Iannis Xenakis (1922-2001): An examination of the implementation of stochastic procedures in selected compositions.

by

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Declaration

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Abstract

The relationship between music and mathematics has been subjected to debate for centuries. There are two schools of thought with the one viewpoint holding that the relationship between mathematics, which is conceived as an abstract and cold discipline, compared to music, which is rich with emotion, must be very limited. Arguments for this view draws, for example, on recent research which indicates that musical talent is not inherently linked to mathematical capability. On the other pole of the debate is the belief that although music and mathematics contribute to different parts of society, there is a very important inter-relationship between the two fields. Of great interest to the latter is the Greek composer Iannis Xenakis whose musical aesthetics incorporates this philosophy.

As composer, Xenakis used mathematical theories as basis for his musical works. He not only incorporated well known mathematical principles such as the Golden Section into his compositions but went further and, for instance, utilized Boolean Algebra, Probability theory and Stochastic processes in his music. His composition method based on these mathematical principles became known under the term *Stochastic* music and forms the focus of this thesis.

The research project concentrates on the early part of Xenakis' life in order to provide insight into the development of his composition methods. The mathematical principles at the centre of his *stochastic* compositions receive specific rationalisation. In doing so, the most significant probability distributions (Linear, Exponential, Poisson and Normal distribution) are defined in terms of their properties and Xenakis' use of them. The application of these distributions is considered by looking at the early works *Metastaseis* (in which Xenakis confronted most of his musical problems and which formed the basis for his musical style) and *Achorripsis* (where he fully developed their implementation).

An in-depth examination of the construction of *Achorripsis* is performed while scrutinizing Xenakis' calculations. Specific attention is drawn to alterations and adjustments made to the calculations. His implementation of them into the final score is furthermore examined and it is shown where he deviated between the calculations and score. The thesis concludes by considering the extent and significance of the adjustments made by the composer in the name of artistic freedom.

Opsomming

Die wisselwerking tussen musiek en wiskunde was reg deur die eeue ‘n tema vir intensiewe debat. Daar bestaan twee denkwyses rakende die onderwerp met een groep wat glo dat die verwantskap tussen wiskunde, wat gesien word as ‘n abstrakte en koue veld, en musiek wat vol van warmte en emosies is, relatief beperk is. Argumente vir hierdie denkwyse word byvoorbeeld gebaseer op onlangse navorsing wat aantoon dat musikale talent nie vanselfsprekend lei tot wiskundige begaafdheid nie. Die teendeel van die debat is die opvatting dat alhoewel wiskunde en musiek uiteenlopende rolle in die samelewing vervul, daar wel ‘n baie belangrike verband tussen die twee rigtings bestaan. Van groot belang vir die laasgenoemde sienswyse, is die Griekse komponis Iannis Xenakis wie se musikale estetika strook met diè standpunt.

As komponis het Xenakis wiskundige beginsels as basis vir sy komposisies gebruik. Hy het egter nie net bekende wiskundige beginsels soos die Goue Verhouding in sy komposisies ingesluit nie maar het verder gegaan en, byvoorbeeld, Boolse Algebra, Waarskynlikheidsteorie en stochastiese prosesse in sy musiek geïnkorporeer. Sy musikale werke wat met behulp van bogenoemde beginsels gekomponeer is, staan later bekend onder die term *Stochastic music* en is die fokuspunt van hierdie tesis.

In die navorsings projek word grotendeels op die vroeë periode van Xenakis se lewe gekonsentreer met die doel om aandag te vestig op die ontwikkeling van sy komposisie metodes. Die wiskundige beginsels wat ‘n betekenisvolle bydra tot sy stochastiese komposisies gelewer het, word beklemtoon en in diepte bespreek. In die proses word die mees insiggewende waarskynlikheidsdistribusies (Eksponensiële, Poisson, Liniêre en Normaal distribusie) gedefinieer in terme van kenmerke en Xenakis se gebruik daarvan. Die toepassing van die distribusies word oorweeg deur te kyk na vroeë werke *Metastaseis* (waarbinne Xenakis meeste van sy musikale probleme aangespreek het en die grondbeginsels neergelê het vir sy musikale styl) en *Achorripsis* (waarin hy volledig die implimentering van die distribusies ontwikkel het).

‘n Omvattende analise van die skeppingsproses van *Achorripsis* word gedoen waarin Xenakis se berekeninge ondersoek word. Aandag word gevestig op veranderinge wat deur hom aangebring is aan die berekeninge. Die implimentering van sy berekeninge in die finale partituur word verder ondersoek en daar word aangedui waar afwykings tussen die twee voorkom. Die tesis eindig deur die impak van die veranderinge in die naam van kunstenaarsvryheid te ondersoek.

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Introduction

I General background

Even though mathematics[1] and music play very different roles in society, they are more closely related than commonly perceived. The multifaceted relationship between music and mathematics has been extensively investigated and research published by Cohen (1961), McClain (1978), Rusin (2004), Beer (2005) and Benson (2006) comes to mind. It is interesting to note that in the past, different aspects of the relationship between music and mathematics received attention.

According to Beer (2005) for example, mathematics and music were closely connected in Ancient Greece. Approximately 1100 to 150 B.C., music was considered a strictly mathematical discipline and placed on the same level as arithmetic, geometry and astronomy. This classification neglected the creative aspects of musical performance as music was merely seen as the science of sound and harmony.

A crucial turning point occurred during the Seventeenth Century when music was changing from a science to an art, and science was moving from the theoretical to the practical domains. Consequently, many connections between mathematics and music can be traced to this period when, for example, the aspects of rhythm and melody began to receive attention. Furthermore, mathematical concepts became a part of musical compositions as shown by Rothwell in *The Phi Factor: Mathematical Proportions in Musical Forms* (1977), in which he reveals arithmetic and geometric reflections in compositions from this period onwards.

Most importantly, Wollenberg, in her essay in Fauvel, Flood & Wilson (2003), showed that during the nineteenth and twentieth centuries the relationship between music and mathematics extended to mathematical approaches in composition. Examples include aleatoric[2] or "chance" compositions by Cage (1912-1992) and serial[3] methods used by Schoenberg (1847-1951). Furthermore, the organizing principles of serialism provided inspiration for mathematical analogues, such as uses of set theory, group theory, operators, and parameterization in the works of, for example Elliott Carter (1908-), Witold Lutosławski (1913-1994) and Iannis Xenakis (1922-2001).

Hoffmann (2007) argues that "Xenakis himself rejected serial method as unsuitable for his compositional objectives" as is evident in Xenakis' article *La crise de la musique sérielle* (1955). Instead he pursued his own vision, employing mathematics and probability theory to create his works. This method of composition became known as *Stochastic* music and will be the focus of this thesis.

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[1] Originally, "mathematics" was a collective term used for geometry, arithmetic and certain physical sciences involving geometrical reasoning. Mathematics is now seen as the science of space, number, quantity, and arrangement whose methods involve logical reasoning with the use of symbolic notation. (Oxford English Dictionary Online (2007)).

[2] Griffiths (2007) states that Aleatoric music relies to a great degree on chance decisions i.e. by the performers of a particular composition.

[3] In music, serialism is a composition technique that uses sets to describe musical elements and allows the manipulation of those sets (Griffiths, 2007).

II Motivation

Although Xenakis's radical compositional theories have influenced many younger composers, his method of *Stochastic* music remains controversial. It is suggested by observers that his own writings in connection with his compositions are intentionally complex and obscure. In this regard, for instance, Pierre Schaeffer (1970) summarised the situation as follows: "As far as Xenakis is concerned, let me emphasize at once that I'd be much more interested in his research if he hadn't set out so obviously to reduce its accessibility and its credibility in a manner which is immediately apparent as soon as you open his book on formal music". Further criticism emanated from musicologists questioning whether algorithmically generated compositions can carry any real artistic importance, since there is potentially little or no human "creativity" involved in the creative process. This resulted in critics, such as Clarendon and Pincherle, labeling Xenakis' music as "anti-music…noisy…crazy".[4]

The criticism and controversy surrounding the work of Xenakis creates the impression that there is still a definite lack of understanding concerning his work and consequently further research is possible. Xenakis summarized the situation by stating in an interview with Bois (1997: 7): "I had written in the programme mathematical formulae. They took fright. Why be frightened? Mathematical formulae are not monsters; one can tame them much more easily than one thinks, provided that one doesn't in advance create a blockage in the mind."

This research project specifically aims to cast light on the mathematical aspects of Xenakis's work. In understanding the "mechanisms" behind his compositions, the lack of understanding surrounding his work can be defused. More specifically, this will be done by considering carefully selected sections of the compositions *Metastaseis* (1953-4) and *Achorripsis* (1956-57). For instance, the specific probability distributions employed by Xenakis can be identified and by considering their characteristics, reasons for their use will become clear.

Another reason for deciding on an investigation of an aspect of Xenakis' work boils down to the fact that following his death in 2001, the increasing international interest in his work did not reverberate locally. That this was not the case became evident during the literature study which revealed no publications by South African scholars.

III Research problem

The term "statistics" refers to a branch of applied mathematics concerned with the use of probability theory to estimate population parameters. A stochastic process operates on a family of random variables which is indexed by another set of variables with compatible probability of distribution. In music, stochastic processes are often used to randomly generate musical parameters while applying strict mathematical processes and can therefore be utilized in the composition of whole musical works.

Arguably, the principal exponent of stochastic processes in musical composition has been Iannis Xenakis. The processes used by him include Exponential distributions, Normal distributions and Poisson distributions. *Metastaseis* laid the foundation for Xenakis' musical career. In this composition he confronted a number of his

[4] This quote comes from an interview with Iannis Xenakis and was published by Mario Bois in *Iannis Xenakis: The man and his music* (1997: 7).

fundamental problems in relation to musical composition. In *Achorripsis*, he further developed his stochastic method and formalized it to a point where it could be automated by means of a computer.

As mentioned earlier, the heavy reliance of Xenakis' music upon mathematics and probability theory in particular, led to criticism and a lack of appreciation by the music community. Childs, in his paper *Achorripsis: A sonification of probability distributions* (2002), attempted to verify the connection between the composition *Achorripsis* and the probability distributions used by Xenakis. The research by Childs led to the formulation of the hypothesis for this research project, which reads as follows:

The compositional method utilized by Iannis Xenakis is not merely an automated process and artistic renderings exist between the stated stochastic processes he employed and his final musical scores.

IV Objectives

As stated above, Childs (2002) examined sections of the composition *Achorripsis* and compared these with publications by Xenakis regarding the work. Childs attempted to establish if any deviations between the mathematics Xenakis employed and his final compositions could be traced.

The first objective of this project is to verify the results published by Childs. This is done by adopting the same methodological approach as Childs in analysing *Achorripsis.* More specifically the analysis of the composition is structured according to the fundamental phases of constructing a musical work as set forth by Xenakis himself.[5]

This project extends Childs' analysis by not only considering Xenakis' implementation of his calculations into the score of *Achorripsis* but by also scrutinizing the calculations themselves. In addition, a different section of the composition *Achorripsis* is analysed. Throughout the thesis, the analysis attempts to establish whether Xenakis made artistic renderings. If these artistic renderings did occur, their impact and rationale are discussed.

V Structure and scope

Chapter 1 provides a biographical background of Iannis Xenakis. The purpose of the chapter is not only to place him into historical context but to also show the development of his compositional methods. Relevant to this study are the works *Metastaseis* and *Achorripsis,* and consequently more weight is given to the early stages of his life until the time of the compositions. *Chapter 2* introduces mathematical concepts relevant to his compositions. Emphasis is given predominantly to Probability theory and the Stochastic distributions which were utilized in *Metastaseis* and *Achorripsis*. *Chapter 3* provides an in depth examination of the composition *Achorripsis*. The analysis is done in accordance to the same phases that Xenakis' employed in composing the work and during each phase his calculations are scrutinised. In this chapter, the research of Childs is also explored. *Chapter 4* expands on the research done in chapter 3 by considering the implementation of

[5] The eight phases are given in Chapter 2, section 2.1.

Xenakis' calculations into his final score. Alterations made by the composer between the two are identified and their significance is discussed and tested. The final conclusions of the thesis are given in *Chapter 5* where further research possibilities are mentioned as well.

CHAPTER 1

Xenakis: Biographical background

1.1 Early childhood

In light of the topic of the thesis it was decided to weigh the introductory biographical chapter in favour of the composers' early musical development and initial exposure to mathematical thought processes.

Born on 29 May 1922 in Braila, Romania, Iannis Xenakis was the oldest child of a Greek businessman.[1] It is interesting to note that his very early musical influences are divided between exposure to Western Classical music and folk or traditional music, principally in the form of gypsy bands. His first experience of Western Classical music came in the form of piano playing by his mother who died when he was almost five years old. Xenakis, as a result, did not like the gypsy and folk music in Braila because to him it "awakens very sad memories" and "reminds me of my mother" (Varga, 1996: 8). It was the fact that he wanted to distance himself from these influences of folk music which resulted in him composing, as he states, "completely original music" (1996: 10). This can be better understood by taking note of the fact that he never intended to write sentimental music because "I do not want to be moved" (1996: 10). He felt that music itself does not contain emotions and therefore his compositions concentrated instead on music's subjective aspects.[2]

The first significant intellectual growth and influences took place from the age of ten to sixteen while Xenakis was at boarding school on the island of Spetsai. There, due to his naive nature and his strange Constantinople accent, he was often tormented by the other boys. He therefore withdrew and found refuge in the library. There he would spend hours reading works by Victor Hugo which included his poetry, Flammarion's work on astronomy, classical Greek poetry and drama (Matossian, 1986: 16). His interest in mathematics and natural sciences also emerged during this time, although he was also drawn ever closer to classical Greek philosophy and literature in which he "found a natural hideout and a substitute for the real world" (1986: 16).

Together with his increased interest in the natural sciences, his musical interest also developed. The music tuition he received at the school was in the form of solfeggio,

[1] This introduction draws on information obtained from Bois (1967), Matossian (1986) and Varga (1996). The selection of authors was motivated by the fact that these sources serve as a cornerstone for the growing body of biographical literature on Xenakis.

[2] The method with which he achieved this will be elaborated on later in this chapter but first a chronological development of his musical influences will be given.

notation and singing, but interestingly enough, no instrumental tuition at all. Xenakis therefore only received piano tuition during his holidays in Romania until 1938. It is important to note the role that the English headmaster, Esmead Noël Paton, played in stimulating his interest in, and fostering a sensitivity towards music. It was through him, by means of a gramophone and a collection of records, that Xenakis came into contact for the first time with important works by European composers which included '…the Brandenburg concertos and some works by Beethoven' (Varga, 1996: 12). Paton, who reportedly took a special interest in Xenakis (Matossian, 1986: 16) would further spark Xenakis' love for music by encouraging the young boy to ask questions about the subject and this resulted in numerous serious debates between the two.

During his time on the island of Spetsai, Greece went through significant political upheaval with rivalry between Monarchists, Communists and Socialists where not one of a series of governments lasted for long. General Metaxas managed to abolish the parliament and constitution in 1936 and establish a police state. Metaxas's dictatorship had a neutral ideology which therefore sometimes sided with Fascism and sometimes leaned towards the ideologies of Britain and France. It was during this anxious political climate that Xenakis chose to go to Athens in 1938, rather than take the opportunity that his father provided of studying naval engineering in England.

In Athens, Xenakis enrolled at a preparatory school in order to prepare himself for the entrance examination at the Athens Polytechnic. On entrance as a student he studied mathematics, ancient literature, law and physics for two years. He also furthered his musical training by initially continuing lessons in piano but then turned his attention to lessons in harmony and counterpoint which he received from Aristotle Koundourov, a former pupil of Ippolitov-Ivanov. When Xenakis successfully passed the entrance examination at the Polytechnic in 1940, his studies were immediately interrupted as the beginning of the academic year coincided with the Italian invasion of Greece. It therefore took him seven years to complete his studies due to constant interruptions during the war.

1.2 Xenakis and the war

The following years, namely 1940-1947, played a very important part in the development of Xenakis' musical style and as he himself stated in Bois (1967: 22): "A large part of my music has its roots in this period" and "my music is of war". For Greece the Second World War commenced with the invasion by Italy on 28 October 1940. This resulted initially in a popular resistance in which students reacted by organising demonstrations and propaganda against the invaders. It is important to take note of the fact that Xenakis was nationalistic and therefore joined in the Resistance with the objective of freeing his country from the foreign invaders. The Germans entered Greece in April 1941 and a month later the Allied forces retreated and Greece was left under the occupation of Italy, Germany and Bulgaria.

Initially Xenakis joined a right-winged group. He, however, felt that they did not accomplish much and thought the movement was a "rather superficial sort of resistance"

(Varga, 1996: 16). He therefore joined the Communist party because he saw what they accomplished and according to him "fought for much more realistic aims" (1996: 16). Xenakis devoted himself to the party's Resistance and in particular to the organisation of demonstrations. As a result, he was imprisoned on several occasions by the Germans and Italians.

During this time it is important to note that even though Xenakis set aside his interest in music and even ancient Greek literature for more urgent political matters, he was still inspired musically by one of his comrades in the Resistance. Nikos Zachariadis, the nephew of the General Secretary of the Communist party, would during these stressful times knock on strangers' doors and ask people if he could use their pianos in order to play some music. On these occasions, Xenakis heard for the first time the music of Debussy, Bartok and Ravel.

By the end of 1944 the Germans left Greece and then the British entered. Initially, they were greeted with enthusiasm by the Resistance fighters, of which Xenakis was a member. The British, however, had the objective of removing the power from the Communist movement, which they saw as a threat, and reinstating the Greek government-in-exile. Further resistance followed during December of 1944, only now against the occupying British forces. At this stage, Xenakis joined a student battalion of the EPON and was furthermore appointed as the political leader of a group named Lord Byron. It was during this time, in one of the skirmishes with the British, that he almost died. He was hit by the shell of a Sherman tank and admitted to hospital on New Year's Day 1945.

In March 1945, when Xenakis was well enough to leave the hospital, the situation in Greece had completely changed and the war was practically over. He returned to the Athens Polytechnic at the beginning of the academic year in 1946 and completed his degree in civil engineering in 1947. This would also be the year in which Xenakis would emigrate for reasons explained by him as follows in an interview in 1980.[3]

> In 1947 the government made a first attempt at organizing an army against the Communists. Previously, suspects had been demobilized, but from 1947 they were sent to concentration camps, to the island of Markronisos and elsewhere. I knew nothing about that. I reported to the army in the hope that they would find me unfit because of my injury but the committee declared me healthy and I was sent to a military camp. It was then that I heard of the concentration camps and discovered that many of my comrades who disappeared had been taken there. I escaped, hid two or three months in Athens (in the meantime a military tribunal sentenced me to death), and fled to Italy.

In Italy Xenakis made contact with the Communist party and succeeded in entering France illegally eventually reaching Paris on 11 November 1947.

[3] The 1980 conversations with Xenakis are published in Varga (1996: 19).

1.3 Xenakis in Paris

Xenakis' arrival in Paris signals the start of a growth spurt in the compositional development of the composer. During his first years in Paris, Xenakis devoted most of his free time to music. As mentioned by Matossian (1986), several of his notebooks from the period indicate that he worked hard at his studies of counterpoint and harmony. During this time he also changed frequently between teachers as he tried to find a teacher who would best suit him. It is important to note that this was a difficult task because many teachers were reluctant to accept him as a student due to his lack of certain basic or formal musical training, as mentioned earlier. Xenakis furthermore, according to Matossian (1986), was reluctant to accept a conventional teacher-student relationship which would "impose on him an authority based on respect for tradition" (1986: 37). This can be illustrated by an encounter between him and Arthur Honneger.

Honneger taught a course in composition, run by the *Ecole Normale*, in which students were given the opportunity to play some of their compositions to Honneger after which he would comment. Xenakis, after attending the class for a while volunteered to play a piece.

> He (Honneger) asked me to play it, and when I was finished he said: 'But there are parallel octaves here!' Yes I know, I replied, but I like them. He became more and more angry and finally said: 'This is no music! Perhaps the first few bars are – but no, even they are not music!' I said nothing but stopped attending his classes (Varga, 1996: 27).

In 1949 Xenakis found a new teacher namely Darius Milhaud who replaced Honneger due to the latter's illness. However, Milhaud, also fell short of Xenakis' expectations.

As Matossian (1986: 38) remarks, Xenakis' notebooks from classes with Milhaud, indicate an "uncompromising refusal to accept what he considered unimportant" and therefore due to Xenakis' meticulous attitude towards his own creativity, the advised changes by Milhaud were deemed insignificant. These lessons with Milhaud, too, soon came to an end.

During his first years in Paris, it is worth noting that Xenakis was not only politically sidelined in Paris, but the contemporary French culture was also alien to him. Therefore, although he was keen to study music, he was not aware of all the different factions of the Parisian musical life, nor how to approach them. One should also note that from around the time of the Second World War, a trend developed which resulted in the production of a vast number of neo-classical[4] works. The musical education at the Conservatoire National in Paris, during Xenakis' first years in Paris, therefore tended towards an ideology of conserving the past and keeping out the present which resulted in the rejection of new compositional developments. It was with this artistic climate in mind that Nadia Boulanger (1887 - 1979), a famous teacher at the Conservatoire who tended towards neo-classical orthodoxy in her tuition, turned down Xenakis as a pupil saying

[4] Neo-classical music refers to a 20th century development in which composers drew inspiration from music of the 18th century.

that although she was interested in him he "wasn't mature enough and that she was too old to start from scratch with a boy my (his) age" (Varga, 1996: 26). Boulanger, however, suggested that Xenakis take lessons with Annette Dieudonné, also a professor at the Conservatoire. He attended a few classes with her, but as he puts it in Varga (1996: 27), "when she saw that the rules of traditional harmony were like fetters for me she said that I'd better see Messiaen". The reasons for the suggestion can be attributed to the fact that, as mentioned in Matossian (1986: 43), the compositional methods of Olivier Messiaen were so unusual that the Conservatoire could not accept them for many years.

Following Dieudonné's advice as well as Le Corbusier's suggestions, Xenakis approached Messiaen with some of his compositions.[5] According to Matossian (1986: 48-49), Messiaen was impressed by the stranger. This was an unusual reaction for Xenakis because of the discouraging responses by previous teachers and he was prepared to start his musical education from scratch. Messiaen, however, replied to Xenakis' question of whether he should begin again to study harmony and counterpoint, by saying: "No, you are almost thirty, you have the good fortune of being Greek, of being an architect and having studied special mathematics. Take advantage of these things. Do them in your music" Matossian (1986: 48).

This interview had a very profound impact on Xenakis and led to a complete transformation in his compositional approach. In addition, Messiaen invited Xenakis to attend his classes whenever possible and show him any works he might want to discuss. In Messiaen, Xenakis finally found "a teacher who tested every musical idea without dogma" (Matossian, 1986: 49), and it was this trait that Xenakis particularly admired. He also added that "for the very first time I saw a musician think in a wide unconventional way". This is remarkable in view of the fact, as stated above, that Xenakis went to Messiaen with a very negative view of his own abilities, and that Messiaen was able to change this opinion into something very positive and full of hope.

As a case in point, the analysis presented by Messiaen of Stravinsky's *Rite of Spring* had a profound influence on Xenakis. Messiaen highlighted the rhythmic cells in the composition and named them "rhythmic characters" in order to compare them to characters in a drama. Matossian (1986: 48-49) argues that this was, to Xenakis, a new way of understanding the role of rhythm and would play an important part in his future compositions. Messiaen, in addition, exposed his students, including Xenakis, to music from the Orient and Asia in his lectures and treated these compositions with the same analytical consideration that he gave to Western music. Matossian (1986: 48-49) furthermore argues that because Xenakis lacked the normal formal musical education acquired at conservatoires, he studied all forms of music from diverse cultures without the evaluative prejudice an educational system might have imposed on him.

It was through the example of Messiaen, in his very "detached approach to music" for which he "produced his own rules", to quote Xenakis (Matossian, 1986: 49), that he himself started composing more frequently. During this time, he experimented often with various instruments and familiarised himself with their sounds. Interestingly, it is by

[5] The exact date of the first meeting between Xenakis and Messiaen is unsure. Matossian (1986) states that it happened in 1951, but in Varga (1996) Xenakis himself states that he studied with Messiaen from 1949-1950.

accident that he discovered a concrete way of experimenting with rhythms. The discovery was made due to a defect in his new tape-recorder. The situation was summarised by him in an interview as follows:

> It had a defect. When you pressed the record button it made a click in the recording. By making a click, then measuring the length of tape until the next click, and reducing the length systematically I was able to make a Fibonacci series. In this way I made some rhythmical studies." (Matossian, 1986: 50).

This discovery led to the composition for chorus and instruments, *Anastenaria* (1953), in which he used such a series of durations which he linked to a sequence of pitches as well.[6]

In Xenakis' unpublished preface to *Anastenaria*, he gave an account of the divine proportion as follows:

> It is one of the laws of biological growth. One finds it in human proportions, in the relation of the height of the head to the solar plexus, in the phalanges of the fingers, the bones of the arms and legs. Musical durations are created by the muscular release which act on human limbs. It is evident that the movement of those limbs have a tendency to produce themselves in time, proportional to the dimensions of those limbs. Hence the consequence – the durations which are in relation to the Golden Section are the most natural for the movement of the human body.

The example of the intellectual grounding involved with *Anastenaria*, illustrates the fact that Xenakis had the ideal of making a set of proportions that apply to different fields just as applicable to music. Matossian (1986) emphasises that this principle of unity was a dream of Xenakis since he was a boy and that at last he was getting closer to realising the dream. The fact that a common formula could be employed by him not only in his music but also in his architecture, excited him. Therefore, during this time he would write up theories, arguments and possible laws and made notes regarding their validity and applicability. Matossian explains why the ideal of unity was of great importance to the young composer: "Firstly for its mathematical beauty and secondly (significant for his later development) for its ubiquity in both man-made and organic structures." The feelings aroused by his latest discoveries can be shown by a letter sent to Francoise Xenakis on 7 April 1952.

> I struggled all morning to see clearly the flute, piano and voice part – I ate at four. Then I attacked a theoretical problem, to find the melodic expression of conic sections, how to put in mathematical formulae a continuous melodic curve. I arrived at a small result, that is to say I found a little path which could guide me later on. We will see later. Sometimes such childish games lead to sensational discoveries. Especially since the mathematical expression of music haunts me since my adolescence. Do you see the consequences of my ideas? I believe on the whole that a new kind of music could be created." (Matossian, 1986: 51).

[6] A more detailed description of the series will be given in the next chapter.

Later in his career Xenakis was able to broaden vastly the scope of mathematics that he was able to apply in his compositions.

1.4 Xenakis and Le Corbusier

It is important not to underestimate the implications of the pivotal role that the architect Charles-Edouard Jeanneret (1887-1965), known professionally as Le Corbusier, played in the development of the young composer during his first years in Paris. It is undoubtedly necessary to consider their relationship while at the same time providing information on the other half of Xenakis' life, that of engineer and architect.

Xenakis' first employment was in Le Corbusier's studio, a position he secured through the help of some of his acquaintances from the Athens Polytechnic. It is interesting to note that at the time that he started working with the renowned architect, Xenakis said that he "wasn't interested in architecture" and "wasn't thinking consciously of architecture" (Varga, 1996: 20 - 21). Therefore, initially he performed merely the tasks required by that of an engineer and after some time, became an advisor in technological questions.

Matossian (1986: 42) states that Le Corbusier's method of integrating the many segments designed by different architects into a unified project was a great influence on Xenakis' musical development. In composition, he would later, as Le Corbusier did in his projects, also keep the comprehensive total idea of a composition while still considering the smaller independent elements and determined how they could function in the whole.

In contrast to Xenakis' other mentor during this period, Messiaen as discussed earlier, Le Corbusier was generally considered "stern, cold and excessively demanding" (1986: 52). It is therefore somewhat surprising that a special bond formed between them during their years of working together. It is noteworthy that both men suffered the same disability since Le Corbusier also lost an eye. Matossian argues that they also shared other similarities in that both "displayed a markedly ascetic lifestyle… a gargantuan capacity for uninterrupted, concentrated work" and that both of them had "restless minds constantly turning over different fields and disciplines". As mentioned earlier, inter-disciplinary thinking and application was very important in the development of Xenakis' compositional method. The influence that Le Corbusier played in this regard was summarised by Xenakis in an interview[7] in 1977:

> I discovered on coming into contact with Le Corbusier that the problems of architecture, as he formulated them, were the same ones I encountered in music. And that way I suddenly acquired an interest in architecture and I did it.

A close relationship developed between the two men during Xenakis' first five years at Le Corbusier's studio. As he became more experienced with the projects undertaken in the studio, Xenakis also became more openly involved with decision-making. He

[7] The 1977 interview with Xenakis is published in Matossian (1986: 53).

became an advisor in technological questions and was in a position to change some of the designs of Le Corbusier's collaborators under the "pretext that their ideas were not technologically feasible" (Varga, 1996: 22). As these changes were made by him more and more often, he finally felt confident enough to approach Le Corbusier in order to be involved first hand with the creation of a design. Le Corbusier responded positively to the idea and started working with Xenakis on the *Couvent de la Tourette*. Matossian (1986: 53) states that Xenakis' inclusion as collaborator in this important project indicated an "unexpected appreciation of the young man's own intellectual temperament". The project was so successful that after its completion, Xenakis was included as one of the designers in Le Corbusier's own team. This can arguably be seen as the start of Xenakis' work as an architect.

1.5 Xenakis emerges as composer

During 1954, Xenakis not only started working on the *Couvent de la Tourette*, but also started exploring ideas for his first major composition, *Metastaseis. Metastaseis* literally means "transformations" and was emotionally inspired by Xenakis' experiences in the war. He wanted to find a way in which to capture the sounds which came from war-like situation such as the following:

> Athens - an anti-Nazi demonstration - hundreds of thousands of people chanting a slogan which reproduces itself like a gigantic rhythm. Then combat with the enemy. The rhythm bursts into an enormous chaos of sharp sounds; the whistling of bullets; the crackling of machine guns. The sounds begin to disperse. Slowly silence falls back on the town. Taken uniquely from an aural point of view and detached from any other aspect these sound events made out of a large number of individual sounds are not separately perceptible, but reunite them again and a new sound is formed which may be perceived in its entirety. It is the same case with the song of the cicadas or the sound of hail or rain, the crashing of waves on the cliffs, the hiss of waves on shingle.[8]

Matossian (1986: 58) argues that Xenakis concluded that the individual sounds such as machine gun fire were not the important aspect but rather the *characteristic distribution of vast numbers of events.* This posed a compositional problem for him which he could not solve with existing serial methods.[9] As mentioned earlier, he was interested in finding methods in which formulas and principles applicable to different fields could be employed in music.

While composing *Metastaseis*, Hoffmann (2008) maintains that Xenakis confronted most of his fundamental musical problems and that the composition therefore formed the basis for his style and aesthetics throughout his musical career. In the work he related the mass movement of the string glissandi to the movement of gas molecules through space as

[8] The quotation comes from an interview with Xenakis as given in Matossian (1986: 58).

[9] His rejection of serialism is discussed in more detail in chapter 2.

determined through the kinetic theory of gasses. As Choong[10] states, Xenakis was able to construct the movement of the glissandi in the composition in such a way that the result was music in which separate voices could not be determined, but the shape of the sound mass that they generate is clear. It is also worth noting that the original planning of the massed glissandi of *Metastaseis* was plotted on a two-dimensional space where the x-axis represented time and the y-axis pitch. This plotting of string glissandi was later used as the curvature for the walls of the Philips Pavilion (figure 1) which was constructed for the 1958 Brussels World Fair. Another mathematical principle, the Fibonacci series, was employed by Xenakis in designing the *Couvent de La Tourette* and he decided to also use it in determining the pitch and duration in *Metastaseis.*[11]

Figure 1 Philips Pavilion[12]

The première of *Metastaseis* was conducted by Hans Rosbaud in Donaueschingen in 1955. Although it was met with heavy criticism, it drew attention from leading musicians of contemporary music. For instance, Messiaen recommended Xenakis and his composition *Metastaseis* strongly to Pierre Schaeffer and through him Xenakis became acquainted with Hermann Scherchen. It was under the baton of Scherchen that Xenakis' second major composition, *Pithoprakta* was premièred in 1957.

Pithoprakta literally means "actions by probabilities" and in the composition for the first time Xenakis developed musical material by purely using Probability theory.[13] This was

[10] From *Iannis Xenakis and Elliott Carter: A Detailed Examination and Comparative Study* (1996) as cited in Zografos (2008).

[11] The Fibonacci series as well as *Metastaseis* will be discussed in detail in Chapter 2.

[12] From Chung (Mathematical and Architectural Concepts Manifested in Iannis Xenakis' Piano Music, 2003: 47).

accomplished by using, for instance, Bernoulli's law of large numbers as well as Gauss, Poisson and Maxwell-Boltzmann distributions. These probability laws also formed the basis for his third major composition *Achorripsis*, and it can therefore be argued that with these two works he truly introduced his style of *Stochastic* music to the musical world.[14] One can furthermore argue that through *Achorripsis, Pithoprakta* and *Metastaseis*, Xenakis assembled the aesthetic means of a mathematically formalized organization of music which formed the foundation for his numerous compositions to follow in the rest of his career.

1.6 Xenakis as established composer

In 1959 Xenakis resigned from Le Corbusier's studio due to "disagreements about aesthetic problems, researches, studies and over money", according to Bois (1967: 5). His standing as a composer during this time, however, was beginning to change as his music was being performed all over the world and he therefore felt confident enough to pursue a musical career. As composer he kept developing stochastic laws in his music which drew more and more attention from audiences and the media alike. This resulted in receiving numerous commissions for works and performances and he consequently completed more than 80 musical compositions between the 1960s and the end of the 1980s.

During this time, Xenakis also authored several books relating to his music composition methods and contributed articles to the journal *Gravesaner Blätter.* These articles later became the sources for his book *Music Formelles*, published in 1963 with the English version Formalized Music: Thought and Mathematics in Music published in 1971.

He furthermore, in the hope of using computers in order to generate music, founded the *Equipe de Mathématique et Automatique Musicales* (EMAMu) in 1966, which in 1972 became *Centre d'Etudes de Mathématique et Automatique Musicales* (CEMAMu). In addition, he held faculty appointments at Indiana University at Bloomington from 1967 until 1972 and at the Sorbonne in Paris from 1973 to 1989.

Some of the distinctions that Xenakis received are the First Prize at the Computer Composed Music competition of the International Federation for Informatic Processing in Edinburgh (1968), *Grand Prix du disque from l'Academie du disque francais* in Paris (1965, 1968 and 1970), the Beethoven Prize in Bonn (1977) and the Edison prize for the best record of contemporary music in Amsterdam (1977).

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[13] *Probability theory* is the branch of mathematics concerned with analysis of random phenomena. The outcome of a random event cannot be determined before it occurs, but it may be any one of several possible outcomes and the actual outcome is considered to be determined by chance. (Encyclopedisa Britannica Online (2008), Weisstein (2008)).

[14] *Stochastic processes* in Probability theory refer to processes involved in the operation of chance. More generally, a stochastic process refers to a family of random variables indexed against some other variable or set of variables. It is one of the most general objects of study in probability. (Encyclopedia Britannica Online (2008) Weisstein (2008)).

Xenakis was married to the writer Françoise Xenakis and they had a daughter, painter and sculptor, Mâkhi Xenakis. He died in Paris in 2001.

1.7 Chapter conclusion

> Iannis Xenakis has passed away but his influence has already passed into the next millennium. Just as Picasso transformed art in the twentieth century and Le Corbusier stamped architecture, it was Xenakis who defined modern music and envisioned the entry of computers before any other composer (Matossian, 2002: 9).

Xenakis was not only able to overcome his complicated war-time childhood and excel in an architectural career, but arguably became one of the most significant composers of the 20th century. He was able to introduce Probability Theory into composition and consequently became the creator of *Stochastic* music. Although his efforts often met with criticism, his extraordinary output as well as his mathematical and technological innovation in the world of modern classical music, secured a warranted degree of fame shared by few other avant-garde composers[15].

[15] Further reading on the topic can be found in Serra (1993), Cappana (2001), Van Maas (2005) and Iliescu (2005).

CHAPTER 2

Xenakis' mathematical grounding

2.1 Introduction

In order to understand the utilization of mathematics in the compositions of Xenakis, it is first necessary to understand how he constructed a musical work. In his book Formalized Music (1992: 22), he divides the process of composition into eight fundamental phases:

1. Initial conceptions. In this phase the composer formulates initial intuitions, proceeds with initial planning and decides on the data that will be used in the composition.

2. Definition of the sonic entities. In this phase, for instance, the sounds of musical instruments or electronic sounds are defined by their potential as well as limitations. Another possibility in this phase is the definition of the sets of elements to be used in the composition, for example the sets of pitches, and whether they are continuous or discreet.

3. Definition of the transformation process. The transformations are on the sonic entities as defined in phase two during the course of the composition. It can be divided into two parts:

Firstly, on a macro-compositional level it refers to the general choice of a logical framework which entails for example elementary algebraic operations and the setting up of relations between entities and sets which were defined in phase two.

Secondly, it refers to the arrangement of these operations on the time-axis of the composition.

4. Micro-composition. This phase requires the detailed description of the functional or stochastic relations between the elements defined in section two in a method which he defines as algebra "outside-time" and "algebra in-time".

5. Sequential programming of phase 3 and 4. This refers to the sculpting of the entire musical work from initial planning in previous phases.

6. Implementation of calculations. This includes the verification, feedback and modifications of the sequential programming.

7. Final symbolic result. In this phase the musical work is converted into common western notation, numerical expressions or graphs.

8. Sonic realization. In this final phase the work is realised by direct orchestral performance or by other means such as computerized synthesis of the sonic entities and their transformations.

Xenakis employed mathematical processes in certain of the above-mentioned phases while composing a musical work. These mathematical principles include Boolean algebra, Stochastic laws, the Arborescence method, Markov chains and Game theory, to name but a few. The aim of this chapter will be a general exploration of a selection of these mathematical principles in order to provide insight into the ideas behind Xenakis' compositions. The mathematical principles to be considered in the next section were selected on the basis that they formed an essential part in the compositions *Metastaseis* and *Achorripsis*

2.2 Background to Mathematical Theory

2.2.1 Golden Section

The first discussion of the mathematical principles that Xenakis employed in his compositions concerns the Fibonacci numbers and the theory of the Golden Section. The Fibonacci Series refers to a sequence of integers which is named after the mathematician Leonardo de Pisa, also known as Fibonacci (1170-1250). The series is constructed of integer numbers in which the sum of the previous two members equals the next member and where the first two numbers are both 1. The Fibonacci series translates into the following:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 \ldots$$

because

$$\{1 + 2 = 3; 2 + 3 = 5; 3 + 5 = 8 \ldots\}$$

It is however important to note that the most significant feature of the series is the fact that as the series continues, the ratio of each two adjacent numbers approximate or converge to the Golden Section. This can be illustrated as follows:

$1 \div 1 = 1$

$1 \div 2 = 0.5$

$2 \div 3 = 0.666666\ldots$

$3 \div 5 = 0.6$

$\ldots$

$55 \div 89 = 0.617977528\ldots$

$89 \div 144 = 0.618055555\ldots$

$\ldots$

As this process continues, the ratio converges to the constant limit of (0.61803398…) which is called the Golden Proportion, Golden Ratio or Golden Section.

Another interpretation of the Golden Section is geometric in nature. It is achieved by the division of a line into two unequal parts (short and long) with a point (GSP) that makes the ratio of the longer segment to the shorter segment the same as the ratio of the longer segment to the whole line. Figure 2 illustrates the division.

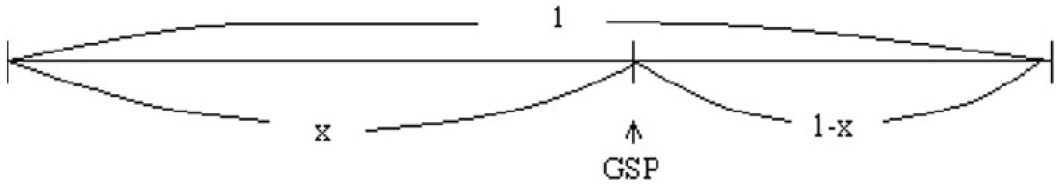


Figure 2: Golden section point[16]

There is only one solution or one point which is called the Golden Section Point (GSP).

The ratio can be expressed as follows:

$(1 - x) \div x = x \div 1$ or $x : (1 - x) = 1 : x$

thus

$x^2 - x + 1 = 0$

solving the equations yields:

[16] From Chung (2003: 24).

$x = 0.618\ldots, 1 - x = 0.382\ldots$

The proportion is not only found in geometric forms, but also in nature as seen in shells and the length of the trunk in relation to the diameter of certain trees. Furthermore, Beer (2005: 4) argues that the Golden Section has found numerous applications in the arts, more specifically painting and photography, where a picture is often divided in length or width (or both) in proportions relating to the Golden Proportion. He further argues that research has discovered that the proportion is also very common in musical compositions where it is used to generate rhythmical inversions or in developing a melodic line.[17]

In order to understand Xenakis' application of the ratio, it is necessary to mention Le Corbusier's influence on the topic. He believed and developed the idea that the very proportions in nature that enable shells to sustain enormous weights and tree trunks to withstand great pressure, could be applied to the construction of roofs, supports and buildings. He was able to relate the Golden Mean to the height of an upright man with his arm raised as shown in figure 3 and called this scale of proportions *Modulor*.

Using the *Modulor*, Le Corbusier was able to make the human scale the constant unit of all the dimensions of a building such as for example the height of a ceiling, the size of a door, the placing of a window and the overall relationship of the structure. Xenakis was familiar with the Golden Section from Greek architecture and therefore had no problem accepting the *Modulor.* It is even suggested by Matossian (1986: 41-42) that the simplicity and elegance of the *Modulor* inspired Xenakis to investigate how the Golden Principle could form part of a musical composition.

[17] Further information on the topic can be found in the thesis *Applying the phi ratio in designing a musical scale* (2005) by Van Zyl Smit.

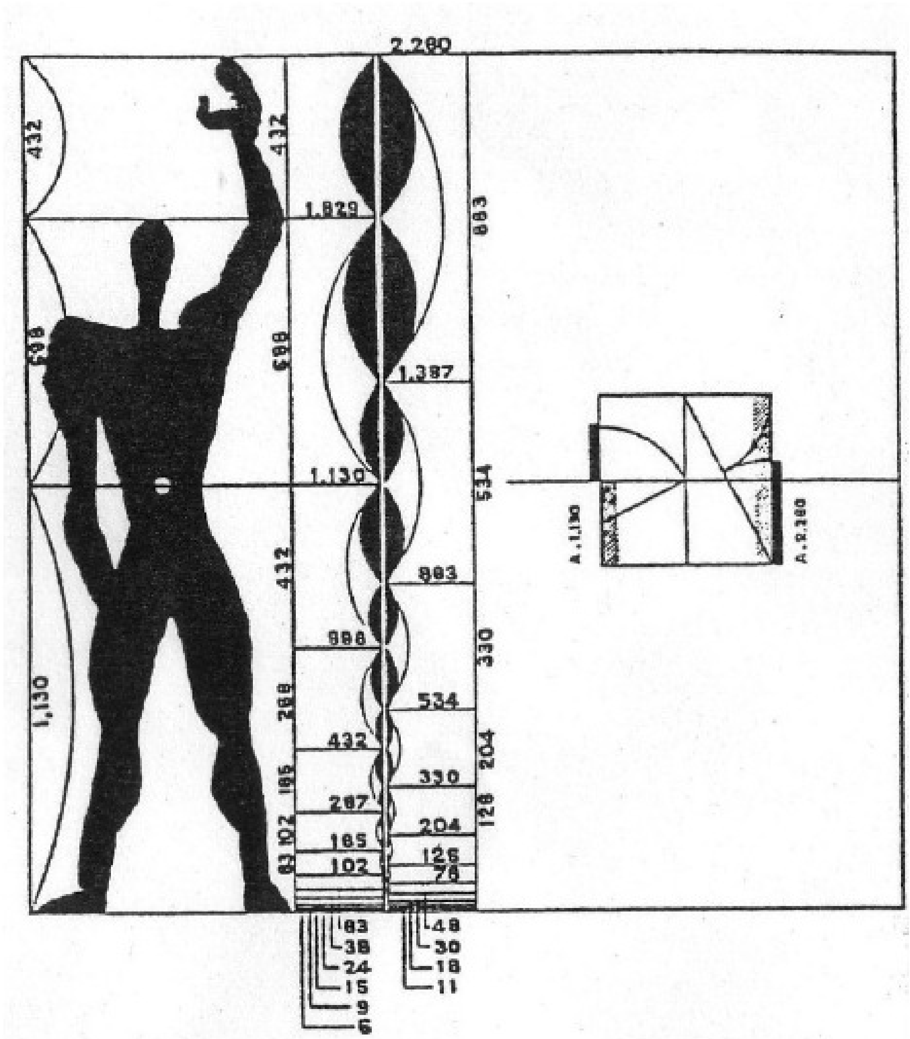


Figure 3: Modular[18]

As an architect, Xenakis employed the principles of Golden Section and the *Modulor* in designing the Couvent de la Tourette. It can be seen in the ratios of the columns on the windowpanes as well as on the horizontal metallic joints that connect two vertical columns which are placed at Golden Section points. As composer, the concept of the Golden Section as used in the structure of the Couvent de la Tourette was transformed into his first major composition *Metastaseis* (1953-54).

The implementation of the Golden Section can be observed by looking at the formal structure of *Metastaseis* as analysed by Baltensperger (1996: 53 -57) and Santana and Santana (2008: 2 - 3). As an illustration, consider the first 55 bars of the composition. The 55 bars are divided into 34 + 21 bars. The reason for this is that there are 34 bars before the first harmony cluster occurs. Thereafter there are 21 bars before the first pause

[18] From Baltensperger (1995: 108).

for the strings occur. The first 34 bars can furthermore be subdivided into 13 + 21 bars respectively with the first 13 bars again subdivided into 8 + 5 bars. The reason for this subdivision is that the first glissando begins in bar 1, the second glissando in bar 8 and the third in bar 13. The second part of the first 55 bars from *Metastaseis,* amounting to 21 bars, can also be subdivided into 13 + 8 bars where the 13 bars are played *non-tremolo* and the 8 bars are played *tremolo*. The 13 *non-tremolo* bars can further be subdivided into 8 + 5 bars where the 8 bars correspond to normal legato playing and the 5 bars correspond to *pizzicato* playing. The changes in dynamics occurring during these 55 bars also coincide exactly with the division and subdivision described above. The structure of the 55 bars is summarized visually in figure 4.

<table>
<tr><th colspan="7">55 Bars</th></tr>
<tr><td colspan="3">34 bars</td><td colspan="4">21 bars</td></tr>
<tr><td colspan="2">pp</td><td>f</td><td colspan="2">f</td><td colspan="2">fff</td></tr>
<tr><td colspan="2">13 bars</td><td>21 bars</td><td colspan="2">13 bars</td><td colspan="2">8 bars</td></tr>
<tr><td colspan="3"></td><td>f</td><td>ff</td><td>ff</td><td>fff</td></tr>
<tr><td>8 bars</td><td>5 bars</td><td></td><td>8 bars</td><td>5 bars</td><td colspan="2"></td></tr>
<tr><td>Gliss. 1</td><td>Gliss. 2</td><td>Gliss. 3</td><td>Normal</td><td>Pizzicato</td><td colspan="2">Tremolo</td></tr>
</table>

Figure 4: Structure of Metastaseis for Bar 1 - 55.

The division of the 55 bars into 34, 21, 8 and 5 bars respectively corresponds not only to Fibonacci numbers but also to the Golden Ratio[19] because

$$\frac{5}{8} \approx \frac{8}{13} \approx \frac{13}{21} \approx \frac{21}{34} \approx \frac{34}{55} \approx golden\ \ ratio \quad .$$

2.2.2 Probability theory in Stochastic music

After completing his first major composition, *Metastaseis* (1953-54), Xenakis further explored the use of mathematics in music and introduced the use of probability distributions in his next two recognized works, namely *Pithoprakta* (1954) and *Achorripsis* (1956-57). He named music composed with the use of probability distributions *Stochastic* music. The next sections of this project will provide a background of probability theory[20] and then consider Xenakis' implementation thereof.

[19] The use of the Golden Section is not only limited to the first 55 bars of the composition, but can be traced throughout the work.

[20] The mathematics in the following section is based on Berry and Lindgren (*Statistics: Theory and Methods*, 1990).

2.2.2.1 Introduction to Probability theory

A stochastic process has direct connotation with Probability theory. Probability theory is concerned with random experiments and their outcomes. For example, it concerns the outcome of actions such as the tossing of a coin or the rolling of a dice and even playing the lottery. Even though these events are random, if we repeat them many times, their results will exhibit certain statistical patterns which can be studied and predicted.

The probability of a certain outcome from a specific event falls in the range from zero to one. If the expected event occurs, it will correspond to a probability of one and a zero will result if the expected result does not happen at all. Mathematically it can be represented as follows:

Suppose Ω is the sample space, in other words, the set of all possible outcomes. The distribution function *m*, consisting of random variables, is defined by Ω (where *x* is an element of Ω) as follows:

$$\sum_{x_i \in \Omega} m(x_i) = 1 \tag{2.1}$$

The probability that events with elements belonging to set A would occur is then as follows:

$$P(A) = \sum_{x \in A} m(x) \tag{2.2}$$

where

$$0 \le P(A) \le 1,\ P(\phi) = 0,\ P(\Omega) = 1\ and\ (\phi : null\ set)$$

The example above refers to discrete numbers or, in other words, to a sample space which contains countable elements. We now consider what happens when the sample space is made up of continuous variables:

Once again assume the sample space is to be Ω. The probability that *X*, which is a random variable, will be less than or equal to *x*, a given number, is given by the cumulative distribution function which we denote by *F*(*x*). It can be mathematically expressed as:

$$P(X \le x) = F(x) \tag{2.3}$$

Now, if the cumulative distribution function is differentiable, we can obtain its probability density function. The probability density function, denoted as f(x), will provide us with the probability that a random variable, say for argument's sake X, belongs to a given set A, where its elements are made up of rational numbers. Mathematically it can be expressed as follows:

$$P(X \in A) = \int_{x \in A} f(x)dx \qquad (2.4)$$

Before concluding this brief introduction to Probability theory, two more concepts, namely *expected values* and the *variance* have to be introduced. The *expected value* is the mean value of the random variable. It is denoted by $E(X)$ or μ and for discrete variables it is calculated as follows:

$$u = E(X) = \sum x_i \, p(x_i) \qquad (2.5)$$

For continuous variables it is calculated:

$$u = E(X) = \int x \, f(x)dx \qquad (2.6)$$

The variance for a random variable measures how widely spread the random variables are around the expected value and can be computed with the following formula:

$$Var(X) = \sigma^2 = E(X - E(X))^2 = E(X^2) - E(X)^2 \qquad (2.7)$$

Central to Probability theory is the *law of large numbers*. The law implies that as the number of experiments increases, the result obtained can be predicted more and more accurately. The probability of certain events happening therefore becomes determinate and tends towards a stable state or *stochos*, from which the adjective 'stochastic' is derived. Due to this law, we are able to predict certain random events with the use of distributions such the Normal, Poisson, Binomial and Exponential distribution, to name but a few.

The distributions that will be explained here were utilized by Xenakis in his compositions. Firstly, their formulas will be listed and a discussion on Xenakis' method of utilization will then follow.

2.2.2.2 The Exponential distribution

The probability distribution function is given as follows:

$$\begin{aligned} f(x) = \delta e^{-\delta x} \quad &, x \geq 0, \\ 0 \quad &, x < 0. \end{aligned} \tag{2.8}$$

[21]

The mean can be computed as follows:

$$mean = E(X) = \frac{1}{\delta} \tag{2.9}$$

[21] e is the base of natural logarithms which is equal to 2.7182…

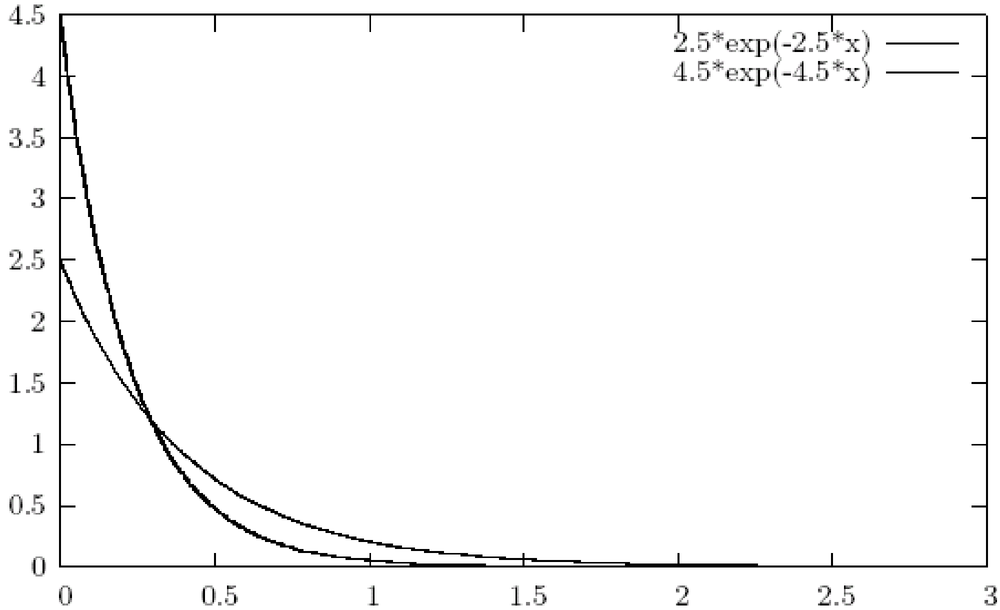


Figure 5: Probability density of the exponential distribution for δ = 2.5 and δ = 4.5[22]

As an example of its everyday use, consider the following situation. The rate of incoming phone calls differs according to the time of day. If we focus, however, on a specific time interval during which the rate stays roughly constant, such as between 14:00 to 17:00 during workdays, then the exponential distribution can be used as a good model for predicting the time until the next phone call arrives.

2.2.2.3 The Poisson distribution

Its probability distribution is as follows:

$$P(X = x) = \frac{m^x}{x!} e^{-m} \qquad (2.10)$$

where

$$x = 0, 1, 2 \ldots n \text{ and } m > 0$$

$x!$ denotes x factorial {where $x! = x \times (x - 1) \times (x - 2) \times (x - 3) \times \ldots \times 1$}and e is the base of natural logarithms which is equal to 2.7182…

22 From Schoner (2003: 8).

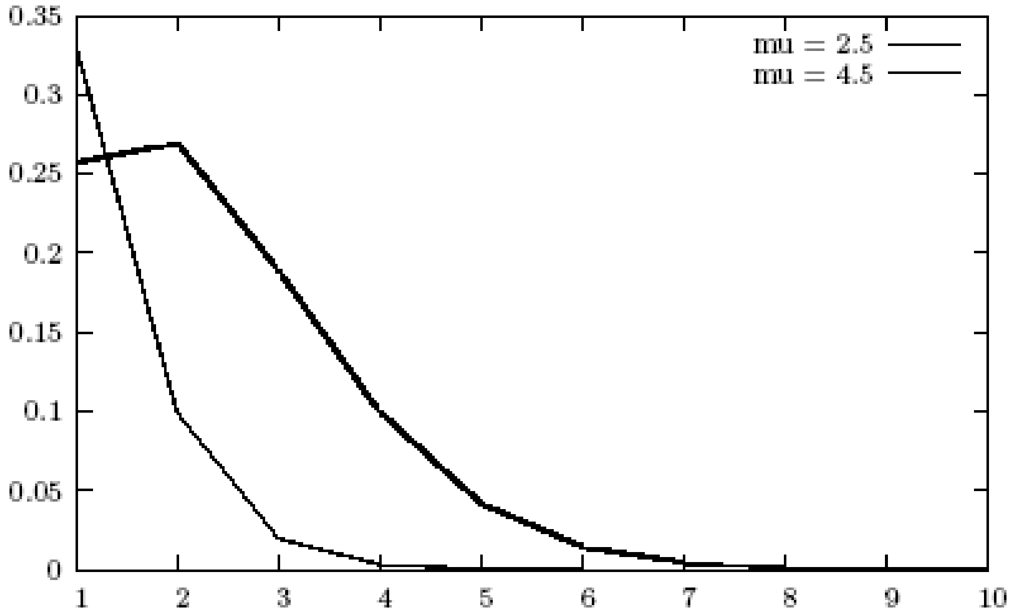


Figure 6: The Poisson distribution with *m* = 2.5 and *m* = 4.5[23].

An everyday use of the distribution is to predict the number of events that occur in a certain time interval. Whereas with the exponential distribution one is able to predict when the next call will arrive, the Poisson distribution can be used to predict the number of calls that will be received between 14:00 to 17:00 during a workday.

2.2.2.4 The Linear distribution

The probability distribution is defined as follows:

$$f(x) = \frac{2}{c}(1 - \frac{x}{c}) \tag{2.11}$$

where

c denotes the maximum interval size

[23] From Schoner (2003: 9).

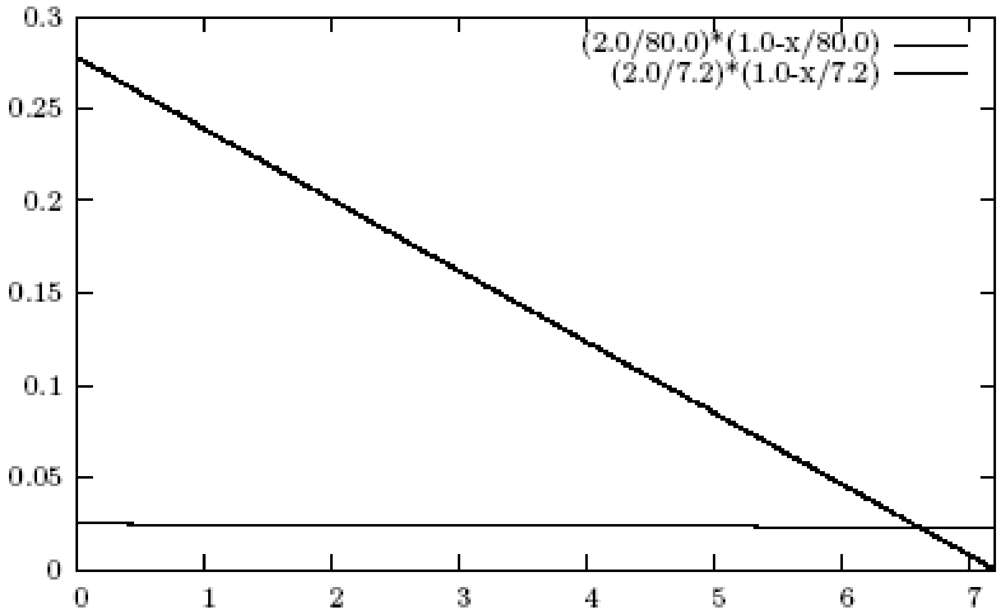


Figure 7: The Linear distribution with *a* = 80 and *a* = 7.2.[24]

2.2.2.5 The normal or Gaussian distribution

Its probability distribution is as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{[-\frac{1}{2}(\frac{x-u}{\sigma})^2]} \tag{2.12}$$

Where *u* is the mean, σ is the standard deviation and *e* is equal to 2.7182…

[24] From Schoner (2003: 10).

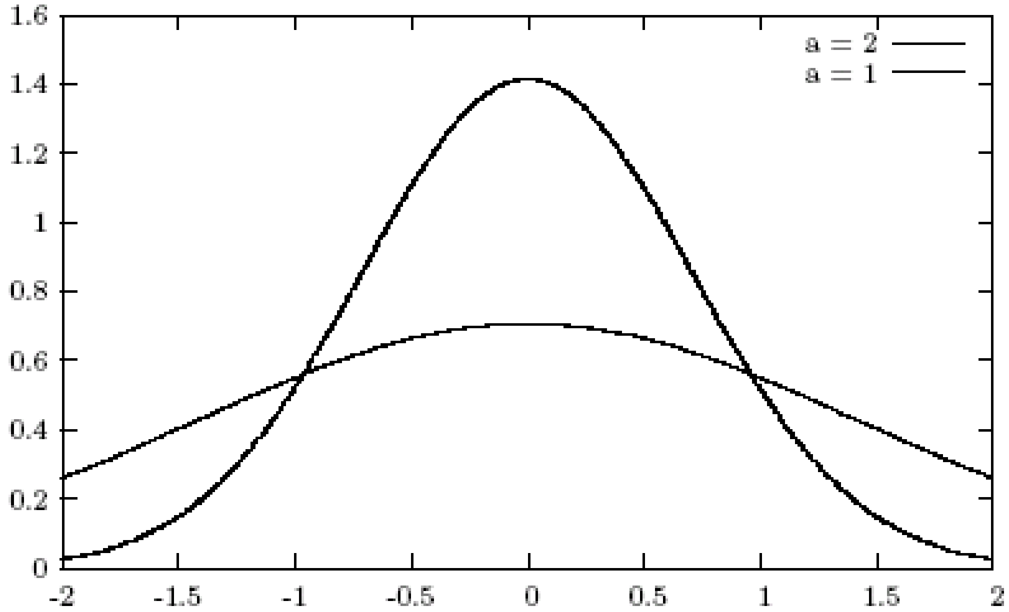


Figure 8: The normal distribution with $\sigma = 2$ and $\sigma = 1$.[25]

For example, the height of a person at a given age for a given gender can be adequately predicted by a Normal distribution. This distribution is the most commonly used of all probability distributions. Xenakis, for instance, used this distribution to govern the 'speed' of glissandi in compositions such as *Metastasis* and *Pithoprakta.*

2.3 Xenakis' implementation of probability distributions in compositions

Before considering how Xenakis employed stochastic formulas in the determination of duration and pitch, one should note his opinion of how the audience observed music. [26] He describes the situation as follows in his article *La crise de la musique sérielle,* published in *Gravesaner Bläter*, no.1 (1955):

> Linear polophony destroys itself by its very complexity; what one hears is in reality nothing but a mass of notes in various registers. The enormous complexity prevents the audience

[25] From Schoner (2003: 11).

[26] The information provided in the next section is based on Xenakis (1992), Baltensperger (1995), Schoner (2003) and on research done by Chung (2003).

> from following the intertwining of the lines and has as its macroscopic effect an irrational and fortuitous dispersion of sounds over the whole extent of the sonic spectrum.

In other words, Xenakis believed that individual notes are not perceived by the audience, but rather groups of notes. He called these groups of notes *clouds* and believed that the whole shape is more meaningful and important than the particular placement of specific notes.

2.3.1 The Exponential distribution

Xenakis showed that note durations and their successions can be determined by an exponential distribution. He did this by considering time as a straight line on which points, corresponding to the variation of the other musical components, are plotted. The interval between two of these points is then identical to duration. Now when one chooses an average or mean number of points on a given length, the probability that the duration will be equal to a certain length can be expressed by the exponential distribution as follows:

As mentioned in the previous section, equation 2.8, the probability distribution function is

$$f(x) = \delta e^{-\delta x}$$

where Xenakis denotes δ as the linear density of points and *x* the length of any segment.

In order to obtain the probability that the length of a note is between two values (say α and β) we integrate the function giving:

$$P(x) = \int_{\alpha}^{\beta} \delta e^{-\delta x} dx \qquad (2.13)$$

2.3.2 The Poisson distribution

The groups of notes or *clouds*, mentioned above, are defined by two characteristics, namely the density and interval relationship. The average density of a musical work depends on the number of actual notes that can be played during a given time period. The aspects such as the number of instrumentalists, the nature of the instrument as well as the technical difficulties should be taken into account when deciding on the density.[27]

The density of any given point within the set of sound events is determined by the number of sound events that occur during a specified time. The density of a *cloud* is equal to δ, and as mentioned earlier its value is chosen by the composer after taking the practical limitations into consideration. Xenakis made use of the Poisson distribution, as given in equation 2.10, in determining the number of events that occur during a given time. In other words, he calculated the probability $P(l)$ that an event occurs l times per time unit, using the Poisson distribution:

$$P(l) = \frac{\delta^{l}}{l!} e^{-\delta} \qquad (2.14)^{28}$$

This was done by first choosing the mean density (δ) and then calculating the probabilities that certain events occur within a given time frame or within given measures based on this chosen mean density. After he calculated the probabilities with this formula, the distribution of events was determined accordingly.

The Poisson distribution, as explained above, was frequently used by Xenakis in composing works that were stochastically constructed. An example of such a composition is *Achorripsis* and in the following chapter a more detailed description will be given.

2.3.3 The Linear distribution

The second characteristic of the groups of notes or *clouds,* namely the interval relationship, is now considered. The probability distribution that Xenakis used in determining the intervals between successive pitches was the Linear distribution:

$$P(\gamma) = f(\gamma)d\gamma = \frac{2}{a}(1 - \frac{\gamma}{a})d\gamma \qquad (2.15)^{29}$$

[27] Xenakis (1992: 136).

[28] Equation 2.14 is directly derived from equation 2.10 by substituting m with δ and x with l.

[29] Equation 2.15 is derived from equation 2.11 by substituting c with a and writing the probability function in terms of γ.

This probability distribution provides the probability that, within a segment of length *a*, a segment with a length between γ and γ + *d*γ will exist, where $0 \le \gamma \le a$. The segment length, *a,* is the maximum interval length as specified by the composer. This maximum interval limit, restricts the occurrence of events in the composition which would be "unnatural sounding or difficult to play" Schoner (2003: 9).

The intervals determine the distribution of *clouds* and can be visually illustrated as follows:

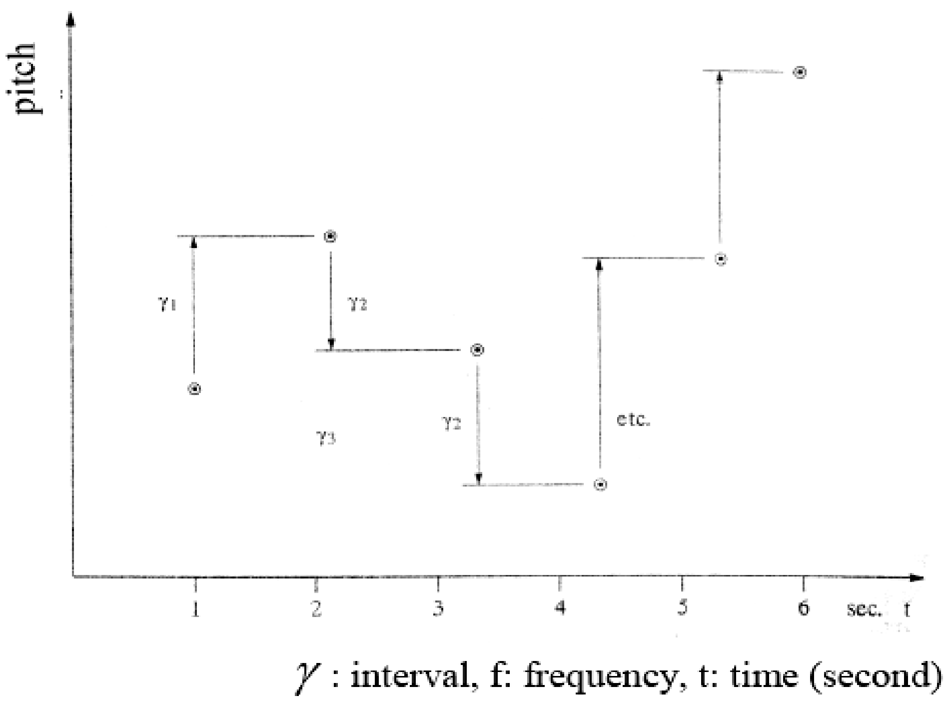


Figure 9: Clouds or groups of notes[30]

2.3.4 The normal or Gaussian distribution

The last distribution that will be introduced in this chapter is the Normal or Gaussian distribution. Xenakis used this probability to characterize the movement of glissandi. He defined the *speed* of glissandi as the change in pitch (*df*) against the change in time (*dt*). This *speed* ($\frac{df}{dt}$) of the glissandi was related to the movement of gas molecules, whose *speed* was shown by Maxwell/Boltzmann to be distributed as a Normal or Gaussian distribution.

[30] Baltensperger (1995: 448).

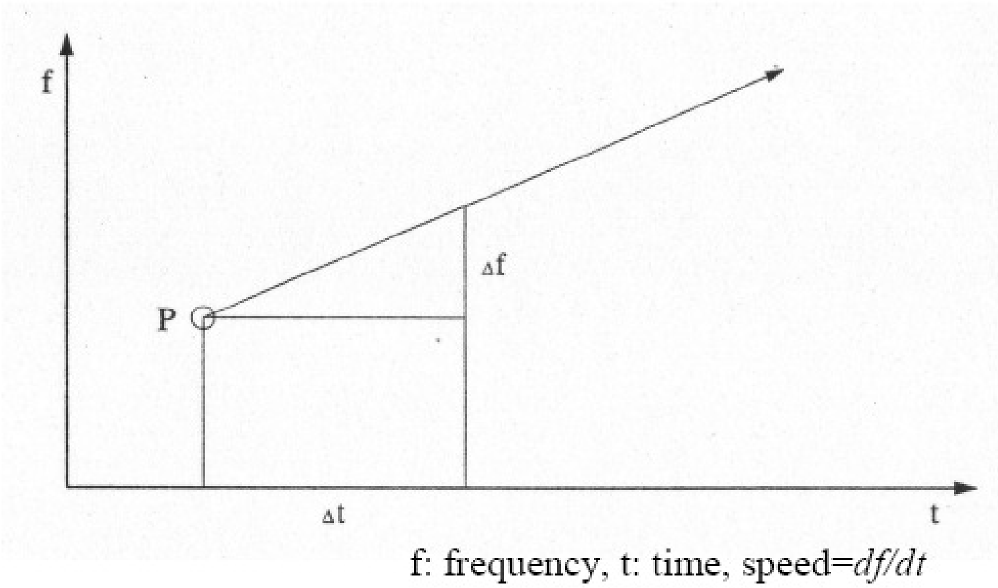


Figure 10: Glissando speed [31]

The probability density function $f(v)$ for the existence of a *speed* is defined by Xenakis (1992: 33) as follows:

$$f(v) = \frac{2}{\alpha\sqrt{\pi}} e^{\frac{-v^2}{\alpha^2}} \tag{2.16}$$

where *v* is defined as the *speed* and α as the quadratic mean of all possible values of *v*.

The probability, *P(λ)*, that a *speed* (*v*) will fall into a specific range, say between *v*1 and *v*2, where *v*2 > *v*1, is given by:

$$P(\lambda) = \theta(\lambda_2) - \theta(\lambda_1) \tag{2.17}$$

[31] Baltensperger (1995: 451).

where

$\lambda_1 = \frac{v_1}{\alpha}$, $\lambda_2 = \frac{v_2}{\alpha}$

and

$$\theta(\lambda) = \frac{2}{\sqrt{\pi}} \int_0^{\lambda} e^{-\lambda^2} d\lambda$$

Compositions in which this method of describing glissandi was used by Xenakis include *Metastaseis* and *Achorripsis*. Figure 11, taken from Xenakis (1992: 3) gives a visual representation of the glissandi in a section of the composition *Metastaseis*.

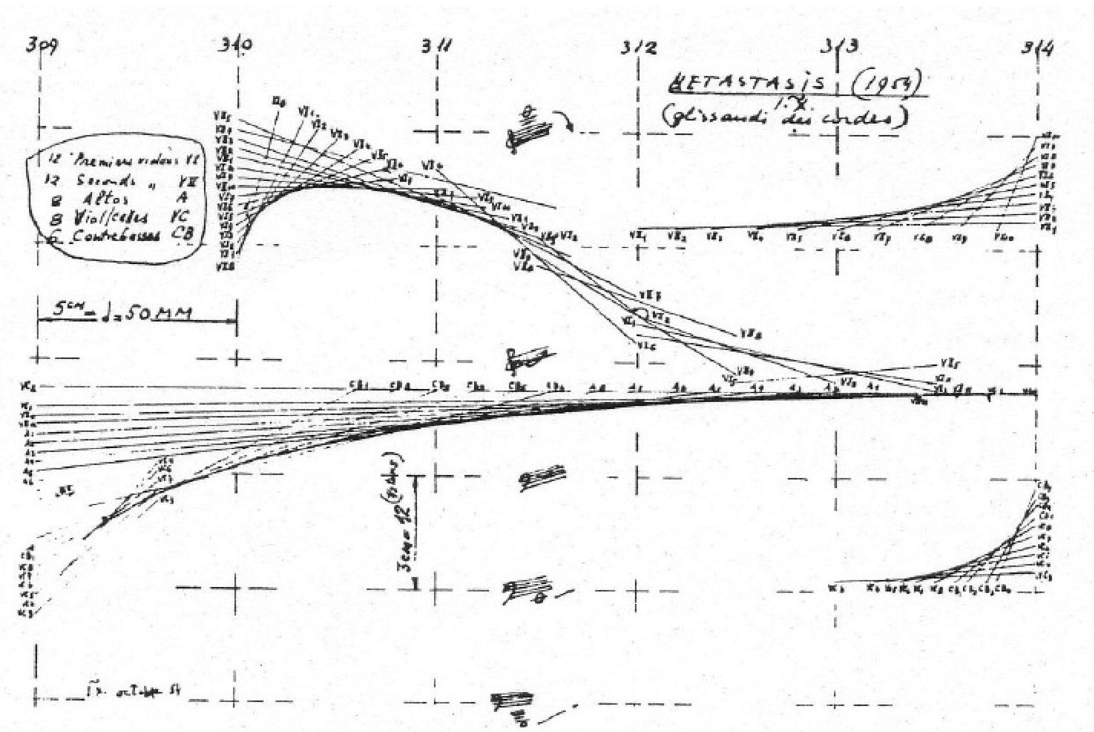


Figure 11: Glissandi in bars 309-314 of Metastaseis[32]

[32] Xenakis (1992: 3).

2.4 Chapter conclusion

This chapter served as an introduction into mathematical concepts utilized in the creation of the compositions *Metastaseis* and *Achorripsis.* Firstly, the Golden Section (Proportion or Ratio) was considered and it was shown how it was used in shaping the overall structure of *Metastaseis.* Attention was then focused on the stochastic processes employed by Xenakis in the two compositions and the Exponential, Linear, Poisson and Normal distributions were explored. This was done by discussing their general properties as well as conventional application and it was lastly shown how and where Xenakis made use of them in his compositions.

Chapter 3

Achorripsis

3.1 Background

The musical work *Achorripsis,* which in Greek means *Jets of Sound*, was Xenakis' third major composition and was composed during 1956-57. It is a significant work because for the first time, as shown by Childs in *Musical Sonification Design* (2003), probability distributions was used by Xenakis in defining the overall structure, the time between musical events, the intervals between successive pitches and the speeds of glissandi. It is furthermore worth mentioning that the fact that his conception of the compositional process was similar to a flow chart,[33] paved the way for algorithmic composition with the aid of computers. This was eventually achieved for the first time in 1961 with the composition *ST/10* where the procedures outlined in *Achorripsis* was implemented.

The first performance of *Achorripsis* was under the direction of Herman Scherchen (1891-1966) in Buenos Aires in 1958. In 1959, Scherchen conducted the work in Cologne, Florence, Hamburg and Paris, with the latter ending in a scandal (Varga, 1996: 37). Further performances followed in America under conductors such as Gunther Schuller, Lukas Foss and Leonard Bernstein. The first major success of the work was during the first all-Xenakis festival at the Salle Gaveau in Paris in May 1965, where *Achorripsis* was received with enthusiasm.

3.2 Choice of composition

The composition *Achorripsis* was selected because it is, as mentioned above, a pioneering work due to the use of probability distributions for the composition. Xenakis in his book Formalized Music, provides a discussion of the composition and it is therefore possible to consider his compositional approach in detail and verify some of his results. Although certain aspects of *Achorripsis* has been mentioned and discussed in broad terms by numerous researchers, only Childs (2003) investigated the finer details of the entire composition.[34]

One of the aims of this chapter will therefore be to explore the method by which Xenakis employed probability distributions in the composition and determine whether any artistic renderings was done by the composer in his final score.

[33] A flow chart is a schematic representation of a sequence of operations and is also called *flow diagram*, *flow sheet*.

[34] It is worth mentioning that Arsenault (2002) also provides a detailed analysis over the initial phases of *Achorripsis*.

Further aims of this chapter will be to follow an approach similar to the one adopted by Childs (2003) in order to verify or contradict his results and to expand on his research by considering other sections of the composition.

3.3 Analysis

As mentioned in the beginning of Chapter 2, Xenakis identified eight distinctive phases in composing a musical work. In this section, *Achorripsis* will be analysed according to each of these phases.

3.3.1 Phase 1 and 2: Initial conceptions

Figure 12 (p. 33) shows the vector matrix of *Achorripsis*, as given in Formalized Music (1992: 28). The matrix shows the formal structure of the composition and is made up of seven rows and twenty-eight columns. Each of the columns represents a fixed unit of time while the rows represent seven distinct timbres. These timbres were defined by Xenakis as follows:

I	Flute (Piccolo, Eb Clarinet, Bass Clarinet in B)
II	Oboe (Oboe, Bassoon, Contrabassoon)
III	String glissando (3 Violins, 3 Cello's, 3 Basses)
IV	Percussion (Xylophone, Wood Block, Bass Drum)
V	Pizzicato (3 Violins, 3 Cello's, 3 Basses)
VI	Brass (2 Trumpets, Trombone)
VII	String arco (3 Violins, 3 Cello's, 3 Basses)

The total length of the piece was chosen subjectively by the composer to last 7 minutes. Therefore each of the 28 columns (units of time) lasts 15 seconds. The tempo of the composition was set at MM = 26 which implied that every column (unit of time) made up 6.5 measures as the time signature is $\frac{2}{2}$. In total there were consequently 182 bars in the score.

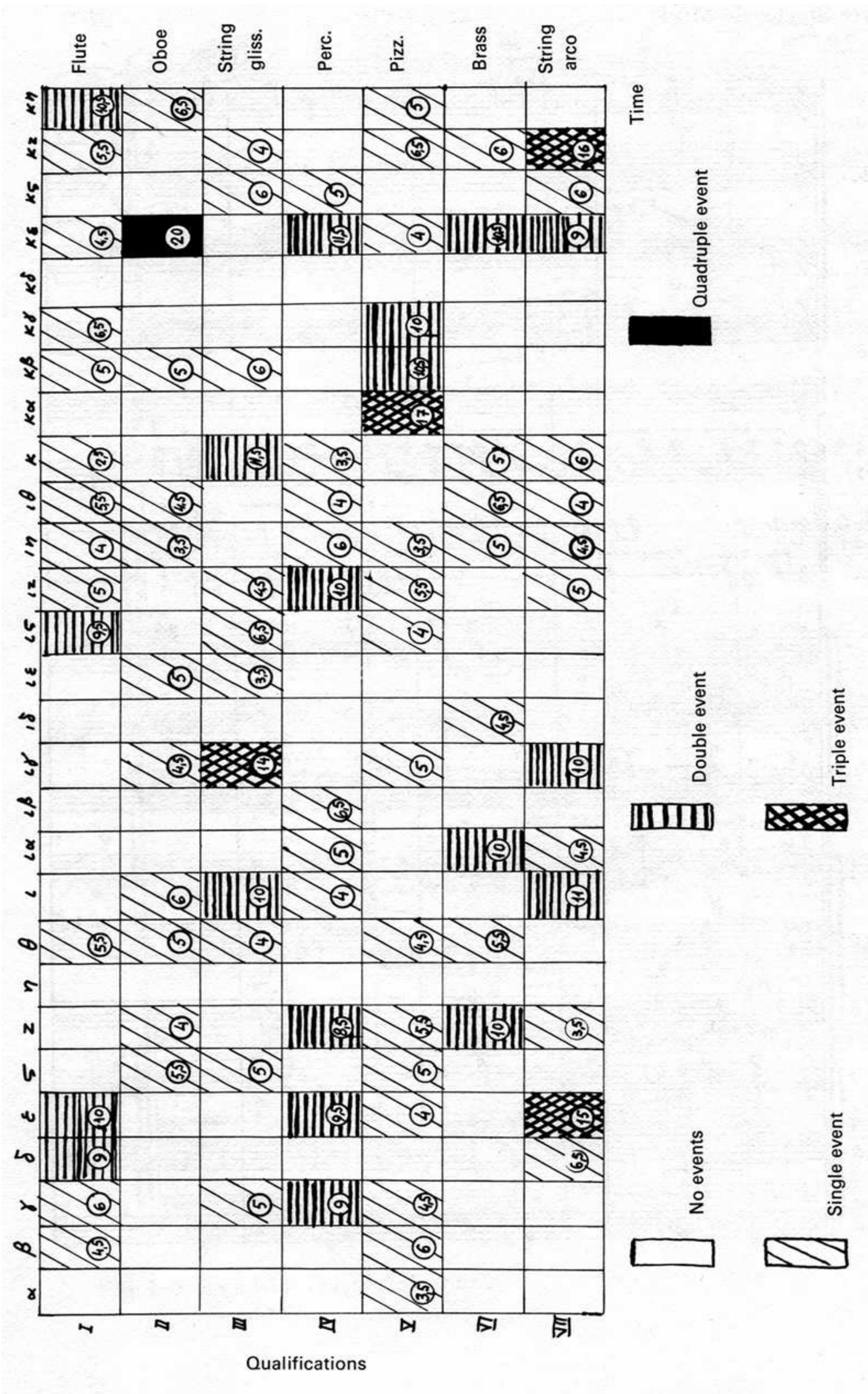


Figure 12: Matrix of Achorripsis[35]

[35] Formalized Music (1992: 28).

3.3.2 Phase 3: Definition of the sonic entities transformations Part 1

3.3.2.1 Calculations

During this phase, Xenakis decided how to allocate "musical events" to the 7 × 28 = 196 cells of the matrix (Childs, 2003: 54). Firstly, he chose a priori, $\delta = 0.6$. This means that on average 0.6 events[36] will take place in each of the cells. The choice made by Xenakis is a subjective one and in Formalized Music (1992: 29) he states that the choice is due to convenience. It is worth mentioning that although all the calculations done for *Achorripsis* can nowadays be completed easily with the assistance of calculators and computers, Xenakis had to do them all by hand. He probably made use of tables, see for example Appendix 2 and Appendix 3, which provide values of the distributions for certain priori values, which would have influenced his choice.

In allocating the events, Xenakis employed the Poisson probability distribution. As mentioned in Section 2.2.2.3, the distribution is generally used to estimate the number of times a specific event occurs within a given time span. For *Achorripsis,* the distribution was used to determine the probability that in any given cell there will be either 0, 1, 2, 3, 4 or 5 events occurring. It was done by using equation 2.14, Chapter 2 section 2.3.2.

From the equation, the probability that 0 events occur in a cell is estimated as follows:

$$P(0) = \frac{0.6^0}{0!} e^{-0.6}$$

$$P(0) = 0.548812$$ [37]

This means that the chance that 0 events occur during a cell is 54,8812%. If we continue in the same manner we find that:

$$P(1) = 0.329287$$
$$P(2) = 0.098786$$
$$P(3) = 0.019757$$
$$P(4) = 0.002964$$
$$P(5) = 0.0004556$$

[36] The definition of what exactly constitutes as an event will be explained in section 3.3.4.

[37] The precision of the calculations is given to six significant digits.

The total sum of all the probabilities is equal to 1 which makes sense if one refers back to Section 2.2.2.1. Now that the probabilities for the events occurring are determined, it is possible to calculate the number of cells that will contain either 0, 1, 2, 3, 4 or 5 events. This is done by multiplying the total number of cells, 196 (7 × 28), with each probability. The results obtained are as follows:

$n(0) = P(0) \times N = 0.548811636 \times 196 = 107.56708$

$n(1) = P(1) \times N = 0.329286981 \times 196 = 64.540248$

$n(2) = P(2) \times N = 0.098786094 \times 196 = 19.362074$

$n(3) = P(3) \times N = 0.019757218 \times 196 = 3.872415$

$n(4) = P(4) \times N = 0.02963582 \times 196 = 0.580862$

$n(5) = P(5) \times N = 0.000455629 \times 196 = 0.08930328$

The sum of the number of cells in which 0, 1, 2, 3, 4 and 5 events occur is equal to 196. As is evident, we are working with continuous variables. For practical reasons they had to be converted to discreet variables by Xenakis. It seems logical that Xenakis rounded these values up which would yield the following:

$$n(0) = 108$$
$$n(1) = 65$$
$$n(2) = 19$$
$$n(3) = 4$$
$$n(4) = 1$$
$$n(5) = 0$$

3.3.2.2 Comparison with results published by Xenakis

If the number of cells is added up, we reach 197 and not 196. In Formalized Music (1992: 29), Xenakis gives the values as follows:

$$n(0) = 107$$
$$n(1) = 65$$
$$n(2) = 19$$
$$n(3) = 4$$
$$n(4) = 1$$

which is exactly what was achieved, apart from n (0) which is equal to 107 and not 108.

If the matrix of *Achorripsis*, figure 12 (p. 33), is compared to the results produced after the calculations, it is evident that Xenakis implemented his calculations precisely. The number of single, double, triple and quadruple events are equal to 65, 19, 4 and 1 and the number of no events are equal to 107 as he calculated in Formalized Music and not 108.

3.3.3 Phase 3: Definition of the sonic entities Part 2

In the previous phase, Xenakis determined the frequencies or total number of zero, single, double, triple and quadruple events. In this phase, he had to decide how to distribute these frequencies across the time-axis of the composition and amongst the timbres (instruments). This was done by assuming that the rows and columns of the matrix, which represents the distinct timbres and units of time respectively as shown in figure 12 (p. 33), follow Poisson's law. Once again it is therefore possible to apply Poisson's formula in determining the number of times for example there are one, two, three, four or five double events in each row and column.

3.3.3.1 Distribution of events across the columns (time blocks)

Consider the number of zero or *no events* which is stated as 107 by Xenakis. We consider now their distribution across the 28 columns. The average number of *no events* (δ) per column is 107÷28=3.821428571≈3.82. If we apply Poisson's equation again we can compute the probability that there will be 0 *no events* which is equal to:

$$P(0) = \frac{3.82^0}{0!} e^{-3.82} = 0.0219278$$

The number of columns (units of time) in which there will be 0 *no events* is therefore equal to 0.0219278×28=0.613978425≈1. If we continue in the same manner we obtain the results summarised in Table 1.

Table 1: Zero or No events for columns

Frequency	Probability	Number of Columns
0	0.021928	1
1	0.083764	2
2	0.15999	4
3	0.20372	6
4	0.194553	5
5	0.148638	4
6	0.094633	3
7	0.051643	1
Totals	1	27[38]

[38] The reason why this value is not 28 is due to the rounding off of the results to round numbers. If the true values are used, they add up to 28.

Now we consider the number of *single events*. Once again they are distributed over the 28 columns and therefore the average (δ) number of *single events* is 65÷28=2.321428571≈2.32. The same procedure as above is followed and the results obtained are given in Table 2.

Table 2: Single events for columns

Frequency	Probability	Number of Columns
0	0.098274	3
1	0.227995	6
2	0.264474	7
3	0.204526	6
4	0.118625	3
5	0.055042	2
6	0.021283	1
7	0.007054	0
Totals	1	28

Table 3 gives the results for *double events* where (δ) is 0.678571428 ≈ 0.679.

Table 3: Double events for columns

Frequency	Probability	Number of Columns
0	0.507124	14
1	0.344337	10
2	0.116902	3
3	0.026459	1
4	0.004491	0
Totals	1	28

Table 4 gives the results for *triple events* where δ is 0.142857142≈ 0.143.

Table 4: Triple events for columns

Frequency	Probability	Number of Columns
0	0.866878	24
1	0.12384	3
2	0.008846	0
Totals	1	27[39]

For *Quadruple events,* δ is 0.035714285 ≈ 0.0357 and Table 5 provides the results.

[39] The reason why this value is not equal to 28 is due to rounding the results to round numbers. If the true values are used, it adds up to 28.

Table 5: Quadruple events for columns

Frequency	Probability	Number of Columns
0	0.96493	27
1	0.034448	1
2	0.000615	0
Totals	1	28

3.3.3.2 Comparison with calculations published by Xenakis and Childs

In Formalized music, Xenakis only provides his results for *single events.* If the results from Table 2 are compared with his results in Formalized music (1992: 30) and with Childs's (2003: 56) results, one observes two differences. Xenakis calculated according to the Poisson distribution that there should be 8 occasions where two *single events* occur in a column (time block), whereas in fact, we determined that there should only be 7. Xenakis also calculated that there should be 5 columns (time blocks) where 3 *single events* occur, but we calculated that there should be 6. Apart from these two differences, the same results for *single events* as Xenakis and Childs are obtained. For *double events*, the results obtained in Table 3 are the same as those published by Childs (2003: 57), but for *triple events* there is one difference between our results given in Table 4 and those of Childs, which can be attributed to the rounding to integer values.

3.3.3.3 Distribution of events across the rows (timbres)

As mentioned earlier, the distribution of *zero, single, double, triple and quadruple* events across the rows or timbres (instruments) was also determined by the Poisson distribution. It is calculated in exactly the same way as was done with the columns and the results as well as their implementation by Xenakis are given in the following tables. In these tables the 'frequency' refers to the number of events that can occur in a row. The 'predicted number of rows' shows our calculated number of rows that should contain the number of events equal to the frequency. The 'actual number of rows' shows the number of rows, which have the number of events equal to the frequency, as given by Xenakis in the Matrix (figure 12, p.33) of *Achorripsis.*

Table 6:[40] Summary of the distribution of Zero Events across the timbres (Rows)

Frequency	Probability	Predicted No. Rows	Actual No. Rows
10	0.044116	0	0
11	0.061304	0	1
12	0.078089	1	0
13	0.091819	1	1
14	0.100252	1	0

[40] Only the most significant portion of the results is presented in the table. The other values of the predicted and actual number of rows, all equal 0.

15	0.102161	1	1
16	0.0976	1	2
17	0.087758	1	1
18	0.074525	1	0
19	0.059956	0	1
20	0.045823	0	0
Total	≈1	7	7

Table 7: Summary of the distribution of Single events across the timbres (Rows)

Frequency	Probability	Predicted No. Rows	Actual No. Rows
5	0.053354	0	0
6	0.082571	1	1
7	0.109533	1	1
8	0.127136	1	1
9	0.131172	1	1
10	0.121802	1	1
11	0.10282	1	1
12	0.079563	1	0
13	0.056831	0	0
14	0.037694	0	1
15	0.023334	0	0
Total	≈1	7	7

Table 8: Summary of the distribution of Double events across the timbres (Rows)

Frequency	Probability	Predicted No. Rows	Actual No. Rows
0	0.066251	0	1
1	0.179826	1	0
2	0.244051	2	2
3	0.220809	2	2
4	0.149835	1	1
5	0.08134	1	1
6	0.036797	0	0
7	0.014268	0	0
Total	≈1	7	7

Table 9: Summary of the distribution of Triple events across the timbres (Rows)

Frequency	Probability	Predicted No. Rows	Actual No. Rows
0	0.564734	4	4
1	0.322689	2	2
2	0.092192	1	1

3	0.01756	0	0
4	0.002508	0	0
Total	≈1	7	7

Table 10: Summary of the distribution of Quadruple events across the timbres (Rows)

Frequency	Probability	Predicted No. Rows	Actual No. Rows
0	0.866875	6	6
1	0.123842	1	1
2	0.008846	0	0
3	0.000421	0	0
Total	≈1	7	7

3.3.4 Phase 3: Definition of Musical Events

3.3.4.1 Calculations

Up to this point the number of *zero, single, double, triple and quadruple* events has been predicted as well as how the events are distributed. There has, however, been no explanation as to what each of these events symbolises. In this phase, Xenakis specified each of these events as follows:

Table 11: Event specification

Event	Sounds per bar
Zero	0
Single	5
Double	10
Triple	15
Quadruple	20

One should note that in defining these events, he took into account the fact he believed that the maximum number of sounds per second that a normal orchestra can produce is ten sounds per second. As mentioned in Section 3.3.1, the tempo of *Achorripsis* was chosen to equal 26 MM with each of the 28 columns (units of time) lasting 15 seconds. The number of sounds per second and per cell of the matrix is as follows:

Table 12: Further event specification

Event	Sounds per second	Sounds per cell
Zero	0	0
Single	2.1666…	32.5
Double	4.333…	65
Triple	6.5..	97.5
Quadruple	8.666	130

It is worth mentioning that if the results from Table 12 are compared to the values given by Xenakis (1992: 32), there are again some inconsistencies. The number of sounds per second given by him for *zero, single, double, triple and quadruple* events are 0, 2.2, 4.4, 6.6 and 8.8 which are different from those obtained by us. The reason for this is probably due to the fact that Xenakis made all his calculations by hand and therefore only approximated the number of sounds per second.

3.3.4.2 Comparison with Xenakis' application

Xenakis let the number of sounds per measure of each event vary and the values given above are not employed as a constant per event. As Childs (2003: 61-62) states, it is not clear how Xenakis decided to assign a specific value for the sounds per measure of each event. One suspects that he once again made use of a stochastic process but no information is given by the composer. In the vector matrix (figure 12, p.33) of *Achorripsis*, these values can be observed as the circled values in each cell.

For ***single*** events he varied the sounds per measure between 2.5 and 6.5. The weighted average of these values equals 4.946 ≈5.

For ***double*** events the sounds per measure varies between 8.5 and 11.5. Their weighted average equals exactly 10.

For ***triple*** events the sounds per measure varies between 14 and 17 with the weighted average being 15.5.

There is only one ***quadruple*** event which is given as 20.

3.3.5 Phase 4: Micro-composition

In this phase, Xenakis turned his attention to the producing of sounds for the events in each of the 196 cells. His rationale of how a glissando sound is produced can be seen in the 7 hypothesis that he developed as mentioned in Formalized Music (1992: 32-34).

Hypothesis 1: The acoustic characteristic of the glissando sound is assimilated to the speed ($v = df/dt$) of a uniformly continuous movement.

Hypothesis 2: The quadratic mean, denoted by α of all possible values of v is proportional to the sonic density δ.

Hypothesis 3: The values of the speeds are distributed according to the most asymmetry (chance). This distribution follows the law of Gauss. In other words, it is a normal distribution. Therefore, the probability $f(v)$ for the existence of the speed v is given by the function

$$f(v) = \frac{2}{a\sqrt{\pi}} e^{-\frac{v^2}{a^2}} \tag{3.1}$$

and that the probability $P(\lambda)$ that v will lie between v_1 and v_2, by the function

$$P(\lambda) = \theta(\lambda_2) - \theta(\lambda_1) \tag{3.2}$$

in which

$$\lambda_1 = v_1 / a \,, \lambda_2 = v_2 / a$$

and

$$\theta(\lambda) = \frac{2}{\sqrt{\pi}} \int_0^{\lambda} e^{-\lambda^2} d\lambda \tag{3.3}$$

Hypothesis 4: A glissando sound is essentially characterized by the moment of its departure (*a*), its speed ($v_m = df / dt$) where $v_1 < v_m < v_2$ and its register (*c*).

Hypothesis 5: Consider time as a line where each moment of departure corresponds to a point on that line. The moment of departure corresponds to a sound. These points then define segments and the probability that the *i*-th segment will have a length x_i between x and $x + dx$ is

$$P(x) = \delta e^{-\delta x} dx \tag{3.4}$$

Hypothesis 6: In defining the pitch of each sound, one should note that the strings have a range of more or less 80 semitones. This can be represented by a line of length a = 80 semitones. Since between two successive or simultaneous glissandi there exist an interval between the pitches at the moments of departure, we can define not only the note of attack for the first glissando, but also the melodic interval which separates the two origins. Therefore, in determining the pitch, the probability that a segment s within a line segment of length a will have a length between j and $j + dj$ where $0 \leq j \leq a$, should be determined. This probability can be solved with the equation:

$$\theta(j)dj = \frac{2}{a}(1-\frac{j}{a})dj \tag{3.5}$$

Hypothesis 7: The three essential characteristics of the glissando sound defined in Hypothesis 4 are independent of each other.

From these seven hypotheses, we can observe Xenakis' theory behind determining the duration of notes, the speed of glissandi and the intervals between pitches. As mentioned in Chapter 2, Section 2.2.2.2, the *Exponential distribution* was used to determine the duration, the *Normal distribution* to govern the speed of the glissandi and the *Linear distribution* to determine the intervals between pitches.

Xenakis provides information on how he used these distributions for cell III, *iz* of the matrix (figure 12, p.33) of Achorripsis. In the following sections, calculation for the duration between the glissandi, speed of glissandi and the intervals between pitches for cell III, *iz* will be given in order to compare the results with Xenakis' own calculations and later to verify them with his final score of the composition.

3.3.5.1 Duration

From hypothesis 5 we are able to calculate the number of durations between glissandi which will fall into certain intervals with the use of the exponential distribution as given by equation 3.4.

Due to the fact that Xenakis had to do his computations by hand, an approximation was made by considering *dx* as a constant factor that is determined as follows.[41]

$$dx = \frac{1-e^{-\delta x}}{\delta} \tag{3.6}$$

and thus

$$dx \approx 0.0805$$

In cell III, *iz* there are 4.5 sounds per measure (δ) at a tempo of MM 26. As mentioned earlier (Section 3.3.1), each cell amounts to 6.5 measures and thus for cell III, *iz,* there will be 29 sounds per cell. These 29 sounds will have 28 durations between them. In determining the durations, Xenakis considered intervals equal to a length of 10% of each measure which means that x=0.10 and that at MM 26 each of these lengths equals (60÷26)×0.1 =0.23076923 seconds.

[41] In order not to deviate too far from our analysis, the mathematical reasoning and proof of this approximation will be provided in the Appendix 1.

Table 13: Duration for cell III, *iz*

x	**Interval In seconds**	**δx**	$e^{-\delta x}$	$\delta e^{-\delta x}dx$	$28\times P(x)$	**Xenakis' results**
0	(0 , 0.2308]	0	1	0.362372	10.15≈10	10
0.1	(0.2308 , 0.4615]	0.45	0.637628	0.231058	6.47≈6	7
0.2	(0.4615 , 0.6923]	0.9	0.40657	0.147329	4.13 ≈4	4
0.3	(0.6923 , 0.9231]	1.35	0.25924	0.093941	2.63≈3	3
0.4	(0.9231 , 1.1538]	1.8	0.165299	0.0599	1.68≈2	2
0.5	(1.1538 , 1.3846]	2.25	0.105399	0.038194	1.07≈1	1
0.6	(1.3846 , 1.6154]	2.7	0.067206	0.024353	0.68≈1	1
0.7	(1.6154 , 1.8462]	3.15	0.042852	0.015528	0.43≈0	0
0.8	(1.8462 , 2.0769]	3.6	0.027324	0.009901	0.28≈0	0
0.9	(2.0769 , 2.3077]	4.05	0.017422	0.006313	0.18≈0	0
1	>2.3077	4.5	0.011109	0.004026	0.11≈0	0
			Totals	**0.992917**	**27**	**28**

If Table 13 is compared with the results given by Xenakis (1992: 34), it can be seen that the majority of the results are the same. The few that differ slightly can be attributed to some approximations made by Xenakis. One important change that Xenakis made is at x=0.1 where in the $28\times P(x)$ column, the value is equal to 6, he states it as 7. This adjustment is made so that the total will sum to 28 durations rather than 27 as in our case. There is also one major difference in calculation, namely the total of the $\delta e^{-\delta x}$ column. When this column is compared with the published column by Xenakis (1992: 34) is can be seen that he states that the total equals 12.415, but if one adds his given values it should equal 12.077. This error influences his value of dx, which would equal 0.803 rather than 0.805. Since he rounded his final values in the $28\times P(x)$ column, the error has no significant influence.

3.3.5.2 Speed

From hypothesis 1 it is clear that Xenakis defined the speed of glissandi as the change in pitch (df) divided by change in time (dt). From hypothesis 3 he states that the probability for the existence of a speed v can be calculated by equation 3.1.

This equation is similar to the equation of a normal distribution given in equation 2.12, chapter 2, section 2.2.2.5

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{[-\frac{1}{2}(\frac{x-u}{\sigma})^2]}$$

which can be standardized by setting $z=\frac{x-u}{\sigma}$ which then yields the standard normal density function

$$f(z)=\frac{1}{\sqrt{2\pi}}e^{[-\frac{1}{2}z^2]} \tag{3.7}$$

For the cumulative standard normal density function,

$$F(z)=\frac{1}{\sqrt{2\pi}}\int e^{-\frac{z^2}{2}}dz \tag{3.8}$$

a statistical table (Appendix 2) exists which can be used to evaluate the function. This is required in calculating the probability $P(\lambda)$ {that v (speed of the glissandi) will lie between v_1 and v_2 } which is calculated as stated by Hypothesis 3 by the function (equation 3.2)

$$P(\lambda)=\theta(\lambda_2)-\theta(\lambda_1)$$

and (equation 3.3)

$$\theta(\lambda)=\frac{2}{\sqrt{\pi}}\int e^{-\lambda^2}d\lambda$$

In order to evaluate the integral by using the table (Appendix 2), a transformation has to be made by setting $\lambda=\frac{z}{\sqrt{2}}$ and therefore $d\lambda=\frac{dz}{\sqrt{2}}$. The transformations yields the function:

$$\theta(z)=\frac{2}{\sqrt{2\pi}}\int e^{-\frac{z^2}{2}}dz \tag{3.9}$$

which is

$$\theta(z)=2\times F(z)$$

In cell III, *iz* there are 4.5 sounds per measure (δ) and each cell amounts to 6.5 measures and thus there will be 29 glissando sounds per cell. The speed of the glissandi (v) is expressed in semitones per measure at 26 MM. The last value that we need in order to calculate the speeds of glissandi in cell III, *iz* is 'α'. 'α' is mentioned by Xenakis as the quadratic mean of the speeds and it is mentioned further in Formalized Music (1992: 15) that it is proportional to the standard deviation ($\alpha = \sqrt{2} \times s$). For this cell Xenakis gives α= 3.88 but as Childs (2003: 67) mentions, Xenakis does not provide an explicit expression of how he calculated α.[42]

As an illustration of the calculations, consider the instance where $v = 6$. $\lambda = \frac{v}{\alpha}$ and thus $\lambda = 1.54639$. $z = \sqrt{2} \times \lambda$ and thus $z = 2.186928$. In order to evaluate $F(z)$, we use the table in Appendix 2 which yields 0.485626167. The function $\theta(z) = 2 \times F(z)$ and thus $\Theta(z) = 0.971252335$. We can then compute $P(\lambda)$ and multiplying it by the total number of glissando sounds per cell (29) which then gives the number of glissando sounds which lies in a specific interval. Table 14 shows the calculations for the other values of v.

Table 14: Glissandi speed for cell III, *iz*

v	$\lambda = \frac{v}{\alpha}$	z	$\Theta(\lambda)$	$P(\lambda) = \theta(\lambda_2) - \theta(\lambda_1)$	$29 \times P(\lambda)$
0	0	0	0	0.284506	8
1	0.257732	0.364488	0.284506275	0.249478	7
2	0.515464	0.728976	0.533983843	0.191826	6
3	0.773196	1.093464	0.725809837	0.129336	4
4	1.030928	1.457952	0.855146183	0.076466	2
5	1.28866	1.82244	0.931611889	0.03964	1
6	1.546392	2.186928	0.971252335	0.018019	1
7	1.804124	2.551416	0.989271339	0.007182	0
8	2.061856	2.915904	0.996453272		

Table 15: Xenakis' own calculation of glissandi speed

v	$\lambda = \frac{v}{\alpha}$	$\Theta(\lambda)$	$P(\lambda) = \theta(\lambda_2) - \theta(\lambda_1)$	$29 \times P(\lambda)$
0	0	0	0.2869	9
1	0.258	0.2869	0.2510	7
2	0.516	0.5379	0.1859	5
3	0.773	0.7238	0.1310	4
4	1.032	0.8548	0.0771	2

[42] The standard deviation for speeds equal to 0, 1, 2, 3, 4, 5, 6, 7 and 8 is 2.738613 which makes $\alpha = \sqrt{2} \times s = 3.87$.

5	1.228	0.9319	0.0397	1
6	1.545	0.9716	0.0179	1
7	1.805	0.9895	0.0071	0

Comparing Table 14 and Table 15, it can be observed that there are numerous small differences. If the final columns are compared, it can be observed that these differences only results in two instances where the final values differs by one.

3.3.5.3 Pitch Interval

From hypothesis 6, Xenakis shows that the linear distribution can be used to calculate the intervals between the pitches of the glissandi by the distribution formula given in equation 3.5:

In order to perform the calculations by hand, an approximation of this formula was used by Xenakis.[43]

$$\theta(j) = \frac{2}{m+1}(1 - \frac{j}{m}) \qquad (3.10)$$

where j = 0, 1, 2, …m.

From hypothesis 6 we also know that the range of the string instruments amounts to 80 semitones. Xenakis decides arbitrarily, to use increments of 4.5 semitones and therefore m= 80 ÷ 4.5 = 17.777…≈ 18. Since for this cell (III, *iz*) there are 4.5 sounds per measure (δ) and each cell amounts to 6.5 measures, there will be 29 glissando sounds per cell. These 29 sounds per cell result in a total of 28 intervals.

It is now possible to calculate, for example, the probability that the interval between two successive pitches falls into the range 0 to 4.5 semitones:

$$\theta(0) = \frac{2}{19}(1 - \frac{0}{18}) = 0.105263$$

Since there are 28 intervals in the cell, there will be 28 × 0.105263 = 2.947 ≈ 3 intervals in the range between 0 and 4.5 semitones. One should note here that in calculating the intervals, Xenakis made a mistake in that he implied that the 29 sounds per cell results in 29 intervals in the cell. He therefore calculated that for the range between 0 and 4.5 semitones there will be 29 × 0.105263 = 3.0526 ≈ 3 intervals.

[43] The mathematical reasoning and proof of this approximation will be provided in Appendix 1.

Table 16: Pitch interval for cell III, *iz*

j	Pitch Range	$\Theta(j)$	$28 \times \Theta(j)$	$29 \times \Theta(j)$	Xenakis' results
0	[0 , 4.5]	0.105263	$2.947 \approx 3$	$3.053 \approx 3$	3
1	[4.5 , 9]	0.099415	$2.784 \approx 3$	$2.883 \approx 3$	3
2	[9 , 13.5]	0.093567	$2.62 \approx 3$	$2.713 \approx 3$	3
3	[13.5 , 18]	0.087719	$2.456 \approx 2$	$2.544 \approx 3$	3
4	[18 , 22.5]	0.081871	$2.292 \approx 2$	$2.374 \approx 2$	2
5	[22.5 , 27]	0.076023	$2.129 \approx 2$	$2.205 \approx 2$	2
6	[27 , 31.5]	0.070175	$1.965 \approx 2$	$2.035 \approx 2$	2
7	[31.5 , 36]	0.064327	$1.801 \approx 2$	$1.865 \approx 2$	2
8	[36 , 40.5]	0.05848	$1.637 \approx 2$	$1.696 \approx 2$	2
9	[40.5 , 45]	0.052632	$1.474 \approx 1$	$1.526 \approx 2$	2
10	[45 , 49.5]	0.046784	$1.31 \approx 1$	$1.357 \approx 1$	1
11	[49.5 , 54]	0.040936	$1.146 \approx 1$	$1.187 \approx 1$	1
12	[54 , 58.5]	0.035088	$0.982 \approx 1$	$1.018 \approx 1$	1
13	[58.5 , 63]	0.02924	$0.819 \approx 1$	$0.848 \approx 1$	1
14	[63 , 67.5]	0.023392	$0.655 \approx 1$	$0.678 \approx 1$	1
15	[67.5 , 72]	0.017544	$0.491 \approx 0$	$0.509 \approx 1$	0
16	[72 , 76.5]	0.011696	$0.327 \approx 0$	$0.339 \approx 0$	0
17	[76.5 , 81]	0.005848	$0.164 \approx 0$	$0.17 \approx 0$	0
18	[81 , 85.5]	0	0	0	0
Totals		1	28	30	29

Table 16 provides the calculations of the intervals between pitches. Comparisons between the calculations and Xenakis' results show similarity to a great extent. There are,however, once again some differences.

3.3.6 Comparison with the score of Achorripsis

In this section, the theoretical calculations of the duration, interval between pitches and glissandi speed will be compared with the score of Achorripsis corresponding to cell (III, *iz*). This cell is located in the seventeenth column of the matrix of Achorripsis (figure 12, p.33) and since each column corresponds to 6.5 measures in the score, this cell represents bar 105 until the half of bar 111. The cell is also located on the third row of the matrix of Achorripsis which corresponds to string glissandi. As can be seen from the score, the string glissandi for these bars is applicable to violin 3, violoncello 2 and double bass 2.

Figure 13: Bars 104 – 107 of *Achorripsis*[44]

[44] These excerpts of *Achorripsis* are borrowed from Childs (2003: 70-71) and are also given in Formalized music (1992: 26-27).

Figure 14: Bars 108 - 111 of *Achorripsis*

3.3.6.1 Comparison between calculated *durations* and score of *Achorripsis*

The first step in comparing the durations of the glissandi, is to calculate the values of the different notes that appear in bar 105 to 111 of the score. As Childs (2003: 69) points out, all the notes can be expressed as combinations of either half, quarter, triplet quarter, eight or quintuplet eight note. As mentioned in section 3.3.1, the tempo of Achorripsis is chosen at 26 MM which means that each bar will last $\frac{60}{26}$ seconds. The time signature is also given as $\frac{2}{2}$. The note values of the different notes are therefore as follows:

Table 17: Note values in *Achorripsis*

Half Note	$\frac{60}{26}\times\frac{1}{2}=1.1538$ seconds
Quarter Note	$\frac{60}{26}\times\frac{1}{4}=0.5769$ seconds
Triplet Quarter Note	$\frac{60}{26}\times\frac{1}{4}\times\frac{2}{3}=0.3846$ seconds
Eighth Note	$\frac{60}{26}\times\frac{1}{8}=0.2885$ seconds
Quintuplet Eight Note	$\frac{60}{26}\times\frac{1}{8}\times\frac{4}{5}=0.2308$ seconds

The glissandi sounds applicable to violin 3, violoncello 2 and Contrabass 2 during bars 105 to 111 can be observed as 30 individual glissando sounds as numbered in the score (figure 13 and 14). One should note that Xenakis' first deviation between his score and calculations occurs here because as shown in Section 3.3.5, there should only be 29 glissandi sounds rather than 30. Each of the 30 glissandi occurs after one another throughout the three voices and we are therefore able to calculate the durations between the start of each of these glissandi as shown in Table 18.

Table 18: Duration between glissandi events in score for cell III, *iz*

Glissando number	Bar number in score	Glissando begins (seconds)	Duration between Glissandi (seconds)
1	105	-0.86538	
2	105	-0.69231	0.173077
3	105	1.153846	1.846154
4	105	1.538462	0.384615
5	106	2.596154	1.057692
6	106	3	0.403846
7	106	3.173077	0.173077

8	106	3.75	0.576923
9	106	4.230769	0.480769
10	106	4.326923	0.096154
11	107	4.903846	0.576923
12	107	5.076923	0.173077
13	107	5.384615	0.307692
14	108	6.923077	1.538462
15	108	7.788462	0.865385
16	108	7.846154	0.057692
17	108	8.076923	0.230769
18	109	9.230769	1.153846
19	109	9.615385	0.384615
20	109	9.692308	0.076923
21	109	9.807692	0.115385
22	109	10.61538	0.807692
23	109	10.76923	0.153846
24	110	12	1.230769
25	110	12.11538	0.115385
26	110	12.69231	0.576923
27	110	13.07692	0.384615
28	110	13.15385	0.076923
29	111	13.84615	0.692308
30	111	14.71154	0.865385

In Table 18, the column *Glissando begins* refers to the length of time, as measured from the beginning of bar 105, that has passed before a new glissandi started. As seen in figure 13, the first two glissandi sounds begin before bar 105 and therefore their times are negative. The duration between glissandi is calculated by subtracting the beginning times of two consecutive glissandi.

The durations between the glissandi as calculated in Table 18 can now be placed into the time intervals as given in Table 13 and be compared with Xenakis' calculations as presented in Section 3.3.5.1, Table 13.

Table 19: Duration frequency between glissandi events in score and Xenakis' calculations, cell III, *iz*

Interval in seconds	Score	Xenakis' calculations
[0 , 0.2308)	10	10
[0.2308 , 0.4615)	6	7
[0.4615 , 0.6923)	4	4
[0.6923 , 0.9231)	4	3
[0.9231 , 1.1538)	2	2
[1.1538 , 1.3846)	1	1
[1.3846 , 1.6154)	1	1
[1.6154 , 1.8462)	0	0

[1.8462 , 2.0769)	1	0
[2.0769 , 2.3077)	0	0
> 2.3077	0	0
Total	29	28

As can be observed in Table 19, Xenakis stayed close to his calculations in the final score of *Achorripsis*. Only in three intervals does he differ from his calculations by a value of one.

3.3.6.2 Comparison between calculated *pitch intervals* with score of *Achorripsis*

This comparison of the pitch intervals between the glissandi will be done in a similar way as done by Childs (2003: 72). If we take middle C as the basis, 0, then we can express all the other pitches in the number of semitones higher or lower than middle C. For example a number of 10 will refer to a pitch 10 semitones higher than middle C, therefore Bb.

Table 20: Pitch interval between glissandi events in score for cell III, *iz*

Glissando number	Bar number in score	Pitch	Pitch interval between glissandi
1	105	-29	
2	105	9	38
3	105	10	1
4	105	16	6
5	106	29	13
6	106	14	-15
7	106	44	30
8	106	9	-35
9	106	-24	-33
10	106	37	61
11	107	43	6
12	107	-3	-46
13	107	-32	-29
14	108	33	65
15	108	48	15
16	108	-3	-51
17	108	-26	-23
18	109	27	53
19	109	21	-6
20	109	-23	-44
21	109	-24	-1
22	109	-22	2
23	109	-31	-9
24	110	-5	26

25	110	31	36
26	110	-10	-41
27	110	1	11
28	110	22	21
29	111	40	18
30	111	48	8

Table 21: Pitch frequency between glissandi events in score and Xenakis' calculations for cell III, *iz*

Interval in semtones	Score	Xenakis' calculations
[0 , 4.5)	3	3
[4.5 , 9)	4	3
[9 , 13.5)	3	3
[13.5 , 18)	2	3
[18 , 22.5)	2	2
[22.5 , 27)	2	2
[27 , 31.5)	2	2
[31.5 , 36)	2	2
[36 , 40.5)	2	2
[40.5 , 45)	2	2
[45 , 49.5)	1	1
[49.5 , 54)	2	1
[54 , 58.5)	0	1
[58.5 , 63)	1	1
[63 , 67.5)	1	1
[67.5 , 72)	0	0
[72 , 76.5)	0	0
[76.5 , 81)	0	0
Totals	29	29

From Table 21, it can be observed that Xenakis, to a certain extent, implemented his calculations rigorously. For four of the intervals, however, there is a difference of a value of one.

3.3.6.3 Comparison between calculated *glissando speeds* and score of *Achorripsis*

As stated earlier, Xenakis defined the speed of glissandi (expressed in semitones per bar) as the change in pitch (df) divided by change in time (dt). In order to determine the change in pitch (df), the pitch at which the glissando starts and ends is needed. This is again done by taking middle C as the basis, 0, and then expressing all the other pitches in the number of semitones higher or lower than middle C. The change in time (dt) is the length of the glissandi expressed as the percentage of a bar. For instance glissandi no 3 is equal to the length of two quarter notes and an eight note and will therefore make up 0.625 of a bar.

As an example the speed of glissando no. 3 is therefore computed as follows:

$$v = \frac{df}{dt} = \frac{12-10}{0.625} = 3.2 \text{ semi tones per bar.}$$

Table 22 shows the speed of the glissandi for cell III, iz.

Table 22: Speed of glissandi in the score of Achorripsis for cell III, *iz*

Glissando number	Bar number in score	Glissando pitch begins	Glissando pitch ends	Duration of bar	Speed of glissandi
1	105	-29	-7	2.208333	9.962264
2	105	9	9	0.8	0
3	105	10	12	0.625	3.2
4	105	16	17	0.633333	1.578947
5	106	29	31	0.25	8
6	106	14	15	0.9	1.111111
7	106	44	43	0.25	4
8	106	9	9	0.25	0
9	106	-24	-23	0.5	2
10	106	37	37	0.25	0
11	107	43	39	0.875	4.571429
12	107	-3	0	1.2	2.5
13	107	-32	-31	1.166667	0.857143
14	108	33	33	0.375	0
15	108	48	46	0.625	3.2
16	108	-3	-4	0.85	1.176471
17	108	-26	-21	0.7	7.142857
18	109	27	27	0.166667	0
19	109	21	25	1.083333	3.692308
20	109	-23	-23	0.466667	0
21	109	-24	-24	0.35	0
22	109	-22	-24	0.6	3.333333
23	109	-31	-29	2.333333	0.857143
24	110	-5	-3	0.3	6.666667
25	110	31	32	0.416667	2.4
26	110	-10	-10	0.2	0
27	110	1	1	0.333333	0
28	110	22	18	1.3	3.076923
29	111	40	41	0.375	2.666667
30	111	48	48	0.625	0

The number of glissandi speeds that fall into specific intervals is given in Table 23 and these frequencies can then be compared with the calculated frequencies by Xenakis.

Table 23: Comparison between glissandi speed in the score and Xenakis' calculations for cell III, *iz*

Interval in seconds	Score	Xenakis' calculations
[0 , 1)	12	9
[1 , 2)	3	7
[2 , 3)	4	5
[3 , 4)	5	4
[4 , 5)	2	2
[5 , 6)	0	1
[6 , 7)	1	1
[7 , 8)	1	0
[8 , 9)	1	0
[9 , 10)	1	0
Total	30	29

As can be observed, there exist numerous deviations between Xenakis' calculations and the final score.

3.3.7 Comparison of Cell I, iz, of the score of *Achorripsis*

In this section, the theoretical calculations of the duration and interval between pitches will be compared with the score of *Achorripsis* corresponding to cell (I, *iz*). This cell is located in the seventeenth column of the matrix of *Achorripsis* (figure 12, p.33) and since each column corresponds to 6.5 measures in the score, this cell represents bar 105 up to the half of bar 111. The cell is also located on the first row of the matrix of *Achorripsis* which corresponds to the timbre 'flute'. In the score this corresponds to the Piccolo, Eb Clarinet and Bass Clarinet in B.

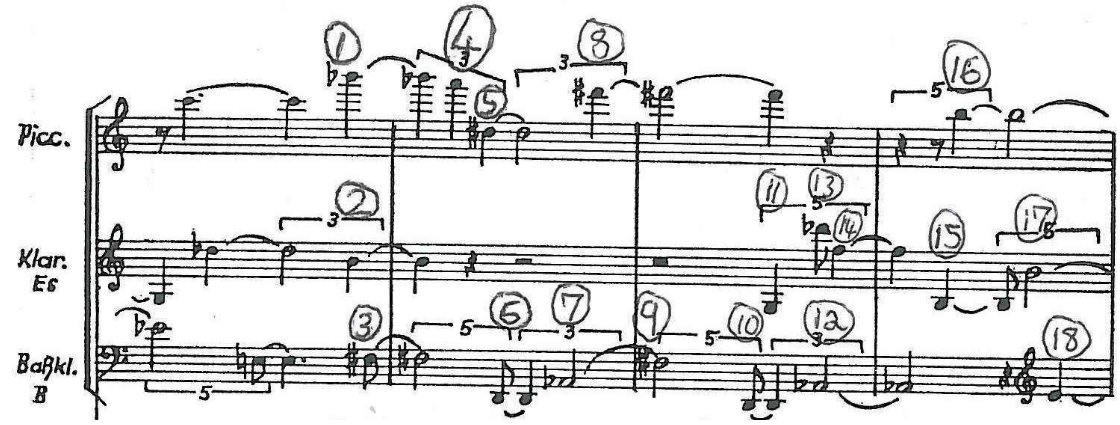


Figure 15: Bar 104 to 107 of *Achorripsis* corresponding to Cell I, *iz*

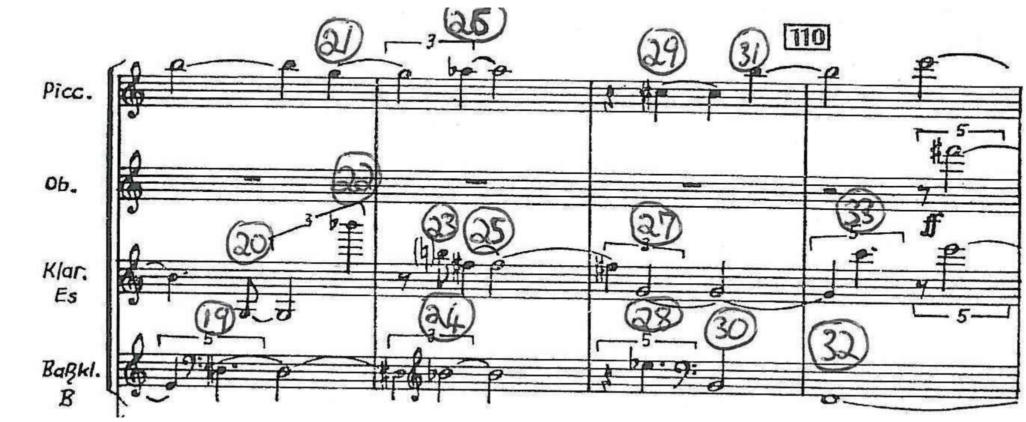


Figure 26: Bar 104 to 107 of *Achorripsis* corresponding to Cell I, *iz*

3.3.7.1 Duration

Computing the duration for Cell I, iz, is done in exactly the same way as for Cell III, iz in section 3.3.5. Xenakis did not provide information about his calculations for this cell and we therefore assume that he once again considered intervals equal to a length of 10% of each measure and thus x=0.10 which amounts to 0.23076923 seconds.

In Cell I, *iz* there are 5 sounds per measure (δ) at a tempo of MM 26. Each cell amounts to 6.5 measures and thus for Cell I, *iz,* there will be 5×6.5= 33 sounds per cell. These 33 sounds will have 32 durations between them. We assume again that dx is approximated by:

$$dx = \frac{1 - e^{-\delta x}}{\delta}$$

and thus

$$dx \approx 0.07869$$

Table 24: Calculation of duration for cell I, *iz*

x	Interval In seconds	δx	$e^{-\delta x}$	$\delta e^{-\delta x}dx$	$32 \times P(x)$
0	[0 , 0.2308)	0	1	0.393469	12.59≈13
0.1	[0.2308 , 0.4615)	0.5	0.606531	0.238651	7.63≈8
0.2	[0.4615 , 0.6923)	1	0.367879	0.144749	4.63≈5
0.3	[0.6923 , 0.9231)	1.5	0.22313	0.087795	2.81≈3
0.4	[0.9231 , 1.1538)	2	0.135335	0.05325	1.7≈2
0.5	[1.1538 , 1.3846)	2.5	0.082085	0.032298	1.03≈1
0.6	[1.3846 , 1.6154)	3	0.049787	0.01959	0.63≈1
0.7	[1.6154 , 1.8462)	3.5	0.030197	0.011882	0.38≈0
0.8	[1.8462 , 2.0769)	4	0.018316	0.007207	0.23≈0
0.9	[2.0769 , 2.3077)	4.5	0.011109	0.004371	0.14≈0
1	>2.3077	5	0.006738	0.002651	0.08≈0
			Totals	0.995913	33[45]

The sounds applicable to Piccolo, Eb Clarinet and Bass Clarinet in B during bars 105 to 111 can be observed as 33 individual sounds as numbered in the score (figure 15 and 16). Each of the 33 sounds occurs one after another throughout the three voices and we are therefore able to calculate the durations between the start of each of these sounds as shown in Table 25.

Table 25: Duration between sounds in score for cell I, *iz*

Sound number	Bar number in score	Sound begins (seconds)	Duration between Sounds (seconds)
1	105	-0.57692	
2	105	-0.38462	0.192308
3	105	-0.28846	0.096154
4	105	0.384615	0.673077
5	105	0.769231	0.384615
6	105	0.923077	0.153846
7	105	1.538462	0.615385
8	105	1.923077	0.384615
9	106	2.307692	0.384615
10	106	3.230769	0.923077
11	106	3.461538	0.230769

[45] We expect this value to equal 32 rather than 33. The reason for the difference is due to the fact that we rounded up.

12	106	3.846154	0.384615
13	106	3.923077	0.076923
14	106	4.153846	0.230769
15	107	5.192308	1.038462
16	107	5.307692	0.115385
17	107	6	0.692308
18	107	6.346154	0.346154
19	108	7.384615	1.038462
20	108	7.788462	0.403846
21	108	8.653846	0.865385
22	108	8.846154	0.192308
23	109	9.519231	0.673077
24	109	9.615385	0.096154
25	109	9.807692	0.192308
26	109	10	0.192308
27	110	11.92308	1.923077
28	110	12	0.076923
29	110	12.11538	0.115385
30	110	12.69231	0.576923
31	110	13.26923	0.576923
32	111	13.84615	0.576923
33	111	14.30769	0.461538

Now that the durations between the start of the 33 different sounds have been determined from the score, it can be divided into intervals and compared with the calculations as given in Table 26.

Table 26: Comparison between the duration in the score and their calculations for cell I, *iz*

Interval in seconds	Score	Calculations
[0 , 0.2308)	11	13
[0.2308 , 0.4615)	8	8
[0.4615 , 0.6923)	7	5
[0.6923 , 0.9231)	2	3
[0.9231 , 1.1538)	3	2
[1.1538 , 1.3846)	0	1
[1.3846 , 1.6154)	0	1
[1.6154 , 1.8462)	0	0
[1.8462 , 2.0769)	1	0
[2.0769 , 2.3077)	0	0
> 2.3077	0	0
Total	32	33

As seen in Table 26, there are some similarities as well as differences concerning the calculated duration between events and the duration between events as observed in the score. This will be discussed in more detail in chapter 4, section 4.5.

3.3.7.2 Pitch

Computing the intervals between pitches for Cell I, iz, is done in exactly the same way as for Cell III, iz in section 3.3.5.

As previously mentioned, Xenakis did not provide information regarding Cell I, iz and it must therefore be assumed that he again used the approximation to the linear distribution as given in equation 3.10.

$$\theta(j) = \frac{2}{m+1}(1 - \frac{j}{m})$$

where j = 0, 1, 2, …m.

We also have to assume that the pitch range for the instrument Piccolo, Clarinet and Bass Clarinet was considered by Xenakis to amount to 80 semitones. If Xenakis decided to use increments of 4.5 semitones again, it would mean that m= 80 ÷ 4.5 = 17.777…≈ 18. Since for this cell (I, *iz*) there are 5 sounds per measure (δ) and each cell amounts to 6.5 measures, there will be 33 sounds per cell. These 33 sounds per cell result in a total of 32 intervals.

It is now possible to calculate, for example, the probability that the interval between two successive pitches falls into the range 4.5 to 9 semitones:

$$\theta(0) = \frac{2}{19}(1 - \frac{1}{18}) = 0.099415$$

Since there are 32 intervals in the cell, there will be 32 × 0.099415 = 3.181 ≈ 3 intervals in the range between 4.5 and 9 semitones. Table 27 shows the calculation for the other intervals.

Table 27: Pitch intervals for cell I, *iz*

j	Pitch Range	*Θ(j)*	32 × *Θ(j)*
0	[0 , 4.5)	0.105263	3.368 ≈ 3
1	[4.5 , 9)	0.099415	3.181 ≈ 3
2	[9 , 13.5)	0.093567	2.994≈ 3
3	[13.5 , 18)	0.087719	2.807 ≈ 3
4	[18 , 22.5)	0.081871	2.62 ≈ 3

5	[22.5 , 27)	0.076023	2.433 ≈ 2
6	[27 , 31.5)	0.070175	2.246 ≈ 2
7	[31.5 , 36)	0.064327	2.058 ≈ 2
8	[36 , 40.5)	0.05848	1.871 ≈ 2
9	[40.5 , 45)	0.052632	1.684 ≈ 2
10	[45 , 49.5)	0.046784	1.497≈ 1
11	[49.5 , 54)	0.040936	1.31 ≈ 1
12	[54 , 58.5)	0.035088	1.122 ≈ 1
13	[58.5 , 63)	0.02924	0.946 ≈ 1
14	[63 , 67.5)	0.023392	0.749 ≈ 1
15	[67.5 , 72)	0.017544	0.561 ≈ 1
16	[72 , 76.5)	0.011696	0.374 ≈ 0
17	[76.5 , 81)	0.005848	0.187 ≈ 0
18	[81 , 85.5)	0	0
Totals		1	31[46]

Turning to the score, the pitches of the 33 sounds are again determined by taking middle C as the basis for the pitch (0) and then expressing all the other pitches in the number of semitones higher or lower than middle C. It must be noted that although we are working with Eb Clarinet and Bass Clarinet in B, Xenakis states that they should sound at the pitch at which they are written in the score. Table 28 gives the pitch intervals between the 33 sound events.

Table 28: Pitch interval between sound events in score for cell I, *iz*

Sound number	**Bar number in score**	**Pitch**	**Pitch interval between sounds**
1	105	46	
2	105	11	35
3	105	-6	17
4	105	45	51
5	105	27	18
6	105	-22	49
7	105	-16	6
8	105	42	58
9	106	-6	48
10	106	-22	16
11	106	-5	17
12	106	-16	11
13	106	25	41
14	106	17	8
15	107	-2	19

[46] We expect this value to equal 32 rather than 31. The reason for the difference is due to the fact that we rounded up.

16	107	35	37
17	107	11	24
18	107	4	7
19	108	-21	25
20	108	-5	16
21	108	31	36
22	108	34	3
23	109	23	11
24	109	10	13
25	109	18	8
26	109	32	14
27	110	7	25
28	110	15	8
29	110	25	10
30	110	-13	38
31	110	35	48
32	111	-19	54
33	111	24	43

Now that intervals between pitches of the 33 different sounds have been determined from the score, they can be divided into intervals and compared with the calculations as given in Table 29.

Table 29: Pitch frequency between sound events in score and calculations for cell I, *iz*

Interval in semtones	Score	Calculations
[0 , 4.5)	1	3
[4.5 , 9)	5	3
[9 , 13.5)	4	3
[13.5 , 18)	5	3
[18 , 22.5)	2	3
[22.5 , 27)	3	2
[27 , 31.5)	0	2
[31.5 , 36)	1	2
[36 , 40.5)	3	2
[40.5 , 45)	2	2
[45 , 49.5)	3	1
[49.5 , 54)	1	1
[54 , 58.5)	2	1
[58.5 , 63)	0	1
[63 , 67.5)	0	1
[67.5 , 72)	0	1
[72 , 76.5)	0	0
[76.5 , 81)	0	0
Totals	32	31

As can be seen in Table 29, there exist quite a number of instances where the actual values in the score differ from their theoretical calculation. The reason for these differences will be discussed in Chapter 4, section 4.5.

3.4 Chapter conclusion

This chapter provided an in-depth examination of mathematical aspects of *Achorripsis*. The analysis was structured according to the fundamental phases of constructing a composition as set forth by Xenakis. For phase 1 and 2, the initial conception of the work was discussed by looking at the instrumentation and length of the work. Phase 3 was subdivided into two parts. During the first part the number of 'musical events' that take place during the work was determined and in the second part it was considered how these 'musical events' should be distributed (on the time-axis and among the instruments) throughout the composition. During phase 4, the micro-compositional level of the work was considered where the duration between glissandi sounds, the speed of glissandi and the intervals between the pitches of the glissandi was analysed. This was firstly done for cell III, *iz* of the composition and the same approach was then performed on Cell I, iz. During the whole chapter, the researcher's own calculations were compared to those of Xenakis as well as Childs and where differences occurred, it was indicated and elaborated upon. For phase 4, the implementation of Xenakis' calculations into the final score of *Achorripsis* was furthermore considered and it was shown where and how he deviated from his calculations.

Chapter 4

The relationship between calculation and score

In Chapter 3, an analysis of the mathematical structure of the composition *Achorripsis* was given. The calculations were compared with Xenakis' own published calculations and deviations between the two were highlighted.

This chapter will focus on the application of his calculations in order to determine whether deviations occurred, and if it is the case, reasons for them will be discussed. Where the application has already been discussed in Chapter 3, their impact will be investigated in this chapter.

4.1 Phase 1 and 2: Initial conceptions

In these two phases, Xenakis determined the overall structure of the composition in terms of the instruments used as well as the length of the work.

When the score is examined, it is observed that Xenakis employed the 21 instruments as stated and that the score consists of 182 bars which results in the work's total length being 7 minutes. There are, however, some additional effects, as mentioned by Childs (2003: 53), which weren't stipulated by Xenakis in his planning:

1. The string *arco* passages all consist of short notes played either all down-bow, all up-bow or as harmonics.
2. The brass instruments all apply mutes from the 14th to the 21st time block.
3. The final string *glissando* passages are *sul ponticello* and *tremolo*.

In addition to this, the score also requires that the clarinets should sound at the pitch written, the piccolo one octave higher, the contrabassoon and bass one octave lower and that the 'c' played by the xylophone should sound two octaves higher. He also indicates in the score that all instruments should play without vibrato.

4.2 Phase 3: Definition of the sonic entities transformations Part 1

In this phase, Xenakis determined the number of each of the “musical events” that should be allocated across the 196 cells of the matrix (figure 12, p.33). As mentioned in Chapter 3, section 3.3.2.2, Xenakis’ published calculations corresponding to this phase of the composition were implemented exactly in his matrix. This is observed by counting the number of cells in which *zero, single, double, triple and quadruple* events occur. By doing this one can see that they indeed equal 107, 65, 19, 4 and 1 respectively.

Regarding his final score of *Achorripsis,* it becomes a bit more complicated to determine how accurately he employed these calculations. The reason for this is that Xenakis does not always rigidly begin and end the cells, corresponding to a total of 6.5 bars. In some instances the sound entries overlap between cells as can be observed for instance in bar 7, Figure 17. Here, according to the matrix, the first Flute (Piccolo, Clarinet and Bass Clarinet) entry should occur exactly in the middle of the bar, in other words, after 1.1538 seconds or a half note rest. This is not the case, as can be seen in the Piccolo and Clarinet entries which both come in earlier. Childs (2003: 58-59) argues that this makes musical sense because composers “often elide phrases or provide transitions from one section to the next, and overlap entrances of a new instrumental group with the fade-out of a previous group”.

Setting aside these artistic adjustments, it can be observed that Xenakis implemented his calculations accurately, in terms of the number of each of the events that should occur during the composition. Nonetheless, there are again some changes made by him. For instance, look at time blocks η (which is the 8^{th} time block) and Κδ (the 24^{th} time block) in figure 12 (p.33). According to the planning of the composition through the vector matrix, during the 6.5 measures there should be *zero* events for all instruments. In other words there should be no notes played for the 6.5 bars. This is not the case as there is some activity, albeit only a few notes, in both instances.

Another major alteration is made by Xenakis for the time block Κη or 28^{th} time block. According to his planning there should be no activity in the form of String arco, String glissandi, Brass and Percussion. The String glissandi, however, is used during these 6.5 bars in the form of *tremolo* glissandi played *sul ponticello.* This deviation from the planning was probably made, as Childs argues, in order to provide an effective ending for the composition.

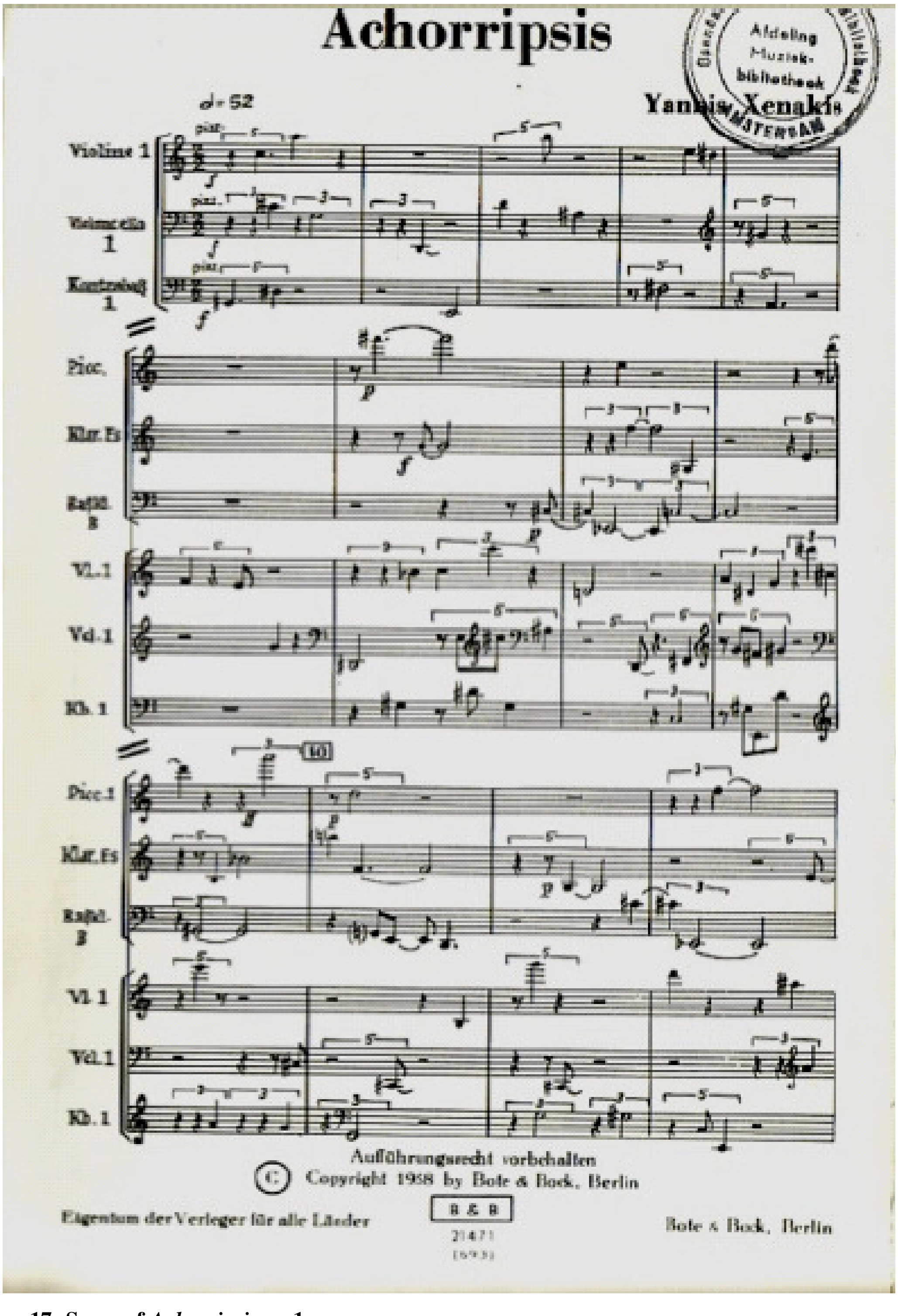


Figure 17: Score of *Achorripsis*, p. 1

4.3 Phase 3: Definition of the sonic entities transformations Part 2

During this phase, it was determined how the number of *zero, single, double, triple and quadruple* events calculated during the previous phase should be distributed across the time-axis of the composition and amongst the timbres (instruments), in other words, how they should be distributed across the columns and rows of the matrix.

In order to determine whether Xenakis stayed true to his calculations, we compare the Vector Matrix of the composition (figure 12, p.33) to our calculations. Firstly consider the columns of the Vector Matrix. To facilitate the comparison, the number of columns of the matrix in which there are 0, 1, 2, 3, 4, 5, 6 and 7 *single* events is counted and compared with the calculation above. The same is done for *zero, double, triple* and *quadruple* events.

In doing this, one can observe that for *single, double, triple and quadruple* events the matrix (figure 12, p.33) corresponds exactly to the calculations published by Xenakis and Childs. It also corresponds to the researcher's calculations, except for the differences mentioned in section 3.3.3.2

For the distribution of *zero* events, however, there are major differences between the calculations and how Xenakis distributed these events as can be seen in Table 30.

Table 30: Comparison with columns

Frequency of Zero events	No. of columns in Matrix	No. of columns calculated
0	0	1
1	2	2
2	6	4
3	5	6
4	5	5
5	4	4
6	4	3
7	2	1

Now consider the rows of the vector matrix. As can be seen in Chapter 3, Table 6 to 10, for *quadruple, triple* and *double* events, Xenakis implemented the predicted values exactly. For *single* and *zero* events, he however deviated to some extent. The reason why certain deviations were made by him is explained as follows: As mentioned in the previous chapters, an important concept in probability theory is the law of large numbers. As the number of rows and columns increases, the distribution of events will become more rigorous. Xenakis also notes that rows of the matrix (figure 12, p.33) which symbolize timbres (instruments) are interchangeable. The columns of the matrix are also interchangeable. Therefore the matrix is only weakly deterministic and therefore, as Xenakis states in Formalised music (1992: 32) "it serves chiefly as basis for thought – for thought which manipulates frequencies of events of all kinds". The fact that there is some indeterminism, gave Xenakis as a composer some subjective or artistic freedom.

Now that the relationship between the calculations and the matrix has been compared, the score of *Achorripsis* can be viewed against the matrix.

Firstly, the columns of the matrix are compared with the score. It can be seen that Xenakis assimilated the columns of the matrix accurately into his score. For example, this can be observed by considering the opening 13 bars of the score which are the columns α and β in the matrix. For column α (bar 1 - 7½), there should be six rows (instrument classes) in which *zero* events occur and there should be one row (Pizzicato) which contains a *single* event. As can be seen in the score, figure 17, this is indeed the case. For column β (bar 7½ - 13) there should be five rows (instrument classes) namely the Brass, String arco, Percussion, String glissandi and Oboe in which *zero* events occur. There should also be two rows (instrument classes) namely Flute and Pizzicato, in which *single* events occur. This was exactly done by Xenakis as seen in figure 17.

When one now looks at the rows of the matrix, it is observed that Xenakis implemented the columns of the matrix accurately into the score as well. For example, consider row six which corresponds to Brass. In the matrix the first six columns (39 bars) are made up of *zero* events and the 7^{th} column contains a *double* event. In the score this is also the case as for the first 39 bars there are only rests for the Brass with the first entry being Trumpet 1 in bar 40 lasting for more or less 6.5 bars. The next *non-zero* events according to his matrix should be a *single* event starting in bar 53, a *double* event starting in bar 66, a *single* event starting in bar 85½, three consecutive *single* events starting in bar 111½, a *double* event starting in bar 157 and a final *single* event starting in bar 170. When this is compared to the score it is seen that this is almost exactly what Xenakis did. The only difference is that the entry of the *single* event in bar 53 comes one bar earlier.

4.4 Phase 3: Definition of Musical Events

During this phase Xenakis described and specified each of the *events*, which were predicted as well as how they should be distributed in the previous phases.

Consider cell III, *iz*: During this cell a single event occurs. Therefore, as specified by Xenakis and mentioned in Chapter 3, section 3.3.4, there should be five sounds per measure. More specifically, this means that there should be five distinct sound entries during each bar. However for single events Xenakis varied the number of sounds per measure between 2.5 and 6.5 while their mean still equalled 5. For cell III, *iz,* the average number of sounds per bar was chosen as 4.5.

Table 31: Sounds per bar in Cell III, *iz*

Bar number	Number of sounds per bar in score
105	4
106	6
107	3
108	4
109	6
110	5

111½	2
Total	30
Average	4.615385

During Cell I, *iz* there also occurs a single event. However, the average number of sounds per bar was chosen to be equal to 5.

Table 32: Sound per bar in Cell I, *iz*

Bar number	Number of sounds per bar in score
105	8
106	6
107	4
108	4
109	4
110	5
111½	2
Total	33
Average	5.076923

As can be observed from Table 31 and Table 32, the average number of sounds per bar for the two cells in the score of Achorripsis is very close to those specified by Xenakis. The reason for the small differences is due to the fact that these two cells, corresponding to a total of 6.5 bars, do not start and end rigidly. This is seen through the fact that in certain instances the sound entries overlap between cells.

4.5 Phase 4: Micro-composition

This phase has already been analysed and compared with the score in Chapter 3. It was found that certain adjustments or alterations were made by Xenakis between his calculations, using the stochastic distributions, and the final score. In this section the significance of these changes will be discussed. This is done with the help of the Chi-Square Test[47].

The chi-square statistic (χ^2) calculates the squared difference between the observed and the expected values relative to the expected value for each category. It is computed as follows:

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected} \tag{4.1}$$

[47] The information is this section is based on Hinders (2004).

The chi-square distribution is furthermore based on the number of degrees of freedom which is determined by the number of categories minus 1 ($df = n - 1$). The critical values for the χ^2-distribution are given in appendix 3. In order to use the test, at least 80% of the counts of the expected frequencies has to be larger than 5.

4.5.1 Cell III, *iz*

4.5.1.1 Pitch intervals

As stated above, in order to use the test, at least 80% of the counts of the expected frequencies has to be larger than 5. The intervals can therefore be regrouped as follows:

Table 33: Regrouping of pitch intervals for cell III, iz

Interval in semtones	Calculations (Expected in %)	Score (Observed in %)
[0 , 4.5)	10.52632	10.34483
[4.5 , 9)	9.94152	13.7931
[9 , 13.5)	9.356725	10.34483
[13.5 , 18)	8.77193	6.896552
[18 , 22.5)	8.187135	6.896552
[22.5 , 27)	7.602339	6.896552
[27 , 31.5)	7.017544	6.896552
[31.5 , 36)	6.432749	6.896552
[36 , 40.5)	5.847953	6.896552
[40.5 , 45)	5.263158	6.896552
[45 , 49.5)	4.678363	3.448276
[49.5 , 54)	4.093567	6.896552
[54, 81)	12.2807	6.896552

H_0: The distribution of pitch is determined according to the Linear distribution
H_1: The distribution of pitch is *not* determined according to the Linear distribution
At a significance level of $\alpha = 0.01$.

T.S.: $\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected} = 7.603298$

D.R.: For a significance level 0.01, reject the null hypothesis if the computed test statistic value $\chi^2 = 7.603298 > \chi^2_{0.01,12}$

Conclusion: Since $\chi^2_{0.01,12} = 26.217$ and 7.603298< 26.217 we do not reject the null hypothesis. Therefore, we cannot reject the fact that the distribution of pitch is determined according to the linear distribution at a significance level of α = 0.01. In other words we can be 99% sure the null hypothesis should not be rejected.

The fact that the hypothesis test indicates that we should not reject the null hypothesis, shows that although Xenakis made changes between his calculations and his final score, these changes were not particularly significant. The overall implementation of his calculations (with regards to the pitch in cell III, *iz*) into his score still shows that they were determined by a linear distribution.

4.5.1.2 Duration

For the test, the intervals are regrouped as follows:

Table 34: Regrouping of duration intervals for cell III, iz

Interval in seconds	Score (Observed in %)	Calculations (expected in %)
[0 , 0.2308)	34.48276	36.2372
[0.2308 , 0.4615)	20.68966	23.1058
[0.4615 , 0.6923)	13.7931	14.7329
[0.6923 , 0.9231)	13.7931	9.3941
[0.9231 , 1.1538)	6.896552	5.99
[1.1538 , 1.3846)	3.448276	3.8194
>1.3846	6.896552	6.0121

H₀: The distribution of duration is determined according to the Exponential distribution
H₁: The distribution of pitch is *not* determined according to the Exponential distribution
At a significance level of α = 0.01.

T.S.: $\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected} = 2.760855$

D.R.: For a significance level 0.01, reject the null hypothesis if the computed test statistic value $\chi^2 = 2.760855 > \chi^2_{0.01,6}$

Conclusion: Since $\chi^2_{0.01,6} = 16.812$ and 2.760855 < 16.812 we do not reject the null hypothesis. Therefore, we cannot reject the fact that the distribution of duration is determined according to the exponential distribution at a significance level of α = 0.01.

The hypothesis test indicates that we should not reject the null hypothesis and thus, although Xenakis made changes between his calculations and his final score, these changes were not particularly significant.

4.5.1.3 Speed

For this test, the intervals are regrouped as follows:

Table 35: Regrouping of speed intervals for cell III, iz

Interval in seconds	Calculations (expected in %)	Score (observed in %)
[0 , 1)	28.69	40
[1 , 2)	25.1	10
[2 , 3)	18.59	13.33333
[3 , 4)	13.1	16.66667
[4 , 5)	7.71	6.666667
[5 , 6)	3.97	0
[6 , 10)	2.5	13.33333

H0: The distribution of speed is determined according to the Normal distribution
H1: The distribution of speed is *not* determined according to the Normal distribution
At a significance level of α = 0.01.

T.S.: $\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected} = 67.05575$

D.R.: For a significance level 0.01, reject the null hypothesis if the computed test statistic value $\chi^2 = 67.05575 > \chi^2_{0.01,6}$

Conclusion: Since $\chi^2_{0.01,6} = 16.812$ and 67.05575 > 16.812 we reject the null hypothesis. Therefore, we reject the fact that the distribution of speed is determined according to the normal distribution at a significance level of α = 0.01.

The hypothesis test indicates that we should reject the null hypothesis and therefore we can say that the changes Xenakis made were significant. We can furthermore argue that although as composer he had the artistic freedom to make adjustments and alterations between his calculations and score, when they are this significant, too many changes have been made. The reason for the argument is that Xenakis' objective was to govern the speed of glissandi by the normal distribution as shown in chapter 3, section 3.3.5.2. With the hypothesis test above, it is, however, shown that his final score does not show traces of the fact that the speed was governed by the normal distribution. Therefore, it is argued that too many changes were made by Xenakis to his calculations with respect to the speed of glissandi.

4.5.2 Cell I, *iz*

In our analysis of cell I, *iz*, in Chapter 3, two assumptions had to be made. For this cell Xenakis did not provide or publish information. It was, however, assumed that he determined the duration between events and the pitch intervals as he had done for cell III, *iz*. In order to determine whether our assumptions were right, two hypothesis tests with the use of a Chi-squared (χ^2) test are carried out.

4.5.2.1 Duration Cell I, *iz*

As mentioned before, in order to use the test, at least 80% of the counts of the expected frequencies has to be larger than 5. We therefore regroup the intervals as follows:

Table 36: Regrouping of duration intervals for Cell I, iz

Interval in seconds	Calculations (Expected %)	Score (Observed %)
[0 , 0.2308)	39.34693	34.375
[0.2308 , 0.4615)	23.86512	25
[0.4615 , 0.6923)	14.47493	21.875
[0.6923 , 0.9231)	8.779488	6.25
[0.9231 , 1.1538)	5.325028	9.375
[1.1538 , 2.3077)	7.799823	3.125

H_0: The distribution of duration is determined according to the Exponential distribution
H_1: The distribution of duration is *not* determined according to the Exponential distribution
At a significance level of $\alpha = 0.01$.

T.S.: $\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected} = 11.07641$

D.R.: For a significance level 0.01, reject the null hypothesis if the computed test statistic value $\chi^2 = 11.07641 > \chi^2_{0.01,5}$

Conclusion: Since $\chi^2_{0.01,5} = 15.086$ and 11.07641 < 15.086 we do not reject the null hypothesis. Therefore, we cannot reject that the distribution of duration is determined according to the exponential distribution at a significance level of α = 0.01. In other words we can be 99% sure the null hypothesis should not be rejected.

4.5.2.2 Pitch intervals Cell I,iz

In order to determine whether our assumption is true that Xenakis used the linear distribution to determine the distribution of pitch intervals for Cell I,iz , we again do a hypothesis test with the use of a Chi-squared (χ^2) test on the data from Cell I, iz. We again have to regroup the intervals.

Table 37: Regrouping of pitch intervals for cell I, iz

Interval in semtones	Calculations (Expected in %)	Score (Observed in %)
[0 , 4.5)	10.52632	3.125
[4.5 , 9)	9.94152	15.625
[9 , 13.5)	9.356725	12.5
[13.5 , 18)	8.77193	15.625
[18 , 22.5)	8.187135	6.25
[22.5 , 27)	7.602339	9.375
[27 , 31.5)	7.017544	0
[31.5 , 36)	6.432749	3.125
[36 , 40.5)	5.847953	9.375
[40.5 , 45)	5.263158	6.25
[45 , 49.5)	4.678363	9.375
[49.5 , 54)	4.093567	3.125
[54, 81)	12.2807	6.25

H₀: The distribution of pitch is determined according to the Linear distribution
H₁: The distribution of pitch is *not* determined according to the Linear distribution
At a significance level of $\alpha = 0.01$.

T.S.: $\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected} = 34.67108$

D.R.: For a significance level 0.01, reject the null hypothesis if the computed test statistic value $\chi^2 = 34.67108 > \chi^2_{0.01,12}$

Conclusion: Since $\chi^2_{0.01,12} = 26.217$ and $34.67108 > 26.217$ we reject the null hypothesis. Therefore, we reject that the distribution of pitch is determined according to the linear distribution at a significance level of $\alpha = 0.01$.

From the hypothesis testing done above, it seems that the first assumption, namely that the duration between events in Cell I, *iz* was determined by Xenakis with the exponential distribution, is correct. The second assumption, namely that Xenakis used the linear distribution to determine the distribution of pitch intervals for Cell I,*iz* was proven incorrect. This can be the result of two possibilities.

The first is that Xenakis determined the distribution of pitch intervals according to another probability distribution for this cell. The second possibility is that the changes made by him between his calculations and his final score, were so considerable that the hypothesis test shows that the pitch intervals were not determined by the linear distribution anymore.

4.6 Chapter conclusion

Chapter 4 considered the relationship between Xenakis' calculations and their implementation into *Achorripsis'* score and also investigated the impact of changes made while applying them to the final score. The chapter was, like Chapter 3, structured according to the fundamental phases of constructing a composition as set forth by Xenakis. Phase 1 to 3 considered the implementation of the calculations into the vector matrix of *Achorripsis* and then scrutinized the application of the calculations into the final score. For Phase 4, the implementation of the calculations into the final score was already discussed in Chapter 3 and in this chapter the rigour of their application was investigated. This was done in the form of hypothesis testing in order to determine whether the changes made by Xenakis were significant. In other words, it was investigated to what degree the boundaries of artistic freedom were pushed.

Chapter 5

Conclusions

5.1 Criticism

Although Xenakis' radical compositional theories have influenced many younger composers,[48] his method of *Stochastic* Music remains controversial. It is suggested by observers that his own writings in connection with his compositions are intentionally complex and obscure. In this regard, for instance, Pierre Schaeffer (1970) summarised the situation as follows:

> As far as Xenakis is concerned, let me emphasize at once that I'd be much more interested in his research if he hadn't set out so obviously to reduce its accessibility and its credibility in a manner which is immediately apparent as soon as you open his book on formal music.

Further criticism came from musicologists questioning whether algorithmically generated compositions have any real artistic importance, since there is potentially little or no human "creativity" involved in the creative process. This resulted in critics, such as Clarendon and Pincherle, labeling Xenakis' music as "anti-music…noisy…crazy".[49]

Even if interest in Xenakis' compositional theories resulted in a scholarly desire to investigate his approach in great detail, music analysts would be confronted with difficulties as stated by Harley in Formal analysis of the music of Iannis Xenakis by means of *sonic events* (2001):

> The theoretical formulations Xenakis put forward in his book Formalized Music are such that the reader could easily conclude that further work would require advanced training in mathematics, something most music students do not receive as part of their musical training. Furthermore, even if the formulas, graphs and matrices do not discourage the potential scholar, it soon becomes apparent that to trace the specific processes by which a musical result was achieved would necessitate having access to a great deal of data used by the composer. Stochastic processes, after all, by their very nature are indeterminate; a specific outcome can only be predicted within a limited degree of probability without having access to all of the numerical inputs and constraints on the mathematical functions.

[48] The best documented followers of Xenakis are Dusapin, Estrada, Maceda, Mâche, Pape and Takahashi as mentioned by Harley (2005, 331-337).

[49] This quote comes from an interview with Iannis Xenakis and published by Mario Bois (1997).

5.2 Rationale for the research

The criticism and controversy surrounding the work of Xenakis creates the impression that there exists a lack of understanding concerning his work and consequently that further research is desirable - and particularly research that would specifically cast light on the mathematical aspects of the composer's work because, as Xenakis stated in an interview with Bois (1997).

> I had written in the programme mathematical formulae. They took fright. Why be frightened? Mathematical formulae are not monsters; one can tame them much more easily than one thinks, provided that one doesn't in advance create a blockage in the mind.

This research project, therefore specifically aimed to provide insight into the mathematical aspects of Xenakis' work. The rationale was that in understanding the "mechanisms" driving his early compositions, the lack of comprehension surrounding them could be addressed. More specifically, this was done by considering the composition *Achorripsis*.

The analysis of *Achorripsis* was furthermore conducted with the objective of answering the following hypothesis:

The compositional method utilized by Iannis Xenakis is not merely an automated process and artistic renderings exist between the stated stochastic processes he employed and his final musical scores.

5.3 The analysis of *Achorripsis*

The analysis of *Achorripsis* was done according to the eight phases of constructing a musical work as stated by Xenakis.

1 and 2. Initial conceptions and definition of the sonic entities. In these two phases it was shown that Xenakis described the overall structure of the composition. The choice of timbres (instruments) for the work was highlighted as well as the choice for the length of the work. It was lastly shown how this formed the rows and columns of the vector matrix.

3. Definition of the transformation process. This phase was described as a three-part procedure. Firstly the employment of the Poisson distribution was discussed which predicted the total number of *zero, single, double, triple and quadruple* events that should occur during the composition. In the second section it was discussed how the Poisson distribution was used in predicting the number of *zero, single, double, triple* and *quadruple* events that should occur in each row and column. During the third part each of the events was defined by the number of sound entries per bar which they symbolised.

4. Micro-composition. For this phase, a detailed description of the use of stochastic procedures in producing sounds was provided. More specifically the method of determining the duration between consecutive notes, the speed of the glissandi and the interval between successive pitches was discussed. In doing so, the use of the Linear, Exponential and Normal or Gaussian distribution was discussed in detail.

5 and 6. Sequential programming and implementation of calculations. These two phases were analysed together with phase 4. After explaining and calculating the amount of time between consecutive notes, the speed of the glissandi and the interval between successive pitches was calculated, it was compared to Xenakis' own published calculations.

7. Final symbolic result. For this phase, sections of the score of *Achorripsis* were selected and put under scrutiny. The final score was compared with the theoretical calculations and planning.

8. Sonic realization. This final phase refers to realisation of the work by means of direct orchestral performance. Analysis of this phase fell outside the scope of this project.

5.4 The results from analysing *Achorripsis*

5.4.1 Comparison with Xenakis' calculations

As part of this project, the calculations done by Xenakis in constructing the composition were recreated. It was done exactly according to his theoretical ideas and specifications and then compared with his own published results in Formalized Music.

What was observed and discussed in detail in Chapter 3 was that for the greater part Xenakis performed his calculations clearly and precisely as described in his theoretical approach. There were, however, also situations in which differences between the calculations performed in this research project and Xenakis' published calculations were noted. For these differences, there were two possible explanations.

On the one hand it is possible that errors were made by him when performing the calculations. This appears to be the case only in a minority of differences identified. Once again, it is important to note that Xenakis performed his calculations without the assistance of a computer. On the other hand it is reasonable to assume that the discrepancies that are there between calculations and their realization in the score were created intentionally by the composer, often for practical reasons. As mentioned in Chapter 3, a problem that was often encountered was the fact that working with probability distributions results in obtaining continuous values whereas for the composition discreet values were required.

5.4.2 Comparison between Xenakis' calculations and the score of *Achorripsis*

After scrutinizing Xenakis' calculations, their realization into the score was examined. This was discussed in detail in Chapter 3 and Chapter 4. It was observed that Xenakis, for the most part, implemented his calculations rigorously. Where divergence between the calculations and score occurred, it could again be explained as due to either mistakes made by the composer or, more likely, that he consciously made changes between the two.

These deliberate changes often appear to have been done for artistic reasons. In Chapter 4, section 4.5, the objective was to determine the significance of the changes made in the name of artistic freedom. Hypothesis testing was used and it was found that for Cell III, *iz,* the alterations made with respect to the pitch and duration were not significant. The changes made to the speed of the glissandi were, however, significant. It was then argued that Xenakis, with regards to the speed of glissandi, pushed the limit of artistic freedom too far.

Hypothesis testing was also done for the pitch and duration of Cell I, *iz* in order to determine whether the assumptions made in Chapter 3, section 3.3.7, about the distributions that Xenakis used, were correct. It was found that the assumption about the duration between events was correct. However, the assumption concerning the distribution of pitch intervals was proven incorrect. The reason for this was either that Xenakis determined the distribution of pitch intervals according to another probability distribution for Cell I, *iz* or that the changes made by him were so considerable that the hypothesis test showed that the pitch intervals were not determined by the Linear distribution anymore. If the latter is correct, then it can again be argued that Xenakis took his artistic freedom too far.

5.5 Comparison with Childs' results

As part of this research project, the research of Childs (2002, 2003) was investigated. His analysis of *Achorripsis* was also conducted according to the eight phases of constructing a composition as set forth by Xenakis. The objectives with his research were to determine whether Xenakis really applied his formulations to generating specific notes and to assess whether or not the scientific concepts underlying *Achorripsis* transfer effectively to sound.

In his research, it was seen that Childs did not scrutinize Xenakis' own calculations but only replicated them in order to provide an understanding of these building blocks of the composition. When he, however, compared the score of *Achorripsis* with Xenakis' calculations, he placed emphasis on how accurately Xenakis employed his own calculations.

Childs' analysis was focused mainly on Cell III, *iz,* beacause Xenakis provided information about this cell and this cell was therefore also examined in this research project. Childs provided excellent analysis of this cell which was compared with the results obtained in this research project. For this cell, the results obtained in this project by comparing the score with Xenakis' calculations corresponded exactly with Childs' results as discussed in Chapter 3. The observable differences, mentioned in Chapter 3, between Childs' results and the results of this project occurred primarily with respect to the calculations done with the stochastic distributions.

This project expanded on Childs' research of *Achorripsis* in a number of ways. Firstly, in providing an analysis of the composition, the calculations provided were not just limited to those for which Xenakis published the results. Other vital instances for the construction of the composition were provided in Chapter 3. Although Xenakis did not publish them, they could be calculated with the assumption that he did them as theoretically stipulated by him. This project extended the analysis to other sections of the composition with specific attention paid to Cell I, *iz*. The analysis for this cell, provided in Chapter 3, was done with the assumption that the method in which Xenakis did calculations for Cell III, *iz,* also applied to Cell I, *iz*. These assumptions were then tested by means of hypothesis testing in Chapter 4.

5.6 Future research possibilities

Although in this project the analysis of *Achorripsis* described the majority of aspects in the construction of the composition, there were certain aspects that were not addressed. They were connected with the construction of the composition at the micro-level.

In the analysis of the composition at the micro-level, Chapter 3, section 3.3.5, it was shown that through the use of the Exponential, Linear and Normal or Gaussian distribution, the intervals between pitches, the time between successive notes as well as the speed of the glissandi could be determined. Aspects that were not examined were the following: the dynamics, the articulation, initial choice of instruments as well as the starting pitches for each of these instruments at the beginning of a cell. The reason for not elaborating on these aspects was due to the fact that Xenakis did not provide any information as to the theoretical reasoning behind them or as to how they were determined by him.

There is therefore the possibility for further research on these topics. In order to conduct the research, however, access has to be gained to Xenakis' own calculations on them or at least his theoretical ideas underpinning them.

5.7 Epilogue

It can be argued that Iannis Xenakis' outlook and approach as a composer developed out of Twentieth Century compositional perspectives. The dodecaphony of the second Viennese school and the Serialism that followed searched for new approaches and pre-compositional means. After openly rejecting Serialism, Xenakis began exploring and developing his own methods. He drew from his background and training as an engineer and architect and started using mathematical principles in his compositions. Mathematics was not only utilized by him as an organizing principle for the overall structure of a composition, but mathematical laws and equations, specifically those pertaining to probability theory, were used in determining the placement of actual phrases and notes. This method of composition would later become known as *Stochastic* music.

The method of *Stochastic* music, however, often met with criticism. A part of the criticism was the result of Xenakis arguably making his compositional methods inaccessible to a musical world which often did not have the necessary mathematical background. Even with mathematical knowledge, understanding his exact procedures is made difficult through his sometimes obscure use of mathematical language as encountered in his book Formalized Music.

The criticism leaves the impression that what is needed is a user-friendly discourse of his procedures as well as compositions. This research project, aimed to contribute in a small way towards this end. The analysis of *Achorripsis* in this project was, however, conducted not only with the objective of providing a better understanding of Xenakis' *stochastic* approach to composition but also with answering the hypothesis:

The compositional method utilized by Iannis Xenakis is not merely an automated process and artistic renderings exist between the stated stochastic processes he employed and his final musical scores.

The research in this project provided enough evidence to prove that the hypothesis is indeed true. It was furthermore observed that Xenakis' ingenuity in the application of his methods of composition was unique and insightful. Viewed in this light, Iannis Xenakis undoubtedly left a meritorious legacy. His originality and independence of mind made him one of the most profound and visionary composers of the Twentieth Century.

References

Arsenault, L. 2002. **Iannis Xenakis's Achorripsis: The Matrix Game**, in: **Computer Music Journal**, vol. 26, pp. 58-72.

Baltensperger, A. 1995. **Iannis Xenakis und die Stochastische Musik: Komposition im Spannungsfeld von Architektur und Mathematik.** Zürich. Paul Haupt.

Beer, M. 2005. **Mathematics and Music: Relating Science to Arts?** http://www.michael.beer.name/research/mathandmusic.html. Accessed 15 March, 2007.

Benson, D. 2006. **Music: A Mathematical Offering.** Cambridge: Cambridge University Press.

Berry, D & Lindgren, W. 1990. **Statistics: Theory and Methods.** California.

Bois, M. 1967. **Iannis Xenakis: The man and his music.** London: Boosey & Hawkes Music Publishers.

Capanna, A. 2001. **Iannis Xenakis: Architect of Light and Sound,** in: Nexus Network Journal, vol. 3, no.1, pp. 19-26.

Childs, E. 2002. **Achorripsis: A sonifications of probability distributions.** International Conference on Auditory display, Kyoto, Japan.

Childs, E. 2003. **Musical Sonification design.** Thesis. Darthmouth College.

Chung, I. 2003. **Mathematical and Architectural Concepts Manifested in Iannis Xenakis's Piano Music.** Ph.D. thesis. University of Texas at Austin.

Cohen, J. E. 1961. **Some Relationships between Music and Mathematics.** Music Educators Journal, vol. 48, no. 1, pp. 104-109.

Fauvel, J., Flood, R. & Wilson, R. 2003. **Music and Mathematics.** Oxford University Press.

Griffiths, P. **Serialism. Grove Music Online ed**. L. Macy. http://www.grovemusic.com. Accessed 15 April 2007.

Griffiths, P. **Aleatory. Grove Music Online ed**. L. Macy. http://www.grovemusic.com. Accessed 15 April 2007.

Harley, J. 2005. **The creative compositional legacy of Iannis Xenakis,** in: **A. Georgaki, M. Solomos (éd.),** ***International Symposium Iannis Xenakis. Conference Proceedings*****, Athens**, pp. 331-337.

Harley, J. 2001. **Formal Analysis of The Music of Iannis Xenakis by Means of Sonic Events: Recent Orchestral Works**, in: **Prèsences de Iannis Xenakis, (Solomos, M., ed.), 16 place del la Fontaine aux Lions, France: Centre dedocumentation de las musique contemporaine.** http://www.cdmc.asso.fr. Accessed 10 April 2008.

Hinders, D. 2004. **5 Steps to a 5 AP Statistics.** Mc Graw-Hill.

Hoffmann, P. **Iannis Xenakis. Grove Music Online ed.** L. Macy. http://www.grovemusic.com. Accessed 5 March 2007.

Iliescu, M. 2005. **Glissandi and traces: A study of the relationship between musical and Extra-Musical; Fields,** in: **A. Georgaki, M. Solomos (éd.),** ***International Symposium Iannis Xenakis. Conference Proceedings*****, Athens**, pp. 49-54.

Nagoski, I. **Do the Math: Iannis Xenakis. http://www.citypaper.com.** Accessed February 19, 2007.

Matossian, N. 1986. **Xenakis.** London: Kahn and Averill.

Matossian, N. 2002. **A composer beyond music,** in **Contemporary Music Review**, vol. 21 pp. 9 - 10.

McClain, E.G. 1978. **The Pythagorean Plato.** Maine: Nicolas-Hays.

Rothwell, J.A. 1977. **The Phi Factor: Mathematical Proportions in Musical Forms.** University of Missouri, Kansas City.

Rusin, D. 2004 **Mathematics and Music.** http://www.math.niu.edu. Accessed 16 March, 2007.

Santana, H.& Santana, M. 2008. ***Metastaseis*** **and** ***Sea Nymphs*** **as scientific educational proposals.** Proceedings of the fourth Conference on Interdisciplinary Musicology (CIM08)Thessaloniki, Greece.

Schaeffer, P. 1970. **Music and Computers**, in **Music and Technology**, UNESCO, La Revue Musicale. Stockholm Meeting June 8-12, 1970.

Schoner, J. 2003. **Iannis Xenakis' Musique Stochastique: System Design and Mathematical Background.** Seminar Computergestützte Komposition, WS 2002/3. Universität des Saarlandes.

Serra, M. 1993. **Stochastic Composition and Stochastic Timbre: Gendy 3 by Iannis Xenakis,** in **Perspectives of New Music,** vol. 31/1.

Solomos, M. 2004. **Xenakis: Thought through his writings**. Journal of New Music Research 33 (2): 125-136.

Solomos, M. **Xenakis.** http://www.Iannis-Xenakis.org. Accessed 13 February, 2007.

Squibbs, R.J. 1996. **An analytical Approach to the Music of Iannis Xenakis: Issues in the Recent Music.** Ph. D. thesis. Yale University.

Van Maas, S. 2005. **The Xenakian Fold,** in **Makis Solomos, Anastasia Georgaki, Giorgos Zervos (ed.), Definitive Proceedings of the** ***International Symposium***.

Iannis Xenakis.

Van Zyl Smit, K. 2005. **Applying the phi ratio in designing a musical scale.** M.Mus. thesis. Stellenbosch University.

Varga, B.A. 1996. **Conversations with Iannis Xenakis.** London: Faber and Faber.

Weisstein, Eric W. **Probability**. From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/Probability.html . Accessed 24 October, 2008.

Weisstein, Eric W. **Stochastic Process**. From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/StochasticProcess.html. Accessed 24 October, 2008.

Xenakis, I. 1958. **Achorripsis.** Berlin: Bote & Bock.

Xenakis, I. 1992. **Formalized Music.** Bloomington: Indiana University Press.

Xenakis, I. 1950. **La crise de la musique sérielle (1955),** pp. 2-4.

Zografos, M. **Iannis Xenakis: The aesthetics of his early works.** Perfect Sound Forever online music magazine. Accessed 16 Augustus 2008.

Probability theory. 2008. In *Encyclopædia Britannica.* Accessed 10 September, 2008, from Encyclopædia Britannica Online: http://www.search.eb.com.

Stochastic process. 2008. In *Encyclopædia Britannica.* Accessed 10 September, 2008, from Encyclopædia Britannica Online: http://www.search.eb.com.

Appendix 1

Second Law

$$f(j)\,dj = \frac{2}{a}\left(1 - \frac{j}{a}\right) dj.$$

Each variable (pitch, intensity, density, etc.) forms an interval (distance) with its predecessor. Each interval is identified with a segment x taken on the axis of the variable. Let there be two points A and B on this axis corresponding to the lower and upper limits of the variable. It is then a matter of drawing at random a segment within AB whose length is included between j and $j + dj$ for $0 \leq j \leq AB$. Then the probability of this event is:

$$P_j = f(j)\,dj = \frac{2}{a}\left(1 - \frac{j}{a}\right) dj \tag{1}$$

for $a = AB$.

APPROXIMATE DEFINITION OF THIS PROBABILITY FOR CALCULATION BY HAND

By taking dj as a constant and j as discontinuous we set $dj = c, j = iv$ with $v = a/m$ for $i = 0, 1, 2, 3, \ldots, m$. Equation (1) becomes

$$P_j = \frac{2}{a}\left(1 - \frac{iv}{a}\right)c. \tag{2}$$

But

$$\sum_{i=0}^{i=m} P_j = \frac{2c}{a}(m + 1) - \frac{2cv}{a^2}\sum_{i=0}^{i=m} i = \frac{2c(m + 1)}{a} - \frac{2cvm(m + 1)}{2a^2} = 1,$$

whence

$$dj = c = \frac{a}{m + 1}.$$

On the other hand P_j must be taken as a function of the decimal approximation required:

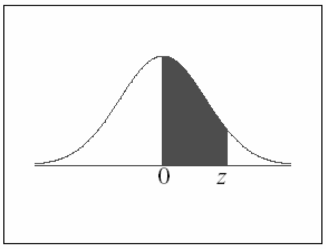
$$P_j = \frac{2}{m + 1}\left(1 - \frac{i}{m}\right) \leq 10^{-n} \qquad (n = 0, 1, 2, 3, \ldots).$$

P_j is at a maximum when $i = 0$, whence $m \geq 2 \cdot 10^n - 1$; so for $m = 2 \cdot 10^n - 1$ we have $v = a/(2 \cdot 10^n - 1)$ and $dj = a/(2 \cdot 10^n)$, and (1) becomes

$$P_j = P_i = \frac{1}{10^n}\left(1 - \frac{i}{2 \cdot 10^n - 1}\right).$$

Appendix 2

Standard Normal Distribution Table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Appendix 3

Table entry for p is the point (χ^2) with probability p lying above it.

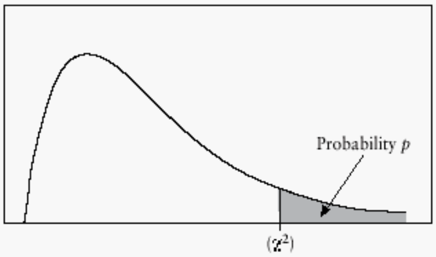


TABLE C χ^2 Critical Values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2