Parabolic curve fitting study subject to Joule heating in MHD thermally stratified mixed convection stagnation point flow of Eyring-Powell fluid induced by an inclined cylindrical surface

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Abstract

The current analysis is carried out to envision the properties of magneto-hydrodynamic boundary layer stagnation point flow of Eyring-Powell (non-Newtonian) fluid induced by an inclined stretching cylindrical surface in the presence of both mixed convection and Joule heating effects. Flow analysis is manifested with temperature stratification phenomena. The strength of temperature adjacent to the cylindrical surface is assumed to be higher in strength as compared to the ambient fluid. A suitable similarity transformations are utilized to convert the flow conducting equations (mathematically modelled) into system of coupled non-linear ordinary differential equations. A fifth order Runge-Kutta algorithmcharted with shooting scheme is used to trace out the numerical additions. It was found that the velocity profile is an increasing function of both mixed convection and curvature parameters. Temperature profile show inciting nature towards Eckert number. In addition, a straight line and parabolic curve fitting way of study is executed to inspect the effect logs of mixed convection parameter, magnetic field parameter, thermal stratification parameter and heat generation parameter on skin friction coefficient and heat rate. It seems to be first attempt in this direction and will serve as a facilitating source for the preceding studies regarding fluid rheology.

Keywords:
Joule heating
Stagnation point
Eyring-Powell fluid
Temperature stratification
Mixed convection
MHD
Heat generation
Parabolic curve fitting
An inclined cylindrical surface

1. Literature survey

The investigation of heat transfer and boundary layer flow of non-Newtonian fluids brought by stretching surfaces are of practical importance and recognized widely by means of number of engineering applications to mention just a few, spinning of fibers, continuous casting, infinite metallic plates cooling, cylindrical wires coating, polymer fiber coating etc. Further, the fluid movement nearby stagnation zone is subjected as stagnation point flow. The heat transfer and fluid pressure are higher in strength under stagnation region. Therefore, boundary layer flows under the region of stagnation point is still a topic of great interest for scientists. Hiemenz (1911) was the earliest to mention stagnation point flow in two dimensional frame of reference by way of Navier-Stokes equations. These types of flow may be viscous or inviscid, two or three dimensional, steady or unsteady, oblique or normal, asymmetric or symmetric, reverse or forward. Furthermore, with the manifestation of stretching surfaces the stagnation point flows owned too much importance regarding industrial and engineering processes. To be more specific, hot rolling, polymer and metal extrusion, wire drawing are the few evidence of such type of flows. Furthermore, the combination of forced and natural convection is called as mixed convection. The applications of mixed convection boundary layer flow of non-Newtonian fluids includes solar power collectors, ocean and atmosphere, fans cooling devices and drying of porous solid. Therefore, it was always remain important for researchers to addressed the effect logs of mixed convection flows for both Newtonian and non-Newtonian fluids namely Gul et al. (2015a,b) discussed mixed convection effects towards MHD nanofluid (contains different shapes of nanoparticles) flow in a channel with saturated porous medium in first case. In second attempt they identified mixed convection effects on ferrofluid flow along a vertical channel. Ullah et al. (2016a) studied MHD mixed convection...
slip flow of Casson fluid towards stretching (non-linearly) sheet through porous medium along with chemical reaction. Magneto-hydrodynamic mixed convection Poiseuille flow of nano-fluid along porous medium in the presence of thermal diffusion, thermal radiation and chemical reaction was taken by Aman et al. (2016). Further, plenty of researchers performed study on forced and natural convection by considering different physical effects see (Ullah et al., 2016b; Zin et al., 2016; Sheikholeslami et al., 2016a, b; Ali et al. (2016), Babu and Sandeep (2016), Sheikholeslami et al. (2015), Ullah et al., 2016c, Babu et al. (2015), Sheikholeslami et al. (2015), Ullah et al. (2016c), Ali et al. (2016), Babu and Sandeep (2016), Sheikholeslami and Shehzad (2016), Sheikholeslami et al. (2016b-d), Sheikholeslami and Vajravelu (2017), Kumaran et al. (2017), Sheikholeslami and Rokni (2017).

The deposition or formation of layers in a flow regime give birth to thermal stratification phenomena. The impact of boundary layer flows along with temperature stratification is significant subject to the heat transfer analysis. The phenomena of temperature stratification arises due to alteration in temperature or combination of various fluids having different densities. As far as practical applications are concern, thermal energy storage (solar ponds system), atmosphere density stratification and production of sheeting mate-rial are the physical significances. In the light of these applications, most of the researchers and scientists probed that the boundary layer flow of Newtonian fluids are not primarily suitable in contrast to non-Newtonian fluid flows. The flow diversity of non-Newtonian fluids in nature is the source of uncertainty regarding rheological features and almost impossible to clip complete physical description by way of single constitutive expression between shear rate against stress. Due to this reason, a variety of models for non-Newtonian fluids (revealing distinct rheological impacts) are offered in the literature. Among those Eyring and Powell proposed a new fluid model in 1944, and it is known as Powell-Eyring fluid model (see Powell and Eyring (1944)). Eyring-Powell model has certain advantages over non-Newtonian model in this sense that it is derived from molecular theory of gases rather than the experimental relation and turn into Newtonian mode at low and high shear rates. Even though it is more complex but advantages of this fluid model overcomes its labouring mathematics. For example it can be used to articulate the flows of modern industrial materials such as ethylene glycol and powdered graphite. Heat diffusion through Eyring-Powell fluid plays a vital role in different geophysical, natural and industrial problems namely, moisture and temperature distribution over agricultural pitches, environmental pollution, underground energy transport etc. Although every non-Newtonian fluid model is important with respect to industrial and engineering point of view, so that the researchers identified different effects namely, magnetic field effect, unsteadiness of flow field, thermal radiation effect, heat generation phenomena, porous medium, melting heat transfer effect over a plane and cylindrical stretching geometry. The fluid flow over a cylindrical surface is treated as a two dimensional if the boundary layer thickness is small as compared to body radius. Whereas, boundary layer thickness is of same order as radius of cylinder for the case of thin or slender cylinder. This implies that fluid flow will be considered as axi-symmetric instead of two dimensional. As far as the Eyring-Powell model is concern, during past time most of the researchers owned the importance of Eyring-Powell model and so investigated diverse effects by considering flow of Eyring-Powell fluid brought by different geometries. Yoon and Ghajar (1987) wrote a note on Eyring-Powell fluid flow and
concluded that Eyring-Powell is truly sensitive towards minor variations for zero shear rate viscosity and moderate sensitive for infinite shear rate viscosity. Sirohi et al. (1987) explored the Powell-Eyring fluid flow near an accelerated plate. They utilized three different methods and developed a comparison among them. Patel and Timol (2009) offered numerical solution of Powell-Eyring fluid flow. They utilized MSABC to obtained numerical computations. Rosca and Popa (2014) investigated the heat transfer effect of Eyring-Powell fluid past a shrinking surface in parallel free stream. Panigrahi et al. (2014) identified MHD impact under mixed convection flow of Eyring-Powell fluid over a non-linear stretching surface. Malik et al. (2013) pointed the Eyring-Powell fluid over a convection flow of Eyring-Powell fluid over a non-linear stretching sheet. Panigrahi et al. (2014) identified MHD impact under mixed convection effects for Eyring-Powell fluid flow over a vertical cylinder under thermal radiation impact. More recently, Khan et al. (2015) reported study on magneto-hydrodynamic boundary layer flow of Eyring-Powell fluid under the region of stagnation point brought about by an inclined cylindrical stretching surface along with temperature stratification, Joule heating and heat generation effects. Thereby an inclined cylindrical stretching surface was taken by Akbar et al. (2015). Hayat et al. (2015) presented Eyring-Powell MHD nanofluid flow brought by thermal radiation impact. The strength of temperature near the cylindrical surface is considered with temperature stratification phenomena. The strength of temperature near the cylindrical surface is considered with temperature stratification phenomena. The strength of temperature near the cylindrical surface is considered with temperature stratification phenomena. The strength of temperature near the cylindrical surface is considered with temperature stratification phenomena. The strength of temperature near the cylindrical surface is considered with temperature stratification phenomena. The strength of temperature near the cylindrical surface is considered with temperature stratification phenomena.

2. Momentum analysis

A steady non-Newtonian boundary layer incompressible fluid flow over an inclined stretching cylindrical geometry of constant radius is assumed. In a two dimensional frame of reference the fluid flow is being deliberated under the region of stagnation point brought about by an inclined cylindrical stretching surface along with temperature stratification, Joule heating and heat generation effects. Further, the flow regime is considered with temperature stratification phenomena. The strength of temperature near the cylindrical surface is supposed to be greater than the ambient fluid. The axial line of cylinder is presumed to be x-axis and radial direction perpendicular to x-axis is taken as r-axis. The central mathematical equation regarding momentum for Eyring-Powell fluid flow is can be written as:

\[
\frac{\partial \mathbf{V}}{\partial t} = \nabla \cdot \mathbf{N} + \nabla \times \mathbf{B},
\]

where

\[
\mathbf{N} = -\rho\mathbf{f} + \tau, \text{ and } \nabla \times \mathbf{B} = \sigma(\nabla \times \mathbf{B}).
\]

Eyring-Powell fluid model stress tensor is defined as:

\[
\tau = \left[ \mu + \frac{1}{\beta \Delta} \sinh \left( \frac{1}{\beta} \Delta \right) \right] \partial \mathbf{u}/\partial \mathbf{x}, \text{ where } \Delta = \sqrt{\frac{\mu(\partial \mathbf{u}/\partial \mathbf{x})^2}{2}}.
\]

A sinh \(-1\) function up to second order approximation is measured as:

\[
\sinh \left( \frac{1}{\beta} \Delta \right) \approx \frac{\Delta^2}{6c^2} + \frac{\Delta}{c^2}, \text{ and } \left| \frac{\Delta}{c} \right| \ll 1.
\]

By way of boundary layer approximation and velocity field vector \( \mathbf{V} = [v(x, r), 0, u(x, r)] \), the mass conservation and momentum equations will take the form:

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial r} = 0,
\]

\[
u \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} \right) = \frac{1}{2} \left( 1 + \frac{1}{\rho} \right) \frac{\partial u}{\partial x} - \frac{1}{2} \frac{\partial v}{\partial r} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} \right) + \left( \frac{1}{\rho} \right) \frac{\partial v}{\partial r} - \frac{1}{2} \frac{\partial v}{\partial r} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} \right)^2 + u_i \frac{\partial \omega}{\partial x} - \frac{\partial \omega}{\partial r} (u - u_i) + g(x(T - T_w)) \cos \omega.
\]

the relevant boundary conditions for flow model are as follows:

\[
u = U(x) = \frac{U_0}{T_r} x, \quad v = 0 \text{ at } r = R \text{ and } u - u_i = a x \text{ as } r \rightarrow \infty.
\]

To find out the solution of Eq. (6) against boundary conditions given by Eq. (7), we have incorporated the following transformations:

\[
\eta = \frac{M_0^2}{\rho_0^2} \left( \frac{1}{\rho_0} \right)^{\frac{1}{2}}, \quad \psi = \left( \frac{1}{\rho_0} \right)^{\frac{1}{2}} \sqrt{\frac{F(\eta)}{\rho_0}},
\]

\[
u = \frac{U_0}{T_r} F(\eta), \quad v = -\frac{u}{T_r} F(\eta).
\]

Note that the Eq. (5) identically satisfies under velocity components in terms of stream function \( \psi \), the components are defined as:

\[
u = \frac{1}{r} \frac{\partial \psi}{\partial x}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x}.
\]

by joining Eqs. (8) and (9) in Eq. (6), the resulting equations are given by:

\[
\begin{align*}
3(1 + 2K_p \eta)(1 + M_p)(F(\eta)^{m+3} + 3F(\eta)(F(\eta)^{m+3} + 6K_p(1 + M_p)F(\eta)^{m+3}) - 3F(\eta)^{m+3} - 4M_pK_p(1 + 2K_p \eta)(F(\eta)^{m+3} - 3M_p \lambda(1 + 2K_p \eta)^2 (F(\eta)^{m+3} - 3\gamma_T^2(F' - A_p) + 3A_p^2 + 3\lambda_T(T(\eta)) \cos \omega = 0,
\end{align*}
\]

the transformed boundary conditions are:

\[
F(\eta) = 1, \quad F(\eta) = 0, \text{ as } \eta \rightarrow 0, \text{ and } F(\eta) \rightarrow A_p, \text{ as } \eta \rightarrow \infty.
\]

The physical parameters involved in Eq. (10) are defined as follows:

\[
K_p = \frac{1}{\beta} \sqrt{\frac{\beta}{2}}, \quad \alpha = \frac{\mu_0}{\rho_0}, \quad M_p = \frac{1}{\beta}, \quad \lambda = \frac{\mu_0 x}{\rho_0}, \quad \gamma_T = \sqrt{\frac{\mu_0 x}{\rho_0}}.
\]

A \( \lambda = \frac{g}{\beta \rho_0}, \quad \lambda_T = \frac{g \rho_0}{\beta \rho_0}, \text{ where } Gr = \frac{\beta(\rho_0 - \rho_0)}{\rho_0}.
\]

The skin friction coefficient at the surface of cylinder is considered as:

\[
C_f = \frac{\tau_w}{\rho_0 \frac{\partial u}{\partial r}}, \quad \tau_w = \left[ \mu \left( \frac{\partial u}{\partial r} \right) + \frac{1}{\beta \rho_0 \frac{\partial u}{\partial r}} \frac{1}{6 \mu_0 c^3} \left( \frac{\partial u}{\partial r} \right)^3 \right]_{r=R}.
\]

the skin friction coefficient (dimensionless form) is prearranged as:
0.5C_lRe^1/2 = (1 + M_p)F'(0) - \frac{M_p}{3} (F'(0))^2, \text{ where } Re = \frac{U_0 x^2}{v L}. \tag{14}

3. Temperature stratification analysis

The fluid flow model is supported by temperature stratification phenomena in the presence of both Joule heating and heat generation effects. The fundamental equation of energy under usual boundary layer assumption takes the form:

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{\sigma R_B^2 \mu^2}{c_p \rho}, \tag{15} \]

the temperature boundary conditions are prescribed as:

\[ T(x, r) = T_w(x) = T_0 + \frac{\eta}{\kappa}, \text{ at } r = R, \]
\[ T(x, r) \rightarrow T_\infty(x) = T_0 + \frac{\eta}{\kappa}, \text{ as } r \rightarrow \infty. \tag{16} \]

To trace out the dimensionless form of Eq. (15), we incorporate the transformation given as:

\[ \eta = \frac{r^2 - R^2}{2R} \left( \frac{U_0}{v L} \right)^{-1/2}, \quad T(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{17} \]

then, the transformed form of Eq. (15) is written as:

\[ (1 + 2K_p \eta) T(\eta)'' + 2K_p T(\eta)' + \text{Pr} \left( F(\eta) T(\eta)'' - F(\eta) T(\eta)' - F(\eta)' S_T + H_p T(\eta) + Ec \eta^2 F(\eta)'' \right) = 0, \tag{18} \]

subjected to the transformed boundary conditions:

\[ T(\eta) = 1 - S_T, \text{ at } \eta = 0, \quad \text{and } T(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty. \tag{19} \]

The physical parameters involved in Eq. (18) are defined by:

\[ K_p = \frac{1}{R} \sqrt{ \frac{v}{\eta} }, \quad \text{Pr} = \frac{v}{\kappa}, \quad S_T = \frac{c}{B}, \quad H_p = \frac{L Q_0}{U_0 \rho c_p}, \tag{20} \]

\[ Ec = \frac{U_0 x}{c_p (T_w - T_\infty) L}. \]

The local Nusselt number can be prescribed as:

\[ \text{Nu}_w = -\frac{q_w}{k (T_w - T_\infty)}, \quad q_w = -k \left( \frac{\partial T}{\partial r} \right)_{r=R}, \tag{21} \]

it can be written in dimensionless form given as:

\[ \text{Nu}_w Re^{1/2} = -T'(0). \tag{22} \]

4. Computational scheme

The Eq. (10), and Eq. (18) with endpoint conditions i-e Eq. (11), and Eq. (19) respectively are the system of governing coupled non-linear ODE’s (ordinary differential equations) and this system is solved by using shooting scheme with the support of fifth order R-K (Runge-Kutta) algorithm. For this purpose as a first step, system of five first order equations are achieved and dropping independent and retain similarity variable \( \eta \) as independent variable. So by permitting

\[ \zeta_1 = F(\eta) ', \]
\[ \zeta_2 = \zeta_2 \]
\[ \zeta_3 = \zeta_3 \]
\[ \zeta_4 = \zeta_4 \]
\[ \zeta_5 = \zeta_5 \]

regarding this the equivalent form of Eqs. (10) and (18) under new variables is given by:

\[ \begin{bmatrix}
\zeta_1' \\
\zeta_2' \\
\zeta_3' \\
\zeta_4' \\
\zeta_5'
\end{bmatrix} = \begin{bmatrix}
\zeta_1 \\
\zeta_2 \\
\zeta_3 \\
\zeta_4 \\
\zeta_5
\end{bmatrix} \tag{23}
\]

the equivalent endpoint conditions in new variables are given as follows:

\[ \zeta_1(0) = 0, \quad \zeta_2(0) = 1, \quad \zeta_3(0) = \text{guess}, \quad \zeta_4(0) = 1 - S_T, \quad \zeta_5(0) = \text{guess}. \tag{24} \]

The system given by Eq. (23) with endpoint condition Eq. (24) fulfils the primary criteria of shooting scheme, but for integration process of Eq. (23) as a initial valued problem (IVP) we must need \( \zeta_1(0) \) as a \( F'(0) \), \( \zeta_5(0) \) as a \( T'(0) \). We have additional boundary conditions:

\[ \zeta_2(\infty) = A_p, \quad \zeta_4(\infty) = 0. \tag{25} \]

5. Physical outcomes

The selection of complimentary values for \( F'(0) \), and \( T'(0) \) are estimated by keeping in mind that the integration of system of first order differential equations carried out in such a way that the conditions in Eq. (25) holds absolutely. Note that the numerical computation up-to four decimal precision as convergence standard are obtained by maintaining \( \Delta \eta = 0.025 \) as a step size.

The adopted parameter values for present computational analysis are given in Table 1. The obtained results are against these values unless indicated on graphs where needed. Table 2 is used to trace out the variations of skin friction coefficient towards Eyring-Powell fluid parameters and curvature of the cylindrical surface. It was found that the skin friction coefficient increases for increasing values of both curvature parameter \( K_p \) and fluid parameter \( M_p \) while it shows decline attitude for larger values of fluid parameter \( l \). The impacts of curvature parameter, Prandtl number, an inclination and Eckert number on heat transfer rate

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Adopted parameters values for computational analysis.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic field parameter</td>
<td>( \gamma_m = 0.1 )</td>
</tr>
<tr>
<td>Thermal Stratification parameter</td>
<td>( S_T = 0.1 )</td>
</tr>
<tr>
<td>Curvature parameter</td>
<td>( K_p = 0.1 )</td>
</tr>
<tr>
<td>Fluid parameters</td>
<td>( \lambda, M_p = 0.1 )</td>
</tr>
<tr>
<td>Velocities ratio parameter</td>
<td>( A_0 = 0 )</td>
</tr>
<tr>
<td>Mixed convection parameter</td>
<td>( \alpha = \xi )</td>
</tr>
<tr>
<td>Inclination</td>
<td>( \gamma = 4 )</td>
</tr>
<tr>
<td>Eckert number</td>
<td>( Ec = 0.1 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Skin friction coefficient for different values of ( \lambda, K_p, \text{ and } M_p ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>( K_p )</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.9423</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.9373</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.9326</td>
</tr>
<tr>
<td>0.3</td>
<td>-1.0168</td>
</tr>
<tr>
<td>0.6</td>
<td>-1.1280</td>
</tr>
<tr>
<td>0.9</td>
<td>-1.2243</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.8728</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.7915</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.7311</td>
</tr>
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</table>
are evaluated and given in Table 3. It was observed that the heat transfer rate is increasing function of both curvature parameter $K_p$ and Prandtl number $Pr$. Whereas, heat transfer rate shows decline nature for Eckert number $Ec$ and an inclination $\omega$ of the cylindrical surface. Figs. 1 and 1a is the physical illustration of flow model and flow chart of numerical method respectively. Whereas, the Figs. 2–10 are plotted to explore the effects logs of involved physical parameters namely, velocities ratio parameter, magnetic field parameter, an inclination, mixed convection parameter, curvature parameter, thermal stratification parameter, heat generation parameter and Eckert number. Particularly, Figs. 2–6 are used demonstrate the impacts of velocities ratio parameter, magnetic field parameter, an inclination, mixed convection parameter and curvature parameter on velocity profile. Figs. 7–10 are established to identify the effects of curvature parameter, thermal stratification parameter, heat generation parameter and Eckert number over a temperature profile. Figs. 11 and 12 are sketched to identify the variation of skin friction coefficient and heat transfer rate through straight line and parabolic curve fitting analysis. To be more specific for Figs. 11 and 12, red lines curves exhibit thermal stratification and mixed convection parameter and blue dash dotted curves depicts the response of heat generation and magnetic field parameters. In detail, Fig. 2 determines the variation of fluid velocity against different values of $A_p$ (velocities ratio parameter). As expected by increasing the velocities ratio parameter the velocity of the fluid increase within a flow regime. The impact of magnetic field parameter over a fluid velocity is depicted in Fig. 3. The applied magnetic field is being consider perpendicular to the fluid flow and has an ability to generate the Lorentz force (drag force), which inclines to oppose the fluid flow and hence horizontal velocity shows decline attitude. Fig. 4 paints the impact of inclination on velocity of the fluid. It was observed that for larger values of an inclination $\omega$ the velocity profile declines. This fact is due to influence of gravity. Because by increasing an inclination $\omega$ relative to $x$-axis, the impact of gravity is reduced which results decline in velocity within a boundary layer. Fig. 5 determines the variation of fluid velocity against mixed convection parameter on velocity

<table>
<thead>
<tr>
<th>$K_p$</th>
<th>$Pr$</th>
<th>$\omega$</th>
<th>$Ec$</th>
<th>$-T(0)$</th>
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<tbody>
<tr>
<td>0.3</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.1683</td>
</tr>
<tr>
<td>0.6</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.2691</td>
</tr>
<tr>
<td>0.9</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1.3699</td>
</tr>
<tr>
<td>–</td>
<td>0.3</td>
<td>–</td>
<td>–</td>
<td>0.4988</td>
</tr>
<tr>
<td>–</td>
<td>0.6</td>
<td>–</td>
<td>–</td>
<td>0.7290</td>
</tr>
<tr>
<td>–</td>
<td>0.9</td>
<td>–</td>
<td>–</td>
<td>0.9275</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>$\pi/6$</td>
<td>–</td>
<td>1.1045</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>$\pi/4$</td>
<td>–</td>
<td>1.1034</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>$\pi/3$</td>
<td>–</td>
<td>1.1019</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>–</td>
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<td>1.1009</td>
</tr>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>0.6</td>
<td>1.0995</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.9</td>
<td>1.0980</td>
</tr>
</tbody>
</table>

Fig. 1. Physical illustration of fluid flow over an inclined stretching cylindrical surface.

Flow chart of shooting method

Fig. 1a. Flow chart for valuation of shooting scheme.
Fig. 2. Effect of velocities ratio parameter $A_p$ on velocity distribution.

Fig. 3. Effect of magnetic parameter $\gamma$ on velocity distribution.

Fig. 4. Effect of inclination $\omega$ on velocity distribution.

Fig. 5. Effect of mixed convection parameter $\lambda_p$ on velocity distribution.

Fig. 6. Effect of curvature parameter $K_p$ on velocity distribution.

Fig. 7. Effect of curvature parameter $K_p$ on temperature distribution.

Fig. 8. Effect of thermal stratification parameter $S_T$ on temperature distribution.

Fig. 9. Effect of heat generation parameter $H_p$ on temperature distribution.
profile. It was found that for higher values of $\lambda_p$ velocity of fluid increases. Physically, it is due to inciting attitude of thermal buoyancy force. Fig. 6 shows that the higher values of curvature parameter is the cause of increase in velocity profile. The curvature parameter $K_p$ has inverse relation with radius of curvature. So when we increase this parameter, the radius of cylinder decreases and hence contact surface area of cylinder with fluid reduces, which offers less resistance to fluid flow. So increase in curvature parameter $K_p$ causes increase in velocity profile. Fig. 7 paints the temperature variation against curvature parameter. It is clearly seen that temperature distribution increases for increasing values of curvature parameter $K_p$. As Kelvin temperature is defined as an average kinetic energy so when we increase curvature of cylindrical, velocity of the fluid increases, resultantly kinetic energy increases and due to this temperature increases. Note that temperature of fluid start decreasing near the cylindrical surface and increases far away with respect to surface. The influence of thermal stratification parameter on temperature of the fluid was sketched in Fig. 8. It show that temperature distribution decreases throughout the flow regime. In view of physical reasoning, this is because of decline in temperature difference between cylindrical surface and ambient fluid therefore, temperature profile decreases. In the view of physical reasoning, this is because of decline in temperature difference between cylindrical surface and ambient fluid therefore, temperature profile decreases. In the view of physical reasoning, this is because of decline in temperature difference between cylindrical surface and ambient fluid therefore, temperature profile decreases.

In this section, we have evaluated the attitude of skin friction coefficient and heat transfer rate by means of straight line and parabolic curve fitting approximation towards mixed convection parameter, magnetic field parameter, thermal stratification parameter and heat generation parameter. The least square method was utilized here and it was proposed by Guass and Legendre. The generalized curve fitting for $m$ degree polynomial is $P(X) = a_0 + a_1X + \ldots + a_m X^m$, where $m \leq n - 1$. Then, $Q$ takes the form $Q = \sum(Y_i - P(X_i))^2$ and depends on $m + 1$ parameters namely, $a_0, a_1, \ldots, a_m$. So, we have $m + 1$ conditions, i-e

$$\frac{\partial Q}{\partial a_0} = 0, \frac{\partial Q}{\partial a_1} = 0, \ldots, \frac{\partial Q}{\partial a_m} = 0,$$

which results a system of $m + 1$ normal equations. The case of quadratic polynomial corresponds $P(X) = a_0 + a_1X + a_2X^2$, and normal equations for quadratic approximations can be written as:

$$\begin{align*}
n &\sum a_0 + a_1 \sum X_i + a_2 \sum X_i^2 = \sum Y_i, \\
a_0 &\sum X_i + a_1 \sum X_i^2 + a_2 \sum X_i^3 = \sum X_i Y_i, \\
a_0 &\sum X_i^2 + a_1 \sum X_i^3 + a_2 \sum X_i^4 = \sum X_i^2 Y_i. \tag{27}
\end{align*}$$

To trace out straight line and parabolic curve fitting for heat transfer rate against thermal stratification and heat generation parameters i-e $S_r$ and $H_p$. Let $X_i = S_{r(l)}$, and $Y_i = (-T(0))_l$ we get,

$$\begin{align*}
\sum (S_{r(l)}) &\approx 0.1; \sum (S_{r(l)}^2) = 0.01000, \\
\sum (-T(0)) &\approx 2.4090; \sum (S_{r(l)}(-T(0))) \approx 0.1184,
\end{align*}$$

by incorporating these values in Eq. (27), we have
by solving system of equations given by Eq. (28), we get
\[ -T'(0) = P(S_T) = a_0 + a_1 S_T \]
(29)
here, \(a_0 = 1.2250\), and \(a_1 = -0.4099\). For parabolic curve fitting, we have
\[ \sum (S_T)^2 = 0.4, \sum (S_T^4) = 0.1000, \sum (S_T^6) = 0.0280, \]
\[ \sum (S_T^8) = 0.0098, \sum (T'(0)) = 3.5108, \sum (S_T^2)(T'(0)) = 0.4489, \]
\[ \sum (S_T^4)(T'(0)) = 0.1110. \]
by incorporating these numeric values into Eq. (27), we obtained
\[ 3a_2 + 0.4a_4 + 0.1000a_3 = 3.5108, \]
\[ 0.4a_2 + 0.1000a_4 + 0.0280a_3 = 0.4489, \]
\[ 0.1000a_3 + 0.0280a_4 = 0.1110. \]
by common algebraic practise this system gives parabolic curve fitting relation for heat transfer rate towards thermal stratification in \(P(S_T) = a_2 + a_3 S_T + a_4 S_T^2\),
\[ -T'(0) = P(S_T) = b_0 + b_1 S_T, \]
(31)
where, \(a_2 = -0.94220, a_3 = 0.20800\), and \(a_4 = -0.10000\). Similarly, line and parabolic curve fitting for heat generation rate for heat generation parameter is entertained as follows.
\[ \sum (H_p)_i = 0.4, \sum (H_p^2)_i = 0.1000, \]
\[ \sum (T'(0))_i = 2.2482, \sum (H_p)(T'(0))_i = 0.4377. \]
\[ 2b_0 + 0.4b_1 = 2.2482, \]
\[ 0.4b_0 + 0.1b_1 = 0.4377. \]
then the straight line approximations towards heat generation parameter is given by:
\[ -T'(0) = P(H_p) = b_0 + b_1 H_p, \]
(33)
where, \(b_0 = 1.2435\) and \(b_1 = -0.5970\). Now for parabolic curve fitting approximations is calculated as follows:
\[ \sum (H_p)_i = 0.9, \sum (H_p^2)_i = 0.3500, \sum (H_p^3)_i = 0.1530, \]
\[ \sum (H_p^4)_i = 0.0707, \sum (T'(0))_i = 3.1695, \]
\[ \sum (H_p)(T'(0))_i = 0.8983, \sum (H_p^2)(T'(0))_i = 0.3379. \]
\[ 3b_2 + 0.9b_3 + 0.3500b_4 = 3.1695, \]
\[ 0.9b_2 + 0.3500b_3 + 0.1530b_4 = 0.8983, \]
\[ 0.3500b_2 + 0.1530b_3 + 0.0707b_4 = 0.3379, \]
than solution of system corresponds
\[ -T'(0) = P(S_T) = b_2 + b_3 S_T + b_4 (S_T^2) \]
(35)
here, \(b_2 = 1.23307, b_3 = -0.46281\), and \(b_4 = -0.32343\). Straight line and Parabolic curve fitting approximations for skin friction coefficient towards magnetic field and mixed convection parameters are processed as follows:
\[ \sum (\gamma_n)_i = 0.4, \sum (\gamma_n^2)_i = 0.1000. \]
\[ 0.5Cf\sqrt{Re_x} = -4.2377, \sum (0.5Cf\sqrt{Re_x})\gamma_n = -0.8542, \]
\[ 2c_0 + 0.4c_1 = -4.2377, \]
\[ 0.4c_0 + 0.1c_1 = -0.8542, \]
after solving this system we obtained
\[ 0.5Cf\sqrt{Re_x} = P(\gamma_n) = c_0 + c_1 \gamma_n \]
under thermal stratification and Joule heating phenomena. In the absence of heat transfer, if we incorporate $K_p = 0$, $\dot{\varphi} = 0$ and $\lambda = 0$, we get

$$F(\eta) = 0, \quad F'(\eta) = 1, \quad \text{as} \quad \eta \to 0,$$

and

$$\frac{1}{2} \left( 1 + M_p \right) F'' - F^2 - M_p F' F^2 - \gamma_n^2 F' = 0.$$  \hspace{1cm} (44)

The Eq. (44) is the mathematical formulation of the physical system in which Eyring-Powell fluid flow is brought by stretching sheet in two dimensional frame, the hidden characteristics regarding this physical flow was already discussed by Akbar et al. (2015) and they justified their remarks by developing comparison with Fathizadeh et al. (2013). Table 4 is constructed to validate our obtained results with Fathizadeh et al. (2013) and Akbar et al. (2015). For this purpose we incorporate $M_p = \dot{\varphi} = 0$ in Eq. (44). It is found that we have excellent match with existing values. This leads to surety of our current computations.

8. Enumerated key findings

The magneto-hydrodynamic mixed convection boundary layer flow of Eyring-Powell fluid brought by an inclined cylindrical surface in the presence of heat generation process under the region of stagnation point was investigated theoretically. The effects logs of involved physical parameters namely, curvature parameter, magnetic field parameter, mixed convection parameter, velocities ratio parameter, thermal stratification parameter, and heat generation parameter on dimensionless velocity and temperature distributions are identified. The variation of skin friction coefficient and heat transfer rate is presented by way of straight line and parabolic curve fitting analysis. The summarized key results of current study are itemized as follows:

- The velocity of fluid is found as decreasing function of angle of inclination and magnetic field parameter while it shows opposite attitude towards curvature, mixed convection and velocities ratio parameters.
- Boundary layer is formed when free stream velocity is larger than stretching velocity, while inverted boundary layer is observed for case of dominancy of stretching velocity.
- The temperature profile shows decline attitude towards thermal stratification parameter while it shows remarkable increment for Eckert number, curvature and heat generation parameters.
- Skin friction coefficient shows decline for positive iterations of thermal stratification and heat generation parameters.
- In the view of absolute frame heat transfer rate is increasing function of magnetic field parameter and paints opposite remarks for mixed convection parameter.
- The impact of pertinent flow controlling parameters was studied first time by way of straight line and parabolic curve fitting approximations. It was trusted that it will serve as a helping hand for the upcoming studies.

### Table 4

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### References


