Stem form, height and volume models for teak in Tanzania

by

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Declaration

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and has not previously been submitted in its entirety or in part at any university for a degree.

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Signature:

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Date:

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Opsomming

Die doel van hierdie studie was om ‘n stel modelle te ontwikkel wat gebruik kan word om volume vir *Tectona grandis* bome en opstande in Tanzanie te bepaal. Modelle wat boom- en dominante opstand hoogte beskryf is as sekondere doelwit ontwikkel.

Totale volume en volume-verhouding modelle is gepas wat onderskeidelik die totale en bemarkbare volume bepaal. Spitsingsmodelle is ontwikkel om die bepaling van volume vir verskillende produkklasse en dimensies toe te laat. Al die data is versamel deur nie-destruktiewe bemonstering met behulp van ‘n Barr en Stroud optiese dendrometer. Hierdie tegniek was beide akkuraat en koste-effektief. Bemonstering stratifikasie is gebaseer op ouderdom en groeiplekkwaliteit en daar is gepoog om so ‘n groot reeks as moontlik te dek om te verseker dat groeiplekke en ouderdomme in Tanzaniese plantasies voldoende verteenwoordig word. ‘n Totaal van 2617 individuele observasies is vanaf 222 bome uit drie plantasies geneem.

Verskeie modelle is geselekteer uit die literatuur om kiaat volume en spitsing te beskryf. Resultate dui aan dat die Schumacher en Hall (1933) volumevergelyking totale volume oor- en onderbas tot n 7.5 cm minimum deursnit die beste beskryf. Bemarkbare volume tot minimum deursnit en hoogte limiete is onderskeidelik die beste beskryf deur die Burkhart (1977) volume verhouding model en sy aangepaste vorm deur Cao en Burkhart (1980). Die veranderlike vorm spitsingsmodel van Perez, Burkhart en Stiff (1990) het die beste resultate gelewer volgens ‘n verskeidenheid kriteria en word aanbeveel om deursnitte tot verskillende hoogtes te bereken, asook, indien ‘n enkele model gebruik word, die bemarkbare volume.

Die dominante hoogte van kiaat opstande is voldoende beskryf deur die Schumacher (1939) groeimodel met die eksponent *k* geskat vanaf die data. ‘n Reeks anamorfiese groeiplek-indekskurwes is op grond hiervan ontwikkel. ‘n Geskikte hoogte-dbh verhouding is verkry deur ‘n eenvoudige liniêre model te gebruik en die voorpellings is verbeter deur ouderdom en groeiplek-indeks by die model te voeg.

Sleutelwoorde: *Tectona grandis,* kiaat, totale volume, bemarkbare volume, spitsing, hoogte, groeiplek-indeks, Barr en Stroud dendrometer

Synopsis

The aim of this study was to develop a set of models that will allow the determination of volume for *Tectona grandis* trees and stands grown in plantation form in Tanzania. As a secondary objective, models describing tree and dominant stand height were developed.

Total volume and volume ratio models were fitted that respectively predict total tree volume and merchantable volume. In order to allow the calculation of volume for different product classes and dimensions, taper models were fitted. All the data were collected by non-destructive sampling methods using a Barr and Stroud optical dendrometer. This proved to be an accurate and inexpensive method of collecting data for developing volume and taper models. Sampling stratification was based on age and site quality and as wide a range as possible was covered to ensure adequate representation of all growing sites and ages present in Tanzanian teak plantations. A total of 2617 individual observations were made from 222 trees at three teak plantations.

Several models were selected from the literature to describe teak volume and shape. Results indicated that the Schumacher and Hall (1933) volume equation best describes total volume over and underbark to a fixed upper limit of 7.5 cm. Merchantable volume to upper stem diameter and height limits were best described by respectively the Burkhart (1977) volume ratio model and the Cao and Burkhart (1980) modification thereof. Many of the fitted taper models were unable to adequately describe stem shape over the whole stem, mainly due to the large range in tree sizes and ages used in model fitting. The variable form taper model by Perez, Burkhart and Stiff (1990) provided the best results according to various criteria and is recommended for predicting teak underbark diameters to various heights and, if only a single model is required, the merchantable volume.

Top height growth of teak stands was adequately described by the generalized Schumacher (1939) model with the value of the exponent *k* estimated from the sample data. From this a series of anamorphic site index curves were developed. Suitable height-dbh curves were obtained by a simple linear model and predictions improved by including stand age and site index as predictor variables.

Keywords: *Tectona grandis,* teak, total volume, merchantable volume, taper, height, site index, Barr and Stroud dendrometer

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1. Introduction

Teak (*Tectona grandis*) is one of the world's premier hardwood timbers. It is indigenous only in India, Myanmar, the Lao People's Democratic Republic and Thailand (Pandey and Brown, 2000).

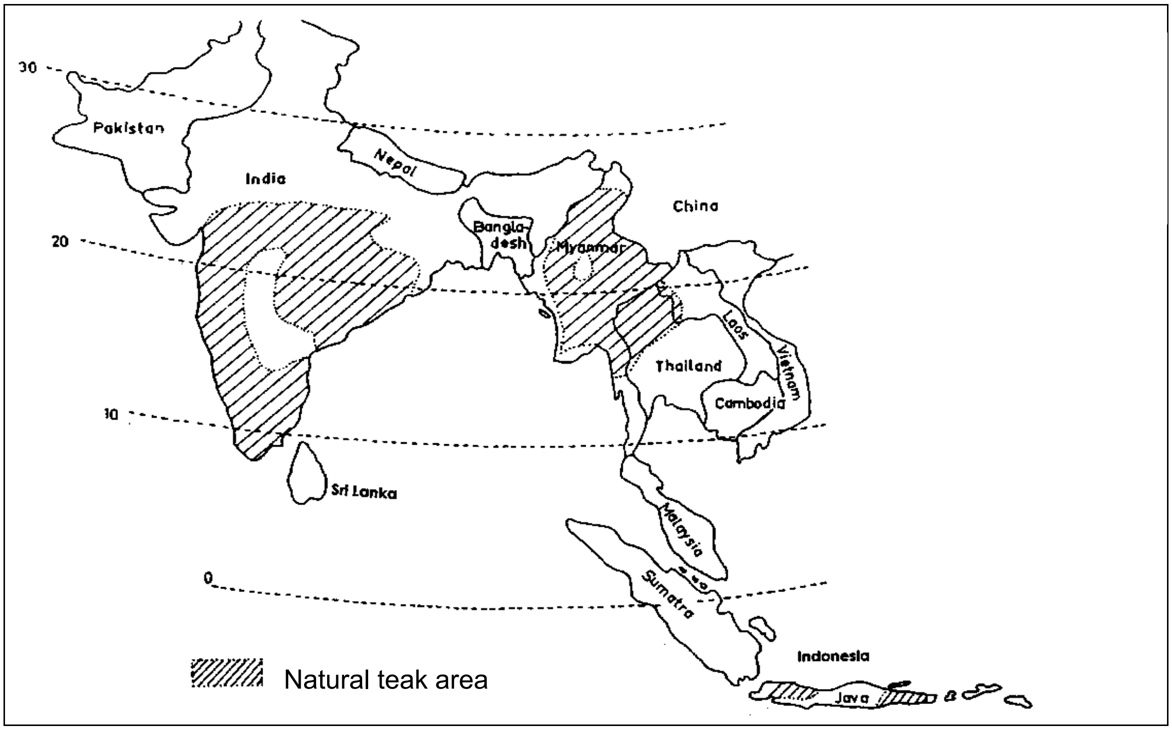


Figure 1. The natural distribution of teak (Pandey and Brown, 2000).

Although relatively unimportant in terms of the total volume of world timber production, teak is the tropical hardwood that is in greatest demand for a specific market of "luxury" applications, including furniture, shipbuilding and decorative building components. Physical qualities such as attractiveness in colour and grain, durability, lightness with strength, ease of seasoning without splitting and cracking, resistance to termite and fungus attack and weathering have established teak as a prime timber worldwide and will ensure that it retains its importance into the future (Kaosa-ard, 1995; Keogh, 1996; Pandey and Brown, 2000). Teak was first planted some 400-600 years ago in Java. Since then it has been widely established in plantations as an exotic species for producing high quality poles and timber, with varied degrees of success (Ball, Pandey and Hirai, 1999). Amongst the quality tropical hardwood species established in

plantation form worldwide, the teak plantation area ranks amongst the top five (Krishnapillay, 2000).
Teak has been established in Tanzania at Longuza and Mtibwa for more than half a century. Increased international demand for this timber has led to a new and more intensive private enterprise plantation establishment programme in the country.

As the objective of timber resource management is to provide the optimum combination of quantity, quality and product sizes of timber that will maximise financial returns, accurate and flexible models are necessary to provide the information required. The variable used most in decision-making with regard to timber management is some measure of volume. This requires that volume be calculated to various specified limits. Total tree volume is predicted by total volume equations that use dbh (diameter at breast height) and height as predictor variables. To estimate the volume to some specified upper stem diameter or height, in other words the merchantable volume, volume ratio equations are used. An alternative, very useful way to estimate the volume for a variety of products is by using a taper equation (Sharma and Burkhart, 2003). Because taper models mathematically describe the shape of the tree, they can provide estimates of the diameter at any height along the stem. It is possible to integrate some taper models, which means that the volume can be directly calculated between the base and any other point on the stem, i.e. the merchantable volume. To calculate merchantable volume by taper models that cannot be integrated, volumes are estimated from calculated underbark diameter and length (numerical integration).

Although volume equations have been developed for teak plantations in Tanzania, they were derived from a very limited range of sites and size classes and are not considered to be representative for intensively managed plantations. Taper equations are non-existent for teak, not only in Tanzania, but globally.

Another important variable used for management decisions and planning is tree height. The dominant tree height (according to a specific definition) is often used as a measure to assess site quality. Tree height is also used as a primary predictor variable for volume computations. It is therefore an important component in most growth and yield prediction systems.

In this study, samples of trees from all teak plantations in Tanzania were measured with a Barr and Stroud optical dendrometer.

These data were used as input for the **primary objective** of this study: the development of total and merchantable volume models that respectively predict the *total tree volume* according to the diameter at breast height and total tree height or the *merchantable volume* to either an upper stem diameter or height limit. Tree shape was modelled by fitting appropriate *taper models* to the measured diameter and height values. Since bark thickness constitutes a relatively large proportion of total tree volume, models were developed for prediction of double bark thickness at breast height and at any given upper stem diameter.

As the **secondary objective**[1] of this study, two models were developed to respectively describe teak height growth with age and to describe the relationship between tree height and dbh using stand parameters age and site index as predictor variables.

[1] The secondary objective models developed in this study are presented in Appendices A and B.

2. Literature review

2.1 Teak in Tanzania

The first teak trees in Tanzania were planted by Germans as trial plantings in 1898 on selected sites in Dar-es-Salaam and Mhoro (Madoffe and Maghembe, 1988). Further experimental trial plots were established at various other suitable locations in the country between 1905 and 1936. Most of these growth trials yielded very positive growth rates and indicated that teak could be planted on a commercial scale in Tanzania. This situation, together with a need to alleviate the pressure on indigenous forest and woodlands in the domestic market and continued demand and good prices of teak timber in the international market, led to the initiation of teak plantation projects in Mtibwa (Morogoro region) and Longuza (Tanga region) during the 1950's (Malende *et al.,* 2001) (see Figure 2). These plantations were all planted with seed from the same provenances in Burma (Bekker, Rance and Monteuuis, 2004). In 1966 a provenance trial containing seed sources from India, Java, New Britain, Nigeria, Sudan, Trinidad, Vietnam and the plantation at Mtibwa were established at Longuza (Madoffe and Maghembe, 1988). The growth rates from this trial have been described by Madoffe and Maghembe (1988) and more recently by Mwihomeke *et al.* (2002)[2]. The trial provides the oldest progeny testing data for teak in the world, making it a very valuable genetic resource, particularly since some of the provenances planted in the trial have disappeared completely from the natural habitat in Asia (Kjær and Foster, 2003).

2.1.1 Mtibwa

The 5000 hectare state-owned Mtibwa plantation lies 120 km north of the town of Morogoro towards the Usambura mountain range, at 460 m above sea level. The area experiences two annual rainfall periods, the "short rains" from November to December and the main rainfall period during March, April and May. The average rainfall (1966 – 1980) is 1260 mm per year.

[2] At ages seventeen and thirty six years respectively.

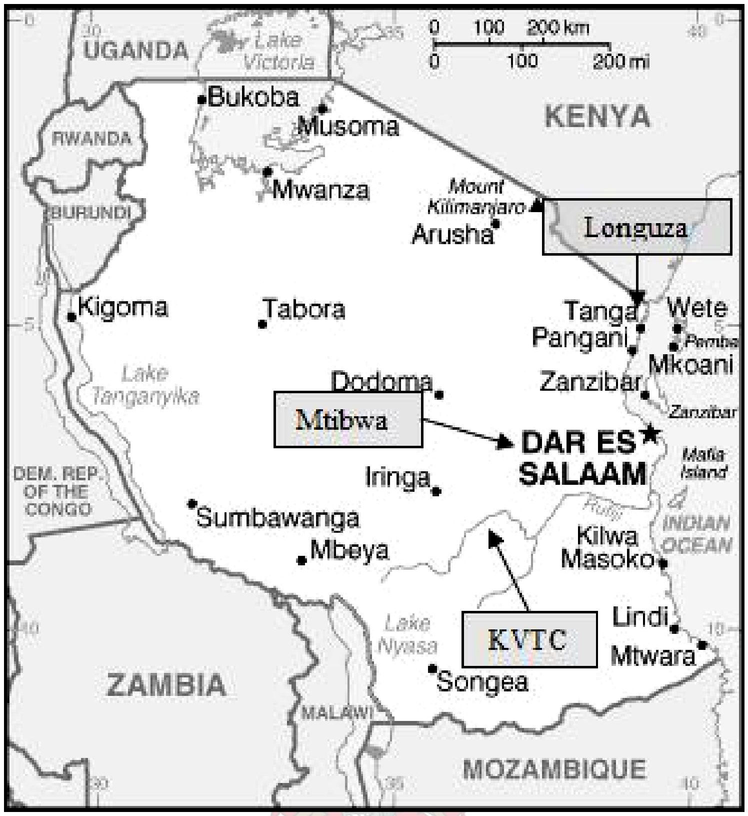


Figure 2. Map of Tanzania indicating the three teak plantations.

The topography of the plantation ranges from completely flat to low lying hills. The soils are generally rich in calcium with a pH range of 5 – 8. In very flat areas drainage is a problem, necessitating the planting of an alternative species (Malende *et al.*, 2001). The region surrounding the Mtibwa plantation has been highly degraded, with formal and informal agriculture covering most of the landscape. Before afforestation with teak, the area was covered with vegetation classified as closed lowland ground water forest with *Sterculia appendiculata* as the dominant species (Malende *et al.*, 2001).

Although management at Mtibwa is faced with some operational problems, silvicultural regimes have to a large extent been followed. Thinnings have generally been applied from above, leading to the best trees being removed. The rotation age is set at sixty years as prescribed by the Tanzanian Forestry Research Institute (TAFORI), but in practise compartments are harvested when funds are needed for the management of the plantation. This leads to better-quality compartments being harvested before the planned age and compartments of inferior quality being harvested long after the planned age. No pruning to improve the wood quality is applied.

2.1.2 Longuza

The other state owned teak plantation in Tanzania, Longuza, lies in the northern region on the foothills of the East Usambura Mountains, 40 km from the port town of Tanga at 180 m above sea level. The natural vegetation of the region is comprised of lowland tropical evergreen forest with *Khaya nyasica* as the dominant species.

The plantation covers an area of 6000 hectares with 1500 hectares planted to teak. Rainfall occurs mainly between October and May with an average of 1500 mm per year (Madoffe and Maghembe, 1988). Although a detailed management regime exists, it is not being followed and has seldom been followed in the past. Some compartments have never been thinned while others have received heavy thinnings from the top. The rotation age is also sixty years as prescribed by TAFORI. As at Mtibwa, no pruning is applied to improve the timber quality.

2.1.3 Kilombero Valley Teak Company

Although the Mtibwa and Longuza plantations are currently the largest teak plantations in Tanzania with mature trees (most compartments are at least 30 years old) and established with well-adapted provenances, planting was discontinued in 1980 due to various reasons, all related to ineffective management and planning (Malimbwi *et al.*, 1998). In 1993 the planting of teak was resumed with the initiation of a new project by the Commonwealth Development Corporation (CDC) and the Tanzanian government in the Kilombero valley which led to the founding of the Kilombero Valley Teak Company (KVTC) (Figure 2). All the KVTC plantations were established with seed from Mtibwa. This provenance is now considered as the "Tanzanian land race", as all the teak plantations in the country are planted from this same provenance (Bekker, Rance and Monteuuis, 2004). Since the first trees were planted in 1993, a total of 4200 hectares have been planted of 28 131 hectares leased from the government. Only 5967 hectares (about 22 %) will be planted with teak, with the rest (miombo and evergreen forest) to be managed as natural conservation areas.

The Kilombero valley lies at an altitude ranging between 280 and 500 meters above sea level. With an annual rainfall of 1000 mm to 1400 mm per year (depending on the area) and with a marked dry season from May to December, it is particularly suitable for teak.

There is a discernable difference between the management regime followed at KVTC and the other two plantations mentioned above. All operations at KVTC are executed very intensively and timeously. In contrast to Longuza and Mtibwa (where no pruning takes place), pruning is applied according to the thickness of the upper stem and thinnings according to an optimum spacing regime. At 32 years, the rotation age is also significantly lower than that of the other two teak plantations. All these factors cause the growth patterns and stem development characteristics of KVTC to differ from that of the plantations at Longuza and Mtibwa.

As a direct consequence of the history of ineffective management at Longuza and Mtibwa, very little research has been conducted at these plantations. Almost no quantitative and qualitative data (apart from the provenance trial at Longuza) have been gathered on either growth or the effect of management regimes on the growth patterns of teak in Tanzania (Malende and Temu, 1990). This makes it extremely difficult to improve and optimise regimes and rotation ages. Although Malende and Temu (1990) developed a preliminary site index classification for teak based on the dominant height development and made an assessment of the volume growth and predicted the biological rotation age[3], no growth and yield model has yet been developed for teak plantations in Tanzania (Isango and Nshubemuki, 1998). This is due to a lack of growth data from either permanent sample plots or spacing and thinning trials in the country (Malimbwi *et al.*, 1998).

2.2 Models for predicting total and merchantable volume and tree shape

A prerequisite for measuring the quantity of timber potentially available from a specific tree, stand or plantation, is the ability to predict the volume of any number of trees to a specified diameter or height. As it is not feasible to **directly** measure the volume of every tree, or even just a sample of trees from every compartment or plantation, **indirect** methods are used. Flexible and accurate models that can determine the volume between two specified points along the tree stem are required. The volume of a tree is largely a function of its height, diameter, shape and bark thickness. These variables can be used to develop indirect volume estimation equations by means of statistical regression analysis.

[3] Only for the Mtibwa plantation.

There are two distinct ways to approach indirect volume estimation. Firstly, by directly measuring tree volume, a relationship can be established with easily measurable tree parameters such as diameter at breast height (dbh) and total tree height. This then results in a *total stem volume equation* or a *merchantable volume equation.* The second approach is to use a *taper equation*. This is a model that describes the entire profile of the stem, also based on simple input variables such as dbh and tree height (Gordon, Lundgren and Hay, 1995).

2.2.1 Volume equations

Numerous equation forms have been developed to predict tree volumes. Some of the most commonly used forms are provided in Table 1[4].

Table 1. Equation forms commonly used for estimation of individual tree volumes.

Name	Equation form	Reference
Logarithmic	$Y = b_0 D^{b_1} H^{b_2}$	Schumacher and Hall (1933)
Constant form factor	$Y = b_0 D^2 H$	Gevorkiantz and Olsen (1955)
Combined variable	$Y = b_0 + b_1 D^2 H$	Spurr (1952); Burkhart (1977)
Generalised combined variable	$Y = b_0 + b_1 D^2 + b_2 H + b_3 D^2 H$	Romancier (1961)
Generalised logarithmic	$Y = b_0 + b_1 D^{b_2} H^{b_3}$	Newnham (1967)

Most of these volume equations have been developed by combining predictor variables in various ways and then regressing predictors on the dependent variable (volume) to find the best fit using least-squares regression (Sharma and Oderwald, 2001).

One of the main factors when developing a volume prediction system is deciding which and how many independent variables will be included in the equation. Two values highly correlated with tree volume are dbh and tree height (Williams and Schreuder, 2000). The measurement of dbh is an easy and accurately repeatable task, while height can be difficult to measure consistently on standing trees. Height measurement therefore leads to an additional cost and could be subjected to considerable measurement error when sampling for volume estimates. Although height is much more difficult to measure than

[4] The volume equations that have been utilized for teak are shown in Section 2.3.

dbh, its inclusion in a volume equation will nevertheless reduce the variance of the predicted volumes. It is only when the measurement error increases to more than about 40% that the volume estimates become biased (Williams and Schreuder, 2000).

As the dbh value alone is only slightly less correlated with tree volume than a combination of dbh and tree height, it is often taken as the only predictor variable in order to reduce sampling expenditure. When only dbh is used as the predictor variable, the equation is known as a *local* or a *single-entry* volume equation (Avery and Burkhart, 2002). Yet, by using dbh as the only predictor variable it is assumed that the parameters of the regression equation are not related to age, site, stand treatment (such as spacing or fertiliser application) and genetic factors (Van Laar and Akça, 1997). When the equation is to be applied to sites with different growing conditions (site quality), ages, or stands managed at different intensities, the assumptions above will no longer hold and more variables are required to reduce the variance of the predicted volumes (Williams and Schreuder, 2000). Such an equation is known as a *standard volume* or a *multiple-entry volume* equation and usually includes the tree dbh, the height and possibly also form or taper variables (Avery and Burkhart, 2002).

2.2.2 Volume ratio equations

In order to determine the merchantable volume (volume from the stump to a fixed upper stem height or diameter limit), *volume ratio equations* (ratio between total volume and volume to the specified merchantability limit) are used. Several forms, with the merchantability limit specified either by an upper stem diameter or height, have been developed (Burkhart, 1977; Cao, Burkhart and Max, 1980; Bi, 1999). A volume ratio equation fitted to predict only the volume to some specified merchantability limit is more accurate than a taper equation[5] (Cao, Burkhart and Max, 1980). Volume ratio equations are better because taper equations are fitted to obtain the most accurate results of diameter (which are then used to calculate the volume of several definitions) whereas volume ratio equations are fitted with the sole purpose to optimize the prediction of merchantable volume to a single limit (e.g. upper stem height or minimum diameter) (Matney and Sullivan, 1982).

[5] Taper equations are introduced in the next section.

2.2.3 Taper equations

A taper equation describes the entire profile of the stem. It is a mathematical description of the relationship between diameter and height and allows the estimation of diameter or height at any point along the stem (from stump to tip), thereby permitting the calculation of volume to any merchantability limit. This ability makes it possible to estimate the total volume, merchantable volume or the volume of individual logs by using a taper equation.

Where a taper equation is continuous and able to be integrated, the volume of the bole can be determined by integration. When this is not possible, the equation can be used to predict the sectional area at 1 or 2 meter intervals and the total stem volume is then determined by adding the volume of the various sections. Since any length or diameter interval can be specified, the volume of sections or products defined by the big and/or small end diameter can be predicted by a taper equation.

Taper equations have been used for more than a century by foresters to express tree form in terms of easily measured characteristics (Perez, Burkhart and Stiff, 1990). Many model forms of varied degrees of complexity have been described by numerous authors. The importance of taper equations becomes evident from the many recent studies and the continued interest and development in the field (Valenti and Cao, 1986; Kozak and Smith, 1993; Bi, 2000; Lee *et al*., 2003; Shaw *et al*., 2003).

2.2.3.1 Classification of taper equations

Taper equations can be divided into two major groups (Lee *et al*., 2003). The simpler group, *single taper models*, describes diameter changes from ground to the tree tip with a single function of different forms (Kozak, 1988). Although easy to fit and integrate for volume calculation, this model form tends to include a large degree of local bias over parts of the stem. In order to overcome the poor performance of single taper models, a new type of model, the "*variable-form*" was introduced (Newnham, 1988, 1992; Kozak, 1988; Perez, Burkhart and Stiff, 1990). This form describes stem shape with a continuous function using a changing exponent to compensate for the neiloid, paraboloid and conic forms of different tree sections (Kozak, 1988). It is based on the assumption that the tree stem form varies continuously along the stem.

The second group, the *segmented taper models,* use different models for various parts of the stem to overcome local bias and join these models in such a way that their first derivatives are equal at the point of intersection (Max and Burkhart, 1976; Brink and Von Gadow, 1986). Models from this group tend to predict diameters with less bias for most parts of the stem, but estimating the parameters and calculating volume can be a complicated task.

Of the general (single and segmented) forms above, several special applications have been introduced and used on a smaller scale. These include the parameter parsimonious taper equation by Lee *et al*. (2003), the similar triangles approach by Shaw *et al.* (2003) and the use of trigonometric function based taper equations (Thomas and Parresol, 1991; Bi, 2000).

Since it is very difficult to formulate general rules that are readily applicable to even a single species, it is impossible to recommend a single model or even a model group as the best for describing the shape of all tree species. Due to the complexity of this problem, a single index cannot be assigned to rank the different models. Kozak and Smith (1993) discuss several practical and statistical considerations that should be taken into account when selecting a model for a specific application. This include factors such as ease of estimating parameters, ease of use, use of readily obtainable variables and applicability to a variety of species.

2.2.3.2 Compatibility of taper equations

In the discussion above (Section 2.2.3), it was shown that taper equations can be integrated to calculate the merchantable volume to any limit. It is also often possible to obtain a volume equation from a taper equation or vice versa (depending on which one is derived first) if equation systems are compatible. The first compatible equations were developed by Demaerschalk (1972) to produce consistent results when retrofitting a taper equation to an existing local or regional volume equation (Gordon, Lundgren and Hay, 1995). *Analytically compatible equation* systems therefore imply that volumes estimated by integration of the taper curve are identical to the volumes obtained from the total volume or appropriate volume ratio equations (Byrne and Reed, 1986). In order for volume estimation systems derived from the integration of taper equations to be compatible (mathematically related), it should be possible to write their coefficients in terms of the taper equation coefficients. When the parameter estimates obtained independently from the taper and volume equations are the same, the equations are *numerically consistent* (Sharma, Oderwald and Amateis, 2002).

Several approaches have been used to derive compatible equations. Demaerschalk (1972) and Goulding and Murray (1976) developed compatible total volume and taper equations by using an existing total volume equation to derive the expression of taper. Volume ratio equations have also been used by Clutter (1980)[6] and Reed and Green (1984) to derive compatible taper equations. More recently, Fang, Borders and Bailey (2000) developed a segmented stem taper model based on a variable form differential equation. This taper model produces compatible total volume and merchantable volume models by direct integration. Sharma and Oderwald (2001) used dimensional analysis techniques to develop a taper and volume equation system that is analytically and numerically compatible by incorporating theoretical information about the dimensional relationships amongst diameter, height and volume of a tree.

A compatible system of equations has several advantages. Obviously, the problem of obtaining inconsistent values from the integrated taper equations and directly from volume equations is circumvented. The key to these systems is the fact that they are generally well specified and better reflect the exhibited underlying biological and physical relationships of the system. The parameters of these compatible systems are also often meaningful and provide information regarding the system being modelled. On the practical side compatible systems of equations do not always yield the most accurate results. Since the prediction objective is not the same (taper equations predict diameters and volume ratio equations predict volume), some precision is lost in the process to ensure that equations are compatible[7] (Cao, Burkhart and Max, 1980).

2.2.3.3 Inclusion of crown characteristics in taper equations

One of the many contributing factors to the variation of tree form, and possibly the most important, is the crown properties (Larson, 1963; Leites and Robinson, 2004). For this reason, *crown ratio*, a measure of the proportion of the stem length within the crown to the total stem length has been successfully incorporated into taper equations to improve the estimate of stem taper (Burkhart and Walton, 1985; Valenti and Cao, 1986; Shaw *et al*., 2003). All these studies report an improvement in fit of taper equations when crown

[6] Developed from the Burkhart (1977) variable top merchantable volume equation.
[7] See the discussion in Section 2.2.1.2 (merchantable volume).

ratio is included as predictor variable. Since there are several other crown related characteristics of a tree that influences its taper, Leites and Robinson (2004) used mixed-effects techniques to include crown shape, crown base diameter and horizontal crown sectional area in addition to the crown ratio when fitting the Max and Burkhart (1976) taper equation. Results show that by substituting the parameters b_1 and b_2 by crown variables the fit and accuracy of prediction of the original equation is improved, but that even more variation can be explained by addition of further crown features.

2.3 Volume model development for teak

The earliest volume tables for teak were developed in India and Burma. In Burma, many “rough” volume tables[8] were developed for natural teak trees and updated regularly before the Second World War (Anon, 1934; Tint and Schneider, 1980). The data used to compile the tables were supplied by the main timber harvesting companies in the Burmese forests and the calculations done largely by the silvicultural staff (Blanford, 1922). These circumstances led to the data collection and yield compilation being rather inaccurate. The authors thus described it as rough and cautioned that no great accuracy should be expected from it. Nevertheless, the tables provided the forest managers with some indication of the volumes being harvested from the forests (Anon, 1934).

More recently, Tint and Schneider (1980) developed single and double entry volume equations for use in a growth and yield model for teak plantations, individual trees and natural teak stands in Burma. Both the equations were derived from data of natural teak trees only. Since the growing conditions and genetic basis of trees growing under natural conditions and in plantations can differ, the use of these volume equations under plantation conditions could lead to biased results.

The single entry equation is presented in Equation 1 and the double entry equation in Equation 2:

$$V = 1.66 \times 10^{-5} \times D^{3.1616} \times 0.9866^{D} \qquad \ldots[1]$$

where: V = tree volume (m^3)

D = dbh (cm)

[8] Burma forest bulletins no. 6, 8, 10, 15, 17 and 31. Burma government printer, Rangoon.

$$V = 1.63 \times 10^{-5} \times D^{1.2577}\left(H + [H - 1.3]\frac{d_i}{D}\right)\left(H - [H - 1.3]\frac{d_i}{D}\right) \quad ...[2]$$

where: V = volume (m^3) to any upper stem diameter, d_i

d_i = upper stem diameter (cm)[9]

H = tree height (m)

The first volume tables for teak in India were developed by Maitland in 1924 (Ram, 1942). Due to the unreliability of these tables and new information becoming available, Ram (1942) developed standard and commercial volume tables for the "central provinces" in India to replace the tables by Maitland. The tables presented give an estimate of the total commercial bole volume. Also included in the study by Ram (1942) are tables that provide the remaining volume after some unutilizable portions (very knotty parts) of the stem had been removed. In addition to these, tables are included that provide the sapwood percentages in the commercial bole of trees for various height and dbh classes, together with tables of bark thickness and percentages for various diameter classes (Ram, 1942).

Tables such as the ones by Ram (1942) were updated on an irregular basis for some of the Indian provinces and therefore provided only scattered information on volume estimation for India.

In a more recent study, Singh (1981) attempted to develop a total tree volume table for teak to also include branch wood in the volume estimates. This was done due to "*pressure on the demand for teak timber which necessitated the calculation of even the small sized branch wood volume together with the stem volume*" (Singh, 1981).

The following equations were used to generate the volume tables:

$$V_1 = 0.19601K_1 + 0.25659D^2H - 0.10787 \quad ...[3]$$

where: V_1 = total tree volume (standard timber) (m^3)

D = dbh (m)

H = tree height (m)

K_1 = 1 if the tree has branch standard timber, else 0

[9] When d_i is zero, the total stem volume will be calculated. For $0<d_i\leq d_{base}$ timber volumes of specified dimensions will result.

$$V_2 = 0.20416K_1 + 0.25514D^2H + 0.10375 \quad \text{...[4]}$$

where: V_2 = total tree volume (m^3)

Since these and other volume equations developed by the Forest Survey of India (FSI) have been based on such divergent information, a more comprehensive set of volume equations that combines the information from numerous previous volume equations were required for local and general volume calculation in India (Chakrbarti and Gaharwar, 1995). This led Chakrbarti and Gaharwar (1995) to develop local volume equations for use in several states of India. Single entry volume equations were developed based on a large number of observations (varying from 30 to 2000) obtained from numerous forest survey reports. These local volume equations were also successfully combined into a single equation for use across India. The following two regression equations were found to give the best fit, with the second equation preferred for use in the whole country:

$$\sqrt{V} = -0.1163 + 2.8013D \quad \text{...[5]}$$

$$V = 0.16571 - 1.235D + 8.0855D^2 \quad \text{...[6]}$$

where: V = volume under bark (m^3)

D = mid point of diameter class (m)

Phillips (1995) developed single and double entry volume equations for use in the growth models developed by him for the commercial teak plantations of Sri Lanka. The total volume calculated by the equations includes the total stem volume overbark to a top diameter of 20 cm as well as smallwood volume, the volume of stem or branchwood longer than 1.5 m with a minimum top diameter of 5 cm.

The single entry equation is provided in Equation 7 and the double entry equation in Equation 8:

$$Ln(V) = -8.685 + 2.429Ln(D) \quad \text{...[7]}$$

$$Ln(V) = -9.733 + 2.055Ln(D) + 0.773Ln(H) \quad \text{...[8]}$$

where: V = total volume overbark (m^3),

D = dbh (cm)

H = tree height (m)

The double entry volume equation is preferred since the growth model allows for a choice of independent height and basal area growth functions. To determine the proportion of timber volume to total tree volume, the following two equations were developed:

$$Vtim / V = -0.699 + 1.504\left(1 - EXP\left(-1.627\left(D^2 H / 10000\right)\right)\right) \quad \text{...[9]}$$

and

$$Vtim / V = 0.949 - \left(2.557 / D^2\right) \quad \text{...[10]}$$

where: *Vtim / V* = ratio of timber volume to total volume

The relationship between the volume of a stand and the mean height and diameter varies according to the site quality of the stand (Munaweera, 1998). Based on dominant height growth variations, three sample population zones were identified from a plantation in Sri Lanka and several sample trees measured in each zone. The data were fitted to four different models by Munaweera (1998) that relate volume to mean tree growth measurements (mean dbh and height) from a stand. It should therefore not be used to determine the volume of individual trees but only to calculate the per hectare volume of a stand from the mean parameters.

The model that gave the best fit was:

$$V = b_0 D^{b_1} H^{b_2} \quad \text{...[11]}$$

where: *V* = volume per hectare (m^3/ha)
D = mean dbh of the stand (cm)
H = mean height of the stand (m)

The first volume tables for teak plantations in Tanzania were developed by Micski and Akchurst (1972). These tables are double entry volume tables, developed from sample trees across the country subjected to different management regimes and climatic conditions. Only a limited amount of data were available from mature plantations. This led to the tables being negatively biased, especially for older and larger trees. This necessitated the construction of local volume tables for use in yield prediction (Malimbwi *et al.* 1998).

A local volume equation was developed by Abdelsalaam (1980) for the Mtibwa teak plantation and used by Malende and Temu (1990) to describe the volume growth of this plantation:

$$V = b_1(D)^2 + b_2(H)^2 + b_3(D)^2 H + b_4(DH^2) \quad \text{...[12]}$$

where: V = volume

D = dbh (cm)

H = height (m)

$b_1 - b_4$ = regression coefficients

New volume equations (Table 2) were developed for teak plantations to include data from Longuza, the other (larger) teak plantation in Tanzania with relatively mature trees and it was recommended that these should replace the biased tables by Micski and Akchurst (1972). The models reported are those that gave the best results (according to their goodness of fit[10]) and are all single entry volume equations (dbh as the independent variable), although double entry (dbh and height) equations were also fitted to the data (Malimbwi *et al.* 1998).

Table 2. Volume equations for teak plantations in Tanzania (Malimbwi *et al.* 1998).

Equations for estimation of over bark volume:	
Total volume over bark	$V = 0.066 + 0.00024D^{2.35}$
Volume over bark to 10 cm minimum top diameter	$V = 0.000906D^2 - 0.0761$
Volume over bark to 15 cm minimum top diameter	$V = 0.0009344D^2 - 0.152$
Volume over bark to 20 cm minimum top diameter	$V = 0.0009709D^2 - 0.2814$
Equations for estimation of under bark volume:	
Total volume under bark	$V = 0.000896D^2 - 0.098$
Volume under bark to 10 cm minimum top diameter	$V = 0.0008534D^2 - 0.084$
Volume under bark to 15 cm minimum top diameter	$V = 0.0008835D^2 - 0.174$
Volume under bark to 20 cm minimum top diameter	$V = 0.0009239D^2 - 0.3181$

10 R^2 value and the standard error.

In an independent study on the hardwood plantations of the East Usambura region in Tanzania, volume equations were developed to aid with the analysis of the inventory results (Pikkarainen, undated). The volume equations were developed from 97 felled trees with a diameter range of 12 cm to 58 cm. Two equations, for the sawlog volume (merchantable volume) and the total volume, were developed:

$$ln\,(V_{Tot}) = -1.05918+(1.921698\times ln(D))+(0.04337129\times H) \qquad ...[13]$$

$$ln(V_{Saw}) = -1.14754+(1.903212\times ln(D))+(0.04393139\times H) \qquad ...[14]$$

where: V_{Saw} = sawlog volume (dm^3)
V_{Tot} = total volume (dm^3)
D = dbh (cm)
H = height (m)

No information about the sampling procedure, intensity or derivation of these functions is provided.

Provisional volume equations for a growth and yield model for teak plantations in Ghana were developed by Nunifu and Murchison (1999). A standard volume equation was developed and recommended for use in the whole country:

$$V = -0.36 + 0.96D - 0.13D^2 + 0.05D^2H \qquad ...[15]$$

where: V = volume (dm^3)
D = dbh (cm)
H = total tree height (m).

A thinning experiment was laid out at the University of Ibadan to develop management regimes for the teak plantations of Nigeria. A volume equation developed from sample tree measurements (Lowe, 1976):

$$V = -2.9737 + 0.1718G + 0.01354G^2 \qquad ...[16]$$

where: V = mean volume tree in the stand (ft^3)
G = girth of the mean girth tree (ft).

A volume equation recommended for use in predicting the volume of teak plantations in the humid tropics of Africa, developed from plantation data in the Ivory Coast by the *Centre Technique Forestier Tropical* (CTFT) is (Dupuy and Mille, 1993):

$$V = 0.0456 - 1.2356D + 11.8011D^2 \quad \text{...[17]}$$

where: V = main stem volume (m^3) to a minimum diameter of 7 cm

D = dbh (m)

Three different volume/height equations from different countries (El Salvador, Jamaica and Trinidad) were used by Keogh (1980) to determine the volume growth of teak in Central America. The stem volume per hectare (m^3) for the respective regions is calculated with the following equations:

$$V = 3.394(H) + 0.344(H^2) - 62.75\ ^{11}, \quad \text{...[18]}$$

$$V = 0.3889\left(H^2\right) - 0.8989\left(H\right) + 0.5031\ ^{12} \text{ and} \quad \text{...[19]}$$

$$V = 72.2 - 14.22\left(H\right) + 0.86\left(H^2\right)^{13} \quad \text{...[20]}$$

where: V = stem volume (m^3/ha)

H = tree height (m).

In order to compare the growth rates of teak plantations in Puerto Rico with the growth of other teak plantations in the Caribbean, Weaver and Francis (1990) summarised the growth of 27 teak plantations in Puerto Rico. To obtain an estimate of merchantable volume, the following volume equation was used:

$$Vc = 3.14\left(D_1 + D_2\right) \times 0.25H \quad \text{...[21]}$$

where: Vc = the commercial volume (m^3)

D_1 and D_2 = dbh and upper stem diameter (cm) respectively

H = height (m) from ground level to top stem diameter and

D_2 is determined by the following relationship:

$$D_2 = 9.512 + 0.63397D_1 - 1.0718Lc \quad \text{...[22]}$$

where: D_1 and D_2 are as defined above

Lc = commercial bole length (m),

[11] Volume under bark, between 0.3 m above ground level and to a minimum diameter of 8 cm under bark. *H* is top height, the mean height of the 100 largest diameter trees/ha.
[12] Volume to 10 cm top diameter. *H* is top height, the mean height of the 100 largest diameter trees/ha.

determined as: *H* – stump height, which is usually assumed to be 0.25 m.

More recently Bermejo, Caneleas and San Miguel (2004) developed growth and yield models for teak plantations in Costa Rica. The volume data from several regions were fitted separately and also combined to several equation forms. The best results were obtained by the combined variable equation[14]:

$$V = -26.7721 + 0.02566D^2H \qquad \text{…[23]}$$

where: V = volume (dm^3 up to a 10 cm upper stem diameter limit underbark)

D = diameter at breast height (cm)

H = total tree height (m)

2.3.1 Discussion of teak volume studies

In the discussion above it was clearly demonstrated that numerous equation forms have been used to estimate the total and merchantable volume of teak trees. There is also an abundance of other equation forms that have been used to estimate the volume of other tree species[15] (Clutter *et al*., 1983). To select a suitable equation from the fitted equations above, great care should be taken. Although some equation forms were shown to be well suited to volume prediction, they were fitted with data from different provenances and growing conditions. Another factor to consider is that the terminology used to define the dependent variable is not always standard (e.g. merchantable volume, utilisable volume and branchwood volume) which can lead to confusion and biased predictions.

The above situation requires that the equations be validated for use in the Tanzanian teak plantations.

[13] Volume under bark. *H* is the mean height of dominant trees.
[14] Coefficients shown are from the results of the combined data from all regions.
[15] See discussion in Section 2.2.1.1.

3. Study data

In this chapter the source of data and methods of data collection are described together with a discussion of the reasoning behind the methods applied. Finally, the characteristics of the sampled data are presented.

3.1 Data gathering rationale

In order to develop an indirect volume estimation equation, it is necessary that the true volume and/or shape of trees of the species of interest be known. There are a number of methods that can be used to directly[16] estimate the volume or shape of a tree. Most often, the data are obtained by sectioning the tree at prefixed lengths and recording the diameter at these points along the stem. This method is laborious and extremely expensive, particularly when the timber value of the species under study, teak, is considered. A method that is comparable in accuracy, but a lot cheaper than the method above is by simply taking upper stem measurements with a dendrometer (James and Kozak, 1984). More specifically, the Barr and Stroud[17] (B & S) optical dendrometer has been proven to be an accurate estimator of tree volume and shape when compared with direct measurements of felled or standing trees (Yocom and Bower, 1975; James and Kozak, 1984).

The Barr and Stroud dendrometer can be described as a "*short-base, split image, coincident type magnifying rangefinder adapted for estimating out of reach diameters*" (Mesavage, 1964). The manufacturers (Barr and Stroud Ltd., 1970) provide very limited information on the theory of the dendrometer operation. The principle and operation of the B & S is thoroughly described and nomograms provided that can be used for field checking of measurement values by De Vries (1971) [18].

The B & S is not a direct reading instrument (Mesavage, 1968). The values read from the B & S are only the true and false coincidence settings of its rotating prisms and the sine of the angle formed by the line of sight and the horizontal plane provided by the inclinometer (Brickell, 1976). In order to derive diameter readings at the respective

[16] The only theoretically unbiased method to directly measure tree volume is by *xylometry* (fluid displacement). *Xylometry* does not require any assumptions to be made about tree shape and thus has no theoretical bias. It is expensive however and thus rarely used outside research applications.
[17] A descriptive illustration of the Barr and Stroud dendrometer is provided in Appendix D, Figures 57 and 58.
[18] In Appendix C a detailed description of how the B & S operates are provided.

heights, exact formulae and constants developed from the basis of operation of B & S are used (Grosenbaugh, 1963)[19]. As these calculations require a lot of time and are ideally done by computer, several useful mechanisms (such as slide rulers and lookup tables) have been developed to allow derivation of diameter and height values directly from the B & S readings (Grosenbaugh, 1963; Mesavage, 1968; De Vries, 1971; Brickell, 1976).

As the B & S readings are made visually only, the derived diameter values are all overbark. In order to calculate the merchantable or total underbark volume and shape, it is necessary that double bark thickness be subtracted from every measurement to derive the underbark diameter. For this reason, bark thickness was measured on opposite sides of all the sample trees at breast height with a Swedish bark gauge[20] (Figure 3).

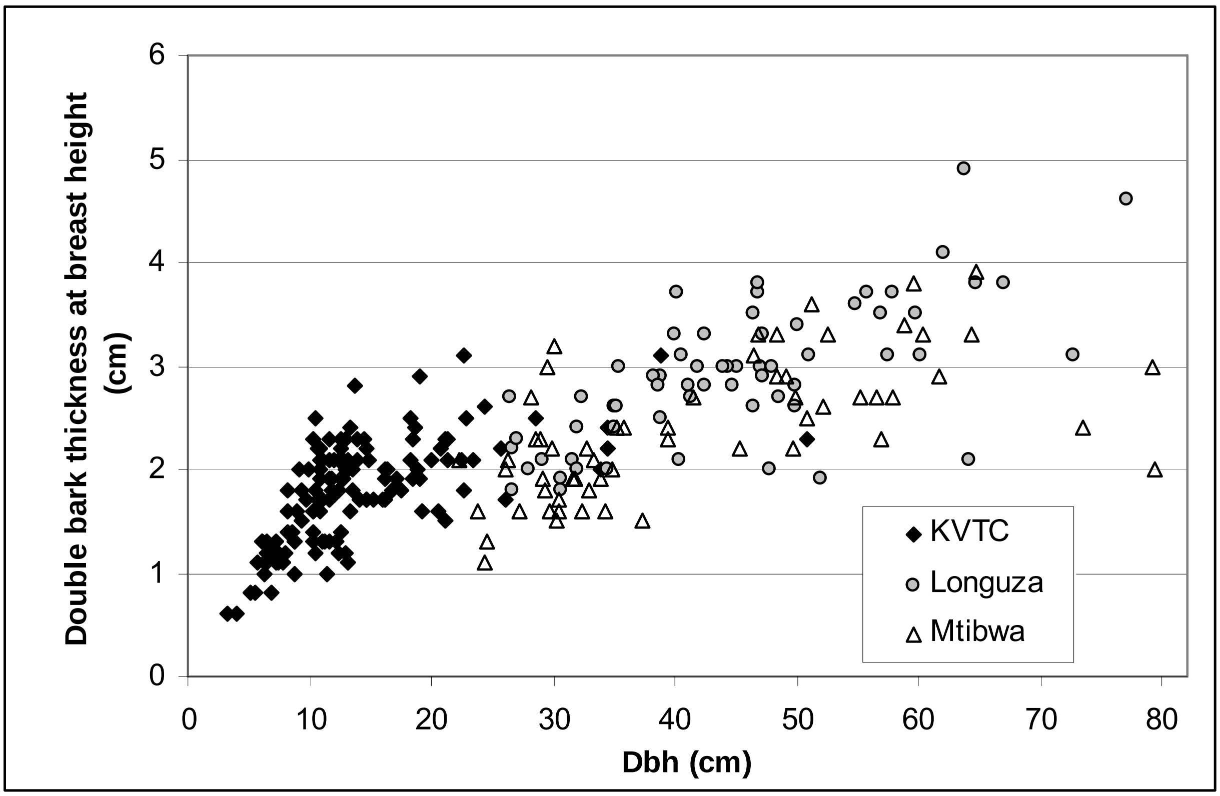


Figure 3. Double bark thickness measured at breast height for all sample trees.

[19] See the example of diameter calculation from the Barr and Stroud settings in Appendix C.
[20] A figure of a Swedish bark gauge is presented in Figure 59, Appendix D.

Bark thickness measurements (n = 76 observations) were also taken at breast height and subsequent one meter intervals along the stem from a small sample[21] of felled trees. This allowed the estimation of the change in relative bark thickness along the tree stem. A model was fitted to this data that allows the calculation and subsequent subtraction of the double bark thickness from every diameter observation along the stem according to the double bark thickness at breast height measured on each tree. From this data the underbark volume and shape was then determined for each tree.

Since the volume is calculated directly (a number of length and diameter measurements taken and the sections summed for each tree) in an indirect way (an equation is used to calculate the volume of the sections that make up the whole tree), some bias might be included. This is due to the tree stem having a very complex geometric shape, so complex that it is impossible to even describe individual sections perfectly with a mathematical expression (Brickell, 1984). The accuracy of the direct volume calculation is determined by two factors: the **nature of the mathematical model** used to represent the stem segments and the **kind and number of measurements** taken on the tree.

The thin and thick end diameters and length of each segment were determined from the respective diameter measurements of the B & S and the length by subtracting the height at each point from the previous point. Of the several equations available to calculate section volumes, the commonly used Smalian equation was used:

$$V_{\log} = \frac{\pi l (d_1^2 + d_2^2)}{8} \quad \text{...[24]}$$

where: V_{log} = volume of the log section (m^3)

l = log length (m)

d_1 = diameter at the thick end of the log (m)

d_2 = diameter at the thin end of the log (m)

This equation was chosen for its ease of use, especially when taking measurements with a dendrometer where an extra measurement in the centre of the log section would be

[21] The objective of the study was to derive all models without felling a single teak tree. This objective was achieved, except for this small sample that was felled to determine the change in bark thickness along the stem.

impossible, as would be required by the slightly more accurate Huber equation (Husch, Miller and Beers, 1982).

The volume of the last section of the tree, the stem section between the last diameter measurement and the tip of the tree, was calculated as the volume of a cone:

$$V_{tip} = l_{tip} \times \frac{s_a}{3} \qquad \text{...[25]}$$

where: V_{tip} = volume of the tip section of the tree (m^3)
l_{tip} = length of the tip section of the tree (m)
s_a = cross sectional area at the base of the conoid (m^2)

The total volume of the tree is hence calculated as:

$$V_{Tree} = \left(\sum_i \frac{\pi l_i (d_{i_1}^2 + d_{i_2}^2)}{8} \right) + \left(l_{tip} \times \frac{s_a}{3} \right) \qquad \text{...[26]}$$

where: V_{Tree} = total tree volume (m^3)
l_i = log length of the i^{th} section (m)
d_{i1} = diameter at the thick end of the i^{th} log (m)
d_{i2} = diameter at the thin end of the i^{th} log (m)

An attempt was made to keep the section lengths of the logs to approximately ten percent of the tree height in order to reduce bias. This technique has been shown to constrain the amount of bias to around one percent when Smalian's equation is used (Brickell, 1984).

An important characteristic in the lower part of the teak stem is fluting and/or buttressing (Streets, 1962). The phenomenon of fluting can be described as irregular involutions and swellings occurring in the butt section of the tree (Krishnapillay, 2000). Since fluting is strongly controlled by provenance, it is possible to minimize its occurrence by only selecting provenances that exhibit very little fluting. A study on all the provenances of teak planted in Tanzania has indicated that they exhibit little fluting (Madoffe and Maghembe, 1989). It is however seldom possible to entirely eliminate fluting by provenance selection, especially since the effect is only detected when the trees get older.

Although the occurrence of fluting in teak has been described, no study for teak has to date described the effect of fluting on the modelling of stem volume or shape. The reason for this might be due to most studies being carried out on plantations with well-selected provenances that exhibit little fluting. Another reason might be that only data from relatively young stands having been used in many studies and hence the effect of fluting or butt swell did not yet pose a problem when the study was done. Nevertheless, the effect of only a slight amount of fluting can lead to biased estimates since the butt section constitutes a large amount of the overall tree volume.

Although no severe fluting was observed from trees measured at the three plantations in this study, the apparent risk of not including some measure of the scale of fluting when modelling teak volume and shape is noted. It was however impossible include the effect of fluting in this study as it cannot be measured with a dendrometer.

3.2 Data collection

Diameter measurements were taken at fixed heights at **0.3 m, 0.6 m, 1.3 m** on each tree. A further eight to ten diameter measurements were taken per tree, spaced at variable distances along the stem according to tree height with the intention to take the readings at 10% height intervals. Table 3 and Figure 4 provide summary statistics for observations taken per tree at each of the plantations.

Table 3. Summary of the total number of observations and the number of observations per tree.

	Number of observations			
	Total	**Average**	**Minimum**	**Maximum**
KVTC	1067	10.67	7	13
Longuza	785	12.46	10	16
Mtibwa	765	12.96	10	16
TOTAL	2617	11.78	7	16

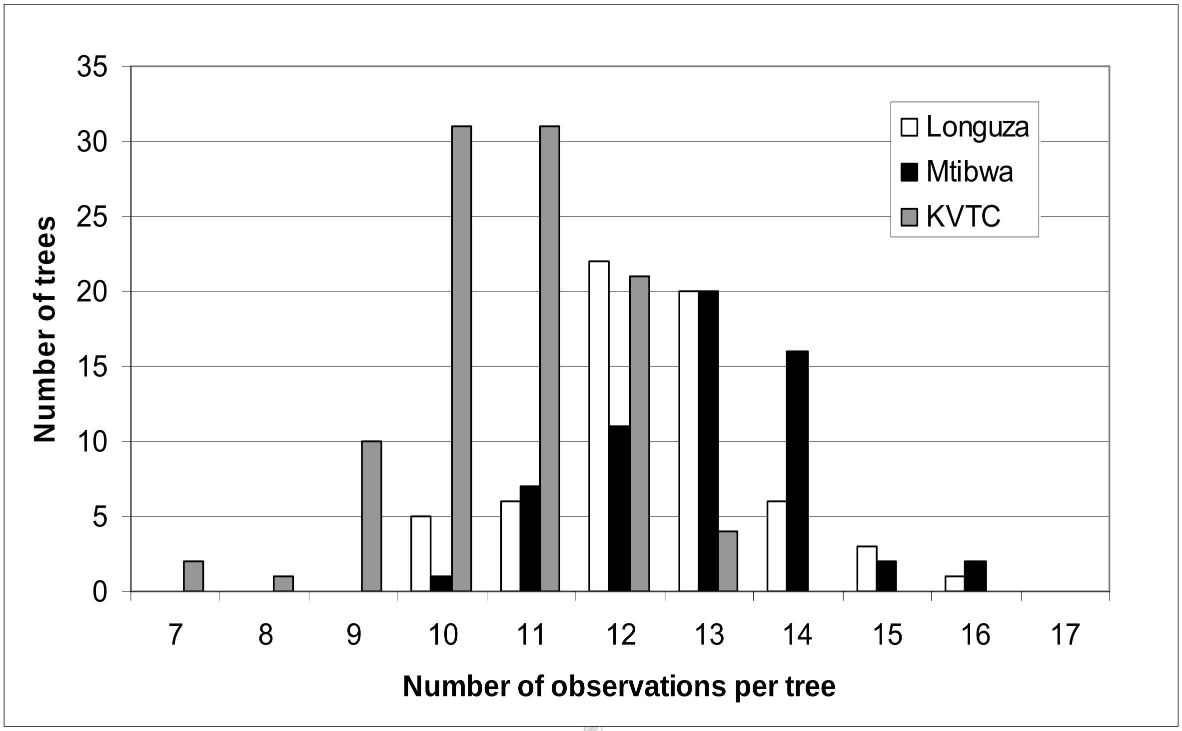


Figure 4. Histogram indicating the number of observations made per tree at each of the sampled plantations.

The B & S dendrometer requires an unobstructed view of the tree stem in order to focus clearly and obtain the true and false coincidence settings that are used to derive the diameter at a specific point along the stem. For this reason, sample plots of specific size were not laid out, but sample trees that would allow a clear view along most of the stem were selected. In each compartment trees from most of the dbh classes were measured. Despite this non-random selection procedure, finding trees inside a compartment with a clear view to most of the stem was almost impossible due to either branches of the tree being measured, or branches and leaves of the surrounding trees obscuring the line of sight. Taking measurements of the bole within the live crown section was made particularly difficult by the very large leaves of teak and the branching habit of the crown. Although the timber within the crown is of lesser importance (since the tip constitutes a very small percentage of the total tree volume), it was attempted to take measurements in the crown that would allow accurate description of the bole within the crown.

Although the assumptions of random sampling were deliberately violated by possibly introducing personal bias, due to subjective selection of the sample trees, the reasons described above made it unavoidable. To improve the objectivity of sample selection and reduce the effect of any possible personal bias, almost all of the trees were measured in the inside of a compartment and only a few on the outer perimeter. After selecting a compartment (of specific age and site quality) and randomly determining a starting point, the first tree found with a clear view to most of the stem was measured. All subsequent trees were measured on the same principle; as soon as a clearly visible tree is found, it is measured.

Another factor to consider when measuring upper stem diameters of trees is the branch whorls. The branches of teak trees develop in distinct whorls, with two branches opposite each other. Where such whorls occur, the nodal swelling leads to a marked increase in stem thickness. When the diameter measurements are taken at constant length intervals (as with most volume/taper studies), it might happen that several of the observations are located at these thickenings. This results in biased data that will lead to the final model overestimating the true volume. It will not be possible to determine the magnitude of such bias. By using the B & S, each observation could be spaced at irregular intervals in such a manner that no observations were taken at the whorls. This was done by viewing the tree through the viewfinder of the B & S and tilting the instrument so that measurements are only taken between whorls, but still close to the intended one tenth of the total tree height.

3.3 Sample data

In order to ensure that the derived models are applicable to all the teak growing sites and ages in Tanzania, it was attempted to include sample trees from all age and site quality classes present at the various plantations where data were collected (Table 4). This was not always possible, especially at Longuza and Mtibwa where no information on the site index of the various compartments is available. Since all measurements were made in even-aged compartments, stratification was inherently applied to the sampled population in terms of age and site quality. The range of sample tree sizes and dbh classes sampled is shown in Figures 5 and 6 and the summary statistics provided by plantation in Table 4.

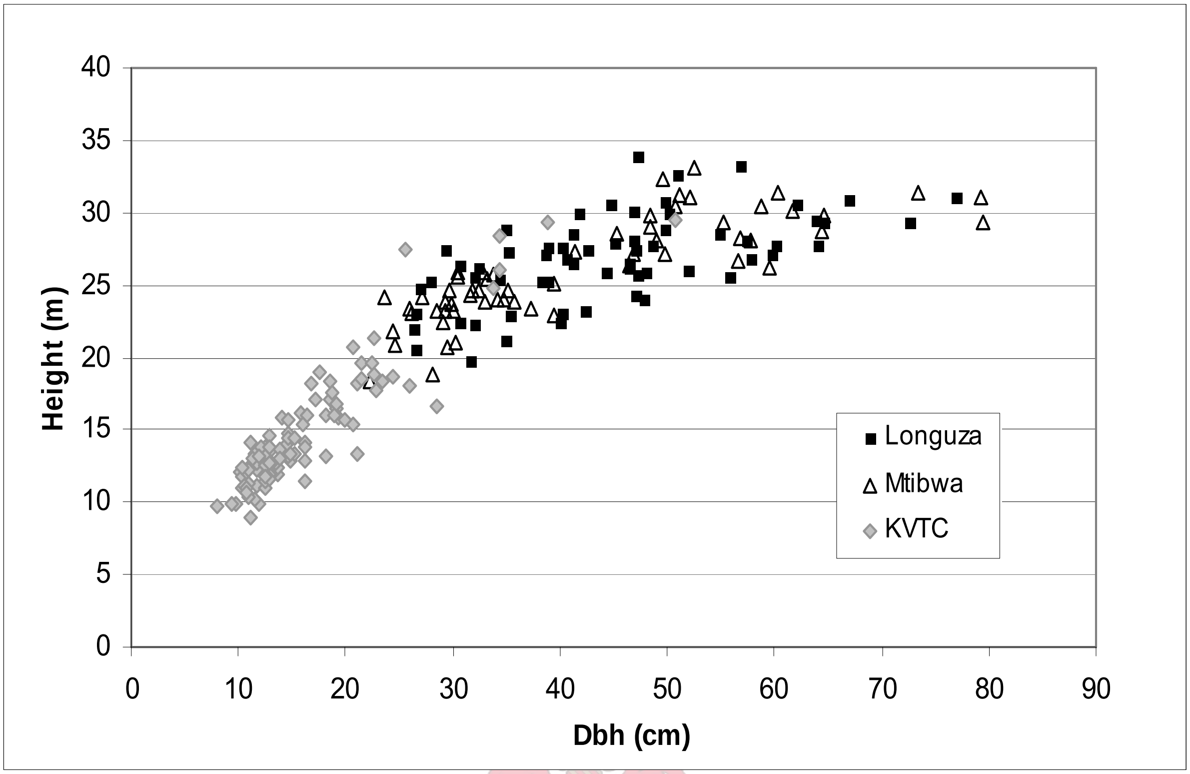


Figure 5. Distribution of sample trees across the entire dbh and height range.

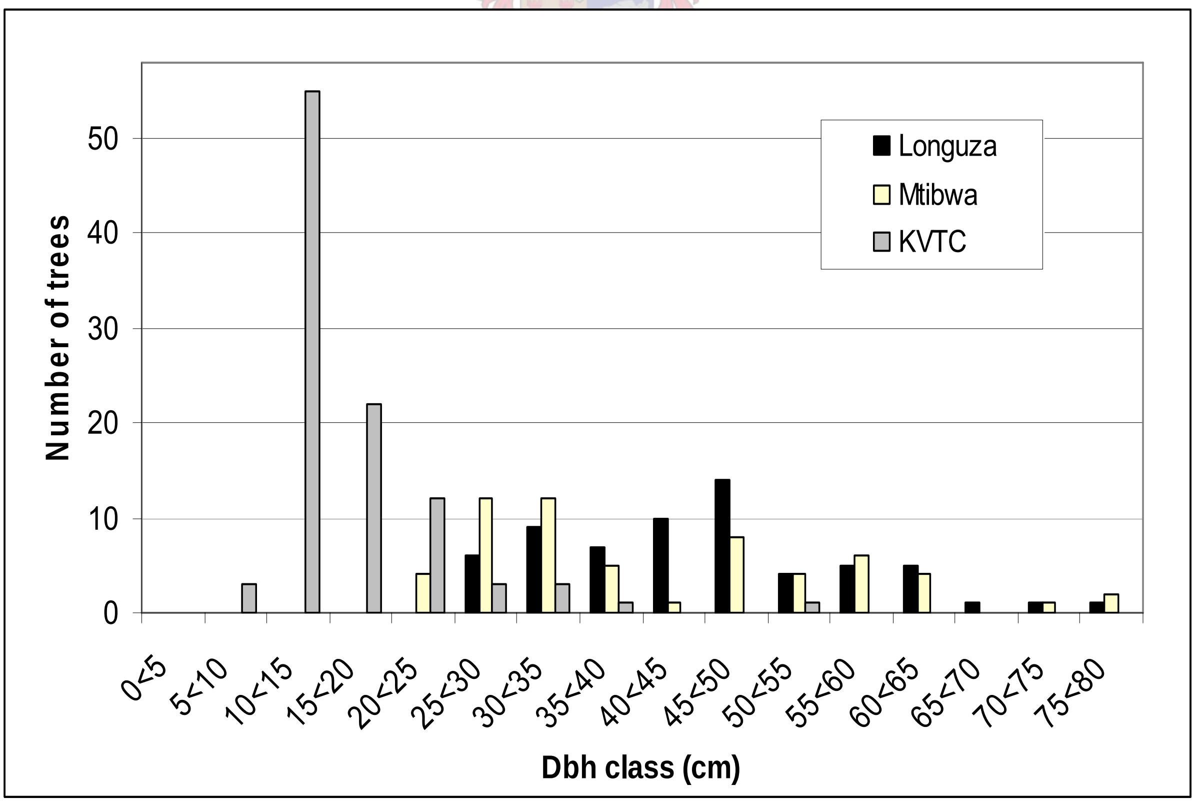


Figure 6. Dbh classes sampled at KVTC, Longuza and Mtibwa.

It is clear that the data from the three sample plantations in Figure 5 follow a similar height-dbh trend and appear to represent a single population. This is in contrast to some species that show distinct differences in this relationship between regions and even plantations (Bredenkamp, 1982). Since there are distinct differences between the growing sites and management at the three plantations, it can be assumed that the height-dbh trend shows this continuity due to the fact that most of the compartments were planted with the same provenances.

From Figures 5 and 6 it can be seen that there were some big trees measured at the KVTC plantation. These trees are all from a small provenance trial planted in the early 1960's. Unfortunately, no management activities were performed and no information gathered from this trial by either the Tanzanian Forestry Research Institute (TAFORI), the responsible party for managing provenance trials in the country or KVTC itself. It is assumed that this trial includes some of the provenances that were also planted at the Mtibwa and Longuza provenance trials.

Table 4. Summary statistics (dbh, total tree height and underbark volume) for each plantation.

KVTC: n = 100				
	Mean	**SD**	**Minimum**	**Maximum**
Dbh (cm)	16.40	6.94	8.10	50.80
Total height (m)	14.94	4.22	8.98	29.51
Total volume under bark (m^3)	0.16	0.27	0.01	2.01

Longuza: n = 63				
	Mean	**SD**	**Minimum**	**Maximum**
Dbh (cm)	45.14	11.87	26.40	77.00
Total height (m)	26.88	2.97	19.68	33.82
Total volume under bark (m^3)	1.79	0.97	0.41	4.55

Mtibwa: n = 59				
	Mean	**SD**	**Minimum**	**Maximum**
Dbh (cm)	41.67	14.38	22.30	79.40
Total height (m)	26.06	3.440	18.35	33.04
Total volume under bark (m^3)	1.60	1.23	0.33	5.94

4. Stem form and volume models for teak in Tanzania

The various models fitted to sample data in this study are introduced and their characteristics and previous use are briefly discussed.

4.1 Volume

4.1.1 Total volume

In order to estimate the total volume[22] of a tree an equation using diameter at breast height and total tree height as the input variables is used. These are easily measurable and highly correlated with volume.

Many such equation forms have been used to predict total tree volume (Avery and Burkhart, 1994). One equation that stands out as being particularly efficient is the logarithmic Schumacher and Hall (1933) equation:

$$V_T = b_0 D^{b_1} H^{b_2} \qquad \text{...[27]}$$

where: V_T = total volume (m^3)

D = diameter at breast height (cm)

H = total tree height (m)

Model 1.1

Logarithmic transformation of the variables in Equation 27 makes it possible to estimate the parameters by linear regression:

$$\ln V_T = b_0 + b_1 \ln D + b_2 \ln H \qquad \text{...[28]}$$

4.1.2 Merchantable volume

The total volume mentioned above is often inadequate to satisfy the needs of the forest manager and planner, essentially because utilization standards tend to change over time. Of more importance is the volume of a particular product such as veneer or saw log, defined by specific limits of utilisation. This requires that volume estimates to various thin end diameters and between given underbark diameters be calculated to cater for a variety of utilisation standards.

[22] Stump, stem and tip volume to a minimum upper stem diameter.

Two strategies can be followed to develop volume prediction models that allow for differing utilisation standards:

- Estimate merchantable volume through a logical relationship to total volume by deriving a volume ratio equation.
- Estimate merchantable volume through the use of a taper equation.

4.1.2.1 Volume ratio equations

Volume ratio equations predict the ratio of the volume to some specified upper stem limit to the total tree volume (calculated from Model 1).

Three equation forms were fitted to the dataset of 1756 observations that included an average of eight diameter-height observations from 222 trees. The total volume for each tree was calculated directly as described in Section 3.1.

Model 1.2.

The Burkhart (1977) volume ratio equation has been shown to be a good predictor of merchantable volume to a specific diameter along the stem as a ratio of the total tree volume:

$$V_M / V_T = (1 - b_0 d_i^{\,b_1} D^{b_2}) \qquad \text{...[29]}$$

where: V_T = total stem volume (m^3)

V_M = volume to any upper stem diameter, d_i (m^3)

D = dbh (cm)

d_i = upper stem diameter (cm)

$b_0 \ldots b_2$ = parameters to be estimated by regression

Model 1.3.

In a similar approach, it is possible to predict the ratio of merchantable volume to any specified height to the total tree volume by the modified Burkhart (1977) equation (Cao and Burkhart, 1980):

$$V_M / V_T = (1 - b_0 \left(\frac{(H-h)^{b_1}}{H^{b_2}} \right) \qquad \text{...[30]}$$

where: V_T = total stem volume (m^3)

V_M = volume to any upper stem height (m^3)

H = total tree height

h = height at which the volume ratio is to be calculated

$b_0 \ldots b_2$ = parameters to be estimated by regression

Both these equations have been shown to be very accurate predictors of merchantable volume to specified upper stem diameters or heights, but only when this is the only purpose for which it is used (Cao, Burkhart and Max, 1980; Byrne and Reed, 1986). The equations are not suited to describe tree shape, especially the butt section of the tree. This is because the equations are derived from least squares estimates of the parameters of volume equations and not tree taper.

Both equations are conditioned so that when d or h, the merchantable diameter or height is zero (at the tip of the tree), the value of V_M/V_T is one. The volume between any two sections is calculated by simply subtracting the volumes at these sections from each other.

Model 1.4.

Another equation form, previously fitted to teak data (Tint and Schneider, 1980), calculates the merchantable volume directly and independently of the total volume. In this case care is taken to ensure that the equation is logically compatible so that the merchantable volume for a 10 cm top diameter is constantly larger than for a 12 cm top diameter.

$$V_M = b_0 \times D^{b_1}\left(H + [H - 1.3]\frac{d_i}{D}\right)\left(H - [H - 1.3]\frac{d_i}{D}\right) \qquad \text{...[31]}$$

where: V_M = volume to any upper stem diameter, d_i (m^3)

D = dbh (cm)

d_i = upper stem diameter (cm)[23]

H = tree height (m)

b_0, b_1 = parameters to be estimated by regression

[23] When d_i is zero, the total stem volume will be given while for $0<d_i \le d_{base}$ (d_{base} = diameter at the base of the tree) timber volumes of specified dimensions will result.

4.1.2.2 Taper models

Model 2.1

A taper model that has been shown to be a good predictor of stem form is the Max and Burkhart (1976) model (Kozak and Smith, 1993). The model is often used as a measure with which newly developed models are compared (Perez, Burkhart and Stiff, 1990; Newnham, 1992; Lee *et al*., 2003). It has also been used as a basis model by many studies where crown factors are incorporated to the taper equation with the aim of improving the accuracy of predictions (Valenti and Cao, 1986; Shaw *et al*., 2003; Leites and Robinson, 2004).

The Max and Burkhart (1976) model is a segmented polynomial model consisting of three sub-models that describe the lower bole, middle and upper sections of the tree stem respectively. The sub-models are joined at two join points, with the model constrained so that the functions are continuous at the join points (Sharma and Burkhart, 2003).

The Max and Burkhart (1976) model can be written in linear form as:

$$Y = b_1(Z-1) + b_2(Z^2-1) + b_3(\alpha_1 - Z)^2 I_1 + b_4(\alpha_2 - Z)^2 I_2 \qquad \text{...[32]}$$

where: $Y = \dfrac{d_i^2}{D^2}$

d_i = upper stem diameter underbark (cm) at height h_i

D = diameter at breast height (cm)

α_1 = upper join point

α_2 = lower join point

$Z = \dfrac{h_i}{H}$

h_i = height at diameter d_i (m)

H = total tree height (m)

$b_1 \ldots b_4$ = parameters to be estimated by regression

$I_i = 1$ if $Z \le \alpha_i$ or $I_i = 0$ if $Z > \alpha_i$

$i = 1, 2$

Model 2.2.

The Kozak (1988) taper model has been shown to be very useful according to several criteria (Kozak and Smith, 1995). In this model, a single continuous function describes the shape of the bole with a changing exponent. When this model was developed, the value *p* (the inflection point) was taken as 0.25. It has been shown that the value of *p* does not have a very large effect on the predictive properties of the model (Perez, Burkhart and Stiff, 1990). In this study the value of *p* was found by refitting the model several times with different values.

The variable exponent Kozak (1988) model is:

$$d_i = b_0 D^{b_1} b_2{}^{D} x^{b_3 Z^2 + b_4 \ln(Z+0.001) + b_5 \sqrt{Z} + b_6 e^{Z} + b_7 \left(\frac{D}{H}\right)} \qquad \text{...[33]}$$

where: d_i = upper stem diameter underbark at height h_i (cm)

D = diameter at breast height (cm)

$x = 1 - \frac{\sqrt{Z}}{\left(1 - \sqrt{p}\right)}$

h_i = height at diameter d_i (m)

H = total tree height (m)

p = proportional height of the inflection point i.t.o. (h_i/H)

$Z = \frac{h_i}{H}$

$b_0 \ldots b_7$ = parameters to be estimated by regression

This taper equation has been shown to perform better than a number of popular taper models according to several selection criteria (Perez, Burkhart and Stiff, 1990; Kozak and Smith, 1993; Bi, 2000; Lee *et al*., 2003). An advantage of the model is that it is easier to obtain parameter estimates than is the case for more complicated models such as the one by Max and Burkhart (1976). On the other hand, volume cannot be calculated by integration of the taper equation. Volume values can only be obtained by calculating the diameter and length of various sections and summing them (numerical integration). Merchantable height for a specified top diameter can also only be obtained by iteration and cannot be calculated directly.

Model 2.3.

In order to reduce the number of parameters that need to be estimated by regression, Perez, Burkhart and Stiff (1990) fitted alternative variable-form taper models and compared the fit with the "full" Kozak (1988) taper model above. By refitting the model with different values of *p,* it was found that the position of the inflection point has very little effect on the predictive ability of the model.

The following model (logarithmically transformed) gave the best fit:

$$\ln(d) = \ln(b_0) + b_1 \ln(D) + b_3 \ln(Z) X^2 + \left(b_4 \ln(X) \ln(Z + 0.001)\right) + b_7 \ln(X)\left(\frac{D}{H}\right) \quad ...[34]$$

where: d = upper stem diameter underbark (cm) at height h_i

D = diameter at breast height (cm)

h_i = height at diameter d_i (m)

H = total tree height (m)

$Z = \frac{h_i}{H}$

$X = \frac{(1-\sqrt{Z})}{(1-\sqrt{p})}$

p = proportional height of the inflection point i.t.o. (h_i/H)

$b_0 \ldots b_4$ = parameters to be estimated by regression

Model 2.4.

A relatively simple variable exponent taper model introduced by Lee *et al.* (2003) describes the changing tree stem by means of a parameter-parsimonious power function. This taper model has been found to compare well with the Max and Burkhart (1976) and Kozak (1988) taper models (Lee *et al.*, 2003).

$$d = b_0 D^{b_1} (1 - Z)^{b_2 Z^2 + b_3 Z + b_4} \quad ...[35]$$

where: d = upper stem diameter underbark at height h_i (cm)

D = diameter at breast height (cm)

$Z = \frac{h_i}{H}$

h_i = height at diameter d_i (m)

H = total tree height (m)

$b_0 \ldots b_4$ = parameters to be estimated by regression

Model 2.5.

As some *Eucalyptus* species are the major commercial species that are similar to teak in terms of taper, the variable form taper model developed by Bi (2000) for Australian *Eucaluptus* plantations was included in this study. The base function of this model is constructed from trigonometric volume ratio equations and the specification for the exponent is based on the Fourier transformation. Variables are included for depicting changes in stem form along the stem and for taking into account the differences in stem form between trees of different sizes. This is done by allowing the base function and exponent to vary with tree size. This means that the point of inflection derived with the trigonometric model can vary with tree size.

$$Rd = \left(\frac{\ln \sin\left(\frac{\pi}{2} Z\right)}{\ln \sin\left(\frac{\pi}{2} RBH\right)} \right)^{b_0 + b_1 \sin\left(\frac{\pi}{2} Z\right) + b_2 \cos\left(\frac{3\pi}{2} Z\right) + \frac{b_3 \sin\left(\frac{\pi}{2} Z\right)}{h} + b_4 D + b_5 Z \sqrt{D} + b_6 Z \sqrt{H}} \quad \text{...[36]}$$

where: $Rd = \dfrac{d_i}{D}$ (relative diameter)

$Z = \dfrac{h_i}{H}$

$RBH = 1.3/H$ (relative breast height)

h_i = height at diameter d_i (m)

H = total tree height (m)

d_i = upper stem diameter underbark at height h_i (cm)

D = dbh (cm)

$b_0 \ldots b_4$ = parameters to be estimated by regression

Model 2.6.

A new segmented taper model was recently developed by Shaw *et al*. (2003) for longleaf pine. Three submodels are represented by three similar triangles and coupled by two join points (expressed in terms of h_i / H). The model performed better than the Max and Burkhart (1976) segmented taper model when fitted with longleaf pine data.

$$Y = b_0 \left[\frac{b_2 \left(H \left(1 + \frac{b_4}{H} - \alpha_1 \right) \right)^{b_2 - 1} \left(H(1 - Z) \right)^{b_1}}{b_1 \left(H + b_4 - 1.3 \right)^{b_2} \left(H(1 - \alpha_1) \right)^{b_1 - 1}} \right] \qquad \text{...[37]}$$

for $1>Z>\alpha_1$,

$$Y = \left[\frac{b_2 b_0 \left(H(1 - \alpha_1) \right) \left(H \left(1 + \frac{b_4}{H} - \alpha_1 \right) \right)^{(b_2)}}{b_1 \left(H + b_4 - 1.3 \right)^{b_2}} \right] - b_0 \left[\frac{\left(H \left(1 + \frac{b_4}{H} - \alpha_1 \right) \right)}{\left(H + b_4 - 1.3 \right)} \right]^{b_2} + b_0 \left[\frac{\left(H \left(1 + \frac{b_4}{H} - Z \right) \right)}{\left(H + b_4 - 1.3 \right)} \right]^{b_2}$$

for $\alpha_1>Z>\alpha_2$, and

$$Y = b_0 \left[\frac{b_2 \left(H(1 - \alpha_1) \right) \left(H \left(1 + \frac{b_4}{H} - \alpha_1 \right) \right)^{(b_2 - 1)}}{b_1 \left(H + b_4 - 1.3 \right)^{b_2}} \right] - b_0 \left[\frac{H \left(1 + \frac{b_4}{H} - \alpha_2 \right)}{\left(H + b_4 - 1.3 \right)} \right]^{b_2} + b_0 \left[\frac{\left(H \left(1 + \frac{b_4}{H} - \alpha_2 \right) \right)}{\left(H + b_4 - 1.3 \right)} \right]^{b_2}$$

$$-b_0\left[\frac{b_2\left(H(1-Z)\right)\left(H\left(1-\frac{b_4}{H}-\alpha_2\right)\right)^{(b_2-1)}}{b_3\left(H+b_4-1.3\right)^{b_2}}\right]$$

$$+b_0\left[\frac{b_2\left(H(1-Z)\right)^{b_3}\left(H\left(1+\frac{b_4}{H}-\alpha_2\right)\right)^{(b_2-1)}}{b_3\left(H+b_4-1.3\right)^{b_2}\left(H(1-\alpha_2)\right)^{(b_3-1)}}\right]$$

for $\alpha_2 > Z > 0$.

where: $Y = \frac{d_i}{D}$

d_i = upper stem diameter underbark at height h_i (cm)

D = diameter at breast height (cm)

α_1 = upper join point

α_2 = lower join point

$Z = \frac{h_i}{H}$

h_i = height at diameter d_i (m)

H = total tree height (m)

$b_0 \ldots b_4$ = parameters to be estimated by regression

4.1.3 Double bark thickness along the stem

In order to determine underbark volume adequately and describe tree shape, the double bark thickness must be subtracted from the upper stem diameter readings as these are determined overbark by optical dendrometers. The calculation of upper stem bark thickness is approached by modelling one of the three broad patterns of the ratio of diameter underbark to diameter overbark along the tree stem (Johnson and Wood, 1987).

1. The ratio remains constant along the stem.
2. The ratio increases curvilinearly above breast height and decreases curvilinearly below breast height.

3. The ratio decreases curvilinearly above breast height and increases curvilinearly below breast height.

Many studies that fit equations to volume and taper data obtained with a dendrometer assume a constant ratio of under and overbark diameter (e.g. James and Kozak, 1984). Although this study obtained data essentially only with a dendrometer, a very small sample of upper stem bark measurements were made in order to make a slight improvement in the calculation of underbark volume over that which would be the case if a constant ratio was assumed.

By applying an exponential transformation to the **relative diameter** (diameter overbark expressed as a ratio of the diameter at breast height) observations in order to linearize it, a simple linear model can easily describe the relationship between the relative diameter and the **relative bark thickness** (bark thickness expressed as a ratio of the bark thickness at breast height) (Figure 7). This model makes it possible to calculate the underbark diameter at any point along the stem by substituting the double bark thickness value at breast height.

Model 3.1.

$$RBT = a_0 + a_1RD^2 \qquad \ldots[38]$$

where: RBT = relative bark thickness

RD = relative diameter

a_0, a_1 = parameters to be estimated by regression

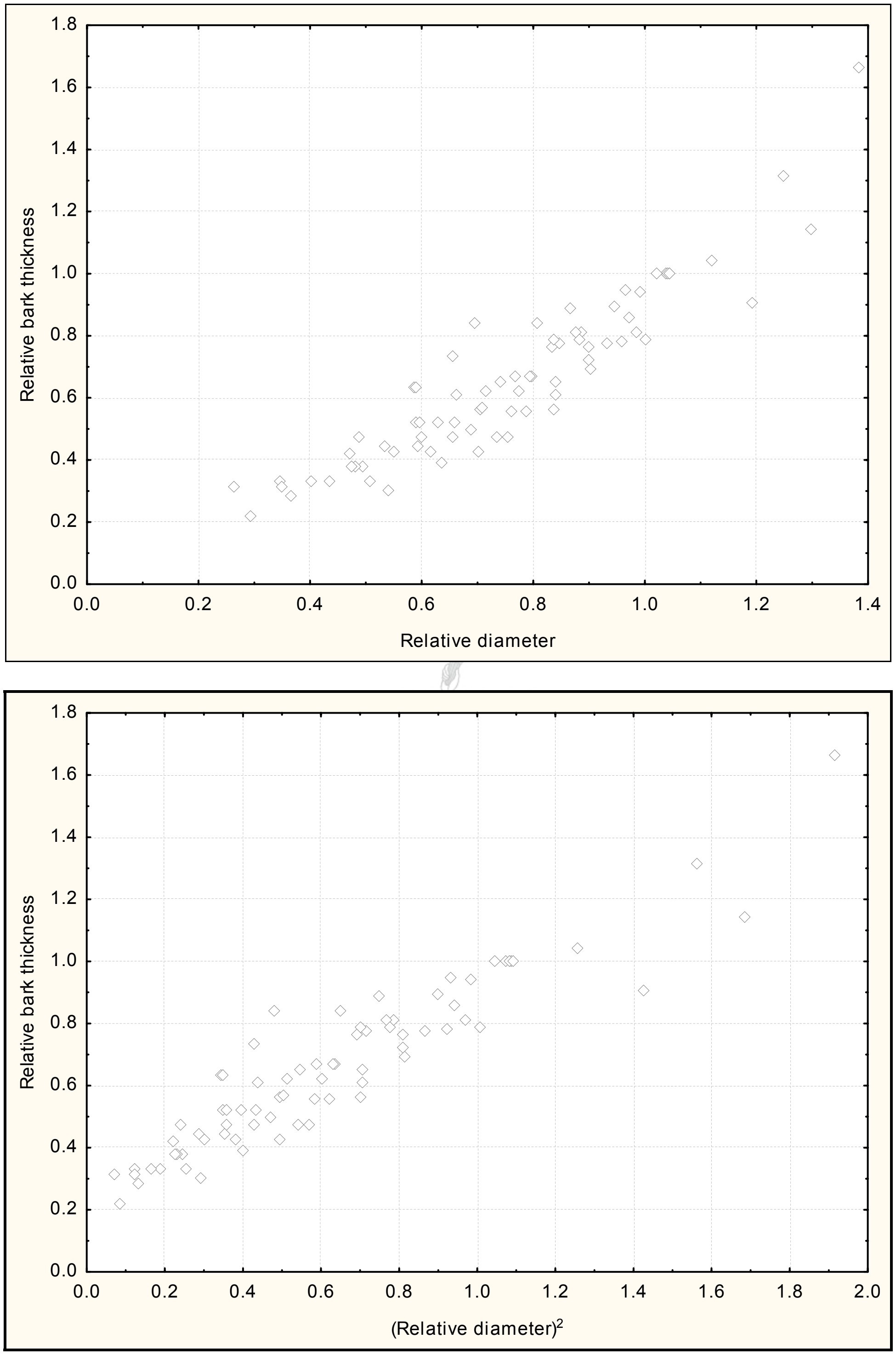


Figure 7. Relative bark thickness plotted over relative diameter (a) and relative diameter squared (b).

4.1.4 Bark thickness at breast height

Although double bark thickness was measured on every sample tree in this study, it will not always be available. A model was fitted to the data that will allow the estimation of double bark thickness at breast height from the diameter at breast height. This value can be used in model 3.1 to estimate bark thickness at an upper stem diameter. The parameter estimates were obtained by linear regression after transformation of the diameter at breast height:

$$DBT_{BH} = a_1 + a_2(\ln(D)) \qquad \text{...[39]}$$

where: DBT_{BH} = double bark thickness at breast height (cm)

D = diameter at breast height (cm)

a_1, a_2 = parameters to be estimated by regression

4.2 Model fitting methodology

All the data were visually inspected for observations that were potentially outliers, possibly as a result of measurement error. Observations so identified were rechecked with the original Barr and Stroud measurements. Further examination of data was performed by the diagnostic tests discussed in the following section.

In order to maximise the predictive accuracy and retain a reliable estimate of that accuracy, all the data were used as both the parameter estimation and the assessment data set for the selected models (Roecker, 1991).

All the models were fitted by means of the appropriate procedures incorporated in the Enterprise Guide SAS statistical software (SAS Institute Inc., 2001).

4.3 Criteria for goodness of fit

- During the model regression procedures, fit statistics including the coefficient of determination (R^2) and mean square error (*MSE*) were calculated. Although these statistics might provide an easy and relatively accurate measure with which different models can be compared, they should be used with discretion. This is because models with different dependent and independent variables and with different numbers of estimated parameters cannot readily be compared. These values are also based on the total fit to the entire tree and make no specification for the accuracy at different portions of the stem.

- In order to better compare the different models, the bias, standard error of estimate (*SEE*) and the respective percentage (relative) bias and *SEE* were calculated. All bias and *SEE* values are reported in terms of the diameter underbark (cm) or total volume underbark (m^3) as these are the real dependent variables of taper models and not the transformed dependent variables used to find the parameters in regression (Kozak and Smith, 1993).

  The most informative measure of mean bias is:

$$B = \frac{\sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2}{n} \qquad ...[40]$$

  The percentage bias can be calculated as:

$$B(\%) = \frac{1}{n}\left[\sum_{i=1}^{n} \frac{\left(Y_i - \hat{Y}_i\right)}{Y_i}\right] \times 100 \qquad ...[41]$$

  where: B = Bias

  $B(\%)$ = Percentage bias

  Y_i = observed value

  $\hat{Y}_i$ = predicted value

  n = number of observations

The *SEE* is the square root of the variance and gives an indication of the spread of the actual observations (Y_i) around the predicted values ($\hat{Y}_i$) (Ott, 1988):

$$SEE = \sqrt{\frac{\sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2}{n-q}} \qquad ...[42]$$

This can be expressed as a percentage as:

$$SEE(\%) = \left[\sqrt{\frac{\sum_{i=1}^{n} \left(\frac{Y_i - \hat{Y}_i}{Y_i}\right)^2}{n-q}}\right] \times 100 \qquad ...[43]$$

where: *SEE* = Standard error of the estimate

SEE(%) = Percentage standard error of the estimate

Y_i = observed value

$\hat{Y}_i$ = predicted value

n = number of observations

q = number of parameters used in the estimation

- Residual plots were used to visually examine the relationship between the predicted and residual values. From these plots it is possible to visually inspect residuals and identify trends that could indicate violation of regression assumptions.
- In order to identify observations that have a significant impact on the estimated parameter values, several diagnostic tests were performed with the **INFLUENCE** procedure in SAS (SAS Institute Inc., 2001) and the observations so identified were further scrutinized.
  - The **student residual** is a measure of the distance of each observation from the regression line. An unusually large residual indicates that the data point is an outlier (in a vertical direction), and that it can thus be influential.
  - The **R-student** is a measure of the influence of a particular observation on the regression. The further a particular data point lies from the fitted line the larger will be the calculated value of R-student, as its omission causes the standard error to become smaller.
  - Observations associated with large **Hat Diagonal** values have the potential to be influential, particularly if they are also outliers with regard to the collection of *y* values. The Hat Diagonal leverage points can increase the efficiency of estimation, but can also indicate that another function would suit the data better.
  - The **Covratio** (covariance ratio) statistic measures the effect of an observation on the variance covariance matrix of the estimated coefficients by calculating the ratio of the determinant of the variance covariance matrix with and without the particular observation. Therefore Covratio measures the effect of the observation on the standard errors, or rather, the precision of the

estimates. An observation for which Covratio > 1 increases the precision, whereas Covratio < 1 indicates that the particular observation decreases the precision. An observation for which Covratio is smaller than 1 – $3q$[24]$/n$ or larger than 1 + $3q/n$, is regarded as influential.

- The **Dffits** is a statistic used to determine the influence of omitting the i^{th} observation on the predicted value or fit of the model. The measure is calculated as the standardized difference between the predicted values with and without the i^{th} observation (= change in fit). It can be shown that the influence of an individual observation on the fit becomes greater with an increase in the number of parameters to be estimated and becomes less with an increase in sample size.

- To determine the influence of the i^{th} observation on the estimates of the regression coefficients themselves, a **Dfbetas** value (difference in beta) can be determined for each of the q coefficients (e.g. one for each of the intercept and the slope in the case of simple linear regression). It can be shown that the influence of an individual observation on the regression coefficients becomes smaller with an increase in sample size, i.e. Dfbetas_j is proportional to $\frac{1}{\sqrt{n}}$. An observation with $\text{Dfbetas}_j > \left|\frac{2}{\sqrt{n}}\right|$ has a large influence on the estimated value of b_j.

[24] q = number of parameters estimated.

5. Results and discussion

5.1 Total volume

The Schumacher and Hall (1933) volume model in transformed logarithmic form provided an excellent fit to the sample data by means of linear regression. In order to allow the calculation of total overbark and underbark volumes, the dependent variable was first taken as the total volume overbark measured and calculated as explained in Section 3.2. Secondly, the total volume underbark determined by subtracting the double bark thickness calculated with Model 3.1 was input as the dependent variable. The estimated parameter values and fit statistics are given in Table 5.

Table 5. Results from fitting Model 1.1.

Model	**Dependent variable**	**Parameter estimates and their standard errors (in brackets)**			**R^2**	***MSE***	***n***
		b_0	b_1	b_2			
1.1	Total volume overbark (m^3)	-9.9185 (0.0796)	1.8889 (0.0373)	1.0090 (0.0619)	0.995	0.0109	225
1.1	Total volume underbark (m^3)	-10.8014 (0.0873)	1.9056 (0.0409)	1.2256 (0.0679)	0.994	0.0131	225

Apart from the very satisfactory results presented in Table 5, Figures 8 and 9 indicate that no apparent residual trend exists. No influential observations were detected by the diagnostic tests.

In Figure 10 the relative amount of bias and *SEE* is shown for dbh classes of the sample data. From these graphs and the residual graph it is clear that the fitted model is very accurate for the small and medium sized tree classes. The model will overestimate volume for the largest size classes, but these classes are so much larger than will be encountered in commercial forestry enterprises that this is of little concern for the present study. The reason for the observed 10 % bias in the 60 cm and 80 cm size class can firstly be ascribed to the fact that many fewer observations were made for this size class than the others (only 15 out of the 222 trees fall in this class). The other reason is due to the fact that, as the trees become larger, variation in most parameters such as diameter and height tend to increase. This again leads to greater variation in tree volume and greater difficulty in accurately modelling these size classes.

Although the model is biased when predicting the volume of very large trees, it is important to note that the size class range used to fit the models in this study is very wide; much wider than has previously been used to model teak volume. This means that volume predictions can now be made more accurately for large trees: some of the largest teak trees found in plantation form in the world having been included in the sample.

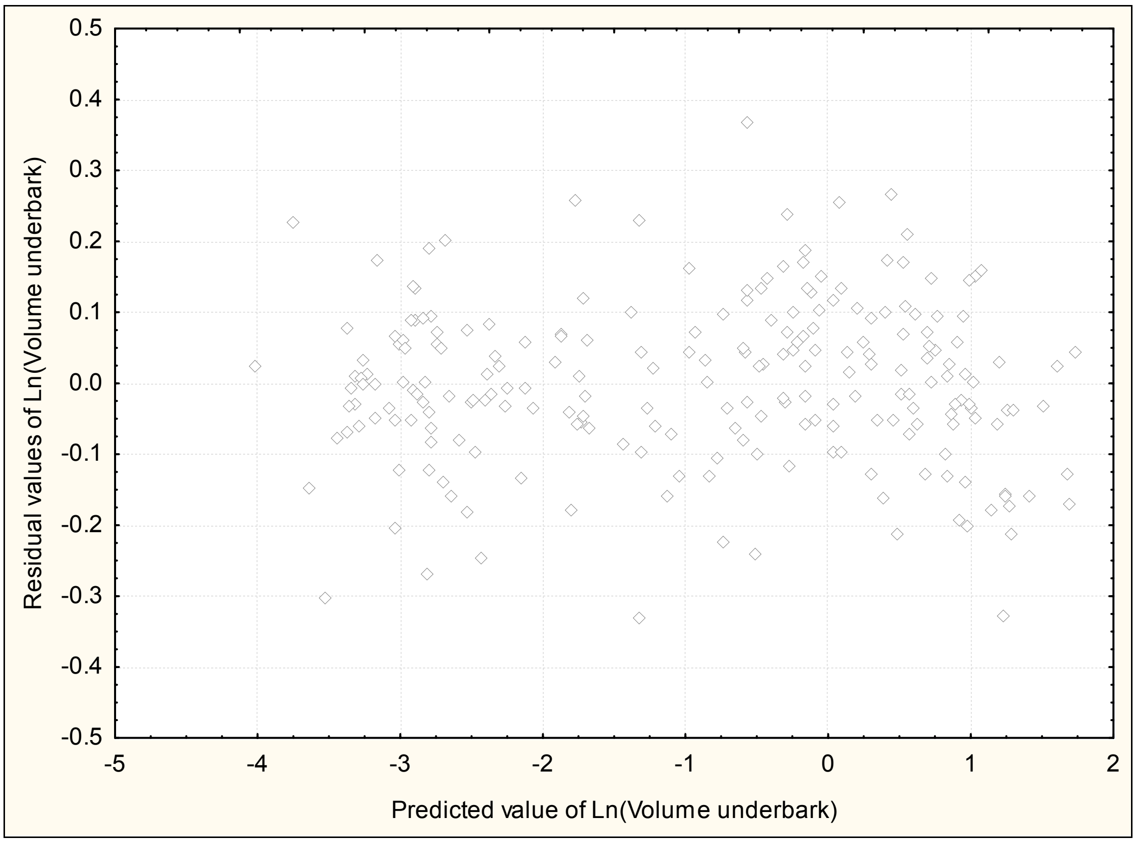


Figure 8. Predicted and residual values of the logarithm of total volume underbark.

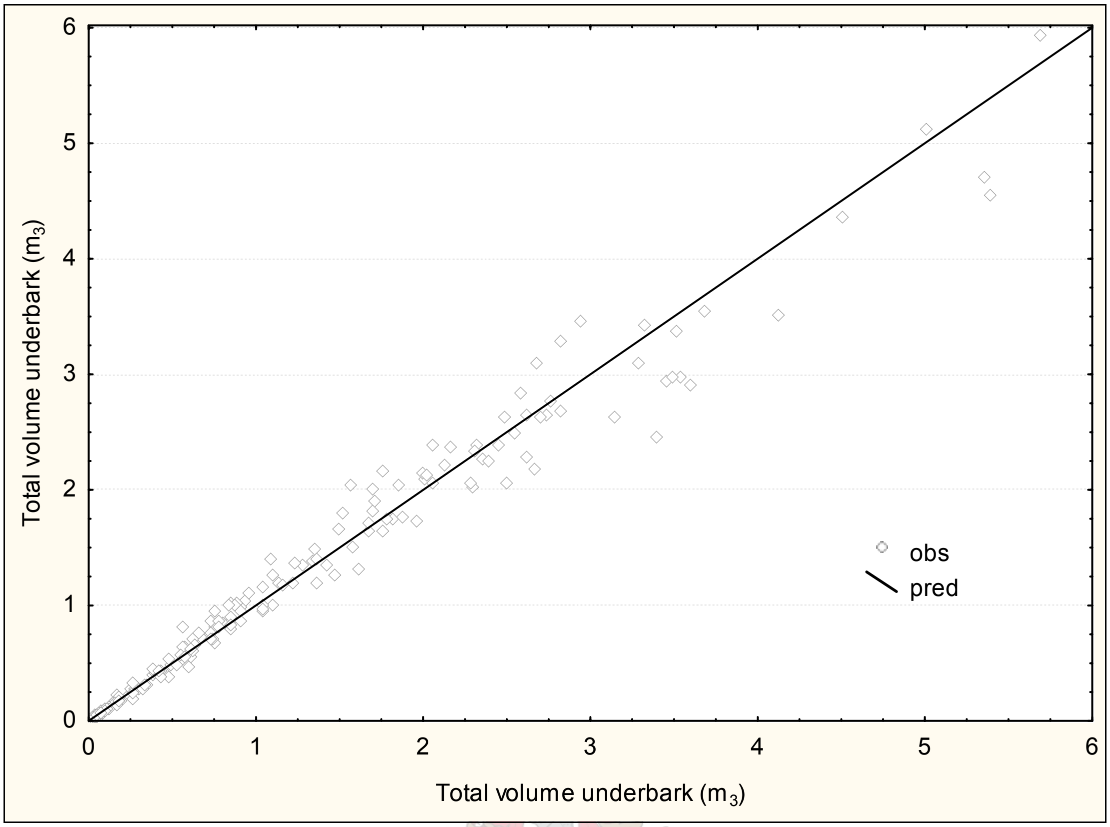
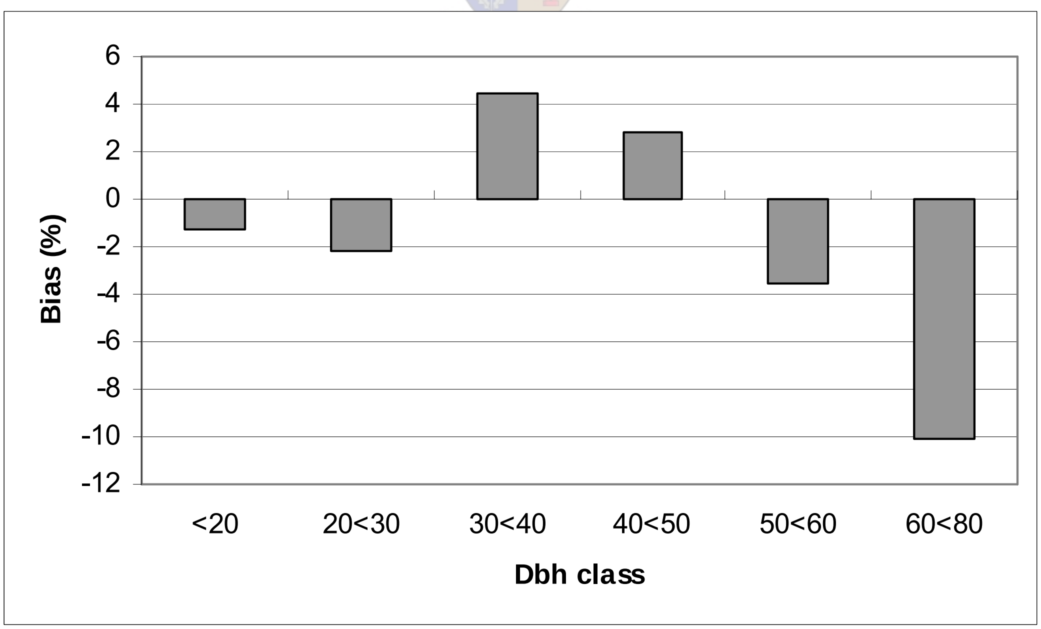


Figure 9. Observed and predicted values of total volume underbark estimated by means of Model 1.1.



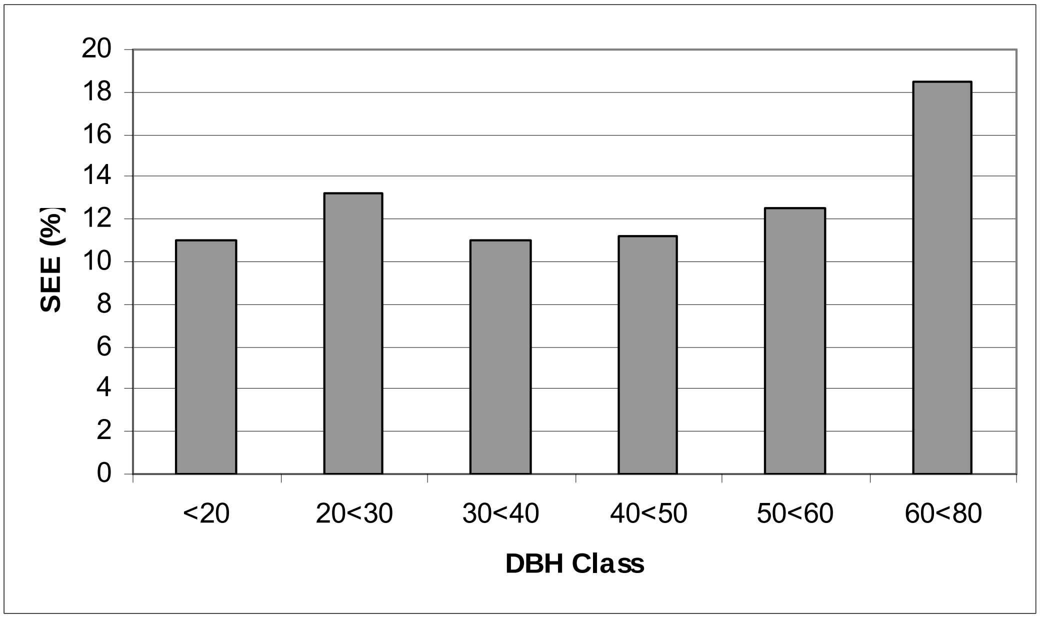


Figure 10. Percentage biases and standard errors of estimate (SEE) of total volume underbark as predicted by Model 1.1 sorted by dbh class.

5.2 Merchantable volume

5.2.1 Merchantable volume to upper stem diameter limit

Models 1.2 and 1.4 were fitted with data from the same sample trees as the total volume equation. To allow estimation of both under and overbark volumes to upper stem diameter limits, parameters were estimated by appropriately altering the dependent variable. As some situations might require that the upper stem diameter limit be underbark, parameters were also estimated for this use.

Table 6. Results from fitting Model 1.2.

Model	Dependent variable	Independent variables	Parameter estimates and their standard errors (in brackets)			R^2	*MSE*	*n*
			b_0	b_1	b_2			
1.2	Merchantable volume overbark	d_{min} overbark and Dbh	0.4234 (0.0130)	3.0654 (0.0345)	-2.8568 (0.0354)	0.921	0.00464	1756
1.2	Merchantable volume underbark	d_{min} overbark and Dbh	0.4430 (0.0135)	3.0653 (0.0343)	-2.8676 (0.0352)	0.922	0.00466	1756
1.2	Merchantable volume underbark	d_{min} underbark and Dbh	0.9771 (0.0345)	3.0396 (0.0355)	-3.0089 (0.0384)	0.915	0.00506	1756

Regression diagnostics did not indicate the presence of influential observations. From the fit statistics in Table 6 it is clear that the model gives a good fit to the sample data. The fit statistics were improved by excluding all data points below the lower inflection point (i.e. where relative diameter greater than one) since the equation does not possess an analytical inflection point.

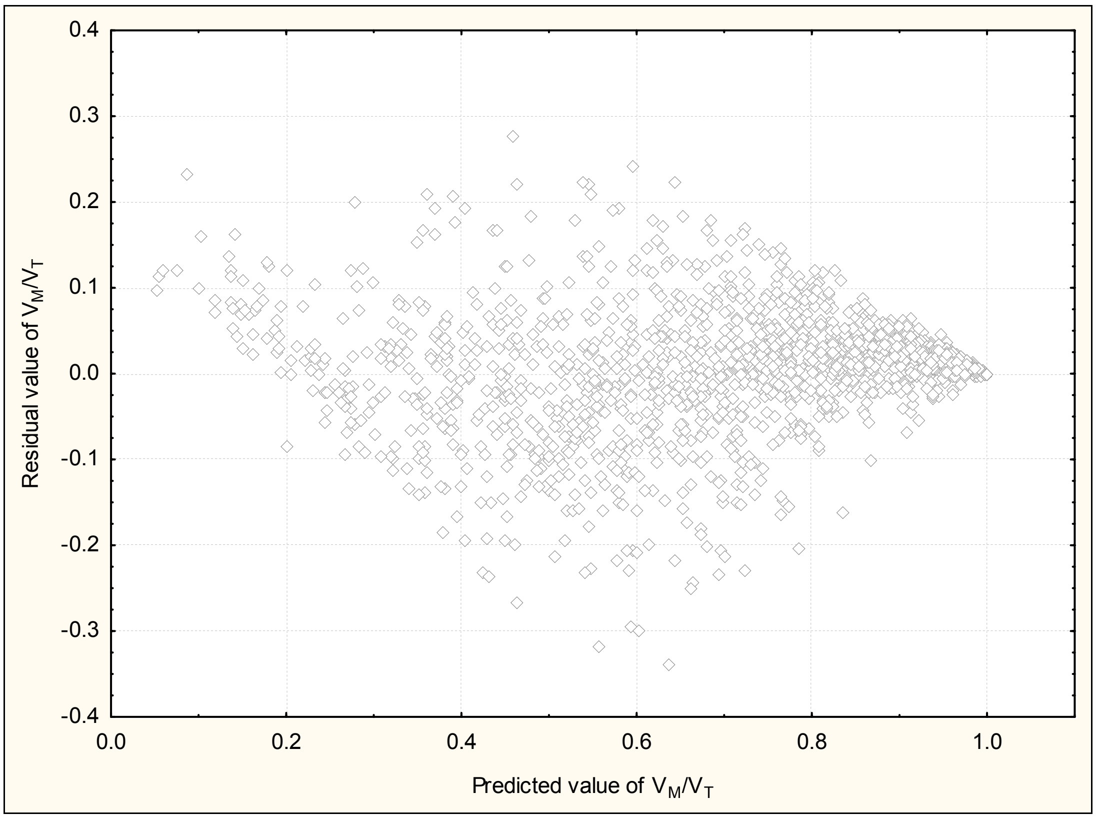


Figure 11. Predicted and residual values of volume ratio (V_M/V_T) for Model 1.2.

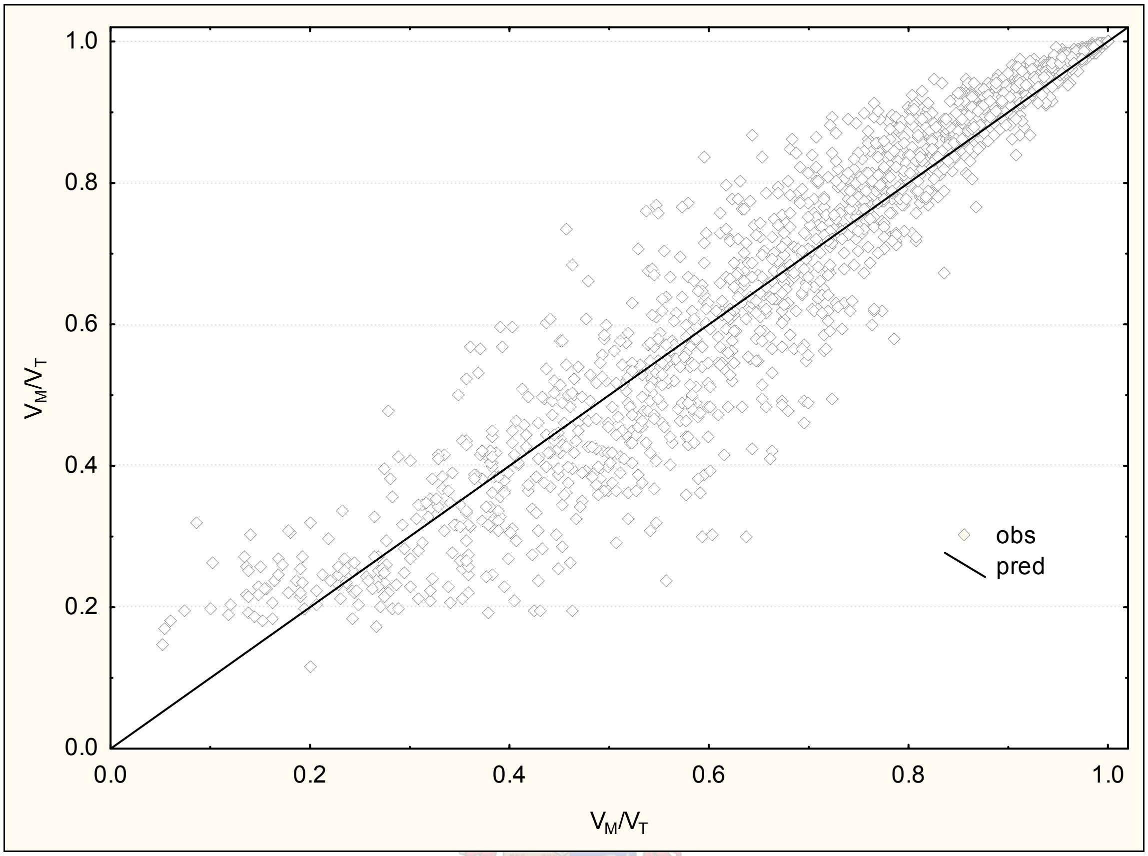


Figure 12. Plot of observed and predicted values of the volume ratio estimated by means of Model 1.2.

It is evident from Figures 11 and 12 that the equation is biased in the lower section of the stem. This is due to the equation being unable to cater for the lower inflection point on the stem profile. Although all data points below breast height were omitted from the sample data, it is still obvious that the volume ratio is overestimated in the lower sections of the stem profile.

Since this equation will seldom be used to estimate volumes below 25 % of the total height of the tree, the bias is not considered to be of much practical importance.

The results from fitting **Model 1.4** are presented in Table 7. From these fit statistics, the model clearly demonstrates the ability to predict merchantable volume to an upper stem diameter from the dbh and height values. In Figure 13 the variance clearly increases with increasing tree size. Although this trend is observed, there is no tendency in the residuals that would indicate that this model is not suitable to predict the volume to an upper stem diameter limit.

Table 7. Results from fitting Model 1.4.

Model	Dependent variable	Independent variables	Parameter estimates and their standard errors (in brackets)		R^2	*MSE*	*n*
			b_0	b_1			
1.4	Merchantable volume underbark	d_{min} ,dbh and total height	0.000007 (0)	1.5266 (0.0194)	0.936	0.0576	1756

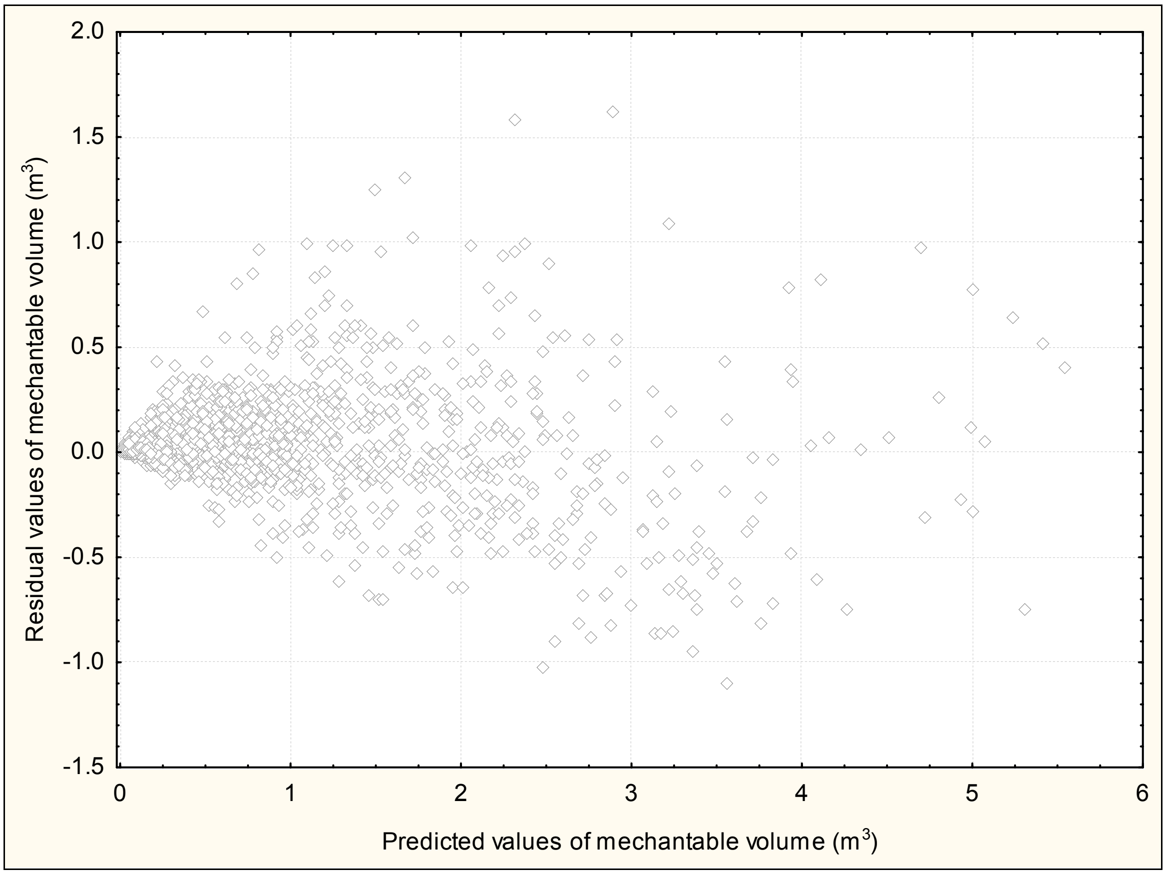


Figure 13. Predicted and residual values of merchantable volume by Model 1.4.

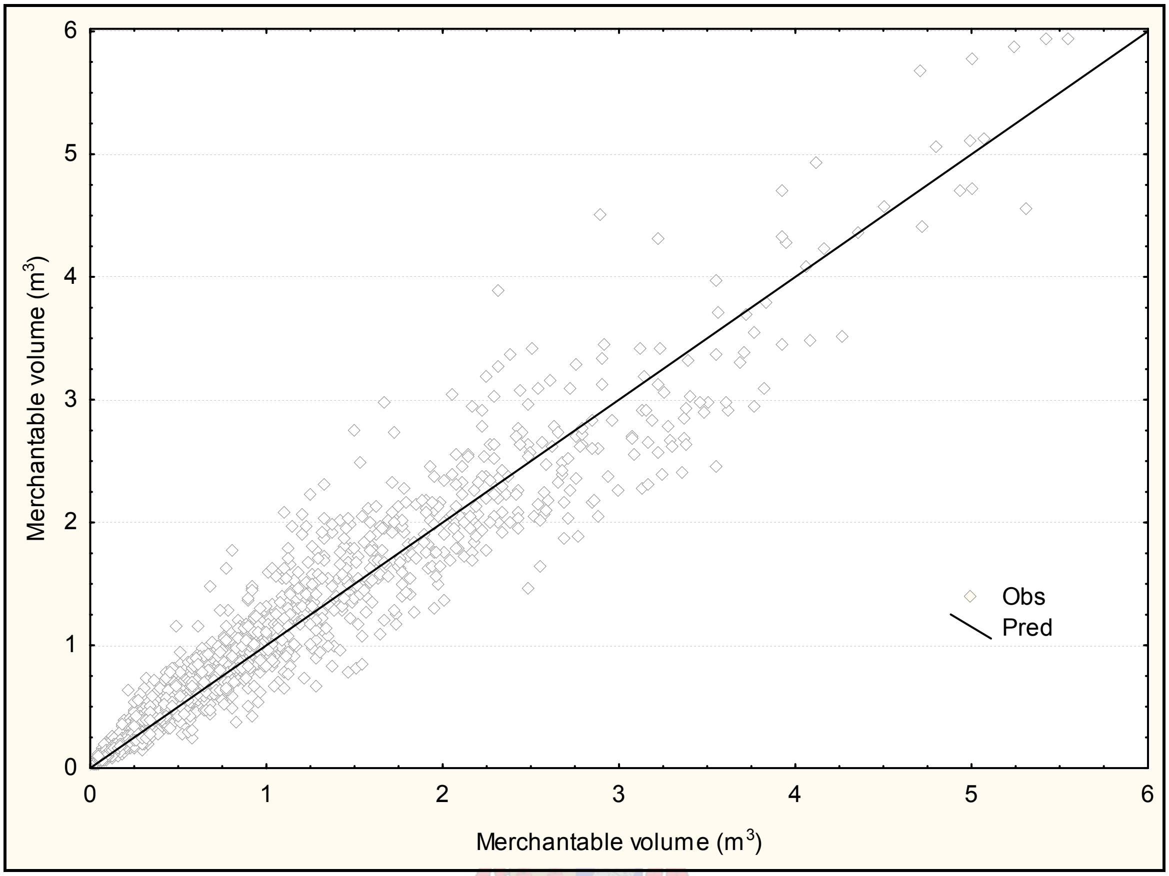


Figure 14. Observed and predicted values of merchantable height estimated by means of Model 1.4.

5.2.2 Merchantable volume to upper stem height limit

The results from fitting various merchantable volumes and upper stem height limits with Model 1.3 are provided in Table 8. The fit statistics indicate that the model fits the data very well and diagnostic tests indicated that no influential observations are present. By studying the residual graph in Figure 15 and the observed and predicted values of the volume ratios in Figure 16, it is evident that this model produces more accurate and precise predictions than the volume ratio to a diameter limit model (Model 1.2). No trends can be discerned, even for very small values of V_M/V_T. These results are consistent with those reported by Byrne and Reed (1986), and Cao, Burkhart and Max (1980).

Table 8. Results from fitting Model 1.3.

Model	Dependent variable	Independent variables	Parameter estimates and their standard errors (in brackets)			R^2	*MSE*	*n*
			b_0	b_1	b_2			
1.3	Merchantable volume overbark	h_{min} and total height	0.7208 (0.0144)	2.3660 (0.0110)	2.2822 (0.0142)	0.986	0.000769	1756
1.3	Merchantable volume underbark	h_{min} and total height	0.7716 (0.0151)	2.3666 (0.0120)	2.3021 (0.0139)	0.987	0.000755	1756

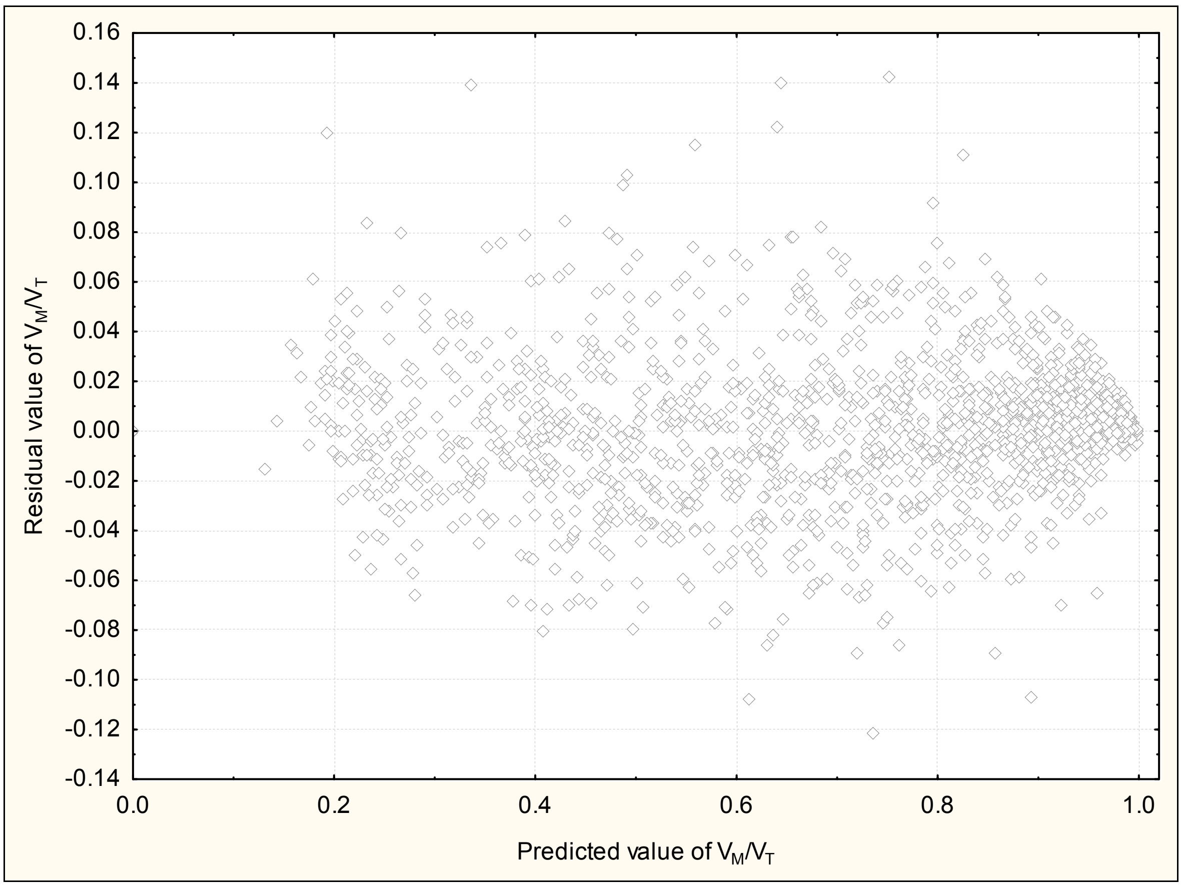


Figure 15. Predicted and residual values of the volume ratio (V_M/V_T) estimated by means of Model 1.3.

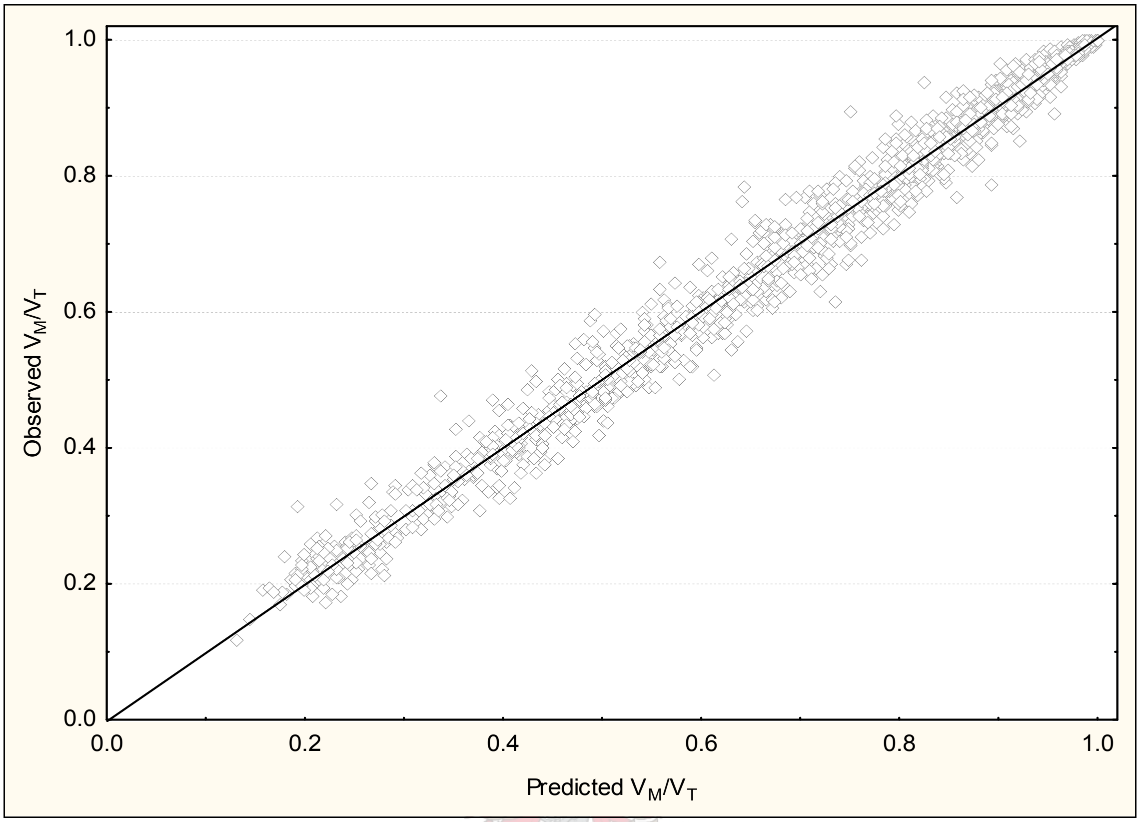


Figure 16. Observed and predicted values of the volume ratio (V_M/V_T) to upper stem heights as estimated by Model 1.3.

5.2.3 Merchantable volume equation to upper stem diameter of 7.5 cm

For every species there is a fixed upper stem diameter limit to which it will be possible to utilise the tree stem economically (the utilisable part made up of different log classes and products). The rest of the tree constitutes the unutilisable tip. Although not the same for all species, this limit has been positioned at 7.5 cm for most of the commercial species in South Africa (Bredenkamp, 2000). Since this upper limit will be in a similar range for teak, a merchantable volume equation with an upper stem diameter limit of 7.5 cm was fitted for teak.

Model 1.1 was fitted with data obtained from subtracting the volume underbark to an upper stem diameter limit of 7.5 cm from the total tree volume underbark. Using Model 1.1 with these parameters allows the direct calculation of underbark volume to a standard limit of utilisation without the need to use the volume ratio equations discussed above. Diagnostic tests indicated that no influential observations were present.

Table 9. Results from fitting Model 1.1 to derive volume underbark to a 7.5 cm upper stem diameter.

Model	Dependent variable	Independent variables	Parameter estimates and their standard errors (in brackets)			R^2	MSE	*n*
			b_0	b_1	b_2			
1.1	Merchantable volume underbark to a 7.5 cm upper stem diameter limit	Dbh and total height	-11.4585 (0.0995)	1.9076 (0.0466)	1.4235 (0.0774)	0.99	0.0175	222

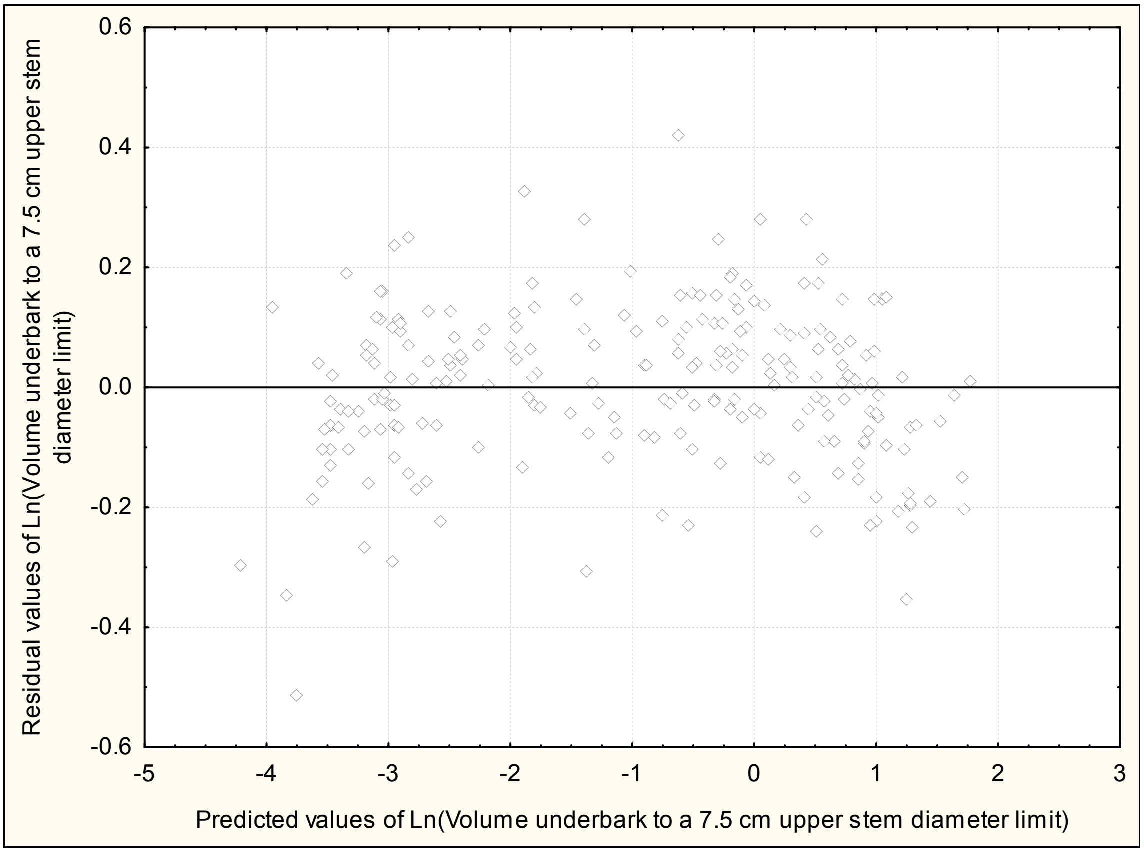


Figure 17. Predicted and residual values of ln(volume underbark to an upper stem diameter limit of 7.5 cm) as estimated by Model 1.1.

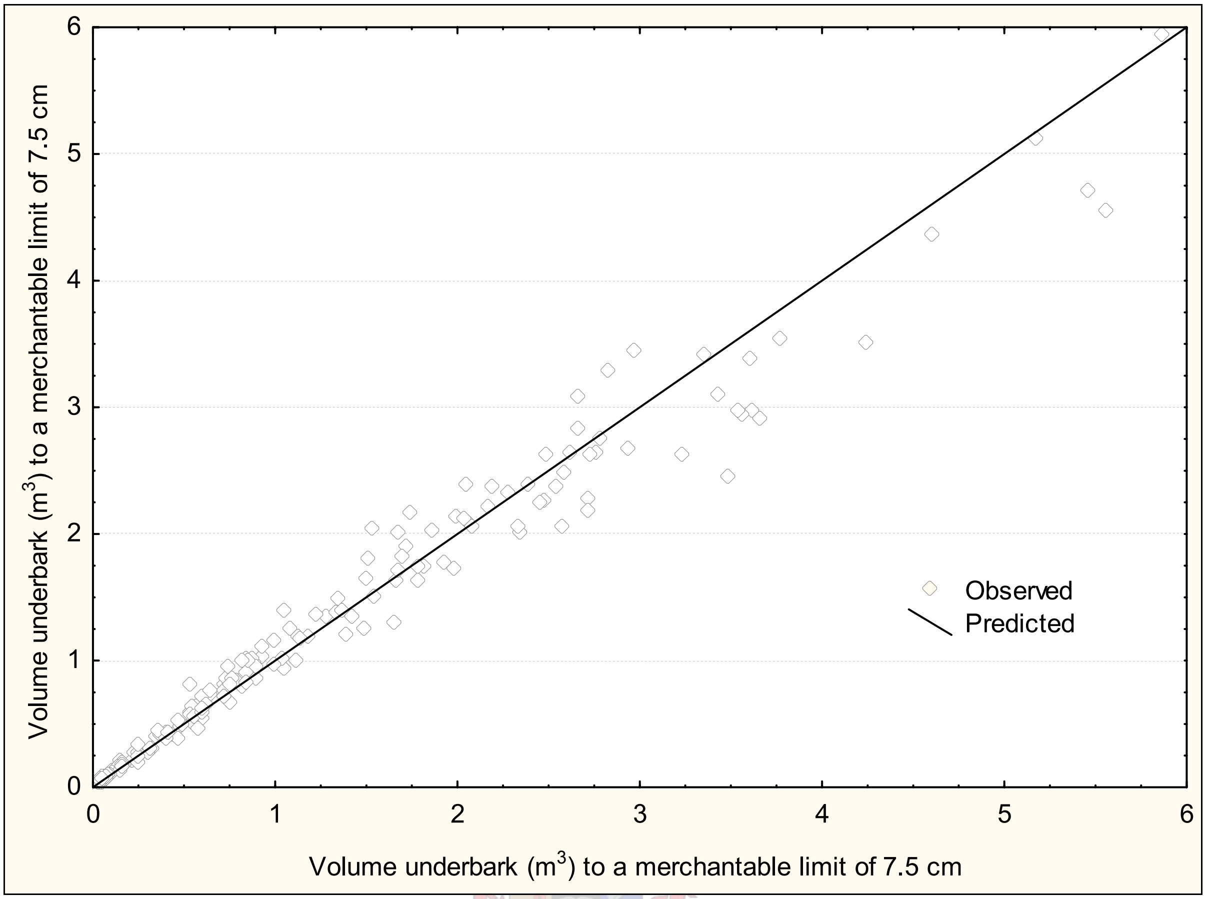


Figure 18. Observed and predicted values of volume underbark to an upper stem diameter limit of 7.5 cm as estimated by Model .1.1.

5.3 Taper models

Models 2.1 to 2.6 were fitted with the same stemprofile data set[25] by means of the **PROC NLIN** procedure in SAS using the **DUD** iteration method where derivatives of models are not required. The **MARQUARD** iteration method, where the derivatives are specified analytically was used only for Model 2.1.

After fitting Model 2.1, plotting the predicted values on the observed values (Figure 26) indicated that the individual model segments are not very flexible and did not track the data adequately, the middle segment in particular. In order to improve the flexibility, the model was refitted by estimating the upper and lower join points visually and fixing these values for the regression procedure.

[25] The method of collection and characteristics of this dataset was described in Sections 3.2 and 3.3.

By iteratively fitting the model with different combinations of fixed join points, it was possible to obtain join points and parameter estimates that minimised the *MSE*. Although the overall *MSE* is similar to the model without the join points fixed, the improvement in flexibility in the middle section of the tree is clearly shown in Figure 27. Model 2.1 was labelled Model 2.1.1 for the version fitted with fixed join points and Model 2.1.2 for the model without fixed join points.

Models 2.2 and 2.3 require that a single inflection point be estimated. The position of this inflection point (p) has been found to have very little effect on the prediction accuracy of the model (Perez, Burkhart and Stiff, 1990; Bi, 2000). In order to determine the position of the inflection point and whether the position thereof affects the accuracy, Models 2.2 and 2.3 were refitted with the inflection point set respectively at relative heights of 0.15, 0.25, 0.35 and 0.45. Results for both models indicated very little change in the *MSE* values, but a value of 0.25 for the inflection point provided the best results in terms of the *MSE*. This is in agreement with results from similar studies (Perez, Burkhart and Stiff, 1990; LeMay *et al.*, 1993).

After fitting Model 2.4 and scrutiny of plots of observed and predicted values (Figure 30), it was clear that this model is unsuitable to describe teak taper. It was therefore not included for further comparison with the rest of the models.

Table 10 presents the parameter estimates, their standard errors and the *MSE* and R^2 values for each model tested. Residual graphs are provided in Figures 19 to 25 for each model and the following observations can be made:

- Although the residuals are distributed evenly at the tip and butt section by **Model 2.1.1** (Figure 19), a clear trend can be observed in the middle section of the tree. This observation is confirmed by the same apparent trend in the middle section of the tree by **Model 2.1.2** (Figure 20).
- The residual graph of **Model 2.2** (Figure 21) indicates no apparent trends, with the variance increasing at the butt end of the tree. Some underestimates might occur at the tip section of the tree.

- The variance of the residual values of **Model 2.3** (Figure 22) increases at the tip of the tree. No apparent trend can be observed.
- The residual values in Figure 23 indicate that **Model 2.4** is not suitable to describe teak diameter along the stem. The variance clearly increases at the tip section and from the middle of the tree downwards.
- The variance of the residuals predicted by **Model 2.5** (Figure 24) remains constant with an increase only occurring below breast height. A clear trend can be observed in the lower stem around breast height.
- **Model 2.6** (Figure 25) predicts diameter very well over the entire stem with only a slight increase in variance below breast height. Slight overestimates may occur in the butt section of the tree.

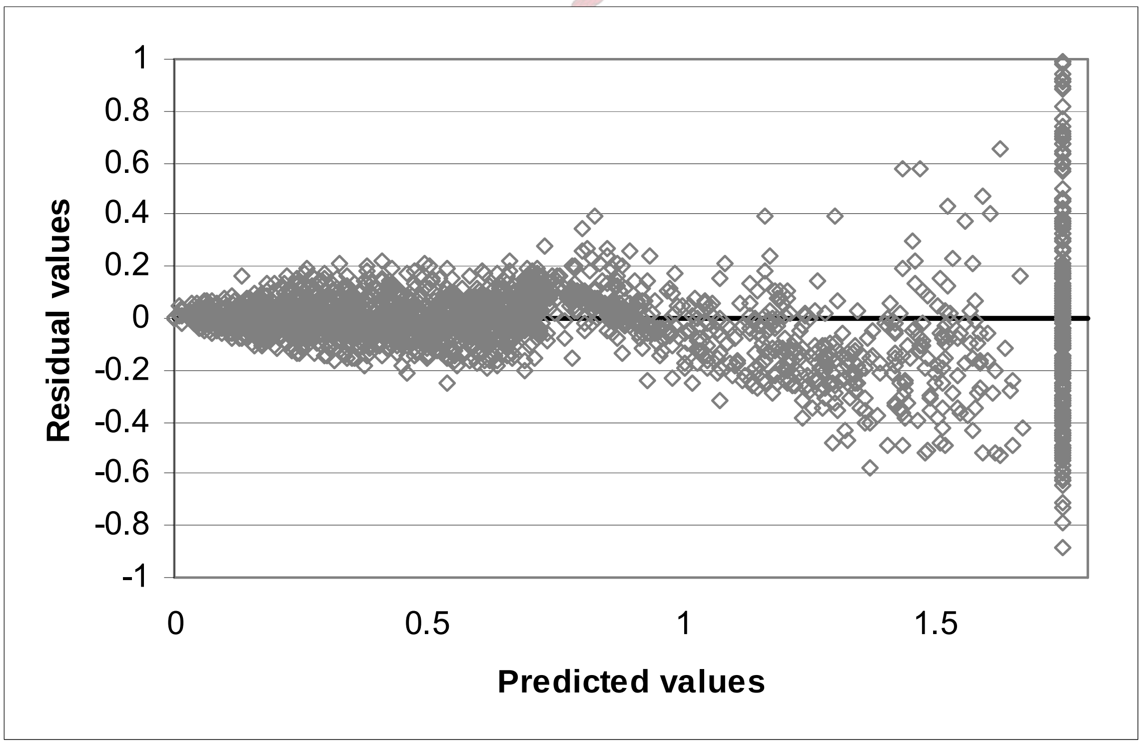


Figure 19. Predicted and residual values for model 2.1.1.

Model	Dependent variable	Parameter estimates									R^2	*MSE*	*n*
		a_1	a_2	b_0	b_1	b_2	b_3	b_4	b_5	b_6			
2.1.1	D_i^2/D^2	0.8	0.07	-4.3733 (1.0756)	2.1901 (0.6293)	-2.1858 (0.7069)	197.8000 (3.7207)				0.879	0.0364	2617
2.1.2	D_i^2/D^2	0.8995 (2.842)	0.0387 (0.0011)	-2.8387 (81.956)	1.2939 (43.1308)	-0.7719 (43.1195)	614.7000 (34.9473)				0.888	0.0336	2617
2.6.	d_i/D	0.2790 (0.177)	0.0546 (0.00261)	0.7785 (0.0405)	0.6793 (0.0118)	0.9281 (0.0112)	66.9528 (3.9686)	2.4141 (0.1237)			0.956	0.0057	2617
2.2.	d_i	0.5426 (0.026)	1.1212 (0.0179)	0.9965 (0.00045)	0.4534 (0.0505)	-0.0468 (0.00813)	-1.0257 (0.137)	0.4503 (0.0615)	0.1250 (0.00918)		0.979	7.6428	2617
2.3.	*ln (d)*			0.6770 (0.0115)	1.0269 (0.00513)	0.6205 (0.0219)	-0.1068 (0.00311)	0.1323 (0.00822)			0.960	0.0211	2617
2.5.	d_i			0.0935 (0.0969)	1.1490 (0.0522)	0.2244 (0.0159)	-0.1439 (0.0575)	0.00373 (0.00016)	-0.0502 (0.00791)	-0.1044 (0.015)	0.911	0.0082	2617
2.4.	d_i			1.2147 (0.0314)	1.0098 (0.00665)	8.3546 (0.2567)	-10.9200 (0.285)	4.4400 (0.0805)			0.956	15.83	2617

Table 10. Parameter estimates and standard errors (in brackets) for the taper Models 2.1.1 to 2.6.

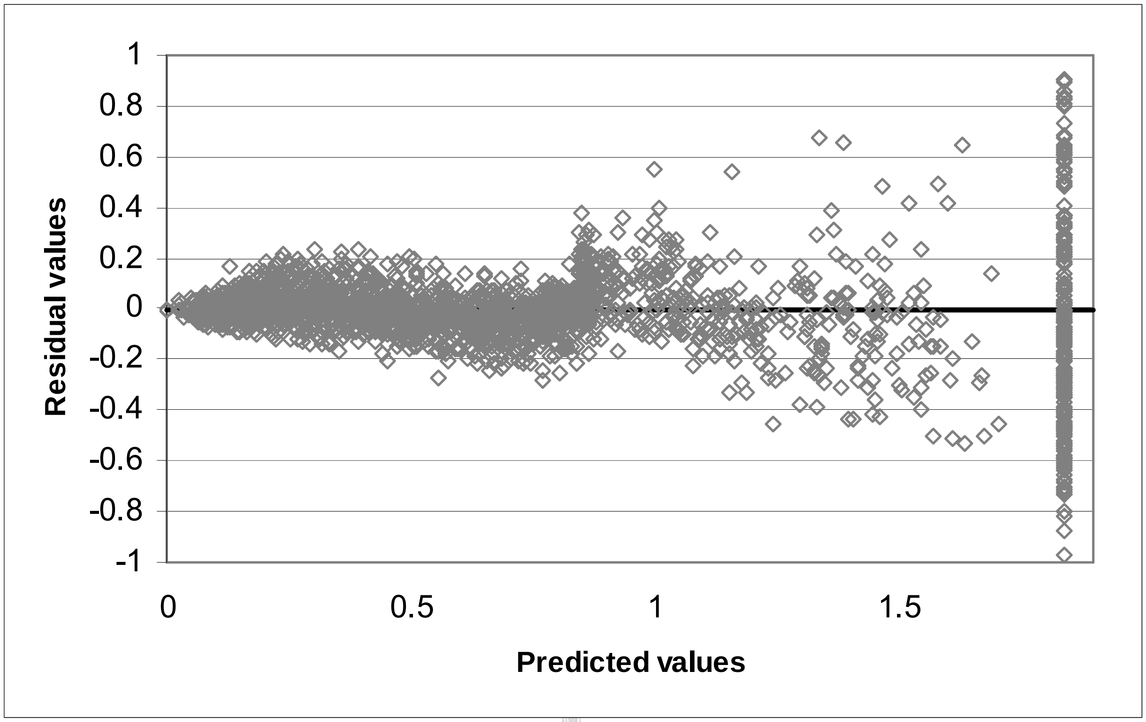


Figure 20. Predicted and residual values for Model 2.1.2.

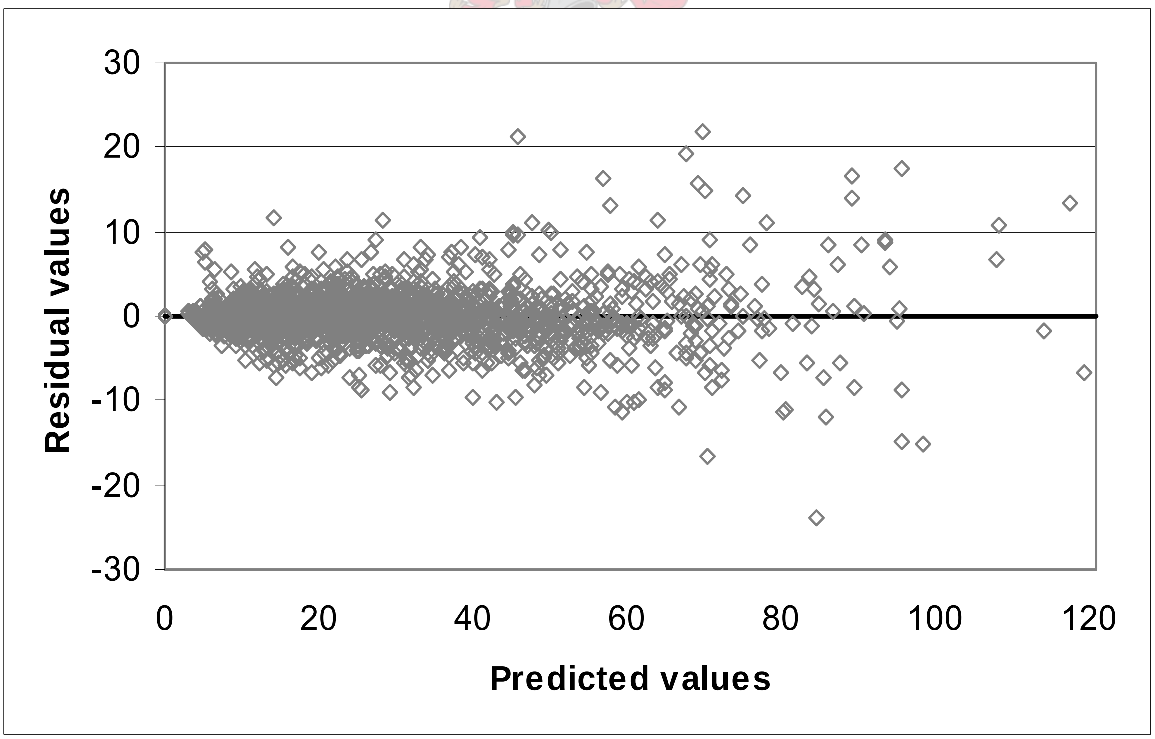


Figure 21. Predicted and residual values for Model 2.2.

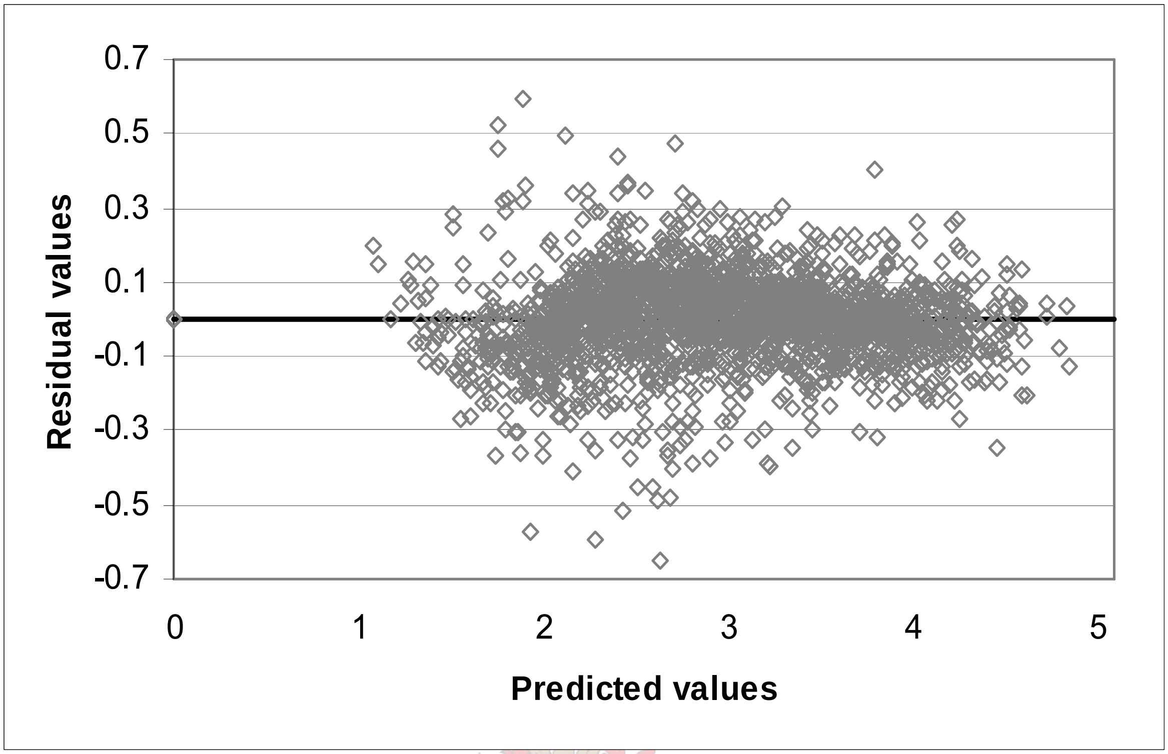


Figure 22. Predicted and residual values for Model 2.3.

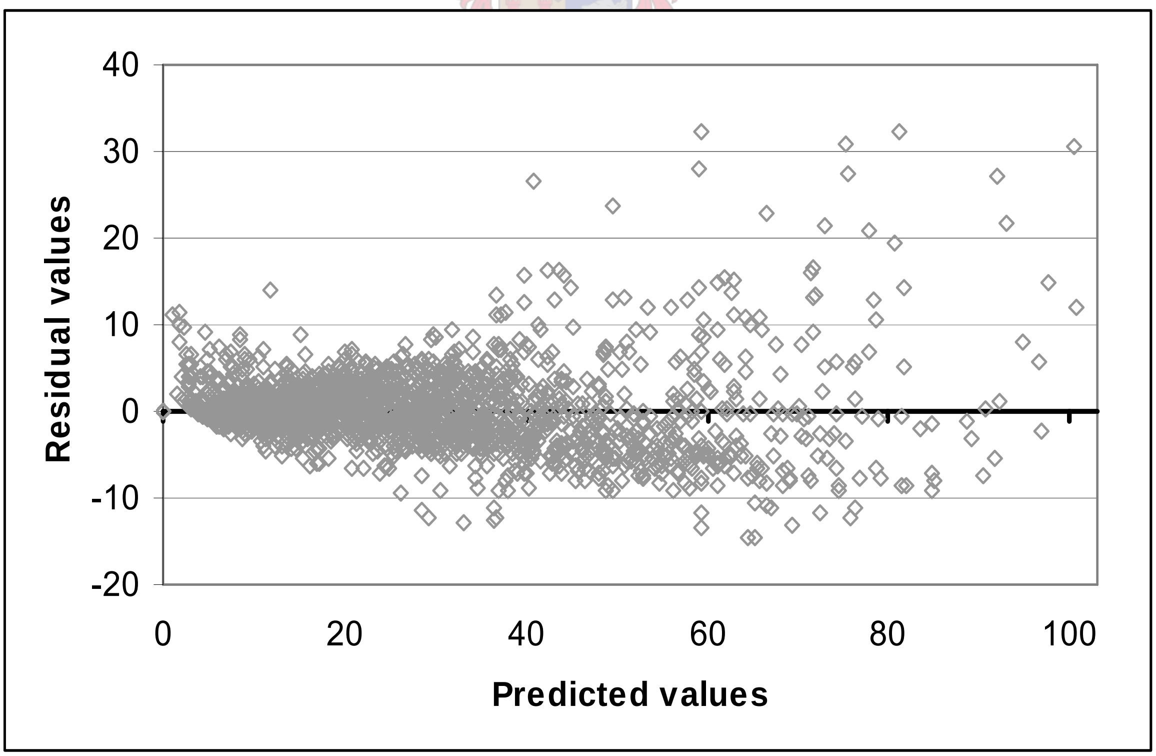


Figure 23. Predicted and residual values for Model 2.4.

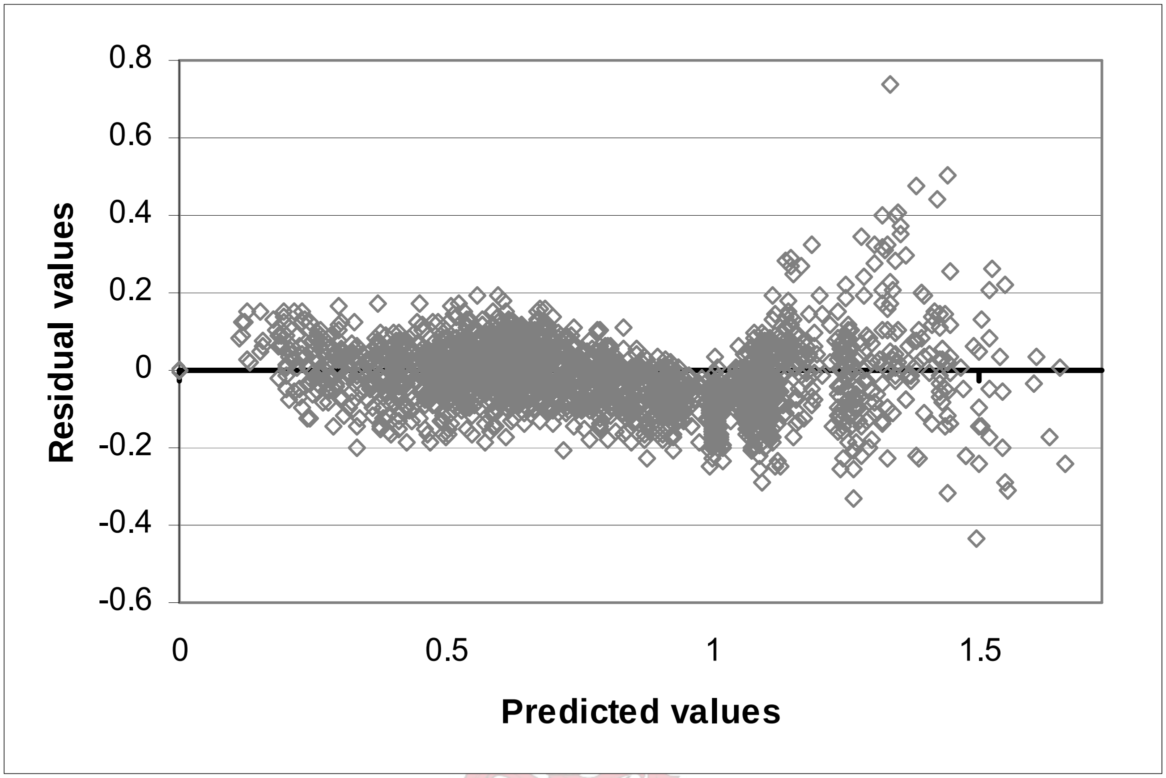


Figure 24. Predicted and residual values for Model 2.5.

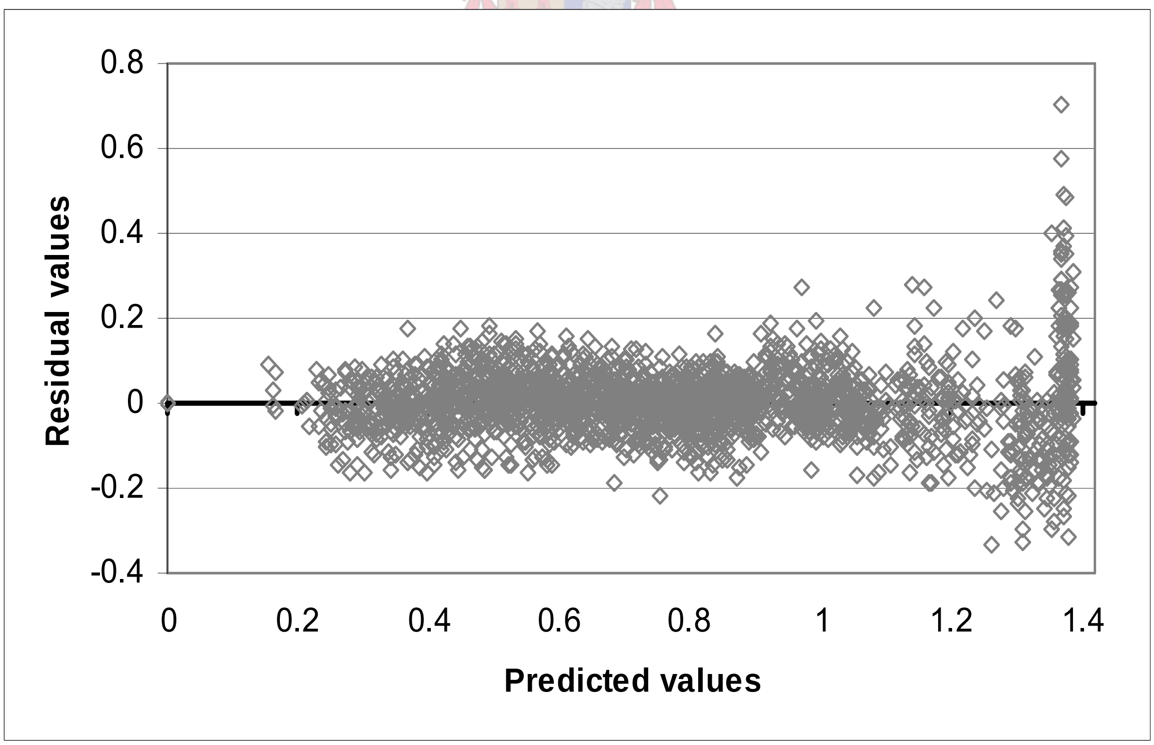


Figure 25. Predicted and residual values for Model 2.6.

5.3.1 Assessment of the ability to predict

In order to asses the predictive ability of the various models; the total bias, total *SEE* and their relative values were calculated for each model (Table 11) according to the predicted total volume underbark and underbark diameters. Rankings[26] of the absolute values of the abovementioned statistics are also included in Table 11 to facilitate comparison of the models. It is clear that Model 2.3 has the lowest ranking and is hence the most accurate in terms of these overall values.

Although it is easy to assess the overall accuracy with these values, Figures 19 to 25 (residuals) and Figures 26 to 32 (observed and predicted) indicate that these values (Table 11) could in fact be useless if not combined with an evaluation of the accuracy along the entire stem and for different size classes (e.g. dbh classes). This is due to the negative and positive biases at different parts of the stem or for different size classes that can average to produce a total bias of close to zero and a low *SEE* value.

For this reason the biases and *SEE* of the estimated underbark diameters were calculated for 10 percent height classes along the stem, from the ground to the tip (Table 12). For total stem volume underbark, the biases and *SEE* were calculated for 10 cm dbh classes (Table 13). The total volume underbark was calculated from the underbark diameter values predicted by Models 2.1.1 to 2.6. As the Newton section volume equation is more accurate than the Smalian equation, it was used to calculate the section volumes from the diameter values and section lengths (Husch, 1963). Section lengths were taken at 0.5 % intervals up to 10 % of total tree height and 5 % intervals for the rest of the tree. The volume from all the sections were added together to yield the total volume of the tree.

The Newton volume equation:

$$V_{li} = \frac{\pi l\left(d_1^2 + 4d_m^2 + d_2^2\right)}{24} \qquad \text{...[44]}$$

where: V_i = volume of the i^{th} section (m^3)

l = length of the i^{th} section (m)

d_1 = thick end diameter (m)

d_m = diameter at mid-length of the log section (m)

d_2 = thin end diameter (m)

[26] Rank one (lowest) is given to the best model and six (highest) to the poorest model.

Table 11. Total bias, *SEE* and their respective percentage values for the ability of each model to predict underbark diameter to upper stem heights and total volume for various dbh classes with ranks according to the bias and *SEE*.

Total volume underbark for various diameter at breast height classes						
Model	**Bias**	***Rank***	**%Bias**	***SEE***	***Rank***	***%SEE***
Model 2.1.1.	0.002	***2***	-0.721	0.180	***3***	12.057
Model 2.1.2.	-0.011	***3***	-2.110	0.186	***4***	12.387
Model 2.2.	0.037	***6***	2.490	0.172	***2***	11.335
Model 2.3.	-0.002	***1***	1.077	0.190	***5***	11.672
Model 2.5.	0.024	***4***	-6.050	0.172	***1***	16.669
Model 2.6.	-0.031	***5***	-0.197	0.208	***6***	11.799

Diameter underbark to various upper stem heights						
Model	**Bias**	***Rank***	**%Bias**	***SEE***	***Rank***	**%SEE**
Model 2.1.1.	-1.561	***5***	-1.561	3.365	***2***	12.577
Model 2.1.2.	-1.527	***4***	-1.527	10.669	***5***	12.769
Model 2.2.	0.653	***2***	0.653	13.069	***6***	13.345
Model 2.3.	0.144	***1***	0.144	3.160	***1***	11.929
Model 2.5.	-2.482	***6***	-2.482	3.612	***3***	14.413
Model 2.6.	-0.967	***3***	-0.967	4.737	***4***	14.220

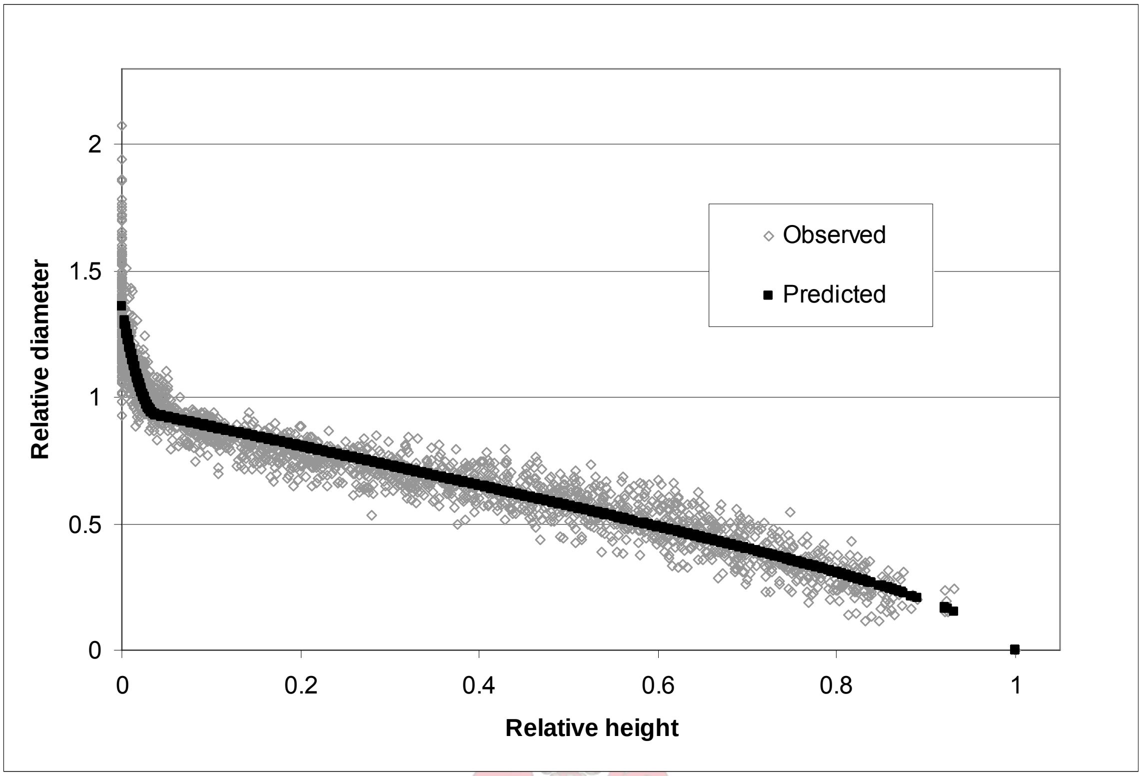


Figure 26. Observed and predicted taper by Model 2.1.2.

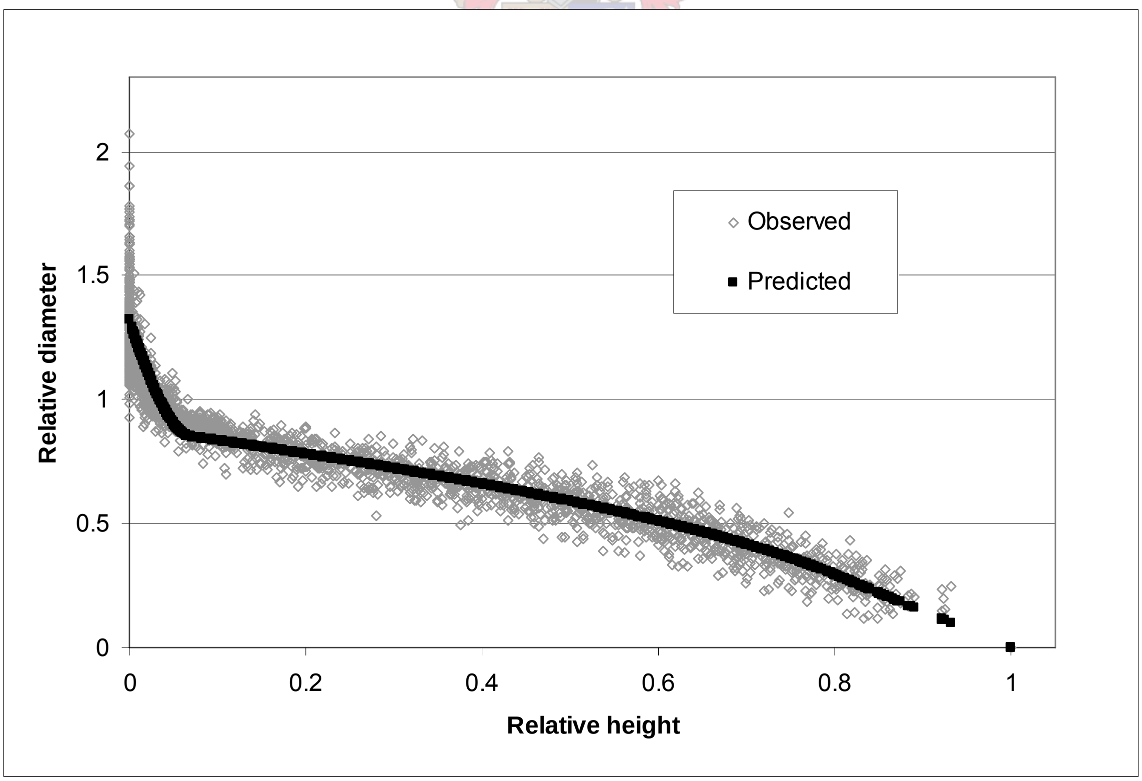


Figure 27. Observed and predicted taper by Model 2.1.1.

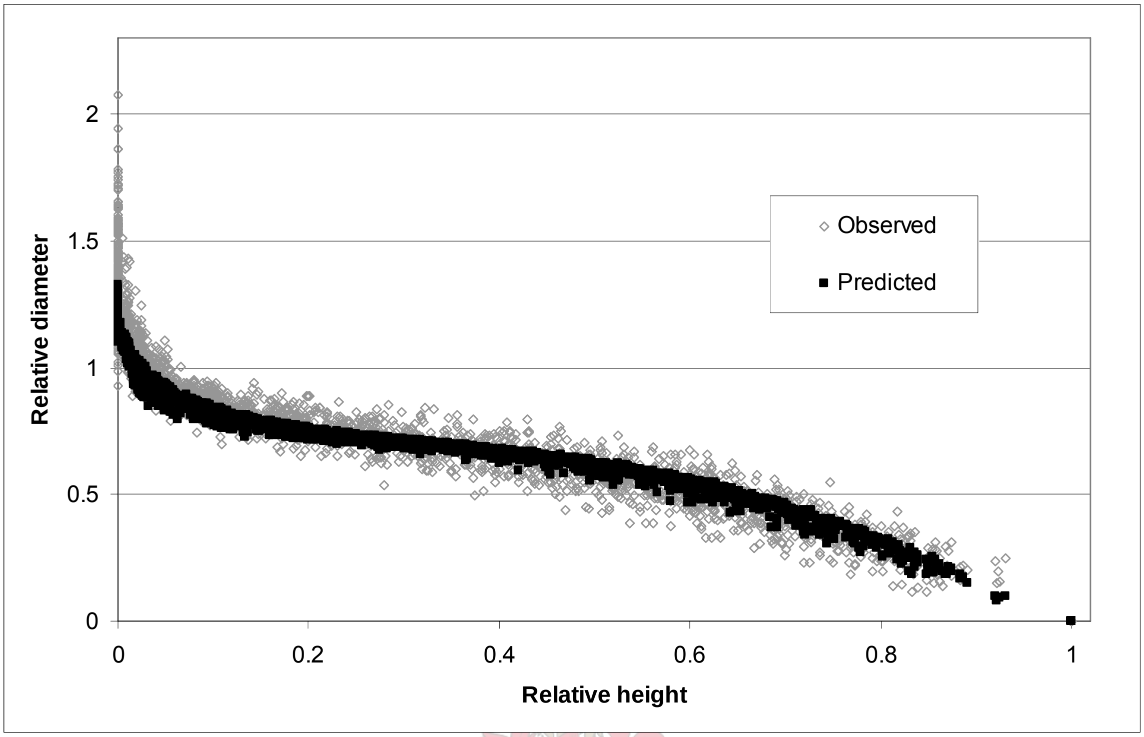


Figure 28. Observed and predicted taper by Model 2.2.

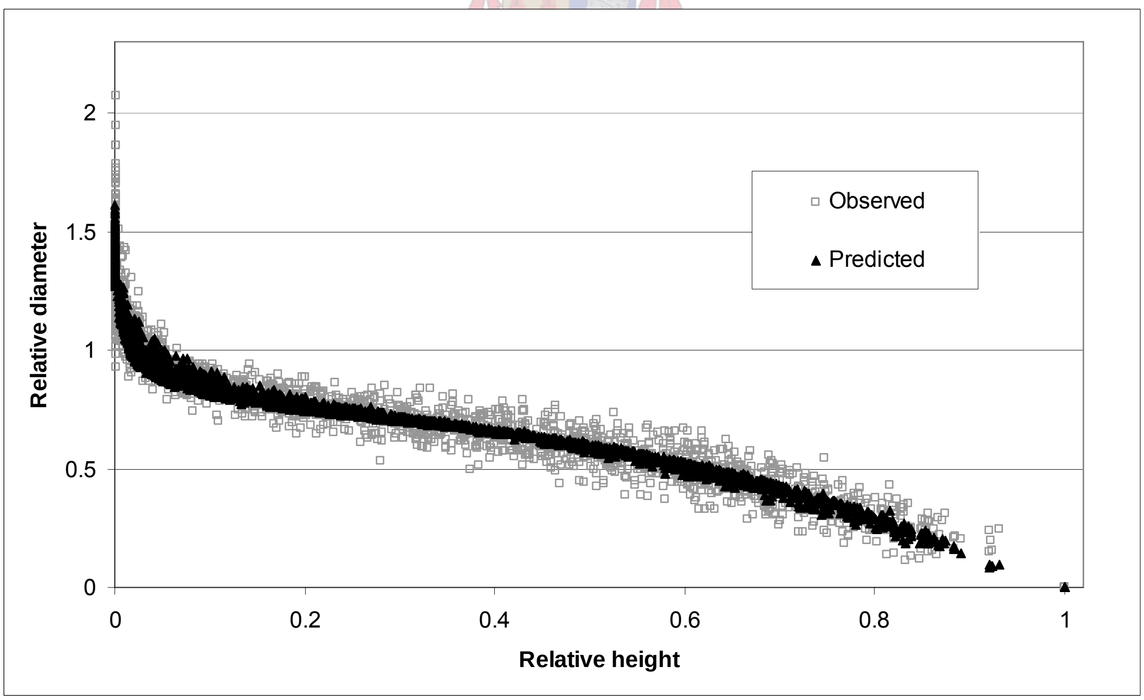


Figure 29. Observed and predicted taper by Model 2.3.

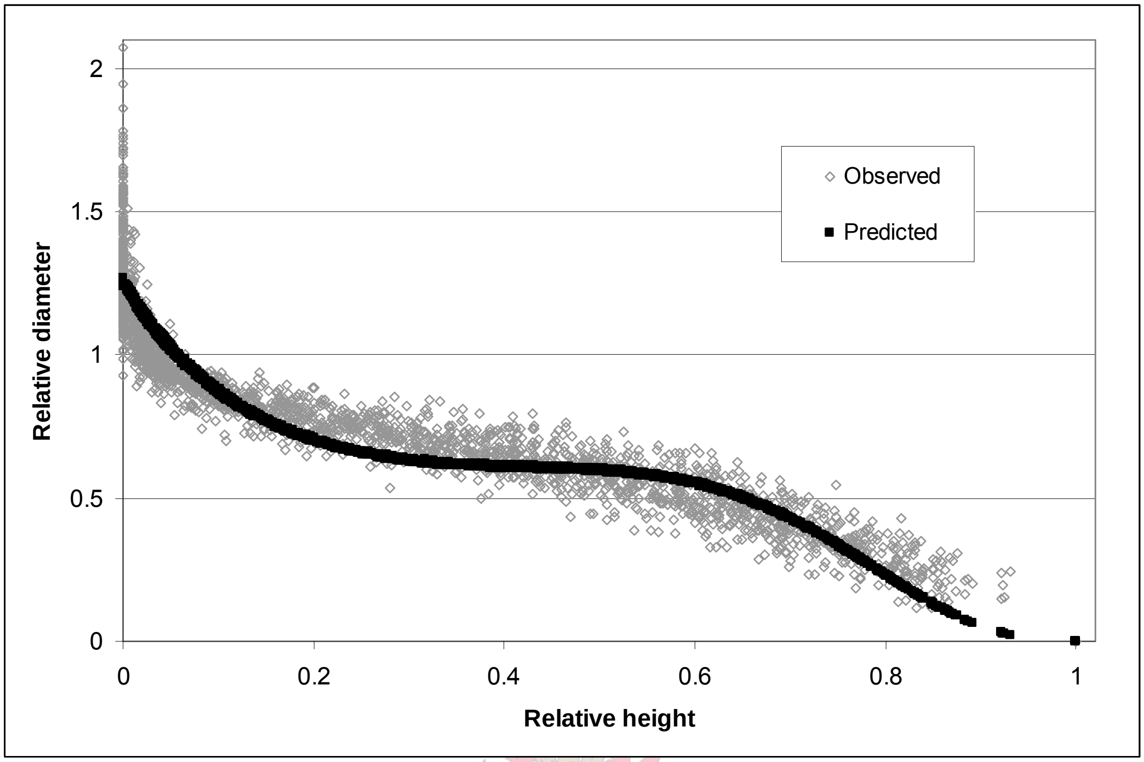


Figure 30. Observed and predicted taper by Model 2.4.

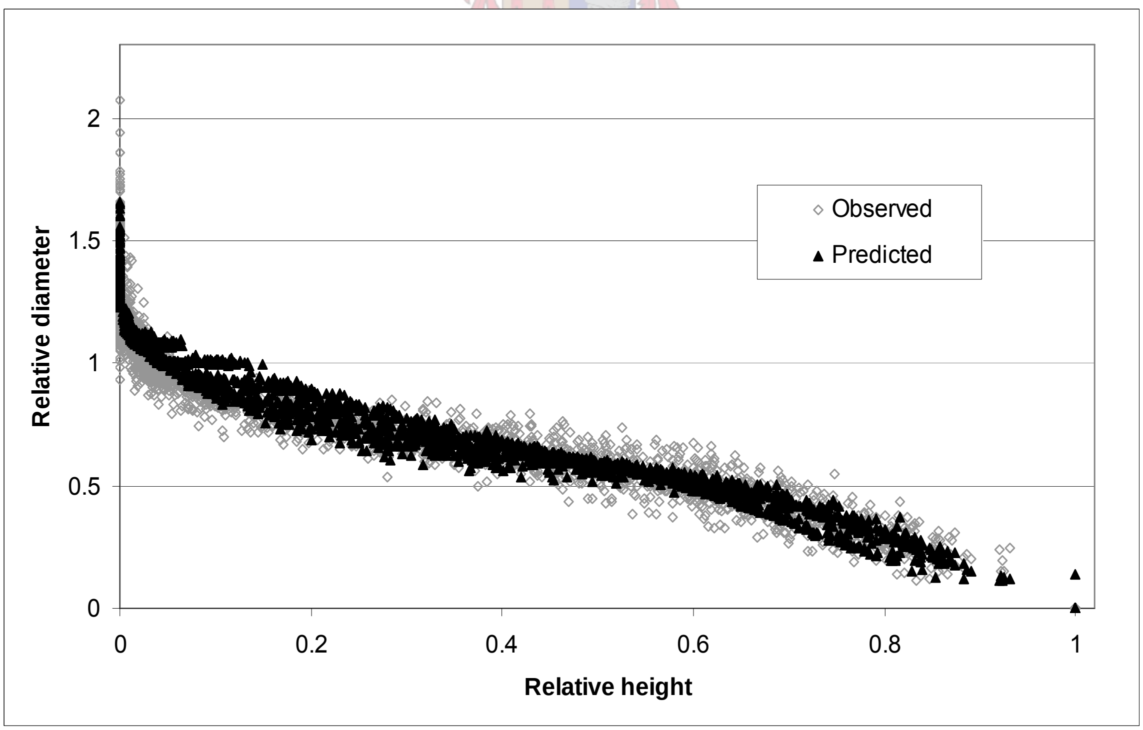


Figure 31. Observed and predicted taper by Model 2.5.

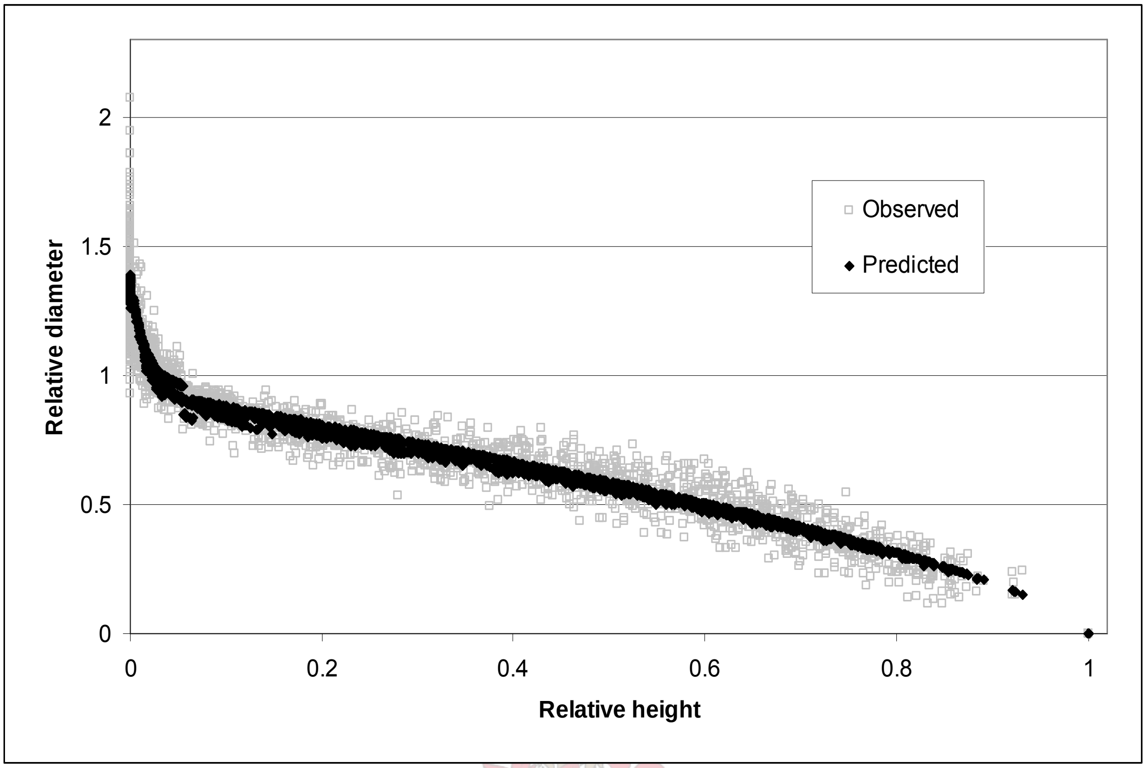


Figure 32. Observed and predicted taper by Model 2.6.

Table 12. Biases and standard errors of estimate of diameter underbark from ground to top.

		Model 2.1.1.				Model 2.1.2.			
Relative height class	*n*	Bias (cm)	% Bias	SEE (cm)	%SEE	Bias (cm)	% Bias	SEE (cm)	%SEE
< 5%	727	-0.057	-2.569	5.014	10.211	0.693	-0.245	5.663	9.647
5%<10%	211	1.281	4.662	2.036	6.447	-0.219	-0.530	1.877	4.940
10%<20%	241	-0.018	0.432	2.025	6.280	-1.139	-4.185	1.965	7.690
20%<30%	192	-0.402	-1.752	2.119	7.732	-0.915	-4.015	1.935	8.680
30%<40%	199	-0.348	-2.536	2.023	8.819	-0.316	-2.413	1.790	8.796
40%<50%	196	0.188	-1.235	2.150	10.209	0.621	1.043	2.218	9.919
50%<60%	187	0.056	-2.075	2.551	13.055	0.746	1.834	2.459	12.535
60%<70%	207	-0.019	-3.223	2.707	16.960	0.615	1.120	2.634	16.025
70%<80%	144	-0.315	-5.885	2.831	21.585	-0.177	-4.888	2.800	21.216
80%<90%	86	0.122	-2.120	2.991	30.944	-0.990	-14.549	2.848	37.265
90%<100%	5	0	0	0	0	0	0	0	0

		Model 2.2.				Model 2.3.			
Relative height class	***n***	**Bias (cm)**	**% Bias**	**SEE (cm)**	**%SEE**	**Bias (cm)**	**% Bias**	**SEE (cm)**	**%SEE**
< 5%	727	2.879	6.558	6.867	10.375	0.964	2.318	4.449	8.766
5%<10%	211	0.919	4.669	1.749	7.139	-0.115	1.576	2.404	6.469
10%<20%	241	0.628	4.189	4.277	7.809	-0.213	1.074	2.447	7.351
20%<30%	192	0.367	2.117	4.269	7.550	-0.212	-0.062	2.201	7.731
30%<40%	199	-0.131	-1.339	2.132	7.954	-0.239	-1.675	1.983	8.343
40%<50%	196	-0.117	-3.450	3.872	11.071	0.182	-1.571	2.153	10.261
50%<60%	187	-0.550	-6.801	2.591	15.167	0.101	-2.810	2.480	13.420
60%<70%	207	-0.589	-9.035	3.852	19.664	0.236	-3.023	2.632	16.738
70%<80%	144	-0.422	-9.800	3.722	23.720	0.287	-3.306	2.811	20.398
80%<90%	86	0.130	-4.298	4.404	28.600	0.598	1.046	2.877	27.062
90%<100%	5	0	0	0	0	0	0	0	0

		Model 2.5.				Model 2.6.			
Relative height class	***n***	**Bias (cm)**	**% Bias**	**SEE (cm)**	**%SEE**	**Bias (cm)**	**% Bias**	**SEE (cm)**	**%SEE**
< 5%	727	0.805	-0.765	4.661	10.266	0.655	-0.126	5.288	11.590
5%<10%	211	-2.023	-8.854	1.761	11.105	-0.034	1.041	2.642	5.235
10%<20%	241	-1.148	-7.919	2.553	11.831	-0.541	-1.029	2.146	9.799
20%<30%	192	0.331	-2.510	2.371	10.661	-0.535	-1.766	2.331	11.053
30%<40%	199	0.741	-0.420	2.024	10.770	-0.392	-1.653	2.310	8.304
40%<50%	196	0.902	0.317	2.269	11.429	0.585	1.373	2.839	11.856
50%<60%	187	0.109	-2.062	2.662	12.617	0.380	0.853	2.440	12.176
60%<70%	207	-0.307	-2.845	2.781	16.769	0.424	0.180	2.775	17.408
70%<80%	144	-0.403	-1.411	2.825	25.108	-0.277	-5.453	3.139	23.753
80%<90%	86	-0.098	-1.228	3.286	38.274	-0.925	-14.844	3.390	40.316
90%<100%	5	0	0	0	0	0	0	0	0

Table 10. Biases and standard errors of estimate of total volume underbark by dbh classes.

		Model 2.1.1.				Model 2.1.2.			
Dbh class	**n**	**Bias (m^3)**	**% Bias**	**SEE (cm)**	**% SEE**	**Bias (m^3)**	**% Bias**	**SEE (cm)**	**% SEE**
10<20	75	-0.002	-4.418	0.010	10.798	-0.003	-5.858	0.010	12.969
20<30	37	0.008	-1.019	0.064	14.667	0.002	-2.412	0.063	14.571
30<40	37	0.066	6.736	0.117	12.294	0.054	5.450	0.110	11.323
40<50	34	0.097	4.742	0.209	12.959	0.074	3.428	0.197	11.129
50<60	20	-0.028	-1.950	0.296	13.478	-0.063	-3.356	0.304	12.418
60<80	15	-0.316	-9.971	0.670	19.707	-0.372	-11.487	0.717	22.307

		Model 2.2.				Model 2.3.			
Dbh class	**n**	**Bias (m^3)**	**% Bias**	**SEE (cm)**	**% SEE**	**Bias (m^3)**	**% Bias**	**SEE (cm)**	**% SEE**
10<20	75	0.002	1.693	0.010	10.798	0.001	0.178	0.010	10.582
20<30	37	0.006	-1.141	0.066	14.667	0.014	0.763	0.065	13.701
30<40	37	0.060	6.035	0.120	12.294	0.070	7.256	0.121	12.429
40<50	34	0.117	5.773	0.239	12.959	0.091	4.435	0.207	11.538
50<60	20	0.068	1.718	0.360	13.478	-0.052	-2.887	0.303	12.283
60<80	15	0.032	-1.157	0.701	19.707	-0.382	-11.672	0.728	22.465

		Model 2.5.				Model 2.6.			
Dbh class	**n**	**Bias (m^3)**	**% Bias**	**SEE (cm)**	**% SEE**	**Bias (m^3)**	**% Bias**	**SEE (cm)**	**% SEE**
10<20	75	-0.011	-17.113	0.015	22.942	0.002	1.571	0.010	10.212
20<30	37	-0.017	-7.429	0.065	17.542	0.003	-1.502	0.061	14.072
30<40	37	0.035	3.259	0.104	10.730	0.043	4.338	0.108	11.296
40<50	34	0.097	4.568	0.220	12.113	0.036	1.383	0.193	11.287
50<60	20	0.060	1.368	0.336	12.641	-0.132	-6.082	0.357	14.980
60<80	15	0.069	-0.256	0.627	17.092	-0.491	-14.687	0.884	27.174

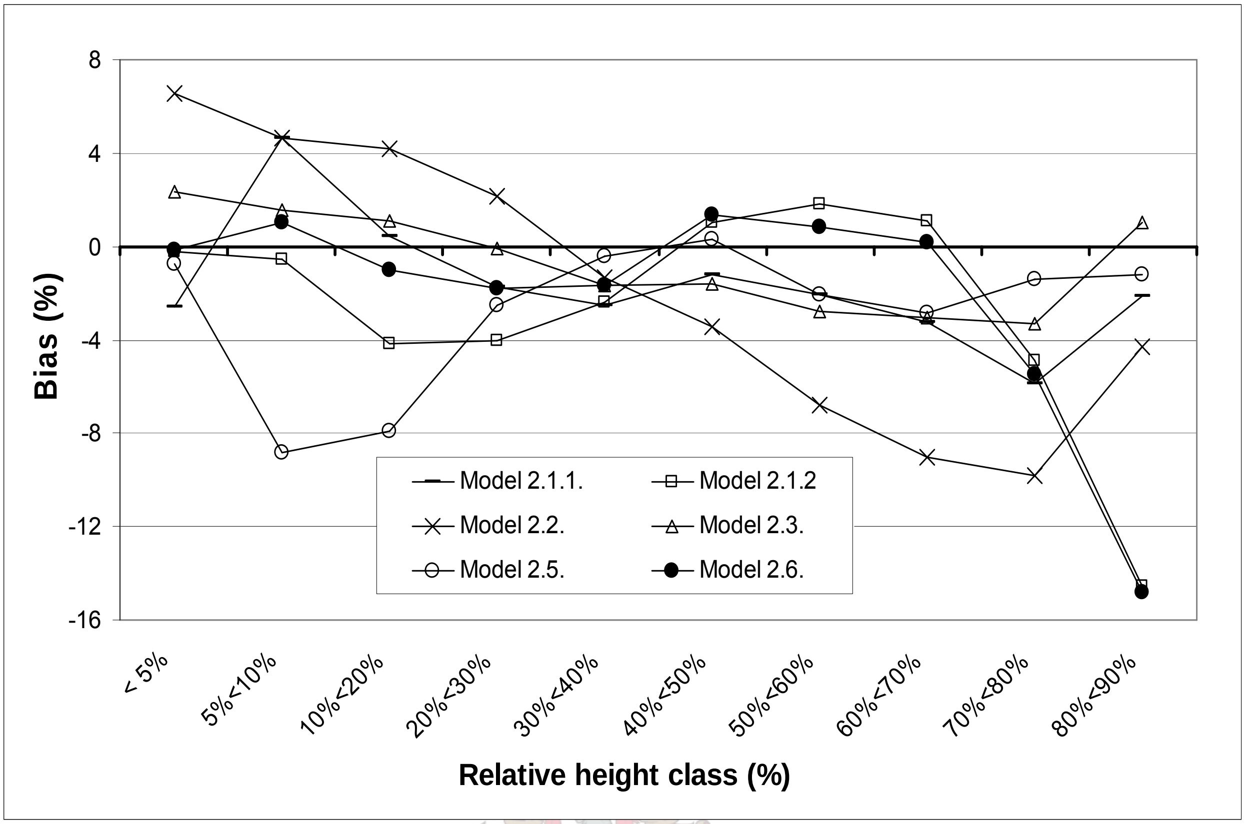


Figure 33. Bias (%)of the estimates of diameter underbark in height classes along the stem by each of the fitted models.

By studying the values in Table 12, Figures 26 to 32 and the percentage bias values from Table 12 in Figure 33 the following deductions were made. Models 2.1.2 and 2.6 predict underbark diameter in the butt section of the tree very well. Models 2.3 and 2.1.1 show similar, less accurate predictive capabilities for the butt section of the tree. For the middle section of the tree all models show similar predictive capabilities with models 2.3 and 2.6 being the most accurate. In the tip section of the tree the amount of relative bias increases for all models except Model 2.3. Models 2.6 and 2.1.2 are clearly unable to describe diameters in the tip section of the tree. It should be taken into account that when relative bias and *SEE* are used for evaluation and ranking, these values increase as size decreases and can thus be misleading for underbark diameters close to the top of the tree (Kozak and Smith, 1993).

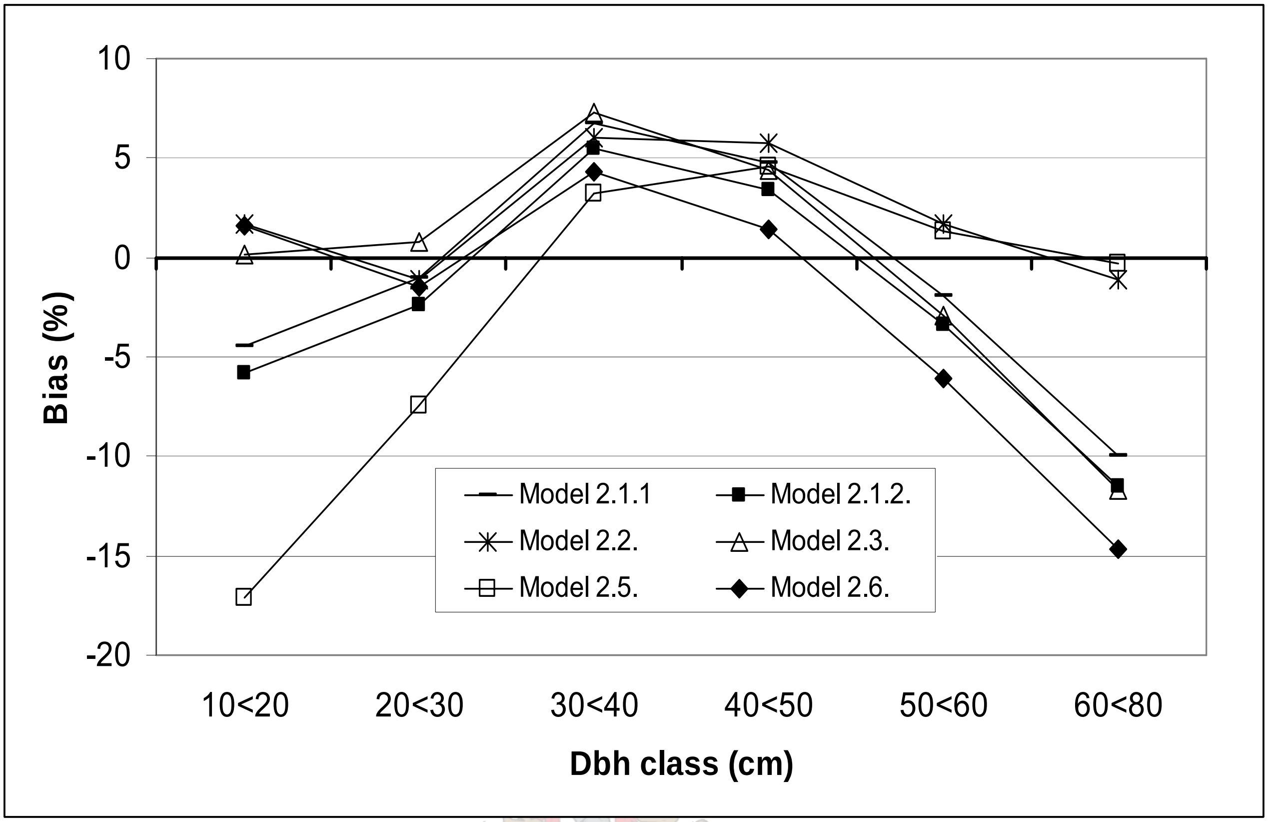


Figure 34. Bias (%) estimation of the total underbark volume estimates by dbh classes for each of the fitted models.

From the values in Table 13 and Figure 34 it is clear that Models 2.1.1, 2.1.2 and 2.5 underestimate the total volume underbark for very small trees. As the tree size increases (between 20 cm and 40 cm dbh classes) all models overestimate the total volume with similar relative bias, Model 2.3 being the most inaccurate. Model 2.6 is the most accurate for tree size classes in the range of 20 cm to 50 cm dbh. As the size classes increase, all models except Models 2.2 and 2.5 increasingly underestimate tree volume. It should be noted that the supposedly “actual” or “observed” values were calculated with the Smalian volume equation which overestimates volume when trees become large and tend to exhibit characteristics such as butt flare (Kozak, 1988). As most of the teak trees in the larger dbh classes (50 cm to 80 cm) in this study exhibited buttressing and fluting near the base of the tree, the relative bias values in Table 13 are not entirely realistic and can be extreme because they are not based on actual values.

5.3.2 Ranking of models

In order to improve objectivity and ease of comparison between the different models, rankings were assigned to the absolute values of the bias and *SEE* values for the diameters at various height classes and total volume underbark predictions of different dbh classes. These rankings in Tables 14 and 15 can now be used together with the residual graphs, the observed and predicted graphs and practical considerations such as ease of estimating the model parameters to select the most suitable taper model. In Table 16 the total combined ranking, calculated as the sum of all the height and dbh class ranks and the total rank for diameters to upper stem heights and total volume for dbh classes are presented. A Friedman test[27] applied to the ranks in Table 16 indicated that at least one of the models differs from the others. This total ranking in Table 16 should however, as with the ranks in Tables 14 and 15, not be used on its own to determine the best model. This is because the ranks given to individual criteria do not take into account the relative difference between the criteria values for two specific models.

The total combined ranks in Table 16 show that Model 2.3 is the best model according to all the various criteria. The discussions in the previous section confirm the results in Table 16, Model 2.3 accurately predicts diameter along the entire stem. Although not as accurate in the butt section as Models 2.6 or 2.1.2, predictions are accurate for the rest of the bole. Volume predictions for smaller dbh classes are very accurate, but as the size classes become more than 60 cm the accuracy decreases. The residual graph (Figure 22) shows no apparent trends, indicating a statistically sound taper model for teak.

The model ranked second in Table 16 is Model 2.1.1. This model is very accurate for predicting the butt section of the tree (Table 14), but the amount of bias increases along the stem. Although Model 2.1.1 compares well with Models 2.3 and 2.6, it is the most inaccurate for the majority of dbh classes among these three. Figure 34 indicates a possible reason for the low total ranking. Similar to the other models, Model 2.1.1 is also highly biased for the larger dbh classes, but slightly less than the other models. This

[27] The Friedman (1936) test is a non-parametric test that uses the ranks of the data rather than the raw values to calculate the statistic. The test statistic for the Friedman test is a Chi-square with $k - 1$ degrees of freedom, where k is the number of repeated measures (Hollander and Wolfe, 1973).

lower bias for the two largest dbh classes means that Model 2.1.1 receives half the rank of Model 2.6 and hence the lower total ranking.

Results from the rankings indicate that, when the ability of the model to fit according to various height and dbh classes is analysed rather than the R^2 and *MSE* values, Model 2.1.1 with the join points estimated visually from the data performs better than Model 2.1.2 where the join points are estimated by nonlinear regression procedures.

The residual graphs in Figures 19 and 20 indicate that this model form is not suited to describe the taper of teak trees due to a clear trend visible in the lower stem around breast height. The fact that both Models 2.1.1 and 2.1.2 show this trend verifies the observation above.

Model 2.6 is ranked third. Although not the most accurate over all sections of the stem, the predictions are consistently accurate over most of the stem, except at the tip. The model is also very accurate for the smaller and middle dbh classes, but very unsuitable for dbh classes larger than 80 cm. This inability to predict at the upper extreme values of height and dbh classes has therefore resulted in a rank higher than would be the case if ranks were only given up to 60 cm dbh classes. The residual graph in Figure 25 indicates no trends, making this model very suitable for describing teak shape and volume, except for trees with a dbh larger than 80 cm. It should be noted that trees at such magnitude fall well outside the usual size spectrum for plantation grown teak.

When all the values in Tables 12 and 13, the rankings in Tables 14, 15 and 16 and the residual graphs are taken into consideration, it is clear that Models 2.3 and 2.6 are the most suitable to describe teak taper and volume. When deciding on a taper model, the ease of use and finding the parameters of the model should be considered in addition to the statistical considerations above (Kozak and Smith, 1993). When these practical factors are considered, Model 2.6 would be inappropriate for most practical purposes since estimating the parameters and then using the model to calculate volume or upper stem diameters can be a daunting task. On the other hand, estimation of the parameters and use of Model 2.3 is relatively easy and possible with much less effort than for Models 2.1, 2.5 and 2.6. This model does however have two drawbacks. Numerical integration must be used to calculate volume and iterative methods must be used to find the merchantable height to a given diameter. These problems are however not difficult to overcome by using computer software.

The results in this chapter and in particular the residual graphs in Figures 19 to 25 show that some of the fitted models are inappropriate to describe teak taper. Although all the models used in this study were selected on the premise that they performed the best in other studies, the contrary seems to be true for some of the models in this study. The reason for this could be manifold. The most probable reason is that the models have only been fitted and tested for a limited number of species and size classes. All the models used in this study, except Model 2.5 were initially fitted and tested for pine species. The situation is similar for many other taper models not used in this study (Kozak, 1988). Due to this reason most taper models fit pine and a limited number of other species very well, especially when the range of tree sizes used in fitting the model is not very large (Kozak and Smith, 1993). This is due to the fact that most pine species have relatively regular form and a relatively small range of dbh and height (Kozak, 1988).

The range of dbh and height used in this study is very wide (5 cm<dbh<80 cm and 9m<height<34m) and the shape of teak is more complex than most pine species, especially due to fluting and buttressing in the lower section of the stem[28]. This could be the reason why a model such as Model 2.1, a very popular model in other studies, is apparently incapable of describing teak taper adequately.

Table 14. Bias and SEE rankings of diameter estimates from the ground to the top.

BIAS Rank						
Height class	**Model 2.1.1.**	**Model 2.1.2.**	**Model 2.2.**	**Model 2.3.**	**Model 2.5.**	**Model 2.6.**
< 5%	1	3	6	5	4	2
5%<10%	5	3	4	2	6	1
10%<20%	1	5	4	2	6	3
20%<30%	4	6	3	1	2	5
30%<40%	4	3	1	2	6	5
40%<50%	3	5	1	2	6	4
50%<60%	1	6	5	2	3	4
60%<70%	1	6	5	2	3	4
70%<80%	4	1	6	3	5	2
80%<90%	2	6	3	4	1	5
Sum	26	44	38	25	42	35
Total rank	**2**	**6**	**4**	**1**	**5**	**3**

[28] The occurrence of fluting and buttressing in teak is discussed in Section 3.1.

SEE **rank**						
Height class	**Model 2.1.1.**	**Model 2.1.2.**	**Model 2.2.**	**Model 2.3.**	**Model 2.5.**	**Model 2.6.**
< 5%	3	5	6	1	2	4
5%<10%	4	3	1	5	2	6
10%<20%	2	1	6	4	5	3
20%<30%	2	1	6	3	5	4
30%<40%	3	1	5	2	4	6
40%<50%	1	3	6	2	4	5
50%<60%	4	2	5	3	6	1
60%<70%	3	2	6	1	5	4
70%<80%	4	1	6	2	3	5
80%<90%	3	1	6	2	4	5
Sum	29	20	53	25	40	43
Total rank	**3**	**1**	**6**	**2**	**4**	**5**

Table 15. Bias and SEE rankings for total volume underbark by dbh classes.

BIAS Rank						
Dbh class	**Model 2.1.1.**	**Model 2.1.2.**	**Model 2.2.**	**Model 2.3.**	**Model 2.5.**	**Model 2.6.**
10<20	2	5	4	1	6	3
20<30	4	1	3	5	6	2
30<40	5	3	4	6	1	2
40<50	5	2	6	3	4	1
50<60	1	4	5	2	3	6
60<80	3	4	1	5	2	6
Sum	20	19	23	22	22	20
Total rank	**2**	**1**	**6**	**4**	**4**	**2**

SEE **rank**						
Dbh class	**Model 2.1.1.**	**Model 2.1.2.**	**Model 2.2.**	**Model 2.3.**	**Model 2.5.**	**Model 2.6.**
10<20	2	4	5	3	6	1
20<30	3	2	6	5	4	1
30<40	4	3	5	6	1	2
40<50	4	2	6	3	5	1
50<60	1	3	6	2	4	5
60<80	2	4	3	5	1	6
Sum	16	18	31	24	21	16
Total rank	**1**	**3**	**6**	**5**	**4**	**1**

Table 16. Sum of the ranks and total ranks based on the rank sums.

Description	Model 2.1.1.	Model 2.1.2.	Model 2.2.	Model 2.3.	Model 2.5.	Model 2.6.
Bias for relative height classes	2	6	4	1	5	3
***SEE* for relative height classes**	3	1	6	2	4	5
Bias for dbh classes	2	1	6	4	4	2
***SEE* for dbh classes**	1	3	6	5	4	1
Total bias for relative height classes	5	4	2	1	6	3
Total *SEE* for relative height classes	2	5	6	1	3	4
Total bias for dbh classes	5	4	2	1	6	3
Total *SEE* for dbh classes	3	4	2	5	1	6
Sum of all the ranks	23	28	34	20	33	27
Total ranking	***2***	***4***	***6***	***1***	***5***	***3***

5.4 Double bark thickness along the stem

After transformation of the relative diameter, simple linear regression of Model 3.1 yielded the following parameter values for estimating the relative bark thickness.

Table 17. Results from fitting Model 3.1.

Model	Dependent variable	Parameter estimates and their standard errors (in brackets)		R^2	*MSE*	*n*
		b_0	b_1			
3.1.	Relative bark thickness	0.250126 (0.0208)	0.647287 (0.2859)	0.8738	0.0086	76

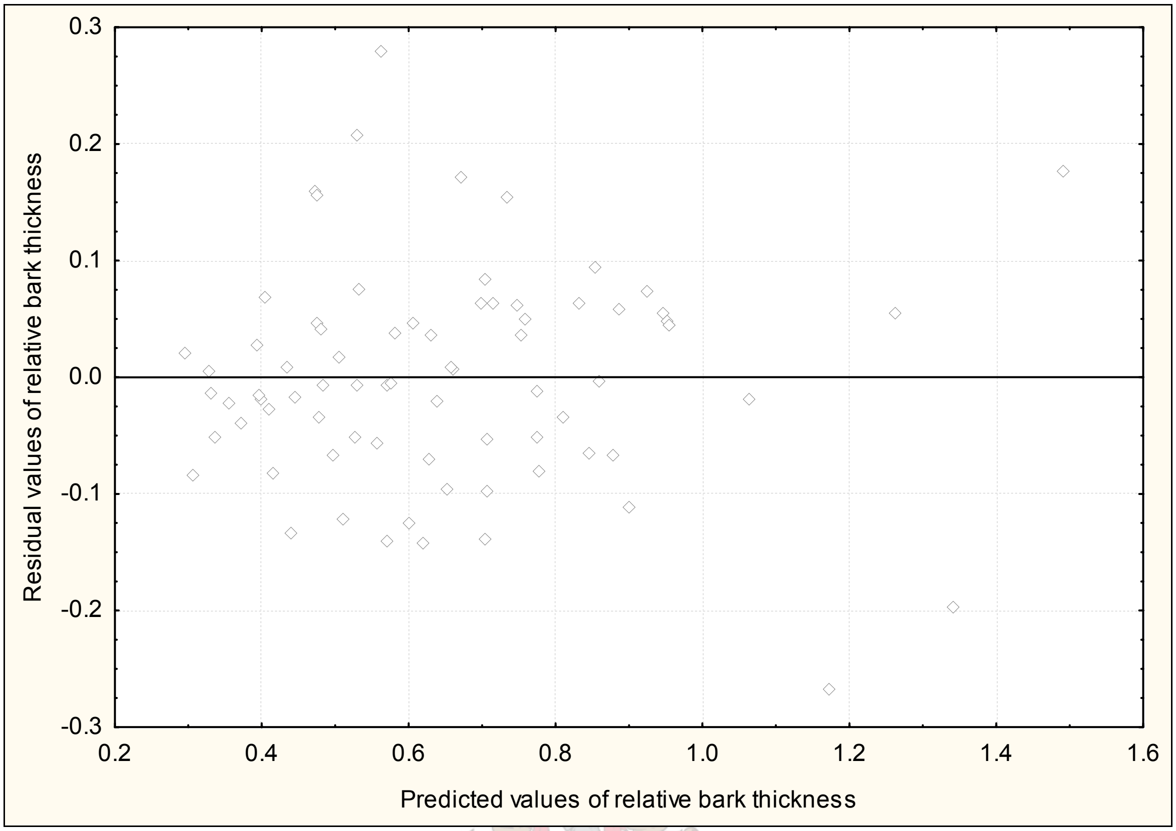


Figure 35. Predicted and residual values of relative bark thickness estimated by means of Model 3.1.

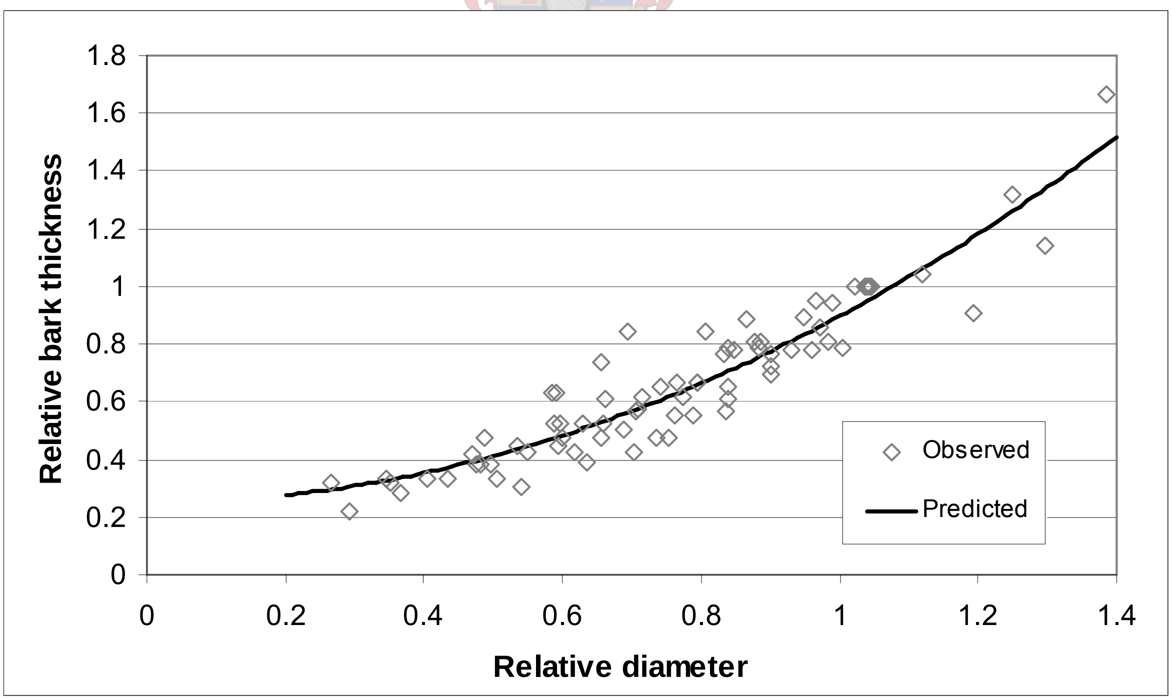


Figure 36. Observed (untransformed) values and predicted relative bark thickness values by Model 3.1.

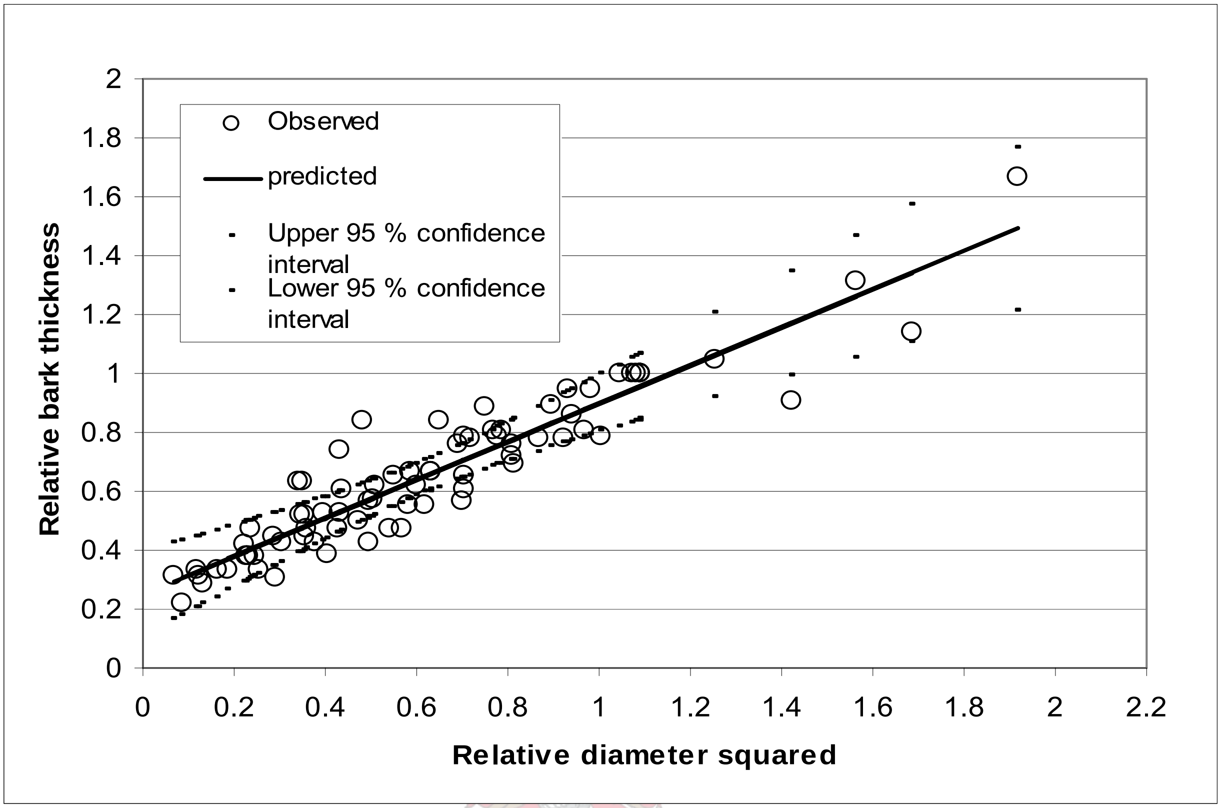


Figure 37. Observed values (transformed) and predicted values with the 95 % confidence interval specified.

From the graphs in Figures 35, 36 and 37, it is clear that after transformation of the relative diameter a simple linear model accurately predicts the relative bark thickness. Diagnostic tests revealed that no influential observations were present.

5.5 Bark thickness at breast height

Model 3.2 was fitted to the bark thickness data after transforming the Dbh values. The parameter estimates and fit statistics are provided in Table 18.

Table 18. Results from fitting Model 3.2.

Model	**Dependent variable**	**Parameter estimates and their standard errors (in brackets)**		R^2	***MSE***	***n***
		b_0	b_1			
3.2.	Double bark thickness at breast height	-0.3289 (0.1324)	0.8157 (0.0415)	0.602	0.224	256

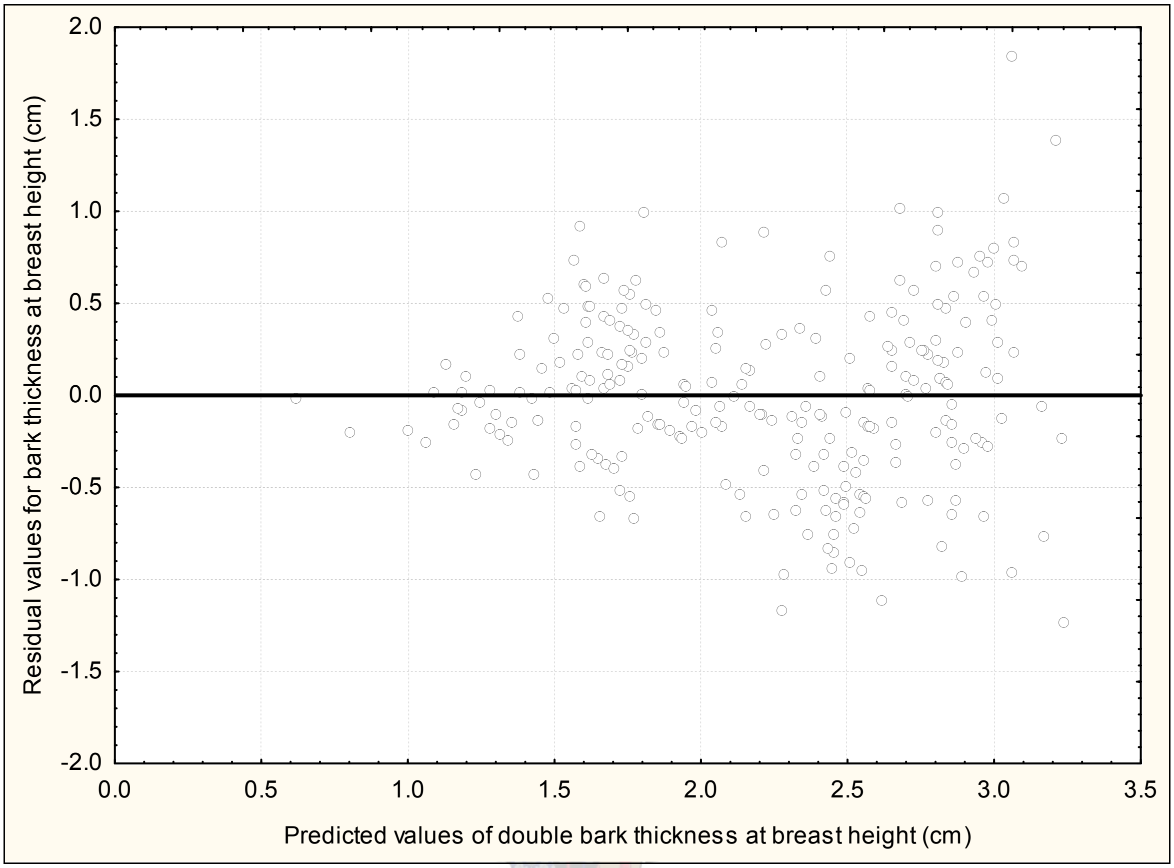


Figure 38. Residual and predicted values for double bark thickness at breast height.

Diagnostic tests detected four influential observations that were subsequently removed from the sample data and the model fitted again without them. The residuals in Figure 38 are distributed very well with no abnormal trends. The model may be overestimating double bark thickness for very small trees, but this will only be for the dbh classes smaller than 10 cm (Figure 39). It is therefore of no consequence since bark thickness will seldom be calculated for trees with a dbh smaller than 10 cm.

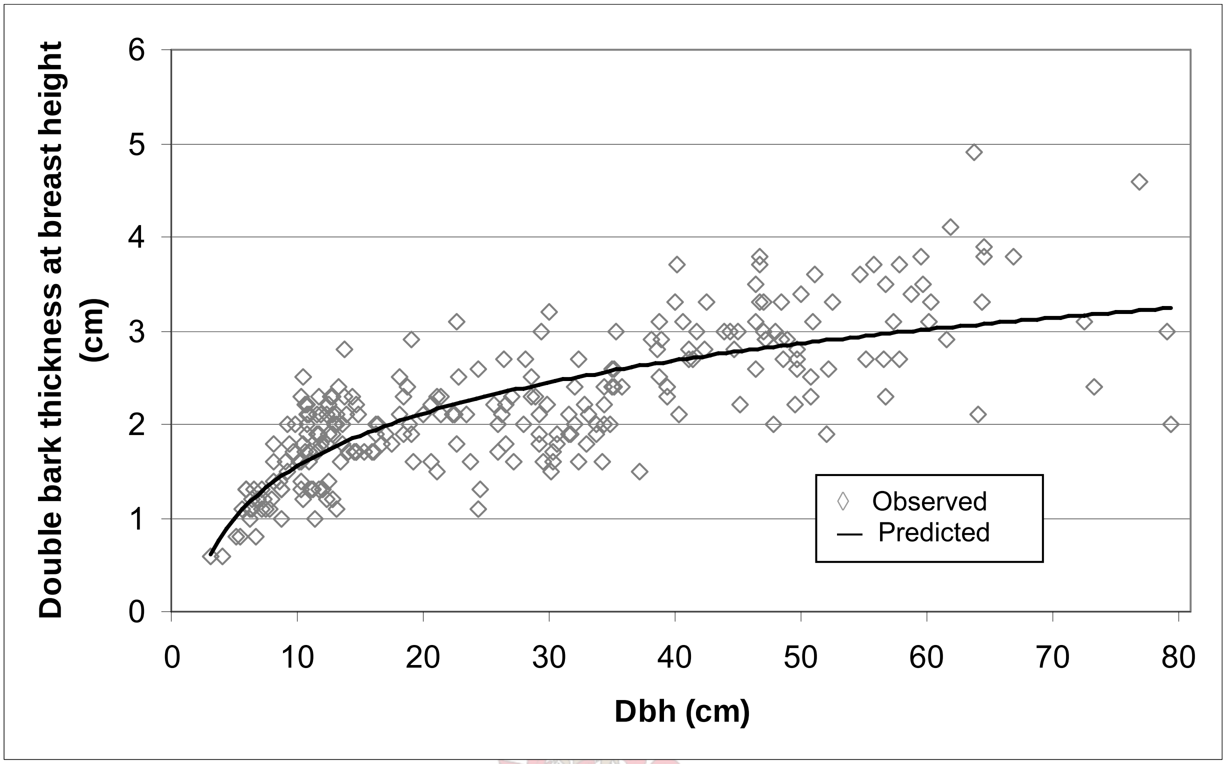


Figure 39. Observed and predicted values of double bark thickness at breast height by means of Model 3.2.

From Figure 39, it is clear that the increase in bark thickness is fast and curvilinear when the trees are small. As the trees get bigger, this rate of increase drops and could possibly become zero (no further increase in bark thickness as the dbh gets more than a certain value). In similar studies for teak, Loetsch *et al*. (1973) in Thailand and Tint and Schneider (1980) in Burma have found that the curvilinear increase in bark thickness continues until the tree reaches a dbh of around 30 cm. After this period Loetsch *et al*. (1973) indicated that the relationship becomes linear (bark thickness remains constant) with increasing dbh whereas Tint and Schneider (1980) show that although bark thickness increments slow dramatically as the tree gets bigger, it continues to increase with increasing dbh. The results in this study are similar to the results from Burma; bark thickness continues to increase (albeit at an ever decreasing rate) as the tree grows.

6. Summary and conclusions

In this study it was shown that although teak is planted globally as a plantation species, only models with limited capabilities exist to predict volumes for various specifications. As the area planted to teak in Tanzania is expanding and current plantations are being managed more intensively, accurate models are required to provide information for management decisions.

The intention of this study was to develop various model components that will enable the determination of teak volume for any required specifications.

As teak is a very valuable timber, an alternative to felling the sample trees during data collection had to be adopted. All measurements on the sample trees were taken with a Barr and Stroud optical dendrometer. This instrument has been shown to be very accurate for measuring the shape of trees. The diameter and corresponding height values as determined with the B & S were used as input for taper model development. The volumes to various defined limits were calculated by summing the volumes of appropriate log sections as calculated by the Smalian equation.

In order to determine underbark volumes, the bark thickness at breast height and along the tree stem were also measured.

The sample size was 222 trees with a total of 2617 observations. Stratification was applied so that data from most ages and site qualities present in Tanzanian teak plantations were included in the sample. This required sampling to be carried out at KVTC, Longuza and Mtibwa; three plantations in distinctly different regions.

Numerous model forms from related studies were considered and potential suitable models included for model development. These models were selected on the basis of their ability to predict total tree volume, merchantable volume to upper stem height and diameter limits and tree taper.

Various criteria were used to evaluate the ability of each model to predict a specified dependent variable. According to these criteria the following models are recommended by this study to describe teak volume and shape according to the relevant specifications:

- **Total tree volume**

Among the several models fitted to the total volume data, the Schumacher and Hall (1933) model provided the best results. In Equations 45 and 46 the model with the newly estimated parameters are provided.

For total volume overbark:

$$LnV = -9.918 + 1.888 \ln D + 1.009 \ln H \qquad ...[45]$$

with the MSE = 0.0109 and R^2 = 0.995

and volume underbark:

$$LnV = -10.801 + 1.905 \ln D + 1.225 \ln H \qquad ...[46]$$

with the MSE = 0.0131 and R^2 = 0.994

The Schumacher and Hall (1933) model was also fitted to directly produce volume estimates to an upper stem diameter limit of 7.5 cm without the use of merchantable volume equations.

$$LnV = -11.4585 + 1.9076 \ln D + 1.4235 \ln H \qquad ...[47]$$

with the MSE = 0.0175 and R^2 = 0.994

- **Merchantable volume to upper stem diameter limit**

When the prediction objective is to determine the volume to a specific upper stem diameter, the Burkhart (1977) volume ratio model provides the best results. Equation 48 presents this model with the parameters applicable to calculating the merchantable volume underbark to an overbark upper stem diameter limit:

$$V_M = V_T (1 - 0.443 d^{3.0653} D^{-2.8676}) \qquad ...[48]$$

with the MSE = 0.00466 and R^2 = 0.922

- **Merchantable volume to upper stem height limit**

The Burkhart (1977) volume ratio equation modified by Cao and Burkhart (1980) provides extremely satisfactory results. In Equation 49 the model is presented with parameters to estimate the merchantable volume underbark to an upper stem height limit:

$$V_M = V_T(1 - 0.7716\left(\frac{(H-h)^{2.3666}}{H^{2.3021}}\right) \quad ...[49]$$

with the *MSE* = 0.000755 and R^2 = 0.987

- **Taper**

The most suitable taper model was selected by ranking the models according to various criteria and analysing the residual graphs. The rankings were based on the overall ability of the model to describe taper and to calculate volume. To improve model selection, rankings were also assigned according to predictions for 10 % height classes and 10 cm dbh classes.

According to these procedures, Model 2.3, the model proposed by Perez, Burkhart and Stiff (1990) provide the best overall results.

Model 2.3 is very accurate for predicting diameters to upper stem heights, but less accurate than the Max and Burkhart (1976) (Model 2.1) or Shaw *et* al. (2003) (Model 2.6) models for predicting tree volume for dbh classes between 30 cm and 60 cm. As Model 2.1 is biased in the middle section of the stem, it is not recommended for volume calculation, although it ranks higher than Model 2.3 for this use. The complex triangle taper model by Shaw *et al.* (2003) (Model 2.6) was ranked third overall and produced very favourable results for both taper estimation and volume calculation. It was only outranked by Model 2.1 due to its inability to predict diameters in the tip of the stem.

Due to the reasons above, the volume ratio model in Equation 48 is recommended when the objective is the prediction of volume to upper stem diameters. When only one model is used for predicting taper and volume, the variable form taper model by Perez, Burkhart and Stiff (1990) with an inflection point (q) of 0.25 is recommended. Equation 50 provides this model with the estimated parameters:

$$\ln(d) = \ln(0.677) + 1.0269\ln(D) + 0.6205\ln(Z)X^2 \quad ...[50]$$
$$+ (-0.1068\ln(Z)\ln(X + 0.001)) + 0.1323\ln(Z)\left(\frac{D}{H}\right)$$

with the *MSE* = 0.0211 and R^2 = 0.96

Several of the taper models fitted in this study demonstrated a clear inability to effectively describe teak taper along the whole stem. The main causes for this inability are that these models were not developed and are seldom used to cover such a wide range in data as used in this study. The complex taper of teak trees, when compared to the original species for which the models were developed, also leads to the inability of these models to describe teak taper in some sections of the stem.

As the **secondary objective** of this study, a height growth model and related site index curves were developed. Results indicated that the Schumacher (1939) growth model provides the best results, but only when the parameter *k* is determined from the sample population. The resulting site index model is presented in Equation 51.

$$\ln H = \ln SI + \left(-3.3843\left(\frac{1}{Age^{0.5}} - \frac{1}{20^{0.5}} \right) \right) \qquad ...[51]$$

with the *MSE* = 0.02943

The height-dbh relationship was predicted well by several models. The accuracy of prediction was greatly improved by including age and site index in the prediction model. The recommended model by this study to predict tree height is presented in Equation 52.

$$Ln\ H = 2.3342 + (-10.0664D^{-1}) + (0.00884 \times A) + (0.0379 \times SI) \qquad ...[52]$$

with the *MSE* = 7.9450

7. Recommendations

In this study, the volume and shape of teak trees have been adequately described by a number of models that can be applied with relative ease. One, totally unmeasured aspect that may influence the results produced by these models is fluting. For this reason, a specific study is required to determine the amount and effect of fluting on volume estimates and to develop some measure whereby the predictions made by the models developed in this study can be appropriately adjusted. Since fluting is highly controlled by provenance and most of the teak plantations (including KVTC) in Tanzania have been established by the same provenance, it is possible to determine the amount of fluting that recently planted trees, showing little or no signs of fluting, will exhibit in the future.

The development of the height growth model as the secondary objective of this study clearly showed that very limited data are available for model development, particularly at the higher ages. This situation can only be improved by the establishment, maintenance and regular measurement of a system of permanent sample plots. It is recommended that spacing trials (such as SSS CCT[29] trials for example) be established that will enable the modelling of not only height, but other related tree and stand parameters.

[29] Correlated curve trend design (Bredenkamp, 1990).

References

Abdelsalaam, A.S. 1980. Inventory of teak at Mtibwa plantations. M.Sc. thesis, University of Dar-es-Salaam, Tanzania. In Malende and Temu (1990).

Adegbehin, J.O. 2002. Growth and yields of *Tectona grandis* (Linn.f) in the Guinea and derived savanna of Northern Nigeria. *Int. For. Rev. 4(1): 66-76.*

Akindele, S.O. 1991. Development of a site index equation for teak plantations in Southwestern Nigeria. *Jnal. of Trop. For. Sci. 4(2): 162-169.*

Amaro, A., Reed, D., Tomé, M., and Themido, I. 1998. Modelling dominant height growth: *Eucalyptus* plantations in Portugal. *For. Sci. 44(1): 37-46.*

Anon, 1934. Rough volume tables for teak (*Tectona grandis*). Burma forest bulletin 31, silvicultural series 15. Burma government printing, Rangoon, Burma. 97 pp.

Avery T.E., and Burkhart, H.E. 2002. *Forest measurements.* McGraw Hill, Boston, USA. 456 pp.

Bailey, R.L., and Clutter, J.L. 1974. Base-age invariant polymorphic site curves. *For. Sci. 20: 155-159.*

Ball, J.B., Pandey, D., and Hirai, S. 1999. *Global overview of teak plantations*. Regional seminar on site, technology and productivity of teak plantations. Chiang Mai, Thailand. Fao, Rome, Italy. 14 pp.

Barr and Stroud Ltd. 1970. *Care and use of the Barr and Stroud Dendrometer Type FP. 15*. Instruction manual no. 1559. Glasgow, U.K. 57 pp.

Bekker, C., Rance, W., and Monteuuis, O. 2004. Teak in Tanzania: II. The Kilombero Valley Teak Company. *Bois et Forêts des Tropiques 279(1): 11-21.*

Bermejo, I., Caneleas, I., and San Miguel, A. 2004. Growth and yield models for teak plantations in Costa Rica. *For. Ecol. and Man. 189: 97-110.*

Bi, H. 1999. Predicting stem volume to any height limit for native tree species in Southern New South Wales and Victoria. *New Zeal. Jnal. of For. Sci.* 29: 318-331.

Bi, H. 2000. Trigonometric variable-form taper equations for Australian eucalypts. *For. Sci. 46(3): 397-409.*

Blanford, H.R. 1922. Rough volume tables for teak. Burma forest bulletin 6, silvicultural series 6. Burma government printing office, Rangoon, Burma. 10 pp.

Bredenkamp, B.V. 1982. Volume regression equations for *Eucalyptus grandis* on the coastal plain of Zululand. *South. Afr. For. Jnal. 122: 1-4.*

Bredenkamp, B.V. 1990. The triple-S CCT design. In von Gadow, K. and Bredenkamp, B.V. (Eds.). Proceedings of a symposium arranged by the Forest Mensuration and Modelling Working Group in collaboration with the Southern African Institute of Forestry and the *Eucalyptus grandis* Research Network on "Management of *Eucalyptus grandis* in South Africa" held in Stellenbosch, South Africa. pp 198-205.

Bredenkamp, B.V. 2000. Volume and mass of logs and standing trees. In: Owen, D.L. (Ed.). *South African Forestry Handbook, Volume 1*. South African Institute of Forestry. V & R Printers, Pretoria. pp 167-174

Brickell, J.E. 1976. Bias and precision of the Barr & Stroud dendrometer under field conditions. USDA For. Serv. Res. Paper INT-186. 46 pp.

Brickell, J.E. 1984. Stem analysis: a conventional approach to volume determination. Proceedings: growth and yield and other mensurational tricks; a regional technical conference. Utah, USA. 61 pp.

Brink C., and Von Gadow, K. 1986. On the growth and decay functions for modelling stem profiles. *EDV Med. Biol. 17(1/2):20-27.*

Burkhart, H.E. 1977. Cubic-foot volume of Loblolly pine to any merchantable top limit. *S. Jnal. of Appl. For. 1: 7-9.*

Burkhart, H.E., and Walton, S. 1985. Incorporating crown ratio into taper equations for Loblolly pine trees. *For. Sci. 31: 478-484.*

Byrne, J.C., and Reed, D.D. 1986. Complex compatible taper and volume estimation systems for Red and Loblolly pine. *For. Sci. 32(2): 423-443.*

Cao, Q.V., and Burkhart, H.E. 1980. Cubic-foot volume of Loblolly pine to any height limit. *S. Jnal. of Appl. For. 4: 166-168.*

Cao, Q.V., Burkhart, H.E., and Max, T.A. 1980. Evaluation of two methods for cubic-volume prediction of Loblolly pine to any merchantable limit. *For. Sci. 26(1): 81-91.*

Cao, Q.V. 1993. Estimating coefficients of base-age-invariant site index equations. *Can. Jnal. For. Res 23: 2343-2347.*

Chakrbarti, S.K., and Gaharwar, K.S. 1995. A study on volume estimation for Indian teak**.** *Indian Forester 121 (6): 503-509.*

Chapman, D.G. 1961. *Statistical problems in population dynamics*. Proceedings of the Fourth Berkeley Symposium on Math Statistics and Probability. University of California Press, Los Angeles. 153-168 pp.

Clutter, J.L. 1980. Development of taper functions from variable top merchantable volume equations. *For. Sci. 26: 117-120.*

Clutter, J.L., Fortson, J.C., Pienaar, L.V., Brister, G.H., and Bailey, R.L. 1983. *Timber management – a quantitative approach*. John Wiley & Sons, New York. 333 pp.

Curtis, R.O. 1967. Height-diameter and height-diameter-age equations for second growth Douglas fir. *For. Sci. 13(4): 365-375.*

Demaerschalk, J.P. 1972. Converting volume equations to compatible taper equations. *For. Sci. 18(3): 241-245.*

De Vries, P.G. 1971. *Nomograms for use with Barr and Stroud dendrometer FP.15*. Mededelingen Landbouwhogeschool Wageningen, Nederland 71(11). 13 pp.

Drechsel, P., and Zech, W. 1994. DRIS evaluation of teak (*Tectona grandis* L.f.) mineral nutrition and effects of nutrition and site quality on teak growth in West Africa. *For. Ecol. and Man. 70: 121-133.*

Dupuy, B., and Mille, G. 1993. *Timber plantations in the humid tropics of Africa*. Centre Technique Forestier Tropical (CTFT), FAO forestry paper 98. Rome, Italy. 190 pp.

Eerikäinen, K. 2003. Predicting the height-diameter pattern of planted *Pinus kesiya* stands in Zambia and Zimbabwe. *For. Ecol. and Man. 175: 355-366.*

Ek, A.R., Birdsall, E.T., and Spears, R.J. 1984. A simple model for estimating total and merchantable tree heights. North central forest experiment station, St. Paul, Minnesota. *Research note NC-309*.

Falkenhagen, E.R. 1980. The law of Eichorn: use, extension and applicability in South Africa. *South Afr. For. Jnal 114: 7-12.*

Fang, Z., and Bailey, R.L. 1998. Height-diameter models for tropical forests on Hainan Island in southern China. *For. Ecol. and Man. 110: 315-327.*

Fang, Z., Borders, B.E., and Bailey, R.L. 2000. Compatible volume-taper models for Loblolly and Slash pine based on a system with segmented-stem form factors. *For. Sci. 46(1): 1-12.*

Friday, K.S. 1987. Site index curves for teak (*Tectona grandis* L.) in the Limestone hill region of Puerto Rico. *Commonw. For. Rev. 66(3): 239-253.*

Friedman, M. 1937. The use of ranks to avoid the assumption of normality implicit in the analysis of variance. *Jnal. of the American Statistical Association 32: 675-701.*

Furnival, G.M., Gregoire, T.G., and Valentine, H.T. 1990. An analysis of three methods for fitting site-index curves. *For. Sci. 36 (2): 464-469.*

Gevorkiantz, S.R., and Olsen, L.P. 1955. Composite volume tables for timber and their application in the Lake States. U.S. Dep. Agric. Tech. Bull. 1104.

Goelz, J.C.G., and Burk, T.E. 1992. Development of a well-behaved site index equation: jack pine in north central Ontario. *Can. Jnal. For. Res. 22: 776-784.*

Gordon, A.D., Lundgren, C., and Hay, E. 1995. Development of a composite taper equation to predict over- and under-bark diameter and volume of *Eucalyptus saligna* in New Zealand. *New Zeal. Jnal. of For. Sci. 25(3):318-327.*

Goulding, C.J., and Murray, J.C. 1976. Polynomial taper equations that are compatible with tree volume equations. *New Zeal. Jnal. of For. Sci. 5: 313-322.*

Grey, D.C. 1982. Site growth models in South Africa – a review. Foris no. 98. University of Stellenbosch, South Africa.

Grosenbaugh, L.R. 1963. Optical dendrometers for out of reach diameters: a conspectus and some new theory. *For. Sci. Monograph. 4*. Washington, USA.

Hollander, M., and Wolfe, D.A. 1973. *Nonparametric statistical methods.* John Wiley and Sons, New York. 503 pp.

Husch, B. 1963. *Forest mensuration and statistics*. Ronald Press, New York. 474 pp.

Husch, B., Miller, C.I., and Beers, T.W. 1982. *Forest Mensuration.* John Wiley and Sons, New York. 402 pp.

Isango, J.A., and Nshubemuki, L. 1998. Management of forest plantations in Tanzania with emphasis on planting stock and growth and yield. University of Joensuu, Faculty of Forestry, Research Notes 68: 25-37.

James, C.A. and Kozak, A. 1984. Fitting taper equations from standing trees. *The Forestry Chronicle 60(3): 157-161.*

Johnson T.S., and Wood, G.B. 1987. Simple linear model reliably predicts bark thickness of radiata pine in the Australian Capital Territory. *For. Ecol. and Man. 22: 173-183.*

Kaosa-ard, A. 1995. Management of Teak Plantations. Teak for the future - proceedings of the second regional seminar on teak, Yangon, Myanmar. Teaknet publication no. 1. Available online at: http://www.fao.org/DOCREP/005/AC773E/ac773e00.htm#Contents.

Keogh, R.M. 1980. Teak (*Tectona grandis* Linn. F.) volume growth and thinning practice in the Caribbean, Central America, Venezuela and Colombia. IUFRO working group S1 07 09. Symposium on wood production in neotropics. Inst. Trop. For. Rio Piedras, Puerto Rico.

Keogh, R.M. 1982. Teak (*Tectona grandis* Linn. f.) provisional site classification chart for the Caribbean, Central America, Venezuela and Colombia. *For. Ecol. and Man. 4: 143-153.*

Keogh, R.M. 1990. Growth rates of teak (*Tectona grandis*) in the Caribbean/Central-American region. *For. Ecol. and Man. 35: 311-314.*

Keogh, R.M. 1996. *Teak 2000*. IIED Forestry and landuse series no. 9. London, UK. 26 pp.

Kjær, E.D., and Foster, G.S. 2003. The economics of tree improvement of teak (*Tectona grandis* L.). Danida Forest Seed Centre, Humlebaek Denmark. *Technical note 50*. Available online at: http://www.dfsc.dk/pdf/Publications/Economics.pdf.

Kozak, A. 1988. A variable-exponent taper equation. *Can. Jnal. For. Res. 18: 1363-1368.*

Kozak, A., and Smith J.H.G. 1993. Standards for evaluating taper estimating systems. *The Forestry Chronicle 69(4): 438-444.*

Krishnapillay, B. 2000. Silviculture and management of teak plantations. *Unasylva* 51(201). Available online at: http://www.fao.org/documents/show_cdr.asp?url_file=/docrep/x4565e/x4565e00.htm.

Lappi, J. 1996. A longitudinal analysis of height/diameter curves. *For. Sci. 43(4): 555-570.*

Larson, P.R. 1963. Stem form development of forest trees. *For. Sci. Monograph 5.*

Laurie, M.V., and Ram, B.S. 1940. Yield and stand tables for teak (Tectona grandis, Linn.F.) plantations in India and Burma. Indian forest records, Silviculture series Vol. IV-A (1). Government of India press, New Delhi. 115 pp.

Lee, W., Seo, J., Son, Y., Lee, K., and Von Gadow, K. 2003. Modelling stem profiles for *Pinus densiflora* in Korea. *For. Ecol. and Man. 172: 69-77.*

Leites, L.P., and Robinson, A.P. 2004. Improving taper equations of Loblolly pine with crown dimensions in a mixed-effects modeling framework. *For. Sci. 50(2): 204-212.*

LeMay, V.M., Kozak, A., Muharewe, C.K., and Kozak, R.A. 1993. Factors affecting the performance of Kozak's (1988) variable-exponent taper functions. Proc. of IUFRO conf. on modern methods of estimating tree and log volume. West Virginia Univ. Publ. Serv., Morgantown, USA. 168 pp.

Loetsch, F., Zöhrer, F., and Haller, K.E. 1973. *Forest inventory, Volume 2*. BLV Verlagsgesellschaft, München, Germany. 469 pp.

Lowe, R.G. 1976. Teak (*Tectona grandis Linn. f.)* thinning experiment in Nigeria. *Commonw. For. Rev. 55(3): 189-202.*

Lynch, T.B., and Murphy, P.A. 1995. A compatible height prediction and projection system for individual trees in natural, even aged Shortleaf pine stands. *For. Sci. 41(1): 194-209.*

Madoffe, S.S., and Maghembe, J.A. 1988. Performance of teak (*Tectona grandis* L.f.) provenances seventeen years after planting at Longuza, Tanzania. *Silvae Genetica 37 (5-6): 175-178.*

Malende, Y.H., and Temu, A.B. 1990. Site index curves and volume growth of teak (*Tectona grandis*) at Mtibwa, Tanzania. *For. Ecol. and Man. 31: 91-99.*

Malende, Y.H., Nzunda, E.F., Malimbwi, R.E., and Shemwetta, D.T.K. 2001. Qualitative and quantitative assessment of the current growing stock of Mtibwa teak plantation, Morogoro, Tanzania. *Tanzania Journal of Forestry and Nature Conservation 74: 93-99*.

Malimbwi, R.E., Mugasha, A.G., Chamshama, S.A.O., and Zahabu, E. 1998. Volume tables for *Tectona grandis* at Mtibwa and Longuza forest plantations, Tanzania. *Sokoine University of Agriculture, Faculty of Forestry and Nature Conservation Record no. 71.* Morogoro. 23 pp

Matney, T.G., and Sullivan, A.D. 1982. Variable top volume and height predictors for Slash pine trees. *For. Sci. 28(2): 274-282.*

Max, T.A., and Burkhart, H.E. 1976. Segmented polynomial regression applied to taper equations. *For. Sci. 22: 283-289.*

Mesavage, C. 1964. Aids for using Barr and Stroud dendrometers. *Proc. Soc. of American Foresters. Denver, USA, 238-244.*

Mesavage, C. 1968. Revised calculator for Barr and Stroud dendrometers. US. For. Serv. Research note SO-84. New Orleans, USA. 4 pp.

Michailoff, I. 1943. Zahlenmässiges verfharen für die ausführung der bestandeshöhenkurven forstw. *Clb. U. Thar. Forstl., Jahrb. 6: 273-279*. In Von Gadow and Hui (2001).

Micski, J., and Ackhurst, P.W. 1972. *Tanzania standard volume table for Tectona grandis*. Forest Division, Ministry of Natural Resources and Tourism, Dar-es-Salaam, Tanzania. 48 pp.

Munaweera, D.P. 1998. Teak volume modelling using mean tree measurements. Forestry and environment symposium. University of Sri Jayawardenpura, Sri Lanka. Available online at: http://lihini.sjp.ac.lk/old/sci/forestry/98sympo/9819muna.htm.

Mwihomeke, S.T., Maliondo, S., Mwang'ingo, P., and Chamshama, S.A.O. 2002. Growth and hardwood percent in different provenances of *Tectona grandis* (Linn. f.) aged 36 years at Longuza forest project, East Usambura, Tanzania. Unpublished report. 14 pp.

Nanang, D.M., and Nunifu, T.K. 1999. Selecting a functional form for anamorphic site index curve estimation. *For. Ecol. and Man. 118: 211-221.*

Newnham, R.M. 1967. A modification to the combined-variable formula for computing tree volumes. *Jnal. of For. 65: 719-720.*

Newnham, R.M. 1988. A variable-form taper function. *For. Can. Petawawa Natl. For. Inst. Inf. Rep. PI-X-83.*

Newnham, R.M. 1992. Variable-form taper functions for four Alberta tree species. *Can. Jnal. For. Res. 22: 210-223*.

Nunifu, T.K., and Murchison, H.G. 1999. Provisional yield models of teak (*Tectona grandis* Linn F.) plantations in northern Ghana. *For. Ecol. and Man. 120: 171-178.*

Ott, R.L. 1988. *An introduction to statistical methods and data analysis*. *Fourth edition.* Duxbury Press, Belmont, California. 1051 pp.

Pandey, D., and Brown, C. 2000. Teak: a global overview. *Unasylva* 51 (201). Available online at: http://www.fao.org/docrep/x4565e/x4565e00.htm#TopOfPage.

Payandeh, B., and Wang, Y. 1994. Modified site index equations for major Canadian timber species. *For. Ecol. and Man. 64: 97-101*.

Peng, C.H. 1999. Nonlinear height-diameter models for nine tree species in Ontario boreal forests. Ministry of Natural Resources, Ontario Forest Research Institute, OFRI-Report: 155.

Perez, D.N., H.E. Burkhart, H.E., and Stiff, C.T. 1990. A variable-form taper function for *Pinus oocarpa* Schiede in Central Honduras. *For. Sci. 36(1): 186-191.*

Phillips, G.B. 1995. Growth functions for teak (*Tectona grandis* Linn. F.) plantations in Sri Lanka. *Commonw. For. Rev. 74(4): 361-375.*

Pienaar, L.V., and Shiver, B.D. 1980. Dominant height growth and site index curves for loblolly pine plantations in the Carolina Flatwoods. *South. J. Appl. For. 4: 54-59.*

Pikkarainen, T. Undated. Description of the applied method for calculating the inventory results of hardwood plantations in the East Usamburas, Tanzania. *Amani forest inventory and management planning project.*

Ram, B.S. 1942. *Standard and commercial volume tables for teak (Tectona grandis, Linn.F.) in the central provinces.* Indian forest records, Silviculture. Vol. 4-A (3). Government of India Press, New Delhi. 145-169 pp.

Reed, D.D., and Green, E.J. 1984. Compatible stem taper and volume ratio equations. *For. Sci. 30: 977-990.*

Richards, F.J. 1959. A flexible growth function for empirical use. *Jnal. of Exp. Botany 10(29): 290-300.*

Roecker, E.B. 1991. Prediction error and its estimation for subset selected models. *Technometrics 33: 459-468.*

Romancier, R.M. 1961. Weight and volume of plantation-grown loblolly pine. USDA For. Serv. Southeast. For. Exp. Stn. Res. Note 161.

SAS Institute Inc., 2001. Enterprise Guide (data analysis software system). Version 1.3.0.161. www.sas.com.

Schumacher, F.X., and Hall, F.D.S. 1933. Logarithmic expression of timber-tree volume. *Journal of Agricultural Research 47: 719–734*.

Schumacher, F.X. 1939. A new growth curve and its application to timber yield studies. *Jnal. of For. 37: 819-820*.

Sharma, M., and Oderwald, R.G. 2001. Dimensionally compatible volume and taper equations. *Can. Jnal. For. Res. 31: 797-803.*

Sharma, M., Oderwald, R.G., and Amateis, R.L. 2002. A consistent system of equations for tree and stand volume. *For. Ecol. and Man. 165: 183-191.*

Sharma, M., and Burkhart, H.E. 2003. Selecting a level of conditioning for the segmented polynomial taper equation. *For. Sci. 49(2):324–330.*

Shaw, D.J., Meldahl, R.S., Kush, J.S., and Somers, G.L. 2003. A tree taper model based on similar triangles and use of crown ratio as a measure of form in taper equations for Longleaf pine. *USDA, Southern research station: General technical report SRS-66.* 8 pp.

Singh, S.P. 1981. Total tree volume table for *Tectona grandis* (teak). *Indian Forester 107: 621-623.*

Soares, P., and Tomé, M. 2002. Height-diameter equation for first rotation eucalypt plantations in Portugal. *For. Ecol. and Man. 166: 99-109.*

Spurr, S.H. 1952. *Forest inventory.* The Ronald Press Company, New York. 476 pp.

Streets, R.J. 1962. *Exotic forest trees in the British Commonwealth.* Clarendon Press, Oxford, UK. 765 pp.

Tewari, V.P., and Von Gadow, K. 1999. Modelling the relationship between tree diameters and heights using SBB distribution. *For. Ecol. and Man. 119: 171-176.*

Thomas, C.E., and Parresol, B.R. 1991. Simple, flexible, trigonometric taper equations. *Can. Jnal. For. Res. 21: 1132-1137.*

Tint, K., and Schneider, T.W. 1980. Dynamic growth and yield models for Burma teak. *Mitteilungen der Bundesforschungsanstalt fur Forst und Holzwirtschaft in Hamburg-Reinbek, no 129.* 93 pp.

Valenti, M.A., and Cao, Q.V. 1986. Use of crown ratio to improve loblolly pine taper equations. *Can. Jnal. For. Res. 16: 1141-1145.*

Van Laar, A., and Akça, A. 1997. *Forest Mensuration*. Cuvillier Verlag, Göttingen, Germany. 418 pp.

Von Gadow, K., and Hui, G. 2001. *Modelling forest development*. Kluwer Academic Publishers, Dordrecht. 213 pp.

Walters, D.K., Gregoire, T.G., and Burkhart, H.E. 1989. Consistent estimation of site index curves fitted to temporary plot data. *Biometrics 45: 23-33.*

Weaver, P.L., and Francis, J.K. 1990. The performance of *Tectona grandis* in Puerto Rico. *Commonw. For. Rev. 69(4): 313-323.*

Williams, M.S., and Schreuder, H.T. 2000. Guidelines for choosing volume equations in the presence of measurement error in height. *Can. Jnal. For. Res. 30: 306–310*.

Yokom, A.H., and Bower, D.R. 1975. Estimating individual tree volumes with Spiegel Relaskop and Barr and Stroud dendrometers. *Jnal. of For. 73(9): 581-583.*

Zang, L., Peng, C., Huang, S., and Zhou, X. 2002. Development and evaluation of ecoregion-based jack pine height-diameter models for Ontario. *The forestry Chronicle 78(4): 530-538.*

Appendix A : Development of a teak site index model

Tree height plays two roles in the growth and yield modelling of forest stands; as a component of stand growth models and for assessing site quality. Firstly, the height growth of dominant trees is used as the basis for site index equations that are used to determine the production potential of a particular stand. Secondly, height/diameter curves are used to compute tree and stand volumes at a given point in time (Lappi, 1996).

The assessment of site quality and growth prediction of stands is commonly based on the law proposed by Eichhorn in 1904. Eichhorn showed that there is a relationship between the average height of a stand and the volume of timber, independent of productivity class (Falkenhagen, 1980). This implies that total volume production can be estimated by prediction of the tree height at that specific age. The use of height of dominant trees in even-aged stands to measure forest productivity and as a predictor variable in forest growth models, is justified due to sensitivity to differences in site quality, strongly correlated with volume growth and weakly correlated with stand density and species composition. These characteristics make height a very good measure of site quality and indicator of plantation productivity (Grey, 1982). Most growth models used for commercial plantations therefore make use of stand height in their predictions. Use is made of top or dominant stand height because it has the advantage of not being affected by thinnings from below where essentially only small and malformed trees are removed.

The relationship between site productivity and height growth is assessed through the use of a family of height growth curves known as site index curves. These curves describe height development patterns and use the height at a specific age as reference (Clutter *et al.,* 1983). The site index can therefore be defined as *the dominant stand height at a specific (index) age, with growth curves that show the height development pattern that a stand will most likely follow during its life.*

The preferred data source used for fitting a model to height-age data is repeated measurements (panel data). These data are obtained from permanent sample plots or from stem analysis of trees. If such data are not available, cross sectional data is used. This type of data is obtained from temporary sample plots where a single measurement of age and height is made in a cross section of plots. The main limitation of using temporary sample plot data is the underlying assumption that the distribution of forest stands with respect to site quality is equal across all ages. When this assumption is not made (several age/site combinations are unavailable or underrepresented) the fitted curves will be biased away from the true average curve of height on age (Walters, Gregoire and Burkhart, 1989).

Plotting height over age for even aged stands results in a generally sigmoid shaped curve. This curve is constructed with regression techniques, often with the equation proposed by Schumacher (1939) (Keogh, 1982; Akindele; 1991; Van Laar and Akça, 1997; Avery and Burkhart, 2002). In logarithmic form the Schumacher equation:

$$\ln H = b_0 + b_1\left(\frac{1}{A}\right) \qquad \text{...[53]}$$

expresses the height (*H*) at age (A).

When the estimated parameters b_0 and b_1 are substituted into the equation, a ln(height)/reciprocal of age line is provided for the sample data. This line is known as the guide curve[30] and is parallel to individual height/age development lines (Clutter *et al.,* 1983). When an index age is substituted into the Equation 53 above, the height is equal to site index (*SI*):

$$\ln SI = b_0 + b_1\left(\frac{1}{A_i}\right) \qquad \text{...[54]}$$

Substitution of the implied definition for b_0 into the original guide curve equation and rearrangement of the equation algebraically, it is possible to calculate the site index of a stand as a function of its height and age:

[30] When temporary sample plot data are used, site index curves can only be developed by the **guide curve** method to generate a system of **anamorphic curves** (Furnival, Gregoire and Valentine, 1990).

$$\ln SI = \ln H_j + b_1\left[\frac{1}{A_j} - \frac{1}{A_i}\right] \qquad (55]$$

where: H_j = height at age A_j (m)

A_i = index age (years).

A popular model used for teak site index curve development is the so-called *log-log* model (Malende & Temu, 1990; Drechsel & Zech, 1994; Nanang & Nunifu, 1999). It is often used in a single variable form ($H = b_0 A^{b_1}$) with height as the independent variable. For estimation with ordinary least squares (*OLS*) regression the equation is transformed into its logarithmic form as:

$$\ln H = \ln b_0 + b_1 \ln A \qquad ...[56]$$

This model was shown to be better than the Schumacher model (Equation 53) for modelling teak height growth (Friday, 1987). This is because the Schumacher curve shows a pattern of rapid early growth, but with much slower later growth and approaching a limit (Figure 40) (Friday, 1987; Keogh, 1990). These comparisons are however based on empirical criteria only, with little regard for the characteristics, assumptions and restrictions imposed by the functional forms of the estimated curves (Nanang and Nunifu, 1999).

An important characteristic of site index curves, determined to some extent by the functional form used to fit the curves is the elasticity[31] of height growth. An evaluation of these two models shows that the *log-log* model assumes constant elasticity (equal to b_0) over the entire site index curve. For many tree species, including teak, this is not true. Tree growth shows varying elasticities at various stages of growth (an initial section of increasing growth that is followed by a section of decreasing growth) (Schumacher, 1939). For mathematical reasons the *log-log* model therefore cannot be used where ranges in the data show positive, constant and negative trends; as observed in tree height growth (Nanang and Nunifu, 1999).

By contrast, the Schumacher model does not display constant elasticity. Instead the elasticity varies with age according to the term b_1/A^k. This implies that as the age increases, the elasticity decreases and hence the percentage change in total height per unit change in age also decreases. In the elasticity term above, the value of k is often accepted simply as 1 (Keogh, 1982; Akindele, 1991; Nanang and Nunifu, 1999). This assumption is not valid since each different dataset will require the calculation of a different value of k. With k = 0.5, the Schumacher model gave a better fit to teak growth data from northern Ghana than both the Schumacher model with k equal to 1 and the *log-log* model (Figure 40) (Nanang and Nunifu, 1999; Nunifu and Murchison, 1999). By estimating k from the data, a better basis is provided to compare the different models.

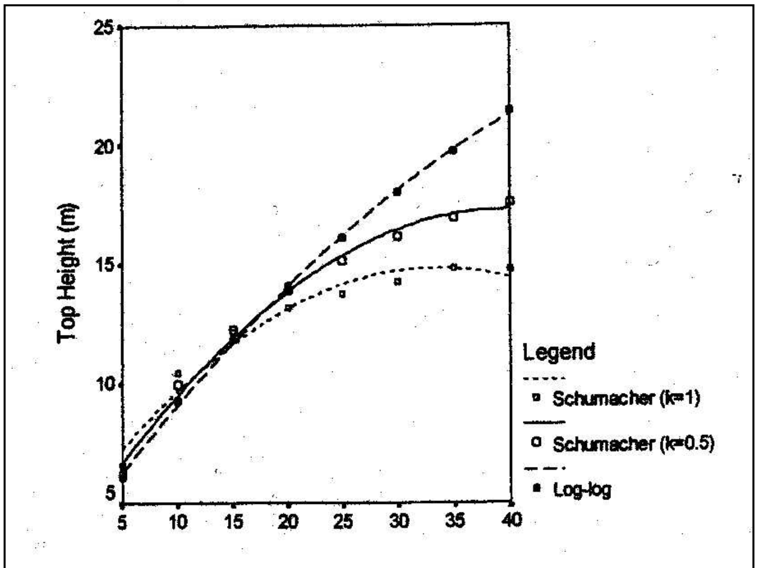


Figure 40. A comparison of estimated top heights by the log-log, Schumacher (k=1) and Schumacher (k=0.5) models for teak (Nanang and Nunifu, 1999).

It is clear that a functional form should be used that allows for combinations of increasing, decreasing or constant elasticity of growth during the life of a stand. This led Nanang and Nunifu (1999) to introduce the *transcendental* and *Spillman* models[32] for use in site index curve development of teak. The transcendental model is a more general form of the *log-log* model and shows non-constant elasticity:

$$\ln H = \ln b_0 + b_1 \ln A + b_2 A \qquad ...[57]$$

Analysis shows that there is little difference between the *log-log* and transcendental models, but the transcendental function is recommended due to the ability it provides to test for constant elasticity[33].

[31] Percentage change in top height corresponding to a 1% change in age.
[32] This was the first study to use these two functions for site index curve development.
[33] When b_2 = 0, the elasticity is constant.

The Spillman model is:

$$H = b_0\left(1 - b_1^A\right) \qquad \text{...[58]}$$

where: H = top height (m)

A = age (years)

b_0 = the maximum height that can be reached on the site provided b_0>0 and b_1 < 1

b_1 = the ratio of successive increments of age to total height

The particular advantage of this model is the ability to test for asymptotic convergence; the maximum height achievable on a specific site[34].

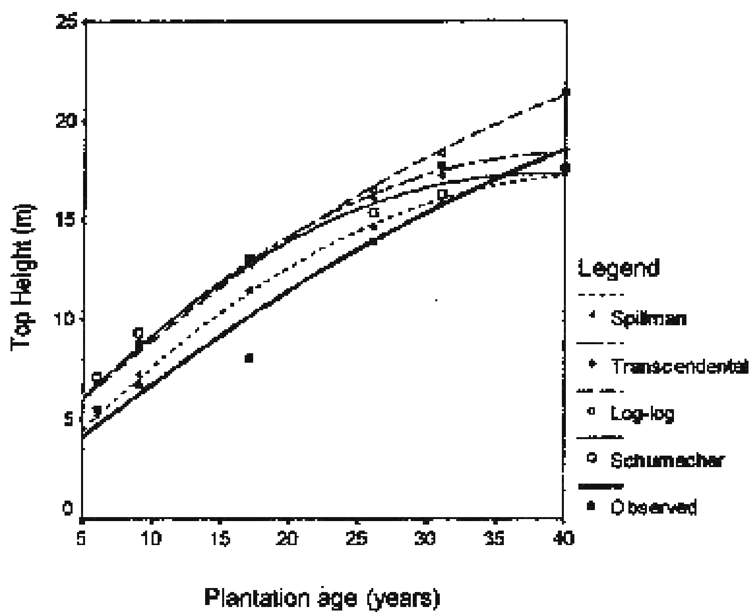


Figure 41. A comparison of observed and predicted heights of teak plantations by the Schumacher (k=0.5), log-log, transcendental and Spillman functions (Nanang and Nunifu, 1999).

In the study by Nanang and Nunifu (1999), the Spillman model fitted the data better[35] (Figure 41) than the *log-log*, Schumacher (with *k* equal to 0.5) and the transcendental models. It is recommended that the Spillman model be used to first test the hypothesis of asymptotic convergence; and the Schumacher model[36] (or transcendental for other datasets) then be used to model height growth with age.

[34] Although it is not assumed that tree growth will cease completely, it is reasonable to assume that at infinite age, the height will reach a limit.
[35] From the data fitting results, it had the lowest χ^2 value.
[36] With *k* estimated from the data.

All the height growth models discussed above generate a system of *anamorphic* height growth curves that show a constant relative growth rate for all sites at a given age. The only study on teak, it seems, where height growth is described with a set of *polymorphic* curves is by Tint and Schneider (1980) who used the same data as Laurie and Ram (1940). In the base-age invariant polymorphic model, derived from the estimation technique described by Bailey and Clutter (1974), the relative height growth rate is shown to be a function of both site and age:

$$H = 10^{b_0}\left(\frac{SI}{10^{b_0}}\right)^{\left(\frac{A_b}{A}\right)^{b_{11}}} \qquad \text{...[59]}$$

where: H = top height (m)

SI = site index, the height at age A_b (m)

A_b = index age (years)

A = age for which H is determined (years)

b_0, b_1 = parameters to be estimated by regression

When the polymorphic curves by Tint and Schneider (1980) are compared with the curves by Laurie and Ram (1940) (Figure 42) it is evident that different methods of curve fitting has led to greatly different results. The polymorphic curves show height growth to be superior and increasing for a longer period (later growth seems unabated though) than indicated by the curves from Laurie and Ram (1940). The curves by Laurie and Ram (1940) are relatively to steep up to around 20 years, but reach an asymptote too soon for the poorer sites in particular (Tint and Schneider, 1980).

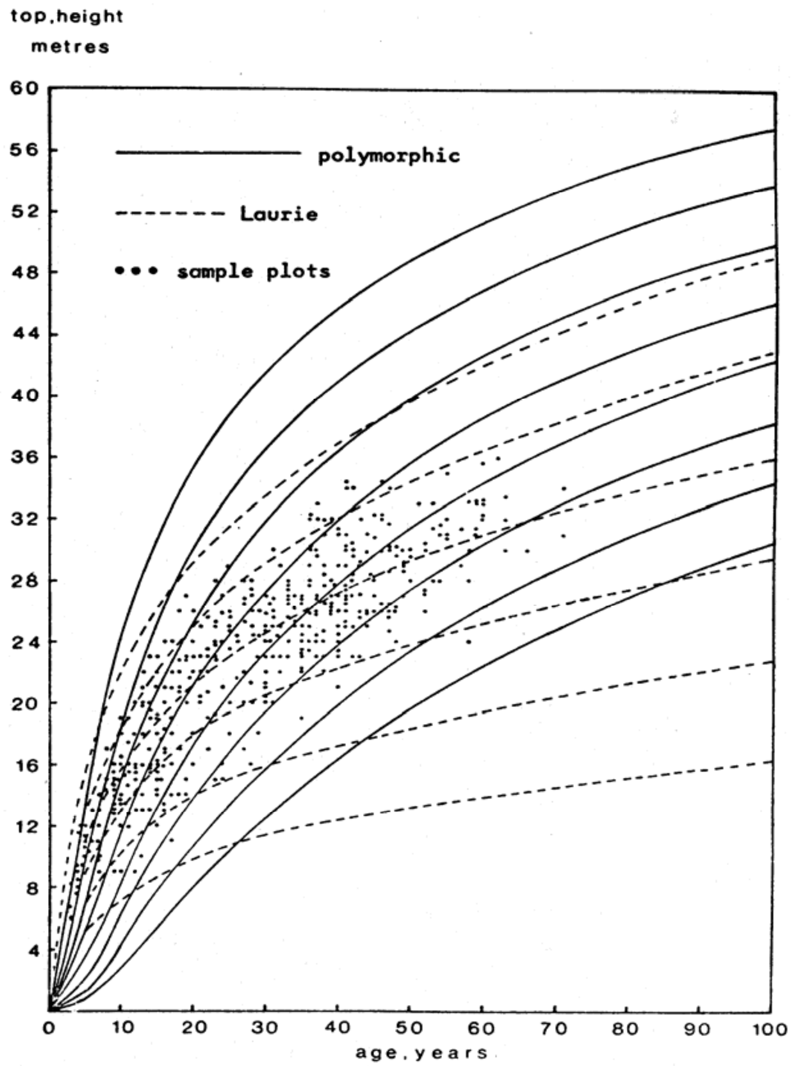


Figure 42. Polymorphic site index curves compared with limiting curves for site qualities produced graphically by Laurie and Ram (1940). The diagram also shows the scatter of sample plots data by top height and age.

Recently, both Phillips (1995) and Adegbehin (2002) used the non-linear Gompertz model to describe teak height growth for plantations from Sri Lanka and Nigeria. Although Phillips (1995) selected the Gompertz model on the basis of minimised *SSD*[37], the resulting curves were almost straight lines, implying indefinite height growth. The problem lies with the data however, since none of the sampled stands were old enough to show a decrease in height growth. This deficiency is only reflected when extrapolating to the higher ages. This necessitated that the height growth curves be

[37] Sum of the squares of the deviations.

constrained according to height growth from other teak growing areas. The constrained curves developed by Philips (1995) are similar to the curves by Keogh (1982). However, it was shown above that the growth curves by Keogh (1982) are negatively biased since the constant *k* in the Schumacher model was simply taken as one.

From this discussion on height growth modelling, it would appear that there is still a great deal of uncertainty on how to accurately describe teak height growth. Some curves have been shown to be mathematically suitable or unsuitable, while others are judged simply on the basis of comparison with other curves believed to best describe height growth. Although Keogh (1982) notes this underestimate in height growth due to the inherent characteristics of the model used and admits that not enough data were available to explain the true trend, Phillips (1995) constrained the Sri Lankan growth curves to match the curves by Keogh (1982)! The reason given is that the curves by Keogh (1982) "*appear to be the only ones fitted mathematically*".

In the study by Nunifu and Murchison (1999), their height growth curves fitted by regression are compared to the graphically fitted curves by Laurie and Ram (1940). The curves are very similar, leading to the conclusion that graphically fitted height growth curves may also be used to compare height growth, sometimes more accurately than with mathematically fitted models.

This apparent confusion on the development of a height growth model for teak is usually blamed on a lack of growth data from representative stands, particularly older stands (Keogh, 1990; Phillips, 1995; Bermejo, Cañellas and San Miguel, 2003). Another factor often overlooked due to the reason of limited data, is the method used for curve fitting. Until such time when adequate data will be available, the methodology used to model teak height growth will remain perplexing.

7.1 Data

Very little growth data are available from the teak plantations in Tanzania. Since limited permanent sample plot data are available only from KVTC and none from the older Longuza and Mtibwa plantations, temporary sample plot data from various sources had to be combined to better represent the spectrum of ages and site qualities. Even by this approach the data do not cover the entire site quality and age range required for unbiased model fitting. In Figure 43 the data from the various sources used in this study are displayed.

The dominant height data from KVTC were obtained from temporary sample plots used in normal plantation inventory. Although covering only a very limited age range, the data from this source encompasses the widest range of site qualities in the whole dataset.

The data from the KVTC permanent sample plots (PSP) are the only repeated measurements available in Tanzania. These permanent plots have been remeasured at least twice but not more than three times. Figure 43 show that these plots are all located in very high quality growing sites.

The data from Longuza covers almost all of the compartments at the plantation. The data were collected from temporary sample plots during an inventory to asses the current growing stock at the plantation. Although not covering all possible combinations of age and site quality, the spread of site qualities is relatively wide[38].

The data from the old compartments were obtained from the measurements discussed in Section 3.1 that were used to develop volume and taper models. The trees from these three compartments are not included according to any measure of dominant height, but instead all trees sampled as described in Section 3.2 were included. Their inclusion is based purely on the premise that by including information on teak height for the widest range of ages as possible, more accurate models will be obtained.

[38] Teak is only planted on suitable sites. This leads to a narrower range in site qualities than would be the case with other species that are more tolerant of poor site qualities.

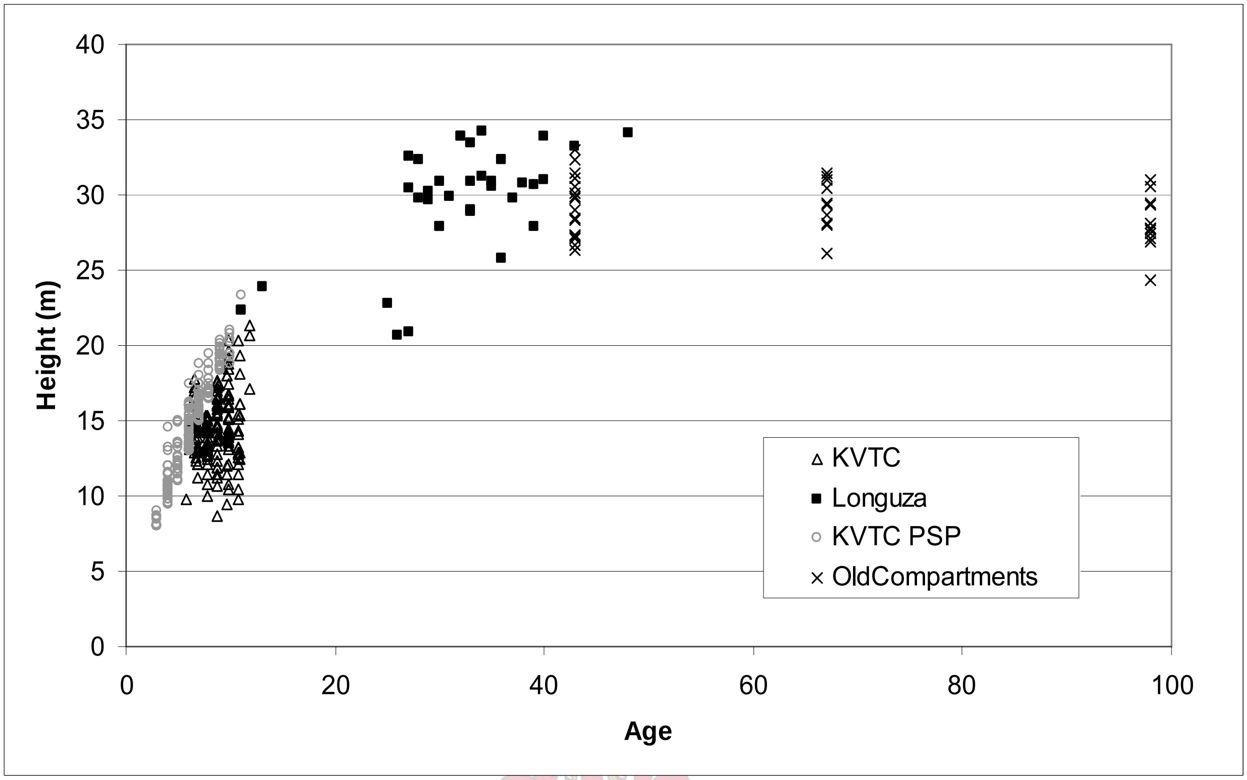


Figure 43. Sample data from the various sources.

7.2 Models

The models selected for use in this study are based on previous studies on teak height growth modelling and on the ability of the model to fit the data set at hand.

The **Schumacher model** is easy to fit and has been shown to be an accurate predictor of tree height at various ages; if the approach by Nanang and Nunifu (1999) is followed:

$$\ln H = b_0 + b_1\left(\frac{1}{A^k}\right) \quad \text{...[60]}$$

where: H = top height (m)

A = age (years)

k = a linearization parameter (often taken to be 1)

$b_0 \dots b_1$ = parameters to be estimated by regression

The site index model for this height growth model is derived by substitution of the index age into the equation and algebraically rearranging it. The height development pattern for a specific site index can then be predicted by:

$$\ln H_j = \ln SI + b_1\left[\frac{1}{A_j^k} - \frac{1}{A_i^k}\right] \qquad \text{...[61]}$$

where: H_j = height at age A_j (m)

A_i = index age (years)[39]

SI = site index (height at age A_i)

A very popular model used to describe height growth is the generalisation of the original Bertalanffy growth model by Richards (1959) and Chapman (1961). The attractiveness of this model is due to its flexibility and the fact that it is based on biological growth theory (Goelz and Burk, 1992). This model has been used in base age invariant form by numerous authors to model biological growth phenomena and especially height growth (Pienaar and Shiver, 1980; Payandeh and Wang, 1994; Amaro *et al.*, 1998).

In this study only the two parameter **Chapman-Richards model** was fitted; the data set did not support the fitting of the three parameter model.

$$H = b_0\left[1 - \exp(b_1 A)\right] \qquad \text{...[62]}$$

where: H = top height (m)

A = age (years)

$b_0 \ldots b_1$ = parameters to be estimated by regression.

This height-age equation can be rewritten into a difference equation (height growth equation) to provide the height development pattern for a specific site index (Cao, 1993):

$$H_j = SI\left[\frac{1 - \exp(b_2 A_j)}{1 - \exp(b_2 A_i)}\right] \qquad \text{...[63]}$$

where: H_j = height at age A_j (m)

A_i = index age (years)[11]

SI = site index (height at age A_i)

[39] An index age of 20 years is used throughout this study.

7.3 Methodology

The parameters for both models were estimated by the appropriate procedures in SAS (SAS Institute Inc., 2001). The parameters for both the Schumacher model and the Chapman Richards model were estimated with the **PROC NLIN** (nonlinear) procedure of SAS.

In order to determine the value of k in the Schumacher model for this dataset, it was first fitted using linear regression and the value of k fixed respectively at 1 and 0.5. The value of k was then allowed to vary by fitting the model with nonlinear regression procedures. Only the value of k was taken from the nonlinear estimate. The other parameters (b_0 and b_1) were then estimated by ordinary least squares (*OLS*) regression with k fixed at the new estimated value. This is done due to the small scale properties of *OLS* and also, as the model is nonlinear in the age variable, the possibility to make *OLS* estimation easy through linearization (Nanang and Nunifu, 1999).

7.4 Results

The parameter estimates, their standard errors and the mean square error (*MSE*) values are provided in Table 19. Figure 44 presents the observed and predicted values by the different models.

From the (*MSE*) values it is clear that the estimated value of k in the Schumacher model is very close to the fixed value of 0.5 and the improvement in *MSE* is inconsequential.

Table 19. Results from fitting the height growth models.

Model	Parameter estimates and standard errors in brackets			*MSE*	*n*
	b_0	b_1	k		
Schumacher	3.4731 (0.0251)	-6.5907 (0.2461)	1	0.03240	243
Schumacher	3.8451 (0.0358)	-3.3843 (0.1187)	0.5	0.02944	243
Schumacher	3.8781 (0.0369)	-3.2680 (0.1167)	0.4788 (0.1284)	0.02943	243
Chapman-Richards	30.5584 (0.4286)	-0.0766 (0.00238)	——	7.80080	243

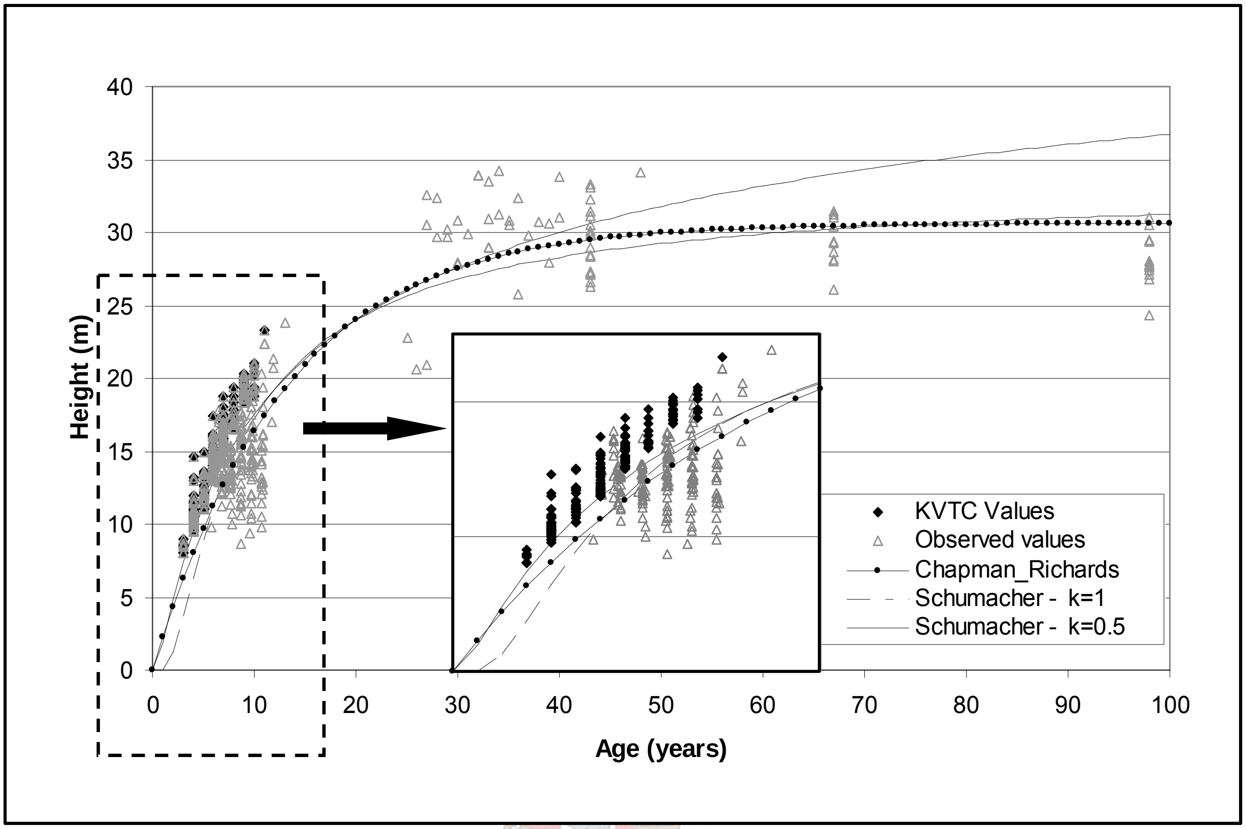


Figure 44. Observed and predicted height values by the Chapman-Richards and Schumacher (*k*: 1 and 0.5) models fitted in this study.

7.5 Discussion

Figure 44 clearly show the similarities between the Schumacher model with k = 1 and the Chapman-Richards model. Both these models reach an asymptotic maximum of around 31 meters at age sixty. The Schumacher model with k = 0.5 does not tend towards an upper asymptote. Instead, it shows height growth continuing to a level far higher than any values observed.

Because four different datasets were used in this study to describe teak height growth, the results will be discussed in terms of each dataset. The values from the old compartments at Mtibwa and Longuza show that all the models overestimate height at high ages, although the Chapman-Richards and Schumacher model with k = 1 reaches an upper asymptote. The data from Longuza (in the age range between 20 and 60 years) show that all three models produce very accurate results for trees in these age classes. For the age classes less than 20 years, it is clear that all the models predict height well, with the Schumacher model with k = 0.5 producing somewhat higher estimates than the other two models. The PSP data from KVTC are, as stated earlier, the only remeasurement data used in this study and represents a very limited range of site quality classes. As this data are from the same plantation as the dominant height enumeration data from KVTC (see Figure 43), it is clear that the PSP data falls in a higher site quality class range than the average site quality of the plantation. This leads to the apparent underestimation of heights by all three models for the PSP data in Figure 44. Therefore, although it might seem that all three models under predict height for the PSP data, this is not the case. In fact, the height growth trend predicted by the Schumacher model with k = 0.5 follow the height growth pattern of the PSP data very well (Figure 45).

The height growth curves in Figure 45 show how the Schumacher model with k = 0.5 accurately follow the height growth pattern for the ages below 20 years. As this is the age range in this dataset with the most observations and also the age range for which most predictions will be made with this model, it can be concluded that the Schumacher model with k = 0.5 is very suitable. Although difficult to asses the prediction accuracy for ages more than 20 years, Figure 45 show that the SI 30 curve accurately follow the highest observations for the age classes less, and more than 20 years. Similarly, the SI 24 and SI 18 curves clearly follow respectively the middle section and bottom section of the data well.

The data from the older stands were included in this study to enable improved assessment of the height-age relationship at extreme ages. Although these data are from very old compartments and present valuable information, it can be seen that these compartments do not give a good representation of a range of site qualities, especially higher than average quality stands (the average height for these three stands actually decrease with age). This characteristic of the data almost defeats the purpose for its inclusion, since it does not give an accurate indication of the upper asymptote that can be expected for teak height growth. What can be inferred from information it provides though, is that teak height growth apparently decreases very rapidly after 35 to 40 years. Although it is unlikely to stop, the rate of increase clearly decreases by a large factor.

From the discussion above it can be concluded that although the Schumacher model with k estimated from the dataset might be the most accurate for younger stands (age < 45 years), it will probably over predict heights for older stands. Nevertheless, it is still more accurate than would be the case if *k* was not estimated from the data or the Chapman-Richards model due to these models reaching an early asymptote. In order to estimate the true value of *k* for teak, more data at higher ages will be required. Unfortunately, very few stands older than 45 years occur in Tanzania.

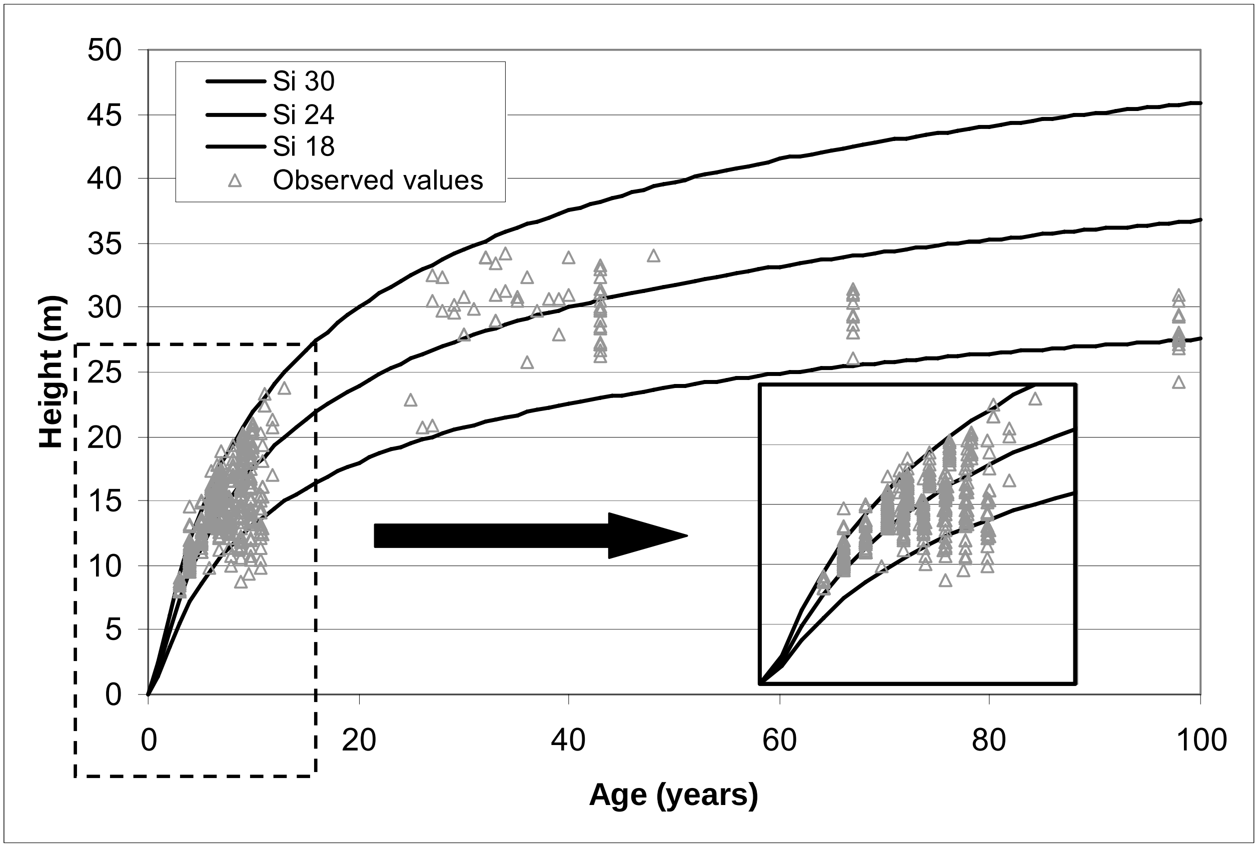


Figure 45. Height development pattern of teak for site indices of 18, 24 and 30 as predicted by the Schumacher model with *k* = 0.5.

Comparative height development patterns (site index = 24) for a range of models is presented in Figure 46. This comparison clearly shows great disparity in the height predictions between the various models. Although some of the dissimilarity can be attributed to differences in growing conditions (e.g. different soils and amount of precipitation), most of the variations can be attributed to the method of fitting the curves.

The disparities between the models become more pronounced at high ages. Some models show that height increases without any decrease in rate as age increase (Tint and Schneider, 1980; Keogh, 1990) while other models reach an asymptote at relatively young ages (Keogh, 1982; Chapman-Richards, this study). Figures 45 and 46 indicate that the Schumacher model (with *k* estimated in this study) predicts height somewhere between these extremes may provide the best results.

It is accepted that comparison of growth curves fitted by different means and for other growing conditions can lead to extremely biased evaluations. The aim should be to obtain data that is more representative of the ages and growing conditions for which projections will be made.

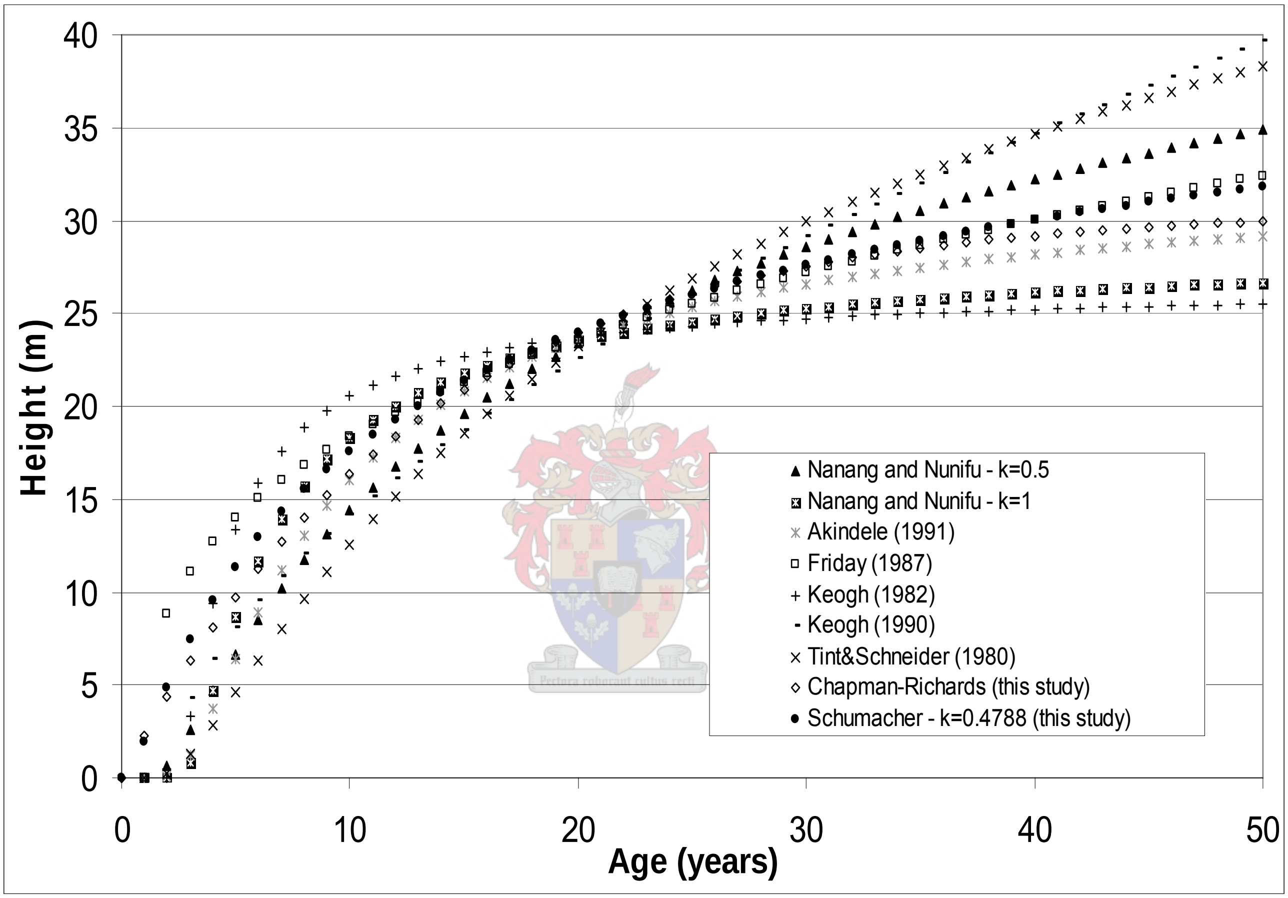


Figure 46. Comparative height development curves for a site index of 24 from models in past studies for teak and the Chapman-Richards and Schumacher models fitted in this study.

The recommended site index equation for Tanzanian teak plantations by this study is presented in Equation 64.

$$\ln H = \ln SI + \left(-3.3843\left(\frac{1}{Age^{0.5}} - \frac{1}{20^{0.5}} \right) \right) \qquad \text{...[64]}$$

Appendix B: Development of a model to predict teak height from dbh, site index and age

Estimates of total tree height and dbh at current and future times are often needed by forest growth and yield prediction systems that predict/project the growth of individual trees or tree size classes (Tewari and Von Gadow, 1999). In yield studies, stand volumes are frequently calculated from tree volumes by using stem volume prediction models derived from the dbh and height distributions of the stand (Eerikäinen, 2003). Dbh and total height are therefore the two most important tree attributes measured in a stand during an inventory.

Unlike tree diameters that are easy to measure infield, tree height is more difficult, time consuming and costly to measure. To overcome this problem, use is often made of an established height-dbh model to predict the "missing" tree heights from field measurements of tree diameters (Zhang, *et al.,* 2002). This height-dbh model presents a height development curve with respect to diameter increase that is useful as a description of the development of a stand over time, for the estimation of mean heights of specific portions of the stand, as an estimation of site index and in the estimation of growth by stand projection methods (Curtis, 1967).

The area of application of a height-dbh model can either be localised, or for more generalised use, dependent on the predictor variables used and the source of the data (Soares and Tomé, 2002). *Local* equations are normally only dependent on tree diameter and should be applied only to the stand/s where the data were collected. A *regional* equation on the other hand, predicts tree height based on additional stand variables such as age, site quality and in some cases a measure of stand density. These can be applied on a regional level and used in conjunction with yield prediction systems to provide stand table estimates at various ages (Lynch and Murphy, 1995). Care must be taken when applying a regional model since local environmental conditions (e.g. soil and climate) play a significant role in affecting the eco-region based height-dbh relationships (Zhang *et al.*, 2002). This factor could cause significant errors when local tree heights and volumes are estimated.

In modelling the height-dbh relationship, an important aspect is the temporal development of this relationship. The temporal development is commonly characterised by two elements: the *form development* and the *asymptotic development* of the height curve (Loetsch, Zöhrer and Haller, 1973; Eerikäinen, 2003). In even aged plantations, the *form development* means that the curves are steeper in younger stands and become more even towards the end of the rotation. This implies that the curve should have a steeper slope for young trees than for more mature ones.

Asymptotic development means that the asymptotic maximum of the height-dbh curve increases as a function of age, even if the model does not have a specific asymptote parameter. The asymptotic development of a height-dbh model can be related to the development of the stand dominant height. This is reasonable since the stand dominant height is frequently a reliable and measurable function of site quality, which makes it a good indicator of tree growth potential (Eerikäinen, 2003).

There are several model forms available for describing the height-dbh relationship. Curtis (1967) presents a comprehensive list of the most common local equations while Lynch and Murphy (1995) provide an overview of height-diameter-age curves. More recent summaries of linear, nonlinear and transformed non-linear model forms are provided by Van Laar and Akça (1997), Soares and Tomé (2002) and Eerikäinen (2003). From the summaries provided by the above-mentioned authors and numerous other studies, it can be seen that many models have been developed to date. Two models stand out as very popular for modelling the relationship between tree height and dbh, the Michailoff (1943) (Curtis, 1967; Soares and Tomé, 2002) and the Chapman-Richards (Ek, Birdsall and Spears, 1984; Lynch and Murphy, 1995; Peng, 1999; Zhang *et al.*, 2002) growth equations.

7.6 Sample data

The data available (n = 2187) for model development were obtained from temporary sample plots during routine forest inventory (sampling intensity ± 5%) on the Longuza teak plantation located in the northern region of Tanzania (as described in Section 2.1). Poor management of this plantation during most of its existence has led to a large variation in diameters and heights, even at the compartment level. This situation is portrayed by the summary statistics for the height and dbh values provided in Table 20 and the visual presentation of the dbh and height sample values in Figure 47.

Table 20. Summary statistics of the 2187 height and dbh observations.

	Dbh (cm)	Height (m)
Mean	38.801	26.788
Standard error	0.198	0.0805
Standard deviation	9.296	3.768
Minimum	12.7	13.0
Maximum	70.0	37.0

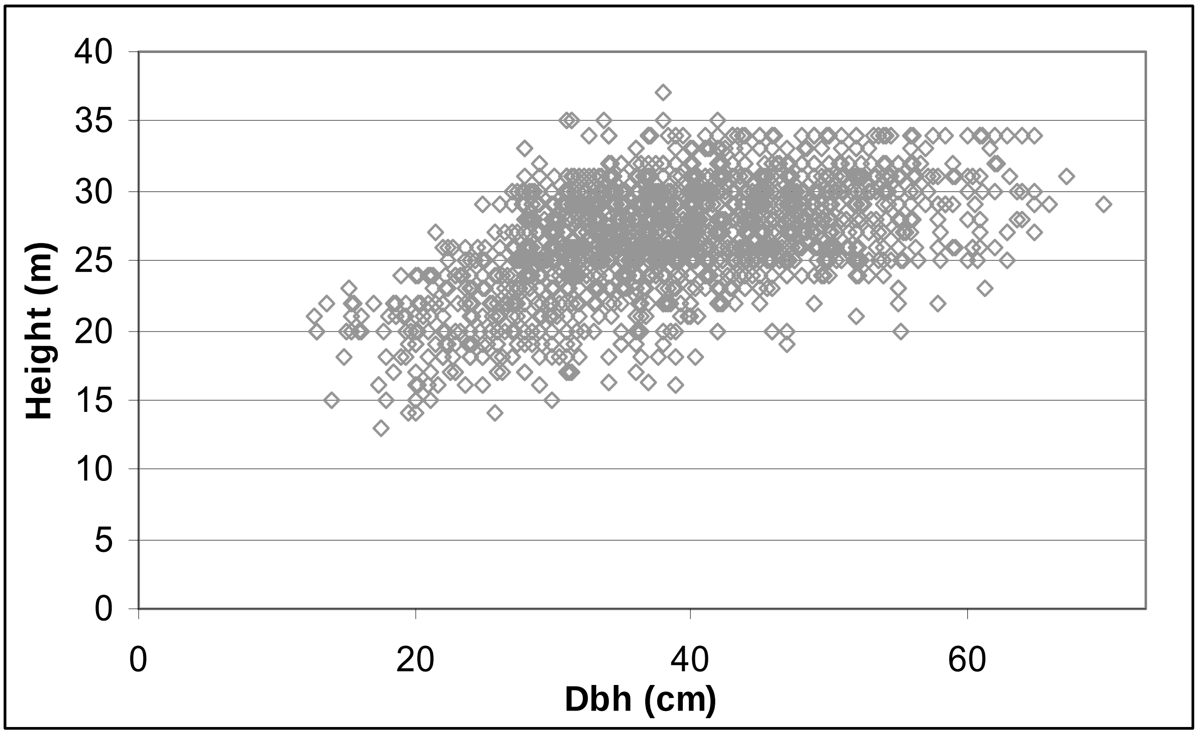


Figure 47. Observed height and dbh values.

The data are unsuitable for modelling the height-dbh relationship; the ideal being successive height-dbh remeasurements from a series of permanent sample plots. Unfortunately, no permanent growth trial in Tanzania that would provide this information exists and since Longuza and Mtibwa are the only teak plantations in Tanzania with mature trees, there is no alternative source of data.

7.7 Rationale

The visual presentation of the sample data in Figure 47 shows a large amount of variation. It is clear that if only dbh is used to predict tree height, a great deal of variation would be unaccounted for. This is further illustrated by Figure 48 where the height and dbh values for trees of the same age, but two different site qualities are shown. The data are evidently grouped in two populations implying an influence of site quality on the height-dbh relationship. In Figure 49 height-dbh values from compartments with the same site index, but different ages are plotted. This indicates that the height-dbh relationship is also influenced by age.

It is thus clear that the stand variables age and site index should be included with dbh in a model to predict height in order to obtain more accurate results.

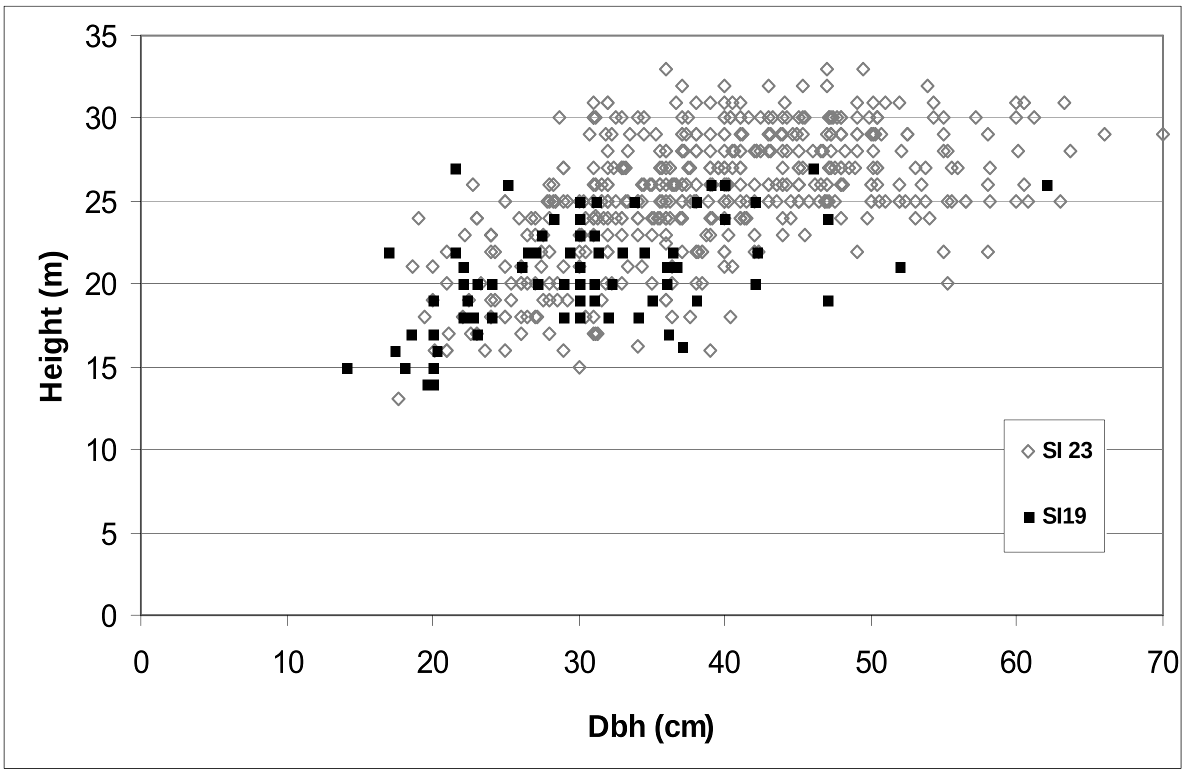


Figure 48. Observed heights for age 31, site indices 19 and 23.

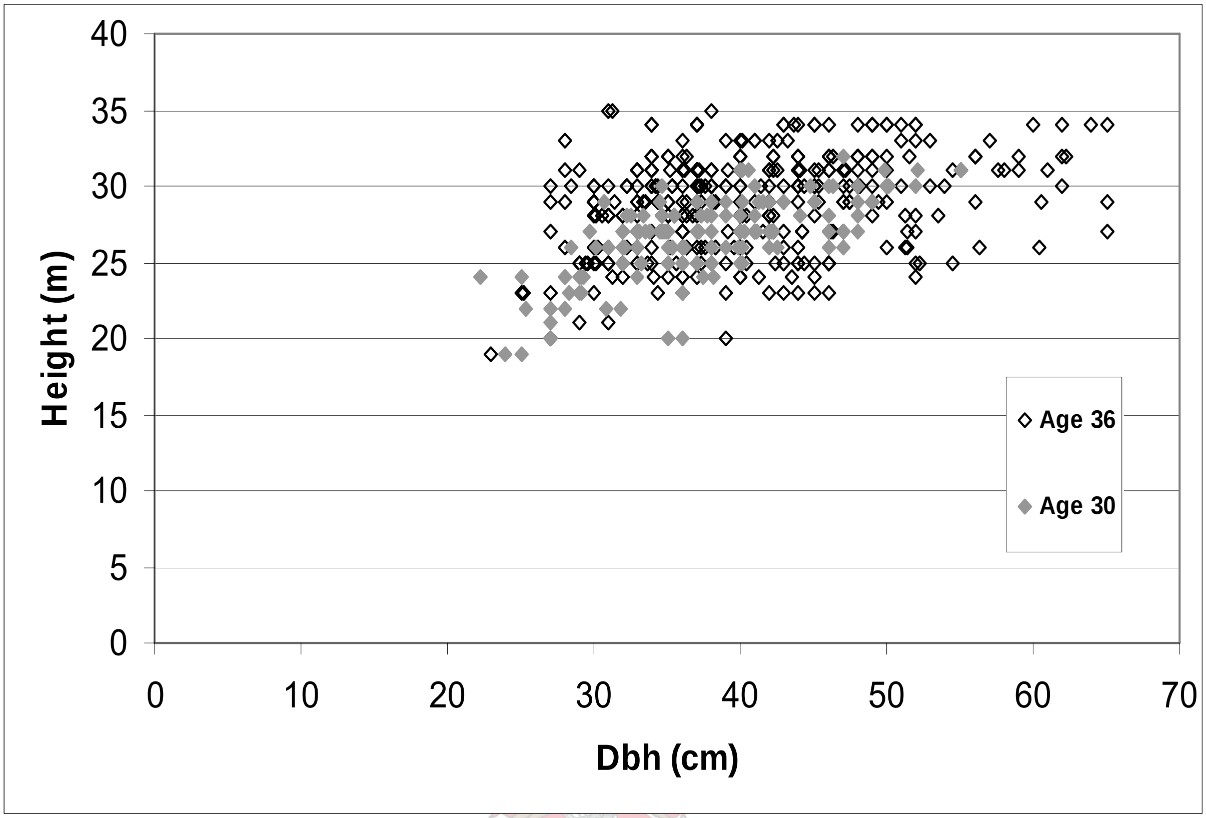


Figure 49. Observed heights for site index 22, ages 30 and 36.

7.8 Models

Model 5.1.

Various non-linear models, including the Chapman-Richards, Weibull, Lundqvist/Korf and exponential model forms were fitted to the sample data. Results obtained by fitting non-linear models are in par with similar studies; all these model forms are similarly well suited to describe the height-dbh relationship (Fang and Bailey, 1998; Peng, 1999). Since the Chapman-Richards model gave a similar fit as the models listed above and is widely known for effectively modelling this relationship due to its flexibility and biologically interpretable coefficients (Zhang *et al.*, 2002), it was selected for further model development. The basic form of the Chapman-Richards three parameter model as used by the studies above is provided in Equation 65.

$$H = 1.3 + b_0\left(1 - e^{-b_1 D}\right)^{b_2} \qquad \text{...[65]}$$

where: H = tree height (m)

D = dbh (cm)

$b_0 \ldots b_4$ = parameters to be estimated by regression

The Chapman-Richards model was first fitted in the form presented in Equation 65. To include the effect of age and site quality on the dbh-height relationship, the approach described by Ek, Birdsall and Spears (1984) was followed. The final model, **Model 5.1** presented in Equation 66 show that correction factors for site index and age are simply added to the basic form of the Chapman-Richards growth model as multipliers.

$$H = 1.3 + b_0\left(1 - e^{-b_1 D}\right)^{b_2} \times SI^{b_2} \times A^{b_4} \qquad ...[66]$$

where: H = tree height (m)

D = dbh (cm)

SI = site index[40]

A = age (years)

$b_0 \ldots b_4$ = parameters to be estimated by regression

Model 5.2.

Probably the most popular model used to describe the relationship between height and diameter is the Schumacher, or Michailoff (1943) model as it is alternatively known (Von Gadow and Hui, 2001). This model is presented in its basic form in Equation 67.

$$Ln\ H = a + bD^{-1} \qquad ...[67]$$

where: H = tree height (m)

D = dbh (cm)

In a similar approach to the inclusion of age and site index as predictor variables to Model 5.1 above, the Schumacher model was first fitted in the form as presented in Equation 67. This model was modified by Curtis (1967) to include the effect of age on the relationship between diameter and height. In this study, a similar methodology as Curtis (1967) was applied; site index was also included as predictor variable along with dbh. Finally, all three predictor variables were fitted together. The final model, **Model 5.2**, with dbh, age and site index included as predictor variables is presented in Equation 68.

$$Ln\ H = b_0 + b_1 D^{-1} + b_2 A + b_3 SI \qquad ...[68]$$

where: H = tree height (m)

D = dbh (cm)

SI = site index

[40] A base age of 20 years is used throughout this study.

A = age (years)

$b_0 \ldots b_4$ = parameters to be estimated by regression

7.9 Methodology

Model 5.1 was fitted by using the **PROC NLIN** (non-linear regression) procedure of SAS (SAS Institute Inc., 2001) to estimate model parameters and statistics. Model 5.2 was fitted by the **PROC GLM** (linear regression) procedure, also in SAS, after appropriately transforming the data.

Initial model selection (between the various non linear models) was based on the *MSE*[41] (mean square error) value. Subsequent selection of the best model was also based on the *MSE* value, together with an evaluation of the appropriate residual graph.

7.10 Results

The parameter values, their standard errors and the *MSE* for the fitted models with different combinations of independent variables are provided in Table 21.

Table 21. Results from fitting Models 5.1 and 5.2.

Model	**Independent variable(s)**	**Parameters**					***MSE***
		b_0	b_1	b_2	b_3	b_4	
5.1 Eq (67)	Dbh, age and SI	0.7654 (0.1409)	0.0662 (0.0070)	1.0976 (0.1733)	0.8842 (0.0444)	0.2399 (0.0189)	7.9306
5.2. Eq (66)	Dbh^{-1}	3.5859 (0.0096)	-11.2216 (0.3340)	—	—	—	9.6080
5.2. Without age	Dbh^{-1} and SI	2.9108 (0.0386)	-11.6890 (0.3129)	0.0278 (0.0015)	—	—	8.6830
5.2. Without SI	Dbh^{-1} and age	3.5233 (0.0268)	-10.8816 (0.3603)	0.0017 (0.000671)	—	—	9.5090
5.2. Eq (68)	Dbh^{-1}, age and SI	2.3342 (0.0580)	-10.0664 (0.3266)	0.00884 (0.00068)	0.0379 (0.0016)	—	7.9450

41 The *MSE* value is calculated by the regression procedures and is a measure of the amount of variation explained by the model, with smaller values indicating a better fit and more variation explained by the model (Ott, 1988).

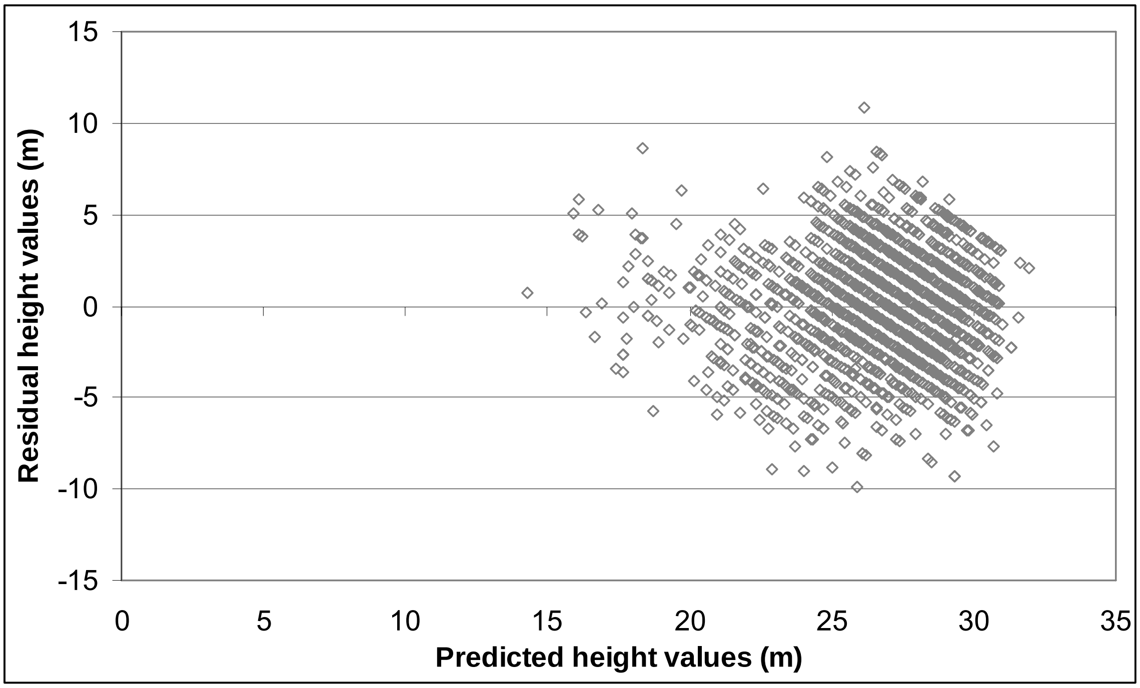


Figure 50. Predicted and residual height values for Model 5.1 with dbh, age and site index as predictor variables.

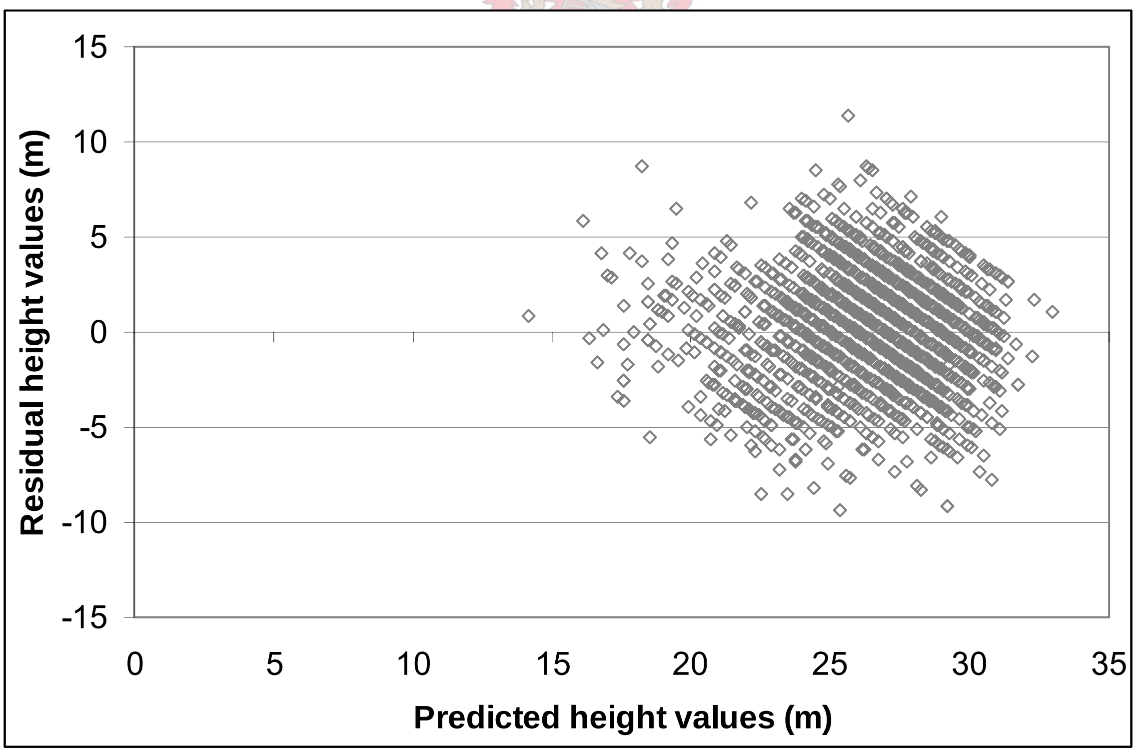


Figure 51. Predicted and residual height values by Model 5.2 with dbh, age and site index as predictor variables.

The residual values for both full models are presented in Figures 50 and 51. Analysis of these graphs show that there is no apparent systematic pattern present for either of the models. The residuals are somewhat greater for larger tree heights, which could be due to the fact that the height of larger trees is more difficult to measure and hence include more measurement error.

7.11 Discussion

From the results in Table 21 it is clear that when the site index and age of a stand are included in addition to tree dbh in a model to predict tree height, the accuracy of predictions is significantly improved as indicated by the *MSE*. The results from fitting the two models with all three independent variables included are very similar. Selection between these two models is therefore based purely on ease of estimating the parameters and using the model. Since Model 5.2 is fitted by linear regression procedures and has fewer parameters, it is recommended. This model is shown with the estimated parameters in Equation 69.

$$Ln\ H = 2.3342 + (-10.0664D^{-1}) + (0.00884 \times A) + (0.0379 \times SI) \quad ...[69]$$

As an indication of how the methodology followed in this study has improved accuracy when predicting tree height from dbh, Figure 52 show the same data as in Figure 48 where two populations of the same age, but different site qualities can be seen. It is apparent that the inclusion of site index has improved the ability to accurately predict tree height.

In Figure 53, the pattern of predicted height over dbh relationship according to Model 5.2 is illustrated when first age and then site index is kept constant. The effect of age is clearly less than the effect of site quality on the predicted height-dbh relationship, but nevertheless sufficiently influential to warrant its inclusion in the model.

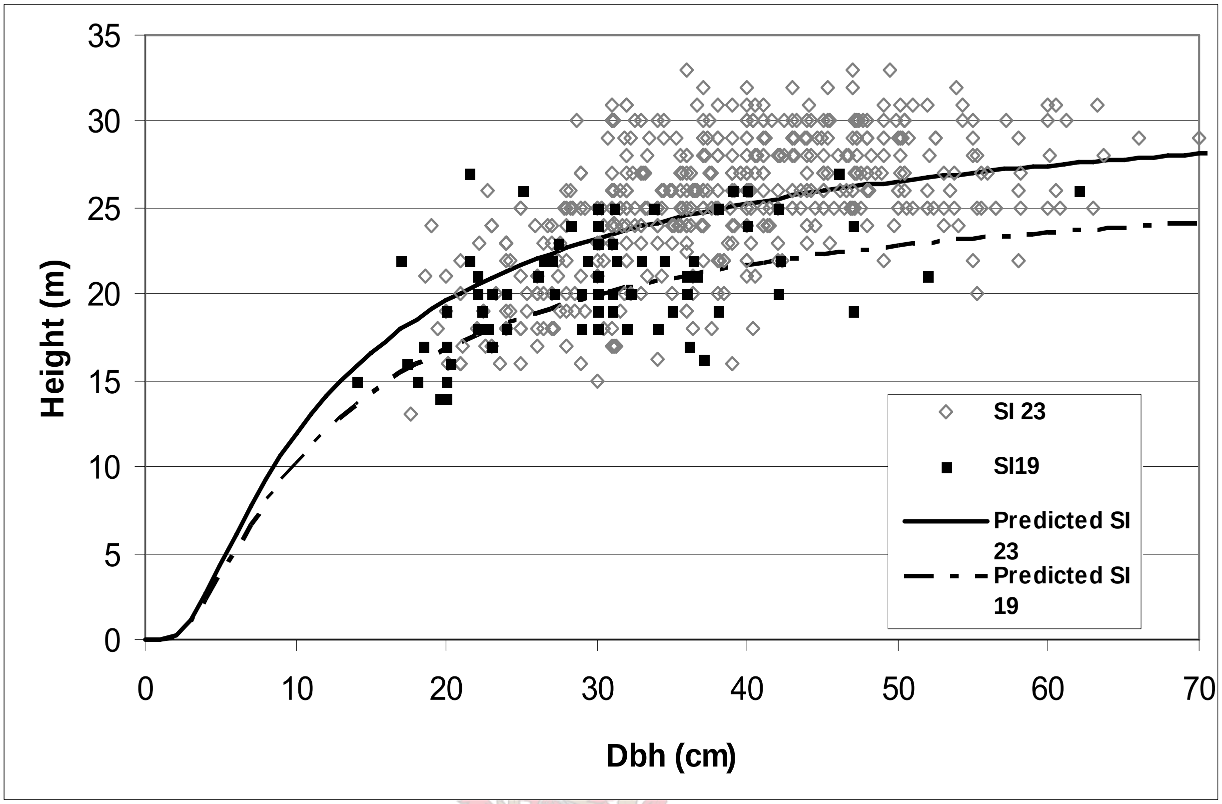


Figure 52. Observed and predicted values by Model 5.2 where age is kept constant at 31 years and site indices of 23 and 19.

According to the results and the visual presentation in Figures 52 and 53, Model 5.2 predicts the *temporal form* of the height-dbh relationship well. Steeper height-dbh curves are obtained for younger stands that develop towards more even curves for older stands. The *asymptotic development* of the curve also shows an increase as a function of both stand age and site quality.

In conclusion, this model may be useful in forest inventory compilations when heights have not been measured.
The model may be used in all the teak plantations in Tanzania with little loss in prediction as compared to equations that do not take into account the effect of age and site quality on the relationship between height and dbh.

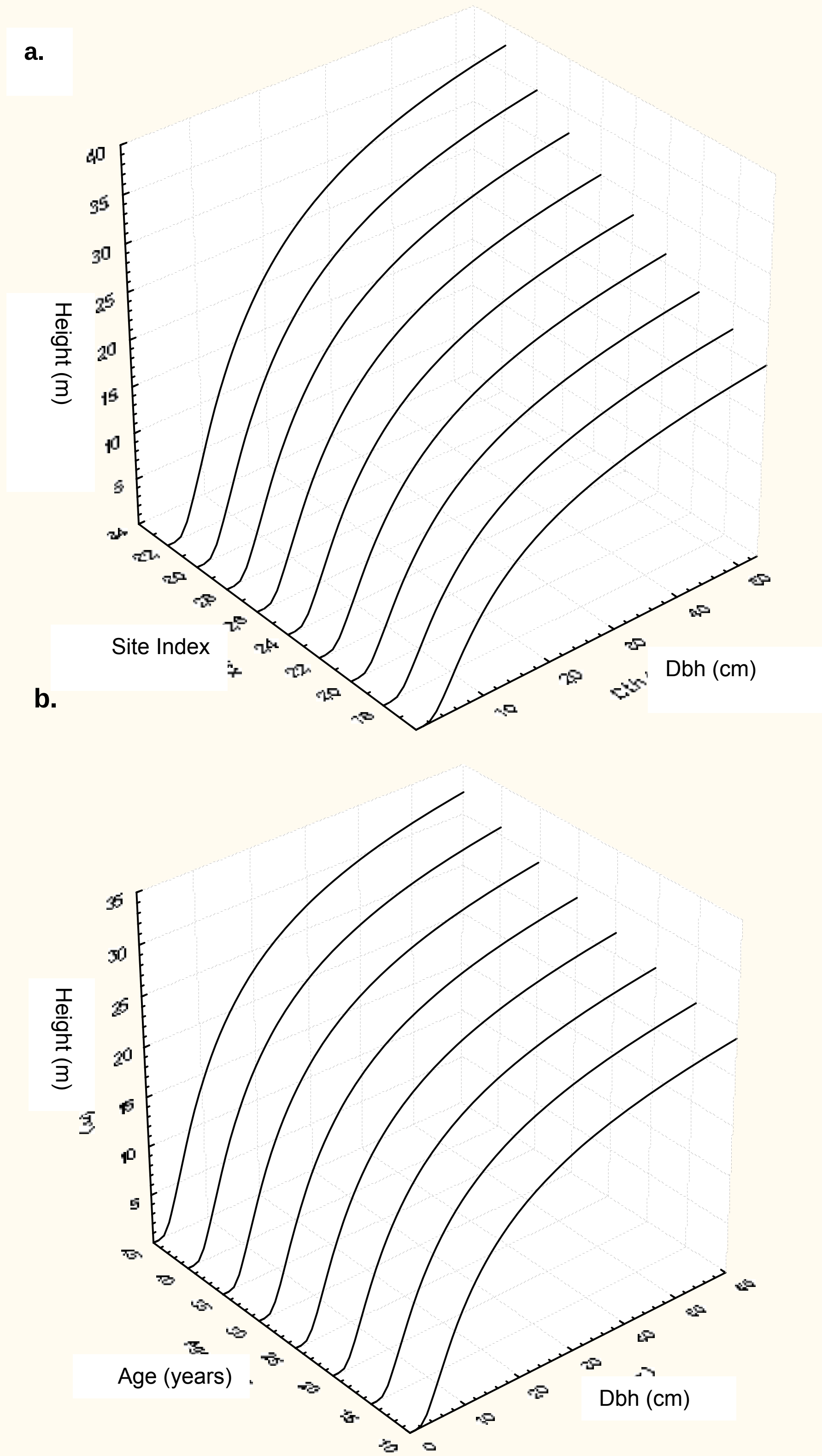


Figure 53. Predicted heights for teak trees by Model 5.2 based on dbh and site index with age constant at 31 years (a) and age with site index constant at 22 (b).

Appendix C:
Principle and operation of the Barr and Stroud dendrometer

The following discussion on the principle and operation of the B & S is taken from DeVries (1971).

A ray of light emerging from point *p* (Figure 56) of (for simplicity's sake) a flat vertical target of width *D* (parallel to *B),* and passing through the left end *l* of, and at right angles with a base line *lr* of fixed length *B,* is reflected by a fixed surface[42] *m11* into a direction parallel to *lr*, after which the likewise fixed surface m12 reflects this ray into a direction perpendicular to *lr*. The image of *p* is *p'* (the optical system by which real images like *p'* are produced is omitted in the sketch). In that situation the image of *q* is formed in *q'*.

A ray emerging from *p* in the direction of *r* hits a system *s* of two counter-rotating prisms. The axis of counter-rotation is in the plane of the figure.

By counter-rotating the prisms the deflective power of the system may be varied continuously from a maximum deviation, via zero, to a maximum deviation in the opposite direction.

Counter-rotation is achieved by turning the instrument's working head, and the angle z by which the head is turned can be read on the main scale or drum with a vernier in tenths of centesimal degrees (grads).

By counter-rotating the prisms the incident ray from *p* can be deflected into a direction perpendicular to *lr*, after which it is reflected by the fixed surface *m21* into a direction parallel to the base. Reflection by the likewise fixed surface *m22* puts the image of *p* in *p"*. In that situation the right-hand optics produce the image q" of q.

Both images *p'q'* and *p"q"* are observed through a magnifying lens *O*. Optical provisions are such that *p'q'* is seen in the upper, and *p"q"* in the lower part of the field of view.

The situation described above, in which *p'* coincides with *p"*, is named 'true coincidence' or 'No. I setting'. The corresponding reading on the main scale in grads is named z_t. From the figure it is evident that the angle of total deviation in case of true coincidence equals the angle of convergence *lpr* = δ_t.

[42] Actually the *'reflecting fixed surfaces*' are pentaprisms.

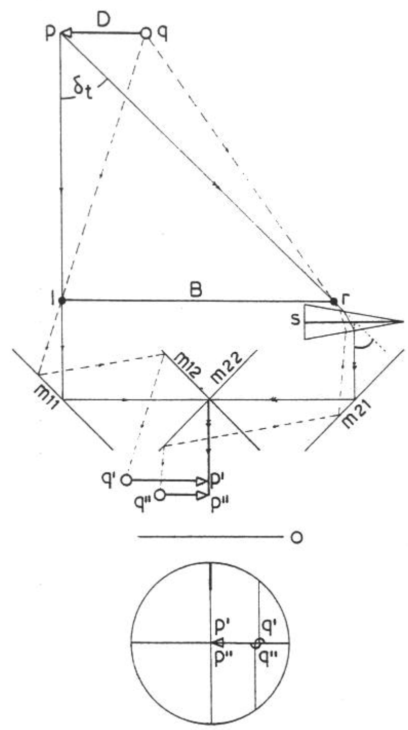


Figure 56. True coincidence (schematically).

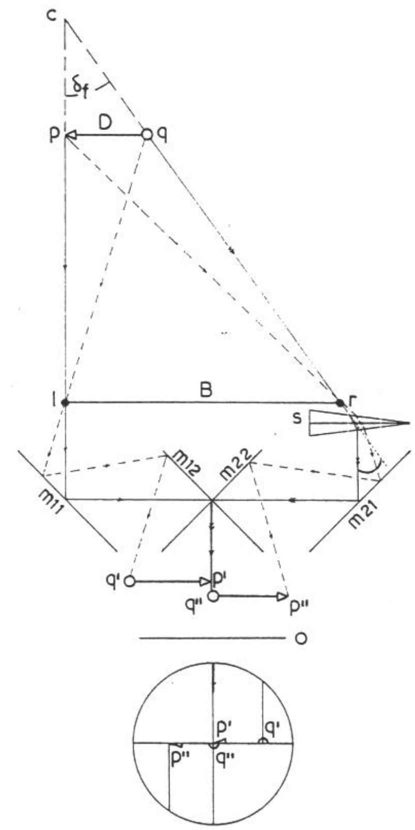


Figure 54. False coincidence (schematically).

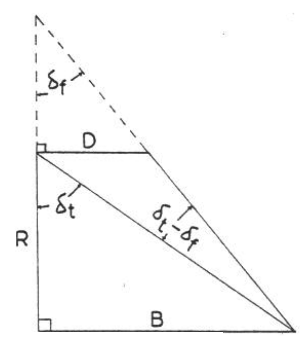


Figure 55. Approximate determination of *R* and *D.*

The angle of deviation caused by the entire optical system depends on:

1. the angle by which the prisms are counter-rotated. Zero counter rotation is when the prisms completely neutralize each other's deflection, which occurs at drum setting 100.0. At drum settings unequal to 100.0 the prisms are in a counter rotated position. Maximum 'positive' counter-rotation is at setting 0.0: the prisms

then cooperate to produce a maximum positive (i.e. clock-wise) counter-rotational deflection. Extreme 'negative' counter-rotation is at setting 156.0: the prisms then cooperate to produce a 'maximum' negative (i.e. anti-clockwise) deflection. The angle of counter-rotation depends on the drum setting and on instrument constants.

2. prism setting, which e.g. may be the setting for minimum-deviation in any situation, or otherwise. Prism setting is a constructional instrument feature.
3. a constant negative bias deflection, caused by the pentaprisms, of M radians, which is also present in the situation of zero counter rotation. Zero total system deviation (parallelism) occurs at drum setting 64.5. In that situation the positive counter-rotational deflection is exactly compensated by the sum of biases of the system.

Given a drum setting of z_t and the relevant instrument constants, the total deviation of the system in case of true coincidence can be calculated, and consequently δ_t is known.

In the situation of true coincidence, the ray from *q* in the direction of *r* is not deflected perpendicularly to *lr*. In order to obtain the latter, i.e. to make *q″* coincide with *p′* (Figure 54), counter-rotation has to be decreased, as a result of which counter rotational deflection also decreases, i.e. the drum setting must become greater than z_t.
The situation in which *q″* coincides with *p′* is named 'false coincidence' or 'No. II setting'. The corresponding reading on the main scale in grads is named z_f, and $z_f > z_t$.
The angle of total deviation in case of false coincidence equals the angle of 'back convergence' $lcr = \delta_f$, and δ_f can be calculated from z_f by the relations indicated above.
Once δ_t and δ_f are known in radians, the range *R* and target width *D* (Figure 55) follow from:

$$R \cong B / \delta_t \qquad \text{...[70]}$$

$$D \cong R(\delta_t - \delta_f) = B(1 - (\delta_f / \delta_t)) \qquad \text{...[71]}$$

The above outline has been given for target widths *D* < *B.* For *D* = *B* (parallelism), the main scale reading in case of false coincidence is 64.5 grads, and total deviation, δ_f> is zero, which is in accordance with (71).
Counting angles of convergence, *δ,* positive in an anti-clockwise direction from the direction *pl*, the angles δ_f become negative for targets *D* > *B.* Hence (71) is generally valid.

As tree diameters are not flat targets, and as in case of inclined line of sight the plane through base *B* and the fixed line of sight *pl* has an elliptical intersection with the stem, the exact formulae for *R* and *D* are rather complex, and the same holds for the formula for the exact height *H* of the target above or below the horizontal. The latter is generally approximated by:

$$H \cong R\sin\alpha = (B\sin\alpha)/\delta_t \qquad \text{...[72]}$$

where α is the angle of inclination (elevation or depression).

The complex formulae for *R, D* and *H* (Grosenbaugh, 1963) call for a computer programme which was supplied in Fortran by the latter author, together with the general theory.

With the normally occurring angles of convergence of only a few degrees, the approximations (70) to (72) however, will do for most practical purposes. In the manual supplied by the manufacturers, tables are given, derived from (70) and (71).

An example of the derivation of diameter values from Barr and Stroud settings.

The true (z_t) and false (z_f) coincidence settings for the observation in this example were respectively 47.9 and 83.9. These values are simply substituted into the following equations to obtain the values of δ_f and δ_t which are then used in Equation 71 to obtain the diameter.

$$Z_t = \cos 0.9\, z_t \qquad \text{...[73]}$$

$$Z_f = \cos 0.9\, z_f \qquad \text{...[74]}$$

$$\delta_{t,f} = L.Z_{t,f} - M \qquad \text{...[75]}$$

$$D = B\left(1 - \left(\delta_f / \delta_t\right)\right) \qquad \text{...[76]}$$

$$\left.\begin{aligned} \text{where: } L &= 0.03929471 \\ M &= 0.02077968 \\ B &= 20.32\text{ cm} \end{aligned}\right\} \text{Constants provided by the manufacturer}$$

$$\therefore D = B\left(1 - \left((L.Z_f - M)/(L.Z_t - M)\right)\right)$$

$$D = 20.32\left(1 - \left(\frac{0.03929471 \times \cos(0.9 \times 83.9) - 0.02077968}{0.03929471 \times \cos(0.9 \times 47.9) - 0.02077968}\right)\right)$$

D = 48.45 cm

The calculated diameter from the true and false settings above is thus **48.45 cm**. The tables provided by the manufacturer in this case yield a diameter of **48.4 cm**.

Appendix D: Illustrations of the instruments used in this study

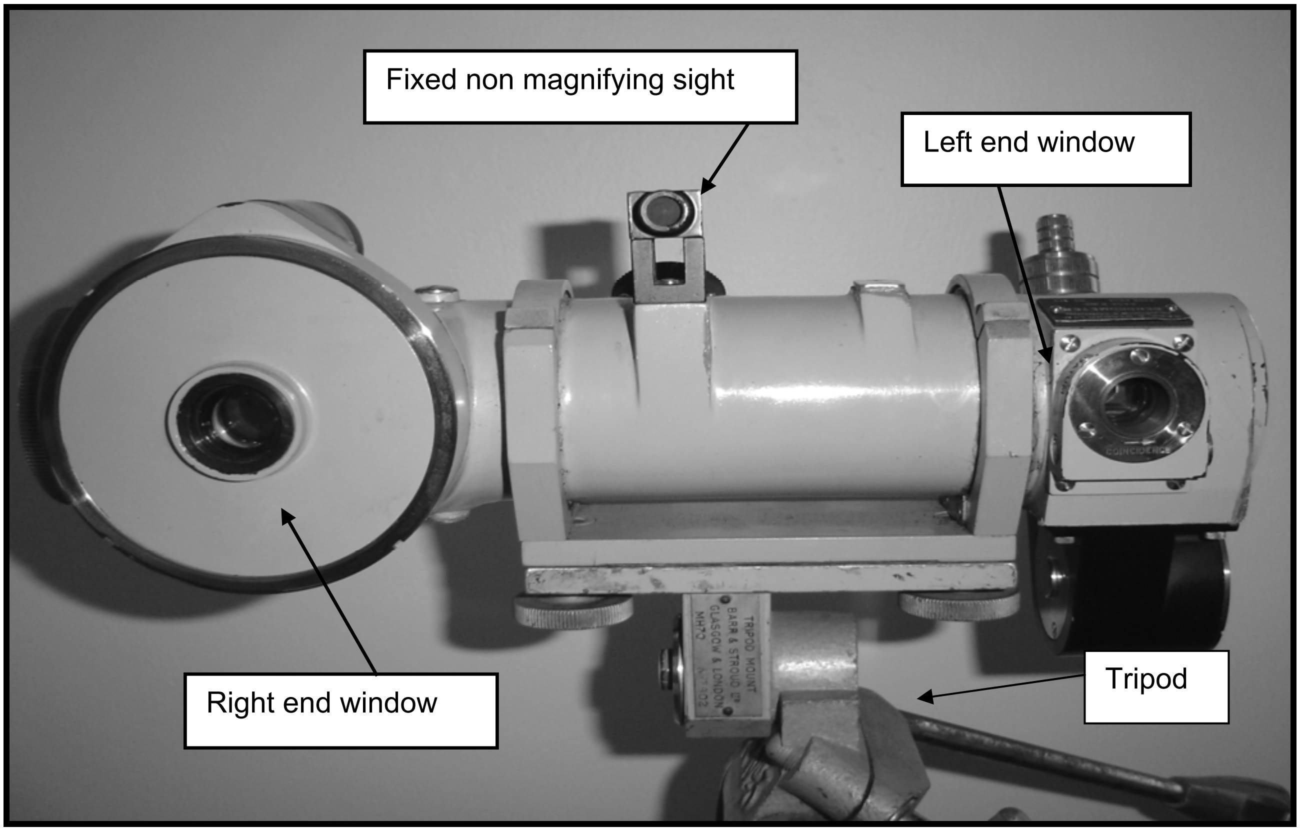


Figure 57. Front view of the Barr and Stroud dendrometer.

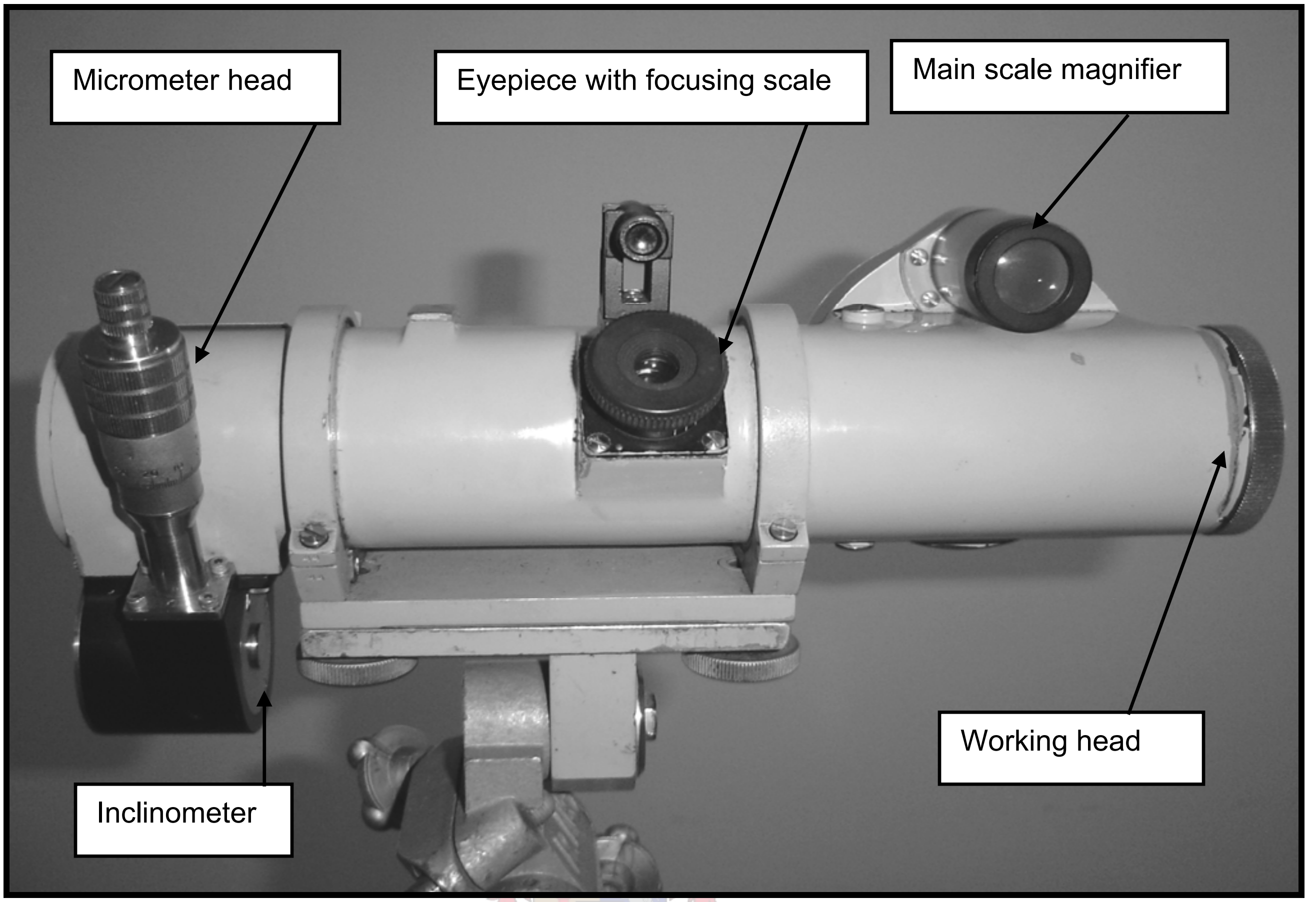


Figure 58. Rear view of the Barr and Stroud dendrometer.

Figure 59. Swedish bark gauge.