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THE EFFECTS OF A JOINT CORRECTION FOR THE ATTENUATING EFFECT OF CRITERION UNRELIABILITY AND CASE 2 RESTRICTION OF RANGE ON THE VALIDITY COEFFICIENT

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OPSOMMING

Hierdie artikel rapporteer die resultate van 'n gedeelte van 'n meer omvattende studie oor die effek van korreksies vir toevallige metingsfout in beide die kriterium sowel as die voorspeller en/of verskeie vorms van inperking van variasiewydte op die parameters [bv., ρ [X,Y], β[Y|X], σ[Y|X]] wat vereis word ten einde 'n seleksieprosedure te spesifiseer en te regverdig. Die doel met die artikel is om die effek van die gesamentlike korreksie vir kriteriumonbetroubaarheid en Tipe 2 inperking van variasiewydte op die geldigheidskoeffisiënt te bepaal. Resultate word grafies voorgestel en omskryf.

ABSTRACT

This paper reports the results of a portion of a more comprehensive study on the effect of correction for random error of measurement in both the criterion and the predictor and/or various forms of restriction of range on the parameters [e.g., ρ[X,Y], β[Y|X], σ[Y|X]] required to specify and justify a selection procedure. The objective of this paper is to determine the effect of a joint correction for criterion unreliability and Case 2 restriction of range on the validity coefficient. Results are depicted graphically and discussed.

Selection, as it is traditionally interpreted, represents a critical human resource intervention in any organisation in so far as it regulates the movement of employees into, through and out of the organisation. As such selection firstly represents a potentially powerful instrument through which the human resource function can add value to the organisation [Boudreau, 1983b; Boudreau & Berger, 1985a; Cascio, 1991b; Cronshaw & Alexander, 1985]. Selection, furthermore, represents a relatively visible mechanism through which access to employment opportunities are regulated. Because of this latter aspect, selection, more than any other human resource intervention, has been singled out for intense scrutiny from the pespective of fairness and affirmative action [Arvey & Faley, 1988; Milkovich & Boudreau, 1994; Singer, 1993]. Two basic criteria are implied in terms of which selection procedures need to be evaluated, namely efficiency and equity [Milkovich & Boudreau, 1994]. The quest for efficient and equitable selection procedures requires periodic psychometric audits to provide the feedback needed to refine the selection procedure to greater efficiency and to provide the evidence required to vindicate the organisation should it be challenged in terms of anti-discriminatory legislation. The empirical evidence needed to meet the aforementioned burden of persuasion is based on a simulation of the actual selection procedure on a sample taken from the applicant population. According to the Guidelines for the validation and use of personnel selection procedures [Society for Industrial Psychology, 1992], the Principles for the validation and use of personnel selection procedures [Society for Industrial and Organisational Psychology, 1987] and the Kleiman and Faley [1985] review of selection litigation, such a psychometric audit of a selection procedure would require the human resource function to demonstrate that:

- the selection procedure has its foundation in a scientifically credible performance theory;
- the selection procedure constitutes a business necessity; and
- the manner in which the selection strategy combines applicant information can be considered fair.

The empirical evidence needed to meet the aforementioned burden of persuasion is acquired through a simulation of the actual selection procedure on a sample taken from the applicant population. Internal and external validity constitute two criteria in terms of which the credibility of the evidence produced by such a simulation would be evaluated. The following two crucial questions are thereby indicated:

- to what extent can the researcher be confident that the research evidence produced by the selection simulation corroborates the latent structure/nomological network postulated by the research hypothesis within the limits set by the specific conditions characterising the simulation?; and
- to what extent can the researcher be confident that the conclusions reached on the basis of the simulation will generalise or transport to the area of actual application?

The conditions under which selection procedures are typically simulated and those prevailing at the eventual use of a selection procedure normally differ to a sufficient extent to challenge the transportability of the validation research evidence. Nevertheless, given the applied nature of selection validation research, an attempt at generalisation is unavoidable. According to Stanley and Campbell [1963] external validity is threatened by the potential specificity of the demonstrated effect of the independent variable[s] on particular features of the research design not shared by the area of application. In selection validation research the effect of the [composite] independent variable on the criterion is captured by the validity coefficient. The area of application is characterised by a sample of actual applicants drawn from the applicant population and measured on a battery of fallible predictors with the aim of "estimating their actual contribution to the organisation [i.e. ultimate criterion scores] and not an indicator of it attenuated by measurement error" [Campbell, 1991, p. 694]. The estimate is derived from a weighted linear composite of predictors derived from a representative sample of the actual applicant population. The question regarding external validity, in the context of selection validation research, essentially represents an inquiry into the unbiasedness of the parametric validity coefficient estimate [i.e. the sample statistic] obtained through the validation study. The parameter of interest is the correlation coefficient obtained when the sample weights derived from a representative sample of subjects are applied to the applicant population and the weighted composite score is correlated with the criterion, unattenuated by measurement error, in the population [Campbell, 1991]. The preceding discussion clearly identifies the term "applicant population" to be of central importance should a sufficiently precise depiction of the area of actual application be desired. The term "applicant population", however, even if defined as the population to which a selection procedure will be applied, still has an annoying impreciseness to it. A more unambiguous definition of the term, however, depends on how the selection procedure is positioned relative to any selection requirements already in use [i.e. whether it would replace, follow on, or be integrated with current selection requirements]. This issue, moreover, is linked to the question regarding the appropriate decision alternative with which to compare the envisaged selection procedure when examining its strategic merit.

In the context of selection validation research, given the aforementioned depiction of the area of application, the following specific threats to external validity can be identified [Campbell, 1991; Lord & Novick, 1968; Tabachnick & Fidell, 1989]:

- the extent to which the actual or operationalised criterion contains random error of measurement;
- the extent to which the actual or operationalised criterion is systematically biased; i.e. the extent to which the actual criterion is deficient and/or contaminated [Blum & Naylor, 1968];
- the extent to which the validation sample is an unrepresentative, biased, sample from the applicant population in terms of homogeneity and specific attributes [e.g. motivation, knowledge/experience];
- the extent to which the sample size and the ratio of sample size to number of predictors allow capitalisation on chance and thus overfitting of the data.

The conditions listed as threats all affect the validity coefficient [Campbell, 1991; Crocker & Algina, 1986; Dobson, 1988; Hakstian, Schroeder & Rogers, 1988; Lord & Novick, 1968; Mendoza & Mumford, 1987; Messick, 1989; Olsen & Becker, 1983; Schepers, 1996], some consistently exerting upward pressure, others downward pressure and for some the direction of influence varies. It thus follows that, to the extent that the aforementioned threats operate in the validation study but do not apply to the actual area of application, the obtained validity coefficient cannot, without formal consideration of these threats, be generalised to the actual area of application. Thus, the obtained validity coefficient cannot, without appropriate corrections, be considered an unbiased estimate of the actual validity coefficient of interest.

Statistical corrections to the validity coefficient are generally available to estimate the validity coefficient that would have been achieved had it been calculated under the condition that characterise that area of actual application [Gulliksen, 1950; Pearson, 1903; Thorndike, 1949]. Campbell [1991, p. 701] consequently recommends that:

> "If the point of central interest is the validity of a specific selection procedure for predicting performance over a relatively long time period for the population of job applicants to follow, then it is necessary to correct for restriction of range, criterion unreliability, and the fitting of error by differential predictor weights. No to do so is to introduce considerable bias into the estimation process."

The remainder of the argument in terms of which a selection procedure is developed and justified could, however, also be biased by any discrepancy between the conditions under which the selection procedure is simulated and those prevailing during the actual use of the selection procedure. Relatively little concern, however, seems to exist for the transportability of the decision function derived from the selection simulation and descriptions/assessments of selection decision utility and fairness. This seems to be a somewhat strange state of affairs. The external validity problems of validation designs are reasonably well documented [Barrett, Phillips & Alexander, 1981; Cook, Campbell & Peracchio, 1992; Guion & Cranny, 1982; Sussman & Roberson, 1986]. It is therefore not as if the psychometric literature is unaware of the problem of generalising validation study research findings to the ultimate area of application. The decision function is probably the pivot of the selection procedure because it firstly captures the underlying performance theory, but more importantly from a practical perspective, because it guides the actual acceptance and rejection choices of applicants [i.e. it forms the basis of the selection strategy matrix]. Restricting the statistical corrections to the validity coefficient would leave the decision function unaltered even though it might also be distorted by the same factors affecting the validity coefficient. Basically the same logic also applies to the evaluation of the decision rule in terms of selection utility and fairness. Correcting only the validity coefficient would leave the "bottom-line" evaluation of the selection procedure unaltered. Restricting the statistical corrections to the validity coefficient basically means that practically speaking nothing really changes.

RESEARCH OBJECTIVES

The general objective of the research reported here is to firstly determine whether specific discrepancies between the conditions under which the selection procedure is simulated and those prevailing during the actual use of the selection procedure produces bias in estimates required to specify and justify the procedure. If bias is found the objective, furthermore, is to delineate appropriate statistical corrections of the validity coefficient, the decision rule and the descriptions/assessments of selection decision utility and fairness, required to align the contexts of evaluation/validation and application. The general objective of the research reported here is, finally, to determine whether the corrections should be applied in validation research. With reference to this latter aspect the following argument is pursued. The evaluation of any personnel intervention in essence constitutes a process where information is obtained and analysed/processed at a cost with the purpose of making a decision [i.e. choosing between two or more treatments] which results in outcomes with a certain value to the decision maker. To add additional information to the evaluation/decision process and/or to extend the analyses of information could be considered rational if it results in an increase in the value of the outcomes at a cost lower than the increase in value. The foregoing argument thus implies that corrections applied to the obtained correlation coefficient are rational to the extent that [Boudreau, 1991]:

- the corrections change decisions concerning:
  - the validity of the research hypothesis [or at least the a priori probability of rejecting H_0 assuming H_0 to be false]; and/or
  - the choice of which applicants to select; and/or
  - the appropriate selection strategy option; and/or
  - the fairness of a particular selection strategy.
- the change in decisions have significant consequences; and
- the cost of applying the statistical corrections are low.

The argument is thus by implications that there is little merit in applying statistical corrections should they not change any part of the total case built by the validation research team in defense of the selection procedure even if the corrections should rectify systematic bias in the obtained estimates.

To cover all of the aforementioned in a single article would, however, constitute a somewhat overly ambitious endeavor. This paper consequently restricts itself to the more modest objective of determining the effect of a joint correction for criterion unreliability and Case 2 restriction of range on the validity coefficient. Case 2 restriction of range refers to the situation were selection occurred [directly/explicitly] on the predictor [or the criterion] through complete truncation on X at X_c [or on Y at Y_c] and both restricted and unrestricted variances are known only for the explicit selection variable X [or Y].

An appropriate notational system is needed to pursue this objective. The conventional Greek symbols will be used to represent population parameters: σ^2 for variance, μ for mean, ρ for correlation. Parameters will carry suitable subscripts to identify the variables involved. The following notation will be used; $\sigma^2[X]$, $\mu[X]$, $\rho[X, Y]$ and $\beta[X, Y]$. Capital letters are used to denote random variables. Let X and Y denote the observed scores on the predictor and criterion respectively. Let T_x, T_y and E_x and E_y denote the true and error score components of the [unrestricted] observed predictor and criterion scores. The true and error score components of the restricted observed predictor and criterion scores will be denoted by corresponding lowercase letters. Let the to be corrected correlation coefficient calculated for the restricted group be indicated as $\rho[x,y]$ and the to be estimated correlation coefficient as $\rho[X,Y]$. Let $\sigma^2[x]$ and $\sigma^2[y]$ represents the calculated [i.e. known] variances for the restricted group and $\sigma^2[X]$ and $\sigma^2[Y]$ the variances for the unrestricted group of which only $\sigma^2[X]$ is known. The capital letter E will be reserved for use as the expected value. The reliability coefficients for the unrestricted criterion and predictor measurements will be denoted as ρ_{tty} and ρ_{ttx} respectively.

THE CORRECTION OF A CORRELATION COEFFICIENT FOR THE JOINT EFFECTS OF ERROR OF MEASUREMENT AND RESTRICTION OF RANGE

Although considerable literature exists regarding the correction of correlation coefficients for the separate attenuating effects of error of measurement and restriction of range [Pearson, 1903, Gulliksen, 1950, Ghiselli, Campbell & Zedeck, 1981; Held & Foley, 1994; Linn, 1983; Olson & Becker, 1983; Ree, Carretta, Earles & Albert, 1994] relatively less attention has been given to the theory underlying the correction of a correlation coefficient for the joint effects of error of measurement and restriction of range [Bobko, 1983; Lee, Miller & Graham, 1982; Mendoza & Mumford, 1987; Schmidt, Hunter & Urry, 1976].

In a typical validation study, restriction of range and criterion unreliability are simultaneously present. Their effects combine to yield an attenuated validity coefficient that could severely underestimate the operational validity [Lee, Miller & Graham, 1982; Schmidt, Hunter & Urry, 1976]. It thus seems to make intuitive sense to double correct an obtained validity coefficient for the attenuating effect of both factors. The APA, however, through their Standards for Educational and Psychological Tests [APA, 1974, p. 41], initially recommended that:

> "It is ordinarily unwise to make sequential corrections, as in applying a correction to a coefficient already corrected for restriction of range. Chains of corrections may be useful in considering possible further research, but their results should not be seriously reported as estimates of population correlation coefficients."

Schmidt, Hunter and Urry [1976], though, consider the APA recommendation to be in error and propose that the obtained validity coefficient should be sequentially corrected for the effects of both restriction of range and criterion unreliability so as to obtain an estimate of the actual operational validity. The revised edition of the Standards for Educational and Psychological Tests [APA, 1985] subsequently also seems to have softened its position on this topic by abstaining from any comment. The stepwise correction procedure suggested by Schmidt, Hunter and Urry [1976] involves first correcting both the obtained validity and reliability coefficients for restriction of range since both coefficients apply only to a restricted applicant group and thus are to a greater or lesser extent negatively biased estimates of the operational reliability and validity coefficients.

Equation 3 is suggested [Feldt & Brennan, 1989; Ghiselli, Campbell & Zedeck, 1981] as an appropriate correction formula to correct the reliability coefficient for the attenuating effect of range restriction if homogeneity of error variance across the range of true criterion scores can be assumed [i.e. the assumption is that applicants were selected in such a manner that the true score variance is reduced whereas the error variance remains unaffected]; Guion, 1965; Gulliksen, 1950; Lee, Miller & Graham, 1982].

From the assumption of homogeneous error variance across the range of true criterion scores it follows that:

$$\sigma[y]\sqrt{(1-\rho_{tty})}=\sigma[Y]\sqrt{(1-\rho_{ttY})} \quad \ldots\ldots 1$$

Squaring Equation 1 and then multiplying by $1/\sigma^2[Y]$, results in:

$$(\sigma^2[y]/\sigma^2[Y])(1-\rho_{tty}) = (1-\rho_{tt}Y) \quad \ldots\ldots 2$$

Isolating the unrestricted reliability coefficient in Equation 2:

$$\rho_{tt}Y = 1 - \{(\sigma[y]/\sigma[Y])^2(1-\rho_{tt}y)\} \quad \ldots\ldots 3$$

The assumption that Equation 3 is based on, however, frequently does not hold [Feldt & Brennan, 1989]. A further problem with Equation 3 in the context of validation research, moreover, is that the criterion variance for the unrestricted group is logically impossible to obtain.

Schmidt, Hunter and Urry [1976] suggest an alternative expression [shown as Equation 4] which avoids the aforementioned problem.

$$\rho_{tt}Y = 1 - (1-_{\rho tty})/(1-\rho[x,y])(1-(\sigma^2[X]/\sigma^2[x])) \quad \ldots\ldots 4$$

Depending on the nature of the selection/restriction of range and the variable for which both the restricted and unrestricted variance is known, the correction of the validity coefficient for the attenuating effect of restriction of range will proceed through the appropriate correction formula. The validity coefficient corrected for restriction of range will then subsequently be corrected for the attenuation effect of criterion unreliability by employing the results of the preceding first two steps [i.e. the reliability and validity coefficients corrected for restriction of range] in the traditional attenuation correction formula for the criterion only.

Lee, Miller and Graham [1982], however, point out that statistical and measurement theory permit a simpler two-step correction. According to the Lee, Miller and Graham [1982] approach the restricted criterion reliability coefficient is used to correct the restricted validity coefficient for the attenuating effect due to the unreliability of the criterion. This partially disattenuated validity coefficient is then subsequently corrected for the attenuating affect of restriction of range. The first step in the Schmidt, Hunter and Urry [1976] procedure is thus disposed of. Although the procedures suggested by Schmidt, Hunter and Urry [1976] and Lee, Miller and Graham [1982] seem to be conceptually distinct, Bobko [1983] points out that these two procedures are in fact arithmetically identical. Combining the two step-approach suggested by Lee, Miller and Graham [1982] into a single equation results in Equation 5 for the double-corrected validity coefficient [assuming Case 2 selection produced the restriction of range] [Bobko, 1983].

$$\rho[X,T_Y]=\sigma[X]\rho[x,y]\rho[y,y]^{-1/2} / \{\sigma^2[X]\rho^2[x,y]\rho[y,y]^{-1}+\sigma^2[x]-\sigma^2[x]\rho^2[x,y]\rho[y,y]^{-1}\}^{1/2} \quad \ldots\ldots 5$$

Similar equations could be derived for the other possible conditions under which correlation estimation bias due to systematic selection could occur.

Mendoza & Mumford [1987] proposed a set of equations in terms of which correlation coefficients can be jointly corrected for:

- range restriction directly on the predictor and unreliability in the predictor and the criterion; or
- range restriction directly on the latent trait measured by the predictor and unreliability in the predictor and the criterion.

Equation 13 shows the appropriate correction formula applicable when range restriction occurs directly on the ability/latent trait measured by the predictor [Mendoza & Mumford, 1987]. The derivation of Equation 13 assumes a linear, homoscedastic regression of the criterion Y on the predictor X in the unrestricted population and in addition makes the two usual restriction of range assumptions, namely that:

- the regression of actual job perforamance [i.e. the ultimate criterion] Y′ on ability will not be affected by explicit selection on the latent trait represented by X; and
- the ultimate criterion variance conditional on X′ will not be altered by explicit selection on the latent trait measured by X [Mendoza & Mumford, 1987].

From the assumption that the regression of actual job performance [i.e. the ultimate criterion] Y′ on ability will not be affected by explicit selection on the latent trait represented by X, it follows that:

$$\beta[t_y|t_x] = \beta[TY|TX] \quad \ldots \quad 6$$

From the assumption that the ultimate criterion variance conditional on X' will not be altered by explicit selection on the latent trait measured by X, it follows that:

$$\sigma^2[t_y][1 - \rho^2[t_x t_y]] = \sigma^2[TY][1 - \rho^2[TX,TY]] \quad \ldots \quad 7$$

However:

$$\begin{aligned}\beta^2[t_y|t_x] &= \rho^2[t_y,t_x][\sigma^2[t_y]/\sigma^2[t_x]] \\ &= \rho^2[t_y,t_x][\sigma^2[y]\rho_{tty}/(\sigma^2[x]\rho_{ttx})] \quad \ldots \quad 8\end{aligned}$$

Similarly:

$$\beta^2[TY|TX] = \rho^2[T_Y,T_X][\sigma^2[Y]\rho_{ttY}/(\sigma^2[X]\rho_{ttX})] \quad \ldots \quad 9$$

Substituting Equations 8 and 9 in Equation 6:

$$[\rho^2[T_Y,T_X][\sigma^2[Y]\rho_{ttY}/(\sigma^2[X]\rho_{ttX})] = [\rho^2[t_y,t_x][\sigma^2[y]\rho_{tty}/(\sigma^2[x]\rho_{ttx})] \quad \ldots \quad 10$$

Isolating the term $\rho^2[T_Y,T_X]$ in Equation 10 by multiplying by $[\sigma^2[X]\rho_{ttX}/\sigma^2[Y]\rho_{ttY}]$

$$\begin{aligned}\rho^2[T_Y,T_X] &= [\rho^2[t_y,t_x][\sigma^2[y]\rho_{tty}/\sigma^2[x]\rho_{ttx}][\sigma^2[X]\rho_{ttX}/\sigma^2[Y]\rho_{ttY}] \\ &= [\rho^2[t_y,t_x][\sigma^2[y]\rho_{tty}\sigma^2[X]\rho_{ttX}]/[\sigma^2[x]\rho_{ttX}\,\sigma^2[Y]\rho_{ttY}] \quad \ldots \quad 11\end{aligned}$$

However, the square of the fully disattenuated validity coefficient can be expressed as:

$$\rho^2[t_x,t_y] = \rho^2[x,y]/(\rho_{ttx}\rho_{tty}) \quad \ldots \quad 12$$

Substituting Equation 12 in Equation 11:

$$\begin{aligned}\rho^2[TX,TY] &= [\rho^2[x,y]/(\rho_{ttx}\rho_{tty})][\sigma^2[y]\rho_{tty}\sigma^2[X]\rho_{ttX}]/[\sigma^2[x]\rho_{ttx}\,\sigma^2[Y]\rho_{ttY}] \\ &= [\rho^2[x,y]\sigma^2[y]\sigma^2[X]\rho_{ttX}]/[\sigma^2[x]\rho^2_{ttx}\sigma^2[Y]\rho_{ttY}] \quad \ldots \quad 13\end{aligned}$$

Equation 13 places rather formidable demands on the analyst in as far as it requires the reliability and variance of both variables in both the restricted and unrestricted groups to be known. This seems to limit the practical value of Equation 13. If it is possible to calculate both $\sigma^2[X]$ and $\sigma[Y]$ [and not only one of the two], it seems more than probable that one would also be able to calculate $\rho[X,Y]$, ρ_{ttX} and ρ_{ttY} and thus estimate $\rho[T_X,TY]$ with the traditional attenuation correction formula [Equation 12]. The need to infer $\rho[T_X,TY]$ indirectly via an equation like Equation 13, would then no longer exists. Mendoza and Mumford [1987] acknowledge the equation's requirement that the reliability of both measures be known in the restricted and unrestricted space, but do not regard this as a problem since the restricted and unrestricted reliabilities are related by Equation 3.

Equation 30 applies to the second, probably more prevalent, situation where restriction of range/selection occurs directly on the predictor [Mendoza & Mumford, 1987]. The derivation of Equation 30 assumes a linear, homoscedastic regression of the criterion Y on the predictor X in the unrestricted population and in addition makes the two usual restriction of range assumptions, namely that:

- the regression of the criterion Y on the predictor will not be affected by explicit selection on the predictor X; and
- the criterion variance conditional on X will not be altered by explicit selection on X [Mendoza & Mumford, 1987].

From the assumption that the regression of the criterion Y on the predictor will not be affected by explicit selection on the predictor X, it follows that:

$$\beta[y|x] = \beta[Y|X] \quad \ldots \quad 14$$

From the assumption that the criterion variance conditional on X will not be altered by explicit selection on the predictor X, it follows that:

$$\sigma^2[y][1 - \rho^2\,[x,y]] = \sigma^2[Y][1 - \rho^2[X,Y]] \quad \ldots \quad 15$$

From Equation 15 it follows that:

$$\rho^2[x,y](\sigma^2[y]/\sigma^2[x]) = \rho^2[X,Y](\sigma^2[Y]/\sigma^2[X]) \quad \ldots \quad 16$$

Isolating the term $\rho^2[X,Y]$ in Equation 16:

$$\rho^2[X,Y] = \rho^2[x,y](\sigma^2[y]\sigma^2[X])/(\sigma^2[x]\sigma^2[Y]) \quad \ldots \quad 17$$

However, the fully disattenuated validity coefficient can be expressed as:

$$\rho[T_X,T_Y] = \rho[X,Y]/(\sqrt{\rho_{ttX}}\sqrt{\rho_{ttY}}) \quad \ldots \quad 18$$

Substituting Equation 17 in the square of Equation 18:

$$\rho^2[T_X,T_Y] = (\rho^2[x,y]\sigma^2[y]\sigma^2[X])/(\sigma^2[x]\sigma^2[Y]\rho_{ttX}\rho_{ttY}) \quad \ldots \quad 19$$

However, $\sigma^2[Y]$ and ρ_{ttY} probably would not be available.

Multiplying Equation 15 by $1/(\sigma^2[Y][1 - \rho^2[x,y]])$:

$$\sigma^2[y]/\,\sigma^2[Y] = [1 - \rho^2[X,Y]]/[1 - \rho^2[x,y]] \quad \ldots \quad 20$$

However, the validity coefficient corrected for Case 2 restriction of range can be expressed as:

$$\rho[X,Y] = (\sigma[X]/\sigma[x])\rho[x,y]/\{(\sigma^2[X]/\sigma^2[x])\rho^2[x,y] + 1 - \rho^2[x,y]\}^{1/2} \quad \ldots \quad 21$$

Squaring Equation 21:

$$\rho^2[X,Y] = (\sigma^2[X]/\sigma^2[x])\rho^2[x,y]/\{(\sigma^2[X]/\sigma^2[x])\rho^2[x,y] + 1 - \rho^2[x,y]\} \quad \ldots \quad 22$$

Let ϕ represent $\sigma^2[X]/\sigma^2[x]$. Equation 22 can then be rewritten as:

$$\rho^2[X,Y] = \phi\ \rho^2[x,y]/\{\phi\ \rho^2[x,y] + 1 - \rho^2[x,y]\} \quad \ldots \quad 23$$

From Equation 23 also:

$$\begin{aligned}1 - \rho^2[X,Y] &= 1 - \phi\ \rho^2[x,y]/\{\phi\ \rho^2[x,y] + 1 - \rho^2[x,y]\} \\ &= \{\phi\ \rho^2[x,y] + 1 - \rho^2[x,y] - \phi\ \rho^2[x,y]\}/\{\phi\rho^2[x,y] + 1 - \rho^2[x,y]\} \\ &= \{1 - \rho^2[x,y]\}/\{\phi\ \rho^2[x,y] + 1 - \rho^2[x,y]\} \quad \ldots \quad 24\end{aligned}$$

Substituting Equation 24 in Equation 20:

$$\begin{aligned}\sigma^2[y]/\sigma^2[Y] &= \{1 - \rho^2[x,y]\}/[\{\phi\ \rho^2[x,y] + 1 - \rho^2[x,y]\}\{1 - \rho^2[x,y]\}] \\ &= \{\phi\ \rho^2[x,y] + 1 - \ \rho^2[x,y]\}^{-1} \quad \ldots \quad 25\end{aligned}$$

Write Equation 19 as:

$$\rho^2[T_X,T_Y] = \rho^2[x,y]\ (\sigma^2[y]/\sigma^2[Y])(\sigma^2[X]/\sigma^2[x])(1/\rho_{ttX})(1/\rho_{ttY}) \quad \ldots \quad 26$$

Substituting Equation 26 in Equation 19:

$$\begin{aligned}\rho^2[T_X,T_Y] &= \rho^2[x,y])\phi\ (1/\rho_{ttX})(1/\rho_{ttY})(\sigma^2[y]/\sigma^2[Y]) \\ &= \{\rho^2[x,y]\ \phi\}/\{(\rho_{ttX}\rho_{ttY})(\phi\ \rho^2[x,y] + 1 - \rho^2[x,y])\} \quad \ldots \quad 27\end{aligned}$$

However, the problem of the unavailability of ρ_{ttY} still exists.

Substituting Equation 25 in Equation 1:

$$\rho_{ttY} = 1 - (\{\phi\ \rho^2[x,y] + 1 - \rho^2[x,y]\}^{-1})(1 - \rho_{tty}) \quad \ldots \quad 28$$

Therefore:

$$\rho_{ttY} = [\{\phi\ \rho^2[x,y] + 1 - \rho^2[x,y]\} - 1 + \rho_{tty}]/\{\phi\ \rho^2[x,y] + 1 - \rho^2[x,y]\}$$

$$= [\phi\, \rho^2[x,y] - \rho^2[x,y] + \rho_{tty}]/\{\phi\, \rho^2[x,y] + 1 - \rho^2[x,y]\} \quad \ldots\ldots 29$$

Substituting Equation 29 in Equation 27 and taking the square root:

$$\rho[T_X,T_Y] = \sqrt{\{\rho^2[x,y]\,\phi\}}/\sqrt{\{\rho_{ttX}[\phi\,\rho^2[x,y]-\rho^2[x,y]+\rho_{tty}]\}} \quad \ldots\ldots 30$$

Equation 30, however, still has rather limited utility in applied validation research. Its primary deficiency lies in the fact that it also corrects the correlation coefficient for the unreliability of predictor variables. Correcting for unreliability in the predictor in a validation context is misleading. It would be of relatively little value to know the validity of a perfectly reliable predictor when such an infallible measuring instrument can never be available for operational use [Lee, Miller & Graham, 1982; Nunnally, 1978; Schmidt, Hunter & Urry, 1976]. This problem can, however, relatively easily be rectified [Schepers, 1996] as shown in Equation 32.

The partially disattenuated validity coefficient can be expressed as:

$$\rho[X,T_y] = \rho[X,Y]/\sqrt{\rho_{ttY}} \quad \ldots\ldots 31$$

By substituting Equation 31 in Equation 17, Equation 32 follows analogously from Equation 17 as Equation 30 followed from Equation 17.

$$\rho[X,T_Y] = \sqrt{\{\rho^2[x,y]\,\phi\}}/\sqrt{\{\phi\,\rho^2[x,y] - \rho^2[x,y] + \rho_{tty}\}} \quad \ldots\ldots 32$$

Equation 32 provides a joint correction of the correlation/validity coefficient for restriction of range directly on the predictor and the unreliability of the criterion. Multiplying the denominator and numerator of Equation 32 by $\sigma[x]/\sqrt{\rho_{tty}}$, it can be shown the Equation 32 is in fact identical to Equation 5 presented by Bokho [1983] based on the two-step procedure suggested by Lee, Miller and Graham [1982]. A hitherto unrecognised agreement between the work of Bobko [1983] and Mendoza and Mumford [1987] on the joint correction of the correlation/validity coefficient is therefore established. The correction formula derived from the work by the Mendoza and Mumford [1987], furthermore, is computationally slightly less cumbersome than the formula suggested by Bobko [1983].

DISCUSSION

How does Equation 32 affect the magnitude of the validity coefficient? The reaction of the double corrected correlation coefficient to changes in $K = \phi$, the reliability coefficient and the attenuated correlation coefficient, is graphically illustrated in FIGURES 1 - 4. The validity coefficient jointly corrected for Case B^2 restriction of range and criterion unreliability was mapped onto a surface defined by $0.05 \leqslant \rho[x,y] \leqslant 0.90$, $0.10 \leqslant \rho_{tty} \leqslant 0.9$ and $1 \leqslant K \leqslant 4$ through a SAS program feeding a selection of surface coordinates into Equation 32. FIGURES 1 - 4 indicate that the amount of benefit derived from Equation 32 increases as K increases and ρ_{tty} decreases. The uncorrected validity coefficient $\rho[x,y]$ [i.e. the observed validity coefficient uncorrected for the attenuating effect of both restriction of range and criterion unreliability] provides a too conservative description of the actual correlation existing between X and T_Y. The extent to which $\rho[x,y]$ underestimate $\rho[X,T_Y]$ increases as the restriction of range becomes more severe and the reliability of the criterion scores declines. The corrected validity coefficient $\rho[X, T_Y]$ seems to be a positive curvilinear function of $\rho[x,y]$, with the degree of curvilinearity diminishing as the attenuated validity coefficient increases. The corrected validity coefficient, similarly, increases curvilinearly with an increase in the attenuated validity coefficient, with the degree of curvilinearity increasing as $K = \sigma^2[X]/\sigma^2[x]$ increases. Relatively more, therefore, is gained by correcting an attenuated validity coefficient observed in the lower region of the validity scale than in the upper region of the scale.

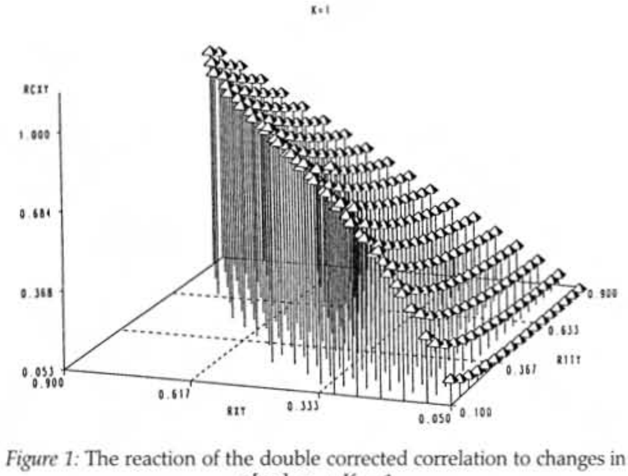

Figure 1: The reaction of the double corrected correlation to changes in $\rho[x,y]$, ρ_{tty}; K = 1.

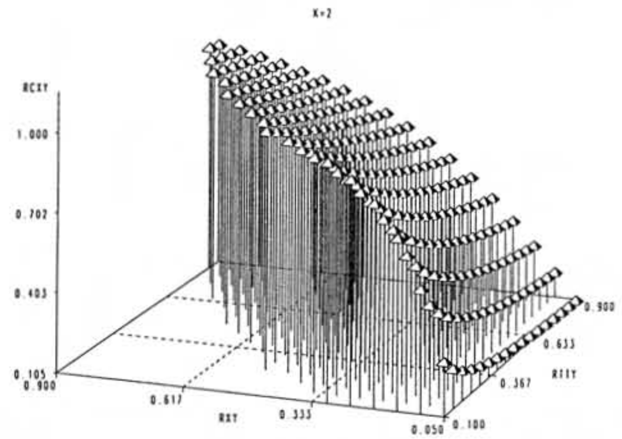

Figure 2: The reaction of the double corrected correlation to changes in $\rho[x,y]$, ρ_{tty}; K = 2.

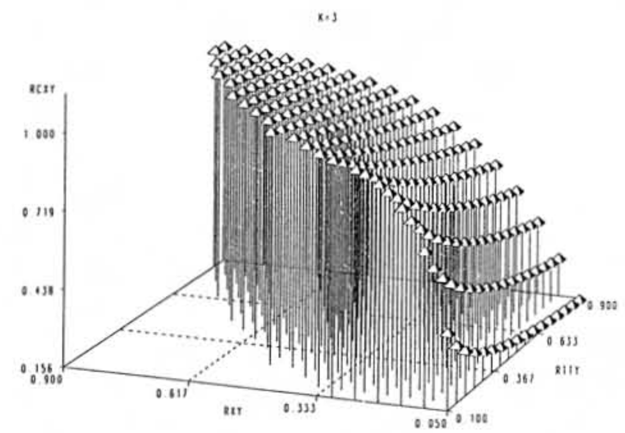

Figure 3: The reaction of the double corrected correlation to changes in $\rho[x,y]$, ρ_{tty}; K = 3.

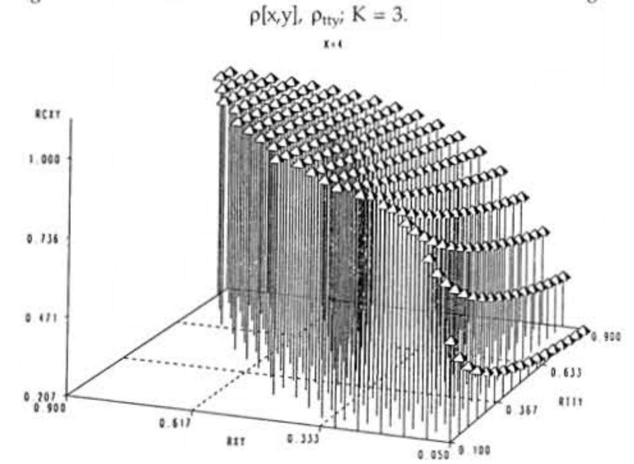

Figure 4: The reaction of the double corrected correlation to changes in $\rho[x,y]$, ρ_{tty}; K = 4.

The findings reported here clearly indicates the dramatic consequence of correcting the observed validity coefficient for the attenuating effect of both restriction of range and criterion unreliability, especially when severe range restriction occurred and the criterion measures suffer from low reliability. Not to correct the observed validity coefficient will severly underestimate the actual validity of the selection procedure for the applicant population. Lee, Miller and Graham [1982], and Bobko [1983] concur that all the available evidence argue in favor of jointly correcting the validity coefficient for the attenuating effect of both range restriction and the unreliability of the criterion. Lee, Miller and Graham [1982] found most corrected validity coefficients to be slight overestimates of the true validity coefficient. In direct contrast to the findings reported by Lee, Miller and Graham [1982], Bobko [1983] concludes that, on average, the double corrected validity coefficient will still underestimate the operational validity coefficient. The research reported here does not permit any comment on bias in the corrected validity coefficient.

A further, less serious, limitation of both Equations 32 and 30 concerns the premise that selection can only occur directly on the predictor. Case C conditions [indirect restriction of range on the predictor and the criterion through direct selection on a third variable] probably constitute the predominant environment in which restriction of range corrections are required. Again, however, this problem can relatively easily be rectified by substituting the Case 2 restriction of range correction formula in the derivation of Equation 30 and Equation 32 with the appropriate Case C correction formula [Gulliksen, 1950; Thorndike, 1949].

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