Contributions to Engineering Electromagnetics

by

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Supervisor: Prof. P. Meyer

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Declaration

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Abstract

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The dissertation presents an overview of the publications of the candidate, and his research group, on engineering electromagnetics — in particular computational electromagnetics (CEM) — which have advanced the field in a number of aspects. They have also impacted materially on local and now international industry. Applications discussed focus initially on primarily defence work, moving through a period of development of advanced CEM methods (many subsequently incorporated in commercial software) to his appointment to the Square Kilometre Array (SKA) South African Research Chair in Engineering Electromagnetics in 2011, in support of South Africa's MeerKAT radio telescope and SKA program. The golden thread of his work has been modelling full-wave electromagnetic fields in increasingly complex environments. This has now expanded to include work on applying these methods to antenna design in the context of radio astronomy. His recent work also addresses other topics in applied engineering electromagnetics, including calibration and imaging for radio astronomy (some of which leverages CEM simulations) as well as antenna metrology and electromagnetic wave propagation modelling.

Uittreksel

D. B. Davidson

Proefskrif: D Ing

Desember 2017

Hierdie proefskrif bied 'n oorsig van die publikasies van die kandidaat, en sy navorsingsgroep, op eletromagnetiese ingenieurswese — spesifiek die rekenaar oplossing van elektromagnetiese probleme — wat die veld in verskeie aspekte bevorder het. Die werk het ook wesenlike impak op plaaslike en internasionale industrie gehad. Toepassings wat bespreek word fokus vir eers meestal op verdegigingswerk, gevolg deur 'n periodie van ontwikkeling van gevorderde rekenaar simulasie tegnieke (waarvan baie later in kommersiële sagteware ingelyf is) tot sy aanstelling in die *Square Kilometre Array (SKA)* Suid Afrikaanse Leerstoel in elektromagnetiese ingenieurswese in 2011, ter ondersteuning van Suid Afrika se MeerKAT radioteleskoop en SKA program. Die goue draad van sy werk is die modelleering van volgolf elektromagnetiese velde in omgewings van toenemende kompleksiteit. Dit het nou uitgebrei om werk op die toepassing van hierdie tegnieke vir antenneontwerp in die konteks van radioastronomie in te sluit. Sy onlangse werk pak ook ander onderwerpe in toegepaste eletromagnetiese ingenieurswese aan, insluitend kalibrasie en beeldvorming vir radio astronomie (van hierdie maak gebruik van rekenaar simulasies) asook antenne metingstegnieke en die modelleering van elektromagnetiese golfvoortplanting.

Acknowledgements

I would like to express my sincere gratitude to the following people and organisations. Looking back over a period of three decades, a list of acknowledgements inevitably becomes rather lengthy.

Firstly, thanks to my family — my wife Amor and my sons Bruce and Ethan; they have had to endure many absences during conferences and other trips around the world. On the occasions when they could accompany me, it has made the trips special.

Secondly, thanks to my postgraduate supervisors, John Cloete and Derek McNamara, and my early-career mentors, Jan Malherbe, who introduced me to electromagnetics in 1981 at Pretoria University and with whom I published my first journal paper in 1984, and Dirk Baker, who gave me my first job at NIAST, CSIR. Also a special word of thanks to Sue Cloete — she and John opened their house to me when I first arrived in Stellenbosch in 1988, and have been very special friends over three decades.

Then, I would like to acknowledge leaders in the field around the world with whom I've been privileged to collaborate with during my career, including Jim Aberle (ASU); Jan Gelart bij de Vaate and Stefan Wijnholds (ASTRON); Tony Brown (Univ Manchester); Ron Ferrari and Ricky Metaxas (Cambridge University); Leo Ligthart and Alex Yaravoy (TU Delft); Rob Maaskant and Marianna Ivashina (Chalmers); Karl Warnick (BYU); and Rick Ziolkowski (Univ of Arizona).

I would also like to acknowledge industrial partners with whom I have worked. At Altair, formerly EMSS-SA, this includes Ulrich Jakobus, Gronum Smith and team; at EMSS-Consulting, Sam Clarke, Frans Meyer, Marnus van Wyk and team; and at EMSS Antennas, Leendert du Toit, Robert Lehmensiek, Isak Theron and their team.

Similarly, I would like to thank SKA-SA for their very generous support over recent years, acknowledging in particular Bernie Fanaroff and Rob Adam as project directors; Justin Jonas, Associate Director Science and Engineering; Kim de Boer, who as Human Capital Development Manager has directly funded many of our students; Jasper Horrell; Francois Kapp and Jason Manley; Carel van der Merwe and Willem Esterhuyse.

The list of graduates whom I have supervised has grown lengthy, and rather than inadvertently omit one of my post-graduates, I refer the reader to Appendix A. It has been a pleasure to supervise each and every one of these students. Much of their work is discussed in this dissertation.

I would also like to acknowledge my colleagues in the Departmental, past and present, in particular Matthys Botha, Johann de Swardt, Dirk de Villiers, Coenrad Fourie, Petrie Meyer, Thomas Niesler, Willie Perold, Howard Reader, Gideon Wiid and PW van der Walt. My thanks to Prof Petrie Meyer for acting as promoter for this D Eng dissertation.

Following the convention of historical works, academic titles and ranks are given contemporaneously with the discussion at hand. Where relevant, footnotes or parenthetic notes indicate subsequent degrees or promotions.

Some parts of this work are based upon research supported by the South African Research Chairs Initiative of the Department of Science and Technology and National Research Foundation, and I also express my gratitude for that support. (Any opinion, findings and conclusions or recommendations expressed in this material are mine; the NRF and DST do not accept any liability with regard thereto.)

Dedication

This dissertation is dedicated to my doctoral supervisor and mentor through much of my career, Johannes Hendrik Cloete, who I met at the University of Pretoria in 1982, and with whom I have walked a very long path — both figuratively in our research, and literally in the mountains of the Western Cape and Scotland.

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List of notation

Throughout this dissertation, the following notation is used. Spatial vectors are indicated as $\boldsymbol{E}$ (in this case, the electric field). Vectors in the linear algebra sense are indicated as $\{x\}$, and matrices as $[A]$. The individual elements of a vector or matrix are of course indicated as x_i or A_{ij} respectively. Otherwise, the notation is as generally encountered in engineering books on this topic. A summary is presented below.

The time convention used for phasor quantities is $e^{j\omega t}$, hence, an e^{-jkr} plane wave propagates in the direction of increasing r. (Note that physics texts often adopt the $e^{-i\omega t}$ convention, in which case the sign also changes in the plane wave exponential factor; engineering texts occasionally use $e^{-j\omega t}$, which has the same effect.)

$\nabla\times$	the curl operation
$\nabla\cdot$	the divergence operation
$\times$	the vector cross product of two vectors
$\boldsymbol{E}$	the (field) vector E
ϵ_0	the permittivity of free space ($\approx 8.854 \times 10^{-12}$ F/m)
ϵ_r	relative permittivity of a dielectric material (dimensionless)
μ_0	the permeability of free space ($4\pi \times 10^{-7}$ H/m)
μ_r	relative permeability of a magnetic material (dimensionless)
c	the speed of light in free space ($\approx 2.9979 \times 10^8$ m/s)
λ	wavelength (m/s)
λ_i	simplex coordinate i
$\mathcal{O}(M^n)$	of the order of M^n, formally, $\mathcal{N} = \mathcal{O}(M^n) \Rightarrow \lim_{M\to\infty} \log \mathcal{N} / \log M = n$

$[A]$	the matrix A
a_{ij}	the ijth element of matrix A
$\{x\}$	the (algebraic) vector x
x_i	the ith element of vector $\{x\}$
$\|\|\{x\}\|\|$	the Euclidean norm of the vector $\{x\}$ of length n, $\|\|\{x\}\|\| \equiv \sqrt{\sum_{i=1}^{n} \|x_i\|^2}$
$\equiv$	is defined as
$\forall$	for all
$\|z\|$	absolute value of z
$\Rightarrow$	implies

Chapter 1

Introduction

My research career started in 1983 as post-graduate student at the University of Pretoria, working on numerical methods for antenna analysis. The research that will be presented in this dissertation spans a career of more than thirty years to date in engineering electromagnetics, with a particular focus on computational electromagnetics through much of this period. As such, there is no more appropriate point to begin than with a brief discussion of Maxwell's equations, before moving on to preview the dissertation and provide some personal background in the rest of this introductory chapter.

1.1 Maxwell's equations

Electromagnetics, the study of electrical and magnetic fields and their interaction, has been one of the core technologies of the twentieth century, and shows every sign of continuing this into the twenty-first. Whilst there are many useful ways of subdividing the field, power frequency versus radio frequency, or alternatively quasi-static versus full-wave, is one of the most insightful here. This dissertation focusses exclusively on radio-frequency, full-wave electromagnetic modelling, as typically encountered in communication systems, remote-sensing applications and radio astronomy telescopes.

The core of modern electromagnetic engineering is of course Maxwell's equations. Written in modern form[1], they are:

$$\nabla \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B} \tag{1.1}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial}{\partial t}\boldsymbol{D} \tag{1.2}$$

$$\nabla \cdot \boldsymbol{D} = \rho \tag{1.3}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{1.4}$$

[1]Maxwell did not actually write his equations in this form; vector analysis was a late nineteenth-century development.

with the associated constitutive equations

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{1.5}$$

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \tag{1.6}$$

Here, I follow the physics convention, regarding $\boldsymbol{B}$ as the primary magnetic field; $\boldsymbol{E}$ and $\boldsymbol{B}$ are the relevant transform pair in the Lorentz transform (Feynmann *et al.*, 1963:Chapter 267, Volume II), and $\boldsymbol{B}$ is the relativistic correction of $\boldsymbol{E}$, as Feynman very elegantly shows (Feynmann *et al.*, 1963:Section 13.6, Volume II). The utility of $\boldsymbol{H}$ in especially quasi-static engineering appliations, such as the design of electrical machines, has led to its widespread usage in the electrical engineering literature as the primary magnetic field.

The actual *solution* of the Maxwell equations is complex, and for realistic problems, approximations are usually required. The numerical approximation of Maxwell's equations, the subject of much of this dissertation, is known as *computational electromagnetics* (CEM). During my career, CEM has emerged from a few (largely US military) research laboratories into widespread deployment in industry, and I have played a part in enabling this.

1.2 Contributions

The main body of work to be presented will be an overview of my own and my research group's extensive publications on computational electromagnetics, which have advanced the field in a number of aspects. (They have also contributed materially to the success of the company Electromagnetic Software and Systems — now Altair.) Application of my research has shifted from primarily defence work (during the late 1980s, when I started my career) to my appointment to the Square Kilometre Array (SKA) South African Research Chair in Engineering Electromagnetics in 2011, in support of South Africa's MeerKAT radio telescope and SKA program. The golden thread of my work has been modelling full-wave electromagnetic fields in increasingly complex environments. This has also now expanded to include work on applying these methods to antenna design in the context of radio astronomy. My work also now addresses other topics in engineering electromagnetics, including calibration and imaging for radio astronomy (much of which leverages CEM simulations) and also antenna metrology and propagation modelling.

My core technical contribution to date has been the establishment of a coherent body of research, embedded in post-graduate training and publication, addressing the theory and application of three main full-wave numerical methods used in RF and microwave engineering - the Method of Moments (MoM), Finite Difference Time Domain method (FDTD) and Finite Element Method (FEM), and significant industrial impact. At present, this includes over 60 peer-reviewed journal articles, over 150 conference presentations and one text book in its 2nd edition. Over 50 post-graduate students and post-doctoral fellows have been supervised or co-supervised by me. A complete list of graduates to date may be found in Appendix A. Two of my doctoral graduates, Prof MM Botha and Prof RH Geschke, hold tenured positions at SU and UCT respectively, and other graduates were instrumental in the successful establishment and growth of Electromagnetic Software and Systems (of which the software business unit is now part of Altair). Details of research funding received may be found in Appendix B.

Particularly important advances and contributions, listed approximately chronologically[2], include:

- Pioneering work on parallel computing for the MoM using "commodity" processors — as opposed to very expensive vector supercomputers — including an efficient parallelized version of the then industry standard program NEC2 running on transputer arrays). Recently, this work has been revisited in the context of BlueGene supercomputers, GPGPUs and ARM processors on smartphones.

- Application of the FDTD to modelling optical devices and frequency selective surfaces.

- A long-running research program on the FEM with significant advances on the use of higher-order elements, error estimation, mesh termination (all of which have been successfully transferred to industry) and the first high-order hybrid explicit-implicit finite element time domain scheme.

- My book on CEM, published by Cambridge University Press, now in its 2nd edition Davidson (2011).

- Work on new methods for the verification of CEM codes, using the method of manufactured solutions.

- Work on efficient CEM analysis methods for sparse arrays.

- Contributions to the SKA mid-frequency aperture array program, in particular via development of a new front-end prototype.

- Work on calibration and imaging, in particular including detailed antenna models into direction and baseline dependent interferometric imaging methods.

- The upgrading of SU's antenna range with entirely new near-field capabilities, and a research program currently running on the evaluation of the chamber.

[2]There is inevitably overlap between substantial research efforts, and some ran in parallel for some time.

These themes will be unpacked, with suitable references, in this dissertation.

1.3 Layout of the dissertation

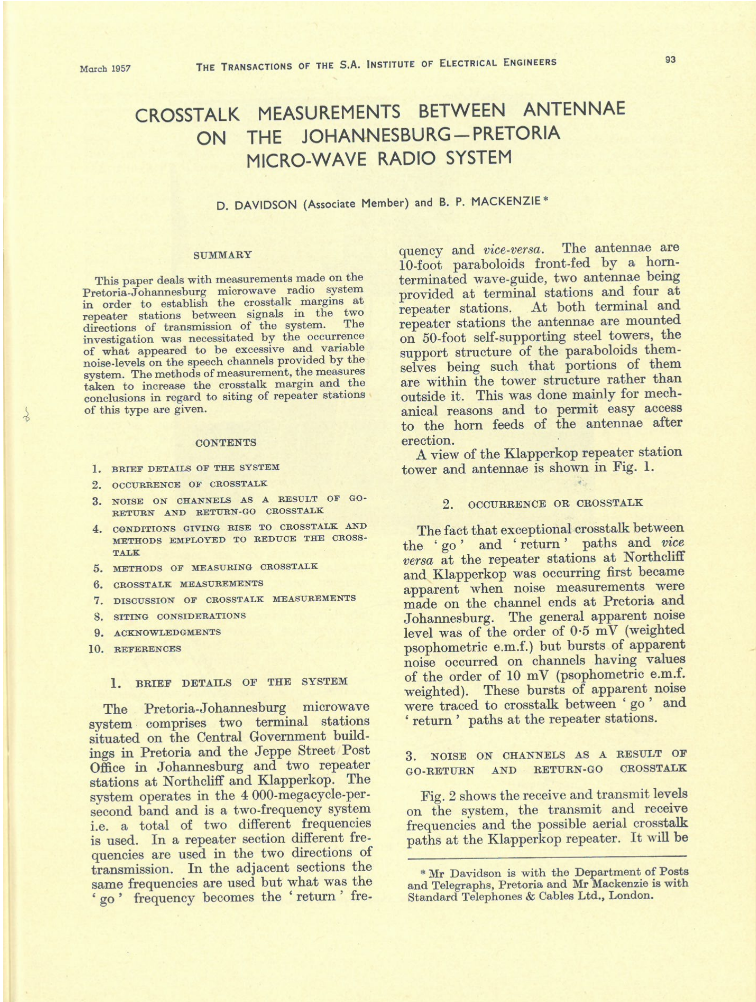
This dissertation is presented in six main chapters. Chapters 2, 3 and 4 cover computational electromagnetics, addressing the MoM, FDTD and FEM respectively. As noted above, this development is approximately chronological, but my work on the MoM has spanned most of my career. Chapter 5 addresses high-performance computing for CEM, and this work has been woven throughout much of my career. Chapter 6 and 7 present recent work, on radio astronomy and antenna metrology and electromagnetic propagation respectively.

1.4 My education, background and work history

As this dissertation spans my entire career, some brief notes on my these topics appears appropriate. I was born in London, England in 1961 of a South African father and English mother. In 1968, my family moved to South Africa, where I was raised and educated, attending Pretoria Boys' High School. My BEng, BEng (Hons) and MEng degrees were obtained at the University of Pretoria in 1982, 1983 and 1986 respectively. My PhD was obtained at Stellenbosch University in 1991.

My very first publications resulted from work I did with Jan Malherbe in 1983 (as an Honours student at the University of Pretoria) on the numerical integration of a mutual impedance integral and subsequent use in the design of a slot array (Malherbe & Davidson, 1984; Malherbe, Cloete, Losch, Robson & Davidson, 1984); these are noted here, as they do not fit very readily into subsequent chapters.

My compulsory national service was spent in the South African Army signals corps (1984-85); following this, I worked at the Council for Scientific and Industrial Research in Pretoria as a research engineer, before being appointed in 1988 at the Dept of Electrical and Electronic Engineering at Stellenbosch University. I subsequently held the posts of Associate Professor (1992–5) and Professor (1996–2014) as Stellenbosch, as well as several temporary academic positions during sabbatical visits. As of January 2011, I have held the Square Kilometre Array Research Chair at Stellenbosch; this is part of the South African Research Chair Initiative (SARCHI) of the Department of Science and Technology (DST) and the NRF. As of July 2014, I also hold the position of Distinguished Professor at Stellenbosch University.



March 1957 THE TRANSACTIONS OF THE S.A. INSTITUTE OF ELECTRICAL ENGINEERS 93

CROSSTALK MEASUREMENTS BETWEEN ANTENNAE ON THE JOHANNESBURG—PRETORIA MICRO-WAVE RADIO SYSTEM

D. DAVIDSON (Associate Member) and B. P. MACKENZIE*

SUMMARY

This paper deals with measurements made on the Pretoria-Johannesburg microwave radio system in order to establish the crosstalk margins at repeater stations between signals in the two directions of transmission of the system. The investigation was necessitated by the occurrence of what appeared to be excessive and variable noise-levels on the speech channels provided by the system. The methods of measurement, the measures taken to increase the crosstalk margin and the conclusions in regard to siting of repeater stations of this type are given.

CONTENTS

1. BRIEF DETAILS OF THE SYSTEM

The Pretoria-Johannesburg microwave system comprises two terminal stations situated on the Central Government buildings in Pretoria and the Jeppe Street Post Office in Johannesburg and two repeater stations at Northcliff and Klapperkop. The system operates in the 4 000-megacycle-per-second band and is a two-frequency system i.e. a total of two different frequencies is used. In a repeater section different frequencies are used in the two directions of transmission. In the adjacent sections the same frequencies are used but what was the 'go' frequency becomes the 'return' frequency and *vice-versa*. The antennae are 10-foot paraboloids front-fed by a horn-terminated wave-guide, two antennae being provided at terminal stations and four at repeater stations. At both terminal and repeater stations the antennae are mounted on 50-foot self-supporting steel towers, the support structure of the paraboloids themselves being such that portions of them are within the tower structure rather than outside it. This was done mainly for mechanical reasons and to permit easy access to the horn feeds of the antennae after erection.

A view of the Klapperkop repeater station tower and antennae is shown in Fig. 1.

2. OCCURRENCE OR CROSSTALK

The fact that exceptional crosstalk between the 'go' and 'return' paths and *vice versa* at the repeater stations at Northcliff and Klapperkop was occurring first became apparent when noise measurements were made on the channel ends at Pretoria and Johannesburg. The general apparent noise level was of the order of 0·5 mV (weighted psophometric e.m.f.) but bursts of apparent noise occurred on channels having values of the order of 10 mV (psophometric e.m.f. weighted). These bursts of apparent noise were traced to crosstalk between 'go' and 'return' paths at the repeater stations.

3. NOISE ON CHANNELS AS A RESULT OF GO-RETURN AND RETURN-GO CROSSTALK

Fig. 2 shows the receive and transmit levels on the system, the transmit and receive frequencies and the possible aerial crosstalk paths at the Klapperkop repeater. It will be

* Mr Davidson is with the Department of Posts and Telegraphs, Pretoria and Mr Mackenzie is with Standard Telephones & Cables Ltd., London.

Figure 1.1: The title page of a paper by my father and my uncle from the Transactions of the SAIEE, March 1957.

On a historical note, my father was also an electronic engineer, and worked in the telecommunications sector in the UK and South Africa for most of his professional life, after serving in the then Union Defence Force from 1942-1946 in the Special Signals Service (who were tasked with operating radars during WWII). I recently came across a paper from 1957 describing work he published with a colleague[3] from the UK on the microwave link from Johannesburg to Pretoria and reproduce the title page as Fig. 1.1.

[3]The second author (Bruce P Mackenzie) would subsequently become my uncle when his sister, Marguerite, married my father in 1959 in London. I take my second name from my uncle, so reproducing their paper here feels appropriate.

1.5 Closing comments

In closing this introductory chapter, the contributions which I will discuss in this dissertation have brought some recognition which I regard as significant. In 1996, I received the President's Award from the NRF as a young researcher; I presently have a B1 rating. In 2012, I was honoured by the IEEE as a Fellow of the institute, with the citation *For contributions to computational electromagnetics*. A more complete list may be found in Appendix C. As of the time of writing, I have approximately 730 citations on Scopus, and 1 340 on Google Scholar[4]. The associated h-indices are 13 and 16 respectively.

This would also be an appropriate point to mention my involvement in the profession. I have been involved in a number of professional activities, in particular via the IEEE, during my career. I chaired the South African IEEE AP/MTT Chapter from 1996–98. I served on the IEEE Antennas and Propagation AdCom (2011-'13), the IEEE APS Awards Committee (2009–2013) and the IEEE APS Fellows Committee (2014–16). I was General Chair of the 8th Finite Elements for Microwave Engineering Workshop, Stellenbosch, 2006 and Chair of the local organizing committee of ICEAA'12-IEEE APWC-EEIS'12, held in Cape Town in September 2012. (Since that 2012 conference, I serve on the steering committee of the ICEAA/IEEE APWC conference series). I am presently an associate editor of both the IEEE Antennas and Propagation Magazine and the IEEE Transactions on Antennas and Propagation. Additionally, I have served on national committees in South Africa, including the NRF's Engineering Assessment Committee in 2001–2002 (in the 2nd year as convenor), and again in 2006. My three-year term on the South African Astronomy Advisory Council, a national body established by the NRF, has just ended.

[4]My long-standing role as Associate Editor of the IEEE AP Magazine results in a number of papers in the column I edit being incorrectly attributed to me on some databases; these have obviously been excluded from these figures as far as possible.

Chapter 2

Contributions to the MoM

2.1 Introduction

The Method of Moments (MoM) was historically the first numerical method widely adopted by antenna engineers. With the first papers appearing in the 1960s[1], the method was ideal for application to highly conducting metallic structures of resonant size. Early applications rigorously solved for the current distribution on a number of canonical antennas, such as log-periodic and Yagi-Uda antennas, widely used in telecommunications and for analogue TV reception (and indeed still in use today). The method was well suited to the limited computational capabilities of that decade. The MoM remains one of the most important numerical methods for antenna engineers to this day.

My work with the MoM, which I will outline in this chapter, has spanned much of my career, from my first graduate work right up to current applications in code testing and radio astronomy antennas. It is most easily presented in two main periods, the 1980s and 1990s, and more recent work (from the late 2000s onwards); during the 2000s, I focussed most of my research on the FEM, which is the topic of Chapter 4. To provide some background, the MoM is briefly reviewed in the following section, drawing on material from (Davidson, 2011:Chapter 4).

2.2 A very brief recap of the MoM

Starting from the decomposition of a time-harmonic electromagnetic field into incident and scattered fields, as

$$\boldsymbol{E}^{\mathrm{tot}} = \boldsymbol{E}^{\mathrm{inc}} + \boldsymbol{E}^{\mathrm{scat}} \tag{2.1}$$

[1] One widely used formulation, due to Pocklington, dates back to 1897, although his paper did not use a numerical method due to the obvious lack of automatic computers.

and using the representation of the electric field in terms of the magnetic vector potential $\boldsymbol{A}$ and electric scalar potential Φ as

$$\boldsymbol{E} = -j\omega\boldsymbol{A} - \nabla\Phi \tag{2.2}$$

it can be shown (see, for example, (Davidson, 2011:Chapter 4)) that for sufficiently thin wires, this can be reduced to the *Pocklington* equation, first introduced in 1897:

$$\begin{aligned} E_z^{scat}(r) &= \frac{1}{j\omega\epsilon_0}\int_{-l/2}^{l/2}\left[\frac{\partial^2\psi(z,z')}{\partial z^2} + k^2\psi(z,z')\right] I_z(z')\,dz' \\ &= -E_z^i(r) \end{aligned} \tag{2.3}$$

This equation is obtained by assuming that we locate the filament on the axis and enforce the boundary condition on the surface (the reciprocal case is sometimes more convenient in deriving this). Although it looks fairly straightforward, the presence of the second derivative of z inside the integral kernel, acting on the Green function, makes this non-trivial to implement. A useful further simplification can be made if the wire is assumed *very* thin ($a \ll \lambda$):

$$\int_{-l/2}^{l/2} I_z(z')\frac{e^{-jkR}}{4\pi R^5}\left[(1+jkR)(2R^2-3a^2)+(kaR)^2\right] dz' = -j\omega\epsilon_0 E_z^i(\rho = a) \tag{2.4}$$

with a the wire radius and $R = \sqrt{a^2 + (z-z')^2}$. This is now a convenient form to program. It appears in numerous texts — see, for example, (Balanis, 1989:p. 720) — and appears to have been first introduced by Richmond (Richmond, 1966), reprinted in (Miller *et al.*, 1992).

The MoM proceeds by approximating the unknown (the axially-directed current $I_z(z')$ in this case) by a finite series approximation

$$I(z') \approx \sum_{n=1}^{N} a_n h_n(z') \tag{2.5}$$

Here, a_n are unknown (but constant) coefficients, and $h_n(z')$ are *basis functions* – also often known as *expansion functions*.

At this stage, linear operator theory provides a convenient framework for developing the formulation (again, see for example, (Davidson, 2011:Section 4.5), from which this material is extracted). We introduce the equation

$$\mathcal{L}f = g \tag{2.6}$$

where $\mathcal{L}$ is the operator which maps function f to function g. In the case of Eq. (2.4), for instance, the function f is the axial current I; the function g is the incident field on the wire; and the linear operator $\mathcal{L}$ is

$$\int_{-l/2}^{l/2} \frac{e^{-jkR}}{4\pi R^5} \left[(1+jkR)(2R^2-3a^2)+(kaR)^2\right](\cdot)dz' \tag{2.7}$$

The bracketed dot is used as a place-holder for the function on which this operator acts. Using this notation, the previous development then produces

$$\mathcal{L}\sum_{n=1}^{N} a_n h_n = g \tag{2.8}$$

where, as before, f has been approximated using the basis functions, viz.

$$f \approx \sum_{n=1}^{N} a_n h_n$$

Using point-matching, the $N \times N$ linear system can be obtained by testing the above at N test points. But now, instead of doing this, we form the *residual* as:

$$\mathcal{R} = \mathcal{L}\sum_{n=1}^{N} a_n h_n - g \tag{2.9}$$

This residual is the difference between the approximate solution and the actual solution. The point-matching procedure forces this residual to zero at N discrete points. A better approach would be to try to obtain some type of average value of the residual over the domain of the problem (the length of the wire in this case), and set this to zero. One can do this in a quite general fashion by introducing the idea of a *weighting function*, which is multiplied by the residual (and hence the name, method of weighted residuals) and integrated over the domain. The weighting function (also often known as a testing function) is also usually expressed as some type of finite series:

$$w = \sum_{m=1}^{M} w_m \tag{2.10}$$

In this case, the equality is appropriate, since we are not approximating this function. Note also that there are no unknown coefficients. Symbolically, the weighted residual method becomes

$$\int_L \mathcal{R}\sum_{m=1}^{M} w_m dz = \int_L \sum_{m=1}^{M} w_m \mathcal{L} \sum_{m=1}^{N} a_n h_n - \int_L \sum_{n=1}^{M} w_n g = 0 \tag{2.11}$$

Usually, the number of basis functions (N) and the number of weighting functions (M) are equal. Because this integration process frequently defines an *inner product*, an equivalent notation frequently encountered is

$$\langle w_m, \mathcal{L} a_n h_n \rangle = \langle w_m, g \rangle \tag{2.12}$$

This is of course the bracket notation widely used in quantum mechanics, for the matrix algebra formulation of Heisenberg. We will not pursue this further, other than to note that the reason for this analogy is that both classical electromagnetics and quantum mechanics are at heart field theories.

It is easy to show that the method of weighted residuals produces a matrix equation:

$$\{V\} = [Z]\{I\} \tag{2.13}$$

with matrix entries

$$\begin{aligned} Z_{mn} &= \langle w_m, \mathcal{L}h_n \rangle \\ V_m &= \langle w_m, g \rangle \\ I_n &= a_n \end{aligned} \tag{2.14}$$

In addition to the question of which type of basis functions to adopt, one now can also choose a variety of weighting functions. This matter has been quite extensively researched. In practice, however, there are two very popular choices. The *Galerkin* procedure uses the same basis and weighting functions. The collocation method, uses Dirac delta functions, which of course reduce to just testing the operator at the sample points.

The computational cost of filling the matrix (i.e. computing the entries of Z_{mn}) is $\mathcal{O}(N^2)$; the cost of factoring the matrix is $\mathcal{O}(N^3)$. The constants in the former can be quite large, depending on the accuracy of numerical integration (quadrature) required. The factorisation operation has constants of the order of unity.

For surfaces, the classic paper (Rao, Wilton *&* Glisson, 1982) introduced the use of vector-based triangular basis functions to solve the Electric Field Integral Equation. These basis functions are widely known in the CEM community as the RWG element (after the authors, Rao, Wilton and Glisson). Following the same lines as the Pocklington equation, integral equations in either the magnetic or electric fields can be derived for problems with currents flowing on surfaces. One integral equation couples the incident electric field to the induced surface current, and is known as the electric field integral equation (EFIE):

$$\begin{aligned} \hat{n} \times \boldsymbol{E}^{\mathrm{inc}}(\boldsymbol{r}) &= \hat{n} \times \int_S \Big[jk\eta \boldsymbol{J}_S(\boldsymbol{r}\,')G(\boldsymbol{r},\boldsymbol{r}\,') \\ &\quad + \frac{\eta}{jk}\{\nabla_s' \cdot \boldsymbol{J}_S(\boldsymbol{r}\,')\}\nabla' G(\boldsymbol{r},\boldsymbol{r}\,')\Big]\, dS\,', \qquad \forall\, \boldsymbol{r},\boldsymbol{r}\,' \in S \end{aligned} \tag{2.15}$$

The ∇' operator implies differentiation in the *source* coordinates. $\hat{n}$ is the unit vector on the surface S. $G(\boldsymbol{r},\boldsymbol{r}\,')$ is the scalar free-space Green function given by

$$G(\boldsymbol{r},\boldsymbol{r}') = \frac{e^{-jkR}}{4\pi R} \tag{2.16}$$

$$R = |\boldsymbol{r}-\boldsymbol{r}'| \tag{2.17}$$

Equation (2.15) is valid for both closed and open surfaces. In the latter case, $\boldsymbol{J}_S$ is the sum of surface currents on both sides of the sheet. A very widely-used form of the EFIE is the mixed potential integral equation (MPIE), which explicitly retains charge as an unknown. From the continuity equation — the time rate of change of charge is the negative of the divergence of the current — charge is of course connected to current, and this is exploited in the MPIE formulation.

A detailed development of an MoM solution using the RWG triangular basis functions may be found in the original paper (Rao, Wilton & Glisson, 1982), and this is reprised with some contemporary insights in (Davidson, 2011:Chapter 6).

2.3 Contributions during the 1980s and 1990s

2.3.1 M.Eng research: 1996

My initial research work in this field, undertaken at Master's level was on radiation from aperture antennas mounted on conducting bodies of revolution (Davidson, 1986). The work was performed at the then National Institute for Aeronautics and Systems Technology (NIAST) of the Council for Scientific Research(CSIR) in Pretoria, South Africa. At that time, NIAST was the leading South African research centre for airborne defence technologies, and there was major interest in accurate modelling of aircraft and missiles, both for antenna positioning and radar cross section prediction, and bodies of revolution have obvious application here.

The BOR formulation is computationally very attractive, as it uses entire-domain Fourier modes to expand the circumferential variation of current, and only the generatrix needs be discretized. For rotationally symmetric excitation, only one Fourier mode is required. For other excitations, a relatively small number generally suffice (details are in the literature and my thesis).

The work was based on existing theoretical formulations using the method of moments (Mautz & Harrington, 1969), but the extension of the method to asymmetrical apertures was rather more complete than other published work; the theoretical work was also carefully supported using both measured results and results computed using the UTD. Additionally, theoretical methods for handling attached wires were considered in detail, as well as original theoretical work on near-field computation of the fields and the computation of coupling between antennas mounted on bodies of revolution. The work led to national and international conference publications, and parts of the work was published nationally as (Davidson & McNamara, 1987) and internationally as (Davidson & McNamara, 1988) and (Davidson & McNamara, 1989).

In terms of assessing this work, my Master's research served as an excellent basis for subsequent research. The work was very well received for a Master's thesis, as evidenced by the mark awarded, and perhaps more significantly from a research viewpoint, in the publications resulting from it.

2.3.2 Doctoral research: 1988–91

The original impetus for this work grew out of the experience obtained with the method of moments during the research for my Master's degree, where problems where encountered with the limited electromagnetic size that was computationally tractable. As noted above, the computational cost is dominated by two terms, the matrix fill — $O(N^2)$ — and the matrix solve — $O(N^3)$ using LU-type solvers. For iterative solvers, the cost is $n_{iter}O(N^2)$ for iterative solvers. (Here, N is the number of degrees of freedom in the simulation and, where relevant, n_{iter} is the number of steps required for the iterative solver to converge adequately[2].)

N is frequency dependent: typically; at least 8–10 unknowns are required per wavelength for wire problems (or this number squared per λ^2 for surface problems). This places limits on the applicability of the method of moments, and for the computers typically available in late 1980s, these limits typically occurred some way before the structure was electromagnetically sufficiently large to use ray-based asymptotic formulations such as the UTD. This problem had practical significance at the time since this "gap" in the coverage of techniques for typical aircraft and ground vehicles fell in the VHF communication band — a most inconvenient place to be unable to predict antenna performance. (As subsequently become evident with later work on hybrid methods, the asymptotic techniques can be problematic to apply even when apparently appropriate, due to the nature of the approximate formulations, which is a further motivation for extending the MoM).

The research approach followed was firstly, to investigate the application of parallel computing to the problem of the matrix fill and matrix solve. The contributions here were efficient parallel conjugate gradient and LU decomposition algorithms. Then major parts of NEC2 were recoded in Occam 2, the "native" language of the T800 transputer (the technology available at the time). The matrix fill was parallelized, used the parallel CG and LU algorithms mentioned above. The culmination of my doctoral research was to demonstrate a version of NEC2 (called PARNEC) that could handle at least double the maximum number of degrees of freedom (segments in NEC2) when compared to that which could previously be handled using the most powerful serial processors

[2]These costs are for conventional methods, not the "fast" methods such as the FMM.

at the University[3]. Smaller problems that had previously required overnight runs could be solved in an hour or two following the doctoral work. This work was presented at five international and three national symposia and was published internationally as (Davidson, 1990, 1992, 1993).

In terms of assessing this contribution, this doctoral work (Davidson, 1991) remains some of the most significant, original research that I have been undertaken *personally*. Subsequent work (with the exception of much of my book, discussed in Section 4.5) has inevitably been undertaken either in a supervisory capacity or collaboratively. With hindsight, the research appears obvious, which can be taken as a measure of the success of the work. When the research was initiated in late 1988, very little had been published on suitable parallel algorithms — indeed, parallel versions of some of the algorithms did not even exist. This research was multi-disciplinary, since it involved electromagnetics, applied mathematics and computer science, and this fact attracted most favourable comment from the examiners. Another important aspect was the careful separation of the underlying principles of the algorithms from the computer technology used to implement the algorithms. Hence the analysis was suitable for the general class of local memory MIMD computers. The demonstrated scaling properties of the algorithms were also significant. Again, this was very favourably received by the examiners. Two invited published tutorials on parallel processing for computational electromagnetics (Davidson, 1990) and (Davidson, 1992), and a journal paper (Davidson, 1993) resulted from this work; the first has been quite widely cited in the CEM literature.

One of the results of the PhD was that the speed-up and efficiency of a parallelized MoM implementation depended fundamentally on the "grain" of the problem, where grain is a measure of the number of unknowns per processor. In most problems of interest, there are far more degrees of freedom that there are processors, so some decomposition (mapping) of the problem is required. For an iterative algorithm, a simple row-block or column-block decomposition is sufficient, but for an LU algorithm, an interleaved decomposition achieves better load balancing (at least in the absence of extensive pivoting).

Examples of measured efficiencies on a transputer array are shown in Fig. 2.1. (The results are shown for slightly different numbers of processors; this was due to different interconnection topologies used for the algorithms.) These data were measured in the early 1990s on a transputer array, hence the problem sizes are small by contemporary standards, but nonetheless, establish the principle. More contemporary results are shown in Fig. 2.2; these results were measured in 2009 using a parallel version of FEKO 5.3, running on the IBM e1350 cluster installed at the Centre for High Performance Computing (CHPC), Cape Town, South Africa. This cluster had 160 nodes, each with two dual-core AMD Opteron 2.6GHz processors, and 16GByte of

[3] The $O(N^3)$ computational dependence should be borne in mind — a problem with twice as many segments requires approximately eight times as long to solve.

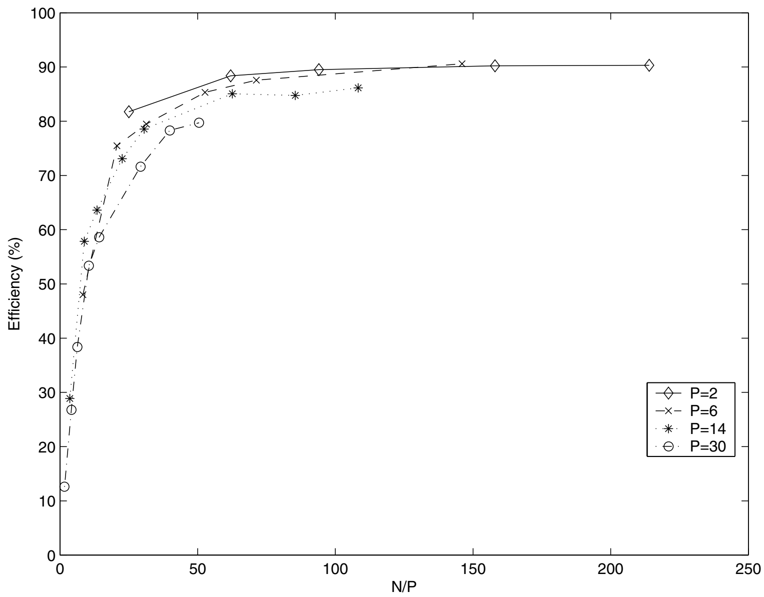


Figure 2.1: Measured efficiency of a parallel CG algorithm on a transputer array, for an MoM problem with a total of N unknowns running on P processors. (After (Davidson, 1993:Fig. 7).)

RAM, giving a total of 640 processors with a peak processing power of around 2.5 TFLOP and 2.56 TBytes of RAM. The system had both 1GBit/s Ethernet and 10GBit/s Infiniband interconnects. (Although faster, the latter was not as widely supported in application programs). The massive increase in unknowns is immediately apparent, driven by two decades of Moore's Law.

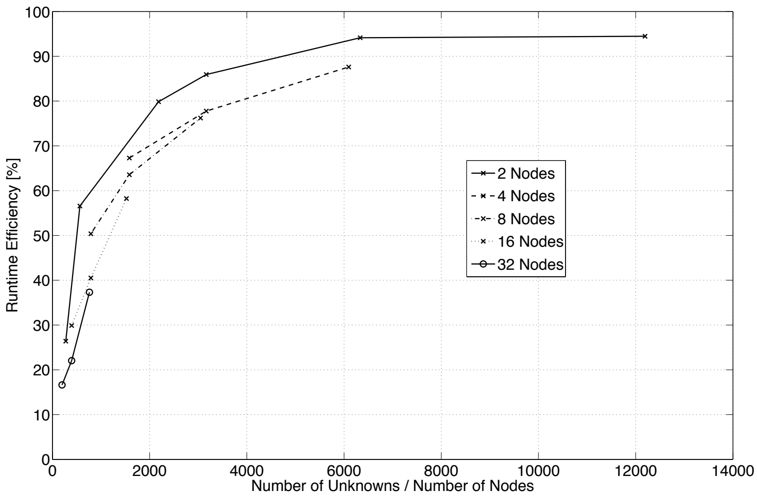


Figure 2.2: Efficiency vs. grain size on on an IBM e1350 cluster, using the Infiniband interconnect. Figure courtesy D. Ludick (Ludick, 2010).

2.3.3 Penetrable BOR modelling: 1990–4

My M.Eng work on the BOR was extended by my doctoral student Pierre Steyn during the period 1990-4. He extended this to include electromagnetically penetrable bodies in his doctoral dissertation (Steyn, 1994). This had important industrial applications for a number of antenna structures such as coated or radome-covered monopoles, for which no analytical or efficient numerical approach previously existed. In assessing this contribution, Dr Steyn did a meticulous piece of work on a difficult problem, and a paper on an aspect of the work won a prize at the local SUPEES '93 symposium. We published the core results of his thesis as (Steyn & Davidson, 1995).

2.3.4 MoM simulation work for industry 1990–1998

During and following the doctoral work, a substantial amount of contract-oriented industrial simulation work was undertaken. This used the MoM tools which had been developed with colleagues and students. A project on using NEC2 and PARNEC for modelling vehicle mounted antennas ran from 1990 to the 1994. This work was primarily for the military, concentrating on ground vehicles and ships; a close relationship was developed with the SA Navy at Simonstown and Silvermine during this period. Much of this work was classified to some or other extent; some unclassified and representative work was reported in (Davidson & du Toit, 1991).

In 1996 this was followed by my successful request via the US DoD for NEC4, which at the time of writing (over 20 years later) is still restricted US military technology. With final-year project student Toit Mouton[4], the use of NEC4 for communication with submerged objects was investigated; a variety of problems were found, the solutions of which led via the complexities of Sommerfeld integrals to a paper on the topic (Davidson & Mouton, 1998).

In assessing this work, it led to a variety of technological innovations, the most significant being the successful NEC pre-processor WIREGRID, originally developed for this research. It was published as (du Toit & Davidson, 1995). WIREGRID went through several subsequent re-writes, and became the first product of start-up company Electromagnetic Software and Systems, EMSS (who eventually discontinued the product, as their focus on FEKO deepened). Over the next twenty years, EMSS would grow to become a very substantial company; my interaction with EMSS will be described in detail at the end of Chapter 4.

2.3.5 Hybrid MoM/UTD work: 1997–1999

During the eighteen month period October 1997 – March 1999, Dr Isak Theron was a post-doctoral associate at the University. He worked with me on hy-

[4]Now a full professor in this Deparment.

brid UTD/MoM formulations — work that also flows from the same type of problems that were addressed in the author's own PhD. This work was done specifically with Dr Ulrich Jakobus, then of the Univ. of Stuttgart, in extending the FEKO MoM simulation program which he originally wrote for his PhD. (This work was also partially sponsored by EMSS.) In (Theron *et al.*, 2000), we presented an extension to the uniform geometrical theory of diffraction (GTD) for reflection from smooth curved surfaces. This approach allowed the source to be much closer to the reflecting surface than the conventional uniform GTD formulation and did not require a Hertzian dipole source. In essence, the field point was mirrored in the plane tangential to the specular (reflection) point; the incident field was then calculated at the mirror point and the uniform GTD reflection coefficients were used to mirror this field to the original field point. This formulation reduced exactly to the conventional uniform GTD if the incident field was ray optical. The application to a hybrid method of moments (MoM)/GTD code was outlined and results computed were presented for a dipole radiating in the vicinity of a cylinder.

This collaboration with Dr Jakobus would subsequently expand into a very substantial research program.

During essentially the same period, an M.Eng student, Sven Keunecke, whose thesis topic was also in this general area, was also supervised by the author. A paper based on some examples from his M.Eng thesis was published as (Davidson & Keunecke, 1999) which was a useful tutorial summary of various various MoM/asymptotic hybrid techniques. Some of this work was revisited in (Davidson, 2011:Chapter 6). Quite recently, I revisited this topic with Siyanda Nazo, who Master's thesis addressed hybrid formulations using large element physical optics (Nazo, 2012).

In assessing this work, Dr Theron and I eventually found the MoM/UTD hybridisation somewhat frustrating; the UTD is a powerful method for a suitable, but very restricted, class of problems, but we found the meaningful extension of it to the rather more arbitrary problems that one wants to address with a general purpose simulation code essentially intractable. Several conference publications resulted from this work; the most important result, for the special case of a dipole radiating near a cylinder, was published as noted above.

2.3.6 Other contributions - spectral domain MoM work, 1997–1999, Sommerfeld formulations, 2002–2003

Whilst my work in the 2000s turned very much to the Finite Element Method, I continued to do some work with the MoM; work on introducing the spectral domain MoM formulation was publshed in (Davidson & Aberle, 2004), several years after a collaboration with Prof Jim Aberle involving a short course initiated the work. More significantly, this led me to study the Sommerfeld formulation in depth, and to present a guide to a single-layer implementation in my textbook. This represented a substantial educational contribution, which will now be briefly described.

Due to its perceived complexity, the topic of stratified media (and the resulting Sommerfeld potentials) is generally regarded as an advanced one, and the coverage tends to be highly theoretical, and frequently impenetrable without lengthy study. One reason for this is that historically, analysis focussed on the problem of a dipole above a dielectric half-space. There are a number of complex issues which this raises, requiring quite sophisticated analytical techniques to understand, in particular for the asymptotic cases where interesting radiation physics can be extracted. However, the analysis of a very important special case, namely the grounded single-layer microstrip line (or patch antenna), can be undertaken without undue complexity, at least for most practical cases where the substrate is relatively thin.

The chapter in my book (Davidson, 2011:Chapter 7) starts with a static analysis of a microstrip transmission line, to demonstrate the basic principles of the spectral domain and the derivation of the Green function. Following this, the electrodynamic analysis is introduced, and the Sommerfeld potentials are derived from first principles. An extensive discussion of the numerical evaluation of the Sommerfeld potentials is presented. This is then brought together in an MoM analysis of a printed dipole using entire domain basis functions; some results are shown in Fig. 2.3. (A number of approximations made in the implementation limit accuracy, and the interested reader is referred to (Davidson, 2011:Fig. 7.13) for further discussion.) Although most the work in this chapter is not original, being based on a synthesis of the literature — in particular (Mosig, 1989) — the presentation in the present format does not appear to have been thus undertaken in other works before mine. I have had good feedback from those dedicated enough to work through the details of the chapter!

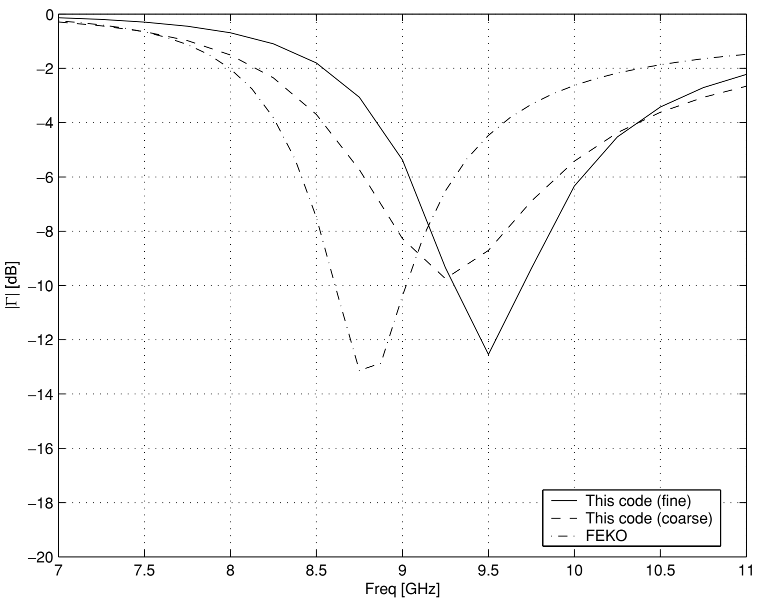


Figure 2.3: Reflection coefficient of a thin printed dipole. After (Davidson, 2011:Fig. 7.13).

2.4 Recent work on MoM — late 2000s onwards

2.4.1 The Method of Manufactured Solutions

My recent focus on radio telescopes led to a renewed and strong interest in the MoM. Marchand's PhD dissertation (2013) represented some very sophisticated work on verifying CEM codes using an approach not previously published in the CEM literature, called the Method of Manufactured Solutions (MMS). In essence, the idea is obvious — provide what is usually the unknown in the problem (eg. the surface current, for a typical MoM code) and then compute from that what would usually be the known driving function or boundary condition (the incident field, ditto). For example, in eq. (2.15), the surface current $\boldsymbol{J}_S(\boldsymbol{r}')$ is chosen, and from the EFIE, the tangential electric field $\hat{n} \times \boldsymbol{E}^{\rm inc}(\boldsymbol{r})$ is computed. This field is then used to drive an MoM solution, from which $\boldsymbol{J}_S(\boldsymbol{r}')$ is computed. This solution can now be compared with the original manufactured solution. It should be noted that this manufactured solution and associated incident field do not need to represent a physically realizable current or field. However, the MS should be chosen appropriately, for instance normally directed current must go to zero at the edge of thin plates.

For the FEM, this turned out to not be too difficult. However, the MoM raised a veritable cornucopia of tough problems, ranging from needing highly accurate quadrature schemes to appropriate norms. (It transpires that when investigating the convergence of MoM solutions, the widely-used L_2 norm is not sufficiently rigorous. One needs to use results from negative fractional Sobolev spaces to better evaluate the norm.) Rates of convergence are also crucial for verifying codes — subtle errors are often only revealed by a careful convergence study — and for the MoM, this is a complex issue. Work over the last two decades in the applied mathematics community has provided results for specific classes of geometry — (Buffa & Christiansen, 2003) and (Christiansen, 2004) are important contributions in this regard — and we used some of these. The core results were published in a special section on Validation of Computational Electromagnetics in the IEEE Trans on EMC (Marchand & Davidson, 2014); in my recent NRF rating application, I rated this as one of my my five best research outputs in that period, and I rate it as one of my career best. I also contributed quite extensively to the actual writing of the paper.

The key advantage of the MMS over canonical solutions is that a much wider range of geometries can be addressed, and hence more code capabilities are exercised than is the case with the classical approach of canonical solutions (which are extremely limited in number). For the types of accuracy required, measured solutions are also often inadequate. An example of the application of this method is the ribbon-line geometry show in Fig. 2.4. The MS is chosen as

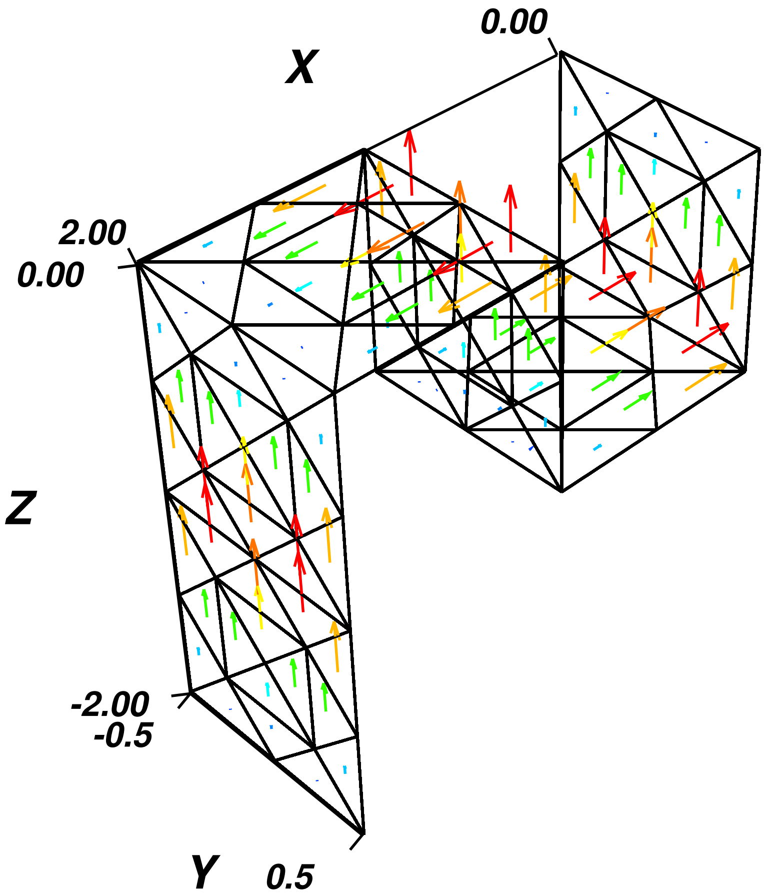


Figure 2.4: Ribbon-like geometry and visual plot of an appropriate manufactured solution (as in the text), after (Marchand & Davidson, 2014:Fig.5).

$$\boldsymbol{J} = \hat{\boldsymbol{n}} \times \boldsymbol{f}_h, \tag{2.18}$$

with

$$\boldsymbol{f}_h = (\cos(x\pi) + \cos((z+1)\pi))\hat{y} \tag{2.19}$$

A comparison of the expected rate of convergence with that actually computed is shown in Fig. 2.5. It should be emphasized that this MS is *not* a physical solution of the Maxwell equations subject to the boundary conditions on the ribbon, but a solution chosen to exercise the code. (In this case, some deficiencies in the MoM implementation were revealed by the process, showcasing the utility of the method. Our application of the MMS also revealed a subtle error in a publicly available FEM package.)

2.4.2 GPU acceleration of the MoM

High-performance computing for CEM (the topic of my own 1991 PhD) was epitomised by the PhD dissertations of (Lezar, 2011) and (Ilgner, 2013), and

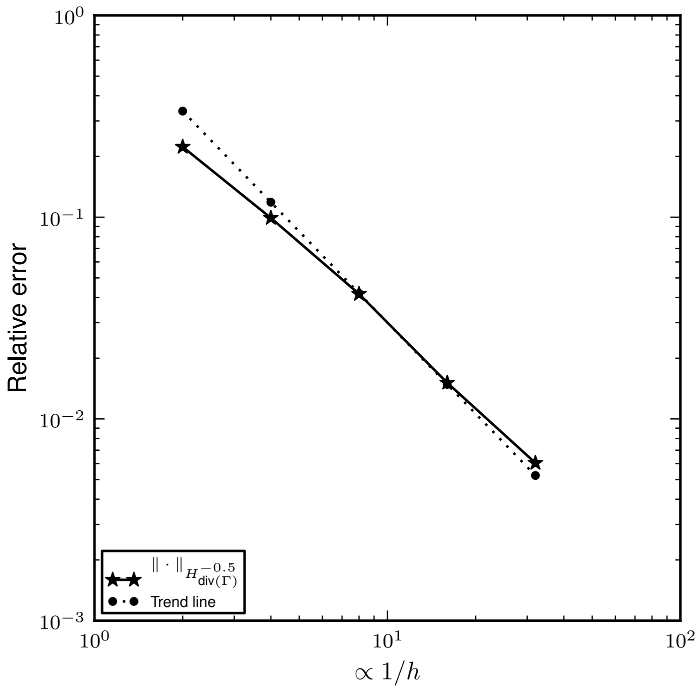


Figure 2.5: Computed convergence rate for the manufactured solution above, compared the expected trend line with slope 3/2. In this figure, the wavenumber multiplied by the maximum mesh element size (k0 h) ranges from 0.16 to 0.01. The upper curve is computed with the correct negative fractional Sobolev norm alluded to in the text. After (Marchand & Davidson, 2014:Fig.6).

this will be revisited in more detail in Chapter 5. Lezar worked on GPU implementations of both MoM and FEM codes; the key results are presented in (Lezar & Davidson, 2010*b*,*a*). At the time of writing, the latter paper is my fifth most highly cited paper, and taken together, these two papers have almost 100 citations on Google Scholar; an excellent example of how early publications on a "hot" topic can generate a large number of citations in short order.

2.4.3 Efficient analysis of large finite arrays - the Domain Green's Function Method

Ludick's PhD dissertation (Ludick, 2014) continued work he originally addressed in his MSc thesis (Ludick, 2010). This work, the first here which specifically focusses on the application of CEM to problems in radio astronomy, addressed the issue of large, but finite unconnected arrays (examples include the MWA, LOFAR and SKA-low). Using a novel method called the Domain Green's Function Method (DGFM) approach, it proved possible to obtain both accurate and computationally cheap solutions for appropriate problems. In its basic form, the DGFM assumes initially that the current on each array element has the same relative spatial distribution, but with potentially different amplitude and phase, as per the feed weighting. Using a block decomposition of the overall MoM matrix, the currents on each array element can then be computed using the "active impedance matrix", which takes both self-coupling and mutual coupling into account, the latter albeit approximately. The solution thus obtained permits the computed currents to depart from the initial assumption of idential spatial distribution. An improved method was also implemented, which relaxed the assumption of identical current shape, and a further iterative extension was proposed in (Ludick *et al.*, 2016). Some of this work was done in collaboration with Chalmers Univ of Technology, and resultant publications have elucidated the connection with the Characteristic Basis Function method. This work has already been incorporated into commercial simulators, and (Ludick *et al.*, 2014) serves as a good reference for the most important results. Fig 2.6 from that paper shows results examining the effect of the "radius of convergence", introduced by Ludick to limit mutual coupling computation (and hence accelerate the method). $R_c = 0\%$ implies no mutual coupling between array elements is taken into account, $R_c = 100\%$ that all mutual coupling terms are (albeit approximately, subject to the DGFM assumptions). The results are computed for a "zig-zag" element, which is has similarities with the current SKA-Low antenna prototype. Fig 2.7 applies the method to a much larger array, more typical of likely station sizes for SKA-Low.

It also showcased increasing international collaboration, including the legendary Prof Raj Mittra, who had been impressed by a presentation by Ludick on his Master's work, and Dr Rob Maaskant, then rapidly establishing a reputation for his work on the CBFM method. Ludick is now continuing further work on this and other antenna-related topics in his post-doc.

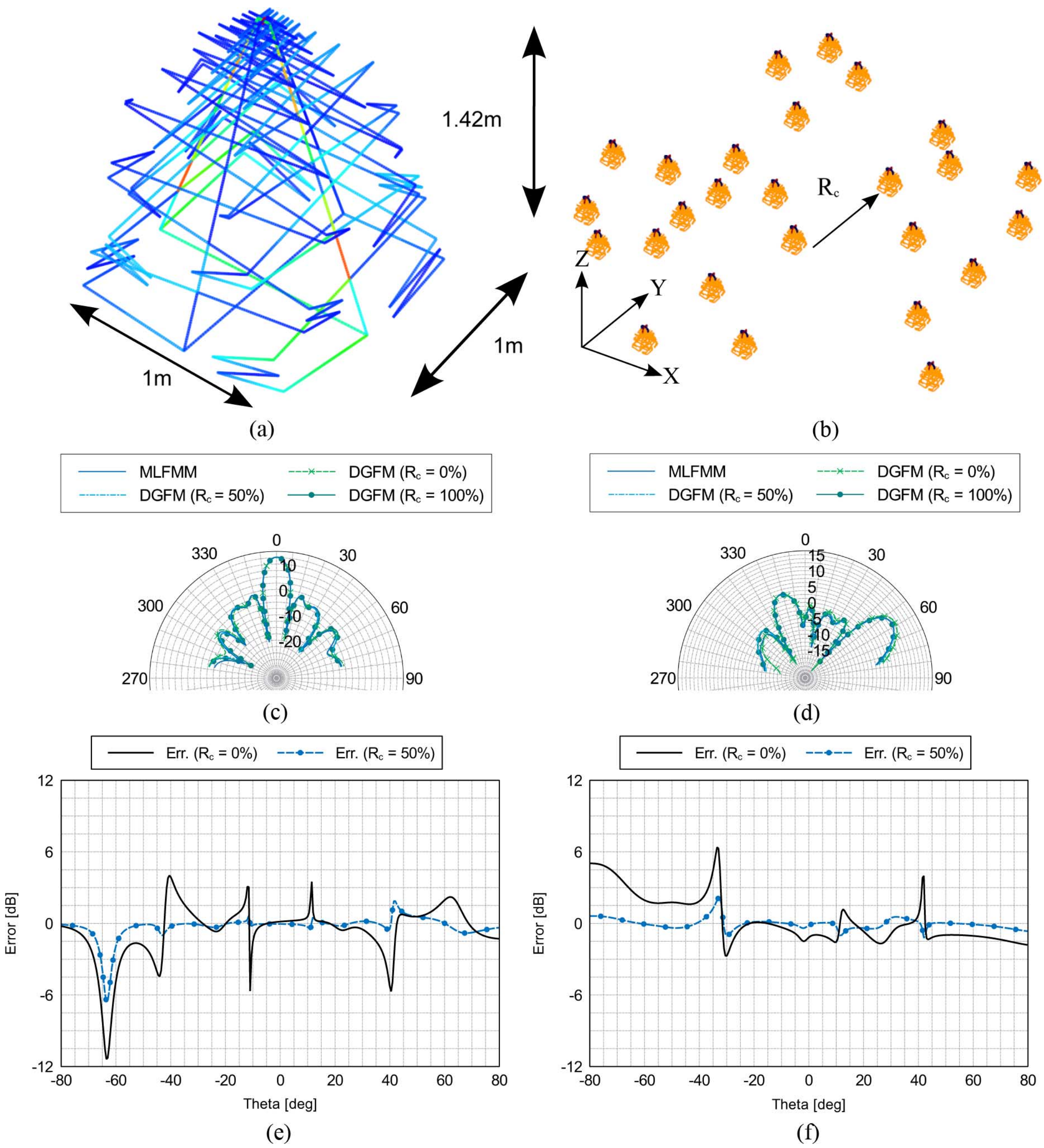


Figure 2.6: Application of the DGFM to a Zig-Zag antenna displayed in (a), in an array configuration displayed in (b). The directivity patterns for scan-angles of $(\theta = 0°, \phi = 0°)$ and $(\theta = 60°, \phi = 0°)$ are presented in (c) and (d), respectively. The errors in the directivity for different R_c values are presented in (e) and (f), where $R_c = 100\%$ is used as reference. All results are obtained for the active array environment where all the elements are excited equally and simultaneously. (a) Dual-polarized Zig-Zag element geometry. (b) Array configuration containing 26 irregularly spaced Zig-Zag elements. The element spacings ranges from $\lambda/2$ to 3λ, at an operating frequency of 70 MHz. (c) Total directivity pattern (dBi) for a scan-angle of $(\theta = 0°, \phi = 0°)$. (d) Total directivity pattern (dBi) for a scan-angle of $(\theta = 60°, \phi = 0°)$. (e) Error (dB) in the calculated directivity for $R_c = 0\%$ and $R_c = 50\%$ compared to $R_c = 100\%$. Considered scan angle: $(\theta = 0°, \phi = 0°)$. (f) Error (dB) in the calculated directivity for $R_c = 0\%$ and $R_c = 50\%$ compared to $R_c = 100\%$. Considered scan angle: $(\theta = 60°, \phi = 0°)$. After (Ludick *et al.*, 2014:Fig.3).

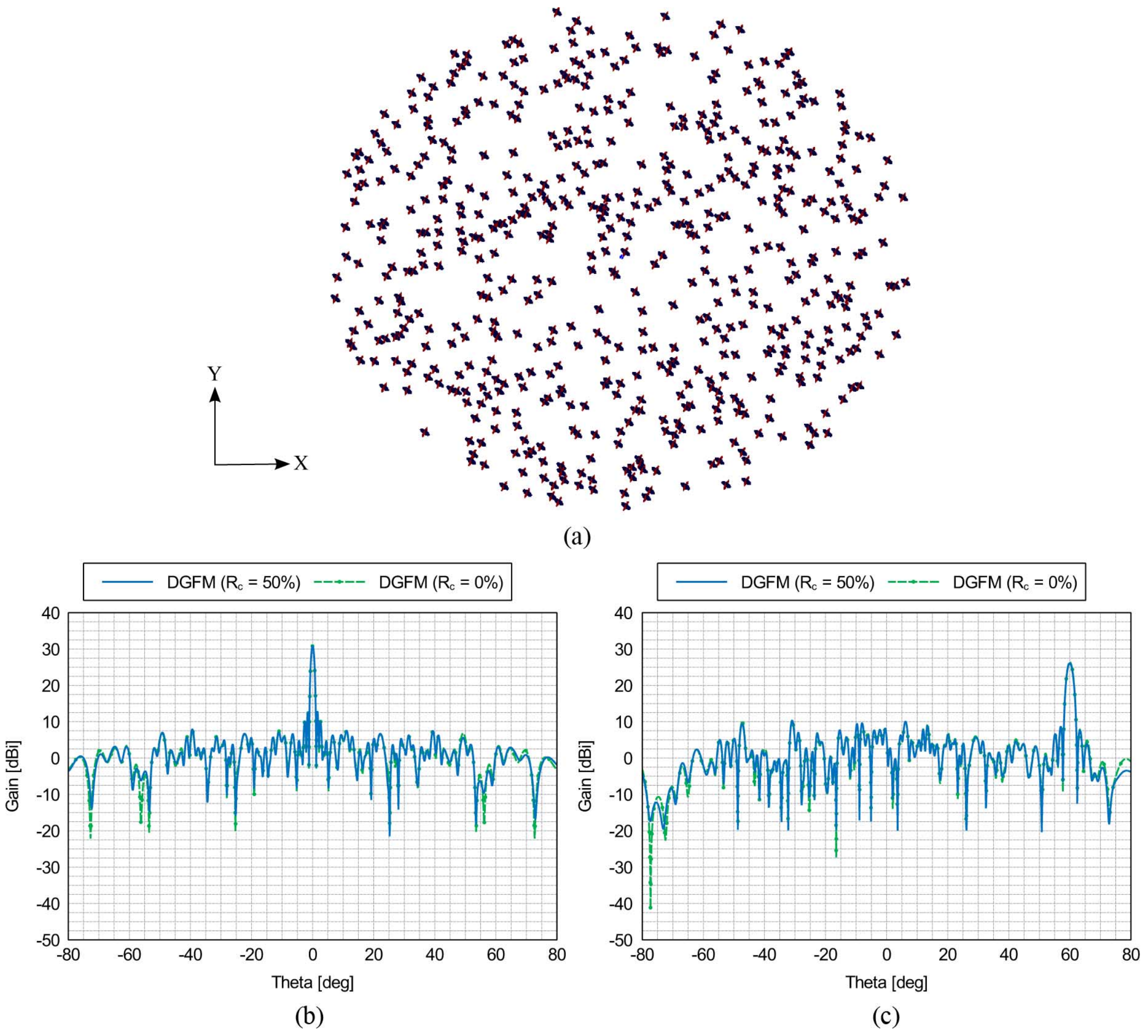


Figure 2.7: Applying the DGFM to a Zig-Zag antenna shown in Fig 2.6a, in a 529-element array configuration. The gain patterns for scan-angles of $(\theta = 0°, \phi = 0°)$ and $(\theta = 60°, \phi = 0°)$ are presented in (b) and (c), respectively. (a) Array configuration containing 529 irregularly spaced Zig-Zag elements. The element spacings ranges from $\lambda/2$ to 3λ, at an operating frequency of 70 MHz. (b) Total gain pattern (dB) for a scan-angle of $(\theta = 0°, \phi = 0°)$. (c) Total gain pattern (dB) for a scan-angle of $(\theta = 60°, \phi = 0°)$. After (Ludick *et al.*, 2014:Fig.5).

2.5 Conclusions

Looking back, my very first encounter with the computational electromagnetics was an introduction to the MoM in 1983, during a post-graduate course taught by Prof Jan Malherbe at the University of Pretoria. Factoring a three-by-three complex matrix on a programmable HP pocket calculator (an HP-41CV) led me to a very early appreciation of computational cost[5], a factor which has driven much of my work in especially the MoM throughout my career to date, with the use of high-performance computing platforms a recurring theme. Related to that is the requirement to improve modeling fidelity, which has been another significant driver — most recently epitomised by the work on the method of manufactured solutions.

In assessing my contributions to the MoM, citations can be useful, and the work Lezar and I published on GPU-acceleration of MoM codes is amongst my most-cited work, but some other very fine work, such as the MMS papers, has only very recently come to the attention of the community. Another significant factor, which citations do not gauge, has been a very close relationship with industry, in particular EMSS-SA. A number of topics originally undertaken as research have been successully commercialised, in particular in the FEKO simulation package. My first boss, Dr Dirk Baker, used to comment that "the best way to transfer technology is on two legs", and a very large number of my former students at EMSS and Altair attest to this.

As one's career progresses, it is most gratifying to see former student become leading researchers in their own right. In the context of the MoM, it is appropriate to mention the career of Dr Matthys Botha[6]. His doctoral work will be described later in this document as it was largely on the FEM, but his later postdoctoral work, and his subsequent work as an independent researcher, increasingly focussed on the MoM, so it is appropriate to note it here. During his post-doc with me in 2004–2005, he initiated work on both volume integral equations and higher-order divergence conforming elements (generalisations of the RWG element discussed earlier in this chapter) and published two excellent papers on these (Botha, 2006, 2007). Dealing with higher-order elements immediately focusses one's attention on the problem of integrating both the singular and near-singular kernels which occur in the MoM formulation — ironically, the latter is often more challenging — and Botha's formidable ability was drawn inexorably to this problem, finally publishing another excellent paper on this topic which synthesised several years' work (Botha, 2013). Currently, he is working on improved Physical Optics formulations, with very promising results.

Whilst the MoM is an extremely powerful method, it is at its best dealing with highly conducting surfaces (or homogenous material structures). Inhomo-

[5]This process took several minutes!

[6]Currently an Associate Professor in our Department.

geneous material structures are often better addressed with differential-based methods, in particular the FDTD and the FEM. After completing my PhD, my own attention was drawn to these methods, as new challenges in dealing with complex material structures arose. I would work on these myself, and also supervise numerous students, more or less up to the present. In the next chapter, I discuss my work on, and contributions to, the FDTD.

Chapter 3

Contributions to the FDTD

3.1 Introduction

During the late 1980s and early 1990s, the Finite Difference Time Domain (FDTD) method became an extremely popular method. This was due to several factors. On the one hand, there were technology pushes, such as algorithmic advances, and the dramatic increases in computational power and RAM available on personal computers (and also workstations); on the other hand, there were application pulls, in particular the widespread deployment of low observable ("stealth") technology in the military, and public concerns about exposure to non-ionizing electromagnetic radiation from cell-phones, the use of which was growing dramatically at that time. Both of these applications involve highly inhomogeneous, electromagnetically penetrable materials, for which the MoM was far from ideally suited. Diffferential-based methods, in particular the FDTD, proved very suitable for this type of problem.

3.2 A brief overview of the FDTD method

As with the preceding chapter, this chapter starts with a brief overview of the method under discussion, which in this case is the FDTD. The following is excerpted from (Davidson, 2011:Chapters 2 & 3).

The finite difference time domain method, usually referred to as the FDTD, is a particular implementation of a general class of methods known as finite difference techniques. The FDTD is so widely used in the CEM community that although finite difference methods cover a wide spectrum of complexity and accuracy, it is the FDTD which is almost always implied in CEM when finite differences are mentioned. Finite difference methods are numerical methods in which derivatives are directly approximated by finite difference quotients. The general class of such methods is the most intuitive numerical approach, and was the first to be extensively developed by the scientific computing community.

To this day, it probably remains the most universally applicable numerical technique and the one most widely used for scientific computation.

The FDTD method was introduced by (Yee, 1966) at much the same time as the original papers on the MoM appeared. However, the computational requirements of the full 3D algorithm were extremely high for 1960s-era computers, due to the requirement to fully discretize both the radiator/scatterer and a substantial volume of free space around it, and it was around a decade before Taflove was to develop the method further, also coining the term FDTD. (The original Yee paper does not use that name).

The FDTD is an initial value method, using a central difference approximation for both the temporal and spatial derivatives. This provides second-order accuracy — but at the cost of two grids, offset in both time and space, which is a complication always to bear in mind with FDTD formulations. For the full three dimensional FDTD all six field components must be considered. The field components are located on the full Yee cell, described in the next section. The field components are offset in both space and time. Details are available in a number of texts; the most widely referenced and comprehensive text remains (Taflove *&* Hagness, 2005).

For a 3D FDTD code, memory is a serious issue; the storage requirements for the six field components (times two, for past and present) and the material arrays (in double precision) become $144N_x \times N_y \times N_z$ bytes. A computational volume with 100 cells on a side will require 144 MB. This will run easily on contemporary personal computers (depending obviously on the amount of memory installed); doubling this to 200 cells in each direction increases the memory requirement to well over 1 Gbyte, still within the scope of most PCs at the time of writing; but doubling this again will most likely exceed available capacity. Double precision is unnecessary for many applications, and one can save storage by storing an integer index rather than the material arrays; similarly, in many applications, the fields can be overwritten immediately, approximately halving storage requirements; but even so, the storage requirement grows very rapidly.

The computational cost associated with fully discertising three dimensions should also be noted. The computational complexity of the algorithm is $\mathcal{O}(N)^4$, but in terms of electromagnetic size, it is $\mathcal{O}(k_{\max}d)^5$–$\mathcal{O}(k_{\max}d)^{5.5}$, due to the necessity of controlling numerical dispersion. Halving the mesh size increases the run time by a factor of 16; doubling the frequency, by between 32 and 45 or so, when numerical dispersion is correctly controlled[1].

It is for these reasons that the development of efficient ABCs was so crucial as the enabling technology which permitted widespread adoption of the FDTD. Highly efficient ABCs permit one to place the scatterer very close

[1]See (Davidson, 2011:Section 1.4), for a discussion of dispersion in FDTD grids, and how controlling it as computational volumes grow in size requires increasing mesh density, as noted above.

Table 3.1: Spatial and temporal location of fields in 3D FDTD algorithm

Field component	x	y	z	t
$E_x(i,j,k,n)$	$x_{i+1/2}$	y_j	z_k	t_n
$E_y(i,j,k,n)$	x_i	$y_{j+1/2}$	z_k	t_n
$E_z(i,j,k,n)$	x_i	y_j	$z_{k+1/2}$	t_n
$H_x(i,j,k,n)$	x_i	$y_{j+1/2}$	$z_{k+1/2}$	$t_{n+1/2}$
$H_y(i,j,k,n)$	$x_{i+1/2}$	y_j	$z_{k+1/2}$	$t_{n+1/2}$
$H_z(i,j,k,n)$	$x_{i+1/2}$	$y_{j+1/2}$	z_k	$t_{n+1/2}$

to the boundary, and one can also obtain scattered fields very close to the boundary without unphysical reflections corrupting the fields.

3.2.1 The Yee cell in 3D

The Yee cell and the FDTD algorithm as proposed by Yee (Yee, 1966) has proven extraordinarily robust. From a coding viewpoint, the update equations are simple to code. From a mathematical viewpoint, there are some additional features which deserve mention, not least that the algorithm implicitly enforces the divergence condition in Gauss's laws, Eq. (1.3) and (1.4) (assuming that the initial conditions are correctly specified). It has also been shown that the algorithm satisfies some deep-lying theoretical requirements for discretization of Maxwell's equations on dual grids (He & Teixeira, 2006). From the perspective of high-performance computing, the matrix-free nature of the algorithm and structured mesh has made the FDTD algorithm the easiest of all the major CEM algorithms to parallelize efficiently, as discussed in Chapter 5.

The original Yee cell positions magnetic field vectors in the centre of faces of the cube, with electric field vectors on the edges, as shown in Fig. 3.1. Taflove chooses a different convention, interchanging the role of the field vectors (Taflove & Hagness, 2005:Fig. 3.1); either convention is valid, but as boundary conditions are most often specified on $\boldsymbol{E}$ fields, it would seem convenient to have the electric fields located on the edges, as in Yee's original scheme which we use here. Although the Yee cell is a very convenient way to visualize the interleaved position of the field vectors, it is important to appreciate that Fig. 3.1 shows fields in some cases associated with a number of indices — for instance, there are three E_x fields shown — and that what is shown on the sketch is actually cell $i-1,j,k$.

For the FDTD algorithm, we associate fields with spatial indices and time steps following Yee's choice, as in Table 3.1.

The associated grid points are defined by

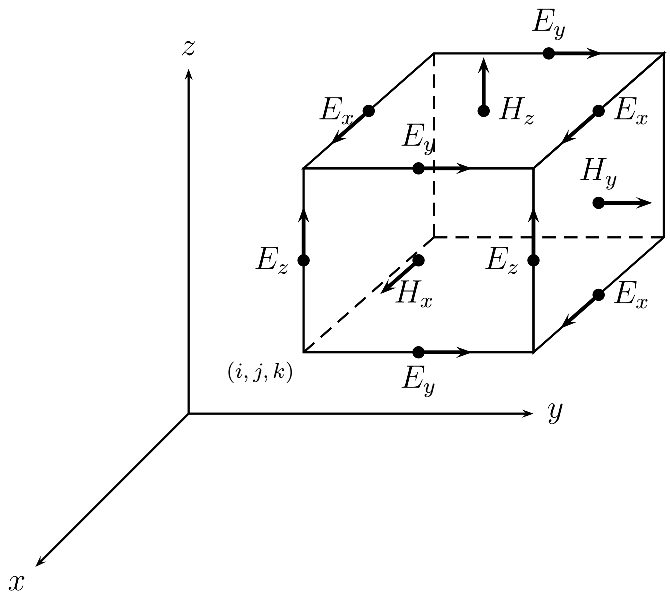


Figure 3.1: The 3D Yee cell, after (Davidson, 2011).

$$x_i = (i-1)\Delta x, \qquad i = 1, 2, \ldots, N_x \tag{3.1}$$
$$y_j = (j-1)\Delta y, \qquad j = 1, 2, \ldots, N_y \tag{3.2}$$
$$z_k = (k-1)\Delta z, \qquad k = 1, 2, \ldots, N_z \tag{3.3}$$
$$t_n = (n-1)\Delta t, \qquad n = 1, 2, 3, \ldots \tag{3.4}$$
$$\tag{3.5}$$

and half-grid points by

$$x_{i+1/2} = (i-1/2)\Delta x, \qquad i = 1, 2, \ldots, N_x \tag{3.6}$$
$$y_{j+1/2} = (j-1/2)\Delta y, \qquad j = 1, 2, \ldots, N_y \tag{3.7}$$
$$z_{k+1/2} = (k-1/2)\Delta y, \qquad k = 1, 2, \ldots, N_y \tag{3.8}$$
$$t_{n+1/2} = (n-1/2)\Delta t, \qquad n = 1, 2, 3, \ldots \tag{3.9}$$

This provides a spatial grid with the appropriate electric fields circulating around each magnetic field and offset in time by $\Delta t/2$. The spatial mesh sizes are given by $\Delta x = \frac{X}{\mathcal{N}_x}$; $\Delta x = \frac{Y}{\mathcal{N}_y}$ and $\Delta x = \frac{Z}{\mathcal{N}_z}$ where $\mathcal{N}_x = N_x - 1$ is the number of Yee cells in the x direction, one less than the number of x nodes (and similarly for y and z). The time step is of course restricted by the Courant limit:

$$\Delta t \leq \frac{1}{c\left[\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}\right]^{1/2}} \tag{3.10}$$

Writing out the Maxwell curl equations Eq. (1.1) and (1.2), and applying central differencing scheme with the spatial and temporal locations as defined above, the update equations for the three magnetic fields in a lossless region are:

$$\begin{aligned} H_x\Big|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} &= H_x\Big|^{n-\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} \\ &+ \frac{\Delta t}{\mu}\left[\frac{E_y|^n_{i,j+\frac{1}{2},k+1} - E_y|^n_{i,j+\frac{1}{2},k}}{\Delta z} - \frac{E_z|^n_{i,j+1,k+\frac{1}{2}} - E_z|^n_{i,j,k+\frac{1}{2}}}{\Delta y}\right] \end{aligned} \tag{3.11}$$

$$\begin{aligned} H_y\Big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} &= H_x\Big|^{n-\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} \\ &+ \frac{\Delta t}{\mu}\left[\frac{E_z|^n_{i+1,j,k+\frac{1}{2}} - E_z|^n_{i,j,k+\frac{1}{2}}}{\Delta x} - \frac{E_x|^n_{i+\frac{1}{2},j,k+1} - E_x|^n_{i+\frac{1}{2},j,k}}{\Delta z}\right] \end{aligned} \tag{3.12}$$

$$\begin{aligned} H_z\Big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} &= H_z\Big|^{n-\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} \\ &+ \frac{\Delta t}{\mu}\left[\frac{E_x|^n_{i+\frac{1}{2},j+1,k} - E_x|^n_{i+\frac{1}{2},j,k}}{\Delta y} - \frac{E_y|^n_{i+1,j+\frac{1}{2},k} - E_y|^n_{i,j+\frac{1}{2},k}}{\Delta x}\right] \end{aligned} \tag{3.13}$$

and for the electric fields,

$$\begin{aligned} E_x\Big|^{n+1}_{i+\frac{1}{2},j,k} &= E_x\Big|^{n}_{i+\frac{1}{2},j,k} \\ &+ \frac{\Delta t}{\varepsilon}\left[\frac{H_z|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z|^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2},k}}{\Delta y} - \frac{H_y|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k-\frac{1}{2}}}{\Delta z}\right] \end{aligned} \tag{3.14}$$

$$\begin{aligned} E_y\Big|^{n+1}_{i,j+\frac{1}{2},k} &= E_y\Big|^{n}_{i,j+\frac{1}{2},k} \\ &+ \frac{\Delta t}{\varepsilon}\left[\frac{H_x|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} - H_x|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k-\frac{1}{2}}}{\Delta z} - \frac{H_z|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z|^{n+\frac{1}{2}}_{i-\frac{1}{2},j+\frac{1}{2},k}}{\Delta x}\right] \end{aligned} \tag{3.15}$$

$$\begin{aligned} E_z\Big|^{n+1}_{i,j,k+\frac{1}{2}} &= E_z\Big|^{n}_{i,j,k+\frac{1}{2}} \\ &+ \frac{\Delta t}{\varepsilon}\left[\frac{H_y|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y|^{n+\frac{1}{2}}_{i-\frac{1}{2},j,k+\frac{1}{2}}}{\Delta x} - \frac{H_x|^{n+\frac{1}{2}}_{i,j+\frac{1}{2},k+\frac{1}{2}} - H_x|^{n+\frac{1}{2}}_{i,j-\frac{1}{2},k+\frac{1}{2}}}{\Delta y}\right]. \end{aligned} \tag{3.16}$$

```
H_x = H_x + Delta_t/mu * (diff(E_y,1,3)/Delta_z - diff(E_z,1,2)/Delta_y);
H_y = H_y + Delta_t/mu * (diff(E_z,1,1)/Delta_x - diff(E_x,1,3)/Delta_z);
H_z = H_z + Delta_t/mu * (diff(E_x,1,2)/Delta_y - diff(E_y,1,1)/Delta_x);
```

Figure 3.2: MATLAB code stub for updating H in 3D.

Here, we have adopted the compact sub- and superscript notation of (Taflove *&* Hagness, 2005; Bondeson *et al.*, 2005), as otherwise the equations become very lengthy, but have still explicitly indicated the half-steps.

An FDTD code proceeds by repetitively updated first the magnetic and then the electric field vectors. Clearly, as an initial value formulation, a source is needed. This can be injected into the grid in various ways; see, for instance, (Davidson, 2011:Chapters 3 & 4).

Regarding coding, the necessity of using vectorized operations for execution speed is well known. MATLAB has a very efficient method of forming differences; this is the `diff` command (Bondeson *et al.*, 2005), and an application to coding Eqs. (3.11)–(3.13) efficiently is shown in Figure 3.2. The first argument of the function is the matrix to be differenced, the second is the order of differentation, and the third is the dimension along which the differencing is performed. Note that the appropriate spatial step ($\frac{1}{\Delta_z}$ and $\frac{1}{\Delta_y}$ respectively, in this case) must still be explicitly included to form the approximation of the relevant partial differential. This code also demonstrates the use of in-place operations, with H_x being overwritten immediately the new value is computed. (In MATLAB, one should be aware that temporary copies of the field arrays are created during such operations, which can result in more memory usage than intended).

When coding the FDTD, the dimensions of the matrices representing the field components will usually be slightly different. The reason is simply that the $\boldsymbol{H}$ fields are positioned in the centre of cell faces (in our, and the original Yee cell), whereas the $\boldsymbol{E}$ fields are positioned on edges. As mentioned, it is usually convenient to have tangential $\boldsymbol{E}$ fields postitioned on the outer boundaries of the region, as very often, a PEC boundary is applied there (in which the relevant field component is simply zeroed). For a cubical region with $\mathcal{N}_x \times \mathcal{N}_y \times \mathcal{N}_z$ Yee cells the dimensions are shown in Fig. 3.3. Looking at one in detail, we see that E_x, which is positioned in the middle of each edge in the x-direction, will have only $\mathcal{N}_x$ samples in that direction, but clearly $\mathcal{N}_{y+1}$ and $\mathcal{N}_{z+1}$ in the y and z directions respectively. One has to be careful to ensure that the dimensions in code such as Fig. 3.2 are compatible (here, they are); for the E_x update, suitable code is given in Fig. 3.4.

When developing an FDTD solver, most of the challenges relate to including sources, correctly treating material boundaries, and of course terminating the mesh. The last has seen some highly efficient boundary conditions, in particular the perfectly matched layer, developed and deployed. The first two

```
E_x = zeros(N_x,  N_y+1,N_z+1);
E_y = zeros(N_x+1,N_y,  N_z+1);
E_z = zeros(N_x+1,N_y+1,N_z);
H_x = zeros(N_x+1,N_y,  N_z);
H_y = zeros(N_x,  N_y+1,N_z);
H_z = zeros(N_x,  N_y,  N_z+1);
```

Figure 3.3: MATLAB code stub dimensioning field arrays. Note that `N_x` in the code corresponds to the number of Yee cells in the x direction, $\mathcal{N}_x$ in the text, and similarly for `N_y` and `N_z`)

```
E_x(:,2:N_y,2:N_z) = E_x(:,2:N_y,2:N_z) + Delta_t/epsilon * ...
  (diff(H_z(:,:,2:N_z),1,2)/Delta_y - diff(H_y(:,2:N_y,:),1,3)/Delta_z);
E_y(2:N_x,:,2:N_z) = E_y(2:N_x,:,2:N_z) + Delta_t/epsilon * ...
  (diff(H_x(2:N_x,:,:),1,3)/Delta_z - diff(H_z(:,:,2:N_z),1,1)/Delta_x);
E_z(2:N_x,2:N_y,:) = E_z(2:N_x,2:N_y,:) + Delta_t/epsilon * ...
  (diff(H_y(:,2:N_y,:),1,1)/Delta_x - diff(H_x(2:N_x,:,:),1,2)/Delta_y);
```

Figure 3.4: MATLAB code stub for updating E in 3D.

are often more problem-specific. This brief introduction will not attempt to address these, but rather efer the reader to the well established literature on the topic, for example (Davidson, 2011; Taflove & Hagness, 2005), and instead will move onto my own contributions to the FDTD.

3.3 Modelling of optics at the University of Arizona, 1993

My first work on the FDTD did not address either stealth technology or cellphones. I was invited to spend January to June 1993 as a Visiting Scholar at the University of Arizona during my first sabbatical. I worked there with Rick Ziolkowski at the Department of Electrical and Computer Engineering. The aim of the research was to work on optical devices using computational electromagnetic methods, with the ultimate aim of developing a suite of computer programs able to provide a full-wave vector model of both linear and non-linear optical devices using Maxwell's equations and various material models. The work we completed served to establish the base for this goal.

I initially developed a 3D FDTD code running on both conventional serial computers (in FORTRAN 77) and also on the Connection Machine-2 (a

SIMD[2] MPP[3] computer) in FORTRAN 90. The computational requirements for solving Maxwell's equations for the 3D case were imposing at the time, hence the Connection Machine code, to permit sizeable problems to be run. Results were obtained for the propagation of very-short-pulse Gaussian beams, showing the spatial and temporal evolution of the pulse from the launching aperture to approximately the Rayleigh distance. We quickly realized that despite the Connection Machine, the optical problems that we ultimately wanted to address would be beyond our computational resources.

At this stage, with the Body of Revolution MoM formulation reasonably fresh in my mind from my Master's work, a BOR-FDTD formulation presented itself as a solution. I developed such a code able to exploit the rotational symmetry of Gaussian beams and typical optical components such as lenses. We then applied it to a number of examples of Gaussian beams interacting with lenses. The time domain formulation permitted us to explore the physics of ultra wide-band pulses (with very short time durations) and also predict novel intensity enhancement effects that are possible by properly shaping the time history of the pulse.

A sequence of results from (Davidson & Ziolkowski, 1994) is shown in Fig. 3.5. These plots show the propagation of a narrow pulse through a lens; in the first figure, the pulse is mainly interacting with the lens; in the second, it has passed through the lens and is focussing, and in the last, it is at the focus. Although perhaps commonplace nowadays, in 1993 this was a remarkable visualization of ultra-short optical phenomena. Rick Ziolkowski would go on to work on non-linear optics, a field where the FDTD is unchallenged (no time-harmonic method can readily handle highly non-linear behaviour), before turning his attention to metamaterials, where he has made major contributions.

Rick Ziolkowski and I worked together very closely during my stay at Arizona. He had extensive experience with the FDTD method and he originally proposed applying the method to optical problems. I did most of the computer program development, with frequent discussions of the progress and results; he did most of the analytical work that guided our numerical experimentation on ultra-short pulses. It was a most enjoyable collaborative effort in the true sense of the word.

In assessing the work, our aim was to make an impression in the optics community with CEM methods. Our papers that appeared in highly regarded optics journals (Davidson & Ziolkowski, 1994) and (Ziolkowski & Davidson, 1994)[4] established a presence in this field. The JOSA paper has been referenced very extensively in the CEM literature; after my book, it is my most cited output and continues to gather citations to this day. I also continued

[2]Single Instruction Multiple Data.

[3]Massively Parallel Processor.

[4]The essence of the work may be found in the latter, and more details on the implementation and the theoretical analysis in the former.

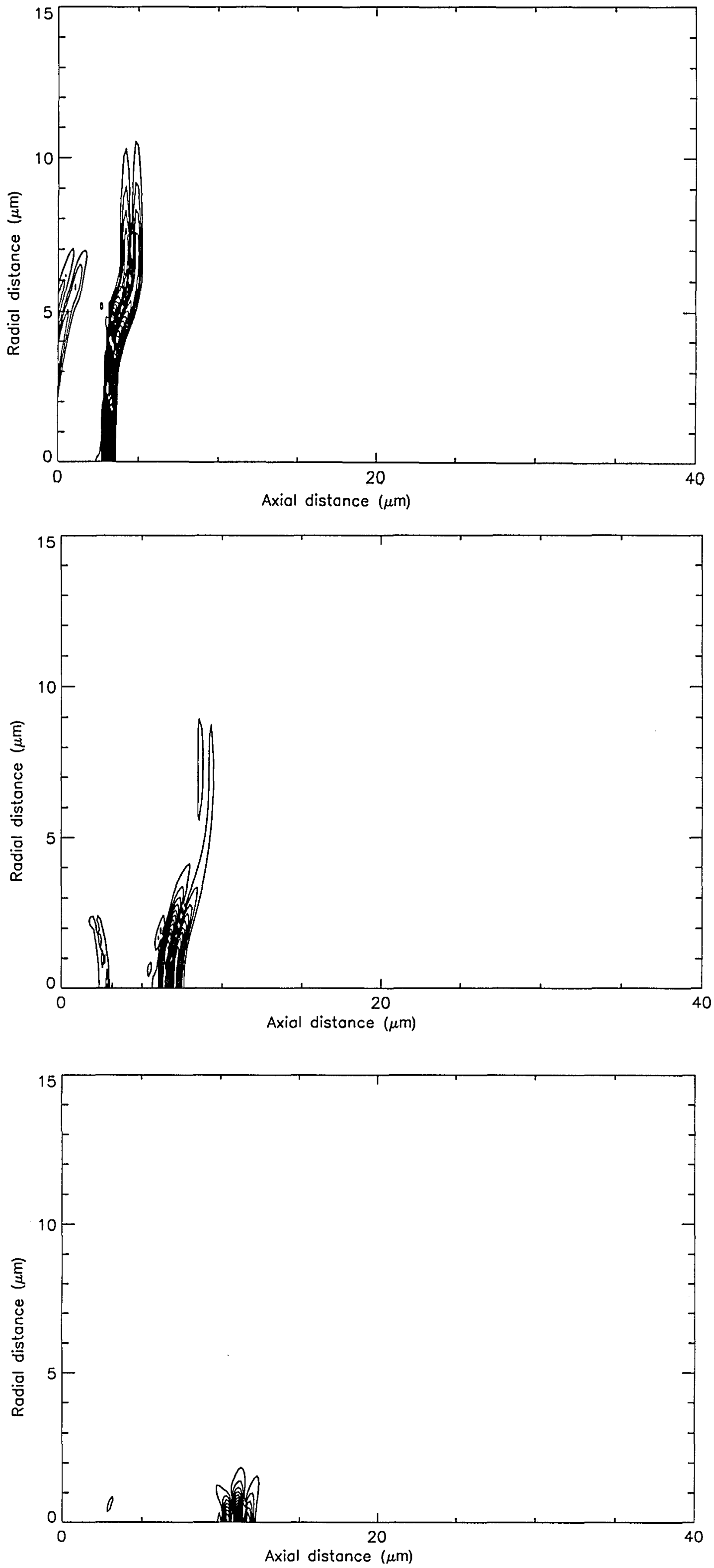


Figure 3.5: The propagation of a narrow pulse through a lens, computed with a BOR-FDTD code — see text for discussion. After (Davidson & Ziolkowski, 1994:Fig. 2–4).

a presence in the international parallel processing literature via (Davidson & Ziolkowski, 1995). As noted above, Rick Ziolkowski continued to work on optics applications for some time, but I re-focussed my work on microwave applications after returning to Stellenbosch.

This work and extended visit had some important spin-offs. In general, it laid the basis for all my subsequent FDTD work. Specifically, it formed the basis of Marianne Bingle's M.Eng thesis (Bingle, 1995) and her subsequent PhD (Bingle, 1998), as well as Wil van der Leij (Van der Leij, 1999), Ernst Burger (Burger, 2000) and Peter Futter's (Futter, 2001) M.Sc. theses. I also met the EM group at Arizona State University during this period. Prof Jim Aberle of ASU visited in 1999 as an invited speaker for Africon'99, and co-presenter — with Ulrich Jakobus, Frans Meyer and myself — of a short course on CEM. This course was the genesis of my book on CEM. Many of these topics are unpacked in more detail subsequently in this dissertation.

3.4 FDTD modelling of frequency selective surfaces, 1993–97

After returning from Arizona, I became involved with the design of frequency selective surfaces (FSS). These are used in aerospace applications to provide electromagnetic "windows" at specific frequencies for the aircraft's own radars etc., whilst reflecting signals of other frequencies. (The application that this work was originally contracted for was the design of "stealth" airframes, and linked in with work by M.Eng student Craig Wilsen (Wilsen, 1996) that I will mention in the next chapter). This work was undertaken for Kentron, and was originally done in collaboration with AMS Polymers of Stellenbosch. A FDTD program developed by me in 1994 was later extended by Johann van Tonder (another of John Cloete's graduates) to include off-normal incidence. Van Tonder also investigated a ray-propagation model hybridised with the FDTD for a curved FSS; preliminary results were promising, but the computational requirements proved completely impractical. Increasingly accurate measurements were made by Gronum Smith (also a former student of John Cloete's). We manufactured a number of FSS test samples, as well as an FSS coated interior hemi-spherical radome (an idea I independently proposed to Kentron). By careful comparison of predicted (FDTD simulation) and measured data, we discovered a variety of problems associated with the manufacture of FSS structures — in particular, the necessity of modelling possible air-inclusions accurately, or conversely, eliminating them during manufacture. Progress on this was reported at a variety of symposia; our ICAP paper in 1997 summarized much of our work (Davidson, Smith & van Tonder, 1997*a*), and our most significant results were published as (Davidson, Smith & van Tonder, 1997*b*).

In assessing this work, there is no question that the apparent simplicity of an FSS is belied by the actual issues involved in real FSS structures, such as finite metallisation thickness, the detuning effects of the material supports, and the doubly-curved nature of structures such as radomes. An example of the first is shown in Fig. 3.6, which shows the detuning effect of a thin air inclusion due to the finite thickness of the metal sheet in which the FSS pattern has been etched.

When the work work which I initiated and summarised in (Davidson *et al.*, 1997*b*) was published in 1997, this was state-of-the-art in South Africa, and (obviously aside from possible "black" facilities in the USA, which I could not then and still cannot assess) competitive in international terms. The main problem which remained was the application of a suitable FSS to a real radome; I moved the focus of this project over to EMSS as the manufacturing and practical engineering issues started to overtake the research problems, and I have not been personally involved since early 1997. This project eventually stopped, due both to a decline in interest in South Africa and the untimely death in an automobile accident of Detlev Grygier of Kentron, who had been

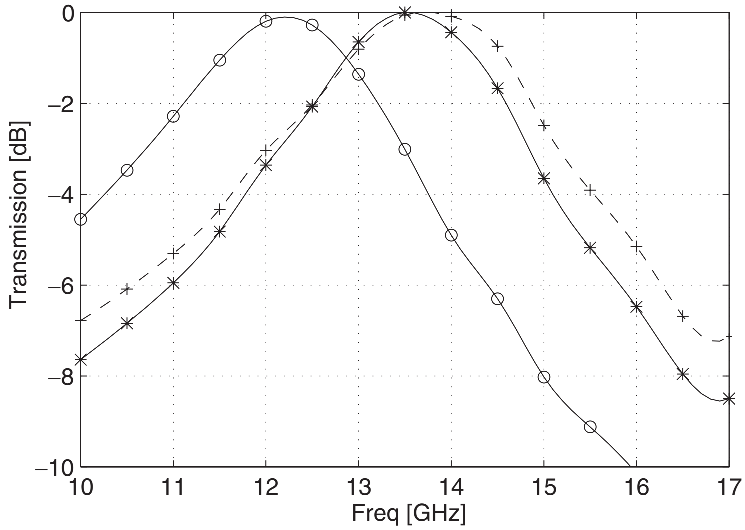


Figure 3.6: Predicted transmission coefficient of an O-ring FSS with one side perspex only. Legend: solid line (o), infinitely thin metal sheet, single-cell perspex in the slot; solid line (x), infinitely thin metal sheet, single-cell air in the slot; dashed line (+), actual 0.26845 mm thick metal sheet; air in the slot. After (Davidson, Smith & van Tonder, 1997*a*).

the major driving force behind the development of the technology in South Africa.

3.5 FDTD work for materials simulation: 1994–2002

Marianne Bingle achieved the distinction in December 1998 of being Stellenbosch University's first female PhD graduate in electronic engineering. Starting in 1994, John Cloete and I jointly supervised first her M.Eng work, and then her doctoral research. For her M.Eng, Marianne Bingle extended my FDTD-BOR code, adding realistic dispersive material models to it; at this time, this was a very "hot" topic in FDTD research, since frequency-dependent materials were only starting to be modelled correctly in FDTD codes (Bingle, 1995).

For her doctorate, she worked on chiral materials (Bingle, 1998). In the late 80's, numerous papers appeared suggesting that electromagnetic absorbers with exceptional properties could be made by utilising the additional "degrees of freedom" afforded by chiral (optically active) materials. Given the interest in RAM (radar absorbent materials) at the time, this has considerable commercial possibilities — were it true. John Cloete, and other researchers in the field, became increasingly suspicious of these claims, and he proposed to us that we try to substantiate or reject these claims. This Marianne Bingle did, using as a test wire hooks. By simply bending these differently, they could exhibit chiral or non-chiral properties, possibly embedded in dielectric hosts. We used the FDTD method, with some elegant extensions for finitely-conducting wires, combined with some experimental validation, for these studies. John Cloete posed the basic physical questions; I provided assistance with the FDTD method; and Marianne Bingle did the actual coding and measurements, as well as contributing extensively theoretically. Our final conclusion was that it is the inclusion of resonant wire structures that provides the absorption; they do not need to be chiral to absorb energy, and we found far more differences between various inter-wire spacings etc. that we did between chiral and non-chiral versions of the same "crystal". We summarised these conclusions in (Cloete, Bingle & Davidson, 2001), and published details on the FDTD simulation in (Bingle, Davidson & Cloete, 2002). Both of these papers have gathered a very respectable number of citations over the years. Fig 3.7 shows results from the latter, validating the thin-wire FDTD formulation and coding. This incorporated the high-frequency internal impedance model of a round ohmic conductor into the subcell thin-wire formulation of the FDTD method, permitting accurate modelling of the the microwave properties of metal wires.

In assessing Dr Bingle's research, a large number of conference papers (six international and three local) were presented in addition to the two journal papers. Her paper on her Master's work won a best-paper prize at our national symposium in 1995. She won two URSI Young Scientist's Grants (the same award that I won in 1992) to present her doctoral research at the URSI 1998 International Symposium on Electromagnetic Theory and then at the XXVI-th General Assembly of URSI in 1999. Following her PhD, she spent two years at Eindhoven University in the Netherlands on a post-doctoral position, and then returned to South Africa to work for EMSS-SA as part of the FEKO kernel development team, where she is still presently working.

3.6 Some other FDTD work in the late 1990s

3.6.1 FDTD work for active antenna array simulation: 1996-98

Contemporaneously with Dr Bingle, I also co-supervised Kevin Williams's doctoral work on active antenna simulation with my departmental colleague Prof Howard Reader. Active antenna arrays distribute the microwave amplifiers to the terminals of each array element; using injection-locking, it it possible to change the phase of each element and thus steer the beam. There were two aspects to this project: firstly, the microwave electronics, and secondly, the electromagnetics of the coupling between arrays, which of course "loads" each array element. My involvement was on the second aspect; Kevin Williams extended an FDTD code to quite successfully compute coupling between microstrip arrays. However, the microwave electronics proved very challenging;

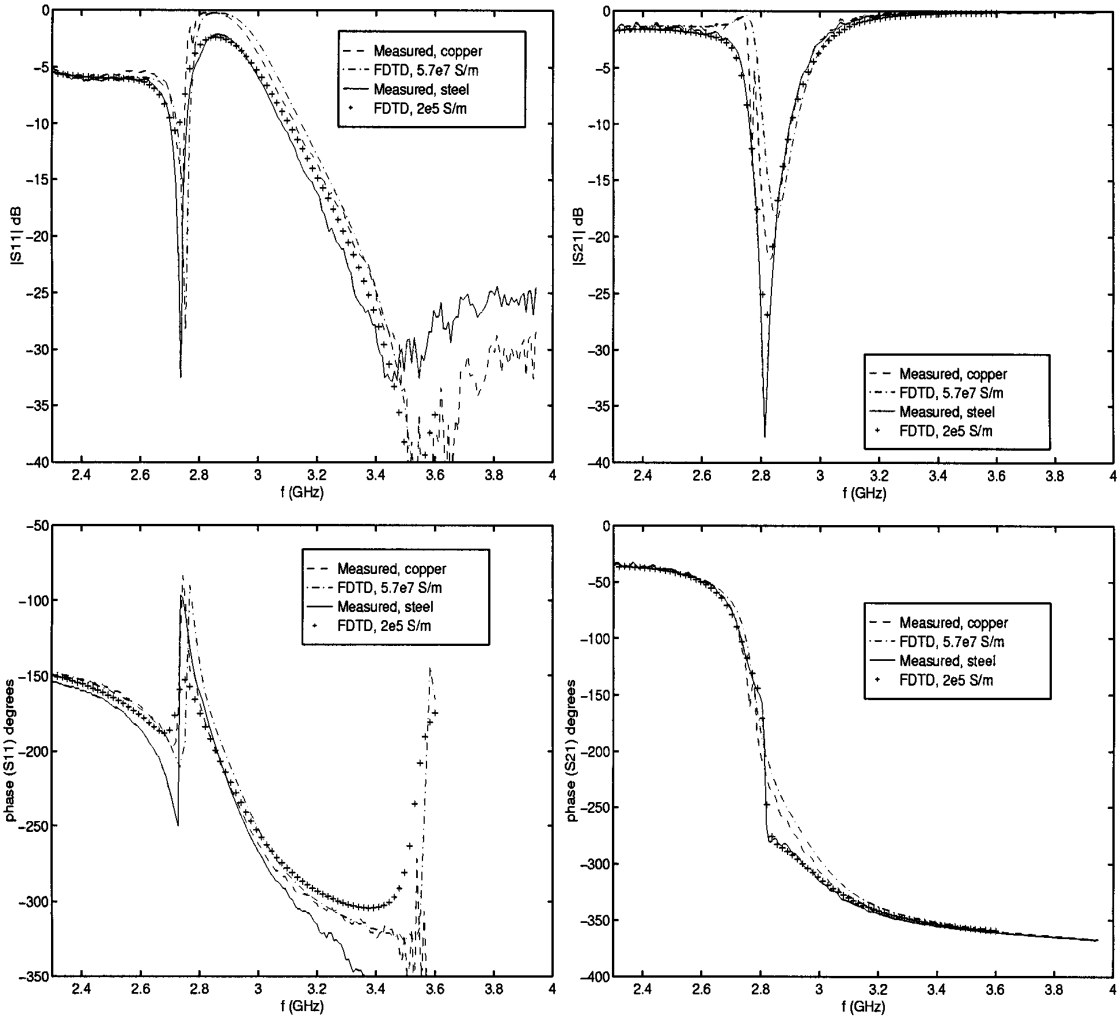


Figure 3.7: Measured and predicted scattering parameters for copper ($\sigma_{wire} = 5.7 \times 10^7$ S/m) and steel ($\sigma_{wire} = 2.0 \times 10^5$ S/m) unit cells of cranks (chiral hooks) in polystyrene foam. The unit cells were illuminated in S-band rectangular waveguide. Calibration was with respect to the front of the sample. After (Bingle *et al.*, 2002).

he was unable to demonstrate a steerable, active array which was the original goal, but nonetheless his PhD demonstrated the competence expected of a doctoral graduate and he made a number of useful contributions. One examiner lauded in particular the multi-disciplinary nature of his work. Whilst a few conference papers followed from the work, there were no journal publications and this work finished at this stage.

3.6.2 FDTD synthetic SAR for humanitarian de-mining: 1998–2000

Another spin-off from the BOR-FDTD code developed in Arizona was a Master's project jointly supervised with my then departmental colleague Prof David Weber[5]. The student, Wil van der Leij, extended my FDTD-BOR code to generate on-axis returns for buried landmines, and worked on signal processing recognition techniques to discriminate landmines from clutter (Van der Leij, 1999). At the time, humanitarian demining was a major challenge as Southern Africa and the Balkans were left dealing with the legacy of undocumented landmines laid during years of guerilla or civil wars. Ironically, the work was not funded, and when David Weber left SU, the work on this also came to an end.

3.6.3 FDTD simulation of borehole radars: 1999-2001

John Cloete initiated a project to develop a borehole radar at around the start of the new millennium. The specific application was for underground mining, in particular for improving mining safety. With a team in the Department, a radar which could fit into the approximately 40 mm diameter borehole widely drilled in the mining industry was developed. With it came an interest in EM simulation tools to assist in the development of imaging algorithms. Ernst Burger worked on this for his Master's thesis (Burger, 2000), and the work was taken further by Peter Futter (Futter, 2001). Although the borehole radar project was successful, the technology was sold to a foreign company and combined with John's retirement and subsequent ill-health, our involvement in supporting the project also ended.

3.7 A return to the FDTD - deployment on major HPC platforms

in 2009, Bob Ilgner joined my group. With a background initially in geophysical exploration, and later in software development, he was mature student. His work was supported by a generously funded "Flagship Project" from the then

[5]He subsequently left SU to pursue a career in industry

newly-established national facility, the Centre for High Performance Computing, entitled "HPC electromagnetic simulation for the MeerKAT and SKA". In his PhD (Ilgner, 2013), he implemented the FDTD on a wide variety of HPC platforms, ranging from low-end GPU cards to then state-of-the-art supercomputers such as the SUN M9000 shared memory processor, SUN Nehalem cluster, and IBM Blue Gene/P systems installed at the CHPC, as well as a Blue Gene/Q available at a German supercomputer centre. The CHPC staff were particularly interested in the comparative results across their platforms. As the code and metrics were consistent, this permitted careful evaluation of the relevant performance/cost issues, published in (Ilgner & Davidson, 2014), Additionally, this has provided new tools for the analysis of propagation on the Karoo SKA site, which were used in Pienaar's PhD (described subsequently). Ilgner continued work on this during his post-doc, and work using ARM processors on smartphones for FDTD simulations was published as (Ilgner & Davidson, 2015).

3.8 Conclusions

This chapter has outlined my contributions to the FDTD, and its applications, largely during the 1990s. The most significant algorithmic development was with Rick Ziolkowski, with the BOR-FDTD implementation. A number of useful applications have been elucidated; of these, the frequency selective surface work and the chiral simulation had the most impact in industry and the research community respectively. Our recent work on evaluating FDTD deployments on a variety of HPC platforms should also serve as a useful reference for future researchers.

During the period when I working largely on the FDTD, I was promoted to full professor; a photograph taken at my inaugural lecture, Fig. 3.8, is a poignant reminder of the academic legacy of Prof John Cloete. Everyone in the photograph has a doctorate in electromagnetics, seven of them supervised by Prof Cloete, and aside from his, the other nine were all awarded between 1991 and 1996 at Stellenbosch University.

The FDTD is a very powerful method, but the Finite Element Method also offers the ability to readily handle inhomogeneous materials, with the additional advantage of not being constrained by the regular cuboidal ("Yee cell") meshing of the FDTD. I had long been interested in the FEM, and from my sabbatical at Cambridge University in 1997 onwards, I was to focus most of my own research effort on the FEM until my appointment to the SKA SARChI chair in 2011. This work is described in the next chapter.

Figure 3.8: Prof Johannes (John) Cloete and a group of the first doctoral graduates from the SU Electromagnetics group. This photo was taken at my inaugural lecture, 13 June 1996. Left to right, front row: Dr Pierre Steyn, Dr Johan van Tonder, Dr Isak Theron and Dr Marius du Plessis. Back row: the author (in the robes), Prof John Cloete, Dr Leendert (LJ) du Toit, and Dr Gronum Smith. Inserts: Dr Frans Meyer (left); Dr Nelis du Toit (right)

Chapter 4

Contributions to the FEM

4.1 Finite Element Work

For around half my time at Stellenbosch, the Finite Element Method (FEM) was the main thrust of my research work. However, from my arrival in Stellenbosch in 1988, it had already formed an important part of the work by my research group; one of the first industrial projects that I undertook was a study of 2D scattering using a very early commercial FE code.

As with the preceding chapters, this chapter starts with a brief review of key FEM concepts for CEM.

4.2 A brief overview of the FEM for full-wave modelling

For electrodynamic problems, subject to the deterministic vector wave equation,

$$\nabla \times \frac{1}{\mu_r} \nabla \times \boldsymbol{E} - k_0^2 \epsilon_r \boldsymbol{E} = -jk_0 Z_0 \boldsymbol{J} \tag{4.1}$$

with $\boldsymbol{J}$ a source internal to domain Ω and k_0 the free-space wavenumber, the equivalent variational functional which must be rendered stationary is:

$$F(\boldsymbol{E}) = \int_\Omega \left[\frac{1}{\mu_r} (\nabla \times \boldsymbol{E}) \cdot (\nabla \times \boldsymbol{E}) - k_0^2 \epsilon_r \boldsymbol{E} \cdot \boldsymbol{E} \right] d\Omega + jk_0 Z_0 \int_\Omega \boldsymbol{E} \cdot \boldsymbol{J} \, d\Omega \tag{4.2}$$

This is the general functional for lossy isotropic materials; see (Jin, 2002:Chapter 6) for further discussion of this and extensions to anistropic media. It assumes either homogeneous Dirichlet or Neumann boundary conditions or a mixture of the two on the boundary of domain Ω.

A closely related functional for the source-free vector wave equation

$$\nabla \times \frac{1}{\mu_r} \nabla \times \boldsymbol{E} - k_i^2 \epsilon_r \boldsymbol{E} = 0 \tag{4.3}$$

is the following:

$$F(\boldsymbol{E}) = \int_\Omega \left[\frac{1}{\mu_r} (\nabla \times \boldsymbol{E}) \cdot \nabla \times \boldsymbol{E}) - k_i^2 \epsilon_r \boldsymbol{E} \cdot \boldsymbol{E} \right] \, d\Omega \tag{4.4}$$

subject to the same boundary conditions. In this case, the solution is the set of eigenvalues k_i and associated eigenvectors $\boldsymbol{E}_i$; if the domain contains lossy materials, the eigenvalues are complex.

In order to show the above properties, one proceeds in a fashion similar to the Poisson equation (Davidson, 2011:Chapter 10), using a vector Green's theorem for the double-curl operator. The details are available in Silvester & Ferrari (1996); Jin (2002).

This form (often called the curl-curl form) has been used for high-frequency FEM analysis for many years. However, although it appears fairly straightforward to discretize, it turned out to have a number of problems which occupied analysts for some years. One of the most important advances was the introduction of vector (edge) elements in the late 1980s, and this occupied much of my own research effort in this field.

For 2D radiation and scattering problems, TE and TM modes permit one to use FEM formulations in terms of only the axial component, which is a scalar equation and can be addressed with nodal elements. This was the approach used in some of our early work, in particular Meyer's research, described later in this chapter.

4.2.1 Vector elements on triangles – the Whitney element

One of the main advantages of the FEM over the FDTD is the geometrical modelling flexibility afforded by triangular and tetrahedral elements in two and three dimensions respectively. Initial work on the FEM used nodal elements to approximate vector fields; for many years, the analysis of waveguides, which seemed like an ideal application of FEM analysis (and was indeed one of the very first back in the 1960s), was plagued by "spurious modes". Such as mode is characterized by an eigenvalue and associated eigenvector in the high-frequency range, but not satisfying the divergence criteria and hence entirely unphysical. Such as mode can be more formally defined as *numerical solutions of the eigenproblem that do not converge to any physical mode of the electromagnetic resonator modelled as the mesh is refined* (Fernandes & Raffetto, 2002).

Vector elements on simplicial elements are defined in terms of simplex coordinates (defined shortly). These have acquired a variety of names during their development, including Whitney, Nedelec, Bossavit or simply edge-based

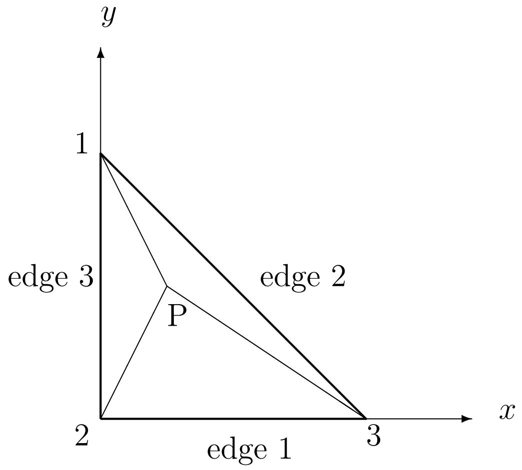


Figure 4.1: The right-angled parent triangle.

elements. As they are crucial to almost all contemporary work on the FEM for full-wave electromagnetics, including much of my own work, the following discussion, based on (Davidson, 2011), provides a brief review.

In its lowest order form, the element has the following definition:

$$\boldsymbol{w}_{ij} = \lambda_i \nabla \lambda_j - \lambda_j \nabla \lambda_i \tag{4.5}$$

There are three such elements per triangle, or six per tetrahedron, each associated with the edge from node i to node j, as will now be demonstrated.

The Whitney element is the basis for *all* vector simplicial elements, both interpolatory and hierarchal, so its properties are of great importance. Additionally, divergence-conforming elements (such as the RWG elements from MoM analysis) may be derived from these. In elucidating the Whitney element, is useful to study the right-angled triangle shown in Fig. 4.1, of unit length along the x- and y-axes. The simplex coordinates are the ratios as follows:

$$\begin{aligned} \lambda_1 &= \frac{\text{area}_{\triangle \text{P23}}}{\text{area}_{\triangle 123}} \\ &= \frac{1/2 \text{ base} \times \text{height}}{1/2} \\ &= y \end{aligned} \tag{4.6}$$

since the area of triangle 123 is 1/2, and the base of triangle P23 is unity and its height is y.

Similarly,

$$\begin{aligned} \lambda_2 &= 1 - (x + y) \\ \lambda_3 &= x \end{aligned} \tag{4.7}$$

The expression for λ_2 is easily derived from the property $\sum_{i=1}^{3} \lambda_i = 1$. With explicit expressions for the simplex coordinates, their gradients follow trivially:

$$\begin{aligned}
\nabla\lambda_1 &= \hat{y} && (4.8)\\
\nabla\lambda_2 &= -\hat{x} - \hat{y} && (4.9)\\
\nabla\lambda_3 &= \hat{x} && (4.10)
\end{aligned}$$

Note that $\nabla\lambda_1$ is normal to edge 1 (that is, the edge opposite node 1), and similarly $\nabla\lambda_2$ and $\nabla\lambda_3$ are normal to edges 2 and 3 respectively.

Now, the Whitney functions can be written in explicit Cartesian form as follows:

$$\begin{aligned}
\boldsymbol{N}_1 = \boldsymbol{w}_{23} &= \lambda_2\nabla\lambda_3 - \lambda_3\nabla\lambda_2\\
&= (1 - x - y)\hat{x} - x(-\hat{x} - \hat{y})\\
&= (1 - y)\hat{x} + x\hat{y}\\
\boldsymbol{N}_2 = -\boldsymbol{w}_{13} &= -y\hat{x} + x\hat{y}\\
\boldsymbol{N}_3 = \boldsymbol{w}_{12} &= -y\hat{x} + (-1 + x)\hat{y} && (4.11)
\end{aligned}$$

These are illustrated in Fig. 4.2.

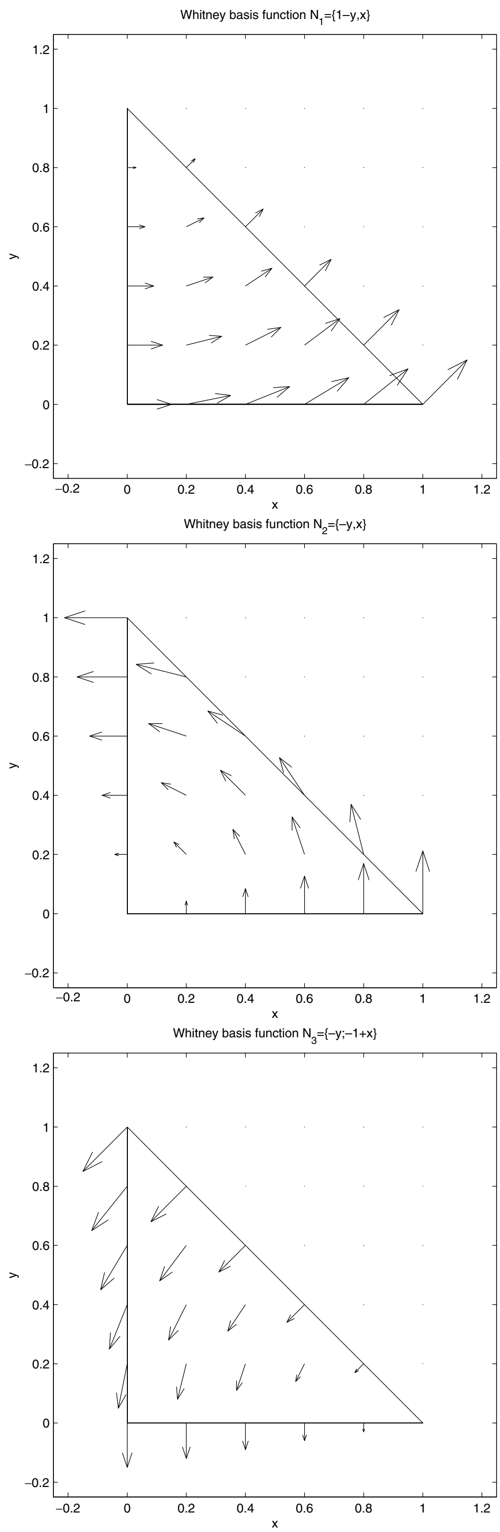


Figure 4.2: The three Whitney basis functions for triangles.

Due to the simple form of these functions on this right-angled parent element, we can immediately establish some of the crucial features of these functions. Let us focus on $\boldsymbol{N}_1 = \boldsymbol{w}_{23}$. Along edges 2 and 3, this function is purely normal, and increases linearly from node 1 to node 2 along edge 3, and similarly from node 1 to node 3 along edge 2. Along edge 1, it has both tangential and normal components. These are easily separated on this right-angled parent element; on edge 1, they are the $\hat{x}$ and $\hat{y}$ components respectively, that is, $(1-y)|_{y=0} = 1$ and x respectively. Thus, on this edge, the tangential component is constant, and the normal component is linear. In short, $\boldsymbol{N}_1 = \boldsymbol{w}_{23}$ is a basis function with a constant tangential component on edge 1, and linear normal components along all the edges. The same is easily shown for the other two basis functions. Hence, this Whitney element has mixed-order CT/LN behavior. Furthermore, suitable degrees of freedom are the average tangential fields along each edge. It is also immediately obvious from Eq. (4.11) that the divergence of the Whitney functions is zero.

An important note: although we have established these properties on a right-angled parent element, they are *generally* true for Whitney elements on *any* triangle. (Some further discussion on the Whitney element may be found in (Davidson, 2011:Appendix A).)

Regarding the normal field component, the boundary condition in this case is normal *flux* continuity; it turns out that this is a natural boundary condition of the variational process, and hence is automatically satisfied at material interfaces (Jin, 2002:Section 5.8.3).

In closing this introductory discussion on Whitney elements, it is very important to note that that the vector field can *only* be recovered by the vector sum of the three vector basis functions and the appropriate amplitudes (the degrees of freedom which the finite element procedure yields); the degrees of freedom lose the convenient interpretation of nodal elements as a field component value at a node. This again is similar to the RWG elements.

Whitney elements revolutionized HF FEM analysis from the mid 1980s on; Ansoft's Eminence package (now HFSS) was one of the first commercial codes to exploit these elements for the three-dimensional finite element analysis of high-frequency devices. Generalizing these elements to higher order was a major thrust of research in the FEM community in electromagnetics throughout the 1990s and early 2000s. A comprehensive publication in the electrical engineering literature is (Webb, 1999); later representative work is (Ingelström, 2006), and a recent and very comprehensive textbook is (Graglia & Peterson, 2016). A substantial number of researchers have made contributions. My own in this regard are discussed subsequently.

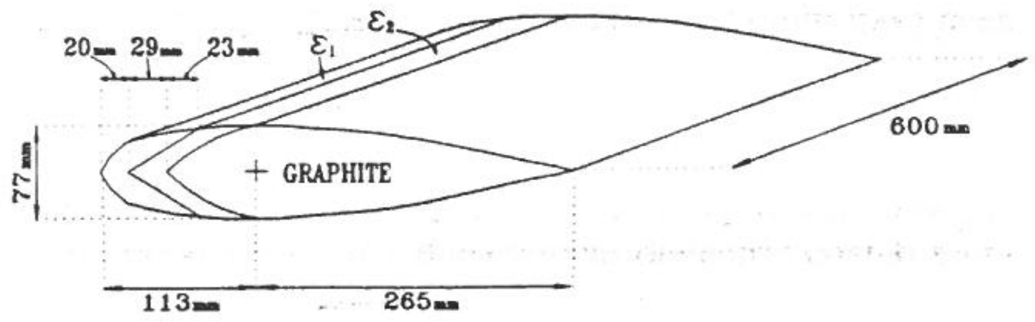


Figure 4.3: A doubly-coated graphite aerofoil. $\epsilon_r 1 = 1.2 - j0.52$; $\epsilon_r 2 = 1.5 - j5.2$ at 5 GHz. After Meyer *&* Davidson (1994*a*).

4.3 Hybrid FEM/BEM 2D modelling: 1989-94

In 1989, Frans Meyer became one of my very first post-graduate students, and started his M.Eng research on hybrid FEM/BEM (equivalently FEM/MoM) techniques. His M.Eng research was very successful (Meyer, 1991), and led directly to his doctoral work. He went deeply into the mathematical basis of the method during his PhD, while preserving a very good feel for the engineering implications of his work. He developed novel methods for error estimation and grid refinement, as well as applying the method to a number of important problems, including radar cross section calculations of coated aerofoils and the electromagnetic interaction of man-pack radios with their operators (Meyer, 1994). An example of the former is shown in Fig. 4.3[1], and the computed and measured RCS values in Fig. 4.4. The dielectric materials[2] were made by AMS Polymers [3], and the measurements were made at Aerotek[4], CSIR, Pretoria.

In assessing this work, Frans Meyer's PhD dissertation was highly praised by his examiners; one of them commented: "I would rate this dissertation amongst the best PhD theses which I have encountered in my field; as well as to himself, it does his Supervisor and Department credit". This work was extensively presented at symposia, and no less than five journal papers resulted from his work, of which (Meyer *&* Davidson, 1994*a*,*b*, 1996*a*,*b*) are the most significant. I was particularly closely involved with writing (Meyer *&* Davidson, 1996*b*). Further evidence of the quality of the work is a comment in the book[5] that Dr Meyer's thesis is the most detailed source to date on the topic

[1] After more than twenty years, the original figures were not available, and these have been scanned from the literature.

[2] Note that in the original paper, an $e^{-j\omega t}$ time convention was used, so the relative permittivities as published are the complex conjugate of those in these figure captions.

[3] This company has ceased trading.

[4] Formerly NIAST; at the time of writing, DPSS.

[5] "Finite Element Software for Microwave Engineering", Itoh et al, Wiley 1996, p381

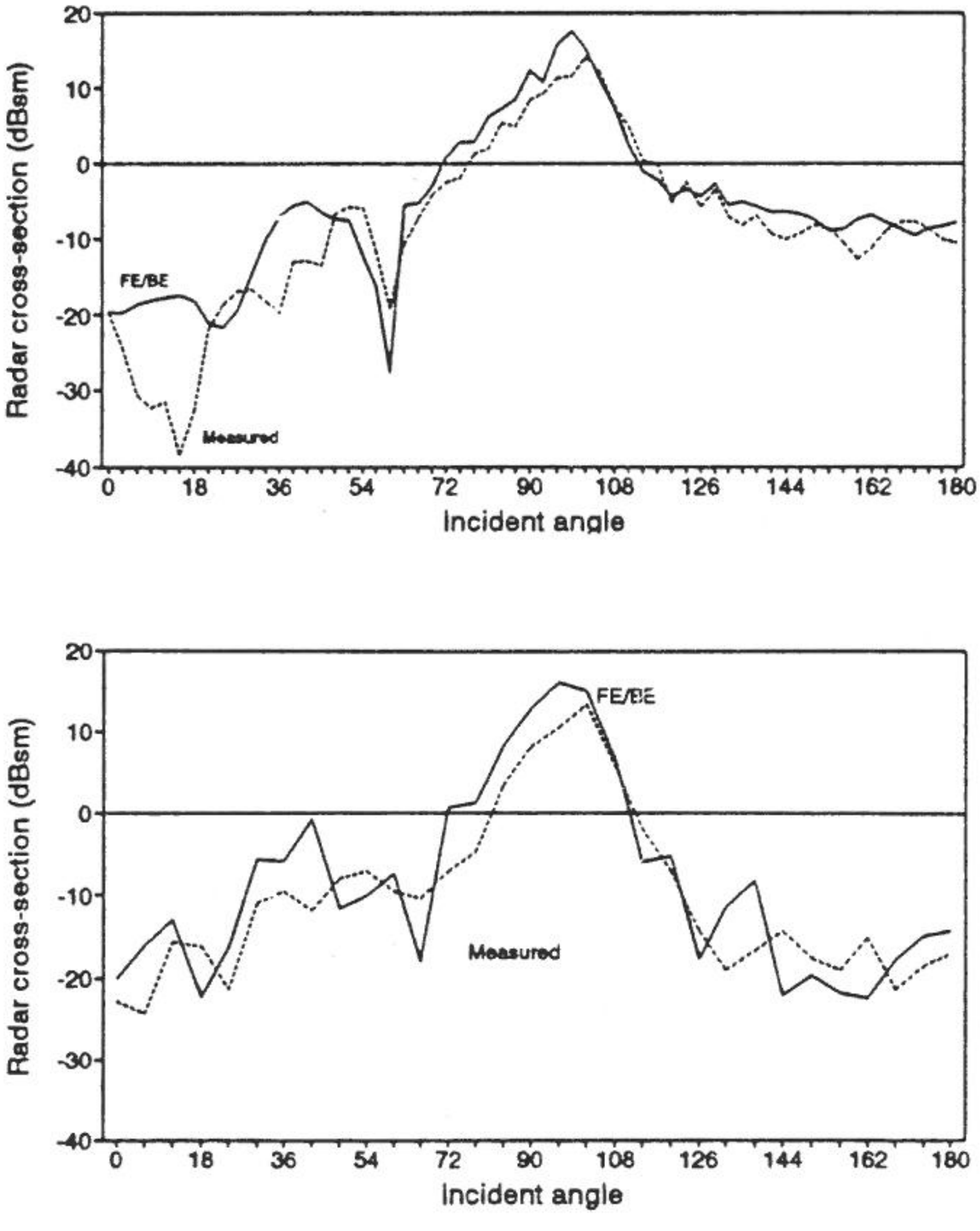


Figure 4.4: The bistatic RCS of the coated aerofoil as a function of incidence angle. Polarizations are TM (E_z) and TE (H_z), respectively; the frequency was 5 GHz and the FE model had 4354 unknowns. After Meyer & Davidson (1994*a*).

of error estimation and mesh adaptation for 2D finite elements.

4.4 3D FEM work: 1994–2011

Dr Meyer and I had hoped to immediately extend his 2D FEM work to 3D, but the demands of a start-up rendered this impossible initially. Hugo Malan started his M.Eng in 1994 under my supervision, and he worked on a specialised 3D hybrid FEM/BEM formulation for cavity backed antennas, using vector (edge-based) elements, which were a major new arrival on the FE scene at this time. Hugo Malan was another very able student indeed, and his thesis resulted in two conference papers, one international (Davidson *et al.*, 1996). He won a scholarship to Cambridge, where he completed a PhD on parallelised FEM — I was to work again with him during my sabbatical at Cambridge in 1997[6]. Craig Wilsen picked up partially from Mr Malan in 1995, working on FEM modelling of microwave antennas, with specific reference to reducing the RCS of the antenna. (This was funded by Kentron, for similar applications as the FSS work). Mr Wilsen used Mr Malan's code, as well as a commercial FEM code, but was not able to significantly extend the code. Mr Wilsen wrote an especially literary M.Eng, and his paper on it at ICAP presents some good computed and measured results, the latter measured at the University of Pretoria's compact range (Wilsen *et al.*, 1997).

In 1997, I took my second sabbatical, spending six months at Trinity College, Cambridge. I collaborated with Dr Ron Ferrari at Trinity and Dr Ricky Metaxas' Electricity Utilisation Group at the Cambridge University Engineering Department, in particular Dr David Dibben, who was doing a post-doc, and Hugo Malan. My work at Cambridge was primarily done on my own; I developed a 3D tetrahedron based FE code during this period, as well as doing some new work on element properties, reported in (Davidson, 1998). I remain in contact with Drs Ferrari and Metaxas to this day.

Also in 1997, Riana Geschke (née Hansmann) started her M.Sc(Eng); she did elegant work on 2D vector elements for her thesis (Hansmann, 1999), presented as (Davidson & Hansmann, 1999). Following this, she started her PhD; as she was also appointed as a lecturer, this proceeded slowly on occasions, but she concluded her doctoral research in 2003 on the implementation and validation of a new FEM/MoM hybrid that Dr Ferrari originally proposed for waveguide formulations; she was jointly supervised by myself, Dr Ferrari and my departmental colleague Prof Petrie Meyer. A detailed paper reporting her work appeared as (Geschke *et al.*, 2006). Although validated in the context of waveguiding problems, this formulation is applicable to computationally costly

[6]Following his PhD, he moved into the private sector, and is currently a manager of a major Sears business unit in the USA after an eventful period at Lehman Brothers immediately preceding the collapse of that bank in 2008 during the global financial crisis of 2007–2008.

problems such as (driven) loaded cavities, as used in microwave heating. Riana is now an Associate Professor at the University of Cape Town.

Matthys Botha, an exceptionally able student, worked under my supervision from 1999-2002. His research was upgraded from MSc to PhD status, and his PhD on a posteriori error estimates for 3D vector finite elements produced very significant results (Botha, 2002). Matthys was able to build on both theoretical work and practical code that I had already developed, but very quickly went on to make fundamental contributions of his own. The main contributions of his very wide-ranging thesis were on implicit and explicit methods of error estimation for the widely-used curl-curl functional form, and p-adaptation based on this. His research resulted in a number of publications, of which the most important were the journal papers (Botha *&* Davidson, 2004, 2005, 2004). Following this, he spent almost two years as a post-doc at the prestigious Centre for CEM Lab at UIUC, where continued his impressive record working with Prof Jianming Jin, publishing a significant paper on hybrid finite element - boundary integral techniques (Botha *&* Jin, 2004). He returned to South African at the end of 2004, and worked with me as a post-doc for another two years, publishing Botha *&* Davidson (2006*b*); Davidson *&* Botha (2007) with me, as well as independently working on the MoM (as mentioned in Chapter 2) and publishing the results as (Botha, 2006, 2007). Also during this period, we hosted the 8th FEM workshop in Stellenbosch in May 2006 at Spier Wine Estate. I served as General Chair, and Matthys was Technical Program Chair.

Sam Clarke completed his Masters degrees using the FEM during this period (Clarke, 2002), and presented his work as (Clarke *&* Davidson, 2002). He moved to EMSS, where within a few years he established himself as a key part of the management team, with a particular flair for new business opportunities. (Sam started MAGUS, an antenna design tool, which following the Altair buy-out of EMSS-SA, is now entirely owned by German-based CST[7]).

During this time, my own personal research considered primarily the properties of hierarchal vector tetrahedral elements. Tetrahedral elements (tets) are the 3D equivalent of triangular elements in 2D, also known as simplicial elements. An example is shown of a rectangular cavity meshed with tets in Fi 4.5. This work was presented at a number of conferences, and published as (Davidson, 2002). It also strongly influenced (Botha *&* Davidson, 2004). These papers applied hierarchal elements to waveguide problems, obtaining very good results and demonstrating the importance of considering both the usual incomplete as well as complete elements — this work appeared in (Davidson, 2003), which is also one of the last papers I solely authored, as the research group grew in size.

As the results in that paper are rather elegant, some are presented here[8].

[7]In their turn recently acquired by Dassault Systémes.

[8]This section is extracted from (Davidson, 2011:Chapter 11).

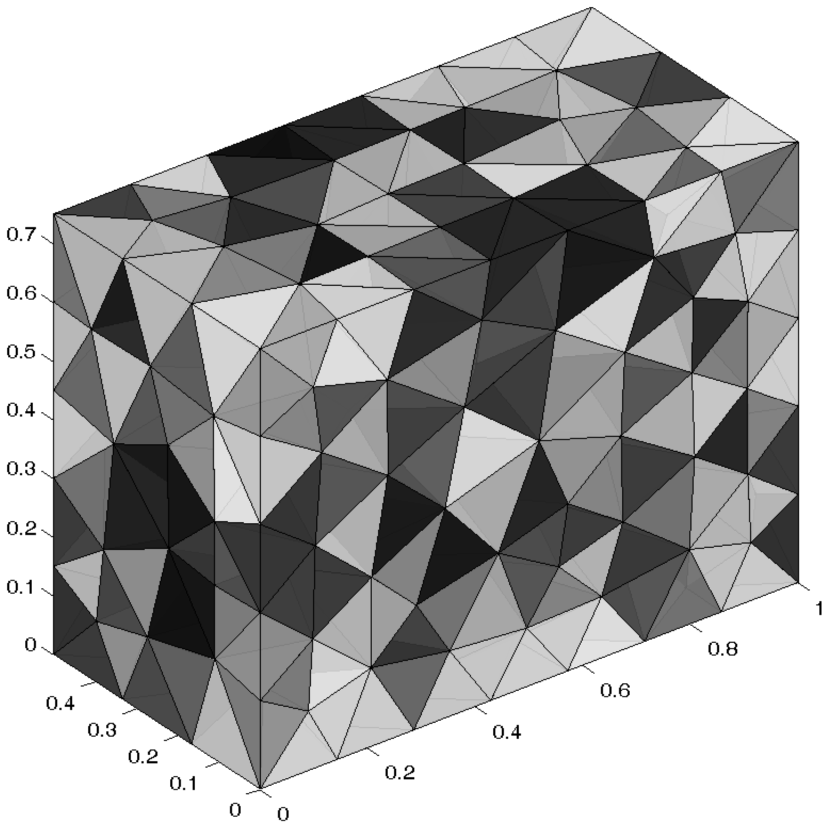


Figure 4.5: A tetrahedral mesh with 1001 elements, generated using gmsh, imported into MATLAB via a custom parsing function, after Davidson (2011).

Following Webb's argument (Webb, 2002), consider that the variational functional which is rendered stationary by the finite element procedure consists (at its simplest) of two terms, one related to the curl of the electric field and one related to the electric field itself. The curl of the electric field is the time rate of change of the magnetic field. The rationale behind mixed-order vector elements is to remove terms from the polynomial approximation of the electric field which do not contribute to the magnetic field. In problems where the electric and magnetic fields are of more or less equal importance, it makes sense only to use the polynomial terms which contribute to both fields, to obtain maximum accuracy for a given number of degrees of freedom. However, there are a number of problems of interest in RF engineering where the fields are dominated by either electric or magnetic fields. In general, a sharp edge will result in a singularity in both the electric and the magnetic fields, but for certain field and discontinuity orientations, such as the capacitive iris problem to be discussed, the singularity is in the electric field alone, and hence the field is dominated by the quasi-static electric field behavior. Hence it can be expected that full-order elements should be useful for such problems.

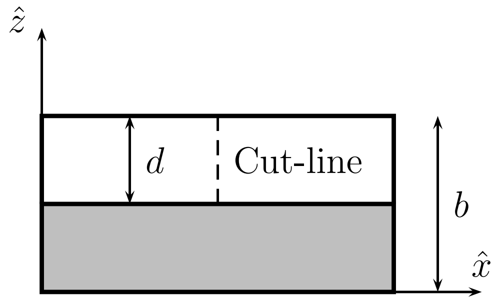


Figure 4.6: The capacitive iris. After (Davidson, 2003).

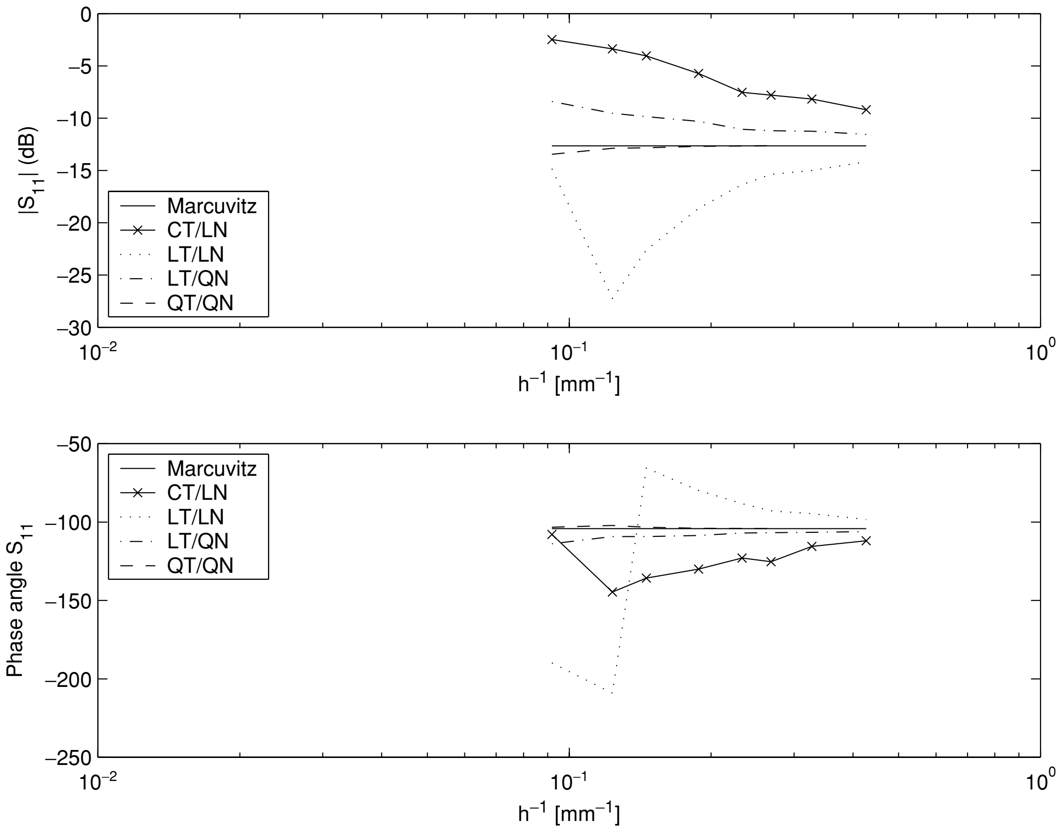


Figure 4.7: Results for a capacitive iris, compared with Marcuvitz's result, as a function of (inverse) mesh size. After (Davidson, 2003).

Here, a capacitive iris is considered[9]. The metallic iris, shown in Fig. 4.6, is half the height of the waveguide, and again, the analysis is performed at X-band. The results shown in Figs. 4.7 and 4.8 were computed at 8.25 GHz, towards the bottom end of the X-band frequency range. A number of different meshes were generated for the problem; the average edge length in the mesh varied from around $h \approx \lambda_g/6$ for the coarsest mesh to $h \approx \lambda_g/25$ for the finest.

Of interest here are the excellent results for the polynomial complete QT/QN elements, which agree very well indeed with Marcuvitz's (approximate) results (Marcuvitz, 1986). (Marcuvitz's models actually give equivalent circuit parameters. A discussion of how to convert these to S-parameters may be found in Davidson (2002).) In the region $4b/\lambda_g < 1$, which is the case at this frequency in X-band waveguide, the error bound on Marcuvitz's results is given as within 1%, a result verified by this QT/QN FEM solution. It is clear that LT/QN elements converge very slowly to the correct solution for this problem. A commercial FEM code using conventional mixed-order elements also produced unconverged results for this problem, despite incorporating adaptive mesh refinement techniques.

I also addressed practical aspects of FEM code development (Davidson, 2000). Work which I did formed the core of the FEM part of the FEM/MoM extension to FEKO undertaken by Dr Frans (Meyer, Davidson, Jakobus & Stuchly, 2003); this paper reported important results computed by Meyer on RF human exposure to GSM base stations using the FEM/MoM hybrid code he developed. The other co-authors (Jakobus and Stuchly) also made significant contributions, and this is another of my most highly cited papers.

Also linked to higher-order elements are curvilinear elements. Although these had been done in CEM, the literature on this was rather incomplete, in particular for vector elements. Neilen Marais worked on 2D curvilinear vector elements (Marais, 2003); key findings were published as (Marais & Davidson, 2006). (Neilen Marais continue a PhD in the field under my supervision after a two-year hiatus teaching English in Taiwan — more shortly). With my post-doc JP Swartz, we studied the issue in 3D, and wrote a comprehensive paper explaining how to implement a particular higher-order element (the 2nd order scheme of (Webb, 1999)) with a 2nd-order geometrical element. This was published as (Swartz & Davidson, 2007). I made far more than the usual 2nd-author contributions to this paper, coding the implementation myself. With the modelling fidelity offered by higher-order vector elements, more efficient approximate boundary conditions became of interest to me. In a closely-linked duo of papers, Matthys Botha (by then a post-doc, as noted earlier) and I explored better schemes than the traditional 1st order ABCs introduced in the late 1980s for vector elements. In (Botha & Davidson, 2006*b*), an elegant method (proposed by Botha) for circumventing a long-standing problem with the 2nd order ABC for curl-conforming vector elements was described and

[9]This example was first published as Davidson (2003).

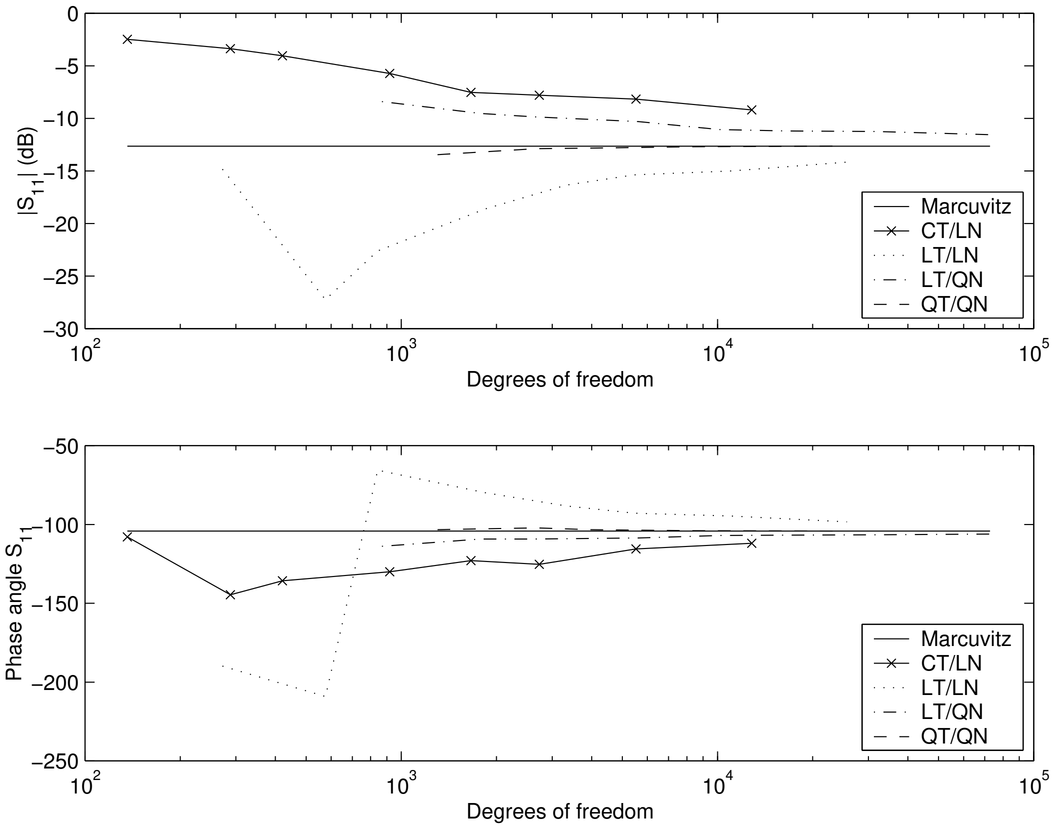


Figure 4.8: Results for a capacitive iris, compared with Marcuvitz's result, as a function of degrees of freedom. After (Davidson, 2003).

evaluated, and in (Davidson & Botha, 2007) an implementation by me of a previously proposed (but unimplemented) spherical ABC was described and evaluated. In the former paper, MMB made the dominant contribution; in the latter, I did.

As higher-order elements became increasingly widely used in the traditional frequency domain FEA for EM, time-domain finite elements were starting to be re-examined in some detail in the early 2000s. (There had been work done on this in the 1990s by several groups, using both Newmark-beta time stepping schemes using the 2nd-order wave equation for the spatial semi-discretization, as well as more FDTD-like potentially explicit schemes). During my sabbatical as a Guest Professor at the IRCTR, TU Delft, I implemented a basic FETD code in 2003, which my PhD student Neilen Marais used as the basis for his very impressive PhD thesis, which developed both the theoretical framework and a working code to hybridize higher-order implicit and explicit FETD schemes. This was a generalization to higher spatial order of the earlier work of Anders and Bondeson (Chalmers, Sweden) which hybridized a time domain wave equation/FDTD approach. This work resulted in a number of publications, including (Marais & Davidson, 2008*a*,*b*, 2010). For the first two papers,

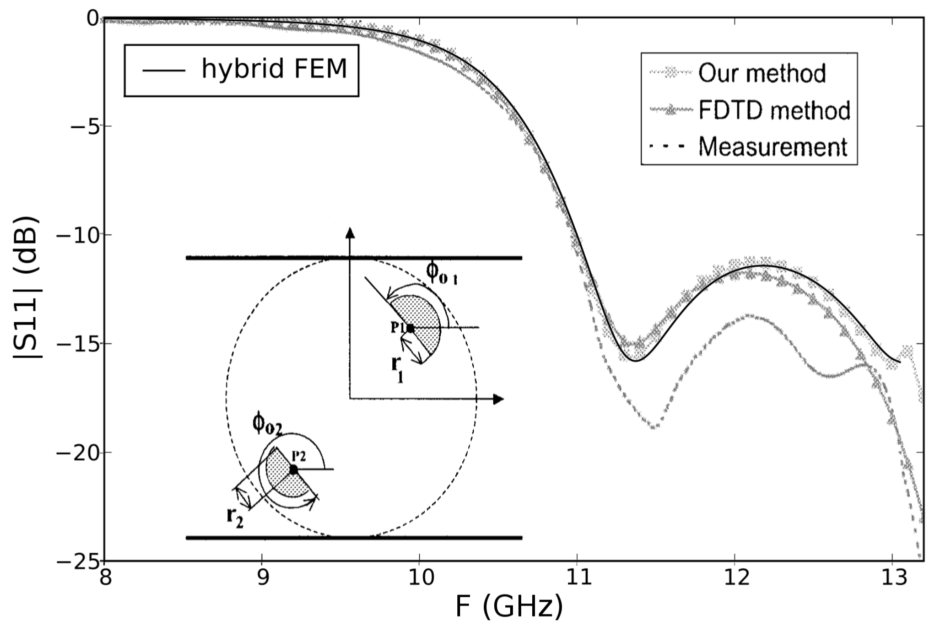


Figure 4.9: Results for the half-cylinder pin filter geometry problem with geometric parameters r_1=4 mm, ϕ_1=150°, r_2=3 mm, ϕ_2=310° mm, P_1=(5,5) mm and P_2=(−5, −5) mm, compared to FDTD results from the literature. After (Marais & Davidson, 2010:Fig. 11).

my contributions were the usual ones of a supervisor; however, I actually wrote the last paper since Marais started a new job immediately on graduating. An example of this hybrid scheme applied to a half-cylinder pin filter geometry is shown in Fig. 4.9. Fig. 4.10 shows the tetrahedral mesh around the pin filter element(s), which is solved using an implicit FETD scheme, transitioning in each case to a regular hexahedral (cubical) mesh region (not shown) in which a highly computationally efficient FDTD-like explicit scheme operates on a relatively coarse mesh.

Although the most widely cited results from Evan Lezar's PhD were on the MoM, already discussed in Chapter 2, he also worked on the FEM for his Masters thesis (Lezar, 2008), which investigated *hp*-mesh adaptation for the analysis of waveguides, as well as during his PhD and follow-on (but brief) post-doc. During the latter period, interesting work on using the open-source FEniCS FEM package was undertaken, which was published in extended form in a book chapter (Lezar & Davidson, 2012). This work taken further by Otto, Marais, Lezar and myself during Otto and Marais's post-docs (Otto, Marais, Lezar & Davidson, 2012). André Young, who doctoral and post-doctoral work will be discussed in Chapter 6, did his Masters research on mesh termination schemes for the FEM (Young, 2007).

In assessing this work, these papers, theses and dissertations form a body of

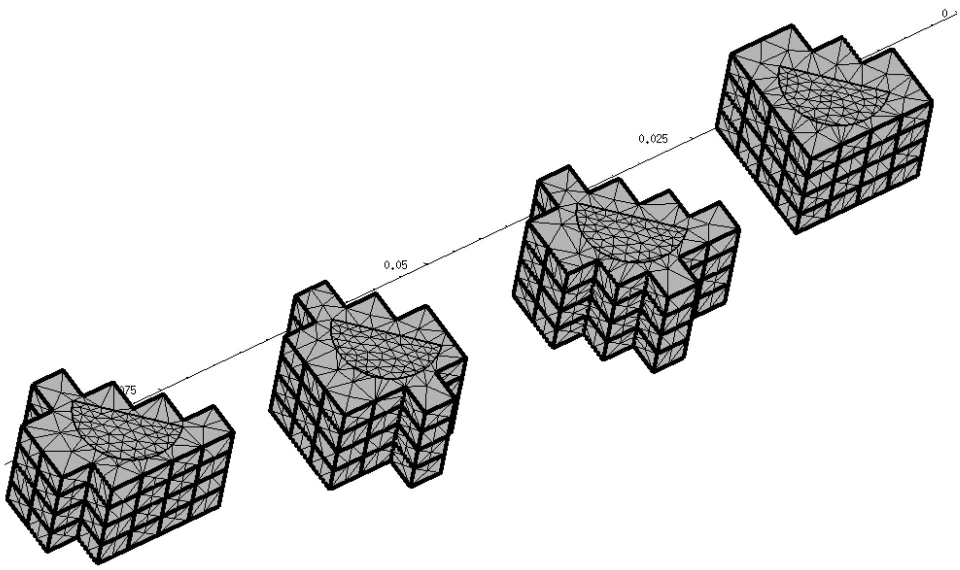


Figure 4.10: Tetrahedral hybrid mesh of the half-cylinder pin filter geometry. The bold lines indicate the boundaries of the hexahedral faces where they meet the tetrahedral mesh; no triangles may cross any bold lines. After (Marais & Davidson, 2010:Fig. 13).

work, largely following the theme of hierarchal vector finite elements, applied to waveguide, radiation, and bio-electromagnetic problems, with increasingly sophisticated and effective analysis techniques, including work on time domain FEM. My research in this field has had important commercial spin-offs; the first hybrid FEM/MoM code (an extension to FEKO) based quite heavily on this work by myself and current and former students was released in 2005. Later releases have incorporated other aspects of the work I have described on the FEM, in particular the waveguide work.

4.5 My textbook on Computational Electromagnetics

Whilst this book covers CEM in general, I have chosen to discuss it in detail in this chapter, both for chronological reasons, and also because it perhaps reflects best my research as well as teaching of the FEM. (One section specifically on stratified media and MoM analysis has already been addressed in Chapter 2).

A substantial contribution to scholarship in this field during this period was my book "Computational Electromagnetics for RF and Microwave Engineering", which required a very considerable investment of time — as did the 2nd edition. Largely during my third sabbatical, at the IRCTR at TU Delft (the Netherlands) in 2003, I prepared the draft for the book; it was published by Cambridge Univ Press in 2005. I spent much of my 2009 sabbatical, some of it spent at the University of Manchester, England, working on a 2nd edition of the book, which was completed in early 2010 and published in 2011.

The book provides a comprehensive introduction to the theory of computational electromagnetics, as well as an extensive discussion of using commercial software, distilling much of my experience in the field. A graduate-level introduction to the most widely used methods in CEM — the FDTD, MoM and FEM — is provided, and the methods are introduced via simple 1D examples. Where possible, the same problems are solved with different formulations, to highlight similarities and differences amongst the formulations. From there, increasingly complex 2D and then 3D problems are addressed, with supporting MATLAB code. At the end of each section, some sophisticated formulations are presented, often drawn from my research work. These include high-performance computing, fast methods for the MoM, the Sommerfeld formulation for a grounded substrate (discussed in Chapter 2), time-domain FEM and higher-order methods (both discussed in the present chapter of this dissertation). This is supported by a large number of examples drawn from RF and microwave engineering practice - primarily antennas and passive microwave devices - illustrating the use of such software, and the many pitfalls to beware of.

This contribution absorbed around two years of my professional life — 2003 and 2009 respectively. (Whilst a case could be made for a 3rd edition, I am not presently planning one). The 2nd edition has grown to 500 pages,with the chapters on the FEM were very substantially revised, reflecting my own research in this field since the first edition. Since its original publication, a number of other texts on CEM have appeared, but mine remains one of the earlier volumes, in particular to reflect not just the theory, but also practical application of codes. As the outgrowth of a quarter of century of research and graduate-level teaching in the field, I regard this as my most significant research output of my career. It is also my most highly cited work, with around 100 citations on Scopus and over 300 on Google Scholar for both editions, and the first edition sold over 1 400 copies internationally.

4.6 Conclusions: the Electromagnetic Software and Systems Group story from 1995–2014

Whilst I have mentioned EMSS on a number of occasions in this dissertation, the conclusion of this chapter is an appropriate place to discuss this in more detail, not least since Dr Frans Meyer, co-founder of what became the EMSS group, was my first PhD graduate, and made significant contributions to the FEM in electromagnetics at the time, as I have outlined in this chapter. Furthermore, my own contributions to FEKO were largely on the FEM modules.

Perhaps fortunately for South Africa, Dr Meyer was unable to secure a university or industry appointment following his graduation in 1994, in the uncertainty of the transition period in the immediate post-apartheid South Africa of that period. With a fellow PhD graduate (another of Prof John Cloete's students), Dr Gronum Smith, very bravely started their own business, Electromagnetic Software and Systems (EMSS). A 2014 photograph of Drs Meyer, Smith and myself is shown in Fig. 4.11. Originally a closed corporation, with just the two founding members operating on a shoe-string budget (I assisted them obtain one of their first substantial contracts, with the SA Navy), they grew dramatically over the next twenty years, eventually splitting into three main business units: EMSS-SA, responsible for FEKO development, marketing and sales; EMSS-Consulting, mainly responsible for cell-phone compliance testing, and EMSS-Antennas, mainly involved with KAT-7, MeerKAT and currently SKA. From 1994 until my appointment as SARChI SKA chair, I worked very closely with EMSS on research projects, and much of research and that of my students was largely directed towards improvements in FEKO. This included SPII programs, as well as THRIP.

In 2014, Altair, a US-based multinational specialising in multi-physics simulation, acquired EMSS-SA — and FEKO. By this time, EMSS-SA also had German, Chinese and USA offices, which were also subsumed into Altair. At that time, the EMSS Group had around one hundred full-time employees (and a number of part-time staff), so not only did the work have sufficient industrial impact to attract a major US corporation, it also contributed significantly to Stellenbosch's reputation for high-tech engineering. A 2014 photograph of the EMSS group's staff is shown in Fig. 4.12.

With this, the EMSS-SA brand ceased, but EMSS-Consulting and EMSS-Antennas were not affected by this. At the time of writing, the EMSS group was involved in a re-branding operation and Alphawave is likely to replace EMSS as the brand name in the near future. Although I have indicated that my involvement came to an end in 2014, this is not entirely the case, as I continue to have some small interests in the group, but my technical involvement largely ended with the Altair buy-out.

Figure 4.11: Left to right: Dr Gronum Smith, Dr Frans Meyer and the author in 2014, at a function celebrating twenty years of EMSS. Photo credits: EMSS-SA/Lea Botha.

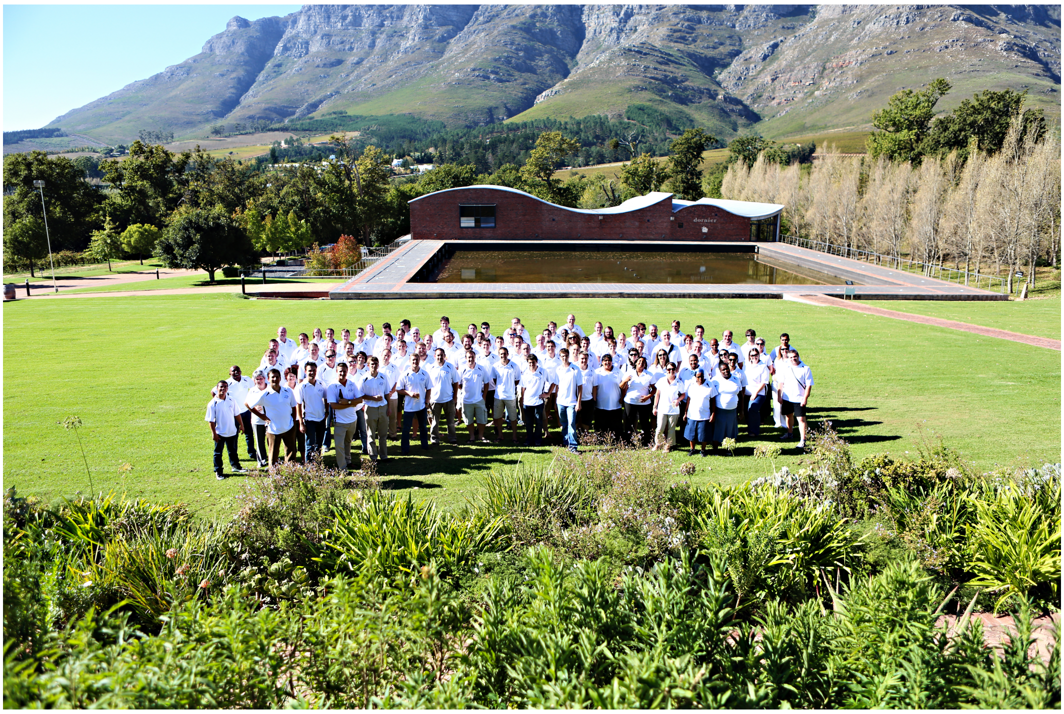


Figure 4.12: The EMSS group in April, 2014, shortly before Altair purchased EMSS-SA. Photo credits: EMSS-SA/Lea Botha.

Chapter 5

Contributions to HPC in CEM

5.1 Introduction

Advances in CEM have been described as driven by two laws — Moore's Law, and what I have dubbed Cendes's Law[1]. Both result on their own in simulation capability doubling approximately every eighteen months, the former due to hardware and the latter due to algorithmic improvements. Taken together of course the increase in simulation capability over the last three decades has been nothing short of spectacular, even given the computational scaling of the algorithms.

Despite implementation issues which remain challenging, parallel processing has emerged as a very useful enabling technology; several commercial codes (such as FEKO) are available in parallelized versions for various platforms. Whilst one does not always appreciate the impact of incremental increases in performance, when compounded over decades the results are deeply impressive. In Fig. 5.1, the time required for direct matrix solution (LU decomposition) on systems capable of sustaining 1 MFLOP, 1 GFLOP, 1 TFLOP and 1 PFLOP respectively are compared.[2] Comparing a 1 MFLOP (typical of the late 1980s) and a 1 GFLOP machine (typical of current systems circa 2010, when this figure was prepared), one notes that for a problem with around 1 000 unknowns, the time had dropped from around an hour to a few seconds. A similar improvement is noted for a 10 000 unknown problem when comparing a 1 GFLOP and a 1 TFLOP machine, the latter typical of entries towards the bottom of the Top 500 list circa 2010. The world's fastest supercomputers have now pushed well in the PFLOPS regime, and the SKA (of which more later) may need in the order of an EFLOP[3] system.

[1]Zoltan Cendes was the driving force behind ANSOFT, and did some of the pioneering work on vector finite elements. He articulated this observation during many talks, but did not associate his name with it.

[2]The operation count for LU decomposition for a matrix of dimension N with complex valued entries is approximately $\frac{8}{3}N^3$ floating point operations.

[3]exaFLOP — 10^{18} floating point operations per second.

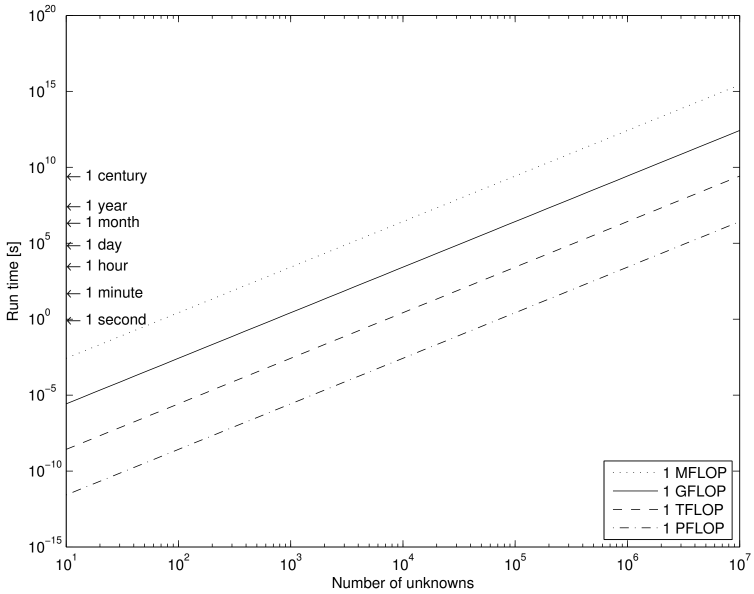


Figure 5.1: Run-times for LU decomposition, compared for systems capable of sustaining 1 MFLOP, 1 GFLOP, 1 TFLOP, and 1 PFLOP, after (Davidson, 2011:Fig. 6.20).

5.2 Transputers and my early work in the field[4]

In the late 1980s, PCs were limited by the 640 kB limitation on RAM imposed by the then dominant operating system, DOS, and clock speeds were low. Supercomputers were (and for that matter still are) extremely expensive. A British company, INMOS, introduced the transputer, one of the first "computers on a chip," incorporating a CPU, floating point unit, memory and communication links. (This was to become quite standard later, but at the time was revolutionary.) The transputer came in several different variants – the T800 model was the one widely used in parallel processing.

The transputer was a 32-bit Reduced Instruction Set Computer (RISC) design, capable of internal operation at up to 30 MHz – this must be seen from the viewpoint of the technology of the time. One T800 transputer was able to produce a peak floating point throughput of 1.5 MFLOP (which Fig. 5.1 puts in contemporary perspective!) A novel feature, still not widely seen on other systems to this day, was the provision of four serial links providing comparatively high-speed communication either with a host processor or with other transputers. Additionally, *all* components could execute concurrently; each of the four links and the floating point processor could perform useful work while the other elements were executing other instructions.

The transputer was a very powerful processor in its own right when introduced, out-performing the microVax, which was then the usual system of

[4]This section is extracted from (Davidson, 2011).

choice for numeric computations in universities and most research laboratories (outside US government research laboratories). However, it was ideally suited for application in parallel processing applications, in particular due to the on-chip links, and a number of experimental prototypes and some commercial products incorporating transputers were produced around the world.

The relentless advance of clock speeds in personal computer CPUs during the 1990s, combined with an over-dependence on a novel but ultimately commercially unsuccessful language-cum-operating system, Occam, eventually consigned the transputer to history. However, its role as an innovative catalyst in affordable parallel processing should not be underestimated; its do-it-yourself bargain-basement philosophy, if not technology, inspired a generation of computational scientists working at institutions unable to afford the extremely expensive supercomputers of the time, and still resonates today in current systems using Linux clusters. The idea resurfaced in the 2000s with the use of graphical processing units (GPUs) for general purpose computing — more on this later in this chapter.

5.3 Subsequent parallel processing work: 1992–8

The enormous pace of technological change has rendered the T800 transputer, a state-of-the-art device in 1989, to the (very large!) scrap-heap of obsolete computer technology a decade later, but the ideas and underlying analysis of parallel MoM algorithms contained in my thesis outlived the transputer. With other researchers, my doctoral research helped open a new research field, namely high-performance computing in CEM. My work continued to leave a footprint, via subsequent work on parallel FDTD algorithms on a CM-2 (see Chapter 3 for more detail), and in the Special Issue of the ACES Journal on Computational Electromagnetics and High Performance Computing, which appeared in July 1998. I co-edited that Special Issue with Tom Cwik of NASA's Jet Propulsion Laboratory, Pasadena, CA, USA. (Dr Cwik was an very influential researcher in this field at the time). We summarised much of the current thinking in this field in our editorial.

5.4 Some observations on the evolution of parallel processing during the 1990s and 2000s

HPC is now well established, and the technology had an enormous shake-out during these two decades. Massively parallel systems — such as the CM-2 — have largely failed to live up to earlier expectations, primarily due to the difficulty of programming them efficiently and moving data around the arrays sufficiently rapidly — indeed, this remains a major bottleneck. An exception in this regard was the IBM BlueGene series of supercomputers, which used very large numbers of relatively cheap and energy efficient processors. For most numerically intensive applications, distributed memory systems using moderate numbers of powerful processors (frequently Intel's Xeon processors), with large numbers of cores, grouped together in nodes with large amounts of RAM, with high-speed interconnects such as InfiniBand, are the systems of choice. (InfiniBand is a proprietary computer-networking communications standard used in high-performance computing that features very high throughput and very low latency; Ethernet is also used, but is usually far slower.) At the time of writing, my group had a dual Power Edge R730 server, each with two 12-core intel Xeon E52670 processors and 512 GB RAM, providing a 48 core, 1 TB compute server, at a cost of around R500 000 in 2016 Rands. This typical of this type of system. Such systems are also very well matched to parallelised, compute- and memory-intensive codes such as FEKO using mainly MPI.

Shared memory systems are rather more challenging to construct, but do offer advantages for algorithms such as the MoM and especially the FEM, which require prodigious amounts of RAM, and in the case of the latter, often with highly non-sequential memory access.

To use these systems efficiently, communications harnesses, epitomised by the Message Passing Interface (MPI), have been developed. MPI is a standardized and portable message-passing system, first released in 1994. It was designed from the outset to be portable to a wide variety of parallel computing architectures. The standard defines the syntax and semantics of a core of library routines, which can be used for writing portable message-passing programs in C, C++, and Fortran. There are several well-tested and efficient implementations of MPI. A number of these are open-source or in the public domain (Wikipedia, 2017*a*).

As architectures have evolved, in particular with greater internal concurrency (multi-core), better fine-grain concurrency control (threading, affinity), and more levels of memory hierarchy, so separate, complementary standards for symmetric multiprocessing, namely OpenMP, has arisen. (Symmetric multiprocessing (SMP) comprises a multiprocessor computer hardware and software architecture, consisting of two or more identical processors which are connected to a single, shared main memory. Additionally, the processors have full access to all the I/O devices, and are controlled by a single operating system instance that treats all processors equally. None are reserved for special purposes (Wikipedia, 2017*b*). This is typical of contemporary CPUs such as the Intel's i7 and Xeon chips.) MPI and OpenMP are not mutually exclusive, and carefully written parallel code can hybridize these systems.

5.5 Recent work on HPC

For around a decade, I did little work on HPC, as I was mainly focussing my efforts on developing the FEM. Furthermore, advances in computing meant that initially workstations (we had a range of Silicon Graphics machines), and later simply PCs, were able to provide enough computational power for at the relatively small problems typically ran to verify and validate new codes.

From time to time in this dissertation, I have mentioned in passing the background of a rapidly changing South Africa. During the 1990s, the South African national science system was almost running on auto-pilot, with the momentum of many years of spending on some very advanced defence systems and infrastructure for energy self-sufficiency carrying the system well into that momentous decade. In 1996, the new government released a carefully researched and well thought out White Paper (DACST, 1996), which would form the basis for science funding in the post-apartheid era. This would result in two projects which would directly impact on my career. The first was the decision in the early 2000s to set up a national Centre for High Performance Computing, and the second was that to bid for the SKA (more on the latter in a subsequent chapter).

In 2008, I was awarded a Flagship Project from the then newly-established national facility, the Centre for High Performance Computing: the title was "HPC electromagnetic simulation for the MeerKAT and SKA". This rekindled my interest in the field, was epitomised by the PhD dissertation of Lezar (Lezar, 2011) and (Ilgner, 2013).

5.5.1 GPU acceleration of the MoM

Lezar worked on GPU implementations of both MoM and FEM codes for his PhD (Lezar, 2011); the key results are presented in (Lezar & Davidson, 2010*b*,*a*). At the time of writing, the latter paper is my fifth most highly cited paper, and taken together, these two papers have almost 100 citations on Google Scholar; an excellent example of how early publications on a "hot" topic can generate a large number of citations in short order.

GPU implementations are almost a step back in time, as one has to again pay close attention to software to hardware mapping, and data locality becomes a crucial issue. For most CEM implementations, the biggest hurdle is the relatively small amount of RAM on most GPU cards. Lezar addressed both FEM and MoM implementations on NVIDIA GPUs for his PhD. An important contribution was an out-of-core algorithm which permitted the efficient solution of MoM systems which were too large to fit onto GPU RAM. In many ways, this was reminiscent of early versions of linear algebra packages such as LINPACK and LAPACK, which had to devise methods to store much of the matrix on (relatively slow) disk, paging the necessary data into and out of (much faster) "core" memory as efficiently as possible. Results are shown in

Fig. 5.2 for an NVIDIA GTX 280 with 1GB of on-board memory, compared to a 2.2GHz AMD Opteron 275 with 16GB of RAM — both of these were powerful systems at the time the work was undertaken, circa 2009–2010. Lezar went on to work for EMSS-SA for several years on GPU acceleration of FEKO.

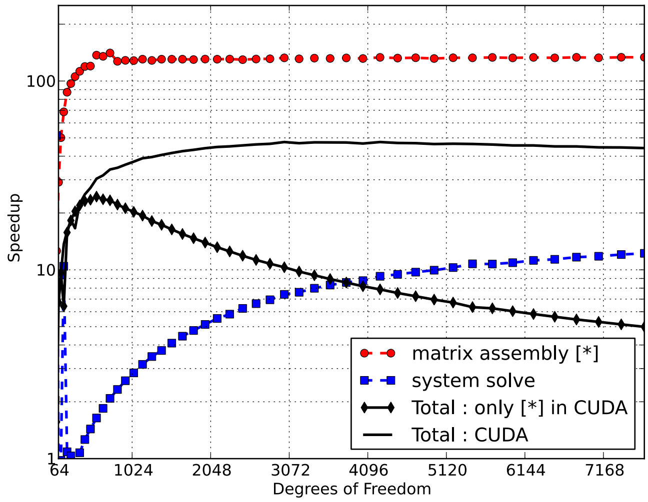


Figure 5.2: The measured speedup of the matrix assembly (●) and solution of the linear system (■) as a function of the number of degrees of freedom. The combined speedup (–) as well as the speedup when only the matrix assembly phase is accelerated (♦) is also shown. A logarithmic scale is used on the vertical axis for better comparison. After (Lezar & Davidson, 2010*a*:Fig. 20).

5.5.2 HPC for the FDTD

For his PhD, Ilgner implemented the FDTD on a wide variety of HPC platforms, ranging from low-end GPU cards to then state-of-the-art supercomputers such as the IBM Blue Gene (Ilgner, 2013). As the code and metrics were consistent, this permitted careful evaluation of the relevant performance/cost issues, as in our 2014 ACES Jnl paper (Ilgner & Davidson, 2014). (Additionally, this has provided new tools for the analysis of propagation on the Karoo SKA site, which were used in Pienaar's PhD, described subsequently). Ilgner continued work on this during his post-doc, and work using ARM processors on smartphones for FDTD simulations appeared as (Ilgner & Davidson, 2015).

Some metrics of the FDTD test code developed by Ilgner for his PhD, deployed on various systems at the Centre for High Performance Computing (CHPC) are shown in Fig. 5.3 and Table. 5.1. The former figure shows the maximum FDTD grid size for the CHPC's systems, which is determined by available memory. For all the HPC platforms (the GPU cluster, the multi-core cluster and the BlueGene/P), problems with billions of cells can be run. (Recall, however, that for a 3D problem, this corresponds to a cubical mesh of only some thousand cells per side). The latter table shows the throughput in millions of (Yee) cells per second (MCPS); this is a function of processing power and interconnect topology and speed. (It is interesting to note that in terms of speed, there is only a factor of approximately three between the fastest and slowest of these systems.) The multi-core clusters make use of their Vector Arithmetic Logic Units, such as the Streaming SIMD Extensions (SSE) for the Xeon-based cluster and the Advanced Vector Extensions (AVX) for the i7 cluster, to achieve good performance for the FDTD, using MPI and/or OpenMP as appropriate.

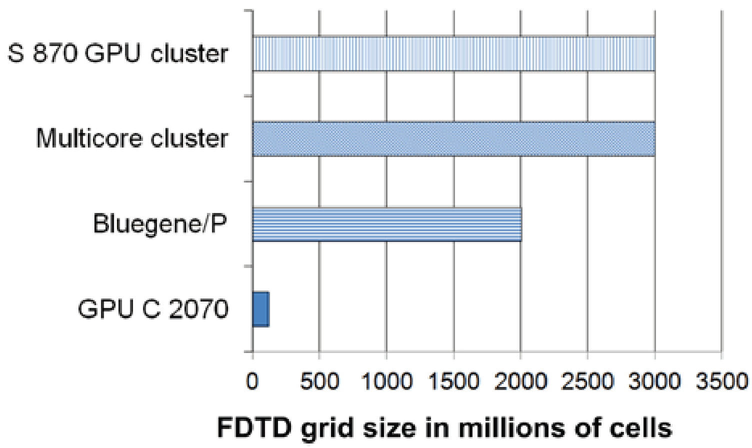


Figure 5.3: A comparison of the processing capability of various systems investigated by Ilgner. After (Ilgner & Davidson, 2014:Fig. 11).

Table 5.1: Computational throughput of the FDTD method on various HPC platforms at the CHPC. Adapted from (Ilgner & Davidson, 2014:Tab. 2).

Platform	Cores	Peak MCPS	Peak MCPS per core	Release date
BlueGene/P	4096	8900	2.2	June 2007
S870 node cluster	8192	12900	1.6	March 2008
Xeon 5670 SSE cluster	100	4155	41.6	March 2010
i7-3960x AVX cluster	36	6900	192	Nov 2011

5.6 Conclusions

This chapter largely completes my story of my contributions to computational electromagnetics, which started in Chapter 2, at least in terms of the development of CEM algorithms and codes primarily as a means in itself. The electromagnetic simulation industry is now a substantial industry worldwide; the purchase of ANSOFT by ANSYS in 2008 was a US $ 832 million deal, and the 2016 acquisition of CST by Dassault Systèmes was valued at approximately 220 million euros. However, for academic researchers, work in CEM per se has perhaps become a victim of its own success. Whilst there are without question some important issues to still resolve, the field is now by and large mature and many advances are now made in industry (rather than at universities), where contribution to knowledge via publication is of course no longer the primary aim. Perhaps one of the more important ways forward for CEM is to apply our ability to compute dynamic fields with high precision for electromagnetically large and/or complex structures to the continuously expanding applications of electromagnetics.

In the remaining chapters of this dissertation, I will outline recent work in radio astronomy, discussing work on radio astronomy beamforming, as well as calibration and imaging in Chapter 6; some of this leverages precisely this power of modern CEM tools. The final chapter will address some contributions to electromagnetic metrology and propagation modelling; once again, the latter work leverages CEM.

Chapter 6

Recent contributions: radio astronomy

6.1 Introduction

In the preceding chapter, I mentioned South Africa's White Paper on Science and Technology (DACST, 1996). A very important result of that was the decision to bid for the Square Kilometer Array (SKA) (Dewdney *et al.*, 2009), and from 2004 onwards, a very impressive programme was started (Jonas, 2009), which at the time of writing has already delivered most of the 64 MeerKAT dishes to be deployed on the Karoo site. I was appointed to the South African SKA Square Research Chair in Engineering Electromagnetics[1] at Stellenbosch University in 2011, following a lengthy process which began in 2009 with a pre-proposal, followed by a full proposal, and finally the candidature proposal. The main thrust of the chair was to develop high-level manpower for the SKA program, with a specific brief in RF engineering. I had been working on SKA topics before my appointment — Ludick's Masters (Ludick, 2010) was funded by SKA-SA — but this occasioned some re-focussing of my work. This included improved methods for interferometric imaging on the current and future generations of radio telescopes, RFI, and front-end processing.

This position has entailed managing a large group. At any one time, somewhere in the region of twenty Masters, PhD and post-docs have been involved with the chair, and whilst I directly supervise or co-supervise only around half, a significant amount of reporting runs through the Chair. Fig. 6.1 shows much of the group at it was in March 2014.

[1]The full title is the Chair in Electromagnetic Systems and EMI Mitigation for SKA.

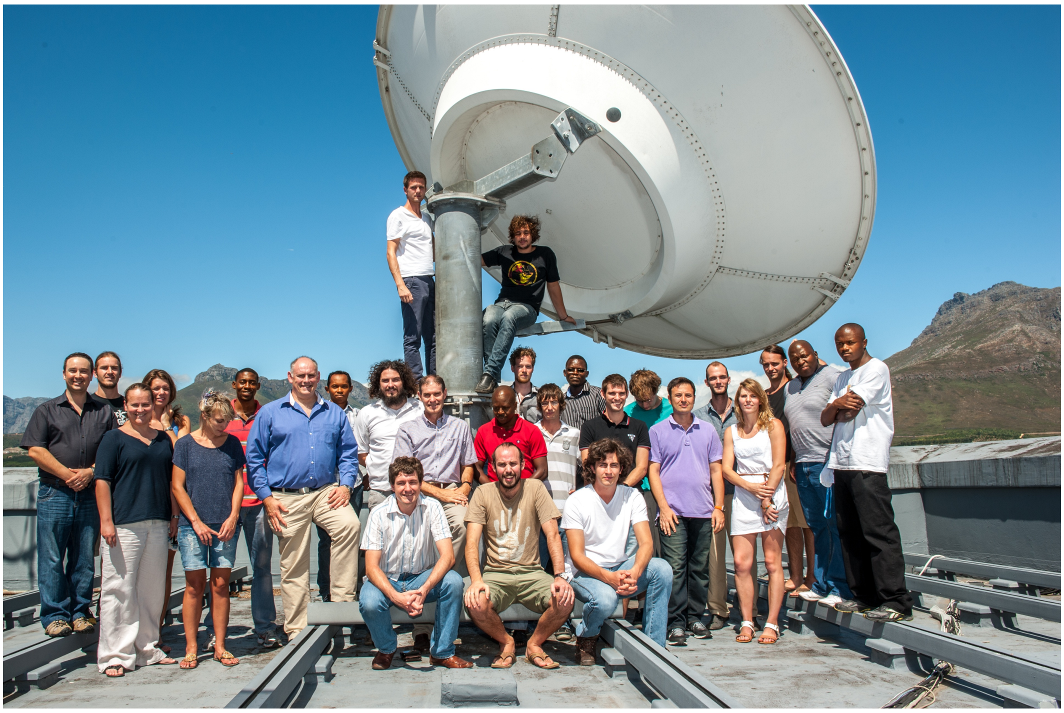


Figure 6.1: Much of the SKA group at Stellenbosch in March 2014. Left to right, front row: Dr André Young, Alex Vermeulen, Stefan Combrink; middle row, Mrs (now Dr) Jacki Gilmore, Ms Vereese van Tonder, the author, Dr (now Prof) Dirk de Villiers, Prof Howard Reader, Mr Stanley Kuja, Dr David Smith, Mr Dewald Botes, Dr Necmi Tezel, Ms Elmien Botes; back row Dr Gideon Wiid, Mr Antheun Botha, Ms Mariet Venter, Mr (now Dr) Temwani Phiri, Mr (now Dr) Joely Andriambeloson, Mr Mark Kleijnen, Mr Emmanuel Mukubwa, Mr Kolijn Wolfaardt, Mr Lee Goodrick, Mr (now Dr) Teunis Beukman, Mr Anathi Hokwana, and Mr (now Dr) Ngoy Mutonkole. On the dish: Mr (now Dr) David Prinsloo and Mr (now Dr) Hardie Pienaar.

6.2 A brief overview of radio interferometry

The basics of sparse synthesis array calibration and interferometric imaging have been known for decades. Here, a brief summary of the basics of radio interferometry (also often known as synthesis imaging) is presented. The following section is based on work currently in preparation for a jointly authored text.

The first two-element interferometric systems were introduced very shortly after the end of WWII, and were used to increase resolution of the instrument. A very comprehensive discussion of these adding interferometers, and later variants such as the phase-switched system, may be found in (Kraus & Marhefka, 2002; Kraus, 1966). Later developments in the early 1960s — pioneered by Martin Ryle at Cambridge — produced "aperture synthesis" techniques, using multiplying interferometers and interferometric imaging. The classic texts on this is (Thompson *et al.*, 2001), and the so-called "White Book", (Taylor, Carilli & Perley, 1999), the latter comprising a large number (33) of contributed chapters by contemporary experts on the topic.

In two dimensional aperture synthesis with a multi-element array, a celestial object (or field) is tracked by the system as the earth rotates, by inserting an appropriate electrical delay to compensate for the changing geometrical delay between antennas as the earth rotates. Each antenna-pair combination (or baseline) contributes a complex visibility as a function of hour angle (right ascension). A map, or image, of the field is then constructed via a Fourier transform of the measured complex visibilities. Good maps require an adequate coverage of baselines — this is often known as *uv* coverage.

Interferometric imaging relies on the inversion of a 3D Fourier-like integral (Thompson *et al.*, 2001:Eq. 3.7) connecting the measured visibilities with the desired sky brightness, with transform variables (u, v, w) and (l, m, n). The (u, v) plane is parallel to the $l - m$ plane, which is tangent to the celestial sphere at the centre of the field to be mapped. w is directed to this field centre. This is also known as the phase centre, or phase reference direction. Subject to what have until recently been very good assumptions, it can be shown that

$$\mathcal{V}(u, v, w = 0) \approx \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{A(l, m)}{\sqrt{1 - l^2 - m^2}} I(l, m) e^{-j2\pi(ul+vm)} dldm \qquad (6.1)$$

Here, the plane (u, v) contains the coordinates of the interferometric baselines normalized to wavelengths; w points toward the source, and u and v are measured in the plane perpendicular to w (also known as the phase reference centre), in respectively the east and north directions. l and m are direction cosines, describing position on the sky. $\mathcal{V}(u, v, w)$ represents the measured visibilities, $I(l, m)$ the desired source brightness (or intensity), and A the beam pattern of the antennas, assumed identical. The use of direction cosines is required to obtain the desired Fourier transform; conveniently, for a small angle between the direction (l, m) and the w axis, l and m are to good approximation the components of this angle in radians in the east-west and north-south directions, respectively (Thompson *et al.*, 2001:§2.4). (This can be shown using a Taylor series expansion).

Frequently, for observations close to the phase reference, Eq. (6.1) is further approximated as

$$\mathcal{V}(u, v) \approx \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(l, m) e^{-j2\pi(ul+vm)} dldm \qquad (6.2)$$

This displays the Fourier transform relationship even more overtly.

Eq. (6.1) is usually derived from the van Cittert-Zernike theorem, originally derived in optics. The derivation and an extensive discussion may be found in (Thompson *et al.*, 2001:Chapter 14). Although Eq. (6.1) (and related forms) comprise the basis of synthesis imaging, there does not appear to be a specific name for the integral. In the contemporary literature, it is sometimes called the van Cittert-Zernike equation, often abbreviated VCZ.

It is important to note that in its ideal form as above, the transform is a function of baseline *spacing* only, and not absolute position. As a result, interferometric arrays laid out in a regular pattern contain repeated (i.e. redundant) baselines. This seems wasteful, as these redundant baselines do not appear to provide useful information; nonetheless, this can be useful for calibration, as the visibilities which should be identical in theory will not be so in practice. A proposal which takes this to the extreme is the proposed "FFT telescope" (Tegmark & Zaldarriaga, 2009), which accepts massive redundancy as the price for a regular grid to which the FFT can be directly applied.

This approximate form has served the radio astronomy community very well for many years. However, Eq. (6.1) (and obviously (6.2)) is actually an approximation of the full three-dimensional form on the celestial sphere:

$$\mathcal{V}(u,v,w) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(l,m)I(l,m)e^{-j2\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]}\frac{dldm}{\sqrt{1-l^2-m^2}} \tag{6.3}$$

Eq. (6.1) is useful approximation of Eq. (6.3) when either the source is close to the phase reference direction (w), or when the baselines are coplanar, as in an East-West array (such as Ryle's Cambridge instruments and the Westerbork Synthesis Radio Telescope in the Netherlands.) For traditional dish-based systems, with narrow primary beams $A(l,m)$, this has been a very serviceable approximation, but for modern systems, with wide primary beams (ideally hemispherical in the case of aperture arrays), the full 3D form of the integral must be taken into account. We return to this shortly.

Applying the 2D transform in Eq. (6.1) appears straightforward, but in practice there are a number of complications. The first is that the visibility plane is both irregularly and sparsely sampled. One can potentially apply directly apply the Fourier transform to this, but of course this is computationally expensive. Most interferometric imaging algorithms rely on the FFT, and this requires that the visibilities be mapped onto a grid. Whilst the most obvious way to do this gridding operation would be via some form of interpolation, it is generally approximated nowadays by a convolution with a kernel of limited support (often 7×7). Furthermore, the sampled visibilities are corrupted by noise. This has stimulated the development of fairly sophisticated iterative algorithms, such as the classic CLEAN algorithm (Högbom, 1974) and its many variants.

Here, only one polarization has been considered. Polarimetry is a core component of radio astronomy; the Radio Interferometry Measurement Equation (RIME) is the contemporary framework within which this is most readily incorporated (Smirnov, 2011).

6.3 Calibration and imaging contributions

Young's PhD dissertation (Young, 2013) was the first to address this new focus at doctoral level. Future radio telescopes will require extensive and continual calibration to achieve their full observing potential. For the Square Kilometre Array, "third generation" calibration techniques will be required: these methods permit calibration of direction-dependent effects. An important example of these is the primary beam pattern, which is usually obtained by measurement in real-time; this is A in the interferometric integrals in the preceding section, eg, Eq. (6.3). The amount of measured data is limited by various factors, and determining how best to use such limited data was the main topic of his research. Traditionally, the assumed primary beams used in the calibration process have been approximations—often Gaussian beams. It is possible to use analytical approximations (for dishes, Bessel-function variants) but recent work has emphasised the use of more accurate physics-based assumed primary beams, known as characteristic basis function patterns (Maaskant, Ivashina, Wijnholds *&* Warnick, 2012). This relies on the generation of a number of patterns capturing both ideal performance, as well as degraded performance due to certain expected types of pattern errors. The latter can often be predicted beforehand. These patterns can either be obtained through measurement or electromagnetic simulation. This was applied to a PAF feed for an offset Gregorian SKA prototype in (Young, Maaskant, Ivashina, de Villiers *&* Davidson, 2013*b*) and extended to address calibration efficiency in (Young, Ivashina, Maaskant, Iupikov *&* Davidson, 2013*a*). Similar methods can be applied to aperture arrays (Young, Carozzi, Maaskant, Ivashina *&* Davidson, 2014).

Young continued this work, now with collaboration expanded to include ASTRON[2]. The resulting *Astronomy and Astrophysics* paper presented a new method ("A-stacking") which can handle both direction-dependent and baseline-dependent effects, using realistic simulation data computed using CEM codes (Young, A., Wijnholds, S. J., Carozzi, T. D., Maaskant, R., Ivashina, M. V. *&* Davidson, D. B., 2015). In this case, the visibilities are not only a function of relative spacing on the ground (i.e. the baseline vectors) but also of the absolute positions (i.e. which baselines). This is typically due to mutual coupling; in particular in sparse, random arrays, this can vary quite substantially across the array. The effect is illustrated in Fig. 6.2 for the LOFAR LBA at Onsala . These patterns were computed using FEKO at 50 MHz. In my opinion this is an important piece of work, and a number of presentations on this for various audiences have elicited a very positive response, in particular the use of CEM modelling to provide accurate date on beam patterns and coupling effects.

[2]The Netherlands Institute for Radio Astronomy.

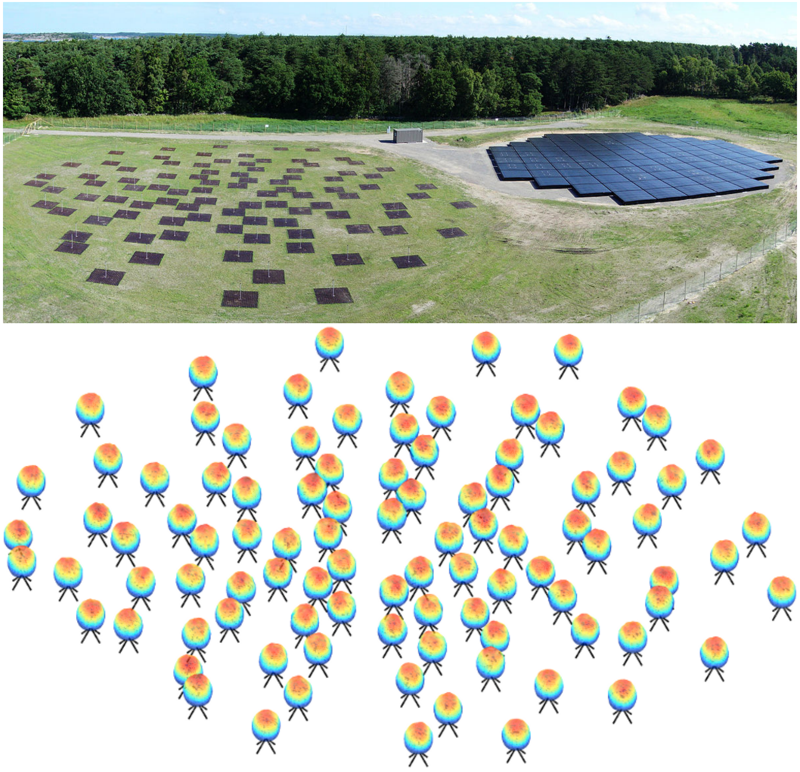


Figure 6.2: a) The LOFAR LBA (left of photo) and HBA (right of photo) stations at Onsala Space Observatory, Sweden. Photograph courtesy of Leif Helldner. (b) FEKO model of the 96 element LBA station showing the radiation patterns (magnitude) of each antenna in the array. The array comprises dual-polarized inverted-V antennas above a ground plane (not shown in model). Generally a larger degree of inter-element variability is observed among the patterns of the antennas that are closely spaced than in the patterns of those that are more isolated. After (Young, A. *et al.*, 2015).

6.4 Array design

The PhD of Gilmore focusses heavily on front-end technology for radio telescopes comprising aperture arrays (Gilmore, 2016). Her topic has been the development of a new prototype for the Mid-frequency Aperture Array (MFAA) component of SKA, presently on the Advanced Instrumentation Program. She has developed a novel dual-polarisation wide-band array using densely packed dipole elements (these are overlapped to broaden their otherwise narrow bandwidth); this had its origins in a discussion I had with Jan Noordam of ASTRON at the formal dinner during the first SKA Engineering Meeting, held in Manchester in 2012 (indicating the importance of social interaction at meetings!) Many years back, he had proposed a "bath-mat" array, and explained the concept to me; on returning home after the meeting, some reading convinced me that the instantaneous bandwidth required for the MFAA would not be achievable using a square array of dipoles, but by overlapping them, mutual coupling would greatly broaden the bandwidth. Gilmore took the idea, developed it most substantially into a working prototype, which she called the Dense Dipole Array (see Fig. 6.3) and also solved a long-standing problem with common mode currents excited on a broad class of such arrays. The work was published in (Gilmore & Davidson, 2015) and work is continuing with two Masters projects currently underway, jointly supervised by Dr Gilmore and myself, and in collaboration with ASTRON. An example of her array in shown in Fig. 6.3, mounted in the spherical near-field scanner (to be described in the next chapter).

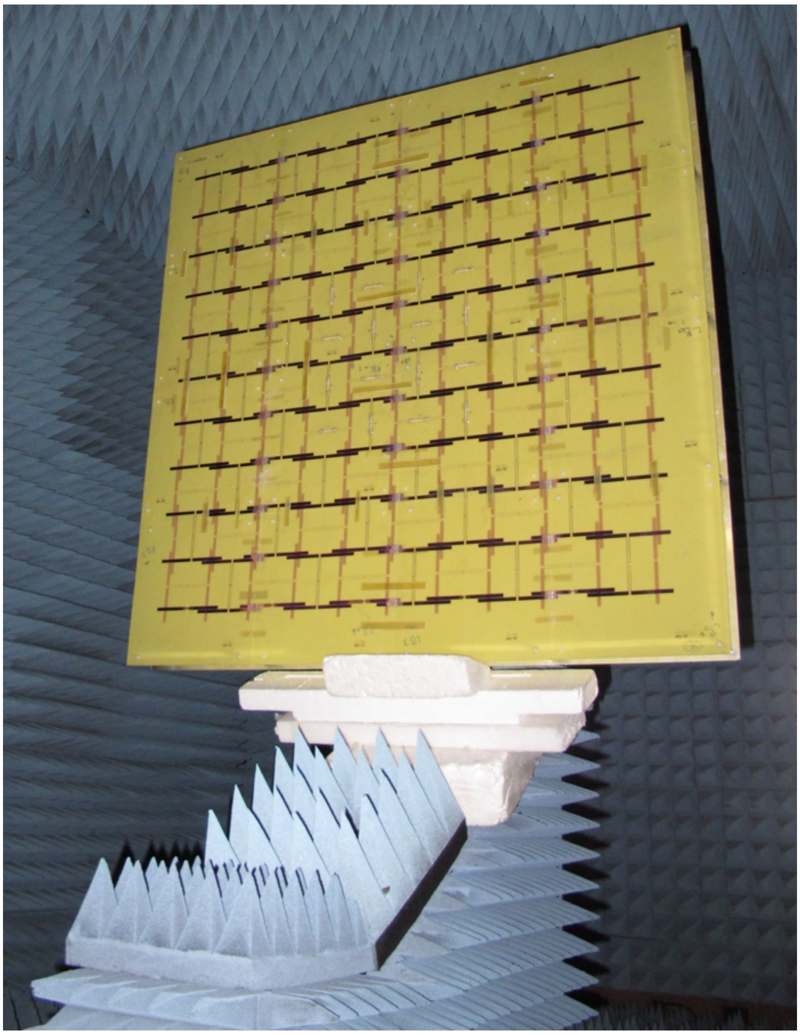


Figure 6.3: The Dense Dipole Array on the Stellenbosch University spherical near-field range. After (Gilmore, 2016).

6.5 Other contributions

Very interesting and exotic doctoral work was undertaken by Volkmann, whose PhD (which I co-supervised) reported ground-breaking work on the possible application of superconducting electronics in the SKA front-end (Volkmann, 2013). (He is now employed by Canadian company D-Wave, whose work on quantum computing leverages his specialist knowledge of superconducting electronics).

The continuing PhDs of Steeb and bij de Vaate should also be mentioned here, as should the collaboration with ASTON and Wijnholds. Steeb's research work started by studying existing spatial filtering techniques to suppress RFI. His Masters was upgraded to a PhD, and Stefan Wijnholds[3] joined as his external supervisor. His current work investigates situations where the usual assumptions, viz. that the RFI source is in the far-field and that the array is calibrated, are no longer valid. In (Steeb *et al.*, 2016), the recovery of astronomical signals from uncalibrated RFI-corrupted LOFAR visibility data using spatial filtering methods was presented. For this demonstration, a near-field continuous-wave RFI source was generated by a hexacopter that was flown around one of the LOFAR LBA (low-band antenna) arrays. Promising results were obtained.

Bij de Vaate's PhD addresses system level aspects of the MFAA design (Gilmore, Davidson *&* Bij de Vaate, 2016). In particular, the MFAA has focussed on dense, regular layouts, such as EMBRACE (Torchinsky, Olofsson, Censier, Karastergiou, Serylak, Picard, Renaud *&* Taffoureau, 2016). The LFAA proposes a sparse, irregular array, where station sidelobes are randomised and largely cancelled. Bij de Vaate is working on sparse, regular arrays; work is presently in progress.

Within the constraints of space, it is not possible to list in detail the contributions of MSc students also involved in my research program, but mention should be made in particular of work that has been extremely useful in developing our expertise in radio astronomy. This includes JP Janse van Rensburg, who built a small interferometer at L-band from COTS components (Jansen van Rensburg, 2012); Vereese van Tonder, who implemented beam-formers on both ROACH and UniBoard platforms (van Tonder, 2014), Nicholas Thompson, who worked on RFI identification techniques (Thompson, 2014), Lee Goodrick (Goodrick, 2015) and Anathi Hokwana (Hokwana, 2017) who worked on aspects of interferometic imaging; Mariet Venter, who worked on feeds for HERA (Venter, 2016) — work published as (Ewall-Wice *et al.*, 2016); Nelis Wolfaardt, who worked on machine learning approaches to identify RFI (Wolfaardt, 2016); and Mourits de Beer, who built a four-element DF system with a novel wide-band antenna (de Beer, 2017).

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[3]At the time of writing, Prof Wijnholds works at ASTRON, and holds an Extraordinary Associate Professor position at SU.

6.6 Conclusions

As is clear from this chapter, my work on radio astronomy is currently still building momentum, so conclusions about its impact would be premature as it is very much work-in-progress.

In the final technical chapter, I will turn my attention to contributions I have made in antenna metrology and propagation.

Chapter 7

Recent contributions: electromagnetic metrology; propagation

7.1 Introduction

This, the last technical chapter, discusses recent contributions to engineering electromagnetics, viz. antenna metrology and propagation. Whilst these have been driven by the requirements of the SKA research chair, they are widely applicable in RF and microwave engineering, and as such are discussed in this separate chapter.

7.2 Upgrade of the Stellenbosch University antenna range

7.2.1 Background

Stellenbosch University's antenna range was originally built over twenty five years ago under the direction of Prof John Cloete, with the majority of components designed and manufactured internally by members of staff (du Toit, Solms, Palmer & Cloete, 1988). Stellenbosch University's facility originally comprised a polarisation axis rotator mounted on an automated planar scanner and cylindrical axis rotator mounted on a manual translation track. That permitted cylindrical and planar near-field measurements, and conventional (direct) measurements of far field cuts. While this facility was still functional, major components were no longer supported nor properly understood, a situation exacerbated by the ill health of the late Prof Keith Palmer, who was able to keep the system running long past its "use-by" date with his insights and keen practical engineering skills. The facility relied upon a venerable HP 8510

VNA, which has not been supported by the manufacturer for several years[1]. Additionally, the control equipment relied upon obsolete software, that was effectively undocumented, and legacy drivers. Finally, a number of crucial components were no longer supported, such as the controller for the stepper motors.

As has been noted earlier in this dissertation, commercial simulation software is now readily available. A good case can be made that it is now the ability to make high quality measurements that differentiates leading research institutes in the field. Access to these facilities is limited, due to the expense of RF measurement equipment, and specialist operators are required to obtain the best results. During the lifespan of this facility preceding the update, well over two hundred post graduate students received training at the facility, providing a number of them with a crucial component in their research. This facility is also used in the final year course on high frequency systems. These considerations made modernising and upgrading the facility a priority.

Near-field metrology has several advantages over far-field metrology, with the most obvious being that the probe is not constrained to being in the far field of the Antenna Under Test (AUT). When the facility was originally commissioned, near-field metrology was in its infancy (Hansen, 1988; Slater, 1991), and facilities were custom-made, both in terms of hardware and software. In the interim, commercial products, with ongoing technical support, have become available (Parini, Gregson, McCormick & van Rensburg, 2015). As our antenna range is a general purpose facility, I decided that upgrading it to a near-field range would provide the most flexibility, although some initial consideration was given to converting the anechoic chamber into a tapered range, similar to that reported by (Baker & Booysen, 2011).

7.2.2 A brief overview of the upgrade

The relevant near-field equipment was sourced from Nearfield Systems, Inc (NSI)[2] of California, USA, and the PNA-X from Keysight Technologies. In Nov 2014, after almost two years of funding applications and planning, the existing planar near-field scanner was upgraded with new NSI-supplied motors, controllers and software, and an entirely new NSI-supplied spherical near-field scanner was installed. The PNA-X Vector network analyser was also successfully integrated with the NSI-supplied computer and the planar and spherical scanners. Work on this was reported in (Smith, Davidson, Bester & Andriambeloson, 2016) and a brief summary is provided here.

The existing mechanical components of the X Track and Y Tower were retained, but with the two motion stages (motors and controllers) replaced with modern motors and control systems, and the polarisation stage redesigned.

[1] Originally Hewlett Packard, then Agilent, presently Keysight Technologies.

[2] Now NSI-MI Technologies following a merger with MI.

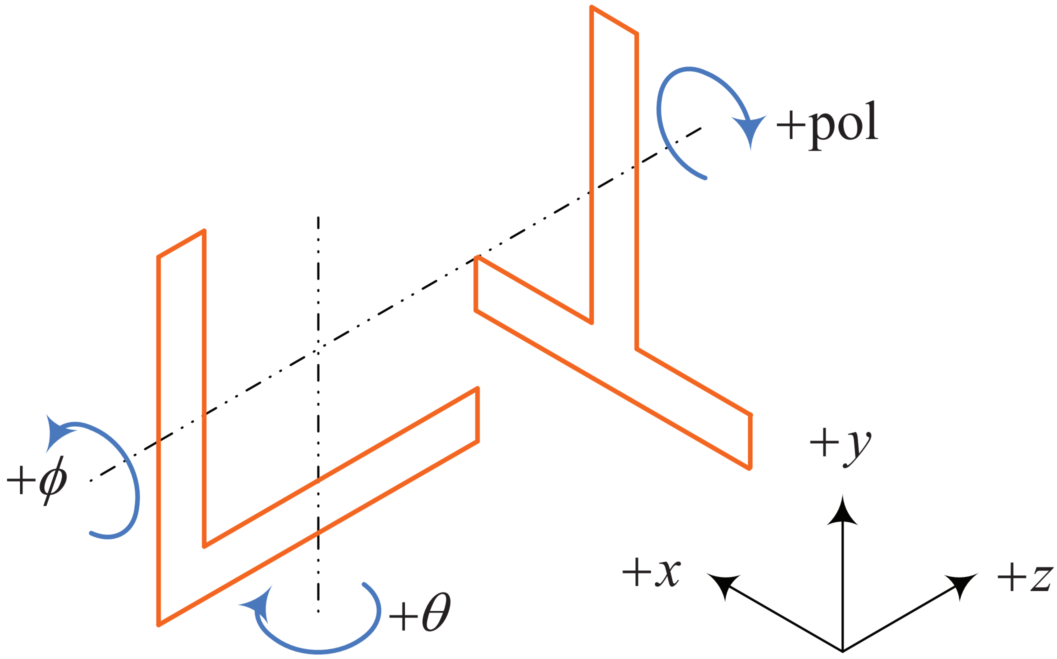

Figure 7.1: The anechoic chamber coordinate system, following the NSI standard. After (Smith, Davidson, Bester & Andriambeloson, 2016).

The coordinate system of the anechoic chamber is shown in Fig. 7.1. Positive θ rotation is clockwise as seen from above the AUT and positive ϕ rotation is counter clockwise as seen from behind the AUT. As seen from in front of the probe, positive x translation is leftward, positive y translation is upward, positive z translation is backward and positive polarisation rotation is clockwise[3].

The planar and spherical near-field scanners are show in Fig 7.2. The planar scanner comprises the absorber-covered X-Y tower on the left, with the rotating polarisation stage immediately behind the horn and absorber, and the spherical scanner consists of the absorber-covered L-bracket and ϕ stage rotator on the right.

The full planar near-field scanning systems comprises the X Track embedded in the floor of the anechoic chamber, the Y Tower mounted on the X Track, and the polarisation stage mounted on the Y Tower. The AUT can be either mounted on the spherical scanner, or a separate custom-built pedestal could be provided for large and/or heavy antennas.

Similarly, the full spherical near-field scanning system comprises the 700S-30 scanner mounted on the scanner base, which is mounted on the Z Track. The AUT is mounted on the scanner, and the probe is mounted on the Y

[3]The definition of θ and ϕ rotations are opposite to those normally used; for our facility, the AUT moves during spherical near-field measurements, whilst the probe remains stationary. (Conventionally in antenna theory, the AUT is stationary, at the centre of the coordinate system, and the fields around it are probed as a function of θ and ϕ.)

Figure 7.2: The antenna range at SU follow the upgrade. See the text for a description of the components. After (Smith, Davidson, Bester & Andriambeloson, 2016).

Tower for convenience. The Z track is embedded in the floor of the anechoic chamber. To move the AUT along the z axis, the spherical near-field scanner is manually translated along the Z Track. To rotate the AUT around the θ axis, the θ stage rotates only the L Bracket of the SNF scanner. To rotate the AUT around the ϕ axis, the ϕ stage rotates the AUT. The existing Z Track was retained, with the spherical near-field base manufactured internally at our university, with the other components forming part of the upgrade.

The anechoic chamber layout is shown in Fig. 7.3. It is a rectangular room of dimension 9.1 m (L) $\times$ 5.5 m (W) $\times$ 3.6 m (H). As is usual with an anechoic chamber, the interior surfaces of the flooring, walls and ceiling are covered with absorber pyramids; these were inspected before the upgrade, and despite their age, were found to be in good condition. The chamber is also screened to some extent, although it is not primarily intended for RFI measurements — there is a separate shielded room for this.

The new system is fully operational. Results measured in this upgraded chamber were used in Pienaar's PhD, and an extensive program of evaluation

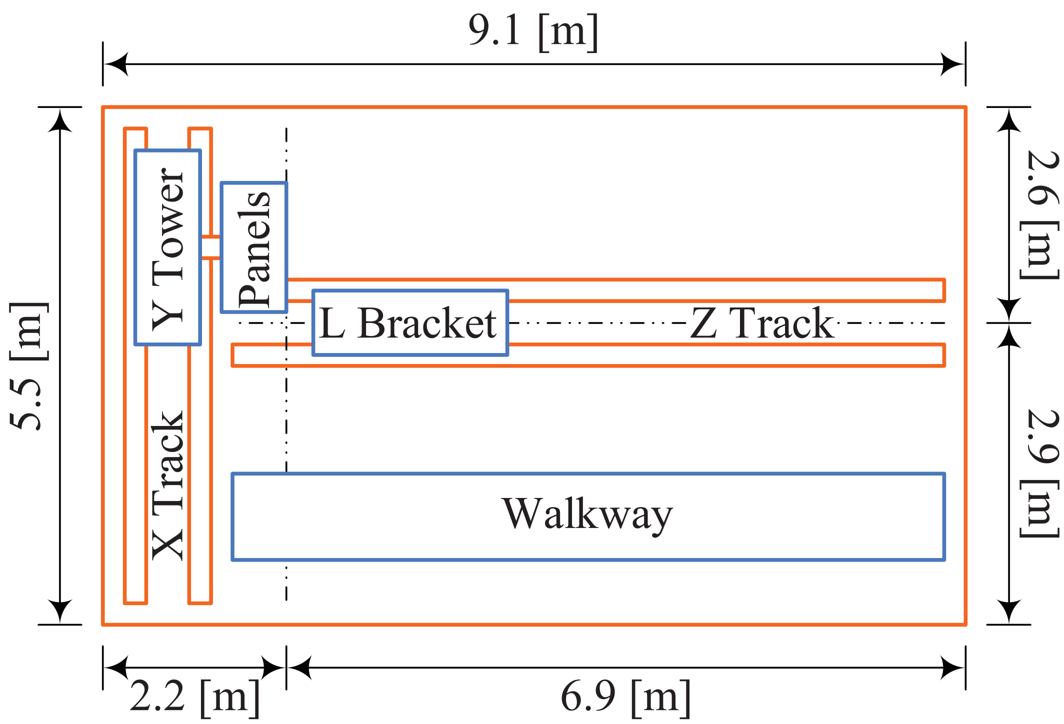

Figure 7.3: A diagram showing the layout of the SU anechoic chamber. See the text for further discuss. After (Smith, Davidson, Bester & Andriambeloson, 2016).

is underway, driven by the antenna range manager Bester, whose salary the research chair largely funds (Smith, Davidson & Bester, 2015; Smith, Davidson, Bester & Andriambeloson, 2016). The recommissioned range has already proven popular with local industry.

In Feb 2016, the range hosted a short course on antenna metrology jointly presented with the University of Cape Town (UCT). Much of the content on the range was presented by Dr Daniel Janse van Rensburg of NSI, who had led the team which upgraded our antenna range in October 2014. We had 26 delegates attend, from several South African universities (SU, UCT, Univ of Kwazulu-Natal, Nelson Mandela Metropolitan University and the Univ of the Witwatersrand) as well as staff from SKA-SA and delegates from the Universities of Zambia and Mauritius. The course was very well received by the students and introduced the range to a broad spectrum of South African and African researchers.

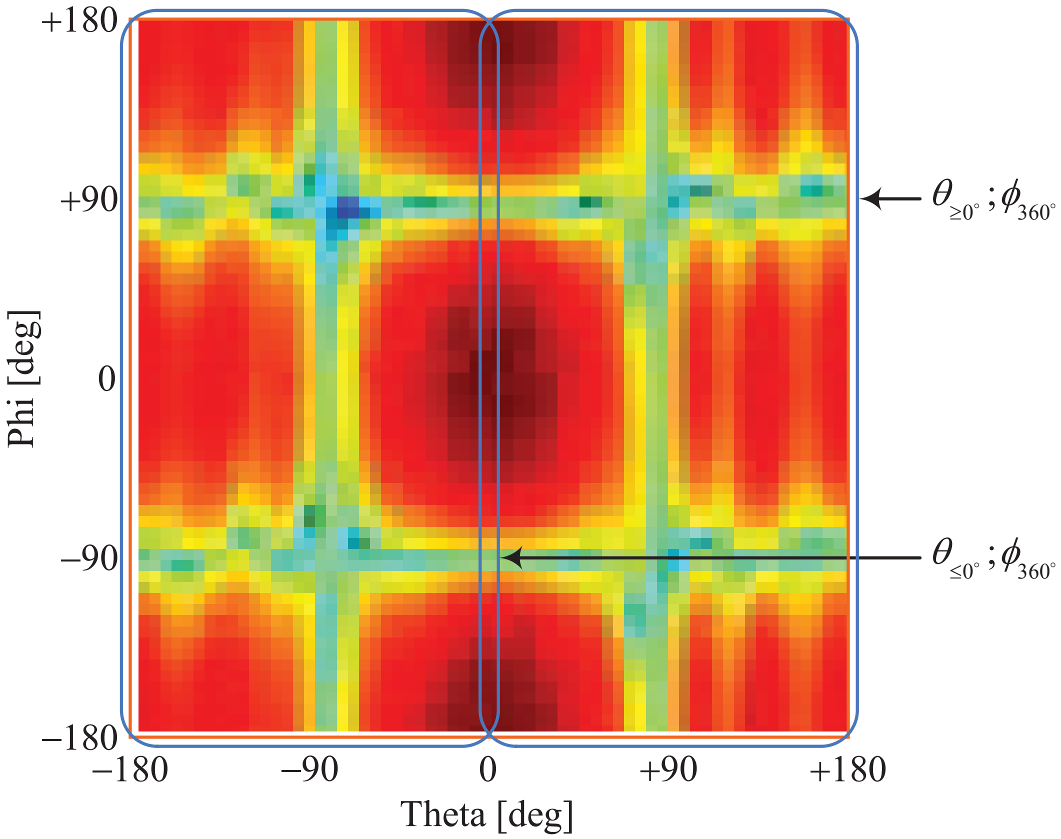

Figure 7.4: An example of a redundant near-field measurement. After (Smith, Davidson, Bester & Andriambeloson, 2016).

7.2.3 An example of chamber evaluation using the spherical near-field system – the walkway and chamber asymmetry

An example of these is an experiment exploiting a particular feature of our indoor antenna range, namely is the ability to take redundant SNF measurements. The AUT can be rotated 360° around both the θ axis and the ϕ axis, resulting in the radiated fields over the full sphere being sampled twice, but with different configurations of the AUT with respect to the chamber. For instance, a full near-field measurement can be performed with the AUT rotated from 0° to +180° around the θ axis, which directs the AUT towards one side of the chamber. A fully redundant near-field measurement can be performed with the AUT rotated from 0° to −180° around the θ axis, which directs the AUT towards the other side of the chamber, as in Fig. 7.4. For both measurements, the AUT is rotated 360° around the ϕ axis.

Theoretically, these two spherical near-field measurements should transform to equivalent far fields patterns. However, the absorptive properties of the absorber material used in our chamber at lower frequencies, and especially below the specified 1 GHz limit, result in radiated fields being reflected off the walls of chamber back towards the probe. The offset position of the SNF scanner (to make room for the walkway) results in a disparity in the signal path on opposite sides of the AUT, which is more pronounced at lower frequencies. The combination of these two phenomena result in a disparity in the radiated fields detected by the probe when the AUT is directed to the opposite sides of the chamber at 0.75 GHz, as shown in Fig. 7.4.

This disparity between the near-field measurements results in a discrepancy in the computed far field patterns. This is more pronounced at lower frequencies than at high frequencies, as in Fig. 7.5. For example, the RMS value between the two principal Azimuth cuts is -25.3 dB at 0.75 GHz, while at 3 GHz it is -32.3 dB. To quantify the disparity between the two sets of redundant spherical measurements, the RMS difference between the two sets of computed far field patterns is calculated over the whole sphere for a range of frequencies, as in Fig. 7.6. As expected, this disparity is larger at lower frequencies, especially below the specified 1 GHz limit. Interestingly, it is not improved by replacing the walkway with pyramids, indicating that the asymmetry of the chamber is the key issue at the bottom end of the chamber's operating frequency band.

Using these calculated RMS values, the measurement uncertainty in the computed far-field patterns at 0.75 GHz is shown in Fig. 7.7. Depending on the measurement configuration, the actual far-field pattern of the AUT would be within the RMS envelop around the computed far field pattern. (The specific AUT in this case is a commercial biconal antenna, with operating band 500 MHz – 3 GHz). Such a plot could be used to determine the (un)certainty with which the measured AUT meets the design requirements.

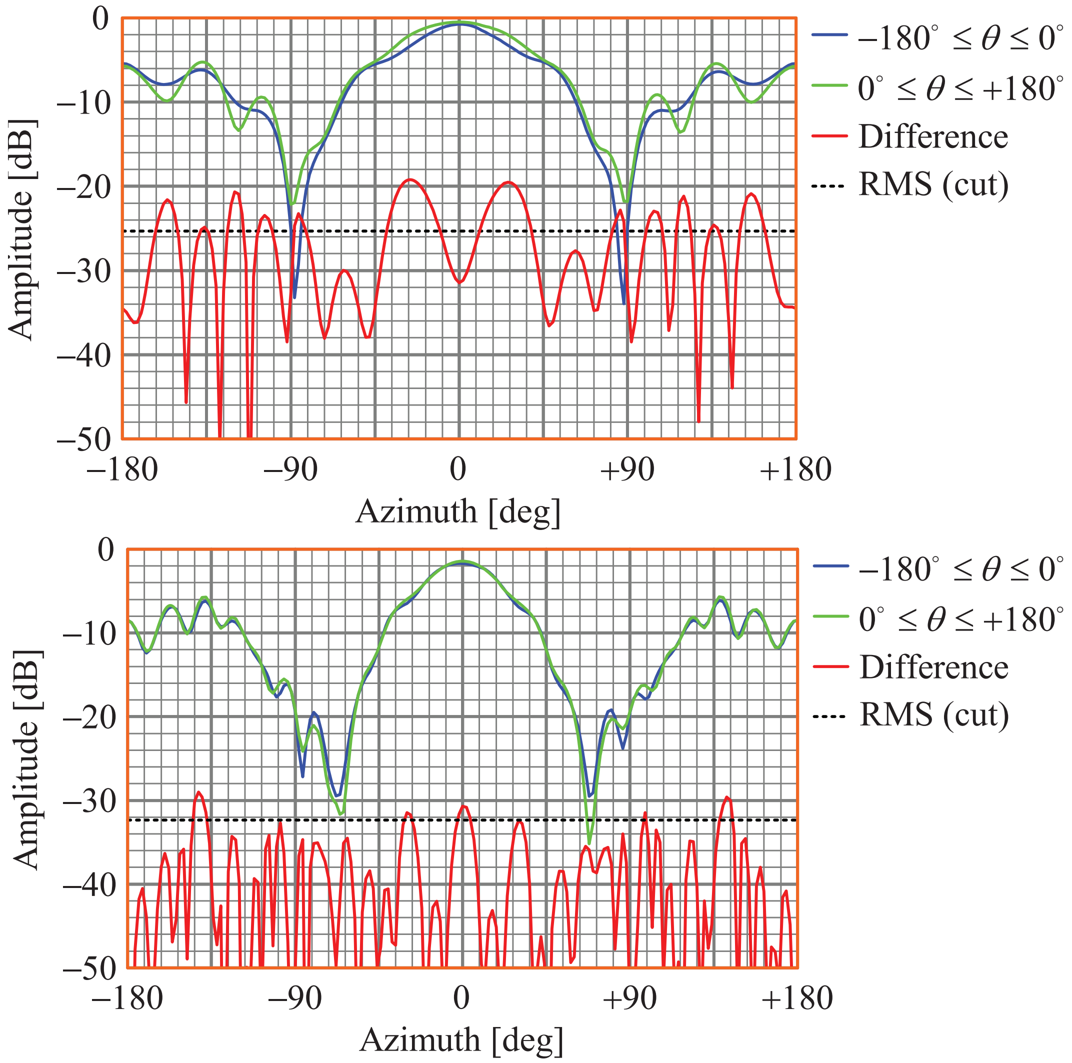

Figure 7.5: Far field patterns corresponding to redundant near-field measurements at 0.75 GHz (top) and 3.0 GHz (bottom). After (Smith, Davidson, Bester & Andriambeloson, 2016).

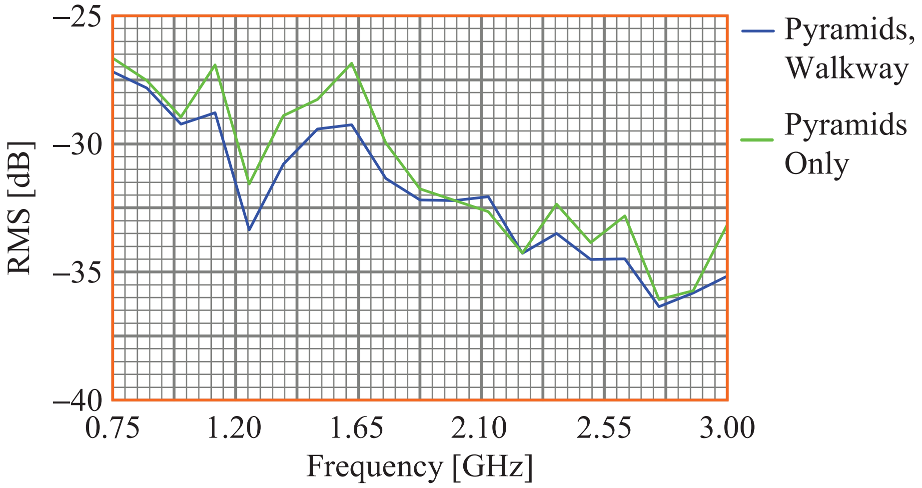

Figure 7.6: RMS disparity between computed far-field patterns. After (Smith, Davidson, Bester & Andriambeloson, 2016).

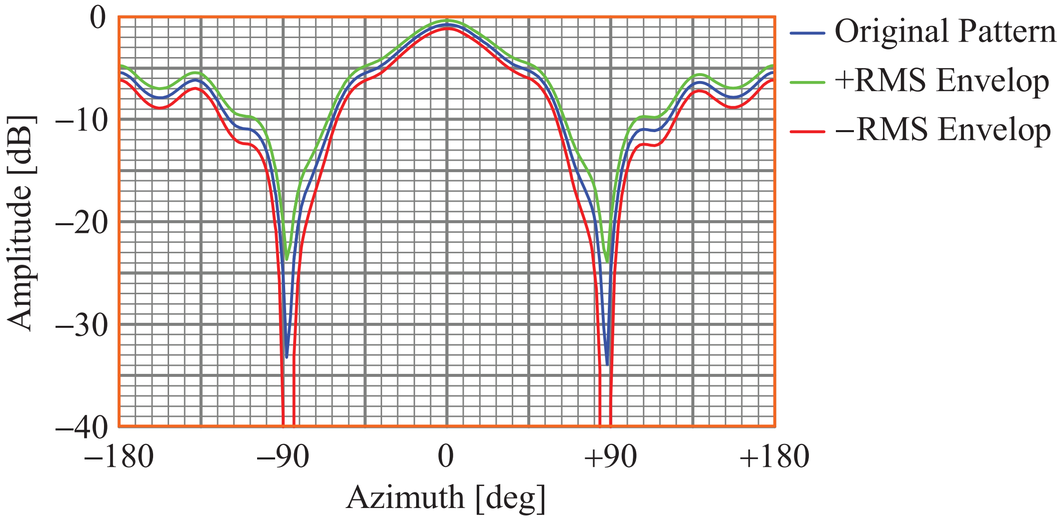

Figure 7.7: Far-field cut with RMS envelop. After (Smith, Davidson, Bester & Andriambeloson, 2016).

Figure 7.8: Contemporary photo of the KAPB, dark roof, taken from the top of Losberg. The berm and the assembly shed can be seen on the left and right of the KAPB respectively, after (Pienaar *et al.*, 2017).

7.3 Propagation modelling on SKA Karoo site

Another topic which involves significant amounts of metrology — and also computation — is recently completed doctoral work on propagation modelling on the SKA Karoo site. Pienaar's PhD addressed the issue of RFI on-site (Pienaar, 2015), looking at the threat posed by necessary infrastructure, especially the Karoo Array Processing Building (KAPB). His dissertation focussed on understanding the shielding and propagation characteristics of both the KAPB building, as well as a man-made soil berm; refer to Fig. 7.8. Scale models, CEM models and on-site measurements using a multi-copter vehicle developed by the candidate were used to investigate the local electromagnetic environment. The work has resulted in a detailed appreciation of RFI shielding levels on the SKA-SA site, and again links CEM and radio astronomy. It was published as (Pienaar, Otto, van der Merwe, Davidson & Reader, 2016*b*) and (Pienaar, Reader & Davidson, 2017). Simulation work on the berm was also reported in (Tezel, Reader & Davidson, 2017).

Recent work has focussed on improving the multi-copter to include full polarimetry — the system used for his PhD was essentially a total power instrument. One application is for antenna pattern measurements, on for instance the SKA mid-frequency aperture array, and a study of positioning accuracy requirements was presented in (Pienaar & Davidson, 2016). Details on shielding the electronics on the multi-copter itself were presented in (Pienaar, Andriambeloson & Davidson, 2016*a*).

Phiri's PhD investigated propagation in the MeerKAT core (Phiri, 2017). Most of the propagation models available at UHF frequencies for rural areas are based on semi-empirical studies from many years ago (when the first generation of TV transmitters was deployed), and these are optimized for good predictive ability over tens of kilometers, not for close-in propagation. Additionally, the large number of MeerKAT dishes in the densely-packed core of the array —

some 40 dishes in a 1 km diameter — means that the dishes themselves impact significantly on propagation, indeed, there are some similarities with the "urban canyons" encountered in cell-phone propagation in cities. Phiri used FEKO simulations to provide coverage maps in the core. He also showed that using free space loss is not reliable for predicting the impact of RFI propagated into (or from within) the array core. Some novel visualizations were presented for attention in the core region; examples are shown in Figs. 7.9 and 7.10. These plots show at show basic transmission loss and reference attention, the deviation from free space path loss, at 1 300 MHz and 3 050 MHz respectively; the latter shows more variation, as well as more at-risk angular sectors. (Note that the colour intensity scales are not the same in the two figures). In some cases, the presence of the dishes results in loss being *less* than predicted by a free-space model, which is usually regarded as conservative for RFI protection planning purposes.

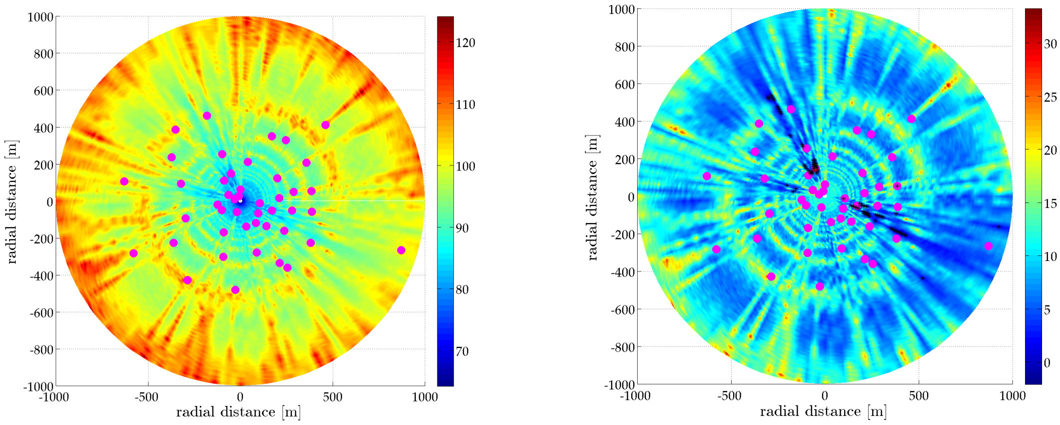

Figure 7.9: Attenuation map for dishes facing 1 (facing southwest), stowed, at 1 300 MHz. The plots show basic transmission loss (left) and reference attention (right). Relative high risk regions of ≤ 2 dB are represented by the asterisks in the right-hand plot. After (Phiri, 2017).

7.4 Conclusions

Taken together with Chapter 6, this chapter represents some of my current work. The focus in this chapter has been on electromagnetic metrology and propagation. Both fields have leveraged significantly off computational techniques in recent years; near-field metrology relies heavily on efficient and rapid near-to-far-field transformations, and propagation prediction is increasingly turning to numerical simulation tools, as demonstrated by recent work on MeerKAT.

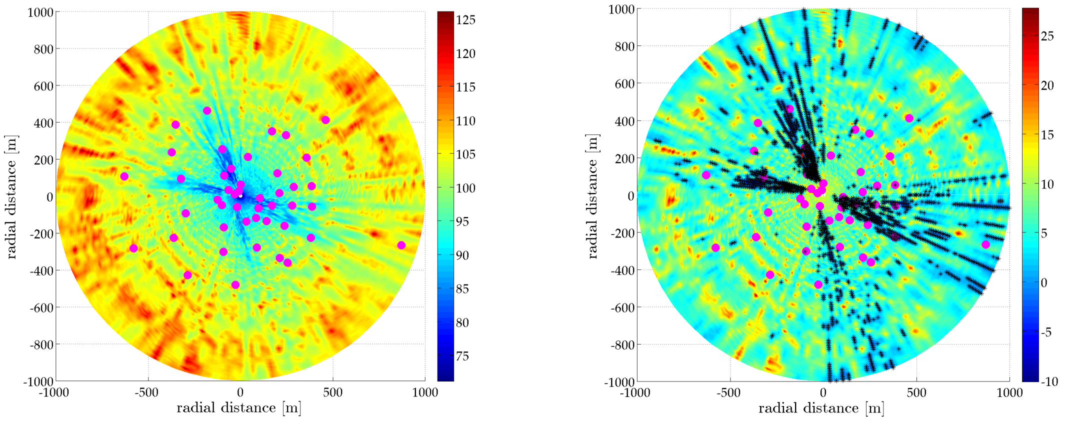

Figure 7.10: Attenuation map for dishes facing 1 (facing southwest), stowed, at 3 050 MHz. The plots show basic transmission loss (left) and reference attention (right). Relative high risk regions of ≤ 1 dB are represented by the asterisks in the right-hand plot. After (Phiri, 2017).

The original antenna range constructed by Prof Johannes Cloete lasted a quarter of a century — and was actually still functional when we undertook the upgrade. It left an enduring legacy of his work via a generation of antenna engineers skilled in antenna metrology; furthermore, a substantial number of components in the refurbished range are those he originally designed in the late 1980s. It will be gratifying if the upgraded chamber can leave a similar heritage.

Chapter 8

Conclusions

This dissertation has covered much ground, with the lion's share addressing computational electromagnetics and enabling technologies such as high-performance computing, but with recent contributions to radio astronomy, antenna metrology and radio wave propagation also being presented.

8.1 Key contributions

To put the key points of this work into perspective, presenting a short list of what I view as my top dozen or so research contributions has merit. The following are motivated by impact (citations[1]), breadth (collaboration), or level of theoretical (or other) contributions, or a combination of these. The list is partially ordered, in that the top few positions are ranked, but thereafter, the order is not especially relevant. As indicated in the introductory chapter, my most substantial contributions have been in the field of CEM, as this is the field I have been active in the longest.

My textbook (Davidson, 2011) With around 300 citations for the 1st and 2nd editions combined, this is by far my most cited work. It is also a work which reflects many of my contributions in the field, and other than one co-authored chapter (the text of which I wrote), it is solely my own work.

My PhD and resulting publications (Davidson, 1990, 1991, 1992, 1993) As noted in Chapter 2, other than my book, this was one of the last research projects I undertook entirely on my own. This work had substantial impact at the time, as the President's Award I received from the NRF a few years thereafter reflected.

Industrial impact I have noted the success of EMSS in the dissertation. Obviously the key role players in a start-up are the full-time personnel who

[1] Google Scholar

are fully committed, but both my research direction over an extended period and the graduates from my group who went to work there attest to my involvement.

My work with Rick Ziolkowski on the FDTD-BOR (Davidson & Ziolkowski, 1994) This is my most cited journal paper, with close on 100 citations. Although it was not the first BOR-FDTD formulation published, it corrected some errors in the relevant Courant limit, and the application was highly novel at the time.

Matthys Botha's PhD and post-doc work As indicated on several occasions, Matthys Botha's work was (and is) of the highest quality and I am proud of the sequence of papers we published during this period (Botha & Davidson, 2005, 2006*a*,*b*; Davidson & Botha, 2007). Each of these papers addresses a significant issue in FEM theory at the time.

The collaboration with Frans Meyer et al on on hybrid MoM/FEM (Meyer, Davidson, Jakobus & Stuchly, 2003) This is another highly cited paper, with around 70 citations. It was also a very comprehensive paper on an important topic at the time in the field of mobile communications and compliance.

Renier Marchand's work on the MMS As indicated in Chapter 2, I view our paper (Marchand & Davidson, 2014) is one of the best of my career. Although the paper currently has limited citations, these are increasing since another group in Spain picked up on the method.

The collaboration with Chalmers Univ, ASTRON and BYU Starting with Young's Phd, a number of important publications have resulted from this, including (Young, Maaskant, Ivashina, de Villiers & Davidson, 2013*b*; Young, Ivashina, Maaskant, Iupikov & Davidson, 2013*a*; Young, Carozzi, Maaskant, Ivashina & Davidson, 2014; Young, A., Wijnholds, S. J., Carozzi, T. D., Maaskant, R., Ivashina, M. V. & Davidson, D. B., 2015) from Young's work; (Warnick, Maaskant, Ivashina, Davidson & Jeffs, 2016) from my own work with the team, and very recently (Steeb, Davidson & Wijnholds, 2016).

Evan Lezar's Phd work on GPUs (Lezar & Davidson, 2010*b*,*a*) With close on 100 citations, these two papers have become standard references on GPUs for the MoM.

Neilen Marais's PhD work The sequence of papers (Marais & Davidson, 2008*a*,*b*, 2010) probably still represents the state of the art in high-order time domain hybrid implicit/explicit FEM Methods for microwave engineering.

Upgrading of the SU antenna range The upgrading of the SU antenna range should leave a lasting legacy for future work here. In this regard, our paper (Smith, Davidson, Bester & Andriambeloson, 2016) is primarily of importance for delineating the status of the laboratory.

Fast array solvers (DGM work) Another highly collaborative effort, (Ludick, Maaskant, Davidson, Jakobus, Mittra & de Villiers, 2014) has attracted a good number of citations given its recent publication, and the method is ripe for further exploitation.

Frequency selective surface work with EMSS, AMS Polymers and Kentron This work (Davidson, Smith & van Tonder, 1997*a*,*b*) was of high quality and represented an investment of some years' work, but the effort terminated due to circumstances outside our control.

Chiral work Our papers on chiral absorbers made useful contributions to this field (Cloete, Bingle & Davidson, 2001; Bingle, Davidson & Cloete, 2002). Taken together, these papers have almost fifty citations (almost equally split) in a very specialist field.

In a research career of over thirty years, a list of this nature inevitably leaves off numerous worthy contributions.

8.2 Quo vadis CEM?

Since CEM has formed the core of much of my research, this is an interesting question to consider, and I attempted to answer it in both the 1st and 2nd editions of my textbook. Whilst my crystal gazing showed some perception, one is immediately humbled by Niels Bohr's comment:

> Prediction is very difficult, especially about the future.

The key feature of the last decade has primarily been non-technical — the continued consolidation of the CEM industry (which as noted earlier, is now a very substantial international niche industry). All three major CEM codes (HFSS – ANSOFT; MWS – CST and FEKO – EMSS-SA) have now been subsumed into much large multi-physics simulation companies (ANSYS, Dassault Systémes and Altair respectively). As well as being commercialised, CEM has also been commoditised, as RF and microwave design tools increasingly include full-wave solvers which are hidden from the users.

Whilst the smaller codes (eg WIPL-D) still have some dedicated followers, it is hard to see how they will compete with the resources of these large multinationals. In the 2nd edition of my textbook, I predicted that public domain solvers might emerge to fill this evolutionary niche; whilst there have been some noble efforts in this regard, e.g. FEniCS, the most prominent such solver is still the venerable NEC2 code. MATLAB have recently released an antenna modelling toolbox based on the MoM, which may open interesting possibilities for university-based research.

The above comments have focussed largely on the CEM industry. From a research perspective, it is reasonable to predict that much work in the field will be driven by the requirements to address larger problems, multi-scale problems and/or multi-physics problems, usually with ever-increasing accuracy. The development of the Multi-Level Fast Multipole Method opened the way for the first, but domain decomposition methods remain important here, and these also impact strongly on multi-scale problems. High performance computing has a crucial role to play here as an enabling technology. Multi-physics applications are challenging and are likely to remain so in the foreseeable future; the demands of meshing a structure such as a large radio-telescope dish for mechanical analysis on the one hand, and electromagnetics analysis on the other, are very different. Better accuracy requires higher-order basis functions, for example (Graglia *&* Peterson, 2016); improved geometrical modelling; and careful attention to numerical integration, in particular when field singularities are present. This also requires highly accurate benchmark solutions.

All of the above are already strong themes of much current CEM research around the world, and several have been addressed in this dissertation. Additionally, advances in applied mathematics impacts fundamentally on CEM; this include both work on the underlying theory of discrete approximations of continuum fields, such as (Sauter *&* Schwab, 2011), as well as new analytical solutions for more complex canonical structures, such as (Vinogradov, Smith *&* Vinogradova, 2002). The former provides important insights into issues such as convergence; the latter provides further benchmarks against which solutions obtained by numerical methods can be verified. (This complements the Method of Manufactured Solutions, which also provides a larger set of benchmarks).

The wider field of electromagnetics seems set to continue as a key component of modern electronic technology. The dominance and capability of commercial codes may sometimes seem discouraging to CEM researchers, although as outlined above, there is no shortage of challenging research problems to address. For those who want to build antennas, microwave devices etc., the power of contemporary CEM codes is wonderfully enabling. Combined with advances in optimisation, new classes of devices and problems can be addressed which we could only dream of when I started my research career.

Appendices

Appendix A

Postgraduate students whom I have supervised

In this appendix, a list (current to March 2017) of my postgraduate students (who have completed their degrees) and post-doctoral fellows (whose fellowships have finished) is provided.

A.1 Supervision of Postgraduate Students

A.1.1 MEng and MSc(Eng) supervision

Supervisor of following graduates:

1. Mr P Steyn 1988-1989 (degree awarded, *cum laude*, in 1989).
2. Mr FJC Meyer 1989-1990 (degree awarded, *cum laude*, Mar 1991).
3. Mr DH Malan, 1994–1995. (degree awarded, *cum laude*, Dec 1995).
4. (Co-supervisor) Ms M Bingle, 1994–1995 (degree awarded, *cum laude*, Dec 1995. Supervisor: Prof JH Cloete)
5. Mr CB Wilsen, 1995–1996 (degree awarded, *cum laude*, Dec 1996).
6. Mr S Keunecke, 1996—1997 (degree awarded Mar 1998).
7. Ms R Hannsman, 1997–1998 (degree awarded, *cum laude*, Mar 1999).
8. Mr WR van der Leij, 1998–1999 (degree awarded, *cum laude*, Dec 1999. Co-supervisor: Prof. DM Weber).
9. Mr MM Botha, 1999–2000 (upgraded to PhD).
10. Mr E Burger, 1999–2000 (degree awarded, Dec 2000).

11. Mr P Futter, 2000–2001 (degree awarded, Dec 2001).

12. Mr N Marais, 2001–2002 (degree awarded, *cum laude*, Mar 2003).

13. Mr SR Clarke, 2001–2002 (degree awarded, *cum laude*, Dec 2002).

14. (Co-supervisor) Ms ML Strydom, 2002-2003 (degree awarded, *cum laude*, Dec 2003. Supervisor: Prof P Meyer).

15. Mr R Marchand 2005–2006 (degree awarded, *cum laude*, Mar 2007. Co-supervisor: Dr MM Botha).

16. Mr E Lezar, 2006–2007 (degree awarded, *cum laude*, Dec 2007).

17. Mr A Young, 2006–2007 (degree awarded, *cum laude*, Mar 2008).

18. D Ludick, 2008–2009 (degree awarded, *cum laude*, March 2010).

19. Mr S Nazo, 2010–2011 (degree awarded, March 2012).

20. Mr JP Jansen van Rensburg, 2011–2012 (degree awarded, *cum laude*, Dec 2012).

21. (Co-supervisor) M Volkmann, 2011–2012 (upgraded to PhD). Main supervisor: Prof C Fourie. Other co-supervisor Prof WJ Perold.

22. Mr NC Thompson, 2012–2013 (degree awarded, *cum laude*, April 2014).

23. Ms V van Tonder, 2013–2014 (degree awarded, *cum laude*, Dec 2014)

24. Mr L Goodrick, 2013–2014 (degree awarded, Mar 2015. Co-supervisor: Dr A Young).

25. Ms M Venter, 2014–2015 (degree awarded, Mar 2016.)

26. Mr CJ Wolfaardt, 2014–2015 (degree awarded, Mar 2016.) (Co-supervisor. Main supervisor: Prof TR Niesler).

27. Mr JW Steeb, 2015-2016 (upgraded to PhD).

28. Mr M de Beer, 2015-2016 (degree awarded, *cum laude*, Mar 2017). (Co-supervisor). Main supervisor: Dr PG Wiid).

29. Mr A Hokwana, 2014-2016 (degree awarded, Mar 2017).

A.1.2 PhD supervision

Promoter of following graduates (main supervisor unless otherwise noted):

1. Mr FJC Meyer: "Hybrid finite element/boundary element methods for three-dimensional electromagnetic problems"; 1992–1994 (degree awarded Dec 1994).

2. Mr P Steyn: "Radiation from antennas mounted on penetrable bodies"; 1992–1994 (degree awarded Dec 1994).

3. (Co-supervisor) Ms M Bingle: "The role of chirality in microwave absorbers"; 1996–9 (Supervisor: Prof JH Cloete; degree awarded Dec 1998).

4. (Co-supervisor) Mr K Williams: "An investigation into the computer-aided modelling of active microstrip patch array"; 1996–1999 (Supervisor: Prof HC Reader; degree awarded Dec 1998).

5. Mr MM Botha: "Efficient finite element electromagnetic analysis of antennas and microwave devices: the FE-BI-FMM formulation and a posteriori error estimation for p adaptive analysis"; 2001–2002 (degree awarded in Dec 2002).

6. Mrs R Geschke: "Application of the extended Huygens' principle to scattering discontinuities in waveguide"; 1999–2003. (Co-supervisors: Prof P Meyer, Dr RL Ferrari (Cambridge Univ), degree awarded in Dec 2003).

7. Mr N Marais: "Efficient High-order Time Domain Finite Element Methods in Electromagnetics"; 2005-2008 (degree awarded Mar 2009).

8. Mr E Lezar: "GPU Acceleration of Matrix-based Methods in Computational Electromagnetics"; 2008-2010 (degree awarded Mar 2011).

9. Mr RG Marchand, "The Method of Manufactured Solutions for the Verification of Computational Electromagnetic Codes", 2010–2012 (degree awarded, Mar 2013).

10. Mr A Young, "Improving the Direction-Dependent Gain Calibration of Reflector Antenna Radio Telescopes", 2008–2013 (Co-supervisors: Prof R Maaskant and Prof MV Ivashina, Chalmers University of Technology, Gothenburg, Sweden; degree awarded, Dec 2013).

11. Mr RG Ilgner, "A comparative analysis of the performance and deployment overhead of parallelised Finite Difference Time Domain (FDTD) algorithms on a selection of high performance multiprocessor systems", 2009–2013 (degree awarded Dec 2013).

12. (Co-supervisor) Mr M Volkmann, "'A Superconducting Software-Defined Radio Frontend for the Square-Kilometre Array'", 2011–2013. (Main supervisor: Prof C Fourie. Other co-supervisor: Prof WJ Perold, degree awarded Dec 2013).
13. Mr D Ludick, "Efficient Numerical Analysis of Finite Antenna Arrays using Domain Decomposition Methods", 2011–2014. (Co-supervisor: Dr U Jakobus, Altair; degree awarded, Dec 2014).
14. Mr H Pienaar, "Karoo Array Telescope Site Shielding: Laboratory, Computational and Multi-copter Studies", 2015. (Co-supervisor: Prof (Emeritus) HC Reader, degree awarded, Dec 2015).
15. Mrs J Gilmore, "Design of a Dual-Polarized Dense Dipole Array for the SKA Mid-Frequency Aperture Array", 2013–2015. (Degree awarded, March 2016).
16. Mr TJ Phiri, "Characterising the Electromagnetic Environment of MeerKAT", 2014–2016. (Co-supervisor Dr PG Wiid, degree awarded March 2017).

A.1.3 Postdoctoral supervision

1. Dr IP Theron, October 1997 – March 1999.
2. Dr MM Botha, September 2004 — December 2005.
3. Dr JP Swartz, April 2005 — June 2006.
4. Dr E Lezar, Feb 2011 – Aug 2011.
5. Dr N Marais, Jan 2011 – Aug 2011.
6. Dr AJ Otto, Jan 2012 – Dec 2012.
7. Dr N Tezel, Jan 2013 – July 2014. (Co-supervisor Prof HC Reader).
8. Dr A Young, June 2013 – Dec 2014.
9. Dr RG Ilgner, Sept 2013 – Feb 2015.
10. Dr DMP Smith, Jan 2014 – Aug 2015.
11. Dr J Andriambeloson, Jan 2015 – Dec 2016. (Co-supervisor Dr PG Wiid).
12. Dr H Pienaar, Jan 2016 – April 2017.
13. Dr D Ludick, March 2015 – June 2017.

Appendix B

Research funding

B.1 A review of some major grants

For good or bad, contemporary academia is heavily driven by funding. This appendix lists some of the major research grants I have received. All figures are given in South African rands, expect where noted, and are not inflation adjusted except for one example.

1996–2010 I was the the Principal Grantholder of an NRF-funded research consortium "HF Electromagnetic and Electronic Engineering", which ran from 1996 to the 2010. Under the auspices of this program, our research group in the Department received annual funding at around R500 000 by 2010[1] and this led to considerable cross-fertilization between smaller research teams within the overall project. Well over a hundred post-graduate students were trained in this field by our group during this very successful program.

2008–2010 In 2008, I was awarded a Flagship Project from the then newly-established national facility, the Centre for High Performance Computing: the title was "HPC electromagnetic simulation for the MeerKAT and SKA". This project was generously funded, with a total grant for the period Apr 2008-Mar 2010 of around R1 400 000.

2011–present My current SKA Research Chair at Stellenbosch has associated with it an annual grant of R4 000 000 currently.

2012-2017 I am the principal grant holder of the project "The MeerKAT High Performance Computing (HPC) for Radio Astronomy Research Programme""; this was a three-year project, renewed to end 2017 with a budget of around R1 500 000 per annum currently.

[1]In 2017 rands, this would be approximately R700 000.

2013–2014 During this period, I raised around R4 000 000 from Stellenbosch University's Strategic Funds and the NRF to upgrade our department's antenna measurement range, as reported in Chapter 7.

2014–2016 I was the principal grant-holder at SU of the MIDPREP EC FP7 IRSES project (the consortium members are ASTRON in the Netherlands, Chalmers Univ of Technology in Sweden, and in South Africa RU, UCT and SU). The annual budget from 2014–6 for SU was around €16 000 of funds for the secondment of researchers.

I have consulted extensively for industry, and my work has contributed directly to commercial simulation packages, in particular FEKO, as noted in the dissertation. I have also been involved with THRIP projects. (THRIP was a joint industry-government funding program in South Africa).

Appendix C

Significant awards received

Here, some significant awards and recognition which I have received during my research career are mentioned.

C.1 Awards

President's Award, 1996 I was a recipient of the NRF (then Foundation for Research Development) President's Award in 1996. The citation accompanying this award is: "Researchers normally younger than 35 years of age who have obtained their doctoral degrees during the past five years and who, on the basis of exceptional potential as researchers during their doctoral studies and early post- doctoral careers, are highly likely to be recognized by the international community as being among the future leaders in their field or as enjoying considerable international recognition as independent researchers of high quality by the next evaluation."

Stellenbosch Universty, 1996 I received the Researcher of the Year award from the Faculty of Engineering at Stellenbosch University in 1995.

Stellenbosch Universty, 2005 I received the Rector's Award for Excellent Research from Stellenbosch University in 2005. At that time, typically only one award was made per faculty annually. (I received a similar award in 2013, for general performance).

IEEE, 2012 As of 2012, I was elevated to Fellow of the IEEE; usually, the number of IEEE Fellows elevated in a year is no more than one-tenth of one percent of the total IEEE voting membership (which is around 400 000). In 2012, 329 new Fellows were elevated worldwide.

IEEE-SAIEE, 2014 I received the inaugural joint IEEE-SAIEE Distinguished Volunteer Award in 2015.

Stellenbosch Universty, 2014 As of July 2014, I also hold the position of Distinguished Professor at Stellenbosch University. The University made a very limited number of these prestigious positions available (around 30–40) and these were applied for competitively.

I was a finalist in the 2014 BHP-Billiton awards. I also received recognition from the Vice Rector (Research) in recognition of my 2013 doctoral supervision.

C.1.1 NRF personal research ratings

South Africa's NRF (and its predecessor, the Foundation for Research Development), maintains an individual research rating process, based largely on international peer review. (This differs from the UK, where institutions are rated, or more usually internationally, project proposals are rated). I received my first rating from the NRF in 1994, a Y rating. In 1996, I received the P award noted above. I received a C1 rating in 2001. In 2006 and 2011, I received B2 ratings. For the most recent cycle, valid January 2017– December 2022, I received a B1 rating. This rating and sub-category is described by the NRF as:

> All reviewers are firmly convinced that the applicant enjoys considerable international recognition for the high quality and impact of his/her recent research outputs, with some of them indicating that he/she is a leading international scholar in the field.

The NRF provides extracts of the peer review feedback (anonymously). From this round, one comment I particularly appreciated was "The applicant does not really need any advice from me. He knows precisely what he is doing, and continues to be a researcher making contributions of the highest quality."

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