaded (Figure 2.7). The first haul truck is fully loaded and drives off to either the plant or the dumpsite/ tipping point where it may have to wait for other trucks from other pits to finish tipping before tipping off its load and then driving back to the shovel to be loaded again.

This cycle has the following possible scenarios: first the cycle is perfectly synchronised and the haul truck does not have to wait at anytime during the cycle (except at loading). This is most likely the case if there are very few trucks deployed which implies the shovel is then waiting. The second extreme is if the shovel is constantly working implying a large number of haul trucks are deployed with the result that there is a significant waiting time. In terms of diesel consumption the former scenario is favourable as the haul truck does not waste fuel with unnecessary idling but it is also not feasible as the production targets would probably never be met. The latter is the least favourable and a compromise must be reached.

From the above paragraph it is clear that there is a need to quantify the performance of different truck-shovel configurations[1] in order to determine the optimal configuration which will meet production requirements and minimise the TCO of the haul trucks. Truck-shovel configuration Scheduling goes hand in hand with the configuration problem.

[1] The term 'truck-shovel configuration' refers to the ratio of haul trucks assigned to a loader(s) and, can include their respective payloads.

Figure 2.7 Shovel Loading Haul Trucks

Figure 2.8 Typical Shovel.

Figure 2.9 DEMAG Shovel

Figure 2.10 Titan Haul Truck.

Figure 2.11 Haul Trucks During Daily Operating Cycle

Determining the optimal truck-shovel configuration would minimise the TCO as follows. It was deduced in section 2.1 that Energy Costs, Repair Costs and Maintenance Costs are elements that can be minimised to reduce TCO. From the preceding discussion of the haul truck operating cycle, truck-shovel configuration will influence the amount of time haul trucks spend idling whilst waiting to load/offload and even during loading. Idling time represents an undesirable energy cost (diesel consumption) and an increase in engine operating time which will effect maintenance and repair costs. Associated productivity losses are also predicted and can be verified by investigating different haul truck-loader configurations.

The haul trucks deployed on opencast mines are appreciably larger than their underground counterparts and convey larger tonnages (150 – 250 tonnes). The haul trucks are powered by a diesel-electric drive train. This means it has a diesel engine that is used to generate power to drive an electric motor(s), mounted in the wheel(s) of the truck, to propel the vehicle. At the Thabazimbi mine, where ore is carried downhill, the drive train of the haul truck is used to retard (brake) the vehicle's descent. On Grootegeluk and Sishen mines the trucks carry the ore or coal uphill and make use of a Trolley Assist Technique commonly referred to as a Pantograph to power the haul truck during the ascent. Pantograph enables the vehicle to draw electricity directly from overhead power supply lines and an extendable trolley mounted on the roof of the haul truck (Figure 2.12). This configuration is similar to that of an electric locomotive. In Figure 2.12 the overhead power lines are clearly visible as well as the red extended trolley on the haul truck's roof.

Pantograph utilisation during the ascent out of the pit reduces diesel consumption (for the obvious reason that electricity is being supplied direct to the motor and the only diesel consumed is that to keep the engine idling) and reduces costs; for a 8 % slope a loaded haul truck operating on Pantograph consumes fuel at approximately 34 litres/hour compared to approximately 450 litres/hour without Pantograph [Hutnyak 2001].

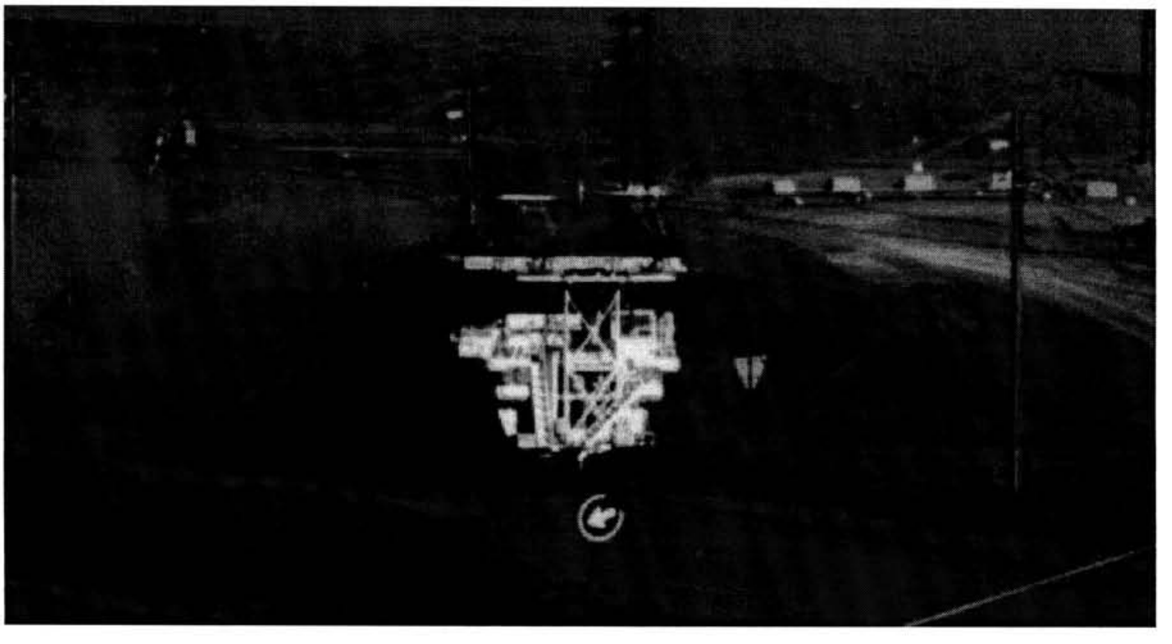

Figure 2.12 Haul Truck utilising Pantograph.

It is evident that utilising a Pantograph yields exceptional energy savings during the loaded ascent of haul trucks. However due to the permanency of the design (overhead power lines), pantograph is only suitable for arterial roads of a permanent nature and thus would not find use throughout the dynamic physical layout of a mine. Also in some mines pantograph is not necessary as the laden haul trucks travel downhill to the plants, as is the case at Thabazimbi (see Table 2.1).

Furthermore, a number of mines have investigated the feasibility of in-pit crushing and/or conveying. In-pit crushing entails the deployment of a crusher in the lower levels of the mine where the ore is being extracted. Conveying refers to the removal of ore from the pit by an inclined conveyor belt [Mitchell et al. 1985]. Haul Trucks are used in conjunction with in-pit crushing and/or conveying to transport the displaced earth to the crusher or conveyor. The implications of such a scheme are that fully laden haul trucks need no longer ascend the steep grades out of the pit to offload and a smaller fleet of haul trucks is needed since the conveyor will fulfil most of the materials handling. In-pit crushing and conveying has been shown to be economically and productively superior [Farquier 1989], however it is not feasible for all mining operations as there are mine design considerations that must be accounted for when adopting in-pit crushing, and, the flexibility and manoeuvrability offered by haul trucks can be a more important consideration to some mines.

Adopting in-pit crushing and conveying would render pantograph obsolete. Whether or not pantograph or in-pit crushing and conveying is applied, there still exists a need for haul trucks in mining operations and thus the current scope of study is still applicable to any opencast mine. The current scope of study is also applicable to military and commercial haulage operations.

The roads on the mines are dirt roads. At the Thabazimbi mine, a portion of the roads have been treated with a bitumen surface called Dust-a-side; with the aim of reducing operating costs. It is hoped that rolling losses are lower on the treated roads and maintenance costs decrease due to the less dust on the new road surface. The effect of Dust-a-side was to be investigated.

2.3 Optimisation, Linear Programming and the Simplex Algorithm.

The theme of this study is the reduction of TCO of haul trucks within the context of mining. This implies that TCO of haul trucks can be minimised but the haul trucks must still meet production requirements. If the mining production requirements are not considered by implicit or explicit means, then the greatest reduction in TCO would be attained by simply not deploying the haul trucks. The theme of study leads to the theory of optimisation because optimisation is the science of finding the 'best' result to a 'given situation'. The theory of optimisation, linear programming and the simplex method is discussed in Appendix G.

The simplex method is a commonly applied method to solve an LP problem and the sensitivity analysis of an LP solution yields greater insights to the effect of the variables on the optimal solution. In chapter 3 it is described how linear programming was used to mathematically model the diesel cost driver model which the simplex method was applied to solve the LP problem. A sensitivity analysis was performed to investigate the nature of the optimal solution to the diesel cost driver model.

2.4 Tyre And Wheel Theory

All the roads in the operational zone of the mine are dirt roads; however frequently used roads at the Thabazimbi mine have been sprayed with a bitumen compound, called Dust-A-Side. Dust-A-Side reduces the amount of dust kicked up by the haul trucks conveying unearthed material. Vehicles travelling on roads treated with Dust-A-Side may sustain a change in rolling losses compared to travelling on unsprayed roads. It is necessary to review some background on the physics of wheel and tyre theory to gain insight into rolling losses and the effect thereof on diesel consumption.

A pneumatic tyre performs 3 functions [Timmons et al. 1997]:

- Support the vehicle's weight.
- To cushion the vehicle on the road surface.
- To transfer power to propel, stop and alter the direction of the vehicle.

Resistive forces occur whilst a tyre performs these functions. These resistive forces acting on a vehicle are power consumers and thus affect diesel consumption. The individual resistive forces acting on a vehicle are [Phelps et al. 1977]:

- Aerodynamic
- Grade

- Cornering
- Inertia
- Chassis friction
- Rolling

Of these resistive forces rolling resistance is the predominant force at speeds below 60 km/h [Thompson et al. 1977] and only at speeds of 95 km/h is the power loss due to aerodynamic drag equal to that caused by rolling resistance [Gusakov et al. 1979]. Hence to achieve any significant changes to fuel consumption on mines, we should address rolling losses as the haul trucks travel at slow speeds [Thompson et al.1977].

Available literature on tyre rolling resistance utilises results from tests with passenger cars and passenger buses, where measurements are taken after tyres are allowed to warm up to reach an equilibrium temperature. A typical warm-up involved driving 30 km at vehicle speeds greater than 50 km/h [Thompson et al. 1977]. This differs from the mining vehicle situation where the vehicles utilised are larger and operating speed and cycle differs. Previous studies utilising coast-down tests recommended research be furthered into different vehicles especially heavier vehicles [Du Plessis 1993].

Tyre rolling resistance is "the force required to push or pull a rotating tyre over a level surface"[Shear 1987]. Rolling resistance is dependant on the driving cycle, for example "stop and go" or sustained speed driving, and can account for as much as 20 % of a vehicle's fuel consumption [Klamp 1977]. Rolling resistance is affected by tyre characteristics, load, inflation pressure, vehicle speed and operating conditions such as environmental conditions, trip length, road type, and occurrences during trip, for example, moderate to severe acceleration and stopping long enough for the tyres to cool [Timmons et al. 1977].

2.4.1 Mechanisms Responsible for Rolling Resistance.

Several mechanisms are responsible for rolling resistance. These include [Willet 1973 and Clark 1982]:

- Increase in potential energy needed for a rigid wheel to surmount an asperity.
- Hysteretic losses within the tyre.
- Inertial distortion of the tyre.
- Windage losses.
- Friction between tyre and road surface.
- Aerodynamic loss of the wheel moving through the atmosphere.

Coulomb was the first to propose that the change in potential energy needed to surmount an asperity led to rolling resistance. This mechanism is no longer considered to be as significant as hysteretic losses, due to the development of pneumatic tyres and well-paved paved roads [Shear 1987].

As a tyre rolls along a flat surface, work is required to flex the tyre as it rotates. The tyre is not perfectly elastic and some of this work is converted to heat [Timmons et al. 1977]. This phenomenon leads to the hysteretic losses referred to above.

Inertial distortion losses occur at high speeds, typically greater than 80 km/h. It involves the reluctance of tread to alter direction due to inertia. This reluctance to directional change increases the distortion of the tyre in the contact zone and adds to the strain cycle of the tyre [Walter et al. 1974].

Rolling losses have also been described by considering the wheels of a vehicle as power transmission devices [Clark et al. 1979]. Power is transmitted from the engine and drive train, to the roadway, through the wheels in order to propel the vehicle. The degree of efficiency of this power transmission can vary between zero and one hundred percent.

Windage losses refers to secondary energy losses incurred at hub mountings and bearings. Windage losses are orders of magnitude smaller than other losses referred to and are thus disregarded in any further analysis.

Aerodynamic losses are dependant on vehicle forward velocity and tyre frontal area. This effect is usually included in the total aerodynamic drag of the vehicle. At speeds of 100 km/h aerodynamic losses are comparable to those due to hysteretic losses within the tyre [Clark 1982].

2.4.2 Factors Affecting Rolling Resistance.

The following factors affect rolling resistance:

Inflation Pressure.

Rolling Resistance of a tyre tends to decrease as inflation pressure increases, and if load is maintained constant, this effect proceeds at a near constant rate above a threshold pressure [Ramshaw et al. 1981]. When a tyre is operated at a higher inflation pressure, tyre deflection is reduced and there is a change in the contact area of the tyre that results in lower rolling resistance. The effect is similar for both cross-ply and radial tyres. For typical passenger car pressures of 200 to 300 kPa, increasing pressure by 50 kPa results in about 10 percent reduction in rolling resistance [Du Plessis 1993].

Tyre Temperature.

Rolling resistance of a pneumatic tyre is dependent on operating temperature. At vehicle start-up, a tyre is cold and gradually warms up during operation. This increase in temperature is from the energy converted to heat by the mechanisms described in section 2.4.1. An increase in tyre temperature also increases the temperature of the air trapped in the tyre cavity and this leads to a higher inflation pressure and thus lower rolling resistance.

The rubber of a cold tyre is relatively stiff and thus takes more energy to distort it, and also loses a greater portion of energy to heat losses. This 'lost' energy warms the tyre which becomes less stiff and thus easier to distort meaning the portion of energy lost to heat loss is less. It is apparent that an equilibrium temperature is reached if the vehicle is operated under steady state conditions. Equilibrium temperature is reached after a period of time called the Warm-up Period. Du Plessis (1993) suggests that equilibrium is reached after 10 to 20 minutes.

Vehicle Speed.

Tyre temperature increases with increased speed, and it has been shown that an increase in tyre temperature reduces rolling resistance. However Gusakov et al. (1979) concluded that there were two velocity effects. Firstly, at constant temperatures, rolling resistance increased with velocity and secondly, at equilibrium temperatures rolling resistance decreased slightly with an increase in velocity. The absolute magnitude of the latter (decrease) was about one third of the former (increase) indicating that increased rolling resistance accompanies an increase in vehicle speed. Du Plessis (1993) further states that it is however "weaker than that of other variables".

Vehicle Load.

Load is considered to be an almost 1 to 1 relationship with rolling resistance, for a given cold inflation pressure [Shear 1987]. This means that for equilibrium running conditions, the rolling resistance coefficient appears to be independent of the load.

Size and Construction of Tyre.

The geometric parameters that affect rolling resistance are tyre radius and aspect ratio (Section height/ Section width as defined in Figure 2.13). All things being equal, increasing tyre radius and decreasing aspect ratio would reduce rolling resistance [Genta 1997]. Decreasing aspect ratio would result in stiffer sidewalls. This would limit deformation of the tyre and therefore the energy required to deflect the tyre and losses due to inelastic deflection are reduced [Genta 1997].

A pneumatic tyre is constructed with an inner carcass of flexible rubber that is strengthened by layers of cord or fabric called a ply. Two basic types of tyre are available and are distinguished by the arrangement of the plies. They are radial ply and bias ply tyres [Dixon 1996].

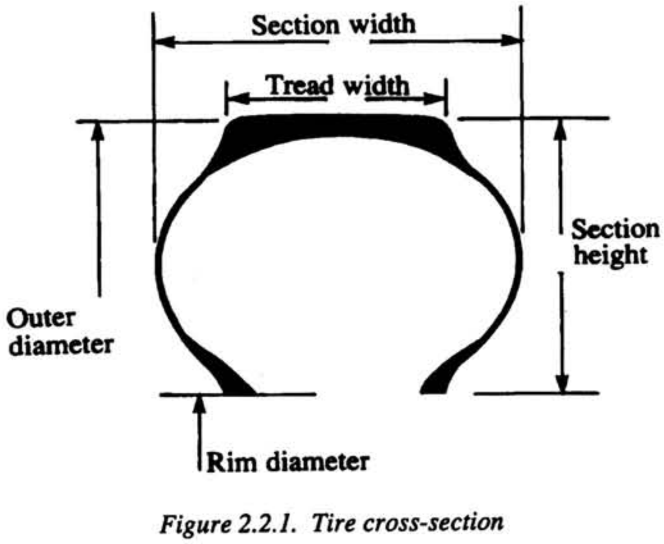

Figure 2.13 Geometric Parameters of Tyre Cross Section [Dixon 1996]

When viewed from the side the plies of a radial-ply tyre are radial and run directly across the tread surface. Radial plies are perpendicular to the circumferential centre line of the tread area. In bias ply tyres the plies are not perpendicular to the centre line and alternate plies are angle left and right of the centre line. When viewed from the side, bias-ply tyres do not have radial plies [Dixon 1996]. The difference in construction of bias-ply and radial-ply tyres is shown Figure 2.14. Bias-ply tyres have a stronger sidewall due to the alignment of the plies and deflect less. [Dixon 1996].

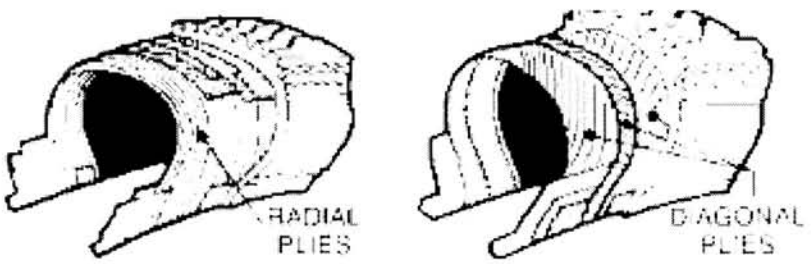

Figure 2.14 The Composition of Radial-Ply and Bias-Ply Tyres [Orbweb 2004]

Bias ply tyres have been shown to have higher rolling resistances than radial ply tyres [Shear 1987]. Gusakov et al. (1979) reported rolling resistance of bias-ply tyres being 50% higher than radial-ply tyres. Because radial-ply tyres deflect more than bias-ply tyres [Shear 1987]; it would have been expected that radial-ply tyres would have greater rolling resistance. It is the same inherent property of the construction of bias-ply that makes it deflect less, that leads to increased rolling resistance. This is because of the scissor effect of alternate plies which increases stiffness between plies and demands more energy to distort a bias-ply tyre than a radial-ply tyre [Chang et al. 1982].

Road Surface.

So far we have only considered the tyre parameters that affect rolling resistance. The interaction between the 'level surface' and the rotating tyre will also affect rolling resistance.

Phelps et al. (1977) suggested that the tyre-road interface losses are composed of the following effects:

- The adhesion effect between molecules of the tyre and road surface.
- Road surface effects resulting from surface texture and roughness.
- Road structure effects due to the deflection of the road structure by the wheel load.

Surface texture varies with aggregate used in road construction and, surface roughness is defined by the vertical displacements of a wheel for a given longitudinal distance. Deflection of the road due to road structure effects will depend on the properties of the road layers, namely strength and stiffness.

Various roads will be constructed with differing road layers and materials and thus varying surface textures and roughness. It is thus conceivable that the magnitude of the above-mentioned effects would differ with road surfaces. Thus road surface will also affect rolling resistance.

Phelps et al. (1997) showed that road surface affects rolling resistance and highlighted coast-down testing as a means of investigating rolling resistance on different road surfaces.

If coast-down testing is performed on similar test sections of different road surfaces, the other resistive forces acting on the test vehicle remain constant. The difference in the behaviour of the test vehicle is attributable to the change in road surface.

DeRaad (1977) also showed changes in rolling resistance due to different road surfaces. Using coast-down testing and the rolling resistance coefficient of a concrete surface as a baseline (100%), DeRaad (1977) determined a range of rolling resistance coefficients from 88% for polished concrete to 133% for coarse asphalt. DeRaad showed that rolling resistance increases with road roughness and texture. Rolling resistance can be more than 200% for the baseline for gravelled roads [DeRaad 1977].

Available literature on the effect of rolling resistance and road surface utilised results from tests with passenger cars and passenger buses [DeRaad 1977, Thompson et al. 1977, Shear 1987, Du Plessis 1993]. This differs from the mining vehicle situation where the vehicles utilised are larger and operating speed and cycle differs. Previous studies utilising coast-down tests recommended research be furthered into different vehicles especially heavier vehicles [Du Plessis 1993].

Coast-down testing is thus a feasible method to determine the effect of roads treated with Dust-a-Side, on rolling resistance. The appropriateness of coast-down testing on vehicles considerably larger than any in previous tests will be investigated in Chapter 5.

2.5 Simulating Operational Cycle of Mining Vehicles.

The operational deployment of haul trucks is extremely dynamic and the effective and efficient performance of a haul truck fleet is sensitive to the variable layout of the mine and the truck-shovel configurations[2] deployed on the mine. As the topography (layout) of the mine changes due to blasting and removal of overburden, the current truck-shovel configuration may lead to under utilisation of haul trucks or loaders and to traffic congestion on the mines. Under utilisation leads to idle time and congestion

[2] The term 'truck-shovel configuration' refers to the ratio of haul trucks assigned to a loader(s) and, can include their respective payloads.

leads to waiting time. Both idle time and waiting time are undesirable as it reflects an energy loss (diesel consumption).

It is therefore imperative to quantify the performance of different truck-shovel configurations in order to determine the optimal configuration that will meet production requirements and minimise the TCO of the haul trucks.

The operating cycle is depicted in Figure 2.15.

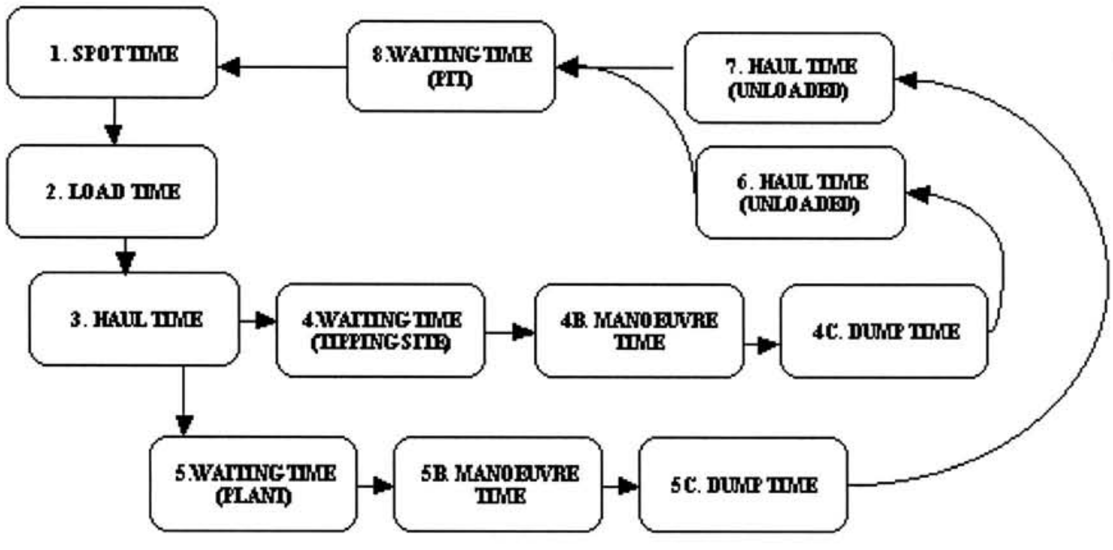

Figure 2.15 Operational Cycle of a Haul Truck

From the preceding discussion of the haul truck operating cycle in section 2.2, truck-shovel configuration will influence the amount of time haul trucks spend idling whilst waiting to load/offload and even during loading. Idling time represents an undesirable energy cost (diesel consumption) and an increase in engine operating time which will effect maintenance and repair costs. Associated productivity losses are also predicted and can be verified by investigating different haul truck-loader configurations.

The first inclination would be to consider this problem as a scheduling problem or as an assignment-routing problem. Rao (1984) describes scheduling as " the fixing of the various jobs or tasks in the time order in which they have to be performed". So the emphasis in scheduling problems is in determining the order in which certain tasks must be completed in order to maximise (minimise) a given objective function [Walsh 1977].

An example of a scheduling problem is the classic "travelling salesman" problem. Here, the problem is to determine the order in which a finite set of points must be "visited", once and only once, which will yield the shortest total distance travelled [Walsh 1977].

In the mining operation the order of tasks is fixed. The haul truck must first be loaded at the pit and then proceed to either the tipping site or the plant. Also the routes travelled by the haul trucks are fixed. Thus the Mining Operation is not amenable to analysis using scheduling theory.

A review of queueing theory [Odoni 2001] indicates that the Operational Cycle depicted in Figure 2.15 can be visualised as a Queue System with individual queues as indicated in Figure 2.16. A queue system is characterised by a finite number of clients, arriving for service, awaiting service if it is not immediately available and leaving after being served.

2.5.1 Nomenclature.

At this time it is convenient to expand on terms/concepts that will be utilised in this study:

Product. Coal/Iron ore.

Work Available. Tons of product and overburden available, at pit, to be handled.

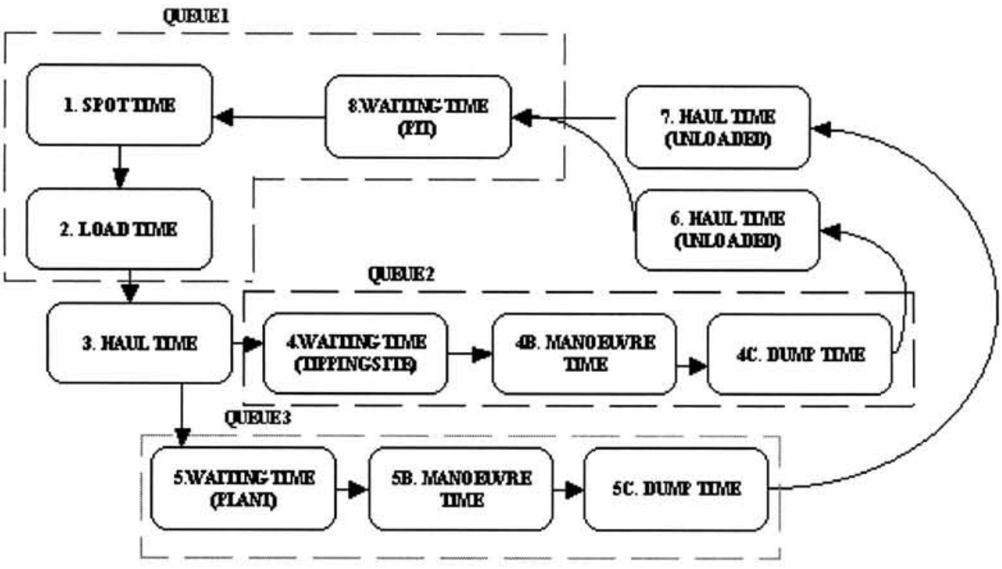

Figure 2.16 Queues in Operational Cycle of Mining Vehicle.

Product Available. Tons of product available, at pit, to be handled.

Production Requirement. Tons of product required by plant. This is a function of economic supply and demand. Note that Product (Work) Requirement is not always equal to Product (Work) Available and Production Requirement is usually coupled with a Production Rate-tons per hour.

Work Requirement. Production Requirement plus tons of overburden that has to be handled to access product requirement.

Strip Ratios. Ratio of Product to Overburden.

Operating Cycle. Events -from the Haul Truck's perspective- from time when haul truck arrives at pit to be loaded until the next time haul truck arrives at pit to be loaded.

Cycle Time. Time for a haul truck to complete one operating cycle.

Cycle Time = Spot Time + Load Time + Haul Loaded + Waiting Time + Manoeuvre Time + Dump Time + Haul Time (unloaded) + Waiting time (pit) + *Delay Time*. (2.1)

Of these times some are of necessity of operation for example, Load Time and are thus unavoidable whilst other parameter times are unwanted. For instance in equation 2.1, an additional term has been added which does not appear on Figure 2.15 or Figure 2.16. This is Delay Time. Waiting Time, although indicated as a separate parameter, is the largest component of delay time while other types of delay time could be due to weather, traffic, driver response and redeployment of loader, etcetera.

We would like to reduce Cycle Time because it could have the following benefits. A decrease in cycle time will mean an increase in production rate. Thus time needed to reach production requirements decreases. Hence vehicles operate for a shorter time, meaning operating costs per ton hauled decrease. Driver fatigue is also reduced due to shorter shift hours. Also if cycle time can be reduced without "speeding up" operation (that is, haul truck travels at faster speeds) less fuel will be consumed.

Queue System. Such a system consists of a finite number of clients arriving for a service, awaiting service (a waiting line), and a service facility with one or more servers. The terms used to describe a queue system are derived from common applications such as a line at a bank. Thus terms such as service and server are to be interpreted in queue theory to mean more than their literal sense.

Consider an intersection on a stretch of road with a set of traffic lights. Traffic is restricted by the traffic lights. The queue is easily visualised as the accumulation of cars waiting to go through the intersection. The intersection in this instance is the server and when they are permitted to travel through the intersection they have been serviced and have left the queue system.

2.5.2 Important Parameters in a Queue System.

A queue system may be characterised by demand, resource and performance parameters [Fishman 1978]. When modelling, demand and resource parameters describe the configuration of the queue system while performance parameters are the behavioural results of the system.

Demand Parameters are:

Arrival Pattern. This describes how clients arise over time. With a deterministic arrival pattern, the times when users arrive is defined with certainty. In a stochastic pattern, arrivals may occur at predetermined times and at random times.

Required Service. A queue may offer a client different types of service.

Resource Requirements. A client may arrive for service that requires one or a combination of specialised resources.

Server Preference. A client may demand service from a particular server. The performance of the system is affected because there could be lesser-preferred servers that are idle whilst a queue exists.

Priority. The need for prompt service may vary amongst clients. Due to the critical nature of certain clients they may enjoy priority over clients already in a queue. High priority demands will then be serviced before serving the existing queue.

Resource Parameters are:

Number of Servers. It was suggested that an increased number of servers would reduce waiting time and queue length but this would affect resource utilisation.

Service Time Characteristics. The time required to service a user may be deterministic or stochastic.

Queue Discipline. This is the specification of which client waiting in a queue will be serviced when a server becomes available. Queue discipline is the fundamental selection rule particular to a queue. Priority is an over-riding exception to the fundamental rule.

Skill Level. A queue system should have a sufficient skill mix to meet the resource requirements of its clients. The configuration of the skill mix can be investigated.

Service Interruption. Client or server failure can be considered as well as the re-deployment of a server to a high priority demand.

Waiting Room. This refers to the upper bound that may be placed on clients waiting in a queue.

Performance Parameters are:

Waiting Demand. The number of users awaiting service. This could be an instantaneous, average or cumulative number of users awaiting service.

Waiting Time. The time a user waits to be serviced. This could refer to an individual, average or cumulative waiting time of users awaiting service.

Resource Utilisation. This refers to total time a server is active, compared to total time it is deployed.

2.5.3 Strengths and Weaknesses of Queueing Theory.

Models derived from Queue theory necessarily involve approximations and simplification of reality. The challenge is thus to express a problem in its simplest form without a loss in generality to the type of problem under consideration. The applicability of the simulation of the operational cycle of haul trucks will be reviewed under this criterion.

Queuing theory provides useful bounds for more general systems at steady state as well as numerical solutions for dynamic systems being easily obtainable [Odoni 2001], compared to other methods of analysis such as Dynamic Programming [Rao 1984]. The latter will become evident when the simplicity and elegance of the results obtained using queueing theory is viewed.

2.5.4 Queue Analysis Using Cumulative Graphs.

Queueing theory usually handles steady state stochastic problems [Vignaux 1997]. In such a problem the assumption is that the potential service rate exceeds the average arrival rate and that resulting queues are due to fluctuations in arrival times and service times [Vignaux 1999].

Considering the previous example of the intersection, the intersection can handle an average of a thousand cars per hour and on any day an average of eight hundred cars arrive per hour at the intersection. So it would seem that there would be no accumulation of cars at the intersection. However rush periods occur where more than a thousand cars arrive per hour at the intersection, and this leads to the formation of a queue which soon dies out after the rush period is over. In such a system the arrival times are random and not influenced by previous occurrences at the queue. So the arrival time of rush traffic is not determined by the arrival time of the previous rush traffic.

In dealing with steady state stochastic systems probability distributions are used in modelling.

The mining operating cycle is a closed cycle; the clients (output) from one queue feeds directly into the next queue and back to the previous queue. This inter-relatedness of queues means that the problem is not stochastic but rather a dynamic deterministic system [Vignaux 1997].

The approach of cumulative graphs is employed for this type of system [Odoni 2001].

The first step is to graph a function of the cumulative number of arrivals at a queue for time ‘t’, that is, the total number of arrivals at a queue by time ‘t’. Since only integer values of clients are permissible, the result is a step function with steps of integer values (Figure 2.17).

The first step (jump) occurs at time t = 13.5 with a step height of 2, meaning that 2 trucks arrived simultaneously at the queue at time t = 13.5 and that a total of 2 trucks has arrived at the queue. The next step (jump) occurs at time t = 16 and the cumulative number of trucks has “jumped” to 4 meaning that two additional trucks have arrived at the queue.

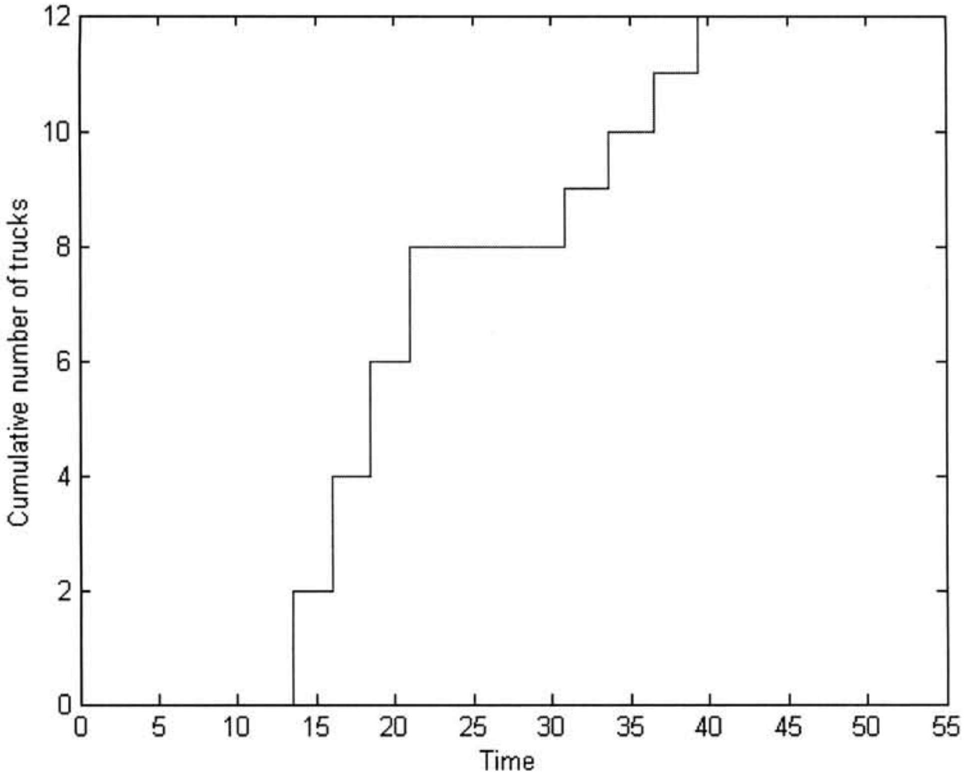

Figure 2.17 Cumulative Number of Arrivals at Queue.

Now on the same graph we plot the cumulative number to enter service and the cumulative number to have left service by time ‘t’; labelled as Service and Departure respectively. Cumulative number of Arrivals is labelled as Arrival. On this new graph (Figure 2.18) it is easy to identify the relevant quantities of interest in queuing theory.

The vertical distance between the Arrival and Service curves indicates the number of trucks in the queue (awaiting service) at that instant (time 't' on the horizontal axis) and the horizontal distance between the Arrival and Service curves gives the waiting time for that relevant truck. This is graphically illustrated, for time = 27.5 (where number of trucks awaiting service = 8-6 = 2) and for the 4th truck (where waiting time = 7), in Figure 2.19.

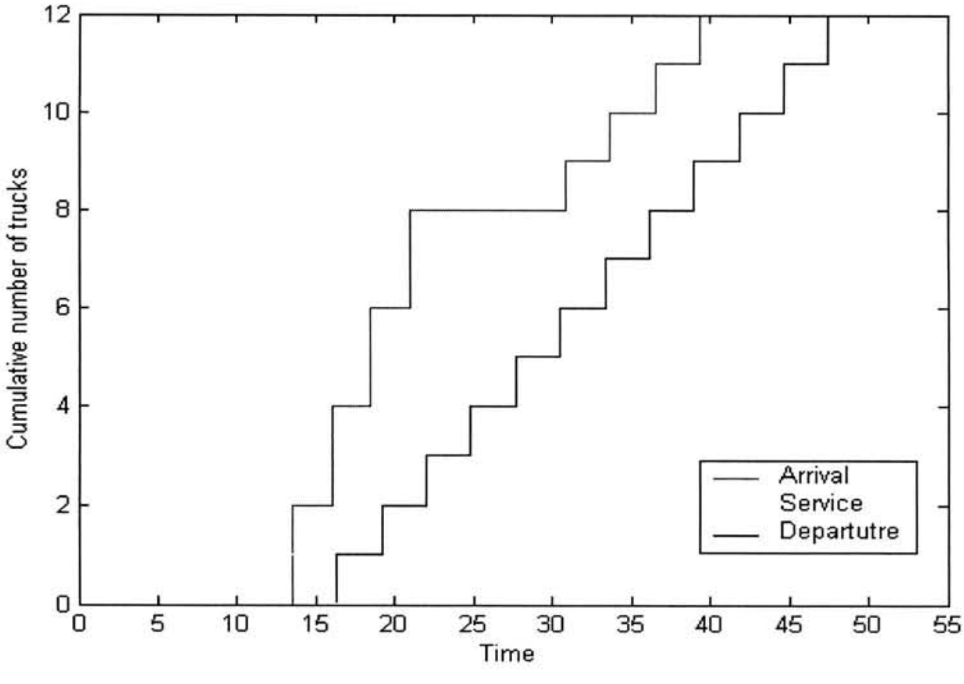

Figure 2.18 Cumulative Arrivals, Departures, Services vs. Time.

Similarly the vertical distance between the Arrival and Departure curves yields total number in the queue system (queue and at servers), and, the vertical distance between the Service and Departure curves yields the number in service which is determined by the number of servers. Figure 2.19 shows that there is only one server.

Also, the horizontal distance between the Arrival and Departure curves yields the total time a truck spends in the queue system, and, the horizontal distance between the Service and Departure curves yields the time it takes to service a truck. Figure 2.19 shows that the service time is constant.

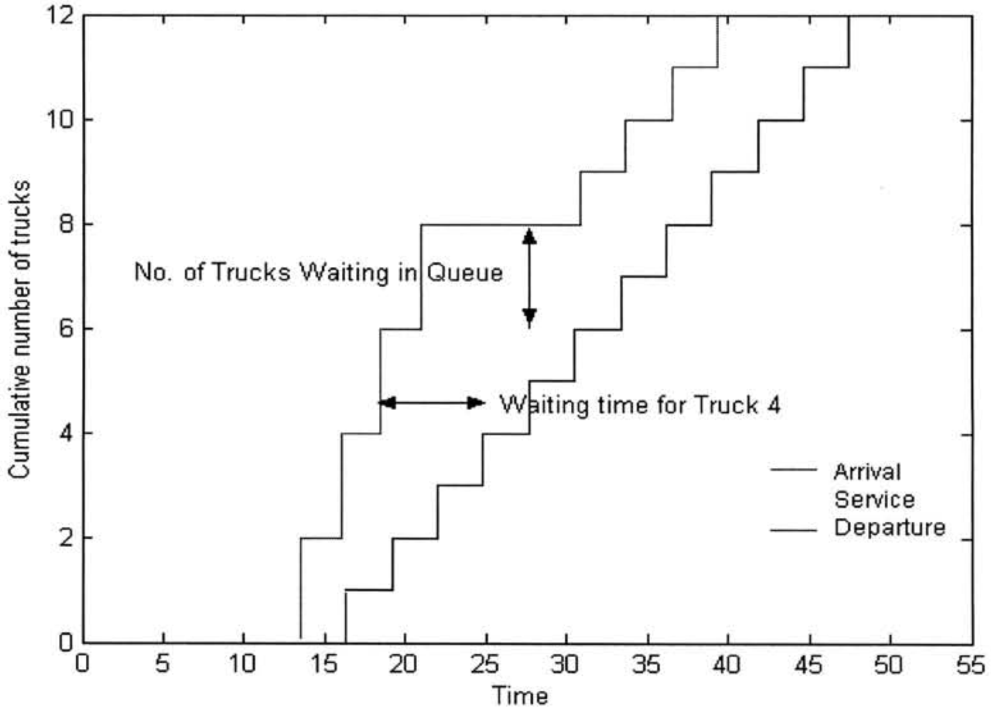

Figure 2.19 Calculation of Waiting Time and Size of Queue.

It must be noted that analysis by means of cumulative graphs is possible if a FIFO queue discipline is employed. This is because FIFO dictates that the first truck to arrive will be the first to be serviced and will be the first truck to depart. Inductively this means that the 'n'th cumulative truck to arrive, will be the 'n'th cumulative truck to be serviced, will be the 'n'th cumulative truck to depart. Thus reading the time values corresponding to y-value of 'n' corresponds to truck 'n'.

If the queue discipline is not FIFO the trucks will not necessarily depart in the same order as they arrived.

The average rate of arrival and departure for a period is given by the slope of the respective curve (Figure 2.20). If the rate of Arrival is greater than the rate of Departure there will be an increasing accumulation of customers awaiting service at the queue. An example of this phenomena is the daily occurrence of rush hour traffic. The queue system will tend to a steady state condition once the excess arrival rate has diminished.

Bearing in mind that the time a truck spends in the queue (or entire queue system) can be determined by the horizontal distance between the Arrival and Service (or Departure) curves, it is possible to determine the total queuing time for a group of trucks. Assume the group of trucks under consideration is from the 'm'th truck up to the 'n'th truck; then the total queuing time is the area bounded by the Arrival and Service curves and horizontal lines corresponding to the 'm'th truck and the 'n'th truck in Figure 2.21.

Then the average time a truck from this group waits in the queue is the Total Queuing Time divided by the number of trucks in the group, namely 'n' –'m'.

Similarly, since the vertical distance between the Arrival and Service curves indicates the number of trucks awaiting service; the average number of trucks in the queue over a time period from t_i to t_j can be determined by the area enclosed by the Arrival and Service curves and the vertical lines $t = t_i$ and $t = t_j$ divided by $(t_j - t_i)$ in Figure 2.22.

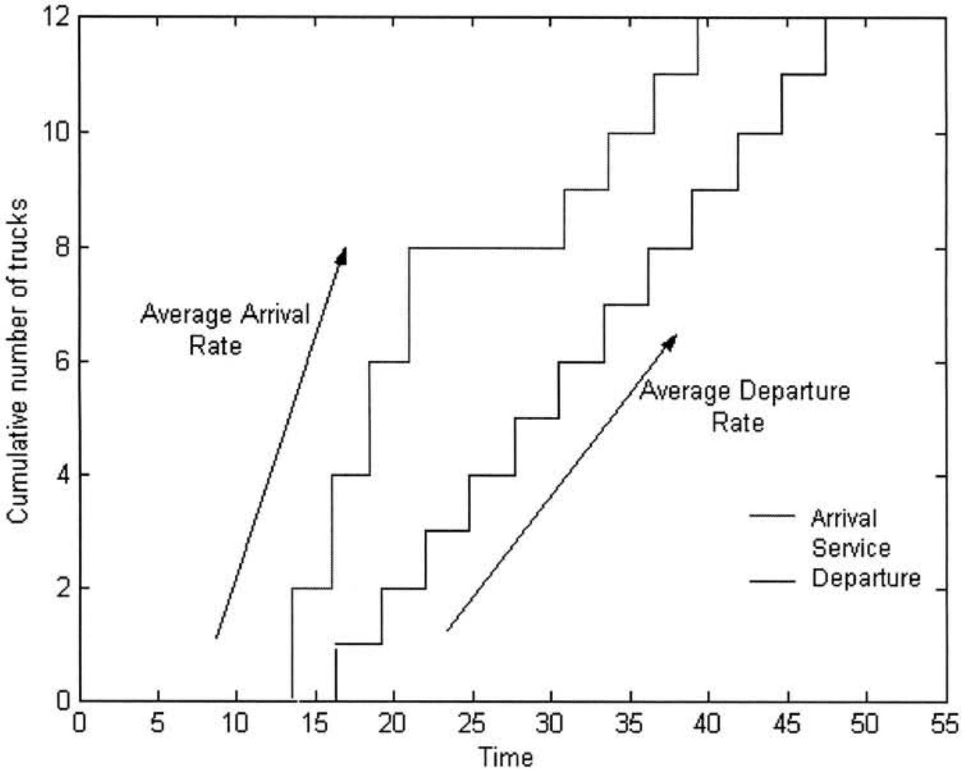

Figure 2.20 Graphical Representation of a Rush Period

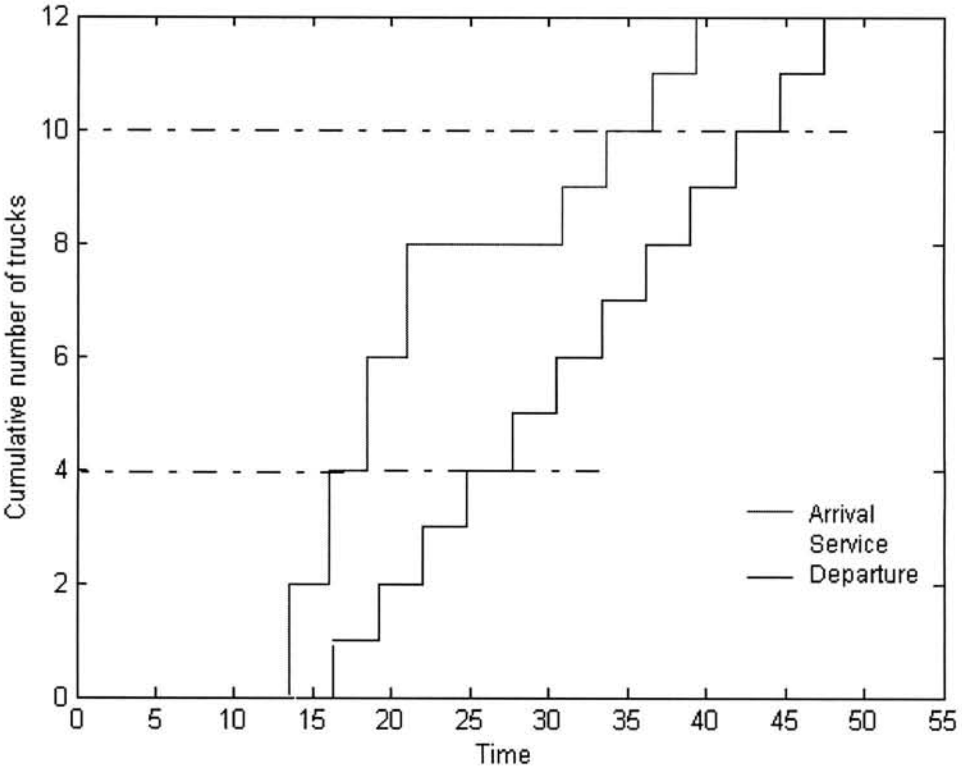

Figure 2.21 Calculation of Total Queuing Time.

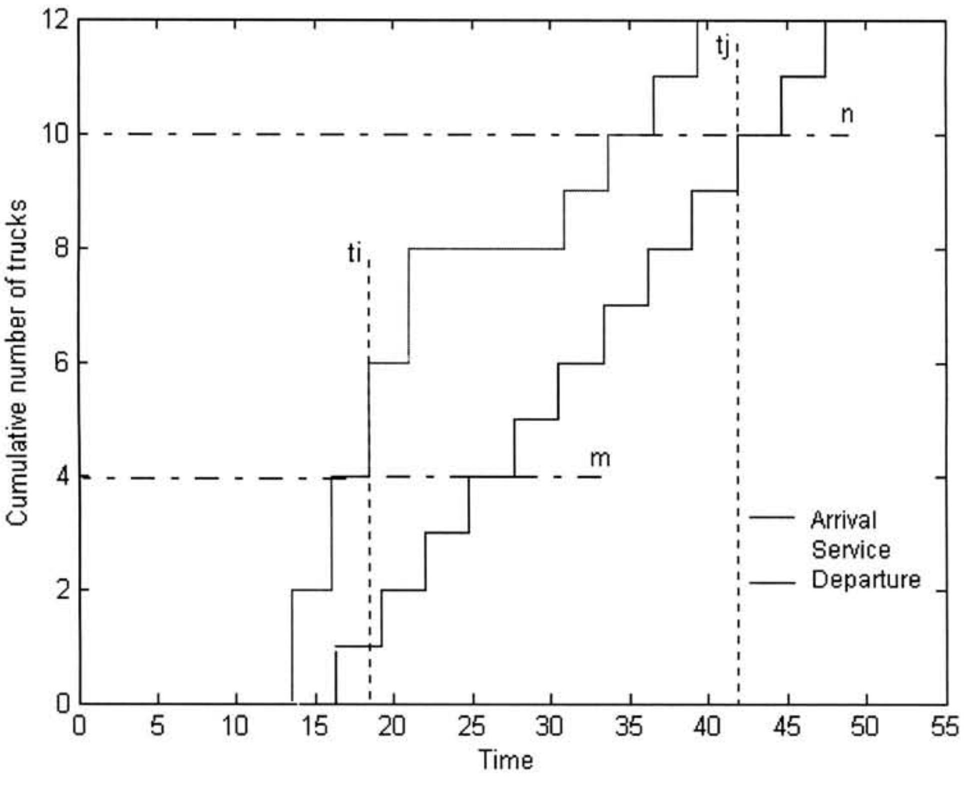

Figure 2.22 Calculation of Average Queue Time and Average Queue Size.

2.5.5 Simulating Operating Cycle Queues.

To perform queue analysis using cumulative graphs, the simulation must advance the queue system forward in simulated time [Watson 1981]. A queue simulation is a discrete event system [Fishman 1978]. A discrete event system is a system where the state variables change at discrete moments of time instead of continuously with time [Winston 1994]. The arrival of a client, the servicing and the departure of that client are all discrete events.

For a queue system to be advanced in simulation, potential events and their effects on the status of the queue system must be identified [Blouin et al 2001]. Events can be divided into two categories. Primary events are those whose occurrence is predetermined by sampling from a distribution and secondary events are those that occur as a result of a primary event [Watson 1981].

A model must be taught how to allow these events to occur. Two common methods are the Next-Event and Fixed-Time Increment Simulations [Watson 1981].

In next-event simulations, the simulation clock is updated to the next event that has been determined to occur. The advantage of this method is that periods of inactivity between discrete events are skipped [Winston 1994]. As the simulation advances from event to event, the actions for each event are executed and future events are scheduled.

For the fixed-time increment simulation, the simulation clock is incremented by a time Δt. For each time increment, a check is performed to determine if any events are scheduled to occur. Actions for the scheduled events are executed and the clock is again advanced by time Δt.

Fixed-time increment simulation is simpler to understand and track the progression but next-event simulation is computationally more efficient [Winston 1994]. This is because performing unnecessary operations checks during obvious periods of inactivity is eliminated.

2.5.6 Incorporating Variability and Monte Carlo Sampling in Queue Systems.

Thus far we have assumed the parameters of a queue system to be constant. However, parameters such as service time, and arrival times may vary. The probability of service interruptions due to mechanical breakdowns and the arrival of high priority demands have relevance if a model is to realistically reflect the operating cycle.

We can incorporate the variability of queue parameters by sampling from probability distributions. Sampling from probability distributions is known as Monte Carlo sampling [Winston 1994].

In Monte Carlo simulations, values for the queue parameters are obtained with probabilities, which require a probability distribution [Watson 1981]. The probability of each event may be determined from the relative frequencies of these events recorded in historical data. But they can be themselves the parameter of interest during the simulation. For example, what probability distribution will yield a required output? Monte Carlo simulations require the generation of a set of random numbers. Random number generators are used that provide for a sequence of numbers that displays no apparent pattern, and satisfies the appropriate conditions for statistical randomness. The uniform distribution is widely used in random number generators, which guarantees that any random number in the set has an equal chance of being selected next. Random numbers in the generated set are associated with the occurrence of a certain event. The probability of an event/value occurring determines the number of values in the set that are allocated to the occurrence of that value. For example, consider an event that has a probability of 0.1 or 10%, we then allocate 10% of the range of random numbers to the occurrence of this event. Now since the sum of the probabilities of all events is equal to1 or 100%, we have allocated the entire range of the random number set to the occurrence of certain events.

As an illustration, consider a set of random numbers generated between 0 and 99 inclusive. The range of possible numbers is thus 100. Now if the probability of event A occurring is $P_A = 0.6$, and event B is $P_B = 0.4$, then we allocate 60% of the range to event A and 40% of the range to event B. This could be done by assigning the range of numbers between 0 and 59 inclusive to event A and the range 60 to 99 inclusive for event B. The range of numbers assigned to either event A or event B need not be sequential. This is because any number has an equal chance of being the next number to occur in our set of randomly generated numbers. The number has no "knowledge" of its numerical value and is analogous to numbered balls in the lottery. So we could assign the ranges of 0 to 9 inclusive and 50 to 99 inclusive and then the range assigned to event B would be 10 to 49 inclusive.

Now assume we have assigned our former ranges of 0 to 59 inclusive for event A and 60 to 99 inclusive for event B and we have generated a sufficiently large set of random numbers between 0 and 99. Table 2.2 displays the first 25 random numbers of such a set.

Table 2.2 Random Integers Between 0 and 99.

19	72	36	55	79
50	63	85	89	61
64	33	33	61	96
83	66	2	82	99
94	77	11	22	80

For the first run of the Monte Carlo simulation, we use the first value of Table 2.2 to determine whether event A or event B will occur. In this instance the value 19 is in the range 0 to 59 inclusive, thus event A will occur. Similarly for the 2^{nd} iteration/run event B will occur based on the value 72. Subsequent iterations are A, A, B, A when reading random numbers row by row from Table 2.2. The random values are used only to determine which event will occur and play no other role in the simulation.

If variability of parameters is incorporated into a simulation model, it is expected that the output generated by the same simulation will differ [Watson 1981 and Winston 1994]. We must determine if the performance of a simulation is acceptable. Let us denote σ, a performance parameter under investigation and σ' the approximation developed by the simulation. We can determine the accuracy of the σ' approximation by the standard deviation. The overall measure of variability is stated in the form of a confidence interval at a certain level [Winston 1994].

Determination of variance estimates can be solved by:

Independent replications,
Spectrum Analysis,
Autoregression Analysis or
The batch-means method.

Independent replications entail several 'runs' of the simulation model. Performance parameters from each replication are then treated as independent observations.

Furthermore, statistical analysis is complicated by autocorrelation, due to events not being independent.

The starting conditions of a simulation will affect the initial behaviour of the queue system. There is a time lapse between start-up and when the queue system reaches a steady-state. This is known as the transient period. Determining the length of transient period for a fixed starting condition will aid the performance analysis of the queue. By knowing the length of the transient period it is possible to differentiate between transient and steady state data and each period can be analysed separately and the effects of continuous shut-down and start-up behaviour can be determined.

Queue theory may be applied to the mining operation to analyse the performance of the mining operation. Queue analysis by means of cumulative graphs may be simulated used a Next-Event simulation. Monte Carlo sampling allows probability distributions to be incorporated in a queue simulation. The simulation of the operating cycle of haul trucks is discussed in Chapter 4.

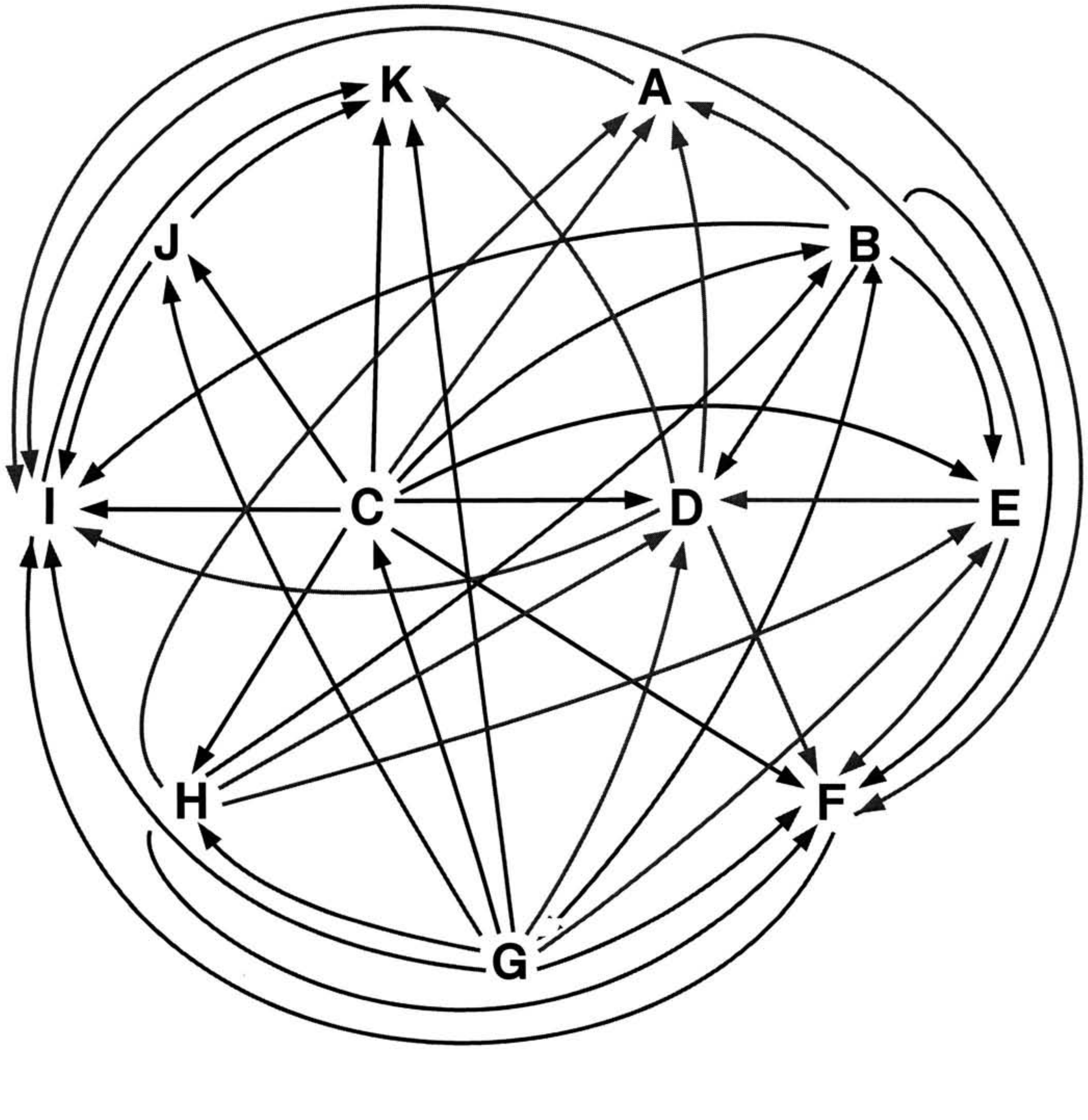

Figure 3.1 IR Diagraph of Factors Affecting Diesel Consumption on Thabazimbi

The constraints for the problem are determined from the IR Diagraph by considering each node and the arrows pointing towards it. Consider node H, depicted hereunder with only the arrows leading to it and the nodes where the arrows originated.

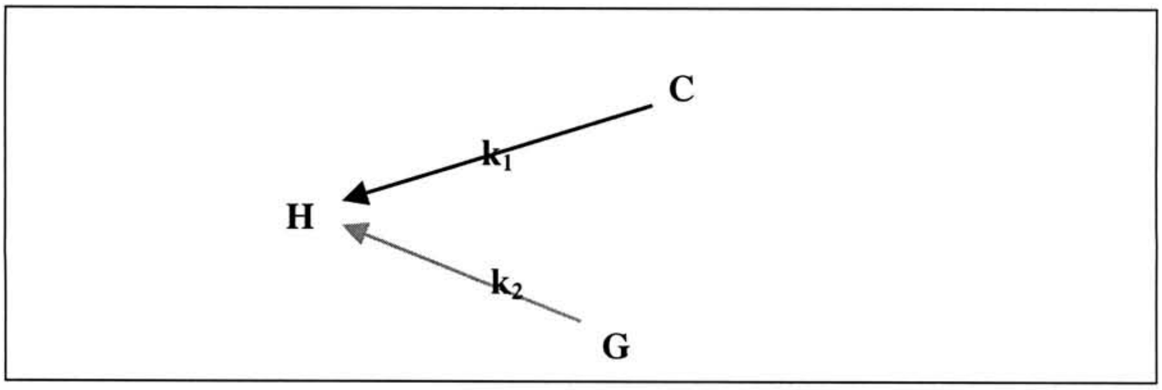

Figure 3.2 IR Graph of Factors Influencing H

It is assumed that the relation between factors is linear and from Figure 3.2 we deduce that

$$H = k_1 .C + k_2 .G \tag{3.2}$$

with k_1 and k_2 numerical constants that will be termed *influence coefficients*.

This provides the first set of constraints for the LP problem. This set of constraints describes how the variables are related to each other but does not place an upper bound on the variables. It is then possible that the solution is unbounded i.e. the optimal solution occurs when one or more variable(s) is (are) infinitely large. This is indeed the case. The number of sub-headings under each factor is used as an initial upper bound for each variable. So, if there had been five sub-headings for our example node H, the next constraint would be

$$H \leq 5. \tag{3.3}$$

The number of sub-headings is used as our first assumption for an upper bound based on the notion that if a factor has several sub-headings that factor encompasses several important aspects and more sub-headings implies more 'importance'. Using the number of sub-headings is thus a means, albeit crude, of quantifying a factor's 'importance'. 'Importance' referring to the number of changes that can possibly be made in this factor (if it is an outcome) or the number of resources it can commit (if it is a driver). This assumption and other alternatives for upper bounds will be discussed later.

With these two sets of constraints it is possible to solve the LP problem by means of the Simplex method. However, the values of the influence coefficients is not known a priori and thus fictitious values will initially be used. It is assumed that the influence coefficients range from zero to one inclusive. Three LP problems will be presented; one for Grootegeluk, one for Thabazimbi and one for Sishen.

3.1 Linear Programming Model for Thabazimbi Diesel Cost-Driver Model

The main factors affecting diesel consumption identified follow in Table 3.1.

Table 3.1 Factors Affecting Diesel Consumption on Thabazimbi

Factor	In*	Out*	Sub-Head∝
A: Optimise Condition of Roads	4	2	7
B: Utilise Dynamic Truck Allocation System	3	5	2
C: Investigate Alternative Transport Technology	1	9	14
D: Ensure Optimal Operator Practices	5	4	9
E: Ensure Optimal Operational Practices	4	3	10
F: Ensure Optimal Tyre Management	7	1	3
G: Availability and Reliability of Information System	0	9	3
H: Optimise Mining Planning Practices (Pit layout)	2	5	8
I: Effective Engine Maintenance Practices	8	1	8
J: Better Commercial Fuel Specifications	2	2	8
K: Effective Loss Control	5	0	2

* Refers to the number of arrows pointing towards/ from that specific node.

∝ Refers to number of Sub-Headings

Utilising Figure 3.1 it is now possible to formulate the LP problem:

$$\text{Max} \qquad Z = A + B + C + D + E + F + G + H + I + J + K \tag{3.4}$$

Subject to

H1: $A = a1 . B + a2 . C + a3 . D + a4 . H$

H2: $B = b1 . C + b2 . G + b3 . H$

H3: $C = c1 . G$

H4: $D = d1 . B + d2 . C + d3 . E + d4 . G + d5 . H$

H5: $E = e1 . B + e2 . C + e3 . G + e4 . H$

H6: $F = f1 . A + f2 . B + f3 . C + f4 . D + f5 . E + f6 . G + f7 . H$

H7: $H = h1 . C + h2 . G$

H8: $I = i1 . A + i2 . B + i3 . C + i4 . D + i5 . E + i6 . F + i7 . G + i8 . J$

H9 : $J = j1 . C + j2 . G$

H10: $K = k1 . C + k2 . D + k3 . G + k4 . I + k5 . J$

G1: A (Optimise Condition of Roads) ≤ 7

G2: B (Utilise Dynamic Truck Allocation System) ≤ 2

G3: C (Investigate Alternative Transport Technology) ≤ 14

G4: D (Ensure Optimal Operator Practices) ≤ 9

G5: E (Ensure Optimal Operational Practices) ≤ 10

G6: F (Ensure Optimal Tyre Management) ≤ 3

G7: G (Availability and Reliability of Information System) ≤ 3

G8: H (Optimise Mining Planning Practices (Pit layout)) ≤ 8

G9: I (Effective Engine Maintenance Practices) ≤ 8

G10: J (Better Commercial Fuel Specifications) ≤ 8

G11: K (Effective Loss Control) ≤ 2

$A,B,C,..,K \geq 0. \qquad a_n, b_n, \ldots, k_n \in R \qquad n = 1,2,3,\ldots 11$

The influence coefficients, denoted by $a_n, b_n, \ldots, k_n$ $n = 1,2,3,\ldots 11$ are unknown constants. However the solution of (3.4) was attempted using simulated coefficients (Appendix A) and it was found that the nature of the solution was the same irrespective of the permutation of coefficients used. This means that despite not knowing the values of the influence coefficients we can still deduce the nature of the solution and embark on a qualitative discussion of the results. Had the influence coefficients been available a qualitative discussion of the results would still have been the only useful outcome as the optimum values of the variables must be viewed relative to each other. In other words a solution of say, Effective Loss Control = 2.4 is meaningless viewed in isolation.

To investigate the nature of the solution the LP problem was first solved by assuming that all the influence coefficients are equal and assuming values between 0 and 1.

The model was solved using the "Optimizer" tool available in Corel Quattro Pro 8 and verified with the Student Version of LINDO Optimisation software.

Firstly, all influence coefficients were assumed to equal 0.3. This assumption is applied to Equation 3.4 and is solved using the student version of LINDO, and the optimal solution is presented in Table 3.2

Table 3.2 The Optimal Solution to Equation 3.4 with Influence Coefficients = 0.3

Variable	Value	Description
A	0.97	Optimise Condition of Roads
B	0.80	Utilise Dynamic Truck Allocation System
C	0.47	Investigate Alternative Transport Technology
D	1.35	Ensure Optimal Operator Practices
E	1.04	Ensure Optimal Operational (Bedryfs) Practices
F	2.04	Ensure Optimal Tyre Management
G	1.57	Availability and Reliability of Information System
H	0.61	Optimise Mynboubeplannings Practices (Pit layout)
I	2.66	Effective Engine Maintenance Practices
J	0.61	Better Commercial Fuel Specifications
K	2.00	Effective Loss Control
Objective	14.13	

A sensitivity analysis was also performed using the student version of LINDO. The results are presented in Table A.2.

At first glance, the optimal solution yields meaningless values for the variables. The optimal solution is of a qualitative nature. So the values assigned to the variables must be compared to each other. In Table 3.3 the optimal solution is compared to the number of arrows pointed to and from that variable, as well as the number of sub-headings of each variable. Table 3.3 is then plotted in Figure 3.3 with the number of arrows pointed to and from that variable, as well as the number of sub-headings plotted on the primary y-axis and the optimal solution is plotted on the secondary axis. It is evident that the number of arrows originating and disseminating from the nodes in Figure 3.3 may explain the behaviour between the optimal solution variables.

Table A.2 highlights the allowable range for the right hand side (RHS) of the constraint equations. The constraints H1-H11 are strictly equal to zero because they are modelled directly from the IR diagraph. The number of sub-headings for each variable defined the RHS of constraints G1-G11. It must be determined if this definition is suitable. From Table A.2 constraints G1-G10 have non-zero slack, implying that the optimal value of these variables is less than the upper bound imposed on them. Therefore increasing the upper bound of one of these constraints will not yield an improvement in the objective function value. This is confirmed by the zero dual prices for the constraints G1-G10 in Table A.2. So we may increase any RHS of G1- G10 infinitely without changing the relation between the optimal variables, but can we increase all the RHS of G1-G10 without changing the nature of the solution?

Consider equation G.14 of Appendix G, for each constraint G1 –G10, $r_j = \Delta b_j / I_j \approx 0$, (because $I_j = \infty$). Thus $\sum r_j \leq 1$ for the constraints G1- G11. The 100% rule for RHS of constraints ensures that the current solution will stay optimal and the nature of the solution will be maintained.

As a result of the above sensitivity analysis, we only need to investigate the effect of decreasing the RHS of G1- G11 by more than the Allowable Decrease. Setting all the constraints G1 – G11 less than or equal to 1 and resolving the LP Problem will suffice in this regard.

Table 3.3 Optimal Solution to Equation 3.4 and the Number of Arrows directed to/from the Nodes.

Variable	Optimal	In	Out	Sub-Head
A	0.97	4	2	7
B	0.80	3	5	2
C	0.47	1	9	14
D	1.35	5	4	9
E	1.04	4	3	10
F	2.04	7	1	3
G	1.57	0	9	3
H	0.61	2	5	8
I	2.66	8	1	8
J	0.61	2	2	8
K	2.00	5	0	2

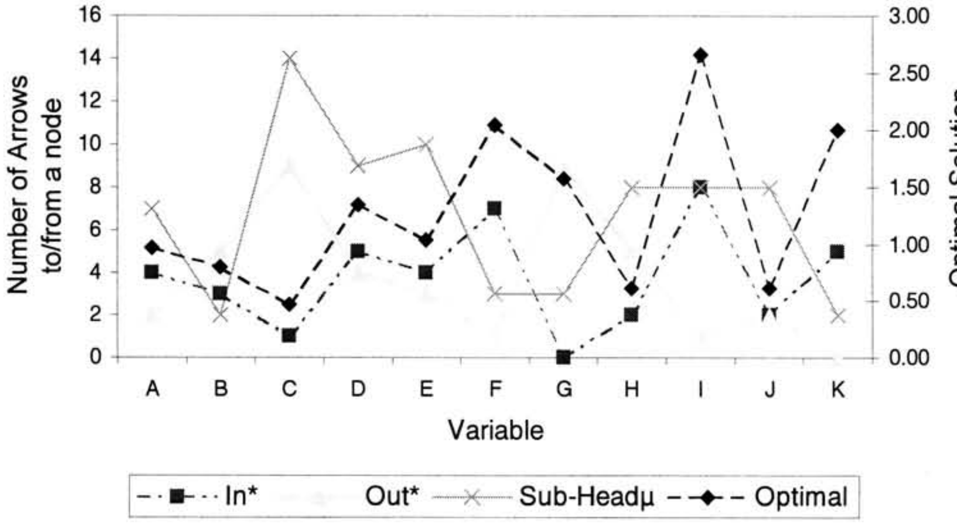

Figure 3.3 Optimal Solution Compared with Arrows To And From A Variable.

The optimal solution for the modified LP Problem of Equation 3.4 is given in Table 3.4 and this solution is compared to the solution of the original LP Problem in Figure 3.4. It is clear from Figure 3.4 that the nature of the optimal solution is unchanged for changes in the RHS of the inequality constraints G1 – G11.

Table 3.4 Optimal Solution to Modified LP Problem of Equation 3.4

Variable	Value	Description
A	0.36	Optimise Condition of Roads
B	0.30	Utilise Dynamic Truck Allocation System
C	0.18	Investigate Alternative Transport Technology
D	0.51	Ensure Optimal Operator Practices
E	0.39	Ensure Optimal Operational (Bedryfs) Practices
F	0.77	Ensure Optimal Tyre Management
G	0.59	Availability and Reliability of Information System
H	0.23	Optimise Mynboubeplannings Practices (Pit layout)
I	1.00	Effective Engine Maintenance Practices
J	0.23	Better Commercial Fuel Specifications
K	0.75	Effective Loss Control
Objective	5.32	

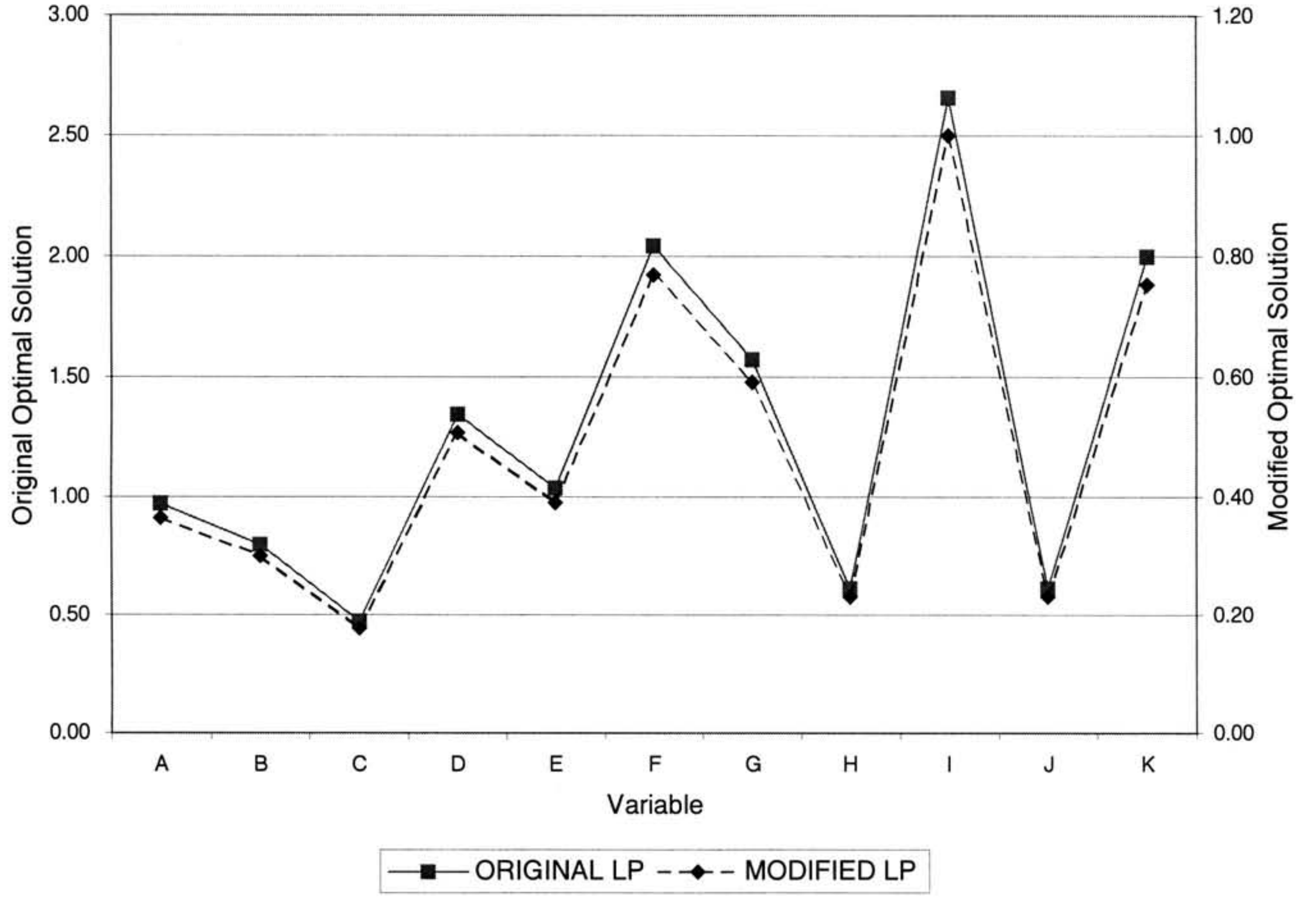

Figure 3.4 Solution of LP Problem 3.4 Compared to Modified LP Problem

Next we must determine if this type of solution is valid for all values of objective coefficients. So we reformulate the LP Problem of Equation 3.4 with all influence coefficients a_i .. k_i = k (see Equation A.1 of Appendix A) and solve the LP Problem for k = 0.1, k = 0.5, k = 0.3 and k = 1. The complete results can be found in Appendix A. Figure 3.5 is a comparison of the optimal solution for the different values of k.

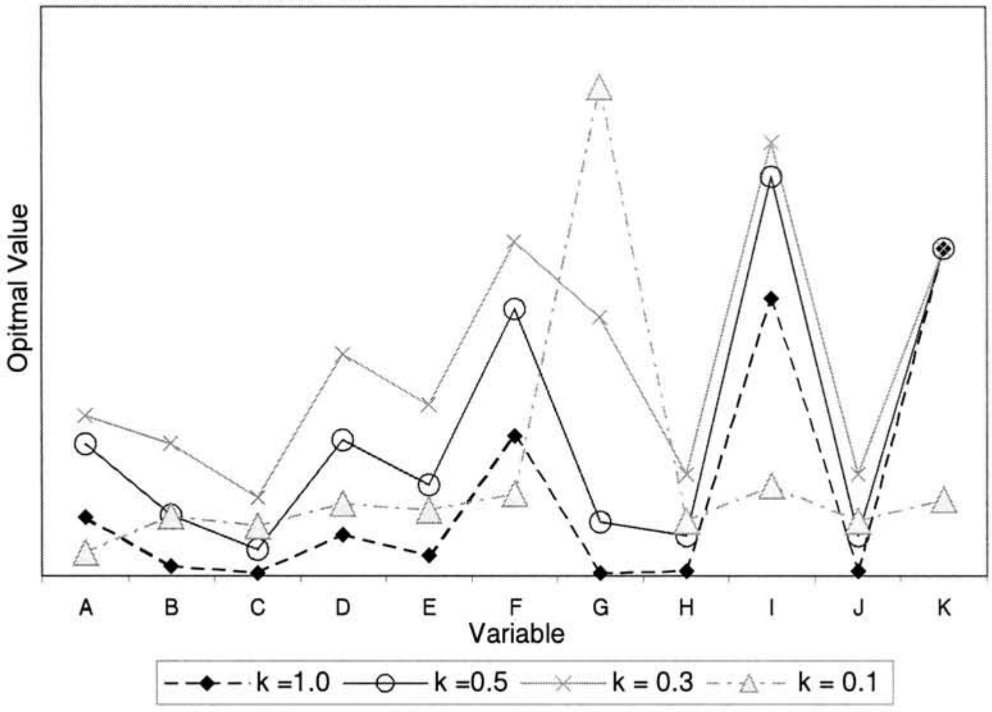

Figure 3.5 Optimal Solution for LP Problem 3.6

3.2 Linear Programming Model for Grootegeluk Diesel Cost-Driver Model

The diesel cost-driver mode for the Grootegeluk mine is shown in Figure 3.6 [Retief 2001-2]. The main factors affecting diesel consumption identified follow in Table 3.5.

Table 3.5 Factors Affecting Diesel Consumption on Grootegeluk

Factor	In*	Out*	Sub-Head∝
A: Availability and Integrity of Management System	0	10	7
K: Influence of Equipment Specification	1	8	1
B: Effective Incentive System	1	7	14
H: Mine Layout and Production Planning	3	4	4
G: Road Conditions and Maintenance	4	3	11
I: Fuel and Lubricants Specifications	2	2	4
J: Minimise Idling	4	2	7
D: Maintenance Planning	7	2	8
C: Level (Quality) of Equipment Maintenance	7	1	6
E: Optimal Development of Operators	3	0	8
F: Availability and Utilisation of Pantograph	7	0	5

Utilising Figure 3.6 and Table 3.5 it was possible to formulate the LP problem:

Maximise $Z = A + B + C + D + E + F + G + H + I + J + K$ (3.5)

Subject to

H1: $B = k.\,A$

H2: $C = k.\,A + k.\,B + k.\,D + k.\,G + k.\,I + k.\,J + k.\,K$

H3: $D = k.\,A + k.\,B + k.\,G + k.\,H + k.\,I + k.\,J + k.\,K$

H4: $E = k.\,A + k.\,B + k.\,K$

H5: $F = k.\,A + k.\,B + k.\,C + k.\,D + k.\,G + k.\,H + k.\,K$

H6: $G = k.\,A + k.\,B + k.\,H + k.K$

H7: $H = k.\,A + k.\,B + k.\,K$

H8: $I = k.\,A + k.\,K$

H9: $J = k.\,A + k.\,B + k.\,H + k.\,K$

H10: $K = k.\,A$

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* Refers to number of Arrows pointing towards/ from that specific node

∝ Refers to number of Sub-Headings

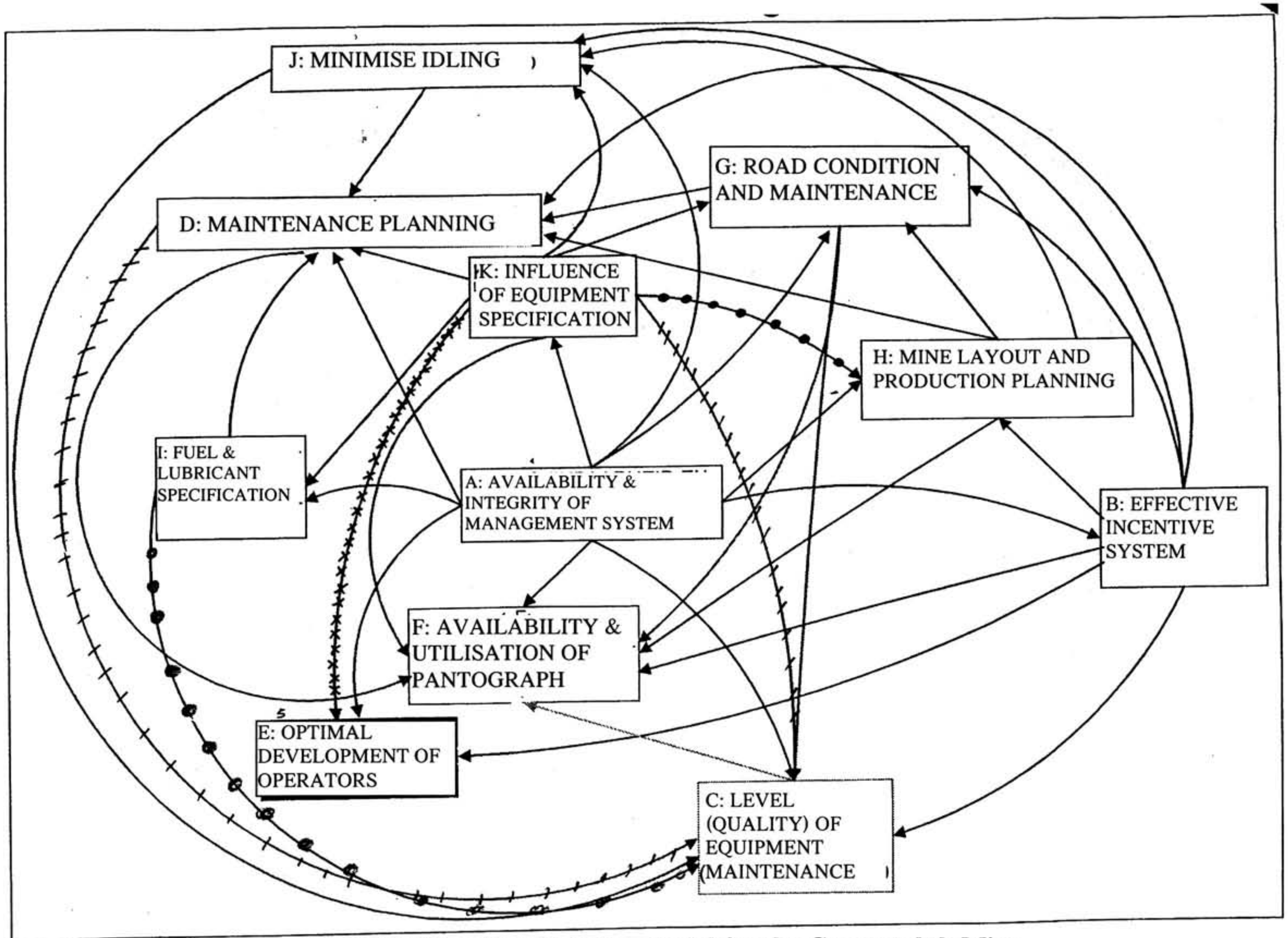

Figure 3.6 Diesel Cost-Driver Model for the Grootegeluk Mine.

G1: $A \le 7$

G2: $B \le 1$

G3: $C \le 14$

G4: $D \le 4$

G5: $E \le 11$

G6: $F \le 4$

G7: $G \le 7$

G8: $H \le 8$

G9: $I \le 6$

G10: $J \le 8$

G11: $K \le 5$

$A, B, C, .., K \ge 0.$

In Equation 3.5, the influence coefficients are assumed to be equal to the constant 'k' where $0 \le k \le 1$. Again, the optimal solution and sensitivity analysis of Equation 3.5 are solved with the LINDO Optimisation Software. The results for $k = 0.3$ is shown in Table B.2.

We must again determine how the optimal solution will differ with different LP parameters. From Table B.2 we conclude that an objective coefficient may be infinitely increased without changing the basis and thus the value of the variables for the optimal solution is unchanged. There will be a change in the objective function value due to the change in objective coefficient but as mentioned earlier we are interested in the qualititative nature of the solution.

The sensitivity analysis of the Grootegeluk diesel cost-driver model is similar to that of the Thabazimbi diesel cost-driver model in Tables A.2. As a result of the sensitivity analysis, we only need to investigate the effect of decreasing the RHS of G1- G11 by more than the allowable decrease. Setting all the constraints G1 – G11 less or equal to 1 and resolving the LP Problem will suffice in this regard.

The results of the modified LP problem is solved and plotted with the original LP problem in Figure 3.7. Note that although the optimal values of the variables change with different constraint values, their relative numerical order remains fairly unchanged. That is, the qualitative nature of the solution is unchanged. Figure 3.8 shows the similarity between the original and the modified LP problem.

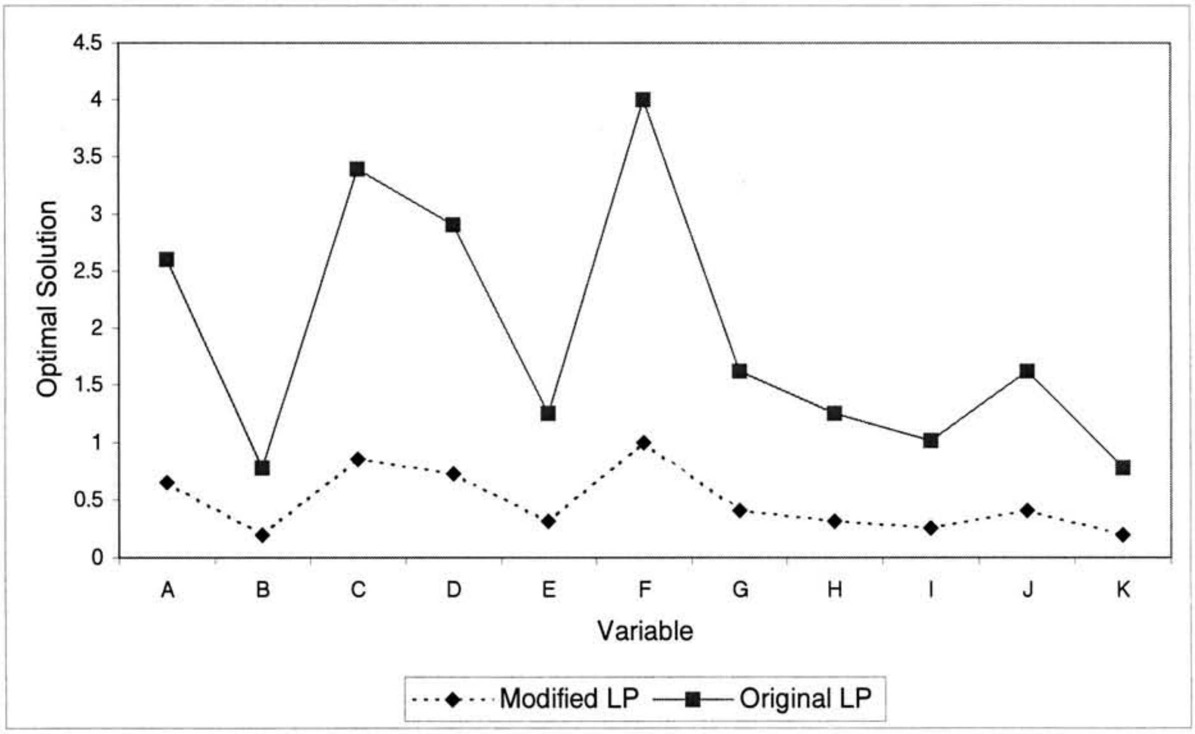

Figure 3.7 Solution of LP Problem 3.5 Compared to Modified LP Problem

We need to determine if the solution changes for different values of 'k', the universal influence coefficient. The detailed results for the solution of Equation 3.5 is given in Appendix B for k = 0.1, 0.3, 0.5 and 1.0. The optimal solutions are summarised in Figure 3.9.

The LP problem 3.5 was also solved by substituting non-equal values for the influence coefficients. One such example is plotted as Figure 3.10.

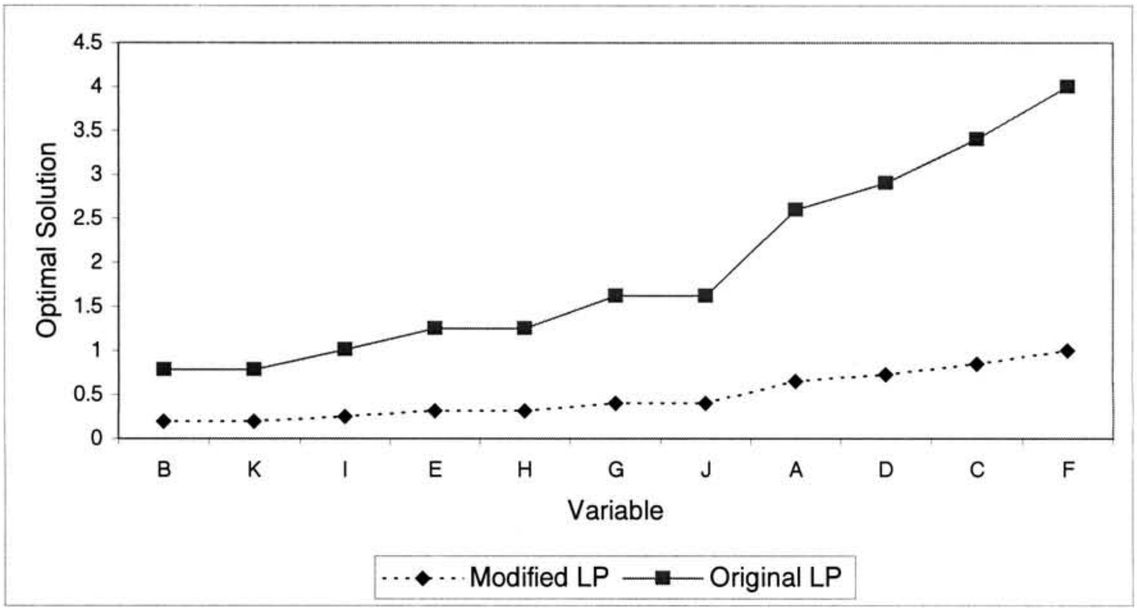

Figure 3.8 Solution of LP Problem 3.5 Compared to Modified LP Problem with Variables in Ascending Numerical Value.

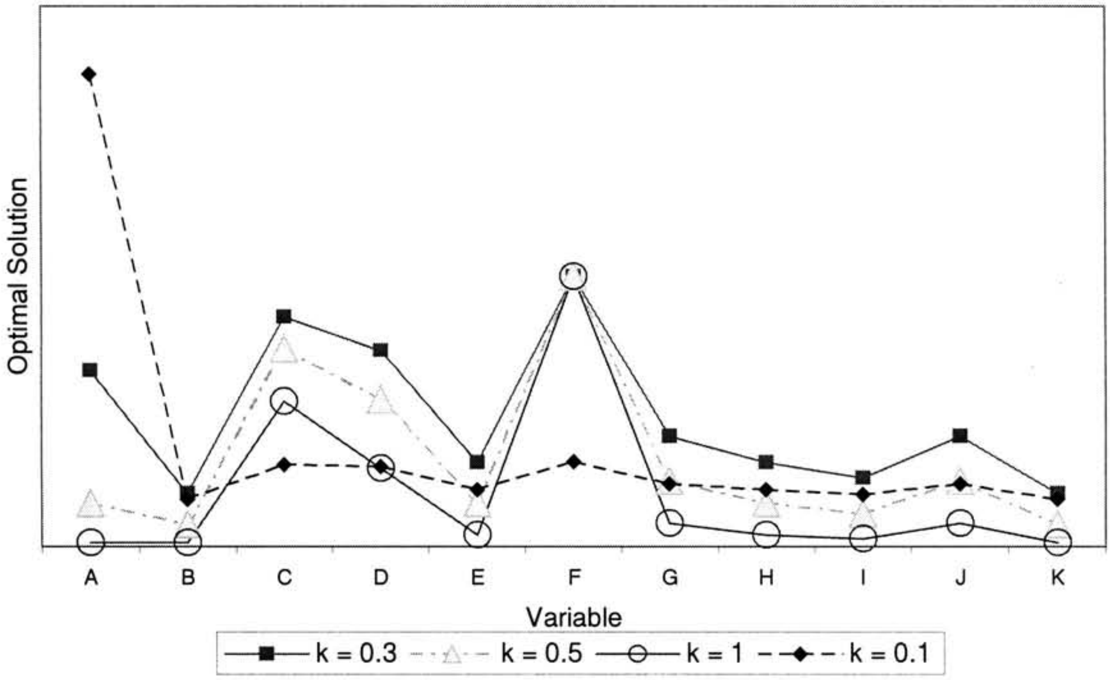

Figure 3.9 Optimal Solution for LP Problem 3.5.

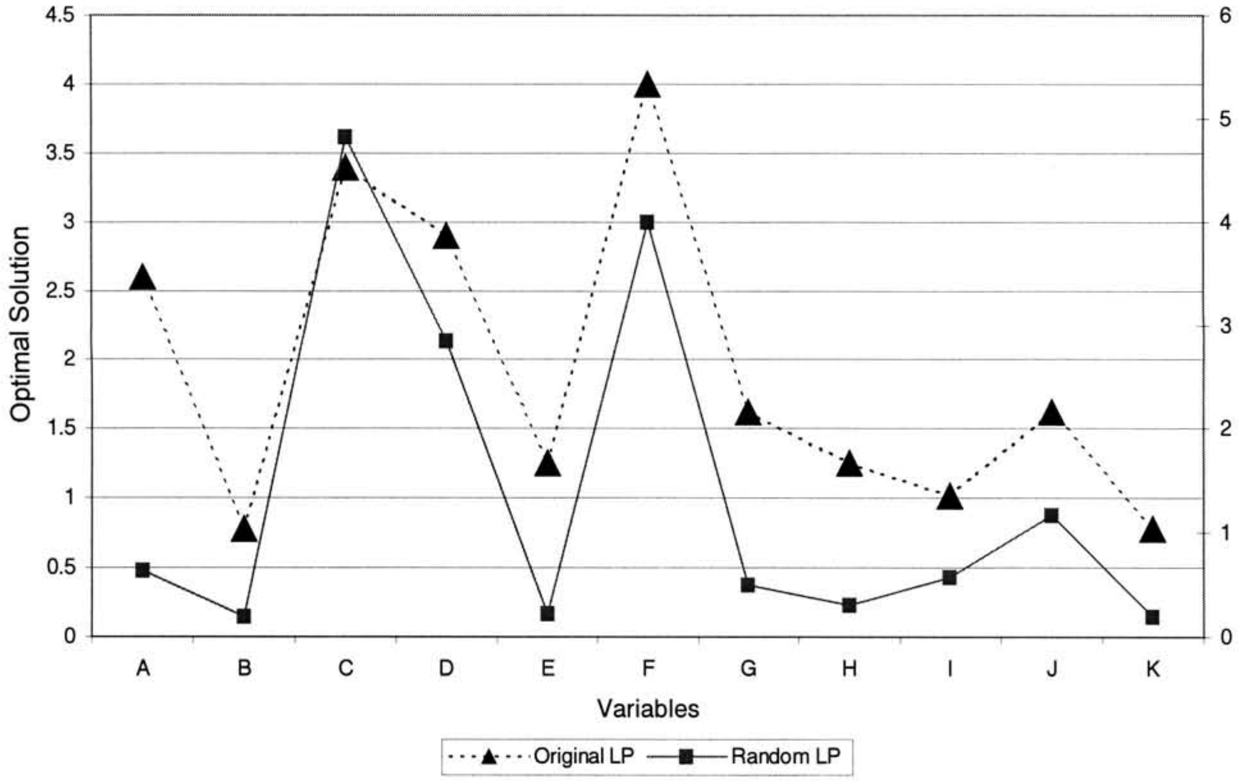

Figure 3.10 Comparison between Optimal LP Solution with Equal Coefficients and Random Coefficients.

3.3 Linear Programming Model for Sishen Diesel Cost-Driver Model

The diesel cost-driver model for the Sishen mine is illustrated in Figure 3.11. Table 3.7 describes the factors each variable represents. Utilising Figure 3.11 and Table 3.6 it is possible to formulate the LP problem for the Sishen diesel-cost driver model:

Maximise $Z = A + B + C + D + E + F + G + H + I + J + K \quad (3.6)$

Subject to

H1: $B = kA + kE + kH + kI$

H2: $C = kA + kB + kD + kE + kF + kG + kI$

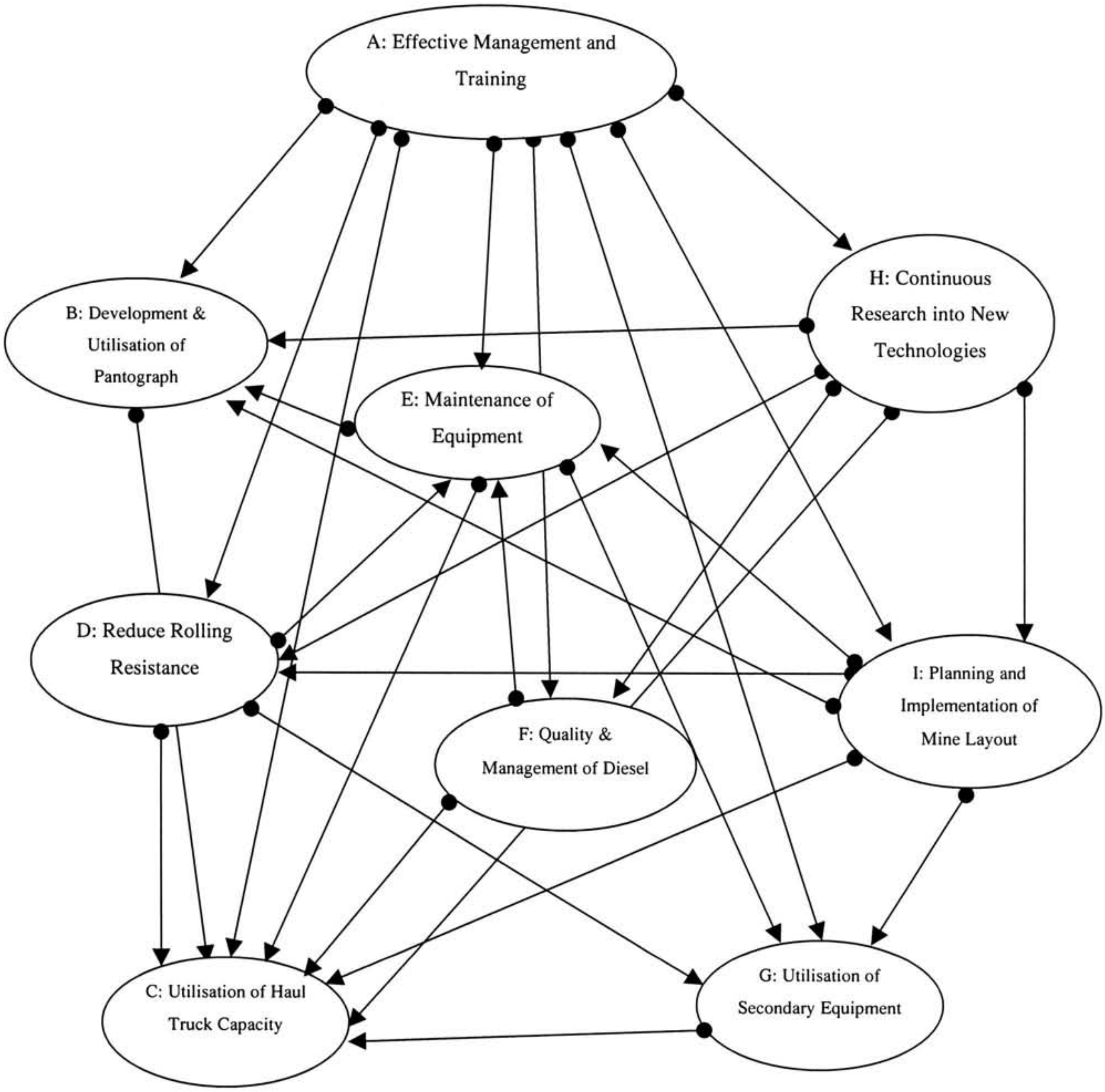

Figure 3.11 Diesel Cost-Driver Model for the Sishen Mine.

Table 3.6 Factors Affecting Diesel Consumption on Sishen

Factor	In*	Out*	Sub-Head∝
A: Effective management and training	0	8	17
B: Development and Utilisation of Pantograph	3	3	8
C: Utilisation of Haul Truck Capacity	6	1	9
D: Reduce Rolling Resistance	3	2	5
E: Maintenance of Equipment	7	1	8
F: Management and Quality of Diesel	3	3	9
G: Utilisation of Secondary Equipment	7	1	3
H: Continuous Investigation of New Technologies	1	7	5
I: Planning and Implementing Mine Layout	2	6	15

H3: $D = kA + kH + kI$

H4: $E = kA + kD + kF + kI$

H5: $F = kA + kH$

H6: $G = kA + kD + kE + kI$

H7: $H = kA$

H8: $I = kA + kH$

G1: $A \leq 17$

G2: $B \leq 8$

G3: $C \leq 9$

G4: $D \leq 5$

G5: $E \leq 8$

G6: $F \leq 9$

G7: $G \leq 3$

G8: $H \leq 5$

G9: $I \leq 15$

---

* Refers to number of Arrows pointing towards/ from that specific node

∝ Refers to number of Sub-Headings

$A, B, C, .., I \geq 0.$

In Equation 3.6, the influence coefficients are assumed to be equal to the constant 'k' where $0 \leq k \leq 1$. This has proven to be a reasonable assumption for the previous two LP problems and its validity for Equation 3.6 will be examined. The optimal solution and sensitivity analysis of Equation 3.6 is solved with the LINDO Optimisation Software and the results for $k = 0.3$ is shown in Table C.2.

We must again determine how the optimal solution will differ with different LP parameters. From Table C.2 we conclude that an objective coefficient may be infinitely increased without changing the basis and thus the value of the variables for the optimal solution is unchanged. There will be a change in the objective function value due to the change in objective coefficient but as mentioned earlier we are interested in the qualititative nature of the solution.

The sensitivity analysis shown in Table C.2 is similar to that of Table A.2 and Table B.2.

By following the same sensitivity analysis as that of the Thabazimbi and Grootegeluk diesel cost-driver models we obtain similar findings. Firstly that, increasing the upper bound of one of these constraints will not yield an improvement in the objective function value. This is confirmed by the zero dual prices for the constraints G1-G9 in Table C.2. Secondly we find that we need only investigate the effect of decreasing the RHS of G1- G11 by more than the Allowable Decrease. Setting all the constraints G1 – G11 less or equal to 1 and resolving the LP Problem will suffice in this regard.

The results of the modified LP problem is solved and plotted with the original LP problem in Figure 3.12. Note that although the optimal values of the variables differs with a change in constraints the relative numerical order of the variables remains unchanged. The Sishen LP model is then solved for $k = 0.1, 0.3, 0.5$ and 1.0. Detailed results may be found in Appendix C. Figure 3.13 is a summary of the results presented in Appendix C. The LP problem is also solved by assigning random values for the influence coefficients. The purpose thereof is to determine the universality of the solution obtained by assuming equal influence coefficients.

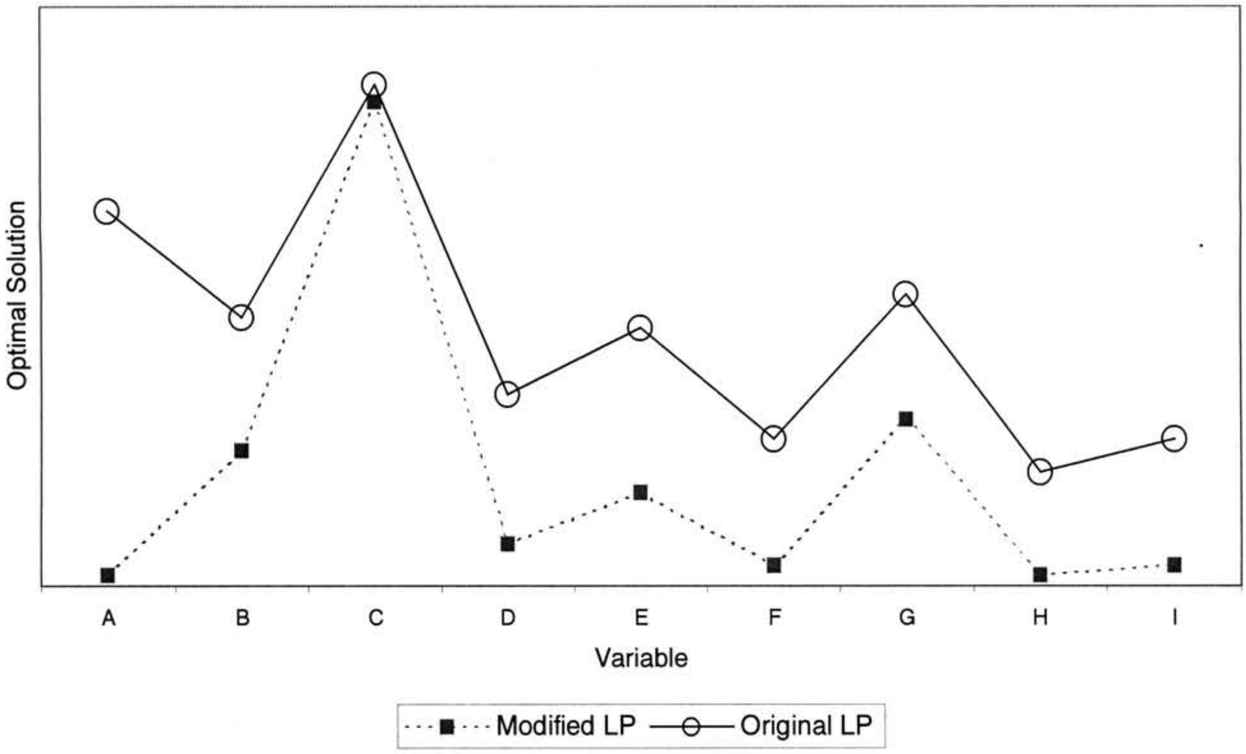

Figure 3.12 Solution of LP Problem 3.9 Compared to a Modified LP Problem

3.4 Discussion of Results of Diesel Cost Driver LP Problem.

A systematic model for diesel consumption on the mines was developed during participative workshops. Qualitative cost drivers and the relative importance thereof, is captured by exploiting role players' expert judgements [Roy et al. 1999-2].

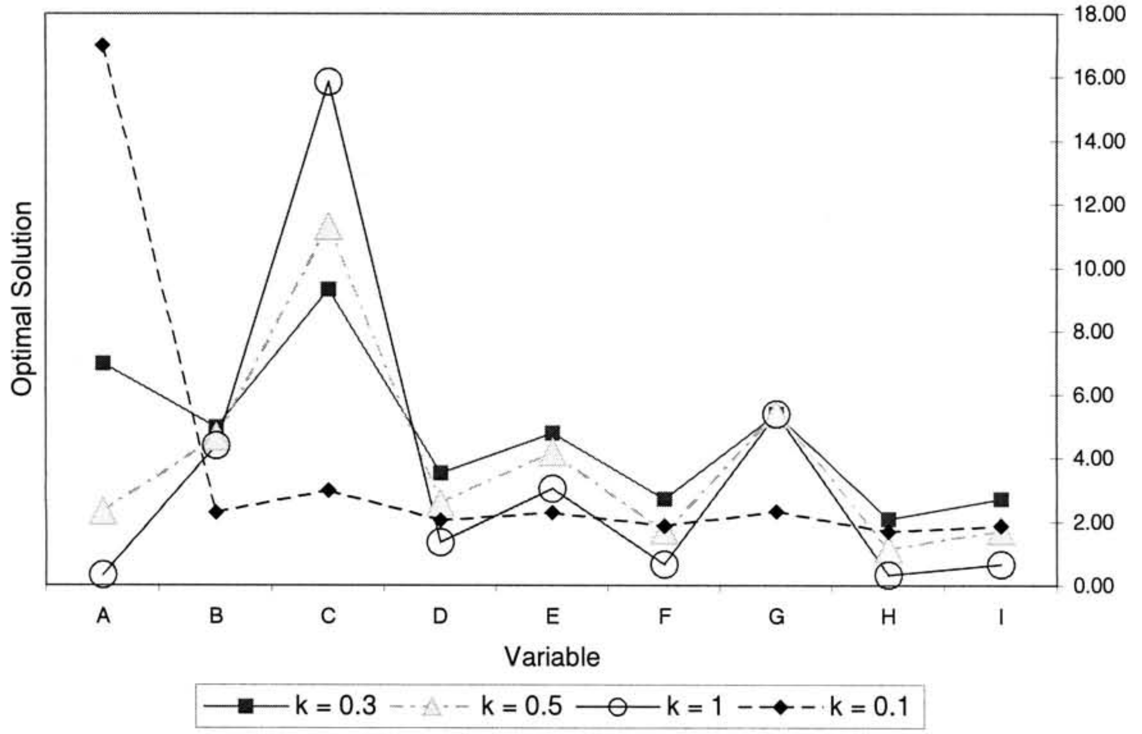

Figure 3.13 Optimal Solution for LP Problem 3.6

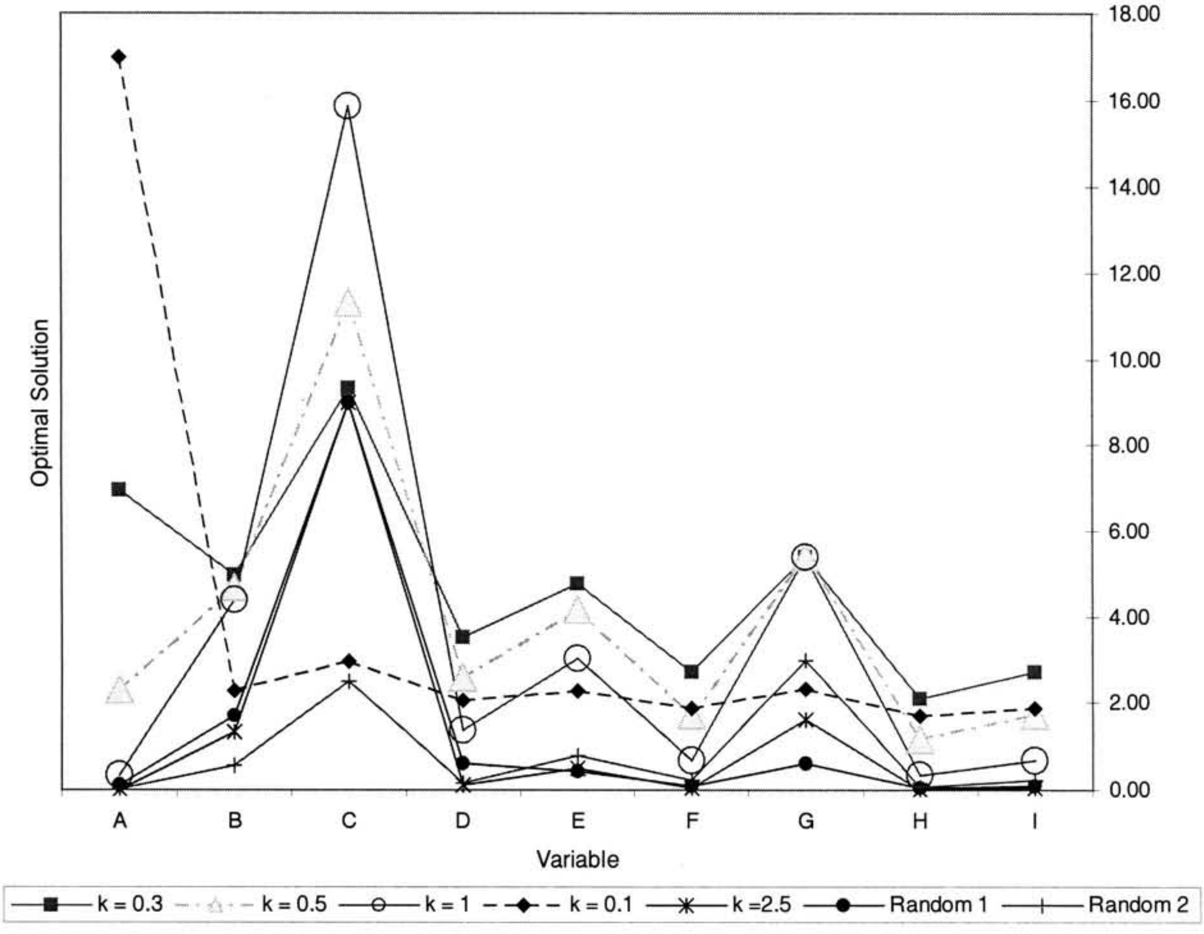

Figure 3.14 Comparison between Optimal LP Solution with Equal Coefficients and Random Coefficients

With the input from mine personnel, factors influencing diesel consumption, that is the cost drivers of diesel consumption, are identified and common factors are grouped under a general heading and then referred to as one cost driver. Then the interrelationship — cause and influence —between these cost drivers is identified. To utilise the cost-driver model it is modelled as an LP problem, to identify where should be focussed to be able to reduce costs associated with diesel consumption.

The solution for the Thambazimbi, Sishen and Grootegeluk models are plotted in order of ascending variables in the Figures 3.15, 3.16 and 3.17.

Consider the assumption that the upper bounds on the variables, as described by the constraints G1, G2, ..Gn, are defined by the number of sub-headings for that factor. This assumption was based on the cost-driver model so the values were not arbitrary. It is based on the premise that if so many ideas were generated on one topic it is a measure of that variable's 'importance'. In other words, a factor with more sub-headings has more of an effect to contribute to the system, if it is a driver, or there is more scope for improvement, if it is an outcome. However this assumption accepts that each sub-heading is equally important and is perception dependant. Another group of respondents could have grouped the factors differently by, for example, making 3 variables of 4 sub-headings instead of 6 variables with 2 sub-headings.

An alternative would be to assume an upper bound of unity for all variables and avoid any bias to a particular factor. This however would undermine the faith in the expert judgement applied in formulating the diesel cost-driver model.

The LP problem was first solved using the number of sub-headings as upper bounds for constraints G1, G2, ..,Gn. From the sensitivity analyses of Tables A.2, B.2 and C.2, it was evident that changing the upper bounds to unity was not within the allowable range for the RHS of the constraints and would require resolving a modified LP problem with the new upper bounds. The optimal solution for the original LP problem is compared to the optimal solution of the modified LP problem in Figures 3.4, 3.8 and 3.12.

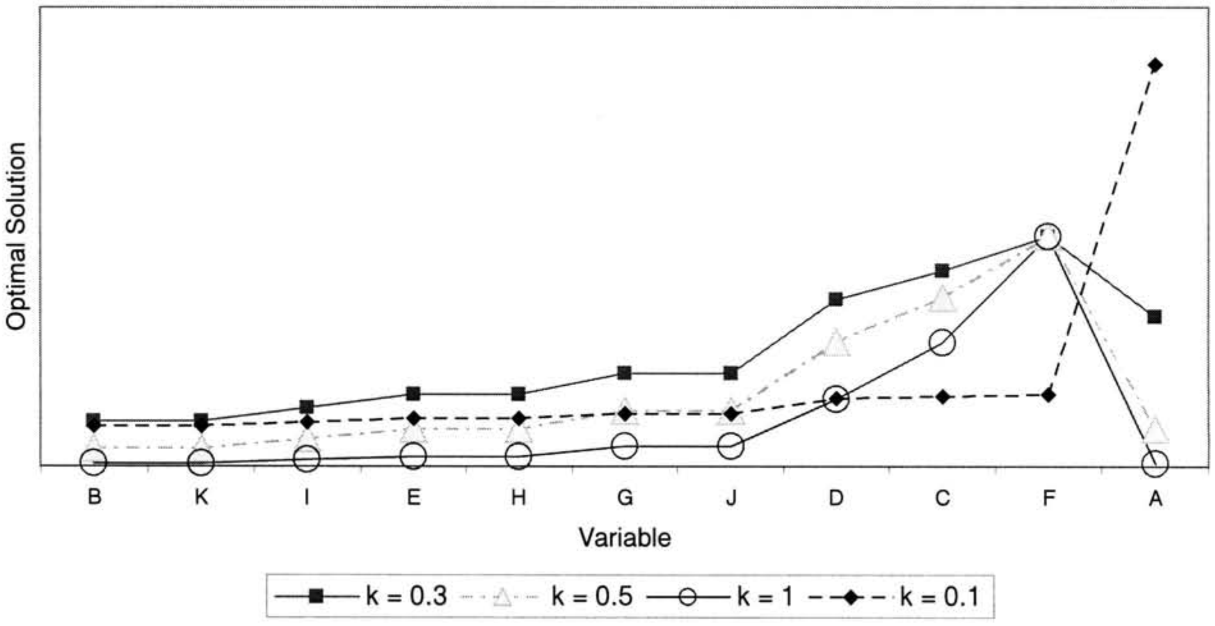

Figure 3.15 Optimal Solution for Grootegeluk LP Problem

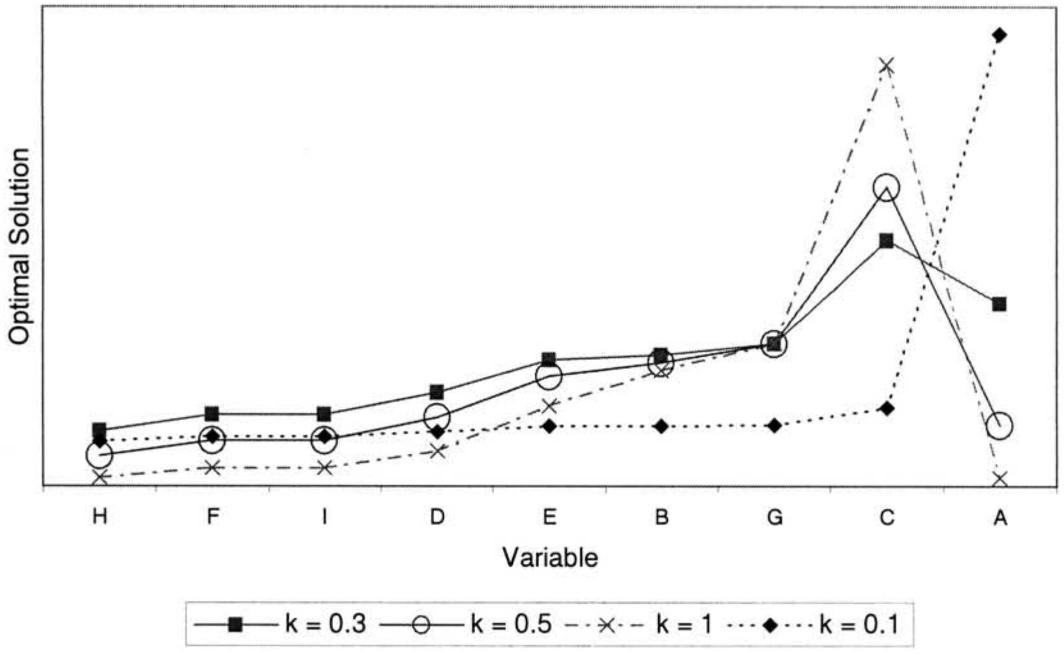

Figure 3.16 Optimal Solution for Sishen LP Problem

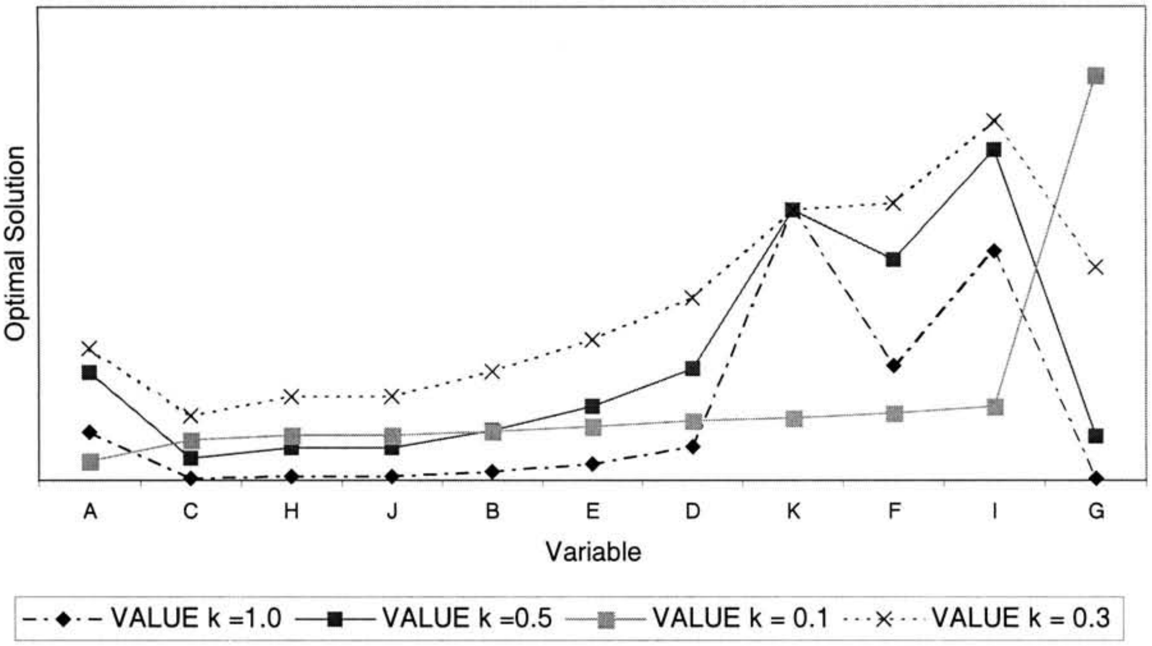

Figure 3.17 Optimal Solution for Thabazimbi LP Problem

From this analysis it was evident that the LP problems for all 3 cost-driver models were insensitive to the upper bounds placed on the inequality constraints G1, G2, .. ,Gn. That is, whether the LP problem had constraints defined by the number of sub-headings or unity, the relative magnitudes of the variables in the optimal solution did not vary. The factor A was still larger than factor C which was still larger than factor G.

The descriptive (fundamental) equations of the LP problem are then the constraints H1, H2, .., Hn. This is confirmed by the fact that these constraints are binding and the optimal solution occurs at the intersection of the binding constraints. The LP model was formulated to identify the cause and effect relationships between factors, and the optimum will occur when a small input yields a large result on the remaining factors. It is thus fitting that the constraints that define the interrelationship between the factors be the equations that ultimately define the character of the solution.

That is not to say that the upper bound constraints are superfluous. Their purpose is to bound the optimal solution. Without the constraints G1, G2, ..,Gn, the system would be unbounded. The need for a finite upperbound is evident in the sensitivity analysis where not all the constraints have an infinite allowable increase for the RHS of constraints G1, G2, ..,Gn.

The LP Problems model the driver-vs-outcome, that is the cause-and-effect relation between factors. Consider the optimal solution(s) for the Grootegeluk model displayed in Figure 3.15. Recall that the influence coefficient describes the positive influence of one variable on another as depicted in Figure 3.2. Influence coefficients are assigned values between zero and one inclusive. A zero influence coefficient implies no causal effect between factors and influence coefficient of one implies maximal effect. Thus for low values of influence coefficient we are damping the effect of the drivers on the outcomes. This is evident for $k = 0.1$ in Figure 3.15. By assigning $k = 0.1$ for all influence coefficients we have 'forced' the system not to be driven by A, our largest driver – with 10 outward directing arrows and 0 inward directing arrows – hence the deflated values of the other variables. A has been forced to assume a relatively large value to obtain a mediocre effect in the other variables and to fulfill the binding constraints. This phenomena is evident for the other two LP problems and their strongest driver, namely, factor G in the Thabazimbi model of Figure 3.17 with 9 outward directing arrows and 0 inward directing arrows; and factor A in the Sishen model of Figure 3.16 with 8 outward directing arrows and 0 inward directing arrows.

As the value of the influence coefficient is increased we see how the dominant driver boosts the other drivers and all the drivers (including the dominant driver) in turn boost the outcomes. As the value of the influence coefficient, in other words the strength of the influence, increases it requires a smaller effort from the dominant driver and the other driver to yield a large effect in the outcomes. This phenomena is due to a 'spill-over' or cascading effect in the LP problem. For the Thabazimbi model of Equation 3.4 and Figure 3.1, an effort in G spills over to C by constraint H3, an effort in C (and also from G both directly and indirectly) spills over to H, and, an effort in H (and also from G and C) spills over into B. Thus a small initial effort is compounded and carried throughout ("cascaded") through the system. This effect is exaggerated by the peaks and troughs of graphing the solutions in a non-ordered fashion such as Figures 3.5, 3.9 and 3.13.

Due to the cascading effect, a small effort in a driver will yield a large result in an outcome. Since the outcome is an effect of multiple compounded small efforts it is the upper bound placed on the outcome which limits the cascading effect achieved.

For example, consider two variables X and Y with $0 < X < 4$ and $0 < Y < 3$. Now suppose $Y = 2X$. So for each input from X we receive double the output from Y. However we are limited to $X < 1.5$ because Y must be less than 3. Thus it is a constraint on an outcome(s) which limits the cumulative effect of the system. We are however concerned with the qualitative input versus output nature of the solution and not the physical results as the values in themselves are meaningless.

3.4.1 Similarities between the models and the definition of a consolidated model.

Consider the optimal solution to the LP problems plotted in Figures 3.15, 3.16 and 3.17. Each LP problem was solved for different values of k and a family of similar curves was obtained. This means the general solution is not dependant on the value of the influence coefficients. For each LP problem we have a 'core' solution where the order of magnitude of the variables in the 'core' is fixed for all values of the influence coefficient 'k'. This core represents the similarity between optimal solutions for different values of 'k' that is evident in Figures 3.15, 3.16 and 3.17. There are also volatile dominant drivers/outcomes in each LP problem which are activated or deactivated by different values of the influence coefficient. These phenomena were discussed earlier. Table 3.7 is a tabular version of Figure 3.16. In Table 3.7 the variables of the optimal solution for the Sishen LP problem are tabulated in ascending order of magnitude. Variable A is the dominant driver and its value relative to the other variables decreases as the influence coefficient increases. If we ignore A and consider the remaining variables, it is evident that the variables are in the same order for all values of k, this set of remaining variables is the core solution.

The variables of the optimal solution to the Grootegeluk and Thabazimbi LP Problems are also tabulated in ascending order in Tables 3.9 and 3.11 and the core solution for each LP problem is determined. In the Thabazimbi model there is also a dominant outcome whose relative value increases as the value of the influence coefficients increases. Although no mention is made of the dominant driver (and the dominant outcome for the Thabazimbi LP problem), it is still part of the optimal solution.

Table 3.7 The Optimal Solution of the Sishen LP Problem

k = 0.1	k = 0.3	k = 0.5	k = 1
H	H	H	H
F	F	F	$\boxed{A}$
I	I	I	F
D	D	$\boxed{A}$	I
E	E	D	D
B	B	E	E
G	G	B	B
C	$\boxed{A}$	G	G
$\boxed{A}$	C	C	C

The core solution of the Sishen LP problem is thus :

Table 3.8 The Core Solution to the Sishen LP Problem

Factor	**In**	**Out**
H: Continuous Investigation of New Technologies	1	7
F: Management and Quality of Diesel	3	3
I: Planning and Implementing Mine Layout	2	6
D: Reduce Rolling Resistance	3	2
E: Maintenance of Equipment	7	1
B: Development and Utilisation of Pantograph	3	3
G: Utilisation of Secondary Equipment	7	1
C: Utilisation of Haul Truck Capacity	6	1

By comparing Tables 3.8, 3.11 and 3.12, it is evident that there are factors that are common to all three diesel cost-driver models. These factors feature in the same region when the variables are sorted by ascending numerical value. This means the relation between common factors of different LP problems is the same. For example, the dominant cost driver in each model is information and management system related (Variables A for the Grootegeluk model, A for the Sishen model and G for the Thabazimbi model) and "Fuel & Lubricant Specifications" has lower value than the Maintenance factor for all 3 models.

Table 3.9 Variables of the Optimal Solution to the GrooteGeluk LP Problem in Ascending Order.

k = 0.1	k = 0.3	k = 0.5	k = 1
B	B	B	B
K	K	K	K
I	I	I	A
E	E	E	I
H	H	H	E
G	G	A	H
J	J	G	G
D	A	J	J
C	D	D	D
F	C	C	C
A	F	F	F

Table 3.10 Variables of the Optimal Solution to the Thabazimbi LP Problem in Ascending Order.

k = 0.1	k = 0.3	k =0.5	k =1.0
A	C	C	C
C	H	H	G
H	J	J	H
J	B	G	J
B	A	B	B
E	E	E	E
D	D	A	D
K	G	D	A
F	K	F	F
I	F	K	I
G	I	I	K

Table 3.11 The Core Solution to the Grootegeluk LP Problem

Factor	Out*	In*
B: Effective Incentive System	7	1
K: Influence of Equipment Specification	8	1
I: Fuel and Lubricants Specifications	2	2
E: Optimal Development of Operators	0	3
H: Mine Layout and Production Planning	4	3
G: Road Conditions and Maintenance	3	4
J: Minimise Idling	2	4
D: Maintenance Planning	2	7
C: Level (Quality) of Equipment Maintenance	1	7

* Refers to number of Arrows pointing towards/ from that specific node

Table 3.12 The Core Solution to the Thabazimbi LP Problem

Factor	Out*	In*
C: Investigate Alternative Transport Technology	9	1
H: Optimise Mynboubeplannings Practices (Pit layout)	5	2
J: Better Commercial Fuel Specifications	2	2
B: Utilise Dynamic Truck Allocation System	5	3
E: Ensure Optimal Operational (Bedryfs) Practices	3	4
D: Ensure Optimal Operator Practices	4	5
F: Ensure Optimal Tyre Management	1	7
I: Effective Engine Maintenance Practices	1	8
D: Maintenance Planning	2	7
C: Level (Quality) of Equipment Maintenance	1	7

* Refers to number of Arrows pointing towards/ from that specific node

The diesel cost-driver models are very similar and differences in the models can be attributed to common closely related topics being grouped under different factors. For example, road condition, rolling resistance and tyre maintenance are closely related and can be considered as separate factors or as only one or two factors. Another reason is the inclusion of pantograph utilisation as a factor for the Sishen and Grootegeluk models whilst Thabazimbi does not utilise pantograph lines.

There is sufficient commonality among the cost-driver models to formulate a consolidated cost-driver model utilising the key factors and relations of the Sishen, Thabazimbi and Grootegeluk models. Table 3.13 highlights the factors adopted for a consolidated cost-driver model using the interrelationships of the established models in Figures 3.1, 3.6 and 3.11. The IR diagram for a consolidated cost-driver is shown in Figure 3.18.

From Figure 3.18 we may define the LP problem for the consolidated diesel cost-driver model. The consolidated diesel cost-driver model is presented in Appendix D.

The student version of LINDO Optimization Software is used to solve Equation D.1 for $k = 0.1$, $k = 0.3$ and $k = 0.5$ and the optimal solution is plotted in Figure 3.19.

It is evident that X1, *Info and Management System*, is the dominant driver of the model. This was also the case for the other three LP problems. The core solution is shown in Table 3.14. Note that drivers of this system are X1, X10, X4 and X7. The variable X3 is an intermediate outcome as it is only influenced by X1 and X10 and is thus affected early in the cascading process. Variable X4 is also influenced by X1 and X10 however it is also a driver along with X7 for other variables. Variable X2 is a suitable choice for an intermediate outcome/driver because it is situated 'mid-stream' of the cascading process.

Thus far the model and the results thereof have been qualitative in nature. We have been able to identify strong or dominant diesel cost-drivers and weak diesel cost-drivers. Dominant cost-drivers are those factors that not only affect diesel consumption but also affect other diesel cost-drivers. By affecting other cost-drivers,

Table 3.13 Factors for a Consolidated Diesel Cost-Driver Model

New Variable	Description/ Factor	Equivalent Factor		
		Thabazimbi	Grootegeluk	Sishen
X1	Info & Management System	G	A	A
X2	Road Condition and Maintenance	A	G	~D
X3	Fuel and Lubrication Specifications	J	I	F
X4	Mine Layout and Planning	H	H	I
X5	Maintenance of Equipment	I	C & D	E
X6	Pantograph Utilisation	n/a	F	B
X7	Operator Development	D	J & ~B/E	~A
X8	Efficient Equipment Utilisation	~E/~B	~K	C & G
X9	Tyre Specification and Maintenance	F	~K	~D
X10	Application of Alternative Technologies			

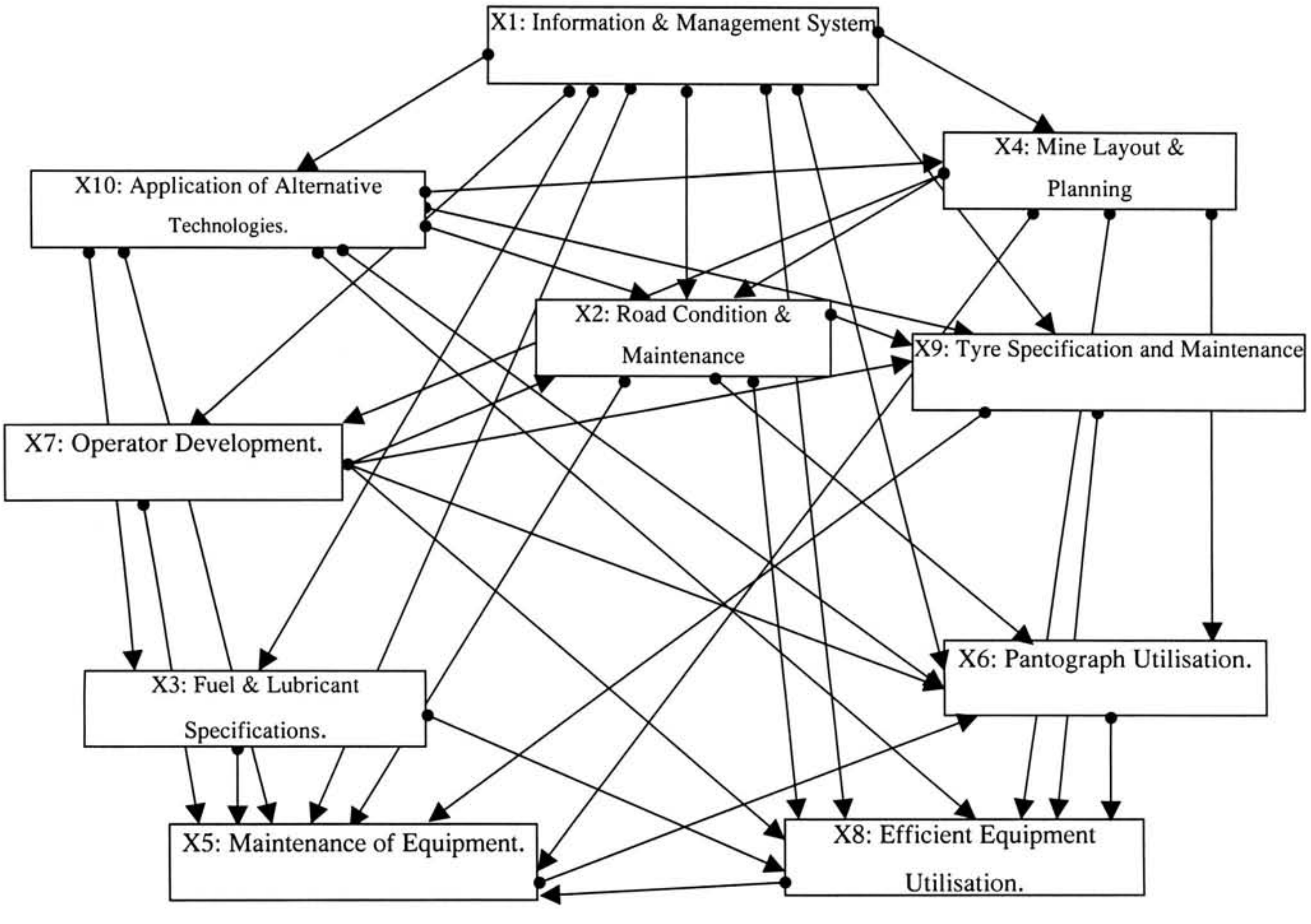

Figure 3.18 IR Diagraph of Consolidated Diesel Cost-Driver Model

drivers. Thus an improvement in a dominant cost-driver will lead to an exponential improvement in the objective function value.

Table 3.14 The Core Solution to the Consolidated LP Problem

Factor	Out*	In*
X10: Application of Alternative Technologies	8	1
X3: Fuel and Lubrication Specifications	2	2
X4: Mine Layout and Planning	5	2
X7: Operator Development	5	3
X2: Road Condition and Maintenance	4	4
X9: Tyre Specification and Maintenance	2	4
X6: Pantograph Utilisation	1	6
X8: Efficient Equipment Utilisation	1	8
X5: Maintenance of Equipment	1	8

* Refers to number of Arrows pointing towards/ from that specific node

A potential pitfall is our lack of knowledge regarding the actual value of influence coefficients. We have shown that as long as the values of the influence coefficients are not negligible the qualitative nature of the solution is certain and therefore direct and indirect influences affect diesel consumption. But, if we do not really know the value of the influence coefficients how can we be certain that the indirect influences are cascaded through the system? Perhaps all the influence coefficients are negligible, that is less than 0.1?

Investigation into the intermediate outcomes/drivers will provide clarity. Intermediate outcomes/drivers are useful in analysing and debugging 'the black box' of the systems process. By defining an intermediate outcome/driver by a quantifiable parameter, we may establish a baseline to quantify the effect of the dominant drivers on the intermediate outcome/driver. We now know how much value the beginning stages of the system have added and we know that we will get an improvement further in the system since the intermediate outcome/driver is also a driver for other factors. We would also like to quantify a final outcome to ascertain if the system did improve overall because it is possible that our intermediate improvement was absorbed by a "faulty" element elsewhere or; that the influence of the intermediate driver/outcome on other factors is negligible.

As previously mentioned X2 – *Road Condition and Maintenance* - is a suitable intermediate outcome/driver. So quantifying the improvement in Road Condition is desirable because we then have a baseline to evaluate any improvements in the more dominant drivers. An improvement in a dominant driver will be manifested in an improvement in this intermediate outcome/driver. This approach is a means to quantify the effect on diesel consumption due to an improvement in a factor of a qualitative nature.

Furthermore, X2 - *Road Condition and Maintenance* - is a driver for other factors so an improvement here will yield improvements in other factors. An improvement in Road Condition may be quantified by rolling resistance coefficient. This is a suitable parameter to quantify the improvement in Road Condition because there exists a relation between rolling resistance coefficient and diesel consumption [du Plessis 1993]. Thus, the quantifiable parameter is relevant to the diesel cost-driver model. To quantify rolling resistance wheel and tyre theory needs to be investigated. This serves as justification for a review of wheel and tyre theory in Chapter 2.

We also need to quantify an outcome of the system as a baseline for evaluating the subsequent effect of improvements in an intermediate outcome/driver and to determine the overall effect of a dominant driver improvement. X8 – *Efficient Equipment Utilisation* is such an outcome and to quantify an improvement in this factor, the daily operating cycle of the mining vehicles will be simulated.

3.5 Conclusion

A cost-driver model for diesel consumption was formulated by exploiting the expert judgement of role players in the mining operations. Cost-driver models were developed for the Sishen, Thabazimbi and Grootegeluk mines. The cost-driver models were then modelled as linear programming problems and solved using the student version of LINDO Optimization Software. The results obtained were of a qualitative nature but highlighted the importance of the cause-effect relation among the various factors. Although formulated independently, the cost-driver models for the different mines, and the optimal solutions thereof were similar. This indicates the universality of the cost-driver models.

The three models were collated into a consolidated cost-driver model. The optimal solution to this model verified the results obtained by the former models. The solution to the LP problem highlighted that the indirect (hidden) benefits of improving a dominant cost driver are more substantial than the direct benefits.

To analyse the indirect effects, if any, of the dominant cost driver on diesel consumption the influence of the dominant cost driver is 'tracked' as it progresses through the cost-driver model. In this regard, the improvement in road conditions and the improvement in the utilisation of equipment must be quantified. One aspect of the 'Improvement in Road Condition' will be quantified by using the rolling resistance coefficient and requires an understanding of wheel and tyre theory, and in order to quantify the utilisation of equipment, the daily operating cycle of the haul trucks will be simulated.

4 SIMULATING OPERATIONAL CYCLE OF MINING VEHICLES.

From Chapter 2.1 it is clear that there is a need to quantify the performance of different truck-shovel configurations[3] in order to determine the optimal configuration which will meet production requirements and minimise the TCO of the haul trucks.

Here we investigate the optimal truck-shovel configuration to minimise TCO? It was deduced in Section 2.1 that Energy Costs, Repair Costs and Maintenance Costs are elements that can be minimised to reduce TCO. From the preceding discussion of the haul truck operating cycle, the truck-shovel configuration will influence the amount of time the haul trucks spend idling whilst waiting to load/offload and even during loading. Idling time represents an undesirable energy cost (diesel consumption) and an increase in engine operating time which will affect maintenance and repair costs. Associated productivity losses are also predicted and can be verified by investigating different haul truck-loader configurations.

Furthermore, it was concluded in Chapter 3 that it is necessary to quantify optimal equipment utilisation in order to fully develop a diesel cost driver model. The quantification of equipment utilisation measures the effect of a change in a dominant cost driver. The utilisation of equipment can be determined by simulating the operating cycle of mining vehicles.

Chapter 2.4 illustrated queue analysis using cumulative graphs and that queue simulation may be modelled as a discrete event system [Fishman 1978].

For a queue system to be advanced in simulation, potential events and their effects on the status of the queue system must be identified [Blouin et al. 2001]. The model must then be taught how to allow these events to occur. Two common methods are the Next-Event and Fixed-Time Increment Simulations [Watson 1981]. The Next-Event and Fixed-Time Increment Simulations were discussed in section 2.5.5.

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[3] The term 'truck-shovel configuration' refers to the ratio of haul trucks assigned to a loader(s) and, can include their respective payloads.

From the discussion in section 2.5.5, we learnt that Fixed-time increment simulation is simpler to understand but next-event simulation is computationally more efficient [Winston 1994]. This is because performing unnecessary operations checks during obvious periods of inactivity is eliminated. Roundoff error is present in a fixed-time increment simulation as a result of representing the continuous time domain with a finite set of time values. For this reason, the Next-Event Technique is used in the simulation algorithms for this chapter.

4.1 Next-Event Technique in Practice: The Development of Q_SIM.

Consider the simplified schematic of the mining operation in Figure 4.1. As an initial approximation, the mining operation can be modelled with a queue at loader, denoted as Queue 1, and then a queue at a dump and a plant denoted as Queue 2 and Queue 3 respectively. Also only one truck may be serviced at a queue at a time and we assign all parameters indicated in Figure 4.1 as constants. Arbitrary initial arrival times at Queue 1 are assigned for 'm' trucks.

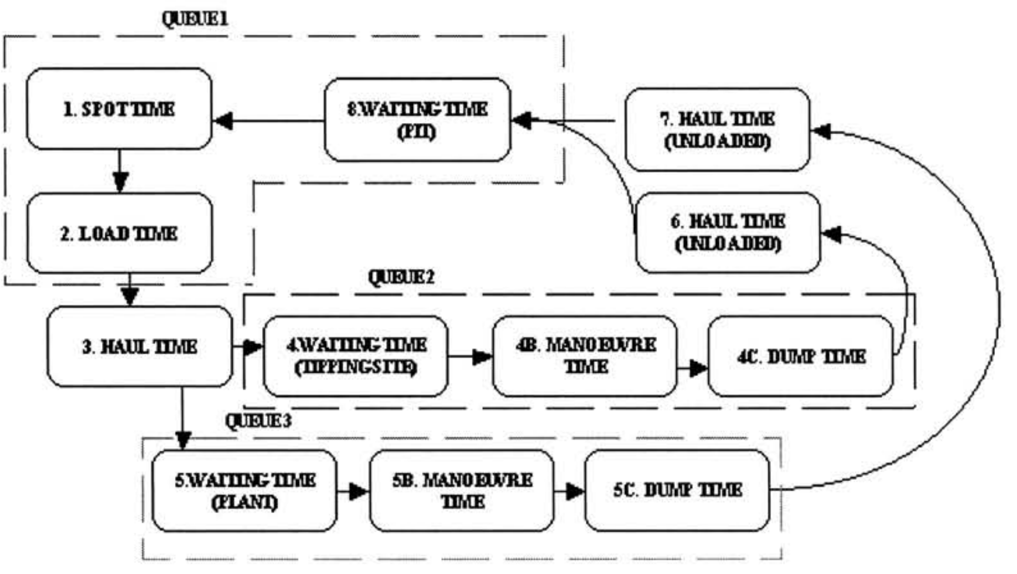

Figure 4.1 Queues in Operational Cycle of Mining Vehicle.

The Next-Event approach is modelled by the utilisation of a 'Tracker' matrix. In the Tracker Matrix, each row denotes a particular truck and each column denotes a discrete state/event of a truck. For Figure 4.1, the following states/events have been identified:

column 1 = waiting in Q1

column 2 = in service at Q1

column 3 = en route from Q1 to Q2

column 4 = en route from Q1 to Q3

column 5 = waiting in Q2

column 6 = in service at Q2

column 7 = waiting in Q3

column 8 = in service at Q3

column 9 = en route from Q2 to Q1

column 10 = en route from Q3 to Q1

Since each truck can only be at one place at a time, each row can only have one non-zero entry at a time. Thus when a truck arrives at a new queue, we can determine if other trucks are already present by looking at the relevant column. If the non-zero entry in each row is the time of completion of that event/state then the minimum of all the non-zero entries is the next event that must occur in the simulation. We thus advance the clock time to the next event identified and perform necessary actions associated with the next event. Necessary actions may include, plotting data on the relevant cumulative graph and moving the particular truck on to its next required event and the time thereof.

The simulation was programmed in the Student Version of MATLAB and is called Q_SIM. Figure 4.2 is the cumulative graphs for the simulation of Figure 4.1 with the following input parameters.

no of haul trucks = 15

arbitrary arrival times

load time + spot time at pit = service time at queue 1 = 7 min

unload time + manoeuvre time at tip site = service time at queue 2 = 7.5 min

unload time + manoeuvre time at plant = service time at queue 3 = 3.5 min

time to travel from pit to tipping site = 5 min

time to travel from pit to plant = 2 min

time to travel to pit from tipping site = 5 min

time to travel to pit from plant = 9 min

Consider the graph of Queue 1. The Arrival curve is initially very steep which reflects that the initial arrival times of the 15 trucks occurred in quick succession. Thus a very steep slope of the arrival curve indicates a 'rush period'. Note that the Service and Departure curves are monotone increasing and the horizontal distance between the two curves is also constant. This is because we assumed Service Times were constant. The so-called rush period is indicative of the model's warm-up period.

Consider the graph of Queue 2. The arrivals at Queue 2 are evenly staggered. This is because of the constant service time and has the effect of staggering the departure from Queue 1 to Queue 2. The arrivals at Queue 2 are also staggered due to constant travel times. From time = 50 min to time ≈ 125 min, the curves are unchanged and horizontal implying no change in the number of arrivals, services and departures. This is characteristic of an idle queue. In this context, the queue is idle whilst the haul trucks travel back to queue 1 to load and travel back to queue 2 where they once again arrive staggered.

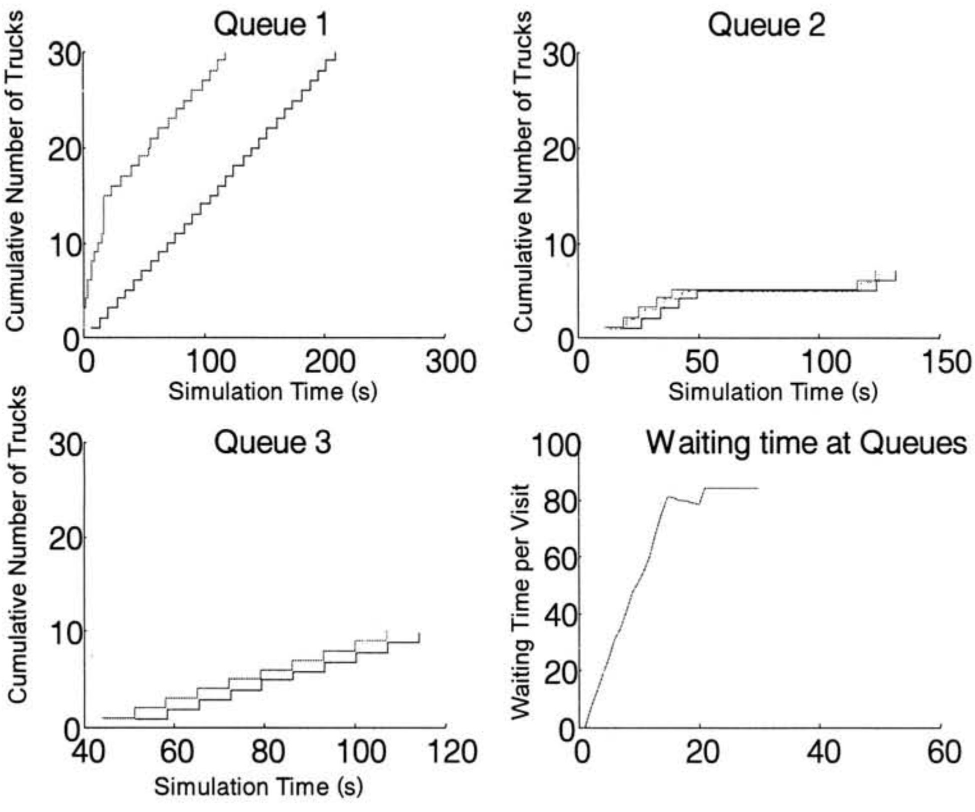

Figure 4.2 Queue Simulation Output for Figure 4.1

With the output of the queue simulation in the form of a cumulative graph it is possible to determine microscopic properties of the queue. However it is often useful to quantify a queue's performance by a macroscopic property. For example, the total waiting time is equal to the area between the arrival and service curves and the total time the server is busy is the area between the service and departure curves. With the total waiting time or the total service time we can determine the average time a truck waits for service as

$$\text{Average Wait Time} = \text{Total Wait Time} / \text{Cumulative no of Trucks.} \tag{4.1}$$

The average service time follows in a similar way:

$$\text{Average Service Time} = \text{Total Service Time} / \text{Cumulative No of Trucks.} \tag{4.2}$$

Another parameter of interest is Server Utilisation, which is defined as the portion of time the server is busy:

$$\text{Utilisation} = \text{Total Service Time} / \text{Total Time.} \tag{4.3}$$

It is also useful to determine average number of trucks awaiting service by

$$\text{Average Queue Size} = \text{Total Wait Time} / \text{Final Clock Time.} \tag{4.4}$$

4.2 Verification of Q_SIM.

After Q_SIM had been developed and debugged, we must verify that Q_SIM performs the functions we expect from it. Testing Q_Sim was done by performing a few iterations of the simulation manually and verifying that the output from Q_SIM was precisely what we intended. Table 4.1 is the manual simulation for the first iteration performed on Microsoft Excel. Table 4.2 is the output from Q_SIM in the same format as Table 4.1. It was evident that Q_SIM performed precisely the functions intended.

Table 4.1 Manual Simulation of Queue System of Figure 4.1.

	Queue 1			Queue 2			Queue 3		
Truck	Arrive	Service	Depart	Arrive	Service	Depart	Arrive	Service	Depart
1	0	0	7	12	12	19.5	--	--	--
2	0	7	14	19	19.5	27	--	--	--
3	1	14	21	26	27	34.5	--	--	--
4	3	21	28	33	34.5	42	--	--	--
5	4	28	35	40	42	49.5	--	--	--
6	4	35	42	--	--	--	44	44	47.5
7	8	42	49	--	--	--	51	51	54.5
8	8	49	56	--	--	--	58	58	61.5
9	9	56	63	--	--	--	65	65	68.5
10	12	63	70	--	--	--	72	72	75.5
11	15	70	77	--	--	--	79	79	82.5
12	17	77	84	--	--	--	86	86	89.5
13	17	84	91	--	--	--	93	93	96.5
14	17	91	98	--	--	--	100	100	103.5
15	17	98	105	--	--	--	107	107	110.5

Table 4.2 Q_SIM Output for Queue System of Figure 4.1.

Arrive Q1	Service Q1	Depart Q1	Arrive Q2	Service Q2	Depart Q2	Arrive Q3	Service Q3	Depart Q3
0	0	7	12	12	19.5	NaN	NaN	NaN
0	7	14	19	19.5	27	NaN	NaN	NaN
1	14	21	26	27	34.5	NaN	NaN	NaN
3	21	28	33	34.5	42	NaN	NaN	NaN
4	28	35	40	42	49.5	NaN	NaN	NaN
4	35	42	NaN	NaN	NaN	44	44	47.5
8	42	49	NaN	NaN	NaN	51	51	54.5
8	49	56	NaN	NaN	NaN	58	58	61.5
9	56	63	NaN	NaN	NaN	65	65	68.5
12	63	70	NaN	NaN	NaN	72	72	75.5
15	70	77	NaN	NaN	NaN	79	79	82.5
17	77	84	NaN	NaN	NaN	86	86	89.5
17	84	91	NaN	NaN	NaN	93	93	96.5
17	91	98	NaN	NaN	NaN	100	100	103.5
17	98	105	NaN	NaN	NaN	107	107	110.5

4.3 Validation of Q_SIM.

It was also determined whether Q_SIM provided a reliable and realistic representation of the real-world queue system. The simulation output may be compared to captured data or an analytical solution to validate the simulation.

This proves to be problematic when simulation is used because capturing data is costly or not yet possible and the system may be too complex to solve analytically using queue theory. Validation is thus done intuitively and by comparing Q_SIM output to a simpler system where an analytical solution or measured data exists.

Winston (1994) provides an example of a single-server queue simulation. The example concerns a fast-food restaurant that has a single drive-in window. The average rate of arrivals is 10 customers per hour and the average service rate is 15 customers per hour. The time between arrivals and, service times are exponentially distributed. The average number of customers present and the utilisation of the drive-in window is desired. This simulation was performed using a spreadsheet [Winston 1994] and compared to the analytical solution derived from queue theory [Vignaux 1999 and Winston 1994].

The above example concerns a single queue whereas Q_SIM models a cyclic network of three queues. By letting the haul times from Queue 1 to Queue 2 and Queue 3, respectively, be extraordinarily large, the haul trucks will not return to Queue 1 during the time the simulation runs and we have a quasi - single queue model from Q_SIM. We assign the following parameter values in Q_SIM.

no of haul trucks = 150
interarrival time = exponential distribution with average of 6 min.
service time at queue 1 = exponential distribution with average of 4 min.
service time at queue 2 = 50000 min
service time at queue 3 = 50000 min
time to travel from pit to tipping site = 50000 min
time to travel from pit to plant = 50000 min
time to travel to pit from tipping site = 50000 min
time to travel to pit from plant = 50000 min

The exponential frequency distribution with an average of 6 min is defined by

$$f(x) = 6\,e^{-6x} \quad \text{where } x > 0. \tag{4.5}$$

It is illustrated in Figure 4.3.

Using the inverse transformation method to generate observations from the general exponential distribution $f(x) = \lambda\ e^{-\lambda x}$ we obtain [Winston 1994]

$$x = -1/\lambda \ln(r) \quad \text{where r is a random number between 0 and 1.} \qquad (4.6)$$

The first arrival is arbitrarily assigned to simulation clock time of zero. Subsequent arrival times are calculated by using equation 4.6 to determine the interarrival times.

Therefore

$$\text{Next Arrival Time} = \text{Previous Arrival Time} - 1/6 \ln(r),\ \ 0 < r \leq 1. \qquad (4.7)$$

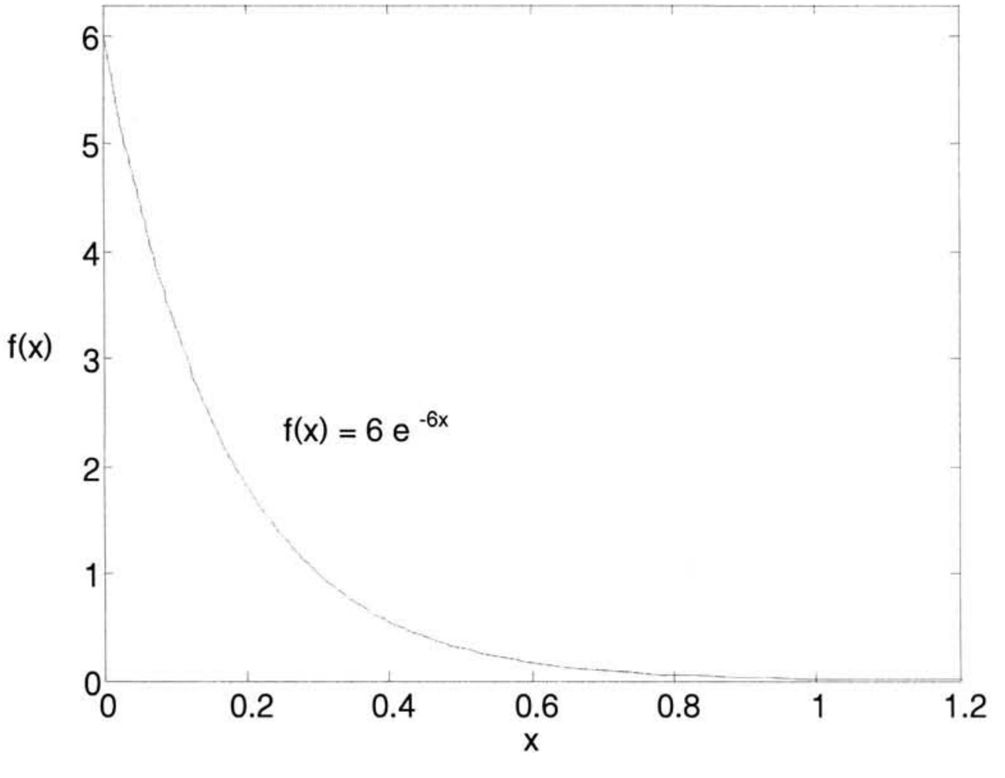

Figure 4.3 Exponential Frequency Distribution for Interarrival Times

Similarly the service times are calculated as

$$\text{Service Time} = -\ 1/4 \ln(r),\ \ 0 < r \leq 1. \qquad (4.8)$$

Equations 4.7 and 4.8 are programmed into Q_SIM and the results of successive runs displayed in Table 4.3. Note that an average of 66.8% for the queue utilisation and an average queue length of 1.4 was calculated. Winston (1994) obtained a queue utilisation of 65.2% and average queue length of 1.64 for one simulation run.

Table 4.3 Results of Single-Server Example Using Q_SIM.

	Run 1	Run 2	Run 3	Run 4	Run 5	Average
Utilisation	0.6684	0.6133	0.674	0.6969	0.6882	0.66816
Mean Waiting Time	9.2197	7.2245	8.7191	6.8767	8.3577	8.07954
Mean Queue Length	1.5438	1.1917	1.5357	1.1724	1.4018	1.36908
Mean_Service Time	3.992	3.718	3.8265	4.0875	4.1031	3.94542

The single-server queue system with exponential interarrival and service times, denoted as an M/M/1 queue, is a common example in queue theory [Winston 1994] and an analytical steady-state solution is readily available. If we denote the arrival rate as λ and the average service rate as μ then

$$\text{Utilisation} = \rho = \lambda / \mu. \tag{4.9}$$

$$\text{Expected number in the system} = L = \rho / (1-\rho). \tag{4.10}$$

$$\text{Expected number in the queue} = L_q = \rho^2 / (1-\rho). \tag{4.11}$$

$$\text{Expected time in the system} = W = L / \lambda = \rho / (\lambda\ (1-\rho)). \tag{4.12}$$

For the single-server example, $\lambda = 10$ arrivals per hour and $\mu = 15$ users per hour and from equations 4.9 – 4.12

$$\text{Utilisation} = \rho = \lambda / \mu = 10/15 = \underline{2/3}. \tag{4.13}$$

$$\begin{aligned}\text{Mean number in the system} = L = \rho / (1-\rho) &= (2/3) / (1 - 2/3) \\ &= \underline{2}.\end{aligned} \tag{4.14}$$

Mean number in the queue = $L_q = \rho^2 / (1-\rho) = (2/3) / (1-2/3)$

$= 4/3$

$= \underline{1.33}$. (4.15)

Mean time in the system = $W = L / \lambda = \rho / \lambda (1-\rho)$

$= 2/10$ hrs

$= \underline{12 \text{ minutes}}$. (4.16)

Mean waiting time = W – mean service time = $W - 60 / \mu$

$= \underline{8 \text{ minutes}}$. (4.17)

Table 4.4 details output of Q_Sim with confidence intervals at the 95% level and Table 4.5 compares the results of Equation 4.13 – 4.17 with the output of Q_SIM. There can be little doubt that the output of Q_SIM is reliable and realistic. The minor discrepancy is explained by the fact that Q_SIM contains transient and steady state values whereas the analytical solution is only the steady state solution.

Table 4.4 Point Estimators and Confidence Intervals of Q_SIM Output.

	Q_SIM	Confidence Interval	
Utilisation	66.8%	65.2%	67.6%
Mean Waiting Time	8.08	7.53	8.42
Mean Queue Length	1.37	1.25	1.41
Mean_Service Time	3.95	3.94	4.04

Table 4.5 Comparison of Q_SIM Output to Analytical Solution.

	Q_SIM	Analytical Solution
Utilisation	66.8%	66.7%
Mean Waiting Time	8.08	8
Mean Queue Length	1.37	1.33
Mean_Service Time	3.95	4

4.4 Diesel Consumption, Production Requirements and Truck Productivity.

The aim of this study is to reduce the TCO of mining vehicles and in particular by reducing diesel consumption. The trivial solution to reduce diesel consumption would be to not deploy any haul trucks. This would yield zero diesel consumption and no production. Production requirements must therefore be considered to avoid the trivial solution. So, the need exists to incorporate diesel consumption and production requirements. That is, reduce diesel consumption whilst maintaining current production requirements or haul greater tonnages for the same amount of diesel consumed. This leads to increased productivity. Productivity is defined as output divided by input. Truck productivity will then be defined as tons hauled divided by mean operating cycle time. Recall that operating cycle time is directly related to diesel consumption. Therefore by considering Truck Productivity we have incorporated diesel consumption, albeit indirectly, and production requirements.

4.5 Trial Run of Q_SIM.

It was emphasized in Chapter 2.1 that there is a need to quantify the performance of different truck-shovel configurations[4] in order to determine the optimal configuration which will meet production requirements and minimise the TCO of the haul trucks. Energy Costs, Repair Costs and Maintenance Costs are elements that can be minimised to reduce TCO. Truck-shovel configuration will influence the amount of time haul trucks spend idling whilst waiting to load/offload and even during loading. Idling time represents an undesirable energy cost (diesel consumption) and an increase in engine operating time which will affect maintenance and repair costs. Associated productivity losses are also predicted and can be verified by investigating different haul truck-loader configurations.

Therefore as illustration of the above we begin by a macroscopic simulation of the mining operation as illustrated in Figure 4.1. The mining operation is modelled by a three-queue network. Queue 1 will represent the pit and Queue 2, represents the

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[4] The term 'truck-shovel configuration' refers to the ratio of haul trucks assigned to a loader(s) and, can include their respective payloads.

tipping site of overburden and Queue 3 represents the crushing plant of ore. The behaviour of the trucks is cyclical, that is a truck will travel from Queue 1 to Queue 2 or Queue 3 and then back to Queue 1 and then from Queue 1 to Queue 2 or Queue 3 and so on.

We shall first assume that a queue may only service one truck at a time and that trucks are served on a 'first come first served' queue discipline. It is also assumed that a truck hauls a constant load of 200 tons. Q_SIM is then employed to investigate the effect of number of haul trucks on production.

Before running Q_SIM it is necessary to determine the number of simulation runs and the number of cycles per run that must be completed to yield results in which one has confidence.

Using the fast food example [Winston 1994] of the preceding section, it was determined that performing 100 cycles per run would eliminate transient effects during queue start-up and that performing 10 runs was sufficient to 'average-out' bias in output.

Ten runs are performed by Q_SIM for each of the different fleet sizes of 6, 12, 18, 24 and 30 trucks. The results of the ten runs are averaged for each fleet size. The simulation is terminated when one of the trucks has completed 99 cycles. The average number of cycles performed by a truck during the simulations was found to be not less than 93. Thus total cycles completed equals average cycles multiplied by number of trucks and exceeds the minimum of 100 cycles required to eliminate start-up effects. However 99 cycles for a truck was selected as the termination criterion to ensure that at least 100 cycles of data is collected as it is unlikely that one truck will complete 99 cycles without another truck having also completed at least one cycle. Furthermore a time elapsed termination criterion was not used because it could be possible that with a large truck fleet we obtain less than one hundred cycles in a pre-determined time interval. The averaged output for each fleet size is presented in Table 4.6 whilst the input parameters and confidence intervals are presented in Appendix E.

From the first column of Table 4.6, the simulation output for a fleet size of 6 trucks is available. For fleet size of 6 trucks, Queue Utilisation is less than 100% for all 3 queues. This implies that there is time when the servers are idle and thus it may be possible to accommodate additional trucks in the system. When the fleet size is increased to 12 trucks, the utilisation of Queue 1 increases to 99%, that is the Queue is occupied 99% of the time. Note that utilisation for the other queues also increases.

From Figure 4.4, we see that as fleet size increases, the total tonnage hauled as well as operating cycle times increase. Increased operating cycle times are a result of increased waiting times caused by congestion of the queue network by too many trucks and increased tonnage does not reflect improved productivity attributed to fleet size but is a result of the terminating criterion based on the number of operating cycles. It is expected that 30 trucks completing an average of 93 cycles will haul greater tonnages than 6 trucks completing an average of 93 cycles. However the elapsed time indicates that the larger fleet size took longer to complete those cycles. Furthermore the inverse of mean operating cycle time is truck productivity expressed as tons per minute. Recall that truck productivity is defined by tons hauled divided by mean operating cycle time. From Table 4.6 truck productivity decreases as fleet size increases.

Although individual truck productivity decreases as fleet size increases, it does not necessarily mean that fleet productivity decreases. Fleet Productivity will be defined as:

$$\text{Fleet Productivity (tons.min}^{-1}\text{)} = \text{Truck Productivity x Fleet Size.} \qquad (4.18)$$

Figure 4.5 illustrates that there is a specific fleet size for which Fleet Productivity is optimum in spite of decreasing truck productivity. From Figure 4.5, we see that Fleet Productivity is optimal when a fleet of 18 trucks is deployed. This shows that optimising the behaviour of an individual does not guarantee optimal behaviour of the system.

Table 4.6 Investigation of Fleet Size and Operating Productivity

	Fleet Size				
	6	12	18	24	30
Tons Hauled:					
Queue 1- Queue 2 – Queue 1	59000	117600	174800	232800	290700
Queue 1- Queue 3 – Queue 1	50100	110900	166200	224200	279900
Time Elapsed (min)	2699.1	4724.7	6826.4	9357.3	11595
Mean Cycle Time (min):					
Queue 1- Queue 2 – Queue 1	27.34	47.57	69.42	94.89	117.54
Queue 1- Queue 3 – Queue 1	32.00	50.14	72.41	97.73	120.84
% Cycle Time spent Idling:					
Queue 1- Queue 2 – Queue 1	23.56	55.99	69.98	78.29	82.94
Queue 1- Queue 3 – Queue 1	18.24	48.22	63.44	73.08	77.92
Truck Productivity (tons per min):					
Queue 1- Queue 2 – Queue 1	7.74	4.41	3.02	2.21	1.78
Queue 1- Queue 3 – Queue 1	6.54	4.24	2.90	2.14	1.72
Queue Utilisation:					
Queue 1	82.28%	99.11%	99.71%	99.78%	99.85%
Queue 2	65.67%	75.10%	77.26%	75.22%	75.83%
Queue 3	46.65%	59.04%	61.27%	60.39%	60.88%
Mean Waiting Time(min):					
Queue 1	5.29	22.54	44.08	69.47	92.41
Queue 2	1.16	4.10	4.50	4.82	5.08
Queue 3	0.55	1.64	1.86	1.95	1.76

So, by deploying additional trucks we sacrifice individual truck efficiency, as measured in mean cycle time or percentage cycle time spent idling, for an increase in overall fleet productivity. The point where the trade off of truck efficiency for fleet productivity is optimum can be determined by employing Q_SIM. This indicates that Q_SIM is a viable tool in reduction of TCO of the mining vehicles.

4.6 Q_SIM Application 1: Economies of Scale.

Trends in open pit mining reflect an increase in the haulage capacities of the haul trucks [Roman et al. 2000]. This trend is motivated by reduced hauls and the number of drivers needed to meet production requirements. Also less energy is consumed per ton hauled and less equipment reduces inventory costs [Roman et al. 2000].

However Roman et al. (2000) indicates that increasing haulage capacities will not guarantee a decrease in production costs, especially when production costs due to unscheduled downtime is considered. Q_SIM will be used to investigate the effect of increasing economies of scale on production costs and diesel consumption.

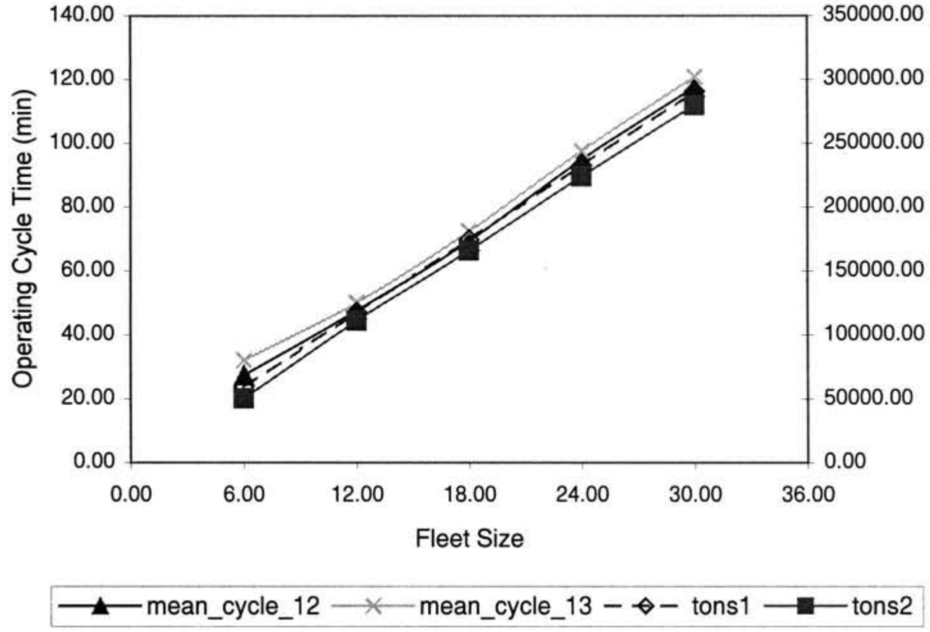

Figure 4.4 The Effect of Increasing Fleet Size on Tonnage Hauled And Operating Cycle Time.

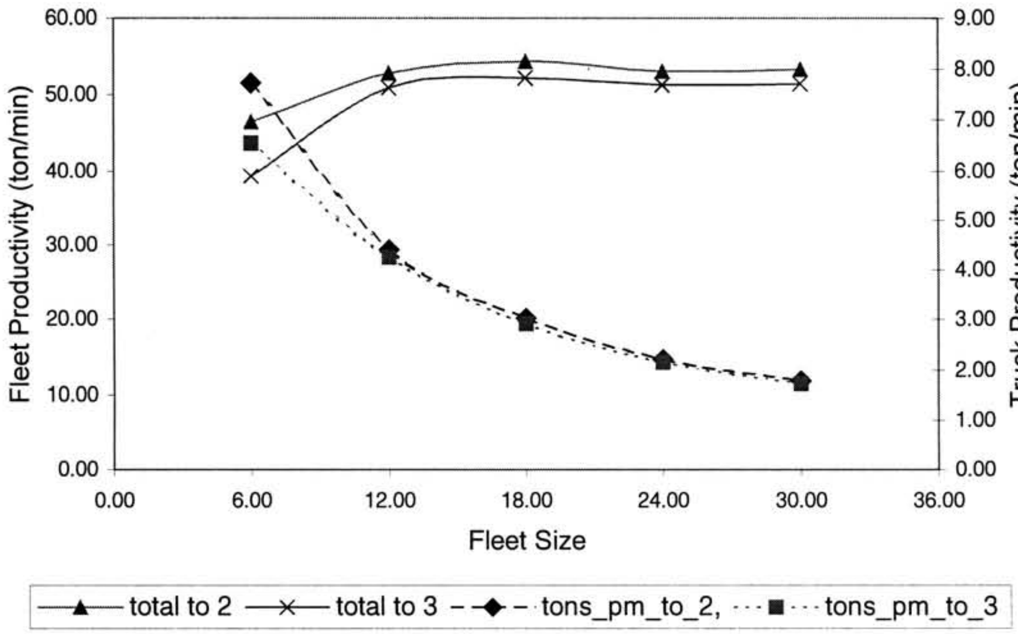

Figure 4.5 Fleet Size and Individual Truck Efficiency and Fleet Productivity

Q_SIM is used to simulate a simplified model of a mining operation illustrated in Figure 4.1. The three queue cyclic network is first modelled with 30 trucks of 200 ton capacity and then by 20 trucks of 300 ton capacity and 1 loader. Ten runs are performed on Q_SIM and the average output follows in Table 4.7

Table 4.7 Q_SIM Output For Simplified Mine Model with 30 x 200 ton Haul Trucks

	Queue 1	Queue 2	Queue 3
Utilisation (%)	99.88	76.37	61.64
Mean Waiting Time (minutes)	91.43	5.14	2.05
Mean Queue Length (trucks)	22.89	0.66	0.25
Mean Service Time (minutes)	3.99	6	5
Mean Cycle Time (minutes):			
Queue 1 – Queue 2 – Queue 1	118.13		
Queue 1 – Queue 3 – Queue 1	121.29		
Mean Truck Idle Time (%)			
Queue 1 – Queue 2 – Queue 1	82.59		
Queue 1 – Queue 3 – Queue 1	77.95		
Tons Hauled			
Queue 1 – Queue 2 – Queue 1	290420		
Queue 1 – Queue 3 – Queue 1	280580		
Total	571000		
Tons Hauled per Minute			
Queue 1 – Queue 2 – Queue 1	1.77		
Queue 1 – Queue 3 – Queue 1	1.71		

For the twenty 300 ton truck scenario, it must be realised that increased haulage capacities would require increased service times and increased travel times. It must be remembered that more powerful Diesel-Electric powertrains are fitted to larger capacity haul trucks and we assume that the shovels are capable of serving these larger capacity haul trucks at a comparable rate. A compromise is reached by increasing service times and travel times for the larger haul trucks by 10%. The averaged output for twenty 300 ton haul trucks is presented in Table 4.8.

Table 4.8 Q_SIM Output For Simplified Mine Model with 20 x 300 ton Haul Trucks

	Queue 1	Queue 2	Queue 3
Utilisation (%)	99.89	68.01	53.81
Mean Waiting Time (minutes)	69.48	3.79	1.64
Mean Queue Length (trucks)	13.97	0.39	0.16
Mean Service Time (minutes)	4.97	6.6	5.5
Mean Cycle Time (minutes):			
Queue 1 – Queue 2 – Queue 1	96.40		
Queue 1 – Queue 3 – Queue 1	100.81		
Mean Truck Idle Time (%)			
Queue 1 – Queue 2 – Queue 1	76.01		
Queue 1 – Queue 3 – Queue 1	70.55		
Tons Hauled			
Queue 1 – Queue 2 – Queue 1	291990		
Queue 1 – Queue 3 – Queue 1	277080		
Total	569070		
Tons Hauled per Minute			
Queue 1 – Queue 2 – Queue 1	3.29		
Queue 1 – Queue 3 – Queue 1	3.13		

Tables 4.7 and 4.8 are consolidated into Table 4.9 to facilitate a comparison between the 2 simulations. Note that due to fewer trucks in the 20 x 300 ton truck scenario, there is less congestion in the network and hence less waiting. This is evident from the reduced idle time compared to deploying 30 x 200 haul trucks; which for the Queue 1 – Queue 2 – Queue 1 route, mean truck idle time was reduced from 82.55% to 76.01%. Reduced truck idle time is due to less waiting and this results in faster cycle times. For the Queue 1 – Queue 2 – Queue 1 route, cycle time decreased from 116.67 minutes to 96.40 minutes. This represents a 17.6% decrease in operating cycle time.

In both instances, the simulation was stopped after one of the trucks had completed 100 cycles whilst the remaining trucks had completed an average of roughly 95 cycles. So for both scenarios each truck completed an average of 95 cycles. Thus the tons hauled in the first scenario would be

$$\begin{aligned}\text{Tons hauled} &= 95 \text{ cycles x } 30 \text{ trucks x } 200 \text{ tons per truck}\\ &= \underline{570\ 000 \text{ tons}}, \end{aligned} \tag{4.19}$$

and tons hauled for the second scenario is

$$\begin{aligned} \text{Tons hauled} &= 95 \text{ cycles x } 20 \text{ trucks x } 300 \text{ tons per truck} \\ &= \underline{570\,000 \text{ tons}}. \end{aligned} \tag{4.20}$$

Equations 4.18 and 4.19 are confirmed by the value of Total Tons Hauled in Table 4.9 and it seems that no advantage was gained in deploying larger capacity trucks. Note however, that the operating cycle time of the larger capacity trucks was 17.6% less than the operating cycle time of the smaller capacity trucks. That means the larger capacity trucks would haul the same load in a shorter time. But what increase in haulage can we expect if the larger capacity trucks where to operate for the same duration as the smaller capacity trucks?

The 'Tons Hauled per Minute' ratio is a measure of production efficiency of one truck. From Table 4.9 one of the twenty 300 ton trucks is hauling ore and overburden at a rate of 3.29 and 3.13 tons per minute, respectively. The thirty 200 ton trucks achieved a rate of 1.79 and 1.76 tons per minute for a single truck. The rate of one 300 ton truck is approximately 1.8 times (3.29 tons per minute / 1.79 tons per minute or 3.13 tons per minute / 1.76 tons per minute) that of a 200 ton trucks.

Thus, the twenty 300 ton trucks will haul product at a rate of

$$20 \text{ x } 3.29 = \underline{65.8 \text{ tons per minute}} \text{ (ore)} \tag{4.21}$$

and,

$$20 \text{ x } 3.13 = \underline{62.6 \text{ tons per minute}} \text{ (overburden)} \tag{4.22}$$

respectively. The thirty 200 ton trucks will haul product at a rate of

$$30 \text{ x } 1.79 = \underline{53.7 \text{ tons per minute}} \text{ (ore)} \tag{4.23}$$

and

$$30 \text{ x } 1.76 = \underline{52.8 \text{ tons per minute}} \text{ (overburden)} \tag{4.24}$$

respectively. This yields a total productivity of

$$65.8 + 62.6 = \underline{128.4 \text{ tons per minute}} \tag{4.25}$$

for the twenty 30 ton trucks and:

$$53.7 + 52.8 = \underline{106.5 \text{ tons per minute}} \tag{4.26}$$

for the thirty 200 ton trucks.

So the haulage capacity was increased by 150%, from 200 tons to 300 tons and service times and travel times were increased by 10% - which would tend to undermine the benefit of increasing capacity. Notwithstanding, the efficiency of an individual truck in the operating cycle was increased by 180%. The total productivity of the queue system increased by 20.6% from 106.5 tons per minute to 128.4 tons per minute. This illustrates the nonlinear nature of queue systems. Deploying a smaller number of large capacity trucks instead of a large number of small capacity trucks improves truck efficiency and increases fleet productivity.

The potential diesel saving that may be derived by judicious deployment of equipment, whilst maintaining or even increasing production rate is evident. Roman et al. (2000) stated that production losses due to scheduled and unscheduled downtime may undermine the economy of scale benefit of deploying larger capacity haul trucks. In the two previous simulations unscheduled downtime was not incorporated and still needs to be investigated.

Scheduled and unscheduled maintenance is incorporated by means of Monte Carlo sampling of haul truck availability. Haul truck availability is incorporated as a probability measure of the haul truck being operationally available to perform the task required. From the trial run of Q_SIM optimal productivity was achieved with eighteen 200 ton haul trucks. Q_SIM is used to simulate the operating cycles of eighteen 200 ton haul trucks and twelve 300 ton haul trucks for decreasing values of availability.

Table 4.9 Consolidation of Table 4.7 and 4.8

	200 ton	300 ton	200 ton	300 ton	200 ton	300 ton
	Queue 1	Queue 1	Queue 2	Queue 2	Queue 3	Queue 3
Utilisation (%)	99.88	99.89	76.37	68.01	61.64	53.81
Mean Waiting Time (minutes)	91.43	69.48	5.14	3.79	2.05	1.64
Mean Queue Length (trucks)	22.89	13.97	0.66	0.39	0.25	0.16
Mean Service Time (minutes)	3.99	4.97	6	6.6	5	5.5
Mean Cycle Time (minutes):						
Queue 1 – Queue 2 – Queue 1	116.97	96.40				
Queue 1 – Queue 3 – Queue 1	119.92	100.81				
Mean Truck Idle Time (%)						
Queue 1 – Queue 2 – Queue 1	82.55	76.01				
Queue 1 – Queue 3 – Queue 1	77.94	70.55				
Tons Hauled						
Queue 1 – Queue 2 – Queue 1	290356	291990				
Queue 1 – Queue 3 – Queue 1	281111	277080				
Total	571467	569070				
Tons Hauled per Minute						
Queue 1 – Queue 2 – Queue 1	1.79	3.29				
Queue 1 – Queue 3 – Queue 1	1.76	3.13				

Ten runs are performed for each value of reliability per fleet and the results then averaged. The relation between fleet productivity and availability is plotted in Figure 4.6. With 100% availability the twelve 300 ton trucks have a greater fleet productivity than the eighteen 200 ton trucks. This is in keeping with our preliminary findings regarding economies of scale. However note that initially, as availability decreases from 100%, fleet productivity actually increases. By 'withdrawing' a limited number of trucks from operation, we have removed a portion of the fleet that congested the queue system, therefore fleet productivity increases. However further decreases in availability will remove non-idle and productive haul trucks from the fleet and productivity decreases with decreasing availability. From Figure 4.6 we notice that the productivity of the twelve 300 ton truck fleet is dependant on availability. It is also noticeable that for haul truck availability less than 0.95, the eighteen 200 ton truck fleet is more productive than the twelve 300 ton truck fleet.

This shows that to obtain the economy of scale benefit of deploying a small group of large capacity haul trucks, availability of the haul truck fleet has to be nearly unity. Costs to maintain this level of availability may be prohibitive. As consolation, as long as availability is above 80%, the productivity of the small fleet of larger trucks will not be less than productivity at 100% availability.

For the large fleet of smaller capacity trucks, that is the eighteen 200 ton truck fleet, the same behaviour between availability and productivity is observed. But note that the eighteen 200 ton trucks are more productive than twelve 300 ton trucks for availability values less than 0.95 and that the maximum value for fleet productivity for the large fleet of small capacity trucks is larger than the maximum value of the twelve 300 ton truck fleet.

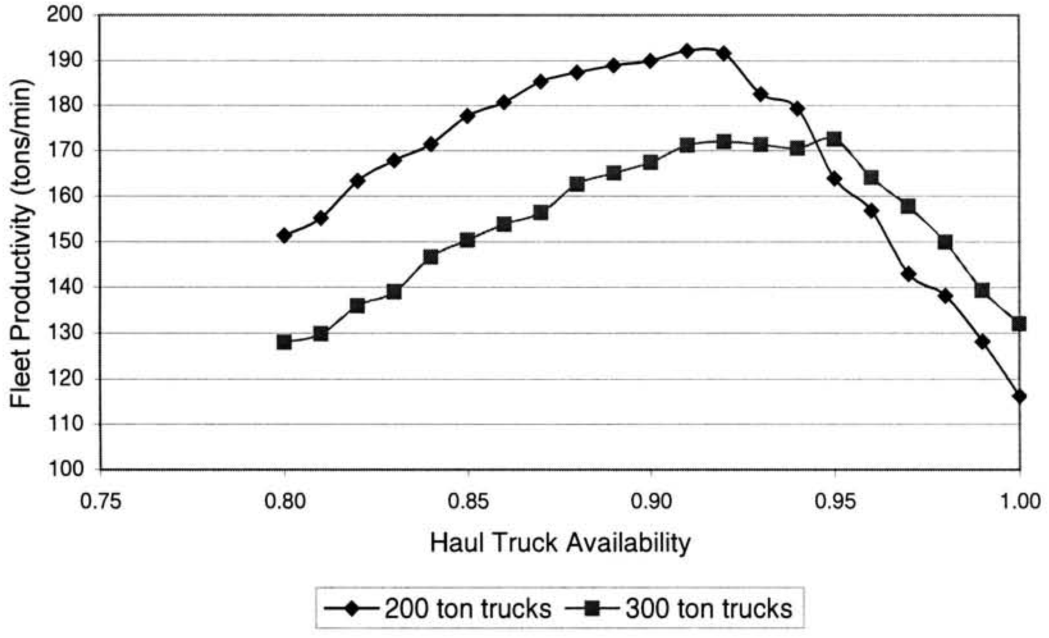

Figure 4.6 The Effect of Availability on Fleet Productivity

Figure 4.6 confirms the statements made by Roman et al. (2000). To obtain benefit of economies of scale in production by deploying a small fleet of larger capacity haul trucks, the availability of the fleet must be kept fairly high. For example, with a availability of 84% we can obtain a fleet productivity of 170 tons per minute from 18 –200 ton trucks. To obtain the same productivity from a fleet of 12 – 300 ton trucks would require an availability of 92%.

A small fleet of large capacity trucks is not as immune to availability changes as a large fleet of small capacity trucks. With a small fleet of large capacity trucks, a single truck represents a greater portion of the entire production capability, thus any downtime, be it a single unscheduled failure or a scheduled maintenance would have more serious consequences than a similar event in a larger fleet. A large fleet of small capacity trucks offers the mining operation more flexibility in deployment and the greater quantity of trucks acts as a buffer against unscheduled failures.

Using simulation such as Q_SIM, the level of availability and thus the required level of maintenance can be determined to obtain the benefits of economies of scale that will reduce TCO of the haul trucks.

4.7 Q_SIM Application 2: 'Bottlenecking' and Queue Cycle Feedback.

From Tables 4.7 and 4.9 we note that as more haul trucks are deployed in the queue system, the utilisation of queues increases until at least one queue reaches maximum utilisation of 100%. Note that once a queue has reached 100% utilisation, the deployment of additional haul trucks will lead to increased congestion that may undermine the increased haulage capabilities of a larger fleet. The remaining queues in the system will not achieve a notable increase in utilisation. Operating cycle times are dictated by the queue with 100% utilisation, that is, the flow of haul trucks through the queue network is determined by the rate of service delivery of the fully utilised queue. Therefore a 'bottleneck' in the queue system has occurred. With a bottleneck in a queue system, the congestion of haul trucks will accumulate at the bottleneck.

Table 4.10 shows that as the fleet size increases from 6 to 18 trucks the utilisation of all the queues has increased, and the utilisation of Queue 1 is approximately 100%. Further increases in fleet size does not improve utilisation of queues but note that the increase in waiting time occurs nearly completely at Queue 1.

Table 4.10 Abridged Form from Table 4.7 Q_SIM Output Investigating the Effect of Fleet Size.

	Fleet Size		
	12	18	24
Queue Utilisation:			
Queue 1	99.11%	99.71%	99.78%
Queue 2	75.10%	77.26%	75.22%
Queue 3	59.04%	61.27%	60.39%
Mean Waiting Time(min):			
Queue 1	22.54	44.08	69.47
Queue 2	4.10	4.50	4.82
Queue 3	1.64	1.86	1.95
Truck Productivity (tons per min):			
Queue 1- Queue 2 – Queue 1	4.41	3.02	2.21
Queue 1- Queue 3 – Queue 1	4.24	2.90	2.14

To alleviate bottle necking at Queue 1 we may deploy an additional shovel at Queue 1. The output from Q_SIM with two shovels is given in Table 4.11. Table 4.11 may be compared to the output for one shovel, which is detailed in Table 4.7. Note that the maximum value of Queue Utilisation is 200% (2 shovels multiplied by 100%). Comparing Tables 4.10 and 4.11 it is clear that by deploying an additional shovel at Queue 1, the Utilisation of Queue 1 has dropped from 99% per shovel to 65% per shovel for two shovels. The utilisation of Queues 2 and 3 has increased as the bottle neck of Queue 1 has been removed. There is also a decrease in mean waiting time at Queue 1 between Tables 4.10 and 4.11. By deploying an additional shovel, truck productivity has also increased. Take note that the mean waiting time for Queues 2 and 3 increase dramatically for an increase in fleet size (see Table 4.11).

Since the haul trucks repeat the operating cycle, a bottleneck restricts the flow of haul trucks through the queue network. Therefore irrespective of the rate of arrivals, a bottleneck will stagger the departure and progression of haul trucks onwards through the network and will result in an ordered arrival pattern at the succeeding queues. The bottleneck queue is a succeeding queue of itself due to the cyclical nature of the mining operation. This is referred to as feedback. Due to feedback, a simple Queue A to Queue B to Queue A network will tend to a steady state solution with relative ease.

Due to the repetitive nature of the haul truck cycle, the upgrading of throughput at a bottleneck will mean that a bottleneck will occur at another point in the network. Being able to identify the point where a bottleneck will occur is possible using Q_SIM. Furthermore Q_SIM allows us to determine how to manipulate the location of a bottleneck to an identified location where queuing space is available to accommodate the waiting trucks and where the cost associated with queuing may be minimised.

Table 4.11 Q_SIM Output for Mining Cycle with 2 Shovels Deployed at Queue 1

	Fleet Size		
	12	18	24
Tons Hauled:			
Queue 1- Queue 2 – Queue 1	103200	147880	196560
Queue 1- Queue 3 – Queue 1	117520	175120	233120
Mean Cycle Time (min):			
Queue 1- Queue 2 – Queue 1	39.49	59.03	78.56
Queue 1- Queue 3 – Queue 1	34.46	49.35	65.69
Truck Productivity (tons per min):			
Queue 1- Queue 2 – Queue 1	5.1	3.4	2.6
Queue 1- Queue 3 – Queue 1	5.9	4.1	3.1
Queue Utilisation (per shovel):			
Queue 1	64.83%	66.28%	66.95%
Queue 2	99.68%	99.68%	99.86%
Queue 3	94.53%	98.30%	98.64%
Mean Waiting Time(min):			
Queue 1	1.01	1.06	1.07
Queue 2	15.79	35.32	54.80
Queue 3	5.29	20.08	36.38

4.8 Q_SIM Application 3: Input Parameters From Production Data.

Thus far, our analyses have illustrated that simulation such as Q_SIM makes it possible to determine an optimum haul truck – server configuration that will reduce the TCO of the haul trucks. Q_SIM input parameters may be determined from production data captured by a computerised recording system mounted on the haul trucks in operation on the Sishen mine. Production data is captured per trip completed by a haul truck. From the available production data, it is apparent that the daily operating cycle of a haul truck is variable and erratic. Haul trucks are routed and re-routed to different locations on a real time basis.

It is not feasible to incorporate such a degree of randomness into Q_SIM. To do so would be resource intensive, entail a loss of generality and universality and would jeopardise simulation repeatability. Therefore the simulation approach is that of assuming the trucks had been routed in a systematic cyclical manner. From such an approach, steady state production parameters can be determined for possible haul truck activities. The actual operating cycle can then be regarded as a piece-wise composition of steady state cycles. This will enable the determination and/or evaluation of a basic set of dispatching strategies for the mining operation.

Table 4.12 is an excerpt from the dispatch tables highlighting the available data. The first row of data describes a trip made by a truck, with the unique despatch code 'T214', to location '46/15/01'. Truck 'T214' arrived at this location at 0:07:32 on 31 January 2004. The zero value for tons indicates that 'T214' was dumping product at '46/15/01' and started dumping at 0:07:35 and completed the dumping at 0:07:54. From the following entry, truck 'T515' loaded 239 tons of product from location '38/13/12'. 'T515' departed at 0:11: 04.

The service time may be calculated by the arithmetic difference between *Time Full or Empty* and *Time Load or Dump.* Similarly, waiting time is *Time Load or Dump* subtract *Time Arrive.* For a specific truck, the difference between the *Time Arrive* of the current entry and the *Time Full or Empty* of the previous entry gives the travel time to the new location from the old location.

Table 4.12 Excerpt of Despatch Production Data

Despatch Code	Date	Time Arrive	Time Load or Dump	Time Full or Empty	Location	Distance	Tons
T214	31-Jan-00	0:07:32	0:07:35	0:07:54	46/15/01	1939	0
T515	31-Jan-00	0:09:17	0:09:22	0:11:04	38/13/12	2700	239
T540	31-Jan-00	0:26:27	0:26:33	0:28:21	62/11/38	3352	0
T524	31-Jan-00	0:24:48	0:25:57	0:28:45	103/05/09	1583	144
T214	31-Jan-00	0:26:57	0:27:12	0:29:23	12/43/01	4310	187

The method used to obtain input parameters for Q_Sim follows. In consultation with staff from Stellenbosch Automotive Engineering (Pty) Ltd (trading as CAE), who had sufficient expertise and experience with the data set, initial 'unreliable' data was neglected. Data was deemed unreliable on the merits of face validation. This data was considered unreliable because despatch was not applied consistently to record events. Hence, all events for a particular haul truck were not captured. The calculation of input parameters is dependant on a reliable event history to obtain travel times and service times. Production data from intervals with an excessive number of failures and with deviant production output parameters was also neglected. Expert judgement was exploited to ascertain what constituted excessive failures and deviant production data.

The remaining production data was analysed by means of database queries in Microsoft Access. The locations to be simulated were selected by the following criteria:

- *Number of visits to each location.* This criterion would determine if there is sufficient data for statistical analysis of this location.

- *Number of trucks to visit each location.* If the number of trucks to visit each location was approximately equal to the number of visits to each location then that location was not part of a defined queue cycle.

- *Number of visits to a particular location per truck.*

By careful inspection and meticulous analysis of the available data the following data was determined and collated into the queue system illustrated in Figure 4.7. Figure 4.7 represents the operational cycle of haul trucks between 3 locations, namely two pits where product is extracted and a plant where the product is tipped. The haul trucks may be divided into two broad categories based on the manufacturer, that is UnitRig and Komatsu. Both categories may be further sub-divided by the manufacturer of the diesel-electric powertrain, but this subdivision was unnecessary as the global mean values used in Figure 4.7, were fairly constant throughout the subdivisions.

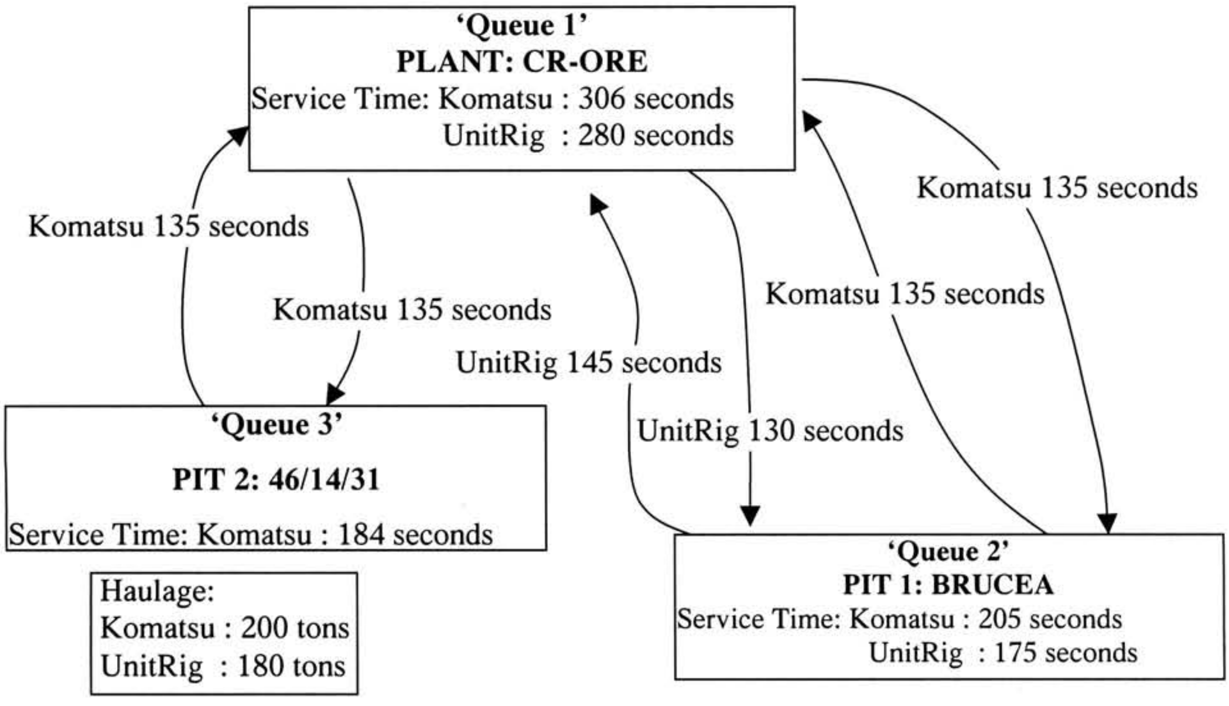

Figure 4.7 Three Queue Network from Sishen Mine Production Data

The data of Figure 4.7 represents the mean input parameters. By constructing the histograms of production data used in constructing Figure 4.7, it was found that the probability density function for these parameters may be approximated by the normal probability distribution N (μ,σ). For N (μ,σ), μ is the mean value and σ is the standard deviation of the population.

Watson (1981) details the Box-Muller method to generate normally distributed parameters. The application of the Box-Muller method and the complete input parameters for Q_Sim are detailed in Appendix F.

The data of Figure 4.7 may now be used to revise the input parameters into Q_SIM.
As an initial approximation we assume that two Komatsu trucks and two UnitRig trucks operate between Pit 1 and the plant and two Komatsu's are deployed between Pit 2 and the Plant. The simulation was terminated after each truck has performed an average of 3 cycles. The small number of haul trucks and operating cycles is chosen to investigate the transient start-up conditions for this system. Figure 4.8 displays the output from the Student Version of Matlab.

Bear in mind that the trucks begin at either Queue 2 or Queue 3 and then travel to queue 1. In the graphs for Queue 1, Queue 2 and Queue 3, there are 3 step functions plotted. These represent, from the vertical axis rightwards, cumulative number of arrivals, cumulative number of services and cumulative number of departures. At start-up there is a rush period of haul trucks to Queue 2, evident in the large initial step in the arrival curve for Queue 2.

In the graph for Waiting Time at Queues the solid line represents the waiting time at Queue 1, the dashed line and the dotted line depicts waiting time for Queue 2 and Queue 3 respectively. Rush periods are associated with pronounced waiting times. The rush period at Queue 2 is visible as a spike in the dashed line for the Waiting time Graph. After the rush had been served at Queue 2, the congestion was carried through to Queue 1, the next destination from Queue 2 where the trucks accumulate awaiting service. The trucks travelling from Queue 3 to Queue 1 add to the congestion. The effect of the congestion is increasing waiting times at Queue 1. This is evident from the steep climbing spike in the solid line in the graph for Waiting Time at Queues. As soon as the majority of trucks has left Queue 1, the rush appears to be over and there is a sharp drop in waiting time at Queue 1. However the trucks are travelling back to either Queue 2 or Queue 3 and the right hand side of the graph for Queue 2 suggests that another rush period will occur.

The sudden spike at the end of the graph for Waiting Time at Queues hints at the possibility that another rush period will occur. Queue 3 exhibits the same trend as Queue 2, however the behaviour is understated due to a smaller number of haul trucks operating at Queue 3.

So it appears as if the fleet of haul trucks are travelling in a congregation from queue to queue. This would lead to successive waiting time spikes. The notion of haul trucks travelling through the system in congregations or bursts is counter-intuitive to the bottleneck phenomena. The simulation is repeated with an extended run time and the Matlab output is depicted in Figure 4.9. In the graph of Queue 1, the service and departure curve is a constant step function. This shows that Queue 1 can only service the haul trucks at a finite rate and therefore haul trucks will depart from Queue 1 at a finite rate.

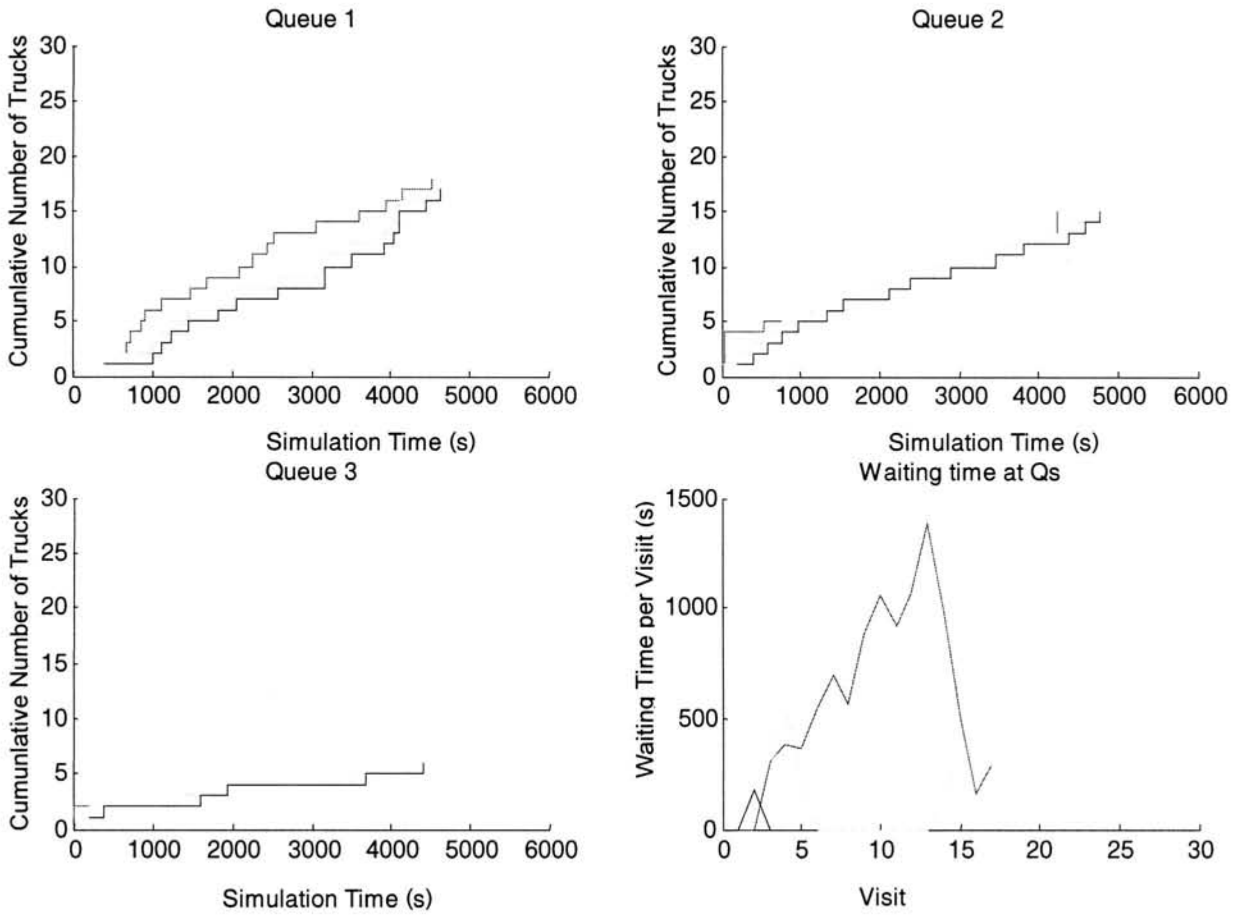

Figure 4.8 Matlab Output of Start-up Conditions of Sishen 3 Queue Network.

This finite rate of departures and services filters out surges in the queue system by staggering the departures from Queue 1, and therefore staggering the arrivals at either Queue 2 or Queue 3. The haul trucks will then accumulate at Queue 1. By extending the run time of the simulation we see that any burst or rush experienced in the system is dampened out by the bottleneck at Queue 1.

From the cyclic pattern of the graphs of Queue 2 and Queue 3 in Figure 4.9, it is evident that the bottleneck at Queue 1, leads to a steady state at these Queues. This is also evident in Figure 4.9 for Waiting Time at the Queues. The oscillation in waiting time for Queue 1 is explained by the arrival of the trucks from Queue 3, that join the trucks from Queue 2 which are already waiting at Queue 1.

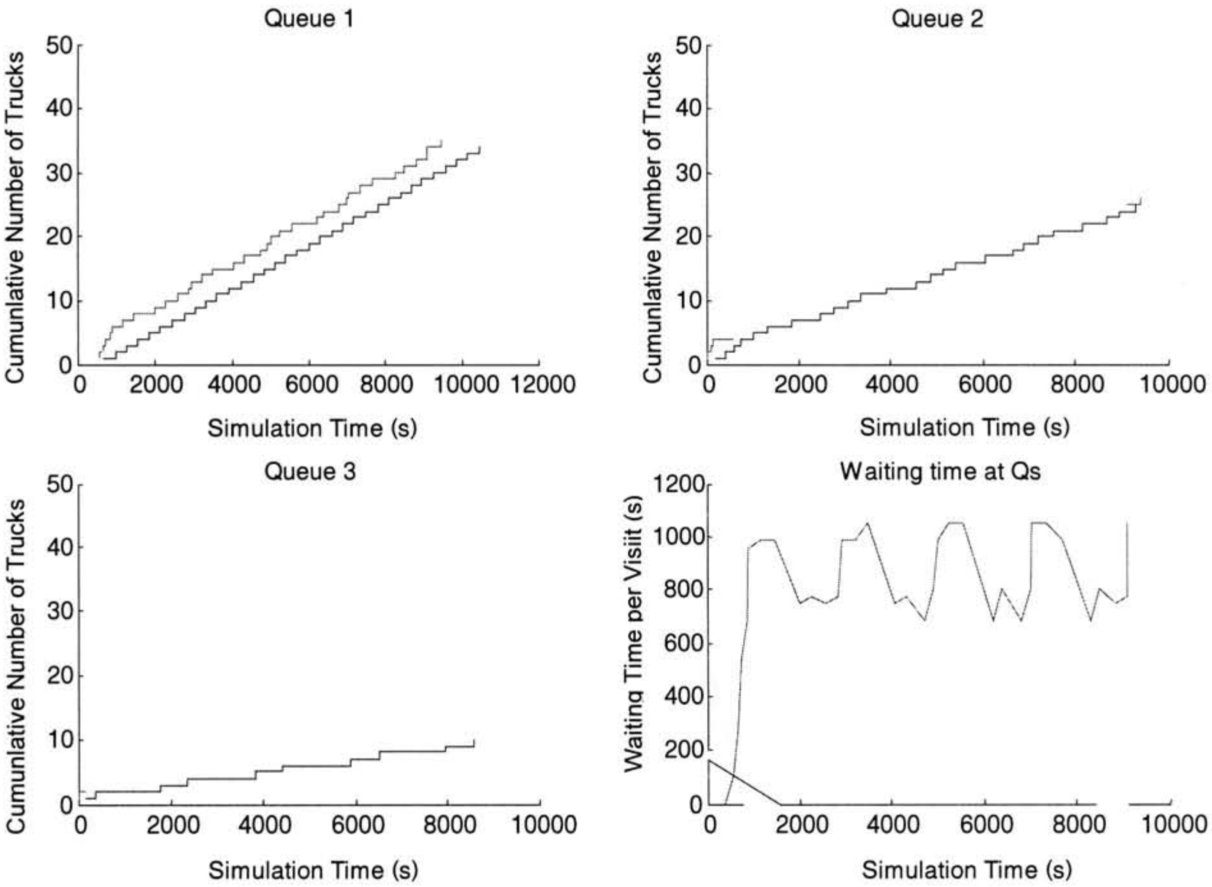

Figure 4.9 Matlab Output of Steady - State Condition of Sishen 3 Queue Network.

By plotting waiting time at Queue 1 and on the same graph as the cumulative graphs of Queue 3 in Figure 4.10, it is evident that waiting time at Queue 1 peaks when Queue 3 is idle. Idle time at Queue 3 is characterised by the extended horizontal distance between an arrival and the previous departure. This takes the shape of long, low steps. Now during this idle time at Queue 3, the waiting time at Queue 1 peaks. The reason Queue 3 is idle is because the haul trucks assigned to Queue 3 are either at Queue 1 or travelling between Queue 1 and Queue 3.

Next, the economies of scale analysis of Chapter 4.5 is applied to the queue network of Figure 4.7. Note that in the queue network of Figure 4.7 both Komatsu and UnitRig haul trucks are deployed between Queue 1 and Queue 2, whilst only Komatsu haul trucks are deployed between Queue 1 and Queue 3. The following 2 operational strategies will be investigated:

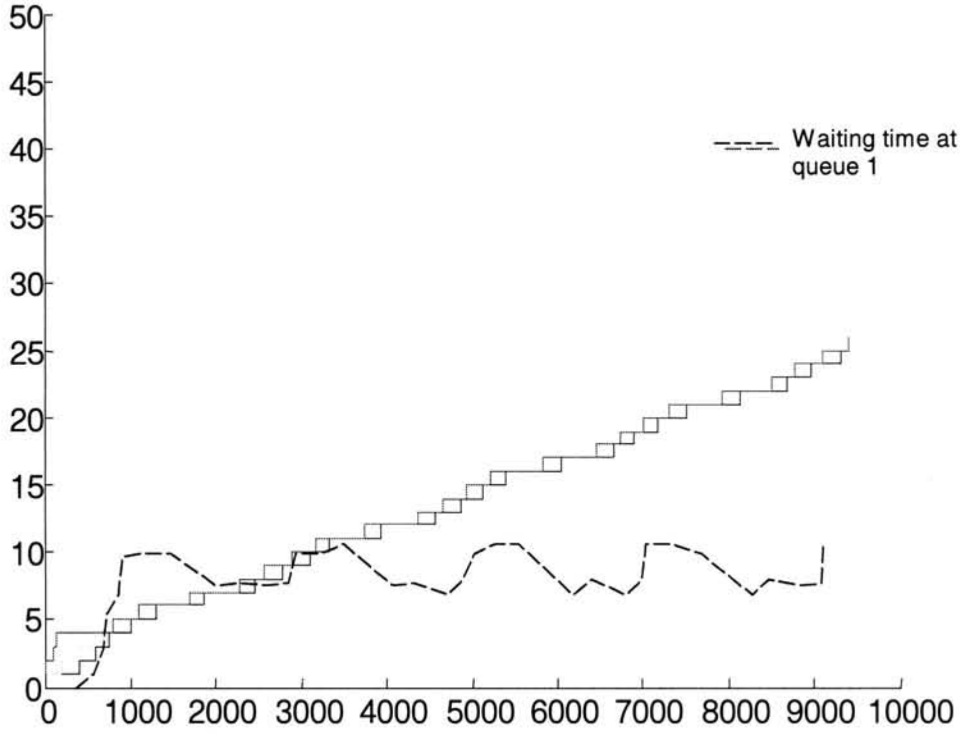

Figure 4.10 The Effect of Haul Trucks Operating Between Queue 1 and Queue 3 on the Waiting Time at Queue 1.

1. Twelve 200 ton Komatsu haul trucks deployed between Queue 1 and Queue 2 and, six 200 ton Komatsu haul trucks deployed between Queue 1 and Queue 3.

2. Fourteen 180 ton UnitRig haul trucks deployed between Queue 1 and Queue 2 and, six 200 ton Komatsu haul trucks deployed between Queue 1 and Queue 3.

Note that the mining operation between Queue 1 and Queue 3 is the same for both scenarios, namely six 200-ton Komatsu haul trucks in service whilst the operation between Queue 1 and Queue 2 differs. This is aimed at evaluating the local or direct effect of economies of scale on an operation and the global or indirect effect of economies of scale on an operation. The local or direct effect is the effect on the operational cycle between Queue 1 and Queue 2, and the global or indirect effect is the effect on the operational cycle between Queue 1 and Queue 3.

The two operational strategies are simulated by Q_Sim. The average of ten runs is utilised. Figure 4.11 presents the effect of haul truck availability on fleet productivity. At 100% availability the economy of scale benefit for operational scenario 1 yields a marginal improvement in total fleet productivity. Furthermore, if the haul truck availability is less than 80% then scenario 2 will provide a higher fleet productivity than that of scenario 1. This phenomena was discussed in Chapter 4.5.

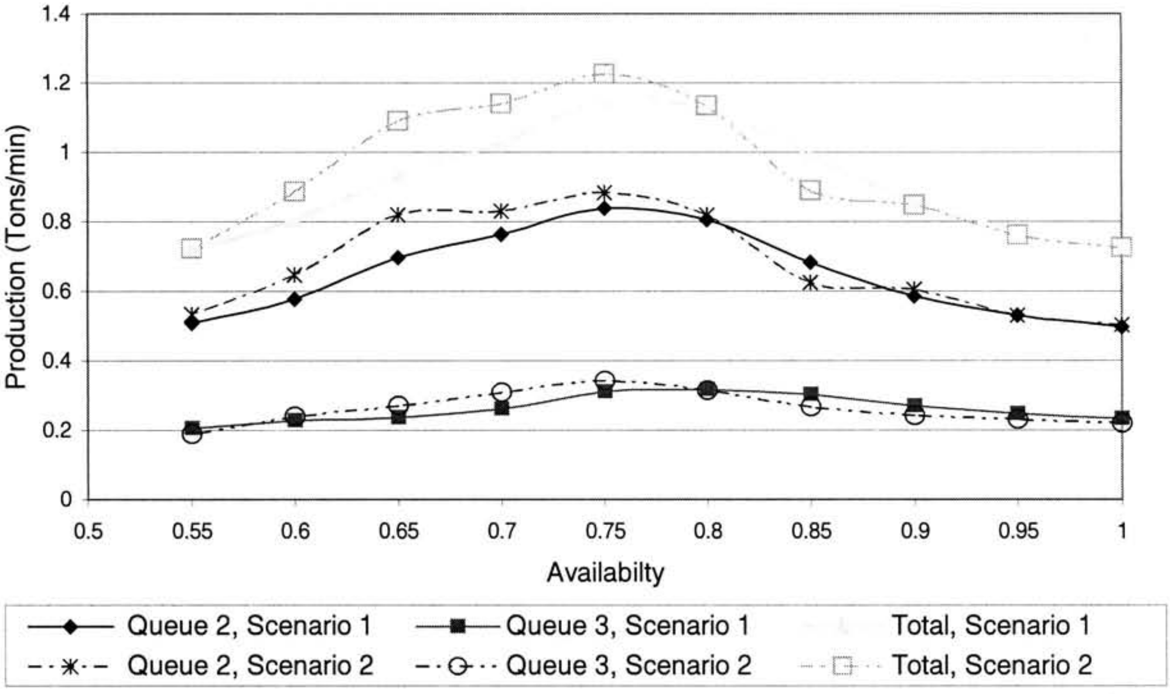

Figure 4.11 The Effect of Haul Truck Availability on Fleet Productivity.

4.9 Conclusion.

The operational cycle of haul trucks was simulated in order to quantify equipment utilisation in order to reduce diesel consumption of the mining vehicles. The operational cycle of haul trucks was modelled utilising queue theory. In its simplest form the mining operation can be modelled by a three queue network with a queue denoting the pit, plant and dump respectively. The simulation of the queue network was implemented in Matlab using the next event advance method.

Q_Sim was verified by manual calculation on spreadsheet and then the output of Q_Sim was validated using a fast-food restaurant case study [Winston 1994] with an analytical steady state solution. The output from Q_Sim had an error of 1-3% which can be accounted for the transient state values during start-up. The output from Q_Sim was therefore realistic and reliable.

Q_Sim enables a user to evaluate operational deployment strategies without having to actually implement that strategy in practice. An optimal deployment strategy can be determined at very little resource cost and in minimal time. For example, a production requirement may be that displaced earth from blasting must be removed in a given time. With Q_Sim, the number of trucks and the truck capacity required to achieve the prescribed production rate can be determined. Real time forecasting of the implications of haul truck configuration changes can be achieved by judicious application of Q_Sim. The implications of haul truck changes are quantified by mean waiting time, mean cycle time of haul trucks and the percentage utilisation of loaders or crushers. These parameters are calculated by Q_Sim.

Q_Sim was applied to a theoretical example to determine optimal fleet size. It was noted that as the fleet size increases, individual haul truck productivity decreases due *to increased* congestion in the queue system. With variation in fleet size, fleet productivity may be optimised. The identification of such an optimal point by Q_Sim indicates that diesel consumption can be reduced whilst still fulfilling production requirements.

The economies of scale of haul truck capacity was also investigated. It was concluded that the capacity of haul trucks and availability thereof affect fleet productivity. This relationship may be determined using Q_Sim. It was shown that to obtain any benefit from economies of scale, a specific level of haul truck availability is required.

The analysis of the benefits of an economies of scale is not limited to reducing diesel consumption of current haul truck fleets. Such an analysis is a valuable tool to evaluate prospective equipment acquisitions. Recall from Chapter 2.1 that Initial Purchase Costs often accounts for only 20 – 30 % of the TCO [Rockwell 2002]. By performing an analysis of economies of scale, the 'hidden' costs necessary to achieve the specified level of availability can be more accurately determined. Q_Sim thus facilitates the minimisation of TCO during acquisition.

The input parameters for Q_Sim were determined from mining production data from the Sishen mine, the output highlights the reduction in diesel consumption that may be obtained by simulating the operating cycle of mining haul trucks.

5 ROLLING LOSSES OF MINE ROADS TREATED WITH DUST-A-SIDE.

In Chapter 3 we concluded that further development and verification of the Diesel Consumption Cost-Driver Model required the quantification of intermediate factors in the model. Rolling resistance was identified as one of those parameters. Furthermore, there is a need to quantify the effect of the Dust-a-Side treatment on the rolling resistance of the haul trucks deployed on the mine. Coast down testing is a popular method employed to quantify rolling resistance, especially between different road surfaces [Du Plessis 1993]. Another method to determine rolling resistance is in a laboratory test utilising a drum tangential to a test tyre. The tyre is either located in the interior or the exterior of the drum. This method is limited by the need for empirical corrections to relate a 'curved roadway' as that of the laboratory drum to the 'flat' roadway in reality [Gusakov et al. 1979]. Furthermore this method is not favourable for the current scope of study as the actual size of the tyres which is approximately 3.6m in diameter, used on the haul truck, is significantly larger than what most laboratory facilities can accommodate.

5.1 Coast-Down Test Planning.

Coast-down testing allows us to determine the change in rolling resistance due to different road surfaces [Phelps et al. 1977]. During coast-down testing, the vehicle is allowed to 'coast' unpowered along a test section. During the unpowered coasting there is no propulsive force to balance the existing resistive forces. Resistive forces will then decelerate the test vehicle as it travels along the test section. The resistive forces were listed in section 2.4 as:

- Aerodynamic
- Grade
- Cornering
- Inertia
- Chassis friction
- Rolling

To determine the change in rolling resistance due to different road surfaces, the other resistive forces must be minimised or kept constant during testing. By selecting straight and level test sections, cornering and grade losses are minimised. By performing all tests using the same vehicle, inertia and chassis friction is kept constant. Aerodynamic losses are proportional to the square of velocity and dependant on the shape of the vehicle. Thompson et al. (1977) states that aerodynamic losses are not significant below 60 km/h. By performing tests in the same speed range and with the same vehicle, aerodynamic losses are kept constant.

Therefore, if two straight and level test sections having different surfaces are selected and the same vehicle is allowed to coast from the same initial speed then the difference in the deceleration is due to the change in rolling resistance associated with different road surfaces.

5.2 Mathematical Formulation of Coast-Down Testing.

Consider the test track below:

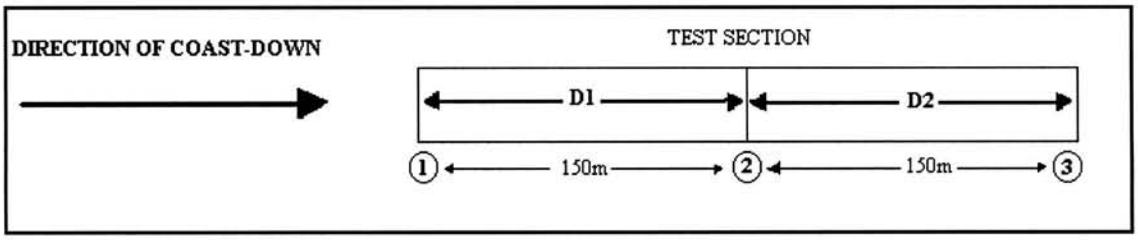

Figure 5.1 Schematic of Test Strip for Coast-Down Testing

At point 1 the vehicle is shifted out of gear thus removing power from the power train. This means that the only forces still acting on the vehicle are those that tend to retard the vehicle. The time to coast from point 1 to point 2 namely, distance D1, is denoted T1, and similarly for distance D2 as time T2. It is assumed that as the vehicle coasts through the course deceleration is constant. This is however not entirely correct but still yields results suitable for analyses as the one intended in this study [Klamp et al. 1977].

Newton's Second law states :

$$F = ma = (W/g).a \tag{5.1}$$

where

F = force (N)

m= mass (kg)

a = acceleration ($m.s^{-2}$)

W = weight (N)

g = acceleration due to gravity ($m.s^{-2}$).

Rolling resistance will be investigated by determining the rolling resistance coefficients for the respective test sections. Rolling resistance is described mathematically as

$$F_r = f_t.W \tag{5.2}$$

where F_r is the retarding force due to rolling resistance (N),

f_t is the rolling resistance coefficient (N/kN),

W is the weight of the vehicle (N).

Now consider the force, F, in Equation 5.1 to be the retarding force, F_r, it follows that:

$$F_r = f_t.W = (W/g).a \tag{5.3}$$

and solving for the rolling resistance coefficient, f_t :

$$f_t = (a/g). \tag{5.4}$$

Let subscript 'i' denote measurements at point 'i'. Thus the change in velocity from point 1 to point 2 is

$$V_2 = V_1 - a.T1 \tag{5.5}$$

and

$$D1 = ((V_1 + V_2)/2).T1 \tag{5.6}$$

which can be stated as

$$V_2 = 2(D1/T1) - V_{1.} \tag{5.7}$$

The addition of equations 5.7 and 5.5 yields

$$2\ V_2 = 2(D1/T1) - a.T1. \tag{5.8}$$

Similarly the change in velocity from point 2 to point 3 is :

$$V_2 = V_3 + a.T2 \tag{5.9}$$

and

$$D2 = ((V_3 + V_2)/2).T2, \tag{5.10}$$

which can be stated as

$$V_2 = 2(D2/T2) - V_{3.} \tag{5.11}$$

The addition of equations 5.9 and 5.11 yields

$$2\ V_2 = 2(D2/T2) + a.T2. \tag{5.12}$$

Solving for acceleration from equations 5.12 and 5.8 we obtain

$$2(D1/T1) - a.T1 = 2(D2/T2) + a.T2 \tag{5.13}$$

$$a\ (T2 + T1) = 2(D1/T1) - 2(D2/T2) \tag{5.14}$$

$$a = [2/(Tt)]\ .\ [(D1/T1) - (D2/T2)] \tag{5.15}$$

where

$$Tt = T1 + T2. \tag{5.16}$$

Having found an expression for acceleration, a, we can determine the rolling loss coefficient, f_t as:

$$f_t = [2/(Tt)] \,.\, [(D1/T1) - (D2/T2)].g^{-1}. \qquad (5.17)$$

Equation 5.17 has been applied in coast-down testing for sedans. Phelps et al. (1997) suggests that a distance of 150 m be used for distances D1 and D2. If a longer distance is used there is an appreciable error due to the assumption of linear acceleration. It is also true that times T1 and T2, measured for a shorter distance are sensitive to the reaction time of the timekeeper(s). This method relies on the accurate measurement of D1 and D2 on the selected test sections and the accurate measurement of times T1 and T2. The driver is required to disengage the drive-train at the exact moment the truck enters the test section. There is a possibility of variability in measurements due to human imprecision. Very limited results are obtained unless multiple sections and timekeepers are deployed for sections D1, D2, .., Dn and times T1, T2, .., Tn.

The alternative is to deploy a 'fifth wheel' measuring device. With such an apparatus, we can accurately measure velocity, distance or time for small intervals and take numerous measurements to test or contradict the linear deceleration assumption.

5.3 Experimental Procedure.

Test sections were identified at the Thabazimbi mine. Three adjoining sections of straight and level road treated by Dust-a-side were identified. A straight, level untreated section in very good condition was also selected. An untreated section in very good condition was selected on the assumption that if the roads were not treated with Dust-a-side, they would still be properly maintained. Selecting a poorly maintained untreated road would bias the coast-down tests in favour of the treated sections. Coast-down 'runs' were performed in both directions along the test sections to average out any minor grade or wind effects.

One haul truck was utilised for all the coast-down testing. This eliminates variability in measurements due to vehicle parameters such as chassis friction, wheel alignment, tyre construction and wear.

Before deploying the fifth wheel, the apparatus must be calibrated. By driving the fifth wheel at defined frequencies and measuring the output voltage, a deterministic relation between voltage and translational (forward) velocity can be determined. By fitting a curve utilising the least squares criterion, forward velocity was described by a cubic expression of voltage.

The fifth wheel was deployed on an unloaded haul truck. From Section 2.4.2, there is a one to one relation between load and rolling resistance, therefore rolling resistance for an unloaded haul truck may be extrapolated for the loaded haul truck scenario. In other words, if a 3 percent difference in rolling resistance is measured for an unloaded haul truck, then there will be a 3 percent difference in rolling resistance for the loaded haul truck. A difference in rolling resistance for the loaded haul truck is a larger absolute value than the same percentage difference in rolling resistance for the unloaded haul truck. Thus performing coast-down tests with a loaded haul truck should provide measurements with less experimental error, however for a given velocity, the momentum of a loaded truck, with a vehicle mass of 105 tons and load of 250 tons, is approximately 3 times that of an unloaded haul truck, with a mass of 105 tons. Thus, longer test sections are needed to obtain sufficient coast down data for a loaded haul truck. The practical implications of utilising a longer test section, namely availability, grade and 'straightness'; do not favour the use of a loaded haul truck in coast down tests.

Tyre temperature also affects rolling resistance and the tyres require an initial warm-up period to allow the tyres to reach equilibrium temperature. Available literature suggests a warm-up period of 10-20 minutes of driving [Du Plessis 1993]. This warm-up period allows tyres, lubricants and suspension components to warm up to operating temperatures. The warm-up period was the driving time to the identified test sections.

Coast-down 'runs' were performed with an initial velocity between 13 km/h and 20 km/h.

5.4 Coast-Down Testing Results.

A velocity-time history, during coast-down 'runs', was captured by a fifth wheel data logger deployed on an unloaded haul truck. Three adjoining treated sections where identified and are denoted as section A, B and C respectively. Coast down data was captured along all 3 sections in one direction and then recorded in the opposite direction. The order of the testing was as follows [Burger 2001]:

Table 5.1 The Order of Coast-Down Runs on Sections Treated with Dust-a-side.

Data-set	Section	Direction	Run
1	A	away	1
2	B	away	1
3	C	away	1
4	C	back	1
5	B	back	1
6	A	back	1
7	A	away	2
8	B	away	2
9	C	away	2
10	C	back	2
11	B	back	2
12	A	back	2
13	A	away	3
14	B	away	3
15	C	away	3
16	C	back	3
17	B	back	3
18	A	back	3

The results are illustrated in Figures 5.2 –5.4 for each treated section [Burger 2001].

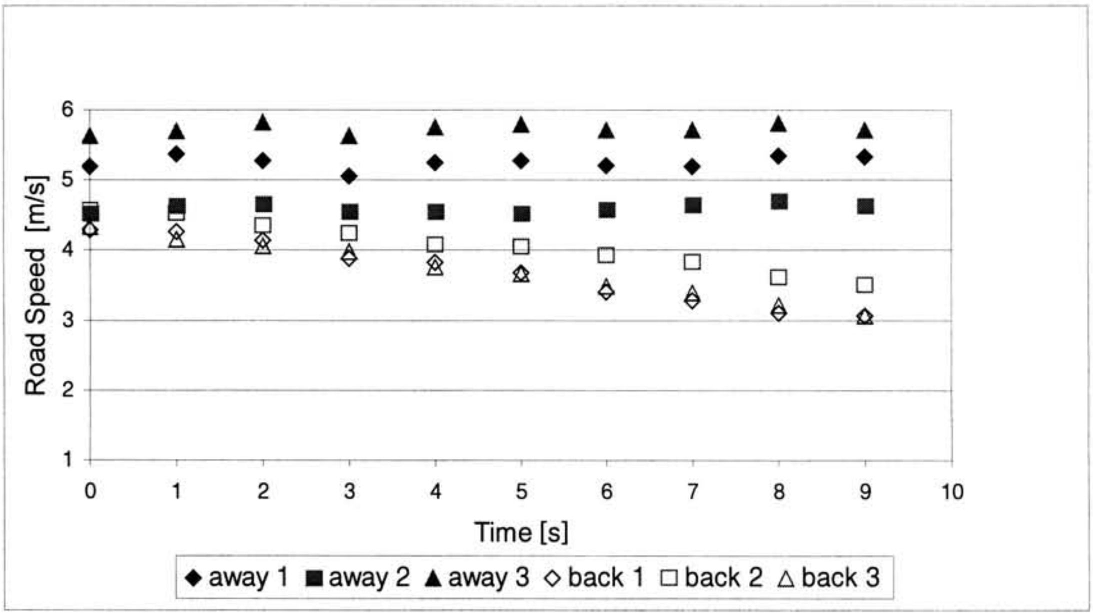

Figure 5.2 Coast-down data for Treated Section A.

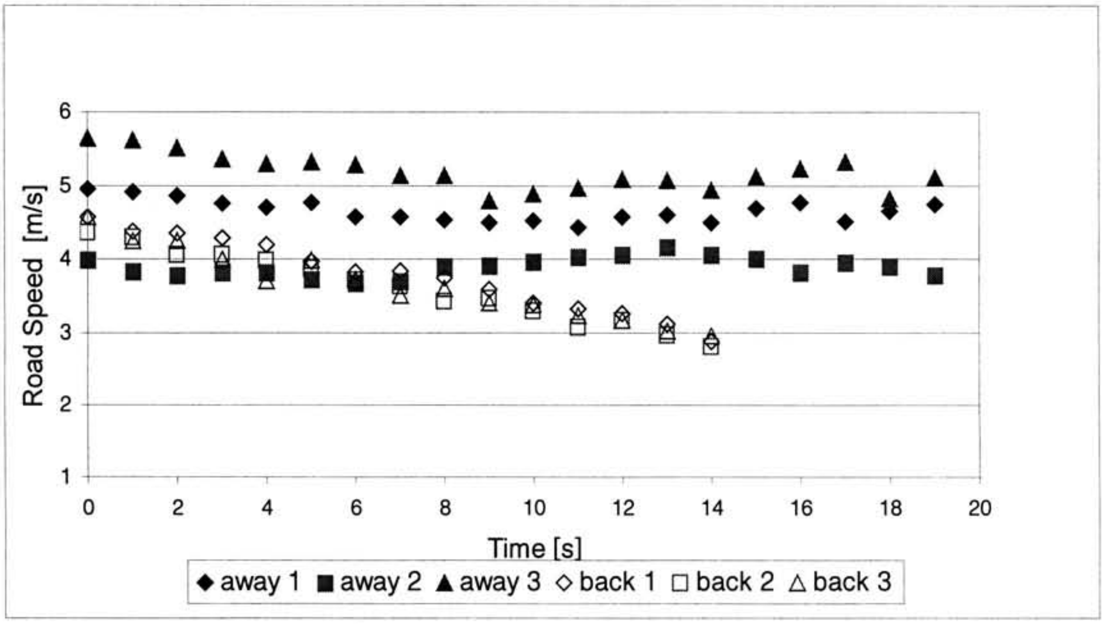

Figure 5.3 Coast-down data for Treated Section B.

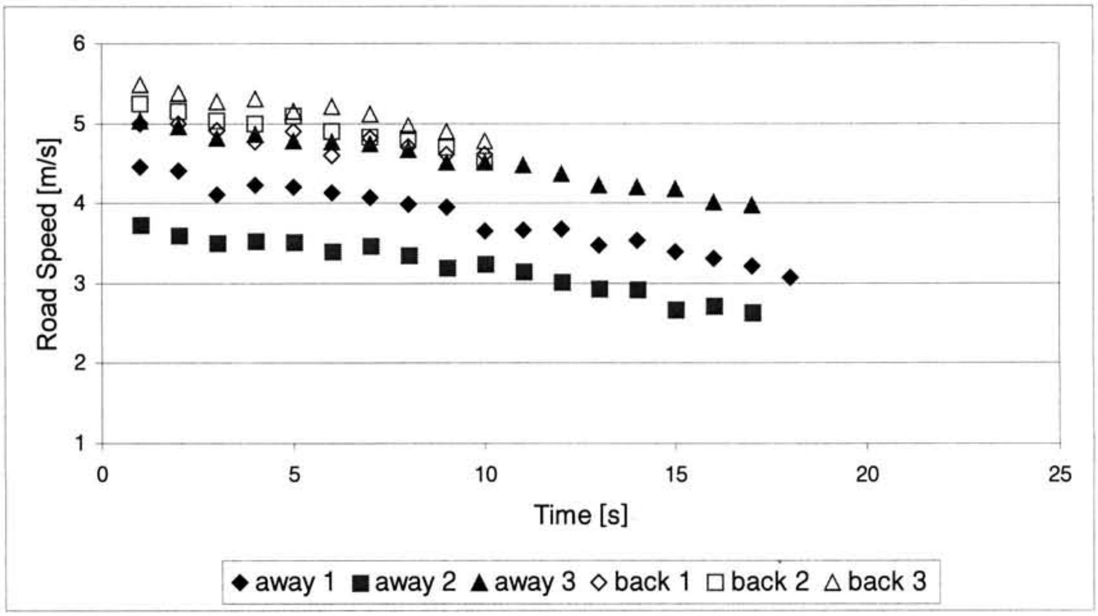

Figure 5.4 Coast-down data for Treated Section C.

Coast-down tests where then performed on the untreated section. Coast down data was captured in one direction and then repeated in the opposite direction. Five 'runs' were performed in each direction to obtain sufficient data to confidently compare the rolling losses between the treated sections and the untreated sections.

The results are illustrated in Figures 5.5 & 5.6 for the untreated section.

5.5 Discussion of results.

5.5.1 Rolling Resistance of the Untreated Section.

Consider the data for the untreated section as illustrated in Figures 5.5 and 5.6. For the 'stopwatch and distance' coast-down method proposed by Phelps et al. (1977), it is assumed that deceleration during the coast down is linear. From Figures 5.5 and acceleration appears to be linear. Linear regression analysis is used to fit a linear equation through the velocity-time data by applying the least squares criterion. The results of the linear regression analysis are plotted on the velocity vs time graphs in Figures 5.7 and 5.8. The Coefficients of Determination (R^2) for each linear equation is also plotted.

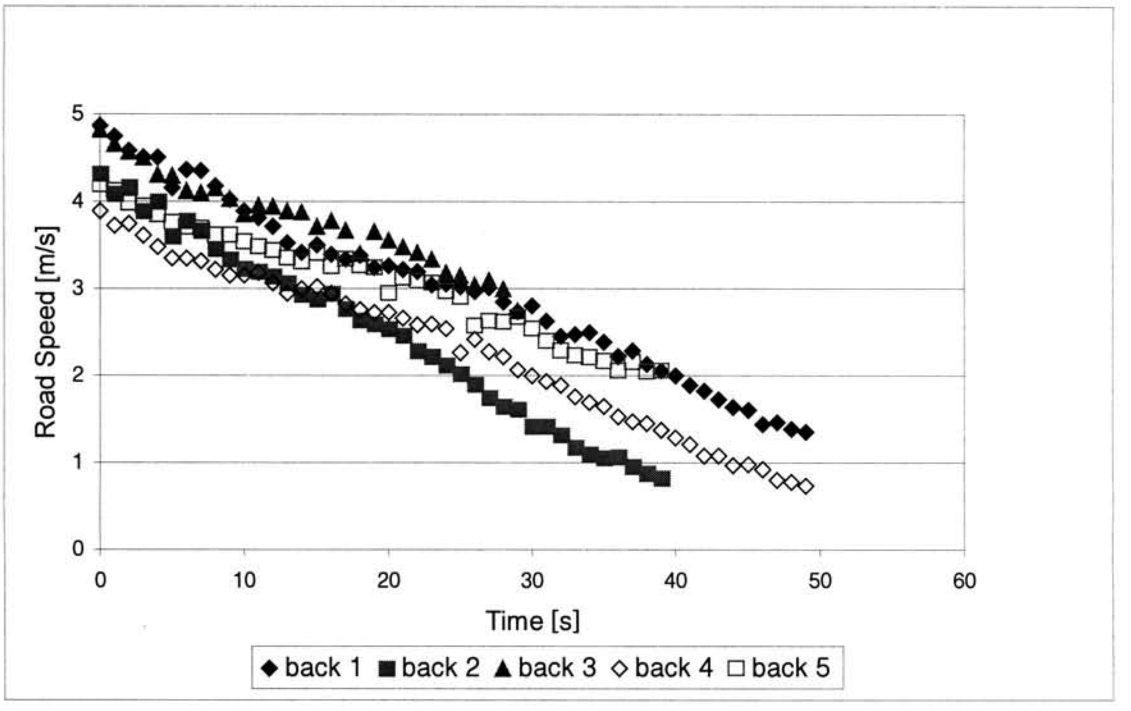

Figure 5.5 Coast-down data for Untreated Section: Back Direction.

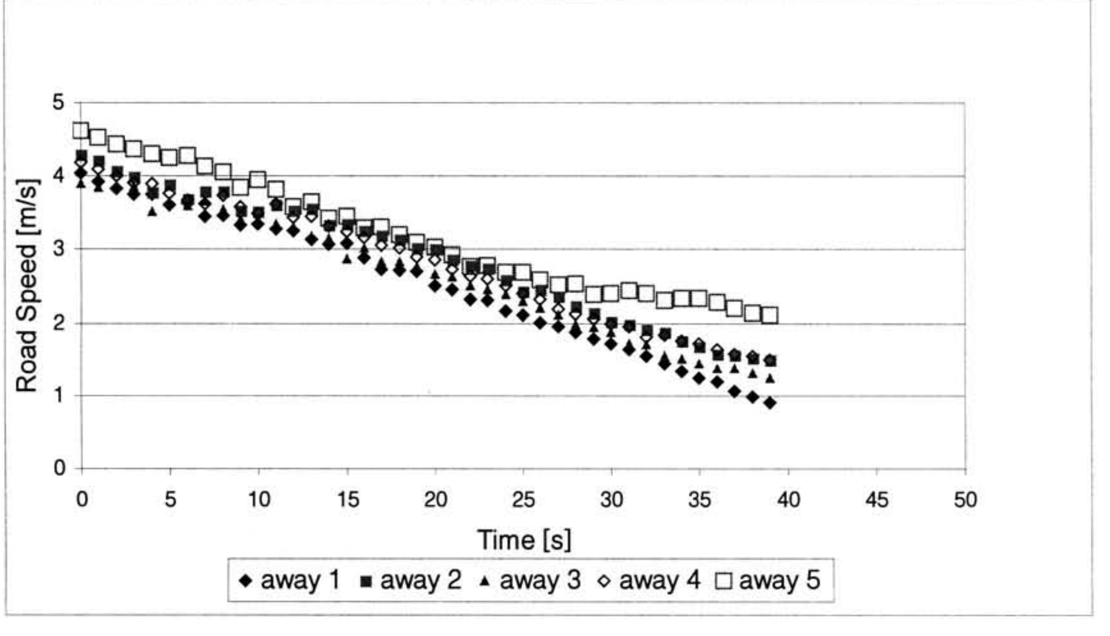

Figure 5.6 Coast-down data for Untreated Section: Away Direction.

The values of R^2 are very close to unity with all values of R^2 (rounded off to two decimal values) greater than or equal to 0.97.

Coast-down data was sampled at a frequency of 1 Hz. We need to verify that this linear velocity-time relation of the discrete sample does indeed represent the entire population of the continuous velocity-time history of a coast-down run. This may be done by the correlation test of hypothesis [Mason et al. 1996].

If we denote the correlation in the entire population as ρ, the correlation of the sample as R and the number of samples as n; then, the null hypothesis, H_0, and alternate hypothesis, H_1, are:

H_0: $\rho = 0$ (There is no linear relation between velocity and time for the entire population)

H_1: $\rho \neq 0$ (There is a linear relation between velocity and time for the entire population)

The test statistic 't' is defined by [Mason et al. 1996]:

$$t = R \left\{ (n-2) \ / \ (1-R^2) \right\}^{1/2}. \tag{5.18}$$

The t-statistic is calculated for the first 4 runs in Table 5.2. The level of significance selected is the 0.1% level. This implies that there is a 0.1% probability of the hypothesis test falsely indicating a linear relation between velocity and time.

Table 5.2 Calculation of the t-Statistic for Correlation Hypothesis Testing of the Untreated Road Section

Run	n	R^2	R	T
Back1	51	0.987	0.994	61.72
Away1	41	0.995	0.997	87.22
Back2	41	0.994	0.997	82.48
Away2	41	0.987	0.993	54.42

The hypothesis test is a two-tailed test and the critical value for the t – distribution at n-2 $\approx$ 40 degrees of freedom [Mason et al. 1996] is 3.551. The decision rule thus states that if the absolute value of the t-statistic is less than or equal to 3.551 then the null hypothesis is not rejected. Thus if the absolute value of the t-statistic is larger than 3.551 then the null hypothesis is rejected meaning there is a linear relation between velocity and time for the coast-down tests on the untreated section.

From Table 5.2, the t-statistic is larger than 3.551 for the first four runs. This means that a linear relation exists and because the 0.1% level was selected it is a 99.9% certainty.

By inspection of the remaining 'runs', the values of R and n are very similar so the values of t for the other runs will be of the same order of magnitude as those previously calculated and clearly larger than 3.551.

Therefore, for the untreated section the assumption of linear deceleration during coast-downs is valid.

Now that it has been confirmed that the deceleration during coast-down is constant, it is necessary to determine if the direction of coast-down affects the value of the deceleration.

Table 5.3 lists the value of the deceleration for each run as determined by the slope of the linear equation fitted through the measured data. The values are grouped in columns 'back' and 'away' which denotes the 2 directions travelled. The mean (average) deceleration differs for the direction travelled. The standard deviation of the results for the 'away' direction is less than that of the 'back' direction; implying that the values for deceleration in the 'away' direction are on average situated closer to their mean value than those of the 'back' direction. This is expressed in non-dimensional form as the coefficient of variation; which states that for the 'away' direction, the deceleration values lie, on average, approximately within 6 % of the mean. The deceleration values for the 'back' direction lie, on average, within 21% of the mean. There is therefore greater variability in the deceleration values for the 'back' direction. This is also evident from the larger range of values for the 'back' direction. By inspection it is clear that the deceleration value for run 2 in the back direction is an outlier.

Table 5.3 Descriptive Statistics of Deceleration during Coast-Down on Untreated Section

Run	Away	Back
1	-0.0799	-0.0673
2	-0.0728	-0.0900
3	-0.0724	-0.0591
4	-0.0716	-0.0631
5	-0.0673	-0.0538
Mean	-0.0728	-0.0667
Standard Deviation	0.0045	0.0140
Range	0.0126	0.0362
Minimum	-0.0799	-0.0900
Maximum	-0.0673	-0.0538
Coefficient of Variation	6.23%	20.95%

The descriptive statistics are recalculated excluding the outlier of run 2 in the 'back' direction and the results are presented in Table 5.4. Note that the results for the 'away' direction and the 'back' direction are similarly dispersed.

Table 5.4 Descriptive Statistics of Deceleration during Coast-Down on Untreated Section Excluding Outliers.

Run	Away	Back
1	-0.0799	-0.0673
2	-0.0728	n/a
3	-0.0724	-0.0591
4	-0.0716	-0.0631
5	-0.0673	-0.0538
Mean	-0.0728	-0.0608
Standard Deviation	0.0045	0.0058
Range	0.0126	0.0135
Minimum	-0.0799	-0.0673
Maximum	-0.0673	-0.0538
Coefficient of Variation	6.23%	9.46%

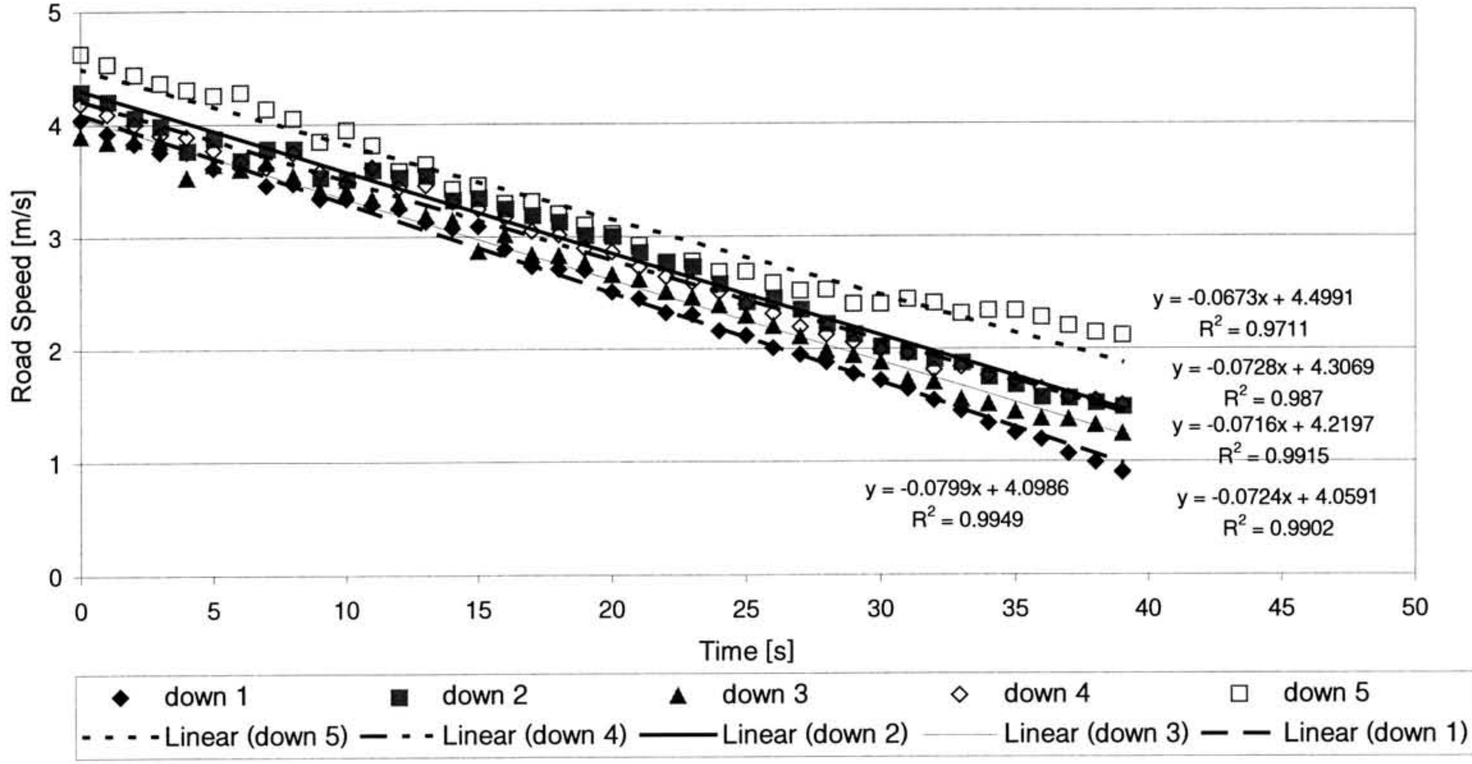

Figure 5.7 Linear Regression Analysis of Coast down data for Untreated Section

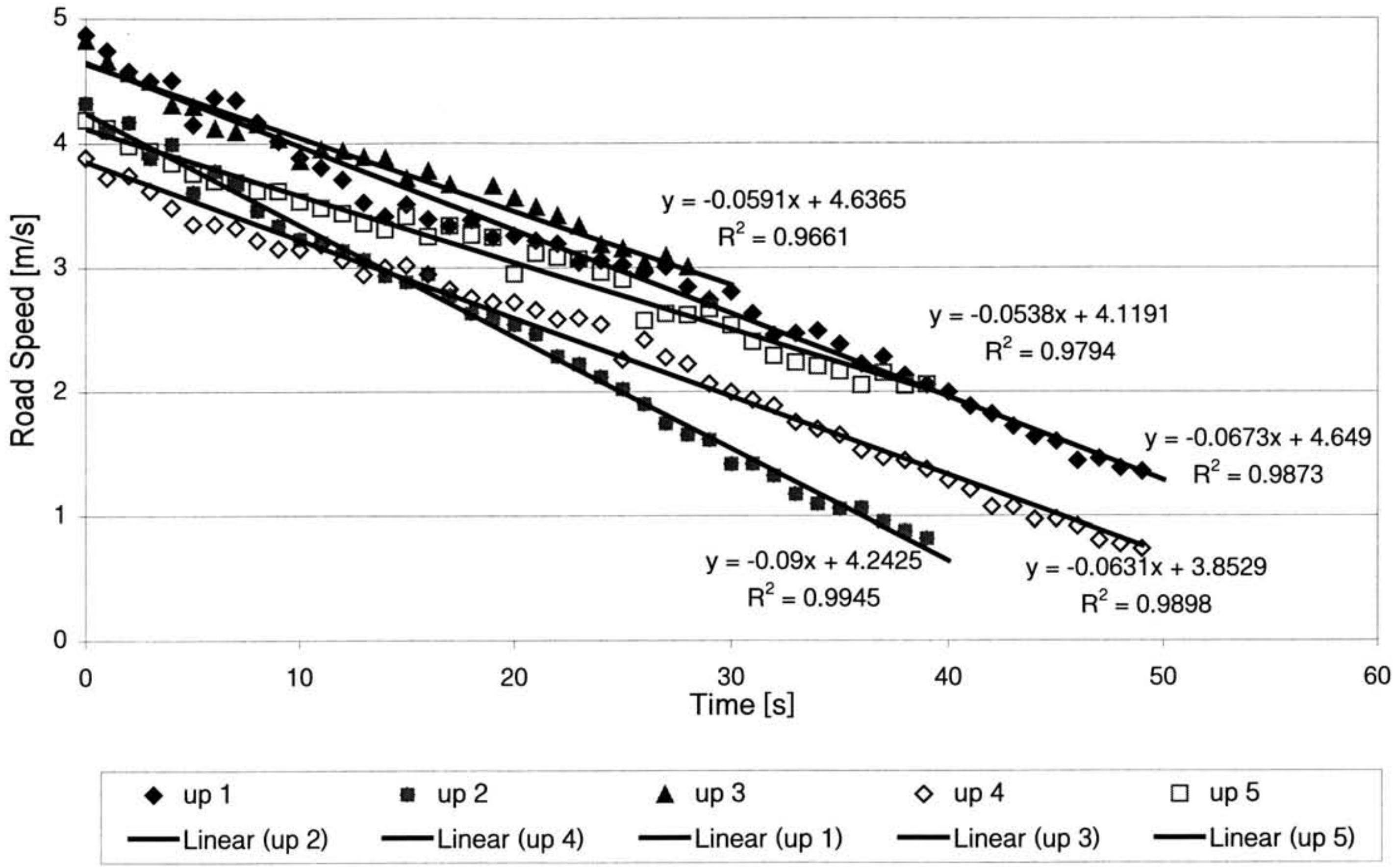

Figure 5.8 Linear Regression Analysis of Coast down data for Untreated Section

To determine if the difference in deceleration values for the 'away' and 'back' directions are statistically significant a paired hypothesis test is applied.

The deceleration values for each run in the 'away' and the 'back' direction are 'paired' and we calculate the difference between the 'away' and 'back' value. We define the difference as the 'back' value subtract the 'away' value. From the scatter plot of Figure 5.9 the difference will be positive. The null hypothesis would be that no change has occurred i.e. the mean of the differences (μ_d) would be less than or equal to zero. The mean of the differences cannot be less than zero because all the differences are at least zero. It is however stated as such for the sake of completeness for a one-tailed test. The alternate hypothesis would be that a positive change has occurred ($\mu_d > 0$).

For a small sample (less than 30 observations), the applicable test statistic is the t – statistic for paired observations:

$$t = \bar{d} \ / \ (S_d / n^{1/2}) \tag{5.19}$$

where:

S_d	is the standard deviation of the differences between the paired observations.
n	is the number of paired observations.
$\bar{d}$	is the mean difference between paired observations.

The 0.5 percent level is selected and the degree of freedom is n-1 = 3. The critical value for the one-tailed test is 5.841 [Mason et al. 1996].

From Table 5.5, t = 10.198 > 5.841. Therefore the difference between the 'away' and 'back' values is statistically significant. This implies that it is necessary to average out the results in both directions to eliminate any effect of gradient or wind that may be present.

Table 5.5 Calculation of t-Test for Paired Observations.

Run	Away	Back	Difference	Difference^2
1	-0.0799	-0.0673	0.0126	0.0001588
3	-0.0724	-0.0591	0.0133	0.0001769
4	-0.0716	-0.0631	0.0085	0.0000723
5	-0.0673	-0.0538	0.0135	0.0001823
		Sum =	**0.0479**	**0.00059015**
		$\bar{d}$ =	0.011975	
		S_d =	0.00235	
		t =	10.198	

The average deceleration for the untreated section is then calculated by pairing data in the two directions for each run, and then determining an average value for each run. The arithmetic mean of the averaged runs is then the deceleration for the untreated section. Dividing the deceleration by g, the gravitational acceleration will yield the rolling resistance coefficient for the untreated section. For the untreated section, the average deceleration is a_U = -0.06801 m.s^{-2}. The rolling resistance coefficient for untreated section is determined from Equation 5.4 as

$$F_{t,U} = a_U / g = -0.06801 \text{ m.s}^{-2} / 9.81 \text{ m.s}^{-2} = -0.00693 = \underline{-6.93 \text{ N/kN}}. \qquad (5.20)$$

Recall that the initial mathematical formulation of coast-down testing, required only the initial and final values for each coast-down run. For greater accuracy and insight into the coast-down phenomenon, the fifth-wheel data logger was deployed. If we had applied the initial formulation we would apply Equation 5.5 to determine the deceleration for each run as final velocity subtract the initial velocity divided by the elapsed time. Table 5.6 compares the values of deceleration between the two approaches. Column 'd (initial)' lists the deceleration values utilising the initial formulation of Equation 5.5 and column 'd (regression)' the values of deceleration obtained from the linear regression analysis. Table 5.6 shows that there is a one percent difference in the mean values of the respective approaches. The variance of each sample is also equal implying equal dispersion of values. These two facts allow us to conclude that there is no significant difference between the 2 approaches if the data is well approximated by a linear function.

Table 5.6 Comparison of the Two Approaches to Determine Coast-Down Deceleration

	d (initial) ($m.s^{-2}$)	d (regression) ($m.s^{-2}$)
up1	0.0709	0.0673
down1	0.0796	0.0799
up2	0.0906	0.09
down2	0.0704	0.0728
up3	0.0700	0.0591
down3	0.0673	0.0724
up4	0.0640	0.0631
down4	0.0681	0.0716
up5	0.0589	0.0538
down5	0.0644	0.0673
mean	0.0704	0.0697
variance	0.0001	0.0001
difference in means		-1.00%

From the analysis of the untreated section we determined that the deceleration during coast-down was constant and that there was a difference in results due to the direction of travel. This difference due to minor changes in gradient and/or wind conditions was averaged out and a baseline was established to quantify changes in rolling resistance due to changes in road surfaces.

5.5.2 Rolling Resistance of the Treated Sections.

From inspection of the scatter plots for Treated Sections A, B and C which are presented as Figures 5.2 - 5.4, there exists data that exhibits the same linear behaviour as the untreated section. Linear regression analysis is used to fit a linear equation through the velocity-time data by applying the least squares criterion. The results of the linear regression analysis are plotted on the velocity vs time graphs in Figures 5.10, 5.11 and 5.12. The coefficient of determination (R^2) for each linear equation is also calculated.

For Section A, the 'away' data does not exhibit a strong linear relation between velocity and time. This is evident from the low values for the R^2 - value. By applying the correlation test of hypothesis we can determine if this discrete sample of data is reflective of the continuous population from which it was extracted.

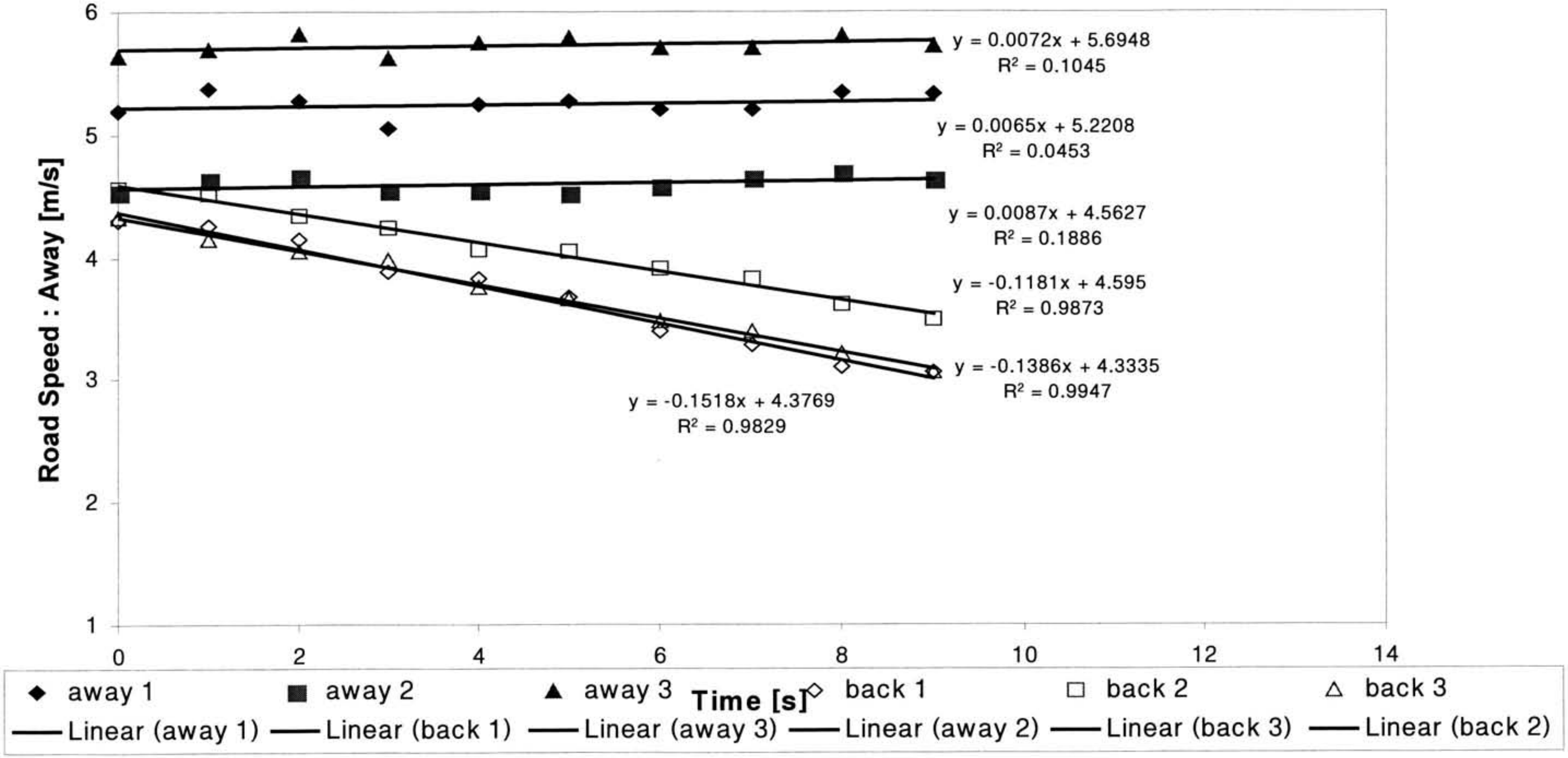

Figure 5.9 Linear Regression Analysis of Coast down data for Treated Section A.

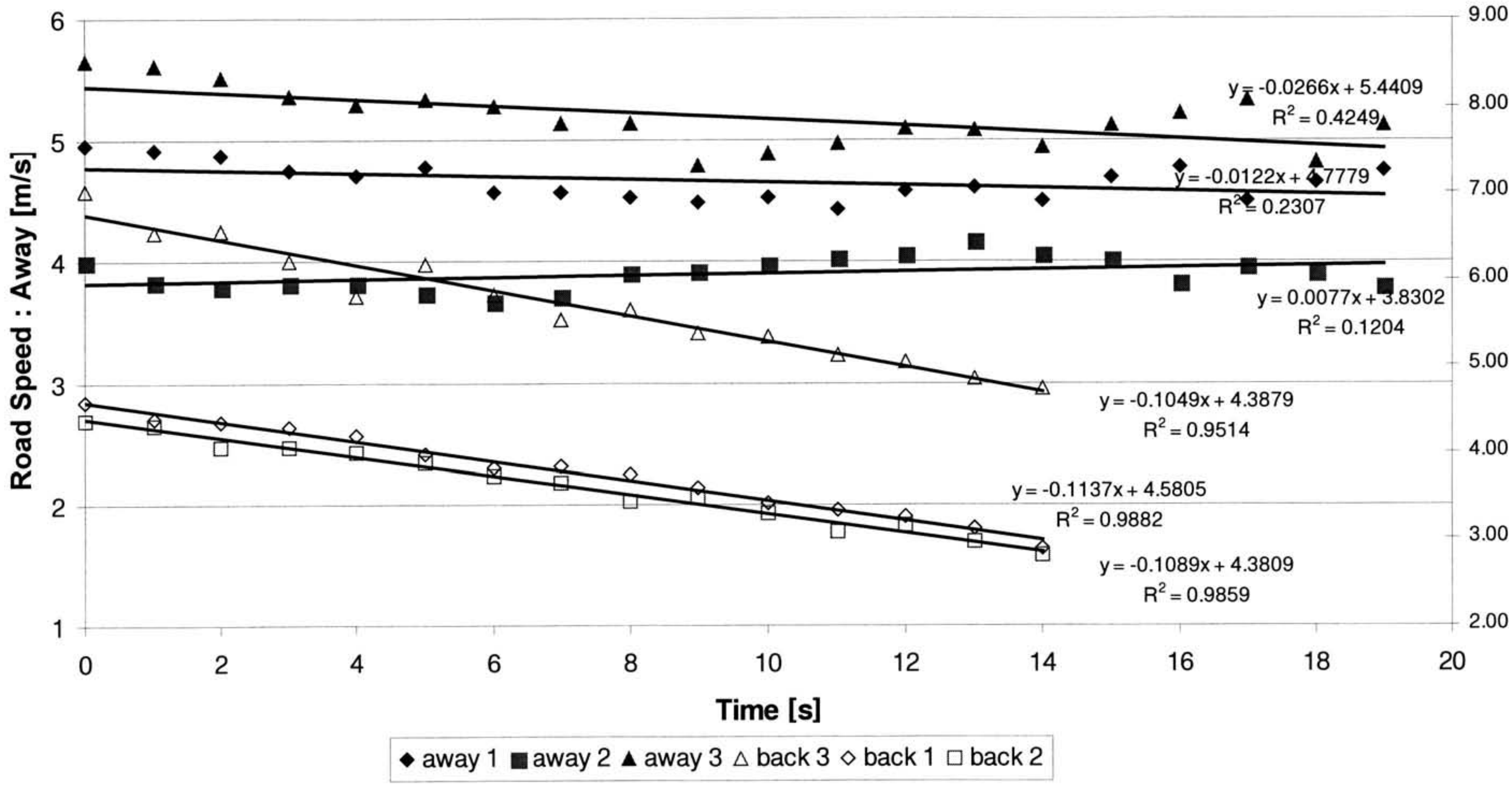

Figure 5.10 Linear Regression Analysis of Coast down data for Treated Section B.

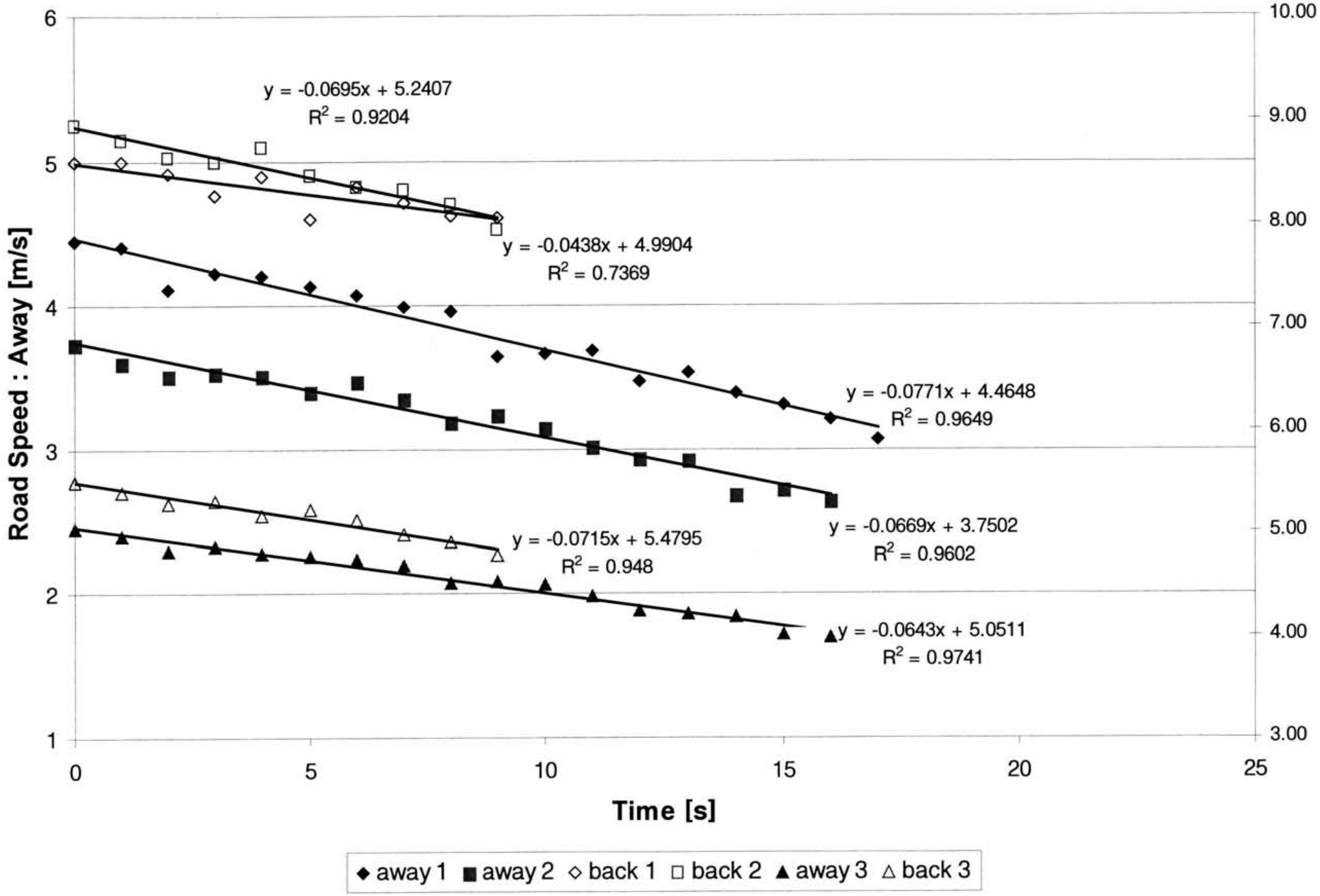

Figure 5.11 Linear Regression Analysis of Coast down data for Treated Section C.

The t-statistic is calculated for the 6 runs on Section A in Table 5.7. The level of significance selected is the 0.1% level.

Table 5.7 Calculation of the t-Statistic for Correlation Hypothesis Testing of the Treated Road Section A

Run	n	R^2	R	t
Away1	10	0.213	0.045	0.616
Away2	10	0.434	0.189	1.364
Away3	10	0.323	0.105	0.966
Back1	10	-0.991	0.983	-21.466
Back2	10	-0.994	0.987	-24.982
Back3	10	-0.997	0.995	-38.729

From Table 5.7, the t-statistic is larger than 5.041 for the 'back' runs only. This means that a linear relation exists and because the 0.1% level was selected it is with 99.9% level of confidence.

The null hypothesis was not rejected for the 'away' runs. This implies that for the 'away' runs performed on section A, there is no linear correlation between velocity and time.

Note that velocity has actually increased during the coasting in the 'away' direction. This indicates that a downhill grade exists in the 'away' direction that tends to accelerate the haul truck in the away direction. This is evident from time t = 6 seconds and onwards for the away direction. The grade will be an uphill in the 'back' direction that will tend to decelerate the haul truck more than due to coasting on a level surface.

Similarly for Section B, the 'away' data does not exhibit a strong linear relation between velocity and time. This is evident from the low R^2 - values. We again apply the correlation test of hypothesis. The results thereof are presented in Table 5.8.

The level of significance selected is the 0.1% level. For the away runs, the critical value for the t – distribution [Mason et al. 1996] is 3.922 at n - 2 = 18 degrees of freedom and for the 'back' runs the critical value is 4.221 at n -2 = 13 degrees of freedom [Mason et al. 1996].

From Table 5.8, the t-statistic is larger than the critical value for the 'back' runs only. This means that a linear relation exists and because the 0.1% level was selected it is a 99.9% level of confidence.

Table 5.8 Calculation of the t-Statistic for Correlation Hypothesis Testing of the Treated Road Section B.

Run	n	R^2	R	t
Away1	20	0.231	0.480	2.323
Away2	20	0.120	0.347	1.570
Away3	20	0.425	0.652	3.647
Back1	15	0.988	0.994	32.995
Back2	15	0.986	0.993	30.149
Back3	15	0.951	0.975	15.953

The linear regression analysis of measured data for Treated Section C is illustrated in Figure 5.11. The Coefficient of Determination (R^2) for each linear equation is also plotted. The majority of values of R^2 are very close to unity indicating a very strong correlation between velocity and time for section C. Table 5.9 describes the calculation of the correlation t – statistic required for correlation hypothesis testing. Correlation Hypothesis testing enables us to determine whether a linear relation between 2 sets of sampled data reflects the relation between the global population of data from which the sample was taken. The 0.1 % level was selected, and the critical values for Away 1 is 4.015 and for Away 2 and 3 is 4.075 and for Back 1,2 and 3 is 5.041 for n - 2 degrees of freedom. Thus for all the runs, except Back 1, the null hypothesis is rejected and there exists a 99.9% level of confidence of a global linear correlation between velocity and time for those runs.

Now that it has been confirmed that the deceleration during coast-down is constant, it is necessary to determine if the direction of coast-down affects the value of the deceleration.

Table 5.9 Calculation of the t-Statistic for Correlation Hypothesis Testing of the Treated Road Section C.

Run	n	R^2	R	t
Away1	18	0.96	0.98	20.97
Away2	17	0.96	0.98	19.02
Away3	17	0.97	0.99	23.75
Back1	10	0.74	0.86	4.73
Back2	10	0.92	0.96	9.62
Back3	10	0.95	0.97	12.08

To determine if the difference in deceleration values for the 'away' and 'back' directions is statistically significant a paired hypothesis test is applied.

The deceleration values for each run in the 'away' and the 'back' direction are 'paired' and we calculate the difference between the 'away' and 'back' value. We define the difference as the 'back' value subtract the 'away' value. The null hypothesis would be that no change has occurred i.e. the mean of the differences (μ_d) would be equal to zero. The alternate hypothesis would be that a change has occurred ($\mu_d \neq 0$).

For a small sample (less than 30 observations), the applicable test statistic is the t – statistic for paired observations which is presented as equation H.9 in Appendix H.

The 1 % level is selected and the critical value for two-tailed test is 9.925 for n-1 = 2 degrees of freedom [Mason et al. 1996].

Table 5.10 Calculation of t-Test for Paired Observations.

Run	Away	Back	Difference	Difference^2
1	-0.0771	-0.0438	0.0333	0.00111
2	-0.0669	-0.0695	-0.0026	0.00001
3	-0.0643	-0.0715	-0.0072	0.00005
		Sum =	**0.0235**	**0.00117**
		$\bar{d}$ =	0.0078	
		S_d =	0.0222	
		t =	0.6199	

From Table 5.10, $t = 0.6199 < 9.925$. Therefore the difference between the 'away' and 'back' values is not statistically significant and the difference between deceleration values is due to sampling variability.

The average deceleration for the treated section is then calculated as the arithmetic mean of the six deceleration values. Thus the deceleration for Treated Section C is $a_C = -\ 0.0655\ m.s^{-2}$. The rolling resistance coefficient for Treated Section C is determined from equation 5.4 as

$$f_{t,C} = a_C / g = -\,0.0655\ m.s^{-2} / 9.81\ m.s^{-2} = -0.00668 = \underline{-6.68\ kN/N}. \qquad (5.21)$$

5.5.3 Comparing the Untreated and Treated Sections.

We have calculated an average deceleration value for both an untreated section and a treated section. From Equation 5.4 rolling resistance coefficient is merely the deceleration divided by g. Thus the terms deceleration and rolling resistance coefficient are virtually interchangeable only differing by a constant factor of $g = 9.81\ m.s^{-2}$.

The change in rolling resistance coefficient is

$$\Delta f_t = (f_{t,C} - f_{t,U}) / f_{t,U}. = \underline{-3.61\%}. \qquad (5.22)$$

It is necessary to determine if the difference in the mean value of rolling resistance coefficient / deceleration for the treated and untreated sections is statistically significant. In other words, can the change in rolling resistance coefficient be attributed to the change in road surface? The other possibility is that the difference in the computed values is purely a result of sampling variability. This would mean that the samples were obtained from a population with the same mean.

The data used in this calculation is the deceleration data for the Treated Section C and the Untreated Section, presented in Table 5.11.

Table 5.11 Deceleration Data for Comparing Treated Section and Untreated Section

	Section C	Untreated Section
	-0.0771	-0.0799
	-0.0669	-0.0728
	-0.0643	-0.0724
	-0.0438	-0.0716
	-0.0695	-0.0673
	-0.0715	-0.0673
		-0.0591
		-0.0631
		-0.0538
Sample Mean	*-0.06552*	*-0.06801*
Sample Variance	*0.00013*	*0.00006*

In order to determine if there is a statistical significance in the different mean values we must determine if they are obtained from the same population. A hypothesis test of the two sample means using the t-distribution and pooled variances will be performed. The theory of hypothesis test of two means using the t-distribution and pooled variances is discussed in Appendix H (refer to Section H.6) and the test statistic is defined by Equation H.11.

If we denote the mean of the entire population for the treated section C as μ_1, and the mean of the entire population for the untreated section as μ_2, then, the null hypothesis, H_0, and alternate hypothesis, H_1, are:

H_0: $\mu_1 = \mu_2$ (There is no difference between population means)

H_1: $\mu_1 \neq \mu_2$ (There is a difference between population means)

By not rejecting the null hypothesis we assume that we may accept the null hypothesis. Accepting the null hypothesis implies that there is no statistical difference between the results obtained for the treated section and the untreated section. Rejecting the null hypothesis implies that the difference between the results for the treated and untreated section is significant and thus a result of different road surfaces.

The pooled variance is defined by Equation H.12 and the number of observations for each sample influences the value of the pooled variance. It was determined that for the Treated Section C, the direction of coast down did not have a statistical effect on the measured data. For this reason all the deceleration values were used to determine the mean deceleration. For the untreated section, direction of coast-down did affect the measured data. To eliminate this effect the results are paired with the value in the other direction, for that particular run, and then the average of the pair is used as an 'averaged-out' deceleration value. The mean deceleration was then determined from the averaged values.

Although this process is mathematically identical to the arithmetic mean of the entire unprocessed sample, for the purposes of this calculation, the untreated section data in Table 5.11 will be paired off before any calculations. This is because the number of observations for each sample affects the pooled variance. It has been shown that the direction of travel did not have an effect on the data from Treated Section C. The data from Treated Section C was accepted at face value. Therefore the Treated Section C data was statistically more 'pure' than the Untreated Section data and thus the pooled variance should be biased towards the 'purer' data. However the volume of data for the Untreated Section which was needed to 'clean' the untreated section data would cause the pooled variance to be biased towards the Untreated Section data. Table 5.12 reflects this change.

Table 5.12 Adjusted Deceleration Data for Comparing Treated Section and Untreated Section

	Section C	Untreated Section
	-0.0771	-0.0736
	-0.0669	-0.06575
	-0.0643	-0.06735
	-0.0438	-0.06055
	-0.0695	-0.0728
	-0.0715	
Sample Mean	*-0.06552*	*-0.06801*
Sample Variance	*0.00013*	*0.00003*

From this Table the t -statistic as defined by equation H.11 is $t = 0.532$.

Now the critical value for the 1 percent interval is 3.169 [Mason et al. 1996]. Since t = 0.532 < 3.169, the null hypothesis is not rejected implying that the 3 percent decrease in the average deceleration (and hence a 3 % decrease in rolling resistance coefficient) when coasting on a surface treated by dust-a-side is due to sampling variability.

5.6 Conclusion.

Coast down tests were performed on 3 sections treated with Dust-a-Side and a well maintained untreated section. The purpose was to determine the effect of the Dust-a-side treatment on rolling resistance coefficient. First the Coast-down data for the untreated section was evaluated. Using statistical analysis techniques a linear correlation between velocity and time was established, this supported the assumption that the deceleration and thus the rolling resistance coefficient was constant for the coast-down. Using hypothesis tests, it was determined that the sample data for the untreated section was representative of the continuous spectrum of velocity vs time, furthermore, it was determined that the results for the untreated section were dependant on the direction of coast-down. The values recorded in opposite directions were paired off to average out any minor inconsistencies and a reliable baseline for the rolling resistance coefficient was obtained.

The data for the Treated Sections was also evaluated using similar statistical analysis techniques. It was found that for sections A and B, the data was direction-dependant implying that a minor a grade had an exaggerated effect on a large vehicle such as a haul truck. This is evident from the increase in velocity whilst coasting on supposedly 'level' sections. The data for treated section C was statistically reliable and it was found that the data from section C was not direction-dependant. Coast-down data served as a good comparison to compare rolling resistance on a treated section with that of an untreated section.

There was a 3.6 percent decrease in rolling resistance coefficient, however this decrease was explained by sampling variability and is not conclusive of any decrease in rolling resistance due to treatment by Dust-a-Side.

The null result occurs because road-surface slip-interaction is orders of magnitude less than hysteretic losses in tyre deformations and also less than energy transfer losses from road to tyre and losses in damping system. Each tyre has to support approximately 50-60 tons (105ton truck+ 250 ton load divided by 6 wheels). These tyres have to be constructed strong enough to sustain high inflation pressures to support this load. So the tyres are designed for strength and load capacity and thus are designed to have minimum deflection therefore a lot of energy is needed to deflect a tyre during its 'crawling along' during travel. So hysteresis losses will be the dominant mechanism in rolling resistance. Available literature indicates that hysteresis losses account for 95% of rolling losses in sedan and truck tyres. This study of significantly larger haul truck tyre reflects a trend similar to sedan and truck tyres. An improvement in tyre-road adhesion is thus negligible.

Although the statistical analysis of coast down data revealed no improvement in rolling resistance, there are other expected benefits of treating road surfaces with Dust-a-side. As the name of the treatment suggests, its purpose is to reduce dust suspension during travel. The expected benefits thereof are longer engine component life, longer maintenance intervals and reduced fuel contamination. Also following distances and operational cycle time may also be minimised. A cost-benefit analysis of Dust-a-side compared to the traditional water- spraying method is beyond the scope of study.

6. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

The aim of this study was to determine suitable advanced analysis techniques and to perform knowledge-based analysis of the fuel consumption and mining production data in order to reduce the TCO of mining vehicles. The approach is thus twofold. Firstly to *determine* suitable techniques for the analysis of fuel consumption and mining production data and, secondly to *perform* knowledge based analysis using the techniques identified.

By conceptualising TCO and then developing an understanding of the mining operation, and its inherent complexities, the following engineering sciences were considered:

- Cost driver Modelling
- Wheel and Tyre Theory
- Resource and Production Scheduling

A review of the related engineering sciences was presented which identified suitable techniques of analysis.

A cost-driver model for diesel consumption was formulated by exploiting the expert judgement of role players in the mining operation. Cost-driver models were developed for the Sishen, Thabazimbi and Grootegeluk mines. The cost-driver models were then modelled as linear programming problems and solved using the student version of LINDO Optimization Software. The results obtained were of a qualitative nature but highlighted the importance of the cause-effect relation between the various factors. Although formulated independantly, the cost-driver models and the optimal solutions thereof were similar. This indicates the universality of the cost-driver models.

The three models were collated into a consolidated cost-driver model. The optimal solution to this model verified the results obtained by the former models. The solution to the LP problem highlighted that the indirect (hidden) benefits of improving a dominant cost driver, namely the Information and Management System, are more substantial than the direct benefits.

To analyse the indirect effects, if any, of the dominant cost driver on diesel consumption the influence of the dominant cost driver is 'tracked' as it progresses through the cost-driver model. In this regard, the improvement in road conditions and the improvement in the utilisation of equipment must be quantified. One aspect of the improvement in road condition was quantified by rolling resistance coefficient and requires an understanding of wheel and tyre theory. In order to quantify the utilisation of equipment, the daily operating cycle of the mining vehicles was simulated.

Further refinement of the diesel cost-driver model is recommended. The consolidated cost-driver model is the product of the first diesel workshops and should be used as a point of departure at subsequent workshops to define the value of the influence coefficients of the model. Thus further research into capturing and exploiting expert judgement for the diesel cost driver model is recommended. An alternative refinement is to quantify all the factors of the cost driver model in a similar approach as 'Utilisation of Equipment' and 'Improvement in road Conditions'. Then the value of the influence coefficients can be determined by analytical techniques. Exploiting expert judgement and the analytical approach should be viewed as complimentary avenues for further research.

The operational cycle of haul trucks was simulated in order to quantify equipment utilisation and reduce diesel consumption of the mining vehicles. The operational cycle of haul trucks was modelled using queueing theory. In its simplest form the mining operation can be modelled by a three queue network with a queue denoting the pit, plant and dump respectively. The simulation of the queue network was implemented in Matlab using the next event advance method and was called Q_Sim.

The output from Q_Sim had an error of 1-3% which can be accounted for the transient state values during start-up. The output from Q_Sim was therefore realistic and reliable.

Simulation and Q_Sim in particular is a 'what-if' tool. Q_Sim enables a user to evaluate operational deployment strategies without having to actually implement that strategy in practice. An optimal deployment strategy can be determined at very little resource cost and in minimal time. For example, a production requirement may be that displaced earth, from blasting, must be removed in a given time. With Q_Sim, the number of trucks and the truck capacity required to achieve the prescribed production rate can be determined. Real time forecasting of the implications of haul truck configuration changes can be achieved by judicious application of Q_Sim. The implications of haul truck changes are quantified by mean waiting time, mean cycle time of haul trucks and the percentage utilisation of loaders or crushers. These parameters are calculated by Q_Sim.

Q_Sim was applied to a theoretical example to determine optimal fleet size. It was noted as fleet size increases, individual haul truck productivity decreases due to increased congestion in the queue system. With an increase in fleet size, fleet productivity may increase and then reach a maximum before it decreases. The identification of such an optimal point by Q_Sim indicates that diesel consumption can be reduced whilst still fulfilling production requirements.

The economies of scale of haul truck capacity were investigated. It was concluded that haul truck capacity and haul truck availability affect fleet productivity. This relationship may be determined using Q_Sim. It was shown that to obtain any benefit from economies of scale, a specific level of haul truck availability is required.

The benefits of an analysis of economies of scale is not limited to reducing diesel consumption of current haul truck fleets. Such an analysis is a valuable tool to evaluate prospective equipment acquisitions. By performing an analysis of economies of scale the 'hidden' costs necessary to achieve the specified level of availability can be more accurately determined. Q_Sim thus facilitates the minimisation of TCO during acquisition.

Q_Sim input parameters were determined from mining production data from the Sishen mine and simulation performed which highlight the reduction in diesel consumption that may be obtained by simulating the operating cycle of mining haul trucks.

Further investigation into the listed potential benefits of Q_Sim is recommended. This would encompass, but not be limited to, the modelling of the entire mine with current data and not historical data, the incorporation of financial data to translate parameters such as tons per minute into comparable format with purchase costs and maintenance costs and optimal dispatching of haul trucks using real time forecasting. The optimisation of the number, and individual capacity, of haul trucks and the number of loaders or crushers per location by means of Taguchi methods requires investigation. Taguchi methods would enable a researcher to investigate a large permutation of simulation parameters by a computationally efficient design of experiments.

Coast down tests were performed on 3 sections treated with Dust-a-Side and a well maintained untreated section. The purpose was to determine the effect of the Dust-a-side treatment on rolling resistance coefficient. First the Coast-down data for the untreated section was evaluated. Using statistical analysis techniques a linear correlation between velocity and time was confirmed, this supported the assumption that the deceleration, and thus rolling resistance coefficient, was constant for the coast-down. Using hypothesis tests, it was determined that the sample data for the untreated section was representative of the continuous spectrum of velocity vs time, furthermore, it was determined that the results for the untreated section were dependant on the direction of coast-down. The values recorded in opposite directions were paired off to average out any minor inconsistencies. A reliable baseline for the rolling resistance coefficient was obtained.

The data for the Treated Sections was also evaluated using similar statistical analysis techniques. It was found that for Sections A and B, the data was direction-dependant. This meant that a minor grade had an exaggerated effect on the coasting of a large vehicle such as a haul truck. This is evident from the increase in velocity whilst coasting on supposedly 'level' sections. The data for Treated Section C was statistically reliable and it was found that the data from section C was not direction-dependant. Coast-down data served as a good comparison to compare rolling resistance on a treated section with that of an untreated section.

There was a 3.6 percent decrease in rolling resistance coefficient, however this decrease is explained by sampling variability and is not conclusive of any decrease in rolling resistance due to treatment by Dust-a-Side.

The null decrease was explained by the fact that hysteretic losses in tyre deformation and energy dissipation from road to tyre and losses in the damping system are more dominant mechanisms of rolling resistance than road-surface slip-interaction. The tyres are designed to have minimum deflection in order to operate under the heavy loading of approximately 50-60 tons per wheel (105 ton truck + 250 ton load divided by 6 wheels). A lot of energy is needed to deflect tyre during its 'crawling along' during travel. This study of significantly larger haul truck tyre reflects a trend similar to sedan and truck tyres. An improvement in tyre-road adhesion provides a negligible decrease in rolling resistance.

The primary purpose of Dust-a-side is to reduce dust suspension during travel. Although no improvement in rolling resistance was attained, the expected benefits due to dust suspension are longer engine component life, longer maintenance intervals and reduced fuel contamination. Following distances and operational cycle time may also be minimised due to dust suspension. The performance of Dust-a-side compared to common water-spraying techniques requires further investigation.

It is hoped that this study has provided insight in techniques to reduce the Total Cost of Ownership of mining vehicles. Moreover, it is believed that the conclusions of this study are not limited in application to the mining operation and these techniques would be appropriate for other logistical operations including the transport sector and the military.

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Appendix A LP Problem For Thabazimbi Cost Driver Model

The results for the general LP problem (Equation A.1) of the Thabazimbi diesel cost driver model is presented for different values for the influence coefficient 'k'.

Maximise $Z = A + B + C + D + E + F + G + H + I + J + K$ (A.1)

Subject to

H1: $A = k\,B + k\,C + k\,D + k\,H$

H2: $B = k\,C + k\,G + k\,H$

H3: $C = k\,G$

H4: $D = k\,B + k\,C + k\,E + k\,G + k\,H$

H5: $E = k\,B + k\,C + k\,G + k\,H$

H6: $F = k\,A + k\,B + k\,C + k\,D + k\,E + k\,G + k\,H$

H7: $H = k\,C + k\,G$

H8: $I = k\,A + k\,B + k\,C + k\,D + k\,E + k\,F + k\,G + k\,J$

H9 : $J = k\,C + k\,G$

H10: $K = k\,C + k\,D + k\,G + k\,I + k\,J$

G1: $A \le 7$

G2: $B \le 2$

G3: $C \le 14$

G4: $D \le 9$

G5: $E \le 10$

G6: $F \le 3$

G7: $G \le 3$

G8: $H \le 8$

G9: $I \le 8$

G10: $J \le 8$

G11: $K \le 2$

$A, B, C, .., K \ge 0.$

Table A.1 Optimal Solution to Equation A.1 for k = 0.1

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	0.14	0
B	0.36	0
C	0.30	0
D	0.44	0
E	0.40	0
F	0.50	0
G	3.00	0
H	0.33	0
I	0.55	0
J	0.33	0
K	0.46	0
Objective	6.81	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	47.56
B	1	INFINITY	18.76
C	1	INFINITY	22.70
D	1	INFINITY	15.51
E	1	INFINITY	17.06
F	1	INFINITY	13.69
G	1	INFINITY	2.27
H	1	INFINITY	20.64
I	1	INFINITY	12.45
J	1	INFINITY	20.64
K	1	INFINITY	14.75

ROW	SLACK	DUAL PRICES
H1)	0	-1.22
H2)	0	-1.62
H3)	0	-2.17
H4)	0	-1.44
H5)	0	-1.37
H6)	0	-1.11
H8)	0	-1.68
H9)	0	-1.10
H10)	0	-1.21
H11)	0	-1.00
G1)	6.86	0
G2)	1.64	0
G3)	13.70	0
G4)	8.56	0
G5)	9.60	0
G6)	2.50	0
G7)	0	2.27
G8)	7.67	0
G9)	7.45	0
G10)	7.67	0
G11)	1.54	0

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	0.14	6.86
H2	0	0.36	1.64
H3	0	0.30	10.92
H4	0	0.44	8.56
H5	0	0.40	9.60
H6	0	0.50	2.50
H8	0	0.33	7.67
H9	0	0.55	7.45
H10	0	0.33	7.67
H11	0	0.46	1.54
G1	7	INFINITY	6.86
G2	2	INFINITY	1.64
G3	14	INFINITY	13.70
G4	9	INFINITY	8.56
G5	10	INFINITY	9.60
G6	3	INFINITY	2.50
G7	3	10.00	3.00
G8	8	INFINITY	7.67
G9	8	INFINITY	7.45
G10	8	INFINITY	7.67
G11	2	INFINITY	1.54

Table A.2 Optimal Solution to Equation A.1 for k = 0.3

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	0.97	0
B	0.80	0
C	0.47	0
D	1.35	0
E	1.04	0
F	2.04	0
G	1.57	0
H	0.61	0
I	2.66	0
J	0.61	0
K	2	0
Objective	14.13	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	14.57
B	1	INFINITY	17.71
C	1	INFINITY	29.92
D	1	INFINITY	10.48
E	1	INFINITY	13.62
F	1	INFINITY	6.91
G	1	INFINITY	8.98
H	1	INFINITY	23.02
I	1	INFINITY	5.32
J	1	INFINITY	23.02
K	1	INFINITY	7.07

ROW	SLACK	DUAL PRICES
H1)	0	-0.98
H2)	0	-1.36
H3)	0	-0.23
H4)	0	0.55
H5)	0	-0.82
H6)	0	-0.75
H8)	0	-2.01
H9)	0	0.82
H10)	0	1.07
H11)	0	6.07
G1)	6.03	0
G2)	1.20	0
G3)	13.53	0
G4)	7.65	0
G5)	8.96	0
G6)	0.96	0
G7)	1.43	0
G8)	7.39	0
G9)	5.34	0
G10)	7.39	0
G11)	0	7.07

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	1.03	5.30
H2	0	0.93	1.41
H3	0	0.62	1.99
H4	0	1.94	4.42
H5	0	1.19	6.02
H6	0	2.25	1.05
H8	0	0.70	2.22
H9	0	3.12	6.67
H10	0	0.70	5.13
H11	0	0.93	2
G1	7	INFINITY	6.03
G2	2	INFINITY	1.20
G3	14	INFINITY	13.53
G4	9	INFINITY	7.65
G5	10	INFINITY	8.96
G6	3	INFINITY	0.96
G7	3	INFINITY	1.43
G8	8	INFINITY	7.39
G9	8	INFINITY	5.34
G10	8	INFINITY	7.39
G11	2	0.93	2

Table A.3 Optimal Solution to Equation A.1 for k = 0.5

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	0.80	0
B	0.37	0
C	0.16	0
D	0.82	0
E	0.55	0
F	1.63	0
G	0.32	0
H	0.24	0
I	2.45	0
J	0.24	0
K	2.00	0
Objective	9.59	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	12.03
B	1	INFINITY	26.23
C	1	INFINITY	59.03
D	1	INFINITY	11.66
E	1	INFINITY	17.49
F	1	INFINITY	5.88
G	1	INFINITY	29.51
H	1	INFINITY	39.35
I	1	INFINITY	3.92
J	1	INFINITY	39.35
K	1	INFINITY	4.79

ROW	SLACK	DUAL PRICES
H1)	0	-0.83
H2)	0	-1.16
H3)	0	-0.28
H4)	0	0.65
H5)	0	-0.50
H6)	0	-0.55
H8)	0	-2.20
H9)	0	0.90
H10)	0	1.34
H11)	0	3.79
G1)	6.20	0
G2)	1.63	0
G3)	13.84	0
G4)	8.18	0
G5)	9.45	0
G6)	1.37	0
G7)	2.68	0
G8)	7.76	0
G9)	5.55	0
G10)	7.76	0
G11)	0	4.79

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	0.94	5.33
H2	0	0.51	1.29
H3	0	0.25	0.47
H4	0	1.46	1.88
H5	0	0.73	2.21
H6	0	2.05	1.72
H8	0	0.33	0.97
H9	0	3.36	4.00
H10	0	0.27	2.67
H11	0	1.68	2.00
G1	7	INFINITY	6.20
G2	2	INFINITY	1.63
G3	14	INFINITY	13.84
G4	9	INFINITY	8.18
G5	10	INFINITY	9.45
G6	3	INFINITY	1.37
G7	3	INFINITY	2.68
G8	8	INFINITY	7.76
G9	8	INFINITY	5.55
G10	8	INFINITY	7.76
G11	2	1.68	2.00

Table A.4 Optimal Solution to Equation A.1 for k = 1.0

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	0.35	0.00
B	0.06	0.00
C	0.02	0.00
D	0.25	0.00
E	0.12	0.00
F	0.85	0.00
G	0.02	0.00
H	0.03	0.00
I	1.69	0.00
J	0.03	0.00
K	2.00	0.00
Objective	5.42	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	15.30
B	1	INFINITY	88.00
C	1	INFINITY	352.00
D	1	INFINITY	22.00
E	1	INFINITY	44.00
F	1	INFINITY	6.40
G	1	INFINITY	352.00
H	1	INFINITY	176.00
I	1	INFINITY	3.20
J	1	INFINITY	176.00
K	1	INFINITY	2.71

ROW	SLACK	DUAL PRICES
H1)	0.00	-0.58
H2)	0.00	-0.68
H3)	0.00	-0.29
H4)	0.00	0.54
H5)	0.00	-0.05
H6)	0.00	-0.29
H8)	0.00	-2.06
H9)	0.00	0.71
H10)	0.00	1.42
H11)	0.00	1.71
G1)	6.65	0.00
G2)	1.94	0.00
G3)	13.98	0.00
G4)	8.75	0.00
G5)	9.88	0.00
G6)	2.15	0.00
G7)	2.98	0.00
G8)	7.97	0.00
G9)	6.31	0.00
G10)	7.97	0.00
G11)	0.00	2.71

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	0.55	1.00
H2	0	0.12	0.13
H3	0	0.03	0.03
H4	0	0.64	0.40
H5	0	0.22	0.29
H6	0	1.47	2.00
H8	0	0.06	0.06
H9	0	5.09	2.00
H10	0	0.03	1.00
H11	0	5.09	2.00
G1	7	INFINITY	6.65
G2	2	INFINITY	1.94
G3	14	INFINITY	13.98
G4	9	INFINITY	8.75
G5	10	INFINITY	9.88
G6	3	INFINITY	2.15
G7	3	INFINITY	2.98
G8	8	INFINITY	7.97
G9	8	INFINITY	6.31
G10	8	INFINITY	7.97
G11	2	5.09	2.00

Appendix B LP Problem For Grootegeluk Cost Driver Model

The results for the general LP problem (Equation B.1) of the Grootegeluk diesel cost driver model is presented for different values for the influence coefficient 'k'.

Maximise $Z = A + B + C + D + E + F + G + H + I + J + K$ (B.1)

Subject to

H1: $B = k A$

H2: $C = k A + k B + k D + k G + k I + k J + k K$

H3: $D = k A + k B + k G + k H + k I + k J + k K$

H4: $E = k A + k B + k K$

H5: $F = k A + k B + k C + k D + k G + k H + k K$

H6: $G = k A + k B + k H + k K$

H7: $H = k A + k B + k K$

H8: $I = k A + k K$

H9: $J = k A + k B + k H + k K$

H10: $K = k A$

G1: $A \leq 7$

G2: $B \leq 1$

G3: $C \leq 14$

G4: $D \leq 4$

G5: $E \leq 11$

G6: $F \leq 4$

G7: $G \leq 7$

G8: $H \leq 8$

G9: $I \leq 6$

G10: $J \leq 8$

G11: $K \leq 5$

$A,B,C,..,K \geq 0.$

Table B.1 Optimal Solution to Equation B.1 for k = 0.1

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	7.00	0
B	0.70	0
C	1.22	0
D	1.19	0
E	0.84	0
F	1.26	0
G	0.92	0
H	0.84	0
I	0.77	0
J	0.92	0
K	0.70	0
Objective	16.36	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	2.34
B	1	INFINITY	23.37
C	1	INFINITY	13.41
D	1	INFINITY	13.80
E	1	INFINITY	19.48
F	1	INFINITY	13.02
G	1	INFINITY	17.71
H	1	INFINITY	19.48
I	1	INFINITY	21.25
J	1	INFINITY	17.71
K	1	INFINITY	23.37

ROW	SLACK	DUAL PRICES
H1)	0.00	-1.83
H2)	0.00	-1.10
H3)	0.00	-1.21
H4)	0.00	-1.00
H5)	0.00	-1.00
H6)	0.00	-1.33
H8)	0.00	-1.48
H9)	0.00	-1.23
H10)	0.00	-1.23
H11)	0.00	-1.96
G1)	0.00	2.34
G2)	0.30	0.00
G3)	12.78	0.00
G4)	2.81	0.00
G5)	10.16	0.00
G6)	2.74	0.00
G7)	6.08	0.00
G8)	7.16	0.00
G9)	5.23	0.00
G10)	7.08	0.00
G11)	4.30	0.00

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	0.70	0.30
H2	0	1.22	12.78
H3	0	1.19	2.81
H4	0	0.84	10.16
H5	0	1.26	2.74
H6	0	0.92	6.08
H8	0	0.84	7.16
H9	0	0.77	5.23
H10	0	0.92	7.08
H11	0	0.70	4.30
G1	7	3.00	7.00
G2	1	INFINITY	0.30
G3	14	INFINITY	12.78
G4	4	INFINITY	2.81
G5	11	INFINITY	10.16
G6	4	INFINITY	2.74
G7	7	INFINITY	6.08
G8	8	INFINITY	7.16
G9	6	INFINITY	5.23
G10	8	INFINITY	7.08
G11	5	INFINITY	4.30

Table B.2 Optimal Solution to Equation B.1 for k = 0.3

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	2.60	0
B	0.78	0
C	3.40	0
D	2.90	0
E	1.25	0
F	4.00	0
G	1.62	0
H	1.25	0
I	1.01	0
J	1.62	0
K	0.78	0
Objective	21.22	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	8.16
B	1	INFINITY	27.19
C	1	INFINITY	6.25
D	1	INFINITY	7.31
E	1	INFINITY	16.99
F	1	INFINITY	5.30
G	1	INFINITY	13.07
H	1	INFINITY	16.99
I	1	INFINITY	20.92
J	1	INFINITY	13.07
K	1	INFINITY	27.19

ROW	SLACK	DUAL PRICES
H1)	0.00	0.19
H2)	0.00	0.29
H3)	0.00	0.38
H4)	0.00	-1.00
H5)	0.00	4.30
H6)	0.00	0.49
H8)	0.00	0.31
H9)	0.00	-0.80
H10)	0.00	-0.80
H11)	0.00	-0.04
G1)	4.40	0.00
G2)	0.22	0.00
G3)	10.60	0.00
G4)	1.10	0.00
G5)	9.75	0.00
G6)	0.00	5.30
G7)	5.38	0.00
G8)	6.75	0.00
G9)	4.99	0.00
G10)	6.38	0.00
G11)	4.22	0.00

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	0.95	0.27
H2	0	3.75	13.33
H3	0	2.89	1.53
H4	0	1.25	9.75
H5	0	1.13	4.00
H6	0	2.04	6.77
H8	0	1.56	6.34
H9	0	1.07	5.26
H10	0	1.77	6.96
H11	0	0.96	4.11
G1	7	INFINITY	4.40
G2	1	INFINITY	0.22
G3	14	INFINITY	10.60
G4	4	INFINITY	1.10
G5	11	INFINITY	9.75
G6	4	1.13	4.00
G7	7	INFINITY	5.38
G8	8	INFINITY	6.75
G9	6	INFINITY	4.99
G10	8	INFINITY	6.38
G11	5	INFINITY	4.22

Table B.3 Optimal Solution to Equation B.1 for k = 0.5

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	0.64	0
B	0.32	0
C	2.93	0
D	2.17	0
E	0.64	0
F	4.00	0
G	0.96	0
H	0.64	0
I	0.48	0
J	0.96	0
K	0.32	0
Objective	14.09	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	21.91
B	1	INFINITY	43.81
C	1	INFINITY	4.80
D	1	INFINITY	6.49
E	1	INFINITY	21.91
F	1	INFINITY	3.52
G	1	INFINITY	14.60
H	1	INFINITY	21.91
I	1	INFINITY	29.21
J	1	INFINITY	14.60
K	1	INFINITY	43.81

ROW	SLACK	DUAL PRICES
H1)	0.00	0.25
H2)	0.00	0.26
H3)	0.00	0.39
H4)	0.00	-1.00
H5)	0.00	2.52
H6)	0.00	0.59
H8)	0.00	0.41
H9)	0.00	-0.67
H10)	0.00	-0.67
H11)	0.00	-0.08
G1)	6.36	0.00
G2)	0.68	0.00
G3)	11.07	0.00
G4)	1.83	0.00
G5)	10.36	0.00
G6)	0.00	3.52
G7)	6.04	0.00
G8)	7.36	0.00
G9)	5.52	0.00
G10)	7.04	0.00
G11)	4.68	0.00

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	0.42	0.88
H2	0	4.63	8.00
H3	0	3.66	3.08
H4	0	0.64	10.36
H5	0	3.37	4.00
H6	0	1.32	3.56
H8	0	0.90	2.29
H9	0	0.52	5.97
H10	0	1.14	6.40
H11	0	0.43	1.25
G1	7	INFINITY	6.36
G2	1	INFINITY	0.68
G3	14	INFINITY	11.07
G4	4	INFINITY	1.83
G5	11	INFINITY	10.36
G6	4	3.37	4.00
G7	7	INFINITY	6.04
G8	8	INFINITY	7.36
G9	6	INFINITY	5.52
G10	8	INFINITY	7.04
G11	5	INFINITY	4.68

Table B.4 Optimal Solution to Equation B.1 for k = 1

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	0.06	0
B	0.06	0
C	2.14	0
D	1.16	0
E	0.17	0
F	4.00	0
G	0.35	0
H	0.17	0
I	0.12	0
J	0.35	0
K	0.06	0
Objective	8.64	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	149.00
B	1	INFINITY	149.00
C	1	INFINITY	4.03
D	1	INFINITY	7.45
E	1	INFINITY	49.67
F	1	INFINITY	2.16
G	1	INFINITY	24.83
H	1	INFINITY	49.67
I	1	INFINITY	74.50
J	1	INFINITY	24.83
K	1	INFINITY	149.00

ROW	SLACK	DUAL PRICES
H1)	0.00	0.35
H2)	0.00	0.16
H3)	0.00	0.32
H4)	0.00	-1.00
H5)	0.00	1.16
H6)	0.00	0.64
H8)	0.00	0.59
H9)	0.00	-0.52
H10)	0.00	-0.52
H11)	0.00	-0.17
G1)	6.94	0.00
G2)	0.94	0.00
G3)	11.86	0.00
G4)	2.84	0.00
G5)	10.83	0.00
G6)	0.00	2.16
G7)	6.65	0.00
G8)	7.83	0.00
G9)	5.88	0.00
G10)	7.65	0.00
G11)	4.94	0.00

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	0.08	0.19
H2	0	4.63	4.00
H3	0	2.76	2.00
H4	0	0.17	10.83
H5	0	9.80	4.00
H6	0	0.53	1.00
H8	0	0.31	0.40
H9	0	0.13	1.33
H10	0	0.47	1.33
H11	0	0.09	0.17
G1	7	INFINITY	6.94
G2	1	INFINITY	0.94
G3	14	INFINITY	11.86
G4	4	INFINITY	2.84
G5	11	INFINITY	10.83
G6	4	9.80	4.00
G7	7	INFINITY	6.65
G8	8	INFINITY	7.83
G9	6	INFINITY	5.88
G10	8	INFINITY	7.65
G11	5	INFINITY	4.94

Appendix C LP Problem For Sishen Cost Driver Model

The LP problem of the Sishen diesel cost driver model is solved, for different values for the influence coefficient 'k', by the student version of LINDO Optimisation Software.

$$\text{Maximise } Z = A + B + C + D + E + F + G + H + I + J + K \tag{C.1}$$

Subject to

H1:	$B = kA + kE + kH + kI$
H2:	$C = kA + kB + kD + kE + kF + kG + kI$
H3:	$D = kA + kH + kI$
H4:	$E = kA + kD + kF + kI$
H5:	$F = kA + kH$
H6:	$G = kA + kD + kE + kI$
H7:	$H = kA$
H8:	$I = kA + kH$
G1:	$A \leq 17$
G2:	$B \leq 8$
G3:	$C \leq 9$
G4:	$D \leq 5$
G5:	$E \leq 8$
G6:	$F \leq 9$
G7:	$G \leq 3$
G8:	$H \leq 5$
G9:	$I \leq 15$

$A,B,C,..,I \geq 0.$

Table C.1 Optimal Solution to Equation C.1 for k = 0.1

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	17.00	0
B	2.28	0
C	2.97	0
D	2.06	0
E	2.28	0
F	1.87	0
G	2.32	0
H	1.70	0
I	1.87	0
Objective	34.35	

ROW	SLACK	DUAL PRICES
H1)	0.00	-1.10
H2)	0.00	-1.00
H3)	0.00	-1.34
H4)	0.00	-1.32
H5)	0.00	-1.23
H6)	0.00	-1.10
H7)	0.00	-1.53
H8)	0.00	-1.59
G1)	0.00	2.02
G2)	5.72	0.00
G3)	6.03	0.00
G4)	2.94	0.00
G5)	5.72	0.00
G6)	7.13	0.00
G7)	0.68	0.00
G8)	3.30	0.00
G9)	13.13	0.00

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	2.020622
B	1	INFINITY	15.033272
C	1	INFINITY	11.572731
D	1	INFINITY	16.699356
E	1	INFINITY	15.068024
F	1	INFINITY	18.369291
G	1	INFINITY	14.802008
H	1	INFINITY	20.206221
I	1	INFINITY	18.369291

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	2.28497	5.71503
H2	0	2.968234	6.031766
H3	0	2.057	2.943
H4	0	2.2797	5.7203
H5	0	1.87	7.13
H6	0	2.32067	0.67933
H7	0	1.7	3.3
H8	0	1.87	5.614297
G1	17	4.976412	17
G2	8	INFINITY	5.71503
G3	9	INFINITY	6.031766
G4	5	INFINITY	2.943
G5	8	INFINITY	5.7203
G6	9	INFINITY	7.13
G7	3	INFINITY	0.67933
G8	5	INFINITY	3.3
G9	15	INFINITY	13.13

Table C.2 Optimal Solution to Equation C.1 for k = 0.3

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	3.87	0
B	2.76	0
C	5.18	0
D	1.96	0
E	2.66	0
F	1.51	0
G	3.00	0
H	1.16	0
I	1.51	0
Objective	23.61	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	6.099118
B	1	INFINITY	8.556203
C	1	INFINITY	4.557505
D	1	INFINITY	12.029819
E	1	INFINITY	8.889546
F	1	INFINITY	15.638763
G	1	INFINITY	7.870541
H	1	INFINITY	20.330393
I	1	INFINITY	15.638763

ROW	SLACK	DUAL PRICES
H1)	0.00	-1.30
H2)	0.00	-1.00
H3)	0.00	0.76
H4)	0.00	0.28
H5)	0.00	-1.22
H6)	0.00	6.57
H7)	0.00	-1.35
H8)	0.00	0.59
G1)	13.13	0.00
G2)	5.24	0.00
G3)	3.82	0.00
G4)	3.04	0.00
G5)	5.34	0.00
G6)	7.49	0.00
G7)	0.00	7.87
G8)	3.84	0.00
G9)	13.49	0.00

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	2.759591	5.240409
H2	0	5.180821	3.819179
H3	0	2.635135	4.077703
H4	0	3.616763	7.276648
H5	0	1.581444	7.845545
H6	0	2.676481	3
H7	0	1.311762	4.335592
H8	0	2.027027	5.91716
G1	17	INFINITY	13.128682
G2	8	INFINITY	5.240409
G3	9	INFINITY	3.819179
G4	5	INFINITY	3.037242
G5	8	INFINITY	5.343889
G6	9	INFINITY	7.490186
G7	3	2.211529	3
G8	5	INFINITY	3.838605
G9	15	INFINITY	13.490186

Table C.3 Optimal Solution to Equation C.1 for k = 0.5

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	1.28	0.00
B	2.60	0.00
C	6.28	0.00
D	1.44	0.00
E	2.32	0.00
F	0.96	0.00
G	3.00	0.00
H	0.64	0.00
I	0.96	0.00
Objective	19.48	

ROW	SLACK	DUAL PRICES
H1)	0.00	-1.50
H2)	0.00	-1.00
H3)	0.00	1.12
H4)	0.00	0.25
H5)	0.00	-1.38
H6)	0.00	4.99
H7)	0.00	-1.41
H8)	0.00	0.93
G1)	15.72	0.00
G2)	5.40	0.00
G3)	2.72	0.00
G4)	3.56	0.00
G5)	5.68	0.00
G6)	8.04	0.00
G7)	0.00	6.49
G8)	4.36	0.00
G9)	14.04	0.00

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	15.21875
B	1	INFINITY	7.492308
C	1	INFINITY	3.101911
D	1	INFINITY	13.527778
E	1	INFINITY	8.396552
F	1	INFINITY	20.291668
G	1	INFINITY	6.493333
H	1	INFINITY	30.4375
I	1	INFINITY	20.291668

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	2.6	5.4
H2	0	6.28	2.72
H3	0	2.25	4
H4	0	3.782609	6
H5	0	1.043478	5.706294
H6	0	1.707113	3
H7	0	0.827586	2.823529
H8	0	1.5	2.666667
G1	17	INFINITY	15.72
G2	8	INFINITY	5.4
G3	9	INFINITY	2.72
G4	5	INFINITY	3.56
G5	8	INFINITY	5.68
G6	9	INFINITY	8.04
G7	3	1.299363	3
G8	5	INFINITY	4.36
G9	15	INFINITY	14.04

Table C.4 Optimal Solution to Equation C.1 for k = 1.0

OPTIMAL SOLUTION

VARIABLE	VALUE	REDUCED COST
A	0.19	0.00
B	2.44	0.00
C	8.81	0.00
D	0.75	0.00
E	1.69	0.00
F	0.38	0.00
G	3.00	0.00
H	0.19	0.00
I	0.38	0.00
Objective	17.81	

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
A	1	INFINITY	95
B	1	INFINITY	7.307693
C	1	INFINITY	2.021277
D	1	INFINITY	23.75
E	1	INFINITY	10.555555
F	1	INFINITY	47.5
G	1	INFINITY	5.9375
H	1	INFINITY	95
I	1	INFINITY	47.5

ROW	SLACK	DUAL PRICES
H1)	0.00	-2.00
H2)	0.00	-1.00
H3)	0.00	1.88
H4)	0.00	-0.06
H5)	0.00	-2.06
H6)	0.00	3.94
H7)	0.00	-1.44
H8)	0.00	1.75
G1)	16.81	0.00
G2)	5.56	0.00
G3)	0.19	0.00
G4)	4.25	0.00
G5)	6.31	0.00
G6)	8.63	0.00
G7)	0.00	5.94
G8)	4.81	0.00
G9)	14.63	0.00

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
H1	0	2.4375	0.1875
H2	0	8.8125	0.1875
H3	0	0.214286	1.5
H4	0	3.857143	3
H5	0	0.428571	0.176471
H6	0	0.096774	3
H7	0	0.333333	0.428571
H8	0	0.25	0.75
G1	17	INFINITY	16.8125
G2	8	INFINITY	5.5625
G3	9	INFINITY	0.1875
G4	5	INFINITY	4.25
G5	8	INFINITY	6.3125
G6	9	INFINITY	8.625
G7	3	0.06383	3
G8	5	INFINITY	4.8125
G9	15	INFINITY	14.625

Appendix D LP Problem For Consolidated Cost Driver Model

The LP problem of the consolidated diesel cost driver model is solved, for different values for the influence coefficient 'k', by the student version of LINDO Optimisation Software.

$$\text{Maximise } F = X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 \tag{D.1}$$

Subject to

H1: $k\,X1 - X2 + k\,X4 + k\,X7 + k\,X10 = 0$

H2: $k\,X1 - X3 + k\,X10 = 0$

H3: $k\,X1 - X4 + k\,X10 = 0$

H4: $k\,X1 + k\,X2 + k\,X3 + k\,X4 - X5 + k\,X7 + k\,X8 + k\,X9 + k\,X10 = 0$

H5: $k\,X1 + k\,X2 + k\,X4 + k\,X5 - X6 + k\,X7 + k\,X10 = 0$

H6: $k\,X1 + k\,X4 - X7 + k\,X10 = 0$

H7: $k\,X1 + k\,X2 + k\,X3 + k\,X4 + k\,X6 + k\,X7 - X8 + k\,X9 + k\,X10 = 0$

H8: $k\,X1 + k\,X2 + k\,X7 - X9 + k\,X10 = 0$

H9: $k\,X1 - X10 = 0$

G1: $X1 < 1$

G2: $X2 < 1$

G3: $X3 < 1$

G4: $X4 < 1$

G5: $X5 < 1$

G6: $X6 < 1$

G7: $X7 < 1$

G8: $X8 < 1$

G9: $X9 < 1$

G10: $X10 < 1$

$X1, X2, X3, \ldots, X10 \geq 0$

Appendix E Input Parameters for Q_Sim Trial Run and Confidence Intervals for Output Parameters.

As an initial approximation the mining operation is modeled with deterministic parameters. The following parameters were used:

arbitrary arrival times
load time + spot time at pit = service time at Queue 1 = 4 min
unload time + manoeuvre time at tip site = service time at Queue 2 = 6 min
unload time + manoeuvre time at plant = service time at Queue 3 = 5 min
time to travel from pit to tipping site = 5 min
time to travel from pit to plant = 7 min
time to travel to pit from tipping site = 6 min
time to travel to pit from plant = 10 min
number of servers = 1
proportion of trucks deployed between Queue 1 and Queue 2 = 50 %
proportion of trucks deployed between Queue 1 and Queue 3 = 50 %
tonnage hauled per truck = 200 tons

The point estimators and confidence intervals at the 95 % level for the output parameters follows:

Table E.1 Output Parameters for Fleet Size of 6 Trucks.

	Mean	Confidence Interval	
Tons Hauled:			
Queue 1- Queue 2 – Queue 1	59000	59002	58918
Queue 1- Queue 3 – Queue 1	50100	50173	49587
Time Elapsed (min)	2699	2744	2645
Mean Cycle Time (min):			
Queue 1- Queue 2 – Queue 1	27.34	27.81	26.81
Queue 1- Queue 3 – Queue 1	32.00	32.55	31.53
Truck Productivity (tons per min):			
Queue 1- Queue 2 – Queue 1	7.74	7.91	7.67
Queue 1- Queue 3 – Queue 1	6.54	6.64	6.48
Queue Utilisation:			
QUEUE 1	82.28%	82.29%	80.21%
Queue 2	65.67%	67.02%	64.63%
Queue 3	46.65%	47.24%	45.76%
Mean Waiting Time(min):			
QUEUE 1	5.29	5.59	4.82
Queue 2	1.16	1.37	1.23
Queue 3	0.55	0.65	0.55

Table E.2 Output Parameters for Fleet Size of 12 Trucks.

	Mean	Confidence Interval	
Tons Hauled:			
Queue 1- Queue 2 – Queue 1	117600	117728	117179
Queue 1- Queue 3 – Queue 1	110900	111308	110079
Time Elapsed (min)	4725	4688	4597
Mean Cycle Time (min):			
Queue 1- Queue 2 – Queue 1	47.57	47.43	46.42
Queue 1- Queue 3 – Queue 1	50.14	50.07	48.74
Truck Productivity (tons per min):			
Queue 1- Queue 2 – Queue 1	4.41	4.52	4.43
Queue 1- Queue 3 – Queue 1	4.24	4.39	4.26
Queue Utilisation:			
QUEUE 1	99.11%	99.20%	98.91%
Queue 2	75.10%	77.14%	75.48%
Queue 3	59.04%	60.77%	59.18%
Mean Waiting Time(min):			
QUEUE 1	22.54	22.35	21.12
Queue 2	4.10	4.44	3.96
Queue 3	1.64	1.73	1.59

Table E.3 Output Parameters for Fleet Size of 18 Trucks.

	Mean	Confidence Interval	
Tons Hauled:			
Queue 1- Queue 2 – Queue 1	174800	176059	175487
Queue 1- Queue 3 – Queue 1	166200	169896	168024
Time Elapsed (min)	6826.4	7026	6899
Mean Cycle Time (min):			
Queue 1- Queue 2 – Queue 1	69.42	71.02	69.72
Queue 1- Queue 3 – Queue 1	72.41	73.47	71.80
Truck Productivity (tons per min):			
Queue 1- Queue 2 – Queue 1	3.02	3.00	2.96
Queue 1- Queue 3 – Queue 1	2.9	2.94	2.87
Queue Utilisation:			
QUEUE 1	99.71%	99.91%	99.80%
Queue 2	77.26%	76.98%	75.55%
Queue 3	61.27%	61.83%	60.44%
Mean Waiting Time(min):			
QUEUE 1	44.08	45.38	43.70
Queue 2	4.50	5.29	4.66
Queue 3	1.86	2.15	1.86

Table E.4 Output Parameters for Fleet Size of 24 Trucks.

	Mean	Confidence Interval	
Tons Hauled:			
Queue 1- Queue 2 – Queue 1	232800	233804	232676
Queue 1- Queue 3 – Queue 1	224200	225336	223944
Time Elapsed (min)	9357	9373	9186
Mean Cycle Time (min):			
Queue 1- Queue 2 – Queue 1	94.89	95.03	93.22
Queue 1- Queue 3 – Queue 1	97.73	97.94	95.85
Truck Productivity (tons per min):			
Queue 1- Queue 2 – Queue 1	2.21	2.24	2.20
Queue 1- Queue 3 – Queue 1	2.14	2.18	2.13
Queue Utilisation:			
QUEUE 1	99.78%	99.92%	99.79%
Queue 2	75.22%	76.65%	75.25%
Queue 3	60.39%	61.60%	60.39%
Mean Waiting Time(min):			
QUEUE 1	69.47	69.57	67.53
Queue 2	4.82	5.25	4.65
Queue 3	1.95	2.04	1.85

Table E.5 Output Parameters for Fleet Size of 30 Trucks.

	Mean	Confidence Interval	
Tons Hauled:			
Queue 1- Queue 2 – Queue 1	290700	291648	290405
Queue 1- Queue 3 – Queue 1	279900	283248	280912
Time Elapsed (min)	11595	11597	11397
Mean Cycle Time (min):			
Queue 1- Queue 2 – Queue 1	117.54	117.52	115.70
Queue 1- Queue 3 – Queue 1	120.84	120.54	118.18
Truck Productivity (tons per min):			
Queue 1- Queue 2 – Queue 1	1.78	1.81	1.78
Queue 1- Queue 3 – Queue 1	1.72	1.76	1.72
Queue Utilisation:			
QUEUE 1	99.85%	99.94%	99.86%
Queue 2	75.83%	77.18%	76.00%
Queue 3	60.88%	62.49%	61.28%
Mean Waiting Time(min):			
QUEUE 1	92.41	91.96	89.56
Queue 2	5.08	5.75	5.10
Queue 3	1.76	2.22	2.00

Appendix F Determination of Input Parameters for Q_Sim Application 3.

The following parameters were calculated from Despatch production data.

SERVICES TIMES.

Queue 1 (PLANT: CR - ORE):

- Komatsu: μ = 306 seconds σ = 45 seconds
- Unitrig: μ = 280 seconds σ = 34 seconds

Queue 2 (PIT1: BRUCEA):

- Komatsu: μ = 205 seconds σ = 30 seconds
- Unitrig: μ = 175 seconds σ = 20 seconds

Queue 3 (PIT2: 46/14/31):

- Komatsu: μ = 184 seconds σ = 40 seconds

TRAVEL TIMES.

Queue 1 to Queue 2

- Komatsu: μ = 135 seconds σ = 15 seconds
- Unitrig: μ = 130 seconds σ = 11 seconds

Queue 1 to Queue 3

- Komatsu: μ = 135 seconds σ = 23 seconds

Queue 3 to Queue 1

- Komatsu: μ = 135 seconds σ = 27.5 seconds

Queue 2 to Queue 1

- Komatsu: μ = 135 seconds σ = 11.5 seconds
- Unitrig: μ = 145 seconds σ = 11 seconds

To generate normally distributed variates using the above mentioned parameters, the Box-Muller method is applied. The Box-Muller method provides an approximation to the normal distribution.

To generate a value x, from the normal distribution, N (μ, σ) let:

$$V_1 = (-2\ln r_1)^{1/2}\cos(2\pi r_2), \quad \text{(F.1)}$$

where r_1 and r_2 are random numbers.

Then

$$x = \mu + V_1\sigma \quad \text{(F.2)}$$

Appendix G Optimisation, Linear Programming and the Simplex Algorithm.

The theme of this study is the reduction of TCO of haul trucks within the context of mining. This implies that TCO of haul trucks can be minimised but the haul trucks must still meet production requirements. If the mining production requirements are not considered by implicit or explicit means, then the greatest reduction in TCO would be attained by simply not deploying the haul trucks. The theme of study leads to the theory of optimisation because optimisation is the science of finding the 'best' result to a 'given situation'.

G.1 Optimisation.

Optimisation is the science of finding the 'best' result to a 'given situation' while it is known that there is no better result. The 'best' result is defined by some performance criteria and, is determined by adjusting certain design settings. The 'best' result is obtained when the selection of design settings which yields the minimum / maximum performance criteria. The 'given situation' defines the context of the design settings.

For example, consider the design of a cardboard box that has the largest volume using a specified amount of cardboard. The performance criteria are the volume of the box and the design settings (although not explicitly mentioned) would be the length, breadth and height of the box. The 'given situation' would be the specified amount of cardboard to be used in the design of the box.

Optimisation problems are usually expressed mathematically. So letting height of box = x_1, length of box = x_2, breadth of box = x_3 the performance criteria would be expressed as

$$\textit{Volume} = x_1 \,.\, x_2 \,.\, x_3$$

The 'given situation' could be expressed as

$$\textit{Amount of cardboard used} = 2\,(x_1 \,.\, x_2 + x_2 \,.\, x_3 + x_1 \,.\, x_3) \leq 50 \text{ cm}^2$$

$$\textit{Dimensions of box cannot be negative:}\ \ x_1 \,,\, x_2 \,,\, x_3 > 0$$

Combining the above expressions we can express the optimisation problem as

Maximise $\quad Volume = x_1 . x_2 . x_3 \quad$ (G.1a)

Subject to

$$2\,(x_1 . x_2 + x_2 . x_3 + x_1 . x_3) \leq 50 \qquad \text{(G.1b)}$$

$$x_1 , x_2 , x_3 > 0 \qquad \text{(G.1c)}$$

Equation G.1a is known as the *objective function* and equations G.1b and G.1c are referred to as the *constraints*. Constraints may have equality or inequality conditions. The terms x_1 , x_2 and x_3 are called the *decision variables* or *design variables* or *variables.*

The set of all values for x_1, x_2 and x_3 that satisfies the constraints are known as the *feasible region* and a set of specific values for x_1, x_2 and x_3 which satisfy the constraints is called a *feasible solution*. In a maximisation (minimisation) problem, the feasible solution which yields the largest (smallest) value for the objective function is known as the *optimal solution.*

G.2 Linear Programming.

Linear Programming (LP) is a commonly applied branch of optimisation problems. In linear programming, the problem is modelled with the objective function and constraints as linear functions of the decision variables. Although more sophisticated non-linear programming algorithms exist, the study of LP is a suitable point of departure in the investigation in how to model the cost drivers of diesel consumption.

Firstly, a non-linear problem can be linearised and solved by linear programming. This produces satisfactory results with less computational complexity than non-linear programming algorithms. Secondly, linear programming methods are used as the basis for more complex methods [Vanderplaats 1999]. The complexities and nuances of general non-linear methods are easily understood when compared to LP [Vanderplaats 1999].

The LP problem can be defined in standard form as:

Minimise : $F(\mathbf{X}) = \sum_{i=1}^{n} c_i X_i$

Subject to :

$$\sum_{i=1}^{n} a_{ji} X_i = b_j \qquad j = 1, m \qquad \text{(G.2)}$$
$$X_i \geq 0 \qquad i = 1, n$$

There are four types of solutions to a LP problem [Winston 1994], namely:

1. A unique solution
2. A nonunique solution.
3. An unbounded solution.
4. No feasible solution

Examples will be used to illustrate the different types of solutions.

Consider the problem

Minimise: $f = -4X_1 - X_2 + 50$ (G.3a)

Subject to: $X_1 - X_2 \leq 2$ (G.3b)

$X_1 + 2X_2 \leq 8$ (G.3c)

$X_1 \geq 0, X_2 \geq 0$ (G.3d)

This problem is illustrated in Figure G.1. From Equation G.3b, the solution must lie above and to the left of the line $X_1 - X_2 = 2$ and from equation G.3c the solution must lie below the line $X_1 + 2X_2 = 8$. Equation G.3d states that the solution must lie in the first quadrant of the Cartesian surface. The dashed lines indicate contours of the objective function and the solid lines bound region ABCD where all the constraints are satisfied. This is known as the feasible region. It is evident that the lowest objective value occurs only at pt B where $X_1 = 4$ and $X_2 = 2$. This is an example of a unique solution.

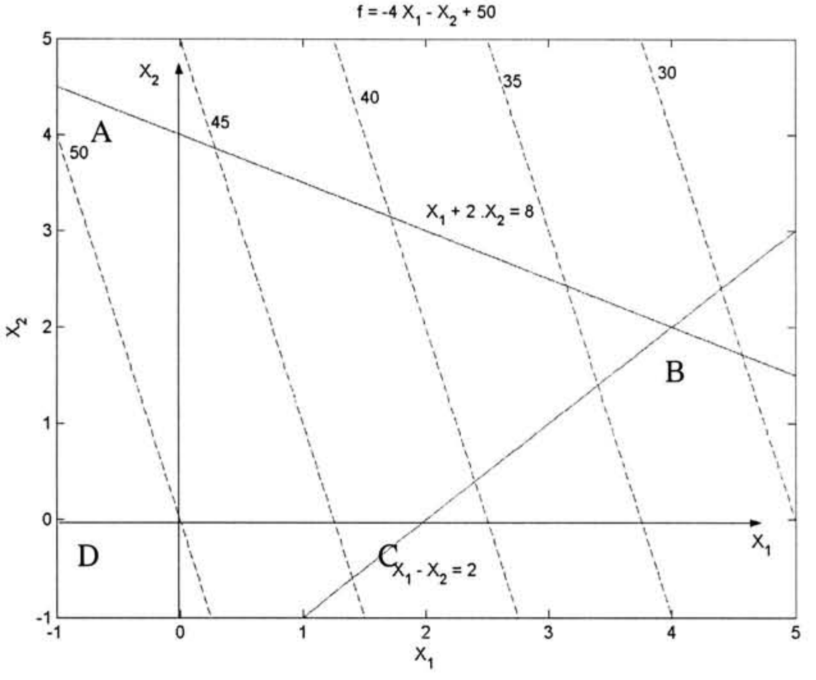

Figure G.1 LP Problem with a Unique Solution

Now consider the following problem

Minimise: $f = -X_1 + 2\,X_2 + 10$ (G.4a)

Subject to: $X_1 - 2X_2 \leq 3$ (G.4b)

$X_1 + 2\,X_2 \leq 8$ (G.4c)

$X_1 \geq 0,\ X_2 \geq 0$ (G.4d)

This is illustrated in Fig G.2. Note that the contours of the objective function are parallel to the first inequality. Consequently any point on the line segment BC is a non-unique optimal solution to the LP problem.

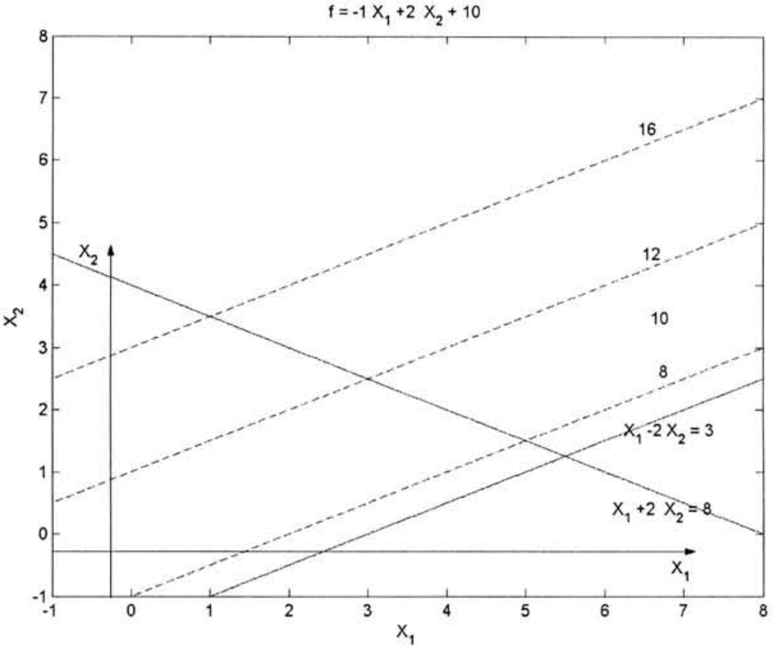

Figure G.2 Non-unique LP Problem

If we were to reconsider the first example (equations G.3a-d) and omit the second inequality (equation G.3c) then we would have an unbounded solution as is illustrated in Figure G.3. The optimal solution is unbounded because the feasible region is unbounded in the direction of decreasing objective function contours, which for this problem is to the top right of Figure G.3.

It may occur that no solution exists that satisfies all the constraints. This means that there is no feasible solution. An example would be the system

Minimise: $f = -4X_1 - X_2 + 50$ (G.5a)

Subject to: $X_1 - X_2 \leq 2$ (G.5b)

$X_1 + 2\,X_2 \leq -1$ (G.5c)

$X_1 \geq 0,\ X_2 \geq 0$ (G.5d)

Equation G.5d states that X_1 and X_2 are non-negative. Equation G.5c cannot be satisfied by non-negative values for X_1 and X_2 hence no feasible solution exists. This is illustrated in Figure G.4.

In the standard form, the constraints of a LP problem are equalities and the variables are non-negative. The above examples (equations G.3-5) were defined with inequality constraints. Inequality constraints can be transformed into equality constraints by the introduction of *slack variables.* Equation G.3 can be expressed in standard form by introducing the slack variable X_3 to equation 2.3b and X_4 to equation G.3c. The system will be

Minimise: $f = -4X_1 - X_2 + 50$ (G.6a)

Subject to: $X_1 - X_2 + X_3 = 2$ (G.6b)

$X_1 + 2X_2 + X_4 = 8$ (G.6c)

$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0, X_4 \geq 0$ (G.6d)

If a constraint was a greater than or equal to constraint, it is transformed into an equality constraint by the subtraction of a non-negative introduced variable called a *surplus variable*.

If an LP problem has an optimal solution, the optimal solution will occur at an extreme point i.e. a corner, of the feasible region [Winston 1999]. If the values of an optimal solution are substituted into a constraint and the left-hand side and the right-hand side, of the constraint, are equal; then the constraint is binding [Winston 1999]. This implies that the optimal point lies on the line defined by the constraint. And, since an optimal solution will occur at an extreme point it means that an optimal solution will occur at an intersection (i.e. an extreme point) of at least two binding constraints.

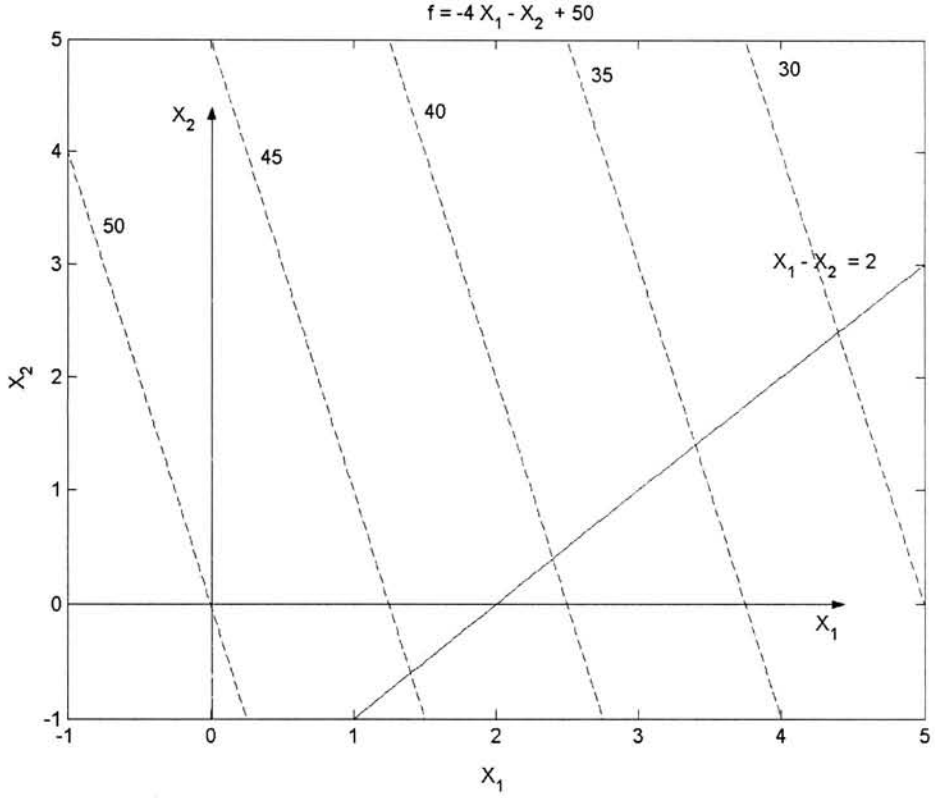

Figure G.3 Unbounded Solution of an LP Problem

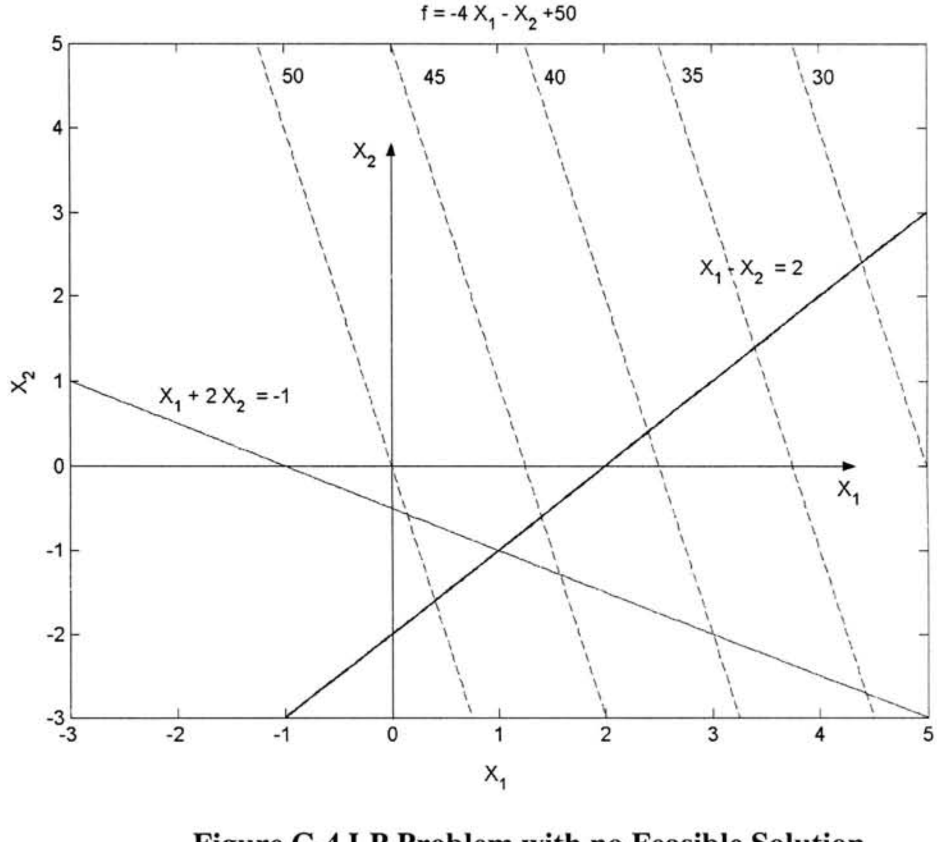

Figure G.4 LP Problem with no Feasible Solution

G.3 The Simplex Method.

In order to solve LP problems by the Simplex Method, which is described below, it is necessary to start with an initial feasible solution. Using the notation of equation G.2, there are "n" variables and "m" constraints in an LP problem. It is possible to find an initial feasible solution by setting "n-m" variables equal to zero and then solving for the remaining "m" variables in the system of "m" linear constraint equations. Let X_1, X_2,.., X_m denote the non-zero variables and X_{m+1}, X_{m+2},..., X_n the variables set to zero. By using forward and backward Gaussian elimination to solve for the "m" non-zero variables, the following system of equations is derived

$$\begin{array}{cccccccccc}
X_1 & X_2 & X_3 & \cdots & X_m & X_{m+1} \cdots & X_n & & B \\
\hline
1 & 0 & 0 & \cdots & 0 & A_{1,m+1} \cdots & a_{1,n} & = & B_1 \\
0 & 1 & 0 & \cdots & 0 & A_{2,m+1} \cdots & a_{2,n} & = & B_2 \\
 & 0 & 1 & & & \vdots & \vdots & \vdots & \vdots \\
\vdots & & . & . & & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & & 0 & 1 & a_{m,m+1} \cdots & a_{m,n} & = & b_m
\end{array} \qquad (G.7)$$

The values of the non-zero variables are equal to the respective values of the constants on the right hand side of the equations i.e. $X_1 = b_1$, $X_2 = b_2$, ..., $X_m = b_m$. The non-zero variables are known as *basic variables* and the set of basic variables is known as a *basis*. The variables set to zero value are known as *non-basic variables.*

If we had included the objective function in the original system and performed the required Gaussian elimination we would have derived equation G.7 with an additional row. This is shown as equation G.8.

$$\begin{array}{ccccccccccc}
X_1 & X_2 & X_3 & \cdots & X_m & X_{m+1} & \cdots & X_n & & b \\
\hline
1 & 0 & 0 & \cdots & 0 & a_{1,m+1} & \cdots & a_{1,n} & = & b_1 \\
0 & 1 & 0 & \cdots & 0 & a_{2,m+1} & \cdots & a_{2,n} & = & b_2 \\
 & 0 & 1 & & & \vdots & & \vdots & \vdots & \vdots \\
\vdots & & . & . & & \vdots & & \vdots & \vdots & \vdots \\
0 & 0 & & 0 & 1 & a_{m,m+1} & \cdots & a_{m,n} & = & b_m \\
0 & 0 & & \cdots & 0 & c_{m+1} & \cdots & c_n & = & f\text{-}f_0
\end{array} \qquad (G.8)$$

The numerical value f_0 denotes the objective value, and the last row expresses the objective as a function of the non-basic variables. Note that for this to be a feasible solution all b_1, b_2, ..., b_m must be non-negative. The solution is then referred to as a *basic feasible solution* and equation G.8 is known as the canonical form [Vanderplaats 1999].

The Simplex Method was developed in the late 1940's and is credited with popularising linear programming [Rao 1984]. The Simplex Method proceeds in 2 phases [Vasek 1983, Vanderplaats 1999]:

Phase I finds a basic feasible solution and Phase II determines the optimal solution by successive improvements of the basic feasible solution.

An optimisation problem must first be written in standard form. To illustrate the Simplex Method, in an understandable manner, we will assume that there exists a basic feasible solution and that it is in canonical form [Rao 1984, Vanderplaats1999].

We will use equation G.3 to illustrate how the simplex method is applied [Vanderplaats 1999]. Slack variables, x_3, x_4 are added to convert the problem into the standard form which gives the simplex tableau:

X_1	X_2	X_3	X_4		B	
1	*-1*	*1*	*0*	=	*2*	(G.9a)
1	*2*	*0*	*1*	=	*8*	(G.9b)
-4	*-1*	*0*	*0*	=	*f-50*	(G.9c)

This is already in canonical form with basic variables of $X_3 = 2$, $X_4 = 8$ and an objective function value of 50. This represents a basic feasible solution. Note that the cost coefficients of both non-basic variables are negative, so any increase in X_1 or X_2 will yield a lower objective function value. Therefore this solution is not the optimal solution.

By comparing the cost coefficients in equation G.9c, increasing X_1 will minimise the objective value more than changing X2 will. This is because the cost coefficient of X_1 is more negative than the cost coefficient of X_2. We are thus going to include X_1 in our basis at the expense of one of the existing basic variables. We need to determine which basic variable X_1 will replace. If we remove X_3 from the basis, $X_1 = 2$ (from equation G.9a) and then $X_4 = 6$ (from equation G.9b). If we remove X_4 from the basis, $X_1 = 8$ (from equation G.9b) and then $X_3 = -6$ (from equation G.9a). This is not feasible as the values of the variables must be non-negative. Therefore X_3 will be replaced by X_1 in the basis.

Next we re-write equation G.9 in canonical form with X_1 and X_4 as the basis. Since X_1 has replaced X_3 a basis, we must now perform row operations till the X_1 column resembles the original X_3 column.

X_1	X_2	X_3	X_4		B	
1	*-1*	*1*	*0*	=	*2*	(G.10a)
0	*3*	*-1*	*1*	=	*6*	(G.10b)
0	*-5*	*4*	*0*	=	*f-42*	(G.10c)

The row operations performed were eqn G.10b ← eqn G.9b – eqn G.9a and eqn G.10c ← eqn G.9b + 4 x eqn G.9a. This is known as pivoting.

The current solution is $X_1 = 2$, $X_2 = 0$, $X_3 = 0$, $X_4 = 6$ and an objective value of $f = 42$.

From equation G.10c, increasing the value of X_2 will minimise the objective value further. If X_2 replaces X_1, then from equation G.10a, $X_2 = -2$. This is not permitted because the variables must be non-negative. So X_2 will replace X_4 in the basis. We thus pivot about the 3 in equation G.10b to obtain the next simplex tableau

X_1	X_2	X_3	X_4		B	
1	*0*	*2/3*	*1/3*	=	*4*	(G.11a)
0	*1*	*- 1/3*	*1/3*	=	*2*	(G.11b)
0	*0*	*2 1/3*	*1 2/3*	=	*f-32*	(G.11c)

The current solution is $X_1 = 4$, $X_2 = 2$, $X_3 = 0$, $X_4 = 0$ and an objective value of $f = 32$.

From equation G.11c, any increase in X_3 or X_4 will not minimise the objective any further. This solution is therefore the optimal solution and corresponds with Figure G.1.

Phase II of the simplex method requires a basic feasible solution expressed in canonical form as input. Phase I of the simplex method finds a basic feasible solution. If the original LP problem only has inequality constraints (as our example using equation G.3 did), then the introduced slack variables serve as a basis and a basic feasible solution is easily obtainable. In Phase I, sufficient artificial variables are added to the constraints to obtain a basis and write the equation in canonical form.

An auxiliary objective function (*w*) is defined as the sum of the artificial variables and Phase II is applied to minimise this new auxiliary function. The optimal solution is then a basic feasible solution for the original LP problem. If an optimal solution is reached for *w*, and *w* is not zero then a feasible solution to the original LP problem does not exist [Chvatal 1983].

The algorithm for the simplex method is [Rao 1984, Winston 1994, Vanderplaats 1999]:

Phase II:

1. Assume a basic feasible solution in canonical form is available.
2. Find c_k, the smallest c_i in the objective function. If $c_k \geq 0$ then go to step 6. Else X_k is the entering basic variable.
3. For all $a_{jk} > 0$ for j =1,..,m, determine $a_{rk} = \min_{\text{for } j=1,\ldots,m} (b_j / a_{jk})$. If $a_{jk} \leq 0$ then the solution is unbounded. Go to step 6.
4. Divide row 'r' by a_{rk}. Then replace (row 'j') with (row 'j' – a_{jk} . row 'r') for j = 1, ..,m+1 $j \neq r$.
5. Go to step 2.
6. Solution complete.

   6a. Unique solution if $c_i > 0$ for all non-basic variables.

   6b. Nonunique Solution if $c_i = 0$ for a non-basic variable.

   6c. Solution is unbounded if a $c_i < 0$.

Phase I:

1. Add artificial variables to obtain LP problem in canonical form.
2. Create auxiliary objective function w = sum of artificial variables.
3. Use Phase II to minimise w.
4. If $w = 0$, then use optimal solution as a basic feasible solution for phase II of original LP problem. Delete all non-basic artificial variables and remove remaining artificial variables when they are removed from the basis. If $w \neq 0$, then no solution to the original problem exists.

It may occur that the basic feasible solution obtained from Phase I has artificial variables in the basis. These variables are then included in Phase II and may be deleted as soon as they are replaced in the basis [Chavtal 1983].

G.4 Sensitivity Analysis.

Once an optimal solution has been calculated, it may occur that the coefficients of the objective function, the coefficients of the constraints or the constant term in the constraints may change. This could result from the problem changing or due to uncertainty in the data used to create the LP problem. The LP problem would have to be solved again using the new parameters. This may often not be a worthwhile option if a few permutations of the coefficients must be solved [Winston 1994]. It would be more efficient to determine the changes, in the coefficients, that will not change the optimal solution. This process is known as *sensitivity analysis.*

To illustrate the scope of sensitivity analysis, consider the 2 variable LP problem defined by equation G.3a-d and the solution thereof depicted in Figure G.5:

Minimise: $f = -4X_1 - X_2 + 50$ (G.3a)

Subject to: $X_1 - X_2 \leq 2$ (G.3b)

$X_1 + 2X_2 \leq 8$ (G.3c)

$X_1 \geq 0, X_2 \geq 0$ (G.3d)

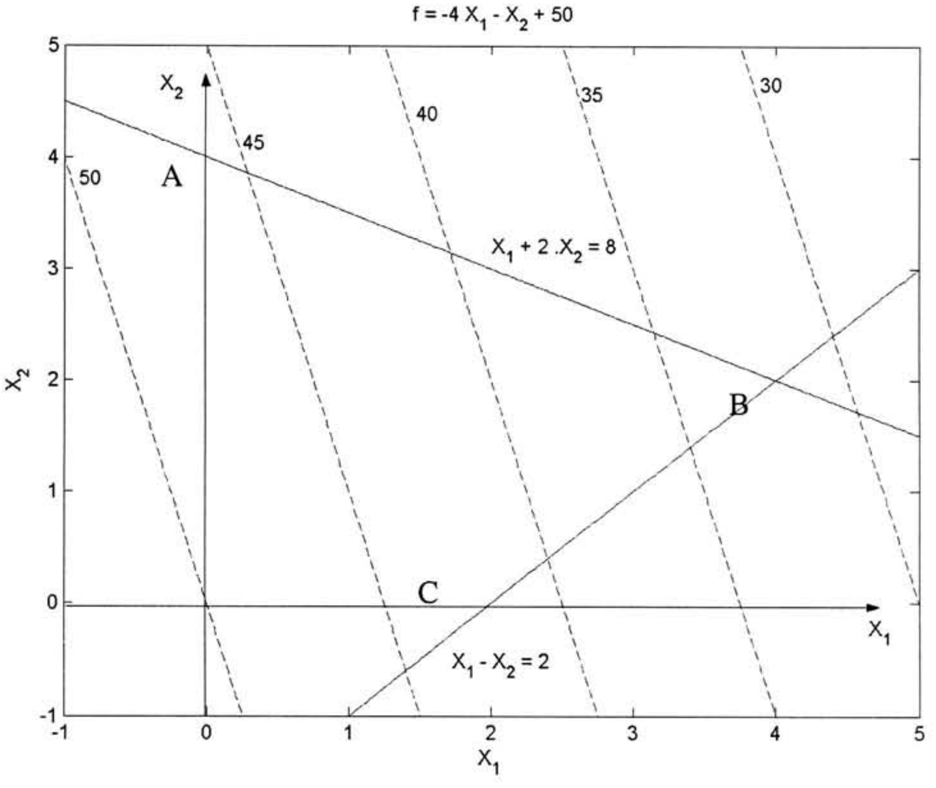

Figure G.5 Optimal Solution to Equation G.3

The first parameter that could be changed is an objective function coefficient. The slope of the objective function contours is determined by the ratio $-c_1/c_2$ (objective coefficient of X_1 divided by objective coefficient of X_2). As the value of c_2 becomes more negative, the slope of the objective function flattens and if $-c_1/c_2 < -0.5$ then the optimal solution would shift from point B ($X_1 = 4$, $X_2 = 2$) to point A ($X_1 = 0$, $X_2 = 4$). This would occur if $c_2 < -8$. Similarly if the value of c_2 is increased, the slope of the objective function will get steeper then become positive and flatten again until the optimal solution would shift to point C ($X_1 = 2$, $X_2 = 0$). This would occur if $c_2 > 4$.

Thus we have determined that for values $-8 > c_2 > 4$, the values of the decision variables calculated to be the optimal solution will be unchanged, but the objective function value may change. A similar analysis may be conducted for values of c_1. Therefore there exists a range for which an objective function coefficient may vary without a change in the optimal solution.

Another parameter of interest is the *reduced cost* of a variable. This is the amount by which the objective coefficient of a non-basic variable must be 'improved' so that the LP problem will be optimal with that variable in the basis [Winston 1999]. 'Improvement' in a minimisation problem would be a decrease in the objective coefficient and, in a maximisation problem would be an increase in the objective coefficient. If the objective coefficient of a non-basic variable is improved by its reduced cost, two alternate optimal solutions will occur: one with that variable still as a non-basic variable and another solution with that variable in the basis. If the objective coefficient is 'improved' by more than the reduced cost a new optimal solution exists with that variable in the basis.

The second parameter that needs to be investigated is that of changing a coefficient in a constraint equation. Recall that the constraints define the feasible region wherein an optimal solution may be located. By changing the value of a coefficient in a constraint the slope of that line is changed and this can alter the shape of the feasible region and thus the location of the optimal point.

The constant value on the right-hand side of a constraint may also change. The effect of this change would be to shift the line of the equation up or down. It was concluded that an optimal solution occurs at the intersection of two or more binding constraints. If we change the constant term for one of the binding constraints then the intersection of the binding constraints will remain the optimal solution provided the intersection remains in the feasible region. This means that the basis will be unchanged but the values of the variables and thus the value of the objective coefficient may change. Hence, there exists a range for each constant such that the current basis will remain unchanged.

To determine the change in objective value to a change in a constant term of a constraint, a shadow or dual price needs to be calculated for each constraint. A shadow price is the 'improvement' in the objective value by increasing the constant term of a constraint by 1. Thus for a change in constant term that falls within the allowable range, the new objective function value can be determined by:

$$z^* = z + s_i \cdot \Delta b_i \qquad \text{(G.12)}$$

where z^* = new optimal function value

z = old optimal function value

s_i = shadow price for constraint "i"

Δb_i = increase in constant term of constraint "i"

If a constraint is non-binding it implies a nonzero surplus/slack variable. The shadow price for a non-binding constraint is zero [Winston1999]. This can be intuitively explained. The existence of a slack/surplus variable implies that the 'resource' modelled by that constraint is not fully utilised. Thus no improvement in objective function would be expected by increasing an 'idle' resource.

Until now, sensitivity analysis dealt with changing only one parameter at a time. What would occur if more than one parameter is changed?

For changes in objective function coefficients, two cases must be considered:

Case 1: All the objective function coefficients that are changed, pertain to variables that have a nonzero reduced cost.

Case2: At least one of the objective function coefficients that is changed, pertains to a variable with a zero reduced cost.

From the definition of reduced cost, a basic variable has a zero reduced cost.

Therefore Case 1 refers to changing the objective coefficient of non-basic variables. If the changes to these objective function coefficients are within the allowable range[1] then the current basis remains optimal.

[1] Allowable range for a parameter is the range of values, for which the basis is unchanged. Allowable range assumes only one parameter is changed.

Case 2, implies that the objective function coefficient of a basic variable is changed. To determine if the current basis remains optimal we apply the 100% rule for objective function coefficients [Winston1994].

We calculate the ratio r_j as:

$$\begin{aligned} &\text{If } \Delta c_j \geq 0, \quad r_j = \Delta c_j / I_j \\ &\text{If } \Delta c_j \leq 0, \quad r_j = -\Delta c_j / D_j \end{aligned} \tag{G.13}$$

where Δc_j = the change in objective function coefficient c_j

I_j = the allowable increase in c_j (current basis stays optimal)

D_j = the allowable decrease in c_j (current basis stays optimal)

Now if $\sum r_j \leq 1$ then the current basis remains optimal but the objective function value may change. If $\sum r_j > 1$ then we cannot determine if the current basis remains optimal.

For changes in the constant term on the right-hand side of a constraint, 2 cases must be considered.

Case 1: The constant terms of only non-binding constraints are changed.

Case 2: The constant term of at least one binding constraint is changed.

For Case 1, if the changes are within the allowable range, then the current basis remains optimal and the objective function value is unchanged.

For Case 2, we apply the 100% Rule for Changing Constants in the Constraints [Winston 1994].

We calculate the ratio r_j as:

$$\begin{aligned} &\text{If } \Delta b_j \geq 0, \quad r_j = \Delta b_j / I_j \\ &\text{If } \Delta b_j \leq 0, \quad r_j = -\Delta b_j / D_j \end{aligned} \tag{G.14}$$

where Δb_j = the change in constant term of constraint j

I_j = the allowable increase in b_j (current basis stays optimal)

D_j = the allowable decrease in b_j (current basis stays optimal)

Now if $\sum r_j \leq 1$ then the current basis remains optimal but the objective function value may change. If $\sum r_j > 1$ then we cannot determine if the current basis remains optimal.

Thus, we have discussed linear programming as a means of optimisation with the aim of reducing TCO of haul trucks. The simplex method is a commonly applied method to solve an LP problem and the sensitivity analysis of an LP solution yields greater insights to the effect of the variables on the optimal solution. In chapter 3 it is described how linear programming was used to mathematically model the diesel cost driver model which the simplex method was applied to solve the LP problem. A sensitivity analysis was performed to investigate the nature of the optimal solution to the diesel cost driver model.

Appendix H Overview of Statistical Analysis.

Statistics may be used to summarise and describe data [Cooper et al. 1983], infer the characteristics of a large population [Hogg et al. 1992] and determine the relationship between multiple sets of data [Mason et al. 1996].

A *Population* is a collection of all possible objects or measurements of interest. In most applications the number of values is prohibitively large or not all the values may be measured. To perform statistical analysis we then select and utilise a portion of the entire population referred to as a *Sample* [Hogg et al. 1992, Mason et al. 1996]. Characteristics of the population are then inferred from the behaviour of the sample.

As an example, the values of a vehicle's forward velocity over a time period t, may be considered a population. If we measured the value of forward velocity at ten intervals during t, then the ten measured values would be a sample of the infinite population. The average velocity of the sample is then inferred to represent the average velocity of the population.

H.1 Summary Statistics.

Summary statistics, the means by which data may be described and summarised, are characterised by measures of central tendency and measures of dispersion [Cooper et al. 1983].

Measures of central tendency describe a value in the 'middle' of the data values. The following are measures of central tendency [Hogg et al. 1992].

- *Sample Mean* ($\underline{x}$) for a given set $x_1, x_2, \ldots, x_n$ is the arithmetic average of the "n" observations i.e.:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{H.1}$$

- *Median* is the middle observation when values are arranged in increasing order of magnitude, if there is an even number of observations , the median is the average of the middle pair.
- *Mode* refers to the most frequent value in a data set.

Measures of dispersion describe how different the values are from each other [Cooper et al. 1983]. Measures of dispersion are also known as measures of variation [Mason et al. 1996] and include:

- The *Range* is the difference between the largest and smallest observation.

- *Sample Variance (s^2).* Deviation of an observation from the sample mean can be expressed as (x_i-$\underline{x}$) the square of this deviation provides some information regarding the variability. The sample variance is a modified average of these *n* squared distances:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 \tag{H.2}$$

- *Sample Standard Deviation(s).* This is the square root of the Sample Variance so it follows that :

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2} \tag{H.3}$$

  This parameter is comparable to the concept of the Root Mean Square. The sample standard deviation indicates that observations are on average approximately "s" units from the mean.

- *Coefficient of Variation (CV).* Sample Standard deviation is a dimension-dependant parameter, the Coefficient of Variation expresses standard deviation as a percentage of the mean:

$$CV = \frac{100s}{\bar{x}} \tag{H.4}$$

This measure says that observations lie approximately within CV percent of the mean.

H.2 Regression Analysis.

It is often desired to evaluate the relation between a response variable to a set of explanatory variables. In reality the relations will not be deterministic (exact) and the relations may be too complicated to be explained by a small number of explanatory variables. The model is no longer exact but rather statistical in nature. The response variable is treated as a random variable and its value varies around a mean value which depends on the explanatory variables. Regression analysis is aimed at developing these approximating functions which describe the main features of the relationships between variables.

To determine an appropriate representation of the data set the least squares criterion is commonly used . Let F denote the approximate function, and x_i denote the ith setting of the explanatory variable x and y_i be the observation at x_i, then the least squares criterion implies we must minimise the sum of all the $(y_i - \mathrm{F}(x_i))^2$. Once an appropriate function is found satisfying the least squares criteria we denote the function values calculated at x_i i.e. $\mathrm{F}(x_i)$ as the fitted values $\hat{y}_i$ and the difference between the fitted value and the observed value is defined as the residual e_i .

The following parameters describe the accuracy of the regression analysis [Mason et al. 1996]:

- *Total Sum of Squares (SSTO).* Ignoring the information that is contained in the explanatory variable we can express the variability in the responses $y_1, y_2, y_3, \ldots, y_n$, as:

$$SSTO = \sum_{i=1}^{n} \left(y_i - \overline{y} \right)^2 \tag{H.5}$$

This variation could be due to orders of magnitude difference in x- values.

- *Regression Sum of Squares (SSR).* The regression model expresses a relation between the explanatory variable and the response variable, which partially "explains" the observation y_i through the fitted value $\hat{y}_i$, the variation of these fitted values about the mean measures the variability that can be explained by the regression model. Thus

$$SSR = \sum_{i=1}^{n} \left(\hat{y}_i - \overline{y} \right)^2 \qquad \text{(H.6)}$$

- *Error Sum of Squares (SSE).* Not all the variation is explained by the regression model and the remainder can be expressed in terms of the residuals.

$$SSE = \sum_{i=1}^{n} e_i^2 \qquad \text{(H.7)}$$

It should be evident that SSTO = SSR + SSE. Consider 2 extremes firstly SSR = 0 i.e. SSTO = SSE; and secondly, SSE = 0 i.e. SSTO = SSR. The first instance implies that the regression model and the explanatory variable explain none of the variability in the observations whereas the second case indicates the opposite i.e. the regression model explains all the variability in Y.

- *Coefficient of Determination* (R^2). This provides a summary statistic that measures how well the regression fits the data. It is given by

$$R^2 = \left(\frac{SSR}{SSTO} \right) = 1 - \frac{SSE}{SSTO} \qquad \text{(H.8)}$$

$R^2=0$ implies that that the model explains none of the regression and $R^2 = 1$ implies a perfect relation by our regression analysis.

H.3 Hypothesis Testing.

A statement about the value of a population parameter is called a hypothesis. Hypothesis testing is a procedure based on sample evidence and probability theory to determine whether a hypothesis is a reasonable statement and should not be rejected or the contrary.

The procedure for testing a hypothesis is portrayed as a flow chart below [Mason et al. 1996]:

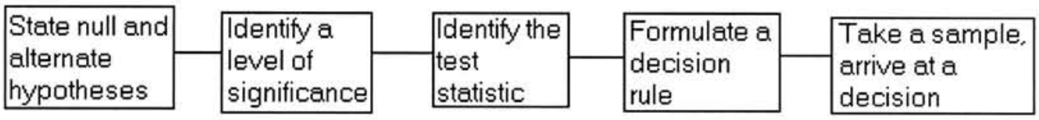

Figure H.1 Procedure For Hypothesis Testing.

The null hypothesis, denoted by H_0, is a tentative assumption made about the value of a population parameter, usually that the parameter has a specific value. The null hypothesis is set up for the purpose of either rejecting it or not. The alternate hypothesis, denoted by H_1, is a statement that will be assumed to be true if our sample data indicates that the null hypothesis is false.

The outcome of a hypothesis test is that either the sample data indicates that the null hypothesis is rejected (false) *or* that the sample data does not provide enough evidence to reject the null hypothesis (possibly not false). The latter does not imply the null hypothesis is true for the entire population. An example of a null hypothesis and alternate hypothesis is:

a. Null hypothesis: Population mean (μ) is 200 implies
   $H_0 : \mu = 200$.

b. Alternate hypothesis: Population mean (μ) is not 200 implies
   $H_1 : \mu \neq 200$.

The Level of Significance (α) is the probability of rejecting the null hypothesis when it is actually true; for this reason it can also be referred to as the level of risk as it is the risk you are taking in rejecting a true null hypothesis. It is the prerogative of the researcher to determine the level of significance before continuing with a hypothesis test. Typical values are 1%, 5%, or 10%.

Rejecting a true null hypothesis is known as a Type I error, hence α is the probability of a type I error and not rejecting a false null hypothesis is known as a Type II error denoted by β. Level of significance can be any value between 0 and 1 but is usually referred to as a percentage (e.g. 0.05 => 5 percent level).

The *Test Statistic* is a value determined from sample data used to determine whether the null hypothesis is rejected or not. The specific parameter is mainly dependant on the size of sample and/or type of probability distribution that is assumed. The following are examples of test statistics:

- z - Statistic: normal probability distribution and "large" sample.
- t - Statistic: normal probability distribution and a sample of 30 or less members.
- F - Statistic: F distribution with 2 or more sample groups.

The *Decision rule* is a statement of the conditions under which the null hypothesis is rejected or not. In this regard a critical value dependant on the type of probability distribution and level of risk is determined. This critical value divides our sampling distribution into a region(s) of rejection and a region of non-rejection and although formulae exist to calculate its value, tables are available which reduce computational effort. The decision rule for a z-statistic is illustrated in Figure 2.29. In this instance the null and alternate hypothesis are:

H_0: $\mu = 200$ (population mean is 200)
H_1: $\mu \neq 200$ (population mean is not 200)
with $\alpha = 0.1$

Note that in the figure, the curve is symmetrical. The alternate hypothesis can also be expressed as H_1: $\mu > 200$ or $\mu < 200$ hence the 2 separate regions of rejection. Had it been only

H_1: $\mu > 200$ (H_1: $\mu < 200$) there is only a region of rejection on the positive (negative) side. The original hypothesis is aptly referred to as a 'two-tailed' test and the latter as a 'one-tailed test'. For a 'two-tailed' test the level of risk is divided equally between both tails.

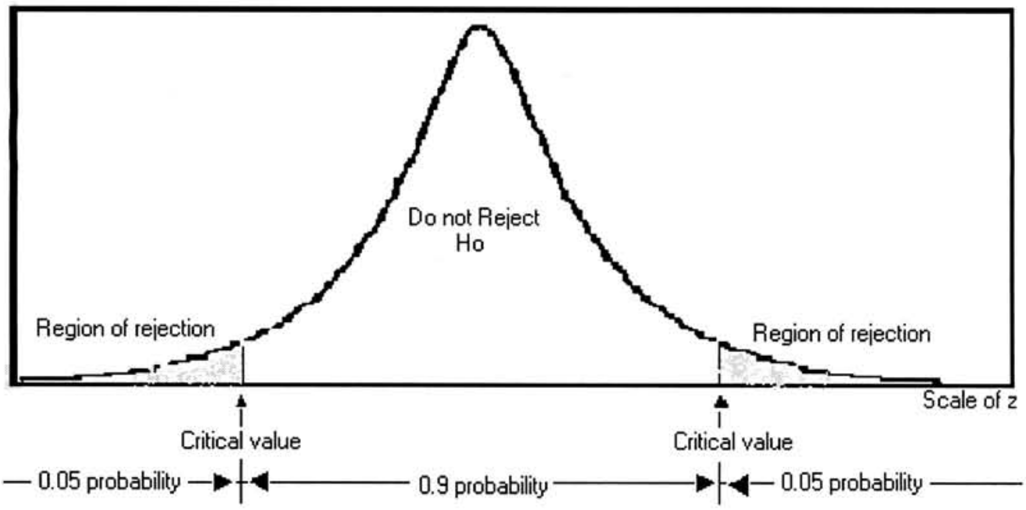

Figure H.2 Sampling Distribution for the Statistic z.

Once a sample has been taken the test statistic can be calculated thereafter compared to the decision rule. If the absolute value of the test statistic is greater than the critical value i.e. test statistic falls in the region of rejection it implies the null hypothesis is rejected.

The following types of hypothesis testing are relevant to this study:

- Hypothesis Testing Involving Paired Observations.
- Hypothesis Test of Correlation
- Hypothesis Testing of Two Means.

H.4 Hypothesis Testing Involving Paired Observations.

There are situations when samples are not independent as in the case of a "before and after" scenario and we say that the before and after samples for each individual are "paired". Then for a test of a hypothesis we only have one sample namely the difference between the before and after values for each member of a sample.

The procedure to test a hypothesis is still applicable and the proper hypotheses and test statistic are given below:

The null hypothesis would be that no change has occurred i.e. the mean of the differences (μ_d) would be zero. The alternate hypothesis could be a positive change has occurred ($\mu_d > 0$) and/or a negative change has occurred ($\mu_d < 0$). It is usually quite pointless to only know that a change has taken place and for that reason a one-tailed test based on the discretion of the researcher is preferred. The possibility of a two-tailed test is however not excluded.

If a small sample (less than 30 observations) is used an applicable test statistic is the t – statistic for paired differences [Mason et al. 1996]:

$$t = \bar{d} \, / \, (S_d \, / \, n^{1/2}) \tag{H.9}$$

where:

- S_d is the standard deviation of the differences between the paired observations.
- n is the number of paired observations.
- $\bar{d}$ is the mean difference between paired observations.

H.5 Hypothesis Test of Correlation.

Consider two sets of sampled data that are linearly related. It implies a correlation value with an absolute value approximately equal to unity. The linear relation between the two samples may occur as a result of a sampling anomaly or it may reflect a global linear relation between the two populations. The Hypothesis Test of Correlation determines if the linear relation between two samples reflects the behaviour of the two populations. That is, does the linear relation between two samples reflect a linear relation between the two populations?

The null hypothesis would be that there is no linear relation between the 2 populations. This implies that the correlation between the populations is zero. Therefore:

H_0: $\rho = 0$

The alternate hypothesis would be that there is a linear relation between the 2 populations.

H_1: $\rho \neq 0$

This is a two-tailed test and if the sample is small the test statistic is the t-statistic for correlation [Mason et al. 1996]:

$$t = R \{ (n-2) \ / \ (1-R^2) \}^{1/2} \qquad \text{(H.10)}$$

where:

R is the Correlation between samples

n is the number of observations in the sample.

H.6 Hypothesis Testing of Two Means.

This test compares the arithmetic means of two samples and determines if there is a statistical difference between the two means. This test is based on the following assumptions [Mason et al. 1996]:

1. The populations must be normally distributed (or approximately normally distributed).
2. The populations must be independent.
3. The population variances must be equal.

If we denote the mean for the two populations as μ_1 and μ_2 respectively, then the null hypothesis, H_0, and alternate hypothesis, H_1, are:

H_0: $\mu_1 = \mu_2$ (There is no difference between population means)

H_1: $\mu_1 \neq \mu_2$ (There is a difference between population means)

Not rejecting the null hypothesis implies that any difference between the means of the 2 samples is attributable to sampling variability and is thus not statistically significant.

The test statistic for a small sample is t, and defined by [Mason et al. 1996]:

$$t = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \tag{H.11}$$

where

$\overline{X_1}$ = sample mean of sample 1.

$\overline{X_2}$ = sample mean of sample 2.

n_1 = number of observations in sample 1.

n_2 = number of observations in sample 2.

S_p^2 = pooled variance for sample 1 and sample 2.

The pooled variance is defined by

$$s_P^2 = [\ (n_1-1)(s_1^2) + (n_2-1)(s_2^2)\ ] \ / \ [n_1 + n_2 - 2]. \tag{H.12}$$

Since the two samples are small, the variance of each sample is pooled to obtain a more realistic measure of sample variance [Mason et al. 1996].

An application of the hypothesis test of two means would be the comparison of mean fuel consumption of 2 vehicles. The frequency distribution of the sample values is illustrated in Figure H.3. Note that equal variance assumption implies similar bellshaped curves for the frequency distribution for fuel consumption of vehicle 1 and 2 respectively. Although the mean values of the 2 samples are different, the large area of overlap for the two samples implies that the samples are samples from a similar population and that the sample means are not statistically different. That means that the average fuel consumption for the vehicles does not differ.

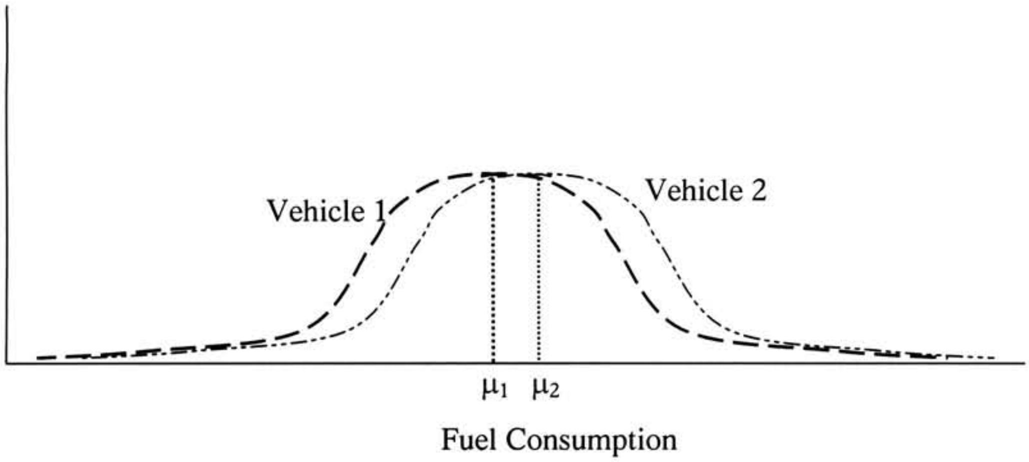

Figure H.3 Normal Frequency Distribution of Two Samples with Equal Variances.