Modelling weapon assignment as a multiobjective decision problem

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Thesis presented in partial fulfilment of the requirements for the degree
MComm (Operations Research)
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March 2012

Declaration

By submitting this dissertation electronically, I declare that the entirety of the work contained therein is my own, original work, that I am the authorship owner thereof (unless to the extent explicitly otherwise stated) and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

Date: March 1, 2012

Abstract

In a *ground-based air defense* (GBAD) military environment, *defended assets* on the ground require protection from enemy aircraft entering the defended airspace. These aircraft are detected by means of a network of sensors and protection is afforded by means of a pre-deployment of various ground-based *weapon systems*. A *fire control officer* is responsible for deciding upon an assignment of weapon systems to those aircraft classified as threats. The problem is therefore to find the best set of weapon systems to assign to the threats, based on some pre-specified criterion or set of criteria. This problem is known as the *weapon assignment problem*.

The conditions under which the fire control officer has to operate are typically extremely stressful. A lack of time is a severely constraining factor, and the fire control officer has to propose an assignment of weapon systems to threats based on his limited knowledge and intuition, with little time for analysis and no room for error. To aid the fire control officer in this difficult decision, a computerised *threat evaluation and weapon assignment* (TEWA) *decision support system* is typically employed. In such a decision support system a *threat evaluation* subsystem is responsible for classifying aircraft in the defended airspace as threats and prioritising them with respect to elimination, whereas a *weapon assignment* subsystem is responsible for proposing weapon assignments to engage these threats.

The aim in this thesis is to model the weapon assignment problem as a multiobjective decision problem. A list of relevant objectives is extracted by means of feedback received from a weapon assignment questionnaire which was completed by a number of military experts. By using two of these objectives, namely the cost of assigning weapon systems and the accumulated *single shot hit probability*, for illustrative purposes, a bi-objective weapon assignment model is derived and solved by means of three multiobjective optimisation methodologies from the literature in the context of a simulated, but realistic, GBAD scenario.

The *analytic hierarchy process* (AHP) is implemented by means of assessments carried out in conjunction with a military expert. The assignment of weapon systems to threats is achieved by means of a greedy assignment heuristic and an AHP assignment model. Both these methods provide plausible results in the form of high quality assignments achieving an acceptable trade-off between the two decision objectives. However, a disadvantage of the AHP approach is that it is inflexible in the sense that a large portion of its pre-assessments have to be reiterated if the set of weapon systems and/or threats is adapted or updated.

A bi-objective additive utility function solution approach to the weapon assignment problem is also developed as a result of various assessments having been carried out in conjunction with a military expert. The assignment of weapon systems to threats is again achieved by means of a greedy assignment heuristic and a utility assignment model. Both these methods again provide high quality assignments of weapon systems to threats, achieving an acceptable trade-off between the two decision objectives. However, a disadvantage of the utility function approach is that if additional weapon systems are added to the current set of weapon systems,

which achieve objective function values outside the current ranges of the values employed, new utility functions have to be determined for the relevant objective function. Moreover, both the AHP and utility function approaches are also constrained by generating only one solution at a time.

A final solution approach considered is the implementation of a multiobjective evolutionary metaheuristic, known as the *Nondominated Sorting Genetic Algorithm II* (NSGA II). This approach provides very promising results with respect to high quality assignments of weapon systems to threats. It is also flexible in the sense that additional weapon systems and threats may be added to the current sets without the need of considerable additional computations or significant model changes. A further advantage of this approach is that it is able to provide an entire front of approximately pareto optimal solutions to the fire control officer.

Uittreksel

In 'n militêre *grond-gebaseerde lugafweeromgewing* vereis bates op die grond beskerming teen vyandelike vliegtuie wat die beskermde lugruim binnedring. Hierdie vliegtuie word deur middel van 'n netwerk van sensors waargeneem en deur middel van 'n ontplooing van 'n verskeidenheid grond-gebaseerde *wapenstelsels* afgeweer. 'n *Afvuur-beheer operateur* is verantwoordelik vir die besluit om wapenstelsels aan vliegtuie wat as bedreigings geklassifiseer is, toe te wys. Die onderliggende probleem is dus om die beste stel wapens, volgens 'n voorafbepaalde kriterium of 'n stel kriteria, aan die bedreigings toe te wys. Hierdie probleem staan as die *wapentoewysingsprobleem* bekend.

Die toestande waaronder die afvuur-beheer operateur besluite ten opsigte van wapentoewysings maak, is besonder stresvol. 'n Gebrek aan tyd is 'n uiters beperkende faktor, en die afvuur-beheer operateur moet gevolglik binne 'n tydspan wat weinige analise en geen ruimte vir foute toelaat, wapentoewysings volgens sy beperkte kennis en intuïsie maak. 'n Gerekenariseerde *bedreigingsafskatting-en-wapentoekenningstelsel* kan gebruik word om die operateur met *besluitsteun* te bedien. In só 'n besluitsteunstelsel is 'n *bedreigingsafskattingdeelstelsel* verantwoordelik om vliegtuie wat die beskermde lugruim binnedring as bedreigings of andersins te klassifiseer en ten opsigte van eliminasie te prioritiseer, terwyl 'n *wapentoewyingsdeelstelsel* verantwoordelik is om wapentoewysings aan die bedreigings voor te stel.

Die hoofdoel in hierdie tesis is om die wapentoewysingsprobleem as 'n multikriteria-besluitnemingsprobleem te modelleer. 'n Lys van relevante doelwitte is met behulp van 'n wapentoewysingsvraelys verkry wat aan militêre kenners vir voltooing uitgestuur is. Twee van hierdie doelwitte, naamlik toewysingskoste en geakkumuleerde enkelskoot-trefwaarskynlikheid, is vir illustratiewe doeleindes gebruik om 'n twee-doelwit wapentoewysingsprobleem te formuleer wat met behulp van drie multikriteria-besluitnemingsmetodologieë uit die literatuur in die konteks van 'n realistiese, gesimuleerde grond-gebaseerde lugafweerscenario opgelos word.

Die *analitiese hiërargiese proses* (AHP) is met behulp van assesserings in samewerking met 'n militêre kenner geïmplementeer. Die toewysing van wapenstelsels is met behulp van 'n gulsige toewysingsheuristiek asook aan die hand van 'n AHP-toewysingsmodel bepaal. Beide hierdie metodes is in staat om resultate van hoë gehalte te behaal wat 'n aanvaarbare afruiling tussen die twee doelwitte verteenwoordig. 'n Nadeel van die AHP is egter dat dit onbuigsaam is in die sin dat 'n groot hoeveelheid vooraf-assesserings herhaal moet word indien meer wapenstelsels en/of bedreigings by die huidige sisteem gevoeg word.

'n *Twee-doelwit additiewe nutsfunksiebenadering* tot die wapentoewysingsprobleem is ook met behulp van velerlei assesserings in samewerking met 'n militêre kenner ontwikkel. Die toewysings is weereens met behulp van 'n gulsige wapentoewysingsheuristiek asook 'n nutstoewysingsmodel bepaal. Beide hierdie metodes is ook in staat om resultate van hoë gehalte te behaal wat 'n aanvaarbare afruiling tussen die twee doelwitte verteenwoordig. 'n Nadeel van die nutsfunksiebenadering is egter dat indien addisionele wapenstelsels by die huidige stel wapenstelsels gevoeg

word, en indien die waardes van hierdie addisionele wapenstelsels buite die grense van die doelfunksiewaardes van die huidige wapenstelsels val, daar 'n nuwe nutsfunksie vir die relevante doelwit van voor af bereken moet word. Beide die AHP- en die nutsfunksiebenaderings is verder tot die lewering van slegs een oplossing op 'n slag beperk.

Laastens is 'n *multikriteria evolusionêre metaheuristiek* (die NSGA II) geïmplementeer wat ook goeie resultate in terme van hoë-gehalte toewysings van wapenstelsels aan bedreigings lewer. Die voordeel van hierdie benadering is dat dit buigsaam is in die sin dat die getal wapenstelsels en bedreigings in die huidige sisteem aangepas kan word sonder om noemenswaardig meer berekeninge of groot modelveranderinge teweeg te bring. 'n Verdere voordeel is dat die metaheuristiese benadering daartoe in staat is om 'n front van benaderde pareto-optimale oplossings gelyktydig te lewer.

Acknowledgements

The author would hereby like to personally acknowledge the following people for their contributions towards the progress of this work:

- I wish to thank my supervisor, Dr I Nieuwoudt, for her dedication, guidance and support as well as the occasional laugh during the past two years. I especially thank her for her accessibility and willingness to help, as well as the kind words of encouragement towards the end of finalising this work.
- I wish to thank my co-supervisor, Prof JH van Vuuren, for his support and guidance throughout this project. I appreciate his enthusiasm, dedication and time. I especially wish to thank him for his patience towards the end of finalising this work. I admire his professionalism in the working environment as well as his hard work to ensure that work of a high standard is delivered.
- I wish to thank the Department of Logistics of the University of Stellenbosch for the use of an excellent research facility as well as the Logistics and Operations Research staff members for their friendliness and assistance during the past two years.
- I wish to thank my family for their loving support and, in particalur, my parents for their considerable moral and financial support during the past years and providing me with the opportunity to attain my tertiary education at Stellenbosch University, as well as my sister for her love and support throughout the past years. I wish to extend my deepest gratitude to Wayne and Linsey for their love and support and especially for providing a home away from home over the past years.
- I wish to thank my friends for their love, support and interest, especially during the past few months.
- I wish to thank my fellow GOReLAB colleagues with whom I shared an office space for the past two years for the great experiences we shared, especially for the sometimes very interesting conversations and laughs during tea times.
- Finally, I wish to thank the Armaments Corporation of South Africa (ARMSCOR) for funding the research reported in this thesis as part of their continued support of the TEWA Centre of Research Development at Stellenbosch University.

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List of Algorithms

List of Acronyms

ACM Air control means

AD Air defense

ADA Air defense artillery

AHP Analytic hierarchy process

AOR Area of responsibility

APM Air picture manager

ASCM Airspace control means

ARMSCOR Armaments corporation of South Africa

CANTCO Can't comply

CE Certainty equivalent

CIWS Close-in weapon system

DA Defended asset

DS Decision support

DSS Decision support system

ECCM Electronic counter counter measures

EEM Engagement efficiency matrix

EU Expected utility

EV Expected values

EW Electronic warfare

FC Fire control

FCO Fire control officer

FNSA Fast nondominated sorting algorithm

FPP Flight path prediction

GA	Genetic Algorithm
GBAD	Ground based air defense
GBADS	Ground based air defense system
HCI	Hostility classification/identification
HMI	Human machine interface
IFF	Identify friend of foe
IPB	Intelligence preparation of the battlefield
k-WAP	k-cardinality Weapon assignment problem
LOS	Line of sight
LRSAM	Long-range surface to air missile
MRSAM	Medium range surface to air missile
NSGA	Non-dominated sorting genetic algorithm
NSGA II	Non-dominated sorting genetic algorithm II
OIL	Operator in the loop
OP	Observation post
PE	Probability equivalent
RP	Risk premium
SAM	Surface to air missile
SHORAD	Short-range air defense system
SSHP	Single shot hit probability
TCI	Type classification/identification
TE	Threat evaluation
TEWA	Threat evaluation and weapon assignment
TM	Track management
VSHORAD	Very short-range air defense system
WA	Weapon assignment
WAP	Weapon assignment problem
WILLCO	Will comply
WS	Weapon system

CHAPTER 1

Introduction

Contents

1.1 Informal problem description

In a typical *Ground-Based Air Defense* (GBAD) military environment, *Defended Assets* (DAs) require protection from enemy aircraft entering the *defended airspace*. Such protection is afforded by means of a number of pre-deployed ground-based *Weapon Systems* (WSs). A network of sensors is responsible for detecting these aircraft after which the aircraft have to be classified according to the perceived level of threat which they pose to the DAs. A *Fire Control Officer* (FCO) is responsible for assigning WSs to engage these threats. The decision problem of assigning available WSs to threats is known as the *Weapon Assignment* (WA) problem.

Not only does the FCO have to propose a high quality assignment of WSs to threats under conditions of severe stress, but his decision also involves a choice with respect to the number of WSs assigned to each threat. Assigning more WSs to a threat may yield an increase in the probability of eliminating the specific threat, but assigning too many WSs to a threat reduces the number of WSs available for assignment at future time instants, which is not desirable in case additional threats enter the defended airspace. The FCO should also consider the cost of assigning these WSs, since the monetary cost of assigning some of these WSs is very high.

Furthermore, the FCO has to assign WSs to engage aerial threats at an appropriate time instant. The problem is: Should he assign a WS to a threat at the current time instant, or should he rather wait for a future time instant when the WS might achieve a larger probability of successfully engaging the threat. Moreover, the WSs achieving a longer range typically involve a higher monetary cost of assignment than do WSs achieving a shorter range.

Another factor contributing towards the stress experienced by the FCO is time. Time is of the essence in the FCO's assignment decision. The speed at which the enemy aircraft approach, leaves little time for analysis or any delay in reaction times. Even a slight delay in reaction time may lead to adverse consequences such as the destruction of one or more of the DAs and/or an increase in casualties [28].

Enemy aircraft may also try to overwhelm the FCO by saturating the defended airspace as a result of employing a large number of aircraft which enter the defended airspace almost simultaneously from different directions. If such a situation occurs, the decision problem of assigning WSs to threats becomes very complex and almost impossible for the FCO to solve optimally in real-time.

It is evident from the above discussion that the decision problem of assigning WSs to threats is not an easy task, and the associated stress factor involved in the decision complicates matters even further. A slight delay in the time to react or a call of poor judgment with respect to evaluating a threat or assigning a WS to a threat may result in adverse consequences. An example of one such incident occurred on 3 July 1988, when the *USS Vincennes* missile cruiser misidentified a commercial airliner as an attacking *F-14 Tomcat fighter aircraft* and accidentally shot down the airliner by firing two radar-guided missiles at it [11, 60]. The result was catastrophic, resulting in the death of all 290 passengers and crew members on board the airliner. After this incident, the United States Office of Naval Research sponsored the development of a program called *Tactical Decision Making Under Stress* (TADMUS) [11]. The aim of this program was twofold, namely to improve decision making skills of operators by means of enhanced training and to provide them with a computerised *Decision Support System* (DSS) [15].

The aim of such a DSS, called a *Threat Evaluation and Weapon Assignment* (TEWA) system, is to provide the FCO with a good alternative or a small number of good alternatives from which he may choose in conjunction with his own judgment, based on experience and training, in order to make WA decisions. A TEWA system typically consists of two subsystems, namely a *Threat Evaluation* (TE) subsystem and a WA subsystem. The TE subsystem is responsible for assessing the level of threat posed to DAs by enemy aircraft, while the WA subsystem is responsible for suggesting good assignments of WSs to engage these threats, taking into account both the efficiencies of WSs with respect to the threats and the priorities of eliminating these threats.

Typical WA models found in the literature involve only single-objective optimisation where the aim is usually to maximise the overall probability of successful engagement of aerial threats by WSs. This overall probability of successful engagement is computed using the *Single Shot Hit Probabilities* (SSHPs) of the various WSs. Stated otherwise, the aim is therefore usually to minimise the accumulated survival probabilities of these threats by assigning appropriate WSs to engage these threats. The aim in this thesis is to investigate the possibility of modelling the WA problem as a multiobjective decision problem. This includes the establishment of a number of fundamental objectives for the purposes of WA and formulating a multiobjective WA decision model by incorporating these objectives. Furthermore, a secondary aim is to identify and suggest suitable solution methodologies for such a multiobjective WA model.

1.2 Scope and objectives pursued in this thesis

The scope of this thesis is restricted to the WA problem of assigning ground-based WSs to targets within a GBADS at a single time instance. Six objectives are pursued in this thesis:

Objective I: To *review* the physical and functional elements typically residing within a GBADS and, in particular, to *describe* the requirements, nature and working of a TEWA DSS employed in a GBADS, while accentuating the requirements for a successful WA subsystem.

Objective II: To *investigate* and *implement* techniques for the extraction of a number of fundamental objectives deemed important from a WA perspective which may be used in the derivation of a multiobjective WA decision model.

Objective III: To *formulate* a multiobjective WA decision model based on the problem objectives defined in Thesis Objective II above.

Objective IV: To *research* and *document* various methodologies from the literature which may be employed to find good solutions to multiobjective decision problems.

Objective V: To *implement* and *illustrate* the workings of the researched methodologies in Thesis Objective IV above, in conjunction with the multiobjective WA model in Thesis Objective III within the context of a simulated but realistic GBADS scenario.

Objective VI: To *suggest* a number of ideas for possible future work in the context of the multiobjective WA problem as possible enhancements to the work contained in this thesis.

1.3 Thesis organisation

This thesis contains six chapters. In Chapter 2 the reader is familiarised with TEWA in a GBADS in fulfilment of Thesis Objective I in §1.2. The chapter opens with a brief introduction to a typical GBADS, including and the hardware and software subsystems residing within such a system. Three such hardware subsystems are described in §2.2–§2.3, namely DAs, sensors and WSs. This is followed by a discussion on three GBAD software subsystems (in §2.4–§2.6), namely the track management subsystem TE subsystem the WA subsystem. The physical conditions under which a GBADS has to operate are described as the tactical environment in §2.7. The chapter closes, in §2.8, with a description of a popular WA model based on the classical assignment problem.

Chapter 3 mainly deals with Thesis Objective IV and contains a literature review on some of the available methodologies for solving multiobjective decision problems. In §3.1 the reader is introduced to multiobjective decision problems in general, including the notion of pareto optimality in §3.1.2. The objectives, goals and attributes of decision problems are discussed in §3.1.3, and this is followed by a description of how to establish the objectives for a multiobjective decision problem. The working of the *Analytic Hierarchy Process* (AHP) is reviewed in §3.2, including a procedure which may be followed to ensure that the decision maker remains consistent (in §3.2.1) and a discussion on the implementation, advantages and disadvantages of the AHP (in §3.2.2). Utility theory in the context of one objective is reviewed in §3.3. This includes a discussion on utility functions under certain as well as uncertain conditions (in §3.3.1) as well as a discussion on qualitative and quantitative characteristics of utility functions. Qualitative characteristics considered include monotonicity (§3.3.2), attitudes towards risk (§3.3.3), the certainty equivalent and risk premium (§3.3.4) and constant, increasing and decreasing attitudes towards risk (§3.3.5). This is followed by a review of methods for evaluating quantitative utility values in §3.3.6 and guidelines which may be followed during the assessment of utility values §3.3.7. Utility functions involving multiple objectives are considered in §3.4. The chapter closes with a discussion on multiobjective evolutionary algorithms in §3.5, focussing on a description of the working of the *Nondominated Sorting Genetic Algorithm II* (NSGA II) attributed to Deb *et al.* [3] in §3.5.5.

Multiobjective decision making approaches towards WA are considered in Chapter 4. A method for identifying objectives in a WA context is described in §4.1, in fulfilment of Thesis Objective II. This is followed by a comprehensive description of a simulated, but realistic, GBADS scenario which is employed to illustrate the working of the various multiobjective WA approaches. In the remainder of the chapter the focus is on Thesis Objective III. The AHP assessments carried out in conjunction with a military decision maker are described in §4.3. This is followed by a discussion on the evaluation of a bi-objective WA utility function in §4.4. This includes a discussion on the assessment of independence between the objectives in §4.4.1, followed by a description of the assessments carried out in conjunction with the decision maker in order to establish qualitative characteristics of the individual utility functions in §4.4.2 and obtaining quantitative utility values in §4.4.3. The assessments carried out during the evaluation of the scaling constants for the objectives are described in §4.4.4. A bi-objective WA utility function is finally presented in §4.4.5. The chapter concludes with a discussion on the computer implemented version of the NSGA II employed to solve the bi-objective WA assignment problem in the context of the scenario presented in §4.2.

Chapter 5 is devoted to the results obtained by means of the methodologies described in Chapter 4 for solving the bi-objective WA decision model for the scenario described in §4.2, in fulfilment of Thesis Objective V. The chapter opens with a summary of the results obtained by means of the AHP in §5.1, followed, in §5.2, by a presentation of the results obtained by means of the bi-objective additive utility function approach. This is followed, in §5.3, by a discussion and interpretation of the results obtained by means of the NSGA II. The chapter closes with various conclusions and recommendations based on the results obtained by each of the solution methodologies.

Finally, Chapter 6 contains a summary of the work contained in this thesis, presented in §6.1. This is followed, in §6.2, by an appraisal of the contributions of this thesis. The chapter closes with a number of ideas with respect to possible future work related to the WA problem within a multiobjective decision context, in fulfilment of Thesis Objective VI.

CHAPTER 2

TEWA in a GBADS: A brief review

Contents

The purpose of this chapter is to introduce the reader to the notion of a *Ground Based Air Defense System* (GBADS) and the subsystems contained within a GBADS. This is achieved by briefly considering the physical elements of a GBADS, namely the *Defended Assets* (DAs) and sensors in §2.2 and *Weapon Systems* (WSs) in §2.3. A discussion on *Track Managament* (TM) of threats follows in §2.4, and descriptions of the workings of the *Threat Evaluation and Weapons Assignment* (TEWA) subsystems follow in §2.5 and §2.6, respectively. The effects of the tactical environment on a GBADS are considered in §2.7 and the chapter concludes with an explanation of a classical WA model in §2.8.

2.1 A ground based air defense system

In a military environment, a GBADS may be defined as a system in which a number of DAs on the ground have to be defended against opposing enemy aircraft. A number of different types of sensors and ground based WSs reside within a GBADS, which may aid in the defense against enemy aircraft. Since WSs offer protection against aerial threats, the volume around the DAs may be considered as the defended airspace. The specific bounded area in which own forces are responsible for planning and conducting operations is known as the *Area Of Responsibility* (AOR) [52]. Sensors are responsible for detecting aircraft entering the defended airspace and WSs are available for possible assignment to and, if necessary, engagement of the aircraft which are classified as threats, in an attempt to protect the DAs.

Once aircraft have been detected in the defended airspace, they have to be labelled and classified according to their hostility and platform type. The labeling and classification of aircraft both form part of a process called TM. After classification, the aircraft may be assessed in terms of the threat which they pose to DAs, in a process known as *Threat Evaluatioin* (TE), which is achieved within a TE subsystem. One or more WSs may be assigned to the aircraft, based on a thorough TE of the aircraft. The process of assigning and engaging ground based WSs is known as WA and is conducted within a WA subsystem.

The WA subsystem relies on output obtained from the TE subsystem, and is therefore initiated once the TE subsystem produces output. Combining these two subsystems yields the larger TEWA system, which serves the purpose of a *Decision Support System* (DSS) providing *decision support* (DS) to human operators in a GBADS. The reason for such DS is that conditions may be very stressful for operators during aerial attacks and the problem of evaluating threats and assigning weapons can easily become very complex and time consuming when a large number of aircraft enter the system.

These subsystems are all contained within a GBADS. Hence, a GBADS may be thought of as a system of subsystems. The subsystems contained within a GBADS may be divided into hardware systems and software systems [44, 41]. The hardware systems consist of all the physical elements such as DAs, sensors and WSs, whereas the software systems consist of TM and the TEWA system.

An important element which may affect the operation of a GBADS is the *tactical environment*. The tactical environment consists of components such as terrain and environmental conditions which are considered at a later stage in this chapter. A typical GBADS is illustrated schematically in Figure 2.1.

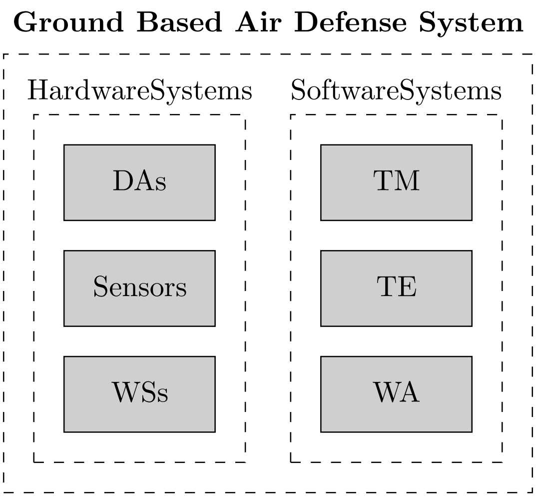

Figure 2.1: *Schematic representation of the components of a typical GBADS.*

2.2 Defended assets and sensors

Two important hardware systems contained in a GBADS are the collection of DAs and the network of sensors. DAs are the objects in a GBADS which require protection from enemy aircraft. The collection of DAs contained in a GBADS may consist of a single DA or may comprise many DAs. Examples of DAs are air bases, crossroads, factories, harbours, main bridges, power plants, *etc.* [43]. The importance of DAs in a GBADS also vary, as some DAs may be deemed more important or critical than others. The entire collection of DAs may therefore be prioritised in a ranked list from most important to least important [43]. The collection of prioritised DAs is integrated with available information on the observed aerial threats as input data to the TE subsystem, whose ouput is provided as input to the WA subsystem. The identification and prioritisation of DAs may be determined pre-operational.

A number of radar sensors reside within a GBADS. These sensors act as the "eyes" of the system, detecting any aircraft which enter the AOR [44]. Sensors are therefore detection subsystems which may be used to determine the altitude, direction, or speed of moving objects [57]. These sensors are usually deployed as a system or network of sensors, called a *sensor grid*. A sensor grid is employed to ensure that at least one sensor in a set of sensors covers a significant portion of the AOR [40]. The sensors of a GBADS are therefore very valuable; Roux [43] even goes as far as stating that sensors form the core of the TE subsystem. Examples of various types of sensors include acoustic, electromagnetic, optical radiation and thermal sensors [43].

Since the emphasis of this thesis is on the WA subsystem and its workings, the interested reader is referred to Roux [43] for a detailed description of DAs and sensors.

2.3 Weapon Systems

Ground based WSs are used in a GBADS to combat approaching threats. They are typically positioned around DAs to provide maximum protection from threats. The physical lay-out of the WSs in a GBADS is known as the WS deployment.

WSs in a GBADS may typically be divided into three categories: *artillery systems*, *missile systems* and *laser systems* [40]. The WSs contained in this trio of categories are typical *hard-kill* WSs, where the aim is to destroy a threat completely. For a more comprehensive discussion on hard-kill WSs, the interested reader is referred to Potgieter [40]. On the other hand, there are also *soft-kill* WSs which aim only to distract or disarm a threat in an attempt to protect DAs [43].

When considering the deployment of WSs, the concept of layered *Air Defense* (AD) is important. Layered AD is a well-known theoretical construct consisting of dividing the defended airspace into different layers based on the capabilities of the various types of WSs [48]. Four different layers reside within the South African *air defense artillery* (ADA) [43]. They are the inner, middle, outer and in-depth layers, as illustrated in Figure 2.2. The figure represents a side view of the divided, defended airspace into the four different layers, based on the range (both horisontal and vertical) of WSs contained in each layer.

The first layer, the inner layer, is where *Close-In WSs* (CIWSs) and *Very Short-Range AD systems* (VSHORADs) are found. CIWSs are known for their short reaction times, high fire rates, short effective ranges (less than 4 000 metres), lengthy deployment procedures (due to allignment requirements), all-weather operation and intensive maintenance procedures [48]. VSHORADs are known for their light weight (man portability), rapid deployment procedures and short effective ranges (less than 6 000 metres) [43].

The second layer of defense, the middle layer, consists of *Short-Range AD systems* (SHORADs). They are distinguished by their extended effective ranges (up to 20 000 metres), possible vertical launch and their ability to operate during day or night time, as well as in all-weather conditions.

The third layer, the outer layer, consists of *Medium-Range Surface to Air Missiles* (MRSAMs) with effective ranges of up to 80 000 metres and engaging altitudes of up to 18 000 metres [43].

The forth and final layer, the in-depth layer, consists of *long-range SAM* (LRSAMs) and interceptor aircraft[1]. LRSAMs are known to have effective ranges of more than 80 000 metres. AD coverage depends on the type of SAM used and the target to be engaged. The coverage provided by an interceptor aircraft depends on the characteristics of the specific aircraft employed [43].

The current WSs artillery employed within a South African GBADS limits the layered AD to only the inner and middle layers. Because of this limitation, only WSs residing within these two layers are considered in this thesis.

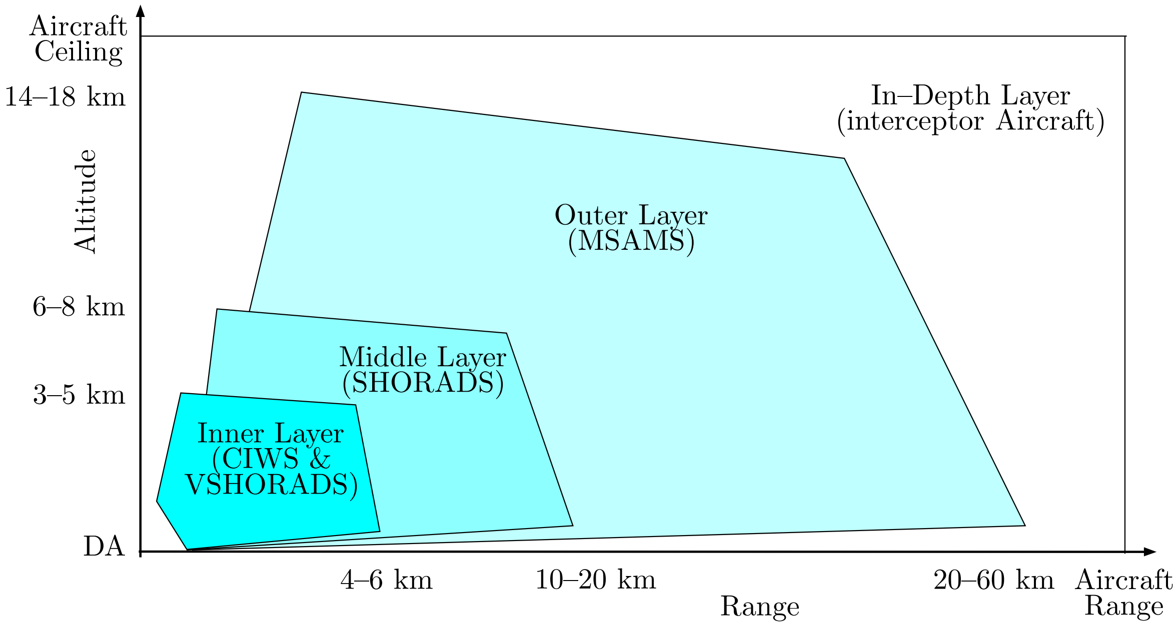

FIGURE 2.2: *Different layers of AD in a GBADS environment [20].*

2.4 Track Management

A central software system in a GBADS is the TM subsystem. Information obtained from the sensor grid is used to form a *system track* for each of the observed aircraft in the TM subsystem. Typical information used in the creation of a system track is aircraft attributes. These attributes include the speed at which the aircraft is travelling, the altitude of the aircraft and the direction in which the aircraft is travelling [40]. The entire set of individual aircraft tracks resides in a TM subsystem and is accessible by both the TE and WA subsystems.

Apart from labeling aircraft, TM also consists of two processes called *Type Classification/ Identification* (TCI) and *Hostility Classification/Identification* (HCI). TCI involves distinguishing between aircraft platform types, such as *rotary wing, fixed wing, cargo, missile, unmanned aerial vehicle, EW platform* or *unknown* [40]. The results of the TCI process are usually based on reports from *Observation Posts* (OPs), as the required information is usually unavailable from the kinematic data of aircraft. HCI involves the classification of system tracks as either *friendly, hostile* or *unknown.* The process of HCI utilises an electronic aircraft interrogation sys-

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[1] Interceptor aircraft are specifically designed to prevent missions of enemy aircraft, and rely on high speed and powerful armament to complete a mission. They are usually employed against enemy aircraft such as bombers and reconnaissance aircraft [55].

tem called *Identity Friend or Foe* (IFF)[2] in conjunction with *Airspace Control Means* (ASCM)[3] or other measures to classify system tracks [1, 40].

The entire set of aircraft tracks generated by the TM subsystem are displayed on the *Human Machine Interface* (HMI) to an *Air Picture Manager* (APM) who is responsible for managing the aerial picture and system tracks. The platform type, hostility classification and raid size of tracks may also be set manually by the APM via the HMI. The decision to alter or augment track information manually, may be based on OP reports or other visual reports [40].

The importance of effective TM lies in the fact that the TE subsystem only considers system tracks which have been classified as hostile or unknown, as discussed in the next section [44].

2.5 The TE subsystem

Another important software system of a GBADS is the TE subsystem. Only system tracks which have been classified as hostile or unknown are considered in the TE subsystem. Information from the sensors as well as the labelled and classified tracks, serve as real-time input to the TE subsystem [44]. These system tracks, now known as threats, are further analysed by the TE subsytem with respect to the threat which they pose to DAs.

Threats are typically analysed according to their capability and intent. The *capability* of a threat is a reflection of its ability to inflict injury or damage to DAs. Factors which influence the capability of a threat include the proximity of the threat to DAs and the characteristics of WSs carried by the threat [45]. The *intent* of a threat refers to the will or determination of the threat to reach DAs in order to inflict damage or injury. Factors which may be used to ascertain the intent of a threat include the velocity of the threat, its course and altitude with respect to DAs and its estimated attack technique (based on its movement) [45].

The capability of a threat is easier to estimate than its intent. It is therefore important for a TE subsystem to be sophisticated enough to explore all the available information with respect to an aircraft which enters the AOR and at the same time to be capable of producing a result based on scant information [40]. A solution to this problem is proposed by Roux and Van Vuuren [43], who suggest the use of a suite of TE models in conjunction with one another. The output of a TE model is a *threat value* allocated to each observed threat. This value is an indication of the level of threat that it poses to each respective DA.

A hierarchical representation of various levels of TE models may be found in Figure 2.3. The simplest models are placed at the top of the figure and the more sophisticated models may be found towards the bottom, consisting of more robust models. The topmost level consists of *flagging models* which are only concerned with sudden changes in aircraft behaviour. These models function on the basis that if any attribute of the detected aircraft deviates significantly from that of their current or past values, an operator is flagged [40]. The middle level of TE models hosts so-called *deterministic models*. These models use measures such as the distance from aircraft to DAs and aircraft bearing with respect to DAs to generate threat values and threat lists [40]. The more sophisticated *probability-based models* in the lower level attempt to rather make use of probability values to express the threat that aircraft pose to the respective DAs. One example of such a probability may be the probability that the aircraft will kill a

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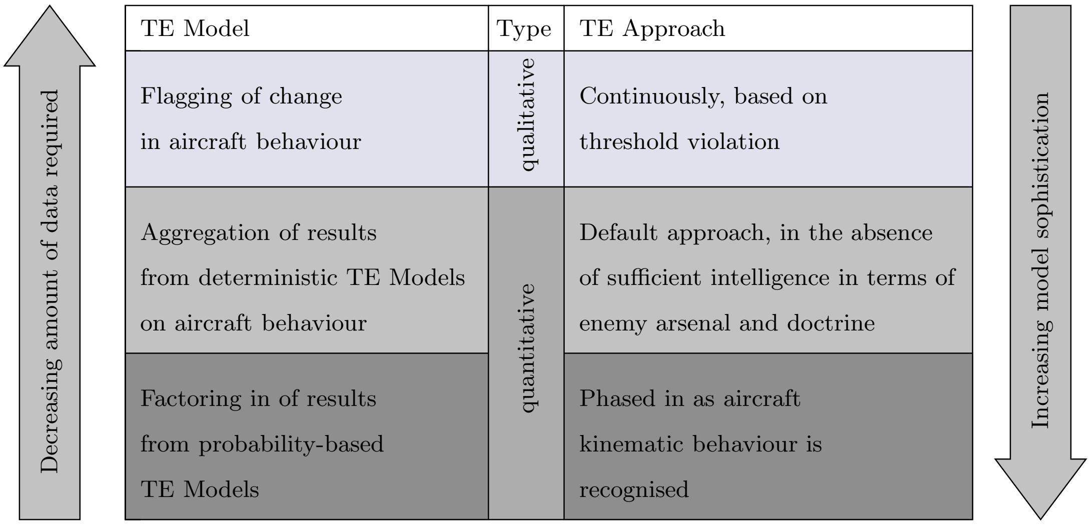
[2] IFF systems have the ability to differentiate between friendly and non-friendly aircraft. The interested reader is referred to [1] for a more comprehensive discussion on modern IFF systems

[3] ASCM comprise guidelines for the use of the airspace inside the AOR. These guidelines are supplied to own aircraft, and aircraft in the AOR which deviate from these guidelines are classified as hostile [40].

DA [40]. Because the research reported in this thesis is mainly concerned with the working of a WA subsystem, the TE subsystem is not discussed in further detail. The interested reader is referred to Roux and Van Vuuren [45] for a full discussion on TE models.

The output obtained from the TE models is stored in what may be called a *static TEWA database*[4]. This output is typically contained in the form of a two-dimensional matrix known as a *threat matrix* representing the estimated threat values for each threat-DA pair. In addition, a *combined threat list* representing the threat that an aircraft poses with respect to the entire collection of DAs may also be derived from the threat matrix [40].



TE Model	Type	TE Approach
Flagging of change in aircraft behaviour	qualitative	Continuously, based on threshold violation
Aggregation of results from deterministic TE Models on aircraft behaviour	quantitative	Default approach, in the absence of sufficient intelligence in terms of enemy arsenal and doctrine
Factoring in of results from probability-based TE Models		Phased in as aircraft kinematic behaviour is recognised

FIGURE 2.3: *A hierarchical illustration of the three different levels of TE models [43].*

2.6 The WA subsystem

The third and final software system of a GBADS is the WA subsystem. WA entails the assignment of available WSs to threats in an attempt to achieve some pre-specified objective. The extent to which the objective is achieved may be used as criterion to evaluate the desirability of WS-to-threat assignments. Examples of such objectives include minimising the expected aggregated damage to DAs, maximising the number of threats engaged or maximising the expected aggregated damage to threats [40]. To complicate matters, a WA subsystem may also operate in different modes of operation. Examples of such modes are *deterrance* and *attrition*. In the former case, a WS may fire at enemy aircraft in an attempt to scare them away[5], while in the latter case the aim is to fire at enemy aircraft in an attempt to maximize the inflicted damage[6] [40].

The person aided by the WA subsystem is a *Fire Control Officer* (FCO). The main responsibility of the FCO is the management of WSs in a WS deployment[7]. The management of WSs include the assignments of WSs to threats as well as communication with the various WS operators [40].

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[4] Static information refers to information which does not change in real-time and may be obtained pre-deployment. This information is contained within a so-called static TEWA database. On the other hand, dynamic information refers to information which may change during real-time and this information is contained within the dynamic TEWA database [40].

[5] In deterrance mode a WS would typically be assigned to a threat as soon as possible.

[6] In attrition mode a WS would typically be assigned to a threat once the threat reaches the area where the WS is most effective with respect to that threat.

[7] The management of WSs in a deployment is also known as *fire control* (FC) [40].

The FCO acts as the human *Operator In the Loop* (OIL) when assignments are proposed and engagements have to be executed [43].

The FCO may use a TEWA system as a tool to aid in the decision to assign WSs. He is therefore authorised to alter certain proposals made by the TEWA system. TE proposals which may be altered include the ranking of the threats in the proposed threat list and resetting the value of a threat to the maximum possible value if desired, in order to invoke a condition called *enhanced reaction*[8]. In extreme cases, and only when absolutely necessary, may a FCO alter the properties of threats or remove them completely from the set threats [40].

The WA subsystem requires the combined threat list output from the TE subsystem as well as a set of data called the *Engagement Efficieny Matrix* (EEM) as input. The EEM consists of a three-dimensional matrix, containing predicted effectiveness values of the WSs with respect to the entire set of observed threats. The EEM has the following dimensions: number of WSs (i), number of threats (j) and number of future time steps (t_s). Hence, the size of the EEM matrix depends on the ranges of values of these three attributes [40]. The EEM and its operations is contained within the EEM component and a graphical representation of an EEM may be found in Figure 2.4. Each WS is associated with an effectiveness value with respect to observed threats, known as the *Single Shot Hit Probability* (SSHP) [40]. SSHP values are normally supplied by the manufacturers of WSs and are based on the characteristics of the specific type of WS in use. The SSHP values are used in the construction of the EEM, and form the core of the EEM.

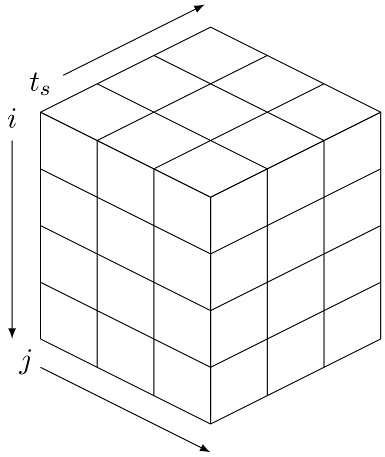

FIGURE 2.4: *Graphical representation of an EEM which serves as input to a WA subsystem.*

Information contained in the EEM may be filtered to make provision for elements contained in the tactical environment [40]. In essence, by filtering the contents of the EEM, the original SSHP values are discounted to make provision for restrictions posed by the tactical environment. The information used to populate the EEM, is stored in the static and dynamic TEWA databases. A *flight path prediction model* (FPP) is used to predict the effectiveness values of WSs at a number of future time steps, if desired, so as to investigate whether the threats will be contained within the SSHP range of WSs within a number of future time steps [40]. A typical flight path prediction model predicts efficiency values for a period up to 120 seconds [41, 44]. However, caution should be taken when predicting these values, as predicting too far into the future may lead to poor estimates which may jeopardise the output result of the WA subsystem.

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[8] Enhanced reaction is a typical *quick* response technique used when a threat is detected very close to a threat for the first time [40].

Once the EEM has been populated, its contents is presented in real-time to the WA model component contained in the WA subsystem. A WA model is responsible for suggesting WS-threat pairings, and it requires both the combined threat list from the TE subsystem as well as the EEM as input [40]. Such a WA model produces output in the form of a *proposed assignment list*. This list contains possible assignments of WSs with respect to the observed threats. The resulting assignment list is also stored in the *dynamic TEWA database*. The components of the WA subsystem may be seen in Figure 2.5.

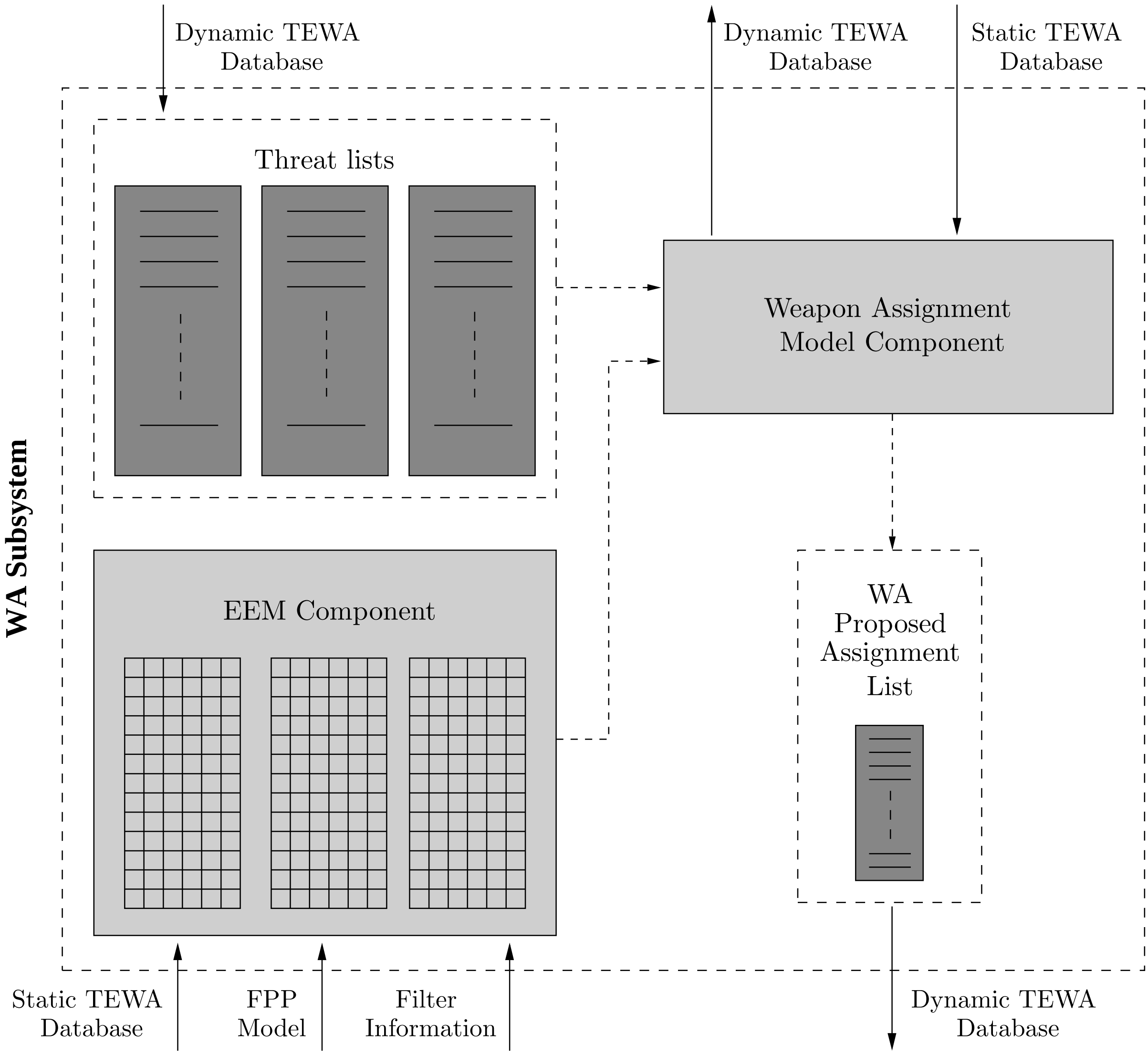

FIGURE 2.5: *The components of a WA subsystem, as well as the flow of information between these components.*

The proposed assignment list is displayed on the HMI, and has to be approved by the FCO. The FCO studies the assignment list, and once he is satisfied with the proposals, he authorises engagements in the assignment list. WSs are then alerted with *Engagement Orders* (EOs). If the FCO is not satisfied with some of the proposals in the assignment list, he may alter the list in order to enforce assignments based on his own judgment [40]. Once an EO is relayed to a WS, it has to reply with either a “WILLCO” (WILL COmply) or “CANTCO” (CAN’T COmply) command. If a “WILLCO” reply is received, the WS is considered engaged. If a

"CANTCO" reply is received, the particular WS has to be replaced by another WS returning a "WILLCO" reply [41, 44]. The individual engagements may be integrated into a so-called *active engagement list.*

The FCO has the authority to create manual engagements and override TEWA proposed engagements. He also has the power to place a hold on an engagement or to cancel an engagement altogether [40]. The former may be achieved by means of a "HOLD FIRE" command and the latter via a "STOP FIRE" command.

Once a WS is engaged, a tracking process is initiated which involves the tracking of the threat by the WS, until it is ready to fire. When a WS finds a suitable opportunity, it fires at the relevant threat. The result of a WS firing may yield a status of either a success or a failure. An executed engagement is deemed a success if the relevant threat is hit or failure if a threat is missed. Once a success or a failure is obtained, the entire TEWA process is repeated for the next time interval. During these last steps, the information of the current status of the WS is relayed continuously to the FCO to keep him informed [41, 44]. The working of the TEWA system, is illustrated in Figure 2.6.

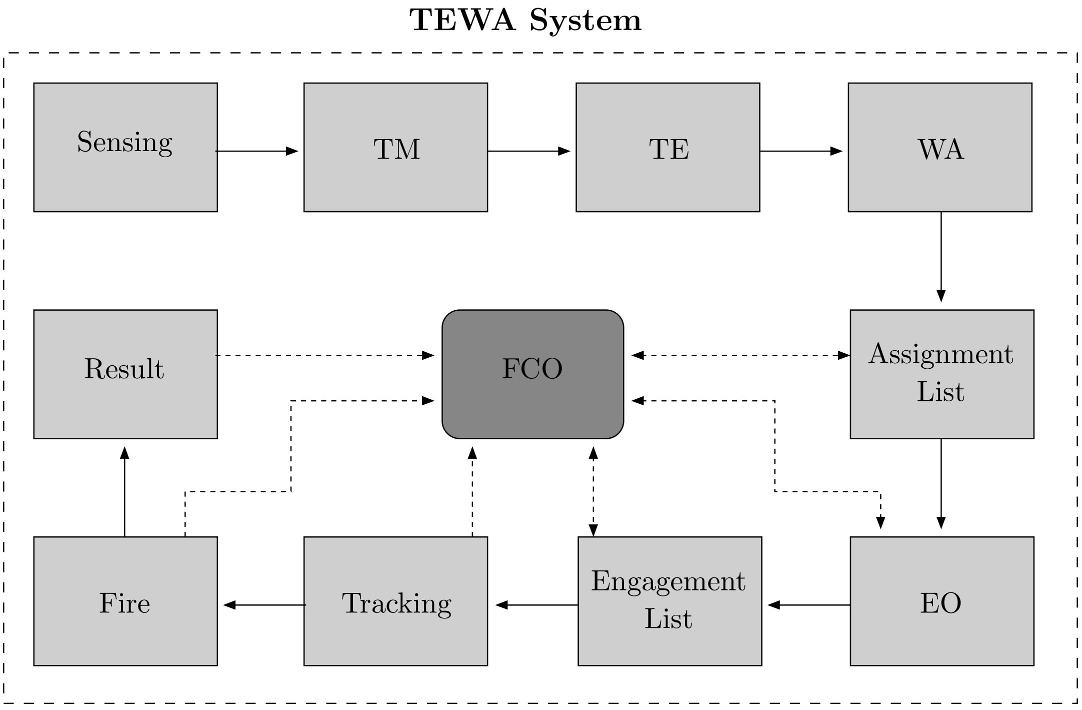

FIGURE 2.6: *A schematic representation of the working TEWA system.*

2.7 The tactical environment

The tactical environment refers to the physical conditions in which a GBADS has to operate [36]. These conditions lead to a reduction in not only the performance of own system operations, but also affects the performance of enemy aircraft. It is therefore essential to assess and understand the impact that these conditions may have on the performance of subsystems operating within a GBADS. The assessment of the tactical environment forms part of the so called *Intelligence Preparation of the Battlefield* (IPB)[9]. The elements of the tactical environment, may be divided

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[9] "IPB is a systematic, continuous process of analysing the threat and environment in a specific area" [27].

into terrain features and environmental conditions [44].

The surrounding terrain in which a GBADS operates, may have a significant impact on not only the ability of sensors to discover threats, but also on the ability of WSs to operate efficiently [40]. A crucial requirement of some sensors, is an electronic or optical *Line Of Sight* (LOS)[10] in order to be able to detect threats and to obtain the necessary information regarding these threats (*e.g.* position, activities and resources of threats) [30].

Some WSs are also limited by terrain to only engage threats that are within LOS, and thus can provide no protection against threats which are concealed behind obstacles residing within the terrain. A WS deployment in an area where the terrain is scattered with various obstacles is therefore highly discouraged [40].

In addition to the constraints placed on systems by terrain, terrain may also be used to the advantage of enemy aircraft in order to approach DAs. Enemy aircraft may use the surrounding terrain to receive cover against WSs [30]. The techniques employed by enemy aircraft to conceal themselves from ground based WSs by using the surrounding environment are called *cover and concealment*[11].

Environmental conditions of the tactical environment refers to the effect that conditions such as weather may have on the subsystems of a GBADS. Environmental conditions may be divided into natural environmental conditions and induced environmental conditions [20]. Natural environmental conditions consist of weather conditions such as atmospheric pressure, cloud cover, dew point temperature, humidity, precipitation, temperature, visibility, wind speed and wind direction [26]. Induced environmental conditions refers to smoke and debris caused by encounters between threats and GBAD WSs [48]. Because of the nature of induced environmental conditions, they are much harder to anticipate than natural environmental conditions, resulting in the focus typically shifting more towards natural environmental conditions [43].

Adverse natural weather conditions affect not only the capabilities of sensors and WSs deployed, but may also have a dramatic impact on the crew operating these systems [40]. Extreme temperatures may cause WSs to malfunction and heavy rainfall, fog or very windy conditions causing sandstorms, may have a significant impact on the LOS required by some sensors and WSs. Strong winds may cause projectiles of ammunition to be steered off course, resulting in a failure of engagements. Finally, adverse weather conditions may make it difficult to deploy certain WSs and sensors as these weather conditions limit the accessibility of the deployment areas [43].

It is important to examine the current as well as predicted future weather conditions as this may be used as an advantage to own forces or to the disadvantage of aerial threats [40]. Furthermore it is, of course, important to exploit the tactical environment to its maximum extent.

2.8 A WA model based on the classical assignment problem

Extensive research has been done on the WA model component of the WA subsystem. For example, Du Toit [21] and Potgieter [40] propose a number of mathematical WA models ranging

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[10] LOS refers to the presence of a visible, distinct path between two objects which is not impeded by terrain features [29].

[11] Typical cover and concealment techniques employed include *contour flying*, *popping-up*, *terrain masking* and *flying with terrain cover*. The interested reader is referred to [30] for a more comprehensive discussion on cover and concealment techniques.

from dynamic[12] to static[13]. The scope of this thesis limits the use of these models to only the static case. However, the interested reader is referred to [21, 40] for an in depth discussion on mathematical WA models.

The model which will serve as a point of departure in later chapters of this thesis, is known as the *k-cardinality WA Problem* (k-WAP). This model is based on the classical WA assignment problem of Ahuja [4] and was adjusted for the assignment of up to k-WSs by Potgieter [40]. In this model, at most k of the $m(\tau)$ WSs available at time τ may be assigned to any of the $n(\tau)$ threats at time τ [40]. Assumptions made in this model include that the SSHP of a WS with respect to a threat depends only on the WS and threat involved, and that the events of a threat surviving engagements by two different WSs are independent [40].

The objective in this model is to minimise the accumulated weighted probability of survival of the observed threats. The probability that a threat will survive, is calculated by taking the product of the probabilities of surviving WS engagements assigned to it, thus representing a product of the efficiencies of all the WSs assigned to it [40]. This probability of survival is then weighted by the priorities of eliminating a threat when a WS-threat engagement proposal is made. Stated mathematically, the problem is therefore to

$$\text{minimise} \sum_{j=0}^{n(\tau)-1} V_j(\tau) \prod_{i=0}^{m(\tau)-1} q_{ij}(\tau)^{x_{ij}(\tau)}, \tag{2.1}$$

$$\begin{aligned}
\text{subject to the constraints} \quad \sum_{j=0}^{n(\tau)-1} x_{ij}(\tau) &\leq 1, & i &= 0,\ldots,m(\tau)-1, & (2.2)\\
\sum_{i=0}^{m(\tau)-1} x_{ij}(\tau) &\leq k, & j &= 0,\ldots,n(\tau)-1, & (2.3)\\
x_{ij}(\tau) &\in \{0,1\}, & i &= 0,\ldots,m(\tau)-1, & (2.4)\\
& & j &= 0,\ldots,n(\tau)-1, & (2.5)
\end{aligned}$$

where $V_j(\tau)$ represents the priority of eliminating threat j at time τ, $q_{ij}(\tau)$ is the probability of survival of threat j when WS i is assigned to threat j at time τ and $x_{ij}(\tau)$ is a binary variable which is equal to one if WS i is assigned to threat j at time interval τ, or zero otherwise [40]. The probability that threat j will survive an engagement by WS i is therefore $1 - p_{ij}(\tau)$, where p_{ij} denotes the SSHP if WS i is assigned to threat j. This model is constrained by two sets of constraints. The first set limits each WS to be assigned at most once, and the second constraint ensures that a maximum of k WSs may be assigned to any specific threat [40].

2.9 Chapter summary

The focus in this chapter was to provide the reader with a brief review on the available literature related to a GBADS and its subsystems. This was achieved by describing a typical the GBADS architecture in §2.1 and further discussing each subsystem of a GBADS. The hardware systems

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[12] A WA problem is considered *dynamic* when the allocation of weapons to threats is considered over some window of time [40].

[13] A WA problem is considered *static* when the position and number of weapons and threats are fixed and known [40].

in a GBADS were discussed first, with a short introduction on DAs and sensors in §2.2, followed by a more comprehensive discussion on WSs in §2.3.

A central software system of a GBADS is the TM subsystem, as discussed in §2.4. This was followed by a brief discussion on the TE subsystem in §2.5. The WA subsystem was described more comprehensively in §2.6 and the tactical environment, which may negatively influence a GBADS, was considered in §2.7. The chapter closed with a review of a classical WA model which is based on the classical assignment problem.

CHAPTER 3

Multiobjective approaches from the literature

Contents

This chapter contains a literature review on decision making analysis and a discussion of three very well-known approaches available in the literature which may be used to solve multiobjective decision making problems. In §3.1, unidimensional decision making theory is briefly mentioned and the reader is familiarised specifically with multiobjective decision analysis.

A first approach considered in solving multiobjective decision making problems is the *Analytic Hierarchy Process* (AHP) discussed in §3.2, which offers the reader a comprehensive study on the working, advantages and limitations placed on the AHP.

This is followed by a discussion on unidimensional utility theory in §3.3, where qualitative characteristics of utility functions are discussed as well as procedures to calculate quantitative values for utility functions. Guidelines for assessing unidimensional utility functions may be found towards the end of the section.

A discussion on multiobjective utility functions and the conditions relating to each of the respective types of utility functions may be found in §3.4 which is followed by a discussion on the techniques which may be used to evaluate values for the scaling constants contained in multiobjective utility functions.

The chapter concludes with a discussion on multiobjective evolutionary algorithms as well as a discussion on the *Nondominated Sorting Genetic Algorithm* II (NSGA II) in §3.5.

3.1 Decision making theory

Decision making occurs in the lives of most individuals on a daily basis. Deciding which clothes to wear each morning, or deciding which job to take from among a number of alternative job offerings, are just some examples of decisions an individual may face. The way in which individuals deal with decisions and the manner in which they make their decisions are the essence of decision making theory. Decision making may be considered to be unidimensional if it is based on one criterion or multi-dimensional if more than one criterion is considered.

3.1.1 Decisions with multiple conflicting objectives

Decision making usually occurs with respect to different criteria. Consider, for instance, the choice in buying a house. An individual may be faced with a number of alternatives, but he may consider decision criteria such as the price, safety in the neighbourhood and distance to nearest schools and work. In order to make a decision, he has to balance these criteria to find the alternative which best satisfies all of them simultaneously [9]. Complicating the problem even further, is the fact that some or all of these criteria may not be completely satisfiable, because they are conflicting. When these conditions are present, the decision falls within the realm of *multicriteria decision making* or *multiobjective decision making.*

Example 1 *Consider a situation where a decision maker is in the market for a new vehicle. After careful consideration, he finds that the price, the life expectancy and the reliability of the vehicle are his main criteria when choosing a suitable vehicle. The decision maker prefers the cheapest vehicle, with the longest life expectancy and, simultaneously, the most reliable. Unfortunately, it is very unlikely to find a vehicle with all of these properties. The three criteria may also be seen as the decision maker's objectives*[1] *in choosing a vehicle. Suppose he can choose between three alternatives with the vehicles exhibiting the properties in Table 3.1.*

Vehicle A is the cheapest with the shortest life expectancy and the least reliable. Vehicle C is the most expensive, but has the longest life expectancy and is very reliable. The properties of vehicle B, are in between those of Vehicle A and C.

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[1] For the sake of consistency, the criteria will be referred to as the objectives of a decision maker in the remainder of this thesis.

Properties of three vehicles	Vehicle A	Vehicle B	Vehicle C
Price	R 25 000	R75 000	R120 000
Life expectancy	5 years	7 years	12 years
Reliability	Unreliable	Reliable	Very reliable

TABLE 3.1: *The properties of three potential vehicles.*

From an objective point of view, it may be said that the decision maker has the need to minimise the price he pays for the vehicle, and maximize the life expectancy and reliability of the vehicle he chooses. Since all of these objectives cannot be met simultaneously, a compromise with respect to some or all of the objectives will have to be made in an attempt to be able to make the particular decision. The reason behind such a compromise is that the above objectives behave in a conflicting manner, since a more reliable vehicle has a higher price tag attached to it, and at the same time the reliability has to be maximised and the cost minimised. ☐

Example 1 highlights some of the concepts involved in multiobjective decision problems. Numerous techniques in the literature may be used as tools to approach and solve multiobjective decision problems. A few examples of such techniques include the analytic hierarchy process, goal programming methods, outranking methods and utility theoretic approaches [9].

3.1.2 The notion of Pareto optimality

The concept of multiobjective decision problems naturally gives rise to the notion of multiobjective optimisation. In single objective optimisation, the focus is aimed at finding the globally optimal solution. However, this is not the case in multiobjective optimisation. In multiobjective optimisation, the focus is aimed at finding a set of solutions each of which yields an extremal value for at least one of the objectives. Due to the conflicting nature of the objectives, there usually does not exist such a single solution which minimises or maximises all of the objectives simultaneously. Hence, the aim is to find a set of solutions which represent acceptable compromises between objectives [12, 17].

The set of feasible solutions usually contain solutions which are superior to other solutions with respect to all the objectives, but are inferior to other solutions with respect to one or more objectives [17]. Such a subset of solutions is known as the *nondominated* set of solutions in the solution space and is also called the *Pareto optimal* solutions. These solutions form the so-called *Pareto optimal frontier* when plotted in solution space. Nondomination implies that none of the solutions contained in the Pareto optimal set are absolutely better than any other solution in the set [17]. Hence, the decision maker is presented with a set of Pareto optimal solutions from which he may choose, based on some personal associated priorities of the objectives. The Pareto optimal set of solutions are considered the best set of solutions to a multiobjective problem.

Since the remaining solutions in the solution space are dominated by the Pareto optimal solution set, they are considered the *dominated solutions*. An example of a dominated solution set and nondominated solution set may be found in Figure 3.1, where the dominated set is represented by the black dots, and the nondominated set is represented by the grey dots.

Suppose that $\boldsymbol{x} = [x_1, \ldots, x_n]$ and $\boldsymbol{y} = [y_1, \ldots, y_n]$ are solution vectors containing criteria elements for each objective i. For a multiobjective optimisation problem in which all objective functions have to be minimised, $\boldsymbol{x}$ strictly dominates $\boldsymbol{y}$, (denoted by $\boldsymbol{x} \prec \boldsymbol{y}$), when each element in $\boldsymbol{x}$ is less than or equal the corresponding element in $\boldsymbol{y}$, and at least one element in $\boldsymbol{x}$ is strictly less than the corresponding element in $\boldsymbol{y}$; that is $\boldsymbol{x}_i \leq \boldsymbol{y}_i$ for all i, and $\boldsymbol{x}_j < \boldsymbol{y}_j$ for some j [12]. Similarly, for a multiobjective optimisation problem in which all the objectives have to be maximised, $\boldsymbol{x}$ strictly dominates $\boldsymbol{y}$ when $\boldsymbol{x}_i \geq \boldsymbol{y}_i$ for all i, and $\boldsymbol{x}_j > \boldsymbol{y}_j$ for some j.

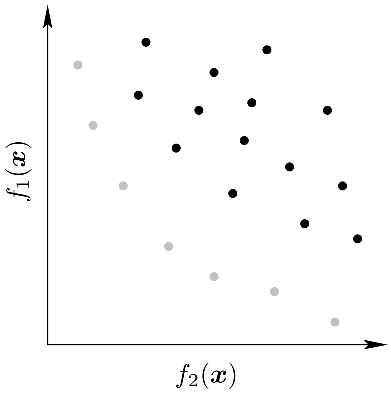

FIGURE 3.1: *Illustration of the solutions contained in the Pareto frontier for a two-objective minimisation problem in which the objective functions are f_1 and f_2.*

3.1.3 Objectives, goals and attributes in decision making applications

When solving a multiobjective decision problem, it is necessary to establish the objectives of the decision maker. This is usually achieved with the help of a decision analyst who is responsible for probing the decision maker to identify the complete set of objectives of the decision maker. The objectives of a decision are usually a representation of the direction in which a decision should be steered for improvement [33]. As mentioned in the previous section, a decision maker is typically faced with a number of objectives that have to optimised simultaneously. Understanding the objectives of a decision is one of the key elements in the decision making initiation phase, as this will act as the foundation of the decision problem. It is therefore important that both the decision analyst as well as the decision maker have a clear view on exactly what each of the objectives reflect. Clemen and Reilly [14] propose that an objectives hierarchy, with all the essential objectives and attributes, should be constructed. This may result in a better understanding and experience of the objectives and their related attributes.

Goals should not be confused with objectives. A goal may be defined as something that has to be achieved. Hence, a goal has the outcomes of either having been achieved or not. The difference between a goal and an objective is that an objective is something the decision maker strives towards, while a goal is achieved or not [33]. Considering as an example, the objective of a courier service to minimise the total time in transit for any given parcel. A goal may be to deliver at least 95% of all parcels within 48 hours from the time of receiving the parcel.

An attribute may be defined as the measure of an objective. If the objective is to maximise profit, the natural attribute may be the amount of profit in rand value [33]. Certain objectives do not have natural attribute scales, and finding appropriate attribute scales for these objectives may be a daunting task, due to the fact that these objectives are often very difficult to quantify; some may not be quantifiable at all. One way of avoiding this problem is to use a different attribute scale as a proxy, which should be closely associated with the original objective [14].

The difference between proxy and natural attributes, is that proxy attributes do not measure the objective directly. Gregory and Keeney [25] provide an example of using a proxy attribute for a manufacturing firm where the objective is to maximise product quality. There may be no obvious way to measure product quality directly, hence, a proxy attribute may be the percentage of products returned. This illustrates that the proxy attribute indirectly measures the product quality.

The objectives in a decision may be separated into *means* and *fundamental* objectives. Means objectives may be defined as objectives which help to achieve other objectives, whereas fundamental objectives may be defined as objectives which reflect on what the decision maker really wants to achieve [14]. Clemen and Reilly [14] gives the example of an objective of a working individual to work fewer hours. Working fewer hours may result in him having more time to spend with his family, at home, or that he has more time available to engage in other activities representing fundamental interests. It is clear that minimising working hours is a means objective, since it helps to achieve the fundamental objectives of spending more time with family or having more time to engage in other activities. Minimising working hours aids the decision maker in achieving the things that are important to him, in other words his fundamental objectives.

Fundamental objectives may be organized into a structure called a *fundamental objectives hierarchy*, consisting of different levels of objectives [14]. The objectives found towards the top levels are considered the universal objectives, while the objectives found towards the bottom are considered those objectives which explain upper level objectives.

Example 2 *Consider an example adapted from Clemen and Reilly [14]. Figure 3.2 depicts a fundamental objectives hierarchy in the context of vehicle safety. The overall objective, found at the top level, is to maximise vehicle safety. Three lower level objectives, namely to minimise loss of life, to minimise serious accidents and to minimise minor injuries, may be found on the next level. These objectives are also considered fundamental objectives in the sense that each of them represents a different sector of safety.*

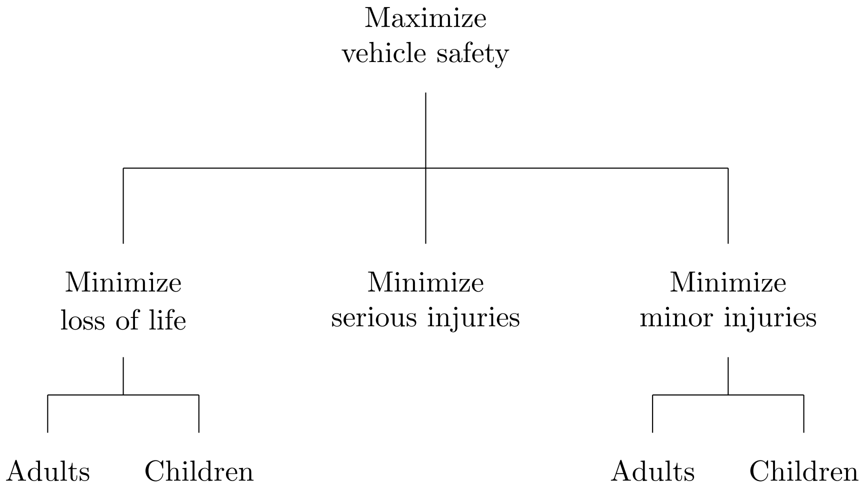

FIGURE 3.2: *An example of a fundamental objectives hierarchy [14].*

If desired, the hierarchy in Figure 3.2 may be expanded further, by including to minimise the loss of lives of children and to minimise loss of lives of adults below the minimise serious injuries objective, for example. ☐

Figure 3.3 contains a general fundamental objectives hierarchy taken from Keeney and Raiffa [33]. The figure depicts a schematic representation of a set of fundamental objectives. The upper level objectives are represented by the symbols X_1 and X_2 and the lower level objectives by $Y_1, Y_2, \ldots, Y_{13}$ respectively. Preferences for objectives may be assessed at the upper or lower level objectives. Here X_1 and X_2 are subjectively assessed composites of $Y_1, \ldots, Y_5$ and $Y_6, \ldots, Y_{13}$, respectively [33]. The purpose of going through all the trouble of expanding the hierarchy as far as possible by including many details may be to obtain a better understanding of the upper level objectives, X_1 and X_2.

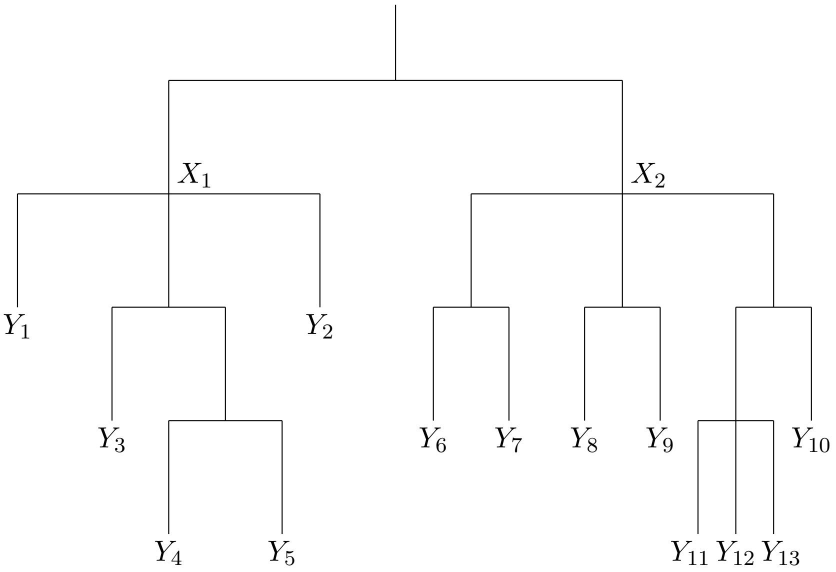

FIGURE 3.3: *A general fundamental objectives hierarchy [33].*

Means objectives are organised into networks, called *means objectives networks* as shown in Figure 3.4, which was taken from [14]. It ties to the previous example of vehicle safety, where means objectives may be to maximise the use of vehicle safety features or to minimise accidents. These objectives assist in the achievement of the maximise safety objective. The network may be expanded even further by adding other means objectives, such as to motivate the purchasing of safety features on vehicles, to maintain vehicles properly and to maximise driving quality. A means objective is not restricted to assist only one other objective. Means objectives may be linked to various other objectives, if applicable. From Figure 3.4, it can be seen that having reasonable traffic laws helps to achieve both the objective of maintaining vehicles properly and maximising driving quality.

Care should be taken not to confuse the network of means objectives with the fundamental objectives hierarchy. Telling fundamental objectives apart from means objectives and structuring them accordingly are considered to be very important in the development process of a multiobjective decision model. Both of these structures have their advantages and disadvantages, but it is the lower level objectives of the fundamental objectives hierarchy which forms the basis with respect to which several outcomes are measured. Means objectives may be considered as a way to help establish fundamental objectives and may sometimes act as substitutes for fundamental objectives in a situation where the fundamental objectives may be very difficult to measure [33].

3.1.4 Establishing objectives

The process of how to isolate or identify objectives usually involves a decision analyst or a facilitator interviewing a decision maker or group of decision makers. During the interview the decision maker may be asked to answer a set of questions which may lead to possible objectives.

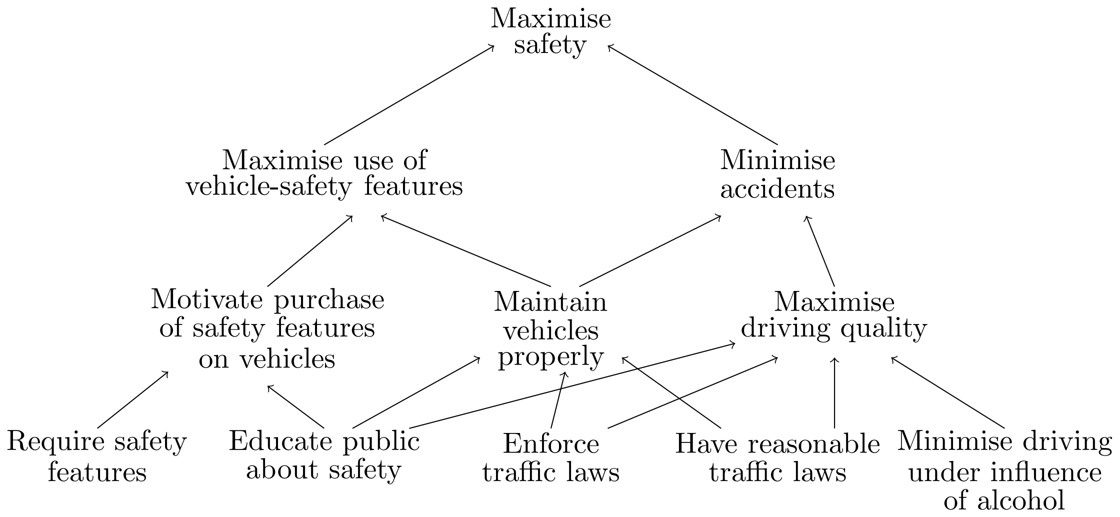

Figure 3.4: *An example of a means objectives network [14].*

The decision maker(s) should be probed in order to determine a good estimation of the objectives which he/they deems important. The replies from the decision maker(s) should be questioned in order to make him/them think harder about their initial reactions. Keeney [32] proposed the following techniques and questions which may be used in such an interview:

1. *Develop a wish list.* What do you want? What do you value? What should you want?

2. *Identify alternatives.* What is a perfect alternative, a terrible alternative, some reasonable alternative? What is good or bad about each?

3. *Consider problems and shortcomings.* What is wrong or right with your organisation? What needs fixing?

4. *Predict consequences.* What has occurred that was good or bad? What might occur that you care about?

5. *Identify goals, constraints and guidelines.* What are your aspirations? What limitations are placed on you?

6. *Consider different perspectives.* What would your competitor or your constituency be concerned about? At some time in future, what would concern you?

7. *Determine strategic objectives.* What are your ultimate objectives? What are your values that are absolutely fundamental?

8. *Determine generic objectives.* What objectives do you have for your customers, your employers, your shareholders and yourself? What environmental, social, economic, or health and safety objectives are important?

9. *Structure objectives.* Follow means-ends relationships: Why is that objective important? How can you achieve it? Be specific: What do you mean by this objective?

10. *Quantify objectives.* How would you measure achievement of this objective? Why is objective A three times as important as objective B?

Objectives may also be obtained from a group of decision makers as a team, especially in large corporations where decisions are typically not made by a single individual. In such a scenario, objectives may be obtained by hosting facilitated workshops with a team of decision makers.

The degree of intensity of these workshops may vary. A simple discussion between the facilitator and team of decision makers may be sufficient to capture the necessary information. However, this procedure may fail to encapsulate the true detail of the problem and is not recommended for complex problems [9]. On the other hand, a workshop may be adapted to include a more comprehensive infusion procedure. In this instance a more detailed set of objectives may be captured [9].

A well-known method which may be applied in a facilitated workshop is one called a *Post-it*[2] session. During such a session, post-it notepads are handed out to the team of decision makers. They are then required to write down all the ideas and facts which they deem relevant to the problem. Team members are then invited to stick their notes to a flat surface, which is visible to other team members, in order for their ideas to be shared and discussed among themselves [9]. In this way it is anticipated that an inclusive set of objectives will be obtained. It should be noted that the efficiency of this technique relies heavily on the abilities and skills of the facilitator. For a more comprehensive discussion on *Post-it* sessions and its extensions, the interested reader is referred to Belton and Stewart [9].

Another popular method is the *Delphi Method*. This method is used to structure the communication process in a group facilitation context [35]. The conventional Delphi method consists of a team of respondents (usually experts in the relevant field, and in this case, the decision makers) who are required to complete a questionnaire, concerning relevant questions relating to the problem at hand [35, 54]. This questionnaire may be developed prior to the process and handed to the decision makers to complete. After completion, the results are summarised and analysed by a facilitator or a team of facilitators. These results are then anonymously presented to the decision makers and they are given a chance to review everybody's original responses based on the feedback from the first round of results [35]. This process is repeated until a stopping criterion is reached, such as reaching a maximum number of rounds or until a consensus is reached [54].

In essence it is believed that the Delphi method works in such a way that it scales down the range of possible objectives, and will eventually converge to the appropriate ones [54]. The above listed techniques by Keeney [32], may be used to develop an appropriate questionnaire in order to identify objectives. This questionnaire may then be implemented by means of the Delphi method so as to obtain an appropriate set of objectives for the decision problem at hand.

Identifying objectives may be a painstaking process, but a well-structured decision model may significantly simplify the decision making process at a later stage, especially when independence between different objectives is evaluated.

3.2 The analytical hierarchy process

A first approach towards multiobjective decision making is the so-called *analytic hierarchy process* (AHP). It was developed by Thomas L. Saaty in the 1970s and has been researched extensively [46]. It is a well-known technique in decision making and has been used by many decision makers world wide in a variety of fields. It was designed to recommend the best solution, considering all the objectives and preferences of the decision maker [51].

Once the objectives of the decision maker have been specified, these objectives are used in the AHP to evaluate all of the alternatives with respect to each objective by means of comparisons

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[2] Post-It refers to a piece of stationary called a post-it notepad containing an adhesive strip on the back to stick to various surfaces [23].

between two alternatives at a time. Each comparison is represented by a numerical value based on an index of priority to the decision maker. These values are then converted into score values, which represent how well each alternative "scores" on each objective. Numerical values are also established for each objective, representing the weight each objective will contribute in the final calculation of the process. These weights are then combined with the numerical score values of the alternatives, with respect to each objective, to obtain a single final score value for each alternative. These values are used to make a recommendation with respect to the best possible alternative, taking into account all the objectives and preferences of the decision maker.

The first step in the AHP is determining a numerical weight value w_i for the i^{th} objective. The weights for each objective may be calculated by constructing an $n \times n$ matrix (known as a *pairwise comparison matrix*), where there are n objectives [9]. The content of this matrix A is a representation of how important one objective is with respect to another. The entry a_{ij} (where i represents a row and j represents a column) is therefore a number representing how much more important objective i is than objective j in the case where objective i is preferred to objective j [62]. These numbers may be seen in Table 3.2 and was taken from Winston [62]; they represent a measurable scale from 1-9 where the number one means that objective i is as important as objective j, while the number nine means that objective i is absolutely more important than objective j [9].

Value of a_{ij}	Interpretation
1	Objectives i and j are of equal importance.
3	Objective i is weakly more important than objective j.
5	Experience and judgment indicate that objective i is strongly more important than objective j.
7	Objective i is very strongly or demonstrably more important than objective j.
9	Objective j is absolutely more important than objective j.
2,4,6,8	Intermediate values mean that the importance is midway between two of the importance values above.

TABLE 3.2: *An interpretation of measurable values used in the construction of a pairwise comparison matrix in the AHP [62].*

When constructing a pairwise comparison matrix, all the entries on the diagonal, must have a value of one [9], because objective i cannot be more important than itself. It is also important, for the sake of consistency, that $a_{ij} = k$ should imply that $a_{ji} = \frac{1}{k}$ [5]. The consistency of a decision maker can be measured by a consistency index which will be discussed later in this section.

Suppose the decision maker is perfectly consistent, that there are n objectives and that w_i represent the weight of objective i. Then, according to Winston [62], the decision maker's pairwise comparison matrix should take the form

$$A = \begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix}. \tag{3.1}$$

To recover the vector $\boldsymbol{w} = [w_1, w_2, \ldots, w_n]$ from A, consider the system of n equations,

$$A\boldsymbol{w}^T = \Delta\boldsymbol{w}^T, \tag{3.2}$$

where $\boldsymbol{w}^T$ is an n-dimensional column vector and Δ is an unknown number. It is clear that the system of equations in (3.2) always has the trivial solution $\boldsymbol{w} = [0\ 0\ \ldots\ 0]$, for any number Δ. If a decision maker is perfectly consistent, and thus has a pairwise comparison matrix of the form A, and $\Delta = 0$ is not allowed, then it can be shown that (3.2) has as only nontrivial solution the eigenvalue $\Delta = n$ and the corresponding eigenvector $\boldsymbol{w} = [w_1\ w_2\ \ldots\ w_n]$. Hence, the weights of a perfectly consistent decision maker can be obtained from the only nontrivial solution to (3.2) [62].

In the case where a decision maker is not perfectly consistent, suppose the largest number for which (3.2) has a nontrivial solution $\boldsymbol{w}_{max}$ is denoted by Δ_{max}. Since the choices of the decision maker exhibit a slight inconsistency, it is expected that Δ_{max} should have a value close to n, and that $\boldsymbol{w}_{max}$ should contain values close to those of $\boldsymbol{w}$. This expectation was verified by Saaty [46], who proposed that $\boldsymbol{w}$ should be approximated by $\boldsymbol{w}_{max}$ and that the consistency of the decision maker should be measured by considering the deviation of Δ_{max} from n. Winston [62] describes a two-step procedure to approximate $\boldsymbol{w}_{max}$. These weights obtained from the pairwise comparisons, as made by the decision maker, reflect the importance of each objective with respect to one another, and are used in the calculation of the final overall score of each of the decision maker's alternatives.

The first step in the process is to normalize the pairwise comparison matrix A by dividing each entry in column j of A by the sum of all the entries in column j [5, 62]. The resulting matrix, called A_{norm}, has the property that the sum of the entries in each column adds up to one. The second step is to approximate $\boldsymbol{w}_{max}$, which will be used as an estimate of $\boldsymbol{w}$ [5, 62]. This may be achieved by taking the average of the entries in the i^{th} row of A_{norm} to estimate the weight w_i for objective i. The process is explained by means of an example.

Example 3 *Suppose a student is to start his first year of university education in a couple of months. He is faced with the decision of which university to attend. Suppose he receives notice of acceptance from four universities, namely Stellenbosch University (SU), the University of Cape Town (UCT), the University of Pretoria (UP) and Northwest University (NWU). His decision will be based on the facilities offered (FO) by the various universities, the cost (C) of attending a university and the resulting proximity to family (PF), which form the objectives in his decision making process. Suppose that after careful assessment the student arrives at the pairwise comparison matrix*

$$A = \begin{array}{c} \\ \mathit{FO} \\ \mathit{C} \\ \mathit{PF} \end{array} \begin{array}{c} \begin{array}{ccc} \mathit{FO} & \mathit{C} & \mathit{PF} \end{array} \\ \left[\begin{array}{ccc} 1 & 5 & 3 \\ \frac{1}{5} & 1 & \frac{1}{2} \\ \frac{1}{3} & 2 & 1 \end{array}\right] \end{array},$$

or after dividing each column entry in A by the sum of that column, the normalized matrix

$$A_{norm} = \begin{array}{c} \\ \mathit{FO} \\ \mathit{C} \\ \mathit{PF} \end{array} \begin{array}{c} \begin{array}{ccc} \mathit{FO} & \mathit{C} & \mathit{PF} \end{array} \\ \left[\begin{array}{ccc} 0.6522 & 0.6250 & 0.6667 \\ 0.1304 & 0.1250 & 0.1111 \\ 0.2174 & 0.2500 & 0.2222 \end{array}\right] \end{array}.$$

The weights for each of the objectives may be computed as

$$w_1 = \frac{0.6522 + 0.6250 + 0.6667}{3} = 0.6479,$$
$$w_2 = \frac{0.1304 + 0.1250 + 0.1111}{3} = 0.1222,$$
$$w_3 = \frac{0.2174 + 0.2500 + 0.2222}{3} = 0.2299.$$

The weight w_1 is the weight associated with facilities offered, w_2 is the weight associated with cost and w_3 is the weight associated with proximity to family. ☐

The next step is to determine a numerical score value s_i for each alternative which is a reflection of how well an alternative scores on a particular objective [5, 62]. The process involves the calculation of score values for each alternative, similar to that of the calculation of the weights of the objectives. The process is initiated by constructing a pairwise comparison matrix for each objective. In this case the rows and columns of the matrix represent the possible alternatives of the decision maker. Each matrix is normalised and the average of each row is calculated. The resulting values are used as an indication on how well each alternative scores with respect to that particular objective.

Example 4 (Continuation of Example 3) *Suppose that after careful consideration, the student constructs the pairwise comparison matrix*

$$B = \begin{array}{c} \\ \mathit{US} \\ \mathit{UCT} \\ \mathit{UP} \\ \mathit{NWU} \end{array} \begin{array}{c} \begin{array}{cccc} \mathit{US} & \mathit{UCT} & \mathit{UP} & \mathit{NWU} \end{array} \\ \left[\begin{array}{cccc} 1 & 2 & 4 & 3 \\ \frac{1}{2} & 1 & 3 & 4 \\ \frac{1}{4} & \frac{1}{3} & 1 & 2 \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{2} & 1 \end{array}\right] \end{array}$$

for the FO objective, and that for the cost and PF objectives, he finds the pairwise comparison matrices

$$C = \begin{array}{c} \\ \mathit{US} \\ \mathit{UCT} \\ \mathit{UP} \\ \mathit{NWU} \end{array} \begin{array}{c} \begin{array}{cccc} \mathit{US} & \mathit{UCT} & \mathit{UP} & \mathit{NWU} \end{array} \\ \left[\begin{array}{cccc} 1 & 3 & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & 1 & \frac{1}{5} & \frac{1}{6} \\ 3 & 5 & 1 & \frac{1}{3} \\ 4 & 6 & 3 & 1 \end{array}\right] \end{array}$$

and

$$D = \begin{array}{c} \\ \mathit{US} \\ \mathit{UCT} \\ \mathit{UP} \\ \mathit{NWU} \end{array} \begin{array}{c} \begin{array}{cccc} \mathit{US} & \mathit{UCT} & \mathit{UP} & \mathit{NWU} \end{array} \\ \left[\begin{array}{cccc} 1 & 2 & \frac{1}{5} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{5} & \frac{1}{4} \\ 5 & 5 & 1 & 2 \\ 4 & 4 & \frac{1}{2} & 1 \end{array}\right] \end{array},$$

respectively.

The resulting scores of each objective with respect to each objective is given in Table 3.3. ☐

Finally, the overall score of each alternative may be computed as

$$\sum_{i=1}^{n} w_i s_i, \tag{3.3}$$

Objective	US	UCT	UP	NWU
FO	0.4522	0.3180	0.1327	0.0974
C	0.1341	0.0615	0.2761	0.5283
PF	0.1096	0.0769	0.4977	0.3158

TABLE 3.3: *Scores for a students' alternatives for each objective.*

where w_i represents the weight associated with objective i, s_i is the score for the particular alternative on objective i, and n is the number of objectives. The alternative rendering the highest score is considered the best alternative, and is recommended as the best choice. When calculating the overall score, more weight is given to an alternative's score on the important objectives [62].

Example 5 (Continuation of Examples 3 and 4) *The calculations*

$$\begin{aligned}
US &= (0.6479 \times 0.4522) + (0.1222 \times 0.1341) + (0.2299 \times 0.1096) = 0.3346,\\
UCT &= (0.6479 \times 0.3180) + (0.1222 \times 0.0615) + (0.2299 \times 0.0769) = 0.2312,\\
UP &= (0.6479 \times 0.1327) + (0.1222 \times 0.2761) + (0.2299 \times 0.4977) = 0.2341, \text{ and}\\
NWU &= (0.6479 \times 0.0971) + (0.1222 \times 0.5283) + (0.2299 \times 0.3158) = 0.2001.
\end{aligned}$$

may be made to find overall scores for each of the student's alternatives. When considering these calculations it seems that the student should choose to attend the University of Stellenbosch for the duration of his studies. Given his comparisons and objectives, this is the alternative with the best outcome yielding the highest overall score. □

3.2.1 Ensuring that the decision maker remains consistent

In order to obtain valuable results, it is important to ensure that the decision maker remains consistent in the assessment of his comparisons. A process which measures the consistency of a decision maker may be found in Albright and Winston [5], Belton and Stewart [9] and Winston [62]. The procedure consists of four steps and makes use of a *random index* (RI), which may be found in Table 3.4.

n	RI
2	0
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.51

TABLE 3.4: *Random index values for different values of* n *[62].*

The first step is to calculate $A\boldsymbol{w}^T$, where $\boldsymbol{w}$ is the n-row vector of the estimated weights obtained from the pairwise comparison matrix A. The vectors $A\boldsymbol{w}^T$ and $\boldsymbol{w}$ are used to determine the ratio,

$$\delta = \frac{1}{n}\sum_{i=1}^{n}\frac{i^{th}\text{ entry in } A\boldsymbol{w}^T}{i^{th}\text{ entry in } \boldsymbol{w}^T}. \tag{3.4}$$

Using δ from (3.4), the *consistency index* (CI) may be calculated as

$$\text{CI} = \frac{\delta - n}{n-1}. \tag{3.5}$$

In the final step the CI is compared to the RI in Table 3.4 for the corresponding value of n. The level of consistency is satisfactory when the value of the ratio CI/RI is smaller than 0.10 [9, 62]. However, if the value of the ratio exceeds 0.10, crucial inconsistencies may exist, jeopardising the results of the AHP.

If a decision maker exhibits perfectly consistent behaviour, then the i^{th} entry in $A\boldsymbol{w}^T$ equals n (i^{th} entry of $\boldsymbol{w}^T$), implying that in this case CI $= 0$. The RI values in Table 3.4 for different values of n are merely average values of CI if the entries in A were to be chosen at random [62]. This, of course, is subject to the constraints that the entries on the diagonal of A must be equal to one, and that

$$a_{ij} = \frac{1}{a_{ji}}, \qquad i, j = 1, \ldots, n. \tag{3.6}$$

The consistency of the decision maker may also be measured during the assessment of the score values of the alternatives to ensure that the scores obtained are accurate. The same procedure may be followed as used with the consistency measurement of weights for the objectives.

Example 6 (Continuation of Example 3-5) *Consider the pairwise comparison matrix A in Example 3. For matrix A, it follows that*

$$A\boldsymbol{w}^T = \begin{bmatrix} 1 & 5 & 3 \\ \frac{1}{5} & 1 & \frac{1}{2} \\ \frac{1}{3} & 2 & 1 \end{bmatrix} \begin{bmatrix} 0.6479 \\ 0.1222 \\ 0.2299 \end{bmatrix} = \begin{bmatrix} 1.9485 \\ 0.3667 \\ 0.6902 \end{bmatrix},$$

and that

$$\delta = \frac{1}{3}\left\{\frac{1.9485}{0.6479} + \frac{0.3667}{0.1222} + \frac{0.6902}{0.2299}\right\} = 3.0037.$$

The CI is calculated by means of (3.5) as

$$CI = \frac{3.0037 - 3}{2} = 0.0018.$$

Since $\frac{CI}{RI} = \frac{0.0018}{0.58} = 0.0032$, which is less than 0.10, the conclusion may be reached that the comparisons made by the student are consistent. It can easily be shown that the decision maker also exhibits consistent behavior in the pairwise comparison matrices B, C and D by using the easy four step procedure explained above. □

3.2.2 Implementation, advantages and disadvantages

The AHP is a very powerful tool in decision making theory, and it is mainly due to its simplicity and computational efficiency that it is often used by decision makers. The AHP may easily be implemented on a spreadsheet, but there also exist specialised computer software which may assist in implementing the AHP. The AHP may become difficult to implement on a spreadsheet when the number of objectives and alternatives are large.

The AHP is very effective when used in situations where groups of people work together on complex decision making problems [51]. Furthermore, many real-life decision problems include factors which may be difficult to quantify or compare. There might also be a communication gap between decision makers due to their difference in perspectives or the different fields in which they operate. One of the advantages of AHP is that it addresses these problems [51]. Another capability of the AHP is that it can compare different alternatives with respect to objectives which are totally incommensurable.

Although use of the AHP has advantages, it also has its disadvantages. A specific disadvantage which received considerable criticism in the literature is a consequence called *rank reversal*. When a new alternative is added to a decision which does not change any of the outcomes of the existing set of alternatives, this addition might change the rank-order of the current set of alternatives as determined by the AHP [9, 51].

3.3 Utility theory in one dimension

Another well-known approach towards making multiobjective decisions in the literature is embodied in the field of *utility theory*. The aim in utility theory is to transform monetary gains or losses of decision makers as a result of uncertainty into so-called *utility values*, and to present these values in the form of a function [5]. Epstein [22] states that a utility value defines a measure of effectiveness of a particular strategy. A utility value consists of a numerical value assigned to a payoff, which may be a wealth gain or loss reflecting the relative utility to the decision maker. Such utility values may then be represented in the form of a *utility function*[3]. A utility function therefore encapsulates a decision maker's attitude towards risk [5]. The aim of a utility function is to provide a mathematical model which reflects the preferences of a decision maker under the different types of risk attitudes which he exhibits. Clemen and Reilly [14] claim that it is not necessary to develop a model which captures the risk attitude of a decision maker perfectly. A model which is able to present preferences with a moderate feeling towards risk may be sufficient.

The birth of utility theory dates back to 1713 in the assessment of risky monetary ventures by Nicholas Bernoulli, called a *expected utility model*. However, this model was only solved in 1738 by Nicholas's cousin, Daniel Bernoulli, as what is known as the *St. Petersburg paradox*[4] [59, 61].

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[3] The Oxford dictionary of statistics defines a utility function as, "A function that takes a numerical value for each possible state of a system (usually an economic system) and is intended as a measure of the benefit or usefulness of that state" [7].

[4] Think of a situation where a game is played by repeatedly flipping a fair coin. The event of the coin landing tails ends the game, which means that the coin is flipped repeatedly until a tail appears. A payoff of R1 is received when the first head appears and is doubled every time a head appears after that, thus resulting in a payoff of 2^n if the coin is flipped n consecutive times, each time landing heads. How much would an individual be willing to pay to partake in this game? A rational decision would be to participate in the game if the fee to pay the game is less than the expected payoff. It can easily be shown that the expected payoff of this game is infinitely large, which means that an individual should be willing to pay any amount of money to play the game.

In its earlier consumer economics forms, the notion of utility was referred to as *final degree of utility, effective utility, specific utility, marginal efficiency or even marginal desirability* [13].

Von Neumann and Morgernstern [33, 62] developed a number of axioms with respect to expected utility[5]. They proved that if a decision maker's preferences satisfy these axioms, then a utility function exists for the decision maker and the expected utility approach is a suitable way for making consistent decisions under conditions of uncertainty. Accepting the Von Neumann and Morgernstern axioms also ensures that the decision maker exhibits consistent behaviour during the decision making process. These axioms are well known in the field of utility theory and are widely available in the literature [19, 22, 62].

Axiom 1(a)—Complete ordering: A decision maker is able to rank the possible outcomes of a decision in terms of preference. That is, for any two outcomes A_i and A_j, the decision maker either prefers A_i to A_j, A_j to A_i or is indifferent between A_i and A_j.

Axiom 1(b)—Transitivity: For any three outcomes A_i, A_j and A_k of a decision, if a decision maker prefers outcome A_i to outcome A_j and outcome A_j to outcome A_k, then the decision maker must prefer outcome A_i to A_k.

Axiom 2(a)—Continuity: For any outcomes A_i, A_j and A_k of a decision, if a decision maker prefers outcome A_i to outcome A_j and outcome A_j to outcome A_k, then there exists a probability $0 < p < 1$, such that the decision maker is indifferent with respect to choosing between a sure outcome A_j and a lottery[6] in which A_i is received with probability p and A_k is received with probability $(1 - p)$, as illustrated in Figure 3.5.

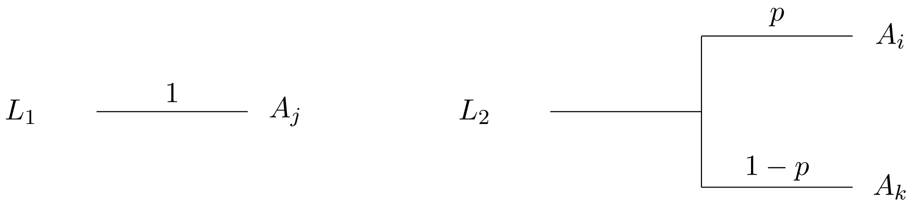

FIGURE 3.5: *A sure outcome L_1 and a lottery L_2 illustrating the continuity axiom.*

Axiom 2(b)—Substitutability: If a decision maker is indifferent with respect to choosing between lotteries L' and L'', then in any probability tree that includes lottery L' as a subtree, the decision maker is also indifferent to substitution of L'' for L'.

Axiom 3—Monotonicity or the unequal probability: For any two outcomes A_i and A_j of a decision, if a decision maker prefers outcome A_i to outcome A_j, the decision maker will prefer a lottery with a higher probability of obtaining A_i.

Axiom 4—Independence: Consider the following sets of outcomes, $A_1, A_2, A_3, \ldots, A_n$ and $B_1, B_2, B_3, \ldots, B_n$ of a decision. If the decision maker prefers A_i to B_i or is indifferent with respect to choosing between A_i and B_i for all $i = 1, \ldots, n$, then an even chance of obtaining A_1 or A_2 or …or A_n is preferred or equivalent to an even chance of obtaining B_1 or B_2 or …or B_n, respectively.

However, this is not the case since some prices are just to high to play the game. This problem is known as the St. Petersburg paradox [47, 59].

[5] The expected utility of a lottery is calculated as the sum of the outcomes, multiplied by their respective probabilities. If a choice has to be made between a number of lotteries, the lottery yielding the highest expected utility is recommended as the best alternative.

[6] A lottery is used as a metaphor to illustrate the condition of uncertainty.

Utility functions may be defined in terms of both as *qualitative* characteristics and *quantitative* characteristics. Qualitative characteristics may be used to indicate a specific attitude towards the preferences with respect to outcomes for a decision maker. These characteristics may be used to derive restrictions on the utility function, and in the process simplify the assessment procedure of the utility function. Qualitative characteristics provide the decision maker with the possibility to perform sensitivity analysis, as well as performing different consistency tests so as to ensure that the preferences of the decision maker are consistent [33]. Qualitative characteristics include, *monotonicity, risk attitudes, the certainty equivalent* and *the risk premium.*

Quantitative characteristics refer to the assessment of utility values for various outcomes [33]. This may be done by using techniques such as the *certainty equivalent approach* or the *probability equivalent approach.* Both the qualitative and the quantitative characteristics may be used to determine a suitable utility function which may yield a good representation of the true preferences of the decision maker.

3.3.1 Certainty versus uncertainty

The events which may occur during decision making may take place under two typical conditions, known as *certainty* and *uncertainty.* When the outcomes in a decision have specific characteristics that one knows for certain before the decision is made, the decision is said to be made under conditions of *certainty.* Decisions made under certainty means that each specific strategy leads to a specific outcome. Examples of decision making under certainty may be found in economic games, especially those including factors such as production schedules, cost functions or time-and-motion considerations [22]. When the outcome of a decision consists of a set of outcomes having a known *a priori* probability distribution, the decision is said to be made under *uncertainty* [22].

Example 7 *Suppose a decision maker wants to purchase a new laptop computer. The attributes affecting the decision includes the price, size and the life expectancy of each computer. The values for each attribute may be found in Table 3.5. If the decision is based only on the information contained in Table 3.5, the decision occurs under certainty. This is due to the fact that all the values of each outcome is known for sure. However, should the life expectancy of the computers follow some probability distribution function, the decision making occurs under conditions of uncertainty.* □

Attribute	Computer A	Computer B
Price	R11 500	R12 000
Size	17 inch	18 inch
Life expectancy	3 years	2 years

Table 3.5: *Characteristics of two different laptop computers.*

Decision making under conditions of certainty may be modelled by *value functions* (also known as *ordinal utility functions*) [14, 62]. Value functions rank the sure outcomes of a decision in a way that remains consistent with the preferences of the decision maker [14].

Decision making under conditions of uncertainty may be modelled by *cardinal utility functions* [14]. Cardinal utility functions employ ranking procedures as do value functions. The difference is that in the cardinal case, lotteries are rank-ordered instead of sure outcomes, whilst maintaining a level of consistency with the risk attitude of the decision maker. The importance of distinguishing between certain and uncertain conditions lies in the methods incorporated to assess a decision maker's utility function. Some methods are more appropriate if used under conditions of certainty, uncertainty or both [14].

3.3.2 Monotonicity

Keeney and Raiffa [33] state that when it is relevant to represent the outcomes of a decision by means of monetary asset position, a large portion of decision makers would prefer a larger amount to a smaller amount, in which case

$$x_1 > x_2 \Leftrightarrow u(x_1) > u(x_2), \tag{3.7}$$

for two outcomes x_1 and x_2, where u represents a utility function. This implies that the utility function is *monotonically increasing*. In the case where a smaller amount is preferred to a larger amount (*e.g.* the response time to calls for a fire brigade) it holds that

$$x_1 > x_2 \Leftrightarrow u(x_2) > u(x_1), \tag{3.8}$$

which means that the utility function is *monotonically decreasing*. A utility function may also be transformed from a monotonically increasing to a monotonically decreasing function by changing the effectiveness measure, also known as the attribute. For further details on the transformation of the effectiveness measure, the interested reader is referred to Keeney and Raiffa [33].

In addition to the possibility of a utility function being either monotonically increasing or monotonically decreasing, it may also be *nonmonotonic*. A nonmonotonic utility function occurs where an outcome with a higher value is preferred up to some point, whereafter a smaller value is preferred to a higher value. An example of such a nonmonotonic utility function is found in medical terms in the level of blood pressure of an individual [33]. There exists an acceptable level of blood pressure for a specific individual. A low level of blood pressure is undesirable and higher values are preferred up to the acceptable level. On the contrary, high levels of blood pressure are also undesirable and lower values are preferred down to the acceptable level. The utility function for the blood pressure of an individual is therefore a nonmonotonic utility function. An example of a nonmonotonic utility function may be seen in Figure 3.6. In this figure the utility function is monotonically increasing up to point, A, whereafter it is monotonically decreasing.

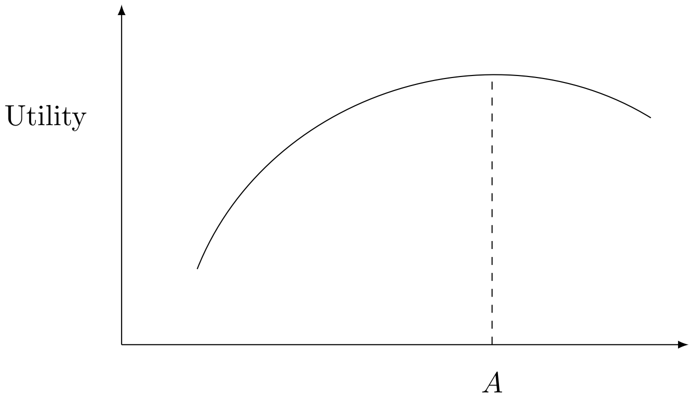

FIGURE 3.6: *An example of a nonmonotonic function.*

3.3.3 Attitudes towards risk

Another qualitative characteristic of utility functions, is the attitude towards risk that a decision maker exhibits. Any real-life decision making situation contains a form of risk — whether it is the choice of stock market to invest in, or a choice among a few different job offerings — it is only the degree of risk that varies.

Example 8 *Suppose a decision maker may choose to play one of two different games. In the first game a fair coin is flipped and the decision maker has the opportunity to either receive* $R\,200$ *or lose* $R\,10$*. In the second game, a fair coin is also flipped, but the decision maker can either receive* $R\,4\,000$ *or lose* $R\,2\,500$*. These games are represented by the decision tree in Figure 3.7. The expected values for each game may be calculated as* $R\,200(0.5) - R\,10(0.5) = R\,95$ *for Game 1, and* $R\,4\,000(0.5) - R\,2\,500(0.5) = R\,750$ *for Game 2.* □

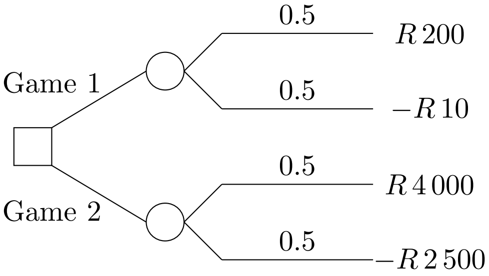

FIGURE 3.7: *A decision tree for a choice between two games having different outcomes.*

If the decision maker in Example 8 considers only the expected values of Game 1 and Game 2 to base his decision on, it is very likely that he will choose the game with the highest expected value (that is, Game 2). When a decision maker considers only the expected value criterion (*i.e.* the average or expected payoff of the alternatives) to base his decision on, it implies that the decision maker does not take into account any risk involved in making a decision [14]. However, most individuals would, in fact, rather prefer to play Game 1 instead of Game 2, because it involves less risk.

On the other hand, there are decision makers who would actually choose to play Game 2 instead of Game 1, regardless of the risk involved. Their choice is not based on the expected value, but because they exhibit a more positive feeling towards risk. This demonstrates that decision makers exhibit different attitudes towards risk. Those decision makers who are sensitive or afraid of risk, are said to be *risk-averse* decision makers and they would typically try and avoid any form of risk. Risk averse decision makers are those who would prefer to play Game 1 in Example 8. Decision makers who are more attracted to riskier choices are said to be *risk-seekers* [14]. Risk seekers are not afraid to take any form of risk involved in a decision, and would much rather prefer riskier choices. These decision makers would prefer to play Game 2 in Example 8. Decision makers who are neither risk-averse nor risk-seeking, are said to be *risk-neutral* [14]. Decision makers who present a risk-neutral behaviour do not care about risk, — all risk aspects involved in their choice of preference are ignored when facing a decision. Since there is no risk involved in their decision, the expected value criterion is an acceptable measure for choosing among alternatives for a risk-neutral decision maker [14].

It was briefly mentioned in the beginning of §3.3 that an individual's utility function encapsulates the attitude towards risk that he/she exhibits. A decision maker's attitude towards risk may be derived from the form of the graph of his/her utility function.

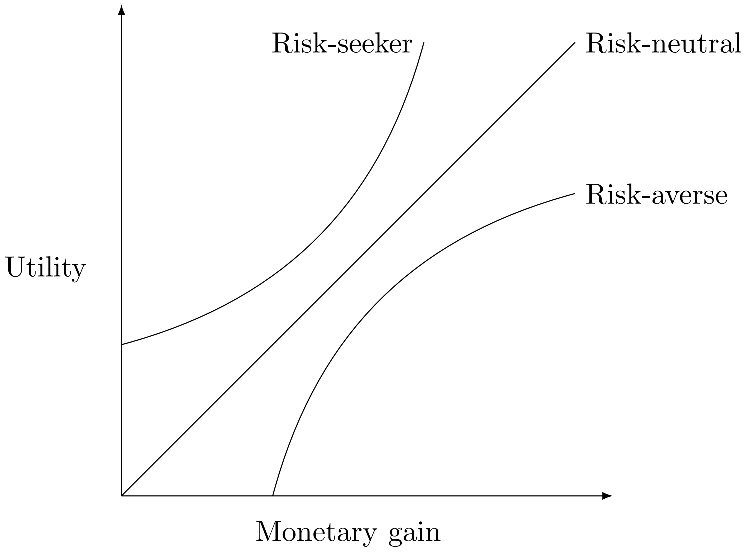

FIGURE 3.8: *The three different attitudes towards risk presented on a two-dimensional utility graph for a monetary gain.*

A decision maker is said to exhibit a risk averse attitude if and only if his utility function has a concave form. The converse of this definition implies that a decision maker exhibits a risk seeking attitude, if and only if his utility function has a convex form. The utility function of a risk neutral decision maker is simply a straight line [14, 33, 62]. The trio of graphs, each representing a different individual's utility function, may be seen graphically in Figure 3.8.

A procedure which may be used to determine a decision maker's attitude towards risk, is to present him with a choice between a lottery having equal outcomes of x_1 and x_2, and a sure outcome consisting of the expected value of the lottery. If the lottery is presented to the decision maker for a number of times with various values of x_1 and x_2, and the decision maker continuously prefers the sure outcome to the lottery, he is considered to be risk averse. If the decision maker continuously prefers the lottery to the sure outcome, he is considered to be risk seeking [33]. And finally, if the decision maker is continuously indifferent between the lottery and the sure outcome, he is considered to be risk neutral.

3.3.4 The certainty equivalent and risk premium

Two further qualitative characteristics of utility functions are the *certainty equivalent* (CE) and *risk premium* (RP). The CE for a given lottery L may be defined as that specific amount x, for which a decision maker is indifferent with respect to choosing between L and the sure amount x [33, 62]. Consider Game 2 in Example 8 again. Suppose the decision maker is indifferent with respect to choosing between a sure amount of $R\,500$ and playing Game 2. The indifference amount of $R\,500$, is then considered to be the decision maker's CE. This implies that if the decision maker is offered an amount which is anything less than $R\,500$, the decision maker will choose to play Game 2, and if the decision maker is offered an amount which is anything more than $R\,500$, the decision maker will choose to take the sure amount.

A very useful property of the CE is that it may be used to determine the risk attitude of a decision maker. Keeney and Raiffa [33] state that a decision maker exhibits a risk averse attitude for a monotonically increasing utility function, if and only if the CE of any nondegenerate[7] lottery, is less than the expected value of that lottery. The converse of this statement implies that a decision maker exhibits a risk seeking attitude for a monotonically increasing utility function, if and only if the CE of any nondegenerate lottery is greater than the expected value of that lottery.

Alternatives may be ranked according to their CEs, which is similar to ranking alternatives according to their expected utilities [14]. Two alternatives having the same CEs, imply that they have the same expected utility values. Hence, if two alternatives have the same CEs, the decision maker should be indifferent with respect to choosing between these alternatives.

Keeney and Raiffa [33] define the RP as "the amount of the attribute that a decision maker is willing to give up from the average, to avoid the risk involved in the relative lottery." The RP may also be seen as the premium that the decision maker is willing to pay to avoid risk [14]. The RP may be calculated by taking the difference between the expected value of a lottery and the CE of that lottery [14, 62].

Example 9 (Continuation of Example 8) *Consider the Game 2 in Example 8. Suppose the CE of the decision maker for Game 2 is* $R\,500$*. His RP may be calculated as* $R\,750 - R\,500 = R\,250$*. This implies that the decision maker is willing to give up* $R\,250$ *in expected value, to avoid the risk involved in playing Game 2.* □

The RP is related to the risk attitude exhibited by the decision maker. Winston [62], and Keeney and Raiffa [33] state, that for any nondegenerate L, a decision maker's attitude towards risk is risk-averse if and only if the RP of L is greater than 0, is risk seeking if and only if the RP of L is less than 0 and risk-neutral if and only if the RP of L is exactly equal to 0.

The notions of the CE and RP of a given lottery may be explained by means of the graph in Figure 3.9. It may be seen from Figure 3.9 that the utility of the CE is equal to the expected utility (EU) value of the lottery, implying that the decision maker should be indifferent with respect to choosing between these two values [14]. To find the CE of the lottery, a horisontal line may be drawn from the EU value until the utility function is reached. Dropping a vertical line from that point to the horisontal axis will yield the CE value. The RP is represented by the distance between the CE and EV as indicated in Figure 3.9. The horisontal line from the EU reaches the utility curve before it reaches the vertical line of the EV, and because the EV is greater than the CE, this implies that the RP for a risk-averse individual should take on a positive value [14]. It can easily be shown that for a risk-seeking individual the converse is true. In this case the RP takes on a negative value, implying that the individual would have to be paid to decline an opportunity.

3.3.5 Constant, decreasing and increasing risk attitudes

The concept of risk attitudes may be expanded further, by investigating how decision makers react towards risk if their monetary wealth position changes. The reason for such an expansion is to determine whether decision makers would rather prefer a riskier alternative if they are in a wealthier position, or whether their decision would not be influenced by their wealth positions [33]. These are very common behaviour amongst decision makers. They may feel, that since they are in a wealthier position, they can afford to take the riskier option.

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[7] A nondegenerate lottery may be defined as a lottery where no one of the outcomes has a probability value of one of occurring [33].

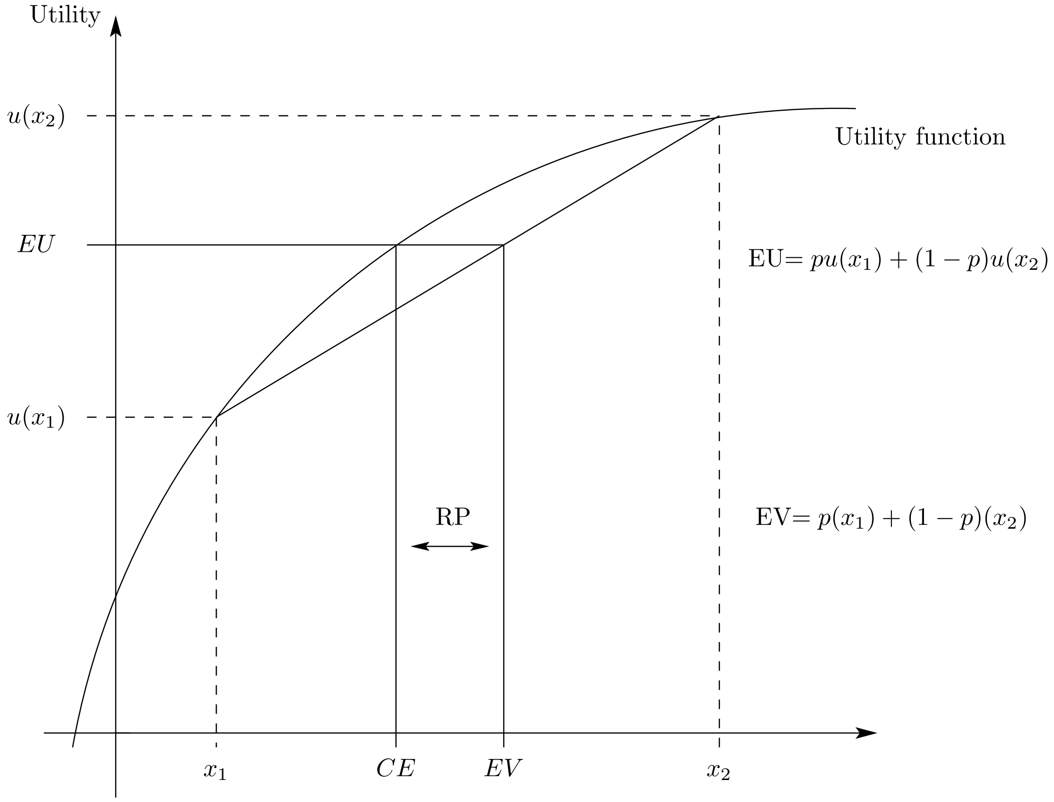

FIGURE 3.9: *The risk premium presented graphically for a risk-averse decision maker [14].*

A reasonable measure is to observe how the value of the RP for a lottery is affected when a constant amount, k is added to the payoffs in the specific lottery [14]. Consider risk averse individuals first. For a decision maker to be consistently risk averse, the RP should remain constant for any value of k [14]. This means that the wealth position of the decision maker does not influence his choice.

For a decision maker to be decreasingly risk averse, the RP of any lottery should decrease as the value of k increases [33]. This means that a decision maker becomes less risk averse as his wealth position increases. For a decision maker to be increasingly risk averse, the RP of any lottery should increase as the value of k increases. This means that the decision maker becomes more risk averse as his wealth position increases.

Decision makers may also exhibit consistently risk seeking, decreasingly risk seeking or increasingly risk seeking behaviour. Consistently risk seeking behaviour is similar to that of consistently risk averse behaviour in the sense that the RP should remain constant for all the values of k. The difference is that the decision maker is risk seeking. For a decision maker to be decreasingly risk seeking, the RP for any lottery should increase as the value of k increases. Finally, for a decision maker to be increasingly risk seeking, the RP for any lottery should decrease as the value of k increases [33].

As will later be explained, certain specific types of utility functions may or may not be appropriate to use when certain risk attitudes are present. Keeney and Raiffa [33] show, for instance, that an exponential utility function is not suitable for a decreasingly risk averse individual. The benefit of being acquainted with these types of risk attitudes of decision makers, is that it may prove useful in choosing an appropriate utility function corresponding to the risk attitude of the decision maker.

3.3.6 Assessing utility values

The assessment of utility values may be seen as a topic of subjective judgement and may be assessed in a number of ways [14]. Techniques which follow subjective procedures, include the *certainty equivalent approach* and the *probability approach*. However, if certain characteristics of the risk attitude of a decision maker are known, utility values may be assessed by using pre-determined mathematical functions [14]. Pre-determined utility functions include the *exponential utility function* and *logarithmic utility function* [33]. One advantage of using a pre-determined function is that the often tedious process of assessments involved in a subjective approach is circumvented [14]. Although assuming a pre-determined functional form for a utility function seems very appealing, it also has its constraints and limitations, such as the limitation placed on the usage of certain utility functions due to the risk attitude exhibited by the decision maker.

When assessing utility values subjectively, the first step is to specify the range of values for the possible outcomes. This helps to narrow down the number of assessments, since values falling outside of the specified range are omitted from the assessment procedure. Specifying a range, may also prevent any confusion on the decision maker's side, since values falling outside the range may have no valuable meaning to the decision maker [33].

The next step is to assign utility values to the best and worst outcomes in the specified range of outcomes denoted by x^+ and x^-, respectively. These extremal outcome values are assigned utility values of 1 and 0, respectively [14, 62]. Utility values of the other possible outcomes are contained in the range of 0 and 1, and may be calculated by using any one of a number of subjective methods or pre-specified utility functional forms.

The certainty equivalent technique

A popular, subjective technique for assessing utility values under conditions of certainty or uncertainty is the CE technique. As the name suggests, the CE technique repeatedly utilises the notion of a CE (see §3.3.4) in order to construct a utility function [14].

The technique involves providing the decision maker with a choice between a lottery, having two outcomes, x^- and x^+, with equal probability as shown in Figure 3.10 [14]. The decision maker is asked to provide his CE for this lottery. Because the decision maker is indifferent with respect to choosing between the lottery and the CE, the expected value of the lottery should be equal to the $u(CE)$ [14]. The utility values $u(x^+)$ and $u(x^-)$ are known and hence, the utility value of the CE may be calculated as

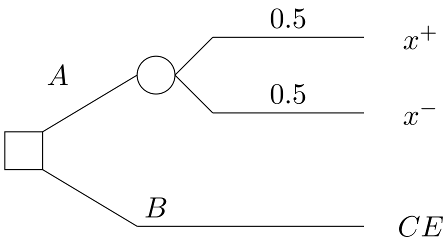

FIGURE 3.10: *A lottery for the assessment of a utility function using the CE technique.*

$$u(CE) = 0.5u(x^+) + 0.5u(x^-). \tag{3.9}$$

A forth utility value may be assessed by providing the decision maker with a lottery containing different payoff values (The utility values of these payoffs should be known). This procedure may be repeated until a reasonable number of utility values are assessed, thereby tracing out the decision maker's utility function.

Example 10 *Suppose that a decision maker is presented with the lottery A in Figure 3.11. The decision maker is asked to provide a CE for the lottery, and suppose that after careful consideration he claims a CE of* $R\,1\,900$. *Since* $R\,5\,000$ *and* $R\,500$ *represent the best and worst outcomes respectively, they are assigned utility values of one and zero respectively. By using Axiom 1(b), the utility value for* $R\,1\,900$ *may be calculated as*

$$\begin{aligned} u(1\,900) &= 0.5\,u(500) + 0.5\,u(5\,000) \\ &= 0.5(0) + 0.5(1) \\ &= 0.5. \end{aligned}$$

Another utility value may by calculated by using a different lottery. In this lottery, the decision maker has to provide a CE for a lottery involving a fair chance in winning either $R\,5\,000$ *or* $R\,1\,900$ *(because the utility values of these outcomes are known). Suppose he claims a CE of* $R\,3\,000$ *for the lottery. The utility value of* $R\,3\,000$ *may be calculated as*

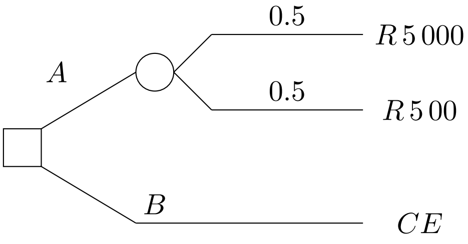

FIGURE 3.11: *An example of the lottery used in the assessment of a utility function using the CE approach in Example 10.*

$$\begin{aligned} u(3\,000) &= 0.5\,u(5\,000) + 0.5\,u(1\,900) \\ &= 0.5(1) + 0.5(0.5) \\ &= 0.75. \end{aligned}$$

Continuing this process, a number of utility values corresponding to different outcomes may be determined. Suppose a utility value of 0.25 *is assessed for* $R\,1\,000$ *and* 0.875 *for* $R\,4\,000$. *Presenting the utility values on a two dimensional graph, with the monetary gain represented by the horisontal axis, and the utility values on the vertical axis, yields the utility function represented in Figure 3.12.* ☐

When the CE technique is applied, the first assessed value corresponding to $x_{0.5}$, has utility value $u(x_{0.5}) = 0.5$, because $u(x^+) = 1$ and $u(x^-) = 0$. It can also be shown that the second utility value assessed during the process described above (associated with an outcome $x_{0.75}$) and the third utility value assessed during the process described above (associated with an outcome $x_{0.25}$) are $u(x_{0.75}) = 0.75$ and $u(x_{0.25}) = 0.25$, respectively [33].

These values may be used to verify the consistency of the decision maker. The decision maker is asked to supply his CE for the lottery in Figure 3.13. Since $u(x_{0.5}) = 0.5$ and

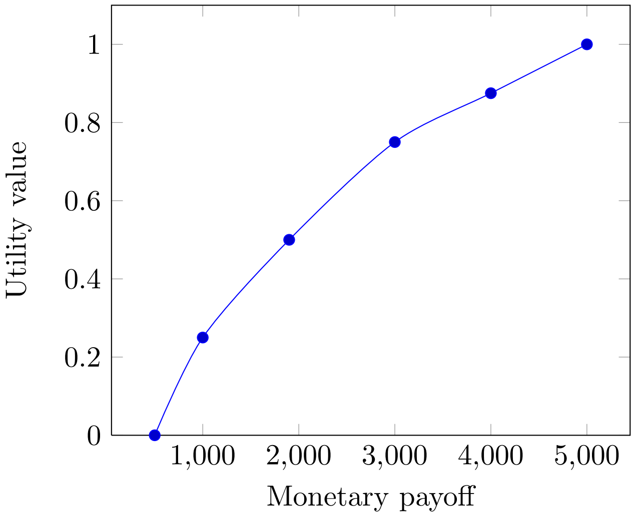

FIGURE 3.12: *The utility function assessed by means of the CE approach for Example 10.*

$$u(CE) = 0.5u(x_{0.25}) + 0.5u(x_{0.75}) = 0.5,$$

the conclusion may be drawn that, in order for a decision maker to be consistent, his CE should be equal to $x_{0.5}$ [33]. Furthermore these values may also be used to confirm whether a utility function is risk averse or risk seeking. Recall from §3.3.4 that an increasing utility function is risk averse if the assessed CE values are less than the expected values of their respective lotteries. If the CE values are larger that the expected values of their respective lotteries, the utility function is risk seeking. One disadvantage of this technique is that assessing CEs may be a tedious process.

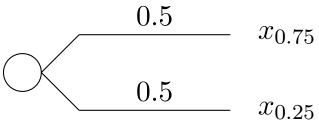

FIGURE 3.13: *A lottery to verify the consistency of a decision maker using CEs.*

The probability equivalent approach

Another technique for utility assessment is called the *Probability Equivalent* (PE) technique, and is suitable for use in decision making under conditions of certainty or uncertainty. The PE technique is similar to the CE approach in the sense that lotteries are used to assess utility values. In the PE technique, the decision maker is presented with a lottery containing the outcomes x^+ and x^-, with probabilities p and $(1-p)$, respectively, and a sure amount y. The value of y represents the amount for which a utility value is sought. This situation is shown graphically in Figure 3.14. The decision maker is asked to identify the probability value p which renders him indifferent with respect to a choice between the lottery and the value of y [14]. This indifference implies that the expected utility of the lottery must be

$$\begin{aligned} u(y) &= p\,u(x^{+}) + (1-p)\,u(x^{-}) \\ &= p(1) + (1-p)(0) \\ &= p. \end{aligned}$$

The value of p which makes him indifferent, is considered his utility value for that particular value of y [14]. This procedure is repeated until a reasonable number of utility values have been assessed, thus tracing out the decision maker's utility function.

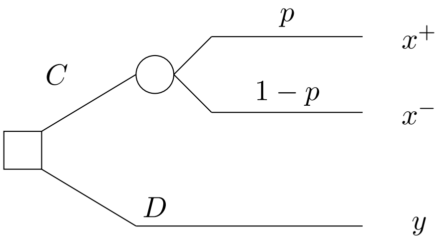

FIGURE 3.14: *An example of the lottery used in the assessment of a utility function using the PE approach.*

Example 11 (Continuation of Example 10) *Consider the same scenario as in Example 10, but with the lottery represented in Figure 3.15. Suppose the decision maker wishes to assess his utility value for a* $R\,1\,500$ *payoff. The decision maker is asked to provide the value of* p *which makes him indifferent with respect to a choice between the lottery and the sure outcome of* $R\,1\,500$*. By using Axiom 2(a), he concludes that a value of* 0.4 *makes him indifferent. Hence,* $u(1\,500) = 0.4$*. By repeating the process, but with different values for the sure outcome, a number of utility values may be assessed for different payoff values. The assessed utility values in conjunction with the payoff values, may then be presented on a graph similar to the one in Figure 3.12, to trace out the utility function.* ☐

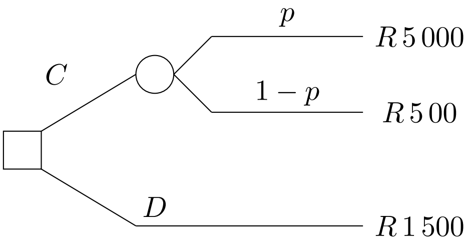

FIGURE 3.15: *The lottery used in the assessment of a utility function using the PE approach in Example 11.*

A very important matter in modelling the preferences of any decision maker, is ensuring that the decision maker remains consistent during the assessment process. Keeney and Raiffa [33] describe one such method when the PE approach is used. Suppose x_1, x_2 and x_3 represent a series, where the decision maker prefers x_2 to x_1 and x_3 to x_2. Furthermore, suppose that x_2 is the certain amount which makes the decision maker indifferent with respect to a choice between the lottery having outcomes x_3 and x_1 with probabilities p and $(1-p)$, respectively. In order for a decision maker to be consistent, the value of p must be such that $u(x_2) = pu(x_3)+(1-p)u(x_1)$, or simplified,

$$p = \frac{u(x_2) - u(x_1)}{u(x_3) - u(x_1)}.$$

It is finally noted that if a decision involves only a few outcomes (possibly up to approximately 50), then the PE approach may be sufficient to assess utility functions, but for larger problems alternative approaches are recommended [33].

An exponential utility function

Pre-determined utility functions exist in the literature, which may be selected to assess utility values for different outcomes. A pre-determined utility function is usually chosen based on the qualitative characteristics displayed by the decision maker, such as a decision maker's attitude towards risk. One pre-determined function which may be used if a decision maker exhibits a form of risk aversion is an *exponential utility function* [14, 33]. An exponential utility function takes the form

$$u(x) = 1 - e^{-x/R}, \tag{3.10}$$

where e is the natural number (*i.e.* the base of the natural logarithm, or approximately 2.71828) and R represents the level of risk aversion displayed by the decision maker, known as the *risk tolerance* [14]. The concern in using an exponential utility function, is to determine the value of R which best represents the level of risk aversion of the decision maker. A small value of R will yield a more curved function, implying a more risk averse decision maker. In contrast, a larger value of R will yield a flatter curve, implying a less risk averse decision maker [14].

The value of R may be evaluated by using a lottery similar to the one in Figure 3.16. The decision maker has a choice between playing the lottery, having an equal chance of receiving an amount of x or losing an amount of $\frac{x}{2}$, or not playing the lottery at all. The largest value of x for which the decision maker is willing to play the lottery is considered his risk tolerance [62].

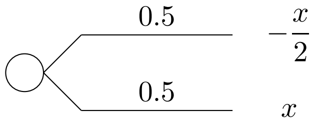

FIGURE 3.16: *An example of a lottery to calculate the risk tolerance of a decision maker in the case of an exponential utility function.*

A significant advantage of using an exponential utility function, is that it is relatively easy to assess the parameter of the function, since it only involves assessing an appropriate value of R. A disadvantage of using the exponential utility function, is that it is limited to decision makers exhibiting risk averse behaviour.

3.3.7 Guidelines for assessing utility functions with one attribute

Since various techniques are available in the literature to assess utility functions, there is no specific set of fixed rules which may be followed during the assessment of a utility function. Keeney and Raiffa [33] state that the reason for this is "because the assessment of utility functions is as much of an art as it is a science." Even though a fixed set of rules does not exist for the process of utility assessment, a set of useful guidelines, proposed by Keeney and Raiffa [33], may be adopted when the preferences of a decision maker is assessed.

It is usually assumed that both the analyst and the decision maker have a clear understanding about the problem at hand, and that they have established the important objectives and attributes, and have structured them accordingly. It is also assumed that the decision maker has been briefed about decision making analysis, and that he understands the reasons behind the evaluation of his preferences [33]. It is important to prepare the decision maker for the assessment procedure. A pre-assessment process involving motivating the decision maker to think hard about his feelings towards the various outcomes and making sure that both the analyst and the decision maker understands that the aim is to assess the preferences of the decision maker [33].

A first step in the utility assessment procedure should be to establish whether the utility function is monotonic or not [33]. This may be achieved by using a line segment of the form in Figure 3.17. To investigate whether the utility function is monotonic or not, the decision maker should be asked whether he prefers outcomes A or B. Suppose he chooses outcome B above A. He should then be asked whether he prefers outcome D or C. Suppose he chooses outcome D above C. A few more of these preference questions may be asked and finally the following general question should be put to the decision maker: “If outcome x_i is greater than x_j, will outcome x_i always be preferred to outcome x_j?” An affirmative answer to this question implies that the utility function is monotonically increasing. If the decision maker indicates that he will always prefer outcome x_i to outcome x_j when x_i is smaller than x_j, the utility function is monotonically decreasing. If no conclusion can be made about the monotonicity of the utility function, this should be explained to the decision maker and he should be given the opportunity to reevaluate his preferences before nonmonotinicity is assumed. This may help to educate the decision maker, rather than to leave him biased [33].

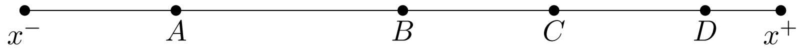

Figure 3.17: *A representation of the outcomes of a decision to assist in the verification of monotonicity.*

The next step in the utility assessment process should be to determine the risk attitude of the decision maker, which will determine whether the utility function is risk averse, risk neutral or risk seeking. The result of this step may be verified by presenting the decision maker with the lottery in Figure 3.18. This lottery presents the decision maker with an option between a sure outcome x and a lottery involving an even chance of gaining amounts of $x - k$ or $x + k$. The decision maker should be asked for his preference between these two outcomes for arbitrary values of x and k. This procedure should be repeated for several different values of x or k, in each case keeping one value fixed and then varying the other, and *vice versa*. The values should include the entire range of possible values of the outcomes.

Alternatively, Keeney and Raiffa [33] suggest that the decision maker should be asked: “If the values of x and k are varied for different amounts over the range of the outcomes, would the sure outcome x always be preferred to the lottery?” An affirmative answer , would be sufficient to assume that the decision maker is risk averse. If x is preferred consistently, it may be safe to assume that the decision maker is risk averse. Finally, if an indifference occurs consistently, a risk neutral attitude may be assumed. Finally if the lottery is preferred consistently, a risk seeking attitude may be assumed [33].

The next step in the utility assessment procedure involves verifying whether the utility function is consistently, decreasingly or increasingly risk averse. The RP may be used for this purpose.

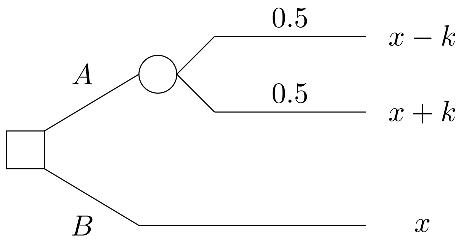

FIGURE 3.18: *A lottery used in the verification of the risk attitude incorporated in a utility function.*

A monotonically increasing utility function is consistently (decreasingly, increasingly, respectively) risk averse if the RP remains constant (decreases, increases, respectively) for the various outcomes [33]. In order to compute the RP, it is necessary to calculate the CE for the lottery with possible outcomes as shown in Figure 3.19, for a number of values i, where x_i increases as the value of i increases.

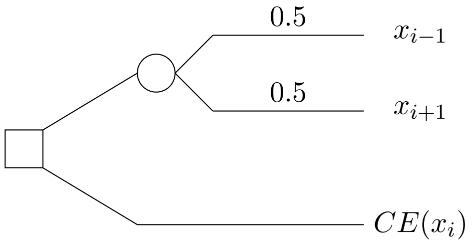

FIGURE 3.19: *A lottery used to determine constant, decreasing or increasing risk aversion.*

An alternative approach may also be used to calculate the RP of a decision maker for the lottery A in Figure 3.18, for specific values of x and k. In this case the decision maker should be asked what the impact may be on this RP if the value of x is increased, while maintaining the fixed value k. If the RP increases as the value of x increases, then it is safe to assume that the utility function is increasingly risk averse. On the other hand, if the RP decreases, it may be assumed that the utility function is decreasingly risk averse [33].

Quantitative values for the utility function may next be assessed. This step is performed by means of the CE approach or PE approach discussed in §3.3.6. The decision maker may then either find a utility function directly (*e.g.* by fitting a curve through the utility values), taking into account that the function should comply with the previous qualitative and quantitative assessments, or the decision maker may use a pre-determined utility functional structure displaying the previous qualitative characteristics and determine relevant parameters in order to obtain an appropriate utility function [33].

Inconsistency of a decision maker during the assessment process should be considered a fatal error, as inconsistencies may jeopordise the outcome of the results. It is thus important to include appropriate measures throughout the entire assessment procedure to maintain a high level of consistency. Whenever any inconsistencies occur, the reasons behind such inconsistencies should be explained to the decision maker which, in turn, should motivate him to rethink his preferences for certain outcomes [33]. This may lead to a possible repetition of certain areas of the assessment.

3.4 Utility functions in multiobjective decision space

In the previous section, a number of ways of assessing single objective utility functions was discussed. Assessing utility functions for decisions in one dimension seems rather straight forward, since the procedures described in §3.3 has to be carried out for only one objective, for each of the alternatives. The question arising, is what happens in the case where multiple objectives are considered simultaneously in a decision problem? In such a case, one utility function is sought which represents all of the objectives and their respective attributes simultaneously.

A number of utility functions for assessing multiple conflicting objectives exist in the literature, and the most popular ones are the *additive utility function*, the *multilinear utility function* and the *multiplicative utility function*. These utility functions simply consist of a scaled combination of the individual utility functions assessed for each objective by means of addition and/or multiplication. This allows for the multiobjective utility function to be broken down into several smaller utility functions (a notion called *separability*). These individual utility functions may then be assessed separately, by using any one of the techniques discussed in §3.3, thus simplifying the assessment of the combined multiobjective utility function significantly [14, 33].

An important phenomenon in multiobjective utility functions, is the interaction between the attributes. In order to utilise any of the multiobjective utility functions, certain conditions have to be satisfied regarding the interaction between the respective attributes [33]. These conditions are known as the *utility independence conditions*, and three of these conditions that the author is aware of are described below. For the sake of simplicity, the case where only two objectives are considered is discussed.

Preferential independence

The first independence condition is called *preferential independence*. Suppose there are two objectives, with their respective attributes denoted by X and Y. Attribute X is considered preferentially independent of attribute Y if preferences for specific outcomes of X do not depend on the level of outcome of the attribute Y [14, 33]. If attribute Y is also preferentially independent of attribute X, the attributes are said to be *mutually preferentially independent*. Mutual preferential independence is considered a necessary condition for a utility function to have the characteristic of separability discussed earlier [14]. To illustrate the concept of preferential independence, consider the following example.

Example 12 *Suppose a decision maker is presented with a choice between different projects, where the cost in Rand value of a project and the time of completion in days of a project are considered as the attributes. Assume that for a specific project the decision maker prefers a time of completion of 10 days to 20 days if the cost of that specific project is* $R\,200$. *If the decision maker also prefers the same time of completion when the cost is* $R\,400$, *then the time of completion attribute is preferentially independent of the cost attribute. This means that the decision maker will always prefer a shorter time of completion regardless of what the cost is.*

Consider the same scenario where the cost is now fixed at $R\,200$, *and the decision maker prefers a 10 day time of completion instead of a 20 day completion time. If the decision maker also prefers a 20 day time of completion to a 30 day completion, then cost is preferentially independent of completion time. Hence, the conclusion can be made that completion time and cost are mutually preferentially independent [14].* $\square$

Preferential independence may easily be verified by presenting the decision maker with a sequence of paired comparisons between different values of attribute X, whilst keeping attribute Y fixed at a specific level. In such a sequence of comparisons the decision maker is asked to give his preference with respect to the outcomes of attributes X in each pair. The procedure is repeated with the same set of outcomes for attribute X, but with different fixed values of attribute Y. If the decision maker prefers the exact same set of outcomes of attribute X at each iteration, it is safe to assume attribute X is preferentially independent of attribute Y [14]. For mutual preferential independence the process should be repeated, but by keeping attribute X fixed at different levels and asking the decision maker for preferences with respect to different values of attribute Y. If attribute Y is also preferentially independent of attribute X, it is safe to assume that the attributes are mutually preferentially independent.

Since preferential independence is concerned with attributes having sure outcomes, it is considered a suitable condition for decision making under conditions of certainty. However, if conditions of uncertainty are present, preferential independence are not quite strong enough; in such cases a stronger independence condition is required [14].

Utility independence

A stronger independence condition than that of preferential independence is the well-known condition of *utility independence*. An attribute X is considered to be utility independent of attribute Y, if preferences for uncertain choices featuring various levels of outcomes of attribute X do not depend on the level of outcome of attribute Y [14]. If it can be shown that attribute Y is also utility independent of attribute X, then the objective corresponding to attributes X and Y are said to be *mutually utility independent*.

Example 13 (Continuation of Example 12) *Consider the attributes of Example 12 again. Assume once again that the cost attribute X is fixed at $R\,200$. Suppose that the decision maker's CE for having a 50% chance of $Y = 10$ and a 50% chance of having $Y = 20$, is assessed at a value of 15. If the decision maker's CE for the probability-based outcomes remains unchanged for various fixed values of X, then Y is considered utility independent of X. If it can be ascertained that X is also utility independent of Y, then the conclusion can be made that X and Y are mutually utility independent.* ☐

When a decision maker's preferences for attributes X and Y exhibit mutual utility independence, a suitable utility function is the *multilinear utility function*

$$u(x,y) = k_X u_X(x) + k_Y u_Y(y) + (1 - k_X - k_Y)u_X(x)u_Y(y), \tag{3.11}$$

where

$$u_X(x) = \text{utility function on } X \text{ scaled so that } u_X(x^-) = 0 \text{ and } u_X(x^+) = 1, \tag{3.12}$$
$$u_Y(y) = \text{utility function on } Y \text{ scaled so that } u_Y(y^-) = 0 \text{ and } u_Y(y^+) = 1, \tag{3.13}$$
$$k_X = u(x^+, y^-), \tag{3.14}$$
$$k_Y = u(x^-, y^+), \tag{3.15}$$

where k_X and k_Y represent the scaling constants for attributes X and Y, respectively [14].

The individual utility functions $u_X(x)$ and $u_Y(y)$ are considered to be *conditional utility functions*, implying that each should be evaluated whilst keeping the outcome of the other fixed at a specific value [14]. The last term, $(1 - k_X - k_Y)u_X(x)u_Y(y)$, in the multilinear utility function allows for the modelling of interaction between the attributes. Keeney and Raiffa [33] describes how the coefficient $(1 - k_X - k_Y)$ may be used to determine whether attributes X and Y are complements of or substitutes for each other. High preference values of attributes X and Y result in high valued conditional utility functions, which will result in a higher overall utility value $u(x, y)$ if the coefficient $(1 - k_X - k_Y)$ is positive. Hence, if the coefficient is positive, attributes X and Y are said to complement each other [14, 33].

However, if the conditional utility values are large and the coefficient $(1 - k_X - k_Y)$ is negative, this will result in a lower overall utility value $u(x, y)$, implying that attributes X and Y work against each other [14].

Finally, if one of the conditional utility values are large and the other small, the effect that a negative coefficient $(1 - k_X - k_Y)$ may have on the overall utility value may not be that large as in the case where both conditional utility values are large. This implies that the attributes X and Y are substitutes.

It can easily be shown that the scaling constants k_X and k_Y are equal to $u(x^+, y^-)$ and $u(x^-, y^+)$, respectively, by substituting the individual utilities from (3.12) and (3.13) into (3.11). The interested reader is referred to Clemen and Reilly [14] for a proof of this result.

The notion of utility independence may seem synonymous with the conditions of preferential independence. However, note that in the case of utility independence, conditions of uncertainty are included in the assessments. Utility independence may easily be verified by presenting the decision maker with the lottery shown in Figure 3.20. The level of attribute Y is fixed at a specific outcome y_k and the decision maker is asked to assess CEs for different values of attribute X. The level of attribute Y is then fixed at a different level of outcome, and the decision maker is asked to assess CEs for the same set of outcomes of attribute X. The process is repeated while each time keeping attribute Y fixed at different levels, covering the entire range of possible values of attribute Y. If each set of assessed CEs are exactly equal to each other, the conclusion may be drawn that attribute X is utility independent of attribute Y.

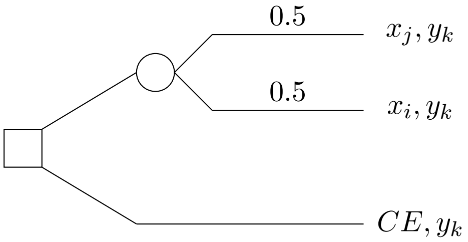

FIGURE 3.20: *A lottery used to verify whether attribute X is utility independent of attribute Y.*

To verify mutual utility independence, the procedure is repeated using a lottery involving different outcomes of attribute Y, whilst repeatedly keeping attribute X fixed at different levels. If attribute Y is also utility independent of attribute X, the attributes are mutually utility independent, and the multilinear utility function may be used to present the preferences of the decision maker.

The question is how many assessments should be carried out before an independence condition may be assumed? Having too few assessments, may compromise the final results and having too many assessments may complicate the procedure unnecessarily. Keeney and Raiffa [33] state that, in practice, a sufficient number of conditions to consider is approximately four spanning the entire range of possible values of attributes X and Y.

Additive independence

The final independence condition is the so-called *additive independence* condition, which is considered the strongest form of independence amongst the independence conditions. When additive independence is present, it can be said that changes in lotteries involving attribute X do not affect preferences for lotteries involving attribute Y [14]. The difference between assessing additive independence and utility independence is that with additive independence uncertain outcomes over both attributes are assessed, whereas in utility independence assessment one of the attributes consists of a sure outcome.

When a decision maker's preferences for the attributes exhibit additive independence, the utility function may be taken as the *additive utility function*

$$u(x, y) = k_X u_X(x) + k_Y u_Y(y), \tag{3.16}$$

where $u_X(x)$ and $u_Y(y)$ are the respective individual utility functions for attributes X and Y, and k_X and k_Y represent scaling constants associated with attributes X and Y, respectively. The values of the scaling constants should be greater than zero and they should sum to one [14]. The best and worst outcomes are again assigned values of one and zero, respectively.

One way of verifying whether additive independence holds, is to first verify whether mutual utility independence holds for the attributes, since mutual utility independence is a necessary condition for additive independence [33]. If attributes X and Y are mutually utility independent, additive independence may be verified by presenting the decision maker with the two lotteries shown in Figure 3.21.

FIGURE 3.21: *Two lotteries to verify the condition of additive independence for attributes X and Y.*

Lottery L_1 represents two possible outcomes which may occur with equal chance, where the first outcome involves the best outcomes for both attributes and the second outcome involves the worst outcomes for both attributes. Lottery L_2 also involves two possible outcomes which may occur with equal chance, but the first outcome involves the best and worst outcomes of attributes X and Y, respectively, while the second outcome involves the worst and best outcomes of attributes X and Y, respectively. If the decision maker has a clear preference for either of Lotteries L_1 or L_2, then additive independence cannot hold [14, 62]. However, if the decision maker is indifferent between Lotteries L_1 and L_2, then it is safe to assume additive independence between attributes X and Y, and the additive utility function may be used to present the preferences of the decision maker.

Failure to comply with the independence conditions

Although it seems relatively easy to verify the independence conditions described above between attributes, it is possible that these conditions cannot be verified for certain attributes. The implication of this is that none of the utility functions discussed can be used as multiobjective utility functions. However, the preferences of the decision maker still have to be modelled.

Keeney and Raiffa [33] discuss a few options which may be used to address this problem. A first alternative is to assess the utility values by means of a direct assessment. This involves assigning utility values of 1 and 0 to the best (x^+, y^+) and worst (x^-, y^-) pairs, respectively, and assessing utility values of other outcomes by using reference lotteries[8].

Another very popular approach is to transform or adjust the attributes of the objectives and continue the assessment with the new set of attributes in the hope that some of the independence conditions may be verified by using the new set of attributes. However, if this transformation procedure is followed, care should be taken that the new set of attributes should still capture the essence of the problem.

Assessing weights for objectives

All the multiobjective models discussed in this section consist of a scaled addition or a scaled addition and multiplication of the individual assessed utility functions of the objectives. Keeney and Raiffa [33] describe a useful technique involving CE assessments to find values for the scaling constants. The aim is to find as much information with respect to indifferences between outcomes and lotteries. This information is then used to derive a set of equations in a number of unknowns, which are solved simultaneously to find values for the scaling constants.

Consider again the multilinear utility function

$$u(x, y) = k_X u_X(x) + k_Y u_Y(y) + (1 - k_X - k_Y) u_X(x) u_Y(y), \tag{3.17}$$

where $u(x^+, y^-) = k_X$ and $u(x^-, y^+) = k_Y$. Suppose the decision maker is indifferent between the outcomes (x_1, y_1) and (x_2, y_2). Then it follows by (3.17) that

$$\begin{aligned} k_X u_X(x_1) + k_Y u_Y(y_1) + (1 - k_X - k_Y) u_X(x_1) u_Y(y_1) = {} & k_X u_X(x_2) + k_Y u_Y(y_2) + \\ & (1 - k_X - k_Y) u_X(x_2) u_Y(y_2). \end{aligned} \tag{3.18}$$

It is assumed that $u_X(x)$ and $u_Y(y)$ have been assessed, implying that equation (3.18) has the two unknowns k_X and k_Y.

Now suppose that the decision maker is indifferent between a sure outcome (x_3, y_3) and a lottery involving an outcome (x_1, y_1) of occurring with probability p and an outcome (x_2, y_2) of occurring with probability $(1 - p)$. This implies that

$$u(x_3, y_3) = p u(x_1, y_1) + (1 - p) u(x_2, y_2). \tag{3.19}$$

By substituting $u(x_3, y_3)$ into (3.17), and by substituting this together with $u(x_1, y_1)$ and $u(x_2, y_2)$ into (3.19) yields the equation

$$\begin{aligned} k_X u_X(x_3) + k_Y u_Y(y_3) + (1 - k_X - k_Y) u_X(x_3) u_Y(y_3) = {} \\ p(k_X u_X(x_1) + k_Y u_Y(y_1) + (1 - k_X - k_Y) u_X(x_1) u_Y(y_1)) + {} \\ (1 - p)(k_X u_X(x_2) + k_Y u_Y(y_2) + (1 - k_X - k_Y) u_X(x_2) u_Y(y_2)). \end{aligned} \tag{3.20}$$

Since $u_X(x)$ and $u_Y(y)$ are known, (3.20) also involves the two unknowns k_X and k_Y. Therefore, (3.18) and (3.20) may be solved simultaneously for the scaling constants k_X and k_Y.

Scaling constants may also be evaluated by using the lottery in Figure 3.22. In this case the decision maker is asked to provide the probability value p for which he is indifferent to the lottery A, involving outcomes (x^+, y^+) with probability p and (x^-, y^-) with probability $(1-p)$, and the

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[8] Clemen and Reilly [14] discuss such a direct assessment in full detail and the interested reader is referred to [14] for further detail.

sure outcome B. By substituting $u(x^+, y^+)$ and $u(x^-, y^-)$ into the multilinear utility function, values of 1 and 0 are obtained, respectively, since $u(x^+)$ and $u(x^-)$ are assigned utility values of 1 and 0, respectively. Recall that $u(x^+, y^-) = k_X$. Since the decision maker is indifferent between the lottery A and the sure outcome B in Figure 3.22, $k_X = p(1) + (1 - p)(0) = p$. It may therefore be concluded that the probability which makes the decision maker indifferent with respect to the lottery A and the sure outcome B in Figure 3.22 is the scaling constant k_X [14, 62].

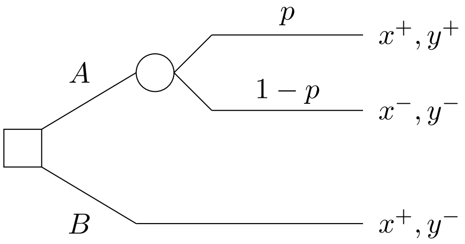


FIGURE 3.22: *A lottery used to determine scaling constants for a multiobjective utility function.*

It may easily be shown in the same way that the scaling constant k_Y is the probability p which makes the decision maker indifferent to the same lottery A in Figure 3.22, but with a different sure outcome B, of (x^-, y^+).

Of course, if the attributes are additive independent, the scaling constants k_X and k_Y should sum to one, as discussed in §3.4. If this is not the case, the reason should be explained to the decision maker, and the assessment procedure repeated.

3.5 Multiobjective evolutionary algorithms

Although classical multiobjective optimisation approaches, such as the AHP and utility theory, seem very appealing to use, they have a few disadvantages. Many classical optimisation methods involve scaling the objectives into a single objective. The problem with such an approach is that solutions are typically very sensitive to the weights associated with these objectives. This requires the decision maker to have an in-depth knowledge about the decision problem [17]. In addition, these methods are subjective in nature, thus depending on the input of a decision maker, which may lead to inconsistencies, such as a decision maker's preferences not remaining constant during the assessment procedure. Therefore consistency tests have to be put into place, which add on to the already tedious assessment process involved in these approaches.

Moreover, classical optimisation methods focus on finding only one solution in the set of pareto optimal solutions at a time [3]. This implies that the method has to be repeated for a number of parameter values in an attempt to find different solutions on the pareto frontier [17].

Multiobjective evolutionary algorithms, which are able to solve problems in a multidimensional decision space, may be used to address this problem. A distinct characteristic of these algorithms is that they are able to find multiple solutions on the pareto frontier in one single run [3].

Evolutionary algorithms, as the name suggests, make use of different evolutionary concepts, also known as operators, to strive to the optimal solution in the solution space. One such evolutionary algorithm is a *Genetic Algorithm* (GA). A GA performs operations on a population of solutions to find better solutions. This makes a GA an ideal candidate for solving multiobjective problems with a view to obtain a number of solutions simultaneously [17].

An early GA designed for the purpose multiobjective optimisation appeared in 1984 an is due to Schaffer [17]. Although the implemented algorithm proved to deliver promising results, it suffered the disadvantage of being biased towards certain solutions on the pareto frontier. Deb and Srinivas [17] addressed this problem by developing the *Nondominated Sorting Genetic Algorithm* (NSGA) which is able to deliver a more thorough distribution of the solutions over the entire pareto frontier region.

The NSGA is very closely related to a basic GA. The difference between these two optimisation methods, lies in the way in which the selection process is executed [17]. The remainder of the operators in the algorithms, such as the crossover and mutation, are the same.

The NSGA is based on the notion of nondominated sorting, as discussed in §3.1.2. This involves a process where all the solutions in the current population are sorted, based on their nondominated status. The solutions which are classified as nondominated, are considered to form the first nondominated front [17]. Each solution in the first nondominated front are assigned a large dummy fitness value. The solutions in this front are then shared with their dummy fitness value, by means of a sharing function, based on the euclidean distance between solutions. The sharing method comprise of adjusting the fitness in such a way that more fitness is assigned to a less crowded region and *vica versa*. This maintains a level of diversity in the population and enables the exploitation of different pareto optimal solutions in a single run [18].

After the sharing procedure is complete, the solutions contained in the first nondominated front are temporarily removed from the population to find the second nondominated front. This is obtained by following the same procedure, but by assigning a lower dummy fitness value than the dummy fitness value assigned to solutions in the first nondominated front [18]. Once the entire population is classified into nondominated fronts, the normal operators of a GA are applied to reproduce the population, based on the dummy fitness values.

Although the NSGA is often able to provide good results, it has received criticism due to the high computational complexity of nondominated sorting, its lack of implementing elitism and the need for specifying a sharing parameter. Deb *et al.* [3] proposed an improved version of the NSGA, called the *Nondominated Sorting Genetic Algorithm II* (NSGA II), to address these problems.

The NSGA II is closely related to the NSGA. The difference is that the NSGA II employs a fast nondominated sorting procedure, an elitist diversity preservation method, and a parameterless niching operator [3]. Since the NSGA II is considered a more efficient multiobjective optimisation method than the original NSGA, it is applied to the multiobjective WA problem later in this thesis. More emphasis will therefore be given to the discussion of the NSGA II in the following sections. Before the working of the NSGA II is discussed, a few concepts relating to evolutionary algorithms are reviewed.

3.5.1 Fitness assignment

In evolutionary algorithms, the variables relating to the problem may be represented in what is called a *chromosome*, consisting of solution values assigned to variables. Each chromosome is considered a solution to the problem, and a set of chromosomes representing a set of solutions to the problem make up what is called a *population*. Various operators may be performed on the population to find better solutions.

A measure of effectiveness of a solution is the *fitness value*. A better solution is one yielding a higher fitness value. In multiobjective evolutionary algorithms, the notion of pareto dominance may be used as a measure of fitness for solutions. Pareto dominance involves calculating the fitness of a chromosome in relation to other chromosomes contained in the population, based on a pareto rank criterion. Various dominance criteria exist which may be used in the fitness assignment. Raad [42] lists three such criteria: *dominance count*, which is the number of solutions that a particular solution is dominated by, *dominance strength*, which is the number of solutions that a particular solution dominates, and *pareto rank*, which is the front in which a solution is contained. The NSGA II uses the pareto rank as a measure of fitness, where a solution having a lower rank is better than a solution having a higher rank. An illustration of the pareto rank fitness assignment may be seen in Figure 3.23 for a problem where two objectives, $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$, are to be minimised.

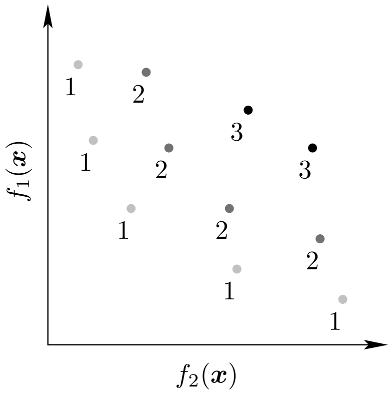


FIGURE 3.23: *Illustration of the pareto rank fitness used in the NSGA II.*

3.5.2 Diversity preservation

One goal of multiobjective evolutionary algorithms is to provide a number of diverse solutions which are uniformly distributed across the pareto frontier. This may be achieved by using information relating to the density of solutions in the objective space [12, 42]. The aim is to favor solutions in less crowded regions of the solutions space, in an attempt to achieve a more thorough search of the solution space [42]. Diversity preservation may be achieved by a number of existing methods, such as *fitness sharing* (which was implemented in the original NSGA), *crowding, restricted mating* and *relaxed domination* [12].

The density measure utilised in the NSGA II, is called *crowding*, which makes use of the distances between solutions. The crowding distance of a solution may be calculated by taking the average distance of its two closest neighboring solutions along each objective axis or direction. Hence, the crowding distance may be seen as the perimeter of the cuboid formed by the neighboring solutions [3]. To calculate the crowding distance requires the sorting of the solutions in ascending order of magnitude along each objective axis. Each side of a solution's neighborhood cuboid represents the distance from a solution to its two neighboring solutions. After normalising these distances, the crowding distance may be calculated by adding together the sides of the cuboid. The reason for normalising the distances, is to prevent differences in magnitude from the objective function values of the solutions [42]. Since solutions with higher crowding distances are more isolated, they are more preferred to solutions with smaller crowding distances. An example of the crowding distance of a solution in an objective space consisting of two objectives, $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$ which are to be minimised, may be seen in Figure 3.24.

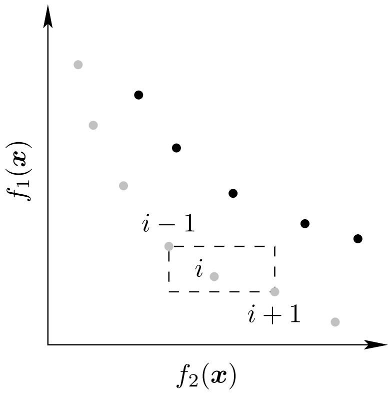


FIGURE 3.24: *Illustration of the cuboid formed around a solution which is used in the calculation of its crowding distance.*

3.5.3 Selection

In evolutionary algorithms a process called *selection* is applied to parent solutions in the population. Selection involves selecting parent solutions from the population to form a mating pool with the aim of choosing solutions which have a better fitness values. The solutions contained in the mating pool are then used to generate child solutions.

One parameter in the selection process, which may be used to favor better solutions, is the *selection pressure*. The selection pressure may be seen as a means of improving the population fitness over succeeding generations. Hence, the selection pressure has an impact on the convergence rate of the algorithm [24]. An increased selection pressure will result in favoring solutions having higher fitness values, which may result in a higher convergence rate. However, it should be noted that the value of selection pressure should be chosen carefully as a high selection pressure may result in the algorithm converging prematurely to a sub-optimal solution, while a low selection pressure may result in unnecessarily slowing down the convergence rate of the algorithm [24]. Various selection procedures exist which may be used in evolutionary algorithms, including *fitness proportion selection*, *truncation selection* or *tournament selection* and the interested reader is referred to [58] for a discussion on the workings of these selection procedures.

It is important to include some form of elitism in the selection procedure. Incorporating elitism into the selection procedure ensures that the best solutions in the current generation are transfered to the next generation [16].

The selection procedure employed in the NSGA II is a binary tournament selection. Tournament selection consists of hosting a tournament among n solutions. The solution having the largest fitness value in the tournament, is considered the winner, and is included into the mating pool. The selection pressure in tournament selection is represented by the value of n, called the *size* of the tournament. An increased value of n will yield an increase in the selection pressure. The reason for this is that the fitness of the winner of a larger tournament will, on average, have a higher fitness than the winner of a smaller tournament [24]. It is recommended in [38] that the selection pressure in the NSGA II may be varied between a value of two and five.

The selection procedure in the NSGA II works in such a way that parent solutions having lower ranks are chosen. If two solutions have the same rank value, the solution having the largest crowding distance is chosen [42]. After child solutions have been created, the parent and child solutions are combined into a larger population of size $2N$. The solutions contained in this larger population are ranked and sorted, and the crowding distance is recalculated for each solution. The selection procedure is then applied in an elitist manner by selecting solutions having rank 1 first, solutions having rank 2 secondly, *etc.* until the population size N is reached.

3.5.4 Crossover and mutation

Once the selection process is complete, the parent solutions are used to create child solutions by using two common GA operators known as *crossover* and *mutation*. Although the selection process already aims to select solutions with the best fitness values, crossover and mutation aims to further explore the solution space in an attempt to find other solutions with better fitness values [42].

Various types of crossover and mutation operators exist, and the type of operator and implementation thereof depends largely on the specific problem at hand, as well as the solution encoding of the variables associated with the problem [31].

Crossover is the process of combining more than one parent solution to create child solutions. The crossover process is stochastic and is hence associated with a probability value of occurring. The crossover probability is usually set to a high value, since crossover is considered an essential concept of introducing variation in the population [16, 42]. Crossover techniques which may be used include *single point crossover*, *two point crossover*, *cut and splice*, and *uniform crossover* [53]. The NSGA II implements a single point crossover for binary coded solutions and a simulated binary crossover for real coded solutions [3]. Since the NSGA II employed later in this thesis consists of binary coded solutions, only the single point crossover operator is explained here. The interested reader is referred to [53] for a discussion on other crossover techniques.

The single point crossover consists of uniformly selecting a single point along the parent solutions and slicing the solutions at this point. Child solutions are obtained by swapping the parent solutions beyond this point. An example of a single point crossover may be seen in Figure 3.25.

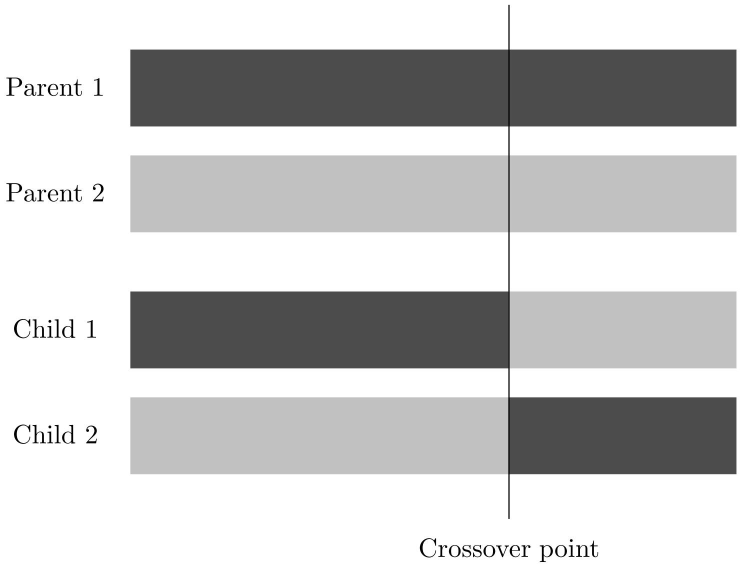


FIGURE 3.25: *Illustration of a single point crossover.*

Mutation consists of randomly altering the value of one or more of the entries in a solution from its original state. The aim of mutation is to diversify the solutions in an attempt to explore new regions of the search space so as to hopefully find better solutions [56, 42]. Mutation techniques which may be used include *bitwise mutation, flip bit mutation, uniform mutation* and *gaussian mutation* [56]. The interested reader is referred to [56] for a discussion on the these mutation techniques.

$$\begin{array}{rl} \text{Child before mutation} & \begin{bmatrix}1 & 0 & 1 & 1 & 0 & 0 & 1 & 0\end{bmatrix} \\ & \begin{matrix} & & & & & \downarrow & & \end{matrix} \\ \text{Child after mutation} & \begin{bmatrix}1 & 0 & 1 & 1 & 0 & 1 & 1 & 0\end{bmatrix} \end{array}$$

FIGURE 3.26: *Illustration of a bitwise mutation.*

The NSGA II implements bitwise mutation, which is usually implemented in problems with binary coded solutions. Bitwise mutation works in such a way that bit values (values of variables in the solution) are flipped from 0 to 1 and *vice versa* with a mutation probability. The mutation probability is usually set at the low value $1/\ell$, where ℓ is the number of variables in the solution, since a large mutation probability may turn the GA into a random search algorithm [16, 42]. An example of a bitwise mutation may be seen in Figure 3.26.

3.5.5 The NSGA II

The first step in the NSGA II is to rank and sort the solutions in a population of size N. This may be achieved by using the *Fast Nondominated Sorting Algorithm* (FNSA) [42] which has a computational complexity of $O(MN^2)$, where M denotes the number of objectives. For each solution $\boldsymbol{i}$, a dominance count $d^c_{\boldsymbol{i}}$ (the number of solutions which dominate $\boldsymbol{i}$), and $S_{\boldsymbol{i}}$, the set of solutions that $\boldsymbol{i}$ dominates, are calculated. This requires $O(MN^2)$ comparisons [3, 42]. All solutions contained in the first nondominated front will have $d^c_{\boldsymbol{i}} = 0$, and are assigned rank 1. The solutions having $d^c_{\boldsymbol{i}} = 0$ are placed in a separate set $\mathcal{F}_1$. For each $\boldsymbol{i}$ in $\mathcal{F}_1$, the algorithm cycles through each solution $\boldsymbol{j}$ in $S_{\boldsymbol{i}}$, and decrements its $d^c_{\boldsymbol{j}}$ value by one, thus discounting the effect of $\boldsymbol{i}$ on $\boldsymbol{j}$'s dominance count [42]. The dominance count of all the rank 2 solutions now have $d^c_{\boldsymbol{i}} = 0$, and are placed in a separate set $\mathcal{F}_2$, for the algorithm to cycle through. The algorithm continues in this fashion until all the solutions are partitioned into ranks. A pseudocode listing of the FNSA is presented as Algorithm 3.1.

The next step in the NSGA II is to calculate the crowding distance density measure. This requires the population to be sorted in ascending order of magnitude along each objective axis. Assume that the number of solutions in a population is denoted by k, and that the objective function value of the i^{th} solution for the h^{th} objective function (in the sorted list) is represented by $X[i]|h$. A crowding distance i_{dist} of infinity is assigned to the boundary solutions, that is the solutions $X[1]|h$ and $X[k]|h$, so as to ensure that they are selected [42]. The crowding distances of the intermediate solutions $\boldsymbol{i}$ are incremented by the normalised distance between their closest neighboring solutions [3, 42]. The normalised distance value may be calculated as $i_{dist}|h + (X[i+1]|h - X[i-1]|h)/(h_{max} - h_{min})$. The overall crowding distance for each objective is seen as the accumulated value of crowding distances of the individual solutions. The crowding distances may now be applied to determine the solution density [42]. A higher value indicates that a solution is more isolated, and a lower value implies that a solution is more crowded by other solutions [3]. A pseudocode listing of the crowding distance procedure is presented as Algorithm 3.2.

A crowding distance operator $\prec_c$ is employed during the selection process of the algorithm. This operator specifies that a more favorable solution between a pair of solutions denoted by $(\boldsymbol{i} \prec_c \boldsymbol{j})$ is the one having the lowest rank, that is $\boldsymbol{i}_{rank} < \boldsymbol{j}_{rank}$. If the solutions reside within the same front, then the solution with the higher crowding distance is chosen as the more favorable one. That is, if $\boldsymbol{i}_{rank} = \boldsymbol{j}_{rank}$, then $\boldsymbol{i}_{dist} > \boldsymbol{j}_{dist}$. In this way, more solutions are explored in less crowded regions of the solution space, which may lead to an approximately uniformly spaced pareto optimal frontier [3, 42].

Algorithm 3.1: Fast Nondominated Sorting Algorithm [42].

Input: A population of solutions $\boldsymbol{P}$, where a solution $\boldsymbol{i}$ in $\boldsymbol{P}$ represents an assignment of values to decision variables $\boldsymbol{x}$, and a vector $\boldsymbol{z}$ containing the objective function values for each solution $\boldsymbol{i}$.

Output: The set of fronts $\mathcal{F}_m$ containing the nondominated solutions for each front.

```
$\mathcal{F}_1 \leftarrow \emptyset$
forall the $\boldsymbol{i} \in \boldsymbol{P}$ do
    $S_{\boldsymbol{i}} \leftarrow \emptyset$
    $d^c_{\boldsymbol{i}} = 0$
    forall the $\boldsymbol{j} \in \boldsymbol{P}$ do
        if $\boldsymbol{i} \prec \boldsymbol{j}$ then
            $S_i \leftarrow S_{\boldsymbol{i}} \cup \{\boldsymbol{j}\}$
        else if $\boldsymbol{j} \prec \boldsymbol{i}$ then
            $d^c_{\boldsymbol{i}} \leftarrow d^c_{\boldsymbol{i}} + 1$
        end
    end
    if $d^c_{\boldsymbol{i}} = 0$ then
        $\boldsymbol{i}_{\text{rank}} \leftarrow 1$
        $\mathcal{F}_1 \leftarrow \mathcal{F}_1 \cup \{\boldsymbol{i}\}$
    end
end
$m \leftarrow 1$
while $\mathcal{F}_m \neq \emptyset$ do
    $\mathcal{A} \leftarrow \emptyset$
    forall the $\boldsymbol{i} \in \mathcal{F}_m$ do
        forall the $\boldsymbol{j} \in S_{\boldsymbol{i}}$ do
            $d^c_{\boldsymbol{j}} \leftarrow d^c_{\boldsymbol{j}} - 1$
            if $d^c_{\boldsymbol{j}} = 0$ then
                $\boldsymbol{j}_{\text{rank}} \leftarrow m + 1$
                $\mathcal{A} \leftarrow \mathcal{A} \cup \{\boldsymbol{j}\}$
            end
        end
    end
    $m \leftarrow m + 1$
    $\mathcal{F}_m \leftarrow \mathcal{A}$
end
```

The main NSGA II procedure may now commence by generating an initial population $\boldsymbol{P}_0$ randomly. $\boldsymbol{P}_0$ is sorted and ranked into fronts, by applying the FNSA. Crowding distances are then calculated for each solution in $\boldsymbol{P}_0$ and a new population $\boldsymbol{Q}_0$ is created by performing binary tournament selection on $\boldsymbol{P}_0$, and applying the crowded comparison, crossover and mutation operators. Once the initial population $\boldsymbol{P}_0$ and the first generation $\boldsymbol{Q}_0$ is obtained, the generation counter t is set to one, and the remainder of the algorithm is iterated until the generation counter reaches a maximum value of $t = G_{\max}$, where $G_{\max}$ denotes the number of iterations [3, 42].

During each iteration or generation, the parent and child solutions are combined to form a population $\boldsymbol{R}_t = \boldsymbol{P}_t \cup \boldsymbol{Q}_t$ of size $2N$. The combined population $\boldsymbol{R}_t$, is then sorted and ranked into nondominated fronts by applying the FNSA. The new population $\boldsymbol{P}_{t+1}$ is created next by adding the solutions in the first front $\mathcal{F}_1$, the second front $\mathcal{F}_2$, and so forth until the population size N is reached. If all the solutions in the next front cannot be included in $\boldsymbol{P}_{t+1}$, the solutions in that front are sorted in descending order with respect to their crowding distance, and the solutions are added to $\boldsymbol{P}_{t+1}$ until a population size of N is reached [3, 42].

Once $\boldsymbol{P}_{t+1}$ has been created, a crowding distance is calculated for each solution in $\boldsymbol{P}_{t+1}$. A new population $\boldsymbol{Q}_{t+1}$ is then created by performing binary tournament selection on the solutions in $\boldsymbol{P}_{t+1}$ to select solutions based on the crowding distance operator $\prec_c$, and applying the crossover and mutation operators on the selected solutions. Crossover may be performed by means of a single point crossover for binary coded solutions, and the simulated binary crossover for real-coded solutions, whilst mutation may be performed by means of bitwise mutation. As mentioned, the mutation probability is taken as $1/\ell$, where ℓ denotes the number of variables in the solution [3]. A pseudocode listing of the NSGA II is represented as Algorithm 3.3, and an illustration of the operation of the NSGA II is shown in Figure 3.27.

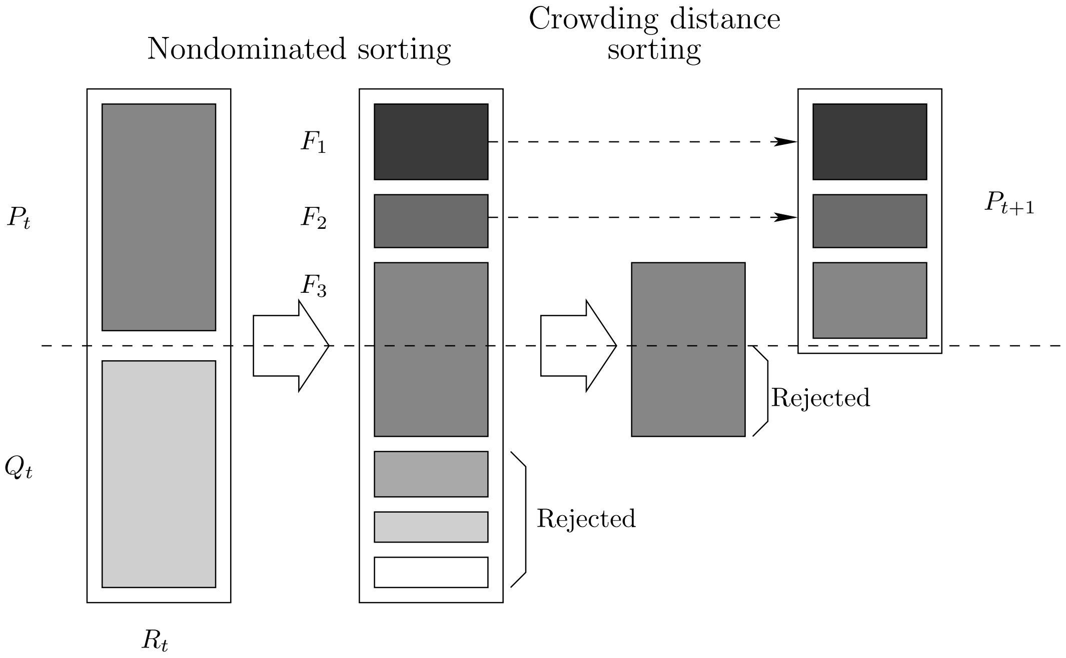


FIGURE 3.27: *The procedures followed in the NSGA II [12].*

Algorithm 3.2: Crowding Distance Assignment Algorithm [42].

```
Input: The population of solutions $\boldsymbol{P}$, and a vector $\boldsymbol{z}$ containing each of the objective
       function's values for each solution.
Output: The crowding distance $\boldsymbol{P}[i]_{\text{dist}}$ for each solution in $\boldsymbol{P}$.
$k = |\boldsymbol{P}|$
forall the $i \in \boldsymbol{P}$ do
    $\boldsymbol{P}[i]_{\text{dist}} \leftarrow 0$
end
forall the $M$ objectives do
    $\boldsymbol{P} = \text{sort}(\boldsymbol{P}, h)$
    $\boldsymbol{P}[1]_{\text{dist}}|h \leftarrow \infty$
    $\boldsymbol{P}[k]_{\text{dist}}|h \leftarrow \infty$
    forall the $i = 2$ to $(k-1)$ do
        $\boldsymbol{P}[i]_{\text{dist}}|h \leftarrow \boldsymbol{P}[i]_{\text{dist}}|h + (\boldsymbol{P}[i+1]|h - \boldsymbol{P}[i-1]|h)/(h_{max} - h_{min})$
    end
end
```

Algorithm 3.3: Nondominated Sorting Genetic Algorithm II [42].

```
Input: A multiobjective problem with variables $\boldsymbol{x}$ and a number of objective functions $M$,
       a vector $\boldsymbol{z}$ containing values for each of the objective functions, a set of constraints
       and constraint violation functions, the size of the population $N$, and the maximum
       number of generations $G_{\text{max}}$.
Output: An approximation of the set of pareto optimal solutions in the multi objective
        space $\mathcal{W}^*$.
Generate an initial solution $\boldsymbol{P}_0$ of size $N$ randomly.
Rank and sort $\boldsymbol{P}_0$ by using the FNSA [Algorithm 3.1].
Calculate the crowding distance for each solution in $\boldsymbol{P}_0$ by using crowding distance
assignment algorithm [Algorithm 3.2].
Create child population $\boldsymbol{Q}_0$ of size $N$ by using binary tournament selection based on the
crowding distance operator $\prec_c$ from $\boldsymbol{P}_0$, and performing crossover and mutation.
$t \leftarrow 0$ while $t < G_{max}$ do
    $\boldsymbol{R}_t \leftarrow \boldsymbol{P}_t \cup \boldsymbol{Q}_t$
    Rank and sort $\boldsymbol{R_t}$ into fronts $\mathcal{F}_1, \mathcal{F}_2, \ldots$ by using FNSA.
    $\boldsymbol{P}_{t+1} \leftarrow \emptyset$ and $m \leftarrow 1$
    while $|\boldsymbol{P}_{t+1}| < N$ do
        if $|\mathcal{F}_m| + |\boldsymbol{P}_{t+1}| <= N$ then
            $\boldsymbol{P}_{t+1} \leftarrow \boldsymbol{P}_{t+1} \cup \mathcal{F}_m$
        else if $|\mathcal{F}_m| + |\boldsymbol{P}_{t+1}| > N$ then
            Calculate the crowding distance for each solution in $\mathcal{F}_m$.
            Sort the solutions in $\mathcal{F}_m$ in descending order, based on crowding distance.
            $\boldsymbol{P}_{t+1} \leftarrow \boldsymbol{P}_{t+1} \cup$ [the first $(N - |\boldsymbol{P}_{t+1}|)$ solutions in $\mathcal{F}_m$]
        end
        $m \leftarrow m + 1$
    end
    Calculate crowding distance for each solution in $\boldsymbol{P}_{t+1}$.
    Create child population $\boldsymbol{Q}_{t+1}$ by using binary tournament selection based on the
    crowding distance operator $\prec_c$, crossover and mutation.
    $t \leftarrow t + 1$
end
$\mathcal{W}^* = \boldsymbol{P}_{G_{\text{max}}}$
```

For constrained optimisation problems, Deb *et al.* [3] propose a constraint handling approach for the NSGA II. In this approach, the manner in which domination is defined between two solutions $\boldsymbol{i}$ and $\boldsymbol{j}$ is modified. From [3], solution $\boldsymbol{i}$ is said to dominate solution $\boldsymbol{j}$ if:

1. Solution $\boldsymbol{i}$ and $\boldsymbol{j}$ are both feasible, and solution $\boldsymbol{i}$ dominates solution $\boldsymbol{j}$ (*i.e.* $\boldsymbol{i} \prec_c \boldsymbol{j}$), or
2. Solution $\boldsymbol{i}$ is feasible and solution $\boldsymbol{j}$ is infeasible, or
3. Solution $\boldsymbol{i}$ and $\boldsymbol{j}$ are both infeasible, and solution i has the smallest overall constraint violation.

The remainder of the procedures contained in the NSGA II remain unchanged and may be applied as usual for constrained optimisation problems. In order to obtain the smallest overall constraint violation, the constraint violations have to be normalised and summed [42]. Since the multiobjective optimisation problem posed later in this thesis does not involve constrained optimisation, this topic will not be discussed further. The interested reader is referred to Raad [42] or to Deb *et al.* [3] for a further discussion on constrained optimisation.

3.6 Chapter summary

The purpose of this chapter was to provide the reader with sufficient information from the literature to understand the concept of multiobjective decision making as well as certain approaches which may be used to solve these problems. This was achieved by discussing multiobjective decision analysis in §3.1 and further discussing three approaches to solve these problems with.

The workings of the AHP were discussed in §3.2, which were followed by a comprehensive discussion on unidimensional utility theory in §3.3. This was followed by guidelines to assess unidimensional utility functions in §3.4, as well as techniques to assess values for the scaling constants in multiobjective utility theory.

The chapter concluded with an introduction to evolutionary algorithms and a discussion on the NSGA II in §3.5 which was specifically developed for multiobjective decision making problems.

CHAPTER 4

Multiobjective decision making approaches towards WA

Contents

This chapter contains the application of the methodologies discussed in Chapter 3. In § 4.1 a method is proposed to identify possible objectives in WA. The implementation of the method is discussed and the results presented. The section concludes with the formulation of a bi-objective WA model. In § 4.2 a comprehensive discussion on a scenario imitating a real-world GBADS scenario is found, which is used later in the solving of the proposed model.

This is followed by a first multiobjective decision making approach towards the bi-objective WA model in §4.3. The section contains a discussion on the procedures involved during the assessment of the score values for the WS-threat pairs, with respect to each time step, as well as the proposal of a AHP assignment model, which may be useful in the assignment of WSs to threats when considering the score values obtained with the AHP.

A utility approach is considered as a next multiobjective decision making approach towards WA in § 4.4. The assessments with respect to the qualitative characteristics as well as the quantitative utility values is discussed, with respect to each objective. This is followed by a description of obtaining scaling constants for each of the respective objectives. The section concludes with a proposal of a utility assignment model, which may be useful in the assignment of WSs to threats when considering the utility values obtained by the bi-objective utility function.

Finally, the chapter concludes with a discussion on the computer implemented NSGA II in § 4.5. The section includes an interpretation of the input as well as output data, and a description of the genetic operators utilised in the NSGA II.

4.1 Identifying objectives from a WA perspective

The first step in approaching any decision making problem, whether single objective or multiobjective, is to identify the complete set of objectives which forms the core of the decision problem. As explained in § 3.1, this may be achieved by incorporating the help of a decision analyst whose primary goal is to interview the decision maker in an attempt to extract important information from the decision maker which may lead to the establishment of fundamental objectives.

The first step in modelling the WA problem as a multiobjective decision problem is therefore to identify the objectives deemed important in the choice of which WSs to assign to which threats. This requires the identification of an individual or group who will act as the decision maker during the assessment procedure. From a military perspective, it is difficult to isolate a single individual who may act as the sole decision maker in the matter of WA; it seems more desirable to identify a number of military experts, residing within the South African military domain, who will collectively act as the decision maker. This is expected to yield a more comprehensive set of objectives with respect to WA.

An ideal approach for the extraction of objectives is to host an interactive workshop presented by an analyst, where interaction between the analyst and the decision making group, and more importantly between the members of the decision making group, is made possible, where ideas and different perspectives of these members may be integrated so as to obtain a final set of objectives. Techniques such as the *Delphi method* or *Post-it* sessions as discussed in §3.1.4, may be implemented to facilitate such a workshop. However, due to time constraints and limitations posed by the geographic placement of these individuals, conducting such an interactive workshop was not a viable option. An alternative is to conduct the objective extraction process by means of an electronic questionnaire or survey, which requires completion by each military expert in the decision making group. A survey, consisting of a set of questions relating to various possible factors which may influence the choice of WSs to assign to threats, was devised for this purpose. Since there are only a small number of military experts in the South African defense force, military expert Visser was asked to help with the identification of military experts. This process involved military expert Visser to contact the head of staff of the South African defense force. Five military experts, who are unknown to the author, were identified by Visser [50] and the head of staff of the South African defense force to participate in such a survey. Due to the sensitivity of the military environment, the survey was completed anonymously by the five military experts and hence, the author was not able to do a follow-up on no responses from the military experts.

A first attempt in preparing the survey was to employ the techniques and questions proposed by Keeney [32], as discussed in §3.1.4. A first-order WA survey, based on these techniques, gave rise to the following set of questions:

1. What are your expectations with respect to the outcome in a situation where you have to assign a WS to a threat, and why is this important?

2. What process do you follow in a situation where you have to decide which WS to assign to a threat, and why is it important?
3. What are the expectations of your superiors in a situation where a WS needs to be assigned to a threat, and why do you think it is important?
4. What would you say are your ultimate objectives when assigning a WS to a threat? What are your values that are absolutely fundamental when assigning a WS? Why do you think these objectives are important and what is meant by each of them?
5. Are any constraints placed on you when deciding which WS to assign to a threat? What are these constraints and which constraints are the most important?
6. Imagine that you are in the shoes of your enemy. What would you be concerned about in a WA situation and why would you be concerned about this?
7. Based on your experience, what other aspects, which are not included in the current process of deciding which WS to assign to a threat, do you think should be included in the evaluation process in order to make a better assignment decision?
8. If a WS (WS_1) is chosen to be assigned to a threat, what aspects/criteria of an alternative WS (WS_2), would make you change your decision from WS_1 to WS_2?
9. From a WS-target assignment perspective, what aspects/criteria of a WS would make it a successful candidate for an assignment? Why do you think these aspects are important?
10. How would you describe WS efficiency? Please motivate your answer by explaining what is meant and stating why you think it is important.
11. What environmental constraints do you think are important in a WA situation and why are they important? Explain what is meant by these constraints.
12. What social constraints do you think are important in a WA situation and why are they important? Explain what is meant by these constraints.
13. Do you think cost elements should be considered in the WA choice? Please motivate your answer. If yes, please state these elements and explain what is meant by each and why you think they are important.
14. What other economic objectives do you think are important in a WA situation and why are they important? Explain what is meant by these objectives.
15. What health and safety objectives regarding your own crew, do you think are important in a WA situation and why are they important? Explain what is meant by these objectives.
16. Which factors would you say should be included in the WA evaluation, and why are they important?

After consulting with TEWA experts Potgieter [41], Roux [44] and military expert Visser [50], the conclusion was drawn that the questions above are well suited for an interactive workshop, where decision makers are in a position to ask questions and discuss different opinions amongst one another. However, due to the lack of communication with other members of the decision making group, and possible misconceptions in terms of terminology (which may lead to confusion and frustration on the decision makers' side), it was advised that this questionnaire is not suited for electronic circulation, and it was decided to revise the survey. A survey which is able to lead the decision making group towards possible objectives may be more appropriate for decision makers with a military background who are not able to interact with one another during the extraction process. This may also result in a smoother execution of a survey conducted electronically.

In order to be able to lead the decision maker towards appropriate objectives a thorough search is required for factors which arise in existing WA procedures. An extensive list of such factors was compiled from material found in the literature and suggestions posed by Potgieter [41] and Roux [44]. These factors are contained in Table 4.1.

Possible factors	
Accessibility of a WS	Lethality of a WS
Ammunition cost	Maintainability of a WS
Ammunition inventory	Maneuverability of a WS
Area covered by a WS	Manpower required to operate a WS
Availability of a WS	Payload of a WS
Compatibility of a WS with respect to other WSs	Range of a WS
Cognitive experience of a WS operator	Reliability of a WS
Cognitive ranking of a WS operator	Safety of the crew operating a WS
Collateral damage and cost of a WS assignment	SSHP of a WS
Cost of deploying and operating a WS	Susceptibility of a WS
Diameter of a WS	Survivability of a WS under attack
WS Downtime during reloading	Transportability of a WS
Effectiveness of a WS	Velocity of a WS's ammunition
Environmental influence on a WS	Vulnerability of a WS
Environmental influence on sensors	Weight of a WS
Ground area required by a WS	
Interoperability of a WS with respect to other WSs	

Table 4.1: *A list of possible factors which may influence the choice of WS to assign to a threat [8, 10, 41, 44].*

The aim of the survey is thus to extract information about factors similar to those presented in Table 4.1 for use in the identification of possible WA objectives.

A second survey was therefore developed, including questions based on a GBADS scenario imitating aerial attacks by threats. The reason for developing a scenario-based survey is that it was anticipated that the military experts may relate easier to the questions if they were presented together with a graphical illustration. The questionnaire was kept simple and the completion process was therefore simplified. It was also expected that the military experts might find the scenario context more appealing, which could result in a positive attitude towards the survey, in turn, having a positive impact on the quality of responses obtained.

The second survey consists of four characteristic-specific scenarios, with a set of questions relating to each scenario. Each scenario consists of one DA requiring protection from one of three possible WSs (*i.e.* a CIWS, a SHORAD and a VSHORAD), deployed in parallel. The aim of using only one of each WS is to avoid unnecessary clutter in the questionnaire, and more importantly to encourage the military experts to think hard about why one WS may be more appropriate to assign to a threat than another. Variation in the scenarios included different surrounding terrain features (*i.e.* flat earth surfaces as well as mountain ranges), different weather conditions (*i.e.* sunny, rainy, windy), single as well as multiple threats approaching DAs, different velocities at which threats travel and the assignments of WSs at varying distances from DAs.

The majority of the questions required the military experts to make a choice between three possible WSs to assign to the approaching threat or threats, located at various distances from the WSs. They also had the opportunity to not assign any WSs at that given moment, implying that the threat may be addressed at a later stage. The questions related to the choice between WSs were followed by a question where the military experts were required to explain their decisions. These explanatory answers may form part of the core purpose of the survey, since it is here where the valuable information is typically provided. The reasoning behind the decision may lead to the identification of further factors in the choice of WS to assign. The complete final survey may be found in Appendix A.1 at the end of the thesis.

The final electronic version of the survey restricted the military experts to alter only their choice of WS by selecting between respective radio buttons, and to insert text inside pre-specified text boxes at the explanatory questions. This was achieved by means of Microsoft Word [39]. An electronic copy of the restricted version of the survey may be found on the compact disc accompanying this thesis. It is located in the directory named `WA_survey` and labeled `WA_restricted_survey.docx`. The survey was distributed via electronic mail to the various military experts.

After approximately two weeks, feedback was received from three of the five military experts to whom the second survey was sent. The feedback from the fourth military expert was incomplete (it only consisted of a set of general comments), while no feedback was received from the last military expert. The feedback from the surveys is summarised in Tables A.1–A.5, which may be found in Appendix A.2 at the end of the thesis.

Ability to engage behind obstacles	Night fight/fire capability
All weather capability	Reaction times of WSs
Available ammunition	Speed of WS ammunition
Cost of ammunition	SSHP
Effective ranges of WSs	Terrain features
Line of sight	Vertical launch capability
Multiple engagements	Weather conditions

TABLE 4.2: *A list of possible factors with respect to WSs which may influence the choice of assignment in the WA process, obtained from the WA survey.*

After examining the responses from the survey, a list of possible factors which may influence the choice of WS to assign to a threat was compiled and this list is presented in Table 4.2. It is evident from Table 4.2 that there are two overarching objectives, that is to maximise the efficiency of the WSs assigned and to minimise the cost of assignment. The only factor contributing to the cost of WSs, is the cost of ammunition. Factors contributing to the efficiency of WSs include the range of a WS, with respect to a threat, the SSHP of a WS, the reaction time of a WS, and the available ammunition of a WS. A few other factors, such as weather conditions, effective ranges and terrain features, may be used to filter the SSHP information in order to populate the EEM.

Due to the lack of more reliable information, it was decided to develop a multiobjective WA model with at most two conflicting objectives, for illustrative purposes. The first objective aims to maximise the overall assignment efficiency by using the SSHP values of WSs, whereas the second objective aims to minimise the overall cost of WSs assigned, by using the cost of ammunition in Rand value. These objectives are conflicting in the sense that when assigning more WSs to a threat, the expected result is a higher overall assignment efficiency, but this will, in turn, result in a higher overall assignment cost. Hence, the aim is to find an acceptable trade-off between the cost and efficiency of WSs assigned.

The k-WA model from Ahuja *et. al.* [4] served as a point of departure for the model adopted here. The first objective remains the same as in the original k-WA model formulation, that is to maximise the weighted expected damage caused to threats by the assignment. This is achievable by minimising the survival probabilities, q_{ij}, of the threats, weighted by the respective threat priorities, V_j.

The second objective aims to minimise the total cost of all the WSs assigned. Denote the cost of using WS$_i$ by C_i, and define the decision variable x_{ij} as a binary variable taking a value of one if WS i is assigned to threat j, or zero otherwise. The objectives are then to

$$\begin{aligned} \text{minimise } & \sum_{j=1}^{n_t} V_j \prod_{i=1}^{n_w} q_{ij}^{x_{ij}}, \qquad (4.1)\\ \text{minimise } & \sum_{i=1}^{n_w} C_i \sum_{j=1}^{n_t} x_{ij}, \end{aligned}$$

subject to the constraints

$$\begin{aligned} \sum_{j=1}^{n_t} x_{ij} &\leq 1, && i = 1, \ldots, n_w && (4.2)\\ x_{ij} &\in \{0, 1\}, && i = 1, \ldots, n_w && (4.3)\\ & && j = 1, \ldots, n_t, && (4.4) \end{aligned}$$

where n_t and n_w represent the number of threats and WSs, respectively. The formulation is constrained by the assumption that a WS may only be assigned to one threat at a time (multiple engagements by a single WS is deemed impossible). However, the number of WSs assigned to a threat is unlimited. A further assumption is that an unlimited amount of funds are available for the assignment of WSs.

Due to the fact that the above model considers only a single time step at a time, a reasonable assumption is that the time steps are independent of one another, and that a WS may only be considered for an assignment for the duration of one time step. This implies that a WS is available for an assignment at each time step. Furthermore, the assignments at a current time instance is not affected by past time instances or does not affect possible future assignments.

4.2 A comprehensive working scenario

The working of the model (4.1)–(4.4) is illustrated by means of a simulated, but realistic GBADS scenario, analysed by Van der Merwe [49]. The scenario mimics a typical GBADS deployment consisting of twelve ground-based WSs providing possible protection to two DAs (represented by the black squares in Figure 4.1), DA$_1$ and DA$_2$, respectively.

The twelve WSs deployed in the scenario, consist of eight VSHORADs (labelled $V_1, \ldots, V_8$, respectively), comprising a middle layer of protection, and four CIWSs (labelled C_1, C_2, C_5, C_4, respectively), comprising an inner layer of protection. The WSs are deployed in evenly spaced concentric circle formations around the DAs, so that the distance from a WS to the DAs is approximately half the distance of the effective range of the various WSs. According to military expert Visser [50], the reason for this close deployment, is to enable WSs to be turned around in a situation where threats may approach from the opposite directions. Nevertheless, a graphical illustration of the deployment of the WSs may be seen in Figure 4.1, where the effective ranges of the VSHORADs and CWISs are represented by the blue and red arcs, respectively. The WSs are each labelled on their respective range arcs to avoid clutter in the center of the figure.

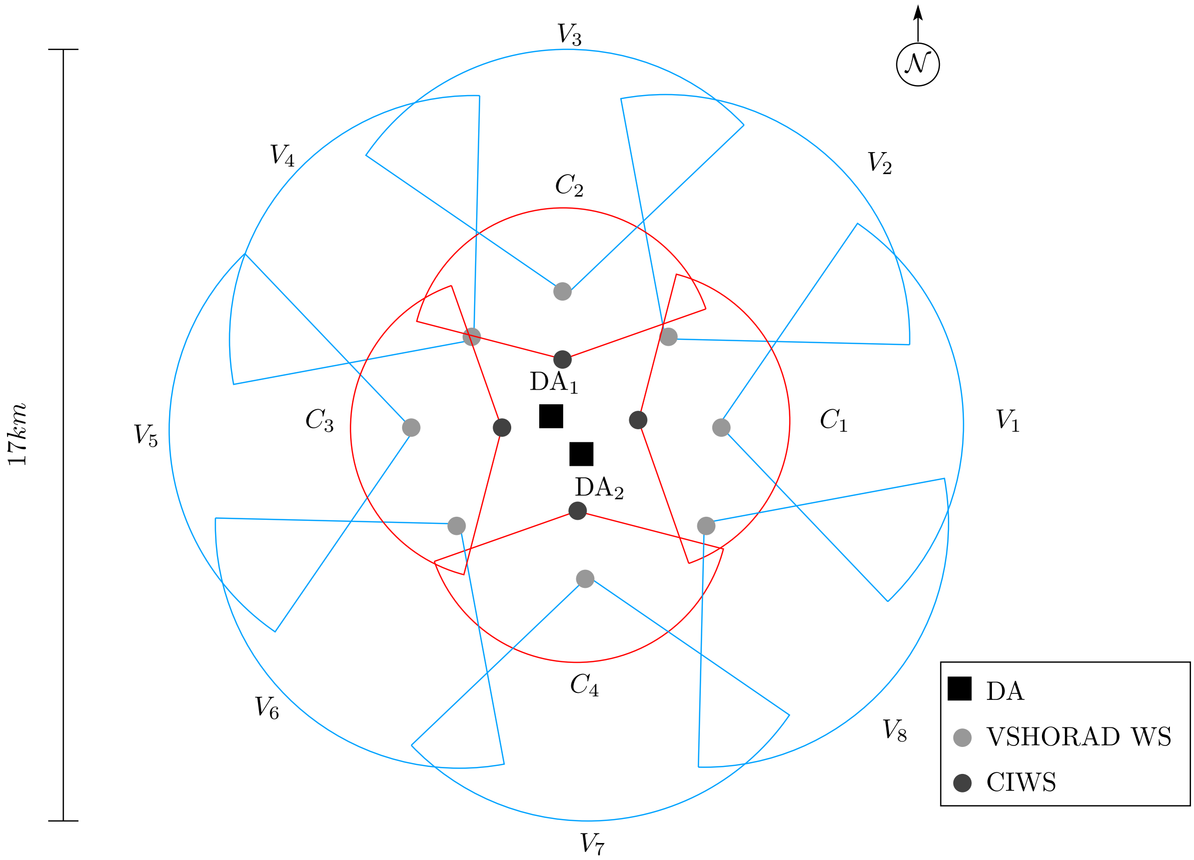


FIGURE 4.1: *The full deployment of WSs with their respective effective ranges.*

Five detected aircraft (labelled T_1, T_2, T_3, T_4, T_5, respectively) act as possible threats to the DAs. They approach the deployment area in a formation of three groups at different time steps. The first group consists of threats one and two (T_1 and T_2, respectively), the second group consists of threats three and four (T_3 and T_4, respectively) and finally threat five (T_5) is unaccompanied.

Threats T_1 and T_2 enter the defended airspace from the north-west and attack DA_2 using a so-called *pitch-and-dive* technique. They fly in a straight line, approaching at low altitude and initially their flight paths do not cross the DAs, until they are at a distance of approximately $7\,500\,m$ from DA_1. Both of them pitch and turn in to DA_2 to dive straight onto DA_2. Their ammunition is delivered at an approximate distance of $800\,m$ from DA_2 [40].

Threats T_3 and T_4 act as decoys and enter the area from the south-west, flying exactly over the DAs at high altitude ($4\,400\,m$), whilst maintaining a constant speed of approximately $250\,ms^{-1}$ until they exit towards the north-east of the DAs.

Threat T_5 enters from the south-west and attacks DA_2 using the so-called *toss bomb technique*. It enters the area in Figure 4.2 in a straight line which does not intersect the DAs. It turns in towards DA$_2$ (to the left) at an approximate distance of $9\,000\,m$ from DA$_2$ to release a bomb at an approximate distance of $8\,000\,m$, from where it exits the system in a south-westerly direction [40]. The five threats, together with an approximation of their respective tracks may be seen in Figure 4.2.

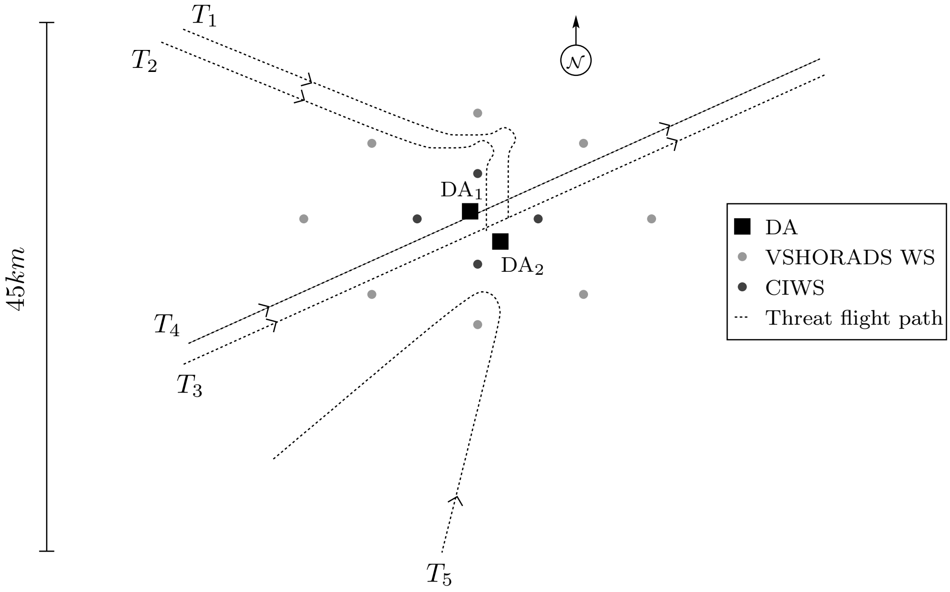


FIGURE 4.2: *The scenario.*

The time continuum of the scenario is subdivided into t_α shorter time steps, each having a duration of four seconds, because the sensors utilised for detection refresh the air picture every four seconds to give an update on the current position of the threats. Since the proposed models from previous sections consider only a single time step at a time, and the time steps are independent of one another, it is redundant to solve the scenario over the entire time continuum. Hence, only time steps t_{20}, t_{35} and t_{39} are considered. These time steps were chosen for their diversity, and an approximation of the location of the threats at each of these respective time steps may be seen in Figures 4.3, 4.4 and 4.5.

It is assumed that the TE process has been completed and that threat values have been evaluated for each of the threats at each time step. The output received from the TE subsystem is in the form of a combined threat list, and the evaluated threat lists for time steps t_{20}, t_{35} and t_{39} may be found in Table 4.3.

Threat	t_{20}	t_{35}	t_{39}
T_1	0.04	0.91	0.99
T_2	0.05	0.94	1
T_3	0.91	0.96	0.76
T_4	0.92	0.95	0.74
T_5	0.06	1	0.5

TABLE 4.3: *The threat values of threats $T_1, \ldots, T_5$ for time steps t_{20}, t_{35} and t_{39}.*

The efficiencies of the WSs are all contained in the EEM and an EEM$_{t_\alpha}$ instance, of the EEM, is generated for each time step t_α. Terrain features and weather conditions are ignored and hence the EEM matrices are not filtered for any terrain or environmental conditions. The EEM

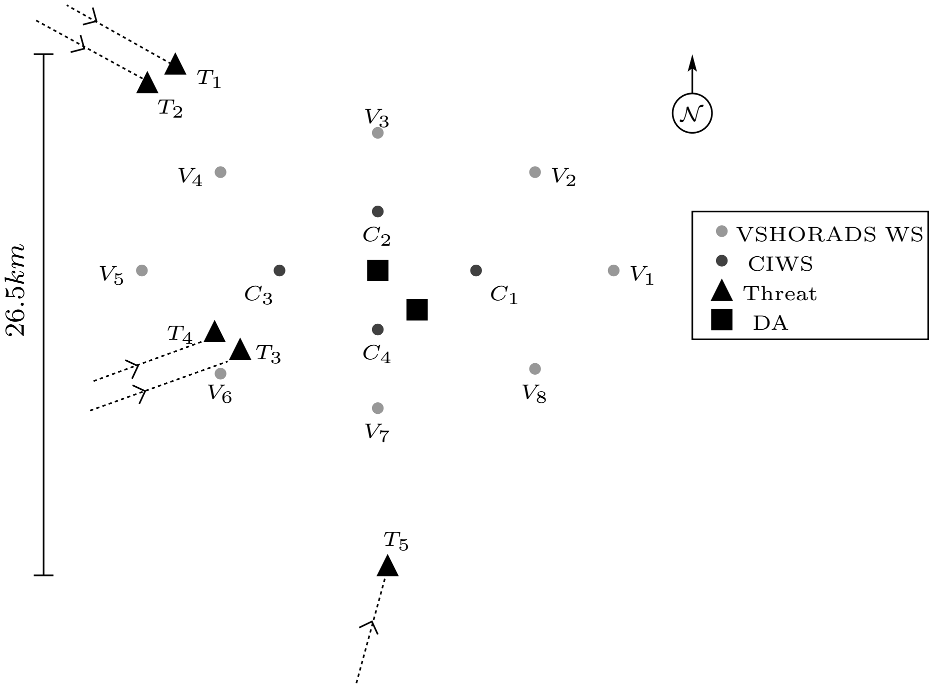


FIGURE 4.3: *The locations of threats* T_1, T_2, T_3, T_4 *and* T_5 *at time step* t_{20}.

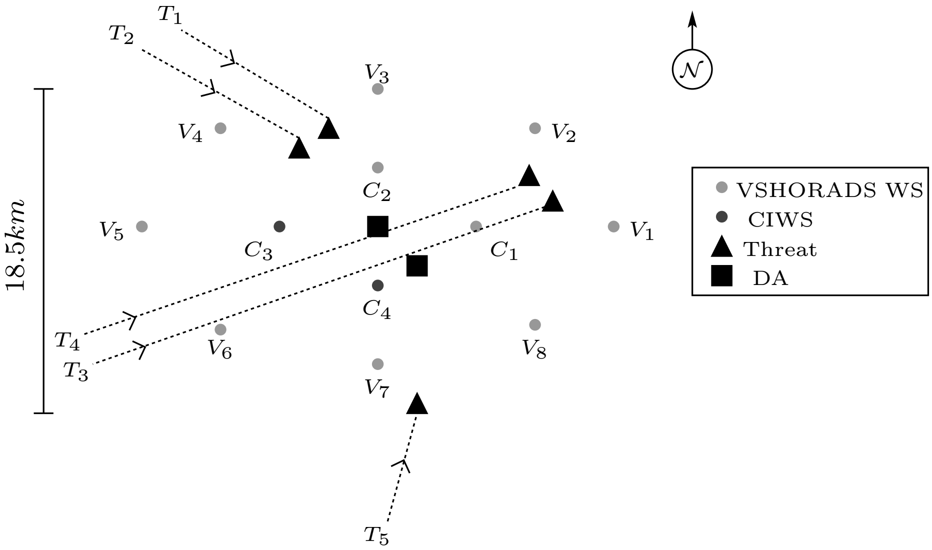


FIGURE 4.4: *The locations of threats* T_1, T_2, T_3, T_4 *and* T_5 *at time step* t_{35}.

therefore contains only SSHP information. The EEM instances for $\text{EEM}_{t_{20}}$, $\text{EEM}_{t_{35}}$ and $\text{EEM}_{t_{39}}$ are shown in Table 4.4.

Finally, it is assumed that the cost of assigning a WS to a threat comprises of only the cost of a single burst of ammunition of that specific WS. After consulting with Visser [50], it is safe to assume that the cost of a VSHORAD is approximately $R\,1\,000\,000$, while the cost of a CIWS is approximately $R\,34\,000$.

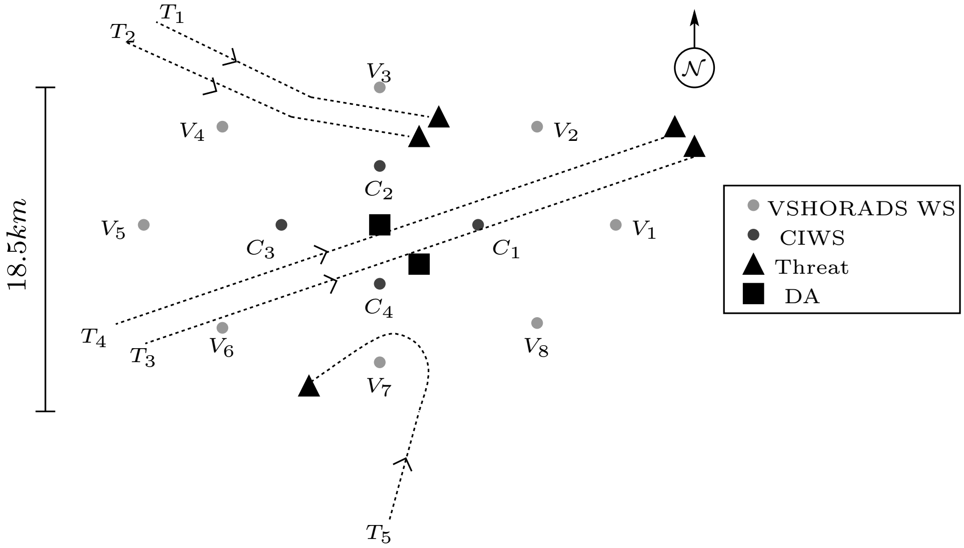


FIGURE 4.5: *The locations of threats T_1, T_2, T_3, T_4 and T_5 at time step t_{39}.*

4.3 The analytic hierarchy process in WA

The AHP assessment procedure requires the input of a decision maker whose main responsibility is to make pairwise comparisons between the different objectives, to obtain weights for each objective, as well as pairwise comparisons between different alternatives with respect to each objective, and to obtain score values for each alternative with respect to each objective. This assessment procedure was carried out by the author together with military expert Visser [50], who will be referred to as the decision maker in the remainder of this chapter.

The two objectives which were considered during the AHP assessment, is to maximise the SSHP of WSs assigned and to minimise the cost of WSs assigned. The assumptions are made that a WS may only be assigned to one threat at each time step and that only one WS may be assigned to a threat at each time step.

After explaining the AHP process to the decision maker, the first step involved the assessment of appropriate weights for each of the objectives. This was achieved by asking the decision maker to complete a pairwise comparison matrix containing the cost and SSHP objectives as presented in Table 4.5.

After careful consideration, the decision maker concluded that SSHP is absolutely more important than cost and by using the index values from Table 3.2, a corresponding value of 9 was obtained when comparing the SSHP to cost. Hence, when comparing the cost to the SSHP, a value of $\frac{1}{9}$ is obtained, which completes the pairwise comparison matrix, yielding the matrix in Table 4.6.

After normalising the matrix in Table 4.6 and taking the average of each row, respectively, weights of 0.9 and 0.1 were obtained for the SSHP and the cost objective, respectively. This indicates that the SSHP objective is considered significantly more important than the cost objective, which is also evident in the feedback obtained from the WA surveys. Since there are only two objectives, only one comparison is involved, which is considered consistent.

The next step was to calculate score values for each of the alternatives with respect to each objective. In view of the scenario described in §4.2, the AHP procedure has to be carried out

	T_1	T_2	T_3	T_4	T_5
V_1	0	0	0	0	0
V_2	0	0	0	0	0
V_3	0	0	0	0	0
V_4	0	0	0	0	0
V_5	0	0	0	0.5	0
V_6	0	0	0.1	0.1	0
V_7	0	0	0	0	0
V_8	0	0	0	0	0
C_1	0	0	0	0	0
C_2	0	0	0	0	0
C_3	0	0	0.1	0.1	0
C_4	0	0	0	0	0

$EEM_{t_{20}}$

	T_1	T_2	T_3	T_4	T_5
V_1	0	0	0.1	0.5	0
V_2	0	0	0.3	0.1	0
V_3	0.1	0.5	0	0	0
V_4	0.1	0.5	0	0	0
V_5	0	0	0	0	0
V_6	0	0	0	0	0
V_7	0	0	0	0	0
V_8	0	0	0	0	0
C_1	0	0	0	0	0
C_2	0.2	0.3	0	0	0
C_3	0	0.1	0	0	0
C_4	0	0	0	0	0

$EEM_{t_{35}}$

	T_1	T_2	T_3	T_4	T_5
V_1	0	0	0.1	0.1	0
V_2	0.7	0	0.5	0.5	0
V_3	0.1	0	0	0	0
V_4	0	0	0	0	0
V_5	0	0	0	0	0
V_6	0	0	0	0	0.1
V_7	0	0	0	0	0.5
V_8	0	0	0	0	0
C_1	0.2	0.4	0	0	0
C_2	0.5	0.9	0	0	0
C_3	0	0	0	0	0
C_4	0	0	0	0	0

$EEM_{t_{39}}$

TABLE 4.4: *The EEMs for time steps t_{20}, t_{35} and t_{39} respectively.*

Objectives	SSHP	Cost
SSHP	1	
Cost		1

TABLE 4.5: *A pairwise comparison matrix for the cost and SSHP objectives.*

Objectives	SSHP	Cost
SSHP	1	9
Cost	1/9	1

TABLE 4.6: *The complete pairwise comparison matrix for the cost and SSHP objectives.*

for each WS-threat pair for each time step t_α. Instead of bothering the decision maker with the assessment of individual pairwise comparison matrices for each WS-threat pair for each time step, a general pairwise comparison imitating a general index for each objective was developed, which may be used to simplify future comparisons of WS-threat pairs.

To find such an index pairwise comparison matrix, the decision maker was asked to complete the pairwise comparison matrix in Table 4.7, consisting of eleven possible WSs with different SSHP values, covering the entire range of possible SSHP values. If, for example, a pairwise comparison is sought for a WS involving a SSHP value of 0.1 and a WS involving a SSHP value of 0.8, the value may be obtained from the completed version of Table 4.7, by using the value corresponding to the WSs involving these SSHP values. In this way, the completed version of Table 4.7 may be used to make comparisons between WSs with respect to the SSHP objective, without asking the decision maker to complete such a matrix each time.

	WS_j	1	2	3	4	5	6	7	8	9	10	11
WS_i	SSHP	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0	1										
2	0.1		1									
3	0.2			1								
4	0.3				1							
5	0.4					1						
6	0.5						1					
7	0.6							1				
8	0.7								1			
9	0.8									1		
10	0.9										1	
11	1											1

TABLE 4.7: *A pairwise comparison matrix, imitating a general index for the SSHP objective.*

After careful consideration, the decision maker completed the pairwise comparison matrix in Table 4.7, and the completed version of this table is presented in Table 4.8. To verify whether the decision maker was consistent when performing his comparisons, the consistency test, as described in §3.2.1, was carried out and a CI/RI-value of 0.04 was obtained, indicating that the pairwise comparisons are consistent. From Table 4.8 it is clear that the decision maker considers a WS involving a SSHP value of 0.9, the same as a WS involving a SSHP value of 1.

	WS_j	1	2	3	4	5	6	7	8	9	10	11
WS_i	SSHP	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0	1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/9	1/9
2	0.1	2	1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/9
3	0.2	3	2	1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/8
4	0.3	4	3	2	1	1/2	1/3	1/4	1/5	1/6	1/7	1/7
5	0.4	5	4	3	2	1	1/2	1/3	1/4	1/5	1/6	1/6
6	0.5	6	5	4	3	2	1	1/2	1/3	1/4	1/5	1/5
7	0.6	7	6	5	4	3	2	1	1/2	1/3	1/4	1/4
8	0.7	8	7	6	5	4	3	2	1	1/2	1/3	1/3
9	0.8	9	8	7	6	5	4	3	2	1	1/2	1/2
10	0.9	9	9	8	7	6	5	4	3	2	1	1
11	1	9	9	8	7	6	5	4	3	2	1	1

TABLE 4.8: *The complete pairwise comparison matrix, imitating a general index for the SSHP objective.*

Once the general index in Table 4.8 had been obtained, a pairwise comparison matrix could be populated for each WS-threat pair for each time step with respect to the scenario, by using the appropriate corresponding values from Table 4.8. For an illustration of how to use the general index for SSHP, consider the following example.

Example 14 *Suppose the SSHP values of WSs* $V_1, \ldots, V_8, C_1, C_2, C_3, C_4$*, with respect to a particular threat, involves the values presented in Table 4.9. A pairwise comparison matrix has to be populated, with respect to the SSHP objective, involving the WSs in Table 4.9. This may be achieved by means of the general index presented in Table 4.8.*

WS	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4
SSHP	0	0	0	0.5	0	0	0.9	0	0	0	0	0

Table 4.9: *The SSHP values for a particular threat, to illustrate the working of the general index in Table 4.8.*

WSs involving similar SSHP values are considered the same, and a value of 1 is used in the comparisons between these WSs. When comparisons between WS V_4 *and the WSs yielding a SSHP value of 0 are considered, a value of 6 should be assigned by considering Table 4.8, since WS* V_4 *involves a SSHP value of 0.5.*

When comparisons between WS V_7 *and the WSs yielding a SSHP of 0 are considered, a value of 9 should be assigned. Finally, when comparing WS* V_4 *and WS* V_7*, a value of 5 should be assigned.*

Completing the pairwise comparison matrix in this way, yields the matrix presented in Table 4.10. This pairwise comparison matrix may be used next to calculate score values with respect to each WS, so as to use in the calculation of the final score values, to recommend the best possible assignment of WSs to threats. □

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4
V_1	1	1	1	1/6	1	1	1/9	1	1	1	1	1
V_2	1	1	1	1/6	1	1	1/9	1	1	1	1	1
V_3	1	1	1	1/6	1	1	1/9	1	1	1	1	1
V_4	6	6	6	1	6	6	1/5	6	6	6	6	6
V_5	1	1	1	1/6	1	1	1/9	1	1	1	1	1
V_6	1	1	1	1/6	1	1	1/9	1	1	1	1	1
V_7	9	9	9	5	9	9	1	9	9	9	9	9
V_8	1	1	1	1/6	1	1	1/9	1	1	1	1	1
C_1	1	1	1	1/6	1	1	1/9	1	1	1	1	1
C_2	1	1	1	1/6	1	1	1/9	1	1	1	1	1
C_3	1	1	1	1/6	1	1	1/9	1	1	1	1	1
C_4	1	1	1	1/6	1	1	1/9	1	1	1	1	1

Table 4.10: *A pairwise comparison matrix for the SSHP objective.*

The same procedure may be followed to obtain a general index with respect to the the cost objective. The decision maker was asked to complete a general pairwise comparison matrix similar to the one presented for the SSHP objective assessment. Since the WSs contained in the scenario vary between a VSHORAD WS and a CIWS, only pairwise comparisons between these two WSs are considered with respect to their cost values.

After careful consideration, the decision maker concluded that a CIWS is considered strongly more important than a VSHORAD WS with respect to cost. By using the index values from Table 3.2 a corresponding value of 5 was obtained, resulting in the complete pairwise comparison matrix in Table 4.11. Since this matrix contains only two alternatives, the pairwise comparisons consist of only one pair, which is considered consistent.

	WS_j	WS_1	WS_2
WS_i	Cost	34 000	1 000 000
WS_1	34 000	1	5
WS_2	1 000 000	1/5	1

Table 4.11: *The complete pairwise comparison matrix, imitating a general index for the cost objective.*

Once the general index in Table 4.11 had been obtained, a pairwise comparison matrix similar to the one in Table 4.10 could be populated by using appropriate corresponding values from the pairwise comparison matrix in Table 4.11. Of course, in this case, the populated matrix is with respect to the cost objective. Since the cost values of WSs remain the same for all the threats, and for each time step of the scenario, the pairwise comparison matrix for the cost objective remains unchanged for the duration of the scenario. The complete pairwise comparison matrix with respect to the cost objective for the scenario is presented in Table 4.12.

Cost	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4
V_1	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5
V_2	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5
V_3	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5
V_4	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5
V_5	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5
V_6	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5
V_7	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5
V_8	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5
C_1	5	5	5	5	5	5	5	5	1	1	1	1
C_2	5	5	5	5	5	5	5	5	1	1	1	1
C_3	5	5	5	5	5	5	5	5	1	1	1	1
C_4	5	5	5	5	5	5	5	5	1	1	1	1

Table 4.12: *The complete pairwise comparison matrix for the cost objective.*

Since the pairwise comparison matrix for the cost objective remains the same for each of the time steps of the scenario, the score values for each WS alternative could be calculated with respect to the cost objective. After normalising the pairwise comparison matrix in Table 4.12, and calculating the average of each row, a score value was obtained for each of the respective WSs as presented in Table 4.13.

Using the pairwise comparison matrices of this section, the weights of each objective may be combined with the score values of each alternative, with respect to each objective, so as to obtain the final score values for each of the WS-threat pairs. Once this is finalised, a choice can to be made regarding which WS to assign to a threat. This seems straight forward if only

WS	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4
Score	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.18	0.18	0.18	0.18

Table 4.13: *Score values with respect to cost, calculated for each of the WSs to be utilised in the calculation of the final score values in the AHP.*

one threat is observed, since the WS yielding the highest overall score is considered as the best alternative. However, in the proposed scenario there are five threats, which implies that WSs have to be assigned to multiple threats simultaneously, and the fact that a WS may only be assigned to one threat at a time, complicates matters even further.

One way of addressing this, is to employ the threat list from the TE subsystem and assigning threats in a greedy manner. This may be achieved by ranking the threats in descending order of their threat values, and assigning WSs to threats in the order of their rankings (*i.e.* the threat with the highest threat value is assigned a WS first, the threat with the second highest threat value is assigned a WS second *etc.*). The threat involving the highest threat value is considered first, and the WS involving the highest score value with respect to that threat is assigned to that threat. The next threat in the ordered threat list is then considered, and the WS involving the highest score value with respect to that threat is assigned to the threat. The process continues until all the threats in the threat list have been assigned WSs. If a proposed WS for a specific threat is already assigned to a higher ranked threat, the second best WS (*i.e.* the WS yielding the second highest overall score value, with respect to that specific threat) is assigned to that threat, and so forth.

An alternative way of assigning the WS to the threats is to develop an assignment model in which the aim is to assign WSs to threats with respect to their final overall score values. The objective of the assignment model is to maximise the total score values of all the assigned WSs, weighted by the priorities of eliminating threats when WS-threat proposals are made.
Stated mathematically, the problem is therefore to

$$\text{maximise } S = \sum_{j=1}^{n_t} V_j \sum_{i=1}^{n_w} s_{ij} x_{ij}, \tag{4.5}$$

subject to the constraints

$$\sum_{j=1}^{n_t} x_{ij} \leq 1, \qquad i = 1, \ldots, n_w, \tag{4.6}$$

$$\sum_{i=0}^{n_w} x_{ij} \leq k, \qquad j = 1, \ldots, n_t, \tag{4.7}$$

$$x_{ij} \in \{0, 1\}, \qquad i = 1, \ldots, n_w, \tag{4.8}$$

$$j = 1, \ldots, n_t, \tag{4.9}$$

where V_j represents the priority of eliminating threat j and s_{ij} represents the final score value of a WS-threat pair. The number of threats and WSs are denoted by n_t and n_w, respectively, and the decision variables are denoted by x_{ij}. The decision variables are binary in nature, and x_{ij} assumes the value 1 if WS i is assigned to threat j, or 0 otherwise. The model is constrained by assuming that a WS may only be assigned to one threat at a time, and that a maximum of k WSs may be assigned to a threat at a time.

The model requires the final set of score values of the WS-threat pairs (in matrix form), as well as the threat list for the same time step, and provides the values of the decision variables as output, which represent the proposed assignments of WSs to threats.

When comparing the WA model in (2.1)–(2.5) with the AHP assignment model in (4.5)–(4.9), it is clear that the objective function in the WA model is nonlinear, whereas the objective function of the AHP assignment model is linear. This implies that the AHP assignment model may easily be solved by using techniques such as the well-known *simplex algorithm* in a branch-and-bound context, whereas the nonlinear WA model may be more difficult to solve, typically by barrier or penalty methods.

The AHP works in such a way that a score value is calculated for each WS-threat pair. Since the AHP aims to maximise the accumulated score value of the WSs assigned, the WS yielding the highest overall score with respect to a specific threat should be recommended for assignment. In the case where a VSHORAD and a CIWS both achieves a SSHP value of 0, the CIWS should therefore be chosen for assignment, since it achieves the highest score value due to the low cost incurred when assigning it. This results in the assignment of a WS which, in effect, achieves an efficiency of 0 with respect to the SSHP objective, and simultaneously results in an increase of the accumulated cost incurred in assigning WSs. To solve this problem of assigning ineffective WSs, a set of dummy WSs was employed. A dummy WS achieves a SSHP value of 0 with respect to the entire set of threats and a cost value of $R\,0$ is incurred if such a WS is assigned to a threat. Assigning a dummy WS therefore imitates the option of not assigning any actual WS. Hence, by adding these dummy WSs to the current set of WSs should result in the model achieving a higher score value than if only the VSHORAD and CIWS were available. The number of dummy WSs to add to the current set of WSs is kn_t. This implies that there should be one dummy WS for each WS that is allowed to be assigned to a threat.

The introduction of the dummy WSs does not have any effect on the AHP pairwise comparison matrices with respect to SSHP. However, since only the outcomes of a VSHORAD and CIWS were considered during the pairwise comparison assessments with respect to cost, a new pairwise comparison matrix has to be populated which includes all three alternatives. Since there is no significant difference between the cost of a CIWS and the cost of a dummy WS, by using the index values from Table 3.2, a pairwise comparison between the dummy WS and the CIWS was assumed to be 2, indicating that a dummy variable is slightly more important than a CIWS with respect to cost. By following the same procedure for a dummy WS and a VSHORAD, a value of 9 was assumed for the comparison between a dummy WS and a VHORADS with respect to cost. The complete pairwise comparison matrix is presented in Table 4.14.

	WS_j	WS_1	WS_2	WS_3
WS_i	Cost	0	34 000	1 000 000
WS_1	0	1	2	9
WS_2	34 000	1/2	1	5
WS_3	1 000 000	1/9	1/5	1

TABLE 4.14: *The complete pairwise comparison matrix with the addition of a dummy WS, imitating a general index for the cost objective.*

In view of the scenario described in §4.2, a complete pairwise comparison matrix was populated, and may be found in Appendix B in Table B.1. The pairwise comparison matrix was verified for consistency by using a RI value of 1.6086 obtained from Alonso and Lamata [6], and a RI/CI value of 0.0002 was obtained, implying that the pairwise comparisons were consistent. The score values corresponding to each of the WSs which were used in the proposed scenario are presented in Table 4.15.

WS	Score
V_1	0.0136
V_2	0.0136
V_3	0.0136
V_4	0.0136
V_5	0.0136
V_6	0.0136
V_7	0.0136
V_8	0.0136
C_1	0.0660
C_2	0.0660
C_3	0.0660
C_4	0.0660
D_1	0.1255
D_2	0.1255
D_3	0.1255
D_4	0.1255
D_5	0.1255

TABLE 4.15: *Score values with respect to cost, calculated for each of the WSs including five dummy WSs to be utilised in the calculation of the final score values in the AHP.*

4.4 A functional utility approach

The AHP and the utility approaches are similar in the sense that both are subjective in nature. This implies that the utility approach also requires the input of a decision maker. Hence, the utility assessments were also carried out in conjunction with military expert Visser [50], acting as the decision maker. A brief introduction to the processes involved during the assessment of utility values was given to the decision maker so as to ensure that he clearly understood what was required of him.

The utility model is restricted by producing a solution for each WS-threat pair at a time, implying that the model has to be solved for each of the WS-threat pairs at each time step. The same two objectives utilised in the AHP approach toward WA are also utilised in the utility approach (*i.e.* to maximise the SSHP of WSs assigned to the threats and to minimise the cost of WSs assigned simultaneously). Two constraints were enforced during the development of the utility model for WA. They are the restrictions of allowing only one WS to be assigned to a threat and of assigning a WS at most once, for each time step.

The guidelines discussed in §3.3.7 were followed in that qualitative characteristics of utility functions were first obtained for each of the objectives (*i.e.* SSHP and cost), after which quantitative values for each of the objectives were computed. Before these assessments could commence, the independence between these two objectives first had to be verified. This was achieved by using the independent conditions discussed in §3.4. Finally, a combined bi-objective utility function was constructed.

4.4.1 Independence between SSHP and cost

The first independence condition to verify is mutual preferential independence. This was achieved by verifying that the SSHP objective is preferentially independent of the cost objective, and that the cost objective is preferentially independent of the SSHP objective. The SSHP and cost values used in the assessment procedures of all the independence conditions were chosen as 0, 0.2, 0.5, 0.7 and 0.9 for the SSHP objective and as $R\,34\,000, R\,300\,000, R\,500\,000$, $R\,750\,000$ and $R\,1\,000\,000$ for the cost objective, respectively, since these values are evenly spaced out and cover the range of possible SSHP and cost values.

To verify whether the cost objective is preferentially independent of the SSHP objective, the decision maker was asked to indicate his preference between different pairs of cost values, whilst keeping the value of SSHP fixed at a certain level.

To render the process comprehensible for the decision maker, the decision maker was asked to imagine that he had a choice between two WSs, WS_1 and WS_2. Both WSs involve a SSHP value of 0.2 (*i.e.* the fixed objective function value) and different cost values. The decision maker was asked to indicate his preference between WS_1 and WS_2, with respect to the cost values presented in Table 4.16. The decision maker indicated that he prefers the WS involving the lower cost value (*i.e.* WS_1). The process was repeated by fixing the SSHP values at 0.5, 0.7 and 0.9, respectively. At each of these assessments, the decision maker indicated that he preferred the WS involving the lower cost value (*i.e.* WS_1). The decision maker concluded by stating that if the SSHP value is fixed (regardless of the value), he will always prefer the lower cost value. Based on these findings, it is safe to assume that the cost objective is preferentially independent of the SSHP objective.

Cost WS_1	Cost WS_2
$R\,34\,000$	$R\,1\,000\,000$
$R\,34\,000$	$R\,500\,000$
$R\,300\,000$	$R\,750\,000$
$R\,500\,000$	$R\,900\,000$
$R\,750\,000$	$R\,1\,000\,000$

Table 4.16: *Different pairs of cost values used to verify whether cost is preferentially independent of SSHP.*

In order to ascertain mutual preferential independence, the SSHP objective should also be preferentially independent of the cost objective. While fixing the cost value at $R\,34\,000$, the decision maker was therefore asked to indicate his preference between two different WSs involving different SSHP values, as presented in Table 4.17. The decision maker indicated that he preferred the WS involving the higher SSHP value (*i.e.* WS_2) between each pair. The process was repeated by fixing the cost value at each of the remaining values that is, $R\,300\,000, R\,500\,000, R\,750\,000$ and $R\,1\,000\,000$, respectively. At each of the assessments the decision maker indicated that he preferred the WS involving the higher SSHP value (*i.e.* WS_2). Finally, he concluded by stating that if the cost value is fixed (no matter what the value), he will always prefer the higher SSHP value.

Based on these findings, is seems reasonable to accept that SSHP is preferentially independent of cost and since both the objectives are preferentially independent of one another, the conclusion can be drawn that they are mutually preferentially independent.

The procedure for verifying the stronger utility independent condition involves similar assessments to those carried out during the preferential independence assessments, with the difference being that the decision maker is asked to assess CE values with respect to a given lottery involving different outcomes in one objective, whilst keeping the value of the other objective fixed at a specific level. A lottery involving the same form as the lottery presented in Figure 3.20

SSHP WS_1	SSHP WS_2
0	0.9
0	0.5
0.2	0.5
0.2	0.9
0.5	0.7
0.5	0.9
0.7	0.9

TABLE 4.17: *Different pairs of SSHP values used to verify whether SSHP is preferentially independent of cost.*

may be used to simplify the assessment.

To verify whether cost is utility independent of SSHP, the lottery presented in Figure 4.6 was used. Once again the decision maker was asked to imagine that he has a choice between WS_1 and WS_2 which both WSs involve a SSHP of 0.2 (*i.e.* the fixed objective function value), where WS_1 incurs a cost value if assigned which is linked to a probability distribution involving two outcomes with an equal chance of occurring and where WS_2 yields a sure unknown outcome. The decision maker was asked to assess his CE which makes him indifferent between WS_1 and WS_2, with respect to the cost value of WS_2. After careful consideration, the decision maker indicated that his CE for the values presented in Figure 4.6 is $R\,517\,000$. The process was repeated with the value of the SSHP fixed at 0.2, but by using different cost values in the lottery. The cost values utilised in the assessment, as well as the corresponding CE values are presented in Table 4.18.

The procedure described above was repeated for lotteries involving the same values presented in Table 4.18, but by fixing the value of the SSHP at 0.5, 0.7 and 0.9, respectively. The decision maker provided the same CE for each of the lotteries, whilst keeping the SSHP values fixed at the specified levels. The decision maker concluded that if the SSHP value is fixed (regardless of the value), his CE will always be equal to the expected value of the lottery, when considering to cost. Hence, it was concluded that cost is utility independent of SSHP.

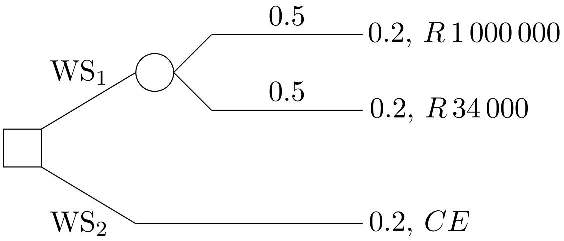


FIGURE 4.6: *A lottery used to verify whether cost is utility independent of SSHP.*

To verify whether SSHP is utility independent of cost, a similar lottery to the one in Figure 4.6 was used, but where the cost values were fixed at $R\,34\,000, R\,300\,000, R\,500\,000, R\,750\,000$ and $R\,1\,000\,000$, respectively, and the SSHP values were varied. The decision maker provided the same CE with respect to SSHP values for each of the lotteries, whilst keeping the cost value fixed at the specified levels. These SSHP values as well as the corresponding CE values are presented in Table 4.19. He concluded that if the cost value is fixed (regardless of the value), his CE assessments for the values presented in Table 4.19, will remain the same. Hence, this verifies that SSHP is utility independent of cost.

50% Cost outcome WS_1	50% cost outcome WS_1	CE WS_2
$R\,34\,000$	$R\,1\,000\,000$	$R\,517\,000$
$R\,34\,000$	$R\,500\,000$	$R\,267\,000$
$R\,300\,000$	$R\,750\,000$	$R\,525\,000$
$R\,500\,000$	$R\,900\,000$	$R\,700\,000$
$R\,750\,000$	$R\,1\,000\,000$	$R\,875\,000$

TABLE 4.18: *Different cost values used in a lottery to verify whether cost is utility independent of SSHP.*

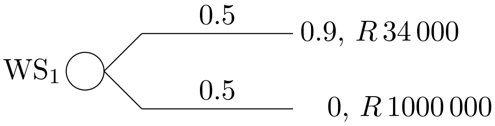
50% SSHP outcome WS_1	50% SSHP outcome WS_1	CE WS_1
0	0.9	0.7
0	0.5	0.4
0.2	0.7	0.6
0.3	0.5	0.4
0.7	0.9	0.8

TABLE 4.19: *Different SSHP values used in a lottery to verify if SSHP is utility independent of cost.*

Since the cost objective and the SSHP objective are also utility independent of one another they are, in fact, mutually utility independent. Hence, the multilinear utility function may be used to model the preferences of the decision maker for these two objectives.

One last utility independent condition should be tested, which may simplify the form of the final multiobjective utility function even further. Since the objectives are mutually utility independent, only one assessment is necessary to verify whether they are additive independent. The decision maker was presented with WS_1 and WS_2, as shown in Figure 4.7, where each WS involves two possible outcomes, each having an equal chance of occurring. WS_1 achieves the best possible values of the SSHP objective and the cost objective in one outcome, and the worst possible values of the SSHP objective and the cost objective in the other outcome. WS_2 achieves the best possible value of the SSHP objective and the worst possible value of the cost objective in one outcome, and the worst possible value of the SSHP objective and the best possible value of the cost objective in the other outcome.

The decision maker was required to indicate his preference between WS_1 and WS_2, or to indicate whether he is indifferent between the WSs. If he indicates, in any sense, that he prefers either of the WSs, the SSHP objective and the cost objective are not additive independent. However, if he is indifferent between these two WSs, the SSHP objective and the cost objective are additive independent.



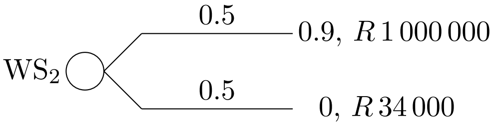


FIGURE 4.7: *Two lotteries to test whether cost and SSHP are additive independent.*

The decision maker indicated that he is indifferent between the two WSs. He concluded that since the SSHP values are the same for both WSs, the only factor contributing to his decision is the cost. Since the expected values with respect to cost are equal for both of the lotteries, the decision maker considers WS_1 and WS_2 as the same.

This result showed that the cost objective and the SSHP objective are additive independent. Hence, the utility function for each objective may be assessed without considering the other objective. These individual utility functions may then be scaled and combined to form an additive utility function in order to represent the preferences of the decision maker for the cost objective and the SSHP objective simultaneously.

4.4.2 Assessing qualitative characteristics for SSHP and cost

Now that the form of the bi-objective utility function for cost and SSHP is known, an individual utility function for each of these objectives could be assessed. Before assessing the quantitative utility values of the individual utility functions, it may be useful to first assess some qualitative characteristics with respect to these objectives. A first qualitative characteristic is to establish whether the utility functions are monotonic or not. During the assessment of preferential independence, the decision maker consistently indicated that he preferred a smaller cost value to a larger cost value among a number of cost value pairs. This may be a good enough reason to suspect that the utility function for the cost objective is monotonically decreasing.

For the SSHP values, the decision maker consistently indicated that he preferred a larger SSHP value to a lower SSHP value among a number of SSHP value pairs. This may be a good enough reason to suspect that the utility function for the SSHP objective is monotonically increasing.

However, these suspicions may be verified by presenting the decision maker with possible outcomes of two potential WSs with respect to both objectives. In each case, he was asked to indicate his preference between the two WSs. The values used during the assessment to verify monotonicity for the cost and SSHP objectives are presented in Table 4.20. For the SSHP values, the decision maker consistently preferred the WS involving the larger SSHP value. Once all the assessments in Table 4.20 were completed, the following question was put to the decision maker: “If WS_i involves a larger SSHP value than WS_j, will WS_i always be preferred to WS_j?” The decision maker answered affirmatively, implying that the utility function for the SSHP objective is monotonically increasing.

For the cost values, the decision maker consistently indicated that he preferred the WS with the lower cost. Once all the assessments in Table 4.20 had been completed, a final question was put to the decision maker: “If WS_i involves a lower cost value than WS_j, will WS_i always be preferred to WS_j?” The decision maker answered affirmatively, implying that the utility function for the cost objective is monotonically decreasing. The corresponding preferences of the decision maker are also presented in Table 4.20.

The next qualitative characteristic involved determining the risk attitude of the decision maker. This is achievable by presenting the decision maker with a number of lotteries and sure outcomes, which are equal to the expected value of each respective lottery, similar to those in Figure 3.18. The decision maker is then required to indicate his preference between the lottery or the sure outcome.

The lotteries were presented to the decision maker in the form of two WSs, WS_1 and WS_2. The outcome of assigning WS_1 is modelled by a lottery involving two outcomes with equal probability, while WS_2 achieves the sure outcome. The values utilised in the lotteries and sure outcomes are summarised in Tables 4.21 and 4.22 for the SSHP and cost objectives, respectively. The first two columns represent the outcomes associated with WS_1, whereas the third column

WS_i	WS_j	Choice
0	0.9	0.9
0.1	0.7	0.7
0.1	0.5	0.5
0	0.3	0.3
0.3	0.9	0.9
0.3	0.7	0.7
0.3	0.5	0.5
0.5	0.9	0.9
0.5	0.7	0.7

SSHP

WS_i	WS_j	Choice
R 34 000	*R* 1 000 000	*R* 34 000
R 34 000	*R* 750 000	*R* 34 000
R 34 000	*R* 300 000	*R* 34 000
R 150 000	*R* 900 000	*R* 150 000
R 300 000	*R* 750 000	*R* 300 000
R 300 000	*R* 1 000 000	*R* 300 000
R 450 000	*R* 600 000	*R* 450 000
R 600 000	*R* 900 000	*R* 600 000
R 750 000	*R* 1 000 000	*R* 750 000

Cost

TABLE 4.20: *Tables to establish whether the decision maker's utility functions are monotonic for the SSHP objective and the cost objective.*

represents the sure outcome, and the fourth column contains the preference of the decision maker.

50% WS_1	50% WS_1	WS_2	Preference
0.9	0.1	0.5	WS_1
0.9	0.3	0.6	WS_1
0.9	0.5	0.7	WS_1
0.9	0.7	0.8	WS_1
0.7	0.3	0.5	WS_1
0.7	0.1	0.4	WS_1
0.6	0.2	0.4	WS_1
0.5	0.3	0.4	WS_1
0.5	0.1	0.3	WS_1

SSHP

TABLE 4.21: *SSHP values for determining the risk attitude of the decision maker with respect to the SSHP objective.*

The decision maker consistently indicated that he preferred the WS involving the lottery to the WS with the sure outcome (*i.e.* prefers WS_1 to WS_2) with respect to the SSHP values. This implies that he exhibits a risk seeking attitude. He explained that from a WA perspective, he would rather assign a WS involving a chance to yield a higher SSHP value than a WS with a lower SSHP value for sure. The decision maker was slightly hesitant when the SSHP values became large, especially in the case where WS_1 involved an even chance of a SSHP of 0.9 and 0.7 and WS_2 involved a sure SSHP value of 0.8. The decision maker was asked to carefully think about his preference, and in this case he concluded that a SSHP value of 0.8 is not significantly better than a SSHP value of 0.7. He therefore decided to still choose WS_1.

The ensure that the decision maker understood the consequences of his preferences, he was asked the following question: "If the expected value of WS_1 is always equal to the value of WS_2, will you always prefer WS_1 to WS_2?" His answer was affirmative. Hence, the conclusion may therefore be drawn that the decision maker exhibits a risk seeking attitude with respect to the SSHP objective. This implies that a utility function for the SSHP objective should take on a convex form.

50% WS_1	50% WS_1	WS_2	Preference
R 34 000	R 1 000 000	R 517 000	Indifferent
R 34 000	R 500 000	R 267 000	Indifferent
R 34 000	R 250 000	R 142 000	Indifferent
R 300 000	R 1 000 000	R 650 000	Indifferent
R 300 000	R 750 000	R 525 000	Indifferent
R 500 000	R 1 000 000	R 750 000	Indifferent
R 500 000	R 750 000	R 625 000	Indifferent
R 250 000	R 1 000 000	R 625 000	Indifferent
R 250 000	R 750 000	R 500 000	Indifferent

Cost

TABLE 4.22: *Cost values for determining the risk attitude of the decision maker with respect to the cost objective.*

For the cost assessments, the decision maker explained that since the cost value of WS_1 is equal to the expected cost value of WS_2 at each assessment, he regards WS_1 and WS_2 the same, with respect to cost values. Hence, he is indifferent between WS_1 and WS_2, despite the fact that certain assessments involved large cost values of up to R 1 000 000. To ensure that the decision maker understood the consequence of his preferences he was asked the following question: "If the expected cost value of WS_1 is always equal to the cost value of WS_2, will you always be indifferent to these two WSs?" His response was affirmative. Hence, the conclusion was drawn that he exhibits a risk neutral behaviour, with respect to the cost values. This implies that his utility function for the cost objective should take on a linear form (*i.e.* a straight line).

4.4.3 Assessing individual utility functions for SSHP and cost

Quantitative utility values were finally assessed for each of the cost and SSHP objectives in order to be presented as a utility function. The CE approach, as discussed in §3.3.6, was followed to assess the utility values of the outcomes with respect to both objectives. The process involves the decision maker assessing his CE for lotteries involving two different outcomes with equal chance. This is achievable by presenting the decision maker with a number of lotteries similar to the one presented in Figure 3.10. The outcomes in the lotteries are varied between the possible outcomes with respect to the two objectives, in order to obtain corresponding CE values. Utility values may then be calculated for the CE values.

A lottery was once again presented to the decision maker as WS_1 and the CE as WS_2. In each assessment the decision maker was required to indicate the SSHP value of WS_2 which will make him indifferent between assigning WS_1 and WS_2. Since the best and worst outcomes of the SSHP objective are 1 and 0, respectively, these outcomes were assigned utility values of 1 and 0, respectively, and they were utilised in the first CE assessment. WS_1 involves two outcomes, each having an equal chance of occurring. The decision maker was asked to think carefully about his CE with respect to WS_1, and he indicated that if WS_2 involves a SSHP value of 0.7, he would be indifferent between WS_1 and WS_2. The utility value corresponding to a SSHP value of 0.7 may thus be calculated as

$$u(0.7) = 0.5u(1) + 0.5u(0) = 0.5(1) + 0.5(0) = 0.5. \tag{4.10}$$

Since the utility values corresponding to SSHP values of 0, 0.7 and 1 are then known, any two of them could be used in the next assessment. The next assessment involved SSHP values of 1 and 0.7, and the decision maker provided a CE value of 0.9 for this lottery. The utility value corresponding to a SSHP value of 0.9 was therefore calculated in the same way as for a SSHP value of 0.7, and the result is $u(0.9) = 0.75$. This process was repeated until a number of values from the range of possible SSHP values had been considered. The SSHP values, as well as the corresponding CE values assessed are presented in Table 4.23. The corresponding utility values are presented in Table 4.24, and illustrated graphically in Figure 4.8.

50% WS$_1$	50% WS$_1$	CE	Expected value
0	0.9	0.7	0.45
0.7	1	0.9	0.85
0.7	0.9	0.8	0.80
0	0.7	0.5	0.35
0.5	0.7	0.6	0.60
0	0.5	0.4	0.25
0	0.4	0.3	0.20
0	0.3	0.2	0.15
0	0.2	0.1	0.10

Table 4.23: *CE values assessed for different SSHP values.*

SSHP value	Utility value
1	1
0.9	0.75
0.8	0.6250
0.7	0.5000
0.6	0.3750
0.5	0.2500
0.4	0.1250
0.3	0.0625
0.2	0.0313
0.1	0.0156
0	0

Table 4.24: *Utility values calculated for the SSHP values presented in Table 4.23.*

By examining Figure 4.8, it is clear that the utility function has a convex form, implying that the decision maker is risk seeking. This verifies the qualitative characteristic that the utility function of the SSHP is convex. The fact that the majority of the CE values are greater than the expected values of their corresponding lotteries in Table 4.23, also verifies the fact that the utility function of the SSHP is risk seeking.

A utility function corresponding to the graph in Figure 4.8 was sought. Since the SSHP objective only contains eleven possible values, these values may either be used directly when solving the WA problem in the context of the scenario discussed in §4.2, or a curve may be fitted through the data points in Table 4.24 to approximate the utility function. The problem with fitting a function is that it is only an approximation of the original set of data points, which may leave room for small errors. Since the number of data points are small, it was decided to rather use a piecewise linear utility function through the data points in Table 4.24. A linear function was obtained between each of the data points. The resulting piecewise linear convex utility function

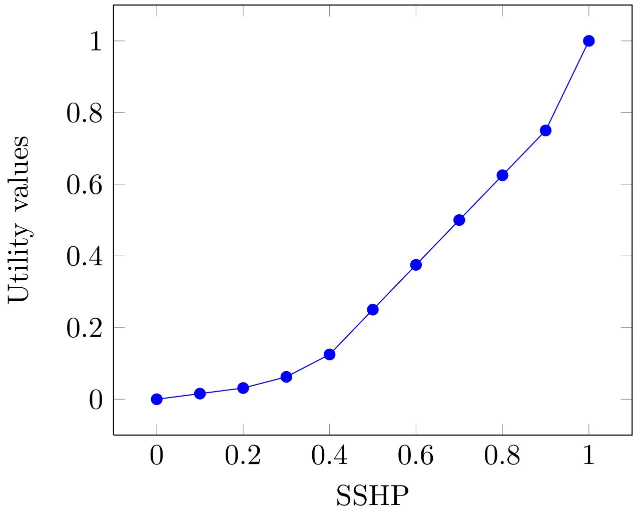


FIGURE 4.8: *The utility function corresponding to the utility values of the SSHP objective.*

for SSHP corresponding to the data points in Table 4.24 is

$$u_{\text{SSHP}}(x) = \begin{cases} 0.156x - 1.839 \times 10^{-18}, & \text{if } 0 < x \leq 0.1, \\ 0.157x - 0.0001, & \text{if } 0.1 < x \leq 0.2, \\ 0.312x - 0.0311, & \text{if } 0.2 < x \leq 0.3, \\ 0.625x - 0.125, & \text{if } 0.3 < x \leq 0.4, \\ 1.25x - 0.375, & \text{if } 0.4 < x \leq 0.9, \\ 2.5x - 1.5, & \text{if } 0.9 < x \leq 1. \end{cases} \tag{4.11}$$

The same procedure may be followed to determine utility values for the cost objective. By presenting the same lottery concept involving two different WSs to the decision maker, he was required to provide his CE values for different cost values covering the possible range of cost values.

The best and worst outcomes for cost, which are $R\,0$ (in the case of no assignment) and $R\,1\,000\,000$, respectively, were assigned utility values of 1 and 0, respectively. Since the utility values of these two outcomes were known, they are employed in the first assessment. The decision maker provided a CE value of $R\,500\,000$ and the corresponding utility value was calculated as

$$u(R\,500\,000) = 0.5u(R\,0) + 0.5u(R\,1\,000\,000) = 0.5(1) + 0.5(0) = 0.5. \tag{4.12}$$

A next assessment was considered involving any one of the outcomes with a known utility value, and the corresponding utility value was calculated accordingly. The process was repeated until a fair number of assessments had been made, covering the full range of possible cost values, and the evaluated utility values may be presented as a utility function. The different cost values used in the assessment, as well as the corresponding CE values, are presented in Table 4.25. The resulting utility values are presented in Table 4.26 and illustrated graphically in Figure 4.9.

By examining the graph in Figure 4.9, it is evident that the utility function for the cost objective is linear, implying a risk neutral attitude with respect to the cost objective. This verifies the qualitative characteristic discussed previously. Since the utility function of the cost objective is a decreasing linear function, it should have the form of a linear function $u_{\text{cost}}(y) = -my + c$, where m denotes the gradient of the graph and c is a constant term. A linear utility function through the cost values and their corresponding utility values is $u_{\text{cost}}(y) = 1 - 1 \times 10^{-6}y$.

50% WS_1	50% WS_1	CE
$R\,0$	$R\,1\,000\,000$	$R\,500\,000$
$R\,0$	$R\,500\,000$	$R\,250\,000$
$R\,500\,000$	$R\,1\,000\,000$	$R\,750\,000$

TABLE 4.25: *CE values assessed for different cost values.*

Cost value	Utility value
$R\,0$	1
$R\,250\,000$	0.75
$R\,500\,000$	0.50
$R\,750\,000$	0.25
$R\,1\,000\,000$	0

TABLE 4.26: *Utility values calculated for the range of cost values.*

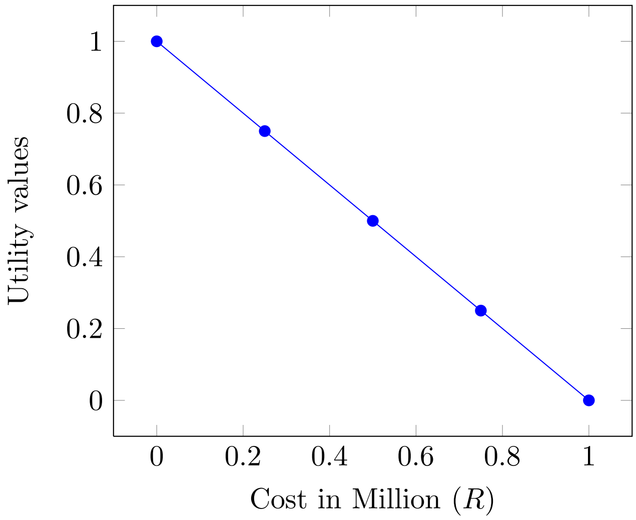


FIGURE 4.9: *The utility function corresponding to the assessed utility values for the cost objective.*

The utility function $u_{\text{cost}}(y)$ is used to evaluate utility values for the range of cost values and compared to the original values to determine the deviation from the original utility values. The original assessed utility values as well as the utility values calculated by means of $u_{\text{cost}}(y)$ with respect to the cost values, are presented in Table 4.27. It is evident from Table 4.27 that the originally assessed utility values are all on the line $u_{\text{cost}}(y) = 1 - 1 \times 10^{-6}y$, so that this function is the utility function which represents the preferences of the decision maker with respect to the cost objective.

4.4.4 Assessing scaling constants for SSHP and cost

The final step, before the multiobjective utility function may be compiled, is to assess scaling constants for each of the objectives. This is achieved by using the lottery process described in §3.4. The decision maker is presented with a lottery A and a sure outcome B, as illustrated in Figure 4.10. The lottery A involves two outcomes, where the first outcome is the best

Cost value	Original utility value	$u_{\text{cost}}(y)$
$R\,0$	1	1
$R\,250\,000$	0.75	0.75
$R\,500\,000$	0.50	0.50
$R\,750\,000$	0.25	0.25
$R\,1\,000\,000$	1	1

TABLE 4.27: *A comparison between the original utility values of cost, and the utility values calculated by means of* $u_{cost}(y)$.

possible outcomes in both objectives with a probability of p and the second outcome involves the worst possible outcomes in both objectives with a probability of $(1-p)$. The sure outcome B, is varied depending on which one of the scaling constants is sought. The decision maker is required to provide the probability p which makes him indifferent between the lottery A and the sure outcome B.

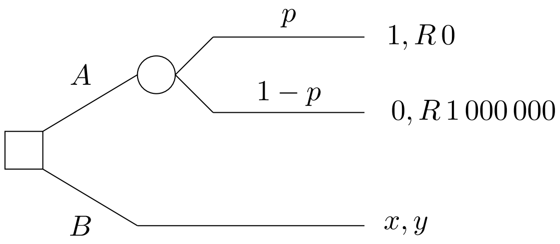


FIGURE 4.10: *Lottery used to determine scaling constants for the SSHP and cost objectives.*

The decision maker is finally asked to imagine that lottery A represents WS_1, involving outcomes linked to a probability distribution and the sure outcome B represents WS_2 involving a sure outcome.

To assess the scaling constant for SSHP, denoted by k_{SSHP}, the decision maker was presented with WS_1 and WS_2 in Figure 4.11. The sure outcomes of WS_2 involve the best outcome in the SSHP objective (*i.e.* the value 1) and the worst outcome in cost objective (*i.e.* the value $R\,1\,000\,000$). The decision maker was required to provide the probability p which makes him indifferent between WS_1 and WS_2.

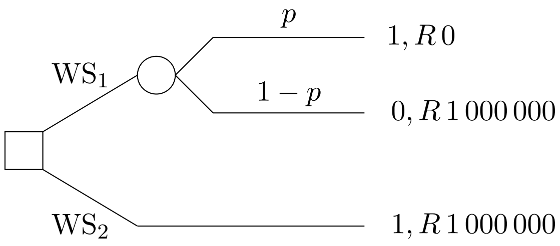


FIGURE 4.11: *Lottery used to determine scaling constant* k_{SSHP}.

To aid the decision maker in the assessment, he was first required to indicate his preference when $p = 0.5$. This yields an expected SSHP value of 0.5 with an expected cost value of $R\,500\,000$, for WS_1. The decision maker explained that although WS_1 involves a possibly smaller cost value, he prefers WS_2 to WS_1, since it involves a much higher sure SSHP value. Since the decision maker had a clear preference, the process have to be repeated for a different value of p. The process was repeated with $p = 0.7$, yielding an expected SSHP value of 0.7 and an expected cost value of $R\,300\,000$, for WS_1. Once again the decision maker indicated that he preferred WS_2 to WS_1, regardless of the possibly lower cost involved in assigning WS_1. When $p = 0.8$, expected SSHP and cost values were calculated as 0.8 and $R\,200\,000$, respectively, and when $p = 0.9$, expected SSHP and cost values were calculated as 0.9 and $R\,100\,000$, respectively, for WS_1.

The decision maker explained that if $p = 0.8$ he would still prefer WS_2 to WS_1 due to the high sure outcome of the SSHP value of WS_2. Finally, he concluded that if $p = 0.9$, he would be indifferent between WS_1 and WS_2 since a SSHP value of 0.9 is considered just as effective as a sure SSHP value of 1. Based on these findings, it was therefore concluded that the scaling constant for the SSHP objective, $k_{\text{SSHP}} = 0.9$.

Since the objectives are additive independent, the scaling constant for the cost objective, denoted by k_{cost}, may be calculated as $k_{\text{cost}} = 1 - k_{\text{SSHP}}$, which yields an answer of 0.1, if $k_{\text{SSHP}} = 0.9$. However, this may be verified by using the same procedure during the assessment of k_{SSHP}, to evaluate the scaling constant for the cost objective k_{cost}.

The decision maker was presented with a lottery involving WS_1 and WS_2, as shown in Figure 4.12. In this case, the outcome of WS_1 remains unchanged, whereas the outcome of WS_2 involves the worst outcome in the SSHP objective and the best outcome in the cost objective.

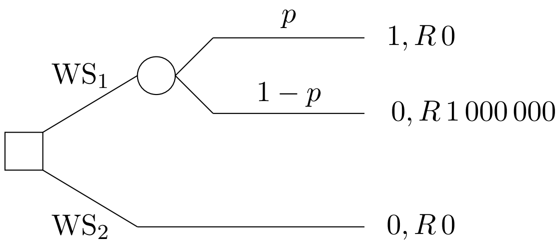


FIGURE 4.12: *A lottery used to determine the scaling constant k_{cost}.*

Since the expected outcomes of both objectives for WS_1 had been calculated when $p = 0.5$, the decision maker was asked to provide his preference between the two WSs. He explained that since WS_1 involves a possible higher SSHP value, he preferred WS_1. When $p = 0.3$, expected values of 0.3 and $R\,700\,000$ were obtained for the SSHP and cost outcomes, respectively, for WS_1. The decision maker indicated that he still preferred WS_1 to WS_2, since he would rather choose a WS involving some chance of hitting a target, than a WS having no chance to do so. When $p = 0.2$ and $p = 0.1$, expected SSHP values of 0.2 and 0.1, were obtained, respectively, and expected cost values of $R\,800\,000$ and $R\,900\,000$, were obtained, respectively.

The decision maker explained that when $p = 0.2$, he still preferred WS_1 to WS_2, since he would rather assign a WS with a SSHP of 0.2, than a WS whose SSHP is 0, even though the cost of assigning this WS is significantly higher. Finally, the decision maker concluded that he will be indifferent between WS_1 and WS_2 if the value of $p = 0.1$.

This verifies that the scaling constant should be $k_{\text{cost}} = 0.1$. Hence the scaling constants for SSHP and cost may be taken as 0.9 and 0.1, respectively, in the compilation of the bi-objective utility function. It is clear from the results obtained during the assessments with the decision maker that he deems the SSHP objective much more important than the cost objective. This is reasonable since the DAs might be worth more than the cost of assigning WSs.

4.4.5 A bi-objective WA utility function

Finally, a bi-objective utility function for the cost and SSHP objectives may be compiled. Combining the individual utility functions of the SSHP and cost objectives using their respective scaling constants, yields the bi-objective additive utility function

$$\begin{aligned} u(x,y) &= 0.9u_{\text{SSHP}}(x) + 0.1u_{\text{cost}}(y) \\ &= 0.9u_{\text{SSHP}}(x) + 0.1(1 - 1 \times 10^{-6}y), \end{aligned} \tag{4.13}$$

where $u_{\text{SSHP}}(x)$ represents the piecewise linear convex utility function for SSHP in (4.11). This utility function may be used to calculate utility values for each WS-threat pair for each time step with respect to the SSHP and cost involved in a specific assignment. The WS yielding the highest utility value with respect to a threat, should then be recommended for possible assignment to that threat.

In the scenario considered in §4.2, WSs have to be assigned to multiple threats simultaneously. By incorporating the threat lists, the same greedy assignment procedure as discussed for the AHP assignment procedure may be used to propose assignments.

An alternative approach is to develop a utility assignment model for recommending possible assignments based on the utility values obtained from the additive utility model above. The objective of such an assignment model should be to maximise the total utility values of all the assigned WSs, weighted by the priorities of eliminating a threat when a WS-threat proposal is made. Therefore, the objective is to

$$\text{maximise } U = \sum_{j=1}^{n_t} V_j \sum_{i=1}^{n_w} u_{ij}x_{ij}, \tag{4.14}$$

subject to the constraints

$$\begin{aligned} \sum_{j=1}^{n_t} x_{ij} &\leq 1, & i &= 1,\ldots,n_w, & (4.15)\\ \sum_{i=0}^{n_w} x_{ij} &\leq k, & j &= 1,\ldots,n_t, & (4.16)\\ x_{ij} &\in \{0,1\}, & i &= 1,\ldots,n_w, & (4.17)\\ & & j &= 1,\ldots,n_t, & (4.18) \end{aligned}$$

where V_j represents the priority of eliminating threat j and u_{ij} represents the combined utility value obtained from $u(x,y)$ in (4.14), involving the SSHP and cost, of a WS-threat pair. The number of threats and WSs are denoted by n_t and n_w, respectively, and the decision variables are denoted by x_{ij}. The decision variables are again binary in nature. The variable x_{ij} takes a value equal to 1 if WS i is assigned to threat j, or zero otherwise. The model is constrained by assuming that a WS may only be assigned to one threat at a time, and that a maximum of k WSs may be assigned to a threat at a time.

The model (4.14)–(4.18) requires the combined set of utility values of the WS-threat pairs (in matrix form) at a time step, as well as the threat list corresponding to the same time step. Thus, the model has to be solved for each time step.

Since the utility approach aims to maximise the accumulated utility value, it will assign a WS to a threat if the utility value of that WS-threat pair achieves the highest positive utility value with respect to other WS-threat pairs. The implication of this is that if a CIWS achieves a SSHP value of zero with respect to a specific threat, a positive utility value will nevertheless be obtained by means of the additive utility function for that specific WS-threat pair. This is due to the $u_{\text{SSHP}}(x)$ achieving a value of 0 and $u_{\text{cost}}(y)$ achieving a value of 0.966. When these values are combined by means of the additive utility model in (4.13) a value of 0.0966 is obtained. If the combined utility values of remaining WSs are 0 with respect to the same threat, the utility model will assign the CIWS. Hence, there is a possibility of assigning WSs to threats which, in effect, achieve zero efficiency with respect to these threats. This results in assigning unnecessary WSs to threats while simultaneously driving up the cost of assigning WSs to threats. This problem was solved by incorporating dummy WSs, denoted by D_ℓ, where ℓ denotes the ℓ^{th} dummy WS. A dummy WS represents an alternative of not assigning a WS. Hence, by adding a dummy WS to the existing set of WSs, three possible choices arise, to assign a VSHORAD WS, to assign a CIWS, or to assign no WS. Since the assignment of dummy WSs involves no assignment of actual WSs, it achieves SSHP values of 0 with respect to the entire set of threats and achieves cost values of $R\,0$. The number of dummy WSs to add to the current set of WSs depends on the value of k and the number of threats n_t, and is given by kn_t.

4.5 The NSGA II

A final approach towards solving the WA problem for the scenario proposed in §4.2 involves a computer implemented version of the NSGA II discussed in §3.5. The NSGA II was coded in the software suite Matlab [37] and is able to solve the WA problem for the complete set of threats in a single run, for each time step. This implies that the NSGA II considers the entire set of WS-threat pairs simultaneously and recommends possible assignments of WSs to threats, with respect to a specific time step. The bi-objective WA model (4.1)–(4.4) described in §4.1, was used in the implementation of the NSGA II.

The NSGA II requires the survival probabilities of the threats (in matrix notation), as well as the threat lists at each time step as input. Since the survival probability of a threat is equal to the difference between a value of one and the corresponding SSHP value, the survival probability for each threat with respect to each WS at time steps t_{20}, t_{35} and t_{39} may be calculated by subtracting each value in the EEMs in Table 4.4 from one. The resulting survival probabilities of the various threats with respect to the WSs, are presented in Table 4.28 for time steps t_{20}, t_{35} and t_{39}. The threat lists presented in Table 4.3 are used as input for each time step. The NSGA II also requires the specification of the number of threats as well as the number of WSs, denoted by n_t and n_w, respectively.

The final requirements, with respect to input data, are the specification of parameter values involving the size of the population, the number of iterations, the tour and pool sizes in the selection procedure and the probability of a mutation occurring. Once the parameter values are specified, the model may be solved for each time step.

The i^{th} solution, or chromosome, p_i to the problem is presented in matrix form with dimensions $n_w \times n_t$. To accommodate the restriction of allowing a WS to be assigned to at most one threat at a time, a solution is allowed to have only one non-zero entry in a row. An example of such a solution may be found in Table 4.29, where WS V_1 is assigned to threat T_2, and WS V_2 is assigned to threat T_5, and so forth. A number, N, of these solutions are contained in the population at each iteration of the NSGA II.

The solutions contained in the population are sorted and ranked on the basis of nondomination,

	T_1	T_2	T_3	T_4	T_5
V_1	1	1	1	1	1
V_2	1	1	1	1	1
V_3	1	1	1	1	1
V_4	1	1	1	1	1
V_5	1	1	1	0.5	1
V_6	1	1	0.9	0.9	1
V_7	1	1	1	1	1
V_8	1	1	1	1	1
C_1	1	1	1	1	1
C_2	1	1	1	1	1
C_3	1	1	0.9	0.9	1
C_4	1	1	1	0	1

Survival probabilities
for time step t_{20}

	T_1	T_2	T_3	T_4	T_5
V_1	1	1	0.9	0.5	1
V_2	1	1	0.7	0.9	1
V_3	0.9	0.5	1	1	1
V_4	0.9	0.5	1	1	1
V_5	1	1	1	1	1
V_6	1	1	1	1	1
V_7	1	1	1	1	1
V_8	1	1	1	1	1
C_1	1	1	1	1	1
C_2	0.8	0.7	1	1	1
C_3	1	0.9	1	1	1
C_4	1	1	1	1	1

Survival probabilities
for time step t_{35}

	T_1	T_2	T_3	T_4	T_5
V_1	1	1	0.9	0.9	1
V_2	0.3	1	0.5	0.5	1
V_3	0.9	1	1	1	1
V_4	1	1	1	1	1
V_5	1	1	1	1	1
V_6	1	1	1	1	0.9
V_7	1	1	1	1	0.5
V_8	1	1	1	1	1
C_1	0.8	0.6	1	1	1
C_2	0.5	0.1	1	1	1
C_3	1	1	1	1	1
C_4	1	1	1	1	1

Survival probabilities
for time step t_{39}

TABLE 4.28: *The survival probabilities of threats T_1, T_2, T_3, T_4 and T_5 for time steps t_{20}, t_{35} and t_{39}, respectively.*

by using the objective function values f_1 and f_2 of the solutions. Objective function value f_1 corresponds to the minimisation of the combined survival probabilities of the threats, weighted by their threat priorities. Since $n_t = 5$ and a threat involves a probability of 1 of surviving, if no WS is assigned to it, the maximum value that f_1 may adopt is 5, implying that f_1 represents a numerical value between 0 and 5.

Objective function value f_2 corresponds to the minimisation of the cost of the WSs assigned to the various threats and may be interpreted as the sum of the cost (in Rand value) of the WSs assigned to the threats. Since there are eight VSHORAD WSs and four CIWS, f_2 may adopt any value in the range from $R\,0$, if no WSs are assigned, to $R\,8\,136\,000$, if the entire set of WSs are assigned. The objective function values are presented in matrix form with dimensions $2 \times N$. An example of the values of f_1 and f_2 corresponding to various candidate solutions is presented in Table 4.30.

The fitness assignment of a solution employed in the NSGA II is based on the pareto rank,

$$\begin{array}{c|ccccc|}
 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 1 & 0 & 0 & 0 \\
V_2 & 0 & 0 & 0 & 0 & 1 \\
V_3 & 1 & 0 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 1 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 1 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 1 & 0 \\
C_1 & 1 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 0 & 1 & 0 & 0 \\
C_3 & 0 & 0 & 1 & 0 & 0 \\
C_4 & 0 & 1 & 0 & 0 & 0
\end{array}$$

TABLE 4.29: *An example of a solution to the bi-objective WA problem utilised in the NSGA II.*

$$\begin{array}{c|ccccc|}
 & p_1 & p_2 & p_3 & \cdots & p_i \\
f_1 & 2.12 & 3.54 & 2.4 & \ldots & 4.2 \\
f_2 & 4\,068\,000 & 2\,034\,0000 & 3\,068\,000 & \ldots & 34\,000
\end{array}$$

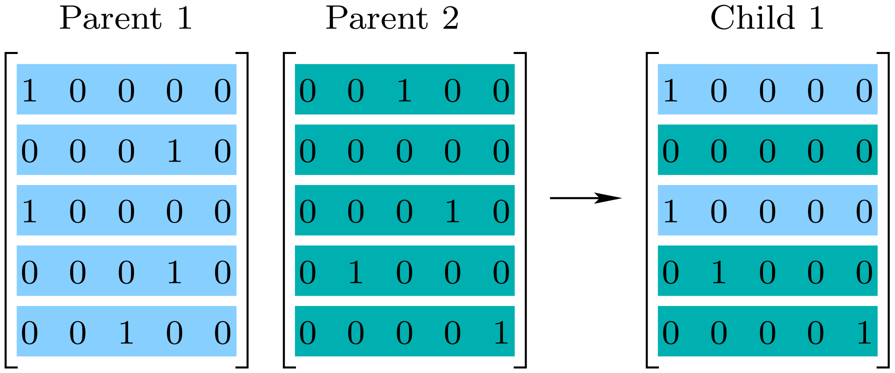
TABLE 4.30: *An example of the objective function values corresponding to different solutions obtained from the NSGA II.*

with respect to a specific solution. If, for example, a solution is contained in rank 1, its fitness value is equal to 1. The fitness values and the crowding distances of solutions are used in the selection process to select parent solutions by using the binary tournament selection procedure described in §3.5.3. In the case where two selected solutions involve the same fitness value, the solution yielding the larger crowding distance is chosen.

Once parent solutions have been selected, the crossover operator may be performed to generate child solutions. This is achieved by considering each corresponding row of two parent solutions at a time, randomly selecting one of these rows to use in the corresponding row of the child solution. The NSGA II was implemented in such a manner that only one child solution is generated during each crossover. This may result in some form of diversity in the child solutions generated. However, the algorithm may take longer to find the approximately pareto optimal solutions, since only one child solution is generated in contrast to the conventional way of generating two child solutions. An example of such a crossover between two parent solutions is presented in Figure 4.13.

Due to the restriction on the row entries of a solution, a bitwise mutation operator is not appropriate, since flipping a random bit (entry) in a solution may possibly lead to a solution with more than one non-zero entry per row, thereby violating constraint (4.2) of the model. A more appropriate mutation operator is therefore to randomly select a row and to change the values of the entries to zero. A random column is then selected, and the entry corresponding to the original selected row is then assigned a value of one. Stated otherwise, a WS is chosen randomly and is unassigned. A threat is then chosen randomly and the randomly chosen WS is assigned to this threat.

The current generation of the population is combined with the child solutions generated from the same generation to form a larger population, which is sorted and ranked. The next generation



$$\begin{bmatrix}1&0&0&0&0\\0&0&0&1&0\\1&0&0&0&0\\0&0&0&1&0\\0&0&1&0&0\end{bmatrix}\begin{bmatrix}0&0&1&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&1&0&0&0\\0&0&0&0&1\end{bmatrix}\longrightarrow\begin{bmatrix}1&0&0&0&0\\0&0&0&0&0\\1&0&0&0&0\\0&1&0&0&0\\0&0&0&0&1\end{bmatrix}$$

FIGURE 4.13: *A graphical illustration of a crossover performed on two parent solutions in the NSGA II.*

is then obtained by selecting parent solutions from this larger population and performing the crossover and mutation operators on these selected solutions. The procedure is repeated until the maximum number of generations is reached.

The results generated by the NSGA II include the solutions contained in each of the nondominated fronts as well as the objective function value of each objective corresponding to each solution, which serves as the output data.

4.6 Chapter summary

The aim in this chapter was to present the reader with an impression of the activities involved in an application of the multiobjective decision methodologies considered in Chapter 3 on a near realistic generated scenario. This was achieved by presenting a method for the extraction of objectives which are deemed important in the choice of which WSs to assign in §4.1. This method was implemented via electronic mail and two of the objectives thus obtained were used to formulate the bi-objective WA model (4.1)–(4.4).

In §4.2 a realistic scenario was comprehensively described. This scenario which was used as an illustrative example for implementation of the model and methodologies. Thereafter the procedures followed during the AHP assessments were described in §4.3 so as to obtain score values for each WS-threat pair. This section closed with a description of a proposed AHP assignment model, which may aid in the assignment of WSs to threats when considering the score values obtained by means of the AHP.

In §4.4 the procedures followed during the assessment of qualitative characteristics as well as quantitative values of the individual utility functions for each objective were described. This section also contains a description of the derivation of a bi-objective additive utility function as well as the assessments of scaling constants for each of the objectives. The section closed with a description of a proposed utility assignment model, which may aid in the assignment of WS to threats when considering utility values.

Finally, the chapter closed in §4.5 with a description of the computer implemented NSGA II in terms of the structure of solutions, input and output data as well as the way in which the genetic operators were performed.

CHAPTER 5

Results and recommendations

Contents

This chapter contains the results obtained by each of the methodologies described in Chapter 4 for solving the WA problem for the scenario of §4.2. The results obtained by means the AHP process are presented and interpreted in §5.1. This is followed, §5.2, by a description and a brief interpretation of the results obtained by means of the bi-objective utility function of §4.4.5. Finally, the results obtained from the output of the NSGA II are presented in §5.3.

5.1 The AHP results

As mentioned in §4.2, the methodologies in this thesis are only applied in the context of the proposed scenario for time steps t_{20}, t_{35} and t_{39}. Since the SSHP values change for each time step, a new pairwise comparison matrix has to be populated for each WS-threat pair, for each of the time steps. Since there are five threats, five pairwise comparison matrices have to be populated for the SSHP objective for each time step by using the index values in Table 4.8. The resulting pairwise comparison matrices may be found in Appendix B. All of these pairwise comparison matrices have been verified for consistency, and the results of these verifications are presented in Appendix B.

Normalising each of the pairwise comparison matrices, and calculating the average of each row yields a score value for each of the WS-threat pairs with respect to the SSHP objective for each time step. The resulting score values are presented in Tables 5.1–5.3, for time steps t_{20}, t_{35} and t_{39}, respectively.

For the cost objective, the score values calculated in §4.3 may be used in the calculation of the final score values. As mentioned previously, these values remain the same for the duration of the scenario; hence, the values remain the same for each time step. Recall that these values may be found in Table 4.15.

Combining the weights of each of the objectives with the score values of each WS for each objective and for each threat, yields five lists of final score values for each WS — one for each

Scores values	T_1	T_2	T_3	T_4	T_5
V_1	0.0588	0.0588	0.0526	0.0411	0.0588
V_2	0.0588	0.0588	0.0526	0.0411	0.0588
V_3	0.0588	0.0588	0.0526	0.0411	0.0588
V_4	0.0588	0.0588	0.0526	0.0411	0.0588
V_5	0.0588	0.0588	0.0526	0.2637	0.0588
V_6	0.0588	0.0588	0.1053	0.0802	0.0588
V_7	0.0588	0.0588	0.0526	0.0411	0.0588
V_8	0.0588	0.0588	0.0526	0.0411	0.0588
C_1	0.0588	0.0588	0.0526	0.0411	0.0588
C_2	0.0588	0.0588	0.0526	0.0411	0.0588
C_3	0.0588	0.0588	0.1053	0.0802	0.0588
C_4	0.0588	0.0588	0.0526	0.0411	0.0588
$D_1 - D_5$	0.0588	0.0588	0.0526	0.0411	0.0588

TABLE 5.1: *Score values with respect to the SSHP objective for time step t_{20}, calculated for each WS-threat pair to be utilised in the calculation of the final score values in the AHP.*

Scores values	T_1	T_2	T_3	T_4	T_5
V_1	0.0475	0.0320	0.0930	0.2679	0.0588
V_2	0.0475	0.0320	0.1950	0.0843	0.0588
V_3	0.0935	0.2056	0.0475	0.0432	0.0588
V_4	0.0935	0.2056	0.0475	0.0432	0.0588
V_5	0.0475	0.0320	0.0475	0.0432	0.0588
V_6	0.0475	0.0320	0.0475	0.0432	0.0588
V_7	0.0475	0.0320	0.0475	0.0432	0.0588
V_8	0.0475	0.0320	0.0475	0.0432	0.0588
C_1	0.0475	0.0320	0.0475	0.0432	0.0588
C_2	0.1479	0.1170	0.0475	0.0432	0.0588
C_3	0.0475	0.0597	0.0475	0.0432	0.0588
C_4	0.0475	0.0324	0.0475	0.0432	0.0588
$D_1 - D_5$	0.0475	0.0324	0.0475	0.0432	0.0588

TABLE 5.2: *Score values with respect to the SSHP objective for time step t_{35}, calculated for each WS-threat pair to be utilised in the calculation of the final score values in the AHP.*

threat at each time step. These results are presented in Tables 5.4–5.6 for time steps t_{20}, t_{35} and t_{39}, respectively. These score values may be used to propose possible assignments of WSs to the threats. Since WSs have to be assigned to multiple threats simultaneously, the threat lists are employed for the purpose of this assignment. The threat lists are ranked in descending order, as shown in Table 5.7 for time steps t_{20}, t_{35} and t_{39}, respectively. The threats are then assigned WSs in order of their threat rankings. Consider, for example, time step t_{20}. Threat T_4 will be assigned a WS first at this time step, followed by threat T_3 and so forth, until the entire set of threats have been assigned WSs.

Consider time step t_{35}. Threat T_5 has the highest threat value at this time step and is therefore assigned a WS first. Considering the fifth column in Table 5.5 it is clear that WSs D_1, D_2, D_3, D_4 and D_5 have the highest score value with respect to threat T_5. Hence, any one of these WSs may be considered for a possible assignment to threat T_5. Suppose WS D_1 is assigned to threat

Scores values	T_1	T_2	T_3	T_4	T_5
V_1	0.0302	0.0339	0.0843	0.0843	0.0432
V_2	0.2773	0.0339	0.2679	0.2679	0.0432
V_3	0.0567	0.0339	0.0432	0.0432	0.0432
V_4	0.0302	0.0339	0.0432	0.0432	0.0432
V_5	0.0302	0.0339	0.0432	0.0432	0.0432
V_6	0.0302	0.0339	0.0432	0.0432	0.0843
V_7	0.0302	0.0339	0.0432	0.0432	0.2679
V_8	0.0302	0.0339	0.0432	0.0432	0.0432
C_1	0.0861	0.1615	0.0432	0.0432	0.0432
C_2	0.1868	0.3299	0.0432	0.0432	0.0432
C_3	0.0302	0.0339	0.0432	0.0432	0.0432
C_4	0.0302	0.0339	0.0432	0.0432	0.0432
$D_1 - D_5$	0.0302	0.0339	0.0432	0.0432	0.0432

TABLE 5.3: *Score values with respect to the SSHP objective for time step t_{39}, calculated for each WS-threat pair to be utilised in the calculation of the final score values in the AHP.*

Scores values	T_1	T_2	T_3	T_4	T_5
V_1	0.0543	0.0543	0.0487	0.0384	0.0543
V_2	0.0543	0.0543	0.0487	0.0384	0.0543
V_3	0.0543	0.0543	0.0487	0.0384	0.0543
V_4	0.0543	0.0543	0.0487	0.0384	0.0543
V_5	0.0543	0.0543	0.0487	0.2387	0.0543
V_6	0.0543	0.0543	0.0961	0.0735	0.0543
V_7	0.0543	0.0543	0.0487	0.0384	0.0543
V_8	0.0543	0.0543	0.0487	0.0384	0.0543
C_1	0.0595	0.0595	0.0540	0.0436	0.0595
C_2	0.0595	0.0595	0.0540	0.0436	0.0595
C_3	0.0595	0.0595	0.1013	0.0436	0.0595
C_4	0.0595	0.0595	0.0540	0.0436	0.0595
$D_1 - D_5$	0.0655	0.0655	0.0599	0.0496	0.0655

TABLE 5.4: *The final score values for each of the WSs for time step t_{20}.*

T_5, implying that no actual WS is assigned to threat T_5.

The second threat to assign a WS to is the one with the second highest threat value, that is threat T_3. Considering the third column in Table 5.5, the WS with the highest overall score value is recommended for assignment. This is WS V_2, which means that WS V_2 may not be considered for any other assignment during time step t_{35}.

The third threat to assign a WS to is threat T_4. By considering column four in Table 5.5, WS V_1 achieves the highest overall score value with respect to threat T_4, and is therefore assigned to threat T_4. By similarly considering column two in Table 5.5 it is clear that WSs V_3 and V_4 yield the highest overall score value with respect to threat T_2 and any one of these WS may therefore be assigned to threat T_2. Finally, by considering column one in Table 5.5, it follows that WS C_2 should be assigned to threat T_1.

The same procedure is followed to assign WSs to threats for time steps t_{20} and t_{35}, respectively.

Scores values	T_1	T_2	T_3	T_4	T_5
V_1	0.0441	0.0301	0.0851	0.2424	0.0543
V_2	0.0441	0.0301	0.1768	0.0772	0.0543
V_3	0.0855	0.1864	0.0441	0.0402	0.0543
V_4	0.0855	0.1864	0.0441	0.0402	0.0543
V_5	0.0441	0.0301	0.0441	0.0402	0.0543
V_6	0.0441	0.0301	0.0441	0.0402	0.0543
V_7	0.0441	0.0301	0.0441	0.0402	0.0543
V_8	0.0441	0.0301	0.0441	0.0402	0.0543
C_1	0.0493	0.0354	0.0493	0.0455	0.0595
C_2	0.1397	0.1062	0.0493	0.0455	0.0595
C_3	0.0493	0.0604	0.0493	0.0455	0.0595
C_4	0.0493	0.0358	0.0493	0.0455	0.0595
$D_1 - D_5$	0.0553	0.0417	0.0553	0.0514	0.0655

TABLE 5.5: *The final score values for each of the WSs for time step t_{35}.*

Scores values	T_1	T_2	T_3	T_4	T_5
V_1	0.0286	0.0319	0.0772	0.0772	0.0402
V_2	0.2509	0.0319	0.2424	0.2424	0.0402
V_3	0.0524	0.0319	0.0402	0.0402	0.0402
V_4	0.0286	0.0319	0.0402	0.0402	0.0402
V_5	0.0286	0.0319	0.0402	0.0402	0.0402
V_6	0.0286	0.0319	0.0402	0.0402	0.0772
V_7	0.0286	0.0319	0.0402	0.0402	0.2424
V_8	0.0286	0.0319	0.0402	0.0402	0.0402
C_1	0.0841	0.1519	0.0455	0.0455	0.0455
C_2	0.1747	0.3035	0.0455	0.0455	0.0455
C_3	0.0338	0.0371	0.0455	0.0455	0.0455
C_4	0.0338	0.0371	0.0455	0.0455	0.0455
$D_1 - D_5$	0.0398	0.0431	0.0514	0.0514	0.0514

TABLE 5.6: *The final score values for each of the WSs for time step t_{39}.*

Time step t_{20}		
Threat	WS	Rank
T_4	0.92	1
T_3	0.91	2
T_5	0.06	3
T_2	0.05	4
T_1	0.04	5

Time step t_{35}		
Threat	WS	Rank
T_5	1	1
T_3	0.96	2
T_4	0.95	3
T_2	0.94	4
T_1	0.91	5

Time step t_{39}		
Threat	WS	Rank
T_2	1	1
T_1	0.99	2
T_3	0.76	3
T_4	0.74	4
T_5	0.50	5

TABLE 5.7: *The ranked threat lists for time steps t_{20}, t_{35} and t_{39}, respectively.*

The assignments of WSs to threats proposed at each time step are summarised in Table 5.8. These assignments are also presented graphically in Figures 5.1–5.3 for time steps t_{20}, t_{35} and t_{39}, respectively.

When considering time step t_{20} and examining Figure 5.1, the assignment of WS V_5 to threat

Time step t_{20}	
Threat	WS
T_4	V_5
T_3	C_3
T_5	D_1/No WS
T_2	D_2/No WS
T_1	D_5/No WS

Time step t_{35}	
Threat	WS
T_5	D_1/No WS
T_3	V_2
T_4	V_1
T_2	V_3 or V_4
T_1	C_2

Time step t_{39}	
Threat	WS
T_2	C_2
T_1	V_2
T_3	V_1
T_4	D_1/No WS
T_5	V_7

TABLE 5.8: *The assignments of WSs proposed by the AHP in conjunction with the greedy assignment heuristic for time steps t_{20}, t_{35} and t_{39}, respectively.*

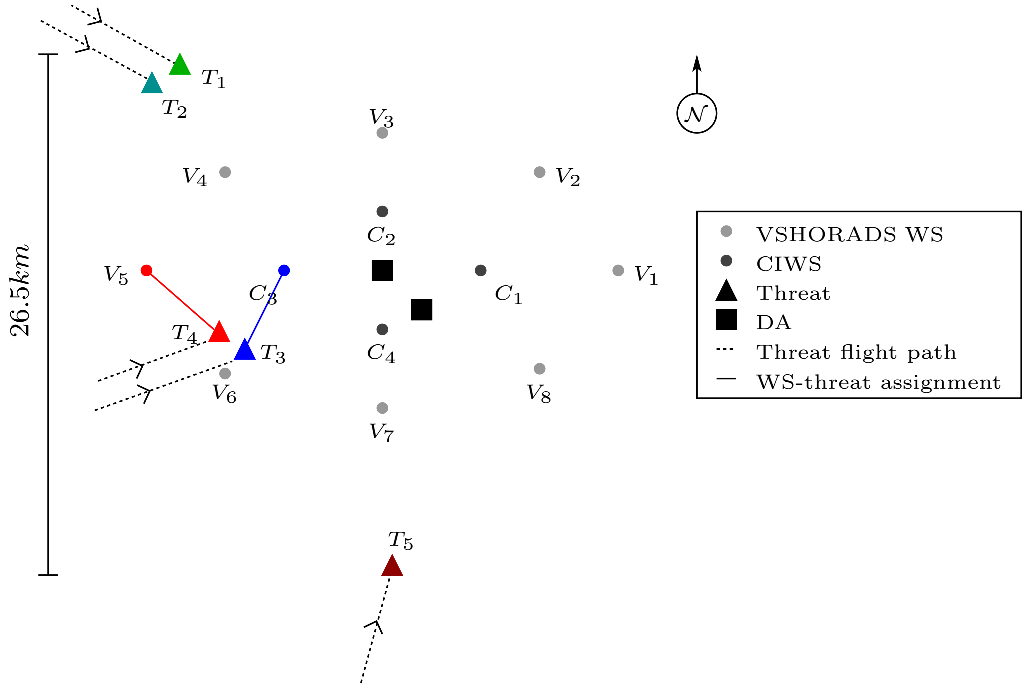


FIGURE 5.1: *A graphical illustration of the recommended assignments of WSs to threats T_1, T_2, T_3, T_4 and T_5 obtained by the AHP in conjunction with the greedy assignment heuristic at time step t_{20}.*

T_4 and the assignment of WS C_3 to threat T_3 seem plausible. Threats T_1, T_2 and T_5 are not assigned any actual WSs.

When considering time step t_{35} and examining Figure 5.2, the assignment of the WSs to the threats seem plausible. However, it seems more intuitive to assign WS V_7 to threat T_5. However, WS V_7 has a SSHP value of zero with respect to threat T_5. This is due to the orientation of WS V_7 (which is parallel to the direction from whence threat T_5 approaches) — threat T_5 is in a so-called *blind spot* of WS V_7, implying a SSHP value of zero.

The assignment model (4.5)–(4.9) discussed in §4.3 may also be used to find possible assignments of WSs with respect to threats T_1, T_2, T_3, T_4 and T_5. The model was coded in the software suite Lingo 11.0 [34], and was solved for time steps t_{20}, t_{35} and t_{39}. To enable a possible comparison between the results obtained by means of the greedy assignment procedure and the AHP assignment model, the parameter value of k is set to the value 1, implying that a maximum of one WS may be assigned to each threat.

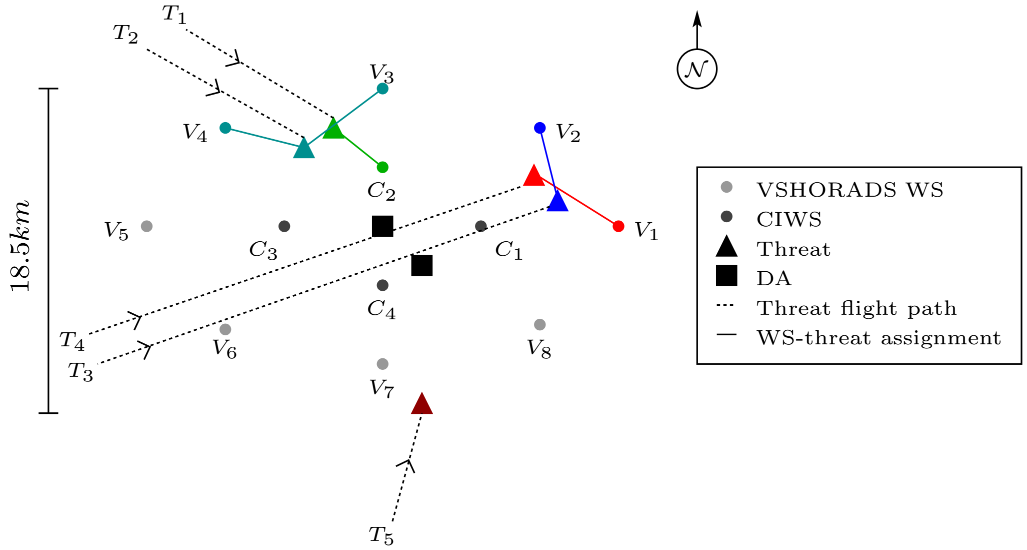


FIGURE 5.2: *A graphical illustration of the recommended assignments of WSs to threats T_1, T_2, T_3, T_4 and T_5 obtained by the AHP in conjunction with the greedy assignment heuristic at time step t_{35}.*

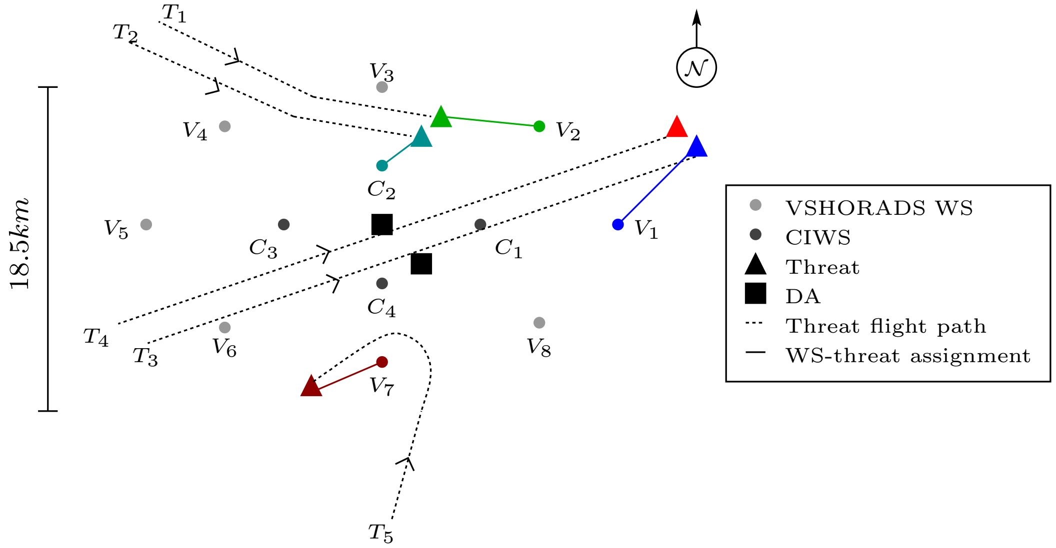


FIGURE 5.3: *A graphical illustration of the recommended assignments of WSs to threats T_1, T_2, T_3, T_4 and T_5 obtained by the AHP in conjunction with the greedy assignment heuristic at time step t_{39}.*

For each time step, the score values in Tables 5.4–Table 5.6 and the corresponding threat list in Table 4.3 are used as input. The proposed assignment lists as well as the corresponding objective function value (denoted by S) and the total cost of the assigned WSs are summarised for each time step in Table 5.9. The assignments are also presented graphically in Figures 5.4–Figure 5.6 for time steps t_{20}, t_{35} and t_{39}, respectively.

When comparing the proposed assignments from the greedy assignment heuristic and the proposed assignments obtained by means of the AHP assignment model (4.5)–(4.9), it is evident from Figures 5.1–5.6 that both methods propose the exact same assignments. This may be due to the small number of WSs and threats present in the system. If a large number of WSs and threats are present, these models may deliver different results.

Time step t_{20}	
Threat	WS
T_1	D_1/No WS
T_2	D_5/No WS
T_3	C_3
T_4	V_5
T_5	D_3/No WS
S	0.3216
Cost	$R\,1\,034\,000$

Time step t_{35}	
Threat	t_{35}
T_1	C_2
T_2	V_4
T_3	V_2
T_4	V_1
T_5	D_1/No WS
S	0.7678
Cost	$R\,3\,034\,000$

Time step t_{39}	
Threat	t_{39}
T_1	V_2
T_2	C_2
T_3	V_1
T_4	D_2/No WS
T_5	V_7
S	0.7679
Cost	$R\,3\,034\,000$

TABLE 5.9: *The assignments of WSs proposed by the AHP assignment model (4.5)–(4.9) for time steps* t_{20}, t_{35} *and* t_{39}.

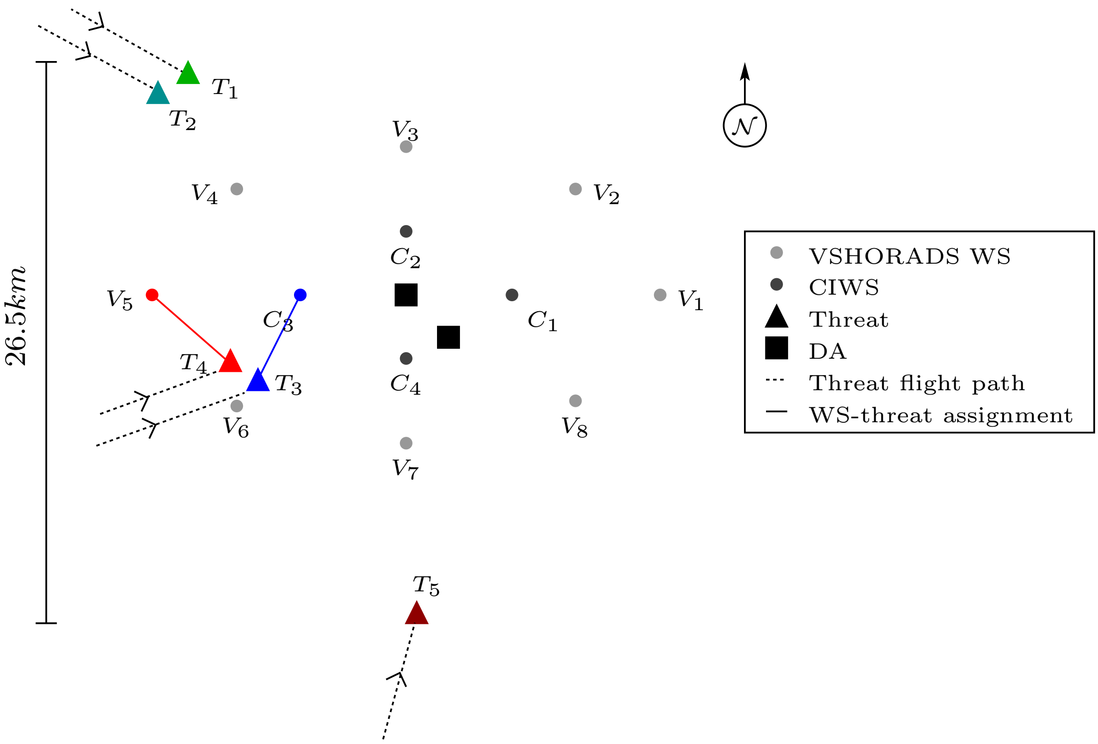


FIGURE 5.4: *A graphical illustration of the recommended assignments of WSs to threats* T_1, T_2, T_3, T_4 *and* T_5 *obtained by the AHP assignment model (4.5)–(4.9) for time step* t_{20}.

All the results presented thus far were generated subject to the constraint that a maximum of $k = 1$ WS may be assigned to a threat. To illustrate how the model works if the value of k is increased, results of the AHP model (4.5)–(4.9) are presented for time step t_{35} with the parameter value $k = 2$. Using the same input values as previously described, the assignments proposed by the model are presented in Table 5.10. These proposed assignments are also presented graphically in Figure 5.7. The model works in such a way that it will assign the maximum number of WSs (denoted by k) to each threat until there are no more WSs available. Hence, in the case where $k = 2$, two WSs are assigned to each threat. In Figure 5.7 it is illustrated that when $k = 2$, the model assigns the same WSs as in the case when $k = 1$ together with an additional WS to each threat. From the results in Table 5.10, it is clear that the additional WSs assigned are all dummy WSs, except for the extra assignment of WS V_4 to threat T_2. These results seem plausible.

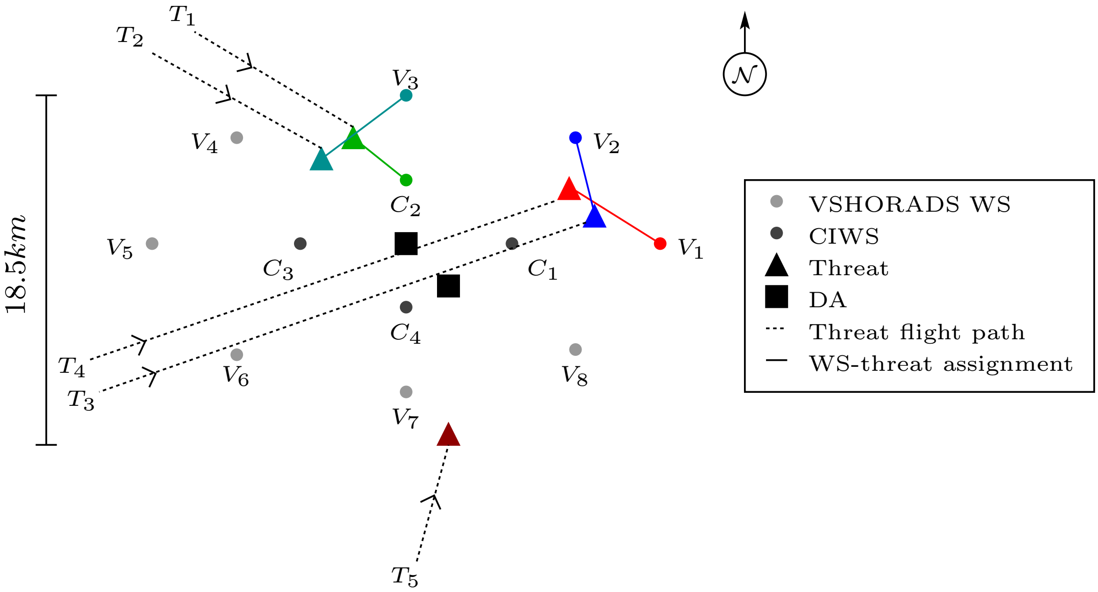


FIGURE 5.5: *A graphical illustration of the recommended assignments of WSs to threats T_1, T_2, T_3, T_4 and T_5 obtained by the AHP assignment model (4.5)–(4.9) for time step t_{35}.*

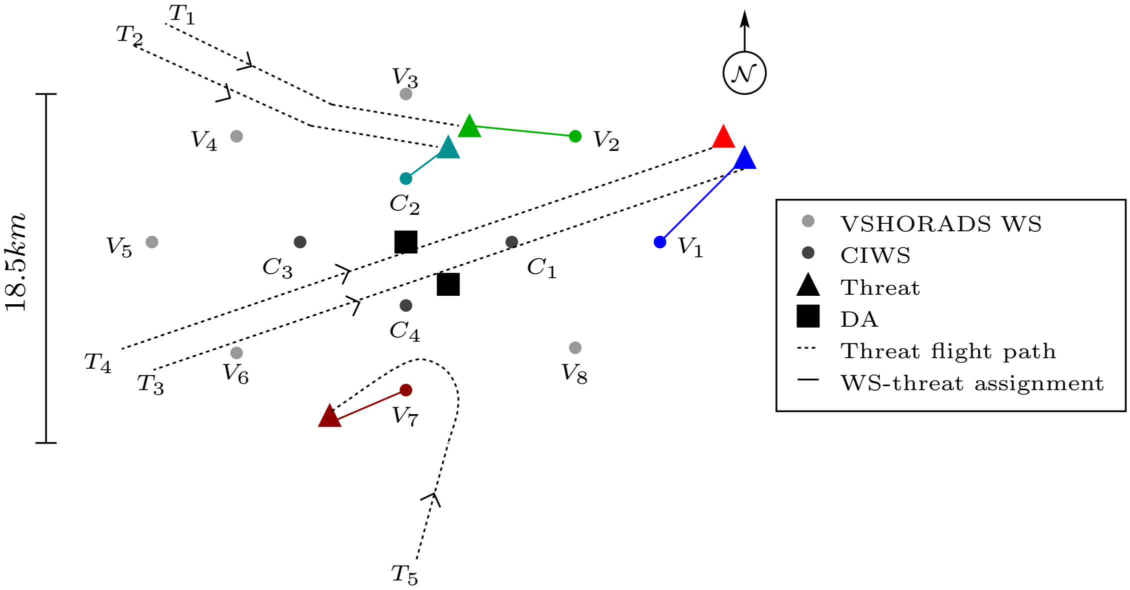


FIGURE 5.6: *A graphical illustration of the recommended assignment of WSs to threats T_1, T_2, T_3, T_4 and T_5 obtained by the AHP assignment model (4.5)–(4.9) for time step t_{39}.*

If the objective function values are considered, there is an increase of 0.1725 in the total score values of the WSs assigned when two WS may be assigned to a threat for time step t_{35}. This is due to the additional assignment of WS V_4 to threat T_2. The extra assignment of WS V_4 results in a better efficiency with respect to threat T_4; however, this results in an increase in the accumulated cost of assigning WSs of $R\,1\,000\,000$.

The results obtained by means of both the greedy assignment heuristic as well as the AHP assignment model in (4.5)–(4.9) are able to achieve a trade-off between the efficiency of WSs assigned and the cost incurred in assigning these WSs, since an increase in the efficiency of WSs results in an increase in the cost incurred in assigning these WSs. Although the AHP seems

Time step t_{35}	
Threat	WS
T_1	C_2 and D_8/No WS
T_2	V_3 and V_4
T_3	V_2 and D_6/No WS
T_4	V_1 and D_9/No WS
T_5	D_1 and D_{10}
S	0.9403
Cost	$R\,4\,034\,000$

TABLE 5.10: *The assignments of WSs proposed by the AHP assignment model (4.5)–(4.9) for time step* t_{35}, *with* $k = 2$.

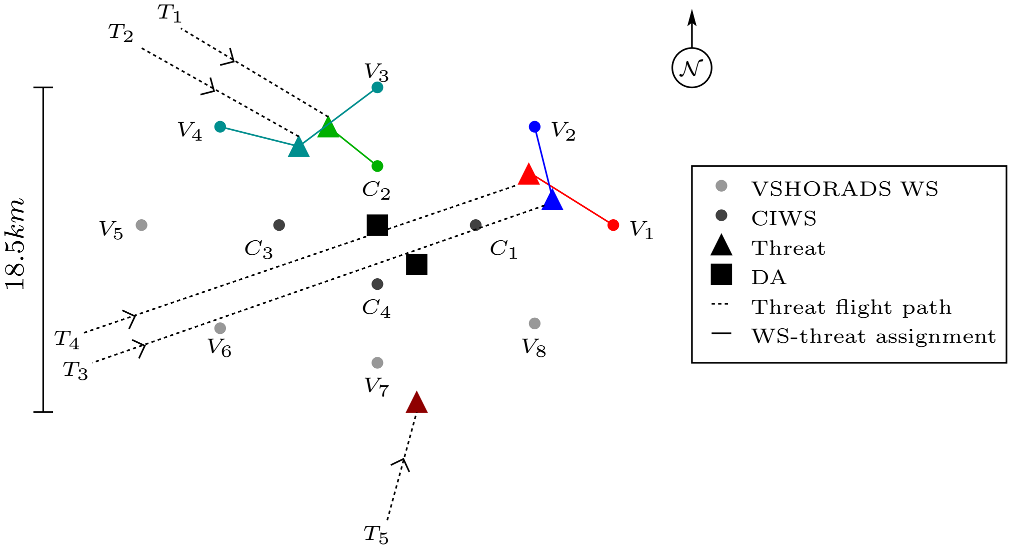


FIGURE 5.7: *A graphical illustration of the recommended assignments of WSs to threats* T_1, T_2, T_3, T_4 *and* T_5 *obtained by the AHP assignment model (4.5)–(4.9) for time step* t_{35}, *with* $k = 2$.

to present plausible results, the processes involved in the assessments of score values are very tedious. Another disadvantage is that if an additional alternative is added to the current set of alternatives, a new pairwise comparison matrix has to be populated from start in order to calculate new score values for each of the alternatives.

5.2 Results obtained by means of the additive utility function

The additive utility function $u(x, y) = 0.9u_{\text{SSHP}}(x) + 0.1u_{\text{cost}}(y)$, derived in §4.4.5, was used to evaluate a combined utility value for each WS-threat pair with respect to the cost and SSHP objectives. Since the SSHP values change during each time step, a new combined utility value has to be evaluated for each WS-threat pair with respect to the SSHP and cost objectives at each time step of the scenario presented in §4.2. Once the combined utility values have been evaluated, the WS-threat pair yielding the highest utility value with respect to a specific threat may be recommended as the best alternative amongst the other WSs for possible assignment.

Again only time steps t_{20}, t_{35} and t_{39} are considered to illustrate the working of WA via the additive utility function. The EEMs in Table 4.4 were used as input to $u_{\text{SSHP}}(x)$ and the cost values corresponding to each WS were used as input to $u_{\text{cost}}(y)$. By using the additive utility function

$$u(x, y) = 0.9(u_{\text{SSHP}}(x)) + 0.1(1 - 1 \times 10^{-6}y), \tag{5.1}$$

combined utility values were evaluated for each WS-threat pair for each time step, as shown in Tables 5.11–5.13 for time steps t_{20}, t_{35} and t_{39}, respectively.

Utility values	T_1	T_2	T_3	T_4	T_5
V_1	0	0	0	0	0
V_2	0	0	0	0	0
V_3	0	0	0	0	0
V_4	0	0	0	0	0
V_5	0	0	0	0.2250	0
V_6	0	0	0.0140	0.0140	0
V_7	0	0	0	0	0
V_8	0	0	0	0	0
C_1	0.0966	0.0966	0.0966	0.0966	0.0966
C_2	0.0966	0.0966	0.0966	0.0966	0.0966
C_3	0.0966	0.0966	0.1106	0.1106	0.0966
C_4	0.0966	0.0966	0.0966	0.0966	0.0966
D_1–D_5	0.1	0.1	0.1	0.1	0.1

TABLE 5.11: *The combined utility values for each of the WS-threat pairs for time step* t_{20}.

Utility values	T_1	T_2	T_3	T_4	T_5
V_1	0	0	0.0140	0.2250	0
V_2	0	0	0.0562	0.0140	0
V_3	0.0140	0.2250	0	0	0
V_4	0.0140	0.2250	0	0	0
V_5	0	0	0	0	0
V_6	0	0	0	0	0
V_7	0	0	0	0	0
V_8	0	0	0	0	0
C_1	0.0966	0.0966	0.0966	0.0966	0.0966
C_2	0.1247	0.1529	0.0966	0.0966	0.0966
C_3	0.0966	0.1106	0.0966	0.0966	0.0966
C_4	0.0966	0.0966	0.0966	0.0966	0.0966
D_1–D_5	0.1	0.1	0.1	0.1	0.1

TABLE 5.12: *The combined utility values for each of the WS-threat pairs for time step* t_{35}.

When only a single threat is observed to which WSs have to be assigned, an assignment may easily be proposed by choosing the WS yielding the highest utility value with respect to that threat. However, when multiple threats are considered simultaneously, the WA problem may be solved by employing the threat priorities of the threats contained in the threat lists. The ranked threat list in Table 5.7 was used again in conjunction with a greedy assignment heuristic to assign WSs to threats in order of their rank. The heuristic involves a process where the

Utility values	T_1	T_2	T_3	T_4	T_5
V_1	0	0	0.0140	0.0140	0
V_2	0.4500	0	0.2250	0.2250	0
V_3	0.0140	0	0	0	0
V_4	0	0	0	0	0
V_5	0	0	0	0	0
V_6	0	0	0	0	0.0140
V_7	0	0	0	0	0.2250
V_8	0	0	0	0	0
C_1	0.1248	0.2091	0.0966	0.0966	0.0966
C_2	0.3216	0.7716	0.0966	0.0966	0.0966
C_3	0.0966	0.0966	0.0966	0.0966	0.0966
C_4	0.0966	0.0966	0.0966	0.0966	0.0966
D_1–D_5	0.1	0.1	0.1	0.1	0.1

TABLE 5.13: *The combined utility values for each of the WS-threat pairs for time step t_{39}.*

highest ranked threat is assigned a WS first by assigning the WS yielding the highest utility value with respect to the specific threat. Thereafter a WS is assigned to the threat involving the second highest rank by assigning the WS yielding the highest utility value with respect to the specific threat provided that this WS has not already been assigned to a higher ranked threat. This process continues until all the threats have been assigned a WS. If it happens that a WS yielding the highest utility value with respect to a specific threat has already been assigned to a higher ranked threat, the WS yielding the second highest utility value may be assigned to that specific threat.

To illustrate the working of the process, consider time step t_{39}. Since threat T_2 involves the lowest rank, it is considered first. When examining column two in Table 5.13, it is found that WS C_2 yields the highest utility value with respect to T_2 and it is therefore assigned to threat T_2.

Since threat T_1 achieves the second highest rank, it is considered next. By examining the first column in Table 5.13, it is found that WS V_2 yields the highest utility value with respect to T_1 and it is therefore assigned to threat T_1. Threat T_3 is considered next, and by examining the third column in Table 5.13, it is found that WS V_2 should be assigned to threat T_3. However, WS V_2 has already been assigned to threat T_1. Hence, the WS yielding the second highest utility value should instead be assigned to threat T_1. WSs D_1–D_5 yield the same second highest utility values with respect to threat T_3. Suppose WS D_1 is assigned to threat T_3, implying that no actual WS is assigned to threat T_3. Threat T_4 is considered next, and by examining the forth column in Table 5.13, it is found that WS V_2 should be assigned to threat T_4, however, WS V_2 is already assigned to threat T_1. The WS yielding the second highest utility value with respect to threat T_4 is WSs D_1–D_5. Since WS D_1 has already been assigned to a higher ranked threat, any one of the remaining dummy WSs may be assigned to threat T_4. Suppose WS D_2 is assigned to threat T_4, again implying that no actual WS is assigned to threat T_4. Finally, threat T_5 is considered, and by examining the fifth column in Table 5.13, it is found that WS V_7 yields the highest utility value with respect to T_5. This WS is therefore assigned to threat T_5.

The greedy assignment heuristic was also applied at time steps t_{20} and t_{35}, respectively, to find possible assignments of WSs to threats. The proposed assignments for all three time steps are summarised in Table 5.14 and are also presented graphically in Figures 5.8–5.10 for time steps

t_{20}, t_{35} and t_{39}, respectively.

Time step t_{20}	
Threat	WS
T_4	V_5
T_3	C_3
T_5	D_1/No WS
T_2	D_2/No WS
T_1	D_3/No WS

Time step t_{35}	
Threat	WS
T_5	D_1/No WS
T_3	D_2/No WS
T_4	V_1
T_2	V_3 or V_4
T_1	C_2

Time step t_{39}	
Threat	WS
T_2	C_2
T_1	V_2
T_3	D_1/No WS
T_4	D_2/No WS
T_5	V_7

TABLE 5.14: *The assignment of WSs by means of the additive utility function $u(x, y)$ in (4.13), in conjunction with the greedy assignment heuristic for time steps t_{20}, t_{35} and t_{39}, respectively.*

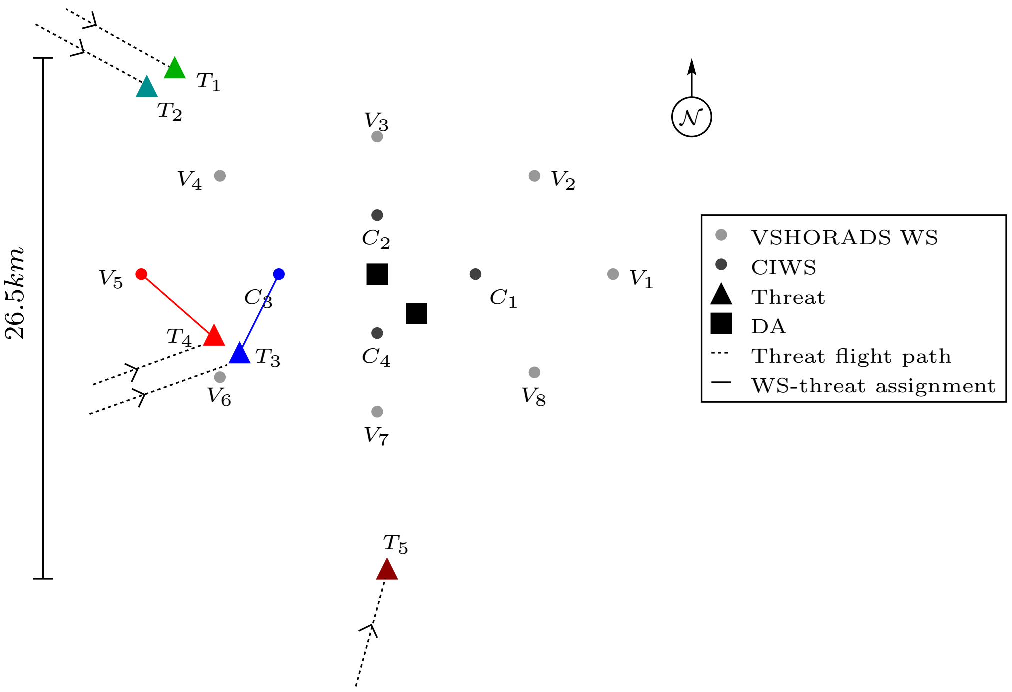


FIGURE 5.8: *Graphical illustration of the assignments of WSs to threats obtained by means of the additive utility function $u(x, y)$ in (4.13), in conjunction with the greedy assignment heuristic for time step t_{20}.*

When considering time step t_{20} and examining Figure 5.8, the assignments of WSs V_5 and C_3 to threats T_4 and T_3, respectively, seem intuitively plausible.

When considering time step t_{35} and examining Figure 5.9, the assignment of WSs C_2, V_3 (or V_4) and V_1 to threats T_1, T_2 and T_4, respectively, also seem acceptable from an intuitive point of view. A strange occurrence is the assignment of WS D_2 to threat T_3, since WS V_2 achieves a SSHP value of 0.3 as opposed to the SSHP value of 0 achieved by WS D_2 with respect to threat T_3. The reason for this assignment is that a VSHORAD WS achieving a SSHP value of 0.3, yields a utility $u_{\text{SSHP}}(0.3) = 0.0625$ and if scaled by $k_{\text{SSHP}} = 0.9$, yields a scaled utility value of 0.0563. Since a VSHORAD achieves the highest cost value in the scenario, its utility value $u_{\text{cost}}(R\,1\,000\,000) = 0$, and when scaling this value by $k_{\text{cost}} = 0.1$, yields the scaled utility value 0. Hence, the combined utility value of a VSHORAD WS achieving a SSHP of 0.3 is evaluated as $u_{x,y}(0.3, R\,1\,000\,000) = 0.0563$.

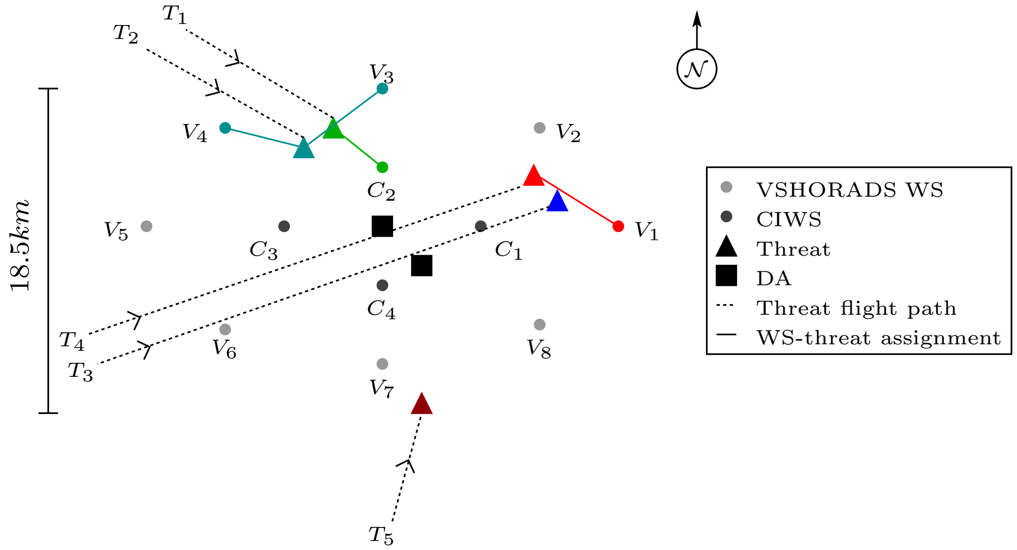


FIGURE 5.9: *Graphical illustration of the assignments of WSs to threats obtained by means of the additive utility function $u(x, y)$ in (4.13), in conjunction with the greedy assignment heuristic for time step t_{35}.*

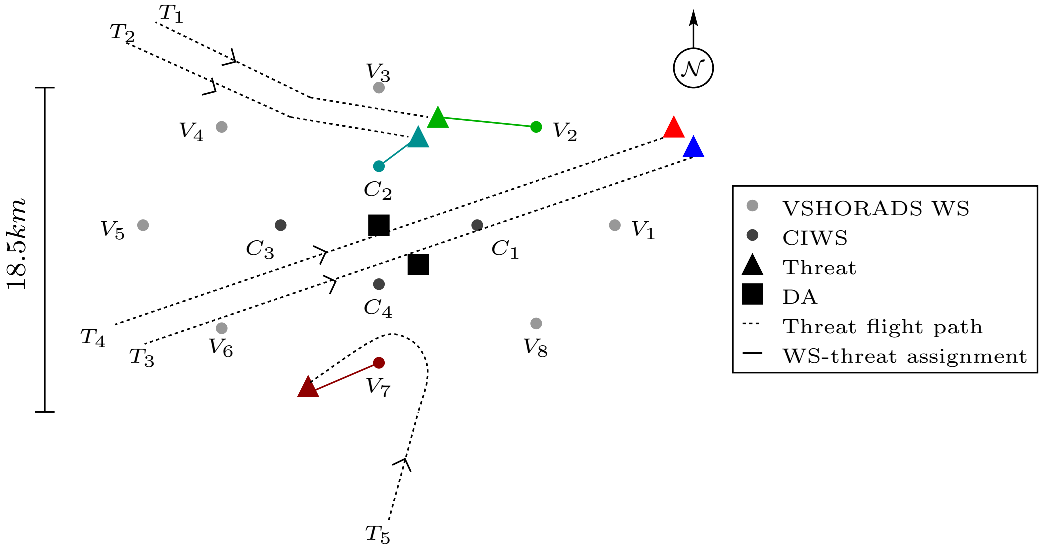


FIGURE 5.10: *Graphical illustration of the assignments of WSs to threats obtained by means the additive utility function $u(x, y)$ in (4.13), in conjunction with the greedy assignment heuristic for time step t_{39}.*

The utility value for a dummy WS (*i.e.* with a SSHP of 0) may be evaluated as $u_{\text{SSHP}}(0) = 0$ and if scaled by $k_{\text{SSHP}} = 0.9$, yields the scaled utility value of 0. Since a dummy WS achieves the lowest cost value with respect to the other WSs in the scenario, the utility value $u_{\text{cost}}(R\,0) = 1$ is evaluated, and if scaled by $k_{\text{cost}} = 0.1$ yields the scaled utility value of 0.1. Hence, the combined utility value of a dummy WS achieving a SSHP of 0 is evaluated as $u_{x,y}(0, R\,0) = 0.0966$. Since the dummy WS achieves a higher utility value, it is recommended for an assignment, implying that no actual WS is assigned.

From these evaluations it is clear that when only the SSHP values are employed in the decision to assign WSs to threats, a VSHORAD WS achieving a SSHP of 0.3 should rather be considered for assignment than no assignment at all. However, when the SSHP values are used in conjunction with the cost values, the VSHORAD is heavily penalized in the utility function for cost, since $x_{\text{cost}}(R\,1\,000\,000) = 0$. On the other hand, the dummy WS is rewarded in the utility function for cost, since $x_{\text{cost}}(R\,0) = 1$. Only if the SSHP value of the VSHORAD increases to a value of 0.4 should the VSHORAD be recommended for an assignment, since a utility value $u_{x,y}(0.4, R\,1\,000\,000) = 0.1125$ is obtained.

Sensitivity analysis with respect to the SSHP values of a CIWS revealed that if a CIWS achieves a SSHP value of as low as 0.1, it should be recommended for an assignment rather than not assigning it, since $u_{x,y}(0.1, R\,34\,000) = 0.1106$.

Sensitivity analysis with respect to the SSHP values of a VSHORAD WS and a CIWS revealed that if a VSHORAD achieves a SSHP value of 0.4 and a CIWS achieves a SSHP value of 0.3, the CIWS should be recommended for assignment, since $u_{x,y}(0.4, R\,1\,000\,000) = 0.1125$ for the VSHORAD WS while $u_{x,y}(0.3, R\,34\,000) = 0.1529$ for the CIWS. The reason for this is also that the VSHORAD is penalized due to its high cost value. However, only if a VSHORAD achieves a SSHP value of 0.5 and a CIWS achieves a SSHP value of 0.4, only then should a VSHORAD WS be recommended for assignment rather than the CIWS, since $u_{x,y}(0.5, R\,1\,000\,000) = 0.2250$ for the VSHORAD WS while $u_{x,y}(0.4, R\,34\,000) = 0.2091$ for the CIWS.

When considering time step t_{39} and examining Figure 5.10, the assignments of WSs V_2, C_2 and V_7 to threats T_1, T_2 and T_5, respectively, all seem plausible.

An alternative approach towards the assignment of WSs to threats is to employ the utility assignment model (4.14)–(4.18) discussed in §4.4.5. The model was coded in the software suite Lingo 11.0 [34] and the model was solved for time steps t_{20}, t_{35} and t_{39}, respectively. The parameter value k was set to a value of 1, implying that only one WS may be assigned to each threat. The reason for this was to allow for possibility of comparing the assignments proposed by the greedy assignment procedure and the assignments proposed by the utility assignment model.

The combined evaluated utility values in Tables 5.11– 5.13, as well as the corresponding threat lists in Table 4.3 are used as input to the utility assignment model for each time step. The output obtained is the proposed assignments of WSs with respect to threats as well as the corresponding total utility value (denoted by U) of the WSs assigned. The proposed assignments together with their corresponding objective function values as well as the cost involved in the proposed assignments are presented in Table 5.15 for each time step. These assignments are also presented graphically in Figures 5.11–5.13 for time steps t_{20}, t_{35} and t_{39}, respectively.

Time step t_{20}	
Threat	WS
T_4	V_5
T_3	C_3
T_5	D_1/No WS
T_2	D_2/No WS
T_1	D_3/No WS
U	0.3227
Cost	$R\,1\,034\,000$

Time step t_{35}	
Threat	WS
T_5	D_1
T_3	D_2
T_4	V_1
T_2	V_3
T_1	C_2
U	0.7348
Cost	$R\,2\,034\,000$

Time step t_{39}	
Threat	WS
T_2	C_2
T_1	V_2
T_3	D_2/No WS
T_4	D_1/No WS
T_5	V_7
U	1.4796
Cost	$R\,2\,034\,000$

TABLE 5.15: *The assignment of WSs by means of the utility assignment model (4.14)–(4.18) for time steps t_{20}, t_{35} and t_{39}, respectively.*

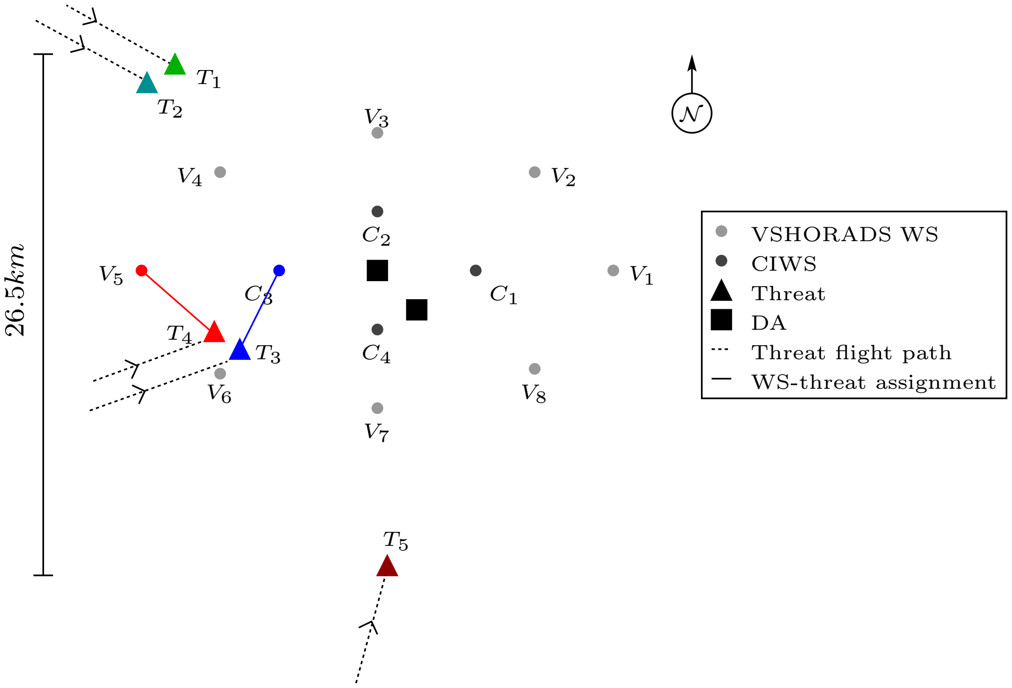


FIGURE 5.11: *Graphical illustration of the recommended assignment of WSs to threats obtained by means of the utility assignment model (4.14)–(4.18) for time step* t_{20}.

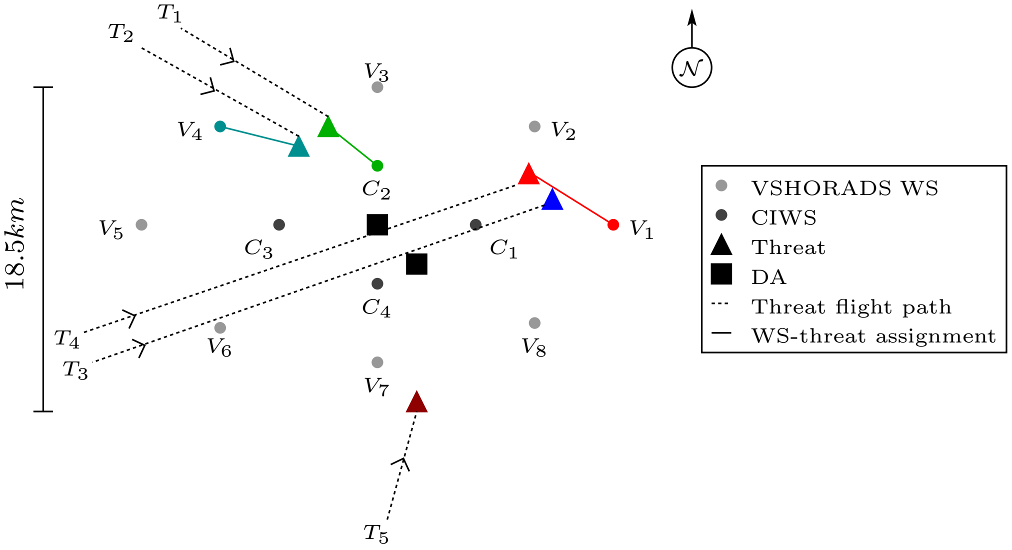


FIGURE 5.12: *Graphical illustration of the recommended assignment of WSs to threats obtained by means of the utility assignment model (4.14)–(4.18) for time step* t_{35}.

When comparing the assignments proposed by the greedy assignment heuristic and the assignments proposed by the utility assignment model (4.14)–(4.18) for all three time steps, it is found that the exact same assignments are proposed at each time step, respectively.

The results presented thus far in this section were obtained subject to the constraint that only one WS may be assigned to a specific threat at a time. To illustrate the working of the utility assignment model (4.14)–(4.18) if this restriction is relaxed to allow a maximum of two WSs to be assigned to a specific threat, consider the utility assignment model for time step t_{39} with

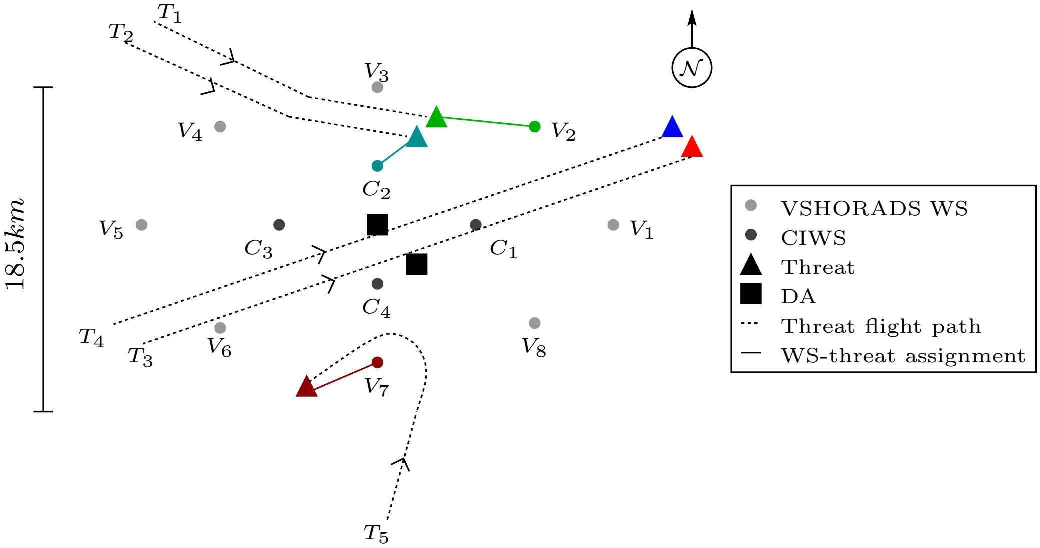


FIGURE 5.13: *Graphical illustration of the recommended assignment of WSs to threats obtained by means of the utility assignment model (4.14)–(4.18) for time step* t_{39}.

$k = 2$. The assignments proposed by the utility assignment model (4.14)–(4.18) with $k = 2$ are presented in Table 5.16 and are also shown graphically in Figure 5.14. The utility assignment model works in such a way that it will assign the maximum specified number of WSs to each threat, until all the available WSs are considered for an assignment. Therefore, if $k = 2$, two WSs are assigned to each threat. When comparing the results obtained when $k = 1$ to the results obtained when $k = 2$, it is found that similar assignments are made, but in the case where $k = 2$ an additional assignment of WS C_1 is made to threat T_2. The reason for this is that WS C_1 achieves a SSHP value of 0.4 with respect to threat T_2. It can therefore contribute towards the accumulated efficiency of WSs with respect to this threat and hence it is assigned. The result of the additional assignment of WS C_1 is an increase in the accumulated utility value U of 0.5081 and an increase in the accumulated cost of $R\,34\,000$.

Time step t_{39}	
Threat	WS
T_1	V_2 and D_3
T_2	C_1 and C_2
T_3	D_7 and D_{10}/No WS
T_4	D_8 and D_9/No WS
T_5	V_7 and D_2
U	1.9877
Cost	$R\,2\,068\,000$

TABLE 5.16: *The assignment of WSs by the utility assignment model (4.14)–(4.18) for time step* t_{39}*, with* $k = 2$.

The conclusion may therefore be drawn that both the greedy assignment heuristic and the utility assignment model (4.14)–(4.18) are able to achieve an acceptable trade-off between the efficiency of WSs assigned to threats and the cost incurred by assigning these WSs, since an increase in the efficiency of WSs with respect to threats results in an increase in the cost incurred by assigning these WSs. Although the utility model is able to achieve a trade-off between the

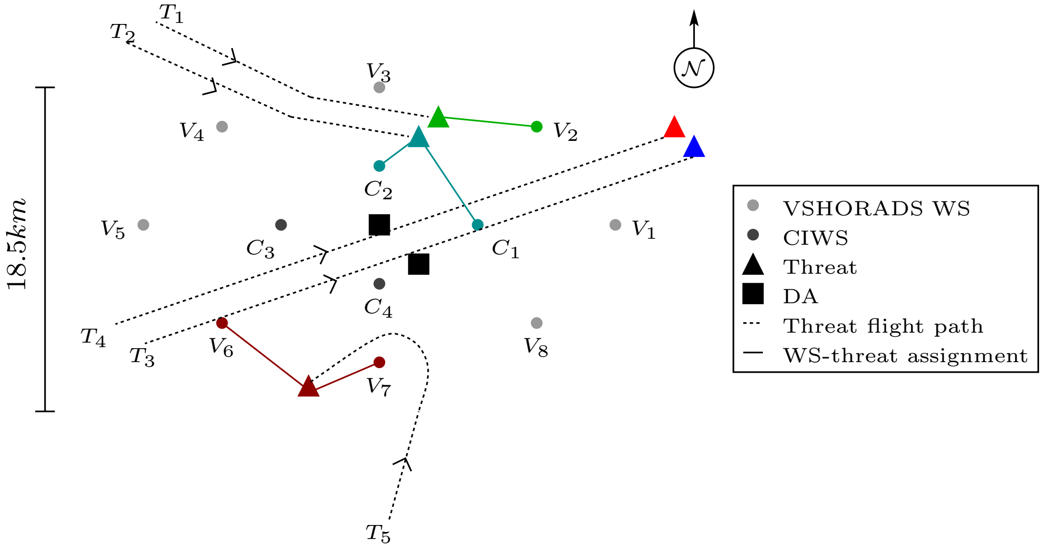


FIGURE 5.14: *Graphical illustration of the recommended assignment of WSs to threats obtained by means of the utility assignment model (4.14)–(4.18) for time step* t_{39}*, with* $k = 2$.

objectives, it only provides a single solution at each time step. A more desirable approach may be to investigate a model which is able to present a number of nondominated solutions for each time step, simultaneously achieving trade-off between the two objectives of all the nondominated solutions. In this way, the decision maker may be presented with a number of good quality solutions from which he may choose.

A disadvantage of the utility functional approach is that the processes involved in finding a utility function for each of the individual objectives, are very tedious. The assessments carried out during the evaluation of a utility function are also subject to the input of a decision maker. The implication is that the assessments carried out in conjunction with the decision maker may differ if the assessments are carried out with a different decision maker. In view of the bi-objective WA model, this implies that different results may be obtained if a different military expert were employed to act as the decision maker.

Another disadvantage of the utility functional approach is that if any additional alternatives are added to the current set of alternatives which achieve values falling outside the current range of values in any one of the objectives, a large portion of the assessments have to be reevaluated in order to obtain a new utility function for the relevant objective.

5.3 NSGA II results

The pareto optimal solutions to the bi-objective WA model (4.1)–(4.4), were obtained for each time step by means of the computer-implemented version of the NSGA II. The model was solved in the context of the proposed scenario in §4.2 for time steps t_{20}, t_{35} and t_{39}, respectively. The survival probabilities of threats with respect to each WS used in conjunction with the threat lists corresponding to each time step served as input to the NSGA II. The initial population of solutions were generated by randomly assigning WSs to threats for each solution.

Another input requirement of the NSGA II is to specify the parameter values for the algorithm. These values include the number of iterations, the size of the population of solutions, the tour size and pool size in the selection procedure, and the probability of a mutation occurring. The aim is to find the set of parameter values which will yield a good spread of pareto optimal solutions. An initial set of parameter values were chosen where the number of iterations were fixed at 350, the size of the population of solutions was 200 and the tour and pool sizes involved in the selection process were 2 and 100, respectively, since a tour size is commonly chosen between 2 and 5 and the pool size is commonly chosen to be half the size of the population [38]. The probability of a mutation occurring should be chosen relatively small; a commonly employed value of $1/n$ was therefore chosen, where n denotes the number of decision variables, yielding a value of 0.014. These initial parameter values used in the NSGA II are summarised in Table 5.17.

Parameter	Value
Number of iterations	350
Population size	200
Tour size	2
Pool size	100
Mutation probability	0.014

TABLE 5.17: *The initial parameter values used in the NSGA II.*

In order to find the set of parameter values which may result in a good spread of pareto optimal solutions, a sensitivity analysis was performed with respect to the parameters in Table 5.17. This was achieved by solving the WA problem for the scenario in §4.2 at time step t_{35} with the parameter values presented in Table 5.17 and varying the values of each of these parameters separately while keeping the remaining parameter values fixed at the initial solutions. After performing a sensitivity analysis in this fashion it was found that when changing the value of the number of iterations from 350, to 500 and to 750, respectively, while keeping the values of the remaining parameters fixed, yields the same spread of solutions on the pareto frontier. It was also found that a number of iterations of 350 yields a better spread of solutions on the pareto frontier than a value of 200. The number of iterations was therefore kept fixed at a maximum of 350 for each time step of the scenario in §4.2.

The size of the population was changed to a value of 300 while keeping the values of the remaining parameters fixed. It was found that the same spread of solutions was obtained as for a population of size 200. Since an increase in the population size yields no significant increase in the quality of the solutions obtained, the size of the population was kept fixed at 200 for each time step of the scenario in §4.2. The tour size and pool size were also changed to values of 4 and 150, respectively, while keeping the values of the remaining parameters fixed. These changes yielded the same spread of pareto optimal solutions as the solutions obtained when the values were 2 and 100, respectively, and the values were therefore kept fixed at 2 and 100, respectively, for each each time step of the scenario in §4.2. Finally, since the probability of mutating should be small, it was decided to keep it fixed at 0.014 for each time step of the scenario in §4.2.

The NSGA II provides a number of solutions simultaneously in the bi-objective solution space. Since one is interested only in the nondominated solutions contained in the first front (*i.e.* the approximately pareto optimal solutions), the focus of the presentation of results in this section is on the solutions contained in the set of approximately pareto optimal solutions. The interested reader is referred to Appendix C for a graphical presentation of the entire set of solutions uncovered in the bi-objective decision space for each time step of the scenario in §4.2.

The entire set of solutions corresponding to time step t_{20} is presented graphically in Figure C.1

in Appendix C. The set of approximately pareto optimal solutions contains five solutions and these solutions together with their corresponding objective function values (accumulated survival probability objective and in rand value (R) for the cost objective) are listed in Table 5.18 and presented graphically in Figure 5.15.

Approximately Pareto optimal solutions for time step t_{20}		
Solution	Accumulated survival probabilities	Cost
1	1.3444	$R\,2\,034\,000$
2	1.4260	$R\,1\,034\,000$
3	1.5166	$R\,1\,000\,000$
4	1.8836	$R\,34\,000$
5	1.9754	$R\,0$

TABLE 5.18: *The set of approximately pareto optimal solutions obtained by the NSGA II for time step t_{20} of the scenario in §4.2.*

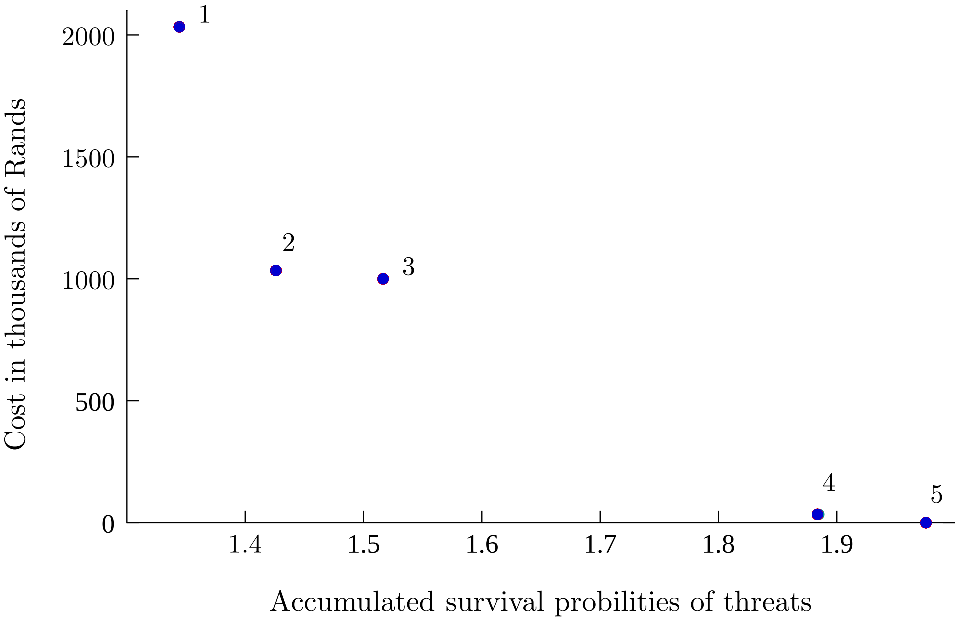


FIGURE 5.15: *Graphical illustration of the set of approximately pareto optimal solutions obtained by means of the NSGA II for time step t_{20} of the scenario in §4.2.*

The proposed assignments of WSs to threats corresponding to each of the solutions in the approximate pareto front are for time step t_{20} presented in Table C.1 in Appendix C and these assignments are summarised in Table 5.19. By examining the results in Table 5.19, it is found that Solution 1 achieves an accumulated survival probability of the threats of 1.3444 and a cost value of $R\,2\,034\,000$, and involves the assignment of WSs V_6 and C_3 to threat T_3 and WS V_5 to threat T_4. The other threats are not assigned any WSs, since the available WSs achieve a SSHP value of zero with respect to these threats. Solution 2 achieves an accumulated survival probability of the threats of 1.4260 and a cost value of $R\,1\,034\,000$, and involves the assignment of WS C_3 to threat T_3 and WS V_5 to threat T_4. Solution 3 achieves an accumulated survival probability of the threats of 1.5166 and a cost value of $R\,1\,000\,000$, and involves the assignment

Solution 1 at t_{20}	
Threat	WS
T_1	None
T_2	None
T_3	V_6 and C_3
T_4	V_5
T_5	None
Accumulated p_i	1.3444
Cost in	$R\,2\,034\,000$

Solution 2 at t_{20}	
Threat	WS
T_1	None
T_2	None
T_3	C_3
T_4	V_5
T_5	None
Accumulated p_i	1.4260
Cost	$R\,1\,034\,000$

Solution 3 at t_{20}	
Threat	WS
T_1	None
T_2	None
T_3	None
T_4	V_5
T_5	None
Accumulated p_i	1.5166
Cost	$R\,1\,000\,000$

Solution 4 at t_{20}	
Threat	WS
T_1	None
T_2	None
T_3	None
T_4	C_3
T_5	None
Accumulated p_i	1.8836
Cost	$R\,34\,000$

Solution 5 at t_{20}	
Threat	WS
T_1	None
T_2	None
T_3	None
T_4	None
T_5	None
Accumulated p_i	1.9754
Cost	$R\,0$

TABLE 5.19: *The assignment of WSs proposed by the NSGA II for each of the fire solutions on the approximate pareto frontier for time step t_{20} shown in Figure 5.15.*

of WS V_5 to threat T_4. Finally, Solution 4 achieves an accumulated survival probability of the threats of 1.8836 and a cost value of $R\,34\,000$, and involves the assignment of WS C_3 to threat T_4. Solution 5 involves no assignment of WSs to threats.

By examining these results, it is clear that in the first solution a WSs is assigned to a threat if it achieves a SSHP value greater than zero with respect to the threat. This solution yields the maximum value in the cost objective and the minimum value in the survival probability objective for the current time step. In the next solution, Solution 2, it is found that the assignment of WSs remains the same, except for assigning only WS C_3 to threat T_3, rather than assigning WSs V_6 and C_3 to threat T_3. The cost therefore decreases, but there is a slight increase in total survival probabilities of the threats. Solutions 3 and 4 follow suite until no WSs are assigned to any of the threats in Solution 5. Solution 5 yields the maximum value of the survival probability objective and the minimum value of the cost objective for time step t_{20}.

Fifteen solutions were obtained in the approximately pareto optimal set of solutions for time step t_{35}. These solutions, together with their corresponding objective function values, are listed in Table 5.20 and presented graphically in Figure 5.16. The entire set of solutions in the bi-objective decision space are presented in Table C.2 in Appendix C. The proposed assignment of WSs to threats corresponding to each of the solutions in the approximately pareto optimal set of solutions, are presented in Tables C.2 and C.3 in Appendix C. A summarised version of these assignments are presented in Table 5.21.

When examining these results it is found that Solution 1 entails assignment of an available WS to a threat if it achieves a SSHP value of greater than zero with respect to the threat. Following the solutions from the top left along the approximate pareto optimal frontier results in similar assignments, except that one fewer assignment is proposed with respect to one of the threats at each of the solutions. This results in a decrease in the overall cost value and a slight increase in the accumulated survival probabilities of the threats.

Finally, ten solutions were obtained in the set of approximately pareto optimal solutions for time step t_{39}. The entire set of solutions in the bi-objective decision space are presented in

Approximately Pareto optimal solutions for time step t_{35}		
Solution	Accumulated survival probabilities	Cost
1	3.0925	R 4 068 000
2	3.1161	R 4 034 000
3	3.2991	R 4 000 000
4	3.3050	R 3 068 000
5	3.3522	R 3 034 000
6	3.5351	R 3 000 000
7	3.5939	R 2 068 000
8	3.6412	R 2 034 000
9	3.8240	R 2 000 000
10	3.9468	R 1 068 000
11	4.0129	R 1 034 000
12	4.2962	R 1 000 000
13	4.4209	R 68 000
14	4.4870	R 34 000
15	4.7703	R 0

TABLE 5.20: *The set of approximately pareto optimal solutions obtained by the NSGA II for time step t_{35} of the scenario in §4.2.*

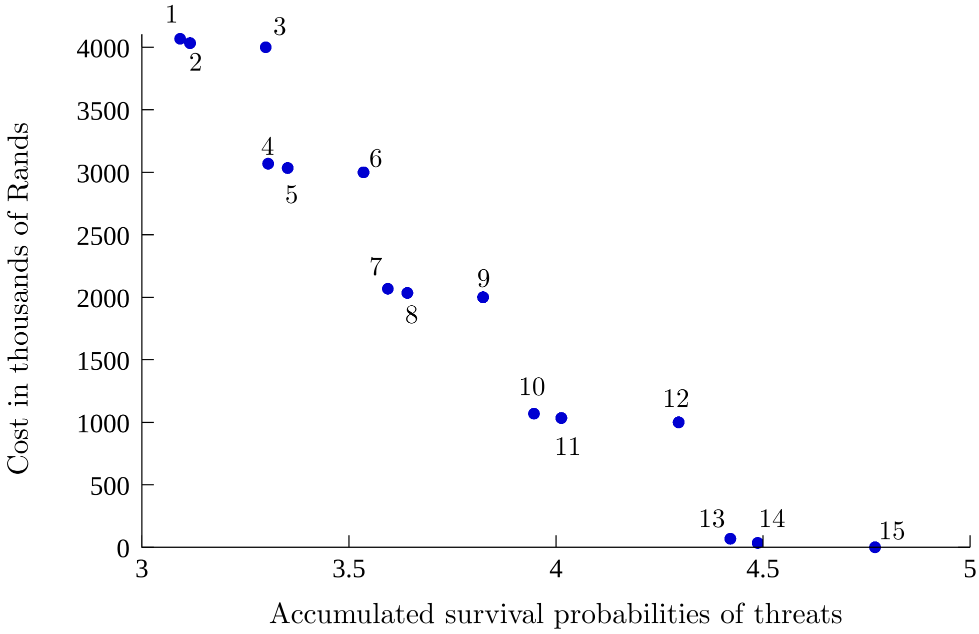


FIGURE 5.16: *Graphical illustration of the set of approximately pareto optimal solutions obtained by means of the NSGA II for time step t_{35} of the scenario in §4.2.*

Figure C.3 in Appendix C. The set of pareto optimal solutions together with their corresponding objective function values for time step t_{39} are listed in Table 5.22 and presented graphically in Figure 5.17. The proposed assignment of WSs to threats corresponding to solutions contained on the approximate pareto optimal frontier are presented in Tables C.4 and C.5 in Appendix C, and these results are summarised in Table 5.23.

Solution 1 at t_{35}	
Threat	WS
T_1	C_2
T_2	V_3, V_4 and C_3
T_3	V_2
T_4	V_1
T_5	None
Accumulated p_i	3.0925
Cost	R 4 068 000

Solution 2 at t_{35}	
Threat	WS
T_1	C_2
T_2	V_3 and V_4
T_3	V_2
T_4	V_1
T_5	None
Accumulated p_i	3.1161
Cost	R 4 034 000

Solution 3 at t_{35}	
Threat	WS
T_1	None
T_2	V_3 and V_4
T_3	V_2
T_4	V_1
T_5	None
Accumulated p_i	3.2991
Cost	R 4 000 000

Solution 4 at t_{35}	
Threat	WS
T_1	C_2
T_2	V_3 and C_3
T_3	V_2
T_4	V_1
T_5	None
Accumulated p_i	3.3050
Cost	R 3 068 000

Solution 5 at t_{35}	
Threat	WS
T_1	C_2
T_2	V_3
T_3	V_2
T_4	V_1
T_5	None
Accumulated p_i	3.3522
Cost	R 3 034 000

Solution 6 at t_{35}	
Threat	WS
T_1	None
T_2	V_3
T_3	V_2
T_4	V_1
T_5	None
Accumulated p_i	3.5351
Cost	R 3 000 000

Solution 7 at t_{35}	
Threat	WS
T_1	C_2
T_2	V_3 and C_3
T_3	None
T_4	V_1
T_5	None
Accumulated p_i	3.5939
Cost	R 2 068 000

Solution 8 at t_{35}	
Threat	WS
T_1	C_2
T_2	V_3
T_3	None
T_4	V_1
T_5	None
Accumulated p_i	3.6412
Cost	R 2 034 000

Solution 9 at t_{35}	
Threat	WS
T_1	None
T_2	V_3
T_3	None
T_4	V_1
T_5	None
Accumulated p_i	3.8240
Cost	R 2 000 000

Solution 10 at t_{35}	
Threat	WS
T_1	None
T_2	C_2 and C_3
T_3	None
T_4	V_1
T_5	None
Accumulated p_i	3.9468
Cost	R 1 068 000

Solution 11 at t_{35}	
Threat	WS
T_1	None
T_2	C_2
T_3	None
T_4	V_1
T_5	None
Accumulated p_i	4.0129
Cost	R 1 034 000

Solution 12 at t_{35}	
Threat	WS
T_1	None
T_2	None
T_3	None
T_4	V_1
T_5	None
Accumulated p_i	4.2962
Cost	R 1 000 000

Solution 13 at t_{35}	
Threat	WS
T_1	None
T_2	C_2 and C_3
T_3	None
T_4	None
T_5	None
Accumulated p_i	4.4209
Cost	R 68 000

Solution 14 at t_{35}	
Threat	WS
T_1	None
T_2	C_2
T_3	None
T_4	None
T_5	None
Accumulated p_i	4.4870
Cost	R 34 000

Solution 15 at t_{35}	
Threat	WS
T_1	None
T_2	None
T_3	None
T_4	None
T_5	None
Accumulated p_i	4.7703
Cost	R 0

TABLE 5.21: *The assignment of WSs proposed by the NSGA II for each of the fifteen fire solutions on the approximate pareto frontier for time step t_{35} shown in Figure 5.16.*

Approximately Pareto optimal solutions for time step t_{39}		
Solution	Accumulated survival probabilities	Cost in $R'000$
1	1.9613	$R\,5\,068\,000$
2	1.9852	$R\,4\,068\,000$
3	2.0102	$R\,3\,068\,000$
4	2.0702	$R\,3\,034\,000$
5	2.0858	$R\,2\,068\,000$
6	2.1457	$R\,2\,034\,000$
7	2.3358	$R\,1\,068\,000$
8	2.3957	$R\,1\,034\,000$
9	2.8952	$R\,68\,000$
10	3.0949	$R\,34\,000$

TABLE 5.22: *The set of approximately pareto optimal solutions obtained by the NSGA II for time step t_{39} of the scenario in §4.2.*

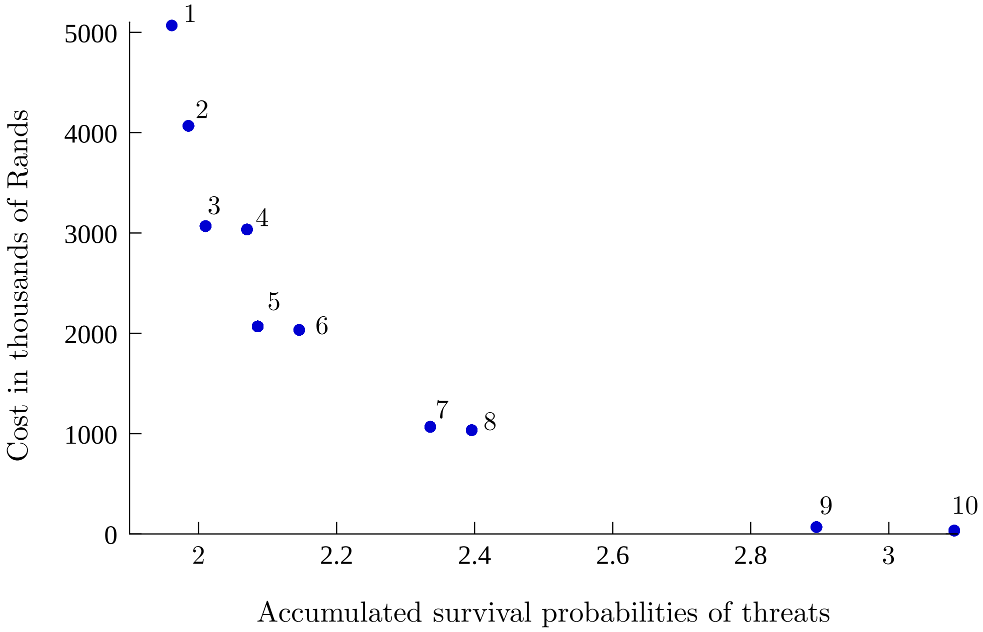


FIGURE 5.17: *Graphical illustration of the set of approximately pareto optimal solutions obtained by means of the NSGA II for time step t_{39} of the scenario in §4.2.*

When considering Solution 1 of the set of approximately pareto optimal solutions for time step t_{39} it is found that WSs V_2, V_3 and C_1 are assigned to threat T_1, WS C_2 is assigned to threat T_2, WS V_1 is assigned to threat T_3 and WSs V_6 and V_7 are assigned to threat T_5. No WSs are assigned to Threat T_4. Since WS V_2 achieves a SSHP value of 0.7 with respect to threat T_1 as opposed to a SSHP value of 0.5 with respect to threat T_4 and the fact that threat T_1 is considered a higher priority threat than threat T_4, WS V_2 is assigned to threat T_1 rather than assigned to threat T_4. The same reason applies for assigning WS V_1 to threat T_3 rather than assigning it to threat T_4. The remaining WSs achieves SSHP of zero with respect to threat T_4. Hence, no WS is assigned to threat T_4. Solution 1 in the set of approximately pareto optimal

Solution 1 at t_{39}	
Threat	WS
T_1	V_2, V_3 and C_1
T_2	C_2
T_3	V_1
T_4	None
T_5	V_6 and V_7
Accumulated p_i	1.9613
Cost in	$R\,5\,068\,000$

Solution 2 at t_{39}	
Threat	WS
T_1	V_2 and C_1
T_2	C_2
T_3	V_1
T_4	None
T_5	V_6 and V_7
Accumulated p_i	1.9852
Cost	$R\,4\,068\,000$

Solution 3 at t_{39}	
Threat	WS
T_1	V_2 and C_1
T_2	C_2
T_3	V_1
T_4	None
T_5	V_7
Accumulated p_i	2.0102
Cost	$R\,3\,068\,000$

Solution 4 at t_{39}	
Threat	WS
T_1	V_2
T_2	C_2
T_3	V_1
T_4	None
T_5	V_7
Accumulated p_i	2.0702
Cost	$R\,3\,034\,000$

Solution 5 at t_{39}	
Threat	WS
T_1	V_2 and C_1
T_2	C_2
T_3	None
T_4	None
T_5	V_7
Accumulated p_i	2.0858
Cost	$R\,2\,068\,000$

Solution 6 at t_{39}	
Threat	WS
T_1	V_2
T_2	C_2
T_3	None
T_4	None
T_5	V_7
Accumulated p_i	2.1457
Cost	$R\,2\,034\,000$

Solution 7 at t_{39}	
Threat	WS
T_1	V_2 and C_1
T_2	C_2
T_3	None
T_4	None
T_5	None
Accumulated p_i	2.3358
Cost	$R\,1\,068\,000$

Solution 8 at t_{39}	
Threat	WS
T_1	V_2
T_2	C_2
T_3	None
T_4	None
T_5	None
Accumulated p_i	2.3957
Cost	$R\,1\,034\,000$

Solution 9 at t_{39}	
Threat	WS
T_1	C_1
T_2	C_2
T_3	None
T_4	None
T_5	None
Accumulated p_i	2.8952
Cost	$R\,68\,000$

Solution 10 at t_{39}	
Threat	WS
T_1	None
T_2	C_2
T_3	None
T_4	None
T_5	None
Accumulated p_i	3.0949
Cost	$R\,34\,000$

TABLE 5.23: *The assignment of WSs proposed by the NSGA II for each of the ten fire solutions on the approximate pareto frontier for time step t_{39} shown in Figure 5.17.*

solutions for time step t_{39} represents the maximum value achieved for the cost objective and the minimum value achieved for the accumulated survival probabilities at time step t_{39}. The assignments of WSs to threats for Solution 1 for time step t_{39} are presented graphically in Figure 5.18.

When considering Solution 3 of the set of approximately pareto optimal solutions for time step t_{39} it is found that similar assignments are proposed as the assignments proposed in Solution 1, except for only assigning WSs V_2 and C_1 to threat T_1, rather than assigning WSs V_2, V_3 and C_1 to threat T_1, and only assigning WS V_7 to threat T_5, rather than assigning WSs V_6 and V_7 to threat T_5. This results in a decrease in the total cost value of $R\,2\,000\,000$ and an increase in the accumulated survival probabilities of threats of 0.0489. The assignments of WSs to threats

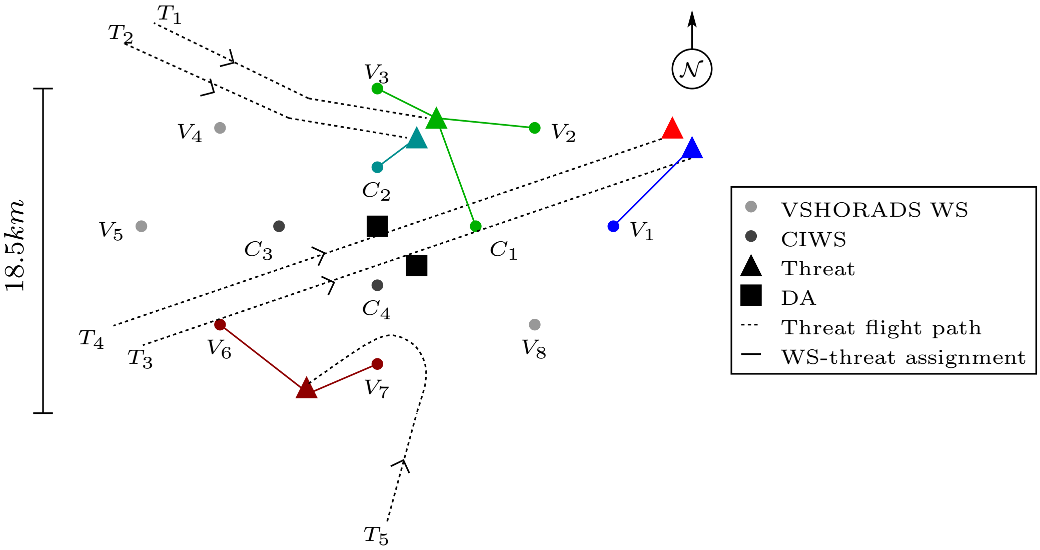


FIGURE 5.18: *Graphical illustration of the assignment of WSs to threats obtained by means of the NSGA II for Solution 1 at time step t_{39}.*

corresponding to Solution 3 in the set of approximately pareto optimal solutions for time step t_{39} are presented graphically in Figure 5.19.

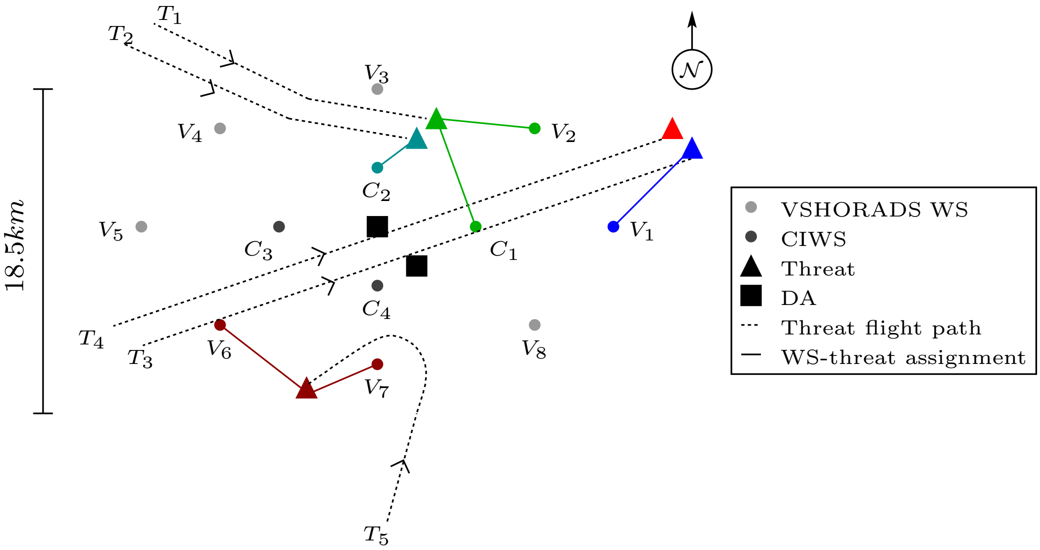


FIGURE 5.19: *Graphical illustration of the assignments of WSs to threats obtained by means of the NSGA II for Solution 3 at time step t_{39}.*

Next, consider Solution 6 of the set of approximately pareto optimal solutions for time step t_{39}. The assignments contained in this solution are similar to the assignments proposed in Solution 3, except for the assignment of only WS V_2 to threat T_1, rather than assigning WSs V_2 and C_2 to threat T_1, while no assignments are made to threat T_3 rather than assigning WS V_1 to threat T_3. The result of this assignment is a decrease in the cost value of $R\,1\,034\,000$ and an increase in the accumulated survival probabilities of threats of 0.1355. The proposed assignments corresponding to Solution 6 for time step t_{39} are presented graphically in Figure 5.20.

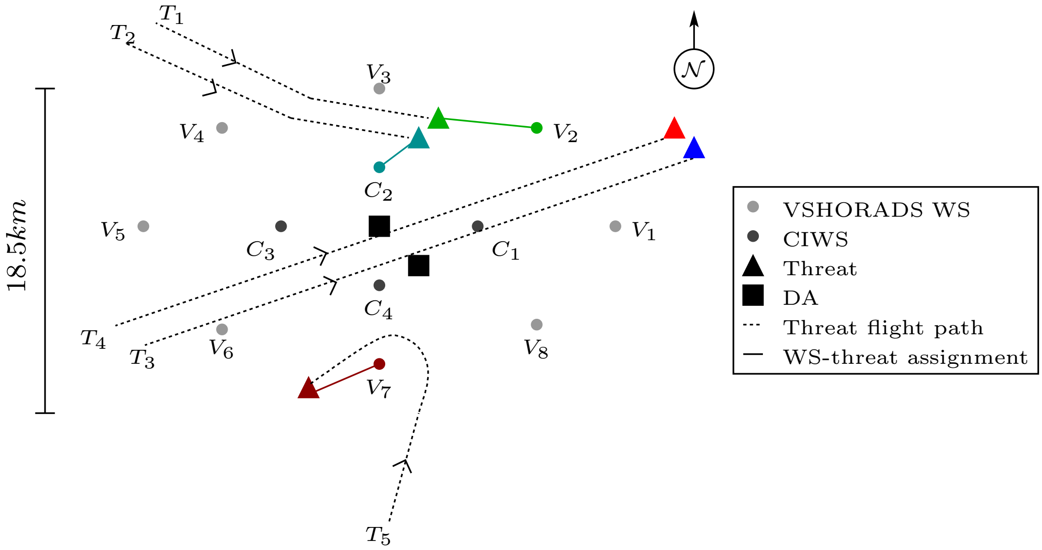


FIGURE 5.20: *Graphical illustration of the assignment of WSs to threats obtained by means of the NSGA II for Solution 6 at time step* t_{39}.

In Solution 8 of the set of approximately pareto optimal solutions for time step t_{39}, similar assignments are proposed as in the assignments proposed in solution 6, except that no WSs are assigned to threat T_5, rather than assigning WS V_7 to threat T_5. This results in a decrease in the cost value of $R\,1\,000\,000$ and an increase in the accumulated survival probabilities of threats of 0.25. The proposed assignments corresponding to Solution 8 in the pareto solutions of time step t_{39} are presented graphically in Figure 5.21.

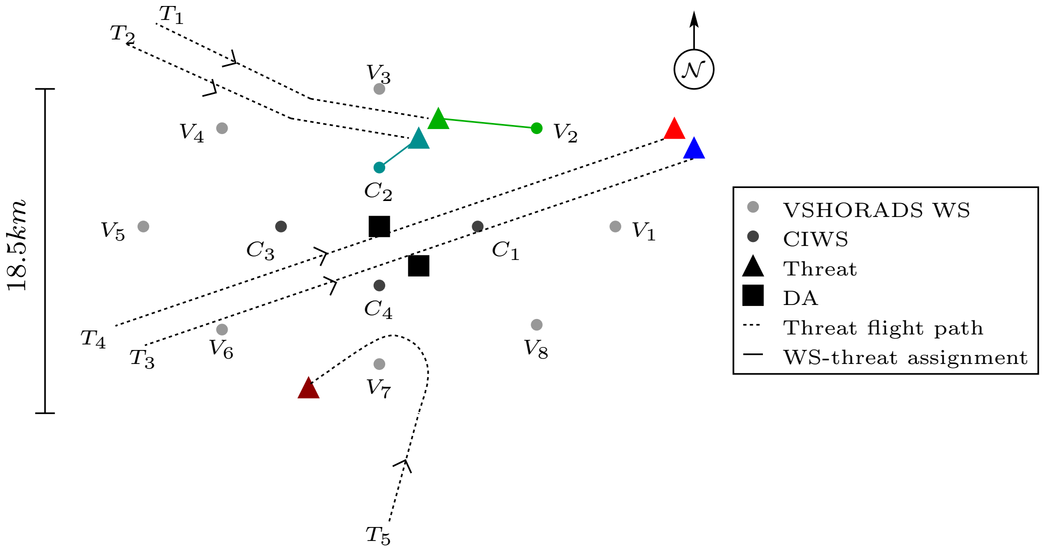


FIGURE 5.21: *Graphical illustration of the assignment of WSs to threats obtained by means of the NSGA II for Solution 8 at time step* t_{39}.

Finally, when considering Solution 10 in the set of approximately pareto optimal solutions for time step t_{39}, it is found that similar assignments are proposed as in the assignments proposed in Solution 8, except that no WSs are assigned to threat T_1 rather than assigning WS V_2 to threat T_1. This results in a further decrease in the total cost value of $R\,1\,000\,000$ and an increase in the total survival probabilities of threats of 0.6992. This solution achieves the minimum value for the cost objective as well as the maximum value for the accumulated survival probability objective. The proposed assignments of WSs to threats of Solution 10 for time step t_{39} are presented graphically in Figure 5.22.

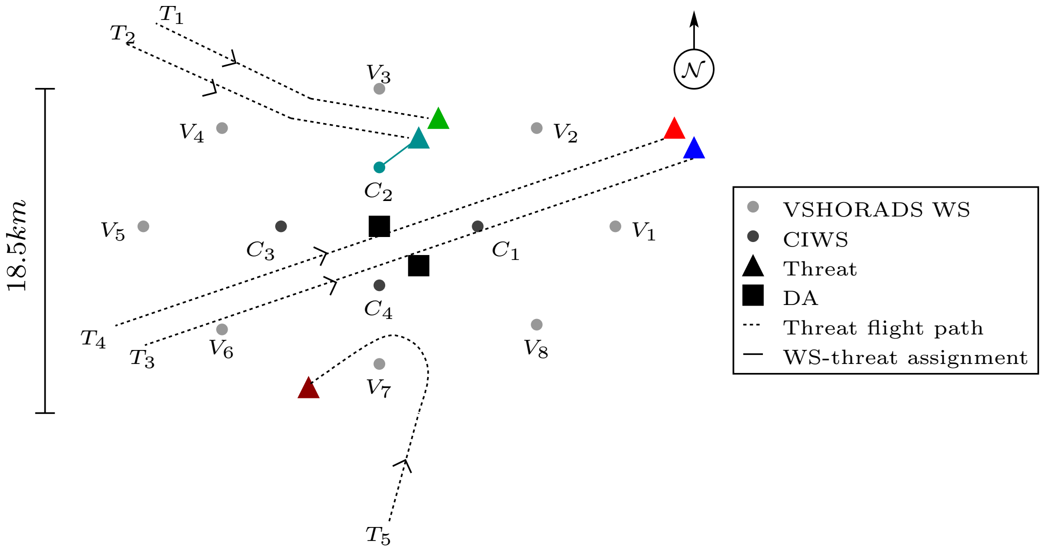


FIGURE 5.22: *Graphical illustration of the assignment of WSs to threats obtained by means of the NSGA II for Solution 10 at time step t_{39}.*

When following the solutions along the set of approximately pareto optimal solutions, it is found that one solution (the one in the top left-hand corner) achieves the worst possible value for the cost objective and achieves the best possible value for the accumulated survival probability objective at a specific time step (*i.e.* Solution 1 at each respective time step). The next solution involves an assignment of one fewer WS with respect to one of the threats resulting in a decrease in the cost objective and an increase in the accumulated survival probability objective. The remaining solutions on the approximate pareto frontier follow suite until the last solution on the pareto front is reached which achieves the best possible value for the cost objective and achieves the worst possible value for the accumulated survival probability objective at a specific time step (*i.e.* Solutions 5, 15 and 10 for times steps t_{20}, t_{35} and t_{39}, respectively).

The solutions contained in the set of pareto optimal solutions are nondominated, implying that no solution in the set of pareto solutions is better than any other solution in the set. The NSGA II therefore works in such a way, that rather than recommending a specific solution to the decision maker, as in the case with the AHP and utility models, a set of solutions are presented to the decision maker simultaneously, from which he may choose the one which best corresponds to his needs and judgment.

The NSGA II also provides a useful trade-off between the cost objective and the survival probability objective by means of the approximately pareto optimal solutions that it produces.

An advantage of the NSGA is that it does not require the input of a decision maker as was the case with the AHP and utility models. It is also flexible in the sense that it is able to accommodate a variation in the number of WSs and threats, without the incorporation of artificial model constraints or excessive numerical re-calculations at each step.

Another significant advantage of the NSGA II, is that the objective functions may be constrained, individually or together, and part of the pareto optimal set of solutions may still be recommended to the decision maker. In context of the scenario in §4.2, suppose the decision maker constrains the cost objective by indicating that the maximum allowable cost of assigning WSs to threats is $R\,3\,500\,000$. This constraint is presented graphically by the horisontal line labelled B in Figure 5.23. The constraint implies that no solution above this line may be recommended for assignments of WSs to threats. This implies that Solutions 1 and 2 may not be recommend for assignments of WSs to threats. Any of the solutions below this line may be recommended for a possible assignment of WSs to threats. The accumulated survival probability objective may be constrained similarly. Suppose that a maximum acceptable value for the accumulated survival probability is 2.1. This constraint is presented graphically by the vertical line labelled A in Figure 5.23. This constraint implies that any solution to the left of line A may be considered as a possible assignment of WSs to threats, while Solutions 6–10 may not be considered as an assignment of WSs to threats.

The possibility also exists that both objectives may be constrained. Suppose that the value of cost is restricted to a maximum of $R\,3\,500\,000$ and that the accumulated survival probability of the threats is restricted to a maximum 2.1. This implies that any solution below line B and to the left of line A may be considered for a possible assignment of WSs to threats. Hence, Solutions 3, 4 and 5 may be considered for an assignment, of which Solution 5 may be considered to be optimal.

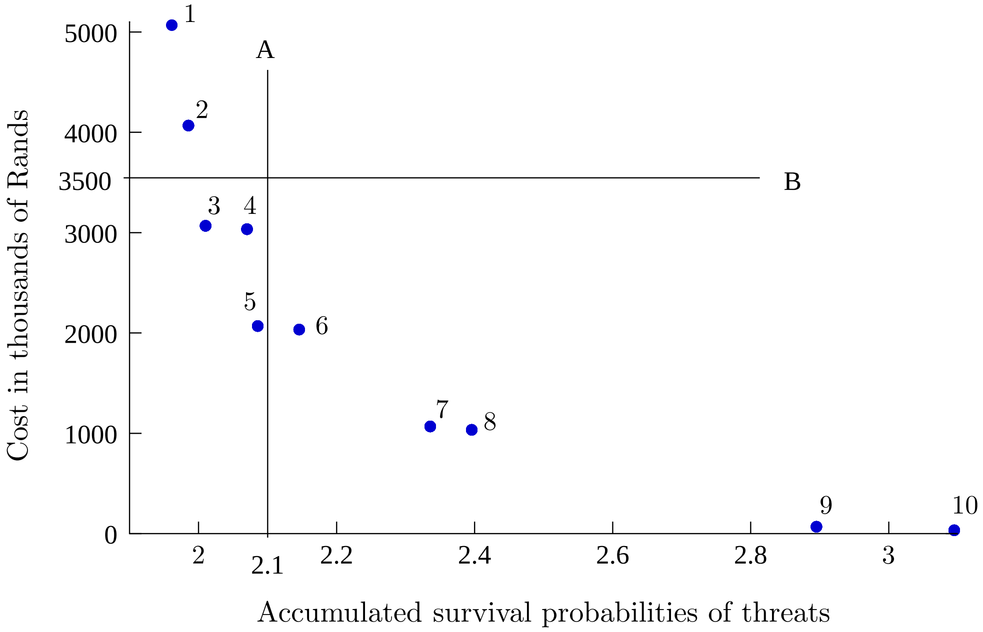


FIGURE 5.23: *Graphical illustration of the set of approximately pareto optimal solutions obtained by means of the NSGA II for time step t_{39} of the scenario in §4.2, when the bi-objective WA problem is constrained.*

To summarise, the NSGA II is able to provide the decision maker with a set of approximately pareto optimal solutions of assignments rather than providing the decision maker with a single solution of possible assignments. This results in shifting the choice of the assignment of WSs to the decision maker, since he may then choose the solution on the approximate pareto frontier which best corresponds to his intuition. The NSGA II also provides a useful trade-off between the cost and survival probability objectives by means of the pareto frontier along which it is clearly illustrated how much of one objective is offered for a gain in the other objective. Finally, the NSGA II permits the possibility of constraining any one of the two objective functions, or both simultaneously, while still providing good quality solutions.

5.4 Chapter summary

The main focus of this chapter was to present the reader with the results obtained from the methodologies described in Chapter 4 for solving the bi-objective WA problem in the context of the proposed scenario described in §4.2. This was achieved by discussing the results of the AHP in §5.1 followed by a discussion on the results obtained by means of the additive utility function in §5.2. The results obtained by means of the NSGA II were discussed in §5.3. Finally, the chapter closed with an overall conclusion on the results obtained by each of the methodologies

CHAPTER 6

Conclusion

Contents

This chapter opens, in §6.1, with a description of the work contained in this thesis. This is followed, in §6.2, by an appraisal of the research contributed in the thesis. The chapter closes, in §6.3, with a discussion on a number of ideas with respect to future work towards the development of more sophisticated multiobjective WA models.

6.1 Thesis summary

The thesis commenced in Chapter 1 with a brief introduction to the role played by the FCO in a GBAD military environment, as well as the conditions under which the FCO has to operate when faced with a choice of assigning WSs to observed aerial threats. This was followed by an informal description of the problem considered in this thesis as well as the objectives pursued in the thesis.

In Chapter 2, a literature review was presented on the various subsystems contained within a GBADS. This included descriptions of the physical elements (such as DAs, sensors and WSs), the central TM software system and the TE and WA subsystems, with an emphasis on the WA subsystem. A discussion on the tactical environment in which a GBADS has to operate and a description of the k-WA model (which is based on the classical assignment problem) concluded the chapter in which Thesis Objective I in §1.2 was therefore achieved.

Thesis Objective II was achieved in §3.1.4, where a literature review was presented with respect to the various procedures which may be employed in order to extract fundamental objectives in a multiobjective decision problem context from a decision maker or group of decision makers. This objective was further pursued in the context of the WA problem in §4.1. The second aim in Chapter 3 was to provide a literature review on available methodologies in the literature for solving multiobjective decision problems. This was achieved by a description of the processes involved in the AHP, followed by an explanation of the assessments involved in capturing qualitative as well as quantitative information for the construction of utility functions and a description of the working a multiobjective evolutionary algorithm, in fulfillment of Thesis Objective IV.

The methodologies described in Chapter 3 were applied in Chapter 4 in the context of a simulated GBADS scenario. A description of the procedures carried out in conjunction with a military expert followed. The chapter closed with a description of the practical implementation of the NSGA II, in fulfillment of Thesis Objective III.

Thesis Objective V was achieved in Chapter 5, in which the results obtained by means of the AHP, a bi-objective utility function and the computer implemented NSGA II were reported.

Thesis Objective VI is achieved at the end of this chapter where a number of ideas for possible future research with respect to the multiobjective WA problem are proposed.

6.2 An appraisal of the work contained in this thesis

The main aim of the research conducted in this thesis was to investigate the possibility of modelling the WA problem as a multiobjective decision problem. A first step towards multiobjective decision problems is to establish the fundamental objectives of the problem which are used in the development of a multiobjective decision model. In the context of WA, a modest first approach towards establishing objectives for the WA problem was to send out a WA survey to a number of military experts residing within the South African military domain. This survey consisted of posing scenario-specific questions to the various military experts in an attempt to extract those objectives which they deem important in the choice of assigning WSs to threats. The feedback received from the survey was analysed in order to obtain a list of possible objectives. Two of these objectives were chosen to employ for illustrative purposes in the context of a multiobjective WA problem in a simulated GBADS scenario. A bi-objective WA model was formulated, based on these two objectives. This model captures two very essential conflicting objectives in the context of the WA problem.

Three general approaches towards solving multiobjective decision making problems were researched and documented in this thesis. The working of these methodologies were demonstrated by solving the bi-objective WA model described above in the context of a realistic GBAD simulated scenario. Two of these approaches are subjective in nature, implying that the input of a decision maker was required. The necessary assessments for these approaches were carried out in conjunction with a military expert. The information captured during these assessment procedures were employed to solve the WA problem by means of the two subjective multiobjective approaches. In this way, the application of both these approaches were illustrated with respect to a real-world WA problem. The bi-objective WA model was also solved by means of a computer implemented version of the NSGA II, a powerful multiobjective optimisation evolutionary metaheuristic. The model provided promising results and one advantage of employing this method is that it is able to present the decision maker with an entire front of approximately pareto optimal solutions. This implies that the decision maker may choose from amongst these good solutions a solution which best corresponds to his intuition. It may happen that a decision maker is not entirely convinced by a solution presented to him. By using the NSGA II, this choice in finding an acceptable trade-off between the conflicting objectives is shifted to the decision maker by presenting him with a number of good quality, nondominated solutions from which he may then choose.

The conclusion was drawn that the AHP is able to generate plausible results and achieve a trade-off between the two objectives. However, a major disadvantage of the AHP is that if the current set of WSs are supplemented by additional WSs or if additional threats enter the defended airspace, a portion of the calculations carried out during the subjective AHP pre-processsing assessment procedure has to be reevaluated.

The bi-objective utility model also seems able to generate plausible results. However, if the set of WSs is supplemented by additional WSs or if the values of these additional WSs exceed the range of the current values employed in the assessment of any of the two objectives, a portion of the assessments carried out in conjunction with the decision maker has to be reevaluated in order to construct a new utility function.

It was further concluded that although both the AHP and utility approaches generate good quality solutions, they are restricted to generating only one solution at time.

Based on these findings, the NSGA II is recommended as the more desirable choice for solving the multiobjective WA problem, since it is able to present the decision maker with a suite of nondominated solutions of good quality, from which he may choose the one which he deems fit. Furthermore, the NSGA II is flexible in the sense that the number of threats and WSs entering the system may be varied without having to make considerable alterations.

6.3 Possible future work

This section contains ideas and suggestions for possible future work with respect to the multiobjective WA problem. Since the scope of any research project is restricted, there is always a possibility for improvement. A number of future developments are discussed which may lead to more sophisticated multiobjective WA models.

6.3.1 Establishing objectives by means of an interactive workshop

Although the feedback received from the surveys sent out to five military experts was deemed sufficient for the purposes of this thesis, a more comprehensive approach towards establishing fundamental objectives for the multiobjective WA problem may be to host an interactive workshop in the presence of a workshop facilitator in conjunction with a number of military experts, spanning a large portion of the hierarchal ranks contained in the South African military domain, so as to obtain the input from a diverse set of individuals, in an environment where the military individuals are free to contribute ideas and question the views of their peers. The idea is to exchange ideas and question proposals made by the military individuals in a more intimate way. The aim of hosting such an interactive workshop may be to reach a possible consensus with respect to the objectives relating to the multiobjective WA problem in a South African GBAD context.

6.3.2 Subjective assessments by means of a group of military individuals

Due to the diverse geographic placement of military personnel in South Africa, the assessments for both the AHP and utility approaches towards solving the multiobjective WA problem were carried out by interviewing only one military expert. A more comprehensive approach may be to capture information regarding these assessments from a group of military individuals, and to use this information to develop multiobjective WA models for both the AHP and utility approaches. This is expected be a very challenging exercise, since a consensus has to be reached regarding assessments based on intuition and knowledge with respect to some of the assessments carried out during these approaches.

6.3.3 Multiobjective WA in the context of other simulated GBAD scenarios

Since the multiobjective WA problem was solved by means of the methodologies in §4 in the context of only one simulated GBAD scenario, a further step in the research towards the multiobjective WA problem may be to solve the problem with the same methodologies in §4, but in the context of various simulated GBAD scenarios.

6.3.4 Multiperiod multiobjective WA optimisation

Since the scope of this thesis was restricted to solving the multiobjective WA problem for a single period at a time, a first step in further research with respect to the bi-objective WA model may be to consider the possibility of modelling the bi-objective WA problem for the current period as well as a number of future time steps, thereby introducing a dynamic, scheduling element into the static WA models of this thesis. A further step may then be to include time windows in the model, during which a WS is available and capable with respect to assignment to a threat.

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APPENDIX A

Weapon assignment survey

Contents

This appendix contains the WA questionnaire which was used to extract possible objectives when a choice has to be made with respect to WA. An electronic version of this survey was sent to five military experts via electronic mail. A hard copy of this survey may be found in §A.1 while the feedback received from the military experts is provided in tabular form in §A.2.

A.1 A WA objectives identification survey

The survey commences by providing the reader with a brief background regarding the survey, as well as an explanation of the purpose of the survey. An introduction to multiobjective optimisation with respect to WA is also provided for further education. In order to comply with military lingo, a *35 mm Gun* is an example of a *Close-In Weapon System* (CIWS) , a *Starstreak* is an example of a *Very Short Range Air Defense System* (VSHORADS) WS, and an *Umkhonto* is an example of a *Short Range Air Defense System* SHORADS WS. The complete survey follows.

Establishing Weapon Assignment (WA) factors

This questionnaire forms part of a greater study at the University of Stellenbosch with respect to *Threat Evaluation and Weapon Assignment* (TEWA) as part of a *Ground Based Air Defence System* (GBADS) environment. For additional information on the project, see the attached section at the end of this questionnaire.

For the purpose of this questionnaire, suppose that all the scenarios are contained within a typical GBADS environment, where *Defended Assets* (DAs) have to be protected against incoming aerial threats. Threats are detected and evaluated by a *Threat Evaluation* (TE) system, and necessary information is transferred to a *Weapon Assignment* (WA) system.

Each set of questions is based on a specific scenario subject to different conditions. Please note that these scenarios are simplified versions of real-world scenarios. Consider only the given conditions and please complete the questions from the perspective of a TEWA system. Please state reasons for your decisions at the Why questions.

The following conditions apply throughout the entire questionnaire, unless explicitly stated otherwise:

1. All detected threats are considered fixed wing aircraft.
2. Weather conditions are considered to be daytime with sunny skies and no wind.
3. The dotted line in each scenario represent the flight paths of threat.
4. Each scenario has one DA represented by a green circle.
5. Each detected threat is travelling at a speed of 250 m/s.
6. In each scenario one of each of the following Weapon Systems (WS), represented by blue squares, are available:
   (a) 35mm cannon section,
   (b) Starstreak system,
   (c) Umkhonto system.
7. All WSs are deployed and ready for action.
8. The threat positions A and B (represented by red dots) in the scenarios are located at various distances from the WSs.
9. A maximum of one WS may be considered for assignment with respect to a threat at each position, *i.e.* either one of the WSs may be selected or none at all, in which case it is decided that the threat should be engaged at a later stage.
10. Note that threat positions A and B are independent of each other, *i.e.* when WSs are considered for assignment to a threat at position B, the proposed assignment at position A may be ignored.
11. In each of the scenarios, threat position A is located 12 km from the WSs and threat position B is located 8 km from the WSs, which is on the OP warning line.
12. At each position, any one of the WSs may be selected or you may choose to wait, which implies that the threat will be engaged at a later stage.

Questions:

Please select your choice of WS by circling the appropriate alternative.

Scenario 1

1.1. Which WS would you like TEWA to propose to the threat at the indicated positions A and B, respectively?

A: Umkhonto Starstreak 35mm Gun None
B: Umkhonto Starstreak 35mm Gun None

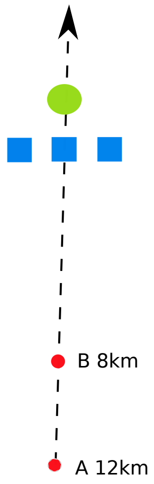

FIGURE A.1: *Scenario 1 for survey questions 1.1–1.12 and 4.1–4.8.*

1.2. Why did you choose these WSs at each of the respective positions?

Now suppose the weather conditions change to heavy rainfall.

1.3. Which WS would you like TEWA to propose to the threat at the indicated positions A and B, respectively?

A: Umkhonto	Starstreak	35mm Gun	None
B: Umkhonto	Starstreak	35mm Gun	None

1.4. Why did you choose these WSs at each of the respective positions?

Now suppose that it is sunny weather conditions, but with a very strong cross wind (> 60 km/h).

1.5. Which WS would you like TEWA to propose to the threat at the indicated positions A and B, respectively?

A: Umkhonto	Starstreak	35mm Gun	None
B: Umkhonto	Starstreak	35mm Gun	None

1.6. Why did you choose these WSs at each of the respective positions?

Now suppose it is night time, with clear skies and no wind.

1.7. Which WS would you like TEWA to propose to the threat at the indicated positions A and B, respectively?

A: Umkhonto	Starstreak	35mm Gun	None
B: Umkhonto	Starstreak	35mm Gun	None

1.8. Why did you choose these WS at each of the respective positions?

Now suppose that the weather is sunny with no wind and the threat is travelling at 150m/s.

1.9. Which WS would you like TEWA to propose to the threat at the indicated positions A and B, respectively?

A: Umkhonto Starstreak 35mm Gun None
B: Umkhonto Starstreak 35mm Gun None

1.10. Why did you choose these WS at each of the respective positions?

Now suppose that the weather is sunny with no wind and the threat is travelling at 300m/s.

1.11. Which WS would you like TEWA to propose to the threat at the indicated positions A and B, respectively?

A: Umkhonto Starstreak 35mm Gun None
B: Umkhonto Starstreak 35mm Gun None

1.12.Why did you choose these WSs at each of the respective positions?

Scenario 2

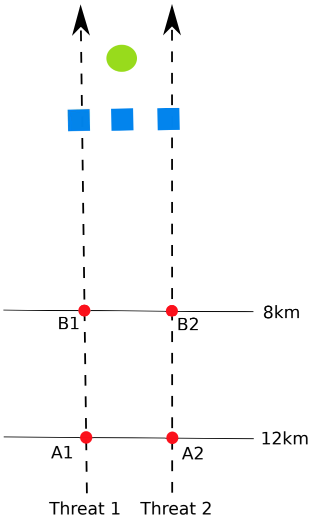

FIGURE A.2: *Scenario 2 for survey Questions 2.1–2.8.*

Suppose that a WS may only be assigned to exactly one threat at a time.

2.1. Which one of the WSs would you like TEWA to assign to the threats at the indicated positions A1, A2, B1 and B2, respectively?

A1: Umkhonto Starstreak 35mm Gun None
A2: Umkhonto Starstreak 35mm Gun None
B1: Umkhonto Starstreak 35mm Gun None
B2: Umkhonto Starstreak 35mm Gun None

2.2. Why did you choose these WSs at each of the respective positions?

Now suppose that threat 1 is travelling at 150m/s, while threat 2 travels at 250m/s.

2.3. Which one of the WSs would you like TEWA to assign to the threats at the indicated positions A1, A2, B1 and B2, respectively?

A1: Umkhonto Starstreak 35mm Gun None
A2: Umkhonto Starstreak 35mm Gun None
B1: Umkhonto Starstreak 35mm Gun None
B2: Umkhonto Starstreak 35mm Gun None

2.4. Why did you choose these WSs at each of the respective points?

Now suppose that threat 1 is only detected at position B1 and travelling at 250m/s, while threat 2 is still detected at position A2.

2.5. Which one of the WSs would you like TEWA to assign to the threats at the indicated positions A2, B1 and B2, respectively?

A2: Umkhonto Starstreak 35mm Gun None
B1: Umkhonto Starstreak 35mm Gun None
B2: Umkhonto Starstreak 35mm Gun None

2.6. Why did you choose these WSs at each of the respective positions?

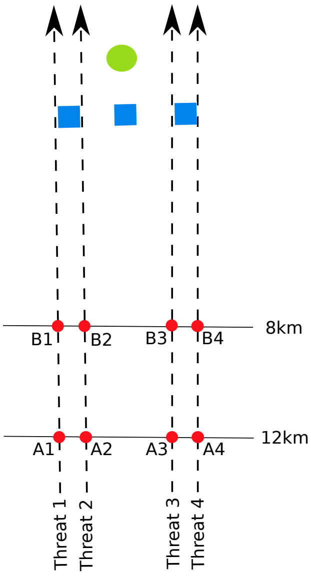

FIGURE A.3: *Scenario 2 for survey Questions 2.9 and 2.10.*

Now suppose that a WS may be assigned to more than one threat at a time.

2.7. Which WSs would you like TEWA to assign to the threats at the indicated positions A1, A2, B1 and B2, respectively?

A1: Umkhonto	Starstreak	35mm Gun	None
A2: Umkhonto	Starstreak	35mm Gun	None
B1: Umkhonto	Starstreak	35mm Gun	None
B2: Umkhonto	Starstreak	35mm Gun	None

2.8. Why did you choose these WSs at each of the respective positions?

Now suppose that there are four detected threats as may be seen in Figure 3, and that a WS may be assigned to more than one threat at a time.

2.9. Which WSs would you like TEWA to assign to the threats at the indicated positions A1, A2, B1 and B2, respectively?

A1: Umkhonto	Starstreak	35mm Gun	None
A2: Umkhonto	Starstreak	35mm Gun	None
A3: Umkhonto	Starstreak	35mm Gun	None
A4: Umkhonto	Starstreak	35mm Gun	None
B1: Umkhonto	Starstreak	35mm Gun	None
B2: Umkhonto	Starstreak	35mm Gun	None
B3: Umkhonto	Starstreak	35mm Gun	None
B4: Umkhonto	Starstreak	35mm Gun	None

2.10. Why did you choose these WS at each of the respective positions?

Scenario 3

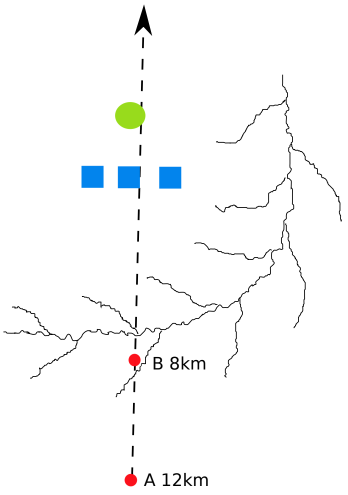

FIGURE A.4: *Scenario 3 for survey questions 3.1 and 3.2.*

In this scenario the WSs and DAs are partially surrounded by a mountain range as indicated in Figure A.4. Due to the mountain range, there is no line of sight between the WSs and the threat at position A; the threat only comes into line of sight at position B. However, it is detected by radar at position A.

3.1. Which one of the three WSs would you choose to assign at the indicated positions A and B, respectively?

A:Umkhonto	Starstreak	35mm Gun	None
B:Umkhonto	Starstreak	35mm Gun	None

3.2. Why did you choose these WSs at each of the respective positions?

Scenario 4

Consider the same situation as in Scenario 1. Additionally, take the monetary cost of one Umkhonto missile as R 2 million, the monetary cost of one Starstreak missile as R 1 million and the monetary cost of a single burst of ammunition for a 35mm Cannon section as R 17 000. Consider the monetary cost of assigning a WS to a threat in answering the following questions.

4.1. Which one of these WS would you like TEWA to propose to the threat at position B?

Umkhonto Starstreak 35mm Gun None

4.2. Why did you choose this WS?

4.3. Are there any other cost factors, other than the monetary cost of ammunition, which should be included in calculations by TEWA when considering WSs against a threat?

Consider the monetary cost of assigning a Starstreak with a predicted Single Shot Hit Probability (SSHP) of 0.9, and a 35mm with a predicted SSHP of 0.7 at position B.

4.4. Which one of these WS would you like TEWA to propose to the threat at position B?

4.5. Under which circumstances would you like TEWA to rather propose a WS which is less capable (smaller SSHP), because it is cheaper to assign?

Suppose the Starstreak has one round of ammunition (hot; and no cold) available, and the 35mm has 50 bursts of ammunition available.

4.6. Would you like TEWA to propose the Starstreak or the 35mm? Why did you choose this WS?

4.7. Under which circumstances would you like TEWA to rather propose a WS with more ammunition than a WS with less ammunition.

4.8. Do you think that ammunition available to a WS should be considered a factor in your decision to assign a WS? Simply answer by stating only YES or NO.

5. Do you think the following factors should be considered when deciding which WSs to propose to threats? A simple YES or NO answer next to each is sufficient.

5.1. SSHP
5.2. Cost of ammunition
5.3. Available ammunition to a WS
5.4. Line of sight
5.5. Weather conditions
5.6. Vertical launch ability of a WS
5.7. Multiple engagements of a WS
5.8. Terrain
5.9. Reaction time of a WS
5.10. Speed of WS ammunition
5.11. Effective range of a WS

6. Can you think of any other factors not covered in this questionnaire that you feel should be included in the decision of assigning WSs to threats?

Additional information regarding the research project.

Thank you for taking the time to participate in this study. The purpose of this study is to model the WA problem as a so-called multiobjective decision making problem. In a typical multiobjective decision making problem, a decision maker has to choose between a number of alternatives (courses of action), where multiple conflicting objectives (factors) have to be achieved (taken into account). An example of conflicting objectives are, for instance, where an investor wants to maximise the return on his investment, but at the same time wants minimise his exposure to risk. Because of their conflicting nature, it is impossible to achieve both at the same time.

In a WA context, the decision maker would have to choose between different WS-threat pairs (the alternatives) where, for instance, the decision maker wants to maximise the capability of the WSs (*e.g.* SSHP) and to simultaneously minimize the cost of assigning those WSs (the conflicting objectives). The first step in any multiobjective decision process is to establish the relevant objectives or factors that are considered when a choice is made with respect to which WS to assign. These factors form the building blocks of the multiobjective models. It is critical that both the decision maker and the consulting analyst have a clear understanding about the problem at hand as well as what the decision maker wants to achieve in the end. The final set of objectives should include all the important aspects of a decision. Failing to perform a thorough analysis on the relevant objectives may result in models delivering inaccurate or inappropriate results.

In practice, these objectives or factors are obtained by interviewing experts in the specific field of relevance to try and extract all the objectives which are important. The purpose of this questionnaire is therefore to try and obtain all the necessary and important objectives that should be included in the decision to assign a WS to an incoming threat.

These factors may then be utilised in the construction of three multiobjective decision support approaches involving utility functions, the *Analytic Hierarchy Process* (AHP) and a *Nondominated Sorting Genetic Algorithm II* (NSGA II). A utility function captures the preferences of the alternatives of a decision maker for each objective. The AHP, on the other hand, makes use of pairwise comparisons between alternatives for each of the objectives. The necessary information needed for the utility function and AHP models will be obtained by means of follow-up interviews and/or questionnaires.

A.2 Feedback obtained from the WA survey

This section contains a summary of the feedback received from the surveys after they were completed by the military experts. Complete feedback was obtained from only three of the five military experts, hence, the tables contain only feedback from these three military experts. A forth military expert did not complete the survey. However, he made a few suggestions and comments, which are captured in the final table, Table A.5. The feedback is captured in five tables labeled as Table A.1 to Table A.5. Each table consists of the question number and the corresponding feedback from each of the military experts, labeled *Result 1* – *Result 3*.

Question nr	Result 1	Result 2	Result 3
1.1 A	Umkhonto	Starstreak	Umkhonto
1.1 B	Starstreak	35mm Gun	35mm Gun
1.2	The range is applicable.	The 35mm will provide a better defence layout and any missile can be used as GAP filler.	A, due to the range. B, due to the ability of the 35 section.
1.3 A	Umkhonto	Umkhonto	Umkhonto
1.3 B	Starstreak	Starstreak	35mm Gun
1.4	The application range.	Can be launched from different shelter positions.	A, the Umkhonto has an all-weather capability. B, the 35 has an all-weather capability.
1.5 A	Starstreak	35mm Gun	Umkhonto
1.5 B	35mm Gun	Umkhonto	35mm Gun
1.6	Enough reaction time for the Guns and the Starstreak.	35 mm layer defence will pose a bigger threat than any of the other.	A, the Umkhonto will lock onto the target when it is close, and therefore the wind will not play a role. B, the 35 has the ability to set the MET data and therefore bring the wind speed into calculation. The Starstreak has to be controlled by the operator and he has to counteract by steering the missile onto the target; that is difficult with a strong wind.
1.7 A	Umkhonto	Starstreak	Umkhonto
1.7 B	Starstreak	Umkhonto	35mm Gun
1.8	Capability to fire at night, even the Guns can be utilised.	Starstreak has a shorter engage distance than the Umkhonto.	A, all weather capability. B, all weather capability.

TABLE A.1: *WA survey feedback for questions 1.1–1.8.*

Question nr	Result 1	Result 2	Result 3
1.9 A	Starstreak	35mm Gun	Umkhonto
1.9 B	35mm Gun	Umkhonto	Starstreak
1.10	Enough time to lock on target.	I think the wind will influence the Starstreak more than the other.	A, distance. B, the operator has the ability to acquire the target and fire.
1.11 A	Umkhonto	35mm Gun	Umkhonto
1.11 B	None	Umkhonto	35mm Gun
1.12	The target will be difficult to track effectively by the Starstreak and Guns.	I think the wind will influence the Starstreak more than the other.	A/B, the ability of the system in both cases. With the Starstreak it is up to the operator to achieve this
2.1 A_1	Umkhonto	Umkhonto	Umkhonto
2.1 A_2	Umkhonto	Umkhonto	None
2.1 B_1	Starstreak	35mm Gun	35mm Gun
2.1 B_2	Starstreak	35mm Gun	Starstreak
2.2	Longer range.	As detterent.	A1, to achieve a hit at this distance. A2 will be addressed by the Umkhoto at the B position. B1/B2 will be addressed at the same time.
2.3 A_1	Starstreak	Umkhonto	Umkhonto
2.3 A_2	Umkhonto	Umkhonto	Umkhonto
2.3 B_1	Starstreak	35mm Gun	Starstreak
2.3 B_2	Umkhonto	35mm Gun	35mm Gun
2.4	The reaction time of the weapon system.	Different ADA defence layers.	Due to the variance in speed, A1 and A2 will be at different positions and can therefore be engaged by the same WS.

Table A.2: *WA survey feedback for questions 1.9–2.4.*

Question nr	Result 1	Result 2	Result 3
2.5 A_2	35mm Gun	Umkhonto	Umkhonto
2.5 B_1	Umkhonto	35mm Gun	35mm Gun
2.5 B_2	Umkhonto	35mm Gun	Starstreak
2.6	The distance and the reaction time of the Guns versus the Starstreak missile.	Different ADA defence layers.	-
2.7 A_1	35mm Gun	Umkhonto	Umkhonto
2.7 A_2	Starstreak	Umkhonto	Umkhonto
2.7 B_1	Umkhonto	35mm Gun	35mm Gun
2.7 B_2	Umkhonto	35mm Gun	Starstreak
2.8	It will give each weapon system enough time to react.	Different ADA defence layers.	A, because of distance. B, because it is easier to assign one at a time.
2.9 A_1	Umkhonto	-	Umkhonto
2.9 A_2	Umkhonto	35mm Gun	Umkhonto
2.9 A_3	Umkhonto	35mm Gun	None
2.9 A_4	Umkhonto	35mm Gun	None
2.9 B_1	Starstreak	35mm Gun	Starstreak
2.9 B_2	Starstreak	35mm Gun	35mm Gun
2.9 B_3	Starstreak	35mm Gun	Starstreak
2.9 B_4	Starstreak	35mm Gun	35mm Gun
2.10	The application distance of each weapon distance will be the determining factor.	Different defence layout.	-
3.1 A	None	Umkhonto	Umkhonto
3.1 B	Umkhonto	-	35mm Gun
3.2	Umkhonto might be in the position to engage at 12km, based on the height. However, the masking will hamper both the Guns and the Starstreak to track at 8km.	Gap filler or flack trap.	The Umkhonto launches vertically and then acquires the target from the radar signal and can therefore be applied against A. B, the reaction and acquiring of the target by the 35 is very quick.

TABLE A.3: *WA survey feedback for questions 2.5–3.2.*

Question nr	Result 1	Result 2	Result 3
4.1	Starstreak	Umkhonto	35mm Gun
4.2	It is the better weapon with the better application range to engage at that track on that distance, irrespective of the cost.	Better hit probability.	If all are equal, then the cheapest applies.
4.3	No, unless the decision is to be made when the target is within range for both Starstreak and Guns.	Hit probability.	-
4.4	Starstreak	Umkhonto	35mm Gun
4.5	When the target is within range for both Starstreak and Guns.	-	-
4.6	35mm, because you have 50 chances to engage on the target than a single shot which might result in a miss.	35mm Gun	35mm Gun
4.7	When the weapon with more ammunition have the same hit probability as the one with less ammunition.	-	The ammo should always try to be levelled if possible. Try not to deplete a system.
4.8	Yes	-	Yes
5.1. SSHP	Yes	-	Yes
5.2. Cost of ammunition	No	-	Yes
5.3. Available ammunition	Yes	Yes	Yes
5.4. Line of sight (LOS)	Yes	Yes	Yes
5.5. Weather conditions	Yes	Yes	Yes
5.6. Vertical launch capability	Yes	Yes	Yes
5.7. Multiple engagements	Yes	Yes	Yes
5.8. Terrain features	Yes	Yes	Yes
5.9. Reaction times	Yes	Yes	Yes
5.10. Speed of WS ammunition	No	Yes	No
5.11. Effective ranges of WSs	Yes	Yes	Yes

Table A.4: *WA survey feedback for questions 4.1–5.*

Question nr	Result 1	Result 2	Result 3
6.	When there is an interference of rotary wings circulating during the engagement, TEWA is to make the decision in terms of fixed or rotary per WS.	No	Targets crossing, targets flying away from the DA and ECCM.
	General comments		
1.	A TEWA system must always consider a combination of WSs or more than one WS to a target. No attacking aircraft will be deterred by only one WS.		
2.	Monetary cost should only be taken into consideration when weapon systems have an equal fire solution.		
3.	The better fire solution should carry more weight than the amount of ammo available. Only if both WSs have the same hit probability, then the WS with more ammo should be designated.		

TABLE A.5: *WA survey feedback for question 6 and general comments made by the military experts.*

APPENDIX B

AHP pairwise comparison matrices

This appendix contains the table presenting the pairwise comparison matrix for the cost objective as well as the tables presenting the AHP pairwise comparison matrices for the SSHP objective, with respect to each WS-threat pair and for each time step in the scenario described in §4.2. The pairwise comparisons with respect to each WS-threat pair (with the inclusion of the dummy WSs) for the cost objective are presented in Table B.1. The pairwise comparisons for threats T_1, T_2, T_3, T_4 and T_5, with respect to time step t_{20} are presented in Tables B.2–B.6, respectively.

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5	1/9	1/9	1/9	1/9	1/9
V_2	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5	1/9	1/9	1/9	1/9	1/9
V_3	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5	1/9	1/9	1/9	1/9	1/9
V_4	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5	1/9	1/9	1/9	1/9	1/9
V_5	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5	1/9	1/9	1/9	1/9	1/9
V_6	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5	1/9	1/9	1/9	1/9	1/9
V_7	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5	1/9	1/9	1/9	1/9	1/9
V_8	1	1	1	1	1	1	1	1	1/5	1/5	1/5	1/5	1/9	1/9	1/9	1/9	1/9
C_1	5	5	5	5	5	5	5	5	1	1	1	1	1/2	1/2	1/2	1/2	1/2
C_2	5	5	5	5	5	5	5	5	1	1	1	1	1/2	1/2	1/2	1/2	1/2
C_3	5	5	5	5	5	5	5	5	1	1	1	1	1/2	1/2	1/2	1/2	1/2
C_4	5	5	5	5	5	5	5	5	1	1	1	1	1/2	1/2	1/2	1/2	1/2
D_1	9	9	9	9	9	9	9	9	2	2	2	2	1	1	1	1	1
D_2	9	9	9	9	9	9	9	9	2	2	2	2	1	1	1	1	1
D_3	9	9	9	9	9	9	9	9	2	2	2	2	1	1	1	1	1
D_4	9	9	9	9	9	9	9	9	2	2	2	2	1	1	1	1	1
D_5	9	9	9	9	9	9	9	9	2	2	2	2	1	1	1	1	1

Table B.1: *The AHP pairwise comparison matrix for the cost objective with respect to each WS-threat with the inclusion of dummy WSs.*

The pairwise comparison matrices were verified for consistency, with respect to each threat. The corresponding CI/RI values for each of the pairwise comparison matrices with respect to time step t_{20} are presented in Table B.7. Since each of the CI/RI values in Table B.7 is less than 0.1, it may be concluded that the pairwise comparisons involved are consistent.

The pairwise comparisons for threats T_1, T_2, T_3, T_4 and T_5 with respect to time step t_{35} are presented in Tables B.8–B.12, respectively.

The corresponding CI/RI values for each of the pairwise comparison matrices with respect to time step t_{35} are presented in Table B.13. Since each of the CI/RI values in Table B.13 is less

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE B.2: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_1 at time step t_{20}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE B.3: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_2 at time step t_{20}.*

than 0.1, it may be concluded that the pairwise comparisons involved are consistent.

Finally, the pairwise comparisons for threats T_1, T_2, T_3, T_4 and T_5 with respect to time step t_{39} are presented in Tables B.14–B.18, respectively.

The corresponding CI/RI values for each of the pairwise comparison matrices with respect to time step t_{39} are presented in Table B.19. Since the CI/RI values in Table B.19 are all less than 0.1, it may be concluded that the pairwise comparisons involved are consistent.

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_2	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_3	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_4	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_5	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_6	2	2	2	2	2	1	2	2	2	2	1	2	2	2	2	2	2
V_7	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_8	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
C_1	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
C_2	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
C_3	2	2	2	2	2	1	2	2	2	2	1	2	2	2	2	2	2
C_4	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_1	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_2	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_3	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_4	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_5	1	1	1	1	1	1/2	1	1	1	1	1/2	1	1	1	1	1	1

TABLE B.4: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_3 at time step t_{20}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_2	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_3	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_4	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_5	6	6	6	6	1	5	6	6	6	6	5	6	6	6	6	6	6
V_6	2	2	2	2	1/5	1	2	2	2	2	1	2	2	2	2	2	2
V_7	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
V_8	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
C_1	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
C_2	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
C_3	2	2	2	2	1/5	1	2	2	2	2	1	2	2	2	2	2	2
C_4	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_1	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_2	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_3	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_4	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1
D_5	1	1	1	1	1/6	1/2	1	1	1	1	1/2	1	1	1	1	1	1

TABLE B.5: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_4 at time step t_{20}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table B.6: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_5 at time step t_{20}.*

Table	CI/RI value
B.2	0
B.3	0
B.4	0
B.5	0.0009
B.6	0

Table B.7: *The CI/RI values for the AHP pairwise comparison matrices with respect to time step t_{20}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
V_2	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
V_3	2	2	1	1	2	2	2	2	2	1/2	2	2	2	2	2	2	2
V_4	2	2	1	1	2	2	2	2	2	1/2	2	2	2	2	2	2	2
V_5	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
V_6	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
V_7	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
V_8	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
C_1	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
C_2	3	3	2	2	3	3	3	3	3	1	3	3	3	3	3	3	3
C_3	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
C_4	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
D_1	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
D_2	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
D_3	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
D_4	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1
D_5	1	1	1/2	1/2	1	1	1	1	1	1/3	1	1	1	1	1	1	1

Table B.8: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_1 at time step t_{35}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1/6	1/6	1	1	1	1	1	1/4	1/2	1	1	1	1	1	1
V_2	1	1	1/6	1/6	1	1	1	1	1	1/4	1/2	1	1	1	1	1	1
V_3	6	6	1	1	6	6	6	6	6	3	5	6	6	6	6	6	6
V_4	6	6	1	1	6	6	6	6	6	3	5	6	6	6	6	6	6
V_5	1	1	1/6	1/6	1	1	1	1	1	1/4	1/2	1	1	1	1	1	1
V_6	1	1	1/6	1/6	1	1	1	1	1	1/4	1/2	1	1	1	1	1	1
V_7	1	1	1/6	1/6	1	1	1	1	1	1/4	1/2	1	1	1	1	1	1
V_8	1	1	1/6	1/6	1	1	1	1	1	1/4	1/2	1	1	1	1	1	1
C_1	1	1	1/6	1/6	1	1	1	1	1	1/4	1/2	1	1	1	1	1	1
C_2	4	4	1/3	1/3	4	4	4	4	4	1	3	3	3	3	3	3	3
C_3	2	2	1/5	1/5	2	2	2	2	2	1/3	1	2	2	2	2	2	2
C_4	1	1	1/6	1/6	1	1	1	1	1	1/3	1/2	1	1	1	1	1	1
D_1	1	1	1/6	1/6	1	1	1	1	1	1/3	1/2	1	1	1	1	1	1
D_2	1	1	1/6	1/6	1	1	1	1	1	1/3	1/2	1	1	1	1	1	1
D_3	1	1	1/6	1/6	1	1	1	1	1	1/3	1/2	1	1	1	1	1	1
D_4	1	1	1/6	1/6	1	1	1	1	1	1/3	1/2	1	1	1	1	1	1
D_5	1	1	1/6	1/6	1	1	1	1	1	1/3	1/2	1	1	1	1	1	1

TABLE B.9: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_2 at time step t_{35}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1/3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
V_2	3	1	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
V_3	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_4	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_5	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_6	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_7	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_8	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_1	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_2	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_1	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_2	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_3	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_4	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_5	1/2	1/4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE B.10: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_3 at time step t_{35}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	5	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
V_2	1/5	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
V_3	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_4	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_5	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_6	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_7	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_8	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_1	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_2	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_1	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_2	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_3	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_4	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_5	1/6	1/2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE B.11: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_4 at time step t_{35}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE B.12: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_5 at time step t_{35}.*

Table	CI/RI value
B.8	0.0003
B.9	0.0032
B.10	0.0003
B.11	0.0005
B.12	0

TABLE B.13: *The CI/RI values for the AHP pairwise comparison matrices with respect to time step t_{35}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1/8	1/5	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
V_2	8	1	7	8	8	8	8	8	6	3	8	8	1	1	1	1	1
V_3	2	1/7	1	2	2	2	2	2	1/2	1/5	2	2	2	2	2	2	2
V_4	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
V_5	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
V_6	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
V_7	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
V_8	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
C_1	3	1/6	2	3	3	3	3	3	1	1/4	3	3	3	3	3	3	3
C_2	6	1/3	5	6	6	6	6	6	4	1	6	6	6	6	6	6	6
C_3	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
C_4	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
D_1	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
D_2	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
D_3	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
D_4	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1
D_5	1	1/8	1/2	1	1	1	1	1	1/3	1/6	1	1	1	1	1	1	1

TABLE B.14: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_1 at time step t_{39}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
V_2	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
V_3	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
V_4	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
V_5	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
V_6	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
V_7	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
V_8	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
C_1	5	5	5	5	5	5	5	5	1	1/6	5	5	5	5	5	5	5
C_2	9	9	9	9	9	9	9	9	6	1	9	9	9	9	9	9	9
C_3	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
C_4	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
D_1	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
D_2	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
D_3	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
D_4	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1
D_5	1	1	1	1	1	1	1	1	1/5	1/9	1	1	1	1	1	1	1

TABLE B.15: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_2 at time step t_{39}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1/5	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
V_2	5	1	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
V_3	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_4	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_5	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_6	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_7	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_8	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_1	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_2	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_1	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_2	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_3	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_4	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_5	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE B.16: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_3 at time step t_{39}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1/5	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
V_2	5	1	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
V_3	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_4	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_5	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_6	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_7	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_8	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_1	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_2	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_1	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_2	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_3	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_4	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D_5	1/2	1/6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE B.17: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat T_4 at time step t_{39}.*

SSHP	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	C_1	C_2	C_3	C_4	D_1	D_2	D_3	D_4	D_5
V_1	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
V_2	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
V_3	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
V_4	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
V_5	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
V_6	2	2	2	2	2	1	1/5	2	2	2	2	2	2	2	2	2	2
V_7	6	6	6	6	6	5	1	6	6	6	6	6	6	6	6	6	6
V_8	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
C_1	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
C_2	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
C_4	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
D_1	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
D_2	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
D_3	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
D_4	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1
D_5	1	1	1	1	1	1/2	1/6	1	1	1	1	1	1	1	1	1	1

TABLE B.18: *The AHP pairwise comparison matrix for the SSHP objective with respect to threat* T_5 *at time step* t_{39}.

Table	CI/RI
B.14	0.0460
B.15	0.0032
B.16	0.0005
B.17	0.0005
B.18	0.0005

TABLE B.19: *The CI/RI values for the pairwise comparison matrices with respect to time step* t_{39}.

APPENDIX C

Solutions obtained by the NSGA II

This appendix contains the assignment of WSs to threats as proposed by the NSGA II for each of the three time steps. The entire set of solutions obtained is illustrated graphically for each of the three time steps. Moreover, the proposed assignments on the approximate pareto front are presented in matrix form, denoted by Sol_r where r represents the r^{th} solution on the approximate pareto frontier.

The entire set of solutions in bi-objective decision space corresponding to time step t_{20} is presented graphically in Figure C.1. There are five solutions on the approximate pareto frontier for time step t_{20}, and the proposed assignments of WSs to the threats corresponding to each of these five solutions are presented in Table C.1.

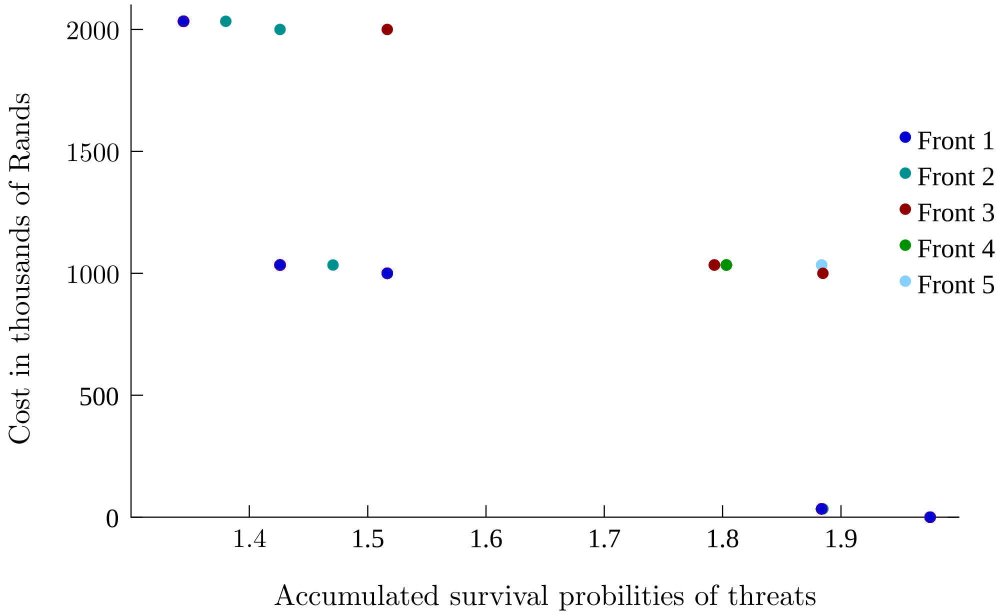

FIGURE C.1: *The set of solutions obtained by means of the NSGA II for time step t_{20}.*

The set of solutions in two-dimensional objective space obtained by the NSGA II for time step t_{35} are presented graphically in Figure C.2. Fifteen solutions are contained in the set of approximately pareto optimal solutions for time step t_{35}. The assignment of WSs to threats corresponding to solutions 1–9 are presented in Table C.2, while the assignment of WSs to threats corresponding to solutions 10–15 are presented in Table C.3.

$$\begin{array}{cc} \text{Sol}_1 & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{cc} \text{Sol}_2 & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{cc} \text{Sol}_3 & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{cc} \text{Sol}_4 & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{cc} \text{Sol}_5 & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

TABLE C.1: *The assignment of WSs to threats for the approximately pareto optimal solutions obtained by means of the NSGA II for time step* t_{20}.

The set of solutions obtained by the NSGA II for time step t_{39} are presented graphically in Figure C.3. There are ten solutions in the set of approximately pareto optimal solutions. The assignment of WSs to threats for the first six approximately pareto optimal solutions are presented in Table C.4, while the assignments corresponding to the remaining four approximately pareto optimal solutions are presented in Table C.5.

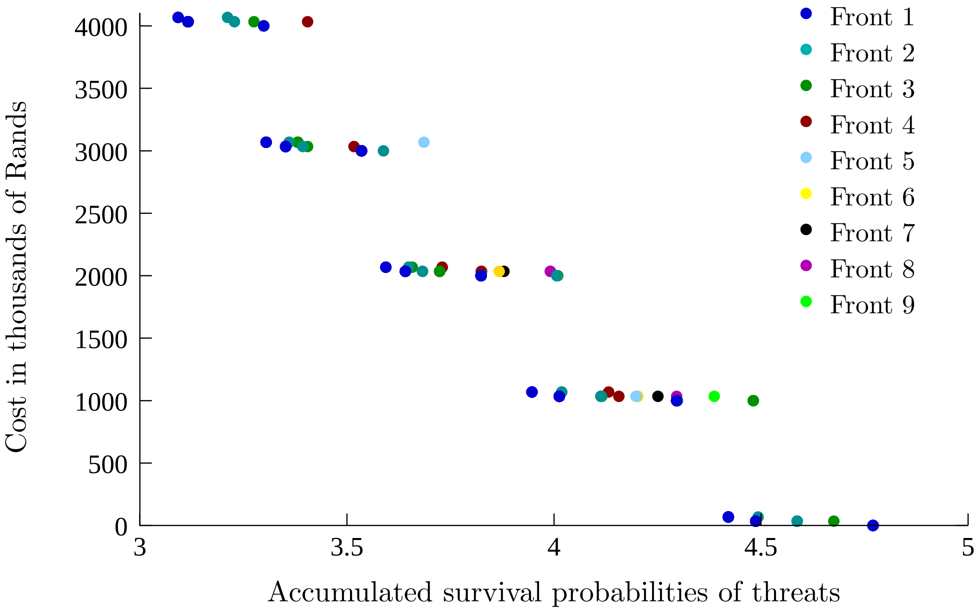

FIGURE C.2: *The set of solutions obtained by means of the NSGA II for time step* t_{35}.

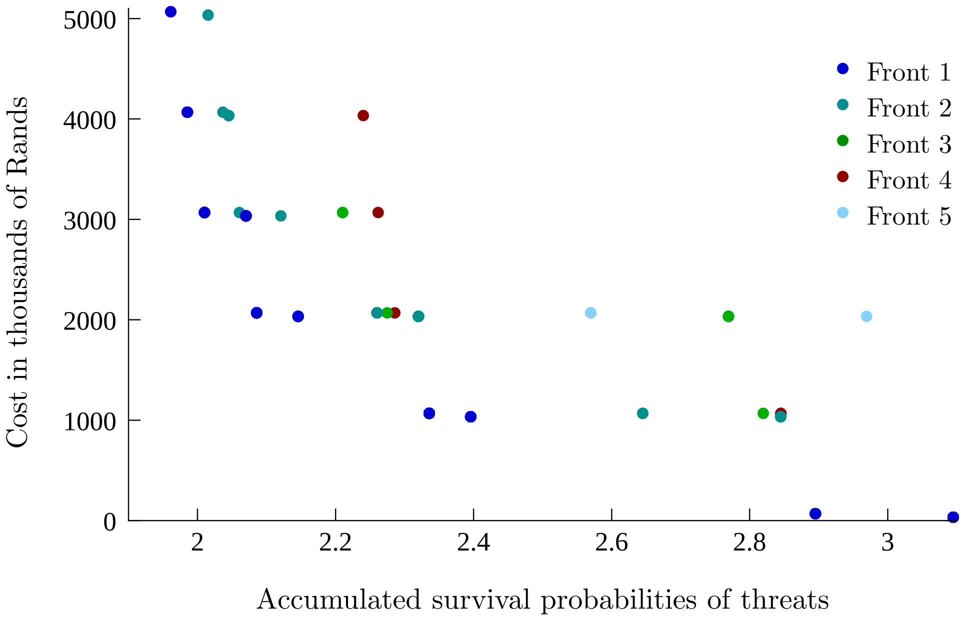

FIGURE C.3: *The set of solutions obtained by means of the NSGA II for time step* t_{39}.

$$\begin{array}{c|ccccc|}
\text{Sol}_1 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 1 & 0 \\
V_2 & 0 & 0 & 1 & 0 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 1 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 1 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 1 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
\text{Sol}_2 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 1 & 0 \\
V_2 & 0 & 0 & 1 & 0 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 1 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 1 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
\text{Sol}_3 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 1 & 0 \\
V_2 & 0 & 0 & 1 & 0 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 1 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
\text{Sol}_4 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 1 & 0 \\
V_2 & 0 & 0 & 1 & 0 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 1 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 1 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
\text{Sol}_5 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 1 & 0 \\
V_2 & 0 & 0 & 1 & 0 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 1 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
\text{Sol}_6 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 1 & 0 \\
V_2 & 0 & 0 & 1 & 0 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
\text{Sol}_7 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 1 & 0 \\
V_2 & 0 & 0 & 0 & 0 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 1 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 1 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
\text{Sol}_8 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 1 & 0 \\
V_2 & 0 & 0 & 0 & 0 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 1 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
\text{Sol}_9 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 1 & 0 \\
V_2 & 0 & 0 & 0 & 0 & 0 \\
V_3 & 0 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

TABLE C.2: *The assignment of WSs to threats for the approximately pareto optimal solutions obtained by means of the NSGA II for solutions 1–9 for time step* t_{35}.

$$\mathrm{Sol}_{10}\quad\begin{array}{cc} & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\mathrm{Sol}_{11}\quad\begin{array}{cc} & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\mathrm{Sol}_{12}\quad\begin{array}{cc} & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\mathrm{Sol}_{13}\quad\begin{array}{cc} & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\mathrm{Sol}_{14}\quad\begin{array}{cc} & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\mathrm{Sol}_{15}\quad\begin{array}{cc} & \begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} & \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

TABLE C.3: *The assignment of WSs to threats for the approximately pareto optimal solutions obtained by means of the NSGA II for solutions 10–15 for time step* t_{35}.

$$\begin{array}{c|ccccc|}
\mathrm{Sol}_1 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 1 & 0 & 0 \\
V_2 & 1 & 0 & 0 & 0 & 0 \\
V_3 & 1 & 0 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 1 \\
V_7 & 0 & 0 & 0 & 0 & 1 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 1 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 1 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}
\quad
\begin{array}{c|ccccc|}
\mathrm{Sol}_2 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 1 & 0 & 0 \\
V_2 & 1 & 0 & 0 & 0 & 0 \\
V_3 & 0 & 0 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 1 \\
V_7 & 0 & 0 & 0 & 0 & 1 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 1 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 1 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}
\quad
\begin{array}{c|ccccc|}
\mathrm{Sol}_3 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 1 & 0 & 0 \\
V_2 & 1 & 0 & 0 & 0 & 0 \\
V_3 & 0 & 0 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 1 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 1 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 1 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccccc|}
\mathrm{Sol}_4 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 1 & 0 & 0 \\
V_2 & 1 & 0 & 0 & 0 & 0 \\
V_3 & 0 & 0 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 1 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 1 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}
\quad
\begin{array}{c|ccccc|}
\mathrm{Sol}_5 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 0 & 0 \\
V_2 & 1 & 0 & 0 & 0 & 0 \\
V_3 & 0 & 0 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 1 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 1 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 1 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}
\quad
\begin{array}{c|ccccc|}
\mathrm{Sol}_6 & T_1 & T_2 & T_3 & T_4 & T_5 \\
V_1 & 0 & 0 & 0 & 0 & 0 \\
V_2 & 1 & 0 & 0 & 0 & 0 \\
V_3 & 0 & 0 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 0 & 0 & 0 & 0 & 0 \\
V_6 & 0 & 0 & 0 & 0 & 0 \\
V_7 & 0 & 0 & 0 & 0 & 1 \\
V_8 & 0 & 0 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 1 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 0 & 0 \\
C_4 & 0 & 0 & 0 & 0 & 0
\end{array}$$

TABLE C.4: *The assignment of WSs to threats for the approximately pareto optimal solutions obtained by means of the NSGA II for solutions 1–6 for time step* t_{39}.

$$\mathrm{Sol}_7 \quad \begin{array}{c|ccccc|} & T_1 & T_2 & T_3 & T_4 & T_5 \\ \hline V_1 & 0 & 0 & 0 & 0 & 0 \\ V_2 & 1 & 0 & 0 & 0 & 0 \\ V_3 & 0 & 0 & 0 & 0 & 0 \\ V_4 & 0 & 0 & 0 & 0 & 0 \\ V_5 & 0 & 0 & 0 & 0 & 0 \\ V_6 & 0 & 0 & 0 & 0 & 0 \\ V_7 & 0 & 0 & 0 & 0 & 0 \\ V_8 & 0 & 0 & 0 & 0 & 0 \\ C_1 & 1 & 0 & 0 & 0 & 0 \\ C_2 & 0 & 1 & 0 & 0 & 0 \\ C_3 & 0 & 0 & 0 & 0 & 0 \\ C_4 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

$$\mathrm{Sol}_8 \quad \begin{array}{c|ccccc|} & T_1 & T_2 & T_3 & T_4 & T_5 \\ \hline V_1 & 0 & 0 & 0 & 0 & 0 \\ V_2 & 1 & 0 & 0 & 0 & 0 \\ V_3 & 0 & 0 & 0 & 0 & 0 \\ V_4 & 0 & 0 & 0 & 0 & 0 \\ V_5 & 0 & 0 & 0 & 0 & 0 \\ V_6 & 0 & 0 & 0 & 0 & 0 \\ V_7 & 0 & 0 & 0 & 0 & 0 \\ V_8 & 0 & 0 & 0 & 0 & 0 \\ C_1 & 0 & 0 & 0 & 0 & 0 \\ C_2 & 0 & 1 & 0 & 0 & 0 \\ C_3 & 0 & 0 & 0 & 0 & 0 \\ C_4 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

$$\mathrm{Sol}_9 \quad \begin{array}{c|ccccc|} & T_1 & T_2 & T_3 & T_4 & T_5 \\ \hline V_1 & 0 & 0 & 0 & 0 & 0 \\ V_2 & 0 & 0 & 0 & 0 & 0 \\ V_3 & 0 & 0 & 0 & 0 & 0 \\ V_4 & 0 & 0 & 0 & 0 & 0 \\ V_5 & 0 & 0 & 0 & 0 & 0 \\ V_6 & 0 & 0 & 0 & 0 & 0 \\ V_7 & 0 & 0 & 0 & 0 & 0 \\ V_8 & 0 & 0 & 0 & 0 & 0 \\ C_1 & 1 & 0 & 0 & 0 & 0 \\ C_2 & 0 & 1 & 0 & 0 & 0 \\ C_3 & 0 & 0 & 0 & 0 & 0 \\ C_4 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

$$\mathrm{Sol}_{10} \quad \begin{array}{c|ccccc|} & T_1 & T_2 & T_3 & T_4 & T_5 \\ \hline V_1 & 0 & 0 & 0 & 0 & 0 \\ V_2 & 0 & 0 & 0 & 0 & 0 \\ V_3 & 0 & 0 & 0 & 0 & 0 \\ V_4 & 0 & 0 & 0 & 0 & 0 \\ V_5 & 0 & 0 & 0 & 0 & 0 \\ V_6 & 0 & 0 & 0 & 0 & 0 \\ V_7 & 0 & 0 & 0 & 0 & 0 \\ V_8 & 0 & 0 & 0 & 0 & 0 \\ C_1 & 0 & 0 & 0 & 0 & 0 \\ C_2 & 0 & 1 & 0 & 0 & 0 \\ C_3 & 0 & 0 & 0 & 0 & 0 \\ C_4 & 0 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

TABLE C.5: *The assignments of WSs to threats for the approximately pareto optimal solutions obtained by means of the NSGA II for solutions 7–10 for time step t_{39}.*

APPENDIX D

Contents of the accompanying compact disc

This appendix contains a brief description of the contents of the compact disc included with this thesis. The compact disc contains an electronic version of the WA questionnaire (described in §4.1) which was sent to military experts for completion. The disc also includes the computer code of the implemented NSGA II, which may be used to reproduce the results obtained by means of the NSGA II in this thesis. It also contains an electronic copy of the thesis itself. The material on the compact disc is organised as follows into three directories:

WA Survey. This directory contains the WA questionnaire, `WA_restricted_survey.docx`, which was sent to five military experts. The user requires Microsoft Office Word [39] to open the file. Furthermore, the user has restricted access to the file and is only allowed to enter text into the relevant textboxes and to make choices with respect to WSs by selecting between various radio buttons.

NSGA II. This directory houses twelve `.m`-files containing the code for the various operators of the NSGA II as well as a file, `input.mat`, containing the input data required. The user requires the software suite Matlab [37] to run the code and is required to specify input data including the EEM data, the cost values c of WSs, the number of threats n_t and the number of WSs n_w. The user is also required to specify various other algorithmic parameter values, including the number of iterations, the population size, the tour size, the pool size and the mutation probability.

Thesis electronic version. This directory contains an electronic copy of the thesis, `Thesis.pdf`. The reader requires Adobe Reader [2] to open this document.