AN INVESTIGATION INTO THE USE OF SPREADSHEET ALGEBRA PROGRAMMES (SAPs) TO INFLUENCE TEACHER CHANGE IN SELECTED TOWNSHIP HIGH SCHOOLS

by

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DECLARATION

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously in its entirety or in part submitted it at any other university for degree purposes. However, I declare that I presented some aspects of this thesis in a paper:"ANALYSIS OF THE USES OF TECHNOLOGY IN THE TEACHING OF MATHEMATICS" at ICME12 in Seoul, Korea in 2012.

DUMISANI MDLALOSE MARCH 2017

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ABSTRACT

This study investigates possible teacher change after the use of Spreadsheet Algebra Programmes (SAPs) in Excel on functions, a 'new form of teaching and learning' in two township high schools from interrelated areas/perspectives, namely: (a) possible changes in teachers' classroom practices, and (b) possible changes in teachers' beliefs. The study was purposely designed such that the entire population ($n = 10$), 6 male and 4 female drawn from the two schools, complete the teachers' beliefs questionnaire to code and analyze teachers' beliefs. There were two respondents, 1 female and 1 male, herein referred to as Teacher A (grade 11) and Teacher B (grade 10), who were interviewed on their practice after an observed lesson on functions. Both would later be part of the ten that would complete the questionnaire. SAPs were prepared for teachers to explore or 'open up' the concept of functions. Teachers were orientated on the use of SAPs beforehand and had an option either to use the SAPs material or prepare their own material on functions in grade 10 and grade 11. Interviews were captured on video, transcribed and analysed for presentation of conclusions and results. In combination with the theoretical framework, these techniques provide an approach to analyze data and re-describe teacher change. The analysis traces the development over the course of one lesson observed in each school and highlights the importance of teachers' beliefs. Notable changes of teacher beliefs and classroom practices were found. It can be discerned that the use of SAPs can save teaching time and assist in re-conceptualisation of functions in high school owing to institutionalization of mathematics and the curriculum. A teaching approach to elicit better understanding of functions in high schools was developed as a contribution to the new body of knowledge. After the use of the SAPs with the two teachers, it became evident that a turnaround teacher development plan is necessary to address issues related to professional support.

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OPSOMMING

Hierdie studie poog om lig te werp op die onderprestasie binne "township" hoërskole spesifiek gekies vir fokus van hierdie studie. Die studie fokus op die invloed van Sigblad (spreadsheets) Algebra Programme (SAP) in 'Excel' op funksies tot die skepping van "'n nuwe vorm van onderrig en leer". Dit (die studie) ondersoek spesifiek moontlike onderwyser veranderinge na die gebruik van die SAP in twee "township" hoërskole vanuit interafhanklik gebiede / perspektiewe, naamlik: (a) moontlike veranderinge in onderwysers se klaskamerpraktyke en (b) moontlike veranderinge in onderwysers se oortuigings ten opsigte van tegnologiese integrasie om resultate te verbeter. Daar was twee respondente, een (1) vrou en een (1) man, waarna in die studie verwys word as Onderwyser A (graad 11) en Onderwyser B (graad 10), met wie onderhoude gevoer is oor hul praktyke na afloop van 'n les oor funksies wat waargeneem is. Beide word later deel van die tien wie die vraelyste voltooi. Die studie is doelgerig ontwerp sodat die hele populasie $(n = 10)$*, 6 manlike en 4 vroulike deelnemers gekies uit die twee skole, om voltooide onderwysersoortuigings vraelyste oor die oortuigings van die onderwysers te kodeer en te analiseer. SAP is vir onderwysers voorberei om die konsep van funksies te verken of te 'ontsluit'. Onderwysers is vooraf georiënteer in die gebruik van SAP en is die opsie gebied om óf die SAP materiaal te gebruik of om hul eie materiaal oor funksies vir graad 10 en graad 11 voor te berei. Die onderhoude is op video vasgelê, getranskribeer en geanaliseer vir die aanbieding van die gevolgtrekkings en resultate. In kombinasie met die teoretiese raamwerk, bied hierdie tegnieke 'n benadering om data te ontleed en te herbeskryf omtrent onderwyser verandering. Die analise ondersoek die ontwikkeling van onderwysers se oortuigings in die loop van 'n les waargeneem in elke skool en beklemtoon die belangrikheid daarvan. Noemenswaardige veranderinge van onderwyser se oortuigings en klaskamerpraktyke gevind. Dit kan beoordeel word dat die gebruik van die SAP onderrigtyd kan bespaar en help in die herkonseptualisering van funksies in die hoërskool as gevolg van die institusionalisering van wiskunde en die kurrikulum. 'n Onderrigbenadering om beter begrip van funksies in hoërskole te ontlok is ontwikkel as 'n bydrae tot die nuwe liggaam van kennis. Na die gebruik van die SAP met die twee onderwysers, het dit duidelik geword dat dit 'n ommekeer in die onderwyser ontwikkelingsplan is wat nodig is om kwessies wat verband hou met professionele ondersteuning aan te spreek.*

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List of abbreviations/acronyms and their explanation

DBE	Department of Basic Education	Refers to the government department responsible for basic education in Grades R-12
WCED	Western Cape Education Department	Refers to Western Cape provincial government department responsible for basic education in Grades R-12
FET	Further Education and Training	Refers to grade 10–12
GET	General Education and Training	Refers to the compulsory band of learning from grade R–12
RNCS	Revised National Curriculum Statement	Refers to electronic or digital learning using computers and other technologies
NCS	National Curriculum Statement	Curriculum underpinning National Senior Certificate
CAPS	Curriculum and Assessment Policy Statement	The DBE introduced CAPS in 2009. It refers to the what, when and how of content to be taught and how it is assessed in schools
NSC	National Senior Certificate	Refers to school-leaving qualification (NQF Level 4 in the GET sub-framework) written after completion of grade 12 learning (exit examination at the end of schooling)
SAPs	Spreadsheet Algebra Programmes	The Excel activities (teaching and learning material) used in this study
TDPs	Teacher Development Programmes	Refers to professional development programmes for teachers
MST	Maths, Science and Technology	Refers to the National Strategy for Mathematics, Science and Technology Education of the Department of Basic Education
ICT	Information Communication Technology	Refers to technology or application (e.g. computer) that enhances learning, instructional and communication
MALATI	Mathematics Learning and Teaching Initiative	Refers to a change agent in curriculum development (1997-1999)
SA	Subject Advisor	Subject Advisors operate in education districts

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CHAPTER 1: INTRODUCTION

1.1 Background

Beginning in 1994, the transformed South African government has been dealing with numerous post-apartheid insecurities and concerns plaguing education. This necessitated a rigorous education restructuring process. As part of transformation, the government formulated policies such as the National Curriculum Statement Grades 10-12 (DBE NCS, 2003), e-Education (2004), MST Strategy (2004). The Western Cape Provincial Government accepted the capacity of technology to model data, promote and encourage higher order thinking in schools. WCED implemented and invested substantial funds in The Khanya Technology in Education Project (Khanya) in 2005-2012 to capitalize on evolving technologies and localise international best practices. When presenting the education Budget to the National Assembly, the then Minister of Education said:

> *All honourable members are fully aware of public concerns about the quality of public education. The concerns range from infrastructure to bus transport, from textbooks delivery to quality of teaching, to quality of passes, to dropouts, to catch up opportunities for youths, to skills training, to higher education success rates. We in education have to answer the nation's question to us: are you ready to excel?... It is very important for us to be honest with those who have gone before us. So we must acknowledge that up to this point we have not yet dealt a blow of death to all the legacies of apartheid education. We do intend to deal decisively with the problem of thousands of poorly performing schools. These schools are located in poorest sections of our society and sadly their inadequacies perpetuate the legacy of disadvantage*. National Education Budget Speech (2006: 2)

In the cited budget speech, the Minister confirmed the presence of a plethora of challenges of education. The policies referred to earlier would have to be designed to address the baggage of the erstwhile apartheid government that preferred inequality over equitable distribution of resources. According to the National Planning Commission: National Development Plan (2011: 282):

> *It is estimated that approximately 80 percent of [our] schools are underperforming. This translates to about 20 000 schools. International experience shows that system wide improvements in education [should] be implemented in a number of ways, including putting together multi-disciplinary teams that [will] assess the functionality of a school, develop a turnaround plan and oversee its implementation*.

It can be discerned from the quotation that according to the commission, the non-performance of schools is largely attributable to discriminatory policies of apartheid. Learners in township high schools are the worst affected by underperformance. There are a number of reasons why learners perform poorly in the NSC examinations in general and in mathematics in particular. The

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experience of the researcher as a mathematics educator, mathematics subject advisor, external moderator and examiner confirms this.

These factors compelled the researcher to become involved in a study of this nature. Among many other reasons why learners perform poorly in mathematics, one is that learners constantly provide inappropriate responses to questions on functions in the NSC examinations. Consistently coming out in the Diagnostic Reports (DBE, 2011–2015) on learner performance in the National Senior Certificate examinations were common errors and misconceptions in mathematics Paper 1. Examples of common errors and misconceptions are lack of exposure to solving equations in the context of graphs (algebraic manipulation), misinterpretation of a question (drawing a curve instead of finding an equation (DBE, 2012)), unfamiliarity with the format for the hyperbola or confusing the values of p and q in $y = \frac{a}{x-p} + q$ (DBE, 2014). CAPS Grades 10-12 mathematics (DBE, 2011: 24) provides this definition:

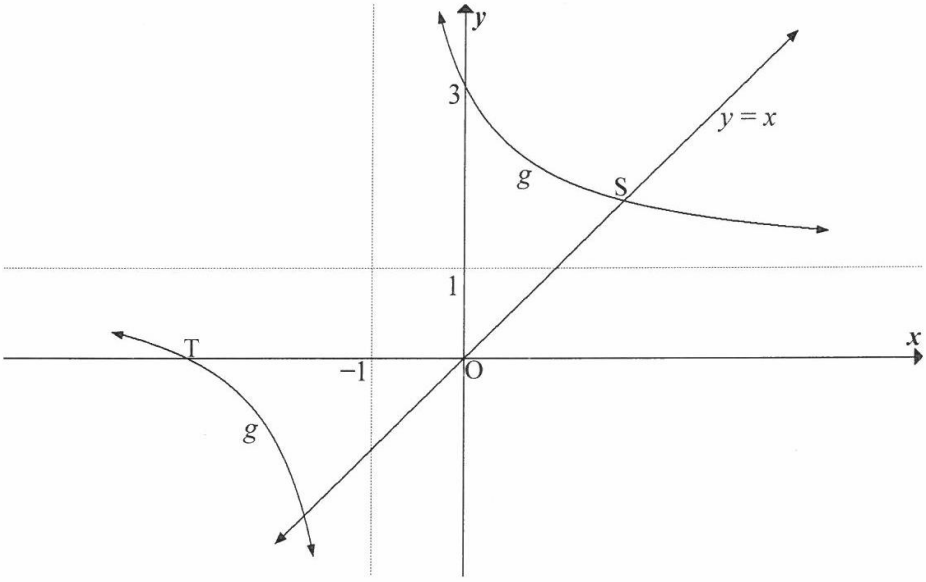
> *The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae).*

Krapfl (1994) contends that the use of multiple representations, interpretation from one representation to another, and analysis which allows learners to relate the graphic, numeric and symbolic information are all critical areas to which learners should be exposed in order to develop a better understanding of functions. This begins to indicate that learners fail to make connections between visual and symbolic representations of functions which is critical to the development of new knowledge and understanding of functions and function sense. For purposes of this study, the researcher focused on common errors and misconceptions on mathematical functions.

The researcher used information on learner performance in NSC examinations in Diagnostic Report (DBE, 2014a-d: 114) and *Illustration 1* below (DBE, NSC Nov 2014 Mathematics Paper 1) to make a claim that learners' inappropriate answers contribute to underperformance in township high schools.

QUESTION 4

The diagram below shows the hyperbola g defined by $g(x) = \frac{2}{x+p} + q$ with asymptotes $y = 1$ and $x = -1$. The graph of g intersects the x-axis at T and the y-axis at $(0 ; 3)$. The line $y = x$ intersects the hyperbola in the first quadrant at S.



4.1 Write down the values of p and q. (2)

4.2 Calculate the x-coordinate of T. (2)

4.3 Write down the equation of the vertical asymptote of the graph of h, if $h(x) = g(x+5)$ (1)

4.4 Calculate the length of OS. (5)

4.5 For which values of k will the equation $g(x) = x + k$ have two real roots that are of opposite signs? (1)

[11]

Figure 1: NSC Nov. 2014 Mathematics Paper 1 (Question 4: Functions, hyperbola)

This is a useful perspective in the context of the study as it lays a foundation for research claim (hypothesis) that underperformance in township high schools is caused, among other factors, by 'inappropriate pedagogy'. There is an ongoing conception among teachers of mathematics that learners lack the ability to understand functions. Bell (2009: 51) who refers to Eisenberg and Dreyfus (1994), and National Research Council (1989) in her study, refutes this, suggesting instead a connection between the quality of pedagogy and learner comprehension of mathematics.

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> *It can therefore be established that the concept of a function and understanding their multiple representations [by teachers] are important in helping learners develop what some researchers refer to a function sense.*

Reiterating the assertion made above, Dick (1992) and Wilson and Krapfl (1994) suggest that:

> *the use of multiple representations, interpretation from one representation to another, and analysis which allows learners to relate the graphic, numeric and symbolic information are critical areas that learners should be exposed to in order to develop a better understanding of functions.*

Taking the discussion forward, Bell (2009: 51) aptly contends that:

> *Although each of these representations is available in a graphing calculator environment, software programmes such as Microsoft Excel also provide an environment for students to explore multiple representations of functions.*

In his study, Porzio (1994) explored the learners' abilities to see or make connections between graphical, numerical, and symbolic representations in the context of problem situations, which can be summarized as follows:

> *Students are better able to see, or make, a connection between different representations when one or more of the representations is emphasized in the instructional approach that they experienced and [underlined by the author] when then instructional approaches includes having students solve problems specifically designed to explore or establish the connection(s) between the representations.*

From what we see in learner responses in the Diagnostic Reports of DBE, as well as from the contention by Porzio (1994), it is clear that the function concept is misunderstood conceptually and mathematically by both teachers and learners. With the influence of Spreadsheet Algebra Programmes (SAPs) as a 'new approach to teaching and learning functions', there is hope that this study can mitigate underperformance in township high schools as a result of possible change in teachers' classroom practices and teachers' beliefs.

1.2 Problem statement

The rapid advance of technology has led to a wide range of technology-mediated instructional tools (e.g. computers) being more readily available to the wider public to improve learner attainment. The emergence and advancement of a computer as an instructional tool, has assisted teachers to improve classroom instruction in particular, and has provided mankind with opportunities to communicate and disseminate information and knowledge rapidly that would otherwise be difficult to attain without it generally. According to Hew and Brush (2006: 224), at the heart of reform-minded instruction lies a belief that technology integration in education can have a positive impact in learners' learning. This belief has led education authorities and many

governments (e.g. Singapore, United States) to create programmes for their schools that encourage teachers to harness technology in teaching and learning in grade R-12. Evidence of this commitment can be seen from rationale of the Primary Mathematics Syllabus of the Ministry of Education in Singapore (MOE, 2007: 5) that:

> *The development of a highly skilled scientifically- and technologically-based manpower requires a strong grounding in mathematics. An emphasis on mathematics will ensure that we have an increasingly competitive workforce to meet the challenges of the 21st century.*

As cited from Ertmer (2005: 26), the No Child Left Behind Act in United States makes provision "to ensure that teachers can integrate technology into the curriculum for purposes of improving students' learning (U.S. DOE, 2001)." The South African government developed a policy on e-Education (DBE, 2003: 13-14) with an aim of using ICT "to extend and enrich educational experiences across the curriculum" over and above the necessary technical skills required in the use of ICT.

The researcher found the research work of Hew and Brush (2006) useful for this study. These researchers analysed 48 existing studies in the period 1995-2006 from databases that reported empirical research findings on 123 barriers and strategies that impact the use of technology in grades R-12 classrooms for instructional purposes across the world. They classified these barriers into 6 categories, namely, (a) Resources, (b) Knowledge and skills, (c) Institution, (d) Attitudes and beliefs, (e) Assessment and (f) Subject culture. Of the literature they reviewed, Hew and Brush (2006) saw uniqueness in the study done by Ertmer et al. (1999) in that it did not only highlight the relationship between barriers but went on and examined the classification of first- and second-order barriers in more detail – see the adapted Table 1 (Hew and Brush, 2006: 240) below:

Classification	**Barrier**	**Strategy**
First-order	Resources	Obtaining the necessary resources
	Institution	Creating a shared vision and technology integration plan
	Subject culture	
	Assessment	Having alternative modes of assessments
Second-order	Attitudes and beliefs	Facilitating attitude change
	Knowledge and skills	Facilitating teacher knowledge and skills

Table 1: First- and second-order barriers and strategies in technology integration

Ertmer (2005) continues to argue that technology (computers) serves as a valuable tool in schools and classrooms where teachers hold personal beliefs aligned to constructivist pedagogy. It therefore stands to reason that to change classroom practice with the use of technology, teachers would have to alter their beliefs. This study therefore aims, among other things, to investigate the extent to which the use of SAPs can influence teachers' classroom practices and their beliefs.

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According to Guskey (1986), the three major outcomes of teacher development are: (a) change in teacher classroom practice, (b) change in teacher beliefs and attitudes, and (c) change in learning outcomes of learners. He asserts that changes in teachers' beliefs and attitudes are likely to take place only after the teachers become aware that changes in student learning have taken place. Thus, the change in beliefs is the *result* and not the cause of the change in teacher practice and the resulting improvement in student outcomes, although further change in practice can occur. In order for teacher beliefs to change significantly, teachers need to persist long enough to observe *demonstrable results in terms of the learning success of a teacher's students*. Teachers must discover that learners can learn from each other by working together on mathematical problems before they can *reposition themselves as orchestrators, rather than conductors, of mathematical inquiry in the classroom* (Goldsmith and Schifter, 1997). According to Goldsmith and Schifter (1997), acquiring a new vocabulary will not help teachers to develop their practice without having the opportunity to experience their classrooms in a new way. Teachers' practice are also influenced by their views of mathematics itself. For example, mathematics can be viewed as a body of knowledge that has been created. Alternatively, learning mathematics can be viewed as a process of conjecturing, discussing, testing etc., and in this case taking time over solving a problem is not viewed as abnormal (Goldsmith and Schifter, 1997). The researcher held the latter view, but some teachers still hold the former. According to Goldsmith and Schifter (1997), if teachers do not have a strong enough grasp of the mathematics they teach, they may not be able to engage learners in an exploration of mathematical ideas beyond calling attention to a variety of possible solution strategies. They themselves may not be able to distinguish valid from invalid reasoning. Schools and educationists need to take cognisance of these facts if young learners are to be prepared adequately for the demands of a highly technological future.

1.3 Research motivation

As Subject Advisor for FET mathematics in the WCED, the researcher conducted numerous workshops on content (e.g. algebra, trigonometry and geometry), pedagogy and technology integration in mathematics prior to doing this study. Hence the limit of the study to two township high schools in the Western Cape. Despite professional teacher support meted out in schools, learners continued to perform poorly in questions on functions. The researcher was deeply concerned about this. The researcher began to question why learners in township high schools in particular continued to provide inappropriate or incorrect answers in questions on functions in NSC examinations. Arguing why learners fail mathematics, some teachers in workshops claimed that the mathematics syllabus is too congested and as a result it takes longer for them to complete it, and that the DBE makes too many changes in the syllabus too often. The researcher was intrigued by these claims. Whilst these claims may be valid, the researcher was of the view (hypothesis) that perhaps teachers' practice and personal beliefs that they hold are not aligned to

constructivist pedagogy as suggested by Ertmer (2005). It would appear that the researcher is not off the mark when arguing that learners fail to make connections between visual and symbolic representations of functions in assessments/examinations owing to mathematical insecurities of their teachers. In his work as Subject Advisor, the researcher was fascinated by the affordances of technology as instructional material that could provide teachers with opportunities to establish **critical teaching moments**[1] that are not easy to do in a traditional 'pen and paper' approach. Hence the choice of SAPs in *Excel* on functions. The SAPs in *Excel* can easily generate tables, graphs and formulas (equations) for learners to interact with them in order to develop and test conjectures, and teachers can then teach or demonstrate the relationships between graphs, tables and formulas. In a traditional 'pen and paper' approach, the concept development of function tends to develop piece-meal knowledge in a fragmented manner, which often leads to different rules for different functions.

In his work as Subject Advisor, the researcher observed that learners in high schools misunderstood the concept of function. Inappropriate learner responses in questions on functions in NSC examinations, and in grades 10 and 11, reveal knowledge gaps and lack a 'function sense'. Drawing from evidence of learners' lack of conceptual understanding of functions in NSC examinations in Diagnostic Reports (DBE, 2011-2015), the researcher identified the following as knowledge and conceptual gaps:

a) Learners confuse rules pertaining to vertical and horizontal translations (shifts), i.e. *Translations* (shifts): $g(x) = f(x - p)$ and $g(x) - p = f(x)$, and combinations of the two;
b) *Scaling* (stretching and shrinking): $g(x) = q.f(x)$ and $g(x) = f\left(\frac{x}{p}\right)$, and combinations of the two;
c) Completing the square and solving equations in the case of quadratic functions;
d) Graphs of *basic functions and* generic *transformation rule* for $y - q = f(x - p)$;
e) For each of the seven function *types* in grades 10-12, learners do not understand that the point $(p;\ q)$ has *physical meaning*, which we must *know* in order to draw the graph:

- $y - q = m(x - p)$ and means a line through the point $(p;\ q)$ parallel to $y = mx$;
- $y - q = a(x - p)^2$ means a parabola with turning point $(p;\ q)$;
- means a hyperbola with asymptotes p and q.

The **function concept vantage point** is the purview of the Subject Advisor or the Mathematics Educator and not necessarily that of the mathematics teacher. The function concept is essential to

[1] A 'teaching moment' is an episode during teaching experience where a teacher engages learners in observations, predictions, generalisations, pattern recognition and qualitative relationships in physical and social phenomena and between mathematical objects themselves (e.g. using multiple representations of functions with the aid of SAPs) to detect errors, misunderstandings and misconceptions in order to replace them with correct forms of knowledge and understanding.

algebra curriculum. It is considered as the most important concept in all of mathematics. According to Fey (1998), functions play a central and unifying role in mathematics and are critical throughout the entire range of mathematics education. Sfard (1987) asserts that the historical definition of function, as relationship between variables, is more relevant and meaningful to learners because it capitalizes on their prior intuitive function concept. Functions can be depicted using a graph, table and equation. The researcher claims that conceptual understanding of functions could ultimately change learners' responses to questions on functions in algebra, calculus and trigonometry in NSC examinations. Furthermore, he contends that with the influence of SAPs in *Excel* on functions, township high school teachers might be able to adjust their level of teaching to elicit conceptual understanding of functions. '**Function sense'** (i.e. ability to make generalisations based on structure of function and working flexibly between tabular, symbolic and graphical representations) is connected to the design features of the SAPs. It appears that learners do not understand that functions behave in a similar manner, and that they (learners) rarely consider the ways in which graphs provide visual insights to the behaviour of functions.

Lack of conceptual understating of functions consequently motivated this study. The researcher argues that lack of conceptual understanding of functions may be an indictment on classroom practice, and personal beliefs that teachers hold about teaching and learning. Furthermore, he argues that mathematics and the curriculum are institutionalised. Institutionalisation of mathematics and the curriculum can be seen from the way CAPS grade 10–12 mathematics (2011) is designed and packaged in preparation for NSC examinations. This can also be seen from the way NSC questions are phrased and question papers structured (see Figure 1), schools as bureaucracies and policy documents related to the curriculum. For example, according to CAPS grade 10-12 mathematics (DBE, 2011: 26; DBE, 2011: 34), functions is a topic for term 2 in grade 10 and 11. The time allocated for teaching functions is 5 weeks and 4 weeks in grade 10 and grade 11 respectively.

The study considers teacher beliefs and classroom practice as having the largest impact on learners' results in township high schools. In this study the researcher argues that teachers have control over instructional decisions (what they say or tell learners in class) and curricular choices (what they do, what resources to use) that they make in a classroom. He asserts that instructional decisions and curricular choices are deeply rooted in teacher beliefs about mathematics, pedagogy and how learners learn mathematics. Further to research question/statement that drives this study, here are the sub-questions:

- How do teachers make these choices?
- What impact do these curricula choices and instructional decisions have on learners with respect to conceptual and mathematical understanding of functions?

- When teachers are given an opportunity to engage with new forms of teaching and learning, will they change the way that they teach functions?

The researcher shared views of Ertmer et al. (2005) that instructional decisions and curricular choices are informed by teacher beliefs, and that these are based on their knowledge of curriculum, content, learners, pedagogy and learning and teaching resources such as educational use of SAPs. For example, a bad instruction results in responses that are infected with misconceptions, errors, mistakes and lack of function sense. As cited from Levin and Wadmany (2006: 158):

> *Beliefs are filters that guide teachers during instructional and curricular decision-making (Pajares, 1992; Prawat, 1992).*

It can be discerned that teachers' instructional decisions and curricular choices have potential to shape a learning culture in a mathematics classroom, which could motivate and inspire learners to perform and produce results in NSC examinations in particular. The study assumes that: (a) any description of a change of teacher practice using SAPs could happen at any stage of lesson presentation, i.e. before, during or after delivering a lesson (b) teacher practices are influenced by teacher beliefs about the nature of mathematics, teaching and understanding, assessment, learner abilities and integration of SAPs in mathematics education, and (c) classroom practices and teacher beliefs are multivariate and interrelated (Levin and Wadmany, 2006).

1.4 Research aims and objectives

Specific objectives of the study include:

a) To document and analyse possible changes in teachers' beliefs;
b) To use of SAPs to engage with new forms of teaching and learning;
c) To reflect on ways SAPs can enable dynamic, inquiry-based mathematics teaching and learning;
d) To rethink the school mathematics curriculum, both in terms of content and teaching approach.

The study has the following elements, (a) investigation of teacher beliefs in respect of assessment, nature of mathematics, teaching and learning, learner ability and calculator use, and (b) classroom practices to investigate (i) possible changes in teachers' classroom observation as a result of use of SAPs or own lesson on functions, and (ii) post-lesson interview to determine possible shifts.

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1.5 Thesis outline

This thesis is made up of 6 chapters. Here is the outline:

Chapter 1: This chapter provides background, problem statement and study objectives.

Chapter 2: This chapter provides literature review on teachers' beliefs and classroom practices.

Chapter 3: This chapter presents research design and methodology followed in the study.

Chapter 4: Data from all sources are presented and analysed in this chapter.

Chapter 5: This chapter presents findings of this study.

Chapter 6: Conclusions and recommendations are presented in this chapter.

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CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter provides literature review on teacher beliefs and classroom practices. The literature reviewed in this chapter will focus on the work done by other researchers in the field of teacher education under discussion.

This chapter is organised in three sections that describe teacher change in terms of classroom practices and teacher beliefs.

a) The researcher will describe SAPs in terms of theoretical framework, and discuss what they are in terms of ICTs and the role they could possibly play in professional teacher development. More importantly, he will also discuss the relationship between the SAPs and the professional development. Further discussions on how the researcher introduced teachers to SAPs, and the criteria used to choose any of the SAPs with the teachers will be discussed in Chapter 3.
b) The conceptual overview of teacher beliefs is presented as a vital contributor towards teacher change, i.e. how they could influence teacher change and suggest how they could be used to facilitate teacher development. Teacher beliefs will be discussed in terms of its definition (i.e. theory that is likely going to get results), role in teacher education and why they are connected to this study. He will also explore the prospects for using change knowledge in further studies on teacher beliefs.
c) In the final section, the researcher will identify some barriers that may stand in the way of moving to a deeper set of strategies.

2.2 Teacher Change

Poor results in NSC examinations as well as South Africa's poor performance in the TIMSS[2] (Trends in International Mathematics and Science Study) have had government and educationists grappling since the dawn of democracy in 1994. For example, in the President's Education Initiative Research Project[3] (1999), which was commissioned by the Teacher Development Centre on behalf of the Department of Education, Taylor and Vinjevold (1999: 159-161) reported that:

> *… reform initiatives aimed at revitalizing teacher education and classroom practices must not only create a new ideological orientation consonant with the goals of the new curriculum in South Africa. They also need to get to grips with what is likely to be a far more intractable problem: the massive upgrading and scaffolding of teachers' conceptual knowledge and skills.*

The need for 'teacher change' in this study is necessitated by poor learner performance in NSC examinations in township high schools in particular and how it (teacher change) could contribute

[2] The South African component of the TIMSS has been assessing mathematics and science achievement among grade 8 and 9 learners since 1995. In South Africa, TIMSS was conducted in 1995, 1999, 2002 2011 and 2015.
[3] The aim of this commissioned research was "to provide a scientific basis for the future planning and delivery of educator development and support programmes."

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to improving the situation as observed by the researcher in his capacity as Subject Advisor for mathematics in the Western Cape Education Department. Lack of clarity about what 'change' requires can be an obstacle to change (Snyder et al., 1992). Fullan (2006: 3) asserts that:

> *Change theory or change knowledge can be very powerful in informing education reform strategies and, in turn, getting results – but only in the hands (and minds, and hearts) of people who have a deep knowledge of the dynamics of how the factors in question operate to get particular results.*

Pang (2012) asserts that teacher change can be described in many ways, such as differences in teacher knowledge, beliefs and instructional practice. According to Pang (2012: 139-140), there are four elements that facilitate teacher change, namely, (a) instructional changes forced by outsiders rather than participating teachers themselves are minimal or superficial (Sowder, 2007), (b) teacher change needs to be confirmed and solidified by learners' results if such change is to be sustained (Smylie, 1988), (c) learning community of teachers is prevalent in effective teacher professional development activities (Borko, 2004), and (d) teachers are more likely to change their teaching approach if it would directly relate to their day-to-day routines or ongoing responsibilities in their classrooms (Guskey, 1986; Smylie, 1988). This study considered (c) and (d) above as the elements of teacher change. Hence the focus on the why (improve pedagogy) and what (teacher practice and beliefs) to change, and offers suggestions on how to implement improvements in township high schools using SAPs. The investigation done in this study describe classroom practices of two teachers, 1 female and 1 male, during the course of a lesson observation as 'antecedents to change' either in learners' learning outcomes or in teachers' beliefs and attitudes (Franke et al. 2001; Guskey, 1986). If we accept that learner performance in township high schools needs to improve, then we must also accept that teachers need professional development to show evidence of learning. For effective change to occur in a setting where teachers work, they must be able to envision what the change will actually involve for their own personal professional behaviour (Elmore, 2004, Lovitt et al., 1991). According to Elmore (2004: 11):

> *The problem [is that] there is almost no opportunity for teachers to engage in continuous and sustained learning about their practice in the settings in which they actually work, observing and being observed by their colleagues in their own classrooms and classrooms of other teachers in other schools confronting similar problems.*

This may be connected to their images of what 'good teaching' entails (Goldsmith and Schifter, 1997). Cole (2004: 3) describes professional development as:

> *the systematic and formal attempts to advance the knowledge, skills and understanding of teachers in ways that lead to changes in their thinking and classroom behaviour. Indeed, as one evaluator observes, education systems have committed vast resources to professional development programs in the belief that participation of teachers in these programs would result in an enhancement of individual practice and in schooling outcomes for students. …… The results of training should be immediate, specific and measurable in terms of how well it has met its purpose of producing improved performance.*

According to Cole (2004), teacher training has a transformation agenda to improve the quality and consistency of teaching and learning, thus produce improved performance in schools. In his discussion of teacher development, Guskey (1986) contends that there are three major outcomes of teacher development. These are, (a) change in teacher classroom practice, (b) change in teacher beliefs and attitudes, and (c) change in learning outcomes of learners. Clearly, instructional improvement requires a change in the prevailing classroom culture of administration and teaching in schools. Rather than top down or change by mandate, change of classroom cultures envisaged in this study depends fundamentally on modeling new values and behavior that should challenge existing ones (Fullan, 2006, Richards, et al., 'n.d.').

2.2.1 Role of Spreadsheet algebra programmes (SAPs) in Excel in TDPs as curriculum materials

In their review and analysis of research on general barriers of technology integration in the period 1995-2006, Hew and Brush (2006) assert that there is no clear definition of the term, technology integration in literature. Levin and Wadmany (2006: 157) assert that when [a computer] has been integrated, clear evidence that it can affect teaching or improve desired learning modes is still lacking (Alexander, 1999). However, in their citation of Cuban, et al. (2001), technology integration is understood and examined in terms of types of computer use by teachers in a classroom, namely low-level (doing internet search) and high-level use (doing multimedia presentation, data analysis in project). The researcher regards low-level to mean pragmatic use of technology (to help teachers to have the job done), and high-level to epistemic use (how the tool makes it work). They cite Hennessy, Ruthven and Brindley (2005) as researchers who understood and examined technology integration "*to carry out familiar activities more reliably and productively and how such use may be reshaping activities*," (Hew and Brush, 2006: 225). Lim et al., (2003) are cited in the study Hew and Brush (2006) as having considered technology integration in terms of teachers using technology to develop students' thinking skills. The study was purposefully tailor-made for the use of SAPs by teachers to purely facilitate learning and teaching of function in a high school. The researcher regards the of use of SAPs as being for epistemic purposes.

SAPs are curriculum material developed by Alwyn Olivier in late 1990s to early 2000s with a potential to help teachers think about their current roles, try out new roles and modify the way they teach by drawing directly on experience of teachers who have developed and tried these materials. Their role lies squarely on their potential for teacher development programmes that could facilitate and enhance teacher change. Hence the claim that traditional 'pen and paper' approach contribute towards institutionalisation of the curriculum and mathematics. SAPs in *Excel* on functions (see *Appendix* (ii)) are computer-mediated learning and teaching material prepared for grades 10-12 teachers. According to Lovitt et al., (1991) and Snyder et al., (1992), instructional

material is not only an important catalyst in curriculum change, but is also a catalyst to support teacher change. In their argument in favour of the use of curriculum-based material, Ball and Cohen, (1996) assert that educative curriculum - curriculum materials designed to address teacher learning as well as student learning - is one potential vehicle to facilitate teacher change. What teachers do in their classrooms depends largely on their knowledge of mathematics, curriculum, knowledge of learners and assessments. Wallace and Louden, (1998); Borko and Putnam (1996) make a claim that teachers need to learn a great deal to be able to enact reform-based curriculum.

SAPs came ready to be used in a lesson for the study. The researcher did not want respondents to design or develop their own SAPs or technology-based instructional material on functions for the research lesson from scratch because this would be too time consuming. The use of SAPs in *Excel* is aligned with the view expressed by Fey (1998: 255) who stated that the use of numerical, graphic and symbol manipulation is a powerful technique for mathematics teaching. The rationale for using *Excel* among other technologies to investigate possible change of classroom practice are:

a) Although SAPs are not widespread or commercially available, Excel is a Microsoft Office suite or application that come installed in computers and ready to use and at no cost to end-users in schools.
b) SAPs were turned into a 'mathematical instrument' and designed to provide an environment in which multiple representations of functions may be investigated.
c) SAPs provided respondents with ample opportunities to extract the useful mathematics and facilitate valuable discussion on functions.

2.2.2 Teacher beliefs

Teacher beliefs that connect to this study are discussed in this section. They are a unit of change of teacher practice. While teacher beliefs about the use SAPs in a mathematics classroom can a facilitator of teaching and learning, they can also be an obstacle.

There seems to be confusion in literature regarding both the labels and definitions used to describe teacher beliefs. Many researchers have different views about the definition of teacher beliefs. For example, Calderhead (1996) and Richardson (1996) define teacher beliefs as premises and suppositions about something that are felt to be true. Calderhead (1996) described teacher beliefs, as well as teacher knowledge and teacher thinking, as comprising the broader concept of teacher cognition. In principle, Richardson (1996) agreed with Pajares (1992) that teachers' educational beliefs tend to influence the nature of their instructional practices. In his review of teacher beliefs, Pajares (1992: 307) labelled them as a "messy construct," noting that:

> *the difficulty in studying teachers' beliefs has been caused by definitional problems, poor conceptualizations, and differing understandings of beliefs and belief structures*.

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Kagan (1990: 420) shares this viewpoint but notes that the term teacher cognition:

> *Is somewhat ambiguous, because researchers invoke the term to refer to different products, including teachers' interactive thoughts during instruction; thoughts during lesson planning; implicit beliefs about learners, classrooms, and learning; [and] reflections about their own teaching performance . . .*

Part of the difficulty in defining teacher beliefs centres on determining if and how they differ from knowledge. In this study, the researcher accepts the distinction suggested by Calderhead (1996: 715) where he argues that:

> *Whereas beliefs generally refer to "suppositions, commitments, and ideologies," knowledge refers to "factual propositions and understandings."*

In their review of the study conducted by Davis et al., (1993), Levin and Wadmany (2006) seem to have accepted their suggestion that the "*challenges of classroom teaching often limit teachers' ability to provide instruction congruent with their beliefs*."

While other researchers have been enthusiastic in their claims that technology can have a positive impact in the teaching and learning of mathematics, in their survey of the use of technology (or computers) in mathematics education, Galbraith and Haines (1998) cite Fey (1989) as cautioning that:

> *It is very difficult to determine the real impact of those ideas and development projects in the daily life of mathematics classrooms, and there is very little solid research evidence validating the nearly boundless optimism of technologies in our field.*

Therefore, after gaining knowledge of a proposition, in the study, the researcher still feels free to accept it as being both true and false. For example, teachers may gain specific knowledge about how to create Excel spreadsheets for keeping learner records and may also know that other teachers have used them successfully in their classes, yet still not believe that spreadsheets offer an effective tool for their classroom use. This might be especially true if, based on previous experiences, they have negative beliefs about their own technical capabilities. Levin and Wadmany (2006: 158) share same viewpoint in their citation of both writers Pajares (1992) and Prawat (1992) who said that:

> *Beliefs are filters that guide teachers during instructional and curricular decision-making.*

In order to understand teachers' beliefs, a framework for teaching mathematics needs to be established. According to TALIS (2009), there are two dimensions of beliefs, namely a constructivist belief about learning and instruction (e.g. learners learn better when they find solutions to problems on their own), and a direct transmission beliefs about learning and instruction (e.g. a quiet classroom is generally needed for effective learning). TALIS (2009: 92) provide the following descriptions of these dimensions of beliefs:

> *The direct transmission view of student learning implies that a teachers' role is to communicate knowledge in a clear and structured way, to explain correct solutions, to give students clear and resolvable problems, and to ensure calm and concentration in the classroom. In contrast, a constructivist view focuses on students not as passive recipients but as active participants in the process of acquiring knowledge. Teachers holding this view emphasise facilitating student inquiry, prefer to give students the chance to develop solutions to problems on their own, and allow students to play active role in instructional activities. Here, the emphasis is on development of cognition and reasoning processes rather than acquisition of specific knowledge.*

From these views, it can be discerned that a balanced approach would be to juxtapose the two methods and use them side-by-side. Ertmer (2005) argues that the decision of whether and how to use technology for instruction ultimately depends on the teachers themselves and the beliefs they hold about technology. She further reports that teacher beliefs may include their educational beliefs about teaching and learning and their beliefs about technology. Hew and Brush (2006) regard beliefs and attitudes as potential barriers to technology integration but also acknowledge the study done by Bodur et al., (2000) who found that beliefs determine a person's attitude. In her citation of Becker (2000), Ertmer (2005: 25) affirms that:

> *Computers serve as a valuable tool in schools and classrooms where teachers have convenient access, are adequately prepared to use them, have some curriculum freedom to integrate computers and hold personal beliefs aligned to constructivist pedagogy.*

It therefore stands to reason that to change classroom practice, teachers would have to alter their beliefs, and supported in the process. Teacher professional development drives teacher change. The perceptions of usefulness and ease of use of SAPs in their teaching mathematics is deeply rooted in teacher beliefs (Ventakesh and Davis, 2000). In broad terms, teacher change requires a fertile ground for a change to occur, i.e. teachers need to create a classroom culture conducive to teaching and learning. This study is premised under a guide that learner responses on functions are to a great extent influenced by instructional decisions (pedagogy) and curricular choices (resources) which are informed by teacher beliefs. Teacher beliefs therefore hold a key to unlock curriculum change from teachers' beliefs about mathematics to how it should be taught.

2.3 Teacher development programmes (TDPs) – as a tool to mitigate misconceptions about teacher beliefs

In this section, the researcher discusses TDPs that could aid in facilitating and aiding teacher change. Cited from National Strategy for Mathematics, Science and Technology Education (2004: 3) at the launch of the MST Strategy, President Thabo Mbeki (2000) is quoted saying:

> *Special attention will need to be given to the compelling evidence that the country has a critical shortage of mathematics, science and language teachers, and to the demands of the new information and communication technologies.*

Mbeki (2000), supports teacher development as sees it as a tool to turn around the shortage of mathematics teachers and to develop capacity to teach mathematics among those that are already in the service. Developing township high school teachers' knowledge of functions and multiple representations thereof holds promise as a vehicle to address teacher professional development. Given South Africa's highly limited economic resources, it stands to reason that professional development will have to be accomplished in a highly cost-effective manner. It is generally accepted that traditional in-service professional development courses fail to empower teachers if not properly planned in terms of resource and time allocation. It is clear there is a need for alternative approaches to the professional development of in-service mathematics teachers. The educational use of technology in the teaching and learning of mathematics in most South African schools seem to focus on procedural proficiency at the expense of conceptual understanding.

It is important to create a platform for a teacher to become the best teacher. A teacher development platform requires a classroom culture conducive to teaching and learning. This classroom culture will enable a teacher to realise a goal of becoming the best teacher. A classroom culture is able to be transformed through TDPs to become a playground for all learning and teaching to take place. It is through support, persuasion and collaboration that a classroom culture could be seen to influence a school culture. Quality teacher professional development will enhance this transformation agenda. According to Garet et al. (2001), Glattenhorn (1987), professional development activities should build on teachers' experiences and should relate directly to their classrooms. Other reasons for teachers to participate in TDPs are to accumulate continuous professional teacher development (CPTD) points that keep them in the profession, ensure continued registration with the South African Council of Educators (SACE), learn new ways of teaching thus stay abreast of new research developments in their subject in particular and share best practices in general. In most functional schools, a healthy school culture rubs off to a classroom culture and ethos.

2.3.1 Necessity and importance of TDPs

Schools systems are informed by culture-based human interactions. According to Lovitt et al. (1991), strategies for change must confront an already-established culture. Teachers and learners already hold knowledge, values and beliefs about mathematics and how it is taught and learned (Lovitt et al, 1991; Nickson, 1992). As a result, a proposed change in teaching approach can meet with resistance from parents and even learners, whose expectations about what constitutes 'proper' mathematics and how it should be taught may conflict with the culture and roles which the teacher is attempting to create (Cooney, 1985; Goldsmith and Schifter, 1997; Nolder, 1990).

It is difficult for the teacher to allow learners to be confused, puzzled and frustrated while solving a mathematics problem if the culture has been such that the learning has seemed 'painless and

progressive' (Goldsmith and Schifter, 1997). To mitigate this risk, thereby making learning seamless, the researcher suggests that teachers should create a classroom culture where learners are able to engage in constructive group discussions. This is a learning culture where learners are able to listen to one another justify and explain their solutions in order to understand key elements of the topic being discussed. Though this is likely going to take time, learners tend to take responsibility for their own learning in such a classroom. The researcher acknowledges that it could be difficult for teachers to listen to learners' solutions, especially when a syllabus has to be completed within a set time frame. It is also difficult to overcome the culture of teacher dependence.

2.3.2 Designing Teacher Development Programmes (TDPs) to influence teacher change

'Design principles' to help teacher change a classroom practice are described in this section.

The TDPs envisaged for this study are inseparable and they are not sequential. They provide an interplay between pedagogy, knowledge and understanding of functions, knowledge of learners (how learners learn mathematics), knowledge of curriculum and knowledge of technology, and inherent beliefs on spheres of knowledge espoused. The researcher drew inspiration and learnings from the work of Fullan (2006) who asserts that needs-based knowledge is a critical ingredient to a kind of support needed to bring about change in the classroom. The model for TDPs that brings about change, according to Fullan (2006), are based on a notion that for teachers to change a classroom practice or to get learners to perform in high-stakes examinations, one could determine their needs. These experiences have led the researcher to extrapolate that applying positive pressure (support in a positive) to meet the needs of a learner are support mechanisms or 'drivers' that yield fruitful change.

The researcher therefore holds a view that teachers are unlikely to change the way they teach unless they change the way they think. Therefore, the quality of teaching is directly proportional to quality of learning, and as such learner high learner performance or achievement is a bi-product of high quality teaching. In his blog, "Why quality professional development of teachers matters" in Edutopia.org (September 16, 2014), Ben Johnson (2014) quotes a Principal who was attended a teacher professional development 'workshop' who said:

> *If we want students to learn, the most critical element is the teacher. So, professional development is the overall most important thing we can do to help students learn.*

Further to that, Johnson (2014) noted that:

> *While schools and teachers have a tremendous influence over student learning, there is nothing the teachers can do to make it happen. It is completely out of the control of teachers to make students learn; the students have to do it by themselves.*

Quattlebaum (2012) citing Richardson (2003) who defines characteristics of an effective professional development as:

> *Statewide, long term with follow-up; encourage collegiality; foster agreement among participants on goals and visions; have a supportive administration; have access to adequate funds for materials, outside speakers, substitute teachers, and so on; encourage and develop agreement among participants; acknowledge participants existing beliefs and practices; and make use of outside facilitator/staff developers.*

In support of the argument advanced by Johnson (2014) above, Quattlebaum (2012) believes that effective teacher development programmes benefit both the teacher and the learner. Teacher development therefore remains a critical instrument of classroom change, a prolonged facet of classroom instruction that is integrated, logical and on-going and incorporates experiences that are consistent with teachers' goals, aligned with standards, assessments, other reform initiatives, and beset by the best research evidence.

2.4 Barriers to integration of technology in education: High school mathematics teacher beliefs about use of computer

This section looks at some of the major impediments and obstacles that can prevent teachers to teach mathematics, and how these can be changed in a classroom. Since teachers do not have control over school management support, therefore the role and responsibility of school management to support and assist teachers and school community to create a conducive environment for teaching and learning is assumed in this study. This is so because a changed school culture will lead to improved learner achievement and performance.

The literature on technology integration has identified numerous factors that are involved with the use of technology in a classroom. It is important to acknowledge these factors and understand their relationship with the learning environment as everything involved in technology integration in a classroom has the potential to be a barrier. In their analysis of 48 research studies on barriers and strategies of technology integration, Hew and Brush (2006) offer a suggestion that teacher development programmes need to remove them if we hope to influence learning outcomes positively. The barriers considered in this study are those that teachers have control over, namely (a) self-esteem or self-belief; (b) teacher confidence; (c) teacher personality; (d) time and (e) availability of SAPs. These are:

(a) ***Self-esteem or self-belief*:** Teachers who have low expectations of learners often have low self-esteem or lack self-belief. They tend to produce poor results. These observable teacher actions result in learners not making an effort to learn. Making insensitive remarks is an observable action of a teacher that shows low expectations of learners. Examples of such remarks are: "*you are good for nothing*"; "*you are stupid*"; "*you won't amount to anything in life*", etc. Teachers' comments

about learners' poor family backgrounds, poverty and biases on their social status, as well as their low academic achievement of family members, lead to a lack of desire to learn. Learners tend not to work hard enough to achieve their aspirations and educational goals when such comments and remarks are directed at them. This perpetuates a cycle of low self-esteem and underperformance. Learners in these classroom situations often present wayward behaviour in class such as ill-discipline in school and lawlessness. Learners tend to get demotivated and end up not doing well at school when they do not have role models or people they can look up to in society. From the words of Mohammed Qahtani (2014) in his video (The Power of Words, Toastmaster International):

> *Words have power to alter someone's belief when said in the right way. They can mend a broken soul. We see this with people they admire. Whatever they say, get accepted. Nobody likes to be threatened or intimidated – pride is at stake.*

Other observable barrier can be seen during teaching when a teacher oversimplifies a concept, e.g. functions. Oversimplifying a concept occur mostly during code switching when an attempt is made to explain a concept. Mathematics is often taught by non-English speaking teachers in township high schools. The meaning or description of a concept can be lost in translation (or 'watered down') from choice of words used to describe it. We also see low self-esteem from questions in school-based assessments when they are poorly structured differentiated into cognitive levels according to the Programme of Assessment in CAPS grade 10-12 Mathematics (2011: 53), i.e. knowledge (20%), routine procedure (35%), complex procedure (30%) and problem solving (15%).

(b) ***Teacher confidence*:** Teachers sometimes claim to believe in something but seem unable to implement it in the researcher's presence. Vacc and Bright (1999) refer to the 'fragile' knowledge about teaching which results in behaviour consistent with the advocated approach in some contexts, but this behaviour may not transfer to all teaching contexts. Lack of knowledge and skills needed to conform to any new role can be a barrier to implementing this role (Snyder et al., 1992). It is difficult for teachers to define their role as 'facilitator' in an inquiry mathematics approach. Goldsmith and Schifter (1997: 32) define this role as follows:

> *As a more knowledgeable, experienced member of the group, and the acknowledged educational leader of the class, it is his or her responsibility to assess students' understanding, monitor their progress, and stimulate continued growth in mathematical understanding.*

Goldsmith and Schifter (1997) continue by saying that teachers must find a balance between valuing students' individual constructions of their mathematical understanding and guiding them towards shared understandings, principles and structures that make up the domain of mathematics.

(c) ***Teacher personality*:** According to Goldsmith and Schifter (1997) and Snyder et al. (1992), motivational and dispositional factors play an important role in initiating the change process and in helping teachers to persevere with their efforts to change. They refer to studies by Schifter and Fosnot (1993) suggesting that encouragement may be an important factor. The ability to reflect is also an important character trait in the process of change.

(d) ***Time*:** Time is an obstacle to the process of teacher development. Time is often taken up by administrative meetings. Teachers experience the transition towards a problem solving approach (through use of SAPs in *Excel*) to be time-consuming, not only in the classroom but also in terms of preparation and assessment (Cooney, 1985; Nolder, 1990). Traditionally, progress has been measured by 'coverage' of the mathematics syllabus (Nolder, 1990). Carter and Richards (1999) refer to the "universal issue/dilemma" of time, and "the teachers' belief that if they do not spend their time 'covering' the 'curriculum' they will be damaging the students". The teacher is accountable to learners, parents and the management of the school, particularly in preparing the students for external exams (Nolder, 1990).

(e) ***Availability of SAPs as instructional tool*:** The other obstacle he mentions is that a substantial number of teachers hold a view that does not make sense to use calculators in a classroom because learners won't be allowed to use them in examination rooms. In his study on technology integration in mathematics education, Olivier (1987) argues that a calculator poses a threat in that teachers believe it is there now to replace their primary duty of teaching arithmetic skills. Olivier (1987) suggests that before teachers could even think of incorporating technology in their teaching, they should be convinced that it will not influence learner's mathematical development negatively and that it holds objective advantages for learner's mathematical development. Venkatesh and Davis (2000: 186) contend that underutilized systems continue to be concerning in spite of rapid advancement of hardware and software capabilities. Bagozzi, Davis and Warshaw (1992) assert that:

> *Because new technologies such as personal computers are complex and an element of uncertainty exists in the minds of decision makers with respect to the successful adoption of them, people form attitudes and intentions toward trying to learn to use the new technology prior to initiating efforts directed at using. Attitudes towards usage and intentions to use may be ill-formed or lacking in conviction or else may occur only after preliminary strivings to learn to use the technology evolve. Thus, actual usage may not be a direct or immediate consequence of such attitudes and intentions.*

To assist respondents in the process of teacher change, the researcher provided them with SAPs in *Excel* and offered one-on-one training on use thereof in line with the assertion above. He also provided them with classroom support and encouragement.

In conclusion, the researcher captured the essence of the discussions in this chapter through the following diagram offered by Manley (2012),:

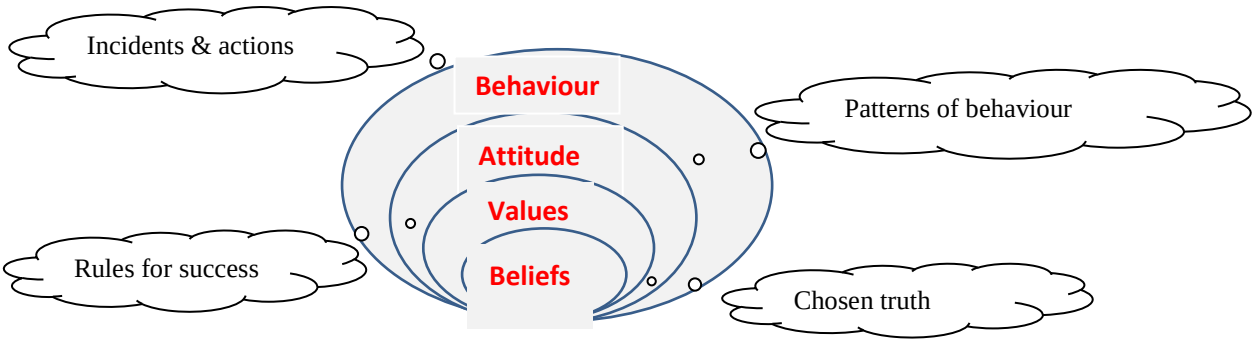

Figure 2.1: Belief to Behavior Manley (2012: 47)

Manley explains:

- The outer ellipse denotes our behaviour – these are observable actions
- The second ellipse, attitudes, determine the way we approach events and situations
- The third ellipse, values, set the rules for living and determine willingness to do or not to do to achieve success.
- The inner ellipse, beliefs denotes who a person really is.

According to Manley (2012), the beliefs carry the truth that one adopts about him/herself on the subject or system. Manley (2012: 46-49) emphasizes that the truth that one adopts can either be a belief about oneself, about one's world or about the system in which one finds him/herself. The illustration above shows the power of the mind in a person. This explanation resonates well with the researcher. To this effect, the researcher adapted Manley's portrayal of 'beliefs to behaviour' to illustrate his claim that to have hope for possible teacher change, teachers need to adjust their beliefs as they have profound influence on their classroom practices.

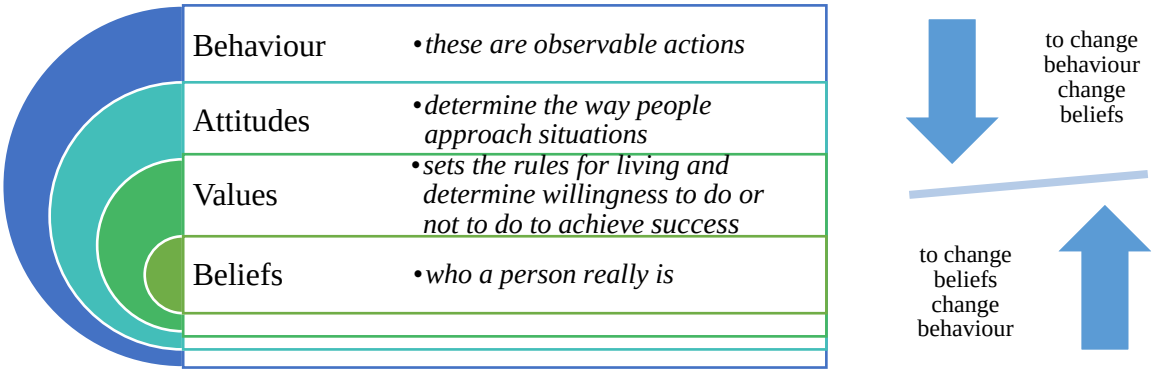

Figure 2.2: Change of behaviour and beliefs

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CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter gives an account of the methodology used in the study and focuses on research design and explanation of the methodology used in this study including the respondents and background of research sites where they teach. It begins with the background, and context which were considered in the design of this study.

This study investigates possible teacher change after the use of SAPs in *Excel* on functions, a 'new form of teaching and learning' in two township high schools from interrelated areas/perspectives, namely: (a) possible changes in teachers' classroom practices, and (b) possible changes in teachers' beliefs. The researcher considered teacher beliefs and classroom practices (i.e. lesson observation and teacher interview) as the units of teacher change in this study. Hence they formed the primary sources of the data.

In the context of this study, instructional decisions and curricular choices constitute teachers' classroom practice, and these are informed by beliefs teachers hold about teaching and learning. As a result, the researcher makes an inference that teachers have control over instructional decisions and curricular choices. Haim Ginott (2003) articulates this claim eloquently in one of his quotes in which he states that:

> *I've come to the frightening conclusion that I am the decisive element in the classroom. It's my personal approach that creates the climate. It's my daily mood that makes the weather. As a teacher, I possess tremendous power to make a learner's life miserable or joyous. I can be a tool of torture or an instrument of inspiration. I can humiliate or humour, hurt or heal. In all situations it is my response that whether a crisis will be escalated or de-escalated and a child humanized or de-humanized.*

The researcher argues that the way in which teachers teach functions in township high schools is greatly influenced by their beliefs on instructional decisions and curricular choices that they make in the classroom. Hence the hypothesis that classroom practices, which are based on teachers' instructional decisions (i.e. subject knowledge, classroom management and organization skills as well as resources including use of technology) and curricular choices (i.e. curricular knowledge, methodology, knowledge of how learners learn) are deeply rooted in their beliefs about teaching functions.

The researcher developed the following diagram of a concept map to illustrate his claim in the study.

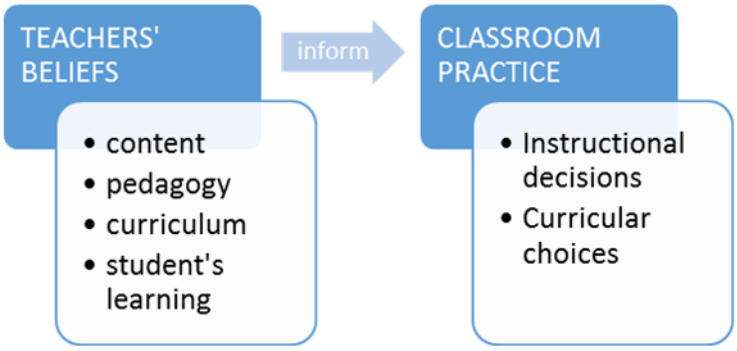

Figure 3: Researcher' view of teachers' beliefs and classroom practice

To drive a possible change of teacher practice, support for teachers is needed. It therefore stands to reason that a well thought-out teacher development activity will help teachers develop the tools that learners need to approach any problem with confidence. Considering the importance of teacher support as a contributing factor in the process of teacher change, the researcher conducted hands-on training sessions with respondents at their schools to give them an experience of working with SAPs. This was done to give them an opportunity to express themselves but also discuss their fears about the use and introduction of the SAPs in *Excel* to teach or challenge learners' conceptual understanding of functions in grade 10 (Teacher B) and grade 11 (Teacher A). Researchers in this study seem to agree that changing teachers' classroom practice and beliefs make it easier for them to hone their teaching skills thus becoming better teachers. By so doing, the researcher was creating an environment where there was interaction between himself and the respondents (participants), but also to create a safe space for teacher professional development at their workplaces. In this way, we were able to share mathematical ideas 'to open up functions' and teaching approaches that could be applied with the use of SAPs, which the study hoped they would emulate in the observed lessons for the study. This required teachers (respondents) to learn and adopt teaching methods that devote a great deal of instructional time to active learning (NCTM, 1991: 52). This viewpoint is supported by Enyart and van Zoest (1998: 150) when they say:

> *Once a teacher has seen students defending their mathematical ideas, questioning other student's ideas, and helping clarify the mathematics to one another, the importance of discourse becomes clear.*

It was the researcher's ambition that these experiences could lead to a reconsideration of teachers' notions about what mathematics is, how learners learn mathematics, and factors that could encourage or hinder such learning (Goldsmith and Schifter, 1997; Murray, Olivier and Human, 1995). The use of SAPs in a mathematics classroom should therefore be seen as a catalyst to reflect on teacher practice and develop teachers professionally. The SAPs were designed for teachers to

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experience a learner's perspective, exploring the functions concepts and making sense of mathematics. Instructions on activities made the role of class discussions very clear. Some of the instructions suggest 'what learners may do' and 'what learners may learn' in order to help the teacher to respond appropriately. This is one aspect of a discourse[4] that sets to open up new dimensions of explaining concepts that are taken for granted (and difficult to do in a traditional 'pen and paper' approach) as either grasped or regarded as social knowledge in teaching, and of mutual engagement in discussions. It was against this backdrop that SAPs were used in this study – a constructivist viewpoint. SAPs required a different approach to teaching and learning. The sooner teachers realise that learners learn more by active experiences than by rote learning (memorizing and retrieving facts), the better. According to Edgar Dale (1946), in this type of interactive learning, learners retain "*90 percent of what they say and do*" after two weeks a lesson. These statistics remind us that it is important that classroom materials should be designed to follow the basic beliefs of providing learners with multiple means of representation, engagement, demonstrations and experience. It is essential for teachers to organize classrooms that foster engagement and collaboration in a classroom, and blend multisensory material to espouse experiences of talking, listening, reading, thinking and writing into the teaching and learning process. In such an environment, it is hoped that learners would be able to reflect and evaluate their ideas, thus improve their conceptual understanding of functions. Such reflections, and de-emphasis on getting 'right' answers can lead to teachers' understanding of learners' needs (function sense and conceptual understanding of functions) which would ultimately improve results in mathematics in assessments. Other teachers who answered the questionnaire[5] but were not observed at school would also begin to notice a change of behaviour of the observed teacher, and begin to reflect on their practice.

3.2 Data collection methods and data analysis

This study investigates possible teacher change after the use of SAPs in *Excel* on functions, a 'new form of teaching and learning' in two township high schools from interrelated areas/perspectives, namely: (a) possible changes in teachers' classroom practices, and (b) possible changes in teachers' beliefs. Two teachers ($n = 2$), 1 female and 1 male, were involved in the study as respondents. In the process of teacher change, the analysis of teachers' conceptual understanding of mathematics was broad-based, including teachers' beliefs, but was restricted content-wise to mathematical functions in grade 10 and 11. The SAPs were used to facilitate the data analysis. The analysis traces the development over the course of one lesson in each grade with Teacher A

[4] In this study, the term discourse refers to an inquiry-based learning culture in a mathematics classroom. It embraces the teacher's and learner's in discourse, SAPs as tools for enhancing discourse and the learning environment in line with professional standards for teaching mathematics according to (NCTM, 1991: 13).
[5] The template is adapted from the MALATI project. It was developed by the Beliefs Action Group at MALATI based on the following sources: Cobb, et al (1991), Newstead (1995), Graven (1997), Pehkonen (1997) and RUMEP (1997).

and Teacher B and highlights the importance of beliefs in the teaching of mathematics and address issues related to teacher professional development. All mathematics teachers from two schools ($n = 10$), 4 female and 6 male including the respondents, completed teacher beliefs questionnaire. The design of the study took account of the suggestion (Levin and Wadmany, 2006: 157) that:

> *….it is worthwhile to investigate teachers' beliefs, and also to explore the implicit link between teachers' views on learning and teaching and their actual classroom practices.*

It is for this reason that the data gathered for this study considered the following elements, (a) investigation of teacher beliefs in respect of assessment, nature of mathematics, teaching and learning, learner ability and calculator use, and (b) classroom practices to investigate (i) possible changes in teachers' classroom observation as a result of use of SAPs or own lesson on functions, and (ii) post-lesson interview to determine possible shifts.

It also took into account that classroom practice and teacher beliefs are being investigated as factors that can contribute towards teacher change in line with Guskey (1986) that there are three major outcomes of teacher development: (a) change in teacher classroom practice, (b) change in teacher beliefs and attitudes, and (c) change in learning outcomes of learners.

Specific objectives of the study include:

a) To document and analyse possible changes in teachers' beliefs;
b) To use of SAPs to engage with new forms of teaching and learning;
c) To reflect on ways SAPs can enable dynamic, inquiry-based mathematics teaching and learning;
d) To rethink the school mathematics curriculum, both in terms of content and teaching approach.

Primary sources of data were:

1. Teacher beliefs questionnaire - to establish underlying beliefs on mathematics, pedagogy and technology,
2. Teacher practices - field notes based on classroom observation,
3. Post-lesson interview - to review the teacher's perspective on observed episodes. All interviews are audiotaped and then transcribed.

The data used in this study were gathered in the third term in July-September 2009 in 15 days (i.e. 11 hours) from two township high schools selected for this study in the Western Cape (WCED). Preliminary data analysis began during data collection to help direct further data collection and analysis, and finalised at the conclusion of the study. The data analysis is qualitative – teachers' responses on the beliefs questionnaire were systematised according to categories that emerged while analysing the data, rather than superimposed beforehand. A qualitative research design has been chosen in this research because of its strength in producing a wealth of detailed in-depth and descriptive data. Descriptions of the categories were then used to describe levels or phases or dimensions of participating teachers' beliefs about teaching and learning mathematics, and how

this had changed during the study. Data analysis was done within a socio-constructivist epistemological framework, and interpreted against the constraints of the socio-political context of participating teachers.

3.3 Sampling

The target group for this study are in-service FET mathematics teachers in two township high schools of Western Cape Education Department. Two teachers ($n = 2$), 1 female and 1 male herein referred to as Teacher A and Teacher B respectively, were respondents in the study. The researcher observed lessons of respondents in their classrooms teaching grade 11 and 10 functions respectively. For the purposes of this survey, the sampling frame was defined as $n = 10$, 4 female and 6 male, including Teacher A (female) and Teacher B (male) who were interviewed following one classroom observation for each teacher at their respective schools.

The populace of both schools is predominantly black. These schools are greatly affected by poor results in mathematics and schools are generally underperforming. Respondents were selected in the study based on:

a) Availability of computers at their schools: in-service teachers teaching grade 10 and grade 11 in two different township high schools who have access to computers.
b) An interest and positive attitude towards computers: in-service grade 10 and 11 mathematics teachers in township high schools in the Western Cape.
c) Experience technical and general support from school: They would have some form of interest in educational technology integration in past or present experiences.
d) Willingness to participate – this includes their willingness to allow the researcher to observe their classroom practices, make notes and record interviews after lesson observations, and to complete teachers' beliefs questionnaires.

This sample was not designed to be representative but to provide indicative evidence of the issues related to mathematics education in a typical township high school. The study was purposely designed such that all mathematics teachers ($n = 10$) in the project schools completed a teachers' beliefs questionnaire. The purpose of involving/implementing all mathematics teachers in two different diverse sites and different grades was (a) to identify teacher beliefs, analyse them and describe unique variables in different contexts and, (b) to generalise findings across different school contexts. Hence a descriptive analysis to reveal patterns. This was done to avoid potential conflict when a respondent would be using technology at a school, and others not. This is based on the theory that teaching should be part of developing a community, and organic growth, i.e. everyone stands to benefit in a vibrant learning environment. Considering that the researcher was working closely with respondents as Subject Advisor, he based his claim for teacher change on what he directly observed as respondents made certain comments in class and post-lesson

interview. Comments made after the interviews, and even in the researcher's informal interactions with respondents long after the research was conducted indicated that their educational beliefs and practices had changed.

All teachers, including principals of the two research schools accepted willingly to have their schools involved in the study. By volunteering to teach a lesson for the study using SAPs, respondents indicated their acceptance of SAPs and that the use thereof could 'open up' functions. This opened a window of opportunity for the researcher to study their beliefs about the integration of computers in mathematics education in general, and the use of SAPs in *Excel* to teach functions in a high mathematics classroom in particular. It is relevant to consider the theory based on a 'technology acceptance model (TAM)' of Venkatesh and Davis (2000: 186-187):

> *TAM theorizes that an individual's behaviour to use a system is determined by two beliefs: perceived usefulness, defined as the extent to which a person believes that using the system will enhance his or her job performance, and perceived ease of use, defined as the extent to which a person believes that using the system will free of effort.*

The researcher gave teachers copies of the SAPs in *Excel* on flash disks a week prior to observation of lessons and data gathered in July–September 2009. Respondents were orientated and trained (one-on-one) on use of the SAPs and showed how they could use sliders to create a classroom culture in which ideas can be shared, evaluated and discussed (see appendix (ii)). The researcher did this to avoid time wastage or at least minimize time required for teachers to develop their own SAPs. With this arrangement, the researcher was giving teachers an opportunity to 'play with it' and establish critical teaching moments to highlight fundamental conceptual gaps as they do so. The SAPs provided respondents with ample opportunities to extract the useful mathematics and facilitate valuable discussion on functions with the use of SAPs in a structured manner rather than to develop isolated, piece-meal knowledge leading to different rules for different functions. The researcher asked them to prepare the lesson for the study based on discussions in orientation session (deepen learner's understanding of functions) and advised that if they choose to develop their own material, they should not deviate from the objectives of the study. These interactions were done with teachers at their schools on different dates prior to scheduled lesson observation. To make up for inadequacies in the teacher notes the researcher was available to help whenever needed, and shared a teaching approach the researcher suggested for the study.

3.4.1 Teacher A and background of school A

The female in-service FET mathematics teacher, Teacher A, had been at School A for two years at the time of the study. Her working experience as an educator spanned over 25 years. Knowing that Teacher B had already offered to teach a grade 10 class, the researcher had to look out for a grade 11 at School A. Teacher A willingly accepted the invitation to be part of the study as

respondent. We agreed on content to be covered, use of SAPs and computer-friendly classroom to use as venue to conduct a lesson for the study, and a post-lesson interview.

In my initial planning meeting at School A in July 2009, the principal said:

> *Working hand in hand with Khanya and Dinaledi, the school moved swiftly to implement their plans to use the technology to consolidate the curriculum by providing revision lessons. The experience is an eye-opener for some of the staff members. Their expectations are surpassed by their rapid progress through the syllabus with the effective use of computer technology.*

School A is a Dinaledi[6] school located in a township outside Cape Town. It is dual-medium. It is a high school with full academic and technical streams, and boasts of a rugby culture. In its long historical background, the school strives to produce balanced, well rounded and suitably prepared learners for schooling in a dynamic society according to its mission statement.

3.4.2 Teacher B and background of school B

The male in-service FET mathematics teacher, herein referred to as Teacher B, is a young Head of Department of mathematics, science and technology. Teacher B began his teaching career at School B in 2007. Teacher B participated in extensive in-service teacher training programmes prior to the study, e.g. school-based interactive whiteboard, and WCED sanctioned *Geometer's Sketchpad* and *Geogebra* that the researcher conducted myself in the district). The researcher also interacted with Teacher B as a Provincial Training Team during orientation and implementation of National Curriculum Statement for FET in 2006-2007 and CAPS for FET mathematics in 2010-2012. In seeking permission for mathematics teachers from the school to participate in the study, it was by coincidence that the researcher approached School B first. Teacher B offered willingly to teach a lesson for the study on functions using a grade 10 class that he teaches.

School B is situated in a low-income township outside Cape Town. It boasts of cultural exchange programmes forming network with schools in England and enjoys financial support of its programmes from public-private partnerships. The researcher's relationship with School B is characterized by mutual trust, co-operation and deep sense of professionalism. On few occasions the researcher was called to observe interview processes of new teacher appointments and assisted in fund-raising for the school programmes. This made it easier to monitor his practice.

[6] Dinaledi is a national DBE initiative whose aims are to maintain and improve high academic standards in mathematics and science to fast track learner performance, participation and attainment in mathematics and science nationally

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3.5 Research instruments

3.5.1 Teacher beliefs questionnaire

The teacher beliefs questionnaire template[7] (see appendix (iii)) used for this study was designed to establish and describe baseline views of mathematics teachers, in the two selected high schools, School A and School B. Teacher responses of 10 teachers (4 female and 6 male), including the respondents Teacher A and Teacher B, from the two schools on beliefs questionnaire were tabulated, coded and analysed according to the categories that emerged during data collection. There were 5 categories that emerged, namely, assessment (19), learner abilities (8), the nature of mathematics (7), teaching and understanding (36), and use of calculators (3) – the numbers in brackets indicate the number of statements in each category. It was important for the study to check usability of the data from the time when it was collected to the conclusion of the study.

3.5.2 Teacher practice and post-lesson interviews

The interview template used in the study was adapted from the Numeracy Centre[8]. Teacher A and Teacher B were interviewed following one classroom observation for each teacher. The classes involved in the study were the grade 10 and 11 functions. It was expected that both respondents would use SAPs in *Excel*, or take heed of design principles and objectives of the SAPs when designing their own lesson for the study. Of particular importance in observations of teacher practices was the role of SAPs in facilitating teacher changes. This research is supplemented with data collected from a beliefs questionnaire that was completed by all mathematics teachers from the two schools including those that formed part of research.

The study is an explanatory qualitative and comparative study that describes teacher change. A qualitative research design has been chosen in this research because of its strength in producing a wealth of detailed in-depth and descriptive data.

In lesson observations the researcher was interested in teachers' instructional decisions and curricular choices – not just whether they have a range of appropriate strategies available, but whether they can make decisions on how and when such strategies can be most effectively employed (Goldsmith and Schifter, 1997). The researcher conducted an open post-lesson interview with respondents to get their perceptions on the observed lesson. Interviews were tape recorded and video-taped, field notes were taken to capture verbatim protocols during interviews and class observations and analyzed for presentation of results. These field notes provide a rich source of data on critical incidents in a lesson. Interviews for Teacher A and Teacher B were semi-structured and lasted for 30 and 45 minutes respectively. The researcher did not expect the

[7] The teacher beliefs questionnaire was developed at MALATI by the Beliefs Group that was led by Karen Newstead. Items or statements were based on the following sources: Cobb et al., (1991); Newstead, (1995); Graven, (1997); Pehkonen, (1997) and RUMEP, (1997

[8] The Numeracy Centre evaluated a computer-based tutorial program at University of Cape Town in 1999.

questions to be asked question by question with teacher's responses recorded. Interviews allowed teachers to expand on episodes in the observed lesson that they found important to them. Probing questions were sometimes asked to bring to light teacher beliefs in respect of (a) perceptions of the teacher, (b) attitudes towards mathematics, and (c) attitudes towards computers in mathematics education. This was done to encourage reflection and conversation on the course making sure that the main thrust of the interview is focused on these three areas of this study.

3.5.3 The spreadsheet algebra programmes (SAPs) in *Excel*

The SAPs in *Excel* were introduced as an instructional tool with power to "influence the mathematics that is taught and enhance students' learning" (NCTM, 2000: 24). The multidisciplinary representation of SAPs creates an opportunity for teachers to engage with new forms of teaching and learning. They were designed such that their dynamic and inquiry-based features could elicit better understanding of functions. The influence of SAPs in 1 lesson observed with each teacher is being investigated and used to describe possible change of teacher practice and beliefs in this study.

The purpose of the study and principles of designing SAPs in *Excel* were discussed. These interactions, including choice of classrooms where lessons were to be observed were done with Teacher A and Teacher B at their schools on different dates, depending on the scheduled lesson observation. Respondents were provided with files containing copies of SAPs in *Excel* on functions on disks beforehand, i.e. a week prior to the lesson observed for the study. Respondents were orientated and shown how they could use sliders (see *Appendix* (ii)) to extract mathematics to deepen learner's understanding of functions from the instructional material. This was to give them an opportunity to 'play with it' (i.e. use sliders on SAPs and begin to generate ideas and formulate conjectures) but emphasized the importance of finding *critical teaching moments* as they play with it. In as much as respondents had liberty to use any instructional material other than SAPs when preparing their lessons for the study, the researcher asked them not to deviate from research aims and objectives. To make up for inadequacies in the teacher notes, the researcher made himself available to help and support respondents with any queries as responses planned and prepared for the 'research lesson'. These discussions were purposefully designed as professional teacher development exercise to support teacher change. These techniques provide an approach to data analysis and re-description of teachers' practice in the lessons observed.

CHAPTER 4: DATA PRESENTATION AND ANALYSIS

4.1 Introduction

In this chapter, data are presented and analysed. At the beginning of the chapter, the researcher overviewed the study which was administered in 11 hours from the two research schools. He then looked at teacher change and present an analysis and findings on teacher beliefs and classroom practice.

4.2 Presentation of data on teacher beliefs and analysis thereof

Responses on the teacher beliefs questionnaire are analysed and results are presented in this section. The study consisted of 10 teachers (6 male and 4 female) in 2 township high schools all of whom completed the teacher beliefs questionnaire prior to the observation of lessons. This sample includes 2 teachers (1 male and 1 female) whose classroom practices were observed and recorded in 1 lesson for each respondent. Data on teacher beliefs is presented and analysed beginning with responses of all mathematics teachers in the participating schools followed by a comparative analysis of beliefs of the respondents. Further analysis of the data on beliefs was done to compare with the investigation of the use of SAPs on classroom practices.

Altogether it took 11 hours in 15 days to collect the data from the 2 participating schools. Preliminary data analysis to check for usability and usefulness of the data began in September 2009. Since data analysis was qualitative, teachers' responses were analysed or grouped according to categories that emerged while analysing the data as opposed to being superimposed beforehand. To help shape and redirect the study, further analysis of the data continued until the conclusion of the study. The 5 categories were then used to describe levels of beliefs of participating teachers. The items or statements (73) were grouped into the following categories, namely, Assessment (19), Learner Ability (8), Nature of Mathematics (7), Teaching and Learning or T & L (36), and Calculators Use (3), where the number in brackets show the number of statements related to a particular category. Respondents were asked to indicate on the response sheet by circling one of the following options for each statement on the 5-point Likert scale (see appendix (iii)):

1.	Strongly disagree	2.	Disagree	3.	Neutral	4.	Agree	5.	Strongly Agree

Figure 4.1 below shows average responses in School A and School B to all statements in each of the five categories in which responses to each statement ranged from strongly disagree (1) to strongly agree (5) were calculated for all participants, $n = 10$.

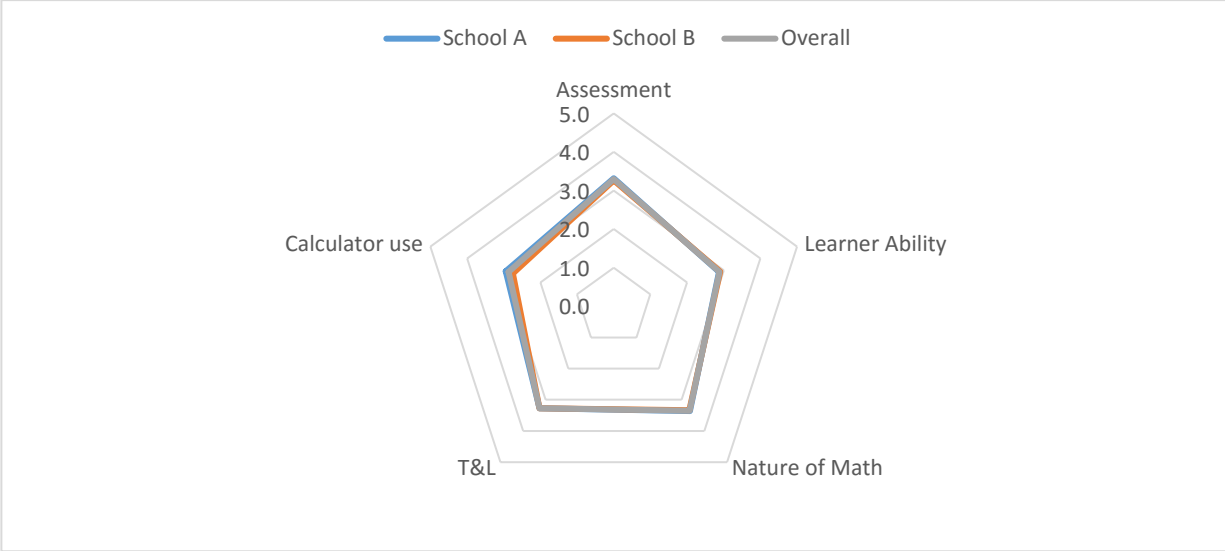

Figure 4.1: Overall Average Beliefs Scores for Schools and all Participants

Figure 4.1 is in the form of a pentagon to highlight interrelationships between the five categories, namely, 'Teaching and Learning (T&L), Calculator use, Leaner ability, the Nature of Mathematics and Assessment'. For example, one possible interrelationship is that for effective teaching and learning to take place, calculator use will need to be adapted to enhance learner abilities with regards to the nature of the mathematics. In this case, there needs to be accompanying 'pen and paper' assessment that incorporate ways the nature of mathematics is configured in the calculator use.

From the data shown in *Figure 4.1* the researcher can conclude that the higher the score, the more positive the belief is towards that statement. To get more insight into teacher responses in the teacher beliefs questionnaire (see appendix (iii)), the researcher displays the responses for each statement by category as given above. He did this to look at each of the statements posed and beliefs recorded beginning with responses of all mathematics teachers $(n = 10)$ in the participating schools followed by those of the respondents $(n = 2)$.

Category 1: Assessment

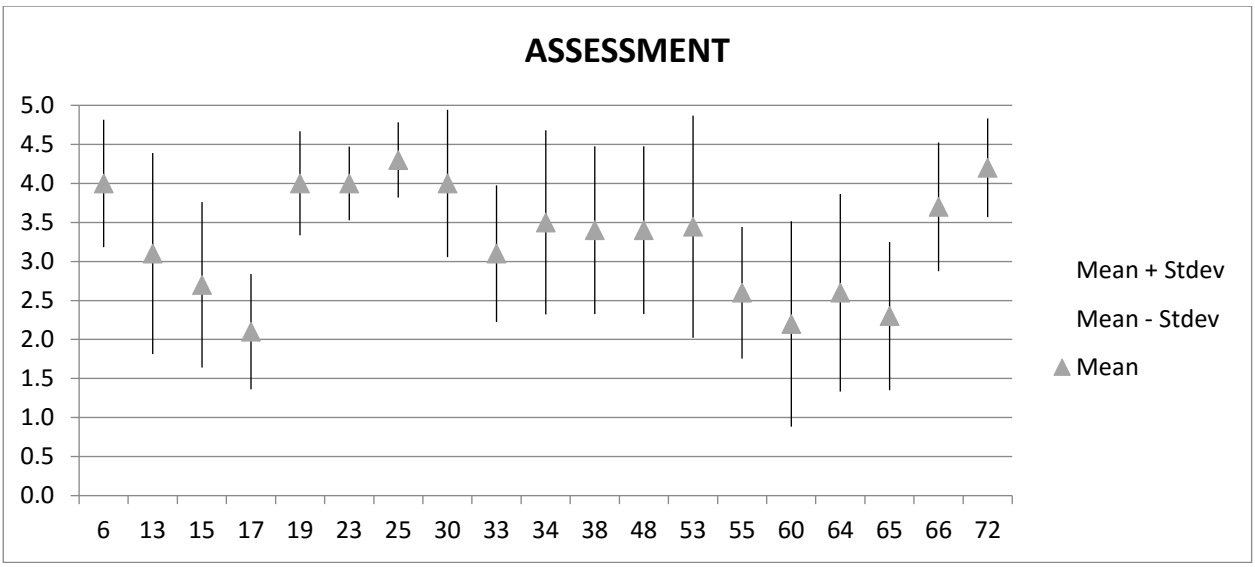

Figure 4.2: Responses to individual statements about Assessment

The *Figure 4.2* above gives the average (mean) response, indicated by an arrow, to each of the statements related to assessment, along with a line through the arrow which gives an indication of the "spread" or "variation" of the responses to the statements, as evidenced by mean + standard deviation and mean – standard deviation. The longer the line, the more varied the beliefs as reflected by the teachers. From *Figure 4.2* it is worth noting that though the overall response to all statements ($n = 10$) related to assessment was around "Neutral" or value 3, the responses to the different statements actually varied a lot, with *Figure 4.2* giving a clear indication of the responses to each statement in this category. It could immediately be noticed that the lowest level of belief relates to question 17, i.e. "*If a learner fails an assessment, the learner is not capable of doing this work*", with the average belief being reflected as "DISAGREE" or a 2 on the scale of 1 (strongly disagree) to 5 (strongly agree). The short line through the statement 17 reflected in *Figure 4.2* further shows that the respondents are quite uniform in their response to this statement, i.e. teachers do not vary a lot about this opinion! Other statements related to Assessment that gave lower levels agreement are statements 60 and 66, or respectively statements "*Only the teacher and school authorities need to know how learners are to be assessed*" and "*If a learner fails, he/she should be given the opportunity to be reassessed*" got lower levels of agreement, in fact, are more on the disagreement scale than on the agreement!

Statements posed about Assessment got responses that indicate the highest level of agreement with statements made were statements 6; 19; 23; 25; 30 and 72 respectively, i.e.
"*Assessment can take place during everyday class activities*"
"*Learners should be given the responsibility of assessing their own progress*"
"*Learners and parents should always know how the learners are to be assessed*"

"*Assessment of group work does not give an accurate reflection of what an individual has learned*"
"*Marks should be assigned for all work that is assessed*"
"*By looking carefully at the results of assessment, the teacher can gain valuable information for the planning of future activities*"

Note that the 25th statement, i.e. "*Assessment of group work does not give an accurate reflection of what an individual has learned*" received the highest level of agreement as can be seen in *Figure 4.2*, with the short line through the arrow in this line indicating that the respondents had a very high level of agreement between them to this response, which on average was agree with a tendency to strongly agree. The longest line through any of the points in *Figure 4.2* relates to statement 53, "*Teachers should assess all learners every lesson*", where the average response indicates that the belief is between neutral and agree ($Mean = 3{,}5$). Teachers generally have strong opposing views, i.e. "Strongly Agree" or "Strongly Disagree" on Assessment. For items: 6; 13; 15; 17; 19; 23; 24; 30; 33; 34; 38; 48; 53; 55; 60; 64; 65; 66; 72, teachers from School A and School B share similar beliefs.

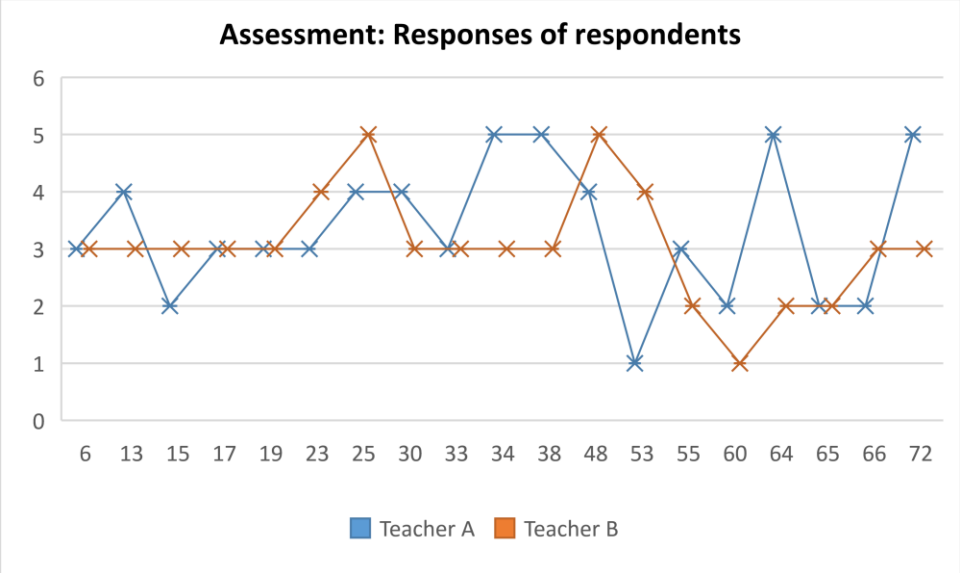

Figure 4.3: Responses of respondents about Assessment

Whilst respondents agree on statements 6; 17; 19; 33 and 65, they have sharp differences with regard to beliefs about statements 53 and 64 as shown in *Figure 4.3*. Teacher B strongly agrees (5) with statement 53 while Teacher A disagrees (2), and, Teacher A strongly agrees (5) with statement 64 while Teacher B disagrees (2). These differences of beliefs of respondents seem to mirror those of all teachers from the two schools as shown in *Figure 4.2*. This necessitates teacher development workshops to synchronise their beliefs on assessment in order to improve mathematics results in high schools.

Category 2: Learner Ability

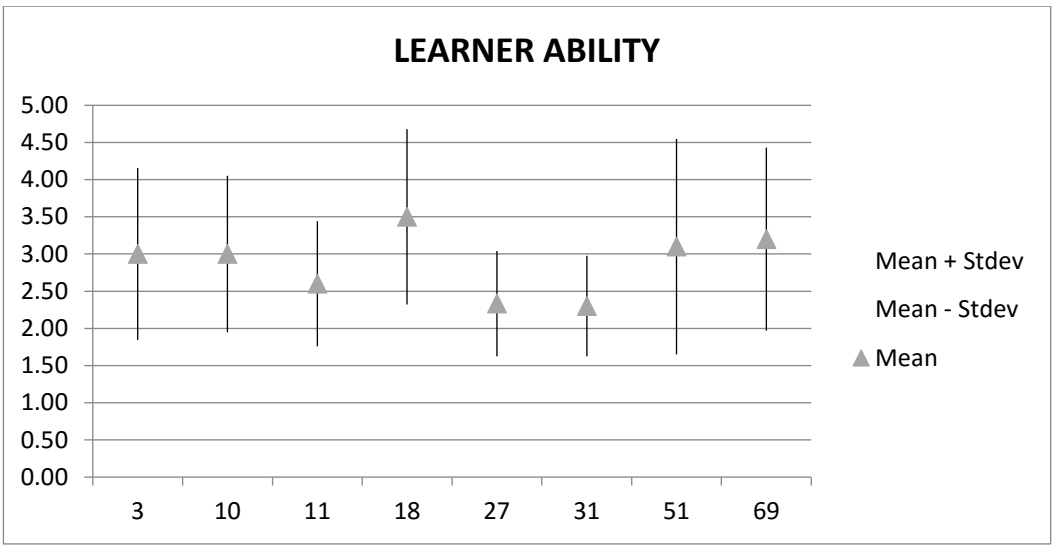

Figure 4.4: Responses to individual statements about Learner Ability

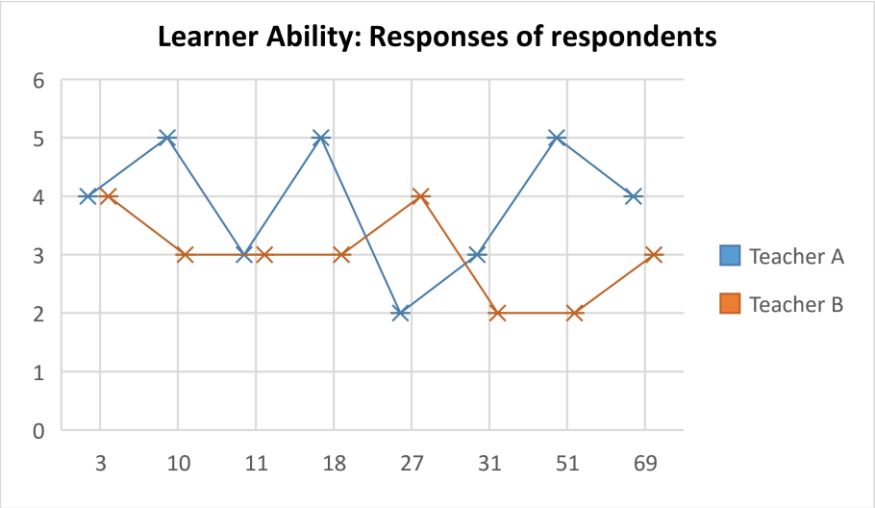

Figure 4.5: Responses of respondents about Learner Ability

Although Teacher A seems to agree ($Mean = 3{,}88$) with items in this category, Teacher A took a neutral stance ($Mean = 3$). Notably, the difference in beliefs about learner ability to comprehend mathematical concepts between Teacher A and Teacher B as shown in *Figure 4.5* above, is insignificant. Therefore Teacher A and Teacher B seem to hold similar beliefs about learner ability to comprehend mathematical concepts in respect of items 3; 10; 11; 18; 31 and 69 respectively.

"Learners cannot reach their mathematical potential unless they are learning mathematics with other learners of the same ability level"

"It is not possible to control a class in which different learners are engaged in different activities"

"If weaker learners are taught separately, they will adjust to the lower expectations and perform poorly"

"The teacher cannot be expected to teach a variety of different ability levels if there is not sufficient material to help him/her"

"Learners cannot learn mathematics effectively in a multicultural classroom"

"Weaker learners benefit from being taught in a mixed-ability class"

There is general consensus between the respondents' beliefs on items listed above.

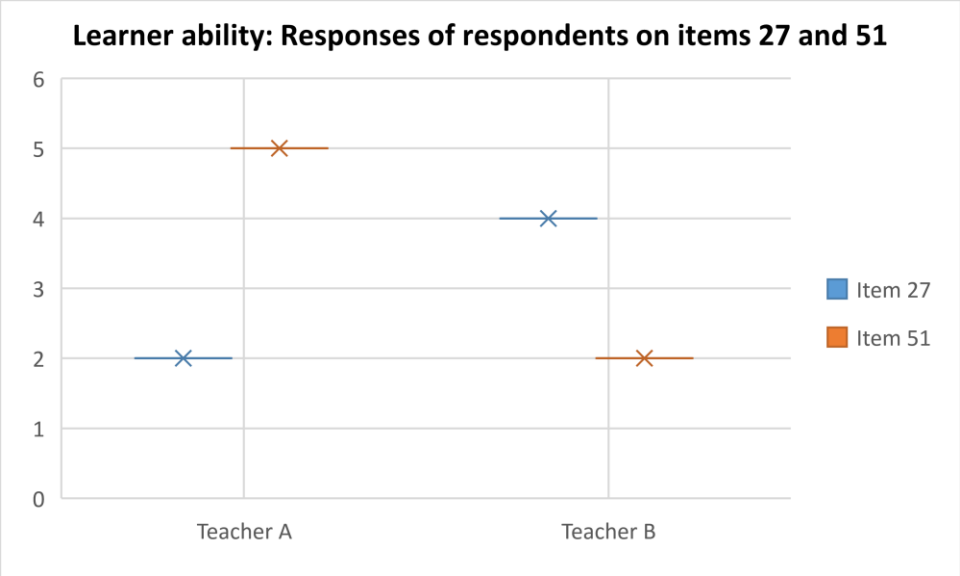

Figure 4.6: Responses of respondents on item 27 and 51

The researcher noted however that the respondents seem to have sharp differences in their responses to the following items 27 and 51 respectively, i.e.

"*Stronger learners benefit from being taught in a mixed-ability class"*

"*Stronger learners' mathematics development will be hampered if they are taught in a mixed-ability class"*

Category 3: Nature of Mathematics

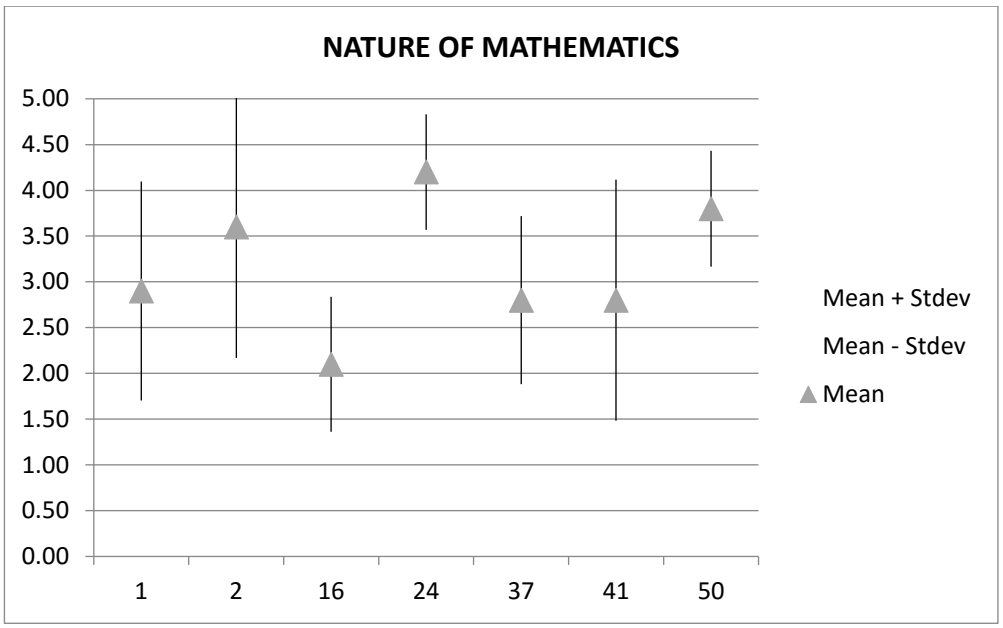

Figure 4.7: Responses to individual statements about Nature of Mathematics

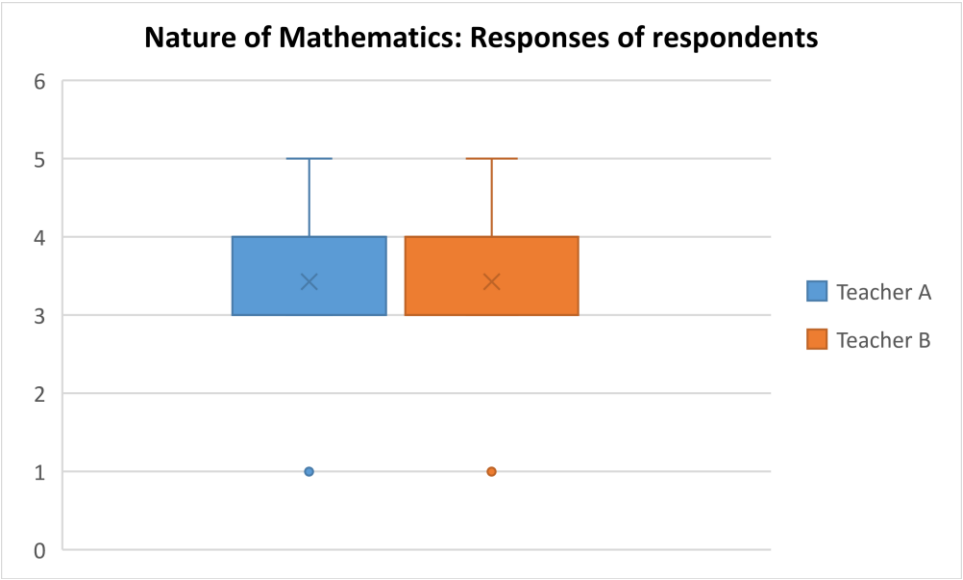

Figure 4.8: Responses of respondents about Nature of Mathematics

It would appear that respondents agree totally on all the items in this category, Nature of Mathematics.

Category 4: Calculator Use

In the context of this study, calculators were included as a category of teacher beliefs to test teachers' acceptance of technology as a tool to enhance the learning and teaching of mathematics because certainly, they offer many exciting possibilities in most mathematics classrooms. As

technology tools, both SAPs in *Excel* and calculators serve different purposes of teaching and learning, and are probably equally relevant in different contexts. For example, calculators can help learners to cope with calculations and quantifications, and teachers can use them to introduce new mathematical concepts (i.e. display table of values, graph and equation).

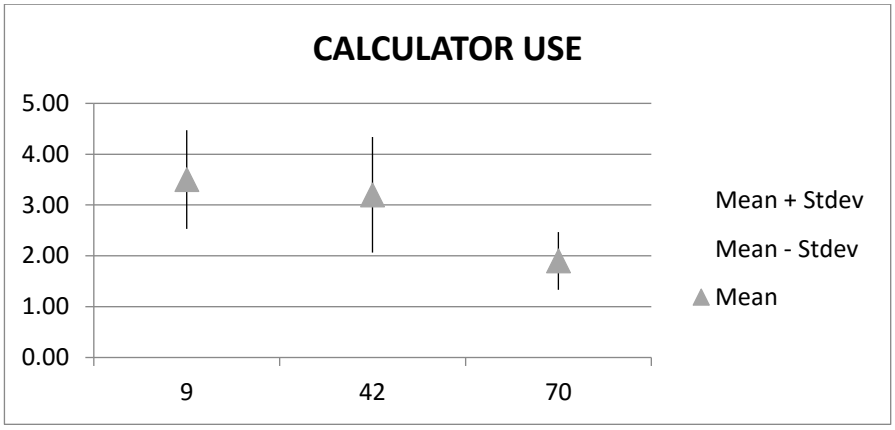

Figure 4.9: Responses to individual statements about Calculator Use

There is general consensus in the responses of all teachers, including respondents that seem to indicate the lowest overall response ($n = 10,\ Mean = 2{,}87$ and $Stdev = 0{,}89$) on the 3 items, namely, 9; 42 and 70 in this category.

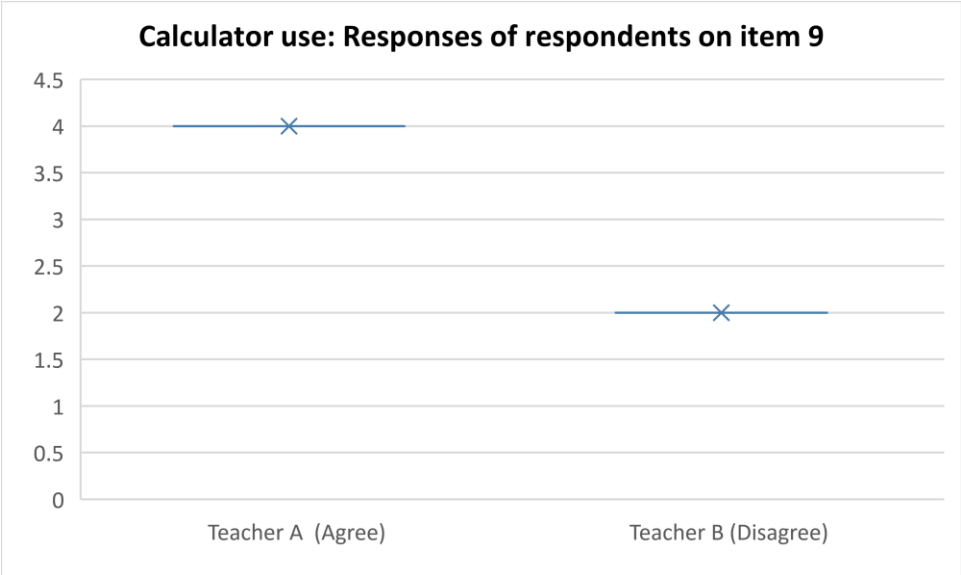

Figure 4.10: Responses of respondents on item 9

Whereas respondents shared the same beliefs about calculator use on item 42 and 70, they differed sharply on item 9, i.e. "*The calculator can be used to introduce learners to important mathematical concepts.*" These contrasting beliefs about calculator use among respondents begin to explain why Teacher B did not use SAPs in *Excel* in the lesson observed for the study.

Category 5: Teaching and Learning

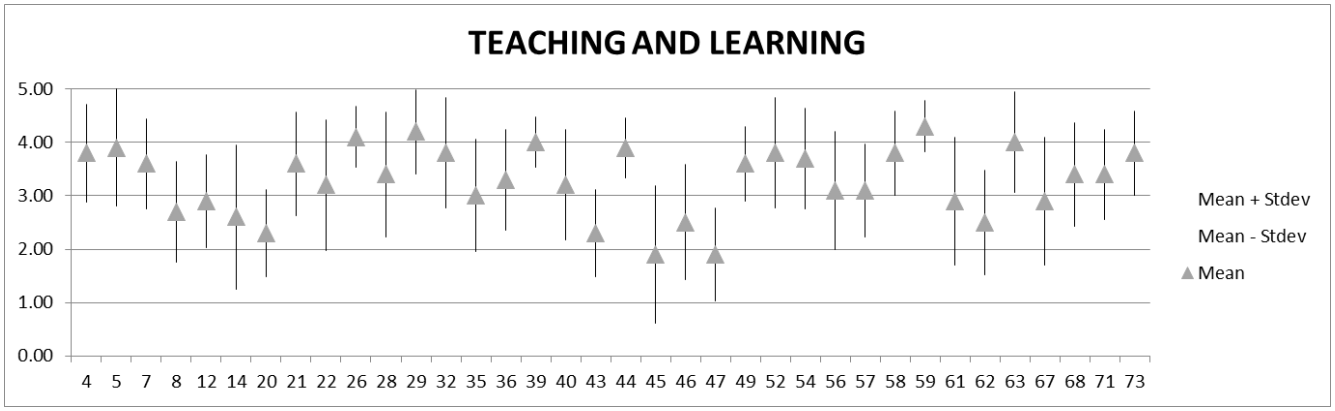

Figure 4.11: Responses to individual statements about Teaching and Learning

Responses ($n = 10$) related to statements about beliefs on Teaching and Learning came out slightly towards "Agree", i.e. just over 3, as is the case for Teaching and Learning. As cited above, the longer the line, the more varied the beliefs as reflected by the teachers.

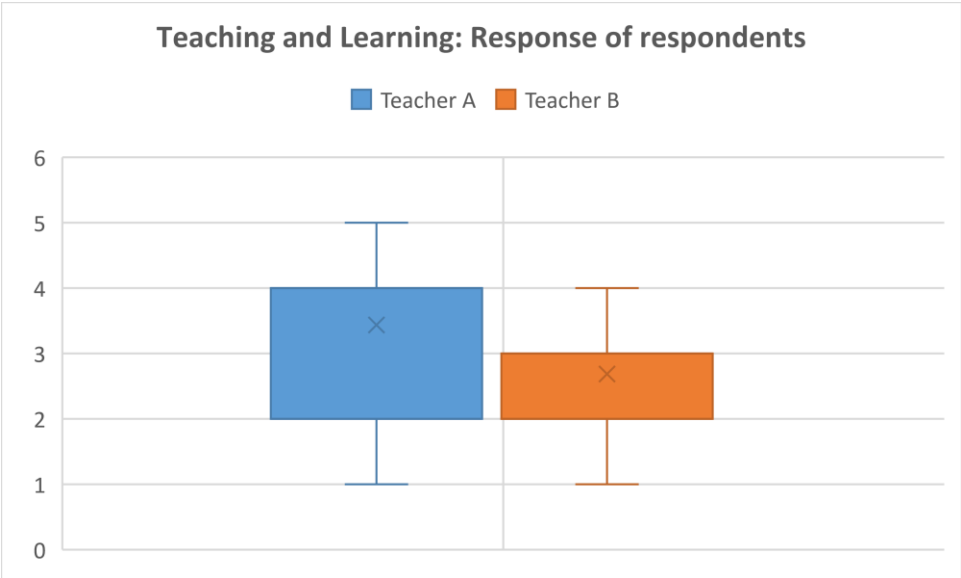

Figure 4.12: Beliefs of respondents on Teaching and Learning

For the listed questionnaire items, Teacher A seems to stay at 4 and 5 ($Mean = 3{,}44$) while Teacher B stays at 2 ($Mean = 2{,}69$). It is at the following items that Teacher A and Teacher B highlight the strong difference of beliefs on teaching and learning, namely, 5; 8; 14; 35; 40; 54 and 61. Therefore the two respondents have contrasting views about teaching and learning. Whereas Teacher A seems to be aligned with the constructivist theory of learning, Teacher B appears to be aligned with the behaviourist (direct transmission) pedagogy.

Teacher beliefs of all mathematics teachers in School A and School B can be summarised as follows:

- The above statistics show that there is very little difference between the beliefs of teachers ($n = 10$) from School A and School B in each of the categories. So not surprisingly, this average response for each of the sections also coincided with the average response for beliefs of all teachers in the study. Owing to this little difference, there is no reason to treat the responses or beliefs of teachers in the two schools separately.
- In all categories of statements, beliefs are on average very close to 3 (neutral), with responses related to statements about beliefs on Assessment coming out slightly towards "Agree", i.e. just over 3, as is the case for Nature of mathematics and T & L. As cited above, the longer the line, the more varied the beliefs as reflected by the teachers. Teachers generally have strong opposing views, i.e. "Strongly Agree" or "Strongly Disagree" on Assessment.
- The lowest overall response seemed to be for beliefs about statements related to Calculator use.

4.3 Presentation of data on teachers' classroom practice and analysis thereof

The researcher analysed classroom practices of Teacher A and Teacher B in this section. The researcher used excerpts from field notes on observed lessons and post-lesson interview which were captured on video, transcribed and analysed for presentation of results to present findings on classroom practices of Teacher A and Teacher B.

The study provides several criteria for teacher change. Significant change of teacher practice cannot be achieved without the opportunity to review teacher beliefs as referred to in the section above, teacher reflection on a practice and without sufficient support for the teacher. These are critical ingredients for teacher development. In their review of the research on shared vision and technology integration plan in schools, Hew and Brush (2005: 235) cite Schiller (2002) as having suggested that:

> *Another issue to be considered in the technology plan is the formulation of monitoring activities to ensure that technology integration is taking place. Examples of monitoring activities used by principals that were found to be significant in ensuring teachers' use of technology include: one-on-one discussions with teachers, observation visits to classrooms, and scrutiny of lesson and program plans.*

The researcher had one-on-one interactions with respondents and provided them with files containing SAPs in *Excel* a week prior to classroom observations. These interactions happened at the research schools. The researcher made himself available to help bridge the gap as a result of inadequacies or unavailability of teacher notes on SAPs. Furthermore, it was important for the researcher to intervene during the course of the lesson whenever it was possible to do so, or when convinced that modelling a strategy for a classroom culture could be advantageous for the whole discussion. Both respondents were comfortable with this.

4.3.1 The observed lesson and interview of Teacher A

This grade 11 lesson observation took place in the third term in September 2009. This was an afternoon lesson in the last period of the day. There were 24 learners in class. The duration of the period was 45 minutes. Teacher A used SAPs in *Excel* on functions in the observed lesson.

a) The classroom observation of Teacher A

The researcher reports on classroom observation of Teacher A in this section. Teacher A used a variety of SAPs in *Excel*, namely Horizontal Translations, Vertical Translation and Translation by a vector. The researcher named them Lesson 1, 2 and 3.

Lesson 1*: Horizontal *Translations

Teacher A started the lesson by greeting learners and introduced the researcher to her grade 11 mathematics class. "*We are going to do transformations of functions in this lesson today*," she told learners. Meanwhile she wrote the topic, Translations on the board. She reminded learners about terminology regarding rigid transformations. She explained the meaning of rigid transformations that "*it is transformations that preserve the shape and size of geometric figures*". She wrote rules of translations on the board, namely:

p (horizontal movement) and the *rule is* $(x; y) \rightarrow (x + p; y)$ and

q (vertical movement) and the rule is $(x; y) \rightarrow (x; y + q)$

She drew Triangle A (ΔA) on the board to illustrate reflection of ΔA to a new position ΔA^1 according to the general rule: $(x; y) \rightarrow (x + 3; y + 3)$.

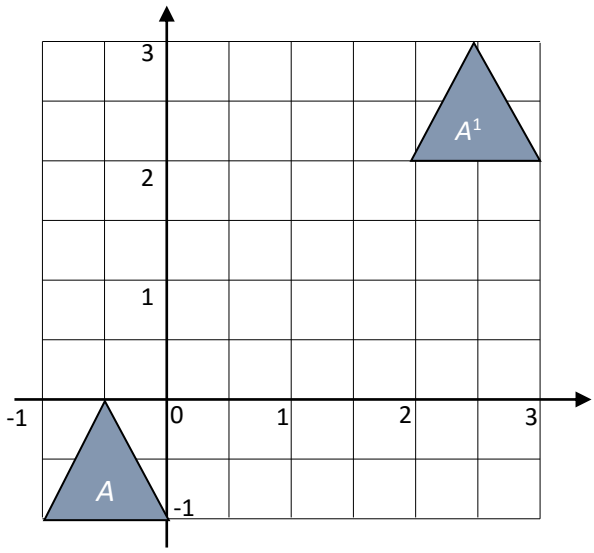

Figure 5: Teacher A's diagram depicting translations of ΔA and its image ΔA^1

She did the same for **reflection** saying that "*rigid figures like the figure below can be reflected with respect to a line of symmetry such as the y- or x-axis*." She drew a rough sketch on the board

depicting reflection of ΔB along a line of reflection horizontally to an image ΔB^1 according to the general rule of reflection $(x; y) \rightarrow (-x; y)$.

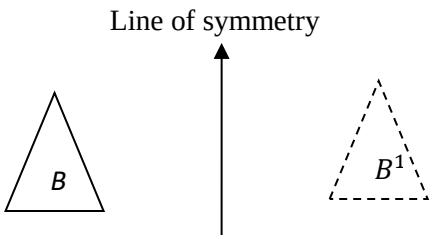

Figure 6: Teacher A's diagram depicting horizontal reflection of a ΔB

She asked learners to name other types of transformation that preserve the shape and size of figures. "*What is the other type of motion that you get*," she asked? One learner said, "*There is reflection along the x-axis Ma'am*." "*That's correct*," said Teacher A. She then wrote the general rule of reflection of a figure along the x-axis that $(x; y) \rightarrow (x; -y)$. She set up the data projector and laptop and began to use SAPs, drawing learner's attention to pentagon ABCDE (see appendix (ii) A: Translation).

She clicked sliders to alter values of parameters p and q on SAPs to show how they affect shape of a graph (translation and scaling). "*Let's look at the effect of p and q on pentagon ABCDE*", she said. The data projector and laptop were already set up in her classroom. "*Do you notice that ABCDE coincides with* $A^{I}B^{I}C^{I}D^{I}E^{I}$?" she asked as she was pointing at the appendix (ii) A: Translation on the data screen. "*Let us focus on a parabola described by* $g(x) = ax^2$ *and* $g(x) = f(x - p)$ *to see what happens when the value of a changes and describe transformation of graphs*," she said. "*What does a represent*?" she continued. "Gradient Ma'am," said a different learner. She used a slider on the SAPs in *Excel* on horizontal translation to change the value of *a*. The slider was designed in such a way that the screen could show changes on the graph, formula and table instantly (see appendix (ii) B: Horizontal Translations).

She asked: "*What will happen when you substitute* x by $x - p$ in $f(x)$? *What will be the new input*? "*What will the graph look like* if $g(x) = f(x - p)^2$?" she continued asking questions. She clicked the sliders making $p = 6; a = 2$ and $n = 2$ of $f(x)$ saying "now $g(x) = a(x - p)^2$." She pointed at the new equation, $g(x) = 2(x - (6))^2 = 2(x - 6)^2$.

"*As you can see, the graph moves 6 units to the right*," pointing at the graph. She did not elaborate. Learners did not ask why the graph moves 6 units, and to the right for that matter. They did not seek clarity on the statement about horizontal translation above, e.g. when will they know which direction the graph will move. Learners did not ask question when they were asked to, which prompted Teacher A to move on to the SAPs on vertical and horizontal translations (Lesson 2) as well as translation by a vector (Lesson 3). As cited above, Teacher A used a variety of SAPs in *Excel*, namely Horizontal Translations, Vertical Translation and Translation by a vector. The researcher named them Lesson 1, 2 and 3.

Summary and analysis: Learners seemed familiar with the topic of transformations. There is a whole section of Transformation Geometry in CAPS Grades 7−9 Mathematics (DBE, 2011: 147). Instructional decisions and curricular choices are focal points of classroom observations. By not elaborating on movement of a parabola by "6 units to the right" in the SAPs on horizontal translations, Teacher A missed a critical teaching moment in which to 'open up' the function concept. Translation of a function is classified as a 'knowledge gap' in this study. The researcher did not get a sense that Teacher A focused her teaching towards addressing knowledge gaps, for example that:

a). learners confused rules pertaining to horizontal translations (shifts) of $g(x) = f(x - p)$,

b). lack of knowledge of the physical meaning of the point $(p;\ q)$, i.e. $y - q = a(x - p)^2$ means a parabola with turning point $(p;\ q)$. Teacher A could have explored translations further by asking learners, for example, to compare parameter p in the function defined by $f(x) = ax^2$ and $g(x) = f(x - p)$ with respect to moving the slider in the opposite direction, describe a pattern, explain the structure and generalize rules governing horizontal movement of functions.

In the process of helping teachers become better at their practice, the researcher was able to make teachers more aware of learners' conceptual and knowledge gaps, which according to this study, emanate from bad mathematical experiences and pedagogy. Teachers need support to be able to identify teaching moments, but also learn how learners learn and provide them with support. Therefore it is important to explain translations of $g(x) = ax^2$ and $g(x) = f(x - p)$ clearly (see appendix (ii) E: Quadratic Functions (Parabola)).

By not elaborating or clarifying why the graph moves 6 units to the right when $p = 6$ and $a = 2$, Teacher A missed a teaching moment. When answering questions on functions in high stakes examinations, candidates seem to confuse the direction of translation of a function. Learner responses stem from a perception that where "+ sign" is used in the equation, e.g. $y = (x + 4)^2$ the graph moves to the right in respect of a number line. The opposite is true for "− sign" in the equation, $y = (x - 4)^2$. SAPs present possibilities for deepening learners' understanding of the pen and paper symbol manipulation. Multiple representations provide teachers with an opportunity to explain the structure of functions clearly.

***Lesson 2*: SAPs in *Excel* is on Vertical Translation of $f(x) = ax^2$ and $g(x) = f(x) + q$**

"You know the rule now. Let's consider translation of $f(x) = ax^2$ to $g(x) = f(x) + q$," she said while opening a worksheet on the SAPs in *Excel* (see appendix (ii) C: Vertical Translation).

She clicked the sliders making $p = -6;\ a = 2$ and $n = 2$ (n represents the highest power of x) to translate $f(x)$ on SAPs. She told learners that: "Since $f(x) = ax^2$, then $g(x) = f(x) + q$, meaning that $g(x) = ax^2 + q = 2x^2 - 6$. The graph of $g(x)$ moves 6 units vertically downwards." She then moved to the next SAPs activity in the next page.

***Lesson 3*: Horizontal and Vertical Translation of $f(x) = ax^2$ and $g(x) = f(x - p) + q$**

In this lesson, Teacher A referred to SAPs in *Excel* focusing on horizontal and vertical translation of $f(x) = ax^2$ and $g(x) = f(x - p) + q$ (see appendix (ii) D: Translation by a vector).

"*Let us now consider what happens when we combine horizontal and vertical movement of graphs in* $g(x) = f(x - p) + q$," said Teacher A. She clicked sliders making $p = 7$; $q = 3$; $a = 2$ and $n = 2$ to translate $f(x)$. The resultant equation was $g(x) - 3 = 2(x - 7)^2$ and the turning point $(7; 3)$. She instructed learners to "*rewrite the equation of a parabola with turning point* $(-3; -5)$". She did not wait for learners to respond, or at least move around to observe what they were doing. She wrote the equation, $y - (-5) = 2(x - (-3))^2$ on the chalkboard and simplified it to $y = 2(x + 3)^2 - 5$. At the sound of the bell to indicate the end of class period at 2:30 PM, she concluded the lesson saying: "*We used SAPs in Excel to look at observed data and modelled data on transformation of functions*." This lesson observation took place in the last period of day.

***Summary and analysis*:** Based on the classroom observation, Teacher A rushed through SAPs. She was able to 'show and tell' learners how the SAPs in *Excel* could enable dynamic, inquiry-based learning. Her confidence in the use of SAPs in *Excel* grew as the lesson progressed.

b) Interview with Teacher A

A post-lesson interview followed immediately after the observed lesson in her classroom. The purpose of the interview was to get the teacher's perspective on observed episodes according to interview guidelines in *Appendix* (i). The interview was audio and videotaped and transcribed, serving as the primary source of data. The interview was about 30 minutes long.

Teacher A's perceptions on the lesson.

Interviewer: What were your expectations of this class prior to the start of the lesson? Were your expectations met? What did you learn from teaching this lesson?

Teacher A: I'm surprised to learn that they've learnt. The dynamic nature of the *Excel* material helps in demonstrating different representations all at the same time. It saves time.

Interviewer: If you were to teach the same lesson again, what would you change in your lesson preparation?

Teacher A: I would start off immediately with the *Excel* instead of writing on the board … give them notes and go over the material slowly. I would then revise tuts [tutorials]."

Interviewer: Have you taught the knowledge and skills that you have applied from this lesson in other classes that you teach? If so, please explain?

Teacher A: I don't really know.

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Summary and analysis: Based on the interview excerpt, she said: "*The dynamic nature of the Excel material helps in demonstrating different representations all at the same time. It saves time."* It appeared that SAPs in *Excel* had positive influence on Teacher A's educational belief. This belief about SAPs appears to have caused a shift into her classroom practice stemming from her statement that: "*I would start off immediately with the Excel instead of writing on the board … give them notes and go over the material slowly.*" Ertmer (2005) asserts that the decision of whether and how to use technology[9] for instruction ultimately depends on the teachers themselves and the beliefs they hold about technology.

This affirmation is aligned to citation of Cuban (2002), Fullan (2001), Fullan (2003), Guskey (2002), Ringstaff and Kelley (2002), and Sandholtz et al. (2002) by Palak and Walls (2009: 417) that:

> *Teachers' beliefs guide the decisions teachers make and actions they take in the classroom.*

This attests to what many researchers in the field of teacher development have said, that teacher support and encouragement are essential catalysts for teacher change. It appeared that teacher training and classroom support in the use SAPs in *Excel* have helped to facilitate and enhance teacher change in the case of Teacher A. The researcher's presence and informal discussion he had with teachers created opportunities for him to sensitively address issues of personal concern raised by teachers.

Teacher A's perceptions on use of SAPs in *Excel*

Interviewer: In terms of using Excel materials and the content, how have you felt about that in this lesson?

Teacher A: A lot more confident this time round. I could select activities taking into account issues of time or class period and important points I wanted to emphasize on the lesson. I could sequence activities knowing how learners have dealt with it [in] the previous lesson. So I had that experience that I could fall back on.

Summary and analysis: Teacher A appeared to be teacher-centred. Based on the lesson observed and interview excerpts, Teacher A was familiar with the concept of transformations but struggled to relate the rules governing translations with functions in the lesson. She rushed through programmes and did not provide opportunities for learners to explore the function concepts further. Her primary concern appeared to be instructional time. Seemingly, this has created a culture of limited engagement in class discussions (e.g. question and answer). There seem to be no time to complete the syllabus due to its demands. According to the pace setter and sequencing

[9] Technology is used in this thesis to mean computers and calculators

of topics in the CAPS grade 10-12 Mathematics, Teacher A was supposed to be teaching *Finance, growth and decay* in week 7 of the 3rd term (DBE, 2011: 19). This is not entirely negative because of the pressure to complete the syllabus. Albeit she seemed more able to independently select SAPs and determine a sequence for structural conception of translations. There was seldom time for reflections and discussions on the function concept. The researcher attributed this to the lesson being for purposes of research and a revision lesson. She had already taught functions in term 2. She felt that the functions concept had already been foregrounded. In other words, her instructional decisions and curricular choices were influenced by her beliefs and perceptions about what learners *could* or *could not* do. This supports the researcher's view that instructional decisions and curricular choices are influenced by teachers' beliefs and an existing culture of teaching and learning in a classroom.

Teacher A was not able to engage learners in meaningful discussions to establish or reinforce the relationship of the graphs of $f(x)$ and $g(x)$ in the SAPs in *Excel*, and understanding of how a formula fit the values in a table. The researcher did not get a sense that learners were enabled to describe emerging patterns, explain structures and make generalizations in respect of known rules of transformations to deepen learners' understanding of functions. SAPs were designed to 'open up' the symbols. Perhaps Teacher A felt she had already grounded the function concept in term 2, and this 'research lesson' did not have a direct impact on assessment. Learners had written the June examinations and had the benefit of learner responses where errors and misconceptions would have been identified. According to Sfard (1987), symbolic representations of functions are 'reifications'. It has been the 'designer's intent' to vary the symbols by clicking sliders on the SAPs in *Excel* as these have the potential to deepen teacher's or learners' understanding of the reified symbols.

The researcher did not ask Teacher A to explain her perception of SAPs that she was feeling "a lot more confident". However, it appeared that her independent selection of the SAPs, the presence of the researcher for classroom support and her acceptance of SAPs, explain this feeling of confidence. It appeared that she does not generally use computers at school because learners tend to be disruptive when in front of a computer. This may be attributed to issues of access, fear and a skills factor (learners are at different skills level). Despite an abundance of technology equipment such as computers, DVDs, smartboards and whiteboards in three computer laboratories and classrooms, she used computers for drill and practice purposes. According to Ertmer (2005), this is low-level usage. In her citation of Becker (1994) and Becker and Riel (1999), Ertmer (2005: 26) asserts that:

> *In general, low-level technology uses tend to be associated with teacher-centred practices while high-level uses tend to associated with [learner]-centred, or constructivist practices.*

An epistemological use of SAPs seem to have renewed her belief that, the use of technology as instructional material, presents possibilities for deepening learners' understanding of the pen and paper symbol manipulation through multiple representations.

4.3.2 The observed lesson and interview of Teacher B

This was a mid-morning lesson in August 2009. There was a morning assembly on the day of the observed lesson. The first three periods at School B were reduced by 10 minutes due to longer time taken up by proceedings in the assembly at school. The class period was 35 minutes long. This was a different classroom than earlier identified for research. There were 48 learners in class, and as a result, there was limited space to move around desks.

a) The lesson observation of Teacher B

Teacher B greeted his grade 10 class, welcomed and introduced the interviewer and researcher to the class. He then introduced the lesson, The Hyperbola. "*This is revision of grade 10 work on the hyperbola*, $f(x) = \frac{k}{x}$", he said. He wrote the following statements on the blackboard:

If $k > 0$, then the graph is in quadrant 1 and 3.

If $k < 0$, then the graph is in quadrant 2 and 4.

"*Now we are not interested in* $\frac{k}{x} = y = f(x)$. *We shall focus on* $f(x) = \frac{a}{x-p} + q$ *to see how this affects the hyperbola*," he said. He wrote $p =$ asymptote (vertical), $q =$ asymptote (horizontal) and $f(x) = \frac{1}{x}$ on the board and instructed a specific learner, Gwendoline (not her real name), to "*come sketch the graph of* $f(x) = \frac{1}{x}$ *for us on the board*." Following the instruction, Gwendoline drew the following rough sketch on the chalkboard (without making any reference to important features to enable us to draw the hyperbola):

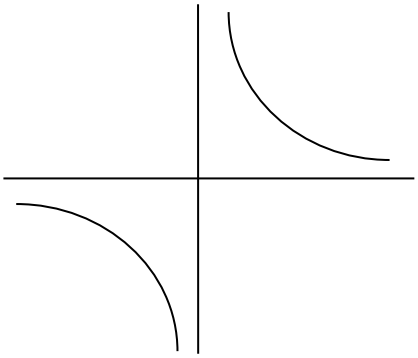

Figure 7: *'Gwendoline's diagram of a hyperbola'* that connects with the analysis of Teacher B

"*Class, is she correct*?" asked Teacher B. Few learners shouted the answer saying, "*she is correct, Sir*." He said: "*Yes she is correct*," pointing at Gwendoline's diagram (Figure 7) of a hyperbola on the chalkboard. He did not probe them further but continued and said: "*Look at the asymptotes*." Teacher B concluded the lesson by comparing the graphs of $f(x)$ and $g(x)$ where $g(x) = \frac{1}{x-1}$ and said, "*now let's look at* $g(x) = \frac{1}{x-1}$." The school bell rang at this time.

Summary: Teacher B used the 'pen and paper' approach in his grade 10 class. He did not use SAPs in *Excel* in the lesson observed. His methodology did not allow him to engage with new forms of teaching and learning.

b) Interview with Teacher B

The observed lesson and interview took place in the same week. The interview took place in Teacher B's office during his 'free period'. The interview took 45 minutes.

Perceptions of the lesson

Interviewer: What were your expectations of learners prior to the lesson?

Teacher B: Well, you will remember that before I could deliver the lesson, I actually related the lesson itself to what they had done in the past. My expectation was that at least they would show some understanding and grasp of [the] concept of functions because I had taught a hyperbola lesson in grade 10 in the 2nd term. The only difference now is that they needed to see the translation of a hyperbola.

Interviewer: Did learners meet your expectations in this lesson?

Teacher B: Firstly, I need to highlight that I have passion for teaching mathematics. My ultimate goal is to see learners achieve their learning outcomes. By achieving in mathematics, I mean I am teaching these learners so that they may pursue further studies in mathematics. For that to be possible, I must make sure they do well in my classroom. Also to change situation in the school pertaining to the low pass rate not only at my school, but actually my aim is current situation nationally.

Analysis: Teacher B's main point of reference in his career and classroom practice seem to be learners' success in mathematics in examinations. This is a view shared by Boardman and Woodruff (2004: 545–557) who assert that:

Most importantly, our results suggest that some teachers may use "high-stakes" assessments as their primary reference point in which to gauge the merit of innovative teaching practices.

Interviewer: Do you think you met your expectations of this lesson?

Teacher B: Given the circumstances, I think that I met my expectations because it was not only about teaching learners. It was also about testing their understanding. I gave them some examples to work on. Learners could come to the board and do a little bit of work there which made me feel that I've actually met my expectation of this lesson.

Summary and analysis: The researcher interpreted the use of the term 'circumstances' in the interview excerpt to mean Teacher B conducted the lesson for the research purposes, and that school management reduced class periods by 10 minutes due to events in the morning assembly. It appeared that the curricular choices of teaching method, content and context were influenced by his 'belief' that learners had full grasp of a hyperbola and that the research lesson had no impact on examinations. In as much as Teacher B encouraged learners to share solutions on the board, there was seemingly no time for correcting mistakes. Correcting learner mistakes appeared to be a crucial instructional decision that impacted negatively on his practice. Teacher B seemingly viewed correcting mistakes as reducing his teaching time. It is a classroom culture that has been established in this way. It appeared that they were not accustomed to making deductions, predictions, and holding discussions in the classroom. Even when a mistake had been identified, it appeared that the decision as to *what to do* or *not to do* with them rests on the shoulders of the teacher.

Interviewer: What would you change in this lesson, if you could?

Teacher B: Well it, it would be exciting if this lesson would be done using a computer because that's where these learners get to see 'these translations live' instead of me telling them, you know. They are able to see this translation live when they look at the computer. I think that would really make a lot of difference to them.

Analysis: Teacher B was probably reminded of the dynamic nature of SAPs in *Excel* to extract useful mathematics that the researcher discussed with him a week prior to the observed lesson.

Interviewer: What's your opinion on use of computers to teach functions? I ask you this question because you did not use SAPs in the lesson for the study.

Teacher B: Look, integration of computers in mathematics education nowadays cannot be ignored! I think it is better when computers are also included in teaching, you know! Besides the lesson topic in class today, there are many other topics that actually require computer knowledge from learners.

Interviewer: Having said the above, would you recommend your method of teaching functions you've just applied to other teachers in your mathematics department?

Teacher B: I would do so because as I have already said, it is the kind of things about a practical approach when one introduces a computer. So it would be best if all educators get computer literacy and also make sure [that] all learners get computer literacy because then also you're giving them an opportunity to do things on their own – obviously with teacher support.

Summary and analysis: It appeared that Teacher B believed strongly that integration of computers in mathematics education can enhance learning and teaching. He expressed this belief in the interview excerpt when he said that: "*... integration of computers in mathematics education nowadays cannot be ignored! I think it is better when computers are also included in teaching, you know!*" It can be concluded from this that Teacher B believed that the use of computers in a mathematics classroom can shape "*conceptions of what we ought to teach and what we can teach*" (Fey, 1998: 199). It appeared though that he does not normally use computers in the classroom, and this can be attributed to his belief that learners lack computer knowledge and skills. It appeared that learners are not exposed to teaching with the use of computers or at least computer-based material at his school. This attest to the researcher's view that instructional decisions and curricular choices are informed by teacher beliefs. According to Ertmer (2005: 27):

> *Ultimately, the decision regarding whether and how to use technology for instruction rests on the shoulders of classroom teachers. If teachers are to achieve fundamental, or second order, changes in classroom teaching practices, we need to examine teachers themselves and the beliefs they hold about teaching, learning and technology.*

This begins to explain that Teacher B's decision not to use SAPs in *Excel* in the observed lesson was influenced by the lack of computer knowledge and skills (computer literacy). By making reference in the interview excerpt that "*all educators get computer literacy*", he implicated himself as needing computer literacy and teacher support. This happened in spite of one-on-one 'training and support session' that was conducted by the researcher at his school prior to the classroom observation. It appeared that he, and other teachers at his school need training and support on the use of computers in mathematics education.

Perceptions on mathematics (i.e. beliefs Teacher B held about mathematics as a subject or nature of the subject)

Interviewer: If you were to use SAPs in *Excel*, do you think your learners would benefit more from integrating technology in teaching functions? What is your view on SAPs in *Excel* that I discussed with you recently?

Teacher B: Yes, I would think so. Excel brings creativity. There's a whole lot of empowerment. When using *Excel*, you can actually save teaching time. A time you can use for revision. At the moment I'm spending about 5 weeks on functions. Learners fail my tests having spent all this time. It has made a lot of difference to me. I need to put my hands on it. We should be using computers at school more. That's why I say there's room for improvement.

Summary and analysis: Teacher B brought up the issue of teaching time in this interview excerpt. This is not the first time that teaching time and curricular issues are being brought up by respondents in this study. These are important issues that speak to the implementation of the

mathematics curriculum. Teacher B was agonizing about that the time allocated on functions (5 weeks) only for his grade 10 learners to fail his tests. CAPS grade 10-12 Mathematics (DBE, 2011: 24) prescribe the following subtopics in grade 10:

- Investigate the effect of a and q on the graphs defined by: $y = a.f(x) + q$ where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x$, $b > 0, b \neq 1$.
- Study the effects of a and q on the graphs defined by: $y = a\sin\theta + q$, $y = a\cos\theta + q$ and $y = a\tan\theta + q$ where $a, q \in Q$ for $\theta \in [0°; 360°]$.

According to the pace setter and sequencing of content in the CAPS grade 10-12 Mathematics (DBE, 2011: 18), Teacher B was supposed to be teaching Statistics (or Trigonometry) in grade 10. Teacher B described SAPs as a creative instructional material. From this description of SAPs, the researcher can conclude that Teacher B was reflecting on his practice by realising and acknowledging the affordances of SAPs, that he could have saved himself some instructional time if he had used SAPs as instructional material in the observed lesson. This could make 'a lot of difference' to his teaching practice and learners' success. His application of the 'pen and paper' approach made it difficult for Teacher B to 'open up' the function concept, at least with respect to the following:

- the *physical meaning* of the point $(p;\ q)$. It is so because understanding the physical meaning of point $(p;\ q)$ enables learners to draw the graph of a hyperbola, i.e. $y - q = \frac{a}{x-p}$ means a hyperbola with asymptotes p and q.
- the rules pertaining to vertical and horizontal translations, i.e. *Translations* (shifts) of graphs defined by $g(x) = f(x - p)$ and $g(x) - p = f(x)$, and combinations of vertical and horizontal translations.
- the rules pertaining to *Scaling* (stretching and shrinking): $g(x) = q.f(x)$ and $g(x) = f\left(\frac{x}{p}\right)$ as well as the combinations of stretching and shrinking.

This is very important in the context of this study as it attests to the researcher's assertion that the institutionalisation of mathematics and the curriculum creates a barrier to learning mathematics. The researcher acknowledges that whilst it is important to listen to others' solutions, it is particularly difficult to achieve this in one lesson, where it is also expected that the syllabus be completed within a set time frame.

Interviewer: Do you design different material for different learners or groups when you design material for teaching?

Teacher B: Learners are not the same. You look at them in terms of their grades but unfortunately if you look at them individually it's very difficult to design material per learner. If you design them according to their abilities, you can't design materials the same way as you would for grade 11, for example. You must design it in such a way that it suits their level.

Summary and analysis: Again an interesting issue of 'time' was being raised. It appeared that instructional time had become a sparse commodity for both respondents and their colleagues in School A and B, taking into account occasions during data collection when teachers were not available to complete the beliefs questionnaire. It appeared that respondents did not have the time to 'develop computer-based instructional material'. This was considered by the researcher. Hence SAPs were made ready-to-use in class and respondents were 'trained' on use thereof, a week prior to scheduled classroom observation. There was no incentive or reward for the respondents to spend time developing new computer-based instructional material from scratch in this small study.

Interviewer: Do you remember asking Gwendoline to come draw the graph of $y = \frac{1}{x}$ on the chalk board?

Teacher B: Oh, yes. I remember that.

Interviewer: She drew a rough sketch and when you asked the class if she was correct, the answer was a unanimous 'yes'. You did not probe her further.

Teacher B: I don't think there is anything wrong in Gwendoline's graph.

Interviewer: That graph can be any hyperbola where $a > 0$, p and $q = 0$. Don't you think she should have written at least two points through which the graph passes? All you referred to were asymptotes.

Teacher B: Yes, both asymptotes were correctly plotted but you know what, you are right. I felt that at that point I wasn't going to finish my lesson on time. Remember, this is revision of grade 10 work done in term 2. The class saw that as a correct answer and I agreed. At the time, I didn't feel equipped to tell her that her graph was correct or incorrect.

Interviewer: When you told the class to move onto the next problem, what was the reason for your decision?

Teacher B: I don't think of any reason other [than] pressure to finish the lesson on time. The lesson was reduced by 10 minutes from a normal class period.

Interviewer: What informed your decision to call a learner, Gwendoline, to draw a graph on the blackboard?

Teacher B: This is the ultimate way of checking whether you're on track. Calling learners to the board means you can see whether they understand, trying very hard to work in front of me. Others won't ever come on board.

Interviewer: If a learner were to make a mistake on the board, how would you approach mistakes that learners make? How will this differ from when using SAPs in *Excel*?

Teacher B: Look, when you call a learner on the board, you must expect them to make mistakes. The question is, how do you make sure that you motivate and encourage them to come to the board after making a mistake because it's not easy? It's not about saying this is right or wrong. It's all about motivating them and showing them in front of their peers how differently they would have done the problem, and for them to learn from their mistakes. That's wonderful.

Summary and analysis: Teacher B seemed to be teacher-centred. In as much as he encouraged learners in the observed grade 10 lesson to share their solution methods on the chalkboard, there seemingly was no time for correcting mistakes. Teacher B viewed correcting learner mistakes and use of SAPs as 'resources' in aid of teaching as reducing his teaching time. Even when a mistake has been identified, a decision about what to do or what not to do with learner mistakes rests with him. When the researcher asked him about curricular issues that could have been dealt with on Gwendoline's diagram (Figure 7: 49), he admitted to having been pressured by time. He said, "*I felt that at that point I wasn't going to finish my lesson on time. Remember, this is revision of grade 10 work done in term 2. The class saw that as a correct answer and I agreed. At the time, I didn't feel equipped to tell her that her graph was correct or incorrect*." This may be due to an existing culture in the classroom that seems to have been established in this way. Despite a special arrangement by the school management to reduce class periods on the day of observed lesson, it emerged from the interview that he was troubled by 'teaching time'. It appeared that he does not normally have the time to 'design instructional material'. Based on the views he held about 'computer literacy' above, he may be needing support.

Following an incident in which Gwendoline used a strategy that brought about what the researcher regards as didactics (content knowledge). Teacher B missed an opportunity to engage her and the class on the drawing. Teacher B did not ask Gwendoline to plot at least two typical points on the graph (or work flexibly between table, graph and equation) and label axes (asymptotes) to indicate the extent of understanding of important features of a hyperbola. Teacher A did not make reference of features of a hyperbola such as domain, range and explain the *physical meaning* of the point $(p;\ q)$ in the context of the graph of $f(x) = \frac{a}{x-p} + q$. However, he referred to a graph of $f(x) = \frac{1}{x}$ as a hyperbola in 1st and 3rd quadrants if $k > 0$, and in the 2nd and 4th quadrants if $k < 0$.

Teacher B had already taught functions in grade 10 in Term 2. It appeared that he viewed the observed lesson as a 'revision lesson', the researcher thought Teacher B assumed that learners already knew and understood that a hyperbola has asymptotes, and what are these asymptotes. He

chose to teach a hyperbola in his grade 10 class in an isolated piecemeal manner, i.e. without structure or coherence. Another approach of teaching a hyperbola that could be used is from a perspective of multipliers and multiplicands. For example: Multipliers and multiplicands whose product is 24:

- A table:

Multipliers (x)	-32	-12	-6	-2	$-\frac{3}{8}$	$\frac{1}{2}$	1	3	12	96
Multiplicands (y)	$-\frac{3}{4}$	-2	-4	-12	-64	48	24	8	2	$\frac{1}{4}$

Table 2: Multipliers and multiplicands

- This function is non-linear whose equation or formula is $xy = 24$ or $y = \frac{24}{x}$.
- As a graph

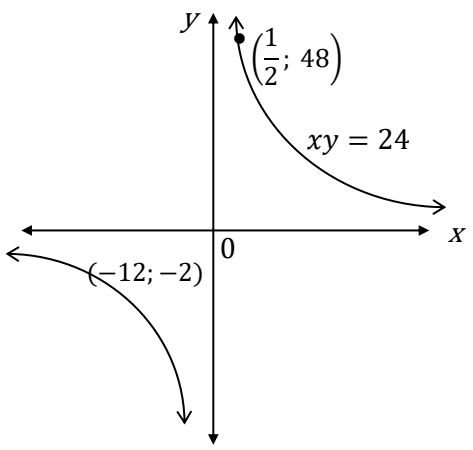

Figure 8: Graph of a hyperbola

- In words we say that the multiplier is *inversely proportional* to the multiplicand.

In mathematics, we describe inverse proportion as:

a) A form $xy = a$ or $y = \frac{a}{x}$, where a is a constant.

b) A graph of an inverse proportion is always a curve. Its shape is called a hyperbola.

c) If x increases, then y decreases, and if x decreases, then y increases.

d) If x doubles, then y halves, and if x halves, then y doubles.

Teachers do not think of hyperbolic functions in this way, and this cannot be attributed to time pressures to complete the syllabus. The researcher observed that Teacher B used the variable k to denote a constant, yet the variable a is used in the syllabus. This has potential to cause confusion for learners.

Interviewer: What were the most important things that your learners said in relation to the lesson on hyperbola in class today?

Teacher B: I was impressed when they could still remember [what] a graph of hyperbola looks like.

Summary and analysis: Teacher B found class visits and lesson observations useful and beneficial to him. In one case, it resulted in a critical change. As head of department, he would recommend SAPs in *Excel* as a mathematical tool to teach functions to his teachers. From the conversation with the researcher after the interview, he mentioned class size (48 learners) as having had greater influence on his decision not to use SAPs. It appeared that he viewed the incorporation of SAPs in teaching functions to mean that 'pen and paper' strategies were going to take a back seat, and fearing that learners would not be able to extract the useful mathematics due to a lack of computer literacy at his school.

Teacher B seemed dissatisfied with the existing learning and attitudes that emerged in his classroom. Etchberger and Shaw (1992) regard 'perturbation' (Goldsmith and Schifter, 1997, after Piaget) as necessary for teacher change, in that it forces the teacher to confront and resolve problematic aspects of their practice. In this instance, teaching time was Teacher B's primary point of reference in which to gauge the merit of a teaching practice (Boardman and Woodruff, 2004). A need for teacher change of practice with the particular SAP under consideration necessitates classroom support. Thus to have hope of teacher change, teachers' classroom practices and teachers' beliefs have to be considered, as these have a great effect on change. As cited from Levin and Wadmany (2006: 158):

> *Beliefs are filters that guide teachers during instructional and curricular decision-making (Pajares, 1992; Prawat, 1992).*

In conclusion, the researcher supports this proclamation as it forms the basis for the choice of area of his study, which looks at the influence of SAPs on teacher change in township high schools. The researcher was of the view that to have hope for a possible change of teacher practice, among other things, teacher's beliefs and teacher's classroom practices should possibly change. Hew and Brush (2006) regard teacher beliefs and attitudes as barriers to technology integration in their acknowledgement of research done by Bodur, Brinberg, and Coupey (2000) who found that beliefs determine a person's attitude. His presence in the classroom and the interview that followed helped to make an ordinary classroom incident critical in the process of change.

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CHAPTER 5: FINDINGS

5.1 Introduction

Findings of this study are presented in this chapter. Findings of possible teacher change of Teacher A and Teacher B with the use of SAPs are based on data gathered and administered in the two schools in a period of 15 days (11 hours), and analysed in Chapter 4. The findings presented are on, (a) teacher beliefs, looking specifically at assessment, nature of mathematics, teaching and learning, learner ability and calculator use, and (b) teacher practice from one lesson observation backed by a post-lesson interview with each teacher. This was done in accordance with the suggestion offered by Levin and Wadmany (2006: 157) that:

> *….it is worthwhile to investigate teachers' beliefs, and also to explore the implicit link between teachers' views on learning and teaching and their actual classroom practices.*

5.2.1 Findings in respect of teacher beliefs

a) Findings with respect to all mathematics teachers $n = 10$, in School A and School B.

- The above statistics show that there is very little difference between the beliefs of teachers from School A and School B in each of the categories, so unsurprisingly, this average response for each of the sections also coincided with the average response for beliefs of all teachers in the study. Owing to this little difference, **there is no reason to treat the responses or beliefs of teachers in the two schools separately.**
- In all categories of statements, beliefs are on average very close to 3 (neutral), with responses related to statements about beliefs on Assessment coming out slightly towards "Agree", i.e. just over 3, as is the case for Nature of mathematics and T & L. As cited above, the longer the line, the more varied the beliefs as reflected by the teachers. Teachers generally have strong opposing views, i.e. "Strongly Agree" or "Strongly Disagree" on Assessment.
- The lowest overall response seemed to be for beliefs about statements related to Calculator use.

b) Findings with respect to respondents ($n = 2$), namely, Teacher A and Teacher B.

Category 1: Assessment

Whilst respondents agree on specific statements on Assessment, there is enough evidence of significant differences among respondents regarding the same. These differences of beliefs between the respondents seem to mirror those of all other teachers from the two schools. **The differences necessitate teacher development workshops to synchronise their beliefs on assessment in order to improve mathematics results in high schools.**

Category 2: Learner Ability

The respondents agreed totally on all the items in this category. The data provided enough evidence to show that respondents shared similar beliefs on Learner Ability.

Category 3: Nature of Mathematics

There is evidence that respondents agree totally on all the items in the Nature of Mathematics.

Category 4: Calculator Use

There were 3 statements in this category. Whereas respondents hold the same beliefs on two-thirds (66,7%) of statements about Calculator Use, they differed sharply on the other statement. Teacher B holds a negative belief about the statement that: "*The calculator can be used to introduce learners to important mathematical concepts*". **This evidence of a contrasting belief between respondents is significant.** Albeit this begins to explain why Teacher B did not use SAPs in *Excel* in the lesson observed for the study. As cited from the interview excerpt, there is evidence to suggest that Teacher B may have had negative beliefs about his own technical capabilities should there be such issues to resolve during the course of the lesson. However, he had positive beliefs about computers. Conflicts between mathematics teachers' beliefs and implementation in the reality of their classroom practice have been widely reported in the literature (e.g. Cooney, 1985; Newstead, 1997; Nesbitt, Vacc and Bright, 1999). In many cases teachers express beliefs about the learning and teaching of mathematics which do not translate into their classroom practice.

There can be various reasons for this lack of consistency, including conflict with the school's culture or with the expectations of students about what constitutes mathematics and how it should be taught (Cooney, 1985). It can, in fact, be questioned whether 'beliefs' expressed in isolation outside the classroom have any relevance, or whether beliefs can only be observed as *situated* in the classroom practice (Hoyles, 1992). This somehow confirms Cuban's (2002) argument that:

> *beliefs by themselves cannot entirely explain how teachers are likely to use technology because teacher practices are inextricably tied to other contextual and organizational factors.*

This seems to confirm the findings of Cox, Abbott, Webb, Blakeley, Beauchamp and Rhodes (2004) who are cited by Levin and Wadmany (2006: 158) that:

> *Without teachers' skilled pedagogical application of educational technology, technology in and of itself cannot provide innovative school practice and educational change.*

Category 5: Teaching and Learning

Whereas Teacher A seems to be aligned with constructivist, Teacher B appears to be aligned with the behaviourist (direct transmission) pedagogy. According to Ertmer (2005), **Teacher A holds personal beliefs aligned with constructivist pedagogy.**

5.2.2 Findings in respect of teacher practice

Findings in respect of one lesson observation and post-lesson interview of respondents ($n = 2$). Teacher A used SAPs in *Excel* to teach a parabola (grade 11). Teacher B on the other hand taught a hyperbola (grade 10) using traditional pen and paper methodology.

Herewith are findings:

a) Teachers' primary concern appeared to be instructional time owing to institutionalisation of the curriculum and mathematics.

Evidence shows that teachers shun educational issues that take teaching time, for example, correcting learner mistakes or discuss mathematics with colleagues at school. Teachers view these as reducing teaching time. Teachers experience the transition towards any new approach as time-consuming, not only in the classroom but also in terms of preparation and assessment (Cooney, 1985; Nolder, 1990). Traditionally, progress has been measured by 'coverage' of the mathematics syllabus (Nolder, 1990). According to Nolder (1990),

> *the teacher is accountable to learners, parents and the management of the school, particularly in preparing the learners for external examinations.*

Carter and Richards (1999) refer to time as the 'universal issue/dilemma' in schools, and that:

> *the teachers' belief that if they do not spend their time 'covering' the 'curriculum' they will be damaging the [learners].*

Schools as bureaucracies contribute greatly to teaching and learning. These bureaucracies influence a culture of teaching and learning in a township high school. According to Lovitt, Stephens, Clarke and Romberg (1991), strategies for change must confront an already-established culture. Lovitt et al (1991) and Nickson (1992) share the same viewpoint that teachers and learners already hold knowledge, values and beliefs about mathematics and how it is taught and learned. As a result, changes in teaching approach can meet with resistance from parents and even learners, whose expectations about what constitutes 'proper' mathematics and how it should be taught, conflict with the culture and roles which the teacher is attempting to create (Cooney, 1985; Nolder, 1990; Goldsmith and Schifter, 1997). It is difficult for the teacher to allow learners to be confused, puzzled and frustrated while solving a mathematics problem if the culture has been such that the learning has seemed 'painless and progressive' (Goldsmith and Schifter, 1997).

b) There is evidence that affordances of SAPs in *Excel* positively influenced on Teacher A's classroom practice.

From the analysis of teacher practice, the study has shown that Teacher A felt a substantial degree of ownership and confidence in the use of SAPs. SAPs in *Excel* had a positive influence on Teacher A's educational belief. Teacher A's perception of SAPs increased her confidence in the use thereof. Her perception of the lesson led her to a new belief that: "*The dynamic nature of the Excel*

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material helps in demonstrating different representations all at the same time. It saves time."

According to Arcavi and Hadas (2000: 25-26):

> *Dynamic environments not only enable students to construct figures with certain properties and thus visualize them, but also allow the user to transform those constructions in real time. This dynamism may contribute towards forming the habit of transforming (either mentally or by means of a tool) a particular instance, in order to study variations, visually suggest invariants, and possibly provide the intuitive basis for formal justifications of conjectures and propositions.*

Teacher A's adoption and confidence in the use of a 'new method of teaching', led her to a conviction that SAPs in *Excel* can 'save teaching time'. Without this conviction, SAPs would not necessarily "*demonstrate different representations all at the same time*". This belief about SAPs caused a shift in her classroom practice. She was 'surprised' that learners learnt about multiple representations of functions in the observed lesson. Asked about what she would change if she had to teach a new lesson, she said that: "*I would start off immediately with the Excel instead of writing on the board … give them notes and go over the material slowly.*" It would appear from this statement that in 'a new lesson', her instructional decisions and curricular choices would be influenced by her beliefs and perceptions about what learners *could* or *could not* do.

Ertmer (2005) asserts that the decision of whether and how to use technology for instruction ultimately depends on the teachers themselves and the beliefs they hold about technology. This affirmation is aligned with citation by Palak and Walls (2009: 417) of Cuban (2002), Fullan (2001), Fullan (2003), Guskey (2002), Ringstaff and Kelley (2002), and Sandholtz et al. (2002) that:

> *Teachers' beliefs guide the decisions teachers make and actions they take in the classroom.*

This also confirms Dick (1992) and Wilson and Krapfl (1994) suggestion in Bell (1999: 51) that:

> *The use of multiple representations, interpretation from one representation to another, and analysis which allows [learners] to relate the graphic, numeric, and symbolic information are critical areas that [learners] should be exposed to in order to develop a better understanding of functions.*

This supports the researcher's view that instructional decisions and curricular choices are influenced by teachers' beliefs and an existing culture of teaching and learning in the classroom. The researcher acknowledges that it takes a courageous and a strong personality to try out new ideas in a classroom.

It is important for teachers to experience the need to change. That both Teacher A and Teacher B willingly committed to taking part in the research is a good motivator for teacher change. The researcher regards the one-on-one interactions with respondents on SAPs prior to classroom observations as an effective teacher development program. The classroom support and encouragement by the researcher, not so much by knowledge of mathematics, were enablers of teacher change. It appeared that teacher training and classroom support in the use SAPs in *Excel* have helped to facilitate and enhance teacher change. This confirms what many researchers in the

field of teacher development have said, that teacher support and encouragement are essential catalysts for teacher change. The researcher's presence and discussions he had with teachers created opportunities for him to sensitively address issues of personal concern raised by teachers.

It was important to demonstrate to teachers the kind of questioning that could elicit understanding. Teachers were comfortable with an interruption by the researcher whenever he was convinced that modelling the kind of questioning to elicit understanding of functions could be of advantage for discussion. For example, when a girl learner in Teacher A's classroom asked the researcher the question in reference to the graphs of parabola, $y = (x + 4)^2$ and $y = (x - 4)^2$: "*Sir, does the graph move to the right or left?"* As cited in Chapter 4, learners seem to confuse rules *of translation* of functions. In as much as they would understand that 4 is a unit of translation from a basic parabola, learners get confused with the direction of the horizontal shift, i.e. "+ sign" and "− sign" in respect of a number line. Hence the learner's quest in the grade 11 class to know which of the graphs between $y = (x + 4)^2$ and $y = (x - 4)^2$ would shift horizontally by 4 units to the left or right of the mother function, $y = x^2$. The researcher interrupted the class and referred the question back to the class for Teacher A to facilitate a whole class discussion. A similar incident happened during the classroom observation of Teacher B. Referring to Gwendolene's diagram (Figure 7: 49), a boy learner asked the researcher the question: "*Sir, is it right*?" The researcher interrupted the class and referred the question back to the class for Teacher B to facilitate a whole class discussion.

c) Evidence suggests that Teacher B has negative beliefs about his own technical capabilities.

Conflict between teacher beliefs and reality of their classroom practice prevailed in the case of Teacher B. Citing Becker (1994) and Becker and Riel (1999), Ertmer (2005: 26) confirms that low-level technology uses generally tend to be associated with teacher-centred practices while high-level uses are associated with constructivist practices. Fear of any kind may have a detrimental effect on the teacher's confidence, class management and control, and outcome of a lesson. It is highly probable for a teacher being observed to deviate from a planned lesson. This could result in lost teaching time leading to an incomplete syllabus. The teacher may even lose learners' confidence and thus cause trouble with the school principal, parents, and the employer (WCED).

d) There is enough evidence of significant differences among respondents regarding their beliefs on assessment

e) There is evidence to show need of content knowledge and computer literacy;

As head of department (HOD), Teacher B could recommend SAPs in *Excel* as a mathematical tool to teach functions to his teachers provided teachers are supported with technology skills to use it. There is evidence that Teacher B believed that integration of computers in mathematics education can enhance learning and teaching. According to Fey (1998); Porzio (1999), and Habre and Abbound (2006), the power of technology lies in its capacity to provide greater and easy access to multiple representations of concepts. Put differently, the unavailability or underutilization of technology to show multiple representations of functions is in itself a means to deprive learners (and teachers in the case of Teacher B) of an opportunity to gain better insight of function sense. The SAPs in *Excel* have the capacity to provide greater and easier access to multiple representations of functions.

In conclusion, the findings in this chapter should be celebrated. They show that it is possible to create a culture in a township high school that focuses on listening to others' solutions, making deductions and predictions is particularly difficult to achieve. Lack of learning opportunities can be seen from how the mathematics curriculum is institutionalised, and as a result opportunities to think about functions in ways that are espoused or advocated in the SAPs is lacking. This could be attributed to a lack of exposure of various methods of teaching functions, that elicit the understanding and beliefs that teachers hold about use of technology in mathematics education.

With a click of sliders in the particular SAPs, Teacher A was able to show her grade 11 class how to investigate the effects of parameters a, b, k, p and q through interplay between different representations of functions as prescribed in the CAPS mathematics grade 10-12 (DBE, 2001: 32) and envisaged in the study. The level of understanding of SAPs conjured a renewed commitment for Teacher A to change her practice as outcome of this study. This change should be celebrated as it is directed towards shaping teachers' classroom practices thereby building a deeper understanding of functions and 'function sense'. Hence a higher level of critical receptiveness for contemporary innovative approaches to mathematics teaching and learning. The development of this study emphasises the influence, role and potential of SAPs in *Excel* to change teacher classroom practice and teacher beliefs. More directly, the findings have implications for ways of fostering academic development in mathematics education at universities in South Africa to increase opportunities for disadvantaged learners and entrench SAPs in *Excel* as part of mainstream academic practice.

In as much as the researcher suggests strategies to help teachers create teaching moments with the use of SAPs in *Excel*, he is aware that this ideal or goal may be difficult to accomplish where teacher dependence is the dominant culture in a mathematics classroom. This goal can be frustrating, difficult to achieve and may take time, especially when a syllabus has to be completed.

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Understandably, teachers have far too many demands on their time. For example, they have to comply with pace setters, complete the syllabus on time, adhere to policy and guidelines on assessments whilst sifting through loads of educational resources and choose a teaching approach suited to helping learners learn mathematics. It is incumbent on teachers to create a culture in which knowledge and understanding are key elements in a mathematics classroom.

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CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

Conclusions and recommendations based on the analysis in Chapter 4 and findings in Chapter 5 are presented in this chapter. The aim of this chapter is to draw the reader to the objectives of the study and what the results reveal about each of those objectives. The researcher also looks at envisaged impact of the study with the view to contribute to the existing body of knowledge and offer suggestions for further research.

6.2 Conclusions from findings of the study

Perhaps if teachers could step outside of their beliefs (personal philosophies) about teaching that have not yielded results and increase classroom support, instead of placing an over emphasis on compliance that is being touted by officials, they could discover the science of what works. It is through appropriate professional development and networking that teachers, and *perhaps* learners, can get opportunities to explore and to experiment with SAPs and polynomial functions in ways that a graph could be seen as providing a visual insight of functions. This will bring about a better understanding of the behaviour of a broader family of functions and contribute towards improving student's achievements in high stakes examinations. Such a process will be in line with Guskey's (1986) assertion that the three major outcomes of teacher development are change in teacher classroom practice, change in teacher beliefs and attitudes and change in learning outcomes for learners. To have hope for teacher change, school management could provide teachers with adequate support in the workplace. According to Garet et al. (2001), Glattenhorn (1987), professional development activities should build on teachers' experiences and should relate directly to their classrooms. As cited above, classroom support and encouragement hold promise for teacher change of practice.

It is generally accepted that traditional in-service workshops fail to empower teachers (Cole, 2004). This is due to lack of support and practice or application of workshop contents on one hand, and duration of the workshop and availability of resources on the other. For some reasons, Principals often send teachers to in-service workshops who are not the specific target group to represent the school, and mostly at short notice. Obviously, those teachers come under-prepared for the workshop or training programme. They simply do not want to be in the workshop. Training becomes an 'add on' which then weakens the provider-workplace linkages (provision of support and needs-based training). Even worse, systems in some schools do not often allow those who attended workshops to report back to relevant departments or staff at school as part of their responsibility, accountability and mandate. As cited earlier, there is a need for alternative approaches to the professional development of in-service mathematics teachers in township high schools.

6.3 Recommendations

The following recommendations are based on findings in Chapter 5. Findings show that teacher professional support systems are necessary.

The recommendations are classified as follows:

<u>Recommendation 1</u>: To address institutionalisation of curriculum (re-conceptualisation of mathematics) and support township high school teachers in the process of change of their classroom practices, a teaching approach that uses SAPs in Excel is recommended

Start with the development of general notions of transformations of *all* or *any* function, i.e. develop understanding and skill of the four rules of transformations of any function $f(x)$:

- Linear function: $f(x) = ax + q$;
- Quadratic function: $f(x) = a(x+p)^2 + q$;
- Hyperbola: $f(x) = \frac{a}{x+p} + q$;
- Exponential function: $f(x) = a.b^{x+p} + q$;
- Sine function: $f(x) = a\sin k(x+p)$;
- Cosine function: $f(x) = a\cos k\,(x+p)$;
- Tangent function: $f(x) = a\tan k\,(x+p)$

1. *Translations* (shifts): $g(x) = f(x+p)$ and $g(x) - p = f(x)$, and combinations of the two;
2. *Scaling* (stretching and shrinking): $g(x) = q.f(x)$ and $g(x) = f\left(\frac{x}{p}\right)$, and combinations of the two;
3. Introduction to transformation of all functions;
4. Introduction to symmetry, asymptotes and turning points, if any;
5. Then apply or repeat 'step' 1 – 4 above for each specific function type;
6. Completing the square and solving equations in the case of quadratic functions.

To facilitate generic, genuine understanding, do not develop isolated, piece-meal knowledge leading to different rules for different functions, but generic, general knowledge that applies to all cases.

Rules for translations

The rules for shifting, i.e. *translating* graphs or axes based on the <u>*operation*</u> in $y = x + a$, $y = (x+a)^2$, $y = \sin(x + 30°)$ as *a* moves to the *left*, and $y = x - a$, $y = (x-a)^2$ and $y = \sin(x + 30°)$ as a moves to the right conflicts with the left-right sign convention of the number line, and a different convention for vertical translations:

$y = x + a$, $y = x^2 + a$, $y = \sin x + a$ as a moves upwards and

$y = x - a$, $y = x^2 - a$, $y = \sin x - a$ as *a* moves *downwards*.

This can be overcome by generalizing the *structure* of the two known cases, i.e.

$y - y_1 = m(x - x_1)$ and $y - q = f(x - p)$

When a function $f(x)$ is shifted p units horizontally and q units vertically, the equation of the new function is given by $y - q = f(x - p)$, i.e. we replace x with $x - p$ and y with $y - q$. The **signs** of p and q determine if the translation is right/up or left/down. And we have the same rule for horizontal and vertical translation. **We should therefore override the notation in the curriculum, e.g. do not study $y = \sin(x + p)$, but rather $y = \sin(x - p)$**. This knowledge should be developed through interplay of different representations, i.e.

Numeric (tables) ⟷ geometric (graphs) ⟷ algebraic (formulas) ⟷ verbal (words)

The SAPs in *Excel* (or calculators) must easily generate the tables, graphs and formulas, and the learner must interact with these to develop and test conjectures, and the teacher must teach or demonstrate the relationships between graphs, tables and formulas. Furthermore, the notion of the transformation of graphs, should be built or connected to the transformation of geometric figures as same ideas or rules apply. Also emphasize that a translation means we have congruent figures, i.e. their size and shape are the same.

Explanation: Try to give an explanation for the structures $y = f(x - p)$ and $y - q = f(x)$ that:
If $f(x)$ is translated by the vector $(p; q)$, then every point $P(x; y)$ on $f(x)$ is translated to a point $P'(x + p; y + q)$ on $g(x)$. So $(x + p; y + q)$ satisfies the equation of $g(x)$:

$$y + q - q = f(x + p - p)$$
$$\therefore y - q = f(x - p)$$

Usefulness: Emphasize or illustrate the meta-perspective that this form or structure is useful in that it allows us to easily draw complicated graphs based on:

- Our knowledge of the graphs of basic functions;
- Our generic transformation rule for $y - q = f(x - p)$.

Emphasize that for each function type, the point $(p; q)$ has physical meaning, which we must know in order to draw the graph, $y - q = f(x - p)$.

The need for new knowledge

The usefulness of the structure to draw graphs, motivates a need to learn how to make the algebraic transformations (completing the square), for example how to transform $y = 2x^2 - 20x + 54$ to the form $y - q = a(x - p)^2$. Also how to solve $f(x) = 0$ to find the x-intercepts of the function. *We need to know how to find the formula from given graphs and tables.*

Rules for scaling (stretching and shrinking)

Again, start with scaling geometric figures (see CAPS grade 10–12 Mathematics (2011: 32)). Distinguish between horizontal and vertical stretching and shrinking. We do not traditionally distinguish between $y = 4x^2$ and $y = (2x)^2$ …. The interpretation of $y = ax$ with a as the gradient, and $y = ax^2$ as narrower and wider depending on a does not connect with the meanings of a in $y = a \sin x$ and $y = \sin(ax)$ at all. Interpret all $g(x) = q.f(x)$ as vertical stretching and shrinking and $g(x) = f\left(\frac{x}{p}\right)$ as horizontal stretching and shrinking. For each function, ask the question: If we multiply a function or this function $f(x)$ by a constant number a, what will be the effect on the graph? Show customized applet for further exploration.

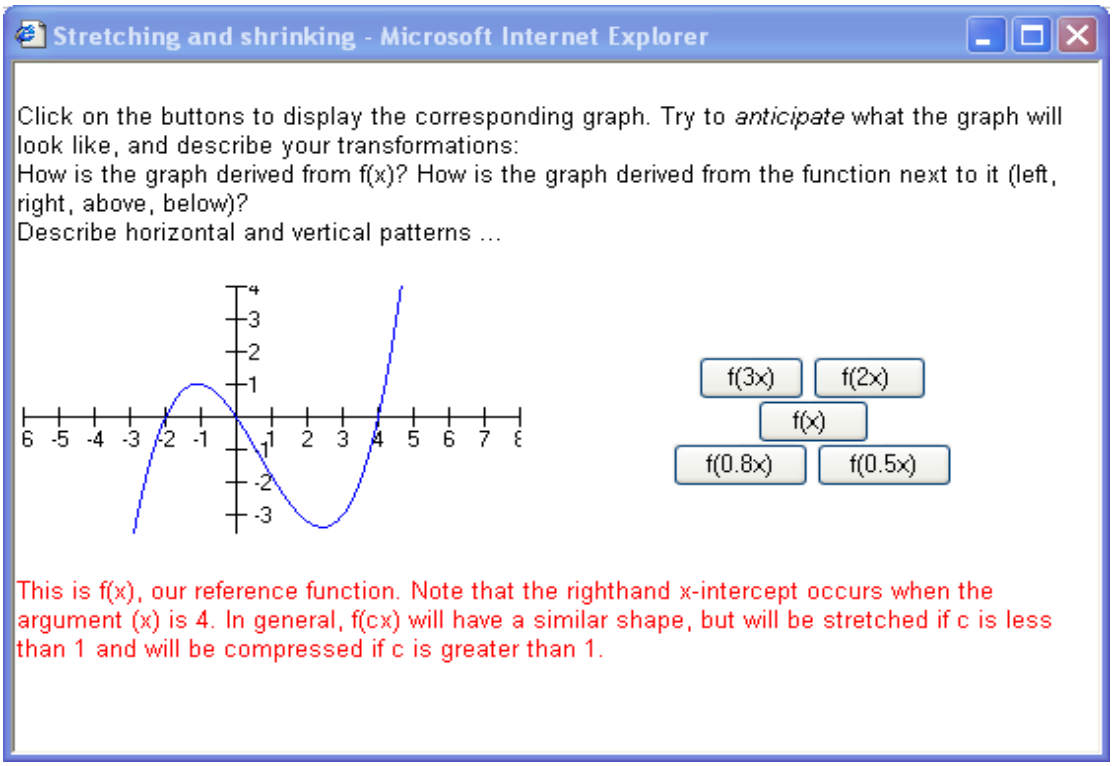

Figure 9: Stretching and shrinking applet

Then *teach* it, preferably *illustrating* on screen with an applet!

<u>Recommendation 2</u>: Teacher support to synchronise the views of teachers on assessment, teacher development programmes are necessary

Teachers in the participating schools made a conscious commitment to take part in the study, which the researcher would assume represents a commitment to change. Such a commitment is considered an important criterion for change (Lovitt et al. 1991), but it is also important that teachers experience the *need* to change. The study has shown that Teacher B was indeed dissatisfied with the existing learning and/or attitudes in their classrooms. Etchberger and Shaw

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(1992) regard 'perturbation' (Goldsmith and Schifter, 1997, after Piaget) as necessary for teacher change, in that it forces the teacher to confront and resolve problematic aspects of their practice.

<u>Recommendation 3</u>: Professional teacher development programmes (TDPs) are necessary to address content knowledge

The knowledge construct of functions is one of observable behavioral patterns in class situation. Knowledge of content makes learning the concept critical. Fey (1998: 107) argues that:

> *There are many different reasons for including topics in the mathematics curricula and for requiring [learners] to study the course. Algebra is a symbol system of unparalleled power for communicating quantitative information and relationships, it is training ground for in careful rule-governed reasoning, its development is a significant thread in the history of mathematics, and its theoretical structure is based on concepts and principles that generalise to provide organizing schema in nearly every branch of mathematics.*

In this citation, Fey (1998) is rationalizing the need to study mathematics and importance of knowledge of content in school mathematics. Fennema and Franke (1992) note research that suggests that teacher content knowledge does influence classroom instruction and the richness of learners' mathematical experiences. For example, bad instructional decisions lead to misconceptions and errors misunderstanding of a function concept. Knowledge of the mathematics as well as ideas that learners bring into the classroom are essential if the teacher is to successfully facilitate conceptual understanding in a mathematics class and challenge students to think about important mathematical connections Goldsmith and Schifter (1997). Such knowledge is a prerequisite for teachers to 'take advantage of teachable moments in mathematics class'.

Lack of knowledge and skills needed to conform to any new role can be a barrier to implementing this role (Snyder et al., 1992). It is difficult for teachers to define their role as 'facilitator' in an inquiry mathematics approach. Goldsmith and Schifter (1997: 32) define this role as follows:

> *As a more knowledgeable, experienced member of the group, and the acknowledged educational leader of the class, it is his or her responsibility to assess students' understanding, monitor their progress, and stimulate continued growth in mathematical understanding.*

Goldsmith and Schifter (1997) continue by saying that teachers must find a balance between valuing students' individual constructions of their mathematical understanding and guiding them towards shared understandings, principles and structures that make up the domain of mathematics. Prior to the study, the researcher had one-on-one interactions with respondents to model strategies to encourage classroom discussions as discourse to help teachers define and implement their facilitation role. However, teachers still found the change in role to be difficult, particularly in the case where learners are not accustomed to discussing, justifying, listening, challenging respectfully and reaching consensus. Teaching time seem to pose a threat in the process of change of teachers' practice.

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The study has shown that teachers were unsure about how and when to intervene, interrupt or distract constructive mathematical discussion. Both teachers had difficulty 'allowing learners to take responsibility for their own learning'. Teacher B did not always realize that he still had a responsibility to monitor the learning process so that he could make decisions about a choice of activities and organization of the class. In addition, for learners to take over this responsibility, it had to be handed to them within a culture in which their obligations as learners are clear – for example obligations to explain, justify, challenge and understand.

Further to the recommendation above, the researcher suggests TDPs at a systemic level of intervention (Hew and Brush, 2006), that provide interplay between pedagogy, knowledge and understanding of functions, knowledge of learners, knowledge of curriculum and knowledge of technology, and inherent beliefs on spheres of knowledge espoused. Knowledge of 'a needs base' is a critical ingredient to a kind of support needed to bring about change. These experiences have taught the researcher that applying positive pressure, responding to needs, providing feedback and reporting are support mechanisms or 'drivers' that yield change.

A suggested Teacher Development Programme developed for the study

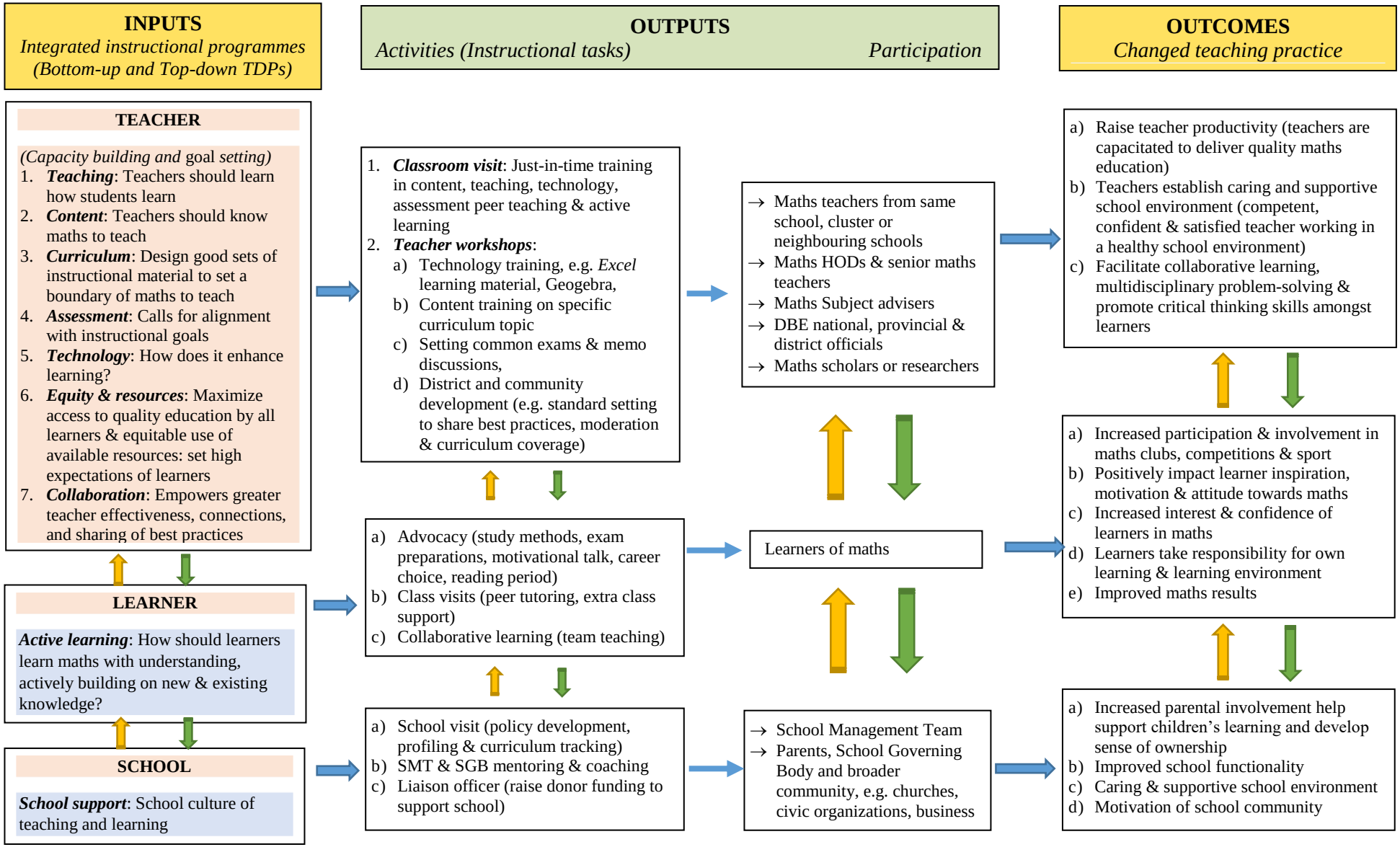

Figure 10: The suggested TDPs model for the study

6.3 Focus of study: What did the researcher do? What is the main idea or unit of analysis

The researcher argues that during the process of teaching, teachers make choices of examples to work out in class, and choose homework exercises that they perceive it will demonstrate a technique of how learners could deepen their knowledge and understanding of a concept.

The researcher asserts that instructional decisions and curricular choices are deeply rooted in teacher beliefs about mathematics, pedagogy and how learners learn mathematics. The researcher then began to ask questions, for example:

- How do teachers make these choices?
- What impact do these curricula choices and instructional decisions have on learners with respect to conceptual and mathematical understanding of functions?
- When teachers are given an opportunity to engage with new forms of teaching and learning, will they change the way that they teach functions?

Teacher support is a critical ingredient for acceptance of SAPs in a mathematics classroom in the process of change. Built-in the design of SAPs in *Excel* is a support programme that was designed to facilitate teacher change. As an instructional tool, SAPs requires support, acceptance and creation of discourse[10] in a classroom - an environment conducive for teaching and learning. A teacher who needs to improve his/her classroom practice could create a classroom culture conducive to fulfilling such a goal. It became apparent that at the heart of teacher change, lies beliefs about teaching and learning, and teacher support. After gathering all the necessary data for analysis, there was a need to fine tune the research question/statement according to the data that emerged, which necessitated shaping and reformulation of the research question in line with the empirical studies in the field of teacher development.

The researcher took a view that to have hope for teacher change of practice, quality TDPs should be developed to support it. These programmes should enable teachers to learn as they teach, and teach as they learn. If we wish to promote radically different beliefs about teaching and mathematics, and create learning communities that embrace reform, then we need different kinds of experiences. These experiences include not only a new set of goals and principles but also new kinds of professional development activities and support.

[10] In a discourse community, learners are able to defend their mathematical ideas, question other learner's ideas and help clarify the mathematics to one another. A teacher, on the other hand is able to employ creative and innovative strategies to enhance learning from his/her knowledge of mathematics.

The researcher formulated a conjecture to depict the notion of change for the study.

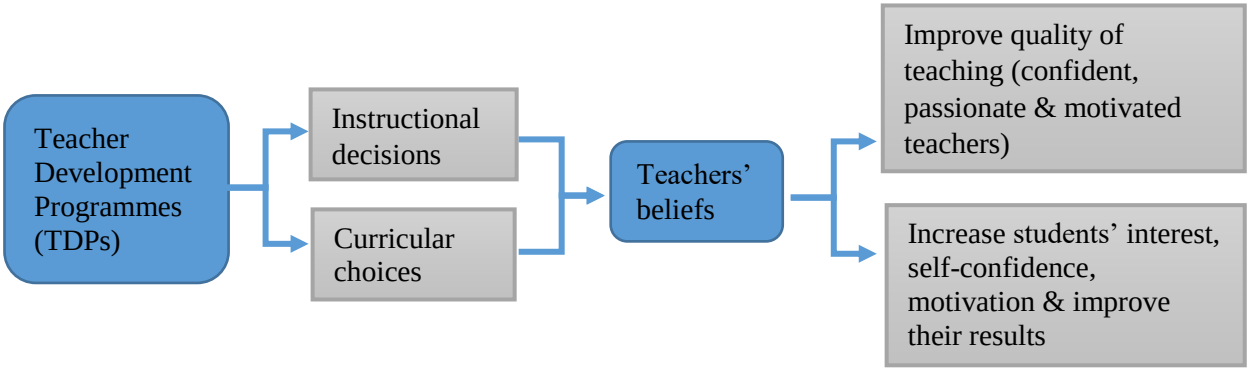

Figure 11: The researcher's Theory of Change

Teacher change should ultimately improve mathematics results and contribute to the future intended curriculum.

6.4 Research capacity building and envisaged impact

In this section, the researcher shares his thoughts spanning from the life of this study to see how they have shaped the study, and possible impact on future research considering internal capacity. Building and strengthening the capacity of mathematics teacher educators or researchers is a high priority in this study. Research findings could lead to advocacy for technology integration in mathematics education, better knowledge and understanding of functions in mathematics classrooms in township high schools. The results of the study could contribute to improved learners' results in NSC examinations in township high schools. Furthermore, it might lead to a re-conceptualization of functions by taking into account technology integration in mathematics education and how best to teach functions to facilitate class discussions in mathematics.

Considering the duration and size of the sample of this study, the researcher feels that there are areas of the research that could be done more intensely. For future and further study, the researcher feels that conducting a pre- and post-teacher beliefs questionnaire can provide valuable data to compare any change of beliefs that can alter teacher practice. The findings from the data collected in this way can assist the researcher to understand teachers' beliefs a little bit better to be able to compare any changes at the beginning and end of study, especially when the study is made bigger and done over a period of 2-3 years (longitudinal study). The pre- and post-teacher beliefs questionnaire can help track beliefs against themselves but also against practices over a reasonable period and sizable sample. Therefore further research into teacher beliefs and classroom practices to improve results is both critical and urgent.

6.5 Contribution to the body of knowledge

The researcher offers suggestions for further research in this section. He does this to contribute to research community on teacher change from a perspective of influence of SAPs in *Excel*. The researcher presents the SAPs in *Excel* and the suggested teaching approach in recommendation 1 as contribution to the body of knowledge both as instructional and teacher professional development tool to address institutionalisation of the curriculum and the re-conceptualization of functions. The following diagram illustrates the researcher's perspective of teacher change.

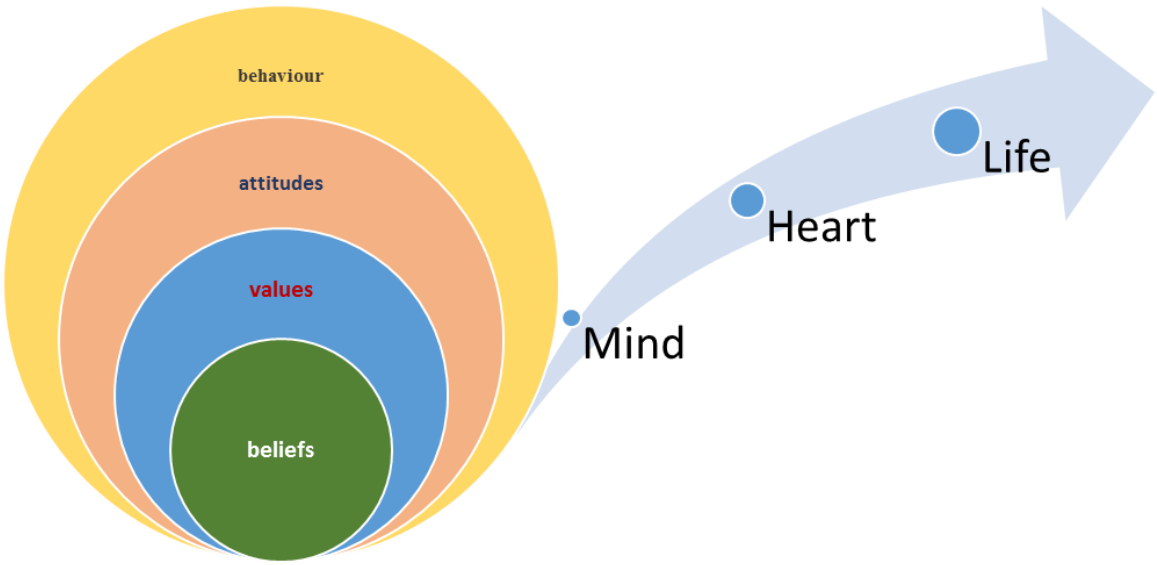

Figure 12: Changing truth about oneself change one's actions

This is an extension of Manley's (2012) depiction of the notion that what one thinks in his/her mind goes to his/her heart, and gets shown by his/her actions. The actions that we "see" from outside of a person, i.e. how a person behaves or teaches, is a manifestation of his/her chosen truth generated from his/her mind. Teacher change in this study is borne out of the researcher's understanding that teachers will not change the way they teach unless they change the way they think. Hence an argument that the way you think reveals what is in your heart and this, in turn, determines how you lead a life.

Furthermore, South Africa requires effective TDPs and support mechanisms that would be accessible to more teachers thus opening doors for further studies, creating opportunities for them to sharpen and hone their teaching skills. If we wish to promote radically different beliefs about teaching and the nature of mathematics, create learning communities that embrace reform, then we need different kinds of experiences. These experiences include not only a new set of goals and principles but also new kinds of professional development and support. These experiences need to be justified and explored by curriculum designers, as teachers are expected to mediate the effects of job-related policies – such as changes in curricula for teachers' initial education or professional development.

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In conclusion, in this study the researcher advocates for a reflective teacher practice as envisaged by Donald Schon in the 1980s in which teachers' beliefs form a solid foundation[11] for sound content knowledge, pedagogy and integration of technology (computers or calculators) in mathematics education. The SAPs in *Excel* on functions (appendix (ii)) and a suggested teaching approach (recommendation 1) have been formulated to provide a classroom culture in which ideas can be shared and discussed, thus creating an opportunity for teachers to reflect on their own practice. There is a likelihood that by creating such an environment, teachers and learners alike can evaluate their ideas, thus improve conceptual understanding of functions in township high schools. The ability to reflect on teacher practice (instructional decisions and curricular choices) is an important character trait in the process of change. That teachers (the respondents) recognized that the use of SAPs as an instructional tool can influence teaching and learning in mathematics education, is an important finding for the study. As Teacher B aptly puts it in the interview excerpt, teachers need 'computer literacy' to help them to reflect on their practice but also to support the process of change.

[11] The expression 'solid foundation' refers to the multiple representations of polynomial functions and how the intentions of the designer of SAPs in *Excel* come into play

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APPENDICES

General Guidelines for Teacher Interviews

1. The researcher will identify teachers from the research schools to be interviewed. Two teachers are to be interviewed, i.e. one from each school. Select third or fourth teacher on the list of participating schools and make sure there are additional teachers selected in case some selected teachers are not available.
2. Interviews will be scheduled half an hour immediately after the observed lesson. Teachers selected for interview will be informed that they may be called on during a free period or after school in July-September should they be needed.
3. Interviews should be about 20-40 minutes long. The time walking between the classroom and interview room can be used for informal discussion on the course and included in the interview write up if appropriate.
4. The researcher will bring recording equipment. The audiotapes will be used as a backup of the interview data. There will be full transcriptions from tape recordings when writing up the interviews.
5. The researcher will take detailed notes where possible and write up the interview using the section headings provided.
6. The researcher will ensure that at least the questions in bold are answered.
7. Other questions can be used to probe and to try to stimulate discussion.
8. This is a semi-structured interview. The researcher do not expect the questions to be asked question by question with teacher's responses recorded but encourage reflection and conversation on the course making sure that the main thrust of the interview is focused on the three areas below.
9. The interview write-ups will capture the main points of discussion under each section heading and may include an interviewer's reflection of additional point(s) of interest.

Appendix (i): Teacher Interviews

Adapted from a tool used to evaluate effectiveness of interactive computer tutorials for an undergraduate mathematical course in Frith et al. (2004).

Teacher's Name: ……..
School: …….
Date of Interview: ……
Interviewer: ……..

1. Perceptions of the teacher

a) What were your expectations of learners prior to the start of the lesson? Did it meet your expectations?

Possible probe: *What did you learn from this lesson? If you were to teach the same lesson again, what would you change?*

b) Have you used the knowledge and skills learned from this lesson in other classes that you are teaching this year? If so, where and how?

Possible probe: *What do you think functions prepares learners for?*

c) Think of a particular incident during the course of the lesson when ……. said ……..…? What was your response? Why did you ignore …….. when s(he) raised her/his hand? Describe an incident that stands out for you in this experience?

Possible probe: *What stands out for you in this lesson?*

- *Would you recommend your teaching approach to other teachers? Why?*

2. Attitudes towards mathematics

a) What do you feel about teaching mathematics?

b) Do you think your attitude to mathematics has changed since using Excel teaching material on functions? If so, how?

c) What do you think mathematics is useful for?

Possible probe: *What do you think mathematics equips you to do?*

- *What kind of people take "or do" mathematics? Why?*

3. Attitudes towards computers in mathematics education

a) What do you feel about using computers to enhance teaching and learning of mathematics?

Possible probe: *Describe a previous experience relating to using computers that you experienced before 2009? Describe an incident that stands out for you in this experience?*

b) Do you think your attitude to computers has changed since teaching functions today? If so, how?

Possible probe: *How did you feel about computers before this course? How do you feel about computers now? Do you think your attitude has changed? If so how?*

c) What functions concept did the Excel spreadsheets help you explain better?

Possible probe: *Describe an incident that stands out for you in this experience?*

- *What do you use computers for at work/school? (now and in the future)*
- *What do you use computers for beyond work/school? (now and in the future).*

Appendix (ii): Spreadsheet Algebra Programme (SAPs)

A: Translations

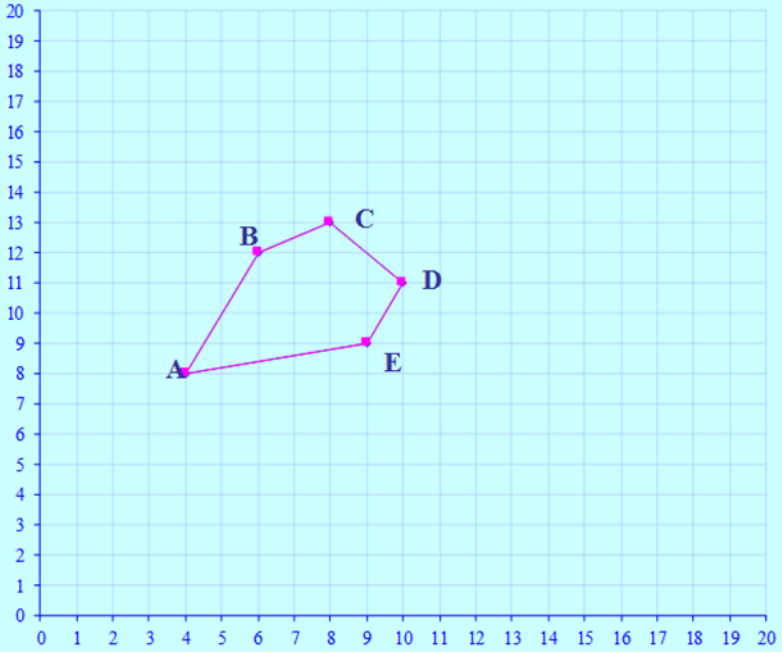
TRANSLATION

Click the sliders to translate the figure ABCDE horizontally and vertically. Describe the relationship between the co-ordinates of corresponding points.

Horizontally p = 0 Vertically q = 0

Original figure	x	y
A	4	8
B	6	12
C	8	13
D	10	11
E	9	9

Tranlated figure	x	y
A'	4	8
B'	6	12
C'	8	13
D'	10	11
E'	9	9



Describe **patterns.** ***Explain*** **the structure!** ***Generalise*** **!**

B: Horizontal Translations

HORIZONTAL TRANSLATION

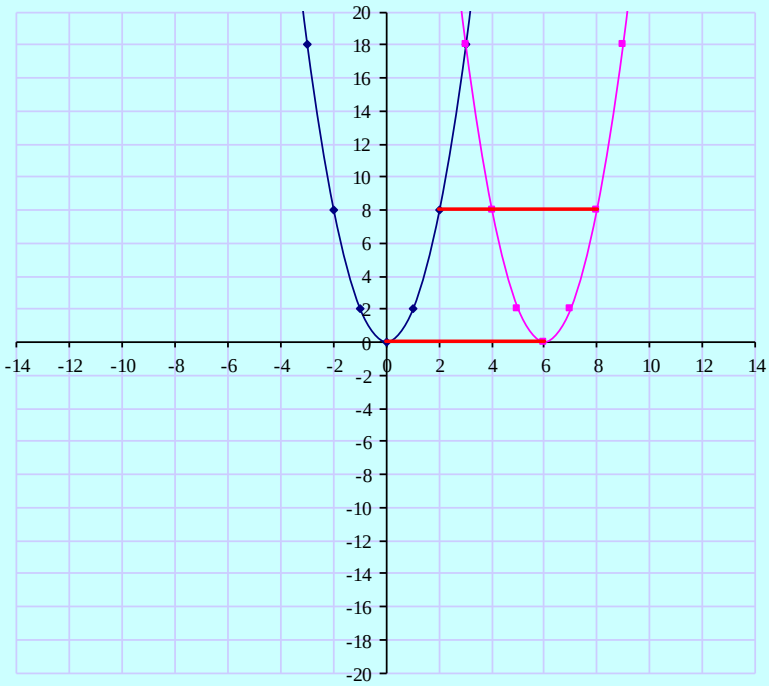
$f(x) = ax^n$ and $g(x) = f(x - p)$

Click the sliders to change p (also a and n). Watch how the graph and the table change! Describe the pattern between the (x, y) co-ordinates of $f(x)$ and $g(x)$.

p = 6 a = 2 n = 2

Function x	f(x)
-4	32
-3	18
-2	8
-1	2
0	0
1	2
2	8
3	18
4	32

Translated function x	g(x)
2	32
3	18
4	8
5	2
6	0
7	2
8	8
9	18
10	32



How are the *formulas* of $f(x)$ and $g(x)$ related?

The formula for f(x): y = 2x^2

The formula for g(x): y = 2(x – 6)^2

Check that the formulae fit the values in the table!

Describe **patterns.** ***Explain*** **the structure!** ***Generalise!***

Connect formula - graph - table!

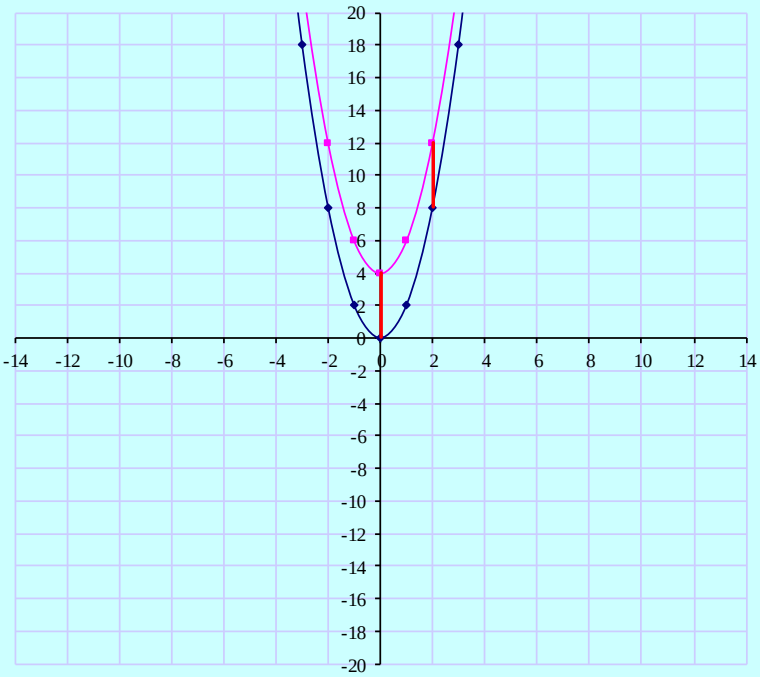
C: Vertical Translation

VERTICAL TRANSLATION

$f(x) = ax^n$ and $g(x) = f(x) + q$

Click the sliders to change q (also a and n).
Watch how the graph and the table change!
Describe the pattern between the (x, y) co-ordinates of $f(x)$ and $g(x)$.

q = 4 a = 2 n = 2



Function

x	f(x)
-4	32
-3	18
-2	8
-1	2
0	0
1	2
2	8
3	18
4	32

Translated function

x	g(x)
-4	36
-3	22
-2	12
-1	6
0	4
1	6
2	12
3	22
4	36

How are the *formulas* of f(x) and g(x) related?

The formula for f(x): y = 2x^2

The formula for g(x): y – 4 = 2x^2

Check that the formulas fit the values in the table!

Describe* patterns. *Explain* the structure! *Generalise!

D: Translation by a vector

$f(x) = ax^n$ is translated by vector (p, q) to g(x).

What is the formula of g(x)?

f(x)

x	y
-4	32
-3	18
-2	8
-1	2
0	0
1	2
2	8
3	18
4	32

g(x)

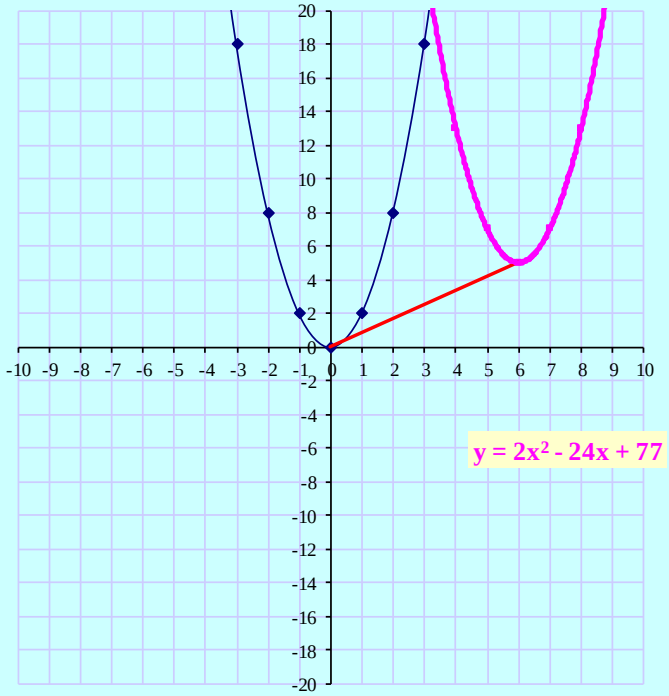
x	y
2	37
3	23
4	13
5	7
6	5
7	7
8	13
9	23
10	37

Click to change a:

a = 2 n = 2

Click the sliders to translate f(x):

p = 6 q = 5



The formula for f(x): y = 2x^2

The formula for g(x): y - 5 = 2(x - 6)^2

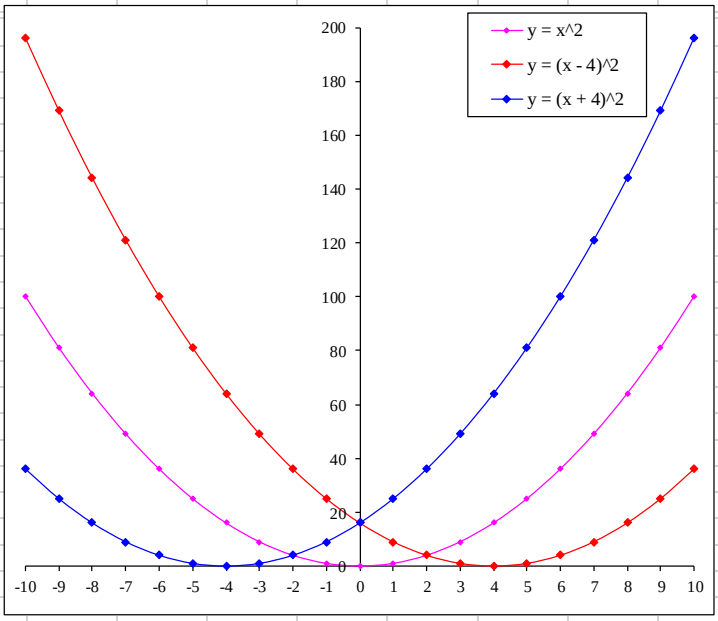
The purple graph is the TRENDLINE. The trendline equation of g(x) is given …

How is this equation related to the equation of g(x)?? Can you deduce it??

E: Quadratic Functions (Parabola)

Formula → Table → Graph

x	y = x^2	y = (x - 4)^2	y = (x + 4)^2
-10	100	196	36
-9	81	169	25
-8	64	144	16
-7	49	121	9
-6	36	100	4
-5	25	81	1
-4	16	64	0
-3	9	49	1
-2	4	36	4
-1	1	25	9
0	0	16	16
1	1	9	25
2	4	4	36
3	9	1	49
4	16	0	64
5	25	1	81
6	36	4	100
7	49	9	121
8	64	16	144
9	81	25	169
10	100	36	196



F: Exponential Function

EXPONENTIAL FUNCTION

$f(x) = ab^x$ and $g(x) = f(x - p) + q$

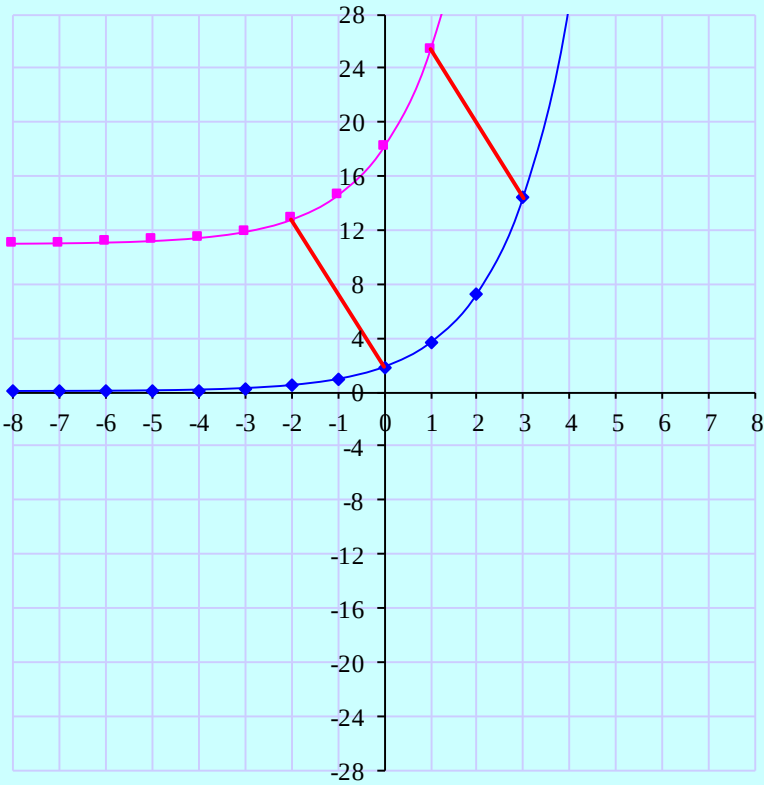
Click the sliders to change p and q..
Watch how the graph and the table and formula change. Describe any patterns!

Horizontally **p = -2** *Vertically* **q = 11**

f(x)		g(x)	
x	*y*	*x*	*y*
-4	0.113	-4	11.45
-3	0.225	-3	11.9
-2	0.45	-2	12.8
-1	0.9	-1	14.6
0	1.8	0	18.2
1	3.6	1	25.4
2	7.2	2	39.8
3	14.4	3	68.6
4	28.8	4	126.2

Change f(x) by clicking values for a and b:

a = 1.8 **b = 2**



Equation of f(x): y = 1.8*2^x

Equation of g(x): y – 11 = 1.8*2^(x – -2)

Describe *patterns* . Explain the *structure! Generalise!*
Connect Equation - graph - table!

G: Constant of trigonometric functions

THE SINE OF AN ANGLE IS CONSTANT!

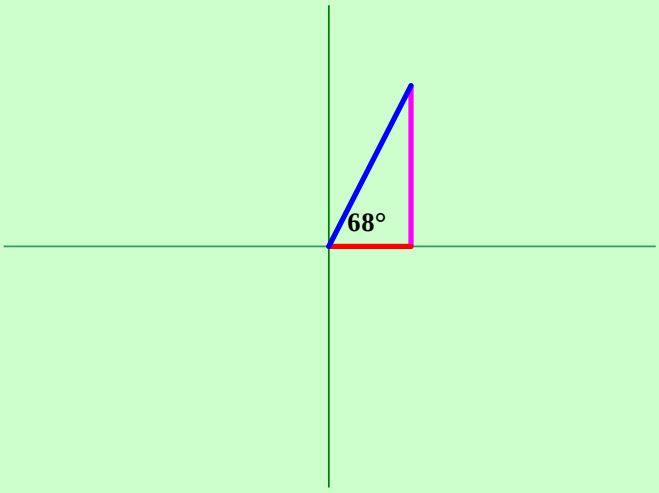
Below are values of x, y, r, sin(68), cos(68) and tan(68).
Click on the arrows to change the *size* of the triangle.
Which values change and which do not? Can you *explain* it?

x	y	r
4.308	10.663	11.500

$$\sin(68) = \frac{y}{r} = \frac{10.663}{11.500} = 0.927$$

$$\cos(68) = \frac{x}{r} = \frac{4.308}{11.500} = 0.375$$

$$\tan(68) = \frac{y}{x} = \frac{10.663}{4.308} = 2.475$$



Is it also true for *other* angles?
Click on the arrows to change the angle and check …

Angle = **68°**

You can make angles from -360° to 360° …

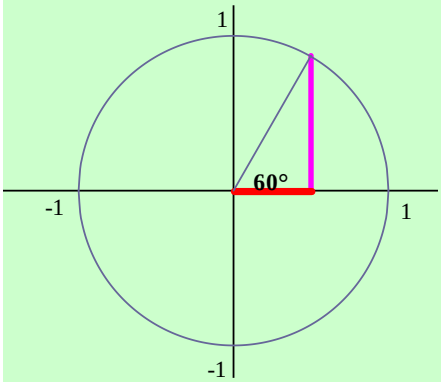
Alwyn Olivier, 2002
aio@sun.ac.za

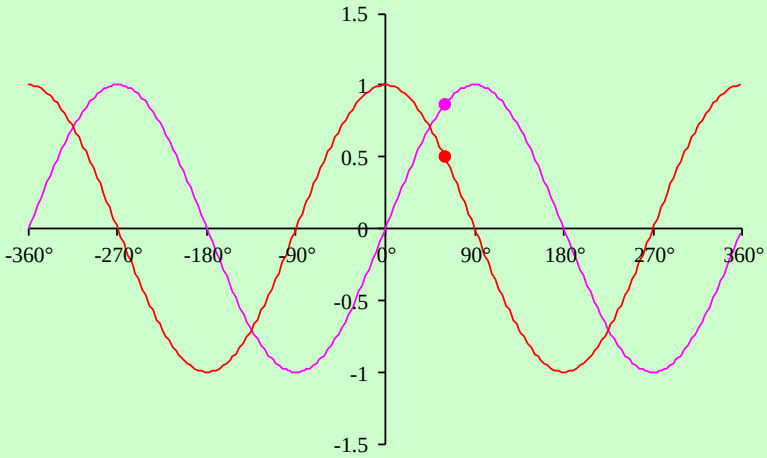
H: The graph of sin θ and cos θ

THE UNIT CIRCLE

Convince yourself: If r =1, then sin(θ) = y = purple segment and cos(θ) = x = red segment …
Click on the arrows to change the angle … (from -360° to 360°)
How does sin(θ) and cos(θ) change as θ change??
Study and compare the sketch, the numbers and the graphs!

Angle = 60° | **sin(θ) = y = 0.866** | **cos(θ) = x = 0.5**





I: The graph of tan θ

TAN(Θ) IN THE UNIT CIRCLE

In the unit circle, the radius vector is positive, so sin(θ) = y/1 = y and cos(θ) = x/1 = x.

But for tan(θ) = y/x, the sign of x must be considered …

Prove in the diagram that tan(θ) = y/x = blue tangent segment. Prove tan(θ) = tan(θ + 180).

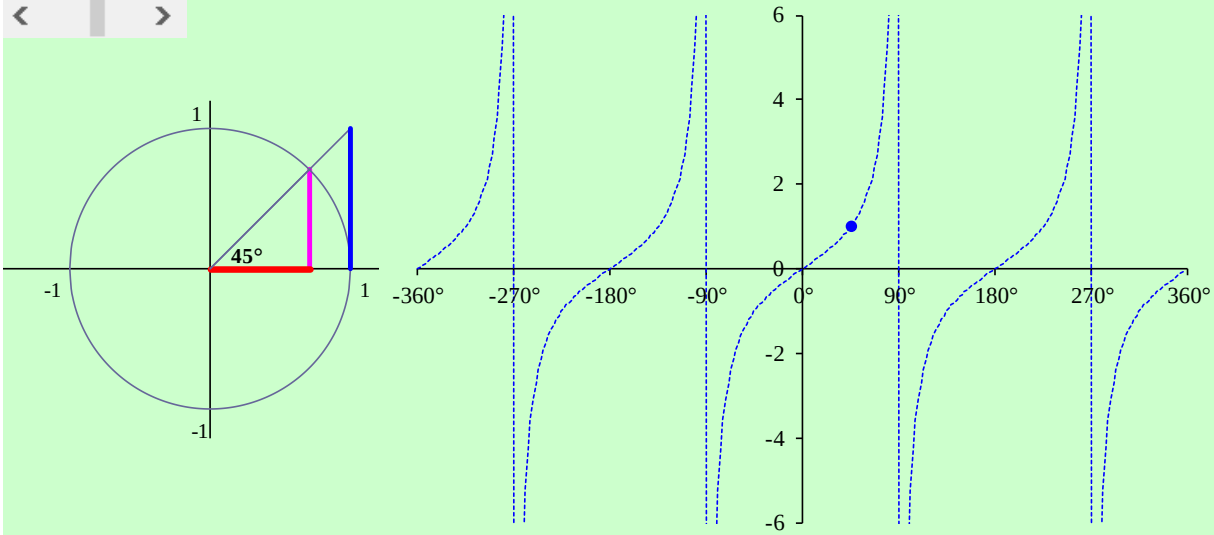
How does tan(θ) change as θ change??

Study and compare the sketch, the numbers and the graphs!

Note: The scales in the circle diagram and graph are not equal ...

Angle = 45°

tan(θ) = 1



Appendix (iii): Teacher Questionnaire

Name: ……..…….
School: ……..……
Grades you are teaching: ……..……

Please answer the following questions as honestly as possible. The correct answer is the one that reflects your true opinion. Please feel free to write any comments on the questionnaire. If you need more space, please use the back of the page. Your answers can be indicated on the response sheet by circling one of the following options for each question:

1. Strongly disagree
2. Disagree
3. Neutral (Please try to avoid this option if at all possible)
4. Agree
5. Strongly agree

THANK YOU FOR YOUR TIME AND COOPERATION

1 Strongly disagree **2** Disagree **3** Neutral **4** Agree **5** Strongly agree

1. Mathematics at school should only be about mathematics which can be used in real life.

1	2	3	4	5

2. The process of doing mathematics is just as important as reaching the right answer.

1	2	3	4	5

3. Learners cannot reach their mathematical potential unless they are learning mathematics with other learners of the same ability level.

1	2	3	4	5

4. In order to develop a good foundation for mathematics at an early age, learners need to be taught mathematics in their home language during the early stages.

1	2	3	4	5

5. Even if learners do not understand how a rule was derived, they should be able to use it.

1	2	3	4	5

6. Assessment can take place during everyday class activities.

1	2	3	4	5

7. Weaker learners should be drilled more than stronger learners.

1	2	3	4	5

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1 Strongly disagree **2** Disagree **3** Neutral **4** Agree **5** Strongly agree

8. When a learner can use a mathematical procedure correctly, he/she understands it.

1	2	3	4	5

9. The calculator can be used to introduce learners to important mathematical concepts.

1	2	3	4	5

10. It is not possible to control a class in which different learners are engaged in different activities.

1	2	3	4	5

11. If weaker learners are taught separately, they will adjust to the lower expectations and perform poorly.

1	2	3	4	5

12. Learners should be taught one mathematical concept at a time.

1	2	3	4	5

13. Valuable information is lost if learners' work is not assessed by their own teacher.

1	2	3	4	5

14. If a learner keeps making the same mistake, the teacher can conclude that he/she has not been listening in class.

1	2	3	4	5

15. It is the teacher's responsibility to decide whether learners' answers to a problem are correct or incorrect.

1	2	3	4	5

16. Mathematical terminology should preferably be introduced only after learners have grasped the relevant concepts.

1	2	3	4	5

17. If a learner fails an assessment, the learner is not capable of doing this work.

1	2	3	4	5

18. The teacher cannot be expected to teach a variety of different ability levels if there is not sufficient material to help him/her.

1	2	3	4	5

19. Learners should be given the responsibility of assessing their own progress.

1	2	3	4	5

20. Learners learn mainly by receiving mathematics knowledge from the teacher.

1	2	3	4	5

18. Teachers should encourage learners' own methods even if they are inefficient.

1	2	3	4	5

22. All learners should reach the same level of knowledge.

1	2	3	4	5

23. Learners and parents should always know how the learners are to be assessed.

1	2	3	4	5

24. Learners' mathematics work books should always be neat and tidy.

1	2	3	4	5

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1 Strongly disagree **2** Disagree **3** Neutral **4** Agree **5** Strongly agree

25. Assessment of group work does not give an accurate reflection of what an individual has learned.

1	2	3	4	5

26. Any mathematics task can be solved in several possible ways.

1	2	3	4	5

27. Stronger learners benefit from being taught in a mixed-ability class.

1	2	3	4	5

28. If the teacher thinks he/she knows what mistakes the learners will make while doing a particular activity, he/she should prevent these mistakes by warning the learners in advance.

1	2	3	4	5

29. It is sometimes important for learners to express their mathematical thinking in their home language.

1	2	3	4	5

30. Marks should be assigned for all work that is assessed.

1	2	3	4	5

31. Learners cannot learn mathematics effectively in a multicultural classroom.

1	2	3	4	5

32. The best way to deal with learners' mistakes and misconceptions is for the teacher to intervene and show them the correct concepts.

1	2	3	4	5

33. A learner should only be assessed for grading when the teacher and learner feel that the learner is ready for it.

1	2	3	4	5

34. Examination and test results enable teachers to sort the learners into ability groups.

1	2	3	4	5

35. Learners can work out effective ways to solve problems without formal instruction.

1	2	3	4	5

36. In mathematics, most learning takes place by memorizing and practicing things.

1	2	3	4	5

37. Doing mathematics does not necessarily imply working clearly and systematically.

1	2	3	4	5

38. Marks for orals, investigations and projects should not count as much as those for tests and examinations as these marks cannot be objective.

1	2	3	4	5

39. Learners should be given problems that integrate several concepts.

1	2	3	4	5

40. In order to provide conditions in which learners can learn mathematics, mathematics classrooms should be relatively silent.

1	2	3	4	5

41. All learners can learn mathematics.

1	2	3	4	5

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1 Strongly disagree **2** Disagree **3** Neutral **4** Agree **5** Strongly agree

42. If learners are allowed to use calculators they will become dependent on the calculator and unable to think for themselves.

1	2	3	4	5

43. If a lesson plan for teaching certain mathematical content has worked well one year, it is guaranteed to succeed the next year.

1	2	3	4	5

44. Problems can be used to introduce new concepts in mathematics.

1	2	3	4	5

45. For most mathematics tasks, there is one correct solution procedure.

1	2	3	4	5

46. If mathematics is taught in a second language, word problems can still be given to all learners regardless of their home language.

1	2	3	4	5

47. The most important thing in mathematics is for learners to get the right answer.

1	2	3	4	5

48. If learners are allowed to assess themselves, the marks will be too high.

1	2	3	4	5

49. The most important factor in planning for mathematics teaching should be the information the teacher has on his/her learners' understandings.

1	2	3	4	5

50. Learners should be given the relevant correct terminology and symbols before they start doing the mathematics concerned.

1	2	3	4	5

51. Stronger learners' mathematics development will be hampered if they are taught in a mixed-ability class.

1	2	3	4	5

52. Learners learn mathematics best by working out their own ways of doing it and discussing various strategies.

1	2	3	4	5

53. Teachers should assess all learners every lesson.

1	2	3	4	5

54. Learners' explanations of their solutions are the best indicators of their mathematical understanding.

1	2	3	4	5

55. The main purpose of assessment is to obtain marks for reporting a learner's progress to others.

1	2	3	4	5

56. Learners should be given different activities according to their different needs.

1	2	3	4	5

57. The mathematics teacher's main role is to have and to demonstrate mathematics knowledge.

1	2	3	4	5

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1 Strongly disagree **2** Disagree **3** Neutral **4** Agree **5** Strongly agree

58. Learners' mistakes and misconceptions should at first be addressed by getting them to share their solutions with each other.

1	2	3	4	5

59. If learners persist in using inefficient methods, the teacher should guide them towards developing more efficient methods.

1	2	3	4	5

60. Only the teacher and school authorities need to know how learners are to be assessed.

1	2	3	4	5

61. Learners cannot solve mathematical problems effectively unless they have been shown how to do them.

1	2	3	4	5

62. If learners learn mathematics in a classroom where there is switching between languages, they will be confused.

1	2	3	4	5

63. Learners should not only solve real life problems but also purely mathematical problems which cannot be directly applied outside school.

1	2	3	4	5

64. The only valid assessment is that which can be marked by all teachers using a given memorandum.

1	2	3	4	5

65. Examination papers which have been used successfully in the past are guaranteed to assess successfully again.

1	2	3	4	5

66. If a learner fails, he/she should be given the opportunity to be reassessed.

1	2	3	4	5

67. If learners are allowed to talk to each other, they will copy from each other.

1	2	3	4	5

68. Mathematics problems should only be given once the necessary mathematics content has been covered.

1	2	3	4	5

69. Weaker learners benefit from being taught in a mixed-ability class.

1	2	3	4	5

70. In mathematics classes, the calculator should be used only to check answers.

1	2	3	4	5

71. Most learners need to be given mathematical procedures, rules and concepts as they cannot develop these on their own.

1	2	3	4	5

72. By looking carefully at the results of assessment, the teacher can gain valuable information for the planning of future activities.

1	2	3	4	5

73. If learners have not mastered the mathematics which has been taught previously, they will not be able to learn more advanced mathematics.

1	2	3	4	5