Maintenance Task and Frequency Optimisation of Single- and Multi-Component Equipment in an FMCG Production Environment

by

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Thesis presented in fulfilment of the requirement for the degree Master of Engineering in the Faculty of Engineering at Stellenbosch University

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Declaration

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Date: March 2017

Abstract

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The aim of this thesis is to provide a structured approach to the cost-optimisation of reliability-centred maintenance (RCM)-based maintenance tasks and frequencies within a fast moving consumer goods (FMCG) production environment, considering non-negligible maintenance times. It will be shown that one of the most challenging tasks in implementing an effective RCM-based methodology approach is the decision-making process of maintenance tasks and frequencies thereof — where sub-optimal determinations of these would result in either (a) over-maintaining of equipment, resulting in avoidable excessive maintenance costs; or (b) under-maintaining of equipment, resulting in unreliability of equipment and, therefore, costly production time losses. The optimisation process utilises mathematical modeling techniques and failure probability distribution parameters in order to develop maintenance cost models for both single- and multi-component systems, where the Monte Carlo simulation approach is used to determine optimal maintenance tasks and frequencies thereof. Ultimately, a structured approach for further utilisation of the proposed models within an FMCG production environment is provided.

An extensive literature study is provided, which covers concepts relevant to the overall study, and which helps to contextualise the problem, revealing the challenges faced by maintenance management in cost-effective decision-making of maintenance tasks and frequencies. A methodology to schedule and develop the mathematical maintenance models, based on RCM scheduling and failure statistics, is presented. Validation of the two cost models makes use of a case study which compares the current incurred cost per unit times to the expected optimal cost per unit times from the models, based on derived optimal maintenance tasks and frequencies, in a current FMCG production facility.

It was shown that the mathematical modeling approach could be utilised to model current failure properties and resulting costs with relatively high accuracies. Based on the cost optimisation simulations, it was shown that there exists significant expected cost-saving potential through the implementation of the proposed optimal maintenance tasks and frequencies. These findings provide an exciting basis on which maintenance management can be assisted in determining RCM-based maintenance tasks and frequencies for equipment in order to optimise bottom-line costs incurred by production facilities.

Uittreksel

Onderhoud Taak en Frekwensie Optimalisering van Enkel- en Multi-Komponent Toerusting in 'n FMCG Produksie Omgewing

("Maintenance Task and Frequency Optimisation of Single- and Multi-Component Equipment in an FMCG Production Environment")

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Die doel met hierdie tesis is om 'n gestruktureerde benadering te verskaf tot die koste-optimalisering van betroubaarheidsgesentreerde onderhoud (BGO) -gebaseerde instandhoudingstake en -frekwensies binne 'n vinnig-bewegende verbruikersgoedere (VBVG) produksie-omgewing, met inagneming van nie-weglaatbare onderhoudstye. Dit sal aangetoon word dat een van die mees uitdagende take in die implementering van ñ doeltreffende BGO-gebaseerde metodologiebenadering is die besluitnemingsproses van instandhoudingstake en frekwensies daarvan — waar sub-optimale bepalings hiervan sal lei tot óf (a) oormatige handhawing van toerusting, wat sal lei tot vermybare buitensporige onderhoudskostes; of (b) ontoereikende handhawing van toerusting, wat sal lei tot onbetroubaarheid van toerusting en dus duur produksietydverliese. Die optimaliseringsproses gebruik wiskundige modelleringstegnieke en mislukkingwaarskynlikheidsverspreiding-parameters ten einde onderhoudskostemodelle vir beide enkel- en multi-komponent stelsels te ontwikkel, waar die Monte Carlo-simulasiebenadering gebruik word om optimale instandhoudingstake en frekwensies daarvan te bepaal. Uiteindelik word ñ gestruktureerde benadering vir verdere gebruik van die voorgestelde modelle binne ñ VBVG produksie-omgewing verskaf.

ñ Uitgebreide literatuurstudie word verskaf, wat konsepte dek wat op die studie as geheel betrekking het en wat help om die probleem te kontekstualiseer. Sodoende word die uitdagings wat die onderhoudsbestuur ten opsigte

van koste-effektiewe besluitneming oor instandhoudingstake en -frekwensies onthul. ñ Metodologie om die wiskundige onderhoudsmodelle te skeduleer en ontwikkel, gebaseer op BGO skedulering- en mislukkingstatistieke, word aangebied. Bevestiging van die twee kostemodelle maak gebruik van ñ gevallestudie wat die huidige koste-per-eenheid tye wat aangegaan is, vergelyk met die verwagte optimale koste-per-eenheid tye van die modelle, wat gebaseer is op afgeleide optimale instandhoudingstake en frekwensies in ñ huidige VBVG produksiefasiliteit.

Daar word bewys dat die wiskundige modelleringsbenadering gebruik kan word om die huidige mislukkings-eienskappe en gevolglike kostes met ñ relatiewe hoë akkuraatheid te modelleer. Op grond van die koste-optimalisering-simulasies, word daar bewys dat deur middel van die implementering van die voorgestelde optimale instandhoudingstake en frekwensies bestaan daar ñ beduidende verwagte koste-besparingspotensiaal. Hierdie bevindinge bied ñ opwindende basis waarop onderhoudsbestuur bygestaan kan word in die bepaling van BGO-gebaseerde instandhoudingstake en frekwensies vir toerusting om grondslag-bedryfskostes wat deur die produksiefasiliteite aangegaan is, te optimaliseer.

Acknowledgements

I would like to express my sincerest appreciation and gratitude to the following people and organisations:

- My supervisor, Dr. J.L. Jooste, for his guidance, encouragement, support and invaluable expertise in ensuring that this study not only remained on the right track, but also provided a contribution to the asset care field.
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- My wife, Elsabé, who has supported me through thick and thin, no matter the circumstances. Elsabé has expressed tremendous patience throughout my studies, and always shown her loving, non-selfish support for me in ensuring that my perseverance never dies out and I never give up on my goal. Without her unwavering love and support, this study may never have been completed.
- South African Breweries (SAB) for their willingness and belief in me, through providing me with the financial means and time that made this study possible.

The Author: December 2016

Dedications

This thesis is dedicated to my son-to-be, who is due to be welcomed to this world on the 11th of January 2017. I will be sure to pass on to you all of the love, support and encouragement that I have received throughout my life. I can't wait to meet you.

Contents

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Acronyms

CCT	Continuous on-condition task
CDF	Cumulative distribution function
CM	Corrective Maintenance
DOD	Department of Defence
EPRI	Electric Power Research Institute
FAA	Federal Aviation Administration
FFA	Functional Failure Analysis
FMCG	Fast Moving Consumer Goods
FMEA	Failure Modes and Effects Analysis
FPT	First Passage Time
FTA	Fault Tree Analysis
IID	Independent and identically distributed
ITM	Inverse transformation method
JSE	Johannesburg Stock Exchange
LCC	Life-cycle Cost
LTA	Logic Tree Analysis
ME	Method of moments

ML	Method of maximum likelihood
MLE	Maximum likelihood estimation
MSG	Maintenance Steering Group
MTBF	Mean time between failures
MTTR	Mean time to repair
NHPP	Non-homogeneous Poisson process
PDF	Probability density function
PdM	Predictive Maintenance
PM	Preventive Maintenance
RBI	Risk-based Inspection
RCM	Reliability-centred Maintenance
RTF	Run to failure
SAB	South African Breweries
SCT	Scheduled on-condition task
SFT	Scheduled function test
SKU	Stock Keeping Unit
SOH	Scheduled overhaul
SRP	Scheduled replacement
SWBS	System Work Breakdown Structure
TPM	Total Productive Maintenance
TQM	Total Quality Management
UK	United Kingdom

USA	United States of America
WCM	World Class Maintenance

Glossary

Condition monitoring

The process of monitoring a parameter of condition in equipment, in order to identify a significant change which is indicative of a developing fault.

Corrective maintenance (CM)

Tasks performed to identify, isolate, and rectify a fault so that the failed equipment, machine, or system can be restored to an acceptable operating condition.

Failure Modes and Effects Analysis (FMEA)

A step-wise approach to identifying all possible failures in a design, a manufacturing or assembly process, or a product or service. "Failure modes" defines the modes in which an item may fail.

Fast Moving Consumer Goods (FMCG)

Products that are sold quickly and at relatively low cost. Examples include goods such as soft drinks, toiletries, over-the-counter drugs, and processed foods.

Predictive maintenance (PdM)

The process of determining the condition of in-service equipment in order to predict when maintenance tasks(s) should be performed.

Preventive maintenance (PM)

The routine repair, replacement, and maintenance of equipment in order to avoid unexpected failure during use.

Reliability-centred Maintenance (RCM)

The systematic approach for identifying effective and efficient maintenance tasks for items in accordance with a specific set of procedures and for establishing intervals between maintenance tasks.

Simulation

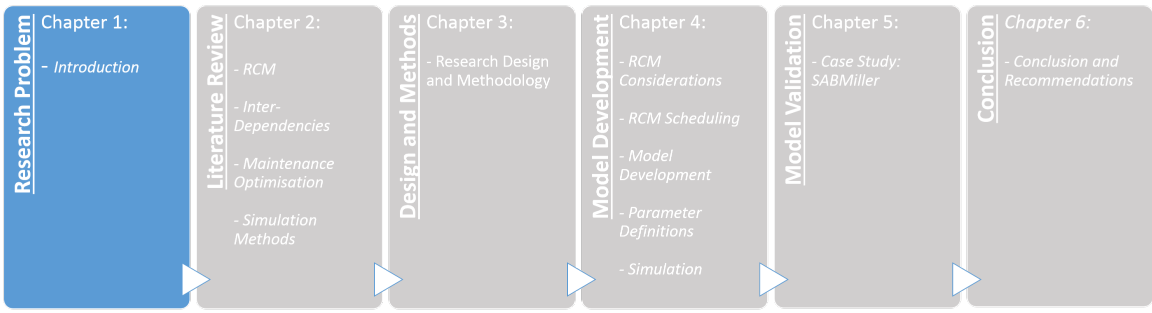
The process of representing a unit or system by some means in order to provide some or all identical inputs, at some interface, for test purposes. A means of prediction.

Socket

A physical, independent testing entity in which a component is placed in order to perform reliability analyses with no dependency to any other socket(s).

Chapter 1

Introduction



1.1 Theoretical Background

Maintenance can be defined as the day-to-day, periodic, or scheduled work required to preserve or restore facilities, systems, and equipment to continually meet or perform according to their designed functions (Dale Johnson, 2002, 5). Due to ageing effects and accumulative wear, most repairable systems do not maintain their perfect functioning state (Kobbacy and Prabhakar Murthy, 2008, 3). In the early stages of industrial development, maintenance practices were simple, primarily of housekeeping and breakdown types (Dale Johnson 2002, 1; Kobbacy and Prabhakar Murthy 2008, 3). Increased mechanisation and automation have increased the capital employed in production equipment, making it increasingly apparent that improvement of maintenance management practices and procedures was essential to achieve efficiency and effectiveness of the maintenance operations (Moubray, 1997, 3). It could be suggested that the principle responsibility of maintenance is to provide a service to an organisation that enhances its ability to make a profit (Davies, 2003, 11).

Preventive Maintenance (PM) was first prescribed to improve safety rather than to increase availability or reduce costs, for example the pressure testing of boilers and re-setting of safety valves (Sherwin, 2000, 145). PM involves the repair, replacement, and maintenance of equipment in order to avoid unexpected failure during use (Lawrence *et al.*, 1995, 46). The objective of any PM programme is the minimisation of the total cost of repair, replacement, and equipment downtime (measured in terms of lost production capacity or reduced product quality) (Sachdeva *et al.*, 2008, 817). The actual implementation of PM varies greatly, with extremely limited programmes such as lubrication and minor adjustments, as well as more comprehensive programmes scheduling repairs, lubrications, adjustments, and inspections (Keith Mobley, 2002, 3). The common denominator for all PM programmes is the scheduling guideline (Osarenren, 2015, 321). The disadvantage of purely time-based PM is that the frequency of PM will most likely to be too high. This frequency can be lowered, without sacrificing reliability when condition monitoring and analysis is used (Keith Mobley, 2002, 71). The decrease in maintenance frequency is offset by the additional costs associated with conducting the condition monitoring.

Predictive Maintenance (PM), or condition monitoring, has many definitions, ranging from monitoring the vibration of rotating machinery in an attempt to detect obscure problems and prevent catastrophic failures, to the infra-red imaging of electrical switchgear, motors and other electrical equipment to detect developing problems (Keith Mobley, 2011, 4). The common premise of PdM is the regular monitoring of the actual condition of systems that will provide the data required to ensure maximum interval between repairs and minimal unplanned breakdowns caused by failures. PdM can be considered as a condition-based preventive maintenance program (Keith Mobley, 2002, 4). Instead of relying on scheduled, time-based maintenance activities, PdM uses direct monitoring of the system's condition to schedule future maintenance activities thus conducting maintenance only when necessary and, in principle, saving resources and system availability (Wang, 2008, 111).

Up until the late 1970s, product development and manufacturing engineering were the dominant technical disciplines in the industrial community, with operations and management often being neglected in the priority of corporate success strategies (Smith and Hinchcliffe, 2004, 1). Recent times has brought about the need and development of maintenance management methods, such as total productive maintenance (TPM) and reliability-centred maintenance (RCM), as the panacea for ineffective maintenance (Keith Mobley, 2002, 6). There are compelling reasons for this, not the least of which is the decisive role that operations and management now plays in issues ranging from safety, liability, and environmental factors to bottom-line profitability (Smith and Hinchcliffe, 2004, 1). RCM-based maintenance strategy originated in the airline industry in the 1960s to counter the ever-increasing cost of maintenance activities in the industry. Nowadays, RCM is probably the global analysis method which is most frequently used for identifying a cost effective maintenance program for a plant (Vatn *et al.*, 1996, 241). It seeks to optimise the maintenance strategy to minimise system failures and, ultimately, increase equipment reliability and availability (Brauer and Brauer, 1987, 17). Rausand and Vatn (2008, 79) defines RCM as a "systematic approach for identifying effective and efficient preventive maintenance tasks for items in accordance with a specific set of procedures and for establishing intervals between maintenance tasks". For the system in question, failure modes and their consequences are identified by using various techniques, for example, failure modes and effects analysis (FMEA); fault tree analysis (FTA); and risk-based inspection (RBI) (MA CMMS [Online], 2016). Cost-effective maintenance techniques that minimise the possibility of system failures can then be determined. Many people share the view that the RCM-based maintenance strategy methodology offers the best available decision strategy for PM optimisation and World Class Maintenance (WCM) (Smith and Hinchcliffe 2004, 1; Deshpande and Modak 2002, 31). Despite the global-wide adoption of the RCM approach to maintenance, there exists no sound foundation for claiming that the maintenance strategy derived from the RCM approach is in any sense 'optimal' (Vatn *et al.*, 1996, 241). Sherwin (2000, 159) argues further that maintenance is principally an economic rather than solely a reliability problem, where RCM attempts at dealing with reliability and maintenance in relative isolation from costs and profits.

Over the last few decades the maintenance of systems has become more and more complex. The associated considerations of plant safety and reliability with issues related to economic performance, such as production revenues and repair and maintenance costs, complicates the management of maintenance and repair activities, especially for complex systems with numerous components (Marseguerra and Zio, 2000, 69). One reason for the increased complexity is that systems consist of many components with interdependencies (Nicolai and Dekker, 2008*a*, 263). The dependence among components can be classified into three different types, namely, *economic dependence* – costs can be saved when jointly maintained; *structural dependence* – if components structurally form a part; *stochastic dependence* – if the state of a component influences the lifetime distribution of other components. Dependence between components can lead to complicated modelling and optimisation of maintenance programmes, but at the same time also creates the opportunity to group maintenance which may save costs (Thomas, 1986, 299).

In the highly competitive environment, to be successful and to achieve world-class manufacturing, organisations must posses both efficient maintenance and effective maintenance strategies (Ahuja and Khamba, 2008, 711). The main question faced by maintenance management is whether its output is produced effectively, in terms of contribution to company profits, and efficiently, in terms of manpower and materials employed (Dekker, 1996, 229). Put in the words of Goldratt (1986, 60), the goal of a company is to "make money by increasing net profit, while simultaneously increasing return on investment, and simultaneously increasing cash flow". The language of upper management is money, and so the costs and values of maintenance should be expressed in cash terms as part of the system of management. A survey published as long ago as 1970 by the then Ministry of Technology in the UK showed that over £3 billion annually was being spent by the manufacturing industry, of which at least 8–10 percent could be saved by some very basic improvements, such as prevention of rust by more effective painting (Sherwin, 2000, 145). Since about 1985, when advanced companies finished installing total quality management (TQM) and started looking for other improvements to life-cycle cost (LCC), there has been more interest in reducing maintenance costs, but not so much in optimising the expenditure for the benefit of the company (Sherwin, 2000, 145). Maintenance costs, as defined by normal accounting procedures, are normally a major portion of the total operating costs in most plants, of which the major contributors to abnormal costs are delays, product rejects, scheduled maintenance downtime, and traditional maintenance costs (for example, labour, overtime, and repair costs) (Keith Mobley, 2011, 1). Another common assumption in developed models is to consider that inspections, repairs, and replacements have negligible task-time and therefore do not affect availability or total system cost (Laggoune *et al.* 2010, 747; Barlow and Proschan 1996, 108; Cléroux *et al.* 1979, 1158; Block *et al.* 1985, 370; Pham and Wang 1996, 851). For many systems, especially mass production manufacturing lines in the fast moving consumer goods (FMCG) industry, the production losses due to downtime can become significantly large, thus rendering this assumption to be invalid in certain cases (Laggoune *et al.*, 2010, 747). In the case of both planned and unplanned maintenance stoppages, the plant is continuously incurring expenses, whether it be in the form of salaries or wages, utilities costs, or equipment costs — during which time there is no actual product being produced which can be sold and, hence, no tangible form of income. Therefore, all downtime or unavailability of a plant can be considered as an expense incurred by the plant at a certain cost per unit of time. In theory, maintenance management, facing these challenges, could benefit from the advent of a large area in operations research called maintenance optimisation (Dekker, 1996, 264). Maintenance optimisation consists in broad terms of those mathematical models aimed at finding either the optimum balance between costs and benefits of maintenance or the most appropriate moment to execute maintenance (Dekker and Scarf 1998, 111; Vatn 2008, 510).

Many researchers have focused on the problem of developing exhaustive models of deteriorating systems (Marseguerra *et al.* 2002, 151; Wang 2002, 469). Markov and semi-Markov models have been widely exploited for achieving analytical results (Hontelez *et al.* 1996, 267; Kopnov 1999, 1; Lam and Yeh 1994, 423; Yeh 1997, 55). Yet, in all these cases, the models had to be built under simplified assumptions. Classical analytical or semi-analytical optimisation approaches, such as those based on the gradient descent methods, for example Vaurio (1995, 23), have been considered, however, these methods generally suffer severe limitations and can be applied only to simple systems with few components (Marseguerra and Zio, 2000). When more complex systems and realistic issues to the system behaviour are to be considered, the system can no longer be described by analytical models, so one has to resort to simulation tools, such as the Monte Carlo method (Marseguerra *et al.* 2002, 151; Grall *et al.* 1998, 381; Berenguer *et al.* 2000, 275; Shahanaghi *et al.* 2008, 230). The Monte Carlo simulation is a powerful modelling tool for the analysis of complex systems, due to its capability to achieving a closer adherence to reality (Zio, 2012, 1). It may generally be defined as a methodology for obtaining estimates of the solution of mathematical problems by means of random numbers. These random numbers can be generated through a roulette-like machine (similar to those used in gambling casinos of the Monte Carlo principalities) — hence the name Monte Carlo (Zio, 2012, 1).

1.2 Problem Statement

In the midst of this theoretical background, most mass production enterprises, specifically in the FMCG industry, implement the RCM-based maintenance strategy to minimise financial losses caused by system failures. Due to the complexity of most systems, as well as the considerations one must heed, determining an 'optimal' RCM-based maintenance strategy proves to be a tremendous challenge to the maintenance management team. The problem that arose is:

Many FMCG production enterprises continue to operate with non-optimised RCM-based maintenance strategy plans, specifically referring to the task and frequency determination. As a result, unnecessary additional costs are incurred.

To address this problem, this research study is conducted to develop a structured approach for FMCG enterprises to assist in determining a generic, mathematically-optimised task- and frequency-based RCM-based maintenance strategy, aiming at minimising the financial losses incurred as a result of unexpected production losses as well as the total maintenance costs.

1.3 Research Questions

Based upon background and the establishment of the problem statement, the primary question for this research is:

How can a structured approach be constructed for assisting maintenance managers in determining an optimal RCM-based maintenance strategy within an FMCG production environment?

In support of the primary research question, the following sub-questions need to be investigated:

(a) What are the foundations of reliability-centred maintenance?

(b) How can these foundations be integrated into a reliability-centred maintenance decision-logic?

(c) How can this decision-logic be optimised?

(d) How can the mathematical maintenance model optimisation be formulated into a structured approach?

(e) How can the 'optimised' reliability-centred maintenance strategy be validated?

1.4 Research Objectives

In order to answer the research questions above, research objectives are formulated to guide the process. The primary objective of the study is:

Develop a structured approach to assist maintenance managers within the FMCG production environment in determining an optimal set of RCM tasks and frequencies.

This objective addresses the need for assistance as indicated in the research question. In an attempt to achieve the primary objective, other manageable sub-objectives are formulated. The sub-objectives of this study are:

1. Establish the fundamentals of reliability-centred maintenance:
    a) Review the historical background of reliability-centred maintenance;
    b) Define corrective maintenance, preventive maintenance, and predictive maintenance;
    c) Identify the fundamental principles of reliability-centred maintenance and the implementation thereof;
2. Identify factors influencing the decision-making process of relative dependencies between components in a system to be maintained:
    a) Define economic, structural, and stochastic dependencies;
    b) Identify factors to be considered when determining the relative dependencies between equipment components;
3. Construct a well defined research methodology;

4. Investigate the academic literature of, and the methodologies founded on, mathematical modelling of maintenance programs;

5. Develop a mathematical model which takes into consideration all costs involved over a component's life-cycle:

    a) Determine all inputs and outputs to be included in the mathematical maintenance model applicable to this research;

    b) Based on the inputs and outputs, construct an applicable mathematical model;

    c) Perform an optimisation of the model in terms of total cost to determine optimal maintenance tasks and frequencies;

6. Formulate a structured approach for the optimisation of the proposed mathematical maintenance models, which can be further utilised on alternative equipment within a fast moving consumer goods production facility;

7. Perform a literature study to investigate simulation tools;

8. Perform a case study on a fast moving consumer goods production facility's maintenance strategy:

    a) Perform data collection of the relative input and output performance of the facility;

    b) Perform a simulation of the theoretical performance of the facility in the case that the optimised maintenance model were to be implemented;

    c) Based on the simulation results, determine the potential cost savings, if the proposed maintenance model were to be implemented;

9. Draw conclusions on the results obtained.

This study seeks to achieve the above-mentioned objectives. The research process is guided by the objectives will be discussed in section 1.7.

1.5 Research Design and Methodology Overview

Owing to the quantitative complexity of this study, a quantitative research design is identified as suitable for this study. This research design is deemed necessary for observing and measuring system reliability data, where an appropriate mathematical procedure, namely mathematical maintenance modeling, is then used for the statistical analysis required for hypothesis testing. In respect of this, the study follows a post-positivist world-view, which is empirical in nature and utilises a deterministic philosophy which determines a specific outcome or effect.

Within the quantitative approach, relationships and dependencies between equipment components are distinguished and developed by means of mathematical modeling and system reliability function probabilities. This is achieved by means of a broad study into current maintenance models. The analysis of these models and their relative areas of applicability ultimately leads to the development of a maintenance model which is relevant to this study. The expected inherent complexity of such a model inevitably leads to the use of simulation in order to determine relative inputs yielding an optimal maintenance output which, in the light of this study, would be the overall maintenance cost.

Validation of the proposed optimised maintenance approach is achieved by means of a case study within an existing FMCG production facility. The crux of the validity of the optimised model lies in the comparison of real-life maintenance costs over a certain time period to the theoretically expected maintenance cost incurred over the same time period if the proposed maintenance model was used.

The quantitative research approach, as well as the post-positivist world-view in this study thus serve as the approach to assist maintenance management in developing a maintenance model that will determine specific maintenance tasks and frequencies resulting in an optimally low maintenance cost.

1.6 Delimitations and Limitations

Since new areas of research are continuously being explored, it is imperative to state the scope, or boundaries, of this study to refine its focus. This section explicitly discusses both the delimitations and limitations of the study. The following primary boundaries are identified and must be addressed:

1. This study attempts at optimising a maintenance programme based on the reliability-centred maintenance policy, and does not consider alternative maintenance policies, such as TPM. Background research has shown that RCM is arguably the most frequently used maintenance strategy in a production plant. In addition, (Dale Johnson 2002, 6; Studebaker; Rosqvist *et al.* 2009, 98) argue that TPM is not a maintenance-specific programme or policy, but rather a culture or philosophy.

2. The study assumes that a functioning RCM-based maintenance strategy has already been implemented within the production environment, and does not encompass a total RCM-based maintenance strategy implementation. The complete new implementation of an effective RCM-based

maintenance strategy is a study within itself and therefore does not fall within the scope of this study.

3. Modern reliability-centred maintenance programs begin in the design phase of systems, and extends throughout the system's service life. To prevent the scope of this research from becoming too large, the optimisation approach will only consider the in-service life of the system, in other words, this research does not consider the design phase of the system in question.

4. The maintenance of equipment used in an FMCG production environment, in particular, is considered. Although the optimisation approach may be applicable in various industries, the study does not seek to provide an overall approach to maintenance of equipment utilised in alternative industries, for example, in equipment-intensive industries.

5. Considering the quantitative nature of this study and given the time-consuming data analysis and interpretation, the time-frame of this study is limited. Thus, the efficacy of the proposed maintenance model will not be based on the implementation thereof, but rather on the comparison between past performance versus the simulated performance of the model. It should be noted that the simulation approach in maintenance has been extensively researched in literature and applied in the field of maintenance.

The above-mentioned delimitations and limitations are considered throughout the execution of this research paper. The outline of the thesis is further elaborated on in the following section.

1.7 Thesis Outline

Based on the objectives listed in section 1.4, this section will describe the structural layout of the research paper. Each chapter is presented in correspondence to the defined research objectives. Table 1.1 illustrates the road map and chapter sequence of the thesis. The thesis is presented in five chapters.

Chapter 1 contextualises the research study. A theoretical background is established, which develops the research problem statement and questions. The specific research objectives of the study are formulated, followed by a brief overview of the research design and methodology. Research delimitations and limitations are stated, where the chapter then concludes with an outline of the thesis document.

Chapter	**Objective**	**Question**
Chapter 2: Literature Review	*1a; 1b; 1c; 2a; 2b; 4;* and *7.*	*a* and *b.*
Chapter 3: Research Design and Methodology	*3.*	
Chapter 4: Maintenance Model Development and Optimisation	*5a; 5b; 5c;* and *6.*	*c* and *d*
Chapter 5: Case Study: SABMiller	*8a; 8b;* and *8c.*	*e.*
Chapter 6: Conclusion and Recommendations	*9.*	

Table 1.1: Summary of chapters and their corresponding objectives and questions

Chapter 2 provides a comprehensive literature review on the theoretical framework relevant to the research. There are three key concepts covered in the chapter: Maintenance policies — *a firm understanding of maintenance policies and strategies which are currently implemented in industry is studied, focussing on the fundamentals of RCM, corrective maintenance (CM), PM, and PdM*; Mathematical modelling — *an extensive literature review in the field of mathematical modelling, specifically the application thereof in maintenance models, is covered*; and Simulation techniques — *the fundamentals of simulation techniques and tools available are studied, with specific focus on simulation techniques currently employed in maintenance optimisation studies.*

Chapter 3 provides a detailed overview of research design and methodologies. An explanation of the philosophical worldview, research design, and research methods followed in the research is provided.

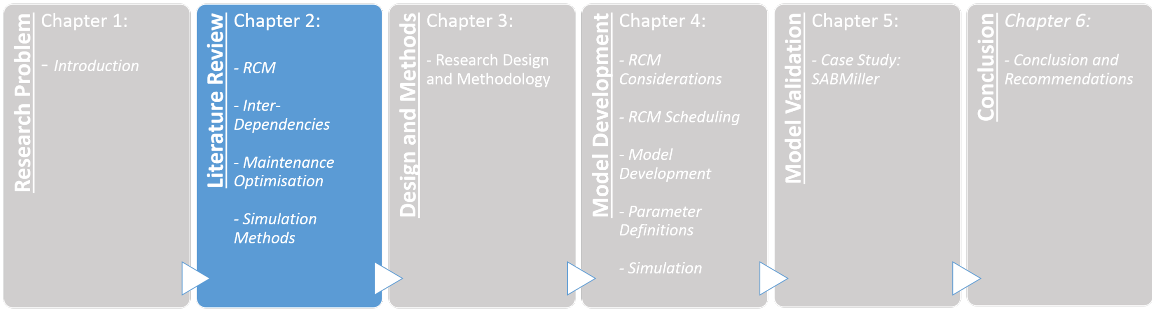
Chapter 4 provides a detailed process description of the development of the single- and multi-component maintenance cost models. The RCM-scheduling approach for maintenance tasks and frequencies is described, on which the maintenance cost models are further developed. Programming of both the single- and multi-component maintenance cost models into Matlab's software is done in order to provide models on which simulation techniques are based, where the output variable is the cost per unit of time under varying maintenance conditions. Maintenance, failure, and cost data factors are defined and described, which lead to the definition and determination of input variables to be used in the proposed maintenance cost models. A summary is provided of the structured sequential steps to be followed in order to optimise maintenance methodologies using the proposed cost models.

Chapter 5 serves to validate the proposed maintenance cost models. Validation is achieved by conducting a case study on one of SABMiller's production facilities. The case study comprises of two sections, where the first section analyses a single component in order to validate the single-component maintenance cost model; and the second section analyses a group of 3 components in order to validate the multi-component maintenance cost model. The crux of the validation of the single- and multi-component cost models lay in the potential cost savings per unit of time for each model — a comparison of the cost per unit of time under current maintenance conditions is thus compared to the cost per unit of time under the proposed optimised maintenance conditions.

Chapter 6 provides the summary and conclusions of the key findings from conducting this study. A brief overview of the study is provided, together with the pre-defined study objectives and how these are met in the current study. Further, recommendations and research possibilities are proposed which could further benefit the engineering and reliability sector.

Chapter 2

Literature Review



This chapter contains the theoretical framework that informed the study, and highlights the need for research into the optimisation of maintenance policies. A thorough overview of RCM is presented, which leads to the specific challenges and, as a result, the objective of the study. The background and impact of relative dependencies between components is discussed, endeavouring to provide an understanding of whether single-component or multi-component modeling criteria should be integrated with the maintenance optimisation objective. An overview and historical background into the concept of mathematical modelling, specifically in the field of maintenance optimisation, is undertaken. Finally, the chapter concludes with a literature study on relevant simulation tools and techniques currently utilised in the field of maintenance.

2.1 Reliability-centred Maintenance

In this section, the history and development of RCM are presented. Key concepts, such as CM, PM, and PdM, forming the fundamental foundations of

RCM are covered. An understanding of the methodology to be followed that constitutes the construction of an RCM policy is also described.

2.1.1 Historical Background of Reliability-centred Maintenance

The background of RCM goes back to the 1960s, where the aviation industry found itself on the threshold of the jumbo jet era. The reality of the 747 jumbo jet was quickly taking shape as hardware at the Boeing factory in Seattle, USA. The licensing of an aircraft type requires that a Federal Aviation Administration (FAA)-approved PM program be specified for use by all owners and operators of the aircraft. The recognised size of the 747 (three times as many passengers as the 707 or DC-8), its new engines, and its many technological advances in structures and avionics led the FAA to initially presume a very extensive PM program on the 747 — so extensive, in fact, that the airlines could not likely operate this aircraft in a profitable fashion (Smith, 1993, 47). In the USA, a task force was formed consisting of representatives from the FAA and the airline companies to investigate the capabilities of PM for aircraft (van der Vet, 1991, 30). Further work during the 1960s showed that more efficient PM programs could be developed through the use of logical decision processes. This work was performed by a Maintenance Steering Group (MSG) consisting of representatives from manufacturers of aircraft-systems, future aircraft operators, and the USA National Aviation Authorities. In July 1968, the handbook MSG-1, "Maintenance Evaluation and Program Development" was issued, which included decision-logic and inter-airline, or manufacturer, procedures for developing a maintenance program for the new Boeing 747 aircraft (van der Vet, 1991, 30). Following the approval by the FAA for the structuring of PM tasks defined in MSG-1 for the Boeing 747, it was clear that the economics of PM on a 747-sized aircraft were quite viable (Smith, 1993, 48). In 1972, these ideas were first applied by United Airlines under Department of Defence (DOD) contract to the Navy P-3 and S-3 aircraft and, in 1974, to the Air Force F-4J. In 1975, DOD directed that the MSG concept be labelled "Reliability-Centred Maintenance", and it could be applied to all major military systems (Smith, 1993, 48). It was in 1983 that the RCM methodology began to branch into various industries, where the Electric Power Research Institute (EPRI) initiated RCM pilot studies on nuclear power plants (Smith and Hinchcliffe, 2004, 63). The implementation of the RCM methodology to the commercial nuclear power industry showed that overall annual savings were in the range of 30–40% reduction in maintenance labour hours and material costs (Deshpande and Modak, 2002, 33). Additional savings were also obtained due to the reduction in forced outages from system failures. The RCM concept was subsequently applied to solar receiving plants to establish the feasibility that it can be applied without modification to non-aviation and non-nuclear systems

(Jones 1995, 10; Wilmeth and Usrey 2000, 26). As the RCM methodology has been applied to different industries, the process has evolved to fit the needs of each application (August, 1999, 55). The proven approach of RCM has led to organisations throughout a wide range of industries introducing RCM to optimise their maintenance operations (Johnston 2002, 511; Vatn 2008, 510).

In nearly every field of human endeavour, RCM is now becoming as fundamental to the responsible custodianship of physical assets as double-entry bookkeeping is to the responsible custodianship of financial assets (Moubray, 1997, 1). In contrast to the traditional approach, current RCM practice focuses on the consequences of failure in a prioritised hierarchical structure and uses a decision logic process to develop an optimum maintenance program (Anderson and Neri, 1990, 16). The following section will cover more detail surrounding the RCM approach.

2.1.2 The Reliability-centred Maintenance Methodology

The IEC (1999) defines RCM as a "systematic approach for identifying effective and efficient preventive maintenance tasks for items in accordance with a specific set of procedures and for establishing intervals between maintenance tasks". A major advantage of the RCM analysis process is a structured, and traceable approach to determine the optimal type of PM. This is achieved through a detailed analysis of failure modes and failure causes (Vatn, 2008, 510). The main objectives of an RCM analysis process are to:

1. Identify effective maintenance tasks

2. Evaluate these tasks by some cost-benefit analysis

3. Prepare a plan for carrying out the identified maintenance tasks at optimal intervals

Smith (1993, 63) defines and characterises RCM in four features, that distinctly sets it apart from traditional PM planning processes:

1. The **primary objective** of RCM, which is the first and most important feature of RCM, is to *preserve equipment function*, as compared to the ingrained notion of PM to *preserve equipment operation*. The expected output, or function, of the equipment must be addressed — preserving this output (function) is essentially the primary task at hand. This feature essentially nullifies the erroneous priori assumption that every item of equipment is equally important.

2. Considering that the primary objective of RCM is to preserve system function, the loss of system function, or **functional failure** is the next item of consideration. Functional failures can occur in many different

forms, and are not always a case of "working versus not-working" states. The many intermediate states that may occur between "working" and "not-working" states must carefully be examined, as these intermediate states may ultimately be of vital importance. A simple example would be the loss of fluid in a hydraulic system, where (a) a minor leak may be qualitatively defined as a drip; (b) a fluid loss to a certain extent may be defined as a basic leak, which is, any leak beyond a predetermined value will produce a negative effect on system function (but not necessarily a total loss); and (c) a total loss of boundary integrity, which may be defined as a catastrophic loss of fluid and loss of function. In this example, a single function, namely to preserve fluid boundary integrity, may have led to three distinctive functional failures. Essentially, the key point of the second feature is *identify specific failure modes that could potentially produce the unwanted functional failures.*

3. With the primary objective of preserving system function, RCM results in an opportunity to decide, in a very systematic way, in what order or priority one wishes to assign in allocating budgets and resources, in other words to **prioritise the importance of the failure modes**.

4. The first three features have essentially formulated a systemic roadmap depicting where (component), what (failure mode), and priority with which to proceed in order to develop specific PM tasks — all of which considers the fundamental premise of preserving system function. Each prioritised failure mode thus identifies candidate PM actions that could be considered. Each potential PM task must be judged according to its inherent **applicability and effectiveness**. The applicability of a task refers to its ability to accomplish one of the three reasons for executing PM, namely to prevent or mitigate failure; to detect the onset of a failure; or discover a hidden failure. The effectiveness of a task refers to the justifiable spend of resources in order to execute the task. If more than one candidate task is judged to be applicable, the least expensive (which is, most effective) task would generally be selected.

The RCM analysis process is carried out as a sequence of activities. The structuring of the RCM process differs in the various standards, guidelines, and textbooks (Vatn, 2008, 510). Anderson and Neri (1990, 16) follows a four-step approach, Smith and Hinchcliffe 2004, 55; Moubray 1997, 71 structure their approach in seven steps, and Vatn (2008, 511) structures it in twelve steps. In essence, each of these structured approaches encompasses the foundational thinking of the RCM methodology. The seven-step approach described by Smith and Hinchcliffe (2004, 55) will be covered in more detail.

Step 1: System selection and information collection

Step 2: System boundary definition

Step 3: System description and functional block diagram

Step 4: System functions and functional failures

Step 5: FMEA

Step 6: Logic Tree Analysis (LTA)

Step 7: Task selection

Satisfactory completion of the above steps will provide a baseline definition of the preferred PM tasks on each system with a well-documented record of exactly how those tasks were selected and why they are considered to be the best selections among competing alternatives (Smith, 1993, 71).

2.1.2.1 Step 1 — System Selection and Information Collection

The overriding motivation of current PM practices is to preserve an equipment's operational condition. Until recently, this has resulted in little, if any, consideration as to why certain PM actions are undertaken, as well as what, if any, priority should be assigned to the expenditure of of PM resources. In the majority of instances, maintenance planning starts directly with the equipment and seeks to specify (as quickly as possible) various tasks that are felt necessary to maintain the operational status of the equipment (Smith and Hinchcliffe, 2004, 49).

Prior to the RCM analysis or implementation, two key questions should be considered:

1. To which systems would the RCM approach be most beneficial?

2. At what level of assembly (plant, system, subsystem) should the approach be implemented?

The number of separately identifiable systems in a plant or facility can vary widely, depending upon plant facility complexity, financial accounting practices, regulatory constraints, and other factors that may be unique to a given industry or organisation (Smith and Hinchcliffe, 2004, 65). Most production plants have therefore developed an assembly hierarchy on production equipment. An assembly hierarchy is essentially an organisation of the system hardware elements into a structure analogous to the root system of a tree (Vatn, 2008, 511). Upon implementation of an RCM policy in a plant, priorities need to be set for systems considering that resources will always be limited and, as a result, the initial RCM analysis should focus on the systems that would presumably benefit most from the RCM methodology. The following terms, sourced from Vatn (2008, 511), provide an understanding of a typical assembly hierarchy:

- *Plant*: A logical grouping of systems that function together to provide an output or product by processing and manipulating various input raw materials and feed stock. A beverage bottling plant may, for example, be considered as a plant. Moubray (1997, 1) refers to a plant as a cost centre.

- *System*: A logical grouping of subsystems that will perform a series of key functions, which often can be summarised as one main function, that is required of a plant (for example, feed water, steam supply, and product injection). It is usually trivial to identify the systems in a plant, seeing as they are used as logical building blocks in the design process. The system may be further broken down into subsystems, and sub-subsystems, and so on.

- *RCM analysis item*: A grouping or collection of components, which together form some identifiable package that will perform at least one significant function as a stand-alone item (for example, pumps, valves, and electric motors). An analysis item is usually repairable, meaning that it can be repaired without replacing the entire item.

- *Component*: The lowest level at which equipment can be disassembled without damage or destruction to the items involved.

When PM planning is approached from the function point of view, experience has shown that the most efficient and meaningful function list for RCM analysis is derived at the system level (Smith and Hinchcliffe, 2004, 75). The system-level approach further raises the question as to which systems to address, and in what order. One of the possible options would be to conduct the analysis to treat all plant or facility systems, however, literature has shown that this approach may not be cost-effective from a maintenance view point in that some systems have neither a history of frequent failures, excessive maintenance costs, nor contributions to forced outages that might warrant a special investigation to "make it better" (Smith, 1993, 76). Various factors such as large PM actions or costs, large CM actions or costs, safety and environmental issues are considered for selection of they system (Deshpande and Modak, 2002, 33).

Considerable time and effort can be saved by researching and collecting some necessary system documents and information that will be needed in subsequent steps (Smith and Hinchcliffe, 2004, 79). Typical documents and information that may be beneficial in the RCM analysis include:

- A system schematic or block diagram.

- Individual vendor manuals for the equipments in the system.

- Equipment history files, which lists actual failures and CM actions that have occurred in the facility or plant.
- System operation manual, which provides valuable details on how the system is intended to function, how it relates to other systems, and what operational limits and ground rules are employed.
- System design specification and description data, which helps identify the system functional description.

The above list is not an all-inclusive view of possible information sources, as there may be unique sources of information depending on the plant or facility being analysed.

2.1.2.2 Step 2 — System Boundary Definition

Some gross system definitions and boundaries usually have been established in the normal course of the plant or facility design — these system definitions have already been established in section 2.1.2.1 as the basis for system selection. These same definitions serve quite well in initially defining the precise boundaries that must be identified for the RCM analysis process (Smith and Hinchcliffe, 2004, 83). The system boundary definition is significantly important in accurately listing components in a system that will not overlap with components in an adjacent system.

Essentially, there are no dead-set rules that precisely govern the establishment of system boundaries, however, there are some *general* rules. One general rule is that the system's operating context should be written in sufficient detail at a level which will help the facilitator lead the group through the RCM process with a maximum understanding of the asset being analysed. Another general rule is that the operating context should be written in sufficient detail to give a reader who may not know anything about the asset enough information to understand how the asset is employed in addition to other information used during the conduct of the analysis (Gehris, 2015, 22).

2.1.2.3 Step 3 — System Description and Functional Block Diagram

The aim of the third step is to identify and document the essential details of the system that are needed to perform the remaining steps in a thorough and technically correct fashion (Smith and Hinchcliffe, 2004, 86). Deshpande and Modak (2002, 34) lists five items of information that are typically developed in step 3:

1. *System description*, revealing the factors such as functional description, redundancy features, and protection features. This will aid in identifying

critical design and operational parameters that frequently play a key role in delineating the degradation or loss of required system functions (Smith and Hinchcliffe, 2004, 88).

2. *Functional block diagram*, indicating top-level representation of major system functions.

3. *IN/OUT interfaces*, establishing what comes in to the system (IN), for example electrical power, and what leaves the system (OUT), for example to support other systems.

4. *System Work Breakdown Structure (SWBS)*, describing the compilation of the equipment lists for each functional subsystem in the functional block diagram.

5. *Equipment history*, typically the history of failures of the system over the past two to three years.

The first two features described by Smith (1993, 83) in the four-feature description in section 2.1.2 indicates that there must exist a thorough understanding of the functions of each asset, together with the associated performance standards. It is a well established principle of value engineering that a function statement should consist of a verb and an object (Moubray, 1997, 22). In some cases, it may be desirable to split system functions into subfunctions on an increasing level of detail, down to functions of analysis items, also known as functional hierarchies. The most common illustrative manner to describe functional hierarchy is the functional block diagram, however, alternative methods such as the reliability block diagram and fault trees can also be used (Vatn, 2008, 511). A simple functional block diagram of a diesel engine is shown in Figure 2.1. Functional hierarchies and functional block diagrams are an essential part of the equipment design process, seeing as design starts with a list of desired functions and designers have to specify an entity (asset or system) which is capable of fulfilling each functional requirement. Functional block diagrams are also useful as a basis for the FMEA, which is discussed later.

2.1.2.4 Step 4 — System Functions and Functional Failures

Step 4 begins by defining system functions, which is done, of course, to satisfy the first RCM principle "to preserve system functions" (Smith and Hinchcliffe, 2004, 65). The process aims at ultimately defining PM tasks that will "preserve" these system functions. The development of the OUT interfaces constitutes the primary source of information for system functions (Deshpande and Modak, 2002, 34). Function statements are developed for each functional subsystem by capturing every output interface. A system function may be subject to a set of performance standards that may be grouped as physical properties,

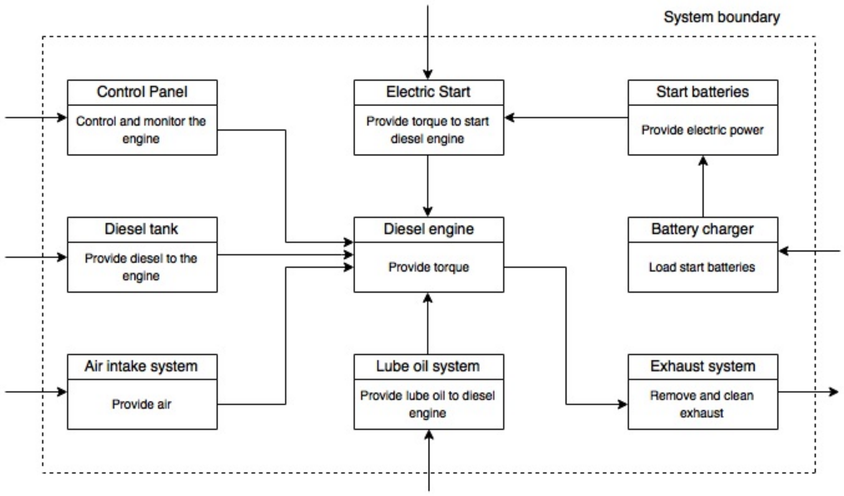


Figure 2.1: Functional block diagram for a diesel engine (Adapted from Vatn (2008, 511))

operational performance properties including output tolerances, and time requirements such as continuous operation or required availability (Vatn, 2008, 511).

Once the system functions have been identified, the definition of functional failures must be executed, seeing as function preservation essentially refers to the avoidance of functional failures. An unacceptable deviation from the defined system functions can be classified as a *functional failure* (Vatn, 2008, 511).

2.1.2.5 Step 5 — Failure Mode and Effects Analysis

Step 5 aims at revealing which component failures have the potential to defeat the principle objective of "preserving function". The FMEA step is of vital importance in the initial analysis and implementation of an effective RCM policy, as Carlson (2012, 5) states that the core of an RCM project is an FMEA on selected manufacturing or operational equipment. Smith and Hinchcliffe (2004, 98) developed the use of a functional failure-equipment matrix for identifying these components, which could play a role in functional failure. This task requires a reasonable knowledge of the system design and operation characteristics (Deshpande and Modak, 2002, 34).

Following the identification of the specific components, the failure modes (how the component must fail in order to produce a functional failure) and the root cause for each failure mode are defined. The root cause refers to the basic reason for the failure mode — that is, why the failure mode occurred. The final stage in the FMEA process is the effects analysis, in which the consequence of the failure mode is determined (Smith and Hinchcliffe, 2004, 51). The two primary reasons for conducting effects analysis are

1. to assure that the failure mode in question does in fact have a potential relationship to the functional failure being studied, and
2. to introduce initial screening of failure modes that are not detrimental (Deshpande and Modak, 2002, 34)

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2.1.2.6 Step 6 — Logic Tree Analysis (LTA)

The purpose of step 6 is to further prioritise the emphasis and resources that should be devoted to each failure mode, recognising that all functions, functional failures and, hence, failure modes are not created equal (Smith, 1993, 109). There exist numerous ranking schemes that could be used to arrive at a priority-list of the failure modes, however, the RCM process uses a simple three-question logic that enables quick and accurate placing of each failure mode into one of four categories (Deshpande and Modak, 2002, 34). The basic LTA uses the decision tree structure shown in Figure 2.2. Based on the decision logic, it is evident that each failure mode will be identified as either

1. safety-related,
2. outage-related, or
3. economics-related

. In addition, the LTA decision logic distinguishes between evident (to the operator) or hidden (Smith and Hinchcliffe, 2004, 109).

The importance of distinguishing evident from hidden failures lies in the identification of hidden failures, which is typical in standby systems where the failure may not be identified until a demand is made. Hidden failures could therefore later give rise to failure-finding PM tasks (Smith and Hinchcliffe, 2004, 21). A safety problem would typically refer to a personnel death or injury, however, safety can be defined according to particular needs. If it is decided that the failure mode does not result in a safety problem, it is evident that the failure mode deals solely with plant or facility economics. According to Figure 2.2, failure modes will be placed into categories “A”, “B”, or “C”, with “D” symbolising *hidden failures*. PMtasks will typically be addressed with decreasing priority from “A” to “B” to “C” (Deshpande and Modak, 2002, 35). Step 6 thus concludes with a prioritised list of failure modes to be addressed in PM task selection, which is discussed in the following section.

2.1.2.7 Step 7 — Task Selection

The first six steps have been directed to delineating those failure modes where a PM task would yield the biggest return for the investment to be made. The

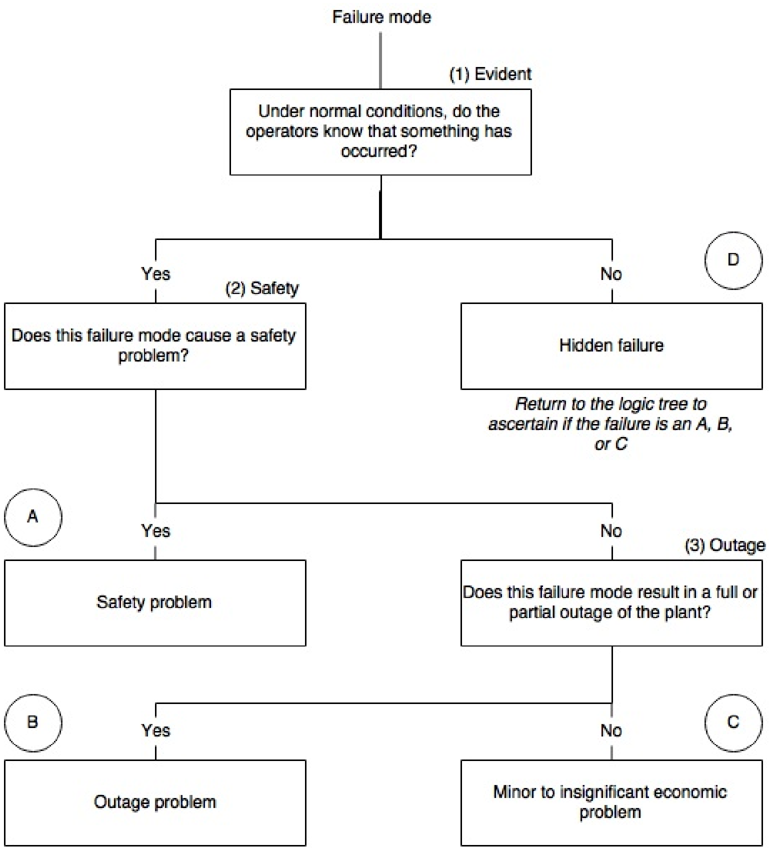

Figure 2.2: Logic tree analysis structure (Adapted from Smith and Hinchcliffe (2004, 109))

aim of step 7 is to decide, for each failure mode, whether a PM task is suitable, or whether it would be best to let the item deliberately run to failure, where a CM task will then be carried out (Vatn, 2008, 511). There are generally three reasons for carrying out PM tasks:

- Prevent a failure
- Detect the onset of a failure
- Reveal a hidden failure

Essentially, for each of prioritised failure modes, a decision must be made as to which of the following basic maintenance tasks is most applicable, as defined by Vatn (2008, 511):

1. Continuous on-condition task (CCT)
2. Scheduled on-condition task (SCT)
3. Scheduled overhaul (SOH)

4. Scheduled replacement (SRP)
5. Scheduled function test (SFT)
6. Run to failure (RTF)

Continuous on-condition task (CCT) is a continuous monitoring of an item to find any potential failures. An on-condition task is only applicable if it is possible to detect reduced failure resistance for a specific failure mode from the measurement of some quality.

Scheduled on-condition task (SCT) is a scheduled inspection of an item at regular intervals to find any potential failures. There are three criteria that must be met for an on-condition task to be applicable:

1. It must be possible to detect reduced failure resistance for a specific failure mode.
2. It must be possible to define a potential failure condition that can be detected by an explicit task.
3. There must be a reasonable consistent age interval between the time of potential failure and time of failure.

There are typically two disadvantages of a scheduled versus a continuous on-condition task:

- The man-hour cost of inspection is often larger than the cost of installing a sensor.
- Since the scheduled inspection is carried out at fixed points of time, one might "miss" situations where the degradation is faster than anticipated.

Scheduled overhaul (SOH) is a scheduled overhaul of an item at or before some specified age limit. An overhaul task can be considered applicable to an item only if the following criteria are met:

1. There must be an identifiable age at which the item shows a rapid increase in the item's failure rate function.
2. A large proportion of the units must survive to that age.
3. It must be possible to restore the original failure resistance of the item by maintaining it.

Scheduled replacement (SRP) is the scheduled discard of an item (or one of its parts) at or before some specified age limit. A scheduled replacement task is applicable only under the following circumstances:

1. The item must be subject to a critical failure.
2. Test data must show that no failures are expected to occur below the specified life limit.
3. The item must be subject to a failure that has major economic (but not safety) consequences.
4. There must be an identifiable age at which the item shows a rapid increase in the failure rate function.
5. A large proportion of the units must survive to that age.

Scheduled function test (SFT) is a scheduled inspection of a hidden function to identify any failure. A scheduled function test task is applicable to an item under the following conditions:

1. The item must be subject to a functional failure that is not evident to the operating crew during the performance of normal duties.
2. The item must be one for which no other type of task is applicable and effective.

Run to failure (RTF) is a deliberate decision to run to failure because the other tasks are not possible or the economics are less favourable.

Developing the candidate list of PM tasks is a crucial step in implementing an effective RCM strategy. Selection of the optimal interval (or frequency) at which to perform a PM task is, by far, the most difficult job (Smith and Hinchcliffe, 2004, 124). Usually, formalised methods for optimisation of maintenance intervals are not a part of the RCM analysis (Vatn, 2008, 511). In order to optimise maintenance intervals, current maintenance optimisation models must be utilised (Vatn, 2008, 511).

2.2 Inter-dependencies of Components

The possibility of dependence among system components is an important consideration in the analysis of system reliability (Nachlas, 2005, 10). As discussed by Nicolai and Dekker (2008*b*, 263), there may exist interdependencies between components within a system, which inevitably leads to increased complexity within the maintenance decision process. Thomas (1986, 299) and Nicolai and Dekker (2008*b*, 263) explain that, although these dependencies lead to complicated modeling and optimisation of maintenance programs, they also create the opportunity to group maintenance which may save costs.

Cho and Parlar (1991, 2) give the following definition of multi-component maintenance models: "*Multi-component maintenance models are concerned* *with optimal maintenance policies for a system consisting of several units of machines or many pieces of equipment, which may or may not depend on each other (economically/stochastically/structurally)*".

2.2.1 Economic Dependence

If the cost or time of replacing two or more units in the system is less than the sum of their individual replacement costs or times, it may be worthwhile to replace a working unit when one is replacing some other failed unit (Thomas 1986, 299; Nicolai and Dekker 2008*b*, 264). Radner and Jorgenson (1962, 184) refers to this as "opportunistic replacement"'. On the one hand, the joint execution of maintenance activities can save costs in some cases (for example due to economies of scale). On the other hand, grouping of maintenance may also lead to higher costs (for example due to manpower restrictions) or may not be allowed (Nicolai and Dekker, 2008*b*, 264). Nicolai and Dekker (2008*b*, 264) therefore subdivides the models with economic dependence into two categories: positive and negative economic dependence.

Positive dependence implies that costs can be saved when several components are jointly, instead of separately, maintained. Compared with the review of Dekker *et al.* (1996, 412), Nicolai and Dekker (2008*b*, 264) refines the concept of (positive) economic dependence and distinguishes the following forms:

- Economies of scale
  - General
  - Single set-up
  - Multiple set-ups
    - Hierarchy of set-ups
- Downtime opportunity

The term economies of scale is often used to indicate that combining maintenance activities is cheaper than performing maintenance on components separately. Economies of scale can result from preparatory or set-up activities that can be shared when several components are maintained simultaneously. For instance, consider a system consisting of two components, which both consist of two sub-components. Maintenance of the sub-components of the components may require a set-up at system level and component level. The set-up cost at component level is therefore paid only once when the maintenance of two sub-components of a component is combined. In addition, the set-up cost at system level is paid only once when all sub-components are maintenance simultaneously. Set-up costs usually come back in the objective function of the maintenance problem — if economies of scale are not explicitly modelled by including set-up costs in the objective functions, the model will then be classified in the "general" category. Component failures can often be regarded as opportunities for PM of non-failed components. In a series system, a component failure results in a non-operating system. In that case, it may be worthwhile to replace other components preventively at the same time (Laggoune *et al.*, 2009, 1501). This way, the system downtime results in cost savings since more components can be replaced simultaneously.

Negative dependence between components occurs when maintaining components simultaneously is more costly than maintaining components individually. Reasons for this include manpower restrictions; safety requirements; and redundancy/production-loss.

Economic dependence is common in most continuous operating systems, such as oil refineries, chemical processing facilities, mass-production manufacturing lines and power generators (Das *et al.* 2007, 163; Laggoune *et al.* 2009, 1501; Vassiliadis and Pistikopoulos 2000, 218).

2.2.2 Stochastic Dependence

Stochastic dependence, also referred to as failure interaction or probabilistic dependence, implies that the state of components can influence the state of other components (Nicolai and Dekker, 2008*b*, 264). Murthy and Nguyen (1985, 240) introduce three different types of failure interaction in a two-component system:

- *Type I* failure implies that the failure of a component can induce a failure of the other component with probability $p(q)$, and has no effect on the other component with probability $1 - p(q)$.
- *Type II* failure interaction in a two-component system is defined as follows: the failure of component 2 can induce a failure of component 1 with probability q, whereas every failure of component 1 acts as a shock to component 2, without inducing an instantaneous failure, but affecting its failure rate.
- *Type III* failure interaction implies that the failure of each component affects the failure rate of the other component. Therefore, every failure of one of the components acts as a shock to the other component.

In general, the maintenance policies considered in the literature on stochastic dependence, are mainly of an opportunistic nature, since the failure of one component is potentially harmful for the other component(s) (Nicolai and Dekker, 2008*b*, 265).

2.2.3 Structural Dependence

Whereas a system's reliability is most affected by the connections between the units — their horizontal dependency — the maintenance and replacement policies of a system are affected by the system's modular structure — its vertical dependency. Thus each item, as well as being considered as a single replaceable unit, can also be considered as a subunit of a large replaceable module at the next level up. The fundamental question is whether it is worth replacing more units than just the ones that have failed and, if so, at what level Thomas 1986, 300; Nicolai and Dekker 2008*b*, 265. Structural dependence means that some operating components have to be replaced or dismantled, before the failed components can be replaced or repaired — therefore the components cannot be maintained independently. Since the failure of a component offers an opportunity to replace other components, opportunistic policies are expected to perform well on systems with structural dependence between components (Nicolai and Dekker, 2008*b*, 265). Dekker and Scarf (1998, 111) provides an example considering road maintenance: several deterioration mechanisms affect roads, for example, longitudinal and transversal unevenness, cracking and ravelling. For each mechanism one may define a virtual component, but if one applies a maintenance action to such a component it also affects the state with respect to the other failure mechanisms.

In a series system, the individual preventive replacements of components improve the global system reliability on the account of its availability, which would be largely penalised due to frequent shut downs for component replacements. For multi-component systems, an optimal maintenance policy must take into account the interactions between the various components of the system (Laggoune *et al.*, 2009, 1501). If any components in the system are dependent upon each other, an optimal decision on the repair or replacement of a single component is not necessarily optimal for the system as a whole. If all components in the system are independent of one another, a maintenance policy for the single component models may be applied to the multi-component maintenance problems (Cho and Parlar, 1991, 3).

2.3 Maintenance Policy Optimisation

Maintenance optimisation consists in broad terms of those mathematical models aimed at finding either the optimum balance between costs and benefits of maintenance or the most appropriate moment to execute maintenance tasks (Dekker and Scarf 1998, 111; Dekker 1996, 231; Maillart and Fang 2006, 804; Thomas 1986, 301). Traditionally, the quantitative approach to determining PM intervals is to consider the total expected cost for a given planning horizon and fix the interval that can minimise that cost. If the system under consideration exhibits ageing characteristics, that is, if the failure rate is increasing with age, and if the failure maintenance (or CM) costs are larger than PM costs, then there exists a finite value of PM interval that will yield the minimum costs (Chareonsuk *et al.*, 1997, 56). The performance and competitiveness of manufacturing companies is dependent on the reliability and productivity of their production facilities (Muchiri *et al.*, 2011, 298). In production systems, particularly in continuous processes, major related costs may be due to production losses during downtime (Chareonsuk *et al.* 1997, 56; Percy 2008, 89; Laggoune *et al.* 2010, 748; Laggoune *et al.* 2009, 1499). In order to ensure specified availability and reliability of production processes, PM should be undertaken during the production process. However, undertaking unscheduled PM can impose high costs to the firm, and adversely decrease the availability of the production line (Ebrahimipour *et al.*, 2013, 111). One way to deal with the challenge of finding the optimal balance between the benefits of planned PM downtime and the resultant production losses incurred is to express the criteria in terms of costs and develop a single objective. This can be achieved by including the production losses due to maintenance downtime in the model. This will force both production and maintenance management to look at the problem at the aggregate level and come to a consensus plan (Chareonsuk *et al.*, 1997, 57). A typical illustration of maintenance costs versus maintenance frequencies, taking into consideration the lost production time, can be seen in Figure 2.3. The challenge faced by management is to balance the costs of PM with the supposed improvements in system reliability. Too few PM actions incurs higher CM costs and lower PM costs, whereas too many PM actions results in lower CM costs and higher PM costs. Unfortunately, there is no simple explanation of how CM and PM affect system reliability. By modelling the failure patterns of systems mathematically, valuable insights can be gained on cost-effective strategies for maintenance decisions (Percy, 2008, 184).

Strategies for scheduling PM are often based on intuition and experience, though considerable improvements in performance can be achieved by fitting mathematical models to observed data (Handlarski 1980, 227; Dagpunar and Jack 2000, 1097; Percy and Kobbacy 2000, 87). For systems comprising few components, and systems comprising many identical components, modelling and analysis using compound renewal processes might be possible, however, many systems comprise a large variety of different components and are far too complicated for applying this methodology (Percy, 2008, 184). These are known as complex repairable systems, which is, any structure of more than one component, which performs a particular function. Typical systems include industrial and domestic machinery, such as production lines, utility supplies, railway operations, and motor vehicles (Percy, 2008, 184). Percy (2008, 184) motivates that for these complex systems, one needs to develop models for failures based on the history of maintenance (PM and CM) available. Once the model has been built, an evaluation can be undertaken to evaluate different PM strategies to determine the optimal solution.

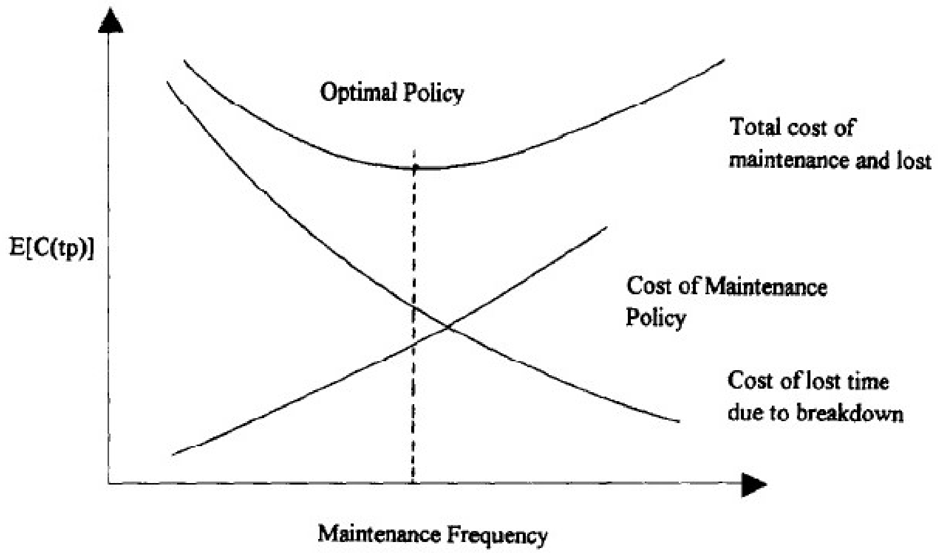

Figure 2.3: Cost comparison between preventive maintenance and corrective maintenance (Adapted from Chareonsuk *et al.* (1997, 56))

2.3.1 Maintenance Policy Models

Dekker (1996, 233) provides a general overview, using four aspects, of what maintenance optimisation models cover:

(i) a description of a technical system, its functions, and its importance;

(ii) a modelling of the deterioration of the system in time and possible consequences for the system;

(iii) a description of the available information about the system and the actions open to management; and

(iv) an objective function and an optimisation technique which helps in finding the best balance.

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Two well-known PM models originating from literature are *age-based* and *block-based maintenance policy models* (Savsar 2011, 681; Wang 2002, 469). Several other maintenance models, based on the fundamentals of these two well-known concepts, have been developed in literature.

Age-based maintenance policy: This is arguably the most common and popular maintenance policy. Under this policy, a unit is replaced at its age T or failure, whichever occurs first, where T is a constant (Barlow and Hunter, 1960, 91). Various extensions and modifications of the age-based maintenance policy have been proposed, largely based on the concepts of minimal repair and imperfect maintenance (Pham and Wang, 1996, 426). The PM performed at T and CM at failure might be either minimal, imperfect, or perfect. Pham and Wang (1996, 426) classifies maintenance activities according to the *degree* in which the item is restored by maintenance as follows:

(a) Perfect repair or imperfect maintenance: a maintenance action which restores the system operating condition to "as good as new". Upon perfect maintenance, a system has the same lifetime distribution and failure rate as a brand new one. Complete overhaul of an engine with a broken connecting rod is an example of perfect repair.

(b) Minimal repair or minimal maintenance: a maintenance action which restores the system to the failure rate it had when it failed. A minimal repair is often referred to as a "bad as old" state. Changing a flat tyre on a car or changing a broken fan belt on an engine are examples of minimal repair, seeing as the overall failure rate of the car is essentially unchanged.

(c) Imperfect repair or imperfect maintenance: a maintenance action does not convert a system to "as good as new", but does repair the item to an improved failure rate state. It is usually assumed that imperfect maintenance restores the system operating state to somewhere between "as good as new" and "as bad as old". Imperfect repair (or maintenance) is a general repair which can include two extreme cases: minimal and perfect repair (or maintenance). Engine tune-up is an example of imperfect maintenance seeing as the engine tune-up may not make an engine "as good as new", but its performance might be significantly improved.

The age-based maintenance policy can be further defined based on the classification of the variable T. If T is a random variable, the policy is referred to as the random age-dependent maintenance policy that is enforced when it is impractical to maintain a unit in a strictly periodic fashion. In this case, the maintenance policy would have to be random one, taking advantage of any free time available to perform maintenance (Wang, 2002, 471). In the age replacement policy, items are replaced if they reach a certain age, which is measured from the time of last replacement. If only minimal repair is undertaken upon failure, the age replacement policy amounts to the "periodic replacement with minimal repair at failure" policy (Wang, 2002, 471). Some researchers have interesting variations of the age replacement policy model, where Tahara and Nishida (1975, 114) introduce the maintenance policy "replace the unit when the first failure after t_0 hours of operation or when the total operating time reaches T $(0 \leq t_0 \leq T)$, whichever occurs first; Failures in $[0,t_0]$ are removed by minimal repair". Note that if $t_0 \equiv 0$, it becomes the age replacement policy, and if $t_0 \equiv T$ it reduced to the "periodic replacement with minimal repair at failure" policy (Wang, 2002, 471).

Nakagawa (1984, 545) extends the age replacement policy to replacing a unit at time T or at number N of failures, whichever occurs first, and undergoes minimal repair at failure between replacements. The decision variables for this policy are T and N. The policy therefore combines the fixed age and the repair number counting ideas. If $N \equiv 1$, this policy reduces to the age replacement policy. The policy is often referred to as the T-N policy.

Sheu *et al.* (1993, 339) examine a generalised age replacement policy, whereby if a unit fails at age $y < t$, it is subject to a perfect repair with probability $p(y)$, or undergoes a minimal repair with probability $q(y) = 1 - p(y)$. Otherwise, the unit is replaced when the first failure after t occurs or the total operating time reaches age T $(0 \leq t \leq T)$, whichever occurs first. The policy decision variables are t and T. If $t \equiv 0$ then this policy becomes the age replacement policy, and if $t \equiv T$ and $q(y) \equiv 1$, it becomes the "periodic replacement with minimal repair at failure" policy.

Block *et al.* (1993, 198) introduce another generalised age replacement policy, *repair replacement policy*, where units are preventively maintained when a certain time has elapsed since their last repair. The items are thus repaired if they fail and are replaced only if they survive beyond a certain fixed time from the last repair of replacement. Units are either minimally or perfectly repaired at failure or replaced if they survive a certain fixed time from the last repair without suffering a CM. If at failure only perfect repair is allowed, then the repair replacement policy reduces to the age replacement policy.

Block-based maintenance policy: In the block-based (or periodic) maintenance policy, a unit is preventively maintained at fixed time intervals kT $(k = 1, 2, \ldots)$ independent of the failure history of the unit, and repaired at intervening failures, where T is a constant. The block replacement policy derives its name from the commonly employed practice of replacing a block (or group) of units in a system at prescribed times kT $(k = 1, 2, \ldots)$ independent of the failure history of the system and is often used in multi-unit systems (Wang, 2002, 471). As with the age-based maintenance policy, many extensions of the block-based maintenance policy have been established, based on the concepts of minimal repair and imperfect maintenance. One expansion of the "periodic replacement with minimal repair at failure" policy is the one where a unit receives imperfect PM every T time unit, intervening failures are subject to minimal repairs, and it is replaced after its age has reached $(O + 1)T$ time units, where O is the number of imperfect PMs which have been done (Liu *et al.*, 1995, 1066). $O = 0$ is allowed in this policy, which means the unit will be replaced whenever it has operated for T time units and there will be no imperfect PM for it. The policy decision variables are O and T. If $O \equiv 0$, this policy becomes the "periodic replacement with minimal repair at failure" policy.

Berg and Epstein (1976, 18) have modified the block replacement policy by setting an age limit. A failed unit is replaced by a new one; however, units whose ages are less than or equal to t_0 $(0 \leq t_0 \leq T)$ at the scheduled replacement times kT $(k = 1, 2, \ldots)$ are not replaced, but remain working until failure or the next scheduled replacement time point. If $t_0 = T$, it reduces to the block replacement policy. This modified block replacement policy was shown to be superior to the block replacement policy in terms of the long-run maintenance cost rate (Wang, 2002, 471).

Nakagawa (1981*b*, 215); and Nakagawa (1981*a*, 166) presents three modifications to the "periodic replacement with minimal repair at failure" policy. The three modifications all establish a reference time T_0 and periodic time T^*. If failure occurs before T_0, then minimal repair occurs. If the unit is operating at time T^*, then replacement occurs at time T^*. If failure occurs between T_0 and T^*, then: (*Policy 1*) the unit is not repaired and remains failed until T^*; (*Policy 2*) the failed unit is replaced by a spare unit as many times as needed until T^*; (*Policy 3*) the failed unit is replaced by a new one. In all three policies, the policy decision variables are T_0 and T^*. If $T_0 \equiv T^*$, all three polices become the "periodic replacement with minimal repair at failure" policy. If $T_0 \equiv 0$, Policy 3 becomes the block replacement policy.

Nakagawa (1986, 539) also makes an expansion to the block replacement policy, whereby the replacement of a unit is scheduled at periodic times kT $(k = 1, 2, \ldots)$ and failure is removed by minimal repair. If the total number of failures is equal to or greater than a specified number n, the replacement should be done at the next scheduled time; otherwise, no maintenance should be done. The decision variables are n and T. If $n = \infty$, this policy becomes the "periodic replacement with minimal repair at failure" policy.

Wang and Pham (1999, 122) extend the block replacement policy to a general case, whereby a unit is imperfectly repaired at failure if the number of repairs is less than N (a positive integer). The repair is imperfect in the sense that the unit has shorter and shorter lifetime upon each repair. Upon the Nth imperfect repair at failure, PM is undertaken on the unit at kT $(k = 1, 2, \ldots)$ where the constant $T > 0$. The PM is imperfect in the sense that after PM the unit is "as good as new" with probability p and "as bad as old" with probability $(1 - p)$. Upon a perfect PM, the maintenance process repeats. The decision variables are N and T. The justification of this policy is that when a new unit is installed, the first N repairs at failure will be performed at a low cost (minor repairs, as the system is generally in a good operating condition). If the repair at failure and PM are perfect and $N \equiv \infty$, this policy reduces to the block replacement policy. If the repair at failure is minimal and PM is perfect and $N \equiv \infty$, this policy amounts to the "periodic replacement with minimal repair at failure" policy.

2.3.2 Mathematical Modeling

The maintenance policy models described in Section 2.3.1 provides a foundational basis on which mathematical models are further developed. The use of mathematical modeling in the maintenance function is certainly not new, and there are numerous reviews available in literature that have been developed and used (Lawrence, 1999, 2). Every maintenance model will somehow try to predict or extrapolate the future performance of the system in question, whether it be in a deterministic or probabilistic fashion (Frangopol *et al.*, 2004, 197). Using reliability, and subsequently probability, as a basis, models that describe equipment performance as a function of maintenance effort provide a means for selecting the most efficient and effective equipment service strategies and policies (Nachlas 2005, 4; Ghosh and Roy 2009, 404). In order to determine optimal maintenance policies, it is required to quantify the effect of inspection and maintenance on reliability and costs. Probabilistic maintenance models are preferably used for this purpose in preventive maintenance studies as well as in RCM approaches, due to their simplicity and the ability to incorporate uncertainties associated with the deterioration of equipment and the outcomes of inspection and maintenance (Abeygunawardane and Jirutitijaroen, 2014, 178). Most models can roughly be divided into two parts: a *deterioration model* which is used to approximate and predict the actual process of ageing in condition or in reliability; and a *decision model* which uses the deterioration model to determine the optimal times of inspection and maintenance (Frangopol *et al.*, 2004, 197).

The effect of inspection and maintenance on reliability and costs is expressed by means of various performance measures. These measures include cost of performing inspection, maintenance and repair (Jirutitijaroen and Singh 2004, 215; Park *et al.* 2000, 106), unavailability (or availability), frequency of failure (Ge and Asgarpoor, 2011, 348), first passage time (FPT) from new state to failure state (Jirutitijaroen and Singh 2004, 215; Heo *et al.* 2011, 2172), cost of production losses (Ge and Asgarpoor, 2011, 348), and cost of lost energy. Typically, optimising the performance measures becomes the objective of maintenance optimisation, where single-objective optimisation formulations usually aim at minimising the cost, although there are numerous cases in literature that aim at maximising availability (Heo *et al.* 2011, 2172; Frangopol *et al.* 2004, 197; Ghosh and Roy 2009, 404). Although there are several performance measures to be considered in the objective of maintenance optimisation, relationships may exist among some of these performance measures.

The two models described by Frangopol *et al.* (2004, 197) are now further explained.

2.3.2.1 Deterioration Model

As described in Section 1.1, equipment systems are subject to failure, which can be defined as any change in equipment condition which causes it to be unable to perform its intended function satisfactorily (see Section 2.1.2.4). Using reliability and probability theory provides a basis for attempting to predict the failure rate and performance of equipment (Nachlas, 2005, 5). A critical input to reliability analysis is the failure rate, which can essentially be considered as the *deterioration* of the system. During the useful life period of a system, a Poisson or negative exponential distribution is usually used to represent its failure rate. This is because the failure rate, which mainly consider natural failures, is assumed constant or steady (Billinton and Allan, 1992, 150). A constant failure rate should however be questioned, as the failure rate is sensitive to variability in the operation or environment of the system. For realistic analyses, variability in parameters representing reliability inputs cannot be ignored and should be included in the model being used to represent a system (Edimu *et al.*, 2011, 916). In the mathematical sense, reliability is measured by the probability that a system or a component will work without failure during a specified time interval $(0, t)$ under given operating conditions and environment, as depicted in Figure 2.4.

Todinov (2005, 21) provides an overall description of the fundamentals of reliability and probability:

Considering the case of a single *continuous* lifetime variable, T, with T being a non-negative random variable representing the lifetimes of components, the probability $P(T > t)$ that the time to failure T will be greater than a specified time t is given by the *reliability function* $R(t) = P(T > t)$, also referred to as the *survival function*. The reliability is a monotonic, non-increasing function, always unity at the start of life $(R(0) = 1, R(\infty) = 0)$. It is linked with the cumulative distribution function (CDF), $F(t)$, of the time to failure by $R(t) = 1 - F(t)$: Reliability = 1−Probability of failure. If T is the time to failure, $F(t)$ gives the probability $P(T \leq t)$ that the time to failure T will be smaller than the specified time t, or in other words, the probability that the system will fail before time t.

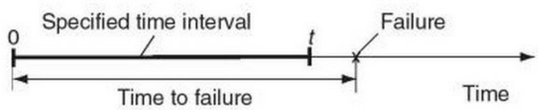


Figure 2.4: Reliability is measured by the probability that the time to failure will be greater than a specified time t (Adapted from Todinov (2005, 2))

The probability density function (PDF) of the time to failure is denoted by $f(t)$. The PDF describes how the failure probability is spread over time. In the infinitesimal interval $[t, t+\mathrm{d}t]$, the probability of failure is $f(t)\mathrm{d}t$. The probability of failure in any specified time interval $t_1 \leq T \leq t_2$ is given by (2.3.1).

$$P(t_1 \leq T \leq t_2) = \int_{t_1}^{t_2} f(t)\mathrm{d}t \tag{2.3.1}$$

Basic properties of the probability density of the time to failure are (i) $f(t)$ is always non-negative; and (ii) the total area beneath $f(t)$ is always equal to one: $\int_0^\infty f(t)\mathrm{d}t = 1$. This is because $f(t)$ is a probability distribution, which is, the probabilities of all outcomes for the time to failure must add up to unity.

The CDF of the time to failure is related to the failure density function by:

$$f(t) = \frac{\mathrm{d}F(t)}{\mathrm{d}t} \tag{2.3.2}$$

From (2.3.2), the probability that the time to failure will be smaller than a specified value t is:

$$F(t) = P(T \leq t) = \int_0^t f(v)\mathrm{d}v \tag{2.3.3}$$

where v is a dummy integration variable; $F(\infty) = \int_0^\infty f(v)\mathrm{d}v = 1$, $F(0) = 0$. Because $f(t)$ is non-negative, its integral $F(t)$ is a monotonic non-decreasing function of t (see Figure 2.5). The value $F(t^*) = \int_0^{t^*} f(v)\mathrm{d}v$ of the CDF at time t^* gives the area beneath the PDF $f(t)$ until time t^*. The link between the reliability function $R(t)$, CDF $F(t)$ and PDF $f(t)$ is illustrated in Figure 2.5.

$P(t_1 < T \leq t_2)$ is the probability of failure between times t_1 and t_2:

$$P(t_1 < T \leq t_2) = \int_{t_1}^{t_2} f(v)\mathrm{d}v = F(t_2) - F(t_1) \tag{2.3.4}$$

The hatched area in Figure 2.6 is equal to the difference $F(t_2) - F(t_1)$ and gives the probability that the time to failure T will be between t_1 and t_2.

A critical input to reliability analyses is the failure rate (Edimu *et al.*, 2011, 916). Assuming a CDF, $F(t)$, and PDF, $f(t)$, the failure rate can be defined as:

$$\lambda(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{\bar{F}(t)} \tag{2.3.5}$$

From (2.3.5), $\lambda(t)\mathrm{d}t$ represents the probability that a component of age t will fail in the time interval $[t, t+\mathrm{d}t]$. An alternative name for the failure rate is the hazard rate. For deteriorating components or systems, the failure rate is increasing (Frangopol *et al.*, 2004, 197).

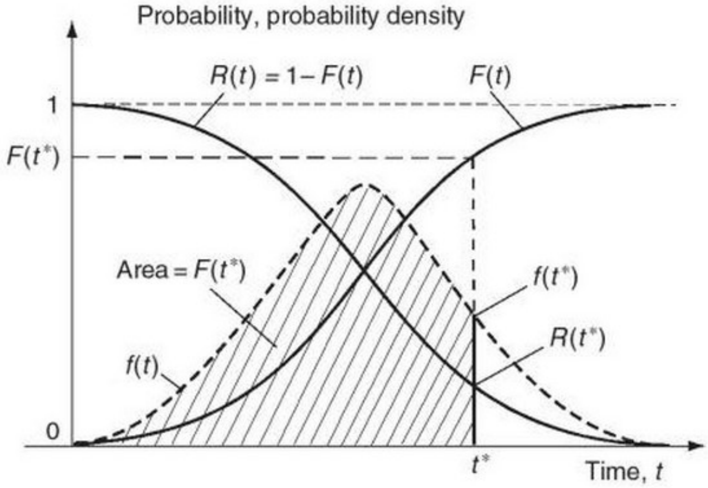


Figure 2.5: Reliability function, cumulative distribution function of the time to failure, and failure density function (Adapted from Todinov (2005, 3))

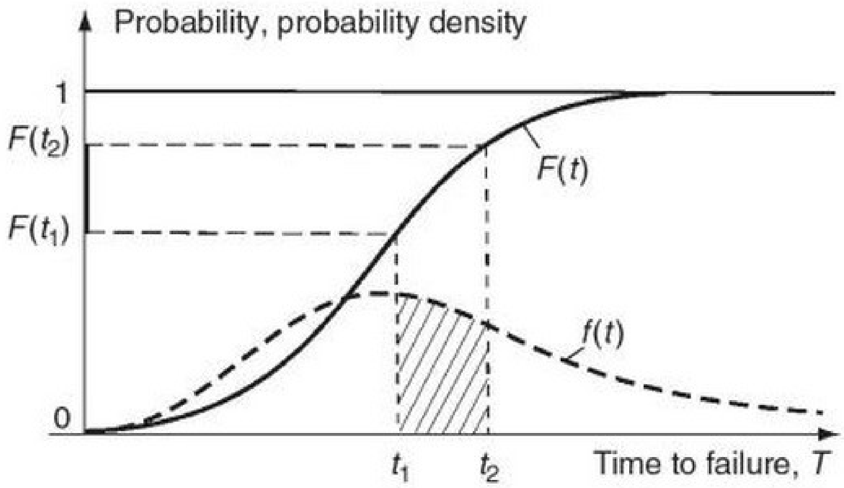


Figure 2.6: Cumulative distribution and probability density function of the time to failure (Adapted from Todinov (2005, 3))

The life of certain systems follows the "bathtub" curve, which is, the failure rate decreases (early life), then stays steady (normal operating life), and then increases (wear-out life), as illustrated in Figure 2.7.

This failure rate is true for certain types of simple equipment, and for some complex items with dominant failure modes. According to Moubray (1997, 12), however, equipment in general is far more complex than twenty years ago, which has led to startling changes in the patterns of failure, as illustrated in Figure 2.8. Pattern A is the well known "bathtub" curve (as illustrated in Figure 2.7); pattern B shows constant or slowly increasing con-

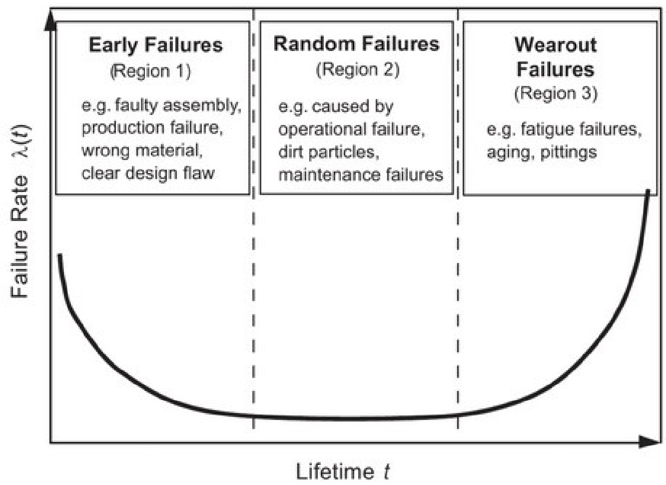


Figure 2.7: The "bathtub curve" (Adapted from Bertsche (2008, 24))

ditional probability of failure, ending in a "wear-out" zone; pattern C shows slowly increasing conditional probability of failure, but there is no identifiable "wear-out" age; pattern D shows low conditional probability of failure when the item is recently installed, followed by a rapid increase to a constant level; pattern E shows a constant conditional probability of failure at all ages (random failure); and pattern F starts with high infant mortality, which drops eventually to a constant or very slowly increasing conditional probability of failure (Moubray, 1997, 13).

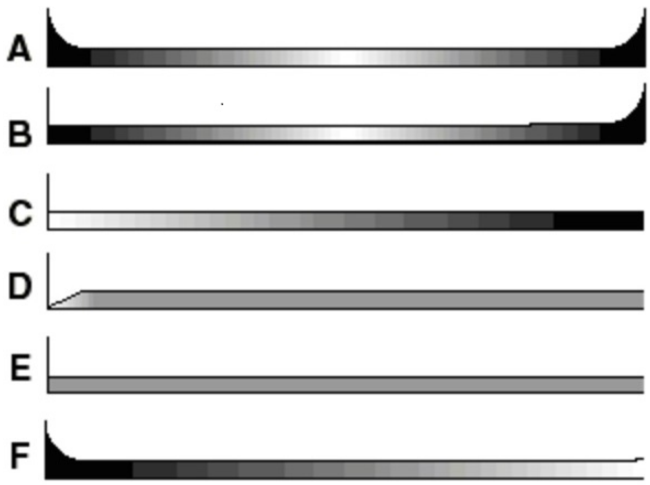


Figure 2.8: Six patterns of failure (Adapted from Nowlan and Heap (1978, 46))

Probably the most common term used in engineering reliability is mean time between failures (MTBF). MTBF refers to the case in which several items, all operating under comparable usage conditions and all sharing the same design features, and if the time since the item was first introduced into service is great, one can expect, on average, to replace one of these items every unit of time equal to the MTBF (Cruse, 1997, 35). MTBF essentially measures how long, on average, before machines stop due to a maintenance problem or failure (Bongers and Gurgenci, 1997, 614). Another common measure in engineering reliability is mean time to repair (MTTR), which measures the ability to diagnose and remedy maintenance delays, or breakdowns, once they have occurred — in essence, the MTTR is a measure of how long, on average, before machines that have failed are returned to operation (Bongers and Gurgenci, 1997, 614). Bongers and Gurgenci (1997) defines MTBF and MTTR as follows:

$$\text{Mean time between failure (MTBF)} = \frac{\text{Actual operating time}}{\text{Number of breakdowns}} \tag{2.3.6}$$

$$\text{Mean time to repair (MTTR)} = \frac{\text{Actual breakdown time}}{\text{Number of breakdowns}} \tag{2.3.7}$$

The PDF, as defined in (2.3.2), can be used to model the failure rate for each stage of the failure curve. Several probability distributions have been identified in literature for use in reliability analysis (Edimu *et al.*, 2011, 919). In practice the parameters that are normally associated with reliability evaluation are described by probability distributions. This can easily be appreciated by considering that all components of a given type, construction, manufacture, and operating condition will not all fail after the same operating time, but will fail at different times in the future. Consequently, these times-to-failure obey a probability distribution which may, or may not, be known and which describes the probability that a given component fails within a certain specified time or survives beyond a certain specified time (Billinton and Allan, 1992, 124). The most popular probability distributions are the binomial, Gaussian, negative exponential, gamma, Weibull and beta distributions (Billinton and Allan 1992, 124; Edimu *et al.* 2011, 919; Cross and Herman 2006, 2; Wangdee and Billinton 2007, 761). Estimation of the reliability function of some equipment is one of the main problems of reliability theory (Soliman, 2002, 337). In all practical cases, the appropriate probability distribution cannot be determined from a knowledge of the geometry of the component, device or system, but must be deduced from sample testing or from a historical data collection scheme associated with the operation of the components, device or systems (Billinton and Allan, 1992, 124).

In order to provide an understanding of the most popular models, a description is provided for each:

Exponential Distribution The distribution function for the exponential distribution is given by (2.3.8).

$$F(t) = 1 - e^{-\lambda t} \tag{2.3.8}$$

for $t \geq 0$ (Blischke and Murthy, 2000, 103). The density function and the failure rate functions are given by (2.3.9).

$$f(t) = \lambda e^{-\lambda t} \tag{2.3.9}$$

and

$$r(t) = \lambda \tag{2.3.10}$$

respectively. Note that the the failure rate is constant and does not change with t (the age of the item). The distribution mean value can be obtained by using (2.3.11).

$$\mu = \frac{1}{\lambda} \tag{2.3.11}$$

Figure 2.9 shows the shapes of $f(t)$ for $\mu = 500, 1000, 1500, 2000,$ and 5000. The exponential distribution has been used to model failures of

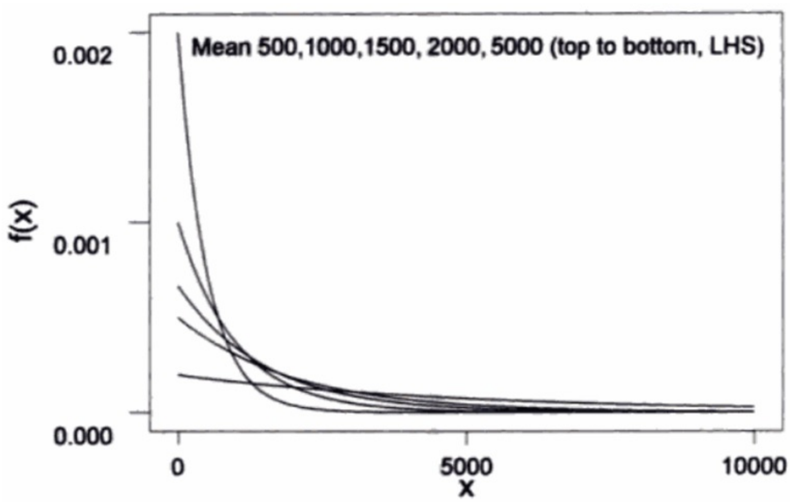


Figure 2.9: Exponential distribution (Adapted from Blischke and Murthy (2000, 104))

electronic and electrical parts and is one of the most widely used failure distributions. The distribution is appropriate whenever failures occur randomly and are not age dependent (Blischke and Murthy, 2000, 104).

Gamma distribution The gamma density function is given by (2.3.12).

$$f(t) = \frac{t^{\alpha-1} e^{-\frac{t}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)} \tag{2.3.12}$$

where $t \geq 0$; $\alpha > 0$ and $\beta > 0$ (Blischke and Murthy, 2000, 105). Here, Γ is the gamma function. The distribution mean value can be obtained by using (2.3.13)

$$\mu = \alpha\beta \tag{2.3.13}$$

Figure 2.10 shows the shapes of $f(t)$ and $r(t)$ for $\alpha = 0.50, 0.75, 1.00, 1.50,$ and 3.00 and $\beta = 0.50$. The gamma distribution has similar properties

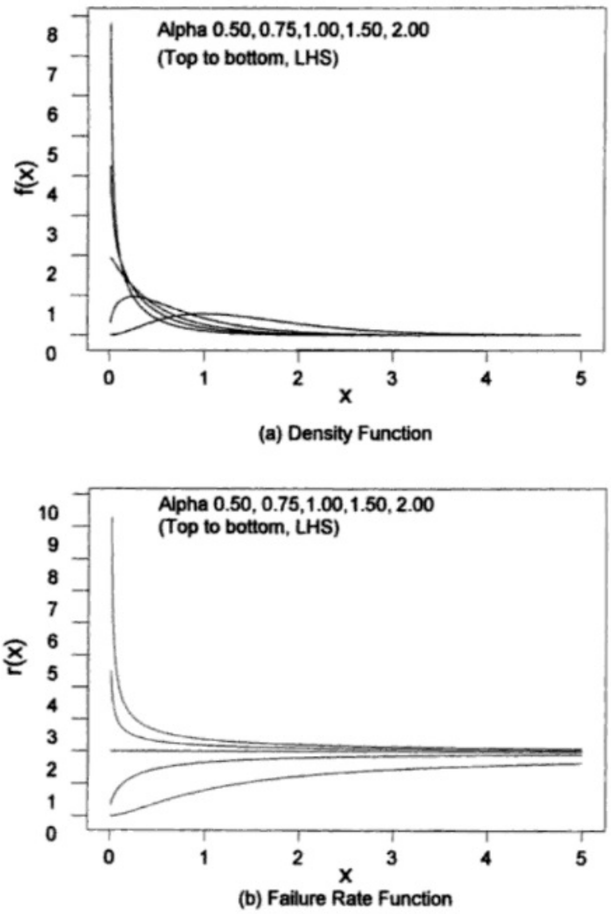


Figure 2.10: Gamma distribution $\beta = 0.5$ (Adapted from Blischke and Murthy (2000, 105))

to that of the Weibull distribution, as it has two parameter distributions (β and α). The gamma distribution has been used to model failure times for many different objects, for example, life expectations of transistors (Blischke and Murthy, 2000, 105). Although a variety of shapes can be produced by varying these two shape parameters, it is generally accepted that the Weibull distribution is often more suited to reliability evaluation (Billinton and Allan, 1992, 125).

Gaussian distribution The Gaussian density function is given by (2.3.14).

$$f(t \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} \tag{2.3.14}$$

where $-\infty < t < \infty$ (Dhillon, 2007, 25). σ and μ are the distribution parameters (standard deviation and mean value, respectively). The distribution mean value can be obtained using (2.3.15).

$$\mu = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} t \exp[-\frac{(t-\mu)}{2\sigma^2}]\mathrm{d}t \tag{2.3.15}$$

Figure 2.11 shows the shapes of $f(t)$ for $\sigma = 0.5, 1.0,$ and 2.0. The

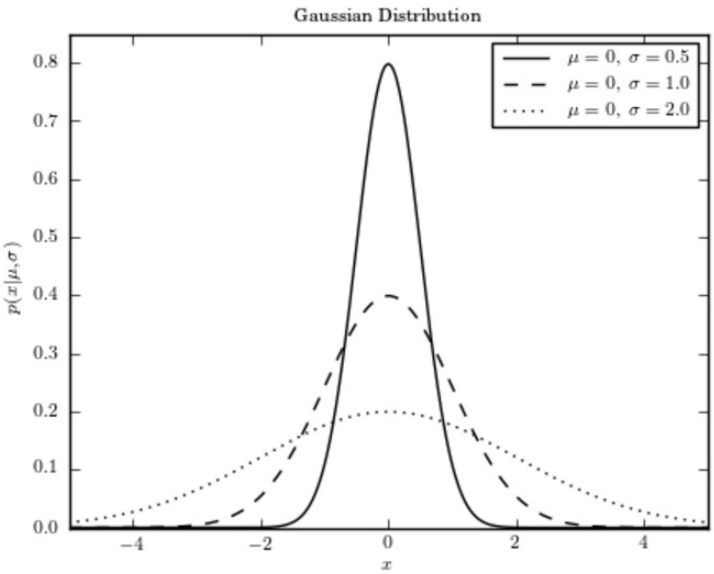


Figure 2.11: Gaussian distribution (Adapted from AstroML [Online] (2016))

Gaussian distribution is also one of, if not the, most widely used distributions in the entire field of statistics and probability (Dhillon 2007, 25; Billinton and Allan 1992, 125). Although having some important applications in reliability evaluation, it is of less significance in this field than many other distributions (Billinton and Allan, 1992, 125). The PDF of the Gaussian distribution is perfectly symmetrical about its mean value and the dispersion about the mean is measured and determined by its standard deviation. By specifying only a mean and standard deviation it is possible that a distribution which is non-normal will be assumed to be normal, simply because no information is available other than the mean value and standard deviation (Billinton and Allan, 1992, 125). The Gaussian distribution is typically used to model the wear-out life of equipment during reliability analyses.

Weibull distribution The Weibull density function is given by (2.3.16)

$$f(t) = \frac{\theta t^{\theta-1}}{\beta^{\theta}} e^{-(\frac{t}{\beta})^{\theta}} \tag{2.3.16}$$

where $t \geq 0$, $\theta > 0$, $\beta > 0$. θ and β are distribution shape and scale parameters, respectively (Dhillon, 2007, 28). The distribution mean value can be obtained using (2.3.17).

$$\mu = \beta\Gamma(1 + \frac{1}{\theta}) \tag{2.3.17}$$

Figure 2.12 shows the shapes of $f(t)$ for various values of θ and β. The

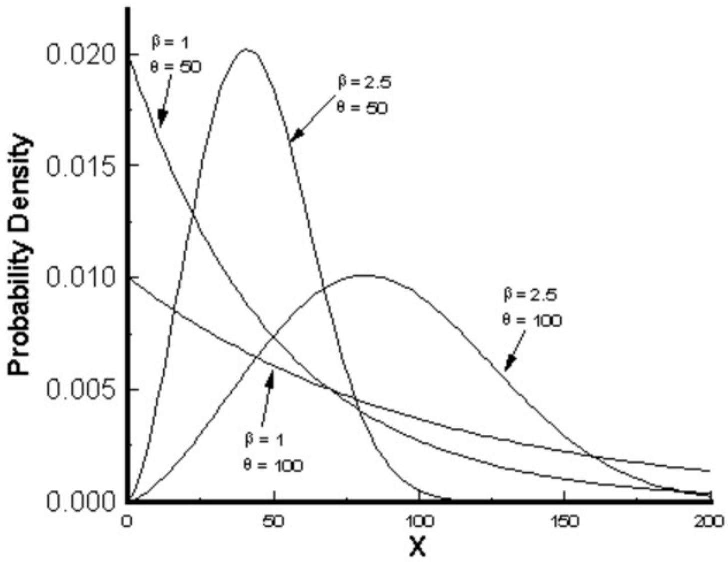


Figure 2.12: Weibull distribution (Adapted from Engineered Software, Inc. [Online] (2016))

Weibull distribution has one very important property; the distribution has no specific characteristic shape – allowing it to be shaped to represent many distributions as well as shaped to fit sets of experimental data that cannot be characterised as a particular distribution other than as a Weibull distribution with certain shaping parameters (Billinton and Allan, 1992, 134). Exponential and Rayleigh distributions are the special cases of the Weibull distribution for $\theta = 1$ and $\theta = 2$, respectively (Dhillon, 2007, 28). The Weibull distribution is widely used in reliability and life data analysis due to its versatility.

The distribution models described above are commonly used in the analysis of repairable systems that possess a constant failure rate (λ), also referred to as a stationary model. A stationary model means that the behaviour, or failure rate, of the system must be the same at all points of time irrespective of the point of time being considered (Billinton and Allan, 1992, 206). Considering the case where components are simply replaced, not repaired, a stationary model may well be applicable, seeing as the components may experience the "stationary" degradation throughout their life cycle. Most complex systems, however, are repaired and not replaced when they fail, in which case if repairs were considered, the stationary models would not be appropriate and a non-stationary model would have to be used (Ascher and Feingold 1984, 163; Crow 1990, 276). The majority of work in literature on non-stationary models has been based on the non-homogeneous Poisson process (NHPP) models (Ascher and Feingold 1984, 163; Barabady and Kumar 2008, 649). The construction of a NHPP model starts with the time process, and is described by Nachlas (2005, 198) as follows: the number of failures, N_t, over the time interval $(0, t)$, can be denoted by (2.3.18).

$$\Lambda(t) = E[N_t] \tag{2.3.18}$$

where $E[N_t]$ represents the expected number of failures over the specified time interval. The failure intensity function, $\lambda(t)$, is then expressed in (2.3.19).

$$\Lambda(t) = \int_0^t \lambda(u)\mathrm{d}u \tag{2.3.19}$$

Note that in the special case when the intensity function, $u(t)$, is constant for all t, the NHPP reduces to the homogeneous Poisson process. Unlike the homogeneous Poisson process failure probability, the intensity, $u(t)\Delta t$, may depend on the age (t) of the system (Crow, 1975, 8). Crow (1975) further explains the basic property of a NHPP as follows: let $X_0 = 0$, and $X_1 < X_2 < X_3 \ldots$ be successive times of of occurrence of events. Let $Y_i = X_i - X_{i-1}$, $i = 1, 2, \ldots$, be the times between successive events. Then the CDF, F_i of Y_i, given that the $(i-1)$st event occurs at time X_{i-1}, is given by (**??**).

$$F_i(y) = \frac{F(X_{i-1} + y) - F(X_{i-1})}{1 - F(X_{i-1})} \tag{2.3.20}$$

where $y > 0$, $i = 1, 2, \ldots$, $F(x) = 1 - exp[-U(x)]$, and $U(x) = \int_0^x u(z)\mathrm{d}z$. Hence, the time between successive events are not independent and identically distributed (IID).

In summary, there is a vast number of types of equipment for which life distributions provide a meaningful model of life duration. The distributions described above are the principle, but not the only distributions used to model life length. Each has advantages, and each has shortcomings. The key is to select one that is appropriate for its application (Nachlas, 2005, 64). In order to fit a statistical model to a life data set, the analyst estimates the parameters of the life distribution that will make the function most closely fit the data. The term "life data" refers to measurements of product life, which can be measured in hours, miles, cycles, or any other metric that applies to the period of successful operation of a particular product. Since time is a common measure of life, life data points are often called "times-to-failure". As discussed in the opening paragraphs of Section 2.3.1, best results for accurate distributions are obtained by fitting mathematical models to observed data. Barabady and Kumar (2008, 649) proposes a basic methodology for model identification, as illustrated in Figure 2.16. There are many sources of data in the maintenance engineering field that provide relevant data for the reliability analysis of systems, including maintenance reports, operational and maintenance information, and data from sensors and equipment (Barabady and Kumar, 2008, 649). Section 2.3.2.3 covers the concept of failure data analysis, as well as the approach to utilise the data in order to determine an appropriate life distribution of the equipment under consideration.

Sometimes, for example, when lifetimes are grouped or measured as a number of cycles of some sort, T may be treated as a discrete random variable. Supposed T can take on values $t_1, t_2, \ldots$, with $0 \leq t_1 < t_2 < \ldots$, and let the probability function be expressed by (2.3.21).

$$f(t_j) = P(T = t_j) \tag{2.3.21}$$

where $j = 1, 2, \ldots$. The survival function is then expressed by (2.3.22).

$$S(t) = P(T \geq t) = \sum_{j;t_j \geq t} f(t_j) \tag{2.3.22}$$

When considered as a function for all $t \geq 0$, $S(t)$ is a left-continuous, non-increasing step function, with $S(0) = 1$ and $S(\infty) = 0$ (Lawless, 2011, 10).

The discrete time hazard (failure) function is defined in (2.3.23).

$$\lambda(t_j) = P(T = t_j | T \geq t_j) = \frac{f(t_j)}{S(t_j)} \tag{2.3.23}$$

where $j = 1, 2, \ldots$.

As in the continuous case, the probability, survivor, and failure functions give equivalent specifications of the distribution of T (Lawless, 2011, 10). Since $f(t_j) = S(t_j) - S(t_{j+1})$, (2.3.23) implies that:

$$\lambda(t_j) = 1 - \frac{S(t_{j+1})}{S(t_j)} \tag{2.3.24}$$

where $j = 1, 2, \ldots$, and thus

$$S(t) = \prod_{j;t_j < t} [1 - \lambda(t_j)] \tag{2.3.25}$$

Most of the standard lifetime data methodology and software is for *continuous* time models, and so even when time is *discrete* (for example, number of cycles to failure), *continuous* models are usually used (Lawless, 2011, 10).

2.3.2.2 Decision Model

Once a suitable solution method for the failure probability, or deterioration, is selected, a decision model has to be formulated (Frangopol *et al.*, 2004, 199) Well defined decision models are the *age-based* and *block-based* policy models (discussed in detail in Section 2.3.1). According to Stewart (2001, 265), there are two most suitable approaches to decision analysis: life-cycle costs and risk ranking. Risk ranking is only useful for the purpose of inspection prioritisation at the time of evaluation or at a fixed time in the future. It does not account for the full life-cycle of a system or component, but only considers its immediate risk. The life-cycle approach is the preferred concept when decision makers are not only concerned with safety, but also with cost (Stewart, 2001, 265). If a component or system is inspected at times $0 \leq t_1 \leq t_2 \leq \ldots$ and it is decided after each inspection to perform a maintenance action or not, then all possible options can be visualised by a decision tree such as the one in Figure 2.13 (Thoft-Christensen and Sø rensen, 1987, 96). After each inspection, the decision is made to perform a maintenance action A (branch labelled with 1) or no action (branch labelled 0). The probability of performing action A will be determined by the state of the component or system. Alternative formulations for preventive replacements, or other types of maintenance actions, can also be defined — for example, if a cost-optimal block-based inspection policy is of interest, the costs associated with the decision tree for each inspection interval can be calculated, where the interval which minimises the expected costs will then be chosen (Frangopol *et al.*, 2004, 199).

2.3.2.3 Statistical Analysis and Utilisation of Failure Data

In practice, a system is frequently represented as a network in which the system components are connected together either in series, parallel, meshed, or a combination of these (Billinton and Allan, 1992, 62) (refer to Section 2.1.2.1 for a description of plant, system, and component). Billinton and Allan (1992, 62) defines series systems and parallel systems, as represented in a reliability network as follows:

Series Systems: The components in a system are said to be in series from a reliability point of view if they must *all* work for system success, or only *one* needs to fail for system failure. A series system typically represents a non-redundant system. For example, let R_A, R_B = probability of successful operation of two components in series, A and B, respectively. (2.3.26) can then be used to give the probability of a series system success

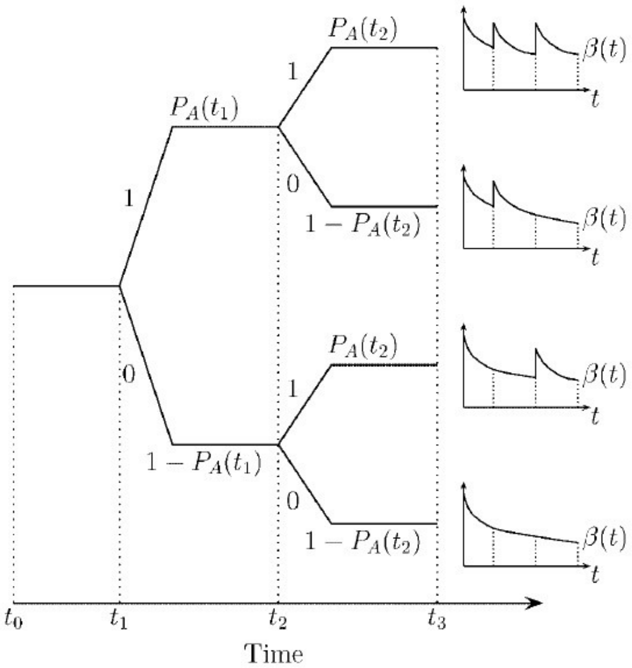


Figure 2.13: A representation of a decision tree as used for optimal life-cycle analysis (Adapted from Frangopol *et al.* (2004, 199))

or reliability. System reliability therefore decreases with an increase in the number of components in series.

$$R_S = R_A \cdot R_B \tag{2.3.26}$$

Parallel Systems The components are said to be in parallel from a reliability point of view if only *one* needs to be working for system success or *all* must fail for system failure. A parallel system represents a redundant system. Considering the relative reliabilities in the series example above, (2.3.27) can be used to give the system reliability of two components in parallel, A and B. System reliability therefore increases with an increase in the number of components in parallel.

$$R_P = R_A + R_B - R_A \cdot R_B \tag{2.3.27}$$

The description above clearly illustrates that the reliability of a system depends on the reliability of its subsystems and on the configuration of the system (Barabady and Kumar, 2008, 649). Prior to analysing data, it is vital that the system configuration be identified.

According to Barabady (2005, 111) and Roy *et al.* (2001, 163), five basic steps must be performed before data can be analysed to determine reliability. These are (i) understanding of the system and identification of subsystems; (ii) collection, sorting and classification of MTBF and MTTR data for each subsystem; (iii) data analysis for verification of the IID assumption; (iv) fitting of the MTBF and MTTR data for subsystems with a theoretical probability distribution; and (v) estimation of the reliability parameters of each subsystem and the system as a whole with a best-fit distribution. IID is defined by Ascher and Feingold (1984, 91) as follows: assume that p nominally identical parts are put on test in p Sockets, under nominally identical conditions, in such a way that no part is affected by the operation or failure of any other part. The p times to failure are therefore assumed to be independent samples from the same distribution function, $F(t)$, which is, that the times to failure are IID. Sample independence, means that the data are free of trends and that each failure is independent of the preceding or succeeding failure. Identically distributed data means that all data in the sample are obtained from the same probability distribution (Roy *et al.*, 2001, 163). Verification of the assumption that the failures or repairs are IID is critical in the sense that if the assumption of IID data is invalid, classical statistical techniques for reliability analysis may not be appropriate (Leemis, 2009, 71). Before any reliability analysis is taken up, tests for trends and serial correlations must be done to check whether the assumption of IID for the data sets are contradicted or not (Kumar and Klefsjö, 1992, 217). Two common methods used to validate the IID assumption are the trend test and the serial correlation test (Barabady and Kumar, 2008, 649). The trend test involves plotting the cumulative failure number against cumulative time between failures. Kumar and Klefsjö (1992, 217) presents a typical trend test whereby the existence of a trend or correlation exists in Figure 2.14; and Roy *et al.* (2001, 163) presents a typical trend test whereby there exists no apparent trend or correlation in Figure 2.15. The serial correlative test is a plot of data pairs (X_i, X_{i+1}) for $i = 1, 2, \dots n$, where n is the total number of failures. If the X are dependent or correlated, the points should lie along a line (Roy *et al.*, 2001, 163).

Once the IID assumption has been successfully verified, the next step according to Figure 2.16 is to determine the best fit distribution for the data. Several methods have been devised to estimate the parameters that will fit a lifetime distribution to a particular data set, such as the method of maximum likelihood (ML), method of moments (ME), and non-linear optimisation (ReliaSoft Corporation [Online] 2016; Raychaudhuri 2008, 96), however, the existence of a vast number of software packages that automate the procedure of fitting relevant lifetime distributions to data sets allows for more efficient and effective means of determining the best fit distributions (Cousineau *et al.* 2004, 742; Lawless 2011, 10). Once the distribution has been fitted, all associated functions (PDF, CDF, and failure functions) are completely determined (Cousineau *et al.*, 2004, 742).

If the assumption that the data are identical is not valid, which is if a trend has been shown to exist, a non-stationary model, such as a NHPP must be fitted (Barabady and Kumar, 2008, 649).

As discussed by Cousineau *et al.* (2004, 742), the fitted distribution com-

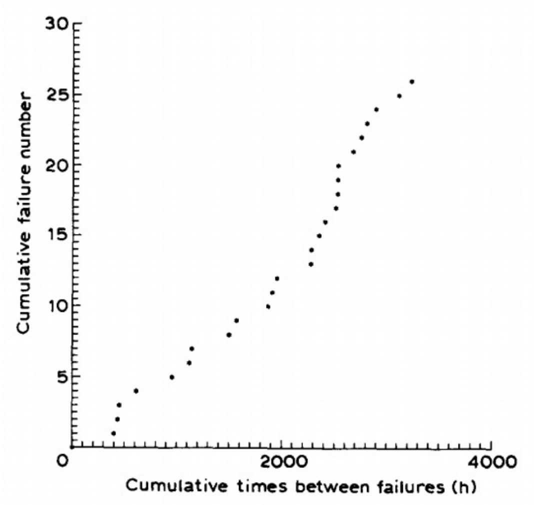


Figure 2.14: Positive trend test for MTBF data (Adapted from Kumar and Klefsjö (1992, 219))

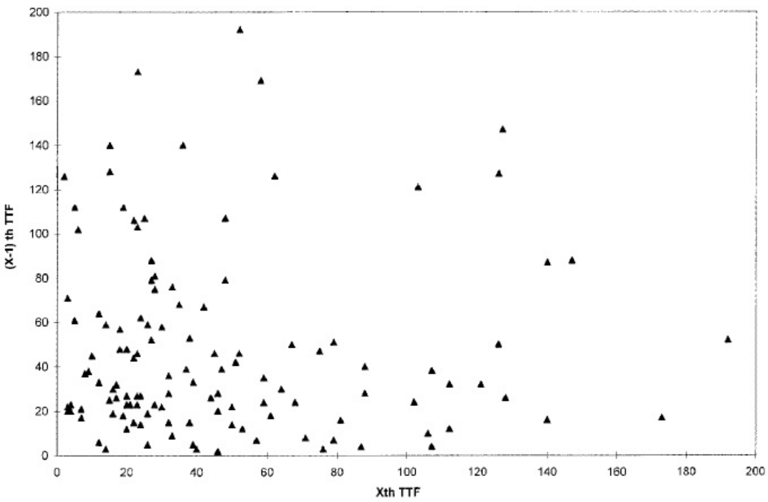


Figure 2.15: Negative trend test for MTBF data (Adapted from Roy *et al.* (2001, 166))

pletely determines all reliability functions for a data set of a component. The reliability analysis, which is the following step in Figure 2.16, encompasses the reliability evaluation of several components (if desired) in unison, which is, of the system as a whole. Depending on the structural arrangement of the components forming the system, the reliability of the system as a whole can be expressed by using (2.3.26) and (2.3.27). There are two fundamental approaches to system reliability evaluation — analytical enumeration and Monte Carlo simulation (Sankarakrishnan and Billinton 1995, 1540; Ge and Asgarpoor 2011, 348). The advantages of the analytical approach include

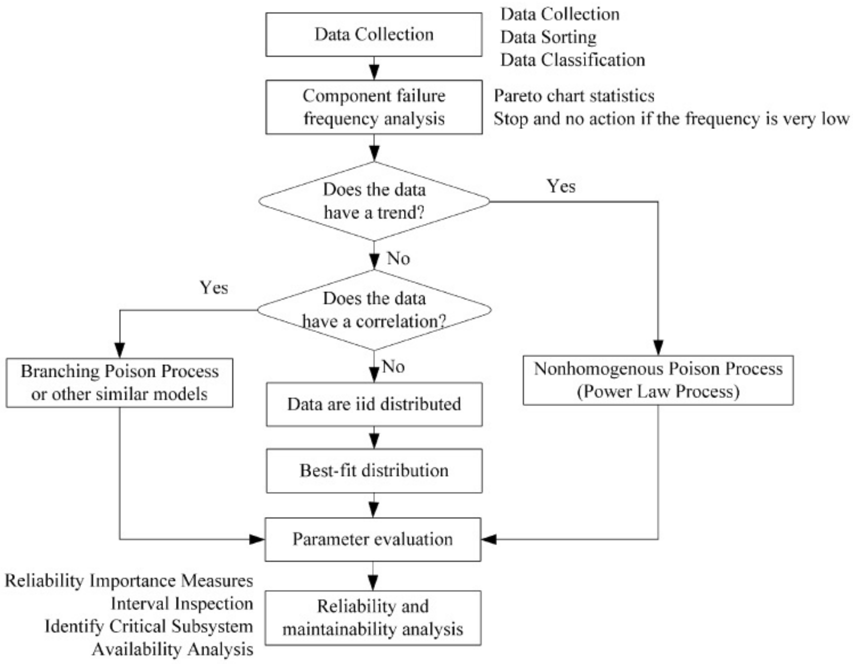


Figure 2.16: Reliability analysis process of a repairable system (Adapted from Barabady and Kumar (2008, 649))

high accuracy and relatively fast computation time; the disadvantages are the limited number of states to be considered, and the inability to provide more reliability information (Ge and Asgarpoor, 2011, 348). Analytical results for single-component deteriorating systems have been established under simplifying conditions, sometimes to an extent that it becomes totally unrealistic (Billinton and Allan 1992, 150; Marseguerra and Zio 2000, 71; Rao and Naikan 2014, 1). As modeling becomes increasingly complex, it therefore becomes evident that an analytical solution of the problem is not possible, and that simulation of the failure and repair process over a given horizon provides the best approach for resolving this difficulty (Percy 2008, 184; Nicolai and Dekker 2008*b*, 189; Barata *et al.* 2002, 255).

2.4 Simulation Methods

The objective of model fitting is to determine the optimal PM scheduled for minimising the expected cost per unit time. Simulation tools are typically needed when treating increasingly complex systems (Barata *et al.* 2002, 255; Rao and Naikan 2014, 1). Simulation has been used as a powerful tool for modeling and analysis of system reliability, which represents the dynamic behaviour of systems in the most realistic sense (Rao and Naikan, 2014, 1).

2.4.1 Monte Carlo Simulation

According to Sankarakrishnan and Billinton (1995, 1540), Monte Carlo simulation is a widely used technique in the probabilistic analysis of engineering systems. It was originated by mathematicians J. Newman and S. Ulam at an early developing stage of nuclear technology, where its applications today have been extended to many areas of science and technology (Wang and Pham, 1997, 187). It is a numerical experimentation technique to obtain the statistics of the output variables of a system computational model, given the statistics of the input variables. In each experiment, the values of the input random variables are sampled based on their distributions, and the output variables are calculated using the computational model. A number of experiments are carried out in this manner, and the results are used to compute the statistics of the output variables (Mahadevan 1997, 123; Raychaudhuri 2008, 95). In short, the Monte Carlo simulation is an empirical method for evaluating statistics (Paxton *et al.*, 2001, 291). Compared to analytical methods, the Monte Carlo simulation approach is a powerful tool that can handle more conditions related to reliability evaluation of systems and, as a result, provides more comprehensive results (Ge and Asgarpoor 2011, 348; Borgonovo *et al.* 2000, 64).

Raychaudhuri (2008, 95) provides a four-step methodology to perform an effective Monte Carlo simulation:

Static Model Generation: Every Monte Carlo simulation initiates with developing a deterministic model which closely resembles the real scenario. Mathematical relationships are then applied which use the values of the input variables, and transform them into the desired output. (The model generation is discussed in more detail in Section 2.3.1).

Input Distribution Identification: Once a deterministic model has been developed, risk components must be added to the model. Since the risks originate from the stochastic nature of the input variables, an attempt must be made to identify the underlying probability distributions which govern the input variables. As discussed in Section 2.3.2.1, this is often referred to as distribution fitting, and can be achieved using historical data for the input variables.

Random Variable Generation: Once the underlying distributions for the input variables have been identified, a set of random numbers (also known as random variates or random samples) must be generated from these distributions. One set of random numbers, consisting of one value for each of the input variables, will be used in the deterministic model, to provide one set of output values. This task is the core of Monte Carlo simulation. The inverse transformation method (ITM) provides the most direct route for generating a random sample from a distribution. The

ITM uses the inverse of the PDF and converts a random number between 0 and 1 to a random value for the input distribution. The process can be mathematically described as follows:
Let X be a continuous random variate (which is to be generated) following a PDF function f. Let the CDF for the variate be denoted by F, which is continuous and strictly increasing in $(0, 1)$. Let F^{-1} denote the inverse of the function F, which is often referred to as the inverse CDF function. Then, the following two steps will generate a random number X from the PDF f:

1. Generate $U \sim U(0, 1)$.
2. Return $X = F^{-1}(U)$.

Note that since $0 \leq U \leq 1$, $F^{-1}(U)$ always exists. The schematic diagram in Figure 2.17 depicts the process, where the curve of a CDF of a certain lognormal distribution is shown on the right hand side, and the left hand side shows a uniform distribution. A randomly generated number (for example 0.65 in the figure), corresponds to 160 at the lognormal CDF curve, which is a random variate from the lognormal distribution. If, for example, 100 of such $U(0,1)$ numbers are generated using the same curve, 100 random variates will be obtained from this distribution.

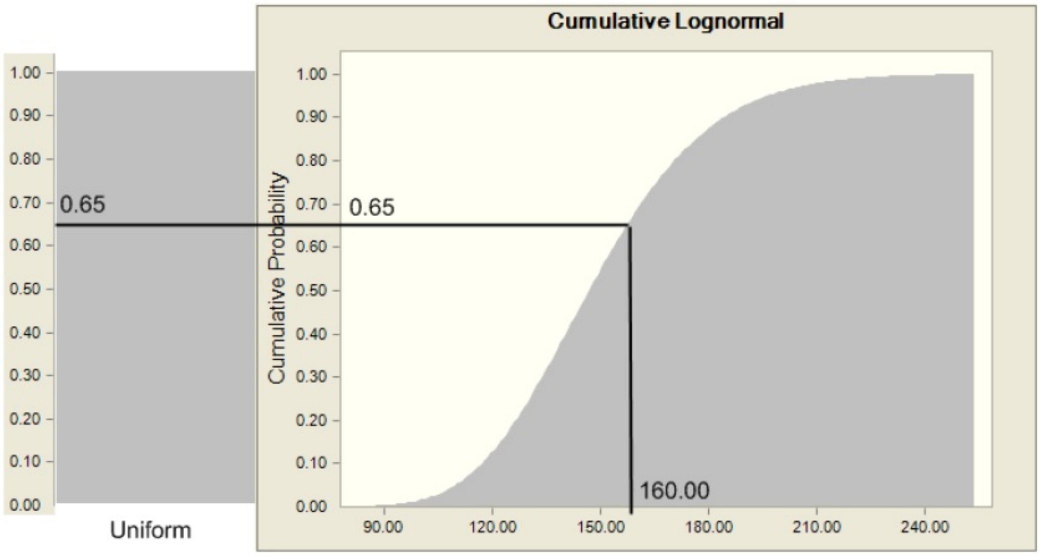


Figure 2.17: Generation of random variates (Adapted from Raychaudhuri (2008, 96))

Analysis and Decision Making: The collection of a sample of output values from the simulation is followed by a statistical analysis of these values. This step provides the statistical confidence for the decisions which might be taken after running the simulation. Averaging trial output values result in an expected value of each of the output variables. Aggregating the output values into groups by size and displaying the values as a frequency histogram provides the approximate shape of the PDF of an output variable. The PDF of the output values can be used for developing confidence bands. The precision of the expected value of the variable and the distribution shape approximations improve as the number of simulation trials increases (Raychaudhuri 2008, 96; Mahadevan 1997, 123).

One of the most common uses of the Monte Carlo simulation in engineering disciplines is to estimate reliability of mechanical components in mechanical engineering (Raychaudhuri, 2008, 96). There exist various options for conducting Monte Carlo simulations using computer software, such as high-level programming languages (C, C++, and Java), as well as using add-ins to popular spreadsheet software such as Microsoft®Excel (Raychaudhuri, 2008, 97).

2.5 Chapter Summary

Over the last few decades there has been a significant change in the approach to maintenance strategies of production facilities, evolving from a purely "corrective" approach, to a more proactive "preventive" approach, whereby maintenance attempts have shifted from correcting failures, to preventing their occurrences in the first place (Dale Johnson 2002, 6; Kobbacy and Prabhakar Murthy 2008, 3). In order to be successful and to achieve world-class manufacturing, organisations must possess both efficient maintenance and effective maintenance strategies (Ahuja and Khamba, 2008, 711). Increased mechanisation and automation have resulted in the maintenance of complex systems becoming increasingly complex (Marseguerra and Zio, 2000, 71). For many systems, especially mass production manufacturing lines in the FMCG industry, both planned and unplanned maintenance stoppages have a significant impact on the economics of the organisation (Dekker *et al.* 1996, 412; Ahuja and Khamba 2008, 711). The increased complexity of manufacturing systems, as well as the impact that maintenance strategies have on economical factors within an organisation, have led to the development of maintenance strategies, of which the RCM methodology is one of, if not the, most widely used maintenance optimisation methodologies in the manufacturing field (Johnston 2002, 512; Vatn 2008, 511).

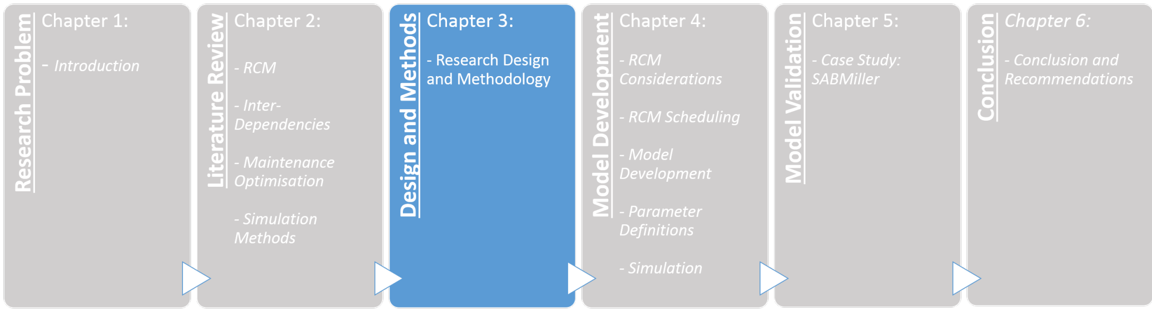
RCM originated in the aviation industry to counter the ever-increasing cost of maintenance activities in the industry (Vatn *et al.*, 1996, 244). The methodology seeks to optimise the maintenance strategy to minimise system failures and, ultimately, increase equipment reliability and availability (Brauer and Brauer, 1987). In order to ensure effective implementation of the RCM methodology, a seven-step approach, as described by Smith and Hinchcliffe (2004, 55), may be followed. Each of the steps fundamentally contributes to the critical step in the process, which is the maintenance task selection. Selection of the optimal interval (or frequency) of these maintenance tasks is by far the biggest challenge for maintenance managers (Smith and Hinchcliffe, 2004, 55). Contributing to the complexity challenge of task frequency optimisation is the inter-dependencies that may exist between components within a system, which may have significant impact on the overall cost of the maintenance strategy employed.

The challenge of maintenance task optimisation has led to the development of numerous mathematical models in literature, which aim at finding either the optimum balance between costs and benefits of maintenance or the most appropriate moment to execute maintenance tasks (Dekker and Scarf 1998, 111; Dekker *et al.* 1996, 231; Maillart and Fang 2006, 804; Thomas 1986, 301). Two of the most well-known PM models originating from literature are the age-based and block-based maintenance policy models, of which numerous extensions and modifications have been proposed. Most models can be divided into two parts: the deterioration model and the decision model. The deterioration model aims at predicting the failure rate and performance of deteriorating equipment by utilising probabilistic methods. The probabilistic methods vary depending on the nature of the equipment under consideration, whereby several distribution functions may be used to predict the future behaviour of the equipment based on historical data. The decision model aims at determining the expected outcome of a maintenance task decision, which was essentially based on the condition of the equipment in the deterioration model. Depending on the structural arrangement of the components forming the system, the reliability of the system as a whole can be determined.

The two fundamental approaches to system reliability evaluation are the analytical approach and the Monte Carlo simulation approach (Sankarakrishnan and Billinton 1995, 1540; Billinton and Allan 1992, 150; Ge and Asgarpoor 2011, 348). As modeling becomes increasingly complex, analytical solutions of the maintenance models become cumbersome and unrealistic, which leads to the simulation approach being the best suited method for complex system analysis (Percy 2008, 184; Nicolai and Dekker 2008*b*, 189; Barata *et al.* 2002, 255). The Monte Carlo simulation is an empirical method for evaluating statistics related to reliability evaluation of systems (Paxton *et al.* 2001, 291; Ge and Asgarpoor 2011, 348; Borgonovo *et al.* 2000, 64). Using the step-wise approach proposed by Raychaudhuri (2008, 95), an effective Monte Carlo simulation can be performed on mathematical maintenance models in order to simulate and, ultimately, determine optimal values (often based on cost) for input variables — such as maintenance tasks and frequencies thereof.

Chapter 3

Research Design and Methodology



This chapter serves to present and discuss the specific research approach, objectives, and data gathering and analysis procedures used to resolve the research problem identified in Chapter 1. The term *research design* is widely used in education, yet it takes on different meanings in different studies — for example, in one study, research design may reflect the entire research process, from conceptualising the problem to the literature review, research questions, methods, and conclusions, whereas in another study, research design refers only to the methodology of a study (Harwell, 2011). In the context of this paper, the latter example provided by Harwell (2011) will be used for further definition and discussion.

Identifying a study's research design is important in the sense that it communicates information about key features of the study, which can differ for qualitative, quantitative, and mixed methods (Harwell, 2011). Creswell (2013) introduces three universal approaches to research:

Qualitative research is an approach for exploring and understanding the meaning individuals or groups ascribe to a social or human problem. The final written report has a flexible structure. Those who engage in this form of inquiry support a way of looking at research that honours an inductive style, a focus on individual meaning, and the importance of rendering the complexity of a situation.

Quantitative research is an approach for testing theories by examining the relationship among variables. These variables, in turn, can be measured so that numbered data can be analysed using statistical procedures. The final written report has a set structure consisting of introduction, literature and theory, methods, results and discussion. Those who engage in this form on inquiry have assumptions about testing theories deductively, building in protections against bias, controlling for alternative explanations, and being able to generalise and replicate the findings.

Mixed methods research is an approach to inquiry involving collecting both quantitative and qualitative data, integrating the two forms of data, and using distinct designs that may involve philosophical assumptions and theoretical frameworks.

In an attempt to select the appropriate research approach, Creswell (2013) suggests using three key components, namely, philosophical assumptions; research design; and specific methods or procedures translating the approach to practice, as depicted in Figure 3.1.

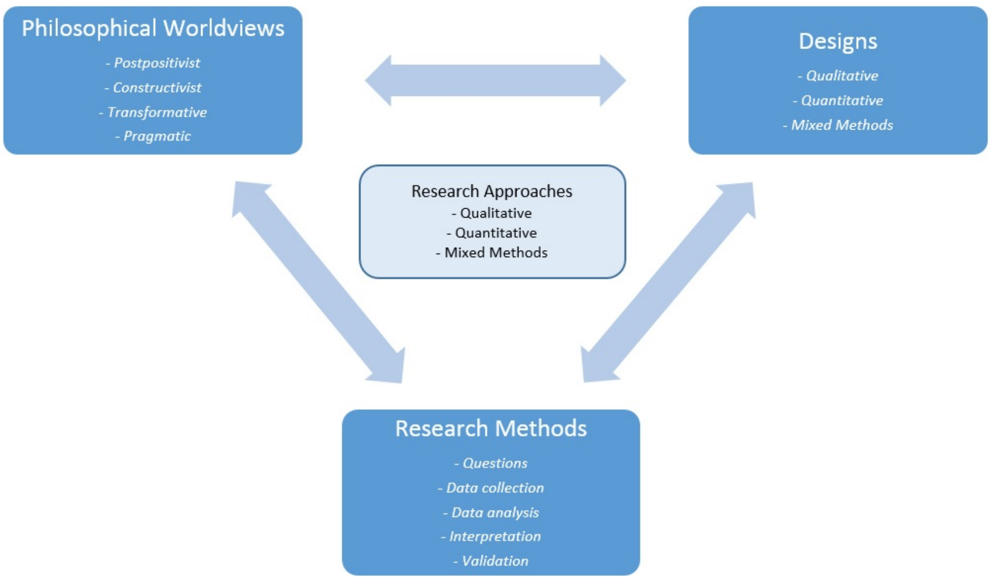


Figure 3.1: A research framework (Adapted from Creswell (2013))

3.1 Philosophical World-views

Creswell (2013) defines *world-view* as "a basic set of beliefs that guide action". World-views can be considered as the general philosophical orientation about the world and the nature of research that a researcher brings to a study (Creswell, 2013). There exists four widely discussed world-views in literature, namely, post-positivism; constructivism; transformative; and pragmatism.

The post-positivist assumptions have represented the traditional form of research, and these assumptions hold true more for quantitative research than qualitative research (Creswell, 2013). Post-positivists hold a deterministic philosophy in which causes determine effects or outcomes (Creswell, 2013). This world-view is often referred to as the *scientific method*, and advocates the use of a scientific approach by developing numeric measures to generate acceptable knowledge (Wahyuni, 2012). The constructivism world-view is seen as an approach to qualitative research. Constructivists believe that individuals seek an understanding of the world in which they live and work (Creswell, 2013), and therefore reject objectivism and a single truth, as proposed in post-positivism, and rather follow a subjective approach (Wahyuni, 2012). A transformative world-view holds that research inquiry needs to be intertwined with politics and a political change agenda to confront social oppression at whatever level it occurs (Creswell, 2013). Thus, the research contains an action agenda for reform that may change lives of the participants, the institutions in which individuals work or live, and the researcher's life (Creswell, 2013). The pragmatic world-view arises out of actions, situations, and consequences, rather than antecedent conditions (as in post-positivism) (Creswell, 2013). Instead of focussing on methods, researchers emphasise the research problem and use all approaches available to understand the problem (Creswell, 2013). Pragmatism is not committed to any one system of philosophy and reality — this applies to mixed methods research in that inquirers draw liberally from both quantitative and qualitative assumptions when they engage their research (Creswell, 2013).

Borrego *et al.* (2009) summarises the philosophical world-views based on their philosophical perspectives, as depicted in Table 3.1. It is clear from these descriptions that the essence of this study will be subjected to the post-positivist world-view, utilising a scientific method with purpose of determining relationships between variables.

3.2 Research Design

Research designs are types of inquiry within qualitative, quantitative, and mixed methods approaches that provide specific direction for procedures in a research design (Creswell, 2013).

Qualitative research is characterised by the collection and analysis of tex-

Theoretical Perspective	**Post-positivism**	**Constructivism**	**Transformative**	**Pragmatism**
View on reality	Single falsifiable reality	Multiple subjective realities	Multiple subjective and political realities	Multiple fragmented realities
Purpose	To find relationships among variables, to define cause and effect	To describe a situation, experience, or phenomenon	To produce a socio-political critique	To de-construct existing 'grand narratives'
Methods	Methods and variables defined in advance, hypothesis driven	Methods and approaches emerge and are to be adjusted during study	Methods and approaches designed to capture inequities	Methods and approaches generated during the study
The role of the researcher	Researcher is detached	Researcher and participants are partners	Researcher and participants are activists	Researcher and participants have various changing roles
Outcome or research product	Context-free generalisation	Situated descriptions	Critical essays, policy changes	Re-conceptualised descriptions of the phenomenon

Table 3.1: Comparison between theoretical perspectives (Adapted from Borrego *et al.* (2009))

tual data (such as surveys, interviews, focus groups, conversational analysis) (Borrego *et al.*, 2009). The research questions that can be answered by qualitative studies are questions such as: *What is occurring? Why does something occur? How does one phenomenon affect another?* (Borrego *et al.*, 2009). While numbers can be used to summarise qualitative data, answering these questions generally requires rich, contextual descriptions of the data (Borrego *et al.*, 2009).

According to Kraska (2010), quantitative research studies produce results that can be used to describe or note numerical changes in measurable characteristics of a population of interest; generalise to other, similar situations; provide explanations of predictions; and explain casual relationships. The quantitative design involves an empirical or theoretical basis for the investigation of populations and samples. Hypotheses must be formulated, and observable and measurable data must be gathered, where appropriate mathematical procedures must then be used for the statistical analyses required for hypothesis testing (Kraska, 2010). Much of engineering research seeks to identify how outcomes (for example mechanical failure) are determined by reducing plausible causes to a discrete set of indicators or variables (Borrego *et al.*, 2009). Creswell (2013) refers to experimental design as having the intent to test an impact of a treatment (or an intervention) on an outcome, controlling for all other factors that might influence that outcome. Conclusions are further derived from collected data and measures of statistical analysis (Creswell, 2013; Thorne and Giesen, 2002).

Mixed methods has been described as the “third methodological movement”

(Teddlie and Tashakkori, 2003). A mixed method approach involves the collection or analysis of both quantitative and/or qualitative data in a single study in which the data are collected concurrently or sequentially, are given a priority, and involve the integration of the data at one or more stages in the process of research (Creswell, 2013).

In the midst of this research study, it is evident that the design of quantitative methods, specifically the experimental design approach, is used whereby scientific methods are utilised to arrive at a single falsifiable reality.

3.3 Research Methodology

The research methodology should aim at utilising the best approach in order to answer the research question. The research methodology is depicted in three interrelated sections, namely: maintenance model development; maintenance model simulation; and validation of the optimised model in the form of a case study.

The *maintenance model* development section will aim at developing a mathematical model using variables typically associated with optimal maintenance decision making. Production facilities that make use of the RCM methodology typically have predefined periods at which PM tasks are undertaken, as explained in Section 2.1. Due to inherent restrictions, including costs and availability of resources, not all equipment components are maintained during a specific interval. In order to determine which components are to be maintained at these predetermined intervals, as well as the type of maintenance task to be executed, inherent reliability functions of components must be included in the maintenance model, with the aim of predicting the most probable failure instances in advance and, ultimately, executing a PM task in order to prevent the failure from occurring. The prediction method is purely based on statistics, making use of historical data of each component.

The development of the maintenance model will inevitably lead to the process of optimisation in the form of *maintenance model simulation*. The aim of the maintenance optimisation in this study is minimise the overall cost of maintenance, hence, the maintenance model will be optimised in terms of the cost function of the maintenance task outcomes. The mathematical model developed revolves around the fact that all maintenance tasks (CM and PM) are coupled with a certain cost — where CM has the disadvantage of unexpected production downtime losses and the risk of unavailable resources; whereas PM has the advantage of preventing CM costs, but also has the disadvantage in terms of the risk of over-maintaining equipment, resulting in unnecessary production downtime as well as over expenditure on typical maintenance costs. Due to the inherent complexity of the maintenance model, an analytical optimisation of the model may not be applicable and, therefore, a simulation approach (namely the Monte Carlo simulation) is proposed. The simulation will aim at determining the maintenance tasks and frequencies for a component resulting in an optimally low maintenance cost for the organisation.

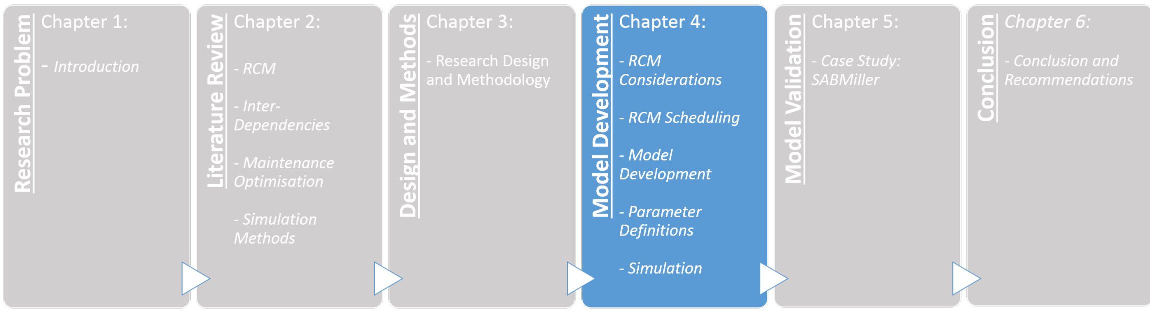
In order to validate the proposed optimised maintenance approach, a *case study* is undertaken. The case study will replicate the methodology used in developing and optimising (simulating) a maintenance model, using historical data in a current FMCG production facility. Relevant historical data collection within the FMCG facility will be undertaken in order provide for all incomes and outcomes of the developed maintenance model. Based on the proposed, optimised maintenance model, it would be possible to determine the resulting theoretical maintenance cost over the predefined time period that the production facility would have incurred if the proposed model was utilised. This theoretical cost can then be compared to the actual maintenance cost that the production facility incurred in reality based on their current maintenance approach. The cost comparison will essentially be the crux of validity of the proposed maintenance strategy, whereby the cost benefit (if any) will be the end measure.

3.4 Chapter Summary

In conclusion, Chapter 3 constructed a well-defined research methodology that is appropriate to the research problem statement and objectives defined in Sections 1.2 and 1.4, respectively. This chapter discussed the various research design methods and philosophical world-views, with emphasis on the applicability of each in various research applications. Based on the definitions of research designs and world-views, it is decided that the quantitative research design, coupled with the post-positivist world-view is most applicable to this study. The scientific approach used in the quantitative research design methodology ws further developed by providing an outline of the methodological approach that is to be utilised throughout the remainder of this study. The guideline proposed by Kraska (2010) is followed, whereby the validation of the experimental results is achieved in the form of a case study of a real-life production facility.

Chapter 4

Maintenance Model Development and Optimisation



4.1 Reliability Centred Maintenance Considerations

The development of the maintenance model used in this study is largely based on the seven step RCM approach described by Smith and Hinchcliffe (2004, 55) in Section 2.1.2. Each step of the seven step approach is considered, specifically focussing on the correlation between the relative step and the proposed maintenance model.

4.1.1 Step 1 — System Selection and Information Collection

Smith and Hinchcliffe (2004, 49) notes that maintenance planning starts directly with the equipment and seeks to specify various tasks that are felt necessary to maintain the operational status of the equipment. All systems may in principle benefit from an RCM analysis, however, with limited resources within a production facility, priorities must be set. A vital step in the RCM methodology application is to fully comprehend the organisational maintenance design and constraints of the facility in consideration as well as the structural set-up of the system for which the model is intended to be used.

A complex system consists of any structure of more than one component, which performs a particular function. Typical systems include industrial machinery such as production lines, utility supplies, and railway operations (Percy, 2008, 184). The complex system may be further broken down into subsystems, which in turn make up the construction of the entire system — the assembly hierarchy described by Vatn (2008, 511) provides an effective mean to assist with identifying the level of the system for which the maintenance model will be developed. The maintenance model developed in this study aims at providing a model that is applicable to both single- and multi-component systems. In the case of a multi-component system, a group of n components is considered, where $n = 1$ in the case of a single-component system.

Depending on the availability of maintenance resources, such as maintenance crews, it is especially advantageous to conduct simultaneous maintenance tasks on components if the components in the system exist in a series set-up, which is, the non-functioning of a single component in the system results in the non-functioning of the entire system. Maintenance resource availability will vary depending on the resource design of the facility in consideration, as certain facilities may have a full compliment of resources on standby during production, whereby other facilities may only have resources dedicated to planned maintenance intervals with a lean set of resources available for any unforeseen equipment breakdowns. The range of possible opportunistic maintenance tasks that a facility can undertake at any given instance is therefore highly dependent on the maintenance resource design of the facility in question. Consider, for example, two production facilities, Facility A and Facility B, which both experience a component failure during a production interval. Facility A's maintenance resource design includes a team of standby maintenance resources which are able to conduct maintenance tasks at any given time, it would therefore be possible to conduct simultaneous opportunistic PM tasks on alternative economically dependent components during this breakdown period. Facility B's maintenance resource design only allows for a full compliment of maintenance resources during planned maintenance intervals, with a lean team of resources dedicated solely to attend to breakdowns and thus to perform CM tasks on the failed component in order to repair the failed component to an operating condition, thus inhibiting the possibility of performing additional PM tasks on alternative economically dependent components simultaneously. It is often economically infeasible for a facility to incur the additional cost of a permanent full-complimented maintenance crew throughout production intervals seeing as this would result in inefficient utilisation of resources. The resource design of Facility B in the aforementioned example would therefore represent the majority of FMCG facilities' maintenance resource designs, and is further assumed to be the design of the facility in consideration for the remainder of the maintenance model development. This assumption essentially results in the constraint that PM tasks can only be performed during planned PM intervals.

4.1.2 Step 2 — System Boundary Definition

The subsystems, described in Section 4.1.1, can be considered a series configuration of multi-component systems, meaning that the failure of any subsystem leads to the non-operation of the entire system (Roy *et al.*, 2001, 404). The entire system's non-operational status can either be in the form of subsequent failure of structurally dependent components in the system, or in the form of subsequent idling of the entire system as a result of the failed component. It must therefore be noted that "non-operation" does not necessarily translate to "failure" of the entire system, but could also refer to "idling" of the entire system — depending on the structural configuration and dependency of the components within the system. Considering that a vast majority of FMCG production facilities are of a continuous-flow, mass-production design, almost any component failure within the production line would eventually result in a "non-operational" status of the production line. Defining the boundary at which the maintenance optimisation process is executed must, therefore, not be undermined, as component dependencies play a major role in maintenance efficacy.

For multi-component systems, an optimal maintenance policy must take into account the interactions between various components (Laggoune *et al.*, 2010, 1501). Structural dependence typically refers to dependency between components whereby one component cannot be maintained unless another component(s) is replaced or dismantled. When defining the system boundary within which the maintenance optimisation process is being done, structural dependence between components will typically result in a compulsory boundary, seeing as the structurally dependent components require the physical intervention of another component(s). An example of structurally dependent components would be an assembly of a shaft and sprocket system, whereby the shaft cannot be maintained unless the sprocket is physically removed form the shaft — in this case, it would be a trivial decision to perform maintenance on the sprocket during the same interval at which maintenance is being performed on the shaft. Economic dependence, which allows for cost or downtime savings by simultaneously maintaining several components, is especially common in mass-production manufacturing lines (Laggoune *et al.*, 2010, 1501). In the majority of cases, structural dependence would therefore result in a coupled economic dependence between components, however, the inverse would not necessarily be true, as economic dependence could exist between components without any structural dependence. Considering the example of the shaft-and-sprocket assembly — if the shaft was driven by a gearbox, it could be economically beneficial to consider replacing the gearbox during the same maintenance interval. Despite the gearbox and shaft not being structurally dependent, which is, the gearbox does not necessarily have to be dismantled to maintain the shaft, it could result in a cost benefit if the gearbox were to be replaced. Economic dependence will largely depend on the production facility's maintenance resource design, as well the structural design of the system under consideration. The last form of dependency that may exist between components is stochastic dependence (Cho and Parlar, 1991, 2). Stochastic dependence implies that the state of components can influence the state of other components. Referring to the shaft-sprocket-gearbox assembly example, the state of the gearbox could influence the state of the shaft, seeing as if there were some degree of wear within the gearbox, it could result in the shaft not running true (which is, not running geometrically concentric), and essentially result in wear on the shaft and ultimately failure thereof. Component dependencies, whether structural, economic, and/or stochastic, therefore play a major role in the maintenance strategy efficacy and should be considered in detail by the production facility in consideration.

The multi-component maintenance model approach aims at grouping dependent components (or subsystems) together in order to plan for simultaneous PM tasks on the group of components during the planned PM intervals — the group of components on which maintenance will be performed simultaneously can be denoted by G_m. When no strong dependence exists between the different components, the traditional single-unit models developed by Barlow and Hunter (1960) can be independently applied to each component. Essentially, defining the system boundary within which the maintenance optimisation process will be conducted depends on the analyst performing the optimisation, however, it is advisable that component dependencies be considered in order to ensure optimum maintenance strategy efficacy.

4.1.3 Step 3 — System Description and Functional Block Diagram

System description typically aims at revealing factors such as functional description, redundancy features, and protections features — all of which would play a significant role in gathering information in order to implement an RCM maintenance strategy within a production facility (Smith and Hinchcliffe, 2004, 88). The correlation between the *system description* and the development of the maintenance model in this study arises in the identification of the system's functional description. For the system under consideration, the functional description would essentially reveal what the intended function of the system is

— hence, by not delivering the intended function of the system, the system can be considered to be *non-operational*. The function of the system under consideration is case-dependent, and would need to be determined by the analyst in order to progress to the next step in Section 4.1.4. Considering the aforementioned shaft-sprocket-gearbox assembly in Section 4.1.2, the function of the shaft would be to transfer a driving force from the gearbox to the sprocket — hence, if the shaft no longer provides a driving force from the gearbox to the sprocket, it can be considered *non-operational*, seeing as the shaft no longer fulfils its intended function. Furthermore, a *functional block diagram* provides an effective mean to visually provide an understanding of the identified system functions, as well as the series-parallel relationships between various system functions.

In the context of the maintenance model development, the *system description* seeks to provide an understanding of the functionality of the system under consideration, as well as the series-parallel relationships of components that may exist within the system. An understanding of the functionality and relationships essentially assists the analyst in deciding on the definition of the system boundary described in Section 4.1.2.

4.1.4 Steps 4, 5, and 6 — Functional Failures, FMEA, and LTA

Steps 4, 5, and 6, described in Section 2.1.2, are combined in the maintenance model development process, as the three steps essentially intend to identify and prioritise failures, as well as specify maintenance tasks that should be carried out in order to prevent these failures from occurring. The *system description* described in Section 4.1.3 provides the basis of identifying *functional failures*. An unacceptable deviation from the defined system function would essentially result in a *functional failure*, seeing as the system no longer performs its intended function. For a given component, there would exist a vast range of failure modes which would result in a functional failure. The task of identifying failure modes requires a reasonable knowledge of the system design and operation characteristics (Deshpande and Modak, 2002, 34). The FMEA process, described in Section 2.1.2.5, provides an effective mean for identifying relevant PM tasks, aimed at preventing the occurrence of the various failure modes. In addition, the LTA further considers the identified potential failures and provides a mean to rank failure modes and, subsequently, to rank maintenance tasks accordingly.

FMEA and LTA should be carried out by any production facility aiming at implementing an RCM strategy, as the processes provide the analyst with valuable insight regarding the specifics around the *detail* of work to be performed within maintenance tasks. Considering the vast array of components in industry and their coupled failure modes, the FMEA and LTA steps would be case-specific and involves a study within itself. Although the FMEA and LTA are essential steps in implementing effective PM tasks, the detailed tasks are not covered in the scope of the maintenance model development in this study. Instead, the tasks performed within the maintenance model are considered to be either CM tasks or PM tasks — the detail of the actual work performed during these tasks is not considered in this study. In the case of CM being performed, which is during a breakdown, the detail of work would inevitably be to restore the component to a functioning state, however, in the case of PM being done, it can be considered that the detail of work would be a separate study which should be performed in parallel with the PM frequency optimisation.

4.1.5 Step 7 — Task Selection

As described by Vatn (2008, 511), task selection involves the decision-making step of whether a PM task for a particular component is cost-feasible, or whether the component should deliberately be run-to-failure, which is, only correctively maintained when necessary. Vatn (2008, 511) provides six alternatives regarding the *type* of maintenance tasks that could be considered (see Section 2.1.2.7):

1. *CCT*: the continuous monitoring of a component is a form of PdM whereby the condition of a component is continuously monitored and, hence, maintenance decisions can be made based on the current condition of the component. Based on the scope of the maintenance model developed in this study, the optimisation of PM tasks are entirely based on statistical reliability analysis and not component-state. For this reason, PdM tasks do not form part of the maintenance task optimisation, and hence, this study will not consider CCT as a maintenance task option.

2. *SCT*: Similar to the CCT, SCT is a task wherein the current state of the component is analysed, however, the process is periodic instead of continuous. As with CCT, SCT is a form of PdM whereby future maintenance tasks are planned based on the condition of the component. SCT is therefore also not considered as a maintenance task option within the scope of this study.

3. *SOH*: the overhaul of a component can be viewed as a planned PM task resulting in an "improved" failure rate as compared to the failure rate just prior to PM being undertaken. According to Vatn (2008, 511), there must be (a) an identifiable age at which the component shows a rapid increase in the component's failure rate; and (b) a large portion of the components – based on historical data – must survive to that specified age. In the case of component age, the MTBF can be used as

an indication for the expected "lifetime" of a component, where the SOH time interval will typically be less than the MTBF to ensure minimal probability of failure within the operating period of the component.

4. *SRP*: a scheduled replacement involves discarding a component and replacing it with a new component. A scheduled replacement can either be performed once the component reaches a certain age, or after the component has undergone a certain number of PM tasks leading into the following planned PM interval. Vatn (2008, 511) advises that, for SRP to be applicable to a component, (a) there must be an identifiable age at which the component shows a rapid increase in the failure rate; and (b) a large portion of the components – based on historical data – must survive to that age. Alternatively, the component can be replaced once it has undergone a certain number of PM tasks.

5. *SFT*: a scheduled function test on a hidden function of a component can be done to identify any failures. According to Vatn (2008, 511), a SFT is applicable to a component that is (a) subject to a functional failure that is not evident to the operating crew during the performance of normal duties; and (b) the item must be one for which no other type of task is applicable and effective. Considering the scope of this study, it is assumed that all failures of components are immediately evident, and that there exists at least one maintenance task that is applicable and effective in preventing the failure of the component — it is therefore considered that SFT maintenance tasks is not applicable within this maintenance model

6. *RTF*: running a component to failure is a deliberate decision to run to failure as a result of other tasks not being possible or the economics are less favourable. Considering the assumption that there exists at least one maintenance task to prevent failure of a component, the only circumstance that would allow for an RTF approach would the consideration whereby it is economically favourable to run a component to failure instead of preventively maintaining it.

4.2 RCM Scheduling of the Maintenance Model

The approach followed for developing maintenance models for both single- and multi-component systems encompasses a diagrammatic time-based illustration of the occurrence of certain events, namely, planned maintenance intervals, planned component replacement intervals, unforeseen *minor* failures, and unforeseen *major* failures.

4.2.1 Scheduling of a Single Component

Section 4.1.1 to Section 4.1.5 has established a comprehensive foundation on which an applicable RCM scheduling approach can be based. As described in Section 2.1, the RCM methodology is based on the premise of scheduled intervals during which planned PM tasks are executed. The planned maintenance interval can either be a fixed interval whereby a facility will undergo PM at pre-determined fixed intervals, or a flexible interval whereby a facility is able adjust interval timing between PM tasks. Based on the common constraint of maintenance resource availability, described in Section 4.1.1, the maintenance model in this study is further developed on the assumption that maintenance intervals are fixed and cannot be changed. The fixed maintenance time interval is denoted as T, which depicts the time period between successive planned PM intervals — each interval occurring at time T_m, $m = 1, 2, \ldots, N$. Figure 4.1 provides an illustration of planned PM intervals for a single component. In Figure 4.1, the first planned PM interval occurs at time T_1, where the following planned PM interval occurs at time T_2 (where time period T separates T_1 and T_2), and so forth. The time between PM intervals is considered to be the time during which the component is in actual operation, which is, *operational time* or *operational periods*. As described in Chapter 1, for many mass production manufacturing lines, the production losses due to downtime, whether planned or unplanned, are significantly large, rendering the assumption of negligible maintenance times to be inaccurate. Therefore, during PM tasks, it is considered that the time to execute planned PM tasks is denoted by ω^p — it is thus assumed that planned PM tasks require a constant amount of time to execute, as shown in Figure 4.1. During the planned PM intervals, an SOH is executed on the component. After $x - 1$ number of planned PM tasks have been executed on a particular component, which is, the component has successfully completed x number of operational intervals in its lifetime, an SRP is executed, whereby the component is discarded and replaced. In Figure 4.1 $x = N$ seeing as the number of operational intervals is equal to the number of planned PM tasks performed until T_N. The time required to execute the planned SRP is denoted as ω^{pR}. The total time of the maintenance cycle, which is, until the component is replaced and considered "good as new", is therefore equal to $NT + \omega^{pR}$.

As explained in Section 4.1.5, an SOH maintenance task is a PM task that results in an improved failure rate of the component. Referring to Section 2.3.1 and the *degree* to which maintenance tasks restore a component described by Wang (2002, 469), the PM tasks in this maintenance model refer to maintenance tasks that restore a component to a condition between "good as new" and "bad as old". The degree to which a component is restored in the maintenance model within this study is similar to the approach of Sheu *et al.* (2012, 1270), whereby the failure rate of the component after a PM task reduces to zero, and then increases more quickly than it did in the previous operational period. The original failure rate of a component, denoted as $\lambda(t)$, would then become $a\lambda(t)$ in the following operational period, where $a \geq 1$ is the improvement factor and $t \geq 0$ represents the time from the previous PM task.

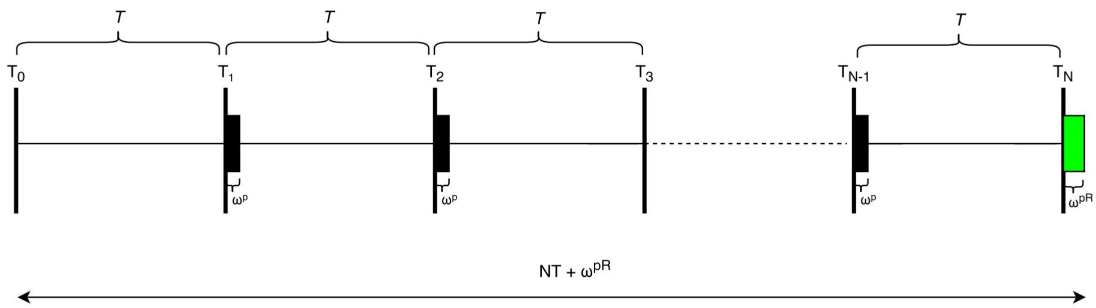


Figure 4.1: Planned PM intervals for a single component.

Between planned PM intervals, which is, during the operational period of the component, unforeseen failures of the component is an inevitable occurrence. Failures are considered to be one of two types, namely, *minor* failures or *major* failures. In the event of a *minor* failure occurring, in order to restore the component to an operating condition, it is necessary to undertake unplanned CM on the component. The CM task undertaken has no effect on the failure rate of the component, thereby implying that the component's state after CM is considered to be "as bad as old", and simply continues to operate at the same failure rate as prior to the CM task. The unforeseen occurrences of *minor* failures are denoted by $S_{op,j}$, where op indicates the operational period in which the failure occurs, and j indicates the number of unforeseen *minor* occurrence(s). Hence, as seen in Figure 4.2, the first and second random unforeseen failures in the first operational period (leading into the first PM interval at time T_1) are denoted by $S_{1,1}$ and $S_{1,2}$, respectively. As with PM tasks, CM will also require a certain amount of time to execute as potential production time is lost during the execution of the CM task. The MTTR is a common indicator of the time required to repair a component to an operating condition. The time required to conduct a CM task is denoted by ω^c. In the event of a *major* failure occurring, denoted by $R_{op,j}$ in Figure 4.2, it is necessary to execute an unplanned SRP, which is, replacement of the component, requiring time ω^{uR} to execute. Seeing as an SRP results in the component being "renewed" ("good as new" state), the operational period immediately after the SRP is considered to be the first operational period of the component.

Following the planned PM task, the component will have an expected lifetime before a *major* failure occurs (denoted by Y_{op}, where op represents the operational interval during which the lifetime of the component is considered) based on the *major* failure probability distribution of the particular component. In Figure 4.2 it can be seen that the expected lifetime of the component in the first, second, and third operational period is illustrated as Y_1, Y_2, and Y_3, respectively. It must be noted that "lifetime" refers to the remainder life of the component in which it can still operate or be repaired, which is, once the component has reached the end of its lifetime, it will need to be replaced. The end of a component's lifetime is brought about by an SRP, whether planned or unplanned, where it can be seen in Figure 4.2 that the lifetime of the component after $R_{3,1}$ "resets" to Y_1, which is, the component begins with its first operational period. Once the component has successfully completed x operational intervals, a planned SRP is scheduled for the next planned PM interval — occurring at interval T_N in Figure 4.2, where the planned SRP task requires time ω^{pR} to execute.

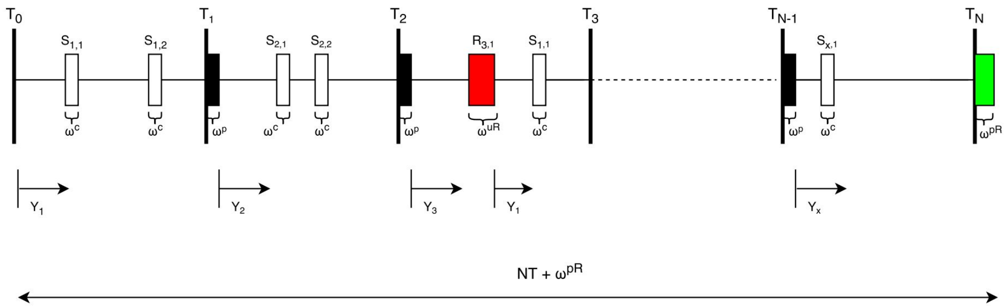


Figure 4.2: Occurrence of unforeseen component failures.

4.2.2 Scheduling of Multi-components

The maintenance task scheduling of more than one component, which is, of a multi-component system, follows a similar approach to that of single component scheduling. Considering a group of n components, a diagram is constructed to illustrate the approach of multi-component scheduling in Figure 4.3. It must be noted that the multi-component system is considered a *series* configuration, meaning that the failure or downtime of any single component results in the downtime of the system as a whole. Each component will have its inherent failure rate and thus result in individual random unforeseen failures, denoted by $S^n_{op,j}$, where n represents the component in consideration, and *op* and j represent the number of unforeseen failures in the operational period and the operational period in which the failure takes place, respectively. The unforeseen failures, $S^n_{op,j}$, are dependent on the component's *minor* failure rate, therefore implying that as the component's state regresses over time, the probability of failure will subsequently increase with time (assuming an increasing failure rate). As with the single component scheduling, planned PM intervals are fixed and separated by the time period T. Seeing as each component will require a unique MTTR, the component's MTTR (or time required to execute CM) is denoted by ω_n^c, where n represents the component in consideration. The time required to execute PM tasks on each component will also be unique, considering that not all components undergo the same maintenance tasks. The time required to execute PM on a particular component is therefore denoted by ω_n^p, where n represents the component in consideration. Another unique property of the components is the expected lifetime of each component, seeing as not all components will exhibit the same failure probability distribution patterns. This results in only certain components having to be maintained during the planned PM intervals. Depending on each component's expected lifetime, the planned PM will fall within one of the PM intervals, which is, T_m. For any given PM interval, there will therefore be a group of components, denoted by G_m, that will be maintained in the same PM interval, for example, at PM interval T_2, G_m will include *component 1* and *component 2*. It can be noted from Figure 4.3 that during PM intervals, there exists the possibility that certain components may experience "un-utilised" downtime, seeing as certain components require longer PM task time compared to others. Therefore, the time required to conduct PM on a group of components (G_m) will be equal to the longest PM task time of all G_m components. The time interval T between successive planned PM intervals must be chosen as the smallest interval among all components' planned PM intervals, thus avoiding the case where all components experience "un-utilised" downtime during a PM interval.

Consideration must be made for unforeseen *major* failures, seeing as there may be instances in which a component experiences a failure which can only be rectified by replacing the component, which is, an unplanned SRP for the component is undertaken. In Figure 4.3 *component 1* experiences a *major* failure during the third operational period (leading into PM interval T_3), where the *major* failure is denoted by $R^1_{3,1}$, which requires an unexpected replacement of *component 1*, where the task thereof requires time ω_1^{ur}. *Component n* also experiences a *major* failure in the second operational interval (leading into PM interval T_2), where the *major* failure is denoted by $R^n_{2,1}$, which requires an unexpected replacement of *component n*, where the task thereof requires time ω_n^{uR}. As discussed in Section 4.1.5, the SRP task results in a "renewal" of the component, where the component is then considered to be "as good as new" thereafter, and the expected lifetime of the component "resets" such that the component is considered to be in its first operational period immediately after the SRP task, which is the same approach to that of single component scheduling. As in the case of single component scheduling, the expected lifetime of the component is denoted by Y^n_{op}, where the additional denotion of n is used to denote the component in consideration.

As with the single component modeling approach, each component will be preventively replaced once the component has successfully reached x_n number of operational periods, where x denotes the number of periods for the com-

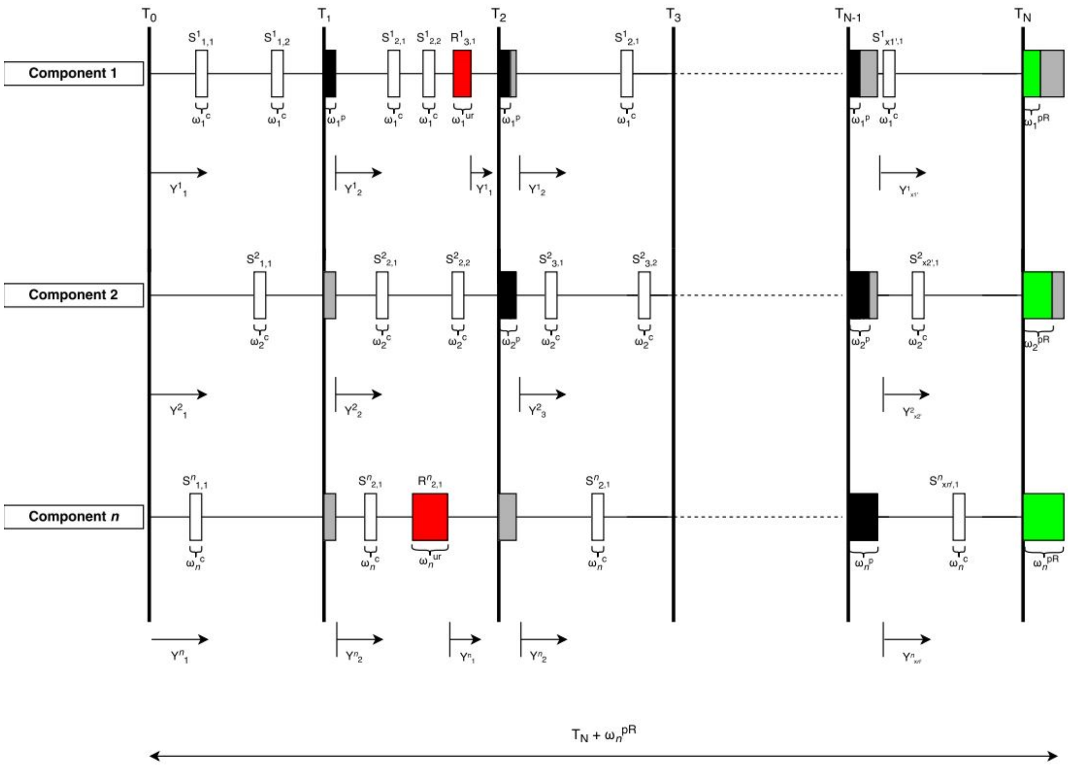


Figure 4.3: Planned PM intervals for multi-components.

ponent, and n denotes the component in consideration. A planned SRP for a component is denoted by PR_n, where n denotes the component in consideration. In order to define a system *cycle*, whereby all components undergo a planned SRP, an overall operational limit is defined in the form of *total system operational intervals*, denoted by X_{TOT}, as shown in (4.2.1). In (4.2.1), x'_i is defined as the optimal number of operational periods which a component successfully completes — the sum of each individual component's optimal operational time (X_{TOT}) provides the *total system operational intervals* which will result in a complete system SRP. The occurrence of an unforeseen *minor* failure in the N-th operational interval is denoted by $S^n_{op,j}$, where $op = x'_n$, considering that the component will be in its x'_n-th operational interval. In Figure 4.3, the PM interval T_N denotes the time instant at which all components undergo a planned SRP task, where the time required to execute the tasks will be equal to the longest SRP task of all components, denoted by ω^{pR}_n in Figure 4.3 (note that the same reasoning is followed for simultaneous component PM tasks, whereby certain components will experience "un-utilised" downtime). Following the system SRP, the multi-component configuration is now considered to "reset" as an entirety, which is, all components in the multi-component configuration are considered "good as new". The total completion time for an entire system cycle is then seen to be $T_N + \omega^{pR\star}_n$, where $\omega^{pR\star}_n$ is equal to the longest planned SRP time required between all components. It must be noted that the possibility exists whereby an unforeseen *major* failure of a component may occur in the N-th operational interval, however, it is considered that in this instance the component will undergo a planned SRP regardless of an unforeseen *major* failure in order to allow for definition of a system cycle.

$$X_{TOT} = \sum_{i=1}^{n} x'_i \tag{4.2.1}$$

4.3 Development of the Maintenance Cost Model

Section 4.2 provides a methodology which clearly defines events within the RCM scheduling and operation of both single- and multi-component systems. Considering that the maintenance model in this study aims at optimising the RCM tasks and frequencies in terms of cost, all events within the scheduling approach must be linked to a cost function. In order to define the cost functions, which relate to the events in the maintenance scheduling, the following notations are defined and further used throughout the development of the cost model:

T	Scheduled interval between successive planned PM periods.
C_{mr}	Total *minor* repair cost over a system cycle.
C_{pr}	Total PM cost over a system cycle.
C_{pR}	Total replacement cost over a system cycle.
G_m	Group of components which undergo planned PM at the m-th PM interval.
c^{ud}	Cost per unit time of unplanned system downtime, which is, during CM tasks.
C^{ud}	Total unplanned downtime cost over a system cycle.
c^{pd}	Cost per unit time of planned system downtime, which is, during planned PM and SRP tasks.
C^{pd}	Total planned downtime cost over a system cycle.
$\bar{c}^{c}_{n}$	Spare part cost to conduct CM on component n (simplifying to $\bar{c}^{c}$ for a single component).
$\bar{c}^{pr}_{n}$	Spare part cost to conduct planned PM on component n (simplifying to $\bar{c}^{pr}$ for a single component).
$\bar{c}^{pR}_{n}$	Spare part replacement cost of component n (simplifying to $\bar{c}^{pR}$ for a single component).

x_n	Scheduled number of maintenances for component n whereupon the component is replaced (simplifying to x for a single component).
x'_n	Total number of operational periods successfully completed by component n, triggering the total system operational periods threshold, X_{TOT} (applicable to the multi-component model only).
X_{TOT}	Threshold of summation of all n components' operational intervals, triggering a planned system SRP in the following planned PM interval (where $X_{TOT} = \sum_{i=1}^{n} x'_n$, and only applicable to the multi-component model).
ω_n^c	Time required to conduct a CM task on component n (simplifying to ω^c for a single component).
ω_n^p	Time required to conduct a PM task on component n (simplifying to ω^p for a single component).
ω_n^{pR}	Time required to conduct a planned SRP on component n (simplifying to ω^{pR} for a single component).
ω_n^{uR}	Time required to conduct an unplanned SRP on component n (simplifying to ω^{uR} for a single component).
$\omega_m^{p\star}$	Time required to conduct planned PM (or planned SRP) during the m-th PM interval (equal to $\omega_{k,m}^{p}\omega_{k,m}^{pR}$, and only applicable to the multi-component model).
$\omega_f^{pR\star}$	Time required to conduct planned SRP of the entire multi-component system (equal to $\max(\omega_f^{pR})$ for $f = 1, 2, \ldots, n$, and only applicable to the multi-component model).
$\lambda_{(op,min)}^n(t)$	*Minor* failure rate function in the *op*-th operational period at time t of component n (simplifying to $\lambda_{(op,min)}(t)$ for a single component).
$f_{(op,min)}^n(t)$	PDF of a *minor* failure for component n in its *op*-th operational period, $op = 1, 2, \ldots, x_n$ (simplifying to $f_{(op,min)}(t)$ for a single component).
$f_{(op,maj)}^n(t)$	PDF of a *major* failure for component n in its *op*-th operational period, $op = 1, 2, \ldots, x_n$ (simplifying to $f_{(op,maj)}(t)$ for a single component).
$F_{(op,min)}^n(t)$	CDF of *minor* failures for component n in its *op*-th operational period, $op = 1, 2, \ldots, x_n$ (simplifying to $F_{(op,min)}(t)$ for a single component).

$F^n_{(op,min)}(t)$	CDF of *minor* failures for component n in its *op*-th operational period, $op = 1, 2, \ldots, x_n$ (simplifying to $F_{(op,min)}(t)$ for a single component).
a_n	Improvement factor in failure rate for component n following a PM task, where $a_n \geq 1$ (simplifying to a for a single component).
A^n_{op}	The failure rate improvement function for component n in its *op*-th operational period, $A^n_{op} = (a_n)^{(op-1)}$, where $A^n_{op} \geq 1$ (simplifying to A_{op} for a single component).
Y^n_{op}	The expected operational lifetime before a *major* failure occurs of component n in the *op*-th operational period, $op = 1, 2, \ldots, x_n$ (simplifying to Y_{op} for a single component).
σ_{op}	The time to the next planned PM (applicable to the single component model only).
$\bar{F}^n_{(op,maj)}(t)$	*Major* failure survival function of Y^n_{op} of component n, $op = 1, 2, \ldots, x_n$ (simplifying to $\bar{F}_{(op,maj)}$ for a single component).
$F^n_{(op,maj)}(t)$	CDF of *major* failures of Y^n_o for component n, $op = 1, 2, \ldots, x_n$; $F^n_{(op,maj)}(t) = 1 - \bar{F}^n_{(op,maj)}(t)$ (simplifying to $F_{(op,maj)}(t)$ for a single component).
$E[R]$	Expected total cost over a renewal cycle.
$E[Z]$	Expected length of a successive renewal cycle.

In addition to the defined notations, the following assumptions are assumed valid throughout the development of the maintenance model:

1. The time required to execute an unplanned replacement of a component is considered to be greater than that of a planned replacement. This assumption seems to be valid based on the logic that a planned replacement will be executed in a more efficient manner, considering that planning of resources and spare part availability has been done prior to the task taking place.

2. The cost per unit of time for unplanned downtime is greater than that of planned downtime. This assumption seems to be valid based on the logic that during unplanned downtime, additional costs are incurred in the forms of un-utilised utilities (for example water, electricity, and steam); and un-utilised labour (in the case of wages being paid for production personnel, which does not translate into actual product being produced).

3. The cost to execute both CM and PM at each occurrence (excluding replacement) are considered to be constant for each component throughout the component's lifetime, and is based on the average cost of CM and PM for the component over its lifetime (using historical data for the particular component).

4. The time to execute CM and PM on a component at each occurrence is considered to be constant for each component throughout the component's lifetime. The time to execute CM is based on the average MTTR of a component over its lifetime (using historical data for the particular component); whereas the time to execute PM is based on the average PM time for a component over its lifetime (using historical data for the component).

5. The *minor* failure rate function becomes $a_n\lambda^n_{op,min}(t)$ for component n in the *op*-th operational period just after executing planned PM on the component, where a_n is the improvement factor in the failure rate after the planned PM task. In the component's first operational period, which is, when the component is in a "good as new" state, the *minor* failure rate simplifies to $\lambda^n_{op,min}(t)$.

6. The component has a *minor* failure rate function $\lambda^n_{op,min}(t) = A_{(n,op)}\lambda^n(t)$ in the *op*-th operational period, where $A_{(n,op)} = (a_n)^{(op-1)}$ and $t \in (0, T_m)$ is the time since the last maintenance.

7. PM tasks exclusively translate into the improvement factor methodology being applied to *minor* failure rates. The occurrence of a *major* failure is determined solely by a normal probability distribution, whereby the MTBF of *major* failures determines the distribution's mean value.

Considering the RCM scheduling approach in Section 4.2, the total cost over a system cycle can be defined by (4.3.1):

$$C(t) = C_{mr} + C_{pr} + C_{pR} + C^{ud} + C^{pd} \tag{4.3.1}$$

According to the *renewal theory* and assuming infinite horizon span, the expected cost per unit of time is given by Barlow and Proschan (1996, 55) and Ross (2013, 52) as:

$$C(T) = \lim_{t\to\infty}\frac{C(t)}{t} = \frac{\textit{Expected cost on one cycle}}{\textit{Expected length of a cycle}} \tag{4.3.2}$$

The planned replacement of the system (single- or multi-component), discussed in Section 4.2, ultimately results in a system "renewal" — considered to depict a system *cycle*. Each of the events described in Section 4.2 incurs an inherent cost, leading to the derivation of an expected cost for the system cycle. Based on the renewal theory (Smith 1958, 244; Barlow and Proschan 1964, 578), and considering an infinite horizon span, it is then possible to determine the expected cost per unit of time for the proposed maintenance model.

4.3.1 Formulation of the Single-component Cost Model

The scheduling of the single component system, illustrated in Figure 4.2, is used to determine the expected cycle cost of the component. As described in Section 4.2.1, the component is replaced either when it has successfully completed its x-th operational interval, or upon a *major* failure. The total expected number of unforeseen *minor* failures between time t_1 and t_2 is denoted by (4.3.3):

$$E[M_{op}(t)] = \int_{t_1}^{t_2} \lambda_{(op,min)}(t)\mathrm{d}t \tag{4.3.3}$$

where $\lambda_{(op,min)}(t)$ denotes the *minor* failure rate of the component in its op-th operational period. Considering the improvement factor (A_{op}), (4.3.3) can be re-written as (4.3.4) for any op-th operational period:

$$E[M_{op}(t)] = a^{op-1} \int_{t_1}^{t_2} \lambda_{min}(t)\mathrm{d}t \tag{4.3.4}$$

Considering the cost incurred due to unplanned CM, planned PM, unplanned SRP, and planned SRP tasks on the component, the cost for each task is defined by (4.3.5), (4.3.6), (4.3.7), and (4.3.8), respectively. The cost for each task consists of (a) the cost of spare part(s) to conduct the task; and (b) the incurred cost based on the task duration multiplied by the cost per unit of time (either planned or unplanned).

$$c^c = \bar{c}^c + (\omega^c \times c^{ud}) \tag{4.3.5}$$

$$c^p = \bar{c}^p + (\omega^p \times c^{pd}) \tag{4.3.6}$$

$$c^{uR} = \bar{c}^{pR} + (\omega^{uR} \times c^{ud}) \tag{4.3.7}$$

$$c^{pR} = \bar{c}^{pR} + (\omega^{pR} \times c^{pd}) \tag{4.3.8}$$

The cost incurred in the component cycle is denoted by (4.3.9).

$$
\begin{aligned}
C = \sum_{op=1}^{x-1} \Bigg\{ & I_{[\sigma_{op},\infty)}(Y_{op}) \Bigg[\Big[c^{pr} + \sum_{j=1}^{M_{op}(\sigma_{op})} c^c S_{(op,j)}\Big] \wedge \big[\sigma_{op} = T^\star\big] \wedge \big[P \leftarrow P+1\big] \Bigg] \\
& + I_{[0,\sigma_{op})}(Y_{op}) \Bigg[\Big[c^{uR} + \sum_{j=1}^{M_{op}(Y_{op})} c^c S_{(op,j)}\Big] \wedge \big[op = 1\big] \wedge \big[\sigma_{op} = T^\star - Y_{op} - \omega^{uR}\big] \Bigg] \Bigg\} \\
& + \Bigg\{ I_{[T^\star,\infty)}(Y_x) \Bigg[c^{pR} + \sum_{j=1}^{M_x(T^\star)} c^c S_{(x,j)} \Bigg] \\
& + I_{[0,T^\star)}(Y_x) \Bigg[c^{uR} + \sum_{j=1}^{M_x(Y_x)} c^c S_{(x,j)} + c^{pR} + \sum_{j=1}^{M_1(T^\star - Y_x - \omega^{uR})} c^c S_{(1,j)} \Bigg] \Bigg\}
\end{aligned}
\tag{4.3.9}
$$

where $I_K(Y)$ is the indicator function of the set K, which is:

$$
I_K(Y) = \begin{cases} 1, & \text{if } Y \in K \\ 0, & \text{otherwise.} \end{cases}
$$

The indicator function essentially translates into the probability that a *major* failure will occur within the designated time period (K), where the probability is expressed using the CDF of the component, where the CDF between time t_1 and t_2 is denoted by (4.3.10):

$$
F_{(op,maj)}(t) = \int_{t_1}^{t_2} f_{(op,maj)} \mathrm{d}t \tag{4.3.10}
$$

The probability of survival, which is, the probability of the component's expected lifetime successfully meeting or exceeding the time to the next planned PM, is expressed using the survival function of the component, denoted by (4.3.11).

$$
\bar{F}_{(op,maj)}(t) = 1 - F_{(op,maj)}(t) \tag{4.3.11}
$$

(4.3.9) can be considered a two-part equation: where the first part equates the expected cost for the component to reach x operational periods; and the second part equates the expected cost in the final operational period(s) leading into the planned SRP for the component.

The first part of (4.3.9) initiates the cost summation process, starting with the first operational period ($op = 1$), and continues to accumulate the costs until the ($x-1$)-th operational period has been reached (inclusive of the ($x-1$)-th operational period). For each operational period of the component, the expected lifetime of the component Y_{op} determines (using the indicator function I_K) the cost to be incurred for the relative operational period. There exists two possible outcomes for each Y_{op}:

Successful operational period: the component's expected lifetime will meet or exceed the time to next planned PM interval, which is $Y_{op} \in [\, \sigma_{op}, \infty\,)$. The cost incurred in the operational interval then consists of the planned PM cost, denoted by c^p, as well as the expected CM cost as a result of *minor* failure(s) throughout the operational period, denoted by $\sum_{j=1}^{M_{op}(\sigma_{op})} c^c S_{(op,j)}$. It must be noted that the initial iteration begins with $\sigma_{op} = T$, where σ_{op} is then further defined by the expected lifetime of the component. A successful operational period results in defining $\sigma_{op} = T^\star$ and $P \leftarrow P + 1$ (P serves as a counting function of PM tasks, which is used in equating the expected cycle time of the component). $T^\star$ is defined as the time to the next planned PM interval during which the component will be in operation, which is, $T^\star = T - \omega^p$, seeing as the operational time will be equal to the time to the next planned PM interval minus the time required to conduct PM on the component. The summation process will then proceed with the following iteration, which is, the $(op + 1)$-th operational period.

Unsuccessful operational period: the component's expected lifetime will to meet or exceed the time to the next planned PM interval, which is $Y_{op} \in [\, 0, \sigma_{op}\,)$. The cost incurred in the operational interval then consists of the unplanned SRP cost, denoted by c^{uR}, as well as the expected CM cost as a result of *minor* failure(s) throughout the operational period leading into the unplanned SRP. An unsuccessful operational period results in defining $op = 1$ (thus "resetting" the component to the first operational period seeing as the component is now considered "good as new") and $\sigma_{op} = T^\star - Y_{op} - \omega^{uR}$ (which defines σ_{op} as the "shortfall" in time to the next planned PM interval). The summation process will then continue with $op = 1$.

The second part of (4.3.9) considers the cost incurred in the final operational period of the component, which is, in the x-th operational period, leading into the planned SRP of the component. Within the x-th operational period, the expected lifetime of the component Y_x, as above, has one of two outcomes:

Successful operational period: the component's expected lifetime will meet or exceed the time to the planned SRP interval, which is, $Y_x \in [\, T^\star, \infty\,)$. The cost incurred in the x-th operational period then consists of the planned SRP cost (denoted by c^{pR}) as well as the expected CM cost as a result of *minor* failure(s) in the x-th operational period. The successful completion of the x-th operational period signifies the end of the component's "cycle".

Unsuccessful operational period: the component's expected lifetime will fail to meet or exceed the time to the planned SRP, which is, $Y_x \in$

$[\,0, T^{\star}\,)$. The cost incurred in the x-th operational period then consists of the unplanned SRP cost (denoted by c^{uR}); the expected CM cost as a result of *minor* failures until the instance of the unplanned SRP; the cost of the planned SRP (denoted by c^{pR}); and the expected CM cost as a result of *minor* failures from the moment the unplanned SRP is completed until the planned SRP commences.

Using (4.3.4), (4.3.10), and (4.3.11), (4.3.9) can be re-written as (4.3.12):

$$
\begin{aligned}
E[C] = \sum_{op=1}^{x-1} \Bigg\{ & \bar{F}_{(op,maj)}(\sigma_{op}) \Bigg[\Big[c^{pr} + (c^{c})(a^{op-1}) \int_{0}^{\sigma_{op}} \lambda_{min}(\sigma_{op}) \mathrm{d}t \Big] \wedge \Big[\sigma_{op} = T^{\star} \Big] \\
& \wedge \Big[P \leftarrow P + 1 \Big] \Bigg] \\
& + F_{(op,maj)}(\sigma_{op}) \Bigg[\Big[c^{uR} + (c^{c})(a^{op-1}) \int_{0}^{Y_{op}} \lambda_{min}(Y_{op}) \mathrm{d}t \Big] \wedge \Big[op = 1 \Big] \\
& \wedge \Big[\sigma_{op} = T^{\star} - Y_{op} - \omega^{uR} \Big] \Bigg] \Bigg\} \\
& + \Bigg\{ \bar{F}_{(op,maj)}(T^{\star}) \Bigg[c^{pR} + (c^{c})(a^{op-1}) \int_{0}^{T^{\star}} \lambda_{min}(T^{\star}) \mathrm{d}t \Bigg] \\
& + F_{(op,maj)}(T^{\star}) \Bigg[c^{uR} + (c^{c})(a^{op-1}) \int_{0}^{Y_{x}} \lambda_{min}(Y_{x}) \mathrm{d}t + c^{pR} \\
& + c^{c} \int_{0}^{T^{\star} - Y_{x} - \omega^{uR}} \lambda_{min}(T^{\star} - Y_{x} - \omega^{uR}) \mathrm{d}t \Bigg] \Bigg\}
\end{aligned}
\tag{4.3.12}
$$

Based on (4.3.2) the expected cost $E[R]$ can be defined by (4.3.9), hence only the expected length of a cycle $E[Z]$ needs to be defined in order to define the expected cost per unit of time for the component. By using the value P, obtained from (4.3.9), it is possible to determine the total number of PM intervals executed on the component. Seeing as each PM interval is separated by fixed time intervals T, the total cycle time for the component is defined in (4.3.13):

$$
E[Z] = (P \times T) + T + \omega^{pR} \tag{4.3.13}
$$

In (4.3.13), the total number of PM intervals is multiplied by the time T between successive PM, adding the interval T until the final PM, as well as the time to execute the planned SRP (ω^{pR}) to determine the total expected length of the component cycle.

4.3.2 Formulation of the Multi-Component Cost Model

The scheduling of the multi-component system, illustrated in Figure 4.3, is used to determine the expected cost of the multi-component system. The total expected number of unforeseen *minor* failures between time t_1 and t_2 is denoted by (4.3.14):

$$E[M^n_{op^n}(t)] = \int_{t_1}^{t_2} \lambda^n_{(op^n,min)}(t)\mathrm{d}t \tag{4.3.14}$$

where $\lambda^n_{(op,min)}(t)$ denotes the *minor* failure rate of component n in the op^n-th operational interval. Considering the improvement factor (A^n_{op}), (4.3.14) can be re-written as (4.3.15) for component n during any op-th operational period:

$$E[M^n_{op^n}(t)] = a_n^{op-1} \int_{t_1}^{t_2} \lambda^n_{min}(t)\mathrm{d}t \tag{4.3.15}$$

Considering the cost incurred due to unplanned CM, planned PM, unplanned SRP, and planned SRP tasks on the components, the cost for each task is defined by (4.3.16), (4.3.17), (4.3.18), and (4.3.19) for component n, respectively. The cost for each task consists of (a) the cost of spare part(s) to conduct the task; and (b) the incurred cost based on the task duration multiplied by the cost per unit of time (either planned or unplanned).

$$c^c_n = \bar{c}^c_n + (\omega^c_n \times c^{ud}) \tag{4.3.16}$$

$$c^p_n = \bar{c}^p_n + (\omega^p_n \times c^{pd}) \tag{4.3.17}$$

$$c^{uR}_n = \bar{c}^{pR}_n + (\omega^{uR}_n \times c^{ud}) \tag{4.3.18}$$

$$c^{pR}_n = \bar{c}^{pR}_n + (\omega^{pR}_n \times c^{pd}) \tag{4.3.19}$$

The expected cost for the multi-component cycle is derived by analysing components' expected lifetimes between each consecutive planned PM interval. The model can be considered a two-part approach, where the first part initiates a summation counter (m) beginning with the first operational period (leading into the first planned PM interval), and continues until the *total system operational intervals* (X_{TOT}) has been reached — essentially accounting for all costs incurred up until the final operational period leading into the planned system SRP. Between each consecutive PM interval (which is, during operational periods) each component is individually analysed in terms of the component's expected lifetime $(Y^n_{op^n})$. Considering the expected lifetime of the component in consideration, the component will either (a) meet or exceed the time to the next planned PM interval; or (b) fail to meet the time to the next planned PM interval.

If the expected lifetime of the component in consideration meets or exceeds the time to the next planned PM interval (denoted by the indicator function $I_{[T^{k\star}+T^{\star}_{m},\infty)}(Y^{k}_{op^k})$), only the CM cost (due to unforeseen *minor* failure(s)) is considered (denoted by $\sum_{j=1}^{M^k_{op^k}(T^{\star}_{m})-M^k_{op^k}(T^{k\star})} c^c_k S^k_{(op^k,j)}$, where $T^{\star}_{m}$ is equal to the operational period). A successful completion of the operational period leading into the next planned PM interval considers the component to remain in its current operational period (denoted by $op^k = op^k$), seeing as no PM has yet been executed on the component. In order to determine whether the component in consideration is due for its planned PM task, the indicator function $I_{[1]}(\frac{T^k}{mT})$ is used, which will result in (a) the component proceeding to its following operational period (denoted by $op^k = op^k + 1$); (b) a positive indication of planned PM time for the component at the m-th PM interval (denoted by $\omega^p_{k,m} = \omega^p_k$); and (c) a negative indication of planned PM time for the component at the m-th PM interval (denoted by $I_{(\infty,1)}(\frac{T^k}{mT})[\omega^p_{k,m} = 0]$).

In the event that the expected lifetime of the component fails to meet the time to the next planned PM interval (denoted by the indicator function $I_{[0,T^{k\star}+T^{\star}_{m})}(Y^{k}_{op^k})$), the expected cost will consist of three inherent cost contributors, as well as a fourth potential cost contributor: (1) the cost of an unplanned replacement of the component (denoted by c^{uR}_k); (2) the CM cost (due to unforeseen *minor* failure(s)) of the component leading into the unforeseen *major* failure (denoted by $\sum_{j=1}^{M^k_{op^k}(Y^k_{op^k})-M^k_{op^k}(T^{k\star})} c^c_k S^k_{op^k,j}$); (3) the CM cost (due to unforeseen *minor* failure(s)) of the component from the moment unplanned replacement is completed on the component until the next planned PM interval (denoted by $\sum_{j=1}^{M^k_1(T^{\star}_m - Y^k_{op^k} + T^{k\star} - \omega^{uR}_k)} c^c_k S^k_{1,j}$); as well as the fourth potential cost contributor (4) which considers whether the component's planned PM task is due (denoted by $I_{[1]}(\frac{T^k}{mT}))[c^p_k]$). As with the successful completion of the operational period, if the component's planned PM task is due, the positive indicator of PM time for the component is defined as $\omega^p_{k,m} = \omega^p_k$ (where the negative indicator will define $\omega^p_{k,m} = 0$). In the event that PM is executed on the component, the component's operational period is considered to continue to the following operational period (denoted by $[op^n \leftarrow op^n + 1]$), where the negative indicator of PM task execution will result in the component continuing with its current operational period. It is thus assumed that, in the case of an unforeseen *major* failure, the component will successfully continue to operate (no further unforeseen *major* failures) until the component's next planned PM interval.

Once the iterative process for all n components in the m-th operational period is complete, the following three properties are defined: (1) the time to complete PM after the m-th operational period (denoted by $\omega^{p\star}_m = \max(\omega^p_{k,m}, \omega^{pR}_{k,m})$); (2) the operational time for the succeeding operational period (denoted by $T^{\star}_{m+1} = T - \omega^{p\star}_m$); and (3) the total system operational periods (denoted by

$X = \sum_{i=1}^{n} op^i$). The initial summation iteration ($\sum_{m=1}^{X=X_{TOT}}$) will continue for each consecutive operational period until the total system operational periods has reached the *total system operational intervals* threshold (X_{TOT}).

The first part of the incurred cycle cost for the multi-component system is defined by (4.3.20):

$$
\begin{aligned}
R_1 = \sum_{m=1}^{X=X_{TOT}} \Bigg\{ \sum_{k=1}^{n} \Bigg\{ \Bigg\{ & I_{[T^{k\star}+T^\star_m,\infty)}(Y^k_{op^k}) \Bigg[\Bigg[\sum_{j=1}^{M^k_{op^k}(T^\star_m)-M^k_{op^k}(T^{k\star})} c^c_k S^k_{(op^k,j)} \Bigg] \\
& \wedge \Big[T^{k\star} = T^{k\star} + T^\star_m \Big] \\
& \wedge [op^k \leftarrow op^k] \\
& + I_{[1]}\left(\frac{T^k}{mT}\right)[c^p_k] \wedge [op^k = op^k + 1] \wedge [\omega^p_{k,m} = \omega^p_k] \wedge \Big[I_{(-\infty,1)}\left(\frac{T^k}{mT}\right)[\omega^p_{k,m} = 0] \Big] \\
& + I_{[0,\infty)}(op^k - x_k)[c^{pR}_k] \wedge [op^k = 1] \wedge [\omega^{pR}_{k,m} = \omega^{pR}_k] \\
& \wedge \Big[I_{(-\infty,0)}(op^k - x_k)[\omega^{pR}_{k,m} = 0] \Big] \wedge [T^{k\star} = 0] \Bigg] \\
& + I_{[0,T^{k\star}+T^\star_m)}(Y^k_{op^k}) \Bigg[\Bigg[c^{uR}_k + \sum_{j=1}^{M^k_{op^k}(Y^k_{op^k})-M^k_{op^k}(T^{k\star})} c^c_k S^k_{(op^k,j)} \\
& + \sum_{j=1}^{M^k_1(T^\star_m - Y^k_{op^k} + T^{k\star} - \omega^{uR}_k)} c^c_k S^k_{(1,j)} \Bigg] \\
& \wedge [\omega^{pR}_{k,m} = 0] \wedge [T^{k\star} = T^\star_m - Y^k_{op^k} + T^{k\star}] \wedge [op^k = 1] \\
& + I_{[1]}\left(\frac{T^k}{mT}\right)[c^p_k] \wedge [op^k = 2] \wedge [\omega^p_{k,m} = \omega^p_k] \wedge \Big[I_{(-\infty,1)}\left(\frac{T^k}{mT}\right)[\omega^p_{k,m} = 0] \Big] \Bigg] \Bigg\} \\
& \wedge [\omega^{p\star}_m = \max(\omega^p_{k,m}, \omega^{pR}_{k,m})] \wedge [T^\star_{m+1} = T - \omega^{p\star}_m] \wedge [X = \sum_{i=1}^{n} op^i] \Bigg\} \Bigg\} \\
& + (\omega^{pR\star}_f \times c^{pd})
\end{aligned}
\tag{4.3.20}
$$

Once the iterative process is completed, which is, once the *total system operational intervals* threshold (X_{TOT}) is reached, the multi-component system commences with its final operational period before a planned system SRP is executed, which initiates the second part of the expected cost for the multi-component system equation. Similar to the logic behind the derivation of (4.3.20), each component is individually analysed in terms of the component's expected lifetime (Y^n_{op}) for the final operational period leading into the planned system SRP interval. Considering the expected lifetime of the component in consideration, the component will either (a) meet or exceed the time to the planned SRP interval; or (b) fail to meet the time to the planned SRP interval.

If the expected lifetime of the component in consideration meets or exceeds the time to planned system SRP interval (denoted by the indicator function $I_{[T^{k\star}+T_m^{\star},\infty)}(Y_{op^k}^k)$), two cost contributors are incurred: (1) the cost of CM (due to unforeseen *minor* failure(s)) of the component up until the execution of the planned SRP begins (denoted by $\sum_{j=1}^{M_{op^k}^k(T_m^{\star})-M_{op^k}^k}(T^{k\star})c_k^c S_{(op^k,j)}^k)$); and (2) the cost of a planned component SRP (denoted by c_k^{pR}).

In the event that the expected lifetime of the component in consideration fails to meet the time to the planned system SRP interval (denoted by the indicator function $I_{[0,T^{k\star}+T_m^{\star})}(Y_{op}^k)$), four cost contributors are incurred: (1) the cost of an unplanned component SRP (denoted by c_k^{uR}); (2) the cost of CM| (due to unforeseen *minor* failure(s)) up until the occurrence of the unplanned component SRP (denoted by $\sum_{j=1}^{M_{op^k}^k(Y_{op^k}^k)-M_{op^k}^k}(T^{k\star})c_k^c S_{(op^k,j)}^k$); (3) the cost of the planned component SRP (denoted by c_k^{pR}); and (4) the cost of CM (due to unforeseen *minor* failure(s)) from the instant that the unplanned component SRP is completed until the planned component SRP initiates (denoted by $\sum_{j=1}^{M_{op^k}^k(T_m^{\star}-Y_{op^k}^k-\omega_k^{uR})} c_k^c S_{(1,j)}^k$). Upon completion of the final operational period (leading into the planned system SRP), the time to complete the system SRP is defined as $\omega_f^{pR} = \max(\omega_k^{pR})$, essentially defining the duration of the system SRP as the maximum time required to execute a planned component SRP (for all n components).

The second part of the incurred cycle cost for the multi-component system is defined by (4.3.21):

$$\begin{aligned} R_2 = \sum_{k=1}^{n} \Bigg\{ & I_{[T^{k\star}+T_m^{\star},\infty)}(Y_{op^k}^k) \left[\sum_{j=1}^{M_{op^k}^k(T_m^{\star})-M_{op^k}^k(T^{k\star})} c_k^c S_{(op^k,j)}^k + c_k^{pR} \right] \\ & + I_{[0,T^{k\star}+T_m^{\star})}(Y_{op^k}^k) \Bigg[c_k^{uR} + \sum_{j=1}^{M_{op^k}^k(Y_{op^k}^k)-M_{op^k}^k(T^{k\star})} c_k^c S_{(op^k,j)}^k + c_k^{pR} \\ & + \sum_{j=1}^{M_1^k(T_m^{\star}-Y_{op^k}^k+T^{k\star}-\omega_k^{uR})} c_k^c S_{(1,j)}^k \Bigg] \Bigg\} + (\omega_f^{pR\star} \times c^{pd}) \end{aligned} \tag{4.3.21}$$

As with the single component model, the probability of component n's expected lifetime meeting or exceeding the time to the next planned PM interval can be obtained using the component's CDF, where the CDF of component n between time t_1 and t_2 is denoted by (4.3.22):

$$F_{(op^n,maj)}^n(t) = \int_{t_1}^{t_2} f_{(op^n,maj)}^n \mathrm{d}t \tag{4.3.22}$$

The probability of survival, which is, the probability of component n's expected lifetime successfully meeting or exceeding the time to the next planned PM, is expressed using the survival function of the component, denoted by (4.3.23):

$$\bar{F}^{n}_{(op^n,maj)}(t) = 1 - F^{n}_{(op^n,maj)}(t) \tag{4.3.23}$$

Using (4.3.15), (4.3.22), and (4.3.23), (4.3.20) and (4.3.21) can be rewritten as (4.3.24) and (4.3.25), respectively:

$$\begin{aligned}
E[R_1] = \sum_{m=1}^{X=X_{TOT}} \Bigg\{ \sum_{k=1}^{n} \Bigg\{ \Bigg\{ & \bar{F}^{k}_{(op^k,maj)}(T^\star_m) \Bigg[\Bigg[(c^c_k)(a^{op-1}_k) \int_{T^{k\star}}^{T^{k\star}+T^\star_m} \lambda^k_{min}(t)\mathrm{d}t \Bigg] \\
& \wedge [op^k \leftarrow op^k] \\
& + I_{[1]}\left(\frac{T^k}{mT}\right)[c^p_k] \wedge [op^k = op^k + 1] \wedge [\omega^p_{k,m} = \omega^p_k] \wedge \Big[I_{(-\infty,1)}\left(\frac{T^k}{mT}\right)[\omega^p_{k,m} = 0]\Big] \\
& + I_{[0,\infty)}(op^k - x_k)[c^{pR}_k] \wedge [op^k = 1] \wedge [\omega^{pR}_{k,m} = \omega^{pR}_k] \wedge \Big[I_{(-\infty,0)}(op^k - x_k)[\omega^{pR}_{k,m} = 0]\Big] \Bigg] \\
& + F^{k}_{(op^k,maj)}(T^\star_m) \Bigg[\Bigg[c^{uR}_k + (c^c_k)(a^{op-1}_k) \int_{T^{k\star}}^{Y^k_{op^k}} \lambda^k_{min}(t)\mathrm{d}t \\
& + (c^c_k) \int_{0}^{T^\star_m - Y^k_{op^k} + T^{k\star} - \omega^{uR}_k} \lambda^k_{min}(t)\mathrm{d}t \Bigg] \\
& \wedge [\omega^{pR}_{k,m} = 0] \wedge [op^k = 1] \\
& + I_{[1]}\left(\frac{T^k}{mT}\right)[c^p_k] \wedge [op^k = 2] \wedge [\omega^p_{k,m} = \omega^p_k] \wedge \Big[I_{(-\infty,1)}\left(\frac{T^k}{mT}\right)[\omega^p_{k,m} = 0]\Big] \Bigg] \Bigg\} \\
& \wedge [\omega^{p\star}_m = \max(\omega^p_{k,m}, \omega^{pR}_{k,m})] \wedge [T^\star_{m+1} = T - \omega^{p\star}_m] \wedge [X = \sum_{i=1}^{n} op^i] \Bigg\} \Bigg\} \\
& + (\omega^{p\star}_m \times c^{pd})
\end{aligned} \tag{4.3.24}$$

$$\begin{aligned}
E[R_2] = \sum_{k=1}^{n} \Bigg\{ & \bar{F}^{k}_{(op^k,maj)}(T^\star_m) \Bigg[(c^c_k)(a^{op-1}_k) \int_{T^{k\star}}^{T^{k\star}+T^\star_m} \lambda^k_{min}(t)\mathrm{d}t + c^{pR}_k \Bigg] \\
& + F^{k}_{(op^k,maj)}(T^\star_m) \Bigg[c^{uR}_k + (c^c_k)(a^{op-1}_k) \int_{T^{k\star}}^{Y^k_{op}} \lambda^k_{min}(t)\mathrm{d}t + c^{pR}_k \\
& + c^c_k \int_{0}^{T^\star_m - Y^k_{op^k} + T^{k\star} - \omega^{uR}_k} \lambda^k_{min}(t)\mathrm{d}t \Bigg] \Bigg\} \\
& + (\omega^{pR\star}_f \times c^{pd})
\end{aligned} \tag{4.3.25}$$

The total expected cost per multi-component system cycle can therefore be defined as the sum of (4.3.24) and (4.3.25):

$$E[R] = E[R_1] + E[R_2] \tag{4.3.26}$$

Based on (4.3.2) the expected cost $E[R]$ can be defined by (4.3.26), hence only the expected length of a cycle $E[Z]$ needs to be defined in order to define the expected cost per unit of time for the multi-component system. By using the value m, obtained from (4.3.24), it is possible to determine the total number of PM intervals executed on the system. Seeing as each PM interval is separated by fixed time intervals T, the total cycle time for the component is defined in (4.3.27):

$$E[Z] = (m \times T) + T + \omega_f^{pR\star} \tag{4.3.27}$$

In (4.3.27), the total number of PM intervals (m) is multiplied by the time T between successive PM intervals, adding the interval T until the total system SRP, as well as the time to execute the planned SRP ($\omega_f^{pR\star}$) to determine the total expected length of the component cycle.

4.4 Definition of Model Parameters

Sections 4.2 and 4.3 provide a comprehensively detailed maintenance scheduling approach for both single- and multi-component systems, yielding the proposed cost models defined by (4.3.12) and (4.3.26), for single- and multi-component systems, respectively. Within both cost models there exist component parameters that need to be defined which, as described in Section 4.1.5, will depend on the particular component's historical data. Referring to (4.3.12) and (4.3.26), this section aims at providing a structured approach in order to define the component-specific parameters that will be needed to commence with optimisation of the aforementioned cost models.

4.4.1 Fixed Component Costs

Considering the component(s) that intend to be analysed, it is assumed that each component (whether part of the single- or multi-component system) will exhibit certain fixed costs for particular tasks that are undertaken throughout the component's defined cycle. The component fixed costs that are defined in (4.3.12) and (4.3.26), are defined as follows:

$\bar{c}_n^{pr}$: This is the fixed cost that is incurred due to the execution of planned PM on the component (simplifying to $\bar{c}^{pr}$ for the single component) in the form of spare parts needed to execute the planned PM. In order to avoid unnecessary complication, it is assumed that the required fixed cost to conduct PM on a component is equal to the average $\bar{c}_n^{pr}$ cost incurred over a component's lifetime. The cost is obtained by analysing PM cost historical data of the component, and thus determining the average incurred cost at each PM task interval.

ω_n^{pr}: This is the fixed amount of time required to execute the PM task on the component (simplifying to ω^{pr} for the single component). As with the fixed PM spare part cost, the fixed amount of time required to execute PM on the component is assumed to be a constant, and is obtained by analysing PM time historical data of the component, and thus determining the average time required to execute PM on the component at each planned PM task interval.

$\bar{c}_n^c$: Similar to the PM spare part cost, the cost incurred to execute a CM task on the component, in the form of spare part cost, is denoted by $\bar{c}_n^c$ (simplifying to $\bar{c}^c$ for the single component). The cost is obtained by analysing CM cost historical data for the component, and determining the average cost incurred at each CM task for the component.

ω_n^c: Each CM task conducted on the component will require a certain amount of time, denoted by ω_n^c (simplifying to ω^c for the single component). The time required per CM task on the component is obtained by analysing historical CM time data, and determining the average time required for each CM task on the component. A common measure that can be used to define ω_n^c is the MTTR of the component, which essentially refers to the average time required to repair the component upon unforeseen *minor* failures.

$\bar{c}_n^{pR}$: The cost incurred due to a planned replacement (SRP) of the component is denoted by $\bar{c}_n^{pR}$ (simplifying to $\bar{c}_n^{pR}$), specifically referring to the cost of the component, seeing as the original component is considered to be discarded and replaced by a new component.

ω_n^{pR}: The time required to conduct a planned replacement (SRP) on the component is denoted by ω_n^{pr} (simplifying to ω^{pR} for the single component). Historical time data of the component is used to determine the average time required to preventively replace the component.

$\bar{c}_n^{uR}$: It is assumed that the cost of the component will be identical to the cost $\bar{c}_n^{pR}$, seeing as the cost of the component will not be altered by the fact that the component was replaced during a planned or unplanned action.

ω_n^{uR}: The time required to conduct an unplanned replacement (SRP) of the component is similar to that of the planned replacement time, and is denoted by ω_n^{uR} (simplifying to ω^{uR} for the single component). However, the difference arises in the assumption that the time required to conduct an unplanned SRP will be greater than the time required to conduct a planned SRP. The assumption therefore results in $\omega_n^{uR} > \omega_n^{pR}$ for all n. Historical time data of the component is analysed to determine the average time required to conduct an unplanned SRP of the component.

Based on the above descriptions and definitions, it is thus possible to obtain all fixed component costs required for proceeding optimisation of (4.3.12) and (4.3.26).

4.4.2 Fixed System Costs

(4.3.12) and (4.3.26) contain certain cost aspects that refer to the system as a whole, which is, costs that are not component-specific. These fixed system costs refer to the system in which the components are in operation, and are defined as follows:

c^{ud}**:** Any unforeseen failure, whether *minor* or *major*, results in the system (considered to be in series configuration) experiencing unplanned downtime. Two factors that potentially contribute to the cost of unplanned downtime of a system are considered: (1) utility costs (such as water, electricity, and steam, which are continuously incurred in the manufacturing facility under consideration); and (2) non-utilised labour costs (such as the cost of production personnel that, during unplanned downtime, do not contribute to actual production). The applicability of the two factors are dependent on the facility in consideration, where the cost per unit time for each of these factors must be obtained in order to define the facility-specific incurred cost per unit of time during any unplanned downtime period.

c^{pd}**:** The cost per unit of time during any planned downtime period, which is, during any planned PM interval, may differ from that of unplanned downtime. The reason for potential difference arising in the cost per unit time of planned versus unplanned downtime exists with the reasoning that, during planned downtime, certain costs may not necessarily be incurred by the facility. Considering the two factors potentially contributing to unplanned downtime costs: utility costs may either be avoided or reduced, seeing as machinery consuming the utility are completely shut down in order to perform planned PM on the system (certain utility costs may still be incurred during PM intervals, albeit to a lesser extent, thus resulting in a reduced incurred utility cost during the PM interval); and labour cost may follow the same reasoning, whereby the facility may plan for production-specific labour to intentionally not report to duty during the PM interval, thereby avoiding the incurred non-utilised labour cost during this period. The applicability of the difference between unplanned and planned downtime cost is therefore also dependent on the facility in consideration, where the cost per unit time for each of these factors must be obtained in order to define the facility-specific incurred cost per unit of time during any planned PM interval.

The descriptions above provide fixed system costs, which essentially contribute to the incurred costs defined in (4.3.12) and (4.3.26).

4.4.3 Component Reliability Functions

Section 2.3.2.3 provides a detailed explanation, using the five steps proposed by Barabady (2005, 111) and Roy *et al.* (2001, 163), of how to effectively analyse data and determine reliability functions of a component or system. The five steps are elaborated on in this section with the intent of providing a clear understanding of how to obtain reliability functions of component(s) used in (4.3.12) and (4.3.26).

i. Understanding of the system: As described in Section 4.1.2, there must exist a thorough understanding of the system under consideration, seeing as non-consideration of dependencies between components may result in inaccurate reliability functions of the system. By clearly defining the system boundary (based on knowledge of the system in consideration) it is thus possible to decide whether to use the single- or multi-component system approach.

ii. Collection & sorting of MTBF and MTTR data: The MTBF and MTTR must be obtained in order to proceed with further reliability analysis. This is achieved by independently analysing each component's historical data in order to determine component-specific MTBF and MTTR values. Considering (4.3.12) and (4.3.26), there will exist different MTBF and MTTR values for the two defined failure modes, namely, *minor* failures and *major* failures.

iii. Determine the existence of IID data: Prior to the reliability analysis proceeding further, tests for trends and serial correlations must be done to check whether the data are IID or not (Kumar and Klefsjö, 1992, 217). The trend test, whereby the cumulative failure number is plotted against the cumulative time between failures, provides an effective mean to determine whether data are IID for a particular component. The positive existence of a trend, which is, non-IID data, translates into a constant failure rate and, hence, the use of a homogeneous Poisson process to define the component's failure probability functions. The negative existence of a trend, which is, IID data, translates into a non-stationary failure rate of the component and, hence, the use of a NHPP to define the component's failure probability functions.

iv. Fitting data for components with a theoretical probability distribution: Based on the outcome of the use of either a homogeneous Poisson process or a NHPP for the component, a best fit probability distribution is defined. As explained in Section 4.1.2, the most efficient and effective means of fitting relevant lifetime distributions to data sets is by utilising one of the vast number of software packages available (for example ReliaSoft) to automate the best fit distribution. Once the best fit distribution for the component is obtained, all associated functions (PDF, CDF, and failure functions) are completely determined (Cousineau *et al.*, 2004, 742).

v. Estimation of the reliability of the system as a whole: In order to determine the reliability of the system as an entirety, the reliability parameters of each component, as well as how these components interact with one another (series or parallel configuration) must be analysed. Based on the approach of the model developed in this study, this final step does not apply seeing as the model compensates for all component failures as a system failure (series configuration), where the system reliability parameters are not necessarily needed in order to optimise the proposed model.

By using the five step approach, it is thus possible, for each component, to determine all inherent reliability functions (failure rate, PDF, and CDF) that are used in (4.3.12) and (4.3.26).

4.4.4 Component Improvement Factors

As described in Section 4.2.1, a similar approach to Wang (2002, 469) is followed in order to define the degree to which a component's failure rate is improved following any planned PM task. The improvement factor a_n (simplifying to a for the single component) exists as a multiplication factor to the failure rate of a component which yields the resultant failure rate of the component in the operational period immediately following the planned PM task. Unlike the approach of Wang (2002, 469), whereby the improvement factor is dependent on the age of the component, the improvement factor of a component is considered to remain constant throughout the component's lifetime. As described in Section 4.2.1, the failure rate of the component after a PM task reduces to zero, and then increases more quickly than it did in the previous operational period. The original failure rate of a component, denoted as $\lambda_n(t)$, would then become $a_n\lambda_n(t)$ in the following operational period, where $a_n \geq 1$ is the improvement factor and $t \geq 0$ represents the time from the previous PM task. Considering that the component's improvement factor remains constant throughout its lifetime, the failure rate of the component at any given time may be defined as $\lambda_{(n,op)}(t) = a_n^{op-1}\lambda_n(t)$. It must be noted that the specific constant improvement factor for a particular component is considered to apply only to the *minor* failure rate. Based on the logic in the development of the maintenance cost models, the

The five step approach in Section 4.4.3 allows for the failure rates of components to be determined. In order to define the improvement factor of a component, historical data is analysed to determine the change in failure rate between successive PM tasks. It is expected that the change in failure rate between successive PM tasks will seldom be identical and, therefore, the average of the change in failure rates over several consecutive PM intervals will yield the anticipated improvement factor for a particular component. (4.4.1) provides an approach to determine the average improvement factor for a component:

$$a_n = \frac{\sum_{op=1}^{x} \left(\frac{\lambda_{op}^n}{\lambda_{(op-1)}^n} \right)}{x} \tag{4.4.1}$$

where n denotes the component in consideration, $op = 1, 2, \ldots, x$, and x is equal to a pre-defined limit over which the failure rates in successive operational periods are analysed. The larger the value of x, which is, the more operational periods that are considered, will yield a more accurate improvement factor for the component.

The improvement factor for the component, obtained form (4.4.1), allows for the determination of the improvement factor that is used in (4.3.12) and (4.3.26).

A typical example of the effect of the improvement factor on the failure rate of a component is depicted in Figure 4.4. In Figure 4.4, the initial failure rate of the component ($\lambda(t)$) becomes $a\lambda(t)$ in the succeeding (second) operational period, and further becomes $a^2\lambda(t)$ in the third operational period. As explained by Wang (2002, 469), following a PM task, the failure rate of the component reduces to zero in the succeeding operational period, and then increase more rapidly (by factor a) in the following operational period.

Based on (4.3.12) and (4.3.26), there exist two failure categories, namely, *minor* and *major* failures. It is thus possible to analyse the failure rates for both *minor* and *major* failures, following the aforementioned approach, in order to derive the improvement factors for the two specific failure categories. In order to avoid unnecessary complexity, it is further assumed in the proposed model in this study that the improvement factor on the *minor* failure rate is also applicable to the *major* failure rate, which is, the both *minor* and *major* failure rates experience the same improvement factor following any given PM task for a particular component.

4.4.5 Component PM Intervals

Planned PM intervals occur at fixed time intervals, denoted by T_m, where each consecutive PM interval is separated by time period T, which is fixed (see

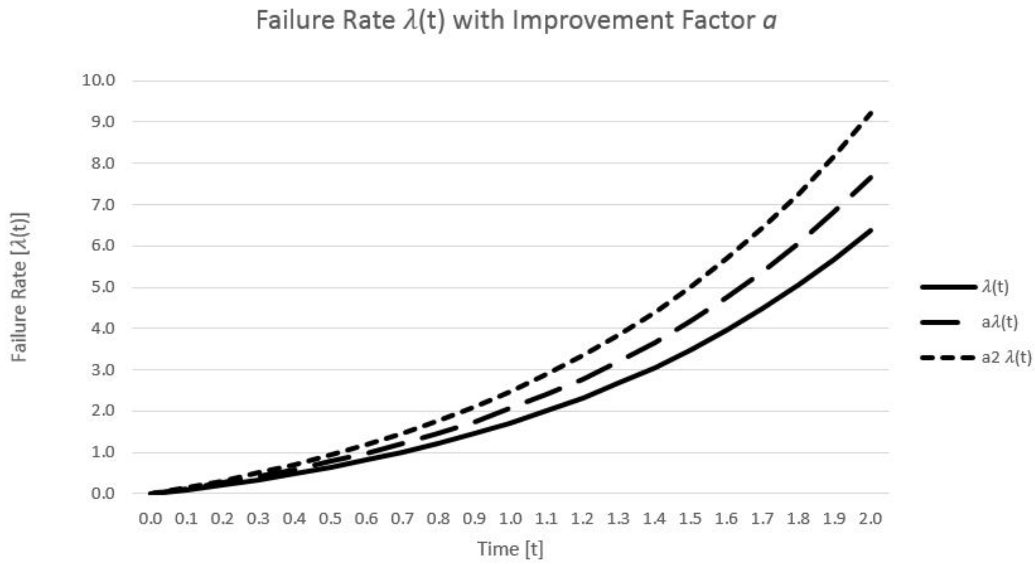


Figure 4.4: An example of the effect of the improvement factor on failure rate.

Section 4.2.1 for detail). The time period T must be chosen so as to ensure that the component with the most frequent *major* failure rate, or least MTBF, is maintained at each consecutive PM interval, thus avoiding frequent unplanned SRP's of the component. In the case of the single component system, the time T between successive PM intervals will simplify to a time period that allows for PM to be executed on the component at a higher frequency than that of its MTBF. In the case of the multi-component system, the component within the system that exhibits the most frequent MTBF will "set the pace" for the occurrence of T_m — where remaining components, exhibiting less frequent MTBF's, will undergo PM at integer factors of T.

The PM intervals for the particular component, denoted by T^n, whereby the component will undergo its planned PM task will therefore depend on the component's inherent MTBF property. A component n that, for example, exhibits a MTBF equal to half of the most frequent MTBF within the multi-component system, will undergo PM at time $2T$, which is, $T^n = 2T$. As with the single component, T^n will always be chosen to be more frequent than the MTBF of the particular component.

Defining the value of T and T^n for all components should typically involve some form of safety factor, with the reasoning that equating T to the exact value of the component's MTBF will involve a significant risk or probability that a *major* failure may occur before the occurrence of the planned PM for the component. The initial decision of "setting the pace" of PM intervals, denoted by T and based on the most frequent MTBF of all components within the system, is thus defined by the analyst at a certain safety factor of the most frequent MTBF. From here on forth, the value of T^n for the remaining components forms part of the optimisation process, which results in the optimised component-specific PM interval frequencies.

4.4.6 Component Operational Period Thresholds

As explained in Sections 4.3.1 and 4.3.2, a component is replaced either upon an unforeseen *major* failure, or once the component has completed a certain threshold of operational periods (denoted by x for the single component, and x_n for the multi-component system). The operational period threshold, which triggers the planned SRP of the component, is a decision variable that is ultimately determined in the optimisation process of the model.

In the case of the multi-component system an additional operational period threshold (X_{TOT}) triggers the final operational period for the system, whereby the system as an entirety will undergo a planned SRP in the next PM interval. The value of X_{TOT} is obtained within the optimisation process of the model.

Optimisation of the proposed cost model for the single- and multi-component systems will therefore determine the operational period thresholds for all components, signifying at which PM intervals the particular components should be preventively replaced in order incur the minimal possible cost.

4.4.7 Programming of the Proposed Cost Model

In order to mathematically model the proposed maintenance cost models for single- and multi-component systems, Matlab software is used. Ultimately, the aim of the modeling process is to model the defined equations (4.3.13) and (4.3.27) for single- and multi-component systems, respectively, in order to simulate the expected cost per unit of time for varying input arguments.

The mathematical model programming of the single- and multi-component systems is seen in Appendices A.1 and A.3, respectively. Each of the programmed models requires definition of fixed and variable parameters, as defined in 4.3, which are dependent on system-specific data and information. As described in Appendices A.1 and A.3, in addition to the fixed system-specific parameters, the developed Matlab models require variable parameters which are iteratively altered in order to simulate the output variable for varying input variable arguments. The variable parameters for the single- and multi-component models are:

Single-component model: Given the definition of the fixed parameters for the single-component system, there are two variable input parameters which are individually, iteratively altered in order to generate the resulting output variable, which is, cost per unit of time. According to the Matlab model in Appendix A.1, the two variable input parameters are (1) time interval between consecutive PM interval(s) (T); and (2) number of operational periods successfully completed by the system whereupon a planned SRP is executed thereafter. Based on the desired number of iterations, each iteration's parameters are stored in a matrix format, ultimately enabling the analyst to determine the specific value of variable input parameters that results in an optimally low cost per unit of time for the system.

Multi-component model: Given the definitions of the fixed parameters for the multi-component system, there are six variable input parameters which are individually, iteratively altered in order to generate the resulting output variable, which is, cost per unit of time. According to the Matlab model in Appendix A.3, the six variable input parameters are: the time interval between consecutive PM intervals for the three components (totalling three variable parameters); and the number of operational periods successfully completed by each of the three components whereupon a planned SRP is executed thereafter (totalling three variable parameters). Based on the desired number of iterations, each iteration's parameters are stored in a matrix format, ultimately enabling the analyst to determine the specific value of variable input parameters that results in an optimally low cost per unit of time for the system.

4.5 Monte Carlo Optimisation Approach

As described in Section 2.4.1 by Mahadevan (1997, 123) and Raychaudhuri (2008, 95), a Monte Carlo simulation is a numerical experimentation technique used to obtain expected output variables of a system computational model, given the statistics of the input variables. In each experiment, the value of the input random variables are sampled based on their distributions, and the output variables are calculated using the computational model. The computational model in the context of this study refers to the proposed cost model, defined by (4.3.12) and (4.3.26). A number of experiments are carried out in this manner, where the results are used to compute the expected value of the output variables, where the output variable in the context of this study is the expected cost per unit of time for the considered system.

Raychaudhuri (2008, 92) provides a conveniently summarised approach to the Monte Carlo simulation, described in four successive steps:

Static model generation: Every Monte Carlo simulation initiates with the development of a deterministic model which closely resembles the "real-life" scenario. In the context of this study, the deterministic model refers to the proposed cost model defined by (4.3.12) and (4.3.26). Initially, the most likely value of the input variables are used in the model. Using the specified value of the input variables and the mathematical relationships defined throughout the proposed cost model, the desired output variable, which in the context of this study refers to the cost per unit of time for the system in consideration, is obtained.

Input distribution identification: Each input variable's probability distribution can be uniquely identified by its parameter set, hence, "distribution fitting" is essentially the same as finding the parameters of a distribution that would generate the given data in question. Although there exists numerous methods to fit historical data to a particular distribution, the most efficient and effective means of fitting relevant lifetime distributions to data sets is by utilising one of the vast number of software packages available to automate the best fit distribution. The software utilised in this study is that of Reliasoft, which thus provides an effective and efficient means of fitting distributions to relevant data parameter sets.

Random variable generation: The identified inherent distributions of the components are used in order to generate a set of random numbers. One set of random numbers, consisting of one value for each of the input variables, is used within the proposed cost model to provide the output variable. This process is repeated by generating more sets of input variables and collecting the values of the output variable. This part forms the core of the Monte Carlo simulation approach.

Analysis and decision making: The set of values collected for the output variable are statistically analysed in order to provide a statistical confidence for the obtained output variable value. In the analysing of simulation output data, a distinction is made between terminating or transient simulations and steady-state simulations (Banks and Carson, 1996, 336). A *terminating* simulation is one that runs for some duration of time T_E, where E is a specified event that stops the simulation. Such a simulated system "opens" at time 0 under well-specified initial conditions and "closes" at the stopping time T_E. A *non-terminating* system is one that runs continuously, or over a very long period of time. Such a simulated system starts at simulation time 0 under initial conditions defined by the analyst and runs for some analyst-specified period of time T_E. Usually the analyst wants to study steady-state, or long-run properties of the system — that is, properties that are not influenced by the initial conditions of the model at time 0.

4.5.1 Measures of Output Data Performance and Estimation

Consider one run of a simulation model over a period of time $[0, T_E]$. Since the model is an input-output transformation, and since some of the model input variables are random variables, it follows that the model output variables are random variables (Banks and Carson, 1996, 338).

Consider the estimation of a performance parameter (θ) of a simulated system. It is desired to have a point estimate and an interval estimate of θ. The length of the interval estimate is a measure of the error in the point estimate. The simulation output data are of the form $Y_1, Y_2, \ldots, Y_n$ for estimating θ; we refer to such output data as *discrete-time data*, because the index n is discrete valued. The point estimator of θ based on the data $Y_1, \ldots, Y_n$ is defined by (4.5.1) (Banks and Carson, 1996, 338).

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} Y_i \tag{4.5.1}$$

where $\hat{\theta}$ is a sample mean based on a sample of size n. The point estimator $\hat{\theta}$ is said to be unbiased for θ if its expected value is θ — that is, if $E(\hat{\theta}) = \theta$. In general, however, $E(\hat{\theta}) \neq \theta$, and $E(\hat{\theta}) - \theta$ is called the *bias* in the point estimator θ.

The natural estimator for θ is the overall sample mean of R independent replications, $\bar{Y} = \sum_{i=1}^{R} \frac{\bar{Y}_i}{R}$, however, $\bar{Y}$ is not θ, it is an estimate, based on a sample, with an inherent error. A confidence interval is a measure of that error. Consider (4.5.2), which is the sample variance across the R replications:

$$S^2 = \frac{1}{R-1}\sum_{i=1}^{R}(Y_i - \bar{Y})^2 \tag{4.5.2}$$

The usual confidence interval, which assumes the Y_i are normally distributed, is shown in (4.5.3):

$$\bar{Y} \pm t_{\alpha/2,R-1}\frac{S}{\sqrt{R}} \tag{4.5.3}$$

where $t_{\alpha/2,R-1}$ is (also defined as H — the confidence interval) the quantile of the t distribution with $R-1$ degrees of freedom that cuts off $\alpha/2$ of the area of each tail. It is not known for certain how far $\bar{Y}$ is from θ, therefore the confidence interval attempts to bound that error. A confidence level, such as 95%, tells the analyst how trustworthy the error between $\bar{Y}$ and θ is bound.

Through conducting several simulations of the proposed maintenance cost model, it is thus possible to determine an estimate of the most likely outcome of the output variable ($\bar{Y}$), as well as the confidence interval ($t_{\alpha/2,R-1}\frac{S}{\sqrt{R}}$) for the output variable.

4.6 Summarised Maintenance Optimisation Approach

Sections 4.1.1 to 4.5 provide a firm understanding of the development and optimisation process to be utilised in order to determine maintenance tasks and frequencies which result in an optimally low bottom-line cost for the implemented maintenance approach on both single- and multi-component systems. As described in Section 1.4, one of the objectives of this study is to formulate a structured approach which may be followed in order to utilise the proposed cost models and, essentially, determine maintenance conditions which result in the optimal cost per unit of time. Based on the developed single- and multi-component models, a summarised structured approach is presented in Table 4.1, which formulates the maintenance optimisation process in five sequential steps.

The five-step approach is further used to conduct a validation study on the proposed cost models by performing a case study on current maintenance methodologies employed in a FMCG environment, as seen in Chapter 5.

4.7 Chapter Summary

This chapter has successfully summarised the RCM considerations to be taken into account in the development of the single- and multi-component maintenance cost models, with specific focus on task definitions and scheduling frequencies. The scheduling approach followed the "fixed interval" approach, whereby planned PM tasks are undertaken at set time intervals, where further consideration was made for inevitable unforeseen *minor* and *major* failures during the operational periods of the system. The definition of a "system cycle", together with sufficient coupled cost factors for each maintenance task (whether planned or unplanned) allowed for the development of the single- and multi-component maintenance cost models, based on the expected cost-per-unit-time of the system. In the case where a system comprises of a single component, which is, a component that does not consist of significant interdependencies with other components, the single-component maintenance cost model has been defined; whereas, in the case of the existence of interdependencies between components, the multi-component maintenance cost model was defined. The process of determining best-fit failure distributions was described, wherein the use of software, such as Reliasoft's Weibull++, together with system historical data should be used. The improvement factor, which was applied to both single- and multi-component maintenance models, compensates for the ageing effect of the system, whereby the system is considered to be in a condition between "as good as new" and "as bad as old". Further clarity was provided in defining each of the model's parameters, which are to be determined for the system under consideration. In order to proceed with

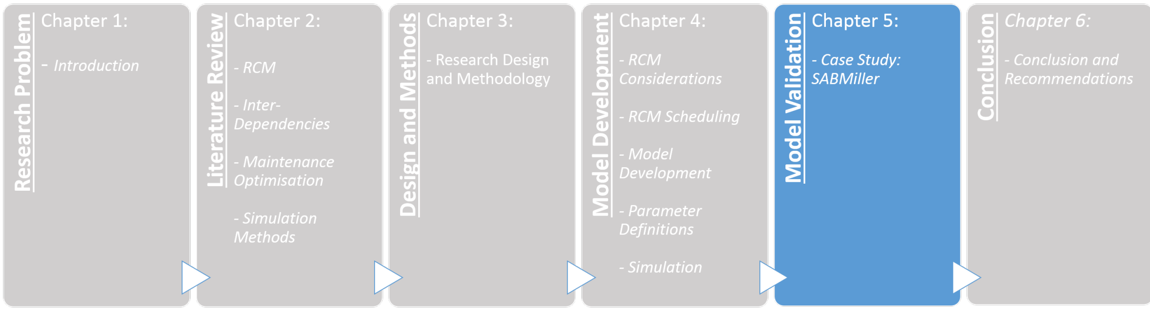
Structured Maintenance Optimisation Approach		
Step	Description	Determined Parameters
1	**Data analysis:** Analysis of historical data for the system under investigation comprises of collection of 3 data categories: (1) Maintenance data — analysis of maintenance records to determine values for PM and SRP frequencies and required times to execute each of these tasks; (2) Failure data — analysis of production downtime records to determine values for MTTR, number of failures, MTBF, and times of these occurrences; and (3) Cost data — analysis of cost records to determine costs for PM and SRP tasks, cost per unit time of unplanned downtime, and cost per unit time of planned downtime.	x_n; ω_n^p; ω_n^{pR}; ω_n^{uR}; T; ω_n^c; $f_{(op,maj)}^n(t)$; a_n; c_n^c; c_n^p; c_n^{uR}; c_n^{pR}; c^{pd}; and c^{ud}.
2	**Best-fit failure distributions:** Using the data from Step one, all *minor* failure occurrences of the component(s) are analysed to determine the best-fit failure rate distribution(s) for the component(s). Software, such as Reliasoft's Weibull++ software, may be utilised. In order to determine the relative accuracy of the proposed best-fit distribution, comparison of the actual number of failures are compared to the theoretical number of failures (determined from the proposed failure rate distribution(s)) within a specified time period.	$\lambda_{(op,min)}^n(t)$.
3	**Parameter definition:** Based on Steps one and two, it is possible to define all input parameters that are further used in the proposed single- and multi-component cost models.	—
4	**Optimisation:** Using the defined parameters from Step three, it is possible to simulate the output variable, which is, the cost per unit of time, using the single- and multi-component cost models. The process is done by iteratively altering the input variables T_n and x_n and ultimately determine the coupled cost per unit time for each of the iterations. In order to determine statistical confidence bounds, the Monte Carlo simulation process is utilised, where the accuracy of the confidence bound is proportional to number of simulations conducted, which is, the more simulations conducted will yield a more accurate depiction of the statistical confidence bound.	*Cost per unit of time.*
5	**Cost comparison:** The simulated optimisation results, obtained from Step four, are used to compare the cost per unit of time for the proposed optimal maintenance tasks and frequency values to the actual cost per unit of time of the system under current maintenance conditions. The comparison essentially determines the potential cost savings that could be obtained if the proposed maintenance tasks and frequencies are to be implemented.	*Potential cost savings.*

Table 4.1: Structured Maintenance Optimisation Approach

the task and frequency optimisation process, the Monte Carlo simulation approach was described in a step-wise methodology. The chapter concluded with the construction of the summarised, structured approach to be followed during the maintenance task and frequency optimisation process.

Chapter 5

Case Study – SABMiller



The case study conducted in this chapter aims to validate the proposed maintenance cost models developed for single- and multi-component systems in Chapter 4. The chapter aims to illustrate the applicability of the proposed cost model within a real-world example, together with the potential theoretical cost benefit to be expected if the proposed model were to be implemented.

5.1 Overview of SABMiller

The case study used here for validation purposes, was conducted in conjunction with South African Breweries (SAB) Limited. SAB Limited is a brewing and bottling company in South Africa, which is a wholly owned subsidiary of SABMiller PLC.

SAB was founded in the year 1895, when Castle Lager was launched from a newly commissioned lager brewery in Johannesburg, South Africa. Shortly thereafter, in the year 1897, SAB listed on the Johannesburg Stock Exchange (JSE) as the first industrial stock exchange. SAB is the dominant brewing company in South Africa, with a market share of approximately 90% (Crotty, A [Online], 2016).

The case study is specifically conducted at SAB's Rosslyn Brewery, located in Pretoria, South Africa. The production facility at Rosslyn Brewery consists of four main departments, namely, brewing, filtration, packaging, and warehousing. In particular, the case study focusses on Rosslyn Brewery's packaging department. The packaging department at Rosslyn Brewery consists of five independent packaging lines — three of the five packaging lines are dedicated returnable bottle packaging lines (categorised as lines one, four, and five); a dedicated non-returnable bottle packaging line (categorised as line two); and a dedicated aluminium can packaging line (categorised as line three).

5.2 Chapter Overview

In the sections that follow the case study is performed on data collected from SAB Rosslyn Brewery's packaging department, with particular focus on line four (returnable bottle packaging line). A brief background of the specific problem is introduced, subsequently leading to the approach and objective of the case study particulars.

The case study focusses on packaging equipment currently used on line four, whereby historical data is used to analyse equipments' failure patterns and maintenance systems. In order to apply the proposed maintenance cost model to current equipment on line four, the case study identifies both single- and multi-component systems, where the analysis steps presented in Table 5.1 are followed.

No.	Step	
	Single-component	**Multi-component**
1	Analysis of failure and maintenance data for the bottle-washer cam system	Analysis of failure and maintenance data for the *TB* conveyor system
2	MTBF calculation for the cam system	MTBF calculations for the chain, sprocket, and wear-strip components
3	Best-fit distribution determination for the failure rate of the bottle-washer cam system	Best-fit distributions determination for the failure rates of the chain, sprocket, and wear-strip components
4	Optimal parameter determination by means of Matlab single-component model execution and Monte Carlo simulation	Optimal parameter determination by means of Matlab multi-component model execution and Monte Carlo simulation
5	Actual versus theoretical cost per unit time comparison	Actual versus theoretical cost per unit time comparison
5	Summary of model validation for the single-component cost model	Summary of model validation for the multi-component cost model

Table 5.1: Analysis steps for single- and multi-components

5.3 The Problem

As stated in Section 1.2, many FMCG production enterprises continue to operate with non-optimised RCM-based maintenance strategy plans, specifically referring to the task and frequency determination. As a result, unnecessary additional costs are incurred in the form of potential over-maintaining of equipment, as well as unnecessary production downtime due to unforeseen equipment failure.

In this specific study, it is to be researched whether an appropriate mathematical maintenance model can be used to assist in determining maintenance tasks and frequencies for both single- and multi-component systems, resulting in an optimal bottom-line cost. SAB's packaging departments currently utilise the RCM-based maintenance strategy on the majority of equipment used in the packaging process. After analysing historical data from Rosslyn Brewery's line four, it was evident that many components either (a) fail to reliably meet expected operational time intervals, resulting in unforeseen production downtime; or (b) are over-maintained, resulting in unnecessary maintenance costs. Two equipment systems were chosen to be used in the case study: (1) bottle washing machine; and (2) a section of the bottle conveyor system. Both equipment systems were essential in ensuring continuous operation of the packaging line as an entirety, seeing as the non-operational status of either of these equipments resulted in the non-operation of the entire packaging line. The bottle washing machine and the bottle conveyor systems were being maintained according to the RCM-based maintenance strategy, whereby each equipment would undergo planned PM at set, pre-defined intervals. Despite having been maintained according to the RCM-based maintenance strategy, historical data evidently showed that these equipments continued to experience unforeseen failures.

5.4 Aims of the Case Study

The aim of this case study is to investigate the feasibility and validity of utilising the proposed maintenance cost model in order to determine maintenance task and frequencies which result in an optimal bottom-line cost. In order to assess the validity of the proposed single-component cost model, a specific component on the bottle washing machine is chosen and its failure properties are determined. The validity of the multi-component cost model is based on three specific components identified within the bottle conveyor system, where failure properties are identified for each component.

The failure properties of the single- and multi-component systems, together with the remainder inputs, as discussed in Section 4.4, can then be used to simulate the expected cost per unit of time for various maintenance tasks and frequencies.

Comparing the expected cost per unit of time for the proposed optimal maintenance tasks and frequencies to the cost per unit of time based on historical data, it can be determined whether the proposed maintenance cost model provides potential bottom-line cost savings for Rosslyn Brewery.

5.5 Study Design

The study design and methodology used is discussed in Chapter 3, where the experimental design approach, coupled with scientific and statistical methods, are utilised to arrive at a single falsifiable reality.

An analysis was conducted on the packaging equipment on line four considering two factors: (1) historical downtime data; and (2) historical maintenance spend. Based on these two factors, it was evident that two common equipments that reflect in the top three contributors for each factor were the bottle washing machine and the bottle conveyor system. Further analysis showed that the top contributor for downtime and spend on the bottle washing machine was the infeed section of the machine, with the top contributor in this particular section being the infeed cam component. The decision was thus made to consider the bottle washing machine's infeed cam component for the validation determination of the single-component maintenance cost model. Considering the identified second equipment, which is, the bottle conveyor system, further analysis showed that the highest contributor in terms of downtime and maintenance cost was a section of conveyors named the *TB* section. Based on the knowledge of the construction of this particular conveyor section, three critical components were identified, namely, the slat chain; wear-strip; and sprocket, which were considered for the validation determination of the multi-component maintenance cost model. Further justification of the decision to consider the three components lay in the existence of economic and structural dependence between the three components, whereby structural intervention is required on all components if it is desired to conduct maintenance on any one of the three components.

The functional operating condition of both identified equipments were critical in the operational condition of the packaging line, as a functional failure of either of the equipments resulted in the non-operating condition of the entire packaging line. The analysis approach presented in Table 5.1 was followed for the single- and multi-component maintenance cost model validation.

5.6 System Boundaries

As stated in Section 5.5, the two systems chosen for the case study were the bottle washing machine's infeed cam and the *TB* section of the bottle conveyor system. The following two sections provide clarity on the definition of system boundaries for both identified systems.

5.6.1 Bottle Washing Machine's Infeed Cam

As discussed in Section 5.5, the bottle washing machine's infeed cam was selected as the component which was used to validate the single-component maintenance cost model. The primary function of the bottle washing is to receive empty bottles that have been returned from trade and, through the functioning processes of the machine, ensure that bottles are washed and sterilised in order to commence with further packaging of the beer into the bottles. The construction of the bottle washing machine can be seen in Figure 5.1. A pivotal step in the bottle washing process is the step whereby bottles are continuously loaded into the machine at the infeed section. This is achieved by means of timed, rotating cam fingers which lift and guide bottles into the machine. Figure 5.2 provides an extract of the process and the relative position of the infeed cam.

Figure 5.1: Bottle Washing Machine

The cam system consists of forty loading cams, where the forty cams are considered to be a single component. The system boundary was taken as the infeed cam system at the bottle washing machine's infeed section. All data relating to the infeed cam in particular were considered for analysis and further model validation.

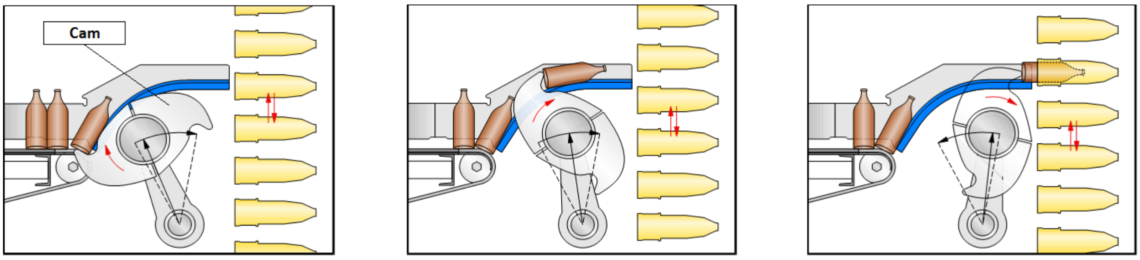


Figure 5.2: Infeed Cam System

5.6.2 *TB* Conveyor System

As discussed in Section 5.5, the *TB* conveyor system was considered for the validation determination of the multi-component maintenance cost model. Three critical components within the *TB* conveyor system, namely, the slat chain, wear-strip, and sprocket, were further considered as the three components that were used for validation determination of the multi-component maintenance cost model. The primary function of the *TB* conveyor system is to transfer filled bottles, in an upright position, from the upstream machine to the downstream machine. The assembly of the three identified components can be seen in Figure 5.3, where the sprocket is illustrated as the "Drive Sprocket".

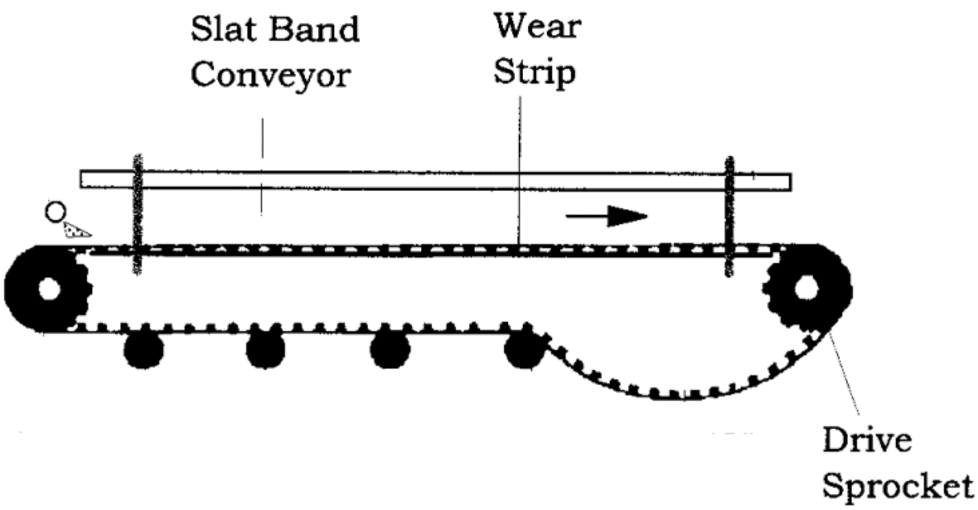


Figure 5.3: Conveyor System Assembly

The slat chain, wear-strip, and sprocket can be seen in Figures 5.4a and 5.4b.

(a) Slat Chain & Wear-strip (b) Sprocket

Figure 5.4: Slat Chain, Wear-strip, and Sprocket

The system boundary was taken as the system boundary of the three components. All data relating to the three identified components in particular were considered for analysis and further model validation.

5.7 Practicalities of Data Gathering

Described in this section are the practicalities of data gathering during the conducted case study.

5.7.1 Data Requirements

In terms of the data requirements needed, three categories of data for the component systems were defined to be used in the case study. The three categories of data are:

1. **Failure data:** For each of the components, failure data, in terms of minor (repairable) and major (non-repairable) failures are required. The time of occurrence of each of these failures is also required, in order to determine the time-dependent failure properties of the particular component.

2. **Maintenance data:** For each of the components the maintenance data is required. Maintenance data includes any CM and PM tasks, coupled with the time of occurrence of each event.

3. **Cost data:** The cost data of each component is required, where the cost data contains any incurred costs due to repair, replacement (planned and unplanned), and cost per unit of time for each of these tasks. Additionally, the historical cost per unit of time for each component is required, in order to define a comparative measure for bottom-line cost determination.

5.7.2 Data Collection

The data that were used in this study were failure data, maintenance data, and cost data. The data was obtained from SAB's electronic data capturing system (SAP software), which records vast amounts of data as captured by production shift personnel. From this system, failure, maintenance, and cost data were pulled and stored in Microsoft Excel over a twelve-month period (ranging from 1 July 2015 to 30 June 2016) for the bottle washer cam system, and over a thirteen-month period (ranging from 1 July 2015 to 31 July 2016) for the *TB* conveyor system. Table 5.2 shows the number of events, based on the three identified categories in Section 5.7.1, found for each component during the stated period.

Component	**Number of Events**		
	Failure	**Maintenance**	**Cost**
Cam	128	131	132
Slat Chain	28	32	33
Wear-strip	35	39	40
Sprocket	17	21	22

Table 5.2: Number of Events Found in Data

The data set obtained from SAB's SAP system consisted of a number of data elements, namely: downtime location (component-specific); downtime instance (date and time); downtime duration; cause of downtime; maintenance instance (date and time); maintenance duration; cost instance (date and time); cost location (component-specific); and cost amount (in Rand value). Using these data elements, it was possible to determine component-specific historical data properties, as stated in the data requirements from Section 5.7.1.

5.7.3 Data Classification

Using the data collection obtained in Section 5.7.2, the category-specific data was analysed, as described in Sections 5.7.3.1 to 5.7.3.3.

5.7.3.1 Failure Data Classification

Each failure occurrence was manually analysed and classified into either *minor* or *major* failures. A *minor* failure was categorised in the event in which repair was undertaken on the component, whereas a *major* failure was categorised in the event in which the component was replaced. Each occurrence was coupled with an instance, where the date, time, and duration was captured.

5.7.3.2 Maintenance Data Classification

Based on the captured maintenance data, the data classification was based on either (a) unplanned maintenance (CM); or (b) planned maintenance (PM). Each occurrence of a maintenance task was manually analysed and categorised as either a CM or a PM task. For both categories of maintenance data, the instance (date and time) as well as duration was captured. It is noted that all times analysed further are based on operating time.

5.7.3.3 Cost Data Classification

Within the defined data capturing period, all costs for the particular components were captured. Costs were classified according to (a) spare part cost; and (b) cost per unit of time for the specific activity. The cost per unit of time for specific activities was based on occurrence of any CM and PM activity. The incurred cost per unit of time during the specific activity was obtained from SAB's cost department, and was based on the cost incurred per unit of time during an unforeseen failure in production time or planned maintenance time. In the event of an unforeseen failure during production time, the cost per unit of time is significantly higher, as compared to planned maintenance time, seeing as costs are continuously being incurred in the form of utilities usage despite realisation of product not being made. In the event of planned maintenance, the cost per unit time is less, seeing as all machines are in an idle state, thus not incurring the extra utilities costs. The cost per unit of time for both measures was defined according to the data obtained for Rosslyn Brewery's line four.

5.8 Analysis of Bottle Washer Cam

In this section the results of the analysis for the bottle washer cam system are shown. The time period used from the data set was from 1 July 2015 to 30 June 2016, which is, a period of twelve consecutive months of operation on line four. Within the 12 month data period, data was analysed to determine at which instance(s) the bottle washer cam system was replaced. All data within the time period between replacement(s) were considered, as this essentially resulted in the identification of a system *cycle*, as described in Section 4.3.1.

5.8.1 Analysis of the Data Set for Bottle Washer Cam

The data for the bottle washer cam system were analysed between consecutive planned SRP intervals conducted on the particular system, initiating from the moment that the system was "as good as new", which is, from the instant that the bottle washer cam system was newly installed, up until the following planned SRP instant. Within the twelve months of data obtained, it was evident that the bottle washer cam system underwent a planned SRP during the production line's bi-annual maintenance shut-down on the 21st of August 2015, where production activities commenced as of 31st August 2015. The following planned SRP was performed on the bottle washer cam system in the following bi-annual maintenance shut-down on the 17th of May 2016. All data ranging from 31st August 2015 to 17th May 2016 were thus considered for further analysis.

As discussed in Section 5.7.3.1, the time instant and duration for each failure, maintenance and cost event was captured as an observation. The data for the bottle washer cam system, spanning over the aforementioned time period, are shown in Table 5.3. By manually sorting the failure data according to the time instant at which the failure occurred (T_i), it was possible to determine the time between failures (S_i) for all failures within the data set. Any preventive maintenance activities were included in the data set (PM_i). For the particular failure observation, the coupled time to repair (TTR) was also captured.

Bottle Washer Cam Data				
Obs. No.	S_i [hrs]	T_i [hrs]	TTR [min]	PM_i
1	115.03	115.03	18	0
2	73.87	188.90	16	0
3	10.13	199.03	15	0
4	13.87	212.90	8	0
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
126	3.00	5 020.42	14	0
127	5.32	5 025.73	12	0
128	57.08	5 082.82	6	0

Table 5.3: Bottle Washer Cam Data

Using the data from Table 5.3 with (2.3.7), it was possible to calculate the MTTR for the bottle washer cam system, shown in (5.8.1).

$$\begin{aligned}\text{Mean time to repair (MTTR)} &= \frac{\text{Actual breakdown time}}{\text{Number of breakdowns}}\\ &= \frac{\Sigma S_i}{\Sigma Observations}\\ &= \frac{925 minutes}{128}\\ &= 7.23 minutes\end{aligned} \tag{5.8.1}$$

5.8.1.1 Failure and Maintenance Data Analysis of the Bottle Washer Cam System

All data relating to failure observations of the bottle washer cam system were considered for further failure distribution property analysis.

In order to determine whether failure data are IID, the trend test, as suggested by Kumar and Klefsjö (1992, 217) (discussed in Section 2.3.2.3), was conducted by means of plotting cumulative failure observations against cumulative times between failures. The plot analysis is shown in Figure 5.5, which showed an increasing failure trend, based on the convex shape of the data trend curve (Asekun and Fourie, 2015, 138) (see Figure 2.14 for comparison), therefore indicative of the data not being IID.

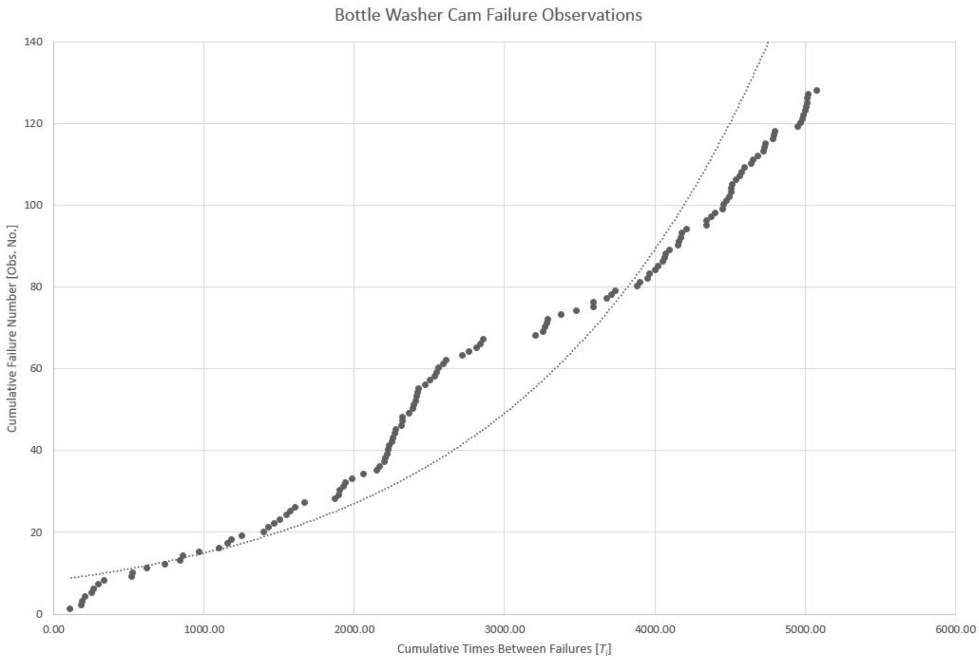


Figure 5.5: Washer Cam Failure Observations versus Cumulative Times Between Failures

Further verification of the existence of a trend was achieved by conducting the Laplace test on the data set. As described by Asekun and Fourie (2015, 138), the Laplace test is used to test a set of data for the null hypothesis of HPP against the alternative of NHPP. The test statistic under the null hypothesis is approximately a standard normal distribution variable. The hypothesis test was as follows:

$$H_0 : HPP$$
$$H_a : NHPP$$

Considering the occurrence of *minor* failures at times t_1, t_2, ..., t_n, and $N(t_i)$ as the total number of failures observed from $T = 0$ — under H_0 and conditioning on t_1, t_2, ..., t_n are uniformly distributed, the test statistic for failure observations terminating at a failure event is presented in (5.8.2):

$$U = \sqrt{12N(t_{n-1})}\left[\frac{\sum_1^{n-1} t_i}{t_n \times N(t_{n-1})} - 0.5\right] \tag{5.8.2}$$

According to Tsang (2012, 6), U is normally distributed with mean = 0 and standard deviation = 1 if the inter-arrival times of failure events are generated from a HPP. When U is significantly small (negative), the null hypothesis of HPP is rejected, indicating evidence of reliability growth; when U is significantly large (positive), the null hypothesis of HPP is rejected as well, indicating evidence of reliability deterioration (Jardine and Tsang, 2005, 264).

The significance level, α, of the test was set at 5%, which is, at 95% confidence, the lower and upper bounds of the test statistic for a two-sided test are -1.96 and 1.96, respectively. If the U value is within this range, a HPP model can be used to characterise the inter-arrival times of the observed failure events.

Using (5.8.2), the test statistic for the bottle washer cam failure data was observed to be equal to 3.17, as seen in (5.8.3).

$$\begin{aligned}
U_{cam} &= \sqrt{12N(t_{n-1})}\left[\frac{\sum_1^{n-1} t_i}{t_n \times N(t_{n-1})} - 0.5\right] \\
&= \sqrt{12 \times 127}\left[\frac{\sum_1^{127} t_i}{5082.82 \times 127} - 0.5\right] \\
&= \sqrt{1524}\left[\frac{375240.03}{645518.14} - 0.5\right] \\
&= 39.04 \times 0.08 \\
&= 3.17
\end{aligned} \tag{5.8.3}$$

Based on the observation that $U > 1.96$, it was deduced that the failure data set of the bottle washer cam system indicated a deterioration in reliability and, therefore, a non-IID data set.

According to Barabady and Kumar (2008, 649), the non-existence of IID data indicated that the system was indeed a repairable system, and resulted in a NHPP distribution set for the data. Reliasoft's *RGA* software was used to identify the best fit distribution for the failure rate of the bottle washer cam system, using the failure data presented in Table 5.3. The resulting best fit distribution of the failure data was determined to be the Power Law distribution, as shown in Figure 5.6. The resulting failure rate distribution parameters were obtained to be $\alpha = 0.001456$ and $\beta = 1.334014$.

Based on the failure rate equation for the Power Law distribution, the failure rate of the bottle washer cam system is shown in (5.8.4).

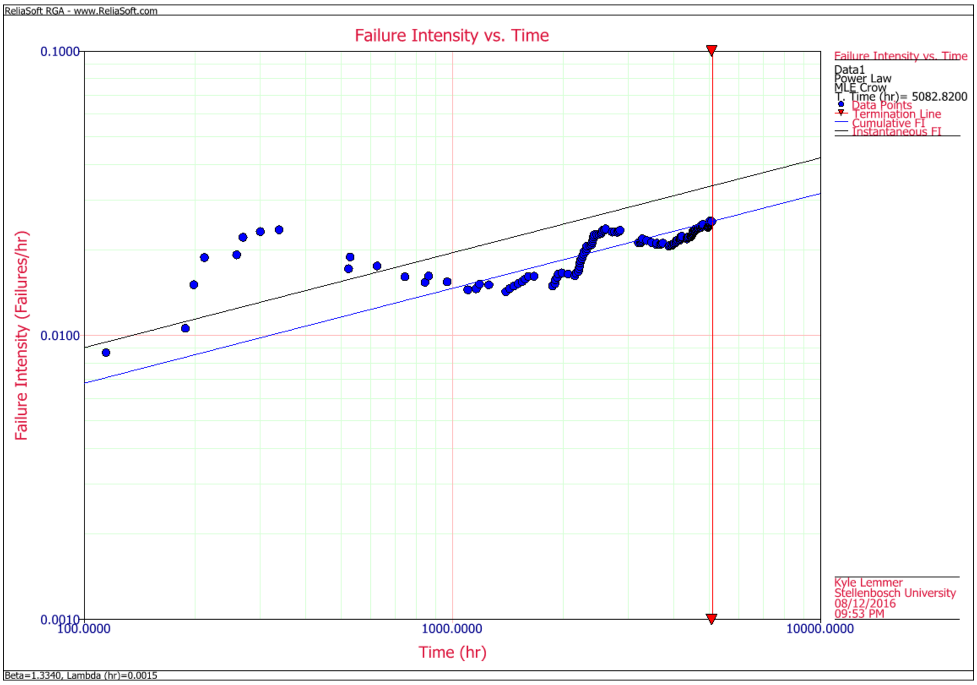


Figure 5.6: Bottle Washer Cam Failure Rate

$$\begin{aligned}\lambda(t) &= \alpha \times \beta \times t^{\beta-1} \\ &= (0.001456)(1.334014)t^{(1.334014-1)} \\ &= 0.00194t^{0.334}\end{aligned} \tag{5.8.4}$$

Verification of the proposed best fit failure rate curve for the bottle washer cam system was determined by conducting the Chi-square goodness-of-fit test, using the total number of actual failures (from data in Table 5.3) and the expected total number of failures. Based on the one hundred and twenty eight failures that were observed, the failures were divided into sixteen intervals, which is, the time of occurrence of the first eight failures was used to determine the expected number of failures at that particular time instant; as well as the following eight number of failures; and so forth. Based on the Chi-square goodness-of-fit test, the null and alternative hypotheses were defined as follows:

$$\begin{aligned} H_0 &: \textit{the specified distribution is an appropriate fit for the sample data} \\ H_a &: \textit{the specified distribution is not an appropriate fit for the sample data}\end{aligned}$$

The total number of failures for a Power Law distribution is obtained by integrating the failure rate equation over the designated time period, shown by (5.8.5). Using the time of occurrence of the eight failures (sixteen intervals), the expected number of failures at these particular time instances were obtained using (5.8.5) - as shown in Table 5.4.

$$F(t) = \int_{t_1}^{t_2} \lambda(t)\mathrm{d}t \tag{5.8.5}$$

Bottle Washer Cam Expected Versus Actual Failures		
Interval Number	Expected Failures	Actual Failures
1	3.60	8.00
2	10.77	8.00
3	9.65	8.00
4	9.17	8.00
5	7.06	8.00
6	9.32	8.00
7	6.99	8.00
8	9.87	8.00
9	7.87	8.00
10	7.66	8.00
11	6.98	8.00
12	8.61	8.00
13	4.57	8.00
14	5.65	8.00
15	11.96	8.00
16	10.26	8.00

Table 5.4: Bottle Washer Cam Expected Versus Actual Failures

The Chi-square statistic equation is shown in (5.8.6).

$$\chi^2 = \sum \frac{(observed - expected)}{expected} \tag{5.8.6}$$

Based on the sixteen failure intervals, the Chi-square statistic for the cam system, based on the proposed Power Law distribution, was calculated in (5.8.7) to be equal to 12.90.

$$\begin{aligned}
\chi^2_{cam} &= \frac{(8.00-3.60)^2}{3.60} + \frac{(8.00-10.77)^2}{10.77} + \frac{(8.00-9.65)^2}{9.65} + \frac{(8.00-9.17)^2}{9.17} + \frac{(8.00-7.06)^2}{7.06} \\
&+ \frac{(8.00-9.32)^2}{9.32} + \frac{(8.00-6.99)^2}{6.99} + \frac{(8.00-9.87)^2}{9.87} + \frac{(8.00-7.87)^2}{7.87} \\
&+ \frac{(8.00-7.66)^2}{7.66} + \frac{(8.00-6.98)^2}{6.98} + \frac{(8.00-8.61)^2}{8.61} + \frac{(8.00-4.57)^2}{4.57} \\
&+ \frac{(8.00-5.65)^2}{5.65} + \frac{(8.00-11.96)^2}{11.96} + \frac{(8.00-10.26)^2}{10.26} \\
&= 5.38 + 0.71 + 0.28 + 0.15 + 0.13 + 0.19 + 0.15 + 0.35 + 0.02 + 0.15 + 0.04 + 2.57 \\
&+ 0.98 + 1.31 + 0.50 \\
&= 12.90
\end{aligned} \tag{5.8.7}$$

Considering a level of significance of $\alpha = 0.05$ and fifteen degrees of freedom (based on the sixteen sample intervals), the critical Chi-square value was obtained from the Chi-square distribution table as $\chi^2_c = 25.00$. Based on the Chi-square goodness-of-fit test methodology, the null hypothesis is rejected if $\chi^2_{cam} > \chi^2_c$ — which was not the case, and therefore it was concluded that there is not sufficient evidence to reject the hypothesis that the specified distribution is an adequate fit for the sample data.

Further analysis of the data in Table 5.3 showed that the two planned PM tasks (PM_1 and PM_2) were executed on the cam system at times $T_i = 2283.97$ hours and $T_i = 4\,382.37$ hours, respectively. Considering that the system's failure rate remained un-changed from time $T_i = 0$ hours until the first planned PM at time $T_i = 2\,283.97$ hours, the total number of failures experienced in this particular time period was considered for validation of the proposed failure rate distribution. Using (5.8.4) and (5.8.5), the theoretical number of failures at time $T_i = 2\,283.97$ hours was determined to be 44.994, as shown in (5.8.8). The actual number of failures experienced during this particular time interval, based on data from Table 5.3, was 45 — therefore indicating that the failure rate distribution accurately represents the actual number of failures within a 99.98% accuracy estimate.

$$\begin{aligned} F(2\,283.97) &= \int_0^{2\,283.97} 0.00194t^{0.334}\mathrm{d}t \\ &= 44.994 \text{ failures} \end{aligned} \tag{5.8.8}$$

Further analysis of the failure data was done in order to determine the improvement factor, which is defined in Section 4.3 as the improvement factor in failure rate of the system following a PM task. Considering that the failure rate, defined in (5.8.4), was used to model the failure rate of the system throughout the system's life cycle, and the assumption that the failure rate of the system "resets" to zero following a PM task and increases more rapidly in the succeeding operational period (discussed in Section 4.4.4), the improvement factor was obtained by comparing the theoretical versus actual number of failures for the time periods leading into the first PM and the time period between the first and second PM. Analysis of the data in Table 5.3 showed that the second PM task was done at time $T_i = 4\,382.37$ hours. The operational time period between the first and second PM task was therefore determined to be the difference between T_i at the two instances of PM, minus the time required to conduct PM at the first PM interval, as shown in (5.8.9) — equal to 2 094.40 hours. The time required to conduct maintenance tasks on the cam system were obtained from maintenance schedules on line four, as shown in Table 5.5.

$$\begin{aligned} T_{op} &= T(PM_2) - T(PM_1) - \omega^p \\ &= 4\,382.37 - 2\,283.97 - 4.00 \\ &= 2\,094.40 \text{ hours} \end{aligned} \tag{5.8.9}$$

Bottle Washer Cam Maintenance Times		
PM [hrs]	Planned SRP [hrs]	Unplanned SRP [hrs]
4.00	8.00	10.00

Table 5.5: Bottle Washer Cam Maintenance Times

The theoretical number of failures in the operational period of $t = 2\,094.40$ hours was determined to be 41.257, as shown in (5.8.10).

$$\begin{aligned} F(2\,094.40) &= \int_0^{2\,094.40} 0.00194t^{0.334}\mathrm{d}t \\ &= 41.257 \text{ failures} \end{aligned} \tag{5.8.10}$$

Failure data from Table 5.3 showed that the actual number of failures in the time interval from $T_i = 2\,283.97$ hours to $T_i = 4\,382.37$ hours was 62. The discrepancy of actual versus theoretical number of failures in this particular time period was used to determine the improvement factor that is to be applied to the failure rate following a PM task, as shown in (5.8.11), where the improvement factor was determined to be $a = 1.503$.

$$\begin{aligned} a &= \frac{\text{Actual number of failures in time period}}{\text{Theoretical number of failures in time period}} \\ &= \frac{62.000}{41.257} \\ &= 1.503 \end{aligned} \tag{5.8.11}$$

5.8.1.2 Cost Data Analysis of the Bottle Washer Cam System

The cost data of the bottle washer cam system was analysed in order to determine several cost inputs that were used in further analysis of the proposed maintenance model. All costs associated with the bottle washer cam system during the prescribed time period (life cycle) of the system was used, which is, from time $T_i = 0$ hours to time $T_i = 5\,082.82$ hours.

The cost analysis involved the consideration of two cost factors, namely, (1) spare part costs; and (2) downtime costs. The spare part costs included any costs incurred over the life cycle of the component, particularly for spare parts used for maintenance activities. It was determined from SAB's cost records that the cost for a single cam was R1 960.13. Further analysis of maintenance and cost data showed that, on average, two cam parts were replaced at each

PM interval, thereby resulting in a PM spare part cost of R3 920.25, as shown in (5.8.12). Considering that the entire cam system (forty cams) is replaced upon an SRP task, the total spare part cost for an SRP task was determined to be R78 405.00, as shown in (5.8.13).

$$\begin{aligned} \text{PM Spare part cost} &= \text{Cost per part} \times \text{Number of parts} \\ &= R1\,960.13 \times 2 \\ &= R3\,920.25 \end{aligned} \tag{5.8.12}$$

$$\begin{aligned} \text{SRP Spare part cost} &= \text{Cost per part} \times \text{Number of parts} \\ &= R1\,960.13 \times 40 \\ &= R78\,405.00 \end{aligned} \tag{5.8.13}$$

The cost of downtime was analysed according to two factors, namely, (1) cost of planned downtime; and (2) cost of unplanned downtime. In the event of planned downtime, only the costs relating to incurred labour costs were considered, seeing as the utilities costs are negligible as all production equipment are in an idle state. The incurred labour cost on line four was obtained from the cost department at SAB's Rosslyn Brewery, which calculates all costs related to labour on line four specifically, at an hourly rate. The hourly cost of labour on line four was determined to be R1 110.70 per hour. The hourly labour rate was applicable to all booked factory hours, whether planned or unplanned downtime is experienced, seeing as all personnel are present on the production line at both downtime instances. The labour cost for the events of planned PM was obtained by multiplying the PM duration by the hourly labour cost rate, as shown in (5.8.14).

$$\begin{aligned} \text{PM labour cost} &= \text{Cost per hour} \times \text{PM duration} \\ &= R1\,110.70 \times 4 \\ &= R4\,442.80 \end{aligned} \tag{5.8.14}$$

Using (5.8.12) and (5.8.14), it was possible to determine the total cost for each planned PM activity on line four's bottle washer cam system, as shown in (5.8.15).

$$\begin{aligned} \text{PM cost} &= \text{PM spare part cost} + \text{PM labour cost} \\ &= R3\,920.25 + R4\,442.80 \\ &= R8\,363.05 \end{aligned} \tag{5.8.15}$$

As with the PM labour cost, the planned SRP labour cost was obtained by multiplying the planned SRP duration by the hourly labour cost rate, as shown in (5.8.16).

$$\begin{aligned}\text{Planned SRP labour cost} &= \text{Cost per hour} \times \text{Planned SRP duration}\\ &= R1\,110.70 \times 8\\ &= R8\,885.60\end{aligned} \tag{5.8.16}$$

Using (5.8.13) and (5.8.16), it was possible to determine the total cost for each planned SRP activity on line four's bottle washer cam system, as shown in (5.8.17).

$$\begin{aligned}\text{Planned SRP cost} &= \text{SRP spare part cost} + \text{Planned SRP labour cost}\\ &= R78\,405.00 + R8\,885.60\\ &= R87\,290.60\end{aligned} \tag{5.8.17}$$

In the event of an unplanned activity, whereby the production line experiences downtime during production times, the hourly rate of production costs were considered, which was obtained from the cost department at SAB's Rosslyn Brewery. The cost incurred at an hourly rate during any unplanned downtime was determined to be R60 222.61 per hour. The total cost of performing an unplanned SRP task on the bottle washer cam system was obtained using the unplanned cost rate, the duration of the unplanned SRP task, and the spare part cost of the SRP activity, as shown in (5.8.18).

$$\begin{aligned}\text{Unplanned SRP cost} &= (\text{Cost per hour} \times \text{Unplanned SRP duration}) + \text{SRP spare part cost}\\ &= \left(\frac{R60\,222.61}{hour} \times 10hours\right) + R78\,405.00\\ &= R680\,631.10\end{aligned} \tag{5.8.18}$$

Using the MTTR of the bottle washer cam system (shown in (5.8.1)) and the cost rate of unplanned downtime, the cost of any CM activity was determined, as shown in (5.8.19). It was assumed, based on historical maintenance records, that no spare parts were used in any CM activity, therefore negating the consideration for spare parts cost in the CM cost calculation.

$$\begin{aligned}\text{CM cost} &= \text{Cost per hour} \times \text{CM duration}\\ &= \frac{R60\,222.61}{hour} \times (7.23minutes) \times \frac{1hour}{60minutes}\\ &= R7\,256.82\end{aligned} \tag{5.8.19}$$

5.8.2 Bottle Washer Cam Modeling and Simulation

Section 5.8 has provided a firm foundation and analysis to assist further modeling and simulation of the proposed maintenance cost model for the bottle washer cam system. This section aims at providing clarity on the determination of the parameters used in the proposed maintenance cost model, as well the simulated results obtained from the model.

5.8.2.1 Bottle Washer Cam Model Parameter Determination

The list of notations listed in Section 4.3 provided a clear understanding of what parameters needed to be determined for use in the maintenance cost model analysis. The listed parameters were identified as either a fixed parameter or a variable parameter. The methods of determination of the fixed parameters for the bottle washer cam system are shown in Table 5.6 (see Section 4.3 for detail on the parameter definition).

The defined fixed parameters listed in Table 5.6 provided a basis on which the variable parameters could be altered and further used as input variables, in order to determine the resulting output variable to be optimised, which is, the cost per unit of time. Referring to the ultimate aim of this study, the parameters to be determined were the (a) maintenance tasks; and (b) frequencies thereof. The determination of the maintenance tasks exist in the decision of conducting either a PM or a SRP task on the component, whereas the frequency thereof exists in the decision of times to conduct PM and SRP tasks. The variable parameters are described in Table 5.7.

5.8.2.2 Bottle Washer Cam Model Optimisation and Simulation

Using the single-component maintenance model, programmed into Matlab's software (see Appendix A.1), the iterative process of altering the variable input parameters was achieved by defining and running the command prompts in Matlab, described in Appendix A.2. For each iteration, which is, for each alteration of a variable input parameter, a resultant output cost per unit time was determined. Once the iteration process was complete, the expected minimum cost per unit time, together with the associated input parameters, was determined.

In order to determine the most probable minimum output result, the Monte Carlo simulation was further utilised. The iterative process described in the preceding paragraph depicted a single simulation of the model for each input argument. Based on Raychaudhuri (2008, 92)'s summarised approach of conducting an effective Monte Carlo simulation, the core of the Monte Carlo simulation lies in the process of repeating the generation of the output variable (cost per unit of time) several times, after which a statistical analysis is conducted in order to provide a statistical confidence obtained for the output variable.

The Monte Carlo simulation approach was achieved by repeating the iterative simulation, described in the first paragraph of Section 5.8.2.2, for a total of one hundred simulations. The resultant minimal cost, together with

Bottle Washer Cam Fixed Parameter Determination	
Parameter	Determination
c^c	The cost of a CM task for the system was obtained from (5.8.19) as R7256.82. c^c is defined from notations $\bar{c}^c$, ω^c, and c^{ud}, as in (4.3.5).
c^p	The cost of a PM task for the system was obtained from (5.8.15) as R8363.05. c^p is defined from notations $\bar{c}^p$, ω^p, and c^{pd}, as in (4.3.6).
c^{uR}	The cost of an unplanned SRP task for the system was obtained from (5.8.18) as R680631.10. c^{uR} is defined from notations $\bar{c}^{pR}$, ω^{uR}, and c^{ud}, as in (4.3.7).
c^{pR}	The cost of a planned SRP task for the system was obtained from (5.8.17) as R87290.60. c^{pR} is defined from notations $\bar{c}^{pR}$, ω^{pR}, and c^{pd}, as in (4.3.8).
ω^c	The time required to conduct a CM task on the system was obtained using the MTTR from (5.8.1) as 7.23 minutes.
ω^p	The time required to conduct a planned PM task on the system was obtained using the maintenance data in Table 5.5 as 4.00 hours.
ω^{pR}	The time required to conduct a planned SRP task on the system was obtained using the maintenance data in Table 5.5 as 8.00 hours.
ω^{uR}	The time required to conduct an unplanned SRP task on the system was obtained using the maintenance data in Table 5.5 as 10.00 hours.
$\lambda_{(op,min)}$	The *minor* failure rate of the system was determined in Section 5.8.1.1, and defined as $\lambda_{(op,min)}(t) = 0.00194t^{0.334}$, as shown in (5.8.4).
$f_{(op,maj)}(t)$	Based on historical data, the system had not experienced a *major* failure in the past five years. In order to determine expected lifetime of the system until a *major* failure occurs, experienced maintenance personnel were consulted, who advised that the system would be able to remain operational for a period of approximately forty weeks before a *major* failure would be expected. Based on one hundred and sixty eight hours in a week, the probability of a *major* failure was assumed to be a normal distribution function, with a mean value of 6 720 hours, and a standard deviation of 168 hours (one week).
a	The expected improvement factor for the cam system, applicable to the *minor* failure rate following a PM task, was obtained to be 1.503, as shown in (5.8.11).
Y_{op}	The expected lifetime, before a *major* failure occurs, of the cam system was generated as a random variable with a value based on the probability distribution of $f_{(op,maj)}(t)$.

Table 5.6: Bottle Washer Cam Fixed Parameter Determination

Bottle Washer Cam Variable Parameter Determination	
Parameter	Determination
T	The intervals at which planned PM tasks were executed was based on the current maintenance strategy implemented on line four, whereby the production line executes PM tasks on various equipment one a one-weekly basis (one hundred and sixty eight hours). The time at which PM was performed on the cam system specifically, was therefore considered as multiples of the one-weekly maintenance stoppage, with the potential maximum frequency of PM therefore being every one hundred and sixty eight hours. The maximum interval between consecutive PM intervals was chosen to be equal to one week less than the system's expected lifetime, seeing as consideration of a lengthier interval would result in an endless iteration process whereby the system statistically fails to successfully complete an operational period. Considering that the system's expected lifetime was determined to be 6 720 hours, the maximum value for T was determined to be 6 552 hours.
x	The operational period threshold, whereupon the system undergoes a planned SRP was determined to range between the minimum value of one (implying a "replace only", which is, no PM tasks to be performed) and a maximum value of thirty nine.

Table 5.7: Bottle Washer Cam Variable Parameter Determination

the associated input variables, were stored in a matrix format which was further used to analyse the statistical confidence of the resulting output variable. Based on the one hundred simulation iterations completed for the Monte Carlo methodology, a point estimate for the cost per unit of time for each alteration of input parameters was determined, as well as the interval estimate, at a 95% confidence level, in order to determine the standard error associated with the point estimate. The point estimate for the one hundred simulations of each parameter alteration was determined using (4.5.1), where the interval estimation (which is, the expected error associated with the point estimation at a 95% confidence level) was determined using (4.5.3). The associated point estimates for each parameter alteration, together with the interval estimates, are shown in Table B.1.

A visual presentation of the results for the simulation is shown in Figure 5.7, where the point estimates for the resulting cost per unit of time under varying values of x and T are illustrated. The associated input parameters that resulted in the optimised minimal cost per unit of time are shown in Table 5.8, together with the interval estimate (H) for the optimal conditions. All one hundred simulations concluded that the optimal cost-per-unit-time was achieved at an operational period of one ($x = 1$), and a planned SRP taking place every one thousand and eight hours ($T = 1\,008$ — equivalent to six weeks), resulting in a cost-per-unit-time equal to R179.52 per hour. The associated interval estimate, at a 95% confidence level, under these specific parameters was determined to be $\pm R1.1386 \times 10^{-14}$. Based on the exhaustive enumeration of input parameters analysed in the simulation, it was concluded that the 'optimal' value (cost per unit of time) had been reached, as the numerical minimum for the exhaustive input parameters was determined.

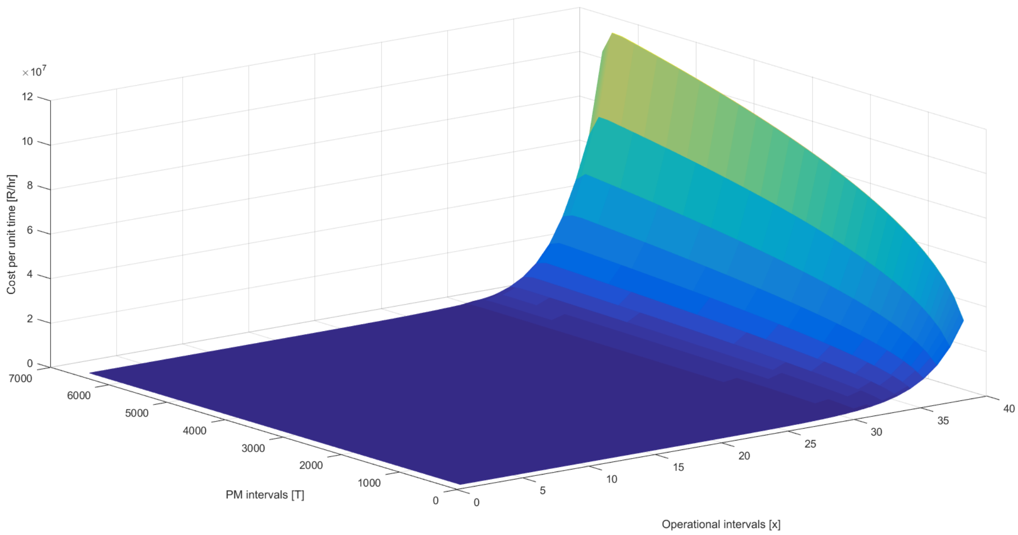


Figure 5.7: Cam System Simulation Results

Bottle Washer Cam Optimal Parameters
$x = 1$ operational periods
$T = 1\,008.00$ hours
Cost $= R179.52$ per hour
H $= \pm R1.1386 \times 10^{-14}$

Table 5.8: Cam System's Optimal Parameters and Resultant Cost

The simulation results indicated that the interval estimate (standard error) for the output parameter (cost-per-unit-of-time) significantly increased as the operational period approached the MTBF for *major* failures of the cam system. This was clear based on the logic that any unforeseen downtime occurrences contributed significantly more cost as compared to *minor* failures and PM activities. By allowing the operational periods of the cam system to approach the MTBF of *major* failures, the probability of a *major* failure increases, thus resulting in the large variances in the output parameter. A visual presentation of the interval estimates for the associated point estimates is illustrated in Figure 5.8.

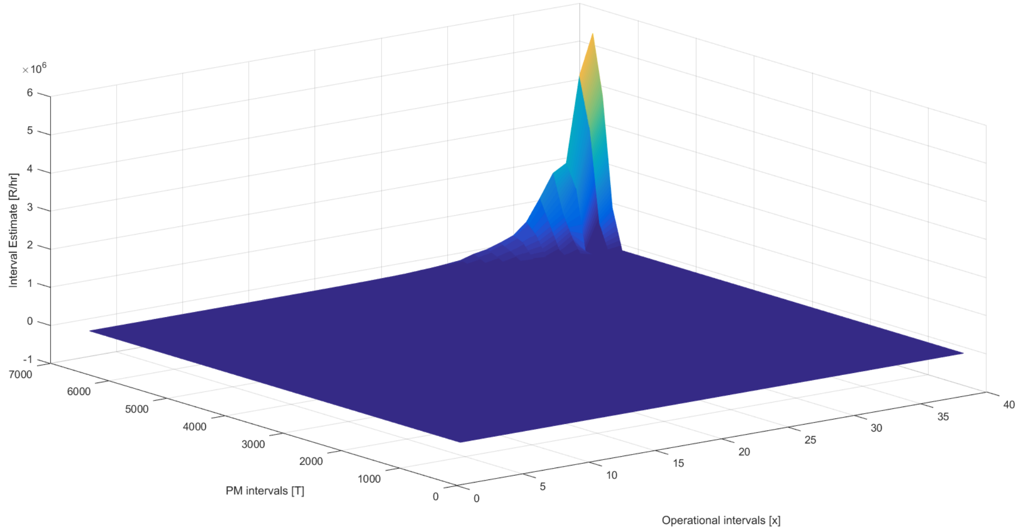


Figure 5.8: Cam System Interval Estimates

5.8.2.3 Bottle Washer Cam Cost Comparison

In order to validate the potential cost benefit that would theoretically arise if the proposed input parameters would be implemented, the actual cost per unit time for the bottle washer cam system was determined, using historical failure, maintenance, and cost data for the system, over the aforementioned period, ranging from 31st August 2015 to 17th May 2016. To ensure consistency between the proposed model and the actual cost incurred, the analysis of the actual incurred cost over the time period considered the following cost factors:

1. PM Cost
2. Planned and unplanned SRP cost
3. CM cost

The actual PM cost was calculated based on labour and spare part cost and determined to be equal to R16 726.10, as shown in Table 5.9.

Bottle Washer Cam Actual PM Cost			
No. of PM Intervals	Labour Cost per Interval	Spare Part Cost per Interval	Total PM Cost
2	R4 442.80	R3 920.25	R16 726.10

Table 5.9: Actual Total PM Cost of Cam System

The actual planned and unplanned SRP cost was calculated based on labour and spare part cost and determined to be equal to R87 290.60, as shown in Table 5.10.

Bottle Washer Cam Actual SRP Cost				
	No. of SRP Intervals	Labour Cost per Interval	Spare Part Cost per Interval	Total SRP Cost
Planned :	1	R8 885.60	R78 405.00	R87 290.60
Unplanned :	0	R11 107.00	R78 405.00	R0.00
			Total SRP Cost:	**R**87 290.60

Table 5.10: Actual Total SRP Cost of Cam System

The actual CM cost was calculated based on the actual unforeseen *minor* failures experienced during the aforementioned time period and the cost per unit time of unplanned downtime, and was determined to be equal to R928 431.90, as shown in Table 5.11.

Bottle Washer Cam Actual CM Cost		
Unplanned Downtime Experienced [$hours$]	Cost per Unit of Unplanned Downtime [R/hr]	Total CM Cost
15.42	R60 222.61	R928 431.90

Table 5.11: Actual Total CM Cost of Cam System

Based on the obtained cost data in Tables 5.9, 5.10, and 5.11, the total actual cost incurred for the bottle washer cam system over the aforementioned time period was calculated to be equal to R1 032 448.60, as shown in (5.8.20).

$$\begin{aligned}\text{Actual Cam Total Cost} &= \text{Total PM Cost} + \text{Total SRP Cost} + \text{Total CM Cost}\\ &= R16\,726.10 + R87\,290.60 + R928\,431.90\\ &= R1\,032\,448.60\end{aligned} \tag{5.8.20}$$

Analysis of the data in the aforementioned time period indicated that the cam system was in operation for a total of 4 967.68 hours. The actual cost per unit of time for the cam system was thereby calculated to be equal to R207.83 per hour, as shown in (5.8.21).

$$\begin{aligned}\text{Actual Cam Cost Rate} &= \frac{\text{Actual Cam Total Cost}}{\text{Total Operational Time}}\\ &= \frac{R1\,032\,448.60}{4\,967.68\text{hours}}\\ &= R207.83 \text{ per hour}\end{aligned} \tag{5.8.21}$$

Comparison of the obtained theoretical cost rate to the actual incurred cost rate of the bottle washer cam system indicated a potential cost savings of R28.31 per hour of operation of the cam system, as shown in (5.8.22).

$$\begin{aligned} \text{Potential Cost Saving Rate } &= \text{Actual Cam Cost Rate} - \text{Model Cost Rate} \\ &= R207.83 - R179.52 \\ &= R28.31 \text{ per hour} \end{aligned} \tag{5.8.22}$$

Based on the aforementioned time period over which data for the cam system were analysed, the total theoretical cost saving is equal to the cost saving rate multiplied by the operational hours during the specified time period, and was determined to be equal to R140 637.85, as shown in (5.8.23).

$$\begin{aligned} \text{Theoretical Cycle Cost Saving } &= \text{Potential Cost Saving Rate} \times \text{Operational Time} \\ &= R28.31 \times 4\,967.78 \text{ hours} \\ &= R140\,637.85 \end{aligned} \tag{5.8.23}$$

5.8.2.4 Summary of Results of Bottle Washer Cam Maintenance Optimisation

Based on historical failure, maintenance, and cost data, the bottle washer cam system was analysed over a cycle, initiating from the “good as new” state, until the next planned SRP task was executed on the system. Using the failure data over this period, the failure rate of the system was determined to be best described by the Power Law distribution (using Reliasoft’s RGA software), with inherent parameters of $\alpha = 0.0015$ and $\beta = 1.334$. In order to verify the goodness-of-fit of the best-fit failure rate distribution, the the Chi-square goodness-of-fit test was conducted, which resulted in acceptance of the null hypothesis (which is, that the proposed distribution accurately represents the sample data), with a level of significance of $\alpha = 0.05$ (which is, with 95% confidence). By analysing the failure rates over two consecutive cycles, the improvement factor, applicable to the failure rate following a PM task, was determined to be $a = 1.503$.

The utilisation of all relevant cost data was used, in conjunction with the developed maintenance cost model, in order to simulate the expected cost per unit of time under varying input parameter conditions. Based on the simulation of the proposed cost model, the optimal cost per unit of time was determined to be equal to R179.52 per hour, with associated parameters x (operational periods) and T (PM intervals) equal to 1 and 1 008 hours, respectively — thus indicative of performing a planned SRP on the cam system every 1 008 hours. By conducting one hundred simulations, using the Monte Carlo methodology, the optimal parameters remained constant at the pre-defined parameter value. The point estimate for the optimal parameters indicated a cost-per-unit-of-time of R179.52, with an interval estimate (standard error) of $\pm R1.1386 \times 10^{-14}$. It was evident from the simulation results' statistical analysis that the interval estimate significantly increased as the operational period approached the MTBF of the cam system — where the logic lay in the fact that the probability of an unforeseen occurrence of a *major* failure significantly increases, which ultimately contributes the largest cost-per-unit-of-time of all failure and maintenance activities. Based on the exhaustive enumeration of input parameters analysed in the simulation, it was concluded that the 'optimal' value (cost per unit of time) had been reached, as the numerical minimum for the exhaustive input parameters was determined.

The validation of the proposed cost model lay in the comparison between actual cost rate and theoretical cost rate. This was done by comparing the actual incurred costs over the same time period (cycle) to the optimal minimum cost obtained from the proposed cost model. The potential theoretical cost saving rate was determined to be R28.31 per hour of operation, which ultimately translated into a potential cycle cost saving of R140 637.85 for the analysed cycle time period.

5.9 Analysis of *TB* Bottle Conveyors

In this section the results of the analysis for the *TB* bottle conveyor system are shown. The time period used for the data was from 1 July 2015 to 31 July 2016, which is, a period of thirteen consecutive months of operation on line four. Within the thirteen months data period, data was analysed to determine at which instance(s) the *TB* conveyor system was replaced. All data within the time period between replacement(s) were considered, as this essentially resulted in the identification of a system *cycle*, as described in Section 4.3.1.

5.9.1 Analysis of the Data Set for the *TB* Conveyors

The data for the *TB* conveyor system were analysed between consecutive planned SRP intervals conducted on the particular system, initiating from the moment that the system was "as good as new", which is, from the instant that *TB* conveyor system was newly installed, up until the following planned SRP instant. Within the thirteen months of data obtained, it was evident that the *TB* conveyor system underwent a planned SRP during the production line's bi-annual maintenance shut-down on the 21st of August 2015, where production activities commenced as of 31st August 2015. The following planned SRP was conducted on 15th July 2016, thus indicating that the planned SRP of the *TB* conveyor system only occurred on every second bi-annual shut-down. All data ranging from 31st August 2015 to 15th July 2016 were thus considered for further analysis.

As discussed in Section 5.7.3.1, the time instant and duration for each failure, maintenance, and cost event was captured as an observation. The data for the *TB* conveyor chain, sprocket, and wear-strip components, spanning over the aforementioned time period, are shown in Tables 5.12, 5.13, and 5.14, respectively. By manually sorting the failure data according to the time instant at which the failure occurred (T_i), it was possible to determine the time between failures (S_i) for all failures within the data sets. Any preventive maintenance activities were included in the data sets (PM_i). For the particular failure observation, the coupled time to repair (TTR) was also captured.

***TB* Conveyor Chain Data**				
Obs. No.	S_i [hrs]	T_i [hrs]	TTR [min]	PM_i
1	337.00	337.00	6	0
2	314.17	651.17	4	0
3	155.63	806.80	5	0
4	152.62	959.42	4	0
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
26	191.37	6 132.20	5	0
27	100.30	6 232.50	5	0
28	30.37	6 262.87	8	0

Table 5.12: *TB* Conveyor Chain Data

***TB* Conveyor Sprocket Data**				
Obs. No.	S_i [hrs]	T_i [hrs]	TTR [min]	PM_i
1	680.90	680.90	40	0
2	505.55	1 186.45	6	0
3	133.00	1 319.45	7	0
4	149.00	1 468.45	7	1
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
15	445.00	5 708.03	8	0
16	344.65	6 052.68	4	0
17	195.72	6 248.40	7	0

Table 5.13: *TB* Conveyor Sprocket Data

Using the data from Tables 5.12, 5.13, and 5.14 with (2.3.7), it was possible to calculate the MTTR for the *TB* conveyor chain, sprocket, and wear-strip components, as shown in (5.9.1), (5.9.2), and (5.9.3), respectively.

***TB* Conveyor Wear-strip Data**				
Obs. No.	S_i [hrs]	T_i [hrs]	TTR [min]	PM_i
1	182.70	182.70	11	0
2	108.95	291.65	4	0
3	10.08	301.73	10	0
4	8.13	309.87	5	1
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
33	192.67	6 134.37	5	0
34	171.15	6 305.52	9	0
35	61.18	6 366.70	8	0

Table 5.14: *TB* Conveyor Wear-strip Data

$$\begin{aligned}\text{Chain mean time to repair (MTTR)} &= \frac{\text{Actual breakdown time}}{\text{Number of breakdowns}}\\ &= \frac{\Sigma S_i}{\Sigma Observations}\\ &= \frac{\text{154 minutes}}{28}\\ &= \text{5.50 minutes}\end{aligned} \tag{5.9.1}$$

$$\begin{aligned}\text{Sprocket mean time to repair (MTTR)} &= \frac{\text{Actual breakdown time}}{\text{Number of breakdowns}}\\ &= \frac{\Sigma S_i}{\Sigma Observations}\\ &= \frac{\text{136 minutes}}{17}\\ &= \text{8.00 minutes}\end{aligned} \tag{5.9.2}$$

$$\begin{aligned}\text{Wear-strip mean time to repair (MTTR)} &= \frac{\text{Actual breakdown time}}{\text{Number of breakdowns}}\\ &= \frac{\Sigma S_i}{\Sigma Observations}\\ &= \frac{\text{238 minutes}}{35}\\ &= \text{6.80 minutes}\end{aligned} \tag{5.9.3}$$

5.9.1.1 Failure and Maintenance Data Analysis of the *TB* Conveyor System

All data relating to failure observations of the *TB* conveyor system were considered for further failure distribution property analysis.

In order to determine whether failure data are IID, the trend test, as suggested by Kumar and Klefsjö (1992, 217) (discussed in Section 2.3.2.3), was conducted by means of plotting cumulative failure observations against cumulative times between failures for the chain, sprocket, and wear-strip components. The plot analyses for the chain, sprocket, and wear-strip are shown in Figures 5.9, 5.10, and 5.11, respectively. The three plots all showed an increasing failure trend, based on the convex shapes of the data trend curves (Asekun and Fourie, 2015, 138) (see Figure 2.14 for comparison), therefore indicative of the data not being IID.

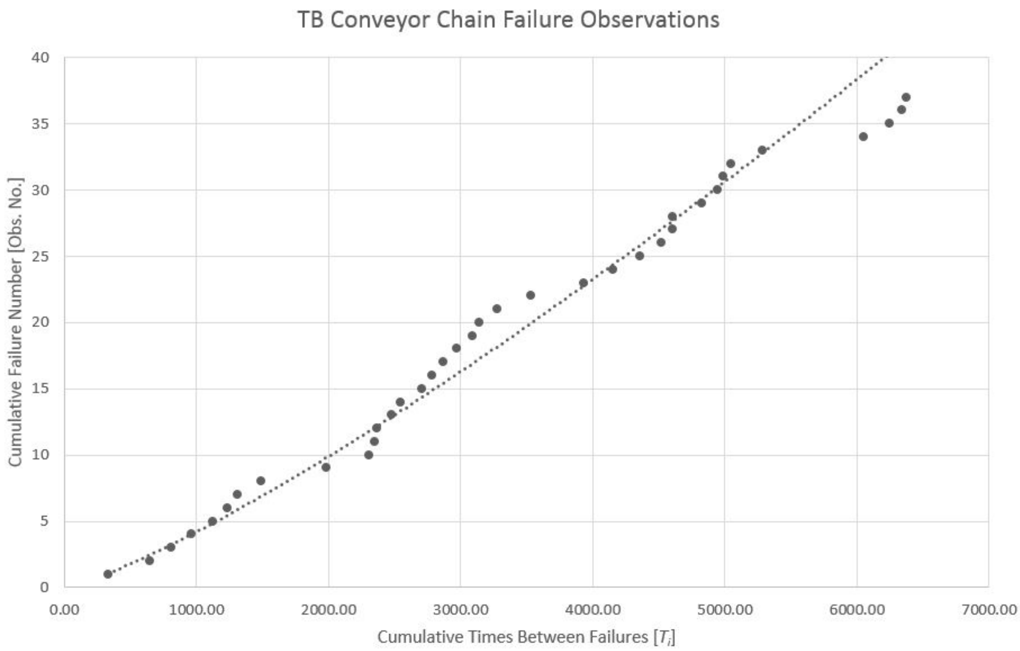


Figure 5.9: TB Chain Failure Observations versus Cumulative Times Between Failures

Further verification of the existence of a trend was achieved by conducting the Laplace test on the three data sets. As with the bottle washer cam data analysis (see Section 5.8.1.1), the hypotheses tests were as follows:

$$H_0 : HPP$$
$$H_a : NHPP$$

Considering the occurrence of *minor* failures at times t_1, t_2, ..., t_n, and $N(t_i)$ as the total number of failures observed from $T = 0$ for the three data sets — under H_0 and conditioning on t_1, t_2, ..., t_n are uniformly distributed, the test statistics for failure observations terminating at a failure event is identical to (5.8.2).

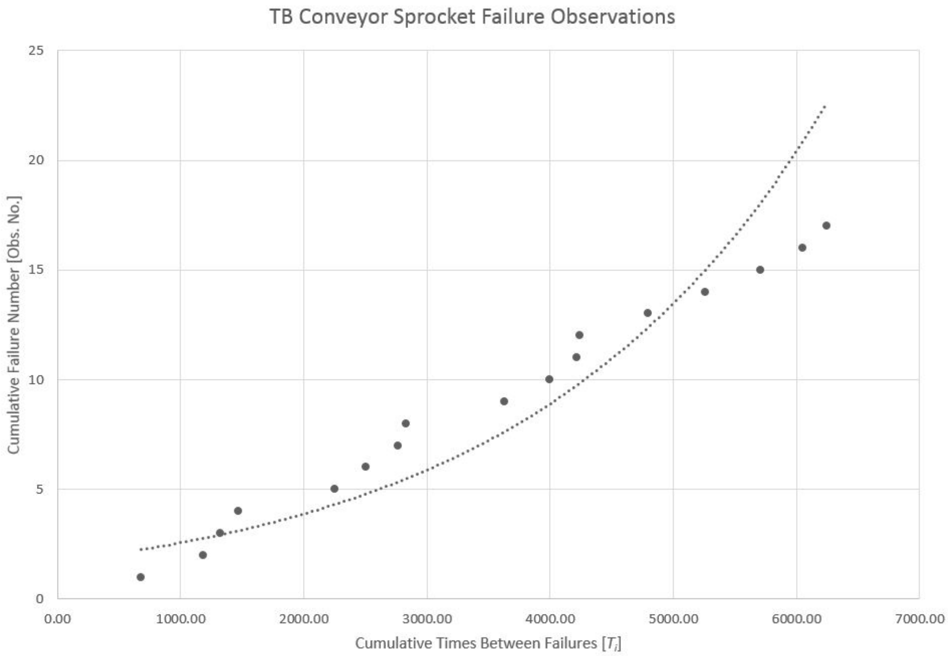


Figure 5.10: TB Sprocket Failure Observations versus Cumulative Times Between Failures

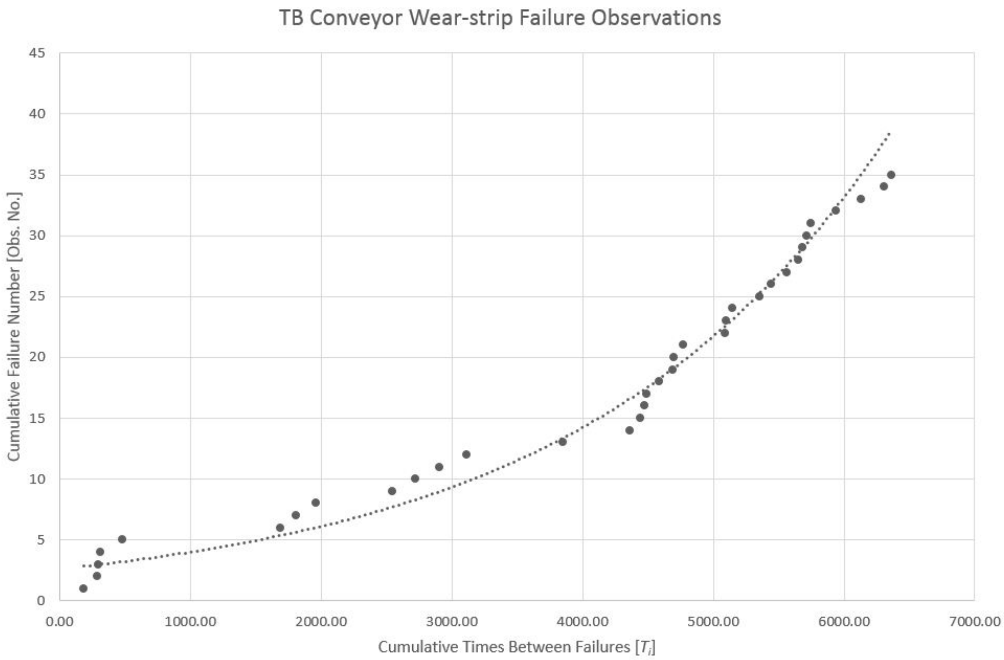


Figure 5.11: TB Wear-strip Failure Observations versus Cumulative Times Between Failures

The significance levels, α, of the tests were set at 5%, which is, at 95% confidence, the lower and upper bounds of the test statistic for a two-sided test are -1.96 and 1.96, respectively. If the U value is within this range, a HPP model can be used to characterise the inter-arrival times of the observed failure events.

Using (5.8.2), the test statistics for the chain, sprocket, and wear-strip failure data were observed to be equal to 0.84, 0.40, and 2.18, as seen in (5.9.4), (5.9.5), and (5.9.6), respectively.

$$\begin{aligned} U_{chain} &= \sqrt{12N(t_{n-1})}\left[\frac{\sum_1^{n-1} t_i}{t_n \times N(t_{n-1})} - 0.5\right] \\ &= \sqrt{12 \times 27}\left[\frac{\sum_1^{27} t_i}{4\,603.92 \times 27} - 0.5\right] \\ &= \sqrt{324}\left[\frac{67\,922.93}{124\,305.84} - 0.5\right] \\ &= 18 \times 0.05 \\ &= 0.84 \end{aligned} \tag{5.9.4}$$

$$\begin{aligned} U_{sprocket} &= \sqrt{12N(t_{n-1})}\left[\frac{\sum_1^{n-1} t_i}{t_n \times N(t_{n-1})} - 0.5\right] \\ &= \sqrt{12 \times 16}\left[\frac{\sum_1^{16} t_i}{6\,248.40 \times 16} - 0.5\right] \\ &= \sqrt{192}\left[\frac{52\,903.43}{99\,974.40} - 0.5\right] \\ &= 13.86 \times 0.03 \\ &= 0.40 \end{aligned} \tag{5.9.5}$$

$$\begin{aligned} U_{wear-strip} &= \sqrt{12N(t_{n-1})}\left[\frac{\sum_1^{n-1} t_i}{t_n \times N(t_{n-1})} - 0.5\right] \\ &= \sqrt{12 \times 34}\left[\frac{\sum_1^{34} t_i}{6\,366.70 \times 34} - 0.5\right] \\ &= \sqrt{408}\left[\frac{131\,585.87}{216\,467.80} - 0.5\right] \\ &= 20.20 \times 0.11 \\ &= 2.18 \end{aligned} \tag{5.9.6}$$

Based on the observations that $-1.96 < U_{chain} < 1.96$; $-1.96 < U_{sprocket} < 1.96$, it was deduced that the failure data sets of the chain and sprocket systems indicated IID data sets; whereas $U_{wear-strip} > 1.96$ indicated a deterioration in reliability and therefore a non-IID data set.

According to Barabady and Kumar (2008, 649), the non-existence of IID data indicated that the wear-strip system was indeed a repairable systems, and resulted in a NHPP distribution set for the data, whereas the existence of IID data for the chain and sprocket components resulted in HPP distributions. Reliasoft's Weibull++ software was used to identify the best fit distribution for the failure rates of the *TB* chain and sprocket components, where Reliasoft's RGA software was used to identify the best fit distribution for the failure rate of the wear-strip component, using the failure data presented in Tables 5.12, 5.13, and 5.14, respectively. In the cases of the chain and sprocket components, the resulting best fit distributions for the failure data were determined to be the Weibull 2-parameter distribution, as shown in Figures 5.12 and 5.13; whereas the resulting best fit distribution for the wear-strip component was determined to be the Power Law distribution, as shown in Figure 5.14. The resulting failure rate distributions' parameters were determined to be the values listed in Table 5.15.

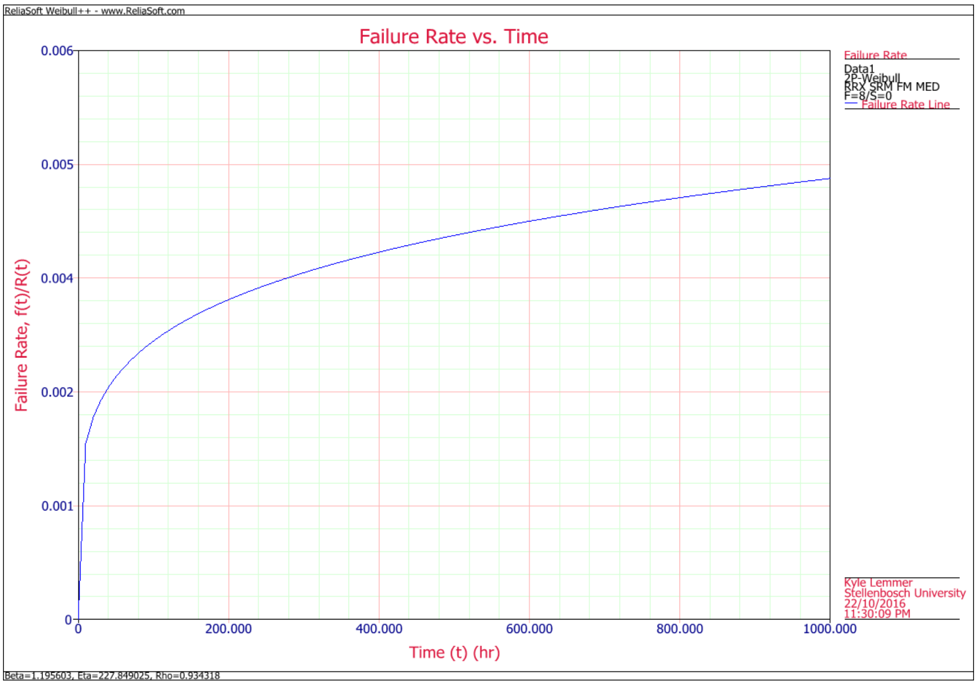


Figure 5.12: *TB* Conveyor Chain Failure Rate

***TB* Conveyor Failure Rate Distribution Parameters**			
	Chain	Sprocket	Wear-strip
Distribution:	Weibull 2-parameter	Weibull 2-parameter	Power Law
Parameters:	$\alpha = 227.849$; $\beta = 1.196$	$\alpha = 428.262$; $\beta = 1.018$	$\alpha = 0.0004$; $\beta = 1.289$

Table 5.15: *TB* Conveyor Failure Rate Distribution Parameters

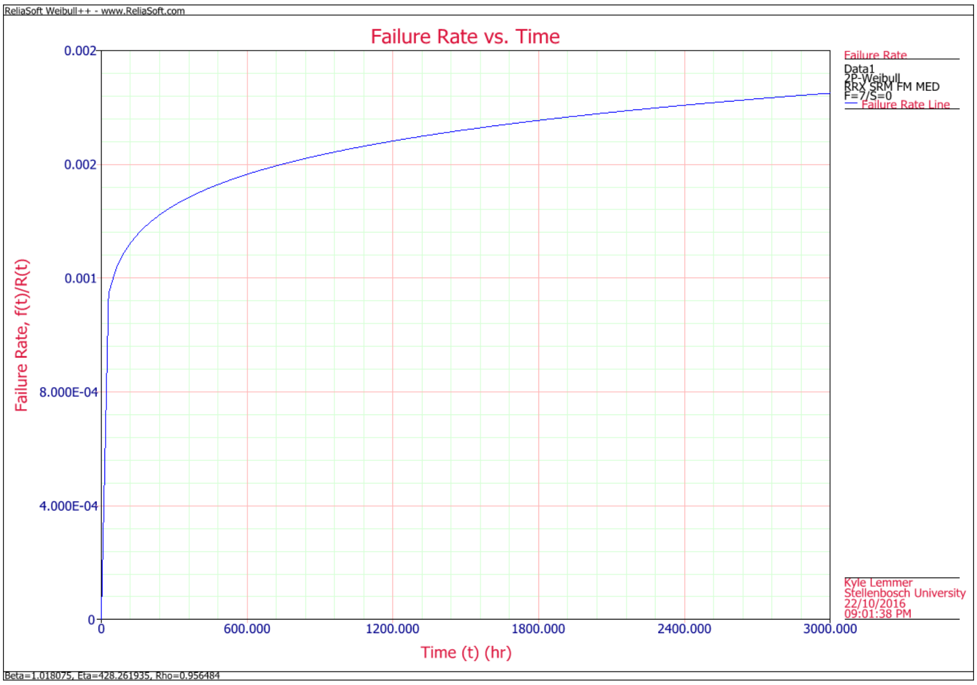


Figure 5.13: *TB* Conveyor Sprocket Failure Rate

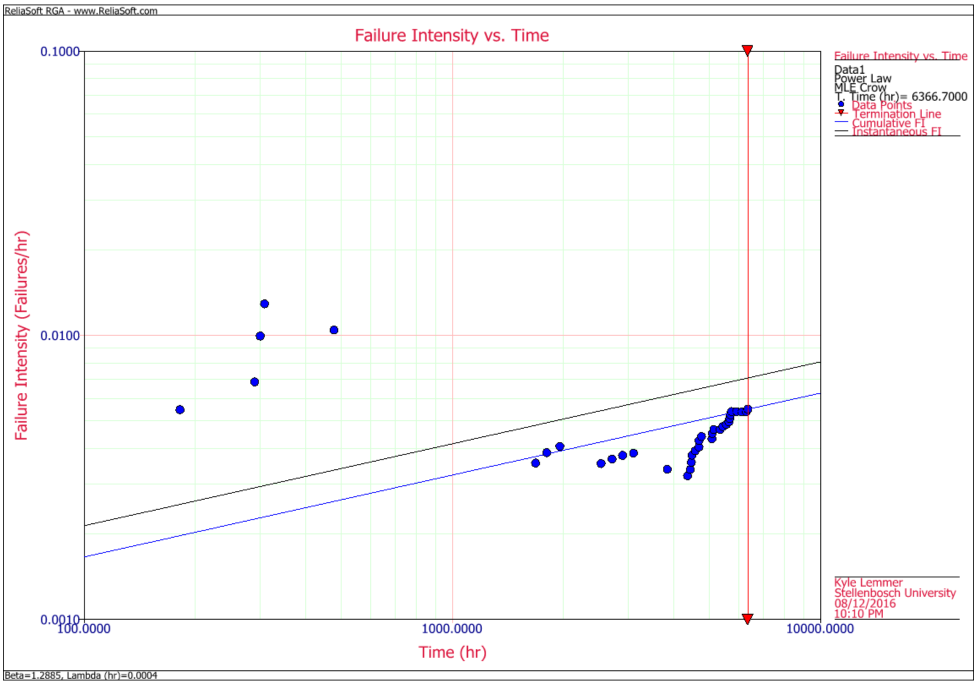


Figure 5.14: *TB* Conveyor Wear-strip Failure Rate

Based on the failure rate equation for the Weibull and Power Law distributions, the failure rates for the *TB* conveyor chain, sprocket, and wear-strip components were determined, as shown in (5.9.7), (5.9.8), and (5.9.9), respectively.

$$\begin{aligned}\lambda_{chain}(t) &= \frac{\beta_{chain}}{\alpha_{chain}}\left(\frac{t}{\alpha_{chain}}\right)^{(\beta_{chain}-1)}\\ &= \frac{1.196}{227.849}\left(\frac{t}{227.849}\right)^{(1-1.196)}\\ &= 0.005\left(\frac{t}{227.849}\right)^{(0.196)}\end{aligned} \tag{5.9.7}$$

$$\begin{aligned}\lambda_{sprocket}(t) &= \frac{\beta_{sprocket}}{\alpha_{sprocket}}\left(\frac{t}{\alpha_{sprocket}}\right)^{(\beta_{sprocket}-1)}\\ &= \frac{1.018}{428.262}\left(\frac{t}{428.262}\right)^{(1-1.018)}\\ &= 0.002\left(\frac{t}{428.262}\right)^{(0.018)}\end{aligned} \tag{5.9.8}$$

$$\begin{aligned}\lambda_{wear-strip}(t) &= \alpha_{wear-strip} \times \beta_{wear-strip} \times t^{\beta_{wear-strip}-1}\\ &= 0.0004 \times 1.289 \times t^{1.289-1}\\ &= 0.0005t^{0.289}\end{aligned} \tag{5.9.9}$$

Verification of the proposed best fit failure rate distributions for the *TB* conveyor system were determined by conducting the Chi-square goodness-of-fit test, using the total number of actual failures (from data in Tables 5.12, 5.13, and 5.14) and the expected total number of failures for each component.

Based on the twenty eight failures that were observed for the chain component, the failures were divided into four intervals, which is, the time of occurrence of the first seven failures was used to determine the expected number of failures at that particular time instant; as well as the following seven number failures; and so forth. Based on the Chi-square goodness-of-fit test, the null and alternative hypotheses were defined as follows:

H_0 : *the specified TB chain distribution is an appropriate fit for the sample data*

H_a : *the specified TB chain distribution is not an appropriate fit for the sample data*

The total number of failures for a Weibull distribution is obtained by integrating the failure rate equation over the designated time period. Using the times of occurrence of the failures (four intervals) for the chain component, the

TB Chain Expected Versus Actual Failures		
Interval Number	Expected Failures	Actual Failures
1	8.13	7.00
2	9.81	7.00
3	6.35	7.00
4	12.13	7.00

Table 5.16: TB Chain Expected Versus Actual Failures

expected number of failures at these particular time instances were obtained using (5.8.5) - as shown in Table 5.16.

The Chi-square statistic equation is shown in (5.8.6). Based on the four failure intervals, the Chi-square statistic for the chain component, based on the proposed Weibull distribution, was calculated in (5.9.10) to be equal to 3.20.

$$\begin{aligned}\chi^2_{chain} &= \frac{(7.00-8.13)^2}{8.13} + \frac{(7.00-9.81)^2}{9.81} + \frac{(7.00-6.35)^2}{6.35} + \frac{(7.00-12.13)^2}{12.13} \\ &= 0.16 + 0.80 + 0.07 + 2.17 \\ &= 3.20\end{aligned} \tag{5.9.10}$$

Considering a level of significance of $\alpha = 0.05$ and three degrees of freedom (based on the four sample intervals), the critical Chi-square value was obtained from the Chi-square distribution table as $\chi^2_c = 7.81$. Based on the Chi-square goodness-of-fit test methodology, the null hypothesis is rejected if $\chi^2_{chain} > \chi^2_c$ — which was not the case, and therefore it was concluded that there is not sufficient evidence to reject the hypothesis that the specified *TB* chain distribution is an adequate fit for the sample data.

Based on the seventeen failures that were observed for the sprocket component, the failures were divided into three intervals as follows: the time of occurrence of the first five failures was used to determine the expected number of failures at that particular time instant; as well as the following five number failures; concluding with a comparison of the final seven failures. Based on the Chi-square goodness-of-fit test, the null and alternative hypotheses were defined as follows:

H_0 : *the specified TB sprocket distribution is an appropriate fit for the sample data*

H_a : *the specified TB sprocket distribution is not an appropriate fit for the sample data*

The total number of failures for a Weibull distribution is obtained by integrating the failure rate equation over the designated time period. Using the times of occurrence of the failures (three intervals) for the sprocket component, the expected number of failures at these particular time instances were obtained using (5.8.5) - as shown in Table 5.17.

TB Sprocket Expected Versus Actual Failures		
Interval Number	Expected Failures	Actual Failures
1	5.41	5.00
2	4.31	5.00
3	5.59	7.00

Table 5.17: TB Sprocket Expected Versus Actual Failures

The Chi-square statistic equation is shown in (5.8.6). Based on the three failure intervals, the Chi-square statistic for the chain component, based on the proposed Weibull distribution, was calculated in (5.9.11) to be equal to 0.50.

$$\begin{aligned} \chi^2_{sprocket} &= \frac{(5.00-5.41)^2}{5.41} + \frac{(5.00-4.31)^2}{4.31} + \frac{(7.00-5.59)^2}{5.59} \\ &= 0.03 + 0.11 + 0.36 \\ &= 0.50 \end{aligned} \tag{5.9.11}$$

Considering a level of significance of $\alpha = 0.05$ and two degrees of freedom (based on the three sample intervals), the critical Chi-square value was obtained from the Chi-square distribution table as $\chi^2_c = 5.99$. Based on the Chi-square goodness-of-fit test methodology, the null hypothesis is rejected if $\chi^2_{sprocket} > \chi^2_c$ — which was not the case, and therefore it was concluded that there is not sufficient evidence to reject the hypothesis that the specified *TB* sprocket distribution is an adequate fit for the sample data.

Based on the thirty five failures that were observed for the wear-strip component, the failures were divided into five intervals, which is, the time of occurrence of the first seven failures was used to determine the expected number of failures at that particular time instant; as well as the following seven number failures; and so forth. Based on the Chi-square goodness-of-fit test, the null and alternative hypotheses were defined as follows:

H_0 : *the specified TB wear-strip distribution is an appropriate fit for the sample data*
H_a : *the specified TB wear-strip distribution is not an appropriate fit for the sample data*

The total number of failures for a Power Law distribution is obtained by integrating the failure rate equation over the designated time period. Using the times of occurrence of the failures (five intervals) for the wear-strip component, the expected number of failures at these particular time instances were obtained using (5.8.5) - as shown in Table 5.18.

TB Wear-strip Expected Versus Actual Failures		
Interval Number	Expected Failures	Actual Failures
1	6.92	7.00
2	11.59	7.00
3	5.58	7.00
4	5.92	7.00
5	4.95	7.00

Table 5.18: TB Wear-strip Expected Versus Actual Failures

The Chi-square statistic equation is shown in (5.8.6). Based on the five failure intervals, the Chi-square statistic for the chain component, based on the proposed Power Law distribution, was calculated in (5.9.12) to be equal to 3.27.

$$\begin{aligned}\chi^2_{wear-strip} &= \frac{(7.00-6.92)^2}{6.92} + \frac{(7.00-11.59)^2}{11.59} + \frac{(7.00-5.58)^2}{5.58} + \frac{(7.00-5.92)^2}{5.92} \\ &+ \frac{(7.00-4.95)^2}{4.95} \\ &= 0.93\times 10^{-3} + 1.82 + 0.36 + 0.20 + 0.85 \\ &= 3.27\end{aligned} \tag{5.9.12}$$

Considering a level of significance of $\alpha = 0.05$ and four degrees of freedom (based on the five sample intervals), the critical Chi-square value was obtained from the Chi-square distribution table as $\chi^2_c = 9.49$. Based on the Chi-square goodness-of-fit test methodology, the null hypothesis is rejected if $\chi^2_{wear-strip} > \chi^2_c$ — which was not the case, and therefore it was concluded that there is not sufficient evidence to reject the hypothesis that the specified *TB* wear-strip distribution is an adequate fit for the sample data.

Further analysis of the failure data was done in order to determine the improvement factors, which is defined in Section 4.3 as the improvement factor in failure rate of the component following a PM task. Considering that the failure rates of the *TB* conveyor chain, sprocket, and wear-strip, defined in (5.9.7), (5.9.8), and (5.9.9), respectively, were used to model the failure rates of the components throughout the components' life cycles, and the assumption that the failure rate of the components "reset" to zero following a PM task and increase more rapidly in the succeeding operational periods (discussed in Section 4.4.4), the relative improvement factors were obtained by comparing the theoretical versus actual number of failures for the time periods leading into the first PM and the time period between the first and second PM. Analysis of the data in Tables 5.12, 5.13, and 5.14 showed that the second PM was done at time $T_i = 3145.73$ hours. The operational time period between the first and second PM was therefore determined to be the difference between T_i at the two instances of PM, minus the time required to conduct PM at the first PM interval, as shown in (5.9.13) — equal to 1646.78 hours. The time required to conduct maintenance on the *TB* conveyor components were obtained from maintenance schedules on line four, as shown in Table 5.19.

$$\begin{aligned} T_{op} &= T(PM_2) - T(PM_1) - \max[\omega^p_{chain}; \omega^p_{sprocket}; \omega^p_{wear-strip}] \\ &= 3\,145.73 - 1\,494.95 - 4.00 \\ &= 1\,646.78 \text{ hours} \end{aligned} \tag{5.9.13}$$

***TB* Conveyor Maintenance Times**			
Component	PM [hrs]	Planned SRP [hrs]	Unplanned SRP [hrs]
Chain	4.00	10.00	12.00
Sprocket	4.00	10.00	12.00
Wear-strip	4.00	10.00	12.00

Table 5.19: *TB* Conveyor Maintenance Times

The theoretical number of failures for the chain, sprocket, and wear-strip components in the operational period of $t = 1646.78$ hours was determined to be 10.650, 3.940, and 5.772, respectively, as shown in (5.9.14), (5.9.15), and (5.9.16).

$$\begin{aligned} F(1\,646.78) &= \int_0^{1\,646.78} 0.005\left(\frac{t}{227.849}\right)^{(0.196)} \mathrm{d}t \\ &= 10.650 \text{ failures} \end{aligned} \tag{5.9.14}$$

$$\begin{aligned} F(1\,646.78) &= \int_0^{1\,646.78} 0.002\left(\frac{t}{428.262}\right)^{(0.018)} \mathrm{d}t \\ &= 3.940 \text{ failures} \end{aligned} \tag{5.9.15}$$

$$\begin{aligned} F(1\,646.78) &= \int_0^{1646.78} 0.0005t^{0.289}\mathrm{d}t \\ &= 5.772 \text{ failures} \end{aligned} \tag{5.9.16}$$

Failure data from Tables 5.12, 5.13, and 5.14 showed that the actual number of failures for the chain, sprocket, and wear-strip components in the time interval from $T_i = 1\,494.95$ hours to $T_i = 3\,145.73$ hours were 12, 4, and 7, respectively. The discrepancy of actual versus theoretical number of failures during this particular time period was used to determine the improvement factors that were to be applied to the relative failure rates following a PM task, as shown in (5.9.17), (5.9.18), and (5.9.19), where the improvement factors of the chain, sprocket, and wear-strip were determined to be 1.127, 1.015, and 1.213, respectively.

$$\begin{aligned} a_{chain} &= \frac{\text{Actual number of failures in time period}}{\text{Theoretical number of failures in time period}} \\ &= \frac{12.000}{10.650} \\ &= 1.127 \end{aligned} \tag{5.9.17}$$

$$\begin{aligned} a_{sprocket} &= \frac{\text{Actual number of failures in time period}}{\text{Theoretical number of failures in time period}} \\ &= \frac{4.000}{3.940} \\ &= 1.015 \end{aligned} \tag{5.9.18}$$

$$\begin{aligned} a_{wear-strip} &= \frac{\text{Actual number of failures in time period}}{\text{Theoretical number of failures in time period}} \\ &= \frac{7.000}{5.772} \\ &= 1.213 \end{aligned} \tag{5.9.19}$$

5.9.1.2 Cost Data Analysis of the *TB* Conveyor System

The cost data of the *TB* conveyor system was analysed in order to determined several cost inputs that were used in further analysis of the proposed multi-component maintenance cost model. All costs associated with the *TB* conveyor system during the defined time period (life cycle) of the system was used, which is, from time $T_i = 0$ hours to time $T_i = 6\,366.70$ hours.

The cost analysis involved the consideration of two cost factors, namely, (1) spare part costs; and (2) downtime costs. The spare part costs included any costs incurred over the life cycle of the chain, sprocket, and wear-strip components, particularly for spare parts used during maintenance activities. Analysis of SAB's cost records assisted in determining the costs for a single chain, sprocket, and wear-strip, as shown in Table 5.20.

Further analysis of maintenance and cost data showed that the average number of chains, sprockets, and wear-strips replaced at each PM was 5, 5, and 8, respectively. The resulting spare costs incurred for each PM task for the relative components was determined, as shown in (5.9.20), (5.9.21), and (5.9.22).

$$\begin{aligned} \text{Chain PM spare part cost} &= \text{Cost per part} \times \text{Number of parts} \\ &= R562.07 \times 5 \\ &= R2\,810.35 \end{aligned} \tag{5.9.20}$$

***TB* Conveyor System Spare Part Costs**	
Component	Cost per Item [Rand]
Chain	562.07
Sprocket	394.20
Wear-strip	247.80

Table 5.20: *TB* Conveyor System Spare Part Costs

$$\begin{aligned}\text{Sprocket PM spare part cost} &= \text{Cost per part} \times \text{Number of parts}\\ &= R394.20 \times 5 \\ &= R1\,971.00\end{aligned} \tag{5.9.21}$$

$$\begin{aligned}\text{Wear-strip PM spare part cost} &= \text{Cost per part} \times \text{Number of parts}\\ &= R247.80 \times 8 \\ &= R1\,982.40\end{aligned} \tag{5.9.22}$$

Analysis of the physical construction of the *TB* conveyor system indicated that the total number of chains, sprockets, and wear-strips in the system were equal to 90, 45, and 180, respectively. Considering that all chains, sprockets, and wear-strips were replaced upon an SRP task, the total spare part cost for each component was determined, as shown in (5.9.23), (5.9.24), and (5.9.25).

$$\begin{aligned}\text{Chain SRP spare part cost} &= \text{Cost per part} \times \text{Number of parts}\\ &= R562.07 \times 90 \\ &= R50\,586.30\end{aligned} \tag{5.9.23}$$

$$\begin{aligned}\text{Sprocket SRP spare part cost} &= \text{Cost per part} \times \text{Number of parts}\\ &= R394.20 \times 45 \\ &= R17\,739.00\end{aligned} \tag{5.9.24}$$

$$\begin{aligned}\text{Wear-strip SRP spare part cost} &= \text{Cost per part} \times \text{Number of parts}\\ &= R247.80 \times 180 \\ &= R44\,604.00\end{aligned} \tag{5.9.25}$$

The cost of downtime was analysed according to two categories: (1) cost of planned downtime; and (2) cost of unplanned downtime. In the event of planned downtime, only the costs relating to incurred labour costs were considered, seeing as the utilities costs are negligible as all production equipment are in an idle state. The incurred labour cost on line four was obtained from the cost department at SAB's Rosslyn Brewery, which calculates all costs related to labour on line four specifically, at an hourly rate. The hourly cost of labour on line four, as defined in Section 5.8.1.2, was R1 110.70 per hour. The hourly labour rate was applicable to all booked factory hours, whether planned or unplanned downtime is experienced, seeing as all personnel are present on the production line at both downtime instances. The labour costs for the events of planned PM was obtained by multiplying PM duration by the hourly labour cost rate, as shown in (5.9.26). In this particular case, the duration of PM is identical for the chain, sprocket, and wear-strip components, thereby implying that the PM labour cost for all three components remains the same.

$$\begin{aligned} TB \text{ conveyor component PM labour cost} &= \text{Cost per hour} \times \text{PM duration} \\ &= R1\,110.70 \times 4 \\ &= R4\,442.80 \end{aligned} \tag{5.9.26}$$

Using (5.9.20), (5.9.21), (5.9.22), and (5.9.26), it was possible to calculate the total cost for each planned PM activity on line four's *TB* conveyor system components, as shown in (5.9.27), (5.9.28), and (5.9.29).

$$\begin{aligned} \text{Chain PM cost} &= \text{Chain PM spare part cost} + \text{Chain PM labour cost} \\ &= R2\,810.35 + R4\,442.80 \\ &= R7\,253.15 \end{aligned} \tag{5.9.27}$$

$$\begin{aligned} \text{Sprocket PM cost} &= \text{Sprocket PM spare part cost} + \text{Sprocket PM labour cost} \\ &= R1\,971.00 + R4\,442.80 \\ &= R6\,413.80 \end{aligned} \tag{5.9.28}$$

$$\begin{aligned} \text{Wear-strip PM cost} &= \text{Wear-strip PM spare part cost} + \text{Wear-strip PM labour cost} \\ &= R1\,982.40 + R4\,442.80 \\ &= R6\,425.20 \end{aligned} \tag{5.9.29}$$

As with the PM labour cost, the planned SRP labour cost was obtained by multiplying the planned SRP duration by the hourly cost rate, as shown in (5.9.30). It must be noted that, in this particular case, the duration of a planned SRP on the chain, sprocket, and wear-strip components is identical, thereby implying that the labour cost of a planned SRP task is identical for all three components.

$$\begin{aligned} TB \text{ conveyor component planned SRP labour cost} &= \text{Cost per hour} \times \text{Planned SRP duration} \\ &= R1\,110.70 \times 10 \\ &= R11\,107.00 \end{aligned} \tag{5.9.30}$$

Using (5.9.23), (5.9.24), (5.9.25), and (5.9.30), it was possible to calculate the total cost for each planned SRP activity on line four's *TB* conveyor system components, as shown in (5.9.31), (5.9.32), and (5.9.33).

$$\begin{aligned} \text{Chain planned SRP cost} &= \text{Chain SRP spare part cost} + \text{Planned SRP labour cost} \\ &= R50\,586.30 + R11\,107.00 \\ &= R61\,693.30 \end{aligned} \tag{5.9.31}$$

$$\begin{aligned} \text{Sprocket planned SRP cost} &= \text{Sprocket SRP spare part cost} + \text{Planned SRP labour cost} \\ &= R17\,739.00 + R11\,107.00 \\ &= R28\,846.00 \end{aligned} \tag{5.9.32}$$

$$\begin{aligned} \text{Wear-strip planned SRP cost} &= \text{Wear-strip SRP spare part cost} + \text{Planned SRP labour cost} \\ &= R44\,604.00 + R11\,107.00 \\ &= R55\,711.00 \end{aligned} \tag{5.9.33}$$

In the event of an unplanned activity, whereby the production line experienced downtime during production times, the hourly rate of production costs were considered, which was obtained from the cost department at SAB's Rosslyn Brewery. As determined in Section 5.8.1.2, the cost incurred at an hourly rate during any unplanned downtime was determined to be R60 222.61 per hour. The total costs of performing unplanned SRP tasks on the chain, sprocket, and wear-strip components were obtained using the unplanned cost rate, the duration of the unplanned SRP task, and the spare part cost of the SRP activity for the chain, sprocket, and wear-strip components, as shown in (5.9.34), (5.9.35), and (5.9.36), respectively.

$$\begin{aligned} \text{Chain unplanned SRP cost} &= (\text{Cost per hour} \times \text{Unplanned SRP duration}) \\ &\quad + \text{Chain SRP spare part cost} \\ &= \left(\frac{R60\,222.61}{hour} \times 12hours\right) + R28\,103.50 \\ &= R750\,774.82 \end{aligned} \tag{5.9.34}$$

$$\begin{aligned}\text{Sprocket unplanned SRP cost} &= (\text{Cost per hour} \times \text{Unplanned SRP duration})\\ &\quad + \text{Sprocket SRP spare part cost}\\ &= \left(\frac{R60\,222.61}{hour} \times 12hours\right) + R17\,739.00\\ &= R740\,410.32\end{aligned} \tag{5.9.35}$$

$$\begin{aligned}\text{Wear-strip unplanned SRP cost} &= (\text{Cost per hour} \times \text{Unplanned SRP duration})\\ &\quad + \text{Wear-strip SRP spare part cost}\\ &= \left(\frac{R60\,222.61}{hour} \times 12hours\right) + R44\,604.00\\ &= R767\,275.32\end{aligned} \tag{5.9.36}$$

Using the MTTR values obtained for the chain, sprocket, and wear-strip components (shown in (5.9.1), (5.9.2), and (5.9.3), respectively) and the cost rate of unplanned downtime, the costs of any CM activities for the three components were determined, as shown in (5.9.37), (5.9.38), and (5.9.39). It was assumed, based on historical maintenance records, that no spare parts were used in any CM activities on the three components, therefore negating the consideration for spare part costs in the CM costs calculations.

$$\begin{aligned}\text{Chain CM cost} &= \text{Cost per hour} \times \text{Chain CM duration}\\ &= \frac{R60\,222.61}{hour} \times (5.50minutes) \times \frac{1hour}{60minutes}\\ &= R5\,520.41\end{aligned} \tag{5.9.37}$$

$$\begin{aligned}\text{Sprocket CM cost} &= \text{Cost per hour} \times \text{Sprocket CM duration}\\ &= \frac{R60\,222.61}{hour} \times (8.00minutes) \times \frac{1hour}{60minutes}\\ &= R8\,029.68\end{aligned} \tag{5.9.38}$$

$$\begin{aligned}\text{Wear-strip CM cost} &= \text{Cost per hour} \times \text{Wear-strip CM duration}\\ &= \frac{R60\,222.61}{hour} \times (6.80minutes) \times \frac{1hour}{60minutes}\\ &= R6\,825.23\end{aligned} \tag{5.9.39}$$

5.9.2 *TB* Conveyor System Modeling and Simulation

Section 5.9 has provided a firm foundation and analysis to assist further modeling and simulation of the proposed multi-component maintenance cost model for the *TB* conveyor system. This section aims at providing clarity on the determination of the parameters used in the proposed maintenance cost model, as well as the simulated results obtained from the model.

5.9.2.1 *TB* Conveyor System Model Parameter Determination

The list of notations listed in Section 4.3 provided a clear understanding of which parameters needed to be determined for use in the maintenance cost model analysis. The listed parameters were identified as either a fixed parameter or a variable parameter. The methods of determination of the fixed parameters for the *TB* conveyor system are shown in Table 5.21 (see Section 4.3 for detail on the parameter definition).

The defined fixed parameters listed in Table 5.21 provided a basis on which the variable parameters could be altered and further used as input variables, in order to determine the resulting output variable to be optimised, which is, the cost per unit of time. Referring to the ultimate aim of this study, the parameters to be determined were the (a) maintenance tasks; and (b) the frequencies thereof. The determination of the maintenance tasks exist in the decision of conducting either a PM or an SRP task on the particular component, whereas the frequency thereof exists in the decision of times to conduct PM and SRP tasks. The variable parameters are described in Table 5.22.

5.9.2.2 *TB* Conveyor System Model Optimisation and Simulation

Using the multi-component maintenance model, programmed into Matlab's software (see Appendix A), the iterative process of altering the variable input parameters was achieved by defining and running the command prompts in Matlab, as described in Appendix B. For each iteration, which is, for each alteration of a variable input parameter, a resultant output cost per unit of time was determined. Once the iteration process was complete, the expected minimum cost per unit of time, together with the associated input parameters, was determined.

In order to determine the most probable minimum output result, the Monte Carlo simulation was further utilised. The iterative process described in the preceding paragraph depicted single simulation of the model for each input argument. Based on Raychaudhuri (2008, 92)'s summarised approach of conducting an effective Monte Carlo simulation, the core of the Monte Carlo simulation lies in the process of repeating the generation of the output variable (cost per unit of time) several times, after which a statistical analysis is

***TB* Conveyor System Fixed Parameter Determination**	
Parameter	Determination
c^{pd}	The cost per unit time of planned downtime, defined earlier in this section as R1110.70.
c_n^c	The cost of a CM task for each component was obtained from (5.9.37), (5.9.38), and (5.9.39) as R5 520.41, R8 029.68, and R6 825.23 for the chain, sprocket, and wear-strip components, respectively. c_n^c is defined from notations $\bar{c}_n^c$, ω_n^c, and c^{ud}, as in (4.3.16).
c_n^p	The cost of a PM task for the each component was obtained from (5.9.20), (5.9.21), and (5.9.22) as R2 810.35, R1 971.00, and R1 982.40 for the chain, sprocket, and wear-strip components, respectively. c_n^p is defined from notations $\bar{c}_n^p$, ω_n^p, and c^{pd}, as in (4.3.17). The PM cost only included the spare part cost, as the labour cost is calculated based on the occurrence of a PM interval, using c^{pd}.
c_n^{uR}	The cost of an unplanned SRP task for each component was obtained from (5.9.34), (5.9.35), and (5.9.36) as R773 257.62, R740 410.32, and R767 275.32 for the chain, sprocket, and wear-strip components, respectively. c_n^{uR} is defined from notations $\bar{c}_n^{pR}$, ω_n^{uR}, and c^{ud}, as in (4.3.18).
c_n^{pR}	The cost of a planned SRP task for each component was obtained from (5.9.23), (5.9.24), and (5.9.25) as R50 586.30, R17 739.00, and R44 604.00 for the chain, sprocket, and wear-strip components, respectively. c_n^{pR} is defined from notations $\bar{c}_n^{pR}$, ω_n^{pR}, and c^{pd}, as in (4.3.19). The planned SRP cost only included the spare part cost, as the labour cost is calculated based on the occurrence of a planned SRP interval, using c^{pd}.
ω_n^c	The time required to conduct a CM task on each component was obtained using the MTTR values from (5.9.1), (5.9.2), and (5.9.3) as 5.50 minutes, 8.00 minutes, and 6.81 minutes for the chain, sprocket, and wear-strip components, respectively.
ω_n^p	The time required to conduct a planned PM task on each component was obtained using the maintenance data in Table 5.19 as 4.00 hours, which was applicable to all three components.
ω_n^{pR}	The time required to conduct a planned SRP task on each component was obtained using the maintenance data in Table 5.19 as 10.00 hours, which was applicable to all three components.
ω_n^{uR}	The time required to conduct an unplanned SRP task on each component was obtained using the maintenance data in Table 5.19 as 12.00 hours, which was applicable to all three components.
$\lambda^n_{(op,min)}$	The *minor* failure rates for each component were determined in Section 5.9.1.1, and defined as $\lambda^c_{(op,min)}(t) = 0.005\left(\frac{t}{227.849}\right)^{(0.196)}$, $\lambda^s_{(op,min)}(t) = 0.002\left(\frac{t}{428.262}\right)^{(0.018)}$, and $\lambda^w_{(op,min)}(t) = 0.0005t^{0.289}$ for the chain, sprocket, and wear-strip components, respectively, as shown in (5.9.7), (5.9.8), and (5.9.9).
$f^n_{(op,maj)}(t)$	Based on historical data, none of the components had experienced a *major* failure, requiring an unplanned SRP task, in the past five years. In order to determine expected lifetime of the components until a *major* failure occurs, experienced maintenance personnel were consulted, who advised that the chain, sprocket, and wear-strip components would be able to remain operational for periods of approximately sixty weeks, forty weeks, and twenty four weeks, respectively, before a *major* failure would be expected. Based on one hundred and sixty eight hours in a week, the probability of *major* failures for the chain, sprocket, and wear-strip components were assumed to be normal distribution functions, with mean values of 10 080 hours, 6 720 hours, and 4 043 hours, and standard deviations of 504 hours (3 weeks), 336 hours (2 weeks), and 168 hours (1 week), respectively.

***TB* Conveyor System Fixed Parameter Determination (*continued*)**	
a_n	The expected improvement factors for the chain, sprocket, and wear-strip components, applicable to the *minor* failure rates following a PM task, were obtained to be 1.127, 1.015, and 1.213, respectively, as shown in (5.9.17), (5.9.18), and (5.9.19).
Y^n_{op}	The expected lifetimes, before *major* failures occurred, of the components were generated as a random variable with a value based on the probability distribution of $f^n_{(op,maj)}(t)$.

Table 5.21: *TB* Conveyor System Fixed Parameter Determination

***TB* Conveyor System Variable Parameter Determination**	
Parameter	Determination
T^k	The intervals at which planned PM tasks were executed on the components were based on the current maintenance strategy implemented on line four, whereby the production line executes PM tasks on various equipment one a one-weekly basis (one hundred and sixty eight hours). The time at which PM was performed on the *TB* conveyor system's specific components, was therefore considered as multiples of the one-weekly maintenance stoppage, with the potential maximum frequency of PM therefore being every one hundred and sixty eight hours. For each component, the maximum interval between consecutive PM intervals were chosen to be equal to the standard deviation of *major* failures less than the particular component's expected lifetime, seeing as consideration of a lengthier interval would statistically result in an endless iteration process whereby the system fails to successfully complete the operational period. Considering that the expected lifetime of the chain, sprocket, and wear-strip components were determined to be 10 080 hours, 6 720 hours, and 4 043 hours, respectively, the maximum interval values for the chain, sprocket, and wear-strip (T^c, T^s, and T^w) were determined to be $T^c = 9\,576$ hours, $T^s = 6\,384$ hours, and $T^w = 3\,875$ hours, respectively.
x_n	The operational period thresholds, whereupon the particular components undergo a planned SRP was determined to range between the minimum value of one (implying a "replace only", which is, no PM tasks to be performed) and a maximum value of five.
X_{TOT}	The threshold for the sum of operational periods successfully completed by the three components was taken to be the sum of the individual defined operational periods' thresholds (x_n). The reason for this selection lay in the fact that any value greater than the sum of the individual operational period thresholds would result in an endless iteration loop in the Matlab programme, seeing as X_{TOT}, being greater than the sum of individual operational periods, would never be triggered. In the event that a cost-optimal value of X_{TOT} be coupled with a value lower than the sum of individual x_n's, this would automatically be determined by analysing the determined x_n values for the specific cost-optimal condition.

Table 5.22: *TB* Conveyor System Variable Parameter Determination

conducted in order to provide a statistical confidence obtained for the output variable.

The Monte Carlo simulation process was achieved by repeating the iterative simulation, described in the first paragraph of Section 5.9.2.2 for a total of twenty simulations. The resultant minimal cost, together with the associated input variables, was stored in a matrix format which was further used to analyse the statistical confidence of the resulting output variable. Based on the twenty simulation iterations completed for the Monte Carlo methodology, a point estimate for the cost-per-unit-of-time for each alteration of input parameters was determined, as well as the interval estimate, at a 95% confidence level, in order to determine the standard error associated with the point estimate. The point estimate for the twenty simulations was determined using (4.5.1), where the interval estimation (which is, the expected error associated with the point estimation at a 95% confidence level) was determined using (4.5.3). The associated point estimates for each parameter, together with the interval estimates, are shown in Table 5.23.

It was seen that, based on the defined input parameters, the optimised minimal cost for the *TB* conveyor system was observed to be equal to R105.38 per hour. The associated input parameters that resulted in the optimised minimal cost per unit of time are shown in Table 5.23 (see Table B.2 in Appendix B.2 for simulated optimal data).

***TB* Conveyor System Optimal Parameters**
$x_{Chain} = 1$ operational period
$x_{Sprocket} = 2$ operational period
$x_{Wear-strip} = 1$ operational periods
$T_{Chain} = 3\,364.00$ hours
$T_{Sprocket} = 1\,682.00$ hours
$T_{Wear-strip} = 3\,364.00$ hours
Cost $= R105.38$ per hour
Interval Estimate $= \pm R1.092 \times 10^{-13}$ per hour

Table 5.23: *TB* Conveyor System's Optimal Parameters and Resultant Cost

The results obtained in Table 5.23 indicated that the optimal minimum cost of R105.38 per hour was obtained $x_{Chain} = 1$ operational periods; $x_{Sprocket} = 2$ operational periods; $x_{Wear-strip} = 1$ operational periods, and $T_{Chain} = 3\,364.00$ hours; $T_{Sprocket} = 1\,682.00$ hours; $T_{Wear-strip} = 3\,364.00$ hours between successive PM intervals. The obtained optimal cost per unit of time for the *TB* conveyor system was thus translated into only performing planned SRP tasks on the chain and wear-strip components without performing PM (a "replace only" maintenance approach) every 3 364.00 hours (effectively performing a planned SRP on the chain and wear-strip components on a 20-weekly basis), whereas the sprocket component requires a PM task every 1 682.00 hours (PM on a 10-weekly basis) and a planned SRP task every 3 364.00 hours (SRP on a

20-weekly basis). It was thus evident that an entire system planned SRP task was to be executed on a 20-weekly basis — ultimately resulting in a theoretical cost per unit time of R105.38 per hour. Based on the exhaustive enumeration of input parameters analysed in the simulation, it was concluded that the 'optimal' value (cost per unit of time) had been reached, as the numerical minimum for the exhaustive input parameters was determined.

5.9.2.3 *TB* Conveyor System Cost Comparison

In order to validate the potential cost benefit that would theoretically arise if the proposed input parameters would be implemented, the actual cost per unit time for the *TB* conveyor system (specifically for the chain, sprocket, and wear-strip components) was determined, using historical failure, maintenance, and cost data for the system, over the aforementioned period, ranging from 31st August 2015 to 15th July 2016. To ensure consistency between the proposed model and the actual cost incurred, the analysis of the actual incurred cost over the time period considered the following cost factors:

1. PM Costs of the chain, sprocket, and wear-strip components
2. Planned and unplanned SRP costs of the chain, sprocket, and wear-strip components
3. CM costs of the chain, sprocket, and wear-strip components

The actual PM cost for the *TB* conveyor system was calculated based on components' labour and spare part costs and determined to be equal to R31 497.80, as shown in Table 5.24. It was noted that the PM was performed simultaneously on the three components, thus only incurring 4 hours of labour cost.

***TB* Conveyor System Actual PM Cost**				
Component	No. of PM Intervals	Labour Cost per Interval	Spare Part Cost per Interval	Component Total PM Cost
Chain	4		R2 810.35	R11 241.40
Sprocket	4	R4 442.80	R1 971.00	R7 884.00
Wear-strip	4		R1 982.40	R7 929.60
			Total PM Cost:	**R**31 497.80

Table 5.24: Actual Total PM Cost of *TB* Conveyor System

The actual planned and unplanned SRP cost of the *TB* conveyor system was calculated based on components' labour and spare part cost and determined to be equal to R124 036.30, as shown in Table 5.25. It was noted that the planned SRP tasks were performed simultaneously on the three components, thus only incurring 10 hours of labour cost.

***TB* Conveyor System Actual SRP Cost**						
Component	No. of SRP Intervals		Labour Cost per Interval		Spare Part Cost per Interval	Component Total SRP Cost
	Planned	Unplanned	Planned	Unplanned		
Chain	1	0	R11 107.00	R13 328.40	R50 586.30	R50 586.30
Sprocket	1	0		R13 328.40	R17 739.00	
Wear-strip	1	0		R13 328.40	R44 604.00	R44 604.00
					Total SRP Cost:	**R**124 036.30

Table 5.25: Actual Total SRP Cost of *TB* Conveyor System

The actual CM cost of the *TB* conveyor system was calculated based on the actual unforeseen *minor* failures experienced by the chain, sprocket, and wear-strip components during the aforementioned time period and the cost per unit time of unplanned downtime, and was determined to be equal to R951 787.24, as shown in Table 5.26.

***TB* Conveyor System Actual CM Cost**			
Component	Unplanned Downtime Experienced [$hours$]	Cost per Unit of Unplanned Downtime [R/hr]	Component Total CM Cost
Chain	5.20	R60 222.61	R313 157.57
Sprocket	3.50	R60 222.61	R210 779.14
Wear-strip	7.10	R60 222.61	R427 850.53
		Total CM Cost:	**R**951 787.24

Table 5.26: Actual Total CM Cost of *TB* Conveyor System

Based on the obtained cost data in Tables 5.24, 5.25, and 5.26, the total actual cost incurred for the *TB* conveyor system over the aforementioned time period was calculated to be equal to R1 107 321.34, as shown in (5.9.40).

$$\begin{aligned}\text{Actual } TB \text{ Total Cost} &= \text{Total PM Cost} + \text{Total SRP Cost} + \text{Total CM Cost}\\ &= R31\,497.80 + R124\,036.30 + R951\,787.24\\ &= R1\,107\,321.34\end{aligned} \tag{5.9.40}$$

Analysis of the data in the aforementioned time period indicated that the *TB* conveyor system was in operation for a total of 6 366.70 hours. The actual cost per unit of time for the *TB* conveyor system was thereby calculated to be equal to R173.92 per hour, as shown in (5.9.41).

$$\begin{aligned}\text{Actual } TB \text{ Cost Rate} &= \frac{\text{Actual } TB \text{ Total Cost}}{\text{Total Operational Time}}\\ &= \frac{R1\,107\,321.34}{6\,366.70\text{hours}}\\ &= R173.92 \text{ per hour}\end{aligned} \tag{5.9.41}$$

Comparison of the obtained theoretical cost rate to the actual incurred cost rate of the *TB* conveyor system indicated a potential cost savings of R68.54 per hour of operation of the cam system, as shown in (5.9.42).

$$\begin{aligned}\text{Potential Cost Saving Rate} &= \text{Actual } TB \text{ Cost Rate} - \text{Model Cost Rate}\\ &= R173.92 - R105.38\\ &= R68.54 \text{ per hour}\end{aligned} \tag{5.9.42}$$

Based on the aforementioned time period over which data for the *TB* conveyor system were analysed, the total theoretical cost saving is equal to the cost saving rate multiplied by the operational hours during the specified time period, and was determined to be equal to R436 373.62, as shown in (5.9.43).

$$\begin{aligned}\text{Theoretical Cycle Cost Saving} &= \text{Potential Cost Saving Rate} \times \text{Operational Time}\\ &= R68.54 \times 6\,366.70 \text{ hours}\\ &= R436\,373.62\end{aligned} \tag{5.9.43}$$

5.9.2.4 Summary of Results of *TB* Conveyor System Maintenance Optimisation

Based on historical failure, maintenance, and cost data, the *TB* conveyor system components were analysed over a cycle, initiating from the "good as new" state, until the next planned SRP task was executed on the system. Using the failure data over this period, the failure rates of the chain and sprocket components were determined to be best described by two-parameter Weibull distributions (using Reliasoft's Weibull++ software), with inherent parameters of $\alpha_{chain} = 227.849$; $\beta_{chain} = 1.196$; $\alpha_{sprocket} = 428.262$; and $\beta_{sprocket} = 1.018$; where the best fit failure distribution for the wear-strip component was determined to be the Power Law (using Reliasoft's RGA software), with parameters $\alpha_{wear-strip} = 0.0004$; and $\beta_{wear-strip} = 1.289$. In order to verify the goodness-of-fit of the best-fit failure rate distributions of the components, the Chi-square goodness-of-fit test was conducted for each component — where all three cases indicate that the null hypotheses (which is, that the proposed best-fit distributions correlate to the sample data with a significance level of 95% confidence) are not rejected. By analysing the failure rates over two consecutive cycles for each component, the improvement factors, applicable to the failure rates following a PM task on a particular component, were determined to be $a_{chain} = 1.127$, $a_{sprocket} = 1.015$, and $a_{wear-strip} = 1.213$.

The utilisation of all relevant cost data were used, in conjunction with the developed maintenance cost model, in order to simulate the expected cost per unit of time under varying input parameter conditions. Based on the simulation of the proposed multi-component cost model, the optimal cost per unit of time was determined to be equal to R105.38 per hour, with coupled parameters $x_{chain} = 1$, $x_{sprocket} = 2$, and $x_{wear-strip} = 1$ (operational periods) and $T_{chain} = 3\,364.00$ hours, $T_{sprocket} = 1\,682.00$ hours, and $T_{wear-strip} = 3\,364.00$ hours (PM intervals) — thus indicative of performing PM only on the sprocket component every 1 682.00 hours, and performing a planned system SRP after two successful completions of operational periods by the sprocket component (the chain and wear-strip components are only replaced, which is, a planned SRP, and no PM is conducted on these two components). The point estimate for the optimal parameters indicated a cost-per-unit-of-time of R105.38, with an interval estimate (standard error) of $\pm R1.092 \times 10^{-13}$. It was evident from the simulation results' statistical analysis that the interval estimate significantly increased as the operational period approached the MTBF of the cam system — where the logic lay in the fact that the probability of an unforeseen occurrence of a *major* failure significantly increases, which ultimately contributes the largest cost-per-unit-of-time of all failure and maintenance activities. Based on the exhaustive enumeration of input parameters analysed in the simulation, it was concluded that the 'optimal' value (cost per unit of time) had been reached, as the numerical minimum for the exhaustive input parameters was determined.

The validation of the proposed cost model lay in the comparison between actual cost rate and theoretical cost rate. This was done by comparing the actual incurred costs over the same time period (cycle) to the optimal minimum cost obtained from the proposed cost model. The potential theoretical cost saving rate was determined to be R68.54 per hour of operation, which ultimately translated into a potential cycle cost saving of R436 373.62 for the analysed cycle time period.

5.10 Final Remarks

The single- and multi-component maintenance cost models have been applied and simulated based on the bottle washer cam system and *TB* conveyor system, respectively, thus allowing for a final comparison to be conducted. Initiating with the analysis of historical data for both systems, the failure data for the cam system indicates that the system exhibits a repairable system behaviour with increasing failure rates (using the Laplace test), where a Power Law distribution proves to be the best-fit distribution for the failure rate (using ReliSoft's RGA software), with the Chi-square goodness-of-fit test indicative of the proposed distribution being a relevant fit with a significance level of 95%. The failure rate for the chain and sprocket components (in the multi-component system), indicates no significant trend (using the Laplace test), where the two-parameter Weibull distribution proves to be the best-fit distribution for the failure rates (using ReliaSoft's Weibull++ software), with the Chi-square goodness-of-fit test indicative of the proposed distributions being relevant fits with a significance level of 95%. The failure rate for the wear-strip component (in the multi-component system), indicates a repairable system behaviour with increasing failure rate (using the Laplace test), where the Power Law distribution proves to be the best-fit distribution for the failure rate (using ReliaSoft's RGA software), with the Chi-square goodness-of-fit test indicative of the proposed distribution being a relevant fit with a significance level of 95%.

Parallel analysis of the maintenance data records indicates that, following a PM task, both systems' failure rates can be modelled using the "improvement factor" methodology suggested by Sheu *et al.* (2012, 1270). The component-specific improvement factors, shown in Table 5.27, suggest that the bottle-washer cam system's failure rate increases at the greatest rate following a planned PM, followed by the *TB* wear-strip, *TB* chain, and *TB* sprocket components.

Component-specific Improvement Factors	
Component	Improvement Factor (a)
Bottle-washer Cam	1.503
TB Chain	1.127
TB Sprocket	1.015
TB Wear-strip	1.213

Table 5.27: Component-specific Improvement Factors

By defining the required parameters and simulating the single- and multi-component models using Matlab software, it is evident that there exist cost-saving potential for SAB Rosslyn Brewery's line four on both the bottle-washer cam system, as well as the *TB* conveyor system. Regarding the bottle-washer cam system, line four currently employs the maintenance approach of executing planned PM on the cam system twice between bi-annual maintenance shut-downs, which is, approximately every 13 weeks, where a planned SRP is executed during every bi-annual maintenance shut-down, which is, approximately every 26 weeks. The resulting actual cost per unit time for the bottle washer cam system, under the current maintenance conditions, is R207.83 per hour of operation. By performing a Monte Carlo simulation the cost-optimal maintenance approach is to perform a planned SRP every six weeks — ultimately resulting in a cost per unit time of R179.52 per hour of operation (based on 95% confidence, the expected error at these conditions is $\pm R1.1386 \times 10^{-14}$). Based on the proposed optimal maintenance methodology, seen in Table 5.28, the potential cost saving for SAB Rosslyn Brewery's line four on the bottle-washer cam system is R28.31 per hour of operation, essentially resulting in a 13.62% cost reduction. Regarding the *TB* conveyor system, line four currently employs the maintenance approach of executing planned PM on the chain, sprocket, and wear-strip components every twelve weeks, where a planned SRP is executed at every second bi-annual maintenance shut-down, which is, every fifty-two weeks. The resulting actual cost per unit of time for the specified *TB* conveyor system, under the current maintenance conditions, is R173.92 per hour of operation. By performing a Monte Carlo simulation the cost-optimal maintenance approach is to perform planned PM only on the sprocket component every ten weeks, with a planned system SRP every twenty weeks — ultimately resulting in a cost per unit time of R105.38 per hour of operation. Table 5.28 illustrates that the potential cost saving for SAB Rosslyn Brewery's line four on the specified *TB* conveyor system is R68.54 per hour of operation, essentially resulting in a 39.41% cost reduction.

Cost Saving Opportunities for Line 4					
Component	Current Maintenance Approach		Proposed Maintenance Approach		Potential Cost Savings [$R/hour$]
	PM Frequency [*weekly*]	SRP Frequency [*weekly*]	PM Frequency [*weekly*]	SRP Frequency [*weekly*]	
Bottle-washer Cam	13	26	—	6	28.31
TB Chain	12	52	—	20	
TB Sprocket	12	52	10	20	68.54
TB Wear-strip	12	52	—	20	

Table 5.28: Cost Saving Opportunities for Line 4

5.11 Chapter Summary

The literature study conducted in Section 2 identifies the current challenge faced by production facilities, specifically in the FMCG environment, regarding cost-inefficiencies in currently-employed RCM strategy methodologies. Despite the global-wide adoption of the RCM approach to maintenance, there exists no sound foundation for claiming that the maintenance strategy derived from the RCM approach is in any sense 'optimal'. Inefficient RCM-based maintenance tasks, as well as frequencies thereof, inevitably results in either costly over-maintaining of equipment leading to avoidable excessive maintenance costs, or, at the other end of the scale, under-maintaining of equipment leading to avoidable unplanned production downtime — where a visual description of this 'trade-off' can be seen in Figure 2.3. Considering the possible inefficiencies in maintenance task and frequency selections, many researchers have focused on the problem of developing mathematical models of deteriorating systems, where more complex systems can no longer be described by analytical models, and therefore leads to the use of simulation methods, such as the Monte Carlo methodology, in order to determine optimal maintenance parameters.

Using the vast amount of research conducted on the mathematical modeling of maintenance decision parameters, it was possible to construct a mathematical model for the relevantly identified parameters in an RCM-based maintenance strategy methodology. Considering the possible interdependencies of components within a system, it was chosen to develop both single- and multi-component mathematical models, where the latter is takes into consideration the possible cost benefits of conducting simultaneous maintenance tasks on a number of components within a particular system.

In order to validate the proposed single- and multi-component maintenance cost models it was chosen to investigate the possible application of the models within the FMCG environment, by conducting a case study on one of the production lines in SABMiller's Rosslyn Brewery. Analysis of the failure, maintenance, and cost historical data for SAB Rosslyn Brewery's line four production line, resulted in the determination of the defined input parameters to be further used in the modeling and simulation tasks. The use of Reliasoft's Weibull++ and RGA software were additionally utilised in order to define the best-fit failure distribution properties of the relative components, where validation of the best-fit failure rate distributions were conducted using the Chi-square goodness-of-fit test — where the null hypotheses of all distributions were not rejected.

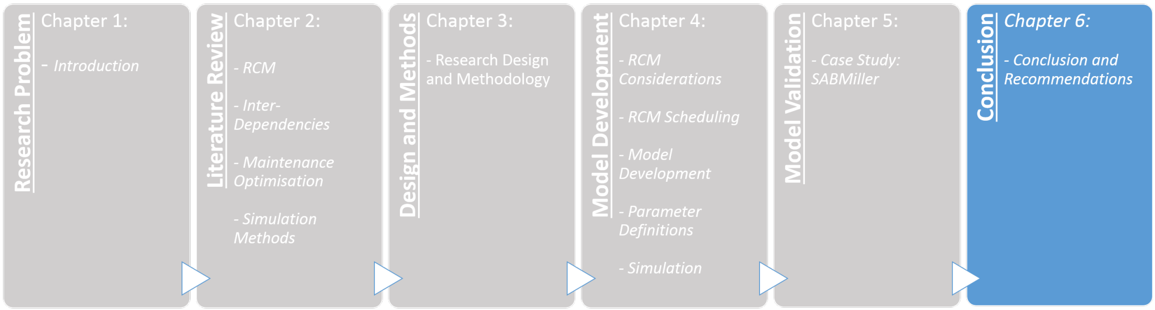
Following the Monte Carlo simulation methodology, suggested by Raychaudhuri (2008, 92), simulation results indicated interval estimates of $\pm R1.1386 \times 10^{-14}$ and $\pm R1.092 \times 10^{-13}$ for the washer cam and TB systems, respectively, at a 95% confidence level. The constant results of the optimal parameters are largely based on the fact that, for a minimal cost per unit time, unplanned SRP tasks should entirely be avoided, seeing as this task, for both single- and multi-component systems incurs the highest cost of all defined tasks. Considering the significantly large contribution that the unplanned SRP tasks have on the cost per unit time, decision of parameters would be chosen to perform PM tasks and ensure frequencies of planned SRP tasks, in order to statistically minimise the probability of any unplanned SRP events — which is, PM and planned SRP tasks are to be conducted at a certain frequency which does not exceed the components' MTBF values for *major* failures.

Selecting the bottle-washer cam system and the TB conveyor system for the validation case study of the single- and multi-component models, respectively, clearly indicated that there exists the possibility to implement changes to currently-employed maintenance methodologies on line four, in order to optimise bottom-line costs incurred by the production facility. In the case of the bottle-washer cam system, a possible 13.62% cost reduction can be obtained if the proposed single-component mathematical model results were to be implemented, which is, to only perform a planned SRP on the system every six weeks. In the case of the TB conveyor system, a possible 39.41% cost reduction can be obtained if the proposed multi-component mathematical model results were to be implemented, which is, to perform planned PM only on the sprocket component every ten weeks and a planned system SRP on the entire defined system every twenty weeks.

It is evident from the above results that by utilising mathematical modeling techniques, simulation processes, and analysis of historical data, it is possible to determine RCM-based tasks and frequencies for components which will ultimately result an optimised bottom-line cost for a production facility.

Chapter 6

Conclusion and Recommendations



6.1 Summary and Conclusion

In today's highly competitive environment, to be successful and to achieve world-class manufacturing, organisations must posses both efficient maintenance and effective maintenance strategies. The main question faced by maintenance management is whether its output is produced effectively, in terms of contribution to company profits, and efficiently, in terms of manpower and materials employed. Maintenance costs form a major portion of the total operating costs in most plants, of which the major contributors to abnormal costs are delays, product rejects, scheduled maintenance downtime, and traditional maintenance costs (for example, labour, overtime, and repair costs). Following the emergence of the RCM-methodology in the aviation industry in the 1960's, the proven approach of RCM has led to organisations throughout a wide range of industries introducing RCM to optimise their maintenance operations. Despite the global-wide adoption of the RCM approach to maintenance, however, there exists no sound foundation for claiming that the maintenance strategy derived from the RCM approach is in any sense 'optimal'. Following the summarised 7-step approach to implementing an effective RCM-based maintenance methodology, the final step, which involves the selection of optimal intervals at which to perform planned maintenance tasks, is considered to be the most challenging step — where inefficient selections thereof instils the risk of either (a) over-maintaining of equipment, thus resulting in excessive maintenance costs; or (b) under-maintaining of equipment, thus resulting in unreliable equipment operation and, ultimately, excessive incurred costs in the form of lost production time. Further complexity arises when consideration is taken for possible inter-dependencies between components, whereby the possibility exists for potential cost savings through simultaneous maintenance task executions on economically or structurally dependent components.

In an attempt to optimise maintenance task and frequency decision making, many researchers have focused on the problem of developing exhaustive mathematical models of deteriorating systems, whereby every maintenance model will try to predict or extrapolate the performance of the system in question. Using reliability, and subsequently probability, as a basis, models that describe system performance as a function of maintenance effort provide a means for selecting the most efficient and effective equipment service strategies and policies. Yet, in all these cases, the models had to be built under simplified assumptions. One of these assumptions is the consideration of maintenance tasks, whether planned or unplanned, to incur negligible times. In the fast-paced, mass-production FMCG environment, however, this assumption proves to be invalid. For this reason, it was deemed necessary to develop a maintenance model for both single- and multi-components that would be applicable in an FMCG environment, which would ultimately assist maintenance management, by following a structured approach, in optimising bottom-line costs through effective maintenance task and frequency determinations.

During the extensive literature study, a historical and holistic understanding of the RCM-based methodology was provided in Chapter 2. It was evident, from this research, that an effective implementation of the RCM-based methodology requires certain pre-requisite steps prior to attempting the optimisation of maintenance tasks and frequencies. The final task, which involves the determination of preventive maintenance tasks and frequencies thereof, was further studied, which essentially formed the foundation of the development of the proposed mathematical model. Due to the aforementioned possibility of potential cost savings, as a result of inter-dependencies between components, a thorough understanding of the type and existence of dependencies was provided in Chapter 2 — with the resulting decision to develop both single- and multi-component maintenance cost models. Included in Chapter 2 was an extensive literature research into current maintenance models, which assisted the further development of the proposed maintenance model, with the additional consideration of maintenance tasks incurring a certain cost factor to the bottom-line cost for the production facility. It was determined in the research study that the probabilistic modeling of equipment under certain maintenance conditions relies heavily on the failure probability distribution parameters of the equipment under consideration. For this reason, a firm foundational understanding of the types of failure parameters was included in Chapter 2 — where the analysis of data and, ultimately, the determination of the existence of an IID data set provides clarity on the application of either a HPP or a NHPP to represent the failure distribution parameters of the equipment. Determination of the best-fit distribution parameter for the failure rates of equipment is dependent on the equipment in consideration, where the use of software, such as Reliasoft's Weibull++ software, simplifies this process and allows the analyst to easily identify the best-fit failure rate distribution for the equipment with the use of historical failure data.

Based on the RCM premise of pre-defined maintenance intervals, the proposed single- and multi-component maintenance models were developed in Chapter 4, with specific consideration of the 'improvement factor' approach, whereby components are considered to be in an intermediate state between a "good as new" and "bad as old" state following a PM task. Only upon an SRP task was the specific component considered to be "as good as new". For both the single- and multi-component systems, it was possible to define a system 'cycle', which in turn provided a basis on which to calculate the cost per unit of time under certain maintenance conditions for the specific component. By programming the proposed maintenance model cycle into Matlab software, together with the coupled inherent cost factors, it was possible to construct a summarised and structured approach to be followed during the maintenance model optimisation process (shown in Table 4.1).

In order to validate the accuracy and efficacy of the proposed models, it was decided to conduct a case study on a current FMCG production facility, namely, SAB Rosslyn Brewery's Line 4 beverage packaging line. Comparison of the expected cost per unit of time with actual data obtained from SAB yielded accuracies of 98.11% and 97.33% for the single- and multi-component models, respectively, thus indicative of relatively high accuracies. Further validation of the efficacy of the cost models through the Monte Carlo simulation process resulted in expected cost per unit time savings of 27.44% and 19.57% for the single- and multi-component models, respectively. The expected cost savings indicated that the current maintenance tasks and frequencies conducted on the specified components are not optimal regarding bottom-line cost and, therefore, the possibility exists to implement the proposed optimal maintenance tasks and frequencies, obtained from the proposed model simulations, in order to benefit on expected bottom-line cost savings for the production facility.

It is evident that all sub-questions to the primary research question were addressed and answered, as seen in Section 1.3 and illustrated in Table 1.1. The first sub-question (a.) was addressed and answered in Chapter 2, which provided a thorough understanding of the RCM-based methodology and its historical background. Chapter 2 provided a 7-step approach assisting in the decision-logic of RCM-based maintenance methodology, where foundation was laid in terms of further development of decision-logic for maintenance tasks and frequencies specifically — thus addressing the sub-question (b.). In order to optimise the decision-logic, raised in sub-question (c.), the field of mathematical modeling and simulation was covered in Chapter 4, which involved an extensive literature background and fundamental understanding of the development, application and techniques used in constructing a relevant mathematical model. Sub-question (d.) required the development of a structured approach to the optimisation of the proposed mathematical models, which was addressed and defined in Table 4.1 in Chapter 4. Ultimately, the validation of the proposed optimisation models was achieved by simulating the programmed models using the Monte Carlo methodology, which covers sub-question (e.) through a case study in Chapter 5.

It is therefore viable to state that this research study successfully achieved all objectives listed in Section 1.4. The following statements were met:

1. The fundamentals of RCM were established in Chapter 2:

   a. The historical background of RCM was reviewed

   b. Definitions of corrective, preventive, and predictive maintenance were successfully established

   c. Identification of the fundamental principles of RCM and the implementation thereof were covered

2. Factors influencing he decision-making process of relative dependencies between components to be maintained were covered in Chapter 2:

   a. Definitions of economic, structural, and stochastic dependencies were provided

   b. Identification of factors to be considered when determining relative dependencies between components was achieved

3. A well defined research methodology was constructed in Chapter 3

4. Investigation of the academic literature and methodologies of mathematical modeling was conducted in Chapter 2

5. The development of a mathematical model, which takes into consideration all costs incurred over a component's life cycle, was conducted in Chapter 4:

   a. All inputs and outputs to be included in the maintenance model were identified

   b. Based on the above inputs and outputs, an applicable mathematical maintenance model was constructed

c. Optimal maintenance tasks and frequencies were determined, based on the pre-constructed models, in Chapter 5

6. A structured approach for the optimisation of the proposed mathematical maintenance models, which can be further utilised on alternative equipment within an FMCG environment, was developed and illustrated in Table 4.1 in Chapter 4

7. A literature study was conducted to investigate simulation tools in Chapter 2

8. A case study on an FMCG production facility's maintenance strategy was conducted in Chapter 5:

    a. Data collection of the relative input and output performance of the facility was performed

    b. Simulations of the theoretical performance of the facility, in the case that the theoretical model were to be implemented, was conducted

    c. The potential cost savings, based on the simulation results, were identified

9. Chapters 5 and 6 conclude results of the study

In conclusion, the successful completion of all the study research objectives enables this study to answer the primary research question, with which the following can be stated:

Optimal RCM-based maintenance tasks and frequencies within an FMCG production environment can be determined by constructing an applicable mathematical model, with the combined utilisation of failure probability distributions and simulation techniques.

During the process of completing this study, some limitations were encountered. These limitations are discussed in Section 6.2.

6.2 Study Limitations

Limitations are encountered during the development and validation of the mathematical model and simulation techniques. It is essential to list these limitations to provide the reader, and potential user of the proposed optimisation methodology, with more comprehensive information on the maintenance task and frequency optimisation process. The aforementioned limitations are:

1. The process of the development of the single- and multi-component cost models is based on the assumption of currently-implemented RCM-based

maintenance methodologies. Both models therefore assume that the relative PM and SRP tasks are relevant and based on an FMEA and resulting task identification. Inaccurate presumption of relevant PM and SRP tasks will ultimately result in inaccuracy of model parameter determinations and simulation techniques.

2. The accuracy of historical data, as well as the number of observations, play a major role in the efficacy and accuracy of the proposed models. Ensuring accuracy and relevancy of failure, maintenance, and cost data for the component under consideration allows for a more accurate modeling and simulation process. Fewer observation data points will also inevitably result in a less accurate representation of the component's failure distribution parameters.

3. The assumption that the improvement factor (a_n) remains constant following each PM task may not always be an accurate assumption, as ageing components may possess a non-linear increase or decrease in failure rate following the PM task.

4. In order to avoid excessive complexity of the proposed models, the *major* failure rate distributions were assumed to be represented by normal distributions with coupled standard deviations. In certain cases, whereby *major* failures are not represented by IID data sets, which is, a trend exists in the occurrence of *major* failures, the assumption of the normal distribution parameter may be inaccurate and will require further development of the proposed models.

5. The proposed models rely on the assumption that planned maintenance intervals are fixed, whereupon further maintenance tasks and frequencies are planned. In the event that maintenance intervals vary with time, the proposed models will not be entirely applicable to the desired optimisation process.

6. The proposed models are based on the assumption that the CM and PM tasks incur a fixed, averaged time and cost, based on historical data. Depending on the variation of these factors in the relevant environment, the accuracy of the models may be affected.

7. The proposed models are based on the assumption that the components under investigation form part of a series-system configuration, thereby implying that any downtime experienced by the particular component results in the downtime of the system as an entirety. In the event that the aforementioned components are not configured in a series configuration, the optimisation methodology of the models may not be applicable.

8. Within the case study conducted at SAB, it is evident that the cost of unplanned production downtime, as compared to that of planned downtime (which is, maintenance time), is significantly larger (comparing the R60 222 cost of unplanned downtime to the R1 110 cost of planned downtime per hour). Although this is expected to be the case in the majority of FMCG environments, mention is made that the large difference between the two downtimes could result in biased decisions being made — leaning to the preference of higher frequencies of planned PM and SRP events.

To address the above-mentioned limitations, recommendations for future research are discussed in Section 6.3.

6.3 Recommendations and Future Research

Through experience gained in completing this study, areas have been identified whereby future research could improve results and extend to a broader range of environments in which the maintenance optimisation approach may be applied. The recommendations and resulting potential areas for future research are listed:

1. A more holistic approach of the optimisation of the RCM-based methodology, whereby the analysis and efficacy of specific maintenance tasks are determined, may provide a more extensive optimisation approach. This would essentially involve optimisation of alternative steps in the 7-step RCM methodology proposed in Section 2.1.2. It could be considered that the potential study would "take a step back" in order to determine the efficacy and effectiveness of specific maintenance tasks prior to the attempt of optimising maintenance tasks and frequencies thereof.

2. The potential improvement of the proposed single- and multi-component maintenance models may be achieved by further developing the definition of the improvement factor applicable to failure rates following a PM task. It is proposed that the scope of historical data be broadened in order to analyse and determine the effect of failure rates over more than two operational periods, which is, considering the observed failure rates experienced between successive PM intervals where the PM interval count is higher than two. The result would be a more concise definition and determination of actual improvement factors applicable to the component and, ultimately, a more accurate representation of expected failures.

3. Further potential improvement of the proposed models could be obtained by conducting additional research on the occurrence of *major* failures experienced by the component under consideration. The resulting study

would determine the failure rates of *major* failures, thus resulting in a more accurate model of the system.

4. Considering that not all production facilities are based on the series configuration of components, an alternative approach may be considered in which the component form part of a parallel system configuration. The study would consider conditional circumstances of 'system' downtime versus 'component' downtime, where the latter does not necessarily result in the former.

5. The proposed models both assume a fixed interval between planned PM tasks. The flexibility of the model may be improved by furthering research into a variable time period between planned PM intervals. This would essentially allow for the analyst to alter intervals between PM tasks and, possibly, provide for further optimisation potential within the modeling approach.

6. This study has provided an understanding of the potential cost savings that could arise as a result of simultaneously maintaining components where inter-dependencies may exist. It is proposed that further studies aim at developing a process whereby dependencies can be identified and, as a result, the potential cost savings of simultaneous maintenance execution may be quantified.

The recommendations listed above may provide interesting areas for potential research in the maintenance optimisation and asset care field.

Appendices

Appendix A

Matlab Coding for Single- and Multi-component Cost Models

A.1 Single-component Cost Model

The Matlab software is used in order to construct the mathematical model to determine the cost per unit of time for the single-component system, given the value of the pre-defined parameters. The constructed mathematical model, as seen below, simulates (4.3.2) and (4.3.13) to determine the expected cost per unit of time for the single-component system, where the cost per unit time is denoted by the variable "cost" in the Matlab code.

In order to proceed with the cost per unit time simulation, a total of 12 input parameters are passed to the function "sing1". The input parameters are defined as follows:

cpr	The total cost of conducting a PM task on the component (equal to denotion c^p in (4.3.6)).
cuR	The total cost of an unplanned SRP on the component (equal to denotion c^{uR} in (4.3.7)).
cc	The total cost of conducting a CM task on the component (equal to denotion c^c in (4.3.5)).
cpR	The total cost of a planned SRP on the component (equal to denotion c^{pR} in (4.3.8)).
x	The number of operational intervals successfully completed by the component, whereupon a planned SRP is executed in the next PM interval.
T	The time period between successive PM intervals.
a	The improvement factor of the component, following a PM task.

funmin	The failure rate of the component (equal to $\lambda(t)$).
pdmaj	The PDF of the expected lifetime of the component, which is, the expected lifetime between *major* failures.
wp	The time required to conduct a PM task on the component.
wpR	The time required to conduct a planned SRP on the component.
wuR	The time required to conduct an unplanned SRP on the component.

The function “sing1”, which receives the input argument parameters is seen below:

```
function [cpm,P,op,fail,maj,cur,cput,Tstar] = sing(cpr,
    cuR,cc,cpR,x,T,a,funmin,pdmaj,wp,wpR,wuR)
    P=0;
    sigmaop=T;
    Tstar=T-wp;
    cpm=0;
    cur=0;
    op=1;
    fail=0;
    maj=0;
    c=0;
    cycle=0;
    cput=0;
    while (op <= (x-1))
        Yop=abs(random(pdmaj));
        if Yop >= sigmaop
            cpm=cpm+cpr+cc*((a.^(op-1))*(integral(funmin
                ,0,sigmaop)));
            fail=fail+(integral(funmin,0,sigmaop));
            sigmaop=Tstar;
            P=P+1;
            op=op+1;
        else
            cur=cur+cuR+cc*((a.^(op-1))*(integral(funmin
                ,0,Yop)));
            op=1;
            sigmaop=Tstar-Yop-wuR;
            op=1;
            maj=maj+1;
        end
```

```
    end
    Yop=abs(random(pdmaj));
    if Yop >= Tstar
        cpm=cpm+cpR+cc*((a.^(op-1))*(integral(funmin,0,Tstar)));
    else
        Tend=Tstar-Yop-wuR;
        cur=cur+cuR+cc*((a.^(op-1))*(integral(funmin,0,Yop)))+cpR+cc*((integral(funmin,0,Tend)));
        maj=maj+1;
    end
    c=cur+cpm;
    cycle=(P*T)+T+wpR;
    cput=c/cycle;
```

A.2 Single-component Cost Model — Bottle Washer Cam Commands

The commands provided in Matlab's command window comprises of step-wise alteration of the parameters T and x, where each is iterated from the value of 1, in steps of 1, to the maximum value of 39. For each iteration, the resultant cost per unit of time is generated as an output variable, and saved in a matrix format, which is ultimately analysed to determine the minimum cost per unit of time. In order to visually display results of the cost per unit for each of the variable parameters, a "mesh" curve is plotted. The Matlab commands are seen below:

```
>> cpra=8363.05;cuRa=680631.1;cca=7256.82;cpRa=87290.6;xa=2;Ta=168;aa=1.503;wpa=4;wpRa=8;wuRa=10;
>> pd=makedist('Normal','mu',67720,'sigma',168);
>> alpha=183.862;beta=1.529;
>>  fun = @(t) (beta/alpha)*(t/alpha).^(beta-1);
>> tableC=zeros(1521,5);
>>  for mc=1:10
for k=1:39
Ta1=Ta*k;
for xa=1:39
[cpm1,P1,op1,fail1 ,maj1,cur1,cost ,Tstar1,cycle1]=sing1(cpra,cuRa,cca,cpRa,xa,Ta1,aa,fun,pd,wpa,wpRa,wuRa);
tableC(xa+(k-1)*39,:)=[xa Ta1 cost cycle1 fail1];
end
end
tableMC(mc,1)=[min(tableC(:,3))];
```

```
end
>> q=tableC(:,1);
w=tableC(:,2);
z=tableC(:,3);
dq=1;dw=1;
q_edge=[floor(min(q)):dq:ceil(max(q))];
w_edge=[floor(min(w)):dw:ceil(max(w))];
[Q,W]=meshgrid(q_edge,w_edge);
Z=griddata(q,w,z,Q,W);
mesh(Q,W,Z)
zlabel('Cost per unit time [R/hr]')
xlabel('Operational periods [x]')
ylabel('PM intervals [T]')
```

A.3 Multi-component Cost Model

The Matlab software is used in order to construct the mathematical model to determine the cost per unit of time for the multi-component system, given the value of the pre-defined parameters. The constructed mathematical model, as seen below, simulates (4.3.2) and (4.3.26) to determine the expected cost per unit of time for the multi-component system, where the cost per unit time is denoted by the variable “cost” in the Matlab code.

In order to proceed with the cost per unit time simulation, a total of 16 input parameters are passed to the function “multi1”. The input parameters are defined as follows:

- n — The total number of components to be simulated in the multi-component system.
- cpr — The total cost of conducting a PM task on each of the n components (equal to denotion c_n^p in (4.3.17)). The values are stored in a $(1 \times n)$ sized matrix format.
- cuR — The total cost of an unplanned SRP on each of the n components (equal to denotion c_n^{uR} in (4.3.18)). The values are stored in a $(1 \times n)$ sized matrix format.
- cc — The total cost of conducting a CM task on each of the n components (equal to denotion c_n^c in (4.3.16)). The values are stored in a $(1 \times n)$ sized matrix format.

cpR The total cost of a planned SRP on each of the n components (equal to denotion c_n^{pR} in (4.3.19)). The values are stored in a $(1 \times n)$ sized matrix format.

cpd The cost per unit of time for planned downtime.

x The number of operational intervals successfully completed by each of the n components, whereupon a planned SRP is executed on the particular component in the next PM interval. The values are stored in a $(1 \times n)$ sized matrix format.

T The time period between successive PM intervals for each of the n components. The values are stored in a $(1 \times n)$ sized matrix format.

a The improvement factor for each of the n components, following a PM task on the particular component. The values are stored in a $(1 \times n)$ sized matrix format.

fun The failure rate for each of the n components (equal to $\lambda^n_{(op,min)}(t)$ for component n). The values are stored in a $(1 \times n)$ sized matrix format.

pdmaj The PDF of the expected lifetime for each of the n components, which is, the expected lifetime between *major* failures for each component. The values are stored in a $(1 \times n)$ sized matrix format.

wp The time required to conduct a PM task on each of the n components. The values are stored in a $(1 \times n)$ sized matrix format.

wpR The time required to conduct a planned SRP on each of the n components. The values are stored in a $(1 \times n)$ sized matrix format.

wuR The time required to conduct an unplanned SRP on each of n the components. The values are stored in a $(1 \times n)$ sized matrix format.

Tmax The maximum value that T may be, considering the *pdmaj* of the particular component. The values are stored in a $(1 \times n)$ sized matrix format.

Xtot The threshold value of the sum of all components' operational periods, whereby a system SRP task is undertaken in the next PM interval.

The function “multi1”, which receives the input argument parameters is seen below:

```
function [Tm,cput,P,TmF,m,x1,x2,x3] = multi1(n,cpr,cuR,
```

```
    cc,cpR,cpd,x,T,a,fun,pdmaj,wp,wpR,wuR,Tmax,Xtot)

Tm=min(T);
cput=0;
cpm=0;
wmpstar=0;
wfpRstar=max(wpR);
Tmstar=Tm-wmpstar;
Tkstar=zeros(n,1);
op=ones(n,1);
P=zeros(n,1);
m=1;
sumop=n;
x1=0;x2=0;x3=0;
mactual=0;

while (op(1,1).*T(1,1)) < Tmax(1,1) && (op(2,1).*T(2,1))
     < Tmax(2,1) && (op(3,1).*T(3,1)) < Tmax(3,1) &&
    sumop < Xtot
    for k=1:n
        pdmajloop=pdmaj(k,1);
        Yoploop=abs(random(pdmajloop));
        if Yoploop >= (Tkstar(k,1)+Tmstar)
            T0=Tkstar(k,1);
            Tend=Tkstar(k,1)+Tmstar;
            cpm=cpm+(cpr(k,1))+((cc(k,1)).*(a(k,1)).^((
                op(k,1))-1).*integral((fun{1,k}),T0,Tend,
                'ArrayValued',true));
            Tkstar(k,1)=Tkstar(k,1)+Tmstar;
            op(k,1)=op(k,1);
            if rem((m*Tm)./T(k,1),1) == 0
                cpm=cpm+cpr(k,1);
                op(k,1)=op(k,1)+1;
                wkmp(k,1)=wp(k,1);
                P(k,1)=P(k,1)+1;
            else
                wkmp(k,1)=0;
            end
            if (op(k,1)-x(k,1)) > 0
                cpm=cpm+cpR(k,1);
                op(k,1)=1;
                wkmpR(k,1)=wpR(k,1);
            else
                wkmpR(k,1)=0;
```

```
                    Tkstar(k,1)=0;
                end
            else
                T0=Tkstar(k,1);
                Tend=(Tmstar+Tkstar(k,1)-Yoploop-wpR(k,1));
                cpm=cpm+cuR(k,1)+(cc(k,1).*(a(k,1).^(op(k,1)
                    -1).*integral(fun{1,k},T0,Yoploop,'
                    ArrayValued',true)+integral(fun{1,k},0,
                    Tend,'ArrayValued',true)));
                wkmpR(k,1)=0;
                op(k,1)=1;
                if rem((m*Tm)./T(k,1),1) == 0
                    cpm=cpm+cpr(k,1);
                    op(k,1)=2;
                    wkmp(k,1)=wp(k,1);
                else
                    wkmp(k,1)=0;
                end
            end
        end
        wmpstar=max([wkmp(:); wkmpR(:)]);
        Tmstar=Tm-wmpstar;
        sumop=sum(op);
        m=m+1;
        x1=op(1,1);
        x2=op(2,1);
        x3=op(3,1);
end

cpm=real(cpm)+(real(wmpstar)*real(cpd));

if m==1
        mactual=1;
else
        mactual=m+1;
end

for k=1:n
        pdmajloop=pdmaj(k,1);
        Yoploop=abs(random(pdmajloop));
        if Yoploop >= (Tkstar(k,1)+Tmstar)
            T0=Tkstar(k,1);
            Tend=(Tkstar(k,1)+Tmstar);
            cpm=cpm+(cc(k,1).*a(k,1).^(op(k,1)-1).*integral(
```

```
                fun{1,k},T0,Tend,'ArrayValued',true)+cpR(k,1)
                );
        else
            T0=Tkstar(k,1);
            Tend=(Tmstar+Tkstar(k,1)-Yoploop-wuR(k,1));
            cpm=cpm+cuR(k,1)+(cc(k,1).*(a(k,1).^(op(k,1)-1)
                .*integral(fun{1,k},T0,Yoploop,'ArrayValued',
                true)+integral(fun{1,k},0,Tend,'ArrayValued',
                true))+cpR(k,1));
        end
end

cpm=real(cpm)+(real(wfpRstar)*real(cpd));
cput=cpm./(((mactual)*Tm)+real(wfpRstar));
TmF=(mactual)*Tm;
```

A.4 Multi-component Cost Model — *TB* Conveyor System Commands

The commands provided in Matlab's command window comprises of step-wise alteration of the parameters T_n and x_n, where each is iterated from the value of 1, in steps of 1, to the maximum value of 39. For each iteration, the resultant cost per unit of time is generated as an output variable, and saved in a matrix format, which is ultimately analysed to determine the minimum cost per unit of time. In order to visually display results of the cost per unit for each of the variable parameters, a "mesh" curve is plotted. The Matlab commands are seen below:

```
for w=1:40
T1=[168;168;168];
T1a(1,1)=w.*T1(1,1);
for b=1:40
T1a(2,1)=b.*T1(2,1);
for d=1:40
T1a(3,1)=d.*T1(3,1);
for i=1:5
x1a=x1;
x1a(3,1)=i;
for j=1:5
x1a(2,1)=j;
for k=1:5
x1a(1,1)=k;
Xtot1=sum(x1a);
```

```
[Tm1, cost ,P1 ,TmF1, m1 , x11 , x12 , x13]=multi1 (n1 , cpr1 , cuR1 ,
    cc1 , cpR1 , cpd1 , x1a , T1a , a1 , fun1 , pdmaj1 , wp1 , wpR1 , wuR1 ,
    Tmax1 , Xtot1 ) ;
c=k+(j -1)*5+(i -1)*25+(d-1)*125+(b-1)*5000+(w-1)*200000;
tableU (c , : ) =[k j i cost T1a(1 ,1) T1a(2 ,1) T1a(3 ,1) TmF1
    m1 x11 x12 x13 ] ;
end
end
end
end
end
end
```

Appendix B

Monte Carlo Simulation Results

B.1 Bottle Washer Cam Monte Carlo Results

As described in Section 5.8.2.2, the Monte Carlo simulation process was achieved by repeating the iterative simulation, described in the first paragraph of Section 5.8.2.2, for a total of one hundred simulations. The resultant point estimate (cost per unit time), together with the associated interval estimate and input variables, was stored in a matrix format, as shown in Table B.1 (only a section of the results are shown here due to the large size of the entire matrix).

B.2 *TB* Conveyor System Monte Carlo Results

As described in Section 5.9.2.2, the Monte Carlo simulation process was achieved by repeating the iterative simulation, described in the first paragraph of Section 5.9.2.2, for a total of twenty simulations. The resultant point estimate (cost per unit time), together with the associated interval estimate and input variables, was stored in a matrix format, as shown in Table B.2 (only a section of the results are shown here due to the large size of the entire matrix).

Bottle Washer Cam Monte Carlo Simulation Results			
Operational Periods $[x]$	PM Intervals $[T(hrs)]$	Cost per Unit of Time $[R/hr]$	Interval Estimate $\pm[R/hr]$
1	168	348.22	1.139×10^{-14}
2	168	292.29	7.970×10^{-14}
3	168	279.90	1.139×10^{-13}
4	168	291.59	1.480×10^{-13}
5	168	323.45	1.366×10^{-13}
6	168	377.29	1.252×10^{-13}
7	168	458.57	1.821×10^{-13}
8	168	576.46	2.277×10^{-13}
9	168	744.74	2.277×10^{-13}
10	168	983.52	9.109×10^{-14}
.	.	.	.
.	.	.	.
.	.	.	.
1	1 008	179.52	1.139×10^{-14}
2	1 008	202.11	1.139×10^{-14}
3	1 008	243.45	5.693×10^{-15}
4	1 008	304.38	2.277×10^{-14}
5	1 008	390.27	2.277×10^{-14}
6	1 008	509.93	1.708×10^{-13}
7	1 008	676.37	1.366×10^{-13}
8	1 008	908.29	1.594×10^{-13}
9	1 008	1 232.45	2.733×10^{-13}
10	1 008	1 687.07	6.832×10^{-13}
.	.	.	.
.	.	.	.
.	.	.	.
1	6 552	256.41	3.935
2	6 552	318.12	3.854
3	6 552	391.66	6.568
4	6 552	502.54	8.903
5	6 552	644.69	14.082
6	6 552	831.71	23.788
7	6 552	1 090.58	32.042
8	6 552	1 438.90	50.947
9	6 552	1 917.64	77.921
10	6 552	2 509.64	117.697
.	.	.	.
.	.	.	.
.	.	.	.
37	6 552	12 842 860.86	2 187 884.26
38	6 552	22 564 451.89	4 424 893.76
39	6 552	28 301 770.47	5 474 190.13

Table B.1: Cam System's Optimal Parameters and Resultant Cost

***TB* Conveyor System Monte Carlo Simulation Results**							
Operational Periods			PM Intervals			Cost per Hour $[R/hr]$	Interval Estimate $\pm[R/hr]$
x_1	x_2	x_3	$T_1(hrs)$	$T_2(hrs)$	$T_3(hrs)$		
1	1	1	168.00	168.00	168.00	948.34	1.092×10^{-13}
2	1	1	168.00	168.00	168.00	638.65	1.092×10^{-13}
3	1	1	168.00	168.00	168.00	700.70	1.092×10^{-13}
4	1	1	168.00	168.00	168.00	739.60	5.459×10^{-14}
5	1	1	168.00	168.00	168.00	766.83	1.638×10^{-13}
1	2	1	168.00	168.00	168.00	704.00	5.459×10^{-14}
2	2	1	168.00	168.00	168.00	582.45	1.092×10^{-13}
3	2	1	168.00	168.00	168.00	761.69	1.092×10^{-13}
4	2	1	168.00	168.00	168.00	671.53	1.092×10^{-13}
5	2	1	168.00	168.00	168.00	783.18	1.092×10^{-13}
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
1	1	1	336.00	336.00	336.00	925.08	1.092×10^{-13}
2	1	1	336.00	336.00	336.00	352.80	1.092×10^{-13}
3	1	1	336.00	336.00	336.00	814.16	1.092×10^{-13}
4	1	1	336.00	336.00	336.00	805.94	1.092×10^{-13}
5	1	1	336.00	336.00	336.00	795.58	1.092×10^{-13}
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
1	2	1	3 364.00	1 682.00	3 364.00	105.38	1.365×10^{-14}
2	2	1	3 364.00	1 682.00	3 364.00	172.97	1.365×10^{-14}
3	2	1	3 364.00	1 682.00	3 364.00	171.91	5.64
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
1	1	1	3 864.00	3 864.00	3 864.00	369.89	23.45

Table B.2: *TB* Conveyor System's Parameters and Resultant Cost

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